BITSAT 2015 Question Paper with Answer Key

Birla Institute of Technology and Science Admission Test

BITSAT : SOLVED PAPER 2015

(memory based)

INSTRUCTIONS

- *This question paper contains total 150 questions divided into four parts:*

  Part I : Physics Q. No. 1 to 40

  Part II : Chemistry Q. No. 41 to 80

  Part III : (A) English Proficiency Q. No. 81 to 95

  (B) Logical Reasoning Q. No. 96 to 105

  Part IV : Mathematics Q. No. 106 to 150
- *All questions are multiple choice questions with four options, only one of them is correct.*
- *Each correct answer awarded 3 marks and –1 for each incorrect answer.*
- *Duration of paper 3 Hours*

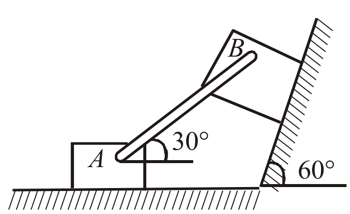
PART - I: PHYSICS

1. An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of the escape velocity from the earth. The height (h) of the satellite above the earth's surface is (Take radius of earth as R_e)

(a) $h = R_e^2$ (b) $h = R_e$

(c) $h = 2R_e$ (d) $h = 4R_e$

2. In figure, two blocks are separated by a uniform strut attached to each block with frictionless pins. Block A weighs 400N, block B weighs 300N, and the strut AB weigh 200N. If $\mu = 0.25$ under B, determine the minimum coefficient of friction under A to prevent motion.



(a) 0.4 (b) 0.2 (c) 0.8 (d) 0.1

3. Two tuning forks with natural frequencies 340 Hz each move relative to a stationary observer. One fork moves away from the observer, while the other moves towards the observer at the same speed. The observer hears beats of frequency 3 Hz. Find the speed of the tuning forks.

(a) 1.5 m/s (b) 2 m/s (c) 1 m/s (d) 2.5 m/s

4. The displacement of a particle is given at time t, by:

$x = A\sin(-2\omega t) + B\sin^2 \omega t$ Then,

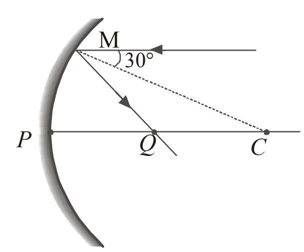
(a) the motion of the particle is SHM with an amplitude of $\sqrt{A^2 + \frac{B^2}{4}}$

(b) the motion of the particle is not SHM, but oscillatory with a time period of $T = \pi/\omega$

(c) the motion of the particle is oscillatory with a time period of $T = \pi/2\omega$

(d) the motion of the particle is a periodic.

5. A ray parallel to principal axis is incident at 30° from normal on concave mirror having radius of curvature R. The point on principal axis where rays are focussed is Q such that PQ is



(a) $\frac{R}{2}$

(b) $\frac{R}{\sqrt{3}}$

(c) $\frac{2\sqrt{R} - R}{\sqrt{2}}$

(d) $R\left(1 - \frac{1}{\sqrt{3}}\right)$

6. A solid sphere of radius R has a charge Q distributed in its volume with a charge density $\rho = kr^a$, where k and a are constants and r is the distance from its centre. If the electric field at $r = \frac{R}{2}$ is $\frac{1}{8}$ times that at $r = R$, the value of a is

(a) 3 (b) 5 (c) 2 (d) 7

7. A charged particle moving in a uniform magnetic field and losses 4% of its kinetic energy. The radius of curvature of its path changes by

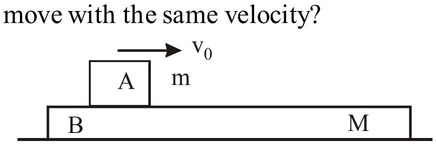
(a) 2% (b) 4% (c) 10% (d) 12%

8. Calculate the wavelength of light used in an interference experiment from the following data : Fringe width = 0.03 cm. Distance between the slits and eyepiece through which the interference pattern is observed is 1m. Distance between the images of the virtual source when a convex lens of focal length 16 cm is used at a distance of 80 cm from the eyepiece is 0.8 cm.

(a) 0.0006 Å (b) 0.0006 m

(c) 600 cm (d) 6000 Å

9. The masses of blocks A and B are m and M respectively. Between A and B, there is a constant frictional force F and B can slide on a smooth horizontal surface. A is set in motion with velocity while B is at rest. What is the distance moved by A relative to B before they move with the same velocity?



(a) $\frac{mMv_0^2}{F(m-M)}$ (b) $\frac{mMv_0^2}{2F(m-M)}$

(c) $\frac{mMv_0^2}{F(m+M)}$ (d) $\frac{mMv_0^2}{2F(M+m)}$

10. An elastic string of unstretched length L and force constant k is stretched by a small length x. It is further stretched by another small length y. The work done in the second stretching is

(a) $1/2\ Ky^2$ (b) $1/2\ Ky(2x + y)$

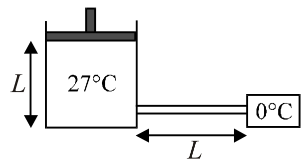
(c) $1/2\ K(x^2 + y^2)$ (d) $1/2\ k\ (x + y)^2$

11. A body is thrown vertically upwards from A, the top of the tower, reaches the ground in time t_1. If it is thrown vertically downwards from A with the same speed, it reaches the ground in time t_2. If it is allowed to fall freely from A, then the time it takes to reach the ground is given by

(a) $t = \frac{t_1 + t_2}{2}$ (b) $t = \frac{t_1 - t_2}{2}$

(c) $t = \sqrt{t_1 t_2}$ (d) $t = \sqrt{\frac{t_1}{t_2}}$

12. 0.5 mole of an ideal gas at constant temperature 27°C kept inside a cylinder of length L and cross-section area A closed by a massless piston.



The cylinder is attached with a conducting rod of length L, cross-section area (1/9) m^2 and thermal conductivity k, whose other end is maintained at 0°C. If piston is moved such that rate of heat flow through the conducing rod is constant then velocity of piston when it is at height $L/2$ from the bottom of cylinder is :

[Neglect any kind of heat loss from system]

(a) $\left(\frac{k}{R}\right)$ m/sec (b) $\left(\frac{k}{10R}\right)$ m/sec

(c) $\left(\frac{k}{100R}\right)$ m/sec (d) $\left(\frac{k}{1000R}\right)$ m/sec

13. A conducting square loop is placed in a magnetic field B with its plane perpendicular to the field. The sides of the loop start shrinking at a constant rate α. The induced emf in the loop at an instant when its side is 'a' is

(a) $2a\alpha B$ (b) $a^2\alpha B$ (c) $2a^2\alpha B$ (d) $a\alpha B$

14. The beam of light has three wavelengths 4144Å, 4972Å and 6216 Å with a total intensity of 3.6×10^{-3} Wm^2 equally distributed amongst the three wavelengths. The beam falls normally on the area 1 cm^2 of a clean metallic surface of work function 2.3 eV. Assume that there is no loss of light by reflection and that each energetically capable photon ejects one electron. Calculate the number of photoelectrons liberated in 2s.

(a) 2×10^9 (b) 1.075×10^{12}

(c) 9×10^8 (d) 3.75×10^6

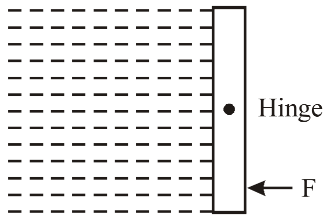
15. A square gate of size 1 m × 1m is hinged at its mid-point. A fluid of density ρ fills the space to the left of the gate. The force F required to hold the gate stationary is

(a) $\frac{\rho g}{3}$

(b) $\frac{\rho g}{2}$

(c) $\frac{\rho g}{6}$

(d) $\frac{\rho g}{8}$



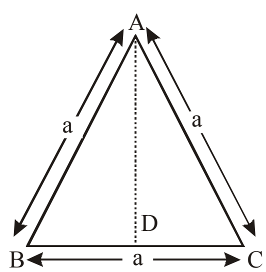
16. When 0.50 Å X-rays strike a material, the photoelectrons from the k shell are observed to move in a circle of radius 23 mm in a magnetic field of 2×10^{-2} tesla acting perpendicularly to the direction of emission of photoelectrons. What is the binding energy of k-shell electrons?

(a) 3.5 keV (b) 6.2 keV
(c) 2.9 keV (d) 5.5 keV

17. In CE transistor amplifier, the audio signal voltage across the collector resistance of 2 kΩ is 2 V. If the base resistance is 1kΩ and the current amplification of the transistor is 100, the input signal voltage is

(a) 2mV (b) 3mV (c) 10mV (d) 0.1 mV

18. At the corners of an equilateral triangle of side a (1 metre), three point charges are placed (each of 0.1 C). If this system is supplied energy at the rate of 1 kw, then calculate the time required to move one of the mid-point of the line joining the other two.



(a) 50 h (b) 60 h (c) 48 h (d) 54 h

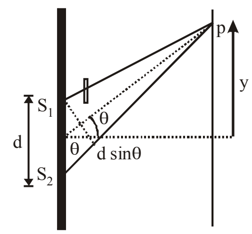
19. A vessel of volume 20L contains a mixture of hydrogen and helium at temperature of 27°C and pressure 2 atm. The mass of mixture is 5g. Assuming the gases to be ideal, the ratio of mass of hydrogen to that of helium in the given mixture will be

(a) 1 : 2 (b) 2 : 3 (c) 2 : 1 (d) 2 : 5

20. The resistance of a wire is *R*. It is bent at the middle by 180° and both the ends are twisted together to make a shorter wire. The resistance of the new wire is

(a) 2*R* (b) *R*/2 (c) *R*/4 (d) *R*/8

21. In a YDSE, the light of wavelength $\lambda = 5000$ Å is used, which emerges in phase from two slits a distance $d = 3 \times 10^{-7}$m apart. A transparent sheet of thickness $t = 1.5 \times 10^{-7}$m refractive index $\mu = 1.17$ is placed over one of the slits. what is the new angular position of the central maxima of the interference pattern, from the centre of the screen? Find the value of y.



(a) $4.9°$ and $\frac{D(\mu-1)t}{2d}$

(b) $4.9°$ and $\frac{D(\mu-1)t}{d}$

(c) $3.9°$ and $\frac{D(\mu+1)t}{d}$

(d) $2.9°$ and $\frac{2D(\mu+1)t}{d}$

22. The position of a projectile launched from the origin at $t = 0$ is given by $\vec{r} = \left(40\hat{i} + 50\hat{j}\right)$m at $t =$ 2s. If the projectile was launched at an angle θ from the horizontal, then θ is
(take $g = 10\ ms^{-2}$)

(a) $\tan^{-1}\frac{2}{3}$ (b) $\tan^{-1}\frac{3}{2}$

(c) $\tan^{-1}\frac{7}{4}$ (d) $\tan^{-1}\frac{4}{5}$

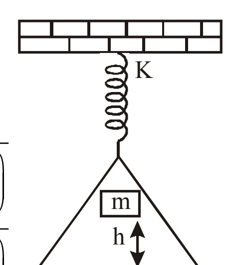
23. Water is flowing on a horizontal fixed surface, such that its flow velocity varies with *y* (vertical direction) as

$v = k\left(\frac{2y^2}{a^2} - \frac{y^3}{a^3}\right)$. If coefficient of viscosity for water is η, what will be shear stress between layers of water at $y = a$.

(a) $\frac{\eta k}{a}$ (b) $\frac{\eta}{ka}$

(c) $\frac{\eta a}{k}$ (d) None of these

24. A load of mass m falls from a height h on to the scale pan hung from the spring as shown in the figure. If the spring constant is k and mass of the scale pan is zero and the mass m does not bounce relative to the pan, then the amplitude of vibration is



(a) mg/d

(b) $\frac{mg}{k}\sqrt{\left(\frac{1+2hk}{mg}\right)}$

(c) $\frac{mg}{k} + \frac{mg}{k}\sqrt{\left(\frac{1+2hk}{mg}\right)}$

(d) $\frac{mg}{k}\sqrt{\left(\frac{1+2hk}{mg} - \frac{mg}{k}\right)}$

25. In an ore containing uranium, the ratio of U^{238} to Pb^{206} is 3. Calculate the age of the ore, assuming that all the lead present in the ore is the final stable product of U^{238}. Take the half-life of U^{238} to be 4.5×10^9 yr.

(a) 1.6×19^3 yr (b) 1.5×10^4 yr
(c) 1.867×10^9 yr (d) 2×10^5 yr

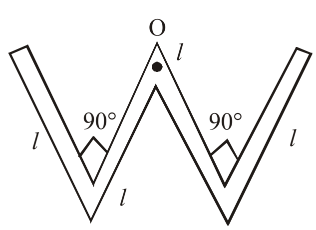
26. A direct current of 5A is superposed on an alternating current $I = 10 \sin \omega t$ flowing through the wire. The effective value of the resulting current will be

(a) (15/2)A (b) $5\sqrt{3}$A
(c) $5\sqrt{5}$A (d) 15 A

27. A planoconvex lens fits exactly into a planoconcave lens. Their plane surface are parallel to each other. If the lenses are made of different materials of refractive indices μ_1 & μ_2 and R is the radius of curvature of the curved surface of the lenses, then focal length of combination is

(a) $\dfrac{R}{\mu_1 - \mu_2}$ (b) $\dfrac{2R}{\mu_1 - \mu_2}$
(c) $\dfrac{R}{2(\mu_1 - \mu_2)}$ (d) $\dfrac{R}{2-(\mu_1 + \mu_2)}$

28. A thin rod of length 4l and mass 4m is bent at the points as shown in figure. What is the moment of inertia of the rod about the axis passes through point O and perpendicular to the plane of paper?



(a) $\dfrac{Ml^2}{3}$
(b) $\dfrac{10Ml^2}{3}$
(c) $\dfrac{Ml^2}{12}$
(d) $\dfrac{Ml^2}{24}$

29. One of the lines in the emission spectrum of Li^{2+} has the same wavelength as that of the 2nd line of Balmer series in hydrogen spectrum. The electronic transition corresponding to this line is $n = 12 \rightarrow n = x$. Find the value of x.

(a) 8 (b) 6 (c) 7 (d) 5

30. Two particles X and Y having equal charges, after being accelerated through the same potential difference, enter a region of uniform magnetic field and describe circular paths of radii R_1 and R_2, respectively. The ratio of masses of X and Y is

(a) $(R_1/R_2)^{1/2}$ (b) (R_2/R_1)
(c) $(R_1/R_2)^2$ (d) (R_1/R_2)

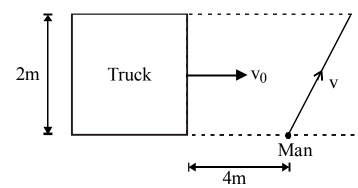
31. A glass capillary tube of internal radius r = 0.25 mm is immersed in water. The top end of the tube projected by 2 cm above the surface of the water. At what angle does the liquid meet the tube? Surface tension of water = 0.7 N/m.

(a) $\theta = 90°$ (b) $\theta = 70°$
(c) $\theta = 45°$ (d) $\theta = 35°$

32. A particle of mass 2 m is projected at an angle of 45° with the horizontal with a velocity of $20\sqrt{2}$ m/s. After 1s, explosion takes place and the particle is broken into two equal pieces. As a result of explosion, one part comes to rest. The maximum height from the ground attained by the other part is

(a) 50 m (b) 25m (c) 40 m (d) 35m

33. A 2 m wide truck is moving with a uniform speed v_0 = 8 m/s along a straight horizontal road. A pedestrain starts to cross the road with a uniform speed v when the truck is 4 m away from him. The minimum value of v so that he can cross the road safely is



(a) 2.62 m/s (b) 4.6 m/s
(c) 3.57 m/s (d) 1.414 m/s

34. A neutron moving with speed v makes a head on collision with a hydrogen atom in ground state kept at rest. The minimum kinetic energy of the neutron for which inelastic collision takes place is

(a) 10.2 eV (b) 20.4 eV
(c) 12.1 eV (d) 16.8 eV

35. Vertical displacement of a Planck with a body of mass m on it is varying according to law $y = \sin \omega t + \sqrt{3} \cos \omega t$. The minimum value of ω for which the mass just breaks off the Planck and the moment it occurs first after t = 0, are given by

(a) $\sqrt{g/2}, \dfrac{\sqrt{2}}{6}\dfrac{\pi}{\sqrt{g}}$ (b) $\dfrac{g}{\sqrt{2}}, \dfrac{2}{3}\sqrt{\pi/g}$
(c) $\sqrt{g/2}, \dfrac{\pi}{3}\sqrt{2/g}$ (d) $\sqrt{2g}, \sqrt{2\pi/3g}$

36. A parallel plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V. Another capacitor of capacitance 2C is similarly charge to a potential difference 2V. The charging battery is now disconnected and the capacitors are connected in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the configuration is

(a) Zero (b) $\frac{3}{2}CV^2$

(c) $\frac{25}{6}CV^2$ (d) $\frac{9}{2}CV^2$

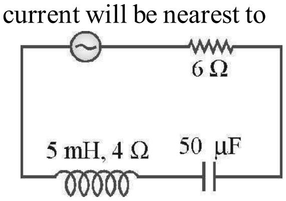
37. In the circuit shown below, the ac source has voltage $V = 20\cos(\omega t)$ volt with $\omega = 2000$ rad/s. The amplitude of the current will be nearest to

(a) 2A

(b) 3.3A

(c) $2/\sqrt{5}$A

(d) $\sqrt{5}$A



38. A constant voltage is applied between the two ends of a uniform metallic wire. Some heat is developed in it. The heat developed is doubled if

(a) both the length and the radius of the wire are halved.

(b) both the length and the radius of the wire are doubled.

(c) the radius of the wire is doubled.

(d) the length of the wire is doubled.

39. The frequency of a sonometer wire is 100 Hz. When the weights producing the tensions are completely immersed in water, the frequency becomes 80 Hz and on immersing the weights in a certain liquid, the frequency becomes 60 Hz. The specific gravity of the liquid is

(a) 1.42 (b) 1.77 (c) 1.82 (d) 1.21

40. A long straight wire along the Z-axis carries a current I in the negative Z-direction. The magnetic vector field $\vec{B}$ at a point having coordinates (x, y) in the $Z = 0$ plane is

(a) $\frac{\mu_0 I(y\hat{i} - x\hat{j})}{2\pi(x^2+y^2)}$ (b) $\frac{\mu_0 I(x\hat{i} + y\hat{j})}{2\pi(x^2+y^2)}$

(c) $\frac{\mu_0 I(x\hat{j} - y\hat{i})}{2\pi(x^2+y^2)}$ (d) $\frac{\mu_0 I(x\hat{i} - y\hat{j})}{2\pi(x^2+y^2)}$

PART - II: CHEMISTRY

41. Which of the following pollutants is main product of automobiles exhaust?

(a) CO (b) CO_2

(c) NO (d) Hydrocarbons

42. The disease caused the high concentration of hydrocarbon pollutants in atmosphere is/are

(a) silicosis (b) TB

(c) cancer (d) asthma

43. The element, with atomic number 118, will be

(a) alkali (b) noble gas

(c) lanthanide (d) transition element

44. Which law of the thermodynamics helps in calculating the absolute entropies of various substances at different temperatures?

(a) First law (b) Second law

(c) Third law (d) Zeroth law

45. The color of $CoCl_3.5NH_3.H_2O$ is

(a) red (b) orange

(c) orange - yellow (d) pink

46. The metal present in vitamin B_{12} is

(a) magnesium (b) cobalt

(c) copper (d) zinc

47. Cobalt (60) isotope is used in the treatment of :

(a) Heart diseases (b) Skin diseases

(c) Diabetes (d) Cancer

48. Polymer used in bullet proof glass is

(a) Lexan (b) PMMA

(c) Nomex (d) Kevlar

49. What is the correct increasing order of Bronsted bases?

(a) $ClO_4^- < ClO_3^- < ClO_2^- < ClO^-$

(b) $ClO_4^- > ClO_3^- > ClO_2^- > ClO^-$

(c) $ClO_3^- < ClO_4^- < ClO_2^- < ClO^-$

(d) $ClO^- > ClO_3^- > ClO_2^- < ClO_4^-$

50. The boiling point of alkyl halide are higher than those of corresponding alkanes because of

(a) dipole-dipole interaction

(b) dipole-induced dipole interaction

(c) H-bonding

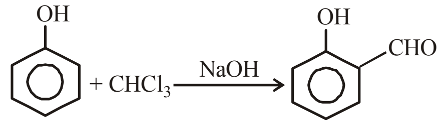
(d) None of the above

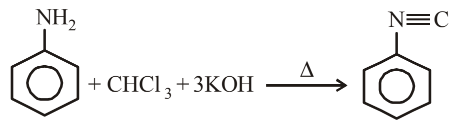
51. Some salts containing two different metallic elements give test for only one of them in solution, such salts are

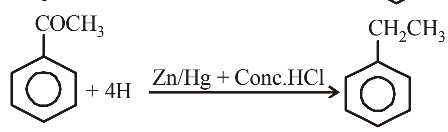
(a) double salts (b) normal salts

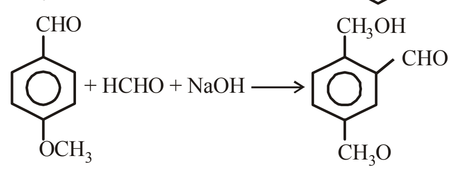
(c) complex salts (d) None of these

52. The carbylamine reaction is

(a) 

(b) 

(c) 

(d) 

53. Laughing gas is
(a) nitrogen pentoxide
(b) nitrous oxide
(c) nitrogen trioxide
(d) nitric oxide

54. The anthracene is purified by
(a) crystallisation (b) filtration
(c) distillation (d) sublimation

55. The common name of $K[PtCl_3(\eta^2.C_2H_4)]$ is
(a) potassium salt (b) Zeise's salt
(c) complex salt (d) None of these

56. The by product of Solvay-ammonia process is
(a) CO_2 (b) NH_3 (c) $CaCl_2$ (d) $CaCO_3$

57. Semiconductor materials like Si and Ge are usually purified by
(a) distillation (b) zone refining
(c) liquation (d) electrolytic refining

58. Which of the following is a strong base?
(a) PH_3 (b) AsH_3 (c) NH_3 (d) SbH_3

59. Ordinary glass is :
(a) Sodium silicate
(b) Calcium silicate
(c) Sodium and calcium silicate
(d) Mixed salt of Na and Ca

60. The prefix 10^{18} is
(a) giga (b) kilo (c) exa (d) nano

61. Which of the following is the most basic oxide?
(a) Sb_2O_3 (b) Bi_2O_3 (c) SeO_2 (d) Al_2O_3

62. Which one of the following does not follow octate rule?
(a) PF_3 (b) BF_3 (c) CO_2 (d) CCl_4

63. Which of the following according to Le-Chatelier's principle is correct?
(a) Increase in temperature favours the endothermic reaction
(b) Increase in temperature favours the exothermic reaction
(c) Increase in pressure shifts the equilibrium in that side in which number of gaseous moles increases
(d) All of the above are true

64. The efficiency of fuel cell is given by the expression, η is
(a) $\eta = -\frac{nFE_{cell}}{\Delta H} \times 100$
(b) $\eta = -\frac{nFE_{cell}}{\Delta S} \times 100$
(c) $\eta = -\frac{nFE_{cell}}{\Delta A} \times 100$
(d) None of the above

65. The mass of the substance deposited when one Faraday of charge is passed through its solution is equal to
(a) relative equivalent weight
(b) gram equivalent weight
(c) specific equivalent weight
(d) None of the above

66. The unit of rate constant for reactions of second order is
(a) $L\ mol^{-1}s^{-1}$ (b) $L^{-1}\ mol\ s^{-1}$
(c) $L\ mol\ s^{-1}$ (d) s^{-1}

67. In a first order reaction with time the concentration of the reactant decreases
(a) linearly (b) exponentially
(c) no change (d) None of these

68. The P—P—P angle in P_4 molecule and S—S—S angle in S_8 molecule is(in degree) respectively
(a) 60°, 107° (b) 107°, 60°
(c) 40°, 60° (d) 60°, 40°

69. The number of elements present in the d-block of the periodic table is
(a) 40 (b) 41 (c) 45 (d) 46

70. Which of the following represents hexadentate ligand?
(a) EDTA (b) DMG
(c) Ethylenediamine (d) None of the above

71. Which one of given elements shows maximum number of different oxidation states in its compounds?
(a) Am (b) Fm (c) La (d) Gd

72. $K_4[Fe(CN)_6]$ is used in detecting.
(a) Fe^{3+} ion (b) Cu^{+} ion
(c) Cu^{3+} ion (d) Fe^{2+} ion

73. A spontaneous reaction is impossible if
(a) both ΔH and ΔS are negative
(b) both ΔH and ΔS are positive
(c) ΔH is negative and ΔS is positive
(d) ΔH is positive and ΔS is negative

74. Which one the following removes temporary hardness of water ?
(a) Slaked lime (b) Plaster of Paris
(c) Epsom (d) Hydrolith

75. Graphite is a
(a) molecular solid (b) covalent solid
(c) ionic solid (d) metallic solid

76. Which of the following ionic substances will be most effective in precipitating the sulphur sol?
(a) KCl (b) $BaCl_2$
(c) $Fe_2(SO_4)_3$ (d) Na_3PO_4

77. Which of the following fluorides of xenon is impossible?
(a) XeF_2 (b) XeF_3 (c) XeF_4 (d) XeF_6

78. Thomas slag is
(a) $Ca_3(PO_4)_2$
(b) $CaSiO_3$
(c) Mixture of (a) and (b)
(d) $FeSiO_3$

79. A sequence of how many nucleotides in messenger RNA makes a codon for an amino acid?
(a) Three (b) Four (c) One (d) Two

80. Which of the following molecule/ion has all the three types of bonds, electrovalent, covalent and co-ordinate :
(a) HCl (b) NH_4^+
(c) Cl^- (d) H_2O_2

PART - III (A): ENGLISH PROFICIENCY

DIRECTIONS (Qs. 81-83): Choose the word which best expreses the meaning of the underlined word in the sentence.

81. Decay is an <u>immutable</u> factor of human life.
(a) important (b) unique
(c) unchangeable (d) awful

82. It was an <u>ignominious</u> defect for the team.
(a) shameful (d) admirable
(c) unaccountable (d) worthy

83. The attitude of western countries towards the third world countries is rather <u>callous</u> to say the least.
(a) cursed (d) unkend
(c) unfeeling (d) passive

DIRECTIONS (Qs. 84-86): Fill in the blank.

84. Freedom and equality are the ______ rights of every human.
(a) inalienable (b) inscrutable
(c) incalculable (d) institutional

85. The team was well trained and strong, but some how their ______ was low.
(a) morale (d) moral
(c) feeling (d) consciousness

86. His speech was disappointing: it ______ all the major issues.
(a) projected (b) revealed
(c) skirted (d) analysed

DIRECTIONS (Qs. 87-89): Choose the word which is closest to the opposite in meaning of the underlined word in the sentence.

87. Hydra is biologically believed to be immortal.
(a) undying (b) perishable
(c) ancient (d) eternal

88. The Gupta rulers <u>patronised</u> all cultural activities and thus Gupta period was called the golden era in Indian History.
(a) criticised (b) rejected
(c) opposed (d) spurned

89. The General Manager is quete <u>tactful</u> and handles the workers union very effectively.
(a) incautious (b) discreet
(c) strict (d) disciplned

DIRECTIONs (Qs. 90-92): In each ot the following questions, out of the four alternatives, choose the one which can be substituted for the given words/ sentence.

90. A person who does not believe in any religion
(a) Philatelist (b) Rationalist
(c) Atheist (d) Pagan

91. A person who believes that pleasure is the chief good
(a) Stoic (d) Hedonist
(c) Epicure (d) Sensual

92. A person who is incharge of museum.
(a) caretaker (b) warden
(c) supervisor (d) curator

DIRECTIONS (Qs. 93-95): Choose the order of the sentences marked A, B, C, D and E to form a logical paragraph.

93. A. Tasty and healthy food can help you bring out their best.
B. One minute they are toddlers and next you see them in their next adventure.
C. Your young ones seem to be growing so fast.

D. Being their loving custodians, you always want to see them doing well.

E. Their eye sparkle with curiosity and endless questions on their tongues.

Codes

(a) DBCEA (b) CADEB
(c) CBEDA (d) ECABD

94. A. It is hoping that overseas friends will bring in big money and lift the morale of the people.

B. But a lot needs to be done to kick start industrial revival.

C. People had big hopes from the new government.

D. So far government has only given an incremental push to existing policies and programmes.

E. Government is to go for big time reforms, which it promised.

Codes

(a) BCDAE (b) EADCB
(c) DABCE (d) CDEAB

95. A : Forecasting the weather has always been a defficult business.

B : During a period of drought, steams and rivers dried up, the cattle died from thirst and were ruined.

C : Many different things affect the weather and we have to study them carefully to make accurate forecast.

D : Ancient egyptians had no need of weather in the Nille valley hardly ever changes.

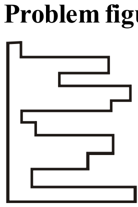
E: In early times, when there were no instruments, such as their mometer or the barometer, a man looked for tell tale signs in the sky.

(a) ABDCE (b) EDCBA
(c) ACBDE (d) BDCAE

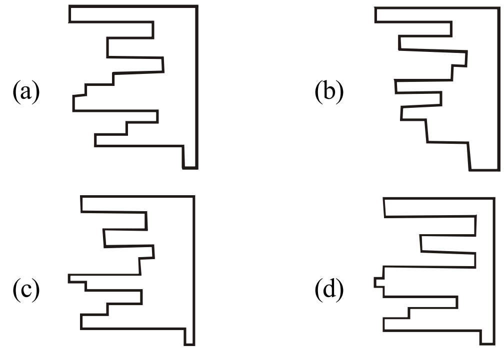
PART - III (B): LOGICAL REASONING

96. Choose the correct answer figure which will make a complete square on joining with the problem figure

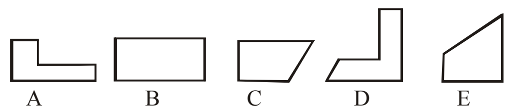
Problem figure



Answer Figures



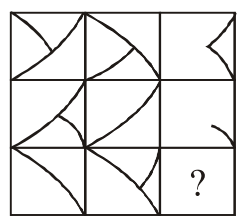
97. In the following question, five figures are given. Out of them, find the three figures that can be joined to form square.



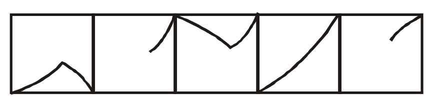
(a) ABC (b) BCD
(c) ACE (d) CDE

98. Choose the answer figure which completes the problem figure matrix.

Problem Figures

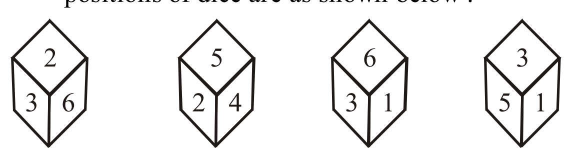


Answer Figures



(a) (b) (c) (d) (e)

99. What is the opposite of 3, if four different positions of dice are as shown below :

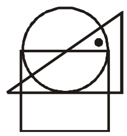


(i) (ii) (iii) (iv)

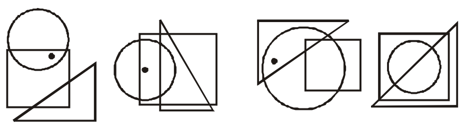
(a) 6 (b) 4 (c) 1 (d) 2

100. In the following questions, one or more dots are placed in the figure marked as (A). The figure is followed by four alternatives marked as (a), (b), (c) and (d). One out of these four options contains region(s) common to the circle, square, triangle, similar to that marked by the dot in figure (A).

Problem Figure



(A)



(a) (b) (c) (d)

101. Complete the series by replaing '? mark

G4T, J9R, M20P, P43N, S90L

(a) S90L (b) V185J (c) M20P (d) P43N

102. Neeraj starts walking towards South. After walking 15 m, he turns towards North. After walking 20 m, he turns towards East and walks 10 m. He then turns towards South and walks 5 m. How far is he from his original position and in which direction?

(a) 10 m, East (b) 10 m, South-East

(c) 10 m, West (d) 10 m, North-East

103. The average age of 8 men is increased by 2 yr when one of them whose age is 20 yr is replaced by a new man. What is the age of the new man

(a) 28 yr (b) 36 yr (c) 34 yr (d) 35 yr

104. Shikha is mother-in-law of Ekta who is sister-in-law of Ankit. Pankaj is father of Sanjay, the only brother of Ankit. How is Shikha related to Ankit?

(a) Mother-in-law (b) Aunt

(c) Wife (d) Mother

105. In a queue of children, Arun is fifth from the left and Suresh is sixth from the right. When they interchange their places among themselves, Arun becomes thirteenth from the left. Then, what will be Suresh's position from the right?

(a) 8th (b) 14th (c) 15th (d) 16th

PART - IV: MATHEMATICS

106. $\lim_{x \to \infty} \dfrac{\int_0^{2x} x e^{x^2} dx}{e^{4x^2}}$ equals

(a) 0 (b) ∞ (c) 2 (d) $\frac{1}{2}$

107. If ω is the complex cube root of unity, then the value of $\omega + \omega^{\left(\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+\frac{27}{128}+....\right)}$ is

(a) -1 (b) 1 (c) $-i$ (d) i

108. The root of the equation

$2(1+i)x^2 - 4(2-i)x - 5 - 3i = 0$ which has greater modulus is

(a) $\frac{3-5i}{2}$ (b) $\frac{5-3i}{2}$

(c) $\frac{3-i}{2}$ (d) none

109. The value of $\frac{3}{4}+\frac{15}{16}+\frac{63}{64}+$... upto n terms is

(a) $n-\frac{4^n}{3}-\frac{1}{3}$ (b) $n+\frac{4^{-n}}{3}-\frac{1}{3}$

(c) $n+\frac{4^n}{3}-\frac{1}{3}$ (d) $n-\frac{4^{-n}}{3}+\frac{1}{3}$

110. The period of tan 3θ is

(a) π (b) 3π/4

(c) π/2 (d) None of these

111. If a function f(x) is given by

$$f(x) = \frac{x}{1+x} + \frac{x}{(x+1)(2x+1)} + \frac{x}{(2x+1)(3x+1)} + ...\infty$$, then at x = 0, f(x)

(a) has no limit

(b) is not continuous

(c) is continuous but not differentiable

(d) is differentiable

112. If g is the inverse of function f and $f'(x) = \sin x$, then $g'(x)$ is equal to

(a) cosec {g(x)} (b) sin {g(x)}

(c) $\frac{1}{\sin\{g(x)\}}$ (d) None of these

113. A bag contains (2n + 1) coins. It is known that n of these coins have a head on both sides, whereas the remaining (n + 1) coins are fair. A coin is picked up at random from the bag and tossed. If the probability that the toss results in a head is 31/42, then n is equal to

(a) 10 (b) 11 (c) 12 (d) 13

114. If $\phi(x)$ is a differential function, then the solution of the differential equation $dy + \{y\phi'(x) - \phi(x)\phi'(x)\}dx = 0$, is

(a) $y = \{\phi(x) - 1\} + Ce^{-\phi(x)}$

(b) $y\phi(x) = \{\phi(x)\}^2 + C$

(c) $ye^{\phi(x)} = \phi(x)e^{\phi(x)} + C$

(d) $y - \phi(x) = \phi(x)e^{-\phi(x)}$

115. The area of the region $R = \{(x, y): |x| \le |y| \text{ and } x^2 + y^2 \le 1\}$ is

(a) $\frac{3\pi}{8}$ sq units (b) $\frac{5\pi}{8}$ sq units

(c) $\frac{\pi}{2}$ sq units (d) $\frac{\pi}{8}$ sq unit

116. Universal set,

$U = \{x \mid x^5 - 6x^4 + 11x^3 - 6x^2 = 0\}$

$A = \{x \mid x^2 - 5x + 6 = 0\}$

$B = \{x \mid x^2 - 3x + 2 = 0\}$

What is $(A \cap B)'$ equal to ?

(a) $\{1, 3\}$ (b) $\{1, 2, 3\}$

(c) $\{0, 1, 3\}$ (d) $\{0, 1, 2, 3\}$

117. If $\cos^{-1} x - \cos^{-1} \frac{y}{2} = \alpha$,

then $4x^2 - 4xy\cos\alpha + y^2$ is equal to

(a) $2\sin 2\alpha$ (b) 4

(c) $4\sin^2\alpha$ (d) $-4\sin^2\alpha$

118. If $\frac{e^x + e^{5x}}{e^{3x}} = a_0 + a_1x + a_2x^2 + a_3x^3 + ...$ then the value of $2a_1 + 2^3a_3 + 2^5a_5 + ...$ is

(a) $e^2 + e^{-2}$ (b) $e^4 - e^{-4}$

(c) $e^4 + e^{-4}$ (d) 0

119 Let a, b and c be three vectors satisfying $a \times b = (a \times c)$, $|a| = |c| = 1$, $|b| = 4$ and $|b \times c| = \sqrt{15}$. If $b - 2c = \lambda a$, then λ equals

(a) 1 (b) – 1 (c) 2 (d) – 4

120. The total number of 4-digit numbers in which the digits are in descending order, is

(a) ${}^{10}C_4 \times 4!$ (b) ${}^{10}C_4$

(c) $\frac{10!}{4!}$ (d) None of these

121. The line which is parallel to X-axis and crosses the curve $y = \sqrt{x}$ at an angle of 45°, is

(a) $x = \frac{1}{4}$ (b) $y = \frac{1}{4}$

(c) $y = \frac{1}{2}$ (d) $y = 1$

122. In a ΔABC, the lengths of the two larger sides are 10 and 9 units, respectively. If the angles are in AP, then the length of the third side can be

(a) $5 \pm \sqrt{6}$ (b) $3\sqrt{3}$

(c) 5 (d) None of these

123. The arithmetic mean of the data 0, 1, 2, ..., n with frequencies 1, ${}^nC_1$, ${}^nC_2$,....., ${}^nC_n$ is

(a) n (b) $\frac{2^n}{n}$ (c) n + 1 (d) $\frac{n}{2}$

124. The mean square deviation of a set of n observation $x_1, x_2, ... x_n$ about a point c is defined as $\frac{1}{n}\sum_{i=1}^{n}(x_i - c)^2$.

The mean square deviations about – 2 and 2 are 18 and 10 respectively, the standard deviation of this set of observations is

(a) 3 (b) 2

(c) 1 (d) None of these

125. Let S be the focus of the parabola $y^2 = 8x$ and PQ be the common chord of the circle $x^2 + y^2 - 2x - 4y = 0$ and the given parabola. The area of ΔPQS is

(a) 4 sq units (b) 3 sq units

(c) 2 sq units (d) 8 sq units

126. The number of real roots of the equation $e^{x-1} + x - 2 = 0$ is

(a) 1 (b) 2 (c) 3 (d) 4

127. Minimise $Z = \sum_{j=1}^{n}\sum_{i=1}^{m} c_{ij} x_{ij}$

Subject to $\sum_{i=1}^{m} x_{ij} = b_j, j = 1, 2, ..., n$

$\sum_{j=1}^{n} x_{ij} = b_j, i = 1, 2, ..., m$ is a LPP with number of constraints

(a) $m - n$ (b) mn (c) $m + n$ (d) $\frac{m}{n}$

128. A bag contains 3 red and 3 white balls. Two balls are drawn one by one. The probability that they are of different colours is.

(a) 3/10 (b) 2/5
(c) 3/5 (d) None of these

129. Let M be a 3 × 3 non-singular matrix with det (M) = α. If $[M^{-1} \text{adj}(\text{adj}(M)] = KI$, then the value of K is

(a) 1 (b) α (c) α^2 (d) α^3

130. Tangents are drawn from the origin to the curve y = cos x. Their points of contact lie on

(a) $x^2y^2 = y^2 - x^2$ (b) $x^2y^2 = x^2 + y^2$
(c) $x^2y^2 = x^2 - y^2$ (d) None of these

131. The slope of the tangent to the curve $y = e^x \cos x$ is minimum at x = α, 0 ≤ a ≤ 2π, then the value of α is

(a) 0 (b) π (c) 2π (d) 3π/2

132. Two lines $L_1 : x = 5, \frac{y}{3-\alpha} = \frac{z}{-2}$

$L_2 : x = \alpha, \frac{y}{-1} = \frac{z}{2-\alpha}$ are coplanar. Then, α can take value (s)

(a) 1, 4, 5 (b) 1, 2, 5
(c) 3, 4, 5 (d) 2, 4, 5

133. The eccentricity of an ellipse, with its centre at the origin, is 1/2. If one of the directrices is x = 4, then the equation of the ellipse is:

(a) $4x^2 + 3y^2 = 1$ (b) $3x^2 + 4y^2 = 12$
(c) $4x^2 + 3y^2 = 12$ (d) $3x^2 + 4y^2 = 1$

134. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minimum at

(a) x = 2 (b) x = –2 (c) x = 0 (d) x = 1

135. If $y = (x + \sqrt{1+x^2})^n$, then $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is

(a) n^2y (b) $-n^2y$ (c) $-y$ (d) $2x^2y$

136. If $\lim_{x\to\infty} x\sin\left(\frac{1}{x}\right) = A$ and $\lim_{x\to 0} x\sin\left(\frac{1}{x}\right) = B$, then which one of the following is correct?

(a) A = 1 and B = 0 (b) A = 0 and B = 1
(c) A = 0 and B = 0 (d) A = 1 and B = 1

137. If a and b are non-zero roots of $x^2 + ax + b = 0$ then the least value of $x^2 + ax + b$ is

(a) $\frac{2}{3}$ (b) $-\frac{9}{4}$ (c) $\frac{9}{4}$ (d) 1

138. If $0 < x < \frac{\pi}{2}$, then

(a) tan x < x < sin x
(b) x < sin x < tan x
(c) sin x < tan x < x
(d) None of the above

139. The degree of the differential equation satisfying

$\sqrt{1-x^2} + \sqrt{1+y^2} = a(x-y)$ is

(a) 1 (b) 2 (c) 3 (d) 4

140. Let f(x) be a polynomial of degree three satisfying f(0) = – 1 and f(1) = 0. Also, 0 is a stationary point of f(x). If f(x) does not have an extremum at x = 0, then the value of $\int \frac{f(x)}{x^3 - 1} dx$ is

(a) $\frac{x^2}{2} + C$ (b) $x + C$
(c) $\frac{x^3}{6} + C$ (d) None of these

141. The domain of the function

$f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is

(a) [1, 2] (b) [2, 3)
(c) [1, 2] (d) [2, 3]

142. If the lines $p_1x + q_1y = 1$, $p_2x + q_2y = 1$ and $p_3x + q_3y = 1$ be concurrent, then the points (p_1, q_1), (p_2, q_2) and (p_3, q_3)

(a) are collinear

(b) form an equilateral triangle

(c) form a scalene triangle

(d) form a right angled triangle

143. Area of the circle in which a chord of length $\sqrt{2}$ makes an angle $\pi/2$ at the centre, is

(a) $\pi/2$ sq units (b) 2π sq units

(c) π sq units (d) $\pi/4$ sq units

144. If $\frac{\cos A}{\cos B} = n$, $\frac{\sin A}{\sin B} = m$, then the value of $(m^2 - n^2)\sin^2 B$ is

(a) $1 + n^2$ (b) $1 - n^2$ (c) n^2 (d) $-n^2$

145. If complex number z_1, z_2 and 0 are vertices of equilateral triangle, then $z_1^2 + z_2^2 - z_1z_2$ is equal to

(a) 0 (b) $z_1 - z_2$ (c) $z_1 + z_2$ (d) 1

146. If $\rho = \{(x, y) \mid x^2 + y^2 = 1; x, y \in R\}$. Then, ρ is

(a) reflexive (b) symmetric

(c) transitive (d) anti-symmetric

147. A line makes the same angle θ with each of the X and Z-axes. If the angle β, which it makes with Y-axis, is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals

(a) 2/5 (b) 1/5 (c) 3/5 (d) 2/3

148. If in a binomial distribution $n = 4$, $P(X = 0) = \frac{16}{81}$, then $P(X = 4)$ equals

(a) $\frac{1}{16}$ (b) $\frac{1}{81}$ (c) $\frac{1}{27}$ (d) $\frac{1}{8}$

149. Let $f : R \to R$ be a function such that

$f(x + y) = f(x) + f(y)$, $\forall\ x, y \in R$

If $f(x)$ is differentiable at $x = 0$, then which one of the following is incorrect?

(a) $f(x)$ is continuous, $\forall x \in R$

(b) $f'(x)$ is constant, $\forall x \in R$

(c) $f(x)$ is differentiable, $\forall x \in R$

(d) $f(x)$ is differentiable only in a finite interval containing zero.

150. If binomial coefficients of three consecutive terms of $(1 + x)^n$ are in HP, then the maximum value of n is

(a) 1 (b) 2

(c) 0 (d) None of these

SOLUTIONS

PART - I: PHYSICS

1. (b) The escape velocity from earth is given by

$v_e = \sqrt{2g R_e}$ … (i)

The orbital velocity of a satellite revolving around earth is given by

$$v_0 = \frac{\sqrt{GM_e}}{(R_e + h)}$$

where, M_e = mass of earth, R_e = radius of earth, h = height of satellite from surface of earth.

By the relation $GM_e = gR_e^2$

So, $v_0 = \frac{\sqrt{gR_e^2}}{(R_e+h)}$ … (ii)

Dividing equation (i) by (ii), we get

$$\frac{v_e}{v_0} = \frac{\sqrt{2(R_e+h)}}{(R_e)}$$

Given, $v_0 = \frac{v_e}{2}$

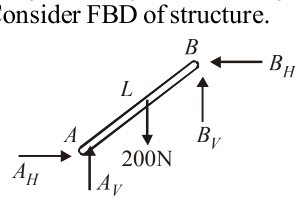
$$\frac{2v_e}{v_e} = \frac{\sqrt{2(R_e+h)}}{R_e}$$

Squaring on both side, we get

$$4 = \frac{2(R_e+h)}{R_e}$$

or $R_e + h = 2R_e$ i.e., $h = R_e$

2. (a) Consider FBD of structure.

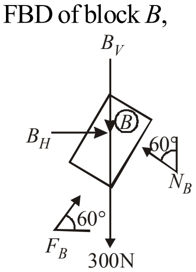


Applying equilibrium equations,

$Av + Bv = 200$ N … (i)

$A_H = B_H$ … (ii)

From FBD of block B,



$B_H + F_B \cos 60° - N_B \sin 60° = 0$

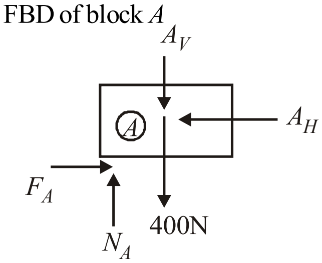
$N_B \cos 60° - B_V - 300 + F_B \sin 60° = 0$

$F_B = 0.25\, N_B$

$B_H - 0.74\, N_B = 0$ … (iii)

$-B_V + 0.71\, N_B = 300$ … (iv)

FBD of block A



$F_A - A_H = 0$

$N_A - A_V = 400$ … (v)

$F_A = \mu_A N_A$

$\therefore \mu_A N_A - A_H = 0$ … (vi)

On solving above equations, we get

$N_A = 650$ N, $F_A = 260$N, $F_A = \mu_A N_A$

$$\therefore \mu_A = \frac{260}{250} = 0.4$$

3. (a) Let v = speed of sound and v_S = speed of tuning forks. Apparent frequency of fork moving towards the observer is

$$n_1 = \left(\frac{v}{v - v_s}\right) n$$

Apparent frequency of the fork moving away from the observer is

$$n_2 = \left(\frac{v}{v + v_s}\right) n$$

If f is the number of beats heard per second. then $f = n_1 - n_2$

$$\Rightarrow f = \left(\frac{v}{v - v_s}\right) n - \left(\frac{v}{v + v_s}\right) n$$

$$\Rightarrow f = \frac{v(v + v_s) - v(v - v_s)}{v^2 - v_s^2} (n)$$

$$\Rightarrow \frac{2vv_s n}{v^2 - v_s^2} = f \Rightarrow 2\left(\frac{v_s}{v}\right) = n = f\{\text{if } v_s << v\}$$

$$\Rightarrow v_s = \frac{fv}{2n}$$

putting v = 340 m/s, f = 3, n = 340 Hz we get,

$$v_s = \frac{340\times 3}{3\times 340} = 1.5\,m/s$$

4. **(a)** The displacement of the particle is given by:

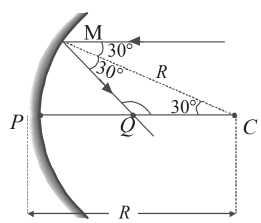
$$x = A\sin(-2\omega t) + B\sin^2\omega t$$

$$= -A\sin 2\omega t + \frac{B}{2}(1-\cos 2\omega t)$$

$$= -(A\sin 2\omega t + \frac{B}{2}\cos 2\omega t) + \frac{B}{2}$$

This motion represents SHM with an amplitude: $\sqrt{A^2 + \frac{B^2}{4}}$, and mean position $\frac{B}{2}$.

5. **(d)** From similar triangles,



$$\frac{QC}{\sin 30°} = \frac{R}{\sin 120°}$$

or $QC = R\times\frac{\sin 30°}{\sin 120°} = \frac{R}{\sqrt{3}}$

Thus

$$PQ = PC - QC = R - \frac{R}{\sqrt{3}} = R\left(1-\frac{1}{\sqrt{3}}\right)$$

6. **(c)** Using Gauss's law, we have

$$\oint \vec{E}.d\vec{A} = \frac{1}{\epsilon_0}\int(\rho dv) = \frac{1}{\epsilon_0}\int_0^R kr^a \times 4\pi r^2 dr$$

or $E\times 4\pi R^2 = \left(\frac{4\pi k}{\epsilon_0}\right)\frac{R^{(a+3)}}{(a+3)}$

$\therefore\quad E_1 = \frac{kR^{(a+1)}}{\epsilon_0\,(a+3)}$

For $r = \frac{R}{2}\cdot\ E_2 = \frac{k\left(\frac{R}{2}\right)^{a+1}}{\epsilon_0\,(a+3)}$

Given, $E_2 = \frac{E_1}{8}$

or $\frac{k\left(\frac{R}{2}\right)^{a+1}}{\epsilon_0\,(a+3)} = \frac{1}{8}\frac{kR^{(a+1)}}{\epsilon_0\,(a+3)}$

$\therefore\quad \frac{1}{2^{a+1}} = \frac{1}{8}$

or $a = 2.$

7. **(a)** As we know $F = qvB = \frac{mv^2}{r}$

$\therefore\ r = \frac{mv}{Bq}$

And $KE = k = \frac{1}{2}mv^2$

$\therefore\ mv = \sqrt{2km}$

$\therefore\ r = \frac{mv}{qB} = \frac{\sqrt{2km}}{qB}$

$\Rightarrow r \propto \sqrt{k}$ or $r = c^{1/2}$ (c is a constant)

$\frac{dr}{dr} = c\frac{dk^{1/2}}{dr}$ or $\frac{c\Delta k}{\Delta r} = 2\sqrt{k}$

or $\frac{\Delta r}{r} = \frac{c\Delta k}{2\sqrt{k}c\sqrt{k}} = \frac{\Delta k}{2k}$

Therefore percentage changes in radius of path,

$$\frac{\Delta r}{r}\times 100 = \frac{\Delta k}{2k}\times 100 = 2\%$$

8. **(d)** Given: fringe with $\beta = 0.03$ cm, D = 1 m = 100 cm

Distance between images of the source = 0.8 cm.

Image distance v = 80 cm

Object distance = u

Using mirror formula,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{60}+\frac{1}{u}=\frac{1}{16}$$

$\Rightarrow u = 20$ cm

Magnification, $m = \frac{v}{u} = \frac{80}{20} = 4$

$$\text{Magnification} = \frac{\text{distances between images of slits}}{\text{distance between slits}}$$

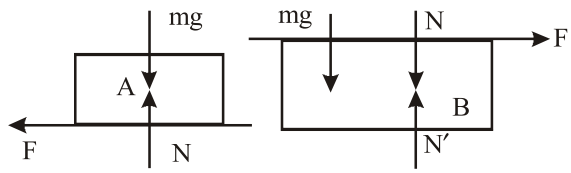
$$= \frac{0.8}{d} = \frac{0.8}{d} = 4 \Rightarrow d = 0.2\text{cm}$$

Fringe width $\beta = \frac{D\lambda}{d}$

or, $\beta = \frac{100\lambda}{0.2} = 0.03\times10^{-2}$

Therefore, wavelength of light used $\lambda = 6000$ Å

9. **(d)** For the blocks A and B FBD as shown below



Equations of motion

$$a_A = \frac{F}{M}(\text{in} - x \text{ direction})$$

$$a_B = \frac{F}{M}(\text{in} + x \text{ direction})$$

Relative acceleration, of A w.r.t. B,

$$a_{A,B} = a_A - a_B = -\frac{F}{m} - \frac{F}{M}$$

$$= -F\left(\frac{M+m}{Mm}\right) \text{ (along} - x \text{ direction)}$$

Initial relative velocity of A w.r.t. B,

$u_{AB} = v_0$

using equation $v^2 = u^2 + 2as$

$$0 = v_0^2 - \frac{2F(m+M)S}{Mm} \Rightarrow S = \frac{Mmv_0^2}{2F(m+M)}$$

i.e., Distance moved by A relative to B

$$S_{AB} = \frac{Mmv_0^2}{2F(m+M)}$$

10. **(b)** In the string elastic force is conservative in nature.

$\therefore W = -\Delta U$

Work done by elastic force of string,

$W = -(U_F - U_i) = U_i - U_F.$

$$W = \frac{1}{2}kx^2 - \frac{k}{2}(x+y)^2$$

$$= \frac{1}{2}kx^2 - \frac{1}{2}k\left(x^2 + y^2 + 2xy\right)$$

$$= \frac{1}{2}kx^2 - \frac{1}{2}ky^2 - \frac{1}{2}kx^2 - \frac{1}{2}k(2xy)$$

$$= -kxy - \frac{1}{2}ky^2$$

Therefore, the work done against elastic force

$$W_{external} = -W = \frac{ky}{2}(2x+y)$$

11. **(c)** Let the body is projected vertically upwards from A with a speed u_0.

Using equation, $s = ut + \frac{1}{2}at^2$

For case (1) $-h = u_0t_1 - \frac{1}{2}gt_1^2$ ...(1)

For case (2) $-h = u_0t_2 - \frac{1}{2}gt_1^2$ ...(2)

Subtracting eq (2) from (1), we get

$$0 = u_0(t_2 + t_1) + \frac{1}{2}g\left(t_2^2 - t_1^2\right)$$

$$\Rightarrow u_0 = \frac{1}{2}g(t_1 - t_2) \quad ...(3)$$

Putting the value of u_0 in eq (2), we get

$$-h = -\left(\frac{1}{2}\right)g(t_1 - t_2)t_2 - \left(\frac{1}{2}\right)gt_2^2$$

$$\Rightarrow h = \frac{1}{2}g(t_1t_2) \quad ...(4)$$

For case 3, $u_0 = 0, t = ?$

$$-h = 0\times t - \left(\frac{1}{2}\right)gt^2$$

$$h = \left(\frac{1}{2}\right)gt^2$$

Comparing eq. (4) and (5), we get

$$\frac{1}{2}gt^2 = \frac{1}{2}gt_1t_2 \;\therefore\; t = \sqrt{t_1t_2}$$

12. **(c)** $\frac{\Delta Q}{\Delta t} = \frac{\Delta W}{\Delta t}$ = work done per unit time = $\frac{k\,a\,\theta}{L}$

$$\frac{dW}{dt} = P\frac{dv}{dt} = k\frac{a\theta}{L},\; P = \frac{nRT}{V}$$

$$\Rightarrow \frac{0.5\,R(300)}{V}A.\frac{d\ell}{dt} = \frac{ka\theta}{L}$$

$$\Rightarrow \frac{0.5\,R(300)}{A.\frac{L}{2}}A.v = \frac{ka\theta}{L}$$

$$\Rightarrow v = \frac{ka}{R}\left(\frac{27}{300}\right) = \frac{k}{100R}$$

13. **(a)** At any time t, the side of the square $a = (a_0 - \alpha t)$, where a_0 = side at $t = 0$.

At this instant, flux through the square :
$\phi = BA\cos 0° = B(a_0 - \alpha t)^2$

$\therefore$ emf induced $E = -\dfrac{d\phi}{dt}$

$\Rightarrow E = -B.2(a_0 - \alpha t)(0 - \alpha) = +2\alpha a B$

14. **(b)** As we know, threshold wavelength

$$(\lambda_0) = \frac{hc}{\phi}$$

$$\Rightarrow \lambda_0 = \frac{(6.63\times10^{-34})\times3\times10^8}{2.3\times(1.6\times10^{-19})} = 5.404\times10^{-7}\,m.$$

$\Rightarrow \lambda_0 = 5404$ Å

Hence, wavelength 4144 Å and 4972 Å will emit electron from the metal surface.

For each wavelength energy incident on the surface per unit time

= intensity of each × area of the surface wavelength

$$= \frac{3.6\times10^{-3}}{3}\times(1\text{cm})^2 = 1.2\times10^{-7}\text{ joule}$$

Therefore, energy incident on the surface for each wavelength in 2s

$E = (1.2\times10^{-7})\times2 = 2.4\times10^{-7}$ J

Number of photons n_1 due to wavelength 4144 Å

$$n_1 = \frac{(2.4\times10^{-7})(4144\times10^{-10})}{(6.63\times10^{-34})(3\times10^8)} = 0.5\times10^{12}$$

Number of photons n_2 due to the wavelength 4972 Å

$$n_2 = \frac{(2.4\times10^{-7})(4972\times10^{-10})}{(6.63\times10^{-34})(3\times10^8)} = 0.572\times10^{12}$$

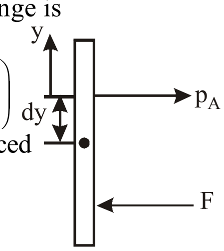
Therefore total number of photoelectrons liberated in 2s,

$$N = n_1 + n_2$$
$$= 0.5\times10^{12} + 0.575\times10^{12}$$
$$= 1.075\times10^{12}$$

15. **(c)** The net force acting on the gate element of width dy at a depth y from the surface of the fluid, is

$$dy = (p_0 + \rho_g y - p_0)\times1dy$$
$$= \rho gydy$$

Torque about the hinge is



$$d\tau = \rho gydy\times\left(\frac{1}{2}-y\right)$$

Net torque experienced by the gate is

$$\tau_{net} = \int d\tau + F\times\frac{1}{2}$$

$$= \int_0^1 \rho g\,ydy\left(\frac{1}{2}-y\right) + F\times\frac{1}{2} = 0$$

$$\Rightarrow\quad F = \frac{\rho g}{6}$$

i.e., The force F required to hold the gate stationary is $\dfrac{pg}{6}$

16. **(b)** As we know,

$$F = qvB = m\frac{v^2}{R} \Rightarrow v = \frac{q}{m}BR$$

The kinetic energy of the photoelectron

$$= \frac{1}{2}mv^2 = \frac{1}{2}\frac{e^2B^2R^2}{m}$$

$$= \frac{1}{2}\frac{(1.6\times10^{-19})^2(2\times10^{-2})^2(23\times10^{-3})^2}{(9.1\times10^{-31})}$$

$= 2.97\times10^{-15}$ J

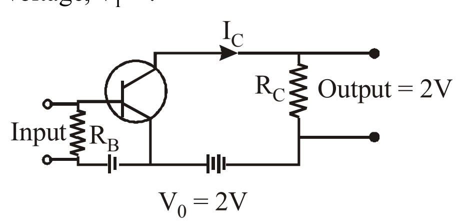
$$= \frac{2.97\times10^{-15}}{1.6\times10^{-19}} = 18.36\,\text{keV}$$

Energy of the incident photon $= \dfrac{hc}{\lambda}$

$$= \frac{12.4}{0.50} = 24.8\,\text{keV}$$

Therefore, Binding energy = 24.8 – 18.6 = 6.2 keV

17. **(c)** Given : Voltage across the collector $V_0 = 2$ V; collector resistance, $R_c = 2\times10^3\ \Omega$; Base resistance $R_B = 1\times10^3\ \Omega$; Input signal voltage, $V_i = ?$



$V_0 = I_C R_C = 2$

$\Rightarrow \quad I_C = \frac{2}{2\times10^3} = 10^{-3}\,A$

Current gain $\alpha = \frac{I_C}{I_B} = 100$

$\Rightarrow \quad I_B = \frac{I_C}{100} = \frac{10^{-3}}{100} = 10^{-5}\,A$

$V_i = R_B I_B \Rightarrow V_i = 1\times10^3\times10^{-5}$

$\Rightarrow \quad V_i = 10^{-2}\,V \Rightarrow V_i = 10\,mV$

18. **(a)** Initial potential energy of the system

$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q^2}{a}+\frac{q^2}{a}+\frac{q^2}{a}\right] = \frac{1}{4\pi\varepsilon_0}\left(\frac{3q^2}{a}\right)$

$= 9\times10^9\left(3\times\frac{(0.1)^2}{1}\right) = 27\times10^7\,J$

Let charge at A is moved to mid-point O, Then final potential energy of thhe system

$U_f = \frac{1}{4\pi\varepsilon_0}\left[\frac{2q^2}{(a/2)}+\frac{q^2}{a}\right]$

$= 5\times\frac{1}{4\pi\varepsilon_0}\left(\frac{q^2}{a^2}\right) = 45\times10^7\,J$

Work done $= U_f - U_i = 18\times10^7\,J$

Also, energy supplied per sec = 1000 J (given)

Time required to move one of the mid-point of the line joining the other two

$t = \frac{18\times10^7}{1000} = 18\times10^4\,s = 50\,h$

19. **(d)** Let there are n_1 moles of hydrogen and n_2 moles of helium in the given mixture. As Pv = nRT

Then the pressure of the mixture

$P = \frac{n_1RT}{V}+\frac{n_2RT}{V} = (n_1+n_2)\frac{RT}{V}$

$\Rightarrow$

$2\times101.3\times10^3 = (n_1+n_2)\times\frac{(8.3\times300)}{20\times10^{-3}}$

or,

$(n_1+n_2) = \frac{2\times101.3\times10^3\times20\times10^{-3}}{(8.3)(300)}$

or, $\quad n_1+n_2 = 1.62 \qquad \ldots(1)$

The mass of the mixture is (in grams)

$n_1\times2+n_2\times4 = 5$

$\Rightarrow \quad (n_1+2n_2) = 2.5 \qquad \ldots(2)$

Solving the eqns. (1) and (2), we get

$n_1 = 0.74$ and $n_2 = 0.88$

Hence, $\frac{m_H}{m_{He}} = \frac{0.74\times2}{0.88\times4} = \frac{1.48}{3.52} = \frac{2}{5}$

20. **(c)** Resistance of wire $(R) = \rho\frac{l}{A}$

If wire is bent in the middle then

$l' = \frac{l}{2}, A' = 2A$

$\therefore$ New resistance, $R' = \rho\frac{l'}{A'} = \frac{\rho\frac{l}{2}}{2A} = \frac{\rho l}{4A} = \frac{R}{4}.$

21. **(b)** The path difference when transparent sheet is introduced $\Delta x = (\mu-1)t$

If the central maxima occupies position of nth fringe, then $(\mu-1)t = n\lambda = d\sin\theta$

$\Rightarrow \sin\theta = \frac{(\mu-1)t}{d} = \frac{(1.17-1)\times1.5\times10^{-7}}{3\times10^{-7}} = 0.085$

Therefore, angular position of central maxima

$\theta = \sin^{-1}(0.085) = 4.88° \approx 4.9$

For small angles, $\sin\theta \approx \theta \approx \tan\theta$

$\Rightarrow \quad \tan\theta = \frac{y}{D}$

$\therefore \quad \frac{y}{D} = \frac{(\mu-1)t}{d} \Rightarrow y = \frac{D(\mu-1)t}{d}$

22. **(c)** From question,

Horizontal velocity (initial),

$u_x = \frac{40}{2} = 20\,m/s$

Vertical velocity (initial), $50 = u_y t + \frac{1}{2}gt^2$

$\Rightarrow \quad u_y\times2+\frac{1}{2}(-10)\times4$

or, $\quad 50 = 2u_y - 20$

or, $\quad u_y = \frac{70}{2} = 35\,m/s$

$\therefore \quad \tan\theta = \frac{u_y}{u_x} = \frac{35}{20} = \frac{7}{4}$

$\Rightarrow \quad$ Angle $\theta = \tan^{-1}\frac{7}{4}$

23. (a) Newton's law of viscosity, $F = \eta A\frac{dv}{dy}$

Stress $= \frac{F}{A} = \eta\left(\frac{dv}{dy}\right) = \eta k\left(\frac{4y}{a^2} - \frac{3y^2}{a^3}\right)$

At $y = a$, stress $= \eta k\left(\frac{4}{a} - \frac{3}{a}\right) = \frac{\eta k}{a}$

24. (b) According to energy conservation principle,

If, x_1 is maximum elongation in the spring when the particle is in its lowest extreme position. Then,

$$mgh = \frac{1}{2}kx_1^2 - mgx_1$$

$$\Rightarrow \frac{1}{2}kx_1^2 - mgx_1 - mgh = 0$$

$$\text{or, } x_1^2 - \frac{2mg}{k}x_1 - \frac{2mg}{k}.h = 0$$

$$\therefore x_1 = \frac{\frac{2mg}{k} \pm \sqrt{\left[\left(\frac{2mg}{k}\right)^2 + 4\times\frac{2mg}{k}h\right]}}{2}$$

Amplitude $A = X_1 - X_0$ (elongation in spring for equilibrium position)

$$A = \frac{mg}{k}\sqrt{\left(1 + \frac{2hk}{mg}\right)}$$

25. (c) Let the initial mass of uranium be M_0

Final mass of uranium after time t,

$$M = \frac{3}{4}M_0$$

According to the law of radioactive disintegration.

$$\frac{M}{M_0} = \left(\frac{1}{2}\right)^{t/T} \Rightarrow \frac{M_0}{M} = (2)^{t/T}$$

$$\therefore \log_{10}\left(\frac{M_0}{M}\right) = \frac{t}{T}\log_{10}(2)$$

$$t = T\frac{\log_{10}\left(\frac{M_0}{M}\right)}{\log_{10}(2)} = \frac{T\log_{10}\left(\frac{4}{3}\right)}{\log_{10}(2)}$$

$$= \frac{T\log_{10}(1.333)}{\log_{10}(2)} = 4.5\times10^9\left(\frac{0.1249}{0.3010}\right)$$

$\Rightarrow \quad t = 1.867 \times 10^9$ yr.

26. (b) Total current, $I = (5 + 10 \sin \omega t)$

$$\Rightarrow I_{eff} = \left[\frac{\int_0^T I^2 dt}{\int_0^T dt}\right]^{1/2}$$

$$= \left[\frac{1}{T}\int_0^T (5 + 10\sin\omega t)^2\, dt\right]^{1/2}$$

$$= \left[\frac{1}{T}\int_0^T \left(25 + 100\sin\omega t + 100\sin^2\omega t\right)\right]^{1/2}$$

But, $\frac{1}{T}\int_0^T \sin\omega t.dt = 0$ and

$$\frac{1}{T}\int_0^T \sin^2\omega t.dt = \frac{1}{2}$$

So, $I_{eff} = \left[25 + \frac{1}{2}\times100\right]^{1/2} = 5\sqrt{3}A$

27. (a) If F be the equivalent focal length, then

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \quad \frac{1}{F} = (\mu_1 - 1)\left(\frac{1}{\infty} + \frac{1}{R}\right)$$

$$+ (\mu_2 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right) = \frac{\mu_1 - \mu_2}{R}$$

$$F = \frac{R}{\mu_1 - \mu_2}$$

28. (b) Total moment of inertia

$= I_1 + I_2 + I_3 + I_4 = 2I_1 + 2I_2$

$= 2(I_1 + I_2)\ [I_3 = I_1, I_1 = I_4]$

Now, $I_2 = I_3 = \frac{Ml^2}{3}$

Using parallel axes theorem, we have

$$I = I_{CM} + Mx^2 \text{ and } x = \sqrt{l^2 + \frac{l^2}{4}}$$

$$I_1 = I_4 = \frac{Ml^2}{12} + M\left[\sqrt{l^2 + \left(\frac{l}{2}\right)^2}\right]^2$$

Putting all values we get

Moment of inertia, $I = 10\left(\frac{Ml^2}{3}\right)$

29. (b) For 2nd line of Balmer series in hydrogen spectrum

$$\frac{1}{\lambda} = R\ (1)\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3}{16}R$$

For Li^{2+} $\left[\frac{1}{\lambda} = R \times 9\left(\frac{1}{x^2} - \frac{1}{12^2}\right) = \frac{3R}{16}\right]$

which is satisfied by $n = 12 \rightarrow n = 6$.

30. (c) When a charge particle is allowed to move in a uniform magnetic field, then it describes spiral or circular path

Centripetal force, $\frac{mv^2}{R} = qvB$ $\therefore$

$v = \left(\frac{qB}{m}R\right)$

Hence, $\sqrt{\frac{2qV}{m}} = \left(\frac{qB}{m}\right)R$ $\left[\because V = \sqrt{\frac{2qV}{m}}\right]$

$\Rightarrow R = \left(\frac{2mV}{q}\right)^{1/2} \times \frac{1}{B}$

or, $m \propto R^2$

[$\because$ V, q and B are constant]

or, $\frac{m_1}{m_2} = \left(\frac{R_1}{R_2}\right)^2$

31. (b) Water wets glass and so the angle of contact is zero.

For full rise, neglecting the small mass in the meniscus

$2\pi rT = \pi r^2 h\rho g \Rightarrow h = \frac{2T}{r\rho g}$ [$\because$ water wets glass, θ=0°]

$= \frac{2 \times 0.07}{0.25 \times 10^{-3} \times 1000 \times 9.8}$

As the tube is only 2 cm above the water and so, water will rise by 2 cm and meet the tube at an angle such that,

$2\pi rT\cos\theta = \pi r^2 h'\rho g$

$\Rightarrow \quad 2T\cos\theta = h'r\rho g$

$\Rightarrow \cos\theta = \frac{h'r\rho g}{2T}$

$= \frac{2 \times 10^{-2} \times 0.25 \times 10^{-3} \times 1000 \times 9.8}{2 \times 0.07}$

The liquid will meet the tube at an angle, $\theta \cong 70°$

32. (d) Given : Initial velocity $u_0 = 20\sqrt{2}$ m/s; angle of projection $\theta = 45°$

Therefore horizontal and vertical components of initial velocity are

$u_x = 20\sqrt{2}\cos 45° = 20 m/s$

and $u_y = 20\sqrt{2}\sin 45° = 20 m/s$

After 1s, horizontal component remains unchanged while the vertical component becomes

$v_y = u_y - gt$

Due to explosion, one part comes to rest. Hence, from the conservation of linear momentum, vertical component of second part will become $v'_y = 20 m/s$.

Therefore, maximum height attained by the second part will be

$H = h_1 + h_2$

Using, $h = ut + \frac{1}{2}at^2$

$\Rightarrow \quad h_1 = (20 \times 1) - \frac{1}{2} \times 10 \times (1)^2 = 15 m$

$a = g = 10 m/s^2$

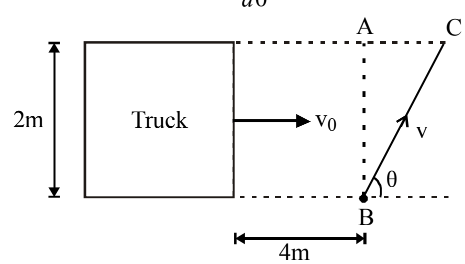
$h_2 = \frac{v_y'^2}{2g} = \frac{(20)^2}{2 \times 10} = 20m$

$H = 20 + 15 = 35 m$

33. (c) Let the man starts crossing the road at an angle θ as shown in figure. For safe crossing the condition is that the man must cross the road by the time the truck describes the distance $4 + AC$ or $4 + 2\cot\theta$.

$\therefore \quad \frac{4 + 2\cot\theta}{8} = \frac{2/\sin\theta}{v}$ or $v = \frac{8}{2\sin\theta + \cos\theta}$ ...(i)

For minimum v, $\frac{dv}{d\theta} = 0$



or $\frac{-8(2\cos\theta - \sin\theta)}{(2\sin\theta + \cos\theta)^2} = 0$ or $2\cos\theta - \sin\theta = 0$

or $\tan\theta = 2$

From equation (i),

$$v_{min} = \frac{8}{2\left(\frac{2}{\sqrt{5}}\right)+\frac{1}{\sqrt{5}}} = \frac{8}{\sqrt{5}} = 3.57 \text{ m/s}$$

34. **(b)** Let speed of neutron before collision = V

Speed of neutron after collision = V_1

Speed of proton or hydrogen atom after collision = V_2

Energy of excitation = ΔE

From the law of conservation of linear momentum,

$mv = mv_1 + mv_2$ ...(1)

And for law of conservation of energy,

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 \quad ...(2)$$

From squaring eq. (i), we get

$$v^2 = v_1^2 + v_2^2 + 2v_1v_2 \quad ...(3)$$

From squaring eq. (ii), we get

$$v^2 = v_1^2 + v_2^2 + \frac{2\Delta E}{m} \quad ...(4)$$

From eqn (3) & (4)

$$\therefore\ 2v_1v_2 = \frac{2\Delta E}{m}$$

$$\therefore\ (v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2 = v^2 - \frac{4\Delta E}{m}$$

As, $v_1 - v_2$ must be real, $v^2 - \frac{4\Delta E}{m} \geq 0$

$$\Rightarrow\ \frac{1}{2}mv^2 \geq 2\Delta E$$

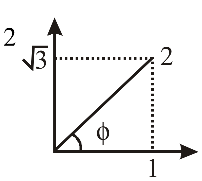
The minimum energy that can be absorbed by the hydrogen atom in the ground state to go into the excited state is 10.2 eV. Therefore, the maximum kinetic energy needed is

$$\frac{1}{2}mv_{min}^2 = 2 \times 10.2 = 20.4 \text{ eV}$$

35. **(a)** From, figure,

$$A_R = \sqrt{\left(\sqrt{3}\right)^2 + (1)^2} = 2$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$$



$$\therefore\ y = 2\sin\left(\omega t + \frac{\pi}{3}\right)$$

$$\frac{d^2y}{dt^2} = a = -2\omega^2 \sin\left(\omega t + \frac{\pi}{3}\right)$$

$$a_{max} = -2\omega^2 = g$$

For which mass just breaks off the plank

$$\omega = \sqrt{g/2}$$

This will be happen for the first time when

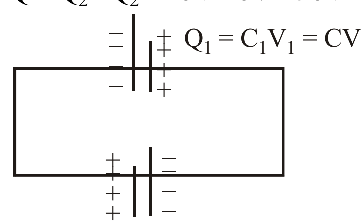
$$\omega t + \frac{\pi}{3} + \frac{\pi}{2} \text{ or } \omega t = \frac{\pi}{6}$$

$$\therefore\ t = \frac{\pi}{6\omega} = \frac{\pi}{6}\sqrt{\frac{2}{g}}$$

36. **(b)** From the figure.

The net charge shared between the two capacitors

$Q' = Q_2 - Q_2 = 4CV - CV = 3CV$



$Q_2 = C_2V_2 = (2C)(2V) = 4CV$

The two capacitors will have some potential, say V'.

The net capacitance of the parallel combination of the two capacitors

$C' = C_1 + C_2 = C + 2C + 3C$

The potential of the capacitors

$$V' = \frac{Q'}{C'} = \frac{3CV}{3C} = V$$

The electrostatic energy of the capacitors

$$E' = \frac{1}{2}C'V'^2 = \frac{1}{2}(3C)V^2 = \frac{3}{2}CV^2$$

37. **(a)**

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= \sqrt{10^2 + \left(2000 \times 5 \times 10^{-3} - \frac{1}{2000 \times 50 \times 10^{-6}}\right)^2}$$

$$= 10\ \Omega$$

$$i = \frac{V_0}{Z} = \frac{20}{10} = 2A.$$

38. **(b)** The heat produced is given by

$$H = \frac{V^2}{R} \text{ and } R = \frac{\rho \ell}{\pi r^2}$$

$$\therefore \quad H = V^2\left(\frac{\pi r^2}{\rho \ell}\right)$$

or $$H = \left(\frac{\pi V^2}{\rho}\right)\frac{r^2}{\ell}$$

Thus heat (H) is doubled if both length (ℓ) and radius (r) are doubled.

39. **(b)** As we know, frequency

$$f \propto \sqrt{mg} \text{ or } f \propto \sqrt{g}$$

In water, $f_w = 0.8 f_{air}$

$$\frac{g'}{g}(0.8)^2 = 0.64$$

$$\Rightarrow 1 - \frac{\rho_w}{\rho_m} = 0.64$$

$$\Rightarrow \frac{\rho_w}{\rho_m} = 0.36 \quad ...(1)$$

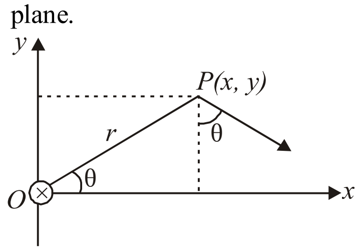
In liquid, $\frac{g'}{g} = (0.6)^2 = 0.36$

$$1 - \frac{\rho_l}{\rho_m} = 0.36 \frac{\rho_l}{\rho_m} = 0.64 \quad ...(2)$$

From eq. (1) and (2)

$$\frac{\rho_l}{\rho_n} = \frac{0.64}{0.36} \quad \therefore \rho_l = 1.77$$

40. **(a)** The wire carries a current I in the negative z-direction. We have to consider the magnetic vector field $\vec{B}$ at (x, y) in the $z = 0$ plane.



Magnetic field $\vec{B}$ is perpendicular to OP.

$$\therefore \vec{B} = B\sin\theta\hat{i} - B\cos\theta\hat{j}$$

$$\sin\theta = \frac{y}{r}, \cos\theta = \frac{x}{r} \quad B = \frac{\mu_0 I}{2\pi r}$$

$$\therefore \quad \vec{B} = \frac{\mu_0 I}{2\pi r^2}\left(y\hat{i} - x\hat{j}\right)$$

or $$\vec{B} = \frac{\mu_0 I\left(y\hat{i} - x\hat{j}\right)}{2\pi\left(x^2 + y^2\right)}.$$

PART - II: CHEMISTRY

41. **(c)** NO pollutant is the main product of automobiles exhaust.

42. **(c)** The high concentration of hydrocarbon pollutants in atmosphere causes cancer.

43. **(b)** Electronic configuration of element with atomic number 118 will be $[Rn]5f^{14}6d^{10}7s^27p^6$. Since its elctronic configuration in the outer most orbit (ns^2np^6) resemble with that of inert or noble gases, therefore it will be noble gas element.

44. **(c)** The third law helps to calculate the absolute entropies of pure substances at different temperature.

The entropy of the substance at different temperature. T may be calculated by the measurement of heat capacity change

$$S_T - S_0 = \Delta S = \int_0^T \frac{C_p.dt}{T}$$

Where S_T = Entropy at T K

S_0 = Entropy at 0K

$$= C_p.\log_e T$$

$$= 2.303 C_p.\log T$$

45. **(d)** $CoCl_3.5NH_3.H_2O$ is pink in colour

46. **(b)** Cobalt is present in vitamin B_{12}.

47. **(d)** Cobalt (60) isotope is used in the treatment of cancer.

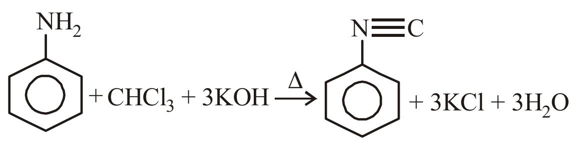
48. **(b)** PMMA is used in bullet proof glass

49. **(a)** $ClO_4^- < ClO_3^- < ClO_2^- < ClO^-$ is the correct increasing order of Bronsted base. With increase in the number of oxygen atoms in the conjugate bases, the delocalisation of the π bond becomes more and more extended. This results in decrease in the electron density. Consequently basicity also decreases.

50. **(a)** Due to dipole-dipole interaction the boiling point of alkyl halide is higher as compared to corresponding alkanes.

51. **(c)** Complex compounds contains two different metallic elements but give test only for one of them. Because complex ions such as $[Fe(CN)_6]^{4-}$ of $K_4[Fe(CN)_6]$, do not dissociate into Fe^{2+} and CN^- ions.

52. **(b)** Primary amines (aromatic or aliphatic) on warming with chloroform and alcoholic KOH, gives carbylamine having offensive smell. This reaction is called carbylamine reaction.



53. **(b)** Nitrous oxide (i.e., N_2O) is the laughing gas.

54. **(d)** Anthracene is purified by sublimation. In sublimation, a solid is converted directly into gaseous state on heating without passing through liquid phase.

55. **(b)** Zeise's salt is common name of $K[PtCl_3(\eta^2 = C_2H_4)]$

56. **(c)** $CaCl_2$ is produced as a by product in solvay ammonia process.

(i) $NaCl + CO_2 + NH_3 + H_2O \longrightarrow NaHCO_3 + NH_4Cl$

(ii) $CaCO_3 \longrightarrow CO_2 + CaO$

(iii) $2NH_2Cl + CaO \longrightarrow 2NH_3 + \underset{\text{By product}}{CaCl_2} + H_2O$

57. **(b)** Semiconductor materials like Si and Ge are usually purified by zone refining. Zone refining is based on the principle of fractional crystallisation i.e. difference in solubilities of impurities in solid and molten states of metal, so that the zones of impurities are formed and finally removed.

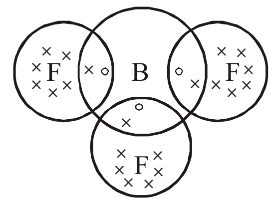
58. **(c)** Order of basic character is $NH_3 > PH_3 > AsH_3 > SbH_3$. Basic-character decreases down the group from N to Bi due to increase in atomic size.

59. **(c)** Normal glass is calcium alkali silicate glass made by fusing the alkali metal carbonate, $CaCO_3$ and SiO_2.

60. **(c)** $Exa = 10^{18}$

61. **(b)** More the oxidation state of the central atom (metal) more is its acidity. Hence SeO_2 (O. S. of Se = +4) is acidic. Further for a given O.S., the basic character of the oxides increases with the increasing size of the central atom. Thus Al_2O_3 and Sb_2O_3 are amphoteric and Bi_2O_3 is basic.

62. **(b)** BF_3 does not follow octate rule because central atom, boron lacks an electron pair. Thus, it also acts as Lewis acid.



63. **(a)** According to Le-Chatelier's principle increase in temperature favours the endothermic reaction while decrease in temperature favour the exothermic reaction. Increase in pressure shifts the equilibrium in that side in which number of gaseous moles decreases.

64. **(a)** Efficiency of fuel cell is:

$$\eta = \frac{-nFE_{cell}}{\Delta H} \times 100$$

65. **(b)** The mass of the substance deposited when one Faraday of charge is passed through its solution is equal to gram equivalent weight.

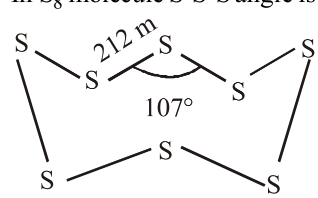
66. **(a)** Unit of rate constant for second order reaction is $L\,mol^{-1}\,sec^{-1}$.

67. **(b)** For first order reaction $[A] = [A_0]e^{-kt}$

$\therefore$ The concentration of reactants will exponentially decreases with time.

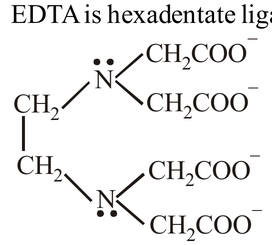
68. **(a)** In P_4 molecule, the four sp^3-hybridised phosphorous atoms lie at the corners of a regular tetrahedron with $\angle PPP = 60°$.

In S_8 molecule S-S-S angle is 107° rings.



69. **(a)** 40 elements are present in d-block.

70. **(a)** EDTA is hexadentate ligand



71. (a) Am shows maximum number of oxidation states, + 3, + 4, + 5, + 6

72. (a) Fe^{3+} ion can be detected by $K_4[Fe(CN)_6]$

$$4Fe^{3+} + 3K_4\left[Fe(CN)_6\right] \longrightarrow Fe_4\left[Fe(CN)_6\right]_3 + 12K^+$$

73. (d) $\Delta G = \Delta H - T\Delta S$; ΔG is positive for a reaction to be non-spontaneous when ΔH is positive and ΔS is negative.

74. (a) This method is known as Clark's process. In this method temporary hardness is removed by adding lime water or milk of lime.

$$Ca(OH)_2 + Ca(HCO_3)_2 \longrightarrow \underset{\text{ppt.}}{2CaCO_3} \downarrow + 2H_2O$$

75. (b) Graphite is covalent solid.

76. (c)

77. (b) $Xe + F_2 \xrightarrow{673K} XeF_2$

2 : 1

$$Xe + 2F_2 \xrightarrow[5-6 Atm]{673K} XeF_4$$

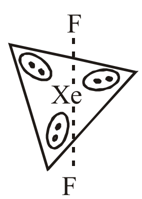
1 : 5

$$Xe + 3F_2 \xrightarrow[50-60\ Atm]{573K} XeF_6$$

1 : 20

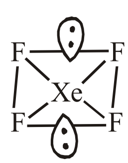
Structures of Xenon fluorides

Xe F_2 : Hybridization sp^3d



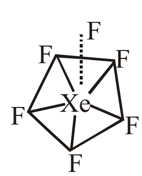
Linear

Xe F_4 : Hybridization sp^3d^2



Square planar

XeF_6 : Hybribisation sp^3d^3



Pentagonal pyramidal or distorted octahedral

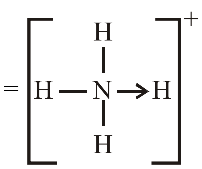
78. (c) Calcium silicophosphate (a mixture of $Ca_3(PO_4)_2$ & Ca_2SiO_4) is called Thomas slag.

79. (a) The sequence of bases in mRNA are read in a serial order in groups of three at a time. Each triplet of nucleotides (having a specific sequence of bases) in known as codon. Each codon specifies one amino acid. Further since, there are four bases. therefore, 4^3 = 64 triplets or codons are possible.

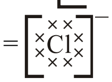
80. (b) Bond structure of molecules are :

$HCl = H^+ - Cl^-$

NH_4^+ =



Cl^- =



$H_2O_2 = H - O - O - H$

hence, clearly NH_4^+ ion contains all three types of bonds.

PART - III (A): ENGLISH PROFICIENCY

81. (c) 'Immutable' means 'unchangeable'. So, option (c) is correct choice.

82. (a) 'ignominious' means 'shameful'. So, option (a) is correct choice.

83. (c) 'callous' means 'showing or having an insensitive and cruel disregard for others'. So, option (c) is correct choice.

84. (a) Option (d) institutional as the word means relating to principles esp. of law, so legally also every human has rights of freedom and equality.

85. (a) **86. (c)**

87. (b) Immortal means living forever, never dying or decaying. So, perishable is the correct opposite to it.

88. (c) Opposed is the correct answer of this. To patronise means favour or pat on the back.

89. (a) Tactful means having or showing skill and senstivity in dealing with others or with defficult essues.

So, in cautious is the correct opposite of factful.

90. (c) Atheist is the best alternative.

91. (c) 'Epicure' is the best alternative.

92. (d)
93. (c) CBEDA
94. (d) CDEAB
95. (c)

PART - III (B): LOGICAL REASONING

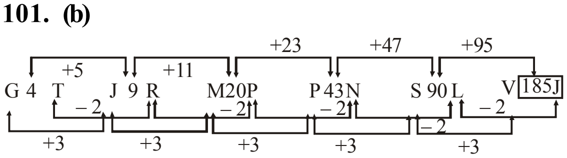
96. (c) **97.** (c)

98. (b) The contents of the third figure in each row (and column) are determined by the contents of the first two figures. Lines are carried forward from the first two figures to the third one, except where two lines appear in the same position, in which they are cancelled out.

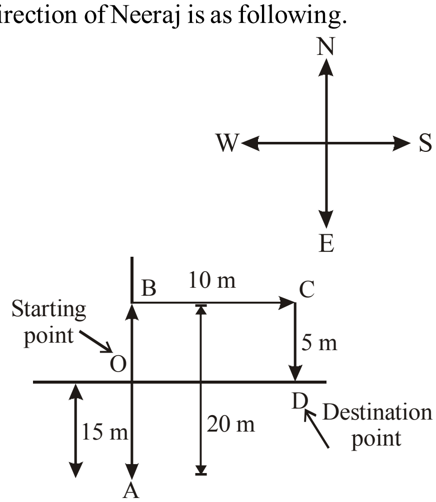
99. (b) From figure, (i), (iii) and (iv), we have concluded that 2, 6, 1 and 5 appear adjacent so 3. clearly, 4 will appear opposite to 3.

100. (c) In figure (A), the dot is placed in the region which is common to the circle and triangle. Now, we have to find similar common region in all the four options. Only in figure (c), we find such a region which is common to the circle and triangle.

101. (b)

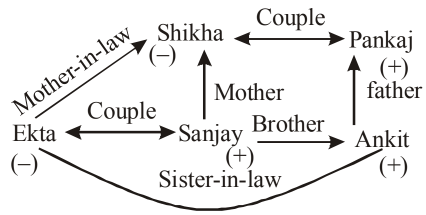


102. (a) According to the given information, the direction of Neeraj is as following.



103. (b) Let the average age of 8 men = x yr
Total age of 8 men = 8 x yr
Now, new average age = (x + 2)yr
Total age = 8(x + 2) yr
Difference of ages = 8(x + 2) – 8x
= 8x + 16 – 8x = 16 yr
∴ Age of new man = 20 + 16 = 36 yr
So, the new man is 16yr older to the man by whom the new man is replaced.

104. (d) The relation is as following:



It is clearly shown that Shikha is the mother of Ankit.

105. (b) Since Arun and Suresh interchange places, so Arun's new position (13th from left) is the same as Suresh's earlier position (6th from right).
So, number of children in the queue = (12 + 1 + 5) = 18.
Now, Suresh's new position is the same as Arun's earlier position fifth from left.
Therefore Suresh's position from the right = (18 – 4) = 14th.

PART - IV: MATHEMATICS

106. (d) Consider $\lim_{x\to\infty} \frac{\int_0^{2x} xe^{x^2} dx}{e^{4x^2}}$

$$= \lim_{x\to\infty} \frac{2\int_0^{2x} xe^{x^2} dx}{2e^{4x^2}}$$

$$= \lim_{x\to\infty} \frac{2\int_0^{2x} e^{x^2} d\left(x^2\right)}{2e^{4x^2}}$$

$$= \lim_{x\to\infty} \frac{[e^{x^2}]_0^{2x}}{e^{4x^2}} = \lim_{x\to\infty} \frac{e^{4x^2}-1}{2e^{4x^2}}$$

$$= \lim_{x\to\infty} \left(\frac{1}{2} - \frac{1}{2e^{4x^2}}\right) = \frac{1}{2}$$

107. (a) Consider $\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+\frac{27}{128}+....$

Which can be written as

$$\frac{3^0}{2^1}+\frac{3^1}{2^3}+\frac{3^2}{2^5}+\frac{3^3}{2^7}+....$$

$$=\frac{1}{2}\left[1+\frac{3}{2^2}+\frac{3^2}{2^4}+\frac{3^3}{2^6}+....\right]$$

Since $\left(1+\frac{3}{2^2}+\frac{3^2}{2^4}+---\right)$ is a G.P. therfore by sum of infinite G.P, we have

$$=\frac{1}{2}\left[\frac{1}{1-\frac{3}{2^2}}\right]=\frac{1}{2}\left[\frac{1}{1-\frac{3}{4}}\right]=2$$

∴ Given expression

$=-1$

$\left[\because 1+\omega+\omega^2=0\right]$

108. (a) Roots $=\frac{4(2-i)\pm\sqrt{16(2-i)^2+8(1+i)(5+3i)}}{4(1+i)}$

$$=\frac{4-i}{1+i} \text{ or } \frac{-i}{1+i}=\frac{3-5i}{2} \text{ or } \frac{-1-i}{2}$$

109. (b) Consider $\frac{3}{4}+\frac{15}{16}+\frac{63}{64}+...\text{upto } n \text{ terms}$

$$=\frac{2^2-1}{2^2}+\frac{2^4-1}{2^4}+\frac{2^6-1}{2^6}\text{upto n terms}$$

$$=\left(1-\frac{1}{2^2}\right)+\left(1-\frac{1}{2^4}\right)+\left(1-\frac{1}{2^6}\right)+...\text{upto n terms}$$

$$=(1+1+1+....\text{upto n terms})$$

$$-\left(\frac{1}{2^2}+\frac{1}{2^4}+\frac{1}{2^6}+...\text{upto n terms}\right)$$

$$=n-\frac{1}{2^2}\left[\frac{1-\left(\frac{1}{2^2}\right)^n}{1-\frac{1}{2^2}}\right]=n-\frac{1}{3}\left(1-4^{-n}\right)$$

$$=n+\frac{4^{-n}}{3}-\frac{1}{3}$$

110. (d) $\tan\theta$ is of period π so that $\tan 3\theta$ is of period $\pi/3$.

111. (b) Let

$$f(x)=\frac{x}{1+x}+\frac{x}{(x+1)(2x+1)}+\frac{x}{(2x+1)(3x+1)}+...\infty$$

$$=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{x}{\left[(r-1)x+1\right](rx+1)}$$

$$=\lim_{x\to\infty}\sum_{r=1}^{n}\left[\frac{x}{\left[(r-1)x+1\right]}-\frac{1}{rx+1}\right]$$

$$=\lim_{n\to\infty}\left[1-\frac{1}{nx+1}\right]=1$$

For $x=0$, we have $f(x)=0$

Thus, we have $f(x)=\begin{cases}1, & x\neq 0\\ 0, & x=0\end{cases}$

Clearly, $\lim_{x\to 0^-}f(x)=\lim_{x\to 0^+}f(x)\neq f(0)$

So, f(x) is not continuous at $x=0$.

112. (c) Since, g is the inverse of function f. Therefore,

$g(x)=f^{-1}(x)$

$\Rightarrow f[g(x)]=x$

$\Rightarrow fog\ (x)=x$, for all x

Differentiate both side,w.r.tx

$\Rightarrow \frac{d}{dx}\{fog\ (x)\}=\frac{d}{dx}(x)$, for all x

$\Rightarrow f'[g(x)]\,g'(x)=1$, for all x

$\Rightarrow \sin\{g(x)\}g'(x)=1$, for all x

(By defn of $f'(x)$)

$$\Rightarrow g'(x)=\frac{1}{\sin\{g(x)\}}$$

113. (a) Total number of coins= 2n+1

Consider the following events:

E_1 = Getting a coin having head on both sides from the bag.

E_2 = Getting a fair coin from the bag

A = Toss results in a head

Given: $P(A)=\frac{31}{42}, P(E_1)=\frac{n}{2n+1}$

and $P(E_2)=\frac{n+1}{2n+1}$

Then,

$$P(A)=P(E_1)P(A/E_1)+P(E_2)P(A/E_2)$$

$$\Rightarrow \frac{31}{42} = \frac{n}{2n+1} \times 1 + \frac{n+1}{2n+1} \times \frac{1}{2}$$

$$\frac{31}{42} = \frac{n}{2n+1} + \frac{n+1}{2(2n+1)}$$

$$\Rightarrow \frac{31}{42} = \frac{3n+1}{2(2n+1)}$$

$$\Rightarrow \frac{31}{21} = \frac{3n+1}{2n+1}$$

$n = 10$

114. (a) Given differential equation is

$$dy + \{y\phi'(x) - \phi(x)\phi'(x)\}dx = 0$$

$$\Rightarrow \frac{dy}{dx} + \phi'(x)\,y = \phi(x)\phi'(x)$$

which is a linear differential equation with

$P = \phi'(x)$, $Q = \phi(x).\phi'(x)$ and

$$I.F = e^{\int \phi'(x)dx} = e^{\phi(x)}$$

$$\therefore \text{Solution is } y.e^{\phi(x)} = \int \phi(x).\phi'(x)e^{\phi(x)}dx + C$$

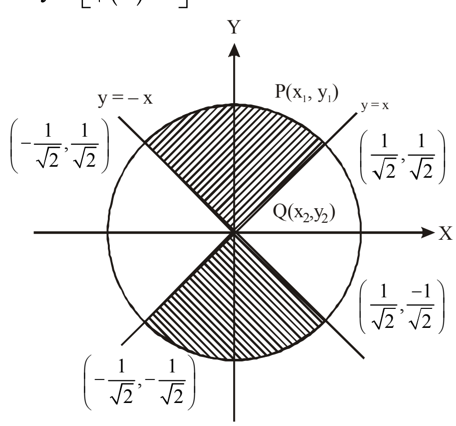
$$\Rightarrow y.e^{\phi(x)} = \int \underset{I}{\phi(x)}.\underset{II}{e^{\phi(x)}\phi'(x)}dx + C$$

$$\Rightarrow y.e^{\phi(x)} = \phi(x)e^{\phi(x)} - \int \phi'(x)\,e^{\phi(x)}\,dx + C$$

$$\Rightarrow y.e^{\phi(x)} = \phi(x)e^{\phi(x)} - e^{\phi(x)} + C$$

$$\Rightarrow y = [\phi(x) - 1] + Ce^{-\phi(x)}$$

115. (c)



Required area = 4 (Area of the shaded region in first quadrant)

$$= 4\int_0^{1\sqrt{2}} (y_1 - y_2)dx = 4\int_0^{1\sqrt{2}} \left(\sqrt{1-x^2} - x\right)dx$$

$$= 4\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x - \frac{x^2}{2}\right]_0^{1/\sqrt{2}}$$

$$= 4\left[\frac{1}{2\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{\pi}{4} - \frac{1}{4}\right]$$

$$= 4\left[\frac{1}{4} + \frac{\pi}{8} - \frac{1}{4}\right] = \frac{4\pi}{8} = \frac{\pi}{2} \text{ sq units}$$

116. (c) $U = \{x : x^5 - 6x^4 + 11x^3 - 6x^2 = 0\}$

Solving for values of x, we get

$U = \{0, 1, 2, 3\}$

$A = \{x : x^2 - 5x + 6 = 0\}$

Solving for values of x, we get

$A = \{2, 3\}$

and $B = \{x : x^2 - 3x + 2 = 0\}$

Solving for values of x, we get

$B = \{2, 1\}$

$A \cap B = \{2\}$

$\therefore (A \cap B)' = U - (A \cap B)$

$= \{0, 1, 2, 3\} - \{2\} = \{0, 1, 3\}$

117. (c) $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$

$$\Rightarrow \cos^{-1}\left(\frac{xy}{2} + \sqrt{(1-x^2)\left(1 - \frac{y^2}{4}\right)}\right) = \alpha$$

$$\Rightarrow \cos^{-1}\left(\frac{xy + \sqrt{4 - y^2 - 4x^2 + x^2y^2}}{2}\right) = \alpha$$

$$\Rightarrow 4 - y^2 - 4x^2 + x^2y^2 =$$

$$4\cos^2\alpha + x^2y^2 - 4xy\cos\alpha$$

$$\Rightarrow 4x^2 + y^2 - 4xy\cos\alpha = 4\sin^2\alpha.$$

118. (d) Let

$$\frac{e^x + e^{5x}}{e^{3x}} = a_0 + a_1x + a_2x^2 + a_3a^3 + ...$$

$$= \frac{e^x}{e^{3x}} + \frac{e^{5x}}{e^{3x}} = a_0 + a_1x + a_2x^2 + ...$$

$$= e^{-2x} + e^{2x} = a_0 + a_1x + a_2x^2 + a_3x^3 + ...$$

By using

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ---- \text{ and}$$

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + ---$$

$$e^{-2x} + e^{2x} = 2\left[1 + \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} + ...\right]$$

$= a_0 + a_1x + a_2x^2 + a_3a^3 + ...$

$= a_1 = a_3 = a_5 = ... = 0$

Hence, $2a_1 + 2^3a_3 + 2^5a_5 + ... = 0$

119. (d) Let θ be the angle between **b** and **c**.

Given, $|b \times c| = \sqrt{15}$

$\Rightarrow |b||c| \sin\theta = \sqrt{15}$

$\Rightarrow \sin\theta = \frac{\sqrt{15}}{|b||c|}$

$\Rightarrow \sin\theta = \sqrt{\frac{15}{4\times1}} = \frac{\sqrt{15}}{4}$

$\therefore \cos\theta = \sqrt{1 - \frac{15}{16}} = \frac{1}{\sqrt{16}} = \frac{1}{4}$

Now given, $b - 2c = \lambda a \Rightarrow \left|b - 2c^2\right| = \left|\lambda a^2\right|$

$\Rightarrow |b|^2 + 4|c| - 4(b.c) = \lambda^2 |a|^2$

$\Rightarrow 16 + 4 - 4|b||c| \cos\theta = \lambda^2$

$\Rightarrow 20 - 16\cos\theta = \lambda^2$

$\Rightarrow 20 - 4 = \lambda^2 \Rightarrow \lambda^2 = 16 \left(\therefore \cos\theta = \frac{1}{4}\right)$

$\Rightarrow \lambda = \pm 4$

120. (b) Total number of arrangements of 10 digits 0, 1, 2, ..., 9 by taking 4 at a time $= {}^{10}C_4 \times 4!$

We observe that in every arrangement of 4 selected digits there is just one arrangement in which the digits are in descending order.

$\therefore$ Required number of 4-digit numbers.

$$= \frac{{}^{10}C_4 \times 4!}{4!} = {}^{10}C_4$$

121. (c) Given equation of a line parallel to X-axis is y = k.

Given equation of the curve is $y = \sqrt{x}$,

On solving equation of line with the equation of curve, we get $x = k^2$

Thus the intersecting point is (k^2, k)

It is given that the line y = k intersect the curve $y = \sqrt{x}$ at an angle of $\pi/4$. This means that the slope of the tangent to $y = \sqrt{x}$ at (k^2, k) is $\tan\left(\pm\frac{\pi}{4}\right) = \pm 1$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(k^2,k)} = \pm 1 \Rightarrow \left(\frac{1}{2\sqrt{x}}\right)_{(k^2,k)} = \pm 1$

$\Rightarrow k = \pm\frac{1}{2}$

122. (a) Let A, B and C be the three angles of ΔABC and

Let a = 10 and b = 9

It is given that the angles are in AP.

$\therefore$ 2B = A + C on adding B both the sides, we get

3B = A + B + C

$\Rightarrow 3B = 180° \Rightarrow B = 60°$

Now, we know $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$\Rightarrow \cos 60° = \frac{10^2 + c^2 - 9^2}{2 \times 10 \times c}$

$\Rightarrow \frac{1}{2} = \frac{100 + c^2 - 81}{20c}$

$\Rightarrow c^2 - 10c + 19 = 0 \Rightarrow c = 5 \pm \sqrt{6}$

123. (d) Since, Mean $= \frac{\sum f_i x_i}{\sum f_i}$ where x_i are observations with frequencies f_i, i = 1, 2, ...n

The required mean is given by

$$\overline{X} = \frac{0.1 + 1.{}^nC_1 + 2.{}^nC_2 + ... + n.{}^nC_n}{1 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n}$$

$$= \frac{\sum_{r=0}^{n} r.{}^nC_r}{\sum_{r=0}^{n} {}^nC_r} = \frac{\sum_{r=1}^{n} r.\frac{n}{r}\,{}^{n-1}C_{r-1}}{\sum_{r=0}^{n} {}^nC_r}$$

$$= \frac{n\sum_{r=1}^{n} {}^{n-1}C_{r-1}}{\sum_{r=0}^{n} {}^nC_r} = \frac{n.2^{n-1}}{2^n} = \frac{n}{2}$$

124. (a) We have $\frac{1}{n}\sum_{i=1}^{n}(x_i + 2)^2 = 18$ and

$\frac{1}{n}\sum_{i=1}^{n}(x_i - 2)^2 = 10$

$\Rightarrow \sum_{i=1}^{n}(x_i + 2)^2 = 18n$ and

$$\sum_{i=1}^{n}(x_i-2)^2=10n$$

$$\Rightarrow \sum_{i=1}^{n}(x_i+2)^2+\sum_{i=1}^{n}(x_i-2)^2=28n$$

and $$\sum_{i=1}^{n}(x_i+2)^2-\sum_{i=1}^{n}(x_i-2)^2=8n$$

$$\Rightarrow 2\sum_{i=1}^{n}(x_i+4)^2=28n \text{ and } 2\sum_{i=1}^{n}4x_i=8n$$

$$\Rightarrow \sum_{i=1}^{n}x_i^2+4n=14n \text{ and } \sum_{i=1}^{n}x_i=n$$

$$\Rightarrow \sum_{i=1}^{n}x_i^2=10n \text{ and } \sum_{i=1}^{n}x_i=n$$

$$\therefore \sigma=\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2-\left(\frac{1}{n}\sum_{i=1'}^{n}x_i\right)^2}=\sqrt{\frac{10n}{n}-\left(\frac{n}{n}\right)^2}=3$$

125. (a) The parametric equations of the parabola $y^2=8x$ are $x=2t^2$ and $y=4t$.

and the given equation of circle is

$x^2+y^2-2x-4y=0$

On putting $x=2t^2$ and $y=4t$ in circle

we get

$4t^4+16t^2-4t^2-16t=0$

$\Rightarrow 4t^2+12t^2-16t=0$

$\Rightarrow 4t(t^3+3t-4)=0$

$\Rightarrow t(t-1)(t^2+t+4)=0$

$\Rightarrow t=0, t=1$

$\left[\because t^2+t+4\neq 0\right]$

Thus the coordinates of points of intersection of the circle and the parabola are Q (0, 0) and P(2, 4). Clearly these are diametrically opposite points on the circle.

The coordinates of the focus S of the parabola are (2, 0) which lies on the circle.

$$\therefore \text{Area of } \Delta PQS=\frac{1}{2}\times QS\times SP=\frac{1}{2}\times 2\times 4$$

$= 4$ sq. units.

126. (a) Let $f(x)=e^{x-1}+x-2$

check for $x=1$

Then, $f(1)=e^0+1-2=0$

So, $x=1$ is a real root of the equation $f(x)=0$

Let $x=\alpha$ be the other root such that $\alpha>1$ or $\alpha<1$. Consider the interval $[1,\alpha]$ or $[\alpha,1]$.

Clearly $f(1)=f(\alpha)=0$

By Rolle's theorem $f'(x)=0$ has a root in $(1,\alpha)$ or in $(\alpha,1)$.

But $f'(x)=e^{x-1}+1>0$, for all x. Thus, $f'(x)\neq 0$, for any $x\in(1,\alpha)$ or $x\in(\alpha,1)$, which is a contradiction.

Hence, $f(x)=0$ has no real root other than 1.

127. (c) Constraints will be

$x_{11}+x_{21}+....+x_{m1}=b_1$

$x_{12}+x_{22}+.....x_{m2}=b_2$

$x_{1n}+x_{2n}+....+x_{mn}=b_n$

$x_{11}+x_{12}+...+x_{1n}=b_1$

$x_{21}+x_{22}+...+x_{2n}=b_2$

$x_{m1}+x_{m2}+...+x_{mn}=b_n$

So, total number of constraints = m + n

128. (c) Let $A\equiv$ event that drawn ball is red

$B\equiv$ event that drawn ball is white

Then AB and BA are two disjoint cases of the given event.

$\therefore P(AB+BA)=P(AB)+P(BA)$

$$=P(A)\ P\left(\frac{B}{A}\right)+P(B)\ P\left(\frac{A}{B}\right)$$

$$=\frac{3}{6}.\frac{3}{5}+\frac{3}{6}.\frac{3}{5}=\frac{3}{5}$$

129. (b) We know that, $M(\text{adj } M)=|M|\,I$

Replacing M by adj M, we get

adj M [adj (adj M) = det (adj M) I

$=\det(M)\ M^{-1}\ [\text{adj}\ (\text{adj}\ M)=\alpha^2 1$

$$\left[\because M^{-1}=\frac{1}{|M|}\text{adj}(M)\right]$$

$\Rightarrow \alpha M^{-1}[\text{adj}(\text{adj } M)]=\alpha^2 I$

$\Rightarrow M^{-1}[\text{adj}(\text{adj } M)]=\alpha I$

But $M^{-1}[\text{adj}(\text{adj } M)]=KI$

Hence, $K=\alpha$

130. (c) Let (x_1, y_1) be one of the points of contact. Given curve is $y = \cos x$

$$\Rightarrow \frac{dy}{dx} = -\sin x$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{(x_1,y_1)} = -\sin x_1$$

Now the equation of the tangent at (x_1,y_1) is

$$y - y_1 \left(\frac{dy}{dx}\right)_{(x_1,y_1)} (x - x_1)$$

$$\Rightarrow \quad y - y_1 = -\sin x_1 (0 - x_1)$$

Since, it is given that equation of tangent passes through origin.

$$\therefore \quad 0 - y_1 = -\sin x_1 (0 - x_1)$$

$$\Rightarrow \quad y_1 = -x_1 \sin x_1 \quad ...(i)$$

Also, point (x_1, y_1) lies on $y = \cos x$.

$$\therefore \quad y_1 = \cos x_1$$

From Eqs. (i), (ii) , we get

$$\sin^2 x_1 + \cos^2 x_1 = \frac{y_1^2}{x_1^2} + y_1^2 = 1$$

$$\Rightarrow \quad x_1^2 = y_1^2 + y_1^2 x_1^2$$

Hence, the locus of (x_1, y_1) is

$$x^2 = y^2 + y^2x^2 \Rightarrow x^2y^2 = x^2 - y^2$$

131. (b) Let m be the slope of the tangent to the curve

$y = e^x \cos x$.

Then, $m = \frac{dy}{dx} = e^x(\cos x - \sin x)$

Diff. w.r.t 'x'

$$\Rightarrow \frac{dm}{dx} = e^x(\cos x - \sin x) + e^x(-\sin x - \cos x)$$

$$= -2e^x \sin x$$

and $\frac{d^2m}{dx^2} = -2e^x(\sin x + \cos x)$

Put $\frac{dm}{dx} = 0 \Rightarrow \sin x = 0 \Rightarrow x = 0, \pi, 2\pi$

Clearly, $\frac{d^2m}{dx^2} > 0$ for $x = \pi$

Thus, y is minimum at $x = \pi$.

Hence the value of $\alpha = \pi$.

132. (a) The equations of given lines can be written as

$$L_1 : x - 5 = \frac{y}{3-\alpha} = \frac{z}{-2}$$

$$L_2 : x - \alpha = \frac{y}{-1} = \frac{z}{2-\alpha}$$

Since, these lines are coplanar.

Therefore, $\begin{vmatrix} 5-\alpha & 0-0 & 0-0 \\ 0 & 3-\alpha & -2 \\ 0 & -1 & 2-\alpha \end{vmatrix} = 0$

$$\Rightarrow \quad (5-\alpha)(3-\alpha)(2-\alpha) - 2 = 0$$

$$\Rightarrow \quad (5-\alpha)(6 - 3\alpha - 2\alpha + \alpha^2 - 2] = 0$$

$$\Rightarrow \quad (5-\alpha)(\alpha^2 - 5\alpha + 4] = 0$$

$$\Rightarrow \quad (5-\alpha)(\alpha-1)(\alpha-4) = 0$$

$$\Rightarrow \quad \alpha = 1, 4, 5$$

133. (b) $e = \frac{1}{2}$. Directrix , $x = \frac{a}{e} = 4$

$$\therefore a = 4 \times \frac{1}{2} = 2 \quad \therefore b = 2\sqrt{1 - \frac{1}{4}} = \sqrt{3}$$

Equation of ellipse is

$$\frac{x^2}{4} + \frac{y^2}{3} = 1 \Rightarrow 3x^2 + 4y^2 = 12$$

134. (d) $\frac{x}{2} + \frac{2}{x}$ is of the form $x + \frac{1}{x} \geq 2$ and equality holds for $x = 1$

135. (a) $y = (x + \sqrt{1+x^2})^n$

$$\frac{dy}{dx} = n(x + \sqrt{1+x^2})^{n-1}\left(1 + \frac{1}{2}(1+x^2)^{-1/2}.\, 2x\right);$$

$$\frac{dy}{dx} = n(x + \sqrt{1+x^2})^{n-1}\frac{(\sqrt{1+x^2} + x)}{\sqrt{1+x^2}}$$

$$= \frac{n(\sqrt{1+x^2} + x)^n}{\sqrt{1+x^2}}$$

or $\sqrt{1+x^2}\frac{dy}{dx}=ny$ or $\sqrt{1+x^2}y_1=ny$

$(y_1=\frac{dy}{dx})$

Squaring, $(1+x^2)y_1^2=n^2y^2$

Differentiating,

$(1+x^2)2y_1y_2+y_1^2.2x=n^2.2yy_1$

or $(1+x^2)y_2+xy_1=n^2y$

136. (a) As given,

$$A=\lim_{x\to\infty}x\sin\left(\frac{1}{x}\right)=\lim_{x\to\infty}\frac{\sin\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)}$$

Let $t=\frac{1}{x}$ when $x\to\alpha,\ \ t\to 0$

$$\Rightarrow A=\lim_{t\to\infty}\frac{\sin t}{t}=1$$

$$\left[\because \lim_{t\to 0}\frac{\sin x}{x}=1\right]$$

and $B=\lim_{x\to 0}x\sin\left(\frac{1}{x}\right)$

$$\Rightarrow B=\lim_{x\to 0}x.\lim_{x\to 0}\sin\left(\frac{1}{x}\right)$$

$\Rightarrow B=0$

$\therefore$ A = 1 and B = 0 is correct

137. (b) As given a and b are the roots of the equation

$x^2+ax+b=0$

$\Rightarrow$ sum of roots, $a+b=-a$

$\Rightarrow$ $b=-2a$ ...(1)

and product of roots, $ab=b$

$\Rightarrow$ $ab-b=0$

$\Rightarrow$ $b(a-1)=0$

if $b=0$ then $a=0$

if $b\neq 0$ then $a=1$ and $b=-2$

so, the expression will be,

$f(x)=x^2+x-2$

$$=x^2+2.\frac{1}{2}x+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2-2$$

$$\Rightarrow f(x)=\left(x+\frac{1}{2}\right)^2-\frac{9}{4}$$

So, f(x) will be minimum, if $\left(x+\frac{1}{2}\right)^2=0$

i.e. when $x=-\frac{1}{2}$

$\Rightarrow$ minimum value of function $=-\frac{9}{4}$

138. (d) Let us assume the functions f(x) and g(x) given by

$f(x)=\tan x-x$ and $g(x)=x-\sin x$, for $0<x<\frac{\pi}{2}$

Now, $f'(x)=\sec^2x-1$ and

$g'(x)=1-\cos x$

$\Rightarrow$ $f'(x)>0$ and $g'(x)>0,\ \forall x\in\left(0,\frac{\pi}{2}\right)$

$\Rightarrow$ $f(x)>f(0)$ and $g(x)>g(0)\ \forall x\in\left(0,\frac{\pi}{2}\right)$

$\Rightarrow$ $\tan x-x>0$ and $x-\sin x>0$, $\forall x\in\left(0,\frac{\pi}{2}\right)$

$\Rightarrow$ $\tan x>x$ and $x>\sin x,\ \forall x\in\left(0,\frac{\pi}{2}\right)$

$\Rightarrow$ $\sin x<x<\tan x,\ \forall x\in\left(0,\frac{\pi}{2}\right)$

139. (a) Put $x=\sin\theta$ and $y=\sin\phi$

$\Rightarrow \cos\theta+\cos\phi=a(\sin\theta-\sin\phi)$

$\Rightarrow$ 2 cos

$$\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2}=2a\cos\frac{\theta+\phi}{2}\sin\frac{\theta-\phi}{2}$$

$\Rightarrow \cot\frac{\theta-\phi}{2}=a\ \ \Rightarrow \theta-\phi=2\cot^{-1}a$

$\Rightarrow \sin^{-1}x-\sin^{-1}y=2\cot^{-1}a$

Differentiate $\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=0$

so the degree is one

140. (b) Let $f(x)=ax^3+bx^2+cx+d$

Put $x=0$ and $x=1$

Then, we get f(0) = –1 and f(1) = 0

$\Rightarrow$ d = – 1 and a + b + c + d = 0

$\Rightarrow$ a + b + c = 1 ...(i)

It is given that x = 0 is a stationary point of f(x), but it is not a point of extremum.

Therefore, $f'(0) = 0 = f''(0)$ and $f'''(0) = 0$

Now, $f(x) = ax^3 + bx^2 + cx + d$

$\Rightarrow f'(x) = 3ax^2 + 2bx + c,$

$f''(x) = 6ax + 2b$ and $f'''(x) = 6a$

$f' = 0,\ f''(0) = 0$ and $f'''(0) = 0 \neq 0$

$\Rightarrow c = 0, b = 0$ and $a \neq 0$

From Eqs. (i) and (ii), we get

a = 1, b = c = 0 and d = – 1

Put these values in f(x)

we get $f(x) = x^3 - 1$

Hence, $\int \frac{f(x)}{x^3 - 1} dx = \int \frac{x^3 - 1}{x^3 - 1} dx = \int 1 dx = x + C$

141. (b) $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9 - x^2}}$ is defined

if (i) $-1 \le x - 3 \le 1 \Rightarrow 2 \le x \le 4$ and

(ii) $9 - x^2 > 0 \Rightarrow -3 < x < 3$

Taking common solution of (i) and (ii), we get $2 \le x < 3$

$\therefore$ Domain = [2, 3)

142. (a) The equations of the lines are

$p_1x + q_1y - 1 = 0$ ...(i)

$p_2x + q_2y - 1 = 0$ ...(ii)

and $p_3x + q_3y - 1 = 0$ ...(iii)

As they are concurrent,

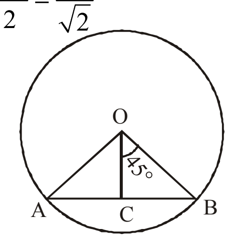
$$\begin{vmatrix} p_1 & q_1 & -1 \\ p_2 & q_2 & -1 \\ p_3 & q_3 & -1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} p_1 & q_1 & 1 \\ p_2 & q_2 & 1 \\ p_3 & q_3 & 1 \end{vmatrix} = 0$$

This is also the condition for the points $(p_1, q_1), (p_2, q_2)$ and (p_3, q_3) to be collinear.

143. (c) Let AB be the chord of length $\sqrt{2}$. Let O be the centre of the circle and let OC be the perpendicular from O on AB.

Then, $AC = BC = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

In ΔOBC, we have



OB = BC cosec 45°

$= \frac{1}{\sqrt{2}} \times \sqrt{2} = 1$

$\therefore$ Area of the circle = $\pi(OB)^2 = \pi$ sq units

144. (b) $\cos A = n \cos B$ and $\sin A = m \sin B$

Squaring and adding, we get

$1 = n^2 \cos^2 B + m^2 \sin^2 B$

$\Rightarrow 1 = n^2 (1 - \sin^2 B) + m^2 \sin^2 B$

$\therefore (m^2 - n^2) \sin^2 B = 1 - n^2$

145. (a) z_1, z_2, 0 are vertices of an equilateral triangle, so we have $z_1^2 + z_2^2 + 0^2 = z_1 z_2 + z_2 . 0 + 0 . z_1$

$\Rightarrow z_1^2 + z_2^2 = z_1 z_2 \Rightarrow z_1^2 + z_2^2 - z_1 z_2 = 0$

146. (b) Obviously, the relation is not reflexive and transitive, but it is symmetric, because

$x^2 + x^2 = 2x^2 \neq 1$

and $x^2 + y^2 = 1, y^2 + z^2 = 1$

$\Rightarrow x^2 + z^2 = 1$

But $x^2 + y^2 = 1 \Rightarrow y^2 + x^2 = 1$

147. (c) Let l, m and n be the direction cosines.

Then, $l = \cos\theta$, m = $\cos\beta$, n = $\cos\theta$

we have $l^2 + m^2 + n^2 = 1$

$\Rightarrow \cos^2\theta + \cos^2\beta + \cos^2\theta = 1$

$\Rightarrow 2\cos^2\theta + 1 - \sin^2\beta = 1$

$\Rightarrow 2\cos^2\theta - \sin^2\beta = 0$

$\Rightarrow 2\cos^2\theta - 3\sin^2\beta = 0$

$[\because \sin^2\beta = 3\sin^2\theta (\text{given})]$

$\Rightarrow \tan^2\theta = 2/3$

$\therefore \cos^2\theta = \frac{1}{1 + \tan^2\theta} = \frac{1}{1 + 2/3} = \frac{3}{5}$

148. (b) Given n = 4 and $P(X = 0) = \frac{16}{81}$

Let p be the probability of success and q that of failure in a trial.

Then, $P(X = 0) = {}^4C_0 p^0 q^4 = \frac{16}{81}$

$\Rightarrow \quad q^4 = \left(\frac{2}{3}\right)^4$

$\left(\because {}^nC_0 = 1\right)$

$\Rightarrow \quad q = \frac{2}{3} \Rightarrow p = \frac{1}{3}$

$\therefore \quad P(X = 4) = {}^4C_4 p^4 q^0 = p^4 = \left(\frac{1}{3}\right)^4 = \frac{1}{81}$

149. (d) Let $f(x + y) = f(x) + f(y), \ \forall\, x, y \in R$

Put $x = 0 = y$

$\Rightarrow f(0) = f(0) + f(0)$

$\Rightarrow f(0) = 0$

Now, $f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$

$f''(0) = \lim_{h \to 0} \frac{f(h)}{h}$

Now, $f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{f(x) + f(h) - f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{f(h)}{h} = f'(0)$

$\Rightarrow f(x) = x f'(0) + C$

But $f(0) = 0$

$\therefore C = 0$

Hence, $f(x) = x f'(0), \ \forall x \in R$

Clearly, f(x) is everywhere continuous and differentiable and f′(x) is constant.

$\forall x \in R$

150. (d) Let the coefficients of rth, (r + 1)th, and (r + 2)th terms be in HP.

Then, $\frac{2}{{}^nC_r} = \frac{1}{{}^nC_{r-1}} + \frac{1}{{}^nC_r + 1}$

$\Rightarrow \quad 2 = \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{{}^nC_r}{{}^nC_{r+1}}$

$\Rightarrow \quad 2 = \frac{n - r + 1}{r} + \frac{r + 1}{n - r}$

$\Rightarrow \quad n^2 - 4nr + 4r^2 + n = 0$

$\Rightarrow \quad (n - 2r)^2 + n = 0$

which is not possible for any value for n.