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Consortium of Medical, Engineering and Dental Colleges of Karnataka
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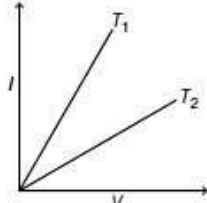
SOLVED PAPER – 2015 (COMEDK)

Instructions

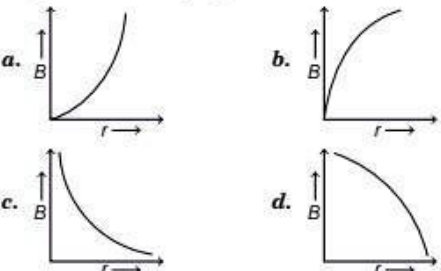- There are 180 questions in all. The number of questions in each section is as given below.

Sections	No. of Questions
Section I : Physics	1–60
Section II : Chemistry	61–120
Section III : Mathematics	121–180

- All the questions are Multiple Choice Questions having four options out of which **ONLY ONE** is correct.
- Candidates will be awarded 1 mark for each correct answer. There will be no negative marking for incorrect answer.
- Time allotted to complete this paper is 3 hrs.

PHYSICS

1. A uniform circular disc of mass 400 g and radius 4.0 cm is rotated about one of its diameter at an angular speed of 10 rot/s. The kinetic energy of the disc is

a. 3.2×10^{-5} J   b. 1.6×10^{-3} J

c. 3.2×10^{-3} J   d. 6.4×10^{-4} J

2. Several lamps of 50 W and 100 V rating are available. How many of them can be connected in parallel across a battery of a 120 V of internal resistance 10 Ω, so that all bulbs glow in full power?

a. 2   b. 4   c. 6   d. 8

3. A stone weight 100 N on the surface of the earth. The ratio of its weight at a height of half the radius of the earth to a depth of half the radius of the earth will be approximately

a. 3.6   b. 2.2   c. 1.8   d. 0.9

4. The gravitational field strength at the surface of a certain planet is g. Which of the following is the gravitational field strength at the surface of a planet with twice the radius and twice the mass?

a. $g/2$ b. g c. $2g$ d. $4g$

5. A crane with a steel cable of length 11 m and radius 2.0 cm is employed to lift a block of concrete of mass 40 tons in a building site. If the Young's modulus of steel is 2.0×10^{11} Pa, what will be roughly the increase in the length of the cable while lifting the block? (Take, $g = 10$ ms^{-2})

a. 0.75 cm   b. 1.25 cm

c. 1.75 cm   d. 2.50 cm

6. I and V are respectively the current and voltage in a metal wire of resistance R. The I-V graph for the two different temperatures T_1 and T_2 given, then

a. $T_1 = T_2$ b. $T_1 > T_2$

c. $T_1 < T_2$ d. $T_1 = 2T_2$

7. A charged particle moves in a uniform magnetic field. The velocity of the particle at some instant makes an acute angle with the magnetic field. The path of the particle will be
a. a straight line
b. a circle
c. a helix with uniform pitch
d. a helix with non-uniform pitch

8. A certain length of insulated wire can be bent to form either a single circular loop (case I) or a double loop of smaller radius (case II). When the same steady current is passed through the wire, the ratio of the magnetic field at the centre in (case I) to that in (case II) is
a. 1   b. 2   c. 1/2   d. 1/4

9. Inside a cyclotron, a charged particle is subjected to both an electric field and a magnetic field. It gains kinetic energy due to the
a. electric field   b. magnetic field
c. Both (a) and (b)   d. None of these

10. The graph showing the variation of the magnetic field strength B with distance r from a long current carrying conductor is

11. The temperature at which centigrade and Fahrenheit scales give the same reading, is
a. $-40°$ b. $40°$ c. $-30°$ d. $30°$

12. The correct definition of Meissner effect is
a. the phenomenon of perfect paramagnetism in superconductors
b. the phenomenon of perfect diamagnetism in superconductors
c. the phenomenon of perfect diamagnetism in semiconductors
d. the phenomenon of perfect ferromagnetism in metals

13. A jet plane is travelling west at 450 m/s. If the horizontal component of earth's magnetic field is 4×10^{-4} T and angle of dip is $30°$, then the vertical component is
a. 3×10^{-4} T   b. 4×10^{-4} T
c. 2.3×10^{-4} T   d. 12.308×10^{-4} T

14. A bicycle generator creates 1.5 V at 15 km/h. The emf generated at 10 km/h is
a. 1.5 V   b. 2 V
c. 0.5 V   d. 1 V

15. The instantaneous voltage of a 50 Hz generator giving peak voltage as 300 V. The generator equation for this voltage is
a. $V = 50\sin 300\pi t$ b. $V = 300\sin 100\pi t$
c. $V = 6\sin 100\pi t$ d. $V = 50\sin 100\pi t$

16. A transformer is used to light a 100 W, 110 V lamp from a 220 V supply. If the supply current is 0.6 A, then the efficiency of the transformer is
a. 66%   b. 76%
c. 86%   d. 96%

17. If instantaneous current is given by $i = 4\cos(\omega t + \phi)$, then the rms value of current is
a. $2\sqrt{2}$ A   b. 4 A
c. $4\sqrt{2}$ A   d. zero

18. If the surface is a perfect reflector, then the change in momentum of the wave after falling on the surface is
a. p b. $2p$
c. $\dfrac{p}{2}$ d. $-2p$

19. A person sitting firmly over a rotating stool has his arms stretched. If he folds his arms, his angular momentum about the axis of the rotation
a. increases   b. decreases
c. remains unchanged   d. doubles

20. The temperature of two bodies A and B are 727°C and 327°C respectively, the ratio $H_A : H_B$ of the rates of heat radiated by them is
a. 727 : 327   b. 5 : 3
c. 25 : 9   d. 625 : 81

21. The refractive index and the permeability of a medium are respectively 1.5 and 5×10^{-7} Hm^{-1}. The relative permittivity of the medium is nearly
a. 25   b. 15   c. 6   d. 10

22. All lights are switched off, except for a bright point-light source kept at the bottom of a swimming pool filled with clear water of refractive index 4/3. As a result, only a circular patch of 6 m diameter of the water surface is visible to spectators standing around the swimming pool. Which of the following gives the nearest value of the depth of the pool?
a. 1.6 m   b. 2.0 m
c. 2.6 m   d. 3.0 m

23. An equiangular glass prism of refractive index 1.6 is kept fully immersed in water of refractive index 4/3, for a certain ray of monochromatic light. What is the closest value for the angle of minimum deviation of the light ray in this setup? (Take, $\sin 37° = 0.6$)
a. 10°   b. 14°
c. 18°   d. 22°

24. A concave mirror gives an image three times as large as its object placed at a distance of 20 cm from it. For the image to be real, the focal length should be
a. 10 cm   b. 15 cm
c. 20 cm   d. 30 cm

25. Two coherent monochromatic light beams of intensities I and $4I$ are superimposed. The maximum and minimum possible resulting intensities are
a. $5I$ and 0   b. $5I$ and $3I$
c. $9I$ and I d. $9I$ and $3I$

26. The momentum of electrons having a wavelength 2Å (given, $h = 6.626 \times 10^{-34}$ J-s, $m = 9.1 \times 10^{-35}$ kg) is
a. 6.313×10^{-24} kg-m s^{-1}
b. 3.313×10^{-24} kg-m s^{-1}
c. 9.313×10^{-25} kg-m s^{-1}
d. 12.313×10^{-24} kg-m s^{-1}

27. The photoelectric current of voltage in a certain experiment is 1.5 V. What is the maximum kinetic energy of photoelectrons emitted?
a. 24×10^{-19} J   b. -24×10^{-19} J
c. 2.4×10^{-19} J   d. 2.04×10^{-19} J

28. The best waves for emission of electrons from a surface are
a. microwaves   b. ultraviolet rays
c. infrared rays   d. X-rays

29. The shortest wavelengths of Paschen, Balmer and Lyman series are in the ratio
a. 9 : 1 : 4   b. 1 : 4 : 9
c. 9 : 4 : 1   d. 1 : 9 : 4

30. A sample of a radioactive element whose half-life is 30 s contains a million nuclei at a certain instant of time. How many nuclei will be left after 10 s?
a. 3.33×10^5 b. 3.78×10^5
c. 1.11×10^5 d. 1.26×10^5

31. A nuclear fission is said to be critical when multiplication factor or k
a. $k = 1$ b. $k > 1$ c. $k < 1$ d. $k = 0$

32. In case of a p-n junction diode at high value of reverse bias, the current rises sharply. The value of reverse bias is known as
a. cut-in   b. zener voltage
c. inverse voltage   d. critical voltage

33. The truth table of a logic gate is given below

Input		Output
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

The logic gate is
a. OR gate   b. AND gate
c. NOR gate   d. NAND gate

34. In a semiconductor diode, the reverse biased current is due to drift of the free electrons and holes caused by
a. thermal expansion only
b. impurity atoms only
c. Both (a) and (b)
d. Neither (a) nor (b)

35. A TV tower is 120 m high. How much more height is to be added to it, if its coverage range is to become double?
a. 120 m   b. 240 m
c. 360 m   d. 480 m

36. A message signal of frequency 10 kHz and peak value of 10 V is used to moderate a carrier of frequency 1 MHz and peak voltage 20 V. The modulation index and side bands produced are
a. 0.4 and 1200 kHz, 990 kHz
b. 0.5 and 1010 kHz, 990 kHz
c. 0.2 and 1010 kHz, 1000 kHz
d. 0.5 and 1500 kHz, 1000 kHz

37. Vector A has a magnitude of 10 units and makes an angle of 30° with the positive X-axis. Vector B has a magnitude of 20 units and makes an angle of 30° with the negative X-axis. What is the magnitude of the resultant between these two vectors?
a. $20\sqrt{3}$ b. 35   c. $15\sqrt{3}$ d. $10\sqrt{3}$

38. A uniform chain of length L is lying partly on a table, the remaining part hanging down from the edge of the table. If the coefficient of friction between the chain and the table is 0.5, what is the minimum length of the chain that should lie on the table, to prevent the chain from slipping down to the ground?

*a.* $L/3$ *b.* $L/2$
c. $2L/3$ *d.* $3L/4$

39. A bomb of mass 18 kg at rest explodes into two pieces of masses 6 kg and 12 kg. The velocity of 12 kg mass is 4 m/s. The kinetic energy of the other mass is

a. 288 J   *b.* 192 J
c. 96 J   *d.* 144 J

40. A soap bubble of radius r is blown up to form a bubble of radius $3r$ under isothermal conditions. If σ is the surface tension of soap solution, then the energy spent in blowing is

a. $3\pi\sigma r^2$ *b.* $6\pi\sigma r^2$
c. $12\pi\sigma r^2$ *d.* $64\pi\sigma r^2$

41. The diagram shows a barrel of weight 1.0×10^3 N on a frictionless slope inclined at $30°$ to the horizontal.
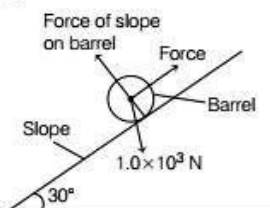
The force is parallel to the slope. What is the work done in moving the barrel a distance of 5.0 m up the slope?

a. 2.5×10^3 J   *b.* 4.3×10^3 J
c. 5.0×10^3 J   *d.* 1.0×10^3 J

42. A 2 kg mass lying on a table is displaced in the horizontal direction through 50 cm. The work done by the normal reaction will be

a. 10 J   *b.* 0
c. 100 erg   *d.* 100 J

43. The dimensions of four wires of the same material are given below. In which wire the increase in length will be maximum when the same tension is applied?

a. Length 100 cm, diameter 1 mm
b. Length 200 cm, diameter 2 mm
c. Length 300 cm, diameter 3 mm
d. Length 50 cm, diameter 0.5 mm

44. Under a constant pressure head, the rate of flow of orderly volume flow of liquid through a capillary tube is V. If the length of the capillary is doubled and the diameter of the bore is halved, then the rate of flow would become

a. $V/4$ *b.* $V/8$ *c.* $16V$ *d.* $V/32$

45. A ball is dropped from the top of the building 100 m high. Simultaneously, another ball is thrown upwards from the bottom of the building with such a velocity that the balls collide exactly mid-way. What is the speed in ms^{-1} with which the second ball is thrown? (Take, $g = 10\ ms^{-2}$)

a. 31.6   *b.* 27.8
c. 22.4   *d.* 19.6

46. In an isochoric process
a. work done is constant
b. volume changes, work done remains same
c. volume remains constant and no work is done by the system
d. Both volume and work done changes

47. The average energy of molecules in a sample of oxygen gas at 300 K is 6.21×10^{-21} J. The corresponding values at 600 K are

a. 12.12×10^{-21} J   *b.* 8.78×10^{-21} J
c. 6.21×10^{-21} J   *d.* 12.42×10^{-21} J

48. A mass M is suspended from a light spring. An additional mass m added displaces the spring further by a distance X. Now, the combined mass will oscillate on the spring with period

a. $T = 2\pi\sqrt{\dfrac{mg}{X(M+m)}}$ *b.* $T = 2\pi\sqrt{\dfrac{(M+m)X}{mg}}$
c. $T = \dfrac{\pi}{2}\sqrt{\dfrac{mg}{X(M+m)}}$ *d.* $T = 2\pi\sqrt{\dfrac{(M+m)}{mg}}$

49. A 10 kg collar is attached to a spring (spring constant 600 N/m), it slides without friction over a horizontal rod. The collar is displaced from its equilibrium position by 20 cm and released. What is the speed of the oscillation?

a. $\sqrt{60} \times 0.2$ m/s   *b.* 60×0.2 m/s
c. 60×2 m/s   *d.* 6×0.2 m/s

50. A body of mass 1 kg starting from origin with initial velocity of $3\hat{i} + 4\hat{j}\ ms^{-1}$. If a constant force of $2\hat{i} - 3\hat{j}$ acts on it, find the coordinate of point it reaches in 3 s.

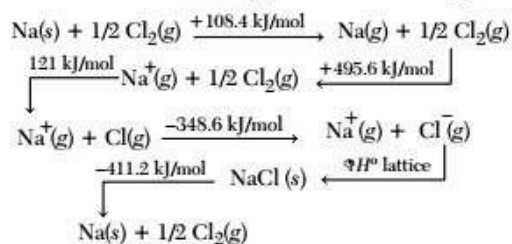*a.* $(18, -1.5)$ *b.* $(9, 1.5)$
c. $(18, 1.5)$ *d.* $(9, -1.5)$

51. A steel wire has a length of 90 cm which is under a constant tension of 100 N. The speed of the transverse waves that can be produced in the wire will be (take, the mass of the steel wire to be 6×10^{-3} kg)

a. 50 m/s
b. 50 cm/s
c. $1 / 3\sqrt{6}$ m/s
d. $50\sqrt{6}$ m/s

52. The number of electrons in 1 C of charges is

a. 6.25×10^{18}
b. 6.25×10^{16}
c. 6.25×10^{10}
d. 6.25×10^{12}

53. An electric dipole consists of two opposite charges, each of magnitude $1.0\,\mu C$ separated by a distance of 2.0 cm. The dipole is placed in an external field of 10^5 NC^{-1}. The maximum torque on the dipole is

a. 0.2×10^{-3} N-m
b. 1×10^{-3} N-m
c. 2×10^{-3} N-m
d. 4×10^{-3} N-m

54. Two small charged spheres A and B have charges $10\,\mu C$ and $40\,\mu C$ respectively and are held at separation of 90 cm from each other. At what distance from A, electric field intensity would be zero?

a. 22.5 cm   b. 18 cm   c. 30 cm   d. 36 cm

55. A charge q is placed at the centre of a cube. The electric flux passing through the cube is

a. $\dfrac{1}{3}\dfrac{q}{\varepsilon_0}$
b. $\dfrac{1}{6}\dfrac{q}{\varepsilon_0}$
c. $\dfrac{q}{\varepsilon_0}$
d. $\dfrac{1}{4}\dfrac{q}{\varepsilon_0}$

56. A parallel plated capacitor has area 2 m^2 separated by 3 dielectric slabs. Their relative permittivity is 2, 3, 6 and thickness is 0.4 mm, 0.6 mm, 1.2 mm, respectively. The capacitance is

a. 5×10^{-8} F
b. 11×10^{-8} F
c. 2.95×10^{-8} F
d. 10×10^{-8} F

57. A particle A has charge $+q$ and particle B has charge $+4q$, each of them having the same mass m. When allowed to fall from rest through the same electrical potential difference, then the ratio of their speeds will become

a. 2 : 1
b. 1 : 2
c. 1 : 4
d. 4 : 1

58. A body accelerates from rest with a uniform acceleration a for a time t. The uncertainty in a is 8% and the uncertainty in t is 4%. The uncertainty in the speed is

a. 32%
b. 12%
c. 8%
d. 2%

59. A parallel plate capacitor has a capacity C. The separation between the plates is doubled and a dielectric medium is inserted between the plates. If the capacity is $3C$, then the dielectric constant of the medium will be

a. 1.5   b. 3   c. 6   d. 12

60. Two resistances A and B have colour codes orange, blue, white and brown, red, green, respectively. Then, ratio of their resistances $A : B$ is

a. 3 : 1
b. 1 : 3
c. $1 : 3 \times 10^4$
d. $3 \times 10^4 : 1$

CHEMISTRY

61. 20 mL of acetic acid reacts with 20 mL of ethyl alcohol to form ethyl acetate. The density of acid and alcohol are 1 g/mL and 0.7 g/mL respectively. The limiting reagent in this reaction is

a. acetic acid
b. ethyl alcohol
c. acetic acid and ethyl alcohol
d. ester

62. The mass of oxygen gas which occupies 5.6 L at STP would be

a. the gram atomic mass of oxygen
b. one-fourth of the gram atomic mass of oxygen
c. double the gram atomic mass of oxygen
d. half of the gram atomic mass of oxygen

63. Energy associated with the first orbit of He$^+$ is

a. 8.72×10^{-18} J
b. 0.872×10^{-18} J
c. -0.872×10^{-18} J
d. -8.72×10^{-18} J

64. Pick the incorrect statement among those given below.

a. Multiple covalent bonds are shorter, than single covalent bonds between same set of atoms.
b. Bond strength varies inversely with bond length.
c. Bond order of isoelectronic species will be same.
d. Bond enthalpy increases with increasing bond length.

65. For one mole of $NaCl(s)$ the lattice enthalpy is

$$Na(s) + 1/2\ Cl_2(g) \xrightarrow{+108.4\ kJ/mol} Na(g) + 1/2\ Cl_2(g)$$

$$\xrightarrow{121\ kJ/mol} Na^+(g) + 1/2\ Cl_2(g) \xleftarrow{+495.6\ kJ/mol}$$

$$Na^+(g) + Cl(g) \xrightarrow{-348.6\ kJ/mol} Na^+(g) + Cl^-(g)$$

$$\xrightarrow{-411.2\ kJ/mol} NaCl(s) \xleftarrow{\Delta H^o\ lattice}$$

$$Na(s) + 1/2\ Cl_2(g)$$

a. $-788\ kJ/mol$ **b.** $+878\ kJ/mol$
c. $+788\ kJ/mol$ **d.** $-878\ kJ/mol$

66. An endothermic reaction is found to have +ve entropy change. The reaction will be
a. possible at high temperature
b. possible only at low temperature
c. not possible at any temperature
d. possible at any temperature

67. For an adiabatic change in a system, the condition which is applicable will be
a. $w = 0$ **b.** $q = -w$
c. $q = w$ **d.** $q = 0$

68. Which of the following equations give ionic product of water?
I. $NH_3(aq) + H_2O \rightleftharpoons NH_4^+(aq) + OH^-(aq)$
II. $NH_4^+(aq) + H_2O \rightleftharpoons H_3O^+ + NH_3(g)$
III. $NH_2^-(aq) + H_2O \rightleftharpoons NH_3(g) + OH^-(aq)$
IV. $NH_3(g) + OH^-(aq) \rightleftharpoons NH_2^-(aq) + H_2O$
a. II and III   **b.** III and IV
c. I and II   **d.** III and I

69. The number of H^+ ions present in 1 mL of a solution whose pH is 13
a. 6.022×10^{10} **b.** 6.022×10^7
c. 6.022×10^{20} **d.** 6.022×10^{23}

70. Strongest reducing agent among the following is
I. $Na^+ + e^- \longrightarrow Na(s)$ $E° = -2.71\ V$
II. $Al^{3+} + 3e^- \longrightarrow Al(s)$ $E° = -1.66\ V$
III. $F_2(g) + 2e^- \longrightarrow 2F^-$ $E° = +2.87\ V$
IV. $2H_2O + 2e^- \longrightarrow H_2(g) + 2OH^-(aq);$
                            $E° = -0.83\ V$
a. only IV   **b.** only III
c. only II   **d.** only I

71. Choose the correct statement applicable for the reaction: $2H_2O + 2F_2 \longrightarrow 4HF + O_2$
a. water is oxidised to O_2.
b. F_2 is oxidised to HF.
c. water is reduced to HF.
d. F_2 acts as reducing agent in the reaction.

72. Decomposition of hydrogen peroxide can be prevented by adding
a. NaOH   **b.** MnO_2
c. urea   **d.** oxalic acid

73. Mg is an important component of
a. haemoglobin   **b.** chlorophyll
c. ATP   **d.** florigen

**74.** $Pb_3O_4 + 4HNO_3 \longrightarrow 2Pb(NO_3)_2 + PbO_2 + 2H_2O$
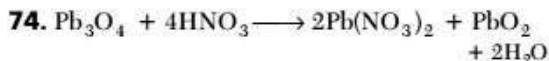
Which is true about the lead ions in Pb_3O_4?
a. Pb^{2+} reacts with HNO_3 to give lead nitrate.
b. Pb^{4+} reacts with HNO_3 to give lead nitrate.
c. Pb^{2+} reacts with HNO_3 to give PbO_2.
d. Pb^{4+} reacts with HNO_3 to give PbO_2.

75. Which of the following chemicals does not liberate hydrogen gas on reaction with aluminium metal?
a. Dil. sulphuric acid
b. Aqueous sodium hydroxide
c. Dil. hydrochloric acid
d. Conc. nitric acid

76. The decreasing order of reactivity towards electrophilic substitution of the following compounds is
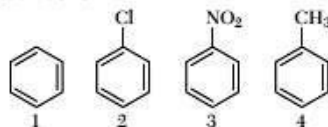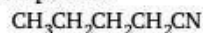

a. $1 > 3 > 4 > 2$ **b.** $4 > 1 > 3 > 2$
c. $4 > 1 > 2 > 3$ **d.** $1 > 4 > 3 > 2$

77. An organic compound weighing 0.15 g gave on Carius estimation, 0.12 g of AgBr.

The percentage of Br in the compound will be close to (Atomic mass of $Ag = 108$, $Br = 80$)
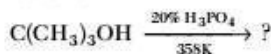**a.** 46%   **b.** 34.1%
c. 3.41%   **d.** 4.6%

78. Identify the correct IUPAC name of the following compound.
        $CH_3CH_2CH_2CH_2CN$
a. Butyl cyanide   **b.** Butanenitrile
c. Pentanenitrile   **d.** Pentyl cyanide

79. Identify the main product in the following reaction.

$$C(CH_3)_3OH \xrightarrow[358K]{20\%\ H_3PO_4} ?$$

a. $(CH_3)_2C = CH_2$ **b.** $(CH_3)_2CHOH$
c. $CH_3CH = CHCH_3$ **d.** $CH_3CH_2CH = CH_2$

80. BOD is a measure of
a. ozone depletion in the stratosphere due to too many free radicals.
b. suspended particulate matter in the air causing lung problems.
c. pesticides which are non-biodegradable in the soil and water.
d. organic material present in water needing dissolved oxygen to break them down depleting oxygen content in water.

81. If same type of atoms are packed in hexagonal close packing and cubic close packing separately, then
a. density of hcp will be greater than ccp
b. density of hcp will be smaller than ccp
c. density of hcp will be equal to ccp
d. density of hcp and ccp will depend upon the temperature of the system

82. Silicon doped with arsenic is
a. p-type semiconductor
b. n-type semiconductor
c. intrinsic semiconductor
d. insulator

83. Which of the following compounds have the same value of van't Hoff's factor (i) as that of $Al_2(SO_4)_3$?
a. $K_4[Fe(CN)_6]$
b. NaCl
c. $Al(NO_3)_3$
d. Na_2SO_4

84. The standard electrode potential for Daniell cell is 1.1 V. What is the standard Gibbs energy for the reaction ?
a. $212.3 \text{ kJ mol}^{-1}$
b. $-212.3 \text{ kJ mol}^{-1}$
c. $106.15 \text{ kJ mol}^{-1}$
d. $-106.15 \text{ kJ mol}^{-1}$

85. The cathode reaction in the dry cell will be
a. $Zn(s) \longrightarrow Zn^{2+} + 2e^-$
b. $MnO_2 + NH_4^+ + e^- \longrightarrow MnO(OH) + NH_3$
c. $Zn(Hg) + 2OH^- \longrightarrow ZnO(s) + H_2O + 2e^-$
d. $MnO(OH) + NH_3 \longrightarrow MnO_2 + NH_4^+ + 2e^-$

86. λ_m° for NH_4Cl, NaOH and NaCl are 130, 248 and $126.5 \ \Omega^{-1} \ cm^2 \ mol^{-1}$ respectively. The λ_m° of NH_4OH will be
a. 251.5
b. 244.5
c. 130
d. 504.5

87. In a chemical reaction i.e., $A + 2B \longrightarrow$ products, when concentration of A is doubled, rate of the reaction increases 4 times and when concentration of B alone is doubled rate continues to be the same. The order of the reaction is
a. 1
b. 2
c. 3
d. 4

88. Zeolites are shape selective structures because of
a. high adsorption capacity
b. three dimensional network of atoms
c. presence of aluminosilicates
d. honey comb like structure

89. Sulphur sol is a type of
a. emulsion
b. multimolecular colloid
c. lyophilic colloid
d. macromolecular colloid

90. The magnetite ore is
a. ferrous oxide
b. ferric oxide
c. ferric hydroxide
d. ferrous-ferric oxide

91. In the metallurgy of aluminium, when the fused matrix is electrolysed, the overall reduction reaction is
a. $Al_2O_3 + 3C \longrightarrow 2Al + 3CO$
b. $Al_2O_3 \longrightarrow 2Al^{3+} + 3O_2$
c. $2Al_2O_3 + 3C \longrightarrow 4Al + 3CO_2$
d. $Al_2O_3 + 2CO \longrightarrow 2Al + 2CO_2$

92. Carbon monoxide forms volatile compound with
a. Ni
b. Cu
c. Al
d. Si

93. Which of the following reactions does not produce PH_3?
a. White phosphorus with water
b. White phosphorus with conc. NaOH
c. Metal phosphide with dil. HCl
d. Metal phosphides with water

94. The decreasing order of stability of oxides of halogen is
a. fluorine > chlorine > iodine > bromine
b. fluorine > chlorine > bromine > iodine
c. iodine > bromine > chlorine > fluorine
d. iodine > chlorine > bromine > fluorine

95. N_2 does not show property of catenation because
a. it has no d-orbitals in the valence shell
b. interelectronic repulsion between non-bonding electrons is greater
c. $N \equiv N$ has high bond enthalpy
d. nitrogen has very high ionisation enthalpy

96. Hybridised state of bromine in bromine pentafluoride is
a. sp^3d
b. d^2sp^3
c. dsp^3
d. sp^3d^2

97. An element belonging to chalcogen group is
    a. carbon
    b. phosphorus
    c. chlorine
    d. sulphur

98. More number of oxidation state are exhibited by the actinoids, than by lanthanoids. The main reason for this is
    a. greater metallic character of lanthanoids than that of their corresponding actinoids
    b. more active nature of actinoids
    c. more energy difference between $5f$ and $6d$-orbitals, than that between $4f$ and $5d$-orbitals
    d. lesser energy difference between $5f$ and $6d$-orbitals, than that between $4f$ and $5d$-orbitals

99. A transition elements with atomic number 24 will have its magnetic moment
    a. 5.92 BM
    b. 6.93 BM
    c. it will be diamagnetic
    d. it will be more than 8 BM

100. In tetrahedral permanganate ion π-bonding type between metal and oxygen is
    a. $p\pi$-$p\pi$
    b. $d\pi$-$p\pi$
    c. $d\pi$-$d\pi$
    d. $d\pi$-s

101. Which of the following pairs has the same size?
    a. Fe^{2+}, Ni^{2+}
    b. Zr^{4+}, Ti^{4+}
    c. Zr^{4+}, Hf^{4+}
    d. Zn^{2+}, Hf^{4+}

102. $[Ti(H_2O)_6]^{2+}$ is a $3d^2$ system, the value of crystal field stabilisation energy decreases by
    a. $4 \times 10^{-1} \Delta_o$
    b. $4.0 \Delta_o$
    c. $8.0 \Delta_o$
    d. $8 \times 10^{-1} \Delta_o$

103. Among the complexes of Ni listed below, the following is the only correct statement about the given compounds.
    I. $[Ni(CO)_4]$
    II. $[Ni(CN)_4]^{2-}$
    III. $[NiCl_4]^{2-}$
    a. all of them have tetrahedral geometry.
    b. I and III are tetrahedral and II has square planar geometry.
    c. I and II are tetrahedral and III is square planar.
    d. all of them have square planar geometry.

104. The formula for sodium trioxalatoaluminate (III) is
    a. $Na_4[Al(C_2O_4)_3]$
    b. $Na_3[Al(C_2O_4)]_3$
    c. $Na_3[Al(C_2O_4)_3]$
    d. $Na_2[Al(C_2O_4)_3]$

105. Which one of the following coordination entity is chiral?
    a. $trans$-$[CrCl_2(ox)_2]^{3-}$
    b. cis-$[Co(NH_3)_4Cl_2]^+$
    c. cis-$[CrCl_2(ox)_2]^{3-}$
    d. $trans$-$[Co(NH_3)_4Cl_2]^+$

106. Which of the following compounds is responsible for depletion of ozone layer?
    a. Freon
    b. Chloroform
    c. DDT
    d. Iodoform

107. Ethanol, when warmed with iodine and sodium carbonate gives a yellow coloured crystalline solid, which is used in preparation of antiseptic ointments. The yellow coloured crystalline substance is
    a. chloroform
    b. iodoform
    c. carbon tetrachloride
    d. dichloromethane

108. Consider the acid strength of the carboxylic acids.
    I. $PhCOOH$
    II. o-$NO_2C_6H_4COOH$
    III. p-$NO_2C_6H_4COOH$
    IV. m-$NO_2C_6H_4COOH$
    a. I > II > III > IV
    b. IV > III > II > I
    c. II > III > IV > I
    d. II > IV > III > I

109. Which of the following is the correct sequence of relative acidic strength?
    a. $FCH_2COOH > ClCH_2COOH > BrCH_2COOH$
    b. $ClCH_2COOH > BrCH_2COOH > FCH_2COOH$
    c. $BrCH_2COOH > ClCH_2COOH > FCH_2COOH$
    d. $ClCH_2COOH > FCH_2COOH > BrCH_2COOH$

110. A mixture of methylamine and dimethyl amine is given to you. The reagents used to separate the components of the mixture are
    a. $CHCl_3$ and HCl
    b. $C_6H_5SO_2Cl$ and KOH
    c. $C_6H_5SO_2Cl$ and HCl
    d. $CHCl_3$ and KOH

111. Gabriel phthalimide synthesis is used for the preparation of
    a. secondary amines
    b. amides
    c. primary amines
    d. tertiary amines

112. The correct increasing order of basic strength of following amines,
    $C_2H_5NH_2$, NH_3, $(C_2H_5)_2NH$, $C_6H_5NH_2$ in gaseous state is
    a. $(C_2H_5)_2NH < C_2H_5NH_2 < NH_3 < C_6H_5NH_2$
    b. $C_6H_5NH_2 < (C_2H_5)_2NH < C_2H_5NH_2 < NH_3$
    c. $C_6H_5NH_2 < NH_3 < C_2H_5NH_2 < (C_2H_5)_2NH$
    d. $(C_2H_5)_2NH < C_2H_5NH_2 < C_6H_5NH_2 < NH_3$

113. The sequence of bases on RNA molecule synthesise on the GCATA strand of DNA is
    a. CGUAU
    b. CGTAT
    c. TACGC
    d. AYCGC

114. α-helix and β-pleated structures of proteins are classified as
    a. primary structure
    b. secondary structure
    c. tertiary structure
    d. quaternary structure

115. Which statement is not true for describing enzymes?
 a. They are polypeptides.
 b. There are glycosidic linkages between the molecules.
 c. They are specific for a particular reaction.
 d. They catalyse biological reactions.

116. Which of the following are monosaccharides?
 a. Glucose and maltose
 b. Glucose and sucrose
 c. Glucose and ribose
 d. None of the above

117. Nylon-6, 6 is an example of a polymer formed by
 a. chain growth polymerisation
 b. crossed linked polymerisation
 c. addition polymerisation
 d. step growth polymerisation

118. Which of the following will form a biodegradable polymer?
 a. Glycine and aminocaproic acid.
 b. Ethylene glycol and phthalic acid.
 c. Hexamethylenediamine and adipic acid.
 d. 1, 3-butadiene and acrylonitrile.

119. Detergents can be made biodegradable and pollution free by
 a. using cyclic hydrocarbon chain
 b. shortening the hydrocarbon chain
 c. using unbranched hydrocarbon chain
 d. hydrocarbons with more branching

120. Sodium rosinate is generally added in the soap because it
 a. prevents rapid drying of soap
 b. produces rich lather
 c. gives antiseptic property
 d. acts as a perfume

MATHEMATICS

121. If two sets A and B have 99 elements in common, then the number of elements common to the sets $A \times B$ and $B \times A$ is
 a. 2^{99} *b.* 99^2
 c. 100 d. 18

122. If $f(x) = e^x$ and $g(x) = \log e^x$, then which of the following is true?
 a. $f\{g(x)\} \neq g\{f(x)\}$
 b. $f\{g(x)\} = g\{f(x)\}$
 c. $f\{g(x)\} + g\{f(x)\} = 0$
 d. $f\{g(x)\} - g\{f(x)\} = 1$

123. The number of bijective functions from the set A to itself, if A contains 108 elements is
 a. 108 *b.* $(108)!$ *c.* $(108)^2$ d. 2^{108}

124. If a class of 175 students the following data shows the number of students opting one or more subjects. Mathematics 100, Physics 70, Chemistry 40. Mathematics and Physics 30, Mathematics and Chemistry 28, Physics and Chemistry 23, Mathematics, Physics and Chemistry 18. The number of students who have opted Mathematics alone is
 a. 35 *b.* 48
 c. 60 d. 22

125. If a set A has 4 elements, then the total number of proper subsets of set A, is
 a. 16 *b.* 14
 c. 15 *d.* 17

126. If $\tan(x + y) = 33$ and $x = \tan^{-1} 3$, then y is
 a. $\dfrac{3}{10}$ *b.* $\dfrac{33}{10}$
 c. $\tan^{-1}\left(\dfrac{1}{3}\right)$ *d.* $\tan^{-1}\left(\dfrac{3}{10}\right)$

127. $\cot^{-1}(21) + \cot^{-1}(13) + \cot^{-1}(-8) =$
 a. 0 *b.* $\cot^{-1} 26$
 c. π *d.* None of these

128. $\tan\left(\cos^{-1}\left(\dfrac{1}{5\sqrt{2}}\right) - \sin^{-1}\left(\dfrac{4}{\sqrt{17}}\right)\right)$ is equal to
 a. $\dfrac{\sqrt{29}}{3}$ *b.* $\dfrac{29}{3}$ *c.* $\dfrac{\sqrt{3}}{29}$ *d.* $\dfrac{3}{29}$

129. The number of solutions of the equation $|\cot x| = \cot x + \dfrac{1}{\sin x}, (0 \le x \le 2\pi)$ is
 a. 0 *b.* 1 *c.* 2 *d.* 3

130. $\dfrac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x}$ is equal to
 a. $-2\sin x$ *b.* $\dfrac{2}{\sin x}$
 c. $\dfrac{1}{\sin x}$ *d.* $2\sin x$

131. In a ΔABC, $\csc A(\sin B \cos C + \cos B \sin C) =$
 a. 0 *b.* 1
 c. -1 *d.* None of these

132. Amplitude of the complex number $i \sin\left(\dfrac{\pi}{19}\right)$ is

a. $\dfrac{\pi}{19}$ b. $-\dfrac{\pi}{19}$ c. $\dfrac{\pi}{2}$ d. $\dfrac{\pi}{2} - \dfrac{\pi}{19}$

133. Given, 5 line segments of lengths 2, 3, 4, 5, 6 units. Then, the number of triangles that can be formed by joining these segments is

a. $^5C_3 - 3$ b. $^5C_3$
c. $^5C_3 - 1$ d. $^5C_3 - 2$

134. How many numbers greater than 1000000 be formed from 2, 3, 0, 3, 4, 2, 3 ?

a. 420   b. 360   c. 400   d. 300

135. The expression

$$\frac{1}{\sqrt{(3x+1)}}\left[\left(\frac{1+\sqrt{3x+1}}{2}\right)^7 - \left(\frac{1-\sqrt{3x+1}}{2}\right)^7\right]$$ is

a polynomial in x of degree

a. 7   b. 5
c. 4   d. 3

136. If in the expansion of $(1+px)^n$, $n \in N$, the coefficient of x and x^2 are 8 and 24, then

a. $n = 3, p = 2$ b. $n = 5, p = 3$
c. $n = 4, p = 3$ d. $n = 4, p = 2$

137. $11^3 + 12^3 + 13^3 + ... + 20^3$ is

a. an even integer.
b. an odd integer divisible by 5.
c. multiple of 10.
d. an odd integer but not a multiple of 5.

138. The medians AD and BE of a triangle with vertices $A(0, b)$, $B(0, 0)$ and $C(a, 0)$ are perpendicular to each other, if

a. $a = \dfrac{b}{2}$ b. $b = \dfrac{a}{2}$
c. $ab = 1$ d. $a = \pm\sqrt{2}b$

139. Let a and b be non-zero real such that $a \neq b$. Then, the equation of the line passing through the origin and the point of intersection of $\dfrac{x}{a} + \dfrac{y}{b} = 1$ and $\dfrac{x}{b} + \dfrac{y}{a} = 1$ is

a. $ax + by = 0$ b. $bx + ay = 0$
c. $y - x = 0$ d. $x + y = 0$

140. A straight lines meets the coordinates axes at A and B, so that the centroid of the ΔOAB is $(1, 2)$. Then, the equation of the line AB is

a. $x + y = 6$ b. $2x + y = 6$
c. $x + 2y = 6$ d. $3x + y = 6$

141. The direction ratios of the line which is perpendicular to the lines

$\dfrac{x-7}{2} = \dfrac{y+17}{-3} = \dfrac{z-6}{1}$ and $\dfrac{x+5}{1} = \dfrac{y+3}{2} = \dfrac{z-4}{-2}$ are

a. $< 4, 5, 7 >$ b. $< 4, -5, 7 >$
c. $< 4, -5, -7 >$ d. $< -4, 5, 7 >$

142. A line making angles $45°$ and $60°$ with the positive directions of the axis of X and Y, makes with the positive direction of Z-axis, an angle of

a. $60°$ b. $120°$
c. $60°$ and $120°$ d. None of these

143. The shortest distance between the lines

$\dfrac{x-3}{3} = \dfrac{y-8}{-1} = \dfrac{z-3}{1}$ and $\dfrac{x+3}{-3} = \dfrac{y+7}{2} = \dfrac{z-6}{4}$ is

a. $\sqrt{30}$ b. $2\sqrt{30}$ c. $5\sqrt{30}$ d. $3\sqrt{30}$

144. $\displaystyle\lim_{x \to \infty} (\sqrt{a^2x^2 + bx + x} - ax) =$

a. $\dfrac{b+1}{2a}$ b. $\dfrac{b}{a}$ c. 0   d. $\dfrac{2b}{a}$

145. If $y = |\cos x| + |\sin x|$, then $\dfrac{dy}{dx}$ at $x = \dfrac{2\pi}{3}$ is

a. $\dfrac{1-\sqrt{3}}{2}$ b. 0

c. $\dfrac{1}{2}(\sqrt{3} - 1)$ d. None of these

146. The slant height of a cone is fixed at 7 cm. If the rate of increase of its height is 0.3 cm/sec, then the rate of increase of its volume when its height is 4 cm is

a. $\dfrac{\pi}{2}$ cc/sec   b. π cc/sec

c. $\dfrac{\pi}{5}$ cc/sec   d. $\dfrac{\pi}{10}$ cc/sec

147. A ladders 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground away from the wall, at the rate of 2m/sec. The speed at which its height on the wall decreases when the foot of the ladder is 4 m away from the wall is

a. $\dfrac{3}{8}$ m/sec   b. $\dfrac{8}{3}$ m/sec

c. $\dfrac{5}{3}$ m/sec   d. $\dfrac{2}{3}$ m/sec

148. The angle between the curve $y^2 = 4ax$ and $ay = 2x^2$ is

a. $\tan^{-1}\dfrac{3}{4}$ b. $\tan^{-1}\dfrac{3}{5}$

c. $\tan^{-1}\dfrac{4}{3}$ d. $\tan^{-1}\dfrac{5}{3}$

149. The maximum area in square units of an isosceles triangle inscribed in an ellipses $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with its vertex at one end of the major axis is

   a. $\sqrt{3}\, ab$ sq units   **b.** $\frac{3\sqrt{3}}{4} ab$ sq units

   c. $\frac{5\sqrt{3}}{4} ab$ sq units   **d.** None of these

150. If $f(x+y) = f(x)\,f(y)$ for all x and y and $f(5) = 2$, $f'(0) = 3$, then $f'(5)$ is

   a. 5   **b.** 6

   c. 0   **d.** None of these

151. If $f(x) = \log_{x^2}(\log x)$, then $f'(x)$ at $x = e$ is

   a. 0   **b.** 1   **c.** $\frac{1}{e}$ **d.** $\frac{1}{2e}$

152. The value of c in Lagrange's mean value theorem for the function $f(x) = \log(\sin x)$ in the interval $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$ is

   a. $\frac{\pi}{4}$ **b.** $\frac{\pi}{2}$

   c. $\frac{2\pi}{3}$ **d.** None of these

153. $\int_{-1}^{1} (x^{27} \cos x + e^x)\,dx =$

   a. $\frac{2e-1}{e}$ **b.** $\frac{e+1}{e}$ **c.** $e - \frac{1}{e}$ **d.** $\frac{1}{e}$

154. The value of the integral $\int_{0}^{\pi} \frac{x \sin^{2n} x}{\sin^{2n} x + \cos^{2n} x}\,dx$ is

   a. π^2 **b.** $2\pi^2$ **c.** $\frac{\pi^2}{4}$ **d.** $\frac{\pi^2}{2}$

155. $\int \frac{x^3 - 1}{x^3 + x}\,dx =$

   a. $x - \log x + \log(x^2 + 1) - \tan^{-1} x + c$

   b. $x - \log x + \frac{1}{2}\log(x^2 + 1) - \tan^{-1} x + c$

   c. $x + \log x + \log(x^2 + 1) - \tan^{-1} x + c$

   d. $x + \log x + \frac{1}{2}\log(x^2 + 1) - \tan^{-1} x + c$

156. If $\int \frac{\cos 8x + 1}{\tan 2x - \cot 2x}\,dx = a\cos 8x + c$, then $a =$

   a. $-\frac{1}{16}$ **b.** $\frac{1}{8}$ **c.** $\frac{1}{16}$ **d.** $-\frac{1}{8}$

157. If $\int_{0}^{\pi} x\, f(\sin x)\,dx = A \int_{0}^{\pi/2} f(\sin x)\,dx$, then A is equal to

   a. 0   **b.** 2π

   c. $\frac{\pi}{4}$ **d.** π

158. Area bounded by the curve $y = x^3$, the X-axis and the ordinates at $x = -2$ and $x = 1$, is

   a. -9 sq units   **b.** $-\frac{15}{4}$ sq units

   c. $\frac{15}{4}$ sq units   **d.** $\frac{17}{4}$ sq units

159. The area in square units bounded by the normal at $(1, 2)$ to the parabola $y^2 = 4x$, X-axis and the curve is given by

   a. $\frac{10}{3}$ **b.** $\frac{7}{3}$

   c. $\frac{4}{3}$ **d.** None of these

160. Area bounded by the Y-axis, $y = \cos x$, $y = \sin x$, when $0 \le x \le \frac{\pi}{2}$ is

   a. $\sqrt{2} - 1$ **b.** $2(\sqrt{2} - 1)$

   c. $(\sqrt{2} + 1)$ **d.** $\sqrt{2}$

161. The area in square units of the region bounded by $y^2 = 9x$ and $y = 3x$ is

   a. 2   **b.** $\frac{1}{4}$

   c. $\frac{1}{2}$ **d.** 1

162. $\int_{1}^{e} \log x \, dx =$

   a. 1   **b.** $e - 1$

   c. $e + 1$ **d.** 0

163. The differential equation of the family of parabolas $y^2 = 4ax$, where a is parameter, is

   a. $\frac{dy}{dx} = \frac{y}{2x}$ **b.** $\frac{dy}{dx} = -\frac{y}{2x}$

   c. $\frac{dy}{dx} = -\frac{2y}{x}$ **d.** $\frac{dy}{dx} = \frac{2y}{x}$

164. If $\frac{dy}{dx} = \frac{y + x \tan \frac{y}{x}}{x}$, then $\sin\left(\frac{y}{x}\right) =$

   a. cx^2 **b.** cx

   c. cx^3 **d.** $\log x$

165. The general solution of the differential equation
$$\left(\frac{dy}{dx}\right) + y \cdot g'(x) = g(x) \cdot g'(x),$$ where $g(x)$ is a given function of x is

a. $g(x) + \log(1 + y + g(x)) = c$
b. $g(x) + \log(1 + y - g(x)) = c$
c. $g(x) - \log(1 + y - g(x)) = c$
d. $g(x) - \log(1 - y + g(x)) = c$

166. The product of the degree and order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 - \left(\frac{dy}{dx}\right)^3 = y^3$ is

a. 4   b. 6
c. 2   d. 3

167. Given, two vectors $\hat{i} - \hat{j}$ and $\hat{i} - 2\hat{j}$. The unit vector, coplanar with the two given vectors and perpendicular to $(\hat{i} - \hat{j})$ is

a. $\frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$ b. $\frac{1}{\sqrt{5}}(2\hat{i} + \hat{j})$
c. $\pm \frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$ d. None of these

168. If a, b, c are three non-zero vectors such that each one of them are perpendicular to the sum of the other two vectors, then the value of $|a + b + c|^2$ is

a. $|a|^2 + |b|^2 + |c|^2$ b. $|a| + |b| + |c|$
c. $2(|a|^2 + |b|^2 + |c|^2)$ d. $\frac{1}{2}(|a|^2 + |b|^2 + |c|^2)$

169. If A and B are 2 matrices such that $AB = A$ and $BA = B$, then B^2 is equal to

a. B b. A
c. zero matrix   d. 1

170. If a, b, c are in AP, then the value of the determinant $\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$ is

a. 0   b. 1
c. x d. $2x$

171. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then det(adj A) is

a. a^{27} b. a^9
c. a^6 d. a^2

172. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $A^8 = aA + bI$, then $(a, b) =$

a. $(8, 7)$ b. $(-7, 8)$
c. $(8, -7)$ d. $(-8, -7)$

173. An urn contains 9 balls, 2 of which are white, 3 blue and 4 black. 3 balls are drawn at random from the urn. The chance that 2 balls will be of the same colour and the third of a different colour is

a. $\frac{45}{84}$ b. $\frac{55}{84}$ c. $\frac{35}{84}$ d. $\frac{25}{84}$

174. 6 boys and 6 girls in a row at random. The probability that all the girls sit together is

a. $\frac{1}{432}$ b. $\frac{12}{431}$
c. $\frac{1}{132}$ d. None of these

175. Three dice are rolled once. The chance of getting a score of 5 is

a. $\frac{5}{216}$ b. $\frac{1}{6}$ c. $\frac{1}{36}$ d. $\frac{1}{7^2}$

176. A bag contains 3 white, 4 black, 2 red balls. If 2 balls are drawn at random, then the probability that both the balls are white is

a. $\frac{1}{18}$ b. $\frac{1}{36}$ c. $\frac{1}{12}$ d. $\frac{1}{24}$

177. The solution of $\frac{6x}{4x-1} < \frac{1}{2}$ is

a. $x < \frac{-1}{8}$ b. $\frac{-1}{8} < x < \frac{1}{4}$
c. $x < \frac{-1}{8}$ and $x > \frac{1}{4}$ d. $x > \frac{1}{8}$

178. If $S^2 = at^2 + 2bt + c$, then acceleration is

a. directly proportional to S.
b. inversely proportional to S.
c. directly proportional to S^2.
d. inversely proportional to S^3.

179. The mean and variance for the data 6, 7, 10, 12, 13, 4, 8, 12 respectively are

a. $8, \sqrt{26.25}$ b. $9, \sqrt{9.25}$
c. $8, 26.25$ d. $9, 9.25$

180. If the conjugate of $(x + iy)(1 - 2i)$ is $1 + i$, then

a. $x - iy = \frac{1-i}{1-2i}$ b. $x + iy = \frac{1-i}{1-2i}$
c. $x = \frac{1}{5}$ d. $x = -\frac{1}{5}$

ANSWERS

Physics

1. (*)	2. (b)	3. (d)	4. (a)	5. (c)	6. (c)	7. (c)	8. (d)	9. (a)	10. (c)
11. (a)	12. (b)	13. (c)	14. (d)	15. (b)	16. (b)	17. (a)	18. (d)	19. (c)	20. (d)
21. (c)	22. (c)	23. (b)	24. (b)	25. (c)	26. (b)	27. (c)	28. (d)	29. (c)	30. (*)
31. (a)	32. (b)	33. (d)	34. (c)	35. (c)	36. (b)	37. (d)	38. (c)	39. (b)	40. (d)
41. (a)	42. (b)	43. (d)	44. (d)	45. (a)	46. (c)	47. (d)	48. (b)	49. (a)	50. (a)
51. (d)	52. (a)	53. (c)	54. (c)	55. (c)	56. (c)	57. (b)	58. (b)	59. (c)	60. (d)

Chemistry

61. (b)	62. (d)	63. (d)	64. (d)	65. (a)	66. (a)	67. (d)	68. (c)	69. (b)	70. (d)
71. (a)	72. (c)	73. (b)	74. (a)	75. (d)	76. (c)	77. (b)	78. (c)	79. (a)	80. (d)
81. (c)	82. (b)	83. (a)	84. (b)	85. (b)	86. (a)	87. (b)	88. (d)	89. (b)	90. (d)
91. (c)	92. (a)	93. (a)	94. (d)	95. (b)	96. (d)	97. (d)	98. (d)	99. (b)	100. (b)
101. (c)	102. (d)	103. (b)	104. (c)	105. (c)	106. (a)	107. (b)	108. (c)	109. (a)	110. (b)
111. (c)	112. (c)	113. (a)	114. (b)	115. (b)	116. (c)	117. (d)	118. (a)	119. (c)	120. (b)

Mathematics

121. (b)	122. (b)	123. (b)	124. (c)	125. (c)	126. (d)	127. (c)	128. (d)	129. (b)	130. (d)
131. (b)	132. (c)	133. (a)	134. (b)	135. (d)	136. (d)	137. (b)	138. (d)	139. (c)	140. (b)
141. (a)	142. (a)	143. (d)	144. (a)	145. (c)	146. (d)	147. (b)	148. (b)	149. (b)	150. (b)
151. (d)	152. (b)	153. (c)	154. (c)	155. (b)	156. (c)	157. (d)	158. (d)	159. (a)	160. (a)
161. (c)	162. (a)	163. (a)	164. (b)	165. (b)	166. (a)	167. (c)	168. (a)	169. (a)	170. (a)
171. (c)	172. (c)	173. (b)	174. (c)	175. (c)	176. (c)	177. (b)	178. (d)	170. (d)	180. (b)

Note (*) None of the option is correct.

HINTS & SOLUTIONS

Physics

1. (*) Given, mass of disc, $M = 400$ g $= 0.4$ kg

Radius, $R = 4$ cm $= 4 \times 10^{-2}$ m

Angular speed, $\omega = 2\pi \times 10$
$$= 20\pi \text{ rad/s}$$
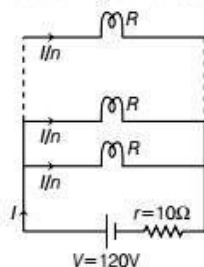
∴ Moment of inertia of disc about its diameter

$$I_d = \frac{1}{4}MR^2$$

∴ Kinetic energy of disc

$$= \frac{1}{2}I_d\omega^2$$
$$= \frac{1}{2} \times \frac{1}{4}MR^2 \times \omega^2 = \frac{1}{8}MR^2\omega^2$$
$$= \frac{1}{8} \times 0.4 \times (4 \times 10^{-2})^2 \times (20\pi)^2$$
$$= 3.2 \times 10^{-1} \text{ J}$$

Hence, no option is correct.

2. (b) According to given condition, let n number of bulbs are connected in parallel as shown below

Resistance of each bulb, $R = \dfrac{V^2}{P}$ $\qquad \left[\because P = \dfrac{V^2}{R}\right]$

$$= \frac{100^2}{50} = 200 \ \Omega$$

Using KVL in the lowest loop,

$$120 - 10I - \frac{I}{n} \times 200 = 0$$
$$\Rightarrow \qquad 12 - I = \frac{20I}{n} \qquad \qquad \text{... (i)}$$

Power delivered by battery = Sum of power used in bulbs

$$(120 - 10I)I = 50n$$
$$\Rightarrow \qquad (12 - I)I = 5n$$
$$\Rightarrow \qquad \frac{20I}{n} \cdot I = 5n \qquad \qquad \text{[from Eq. (i)]}$$
$$\Rightarrow \qquad 4I^2 = n^2$$
$$\Rightarrow \qquad I = \frac{n}{2} \qquad \qquad \text{... (ii)}$$

From Eqs. (i) and (ii), we get

$$12 - \frac{n}{2} = \frac{20}{n} \times \frac{n}{2} \Rightarrow n = 4$$

3. (d) Weight of stone at the surface of the earth, $w = 100$ N

At height h from the surface of earth, gravitational acceleration,

$$g_h = \frac{g}{\left(1 + \dfrac{h}{R}\right)^2}$$

At depth d from the surface of the earth, gravitational acceleration,

$$g_d = g\left(1 - \frac{d}{R}\right)$$

According to question,

$$\frac{\text{Weight at } h = R/2}{\text{Weight at } d = R/2} = \frac{mg_{h\,(\text{at } h = R/2)}}{mg_{d\,(\text{at } h = R/2)}}$$

$$= \frac{\dfrac{g}{\left(1 + \dfrac{h}{R}\right)^2} \ (\text{at } h = R/2)}{g\left(1 - \dfrac{d}{R}\right) \text{ at}(d = R/2)}$$

$$= \frac{\dfrac{1}{\left(1 + \dfrac{R/2}{R}\right)^2}}{1 - \dfrac{R/2}{R}} = \frac{4/9}{1/2} = \frac{8}{9} = 0.9$$

4. (a) At the surface of the planet, gravitational acceleration,

$$g = \frac{GM}{R^2} \qquad \qquad \text{... (i)}$$

where, M = mass of the planet
R = radius of the planet.

When $M' = 2M$ and $R' = 2R$, then

$$g' = \frac{GM'}{R'^2}$$
$$= \frac{G \cdot 2M}{(2R)^2} = \frac{GM}{2R^2} = \frac{1}{2} \cdot g \qquad \text{[from Eq. (i)]}$$
$$\Rightarrow \qquad g' = \frac{g}{2}$$

5. (c) Given, length, $l = 11$ m

Radius, $\quad r = 2$ cm
$$= 2 \times 10^{-2} \text{ m}$$

Mass, $m = 40$ tons
$$= 4 \times 10^4 \text{ kg}$$

Increase in length, $\Delta l = ?$

We know that, Young's modulus,

$$Y = \frac{mgl}{\pi r^2 (\Delta l)}$$

$$\Rightarrow \qquad \Delta l = \frac{mgl}{\pi r^2 Y}$$

$$= \frac{4 \times 10^4 \times 10 \times 11}{3.14 \times (2 \times 10^{-2})^2 \times 2 \times 10^{11}}$$

$$= 1.75 \times 10^{-2}\ \text{m}$$

$$= 1.75\ \text{cm}$$

6. (c) From the I-V graph,

Slope of the line $= \dfrac{1}{R}$

$$\Rightarrow \qquad R = \frac{1}{\text{slope}}$$

Since, slope of line having temperature T_1 is greater than slope of line having temperature T_2.

Hence, $\qquad R_2 > R_1$

Since, on increasing temperature of a conductor, its resistance increases. Hence, $T_2 > T_1$

7. (c) When a charged particle moves in uniform magnetic field making an acute angle with the direction of magnetic field, then it moves on helical path with uniform pitch.

8. (d) Magnetic field at the centre due to current carrying circular loop having N turns is given as

$$B = \frac{\mu_0 NI}{2r}$$

If l be the length of wire, then for the first case,

$$2\pi r_1 = l$$

$$\Rightarrow \qquad r_1 = \frac{l}{2\pi}$$

$\therefore$ Magnetic field, $B_1 = \dfrac{\mu_0 \cdot 1 \cdot I}{2 \cdot r_1} = \dfrac{\mu_0 I}{2 \cdot l / 2\pi}$

$$B_1 = \frac{\mu_0 I \pi}{l} \qquad \dots \text{(i)}$$

For the second case,

$$2 \times 2\pi r_2 = l$$

$$\Rightarrow \qquad r_2 = \frac{l}{4\pi}$$

$\therefore \qquad B_2 = \dfrac{\mu_0 \cdot 2 \cdot I}{2 r_2} = \dfrac{2\mu_0 I}{2 \cdot \dfrac{l}{4\pi}}$

$$\Rightarrow \qquad B_2 = \frac{4\mu_0 I \pi}{l} = 4 \times B_1 \qquad \text{[from Eq. (i)]}$$

$$\Rightarrow \qquad \frac{B_1}{B_2} = \frac{1}{4}$$

9. (a) Inside a cyclotron, charged particle gains kinetic energy only due to electric field, because in magnetic field, its kinetic energy remains unchanged.

10. (c) Magnetic field strength B due to long straight current carrying conductor at a distance r is given as

$$B = \frac{\mu_0}{2\pi} \cdot \frac{I}{r}$$

$$\Rightarrow \qquad B \propto \frac{1}{r}$$

Hence, correct variation of B with r is shown in the graph given in option (c).

11. (a) Using relation,

$$\frac{F - 32}{180} = \frac{C}{100} \qquad \dots \text{(i)}$$

If $F = C = x$, then substituting value of Eq. (i), we get

$$\Rightarrow \qquad \frac{x - 32}{180} = \frac{x}{100} \Rightarrow \frac{x}{5} = \frac{x - 32}{9}$$

$$\Rightarrow \qquad 9x = 5x - 160 \Rightarrow 160 = -4x$$

$$\therefore \qquad x = -40°$$

This is the required temperature at which centigrade and Fahrenheit scale give same reading.

12. (b) Superconductors in the Meissner state exhibit perfect diamagnetism or superdiamagnetism, meaning that the total magnetic field is very close to zero deep inside them.

13. (c) Given, $B_H = 4 \times 10^{-4}\ \text{T}$

Angle of dip, $\delta = 30°$

$$B_V = ?$$

We know that,

$$\tan\delta = \frac{B_V}{B_H}$$

$$\Rightarrow \qquad B_V = B_H \tan\delta$$

$$= 4 \times 10^{-4} \tan 30°$$

$$= \frac{4}{\sqrt{3}} \times 10^{-4}$$

$$= 2.3 \times 10^{-4}\ \text{T}$$

14. (d) Initial speed of bicycle, $v_1 = 15\ \text{km/h}$

Emf produced, $e_1 = 1.5\ \text{V}$

Final speed of bicycle, $v_2 = 10\ \text{km/h}$

If emf generated at speed v_2 be e_2, then

$$e = Bvl$$

$$\Rightarrow \qquad e \propto v$$

$$\Rightarrow \qquad \frac{e_2}{e_1} = \frac{v_2}{v_1} = \frac{10}{15}$$

$$\Rightarrow \qquad \frac{e_2}{e_1} = \frac{2}{3} \Rightarrow e_2 = \frac{2}{3} e_1 = \frac{2}{3} \times 1.5 = 1\ \text{V}$$

15. (b) Given, frequency of AC, $\nu = 50\ \text{Hz}$

Peak voltage, $V_0 = 300\ \text{V}$

Generator equation for the voltage is given as

$$V = V_0 \sin\omega t$$

$$= V_0 \sin 2\pi \nu t$$

$$= 300 \sin(2\pi \times 50 \times t)$$

$$= 300 \sin 100\pi t$$

16. (b) For a transformer,
Input voltage, $V_1 = 220$ V
Input current, $I_1 = 0.6$ A
Output power, $P_2 = 100$ W
From, input power, $P_1 = V_1 I_1$
$$= 220 \times 0.6$$
$$= 132 \text{ W}$$
We know that, efficiency of transformer,
$$\eta = \frac{P_2}{P_1} = \frac{100}{132} = 0.7576$$
$$= 75.76\% \simeq 76\%$$

17. (a) Instantaneous current,
$$i = 4\cos(\omega t + \phi)$$
Comparing with standard equation, $i = i_0 \cos(\omega t + \phi)$, we get
$\therefore$ Peak current, $i_0 = 4$ A
Hence, $i_{rms} = \frac{i_0}{\sqrt{2}}$
$$= \frac{4}{\sqrt{2}} = 2\sqrt{2} \text{ A}$$

18. (d) p is the momentum of incident wave.
$\therefore$ Momentum of reflected wave after reflecting from perfect reflector $= -p$
$\therefore$ Change in momentum = Final momentum − Initial momentum
$$= -p - p = -2p$$

19. (c) According to conservation of angular momentum, when no external torque is applied on the rotating system, then its angular momentum remains unchanged.

20. (d) Given, $T_A = 727°C$ and $T_B = 327°C$
As we know, rate of heat radiated,
$$Q \propto T^4$$
$$\Rightarrow \quad \frac{Q_A}{Q_B} = \frac{H_A}{H_B} = \left(\frac{273 + 727}{273 + 327}\right)^4 = \frac{625}{81}$$

21. (c) Given, refractive index of the medium, $n = 1.5$
Permeability of the medium,
$$\mu = 5 \times 10^{-7} \text{ Hm}^{-1}$$
Relative permeability of the medium,
$$\mu_r = \frac{\mu}{\mu_0} = \frac{5 \times 10^{-7}}{4\pi \times 10^{-7}} = \frac{5}{4\pi}$$
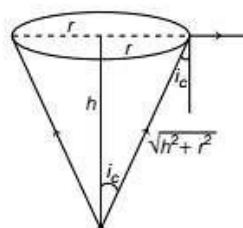We know that, refractive index,
$$n = \sqrt{\varepsilon_r \mu_r}$$
$$n^2 = \varepsilon_r \mu_r$$
$$\Rightarrow \quad \varepsilon_r = \frac{n^2}{\mu_r} = \frac{(1.5)^2}{5/4\pi}$$
$$= \frac{(1.5)^2 \times 4\pi}{5} = 5.7 \simeq 6$$

22. (c) For the given situation, the ray diagram is shown below.

Source of light

Refractive index of water, $\mu_w = \frac{4}{3}$
$$r = \frac{D}{2} = \frac{6}{2} = 3 \text{ m}$$
The given situation is possible only for the case of total internal reflection. If i_C be the critical angle, then
$$\sin i_C = \frac{1}{\mu_w}$$
$$\Rightarrow \quad \sin i_C = \frac{1}{4/3}$$
$$\Rightarrow \quad \sin i_C = \frac{3}{4}$$
$$\Rightarrow \quad \frac{r}{\sqrt{r^2 + h^2}} = \frac{3}{4} \qquad \left[\because \sin i_C = \frac{r}{\sqrt{r^2 + h^2}}\right]$$
$$\Rightarrow \quad \frac{3}{\sqrt{3^2 + h^2}} = \frac{3}{4}$$
$$\Rightarrow \quad 9 + h^2 = 16$$
$$\Rightarrow \quad h^2 = 7, h = \sqrt{7} = 2.6 \text{ m}$$

23. (b) For equiangular glass prism, $A = 60°$
Refractive index of prism, $\mu_p = 1.6$
Refractive index of water, $\mu_w = \frac{4}{3}$
We know that,
$$^w\mu_p = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin A/2}$$
$$\frac{\mu_p}{\mu_w} = \frac{\sin\left(\frac{60 + \delta_m}{2}\right)}{\sin 60/2}$$
$$\Rightarrow \quad \frac{1.6}{4/3} = \frac{\sin\left(\frac{60 + \delta_m}{2}\right)}{\sin 30°}$$
$$\Rightarrow \quad \sin\left(\frac{A + \delta_m}{2}\right) = \frac{1.6 \times 3}{4} \times \sin 30°$$
$$\Rightarrow \quad \sin\left(\frac{A + \delta_m}{2}\right) = 0.6$$
$$\Rightarrow \quad \sin\frac{A + \delta_m}{2} = \sin 37° \qquad [\because \sin 37 = 0.6]$$

$$\Rightarrow \quad \frac{A + \delta_m}{2} = 37°$$
$$\Rightarrow \quad 60 + \delta_m = 2 \times 37 = 74°$$
$$\Rightarrow \quad \delta_m = 74 - 60 = 14°$$

24. (b) Given, size of image formed by concave mirror,
$$h_2 = 3h_1$$
where, h_1 is the size of object.
$$m = \frac{h_2}{h_1} = \frac{3h_1}{h_1} = 3$$
But for real image, magnification m will be negative.
$$\therefore \quad m = -3$$
$$\Rightarrow \quad -\frac{v}{u} = -3$$
$$\Rightarrow \quad v = 3u$$
Here, $u = -20$ cm,
$$\therefore \quad v = 3(-20) = -60 \text{ cm}$$
By using mirror formula,
$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$
$$= \frac{1}{-20} + \frac{1}{-60} = -\frac{1}{15}$$
$$\Rightarrow \quad f = -15 \text{ cm}$$

25. (c) Intensities of two monochromatic sources,
$$I_1 = I \text{ and } I_2 = 4I$$
Maximum intensity,
$$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$$
$$= I_1 + I_2 + 2\sqrt{I_1 I_2}$$
$$= I + 4I + 2\sqrt{I}\sqrt{4I} = 9I$$
Minimum intensity,
$$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$$
$$= I_1 + I_2 - 2\sqrt{I_1}\sqrt{I_2}$$
$$= I + 4I - 2\sqrt{I}\sqrt{4I}$$
$$= 5I - 4I = I$$

26. (b) Given, $h = 6.626 \times 10^{-34}$ J-s
$$m = 9.1 \times 10^{-35} \text{ kg}$$
Wavelength, $\lambda = 2 \text{ Å}$
$$= 2 \times 10^{-10} \text{ m}$$
$\therefore$ Momentum of electron is given as
$$p = \frac{h}{\lambda} \qquad \left[\because p = \frac{h}{\lambda} \right]$$
$$= \frac{6.626 \times 10^{-34}}{2 \times 10^{-10}} = 3.313 \times 10^{-24} \text{ kg-ms}^{-1}$$

27. (c) Given, $V_0 = 1.5$ V
Maximum kinetic energy of emitted photoelectron,
$$K_{max} = eV_0$$
$$= 1.6 \times 10^{-19} \times 1.5 = 2.4 \times 10^{-19} \text{ J}$$

28. (d) The best waves for emission of electrons from a surface are X-rays because when X-ray photon strikes a metal surface, then produce Compton or photoelectric effect more suitably.

29. (c) Wavelength λ of spectral lines in H-atom is given as
$$\frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \qquad \ldots \text{(i)}$$
For shortest wavelength of Paschen series,
$$n_1 = 3 \text{ and } n_2 = \infty$$
$\therefore$ From Eq. (i), we get
$$\frac{1}{\lambda_{PS}} = R \left(\frac{1}{3^2} - \frac{1}{\infty} \right) = \frac{R}{9}$$
$$\Rightarrow \quad \lambda_{PS} = \frac{9}{R} \qquad \ldots \text{(ii)}$$
For shortest wavelength in Balmer series,
$$n_1 = 2, n_2 = \infty$$
$\therefore$ From Eq. (i), we get
$$\frac{1}{\lambda_{BS}} = R \left(\frac{1}{2^2} - \frac{1}{\infty} \right) = \frac{R}{4}$$
$$\Rightarrow \quad \lambda_{BS} = \frac{4}{R} \qquad \ldots \text{(iii)}$$
For shortest wavelength of Lyman series,
$$n_1 = 1 \text{ and } n_2 = \infty$$
$\therefore$ From Eq. (i), we get
$$\frac{1}{\lambda_{LS}} = R \left(\frac{1}{1^2} - \frac{1}{\infty} \right) = R$$
$$\Rightarrow \quad \lambda_{LS} = \frac{1}{R}$$
Hence, from Eq. (i), (ii) and (iii), we have
$$\lambda_{PS} : \lambda_{BS} : \lambda_{LS} = \frac{9}{R} : \frac{4}{R} : \frac{1}{R}$$
$$= 9 : 4 : 1$$

30. (*) Half-life of radioactive sample,
$$T_{1/2} = 30 \text{ s}, N_0 = 10^6$$
If N be the remaining number of nuclei after $t = 10$ s, then according to radioactive decay law,
$$N = N_0 \left(\frac{1}{2} \right)^n = 10^6 \left(\frac{1}{2} \right)^{t/T_{1/2}}$$
$$= 10^6 \left(\frac{1}{2} \right)^{\frac{10}{30}} = 10^6 \left(\frac{1}{2} \right)^{1/3}$$
$$= 10^6 (0.796) = 7.96 \times 10^5$$

31. (a) A nuclear fission is said to be critical when multiplication factor k will be equal to unity.
i.e., $\quad k = 1$

32. (b) In case of a p-n junction diode at high value of reverse bias voltage, the current rises sharply due to zener breakdown. At zener voltage, zener breakdown occurs and current rises sharply.

33. (d) According to given truth table, logic expression of the output Y for the given input A and B is given as

$$Y = \overline{AB}$$

which is the output expression for NAND gate.

34. (c) In a semiconductor diode, the reverse biased current is due to drift of the free electrons and holes which are produced by thermal expansion, impurity atoms etc.

35. (c) Initial height of TV tower, $h_1 = 120$ m

Coverage range of TV tower is given as

$$d = \sqrt{2Rh}$$

where, R is radius of the earth.

$$\Rightarrow \qquad \frac{d_1}{d_2} = \sqrt{\frac{h_1}{h_2}}$$

$$\Rightarrow \qquad \frac{d_1}{2d_1} = \sqrt{\frac{120}{h_2}} \qquad [\because d_2 = 2d_1]$$

$$\Rightarrow \qquad \frac{1}{4} = \frac{120}{h_2}$$

$$\Rightarrow \qquad h_2 = 480 \text{ m}$$

$\therefore$ Height to be added $= h_2 - h_1$

$$= 480 - 120 = 360 \text{ m}$$

36. (b) Frequency of message signal, $f_m = 10$ kHz

Carrier frequency, $f_c = 1$ MHz $= 1000$ kHz
Peak value of message signal, $V_m = 10$ V
Peak value of carrier signal, $V_c = 20$ V

$\therefore$ Modulation index, $\mu = \dfrac{V_m}{V_c} = \dfrac{10}{20} = 0.5$

Upper side band frequency,
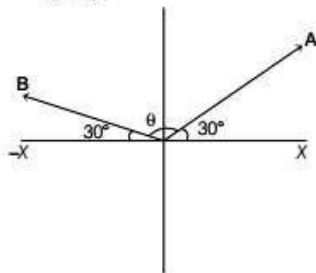$$f_{USB} = f_c + f_m = 1000 + 10$$
$$= 1010 \text{ kHz}$$
Lower side band frequency,
$$f_{LSB} = f_c - f_m = 1000 - 10 = 990 \text{ kHz}$$

37. (d) Given, $A = 10$ units
$\qquad\qquad B = 20$ units
$\qquad\qquad \theta = 30°$

The given situation is shown in the figure. The angle between A and B,

$$\theta = 180° - 30° - 30° = 120°$$

$\therefore$ Magnitude of the resultant vector,

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$
$$= \sqrt{10^2 + 20^2 + 2 \times 10 \times 20 \cos 120°}$$
$$= \sqrt{100 + 400 - 200}$$
$$= 10\sqrt{3}$$

38. (c) Let length of chain lying the table be y.

Mass per unit length of the chain $= \dfrac{M}{L}$

Mass of the chain lying on the table, $M' = \dfrac{M}{L} \cdot y$

Mass of the length $(L - y)$ of hanging chain $= \dfrac{M}{L}(L - y)$

At equilibrium,
Friction force between table and chain
$\qquad\qquad =$ Weight of hanging part of the chain

$$\mu\left(\frac{M}{L}y\right)g = \frac{M}{L}(L - y)g$$

$$\Rightarrow \qquad \mu y = L - y$$
$$\Rightarrow \qquad 0.5y = L - y \qquad [\because \mu = 0.5]$$
$$\Rightarrow \qquad y = \frac{2L}{3}$$

39. (b) Mass of bomb, $M = 18$ kg

Mass of pieces, $m_1 = 6$ kg, $m_2 = 12$ kg
Velocity of 12 kg mass, $v_2 = 4$ m/s
Let v_1 be the velocity of 6 kg mass.
According to conservation of linear momentum,
$$M \times 0 = m_1 v_1 + m_2 v_2$$
$$\Rightarrow \qquad m_1 v_1 = -m_2 v_2$$
$$6v_1 = -12 \times 4$$
$$\Rightarrow \qquad v_1 = -8 \text{ m/s}$$
Kinetic energy of other (6 kg) mass,

$$K = \frac{1}{2}m_1 v_1^2$$
$$= \frac{1}{2} \times 6 \times (-8)^2 = 3 \times 64 = 192 \text{ J}$$

40. (d) Surface area of bubble of radius, $r = 4\pi r^2$

Surface area of bubble of radius, $3r = 4\pi(3r)^2 = 36\pi r^2$

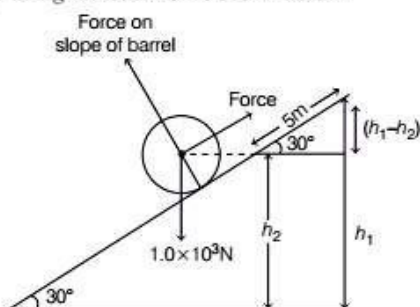Therefore, increase in surface area
$$= 36\pi r^2 - 4\pi r^2 = 32\pi r^2$$

Since, a bubble has two surfaces, the total increase in surface area $= 2 \times 32\pi r^2 = 64\pi r^2$

$\therefore$ Energy spent = Work done = Surface tension $\times$ Total increase in surface area $= 64\pi\sigma r^2$

41. (a) The given situation is shown below

Work done in moving the barrel on the frictionless slope is equal to change in potential energy.

i.e. $\qquad W = mg(h_1 - h_2)$ ... (i)

Here, $\qquad mg = 1 \times 10^3$ N

From figure, $\sin 30° = \dfrac{h_1 - h_2}{5}$

$\Rightarrow \qquad h_1 - h_2 = 5 \sin 30° = \dfrac{5}{2} = 2.5$ m

Putting these values in Eq. (i), we get

$\qquad W = 1 \times 10^3 (2.5) = 2.5 \times 10^3$ J

42. (b) Since, displacement on the body of mass 2 kg is in the horizontal direction while normal reaction acts on the body in perpendicular to the displacement.

Hence, $\qquad \theta = 0$

$\therefore$ Work done, $W = Fs \cos 90° = 0$

43. (d) We know that, Young's modulus of elasticity,

$\qquad Y = \dfrac{FL}{\pi r^2 l}$ [where, l is change in length]

$\Rightarrow \qquad l = \dfrac{FL}{\pi r^2 Y}$

$\Rightarrow \qquad l \propto \dfrac{L}{r^2} \propto \dfrac{L}{d^2}$ [where, $d =$ diameter]

$\Rightarrow \qquad l \propto \dfrac{L}{d^2}$

Among the given options, $\dfrac{L}{d^2}$ is maximum for the values of $L = 50$ cm and $d = 0.5$ mm.

44. (d) Given, $V_1 = V$, $l_1 = l$, $r_1 = r$

$\qquad l_2 = 2l_1 = 2l$

$\qquad r_2 = \dfrac{r_1}{2} = \dfrac{r}{2}$

$\qquad V_2 = ?$

According to Poiseuille's formula, the rate of flow of liquid through a narrow tube is given as
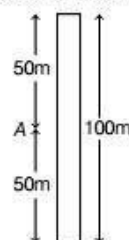
$\qquad V = \dfrac{\pi p r^4}{8 \eta l}$

$\Rightarrow \qquad V \propto \dfrac{r^4}{l}$

$\Rightarrow \qquad \dfrac{V_2}{V_1} = \left(\dfrac{r_2}{r_1}\right)^4 \cdot \dfrac{l_1}{l_2}$

$\qquad = \left(\dfrac{r/2}{r}\right)^4 \left(\dfrac{l}{2l}\right) = \left(\dfrac{1}{2}\right)^4 \cdot \dfrac{1}{2} = \dfrac{1}{32}$

$\Rightarrow \qquad V_2 = \dfrac{V_1}{32} = \dfrac{V}{32}$

45. (a) The given situation is shown in the figure.

For the first ball, time taken to reach at mid point A,

$\qquad t = \sqrt{\dfrac{2h}{g}}$

$\qquad = \sqrt{\dfrac{2 \times 50}{10}} = \sqrt{10}$ s

For the second ball,

$\qquad v = 0, u = ?, a = -g$

$\therefore$ From equation, $v = u + gt$

$\qquad 0 = u - gt_1$

$\Rightarrow \qquad t_1 = \dfrac{u}{g}$

Since, both balls collide at some time,

Hence, $\qquad t = t_1$

$\qquad \sqrt{10} = \dfrac{u}{g}$

$\Rightarrow \qquad u = 10\sqrt{10}$ m/s $= 31.6$ m/s

46. (c) In isochoric process, volume of the system remains constant.

i.e. $\qquad \Delta V = V_1 - V_2 = V - V = 0 [\because V_1 = V_2 = V]$

Hence, work done, $W = p \times \Delta V = p \times 0 = 0$

47. (d) At temperature, $T_1 = 300$ K

Average kinetic energy, $E_1 = 6.21 \times 10^{-21}$ J

$\therefore \qquad T_2 = 600$ K, $E_2 = ?$

We know that, average kinetic energy of gas molecule,

$\qquad E \propto T$

$\Rightarrow \qquad \dfrac{E_2}{E_1} = \dfrac{T_2}{T_1} \Rightarrow E_2 = \dfrac{T_2}{T_1} \times E_1$

$\qquad = \dfrac{600}{300} \times 6.21 \times 10^{-21}$

$\qquad = 2 \times 6.21 \times 10^{-21}$

$\qquad = 12.42 \times 10^{-21}$ J

48. (b) At initial condition of spring,

$\qquad Mg = kX_0$ ... (i)

When mass m is added, then at equilibrium,

$\qquad (M + m)g = k(X + X_0)$

$\Rightarrow \quad Mg + mg = kX + kX_0$

$\Rightarrow \quad Mg + mg = kX + Mg \qquad$ [from Eq. (i)]

$\Rightarrow \qquad mg = kX$

$\Rightarrow \qquad k = \dfrac{mg}{X} \qquad\qquad$ ... (ii)

$\therefore$ Time period,

$$T = 2\pi \sqrt{\dfrac{\text{Total mass of the block}}{\text{Force constant } k}}$$

$$= 2\pi \sqrt{\dfrac{M + m}{mg/X}}$$

$$= 2\pi \sqrt{\dfrac{(M + m)X}{mg}}$$

49. *(a)* Given, amplitude, $A = 20$ cm $= 0.2$ m

Force constant, $k = 600$ Nm^{-1}

$$m = 10 \text{ kg}$$

Maximum speed of oscillation,

$$v_{max} = A\omega$$

$$= A\sqrt{\dfrac{k}{m}} \qquad \left[\because \omega = \sqrt{\dfrac{k}{m}}\right]$$

$$= 0.2\sqrt{\dfrac{600}{10}} = 0.2\sqrt{60} \text{ m/s}$$

50. *(a)* Given, initial velocity, $u = 3\hat{i} + 4\hat{j}$

Force, $F = 2\hat{i} - 3\hat{j}$

Initial velocity along X-axis and Y-axis are $u_x = 3$ ms^{-1} and $u_y = 4$ ms^{-1}, respectively.

Similarly, $F_x = 2$ N and $F_y = -3$ N

If acceleration along X and Y-axes are a_x and a_y, then

$$a_x = \dfrac{F_x}{m} = \dfrac{2}{1} = 2 \text{ m/s}^2 \qquad [\because F = ma]$$

$$a_y = \dfrac{F_y}{m} = \dfrac{-3}{1} = -3 \text{ m/s}^2$$

$$s_x = u_x t + \dfrac{1}{2}a_x t^2 = 3 \times 3 + \dfrac{1}{2} \times 2 \times 9 = 18 \text{ m}$$

$$s_y = u_y t + \dfrac{1}{2}a_y t^2 = 4 \times 3 + \dfrac{1}{2} \times (-3) \times 9$$

$$= 12 - 13.5 = -1.5 \text{ m}$$

So, required x and y-coordinates of point = $(18, -1.5)$

51. *(d)* Given, tension is the wire, $T = 100$ N

Mass of steel wire, $m = 6 \times 10^{-3}$ kg

Length, $l = 90$ cm $= 0.9$ m

Mass per unit length of steel wire,

$$\mu = \dfrac{m}{l}$$

$$= \dfrac{6 \times 10^{-3}}{0.9}$$

$$= \dfrac{2}{3} \times 10^{-2} \text{ kg-m}^{-1}$$

Speed of transverse wave,

$$v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{100}{\left(\dfrac{2}{3}\right) \times 10^{-2}}}$$

$$= \dfrac{300}{\sqrt{6}} = 50\sqrt{6} \text{ m/s}$$

52. *(a)* Charge, $q = 1$ C

If n be the total number of electrons in 1 C of charge, then $q = ne$

$$\Rightarrow \qquad n = \dfrac{q}{e}$$

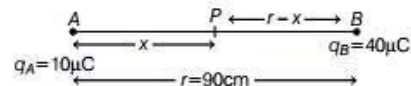$$= \dfrac{1}{1.6 \times 10^{-19}}$$

$$= 6.25 \times 10^{18} \text{ electrons}$$

53. *(c)* $|q_1| = |q_2| = 1\mu$C $= 10^{-6}$ C

$$2l = 2 \text{ cm} = 2 \times 10^{-2} \text{ m}$$

$$E = 10^5 \text{ NC}^{-1}$$

$\therefore$ Maximum torque, $\tau_{max} = pE\sin 90°$

$$= pE = q(2l)E$$

$$= 10^{-6} \times 2 \times 10^{-2} \times 10^5$$

$$= 2 \times 10^{-3} \text{ N-m}$$

54. *(c)* The given situation is shown below,

A •——————— P ←—$r-x$—→ B
$q_A = 10\mu$C ←—x—→ $\quad\quad\quad q_B = 40\mu$C
←——————$r = 90$cm——————→

Let electric field at point P at a distance x from A is zero.

At point P, $E_A = E_B$

$$\Rightarrow \quad \dfrac{1}{4\pi\varepsilon_0} \dfrac{q_A}{x^2} = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q_B}{(r-x)^2}$$

$$\Rightarrow \quad \dfrac{10 \times 10^{-6}}{x^2} = \dfrac{40 \times 10^{-6}}{(r-x)^2}$$

$$\Rightarrow \quad \left(\dfrac{1}{x}\right)^2 = \left(\dfrac{2}{r-x}\right)^2$$

$$\Rightarrow \quad \dfrac{1}{x} = \dfrac{2}{r-x}$$

$$\Rightarrow \quad 2x = r - x$$

$$\Rightarrow \quad x = \dfrac{r}{3}$$

$$= \dfrac{90}{3} = 30 \text{ cm}$$

55. *(c)* According to Gauss's law, total electric flux passing through the cube,

$$\phi = \dfrac{\text{total enclosed charge}}{\varepsilon_0} = \dfrac{q}{\varepsilon_0}$$

56. *(c)* The given situation is shown below,
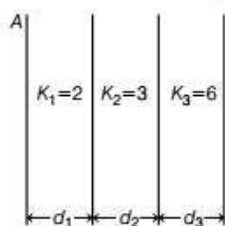
$$d_1 = 0.4 \text{ mm} = 4 \times 10^{-4} \text{ m}$$
$$d_2 = 0.6 \text{ mm} = 6 \times 10^{-4} \text{ m}$$
$$d_3 = 1.2 \text{ mm} = 1.2 \times 10^{-3} \text{ m}$$

The given arrangement will be equal to three capacitors connected in series.

i.e. $\quad C_1 = \dfrac{\varepsilon_0 K_1 A}{d_1}, C_2 = \dfrac{\varepsilon_0 K_2 A}{d_2}$

and $\quad C_3 = \dfrac{\varepsilon_0 K_3 A}{d_3}$

Equivalent capacitance is given as,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{\varepsilon_0 A}\left(\frac{d_1}{K_1} + \frac{d_2}{K_2} + \frac{d_3}{K_3}\right)$$

$$\frac{1}{C_{eq}} = \frac{1}{\varepsilon_0 A}\left[\frac{4 \times 10^{-4}}{2} + \frac{6 \times 10^{-4}}{3} + \frac{1.2 \times 10^{-3}}{6}\right]$$

$$\Rightarrow \frac{1}{C_{eq}} = \frac{1}{\varepsilon_0 A}[6 \times 10^{-4}]$$

$$C_{eq} = \frac{\varepsilon_0 A}{6 \times 10^{-4}} = \frac{8.85 \times 10^{-12} \times 2}{6 \times 10^{-4}} = 2.95 \times 10^{-8} \text{ F}$$

57. *(b)* Given, $m_1 = m_2 = m$
$$q_1 = q$$
$$q_2 = 4q$$

We know that

$$E = \frac{1}{2}mv^2 = qV$$

$$\Rightarrow \quad v^2 \propto q$$

[For same value of potential difference V]

$$\Rightarrow \quad v \propto \sqrt{q}$$

$$\Rightarrow \quad \frac{v_1}{v_2} = \sqrt{\frac{q_1}{q_2}}$$

$$= \sqrt{\frac{q}{4q}} = \frac{1}{2}$$

$$\Rightarrow \quad v_1 : v_2 = 1 : 2$$

58. *(b)* Given, uncertainity in acceleration, $\dfrac{\Delta a}{a} \times 100 = 8\%$

Uncertainity in time, $\dfrac{\Delta t}{t} \times 100 = 4\%$

Since, body starts motion from rest, hence by the first equation of motion,

$$v = u + at$$
$$v = 0 + at$$

$$\Rightarrow \quad \frac{\Delta v}{v} = \frac{\Delta a}{a} + \frac{\Delta t}{t}$$

$$\Rightarrow \quad \frac{\Delta v}{v} \times 100 = \frac{\Delta a}{a} \times 100 + \frac{\Delta t}{t} \times 100$$

$$= 8\% + 4\% = 12\%$$

59. *(c)* We know that, capacitance of a parallel plate capacitor,

$$C = \frac{\varepsilon_0 A}{d} \qquad \dots \text{(i)}$$

where, A = area of plate

d = separation between the plates.

When $d' = 2d$, then

$$C' = \frac{\varepsilon_0 A K}{d'}$$

$$\Rightarrow \quad 3C = \frac{\varepsilon_0 A K}{2d} \qquad [\because C' = 3C]$$

$$\Rightarrow \quad 3C = \frac{\varepsilon_0 A}{d} \cdot \frac{K}{2}$$

$$\Rightarrow \quad 3C = C \cdot \frac{K}{2} \qquad [\text{from Eq. (i)}]$$

$$\Rightarrow \quad 3 = \frac{K}{2}$$

$$\Rightarrow \quad K = 6$$

60. *(d)* Using colour code for carbon resistor, (For A)

Orange	Blue	White
↓	↓	↓
3	6	9

$$\therefore \quad R_A = 36 \times 10^9 \, \Omega$$

Similarly, (For B)

Brown	Red	Green
↓	↓	↓
1	2	5

$$\therefore \quad R_B = 12 \times 10^5 \, \Omega$$

$$\therefore \quad \frac{R_A}{R_B} = \frac{36 \times 10^9}{12 \times 10^5}$$

$$= 3 \times 10^4$$

$$\therefore \quad R_A : R_B = 3 \times 10^4 : 1$$

Chemistry

61. (b) Mass of 20 mL of acetic acid = $20 \times 1 = 20$ g

Mass of 20 mL of ethyl alcohol = $20 \times 0.7 = 14$ g

For the reaction,

$$\underset{60\text{ g}}{CH_3COOH} + \underset{46\text{ g}}{C_2H_5OH} \xrightarrow[-H_2O]{} CH_3COOC_2H_5$$

60 g of acetic acid requires = 46 g of ethyl alcohol

20 g of acetic acid requires = $\dfrac{46}{60} \times 20 = 15.3$ g of ethyl alcohol

Therefore, ethyl alcohol is the limiting reagent.

62. (d) Mass of oxygen in 22.4 L at STP = 32 g

Mass of oxygen in 5.6 L at STP = $\dfrac{32}{22.4} \times 5.6 = 8$ g

$= \dfrac{1}{2} \times$ gram atomic mass of oxygen

Therefore, mass of oxygen gas in 5.6 L at STP would be the half of the gram atomic mass of oxygen.

63. (d) For H-like particles,

$$E_n = \dfrac{-21.78 \times 10^{-19}}{n^2} Z^2 \text{ J}$$

$$= \dfrac{-21.78 \times 10^{-19}}{1^2} \times (2)^2 \text{ [For helium, } Z = 2, n = 1]$$

$$= -8.72 \times 10^{-18} \text{ J}$$

64. (d) Bond enthalpy decreases with increasing bond length as the intermolecular force of attraction decreases with increase in the distance between the constituent particles.

65. (a)

$$Na(s) + 1/2\, Cl_2(g) \xrightarrow{+108.4\text{ kJ/mol}} Na(g) + 1/2\, Cl_2(g)$$

$$\xrightarrow[S]{121\text{ kJ/mol}} Na^+(g) + 1/2\, Cl_2(g) \xleftarrow[IE]{+495.6\text{ kJ/mol}}$$

$$Na^+(g) + Cl_2(g) \xrightarrow[EA]{-348.6\text{ kJ/mol}} Na^+(g) + Cl^-(g)$$

$$\xrightarrow[\Delta_f H^o]{-411.2\text{ kJ/mol}} NaCl(s) \xleftarrow{\Delta H^o \text{ lattice}}$$

$$Na(s) + 1/2\, Cl_2(g)$$

According to Hess law,

$$\Delta_f H^\bullet = S + D + IE + EA + U$$

$$-411.2 = 108.4 + 121 + 495.6 - 348.6 + U$$

$$U = -787.6 \text{ kJ/mol} \simeq -788 \text{ kJ/mol}$$

66. (a) Given, $\Delta S = +$ ve and $\Delta H = +$ ve (For endothermic reaction)

Now, $\qquad \Delta G = \Delta H - T\Delta S$

For a reaction to be spontaneous, ΔG should be $-$ve which is possible only at high temperature i.e., when $T\Delta S > \Delta H$.

67. (d) For an adiabatic change in a system, no heat is exchanged between the system and the surroundings i.e., $q = 0$.

68. (c) Ionic product of water is given by following reaction,

I. $NH_3(aq) + H_2O \rightleftharpoons NH_4^+ + OH^-(aq)$

II. $NH_4^+(aq) + H_2O \rightleftharpoons H_3O^+ + NH_3(g)$

69. (b) pH = 13

$$[H]^+ = 10^{-pH} = 10^{-13} \text{ mol L}^{-1}$$

$\Rightarrow$ 1000 mL solution contains

$$= 10^{-13} \times 6.022 \times 10^{23} \text{ H}^+ \text{ ions}$$

$$= 6.022 \times 10^{10} \text{ H}^+ \text{ ions}$$

1 mL solution contains

$$= \dfrac{6.022 \times 10^{10}}{1000} = 6.022 \times 10^{7} \text{ H}^+ \text{ ions.}$$

70. (d) Lower the E° value (more negative), stronger is the reducing agent.

$\therefore$ Na^+ is the strongest reducing agent.

71. (a) Water is oxidised to O_2 and F_2 is reduced to HF as follows.

72. (c) H_2O_2 decomposes slowly on exposure to light.

$$2H_2O_2(l) \longrightarrow 2H_2O(l) + O_2(g)$$

Its decomposition can be prevented by adding urea as a stabiliser.

73. (b) Mg is an important component of chlorophyll which is used in synthesis of food by plants in presence of sunlight.

74. (a) Pb_3O_4 is a mixture of $PbO_2 + PbO$ and only PbO i.e., Pb^{2+} reacts with HNO_3 to form $Pb(NO_3)_2$.

75. (d) Dil. and conc. HNO_3 has no effect on Al.

Aluminium is rendered passive by nitric acid due to the formation of a thin film of oxide (Al_2O_3) on its surface.

$$2Al + 6HNO_3(conc.) \longrightarrow Al_2O_3 + 6NO_2 + 3H_2O$$

Aluminium reacts with H_2SO_4 to liberate H_2 gas.

$$2Al + 3H_2SO_4 \longrightarrow Al_2(SO_4)_3 + 3H_2\uparrow$$

Aluminium also reacts with aqueous alkali and liberates dihydrogen.

$$2Al(s) + 2NaOH(aq) + 6H_2O(l) \longrightarrow 2Na[Al(OH)_4](aq) + 3H_2(g)$$

Aluminium dissolves in dil. HCl and liberates dihydrogen.

$$2Al(s) + 6HCl(aq) \longrightarrow 2Al^{3+}(aq) + 6Cl^-(aq) + 3H_2(g)$$

76. (c) Due to $+I$-effect of —CH_3 group (4) has higher electron density, than benzene (1). Due to strong $-I$-effect of —Cl group and strong $-I$ and $-M$-effect of

—NO_2 group, both (2) and (3) have lower electron density, than benzene (1).

As $-I$-effect of —Cl group is weaker, than that of —NO_2 (3) has much lower electron density than (2).

Hence, nitrobenzene will have high reactivity towards electrophilic substitution due to lower electron density on benzene ring.

Thus, the overall reactivity order towards electrophilic substitution is $4 > 1 > 2 > 3$.
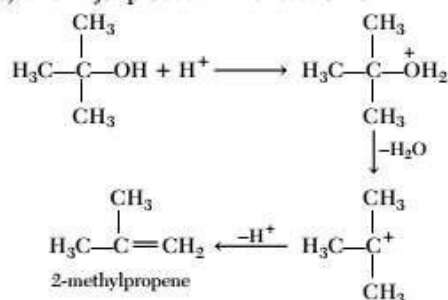
77. *(b)* Percentage of bromine in the compound

$$= \frac{80}{188} \times \frac{\text{Mass of AgBr formed}}{\text{Mass of substance taken}} \times 100$$

$$= \frac{80}{188} \times \frac{0.12}{0.15} \times 100 = 34.04\%$$

78. *(c)* The IUPAC name of compound

$$\overset{5}{CH_3}\overset{4}{CH_2}\overset{3}{CH_2}\overset{2}{CH_2}\overset{1}{CN} \text{ is pentane nitrile.}$$

79. *(a)* The major product in the reaction is

80. *(d)* BOD is a measure of organic material present in water needing dissolved oxygen for decomposition Hence, depleting the oxygen content in water.

81. *(c)* Hexagonal close packing and cubic close packing are equally efficient.

∴ Density of both the packings will be same.

82. *(b)* When, Si (having 4 valence electrons) is doped with As (having 5 valence electrons), 4 electrons form covalent bonds, while the fifth electron remains free and increase conductivity. Hence, Si doped with As is n-type semiconductor.

83. *(a)* The dissociation of compounds take place as follows:

$$Al_2(SO_4)_3 \longrightarrow 2Al^{3+} + 3SO_4^{2-}, i = 5$$
$$K_4[Fe(CN)_6] \longrightarrow 4K^+ + [Fe(CN)_6]^{4-}, i = 5$$
$$NaCl \longrightarrow Na^+ + Cl^-, i = 2$$
$$Al(NO_3)_3 \longrightarrow Al^{3+} + 3NO_3^-, i = 4$$
$$Na_2SO_4 \longrightarrow 2Na^+ + SO_4^{2-}, i = 3$$
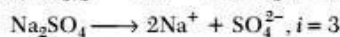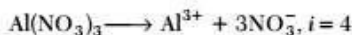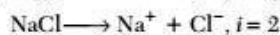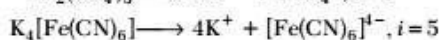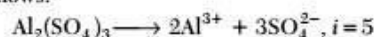
84. *(b)* Net reaction occuring in a Daniell cell is

$$Zn(s) + Cu^{2+}(aq) \longrightarrow Zn^{2+}(aq) + Cu(s)$$

$$\Delta G = -nFE° = -2 \times 96500 \times 1.1$$
$$= -212300 \text{ J mol}^{-1} = -212.3 \text{ kJ mol}^{-1}$$

85. *(b)* The cathode reaction in dry cell is

$$MnO_2 + NH_4^+ + e^- \longrightarrow MnO(OH) + NH_3$$

86. *(a)* $\lambda°_{m(NH_4OH)} = \lambda°_{m(NH_4Cl)} + \lambda°_{m(NaOH)} - \lambda°_{m(NaCl)}$

$$= 130 + 248 - 126.5 = 251.5 \ \Omega^{-1}\text{cm}^2 \text{ mol}^{-1}$$

87. *(b)* Let the order of reaction w.r.t. A is x and w.r.t. B is y.

$$r_1 = k[A]^x[B]^y \qquad \text{...(i)}$$
$$r_2 = k[2A]^x[B]^y \qquad \text{...(ii)}$$
$$r_3 = k[A]^x[2B]^y \qquad \text{...(iii)}$$

$$\frac{r_1}{r_2} = \frac{k[A]^x[B]^y}{k[2A]^x[B]^y} \Rightarrow \frac{1}{4} = \left(\frac{1}{2}\right)^x$$

$$\Rightarrow \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^x$$

$$\Rightarrow \qquad x = 2$$

Similarly,

$$\frac{r_1}{r_3} = \frac{k[A]^x[B]^y}{k[A]^x[2B]^y} \Rightarrow 1 = \left(\frac{1}{2}\right)^y$$

$$\Rightarrow \qquad \left(\frac{1}{2}\right)^0 = \left(\frac{1}{2}\right)^y$$

$$y = 0$$

Hence, the rate of equation is

$$\text{Rate} = k[A]^2[B]^0$$

Order of reaction $= 2$

88. *(d)* The reactions taking place in zeolites depend upon the size and shape of the reactant and product molecules and upon the pores and cavities of the zeolities. Hence, they are shape-selective catalyst.

89. *(b)* Sulphur sol is a type of multimolecular colloid as it consists of particles containing a thousand or more of S_8 sulphur molecules.
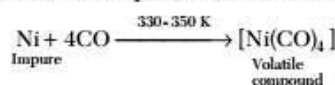
90. *(d)* Chemical formula of magnetite is Fe_3O_4.

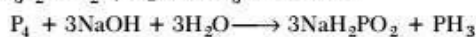The IUPAC name is iron (II, III) oxide and common chemical name is ferrous-ferric oxide.

91. *(c)* The overall reduction reaction of fused matrix is

$$2Al_2O_3 + 3C \longrightarrow 4Al + 3CO_2$$

92. *(a)* Nickel on heating in a stream of carbon monoxide forms a volatile complex, nickel tetracarbonyl.

$$\underset{\text{Impure}}{Ni} + 4CO \xrightarrow{330\text{-}350 \text{ K}} \underset{\substack{\text{Volatile} \\ \text{compound}}}{[Ni(CO)_4]}$$

93. *(a)* White phosphorus (P_4) with water do not produce PH_3, while with NaOH, $Ca_3P_2 + HCl$ and ($Ca_3P_2 + H_2O$) it gives PH_3 as follows :

$$P_4 + 3NaOH + 3H_2O \longrightarrow 3NaH_2PO_2 + PH_3$$
$$Ca_3P_2 + 6HCl \longrightarrow 3CaCl_2 + 2PH_3$$
$$Ca_3P_2 + 6H_2O \longrightarrow 3Ca(OH)_2 + 2PH_3$$

94. *(d)* In oxides of halogen, the bonds are mainly covalent due to small difference in electronegativity between

the halogens and oxygen. The bond polarity, however, increases as we move from F to I.

The stability of oxides of iodine is greater than those of chlorine, while bromine oxides are the least stable. Iodine-oxygen bond is stable due to greater polarity of the bond, while the stability of the chlorine-oxygen bond is due to multiple bond formation involving d-orbitals of the chlorine atom. Bromine being in between lacks both these characteristics. Thus, the stability of oxides of halogens decreases in the following order

$$I > Cl > Br > F$$

95. (b) Nitrogen does not show property of catenation. Since, N—N single bond is very weak due to large interelectronic repulsions between the lone pairs of electrons present on the N-atoms of N—N bond having small bond length.

96. (d) Hybridisation = number of σ bond + lone pair + coordinate bond

$$= 5 + 1 + 0 = 6\,(sp^3d^2)$$
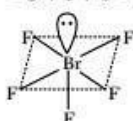
Distarted octahedral
or square pyramidal

97. (d) Group 16 elements are called chalcogens.
∴ Sulphur belongs to chalcogen family.

98. (d) More number of oxidation state are exhibited by the actinoids, than by lanthanoids. The main reason for this is lesser energy difference between $5f$ and $6d$-orbitals, than that between $4f$ and $5d$-orbitals.

99. (b) Outer electronic configuration of the element with atomic number 24 is $3d^5\,4s^1$.
Number of unpaired electrons = 6

$$\mu = \sqrt{n(n+2)}\ \text{BM} = \sqrt{6(6+2)} = \sqrt{48}$$
$$= 6.928 = 6.93\ \text{BM}$$

100. (b) The π-bonding type between metal and oxygen is $d\pi$-$p\pi$ bonding. (d-orbital of Mn and p-orbital of oxygen)

sp^3-hybridisation
(Tetrahedral)
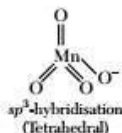
101. (c) The similarity in the size of ions of the elements belonging to the same group of the second and third transition series is due to the effect of lanthanoid contraction. Thus, Zr^{4+} and Hf^{4+} has same size.

102. (d) For a $3d^2$ system, electrons will go into more stable t_{2g} orbitals and each electron going into a t_{2g} orbital is

stabilised by $0.4\Delta_o$ i.e., 2 electrons will be stabilised by $0.8\Delta_o$ or $8 \times 10^{-1}\Delta_o$.
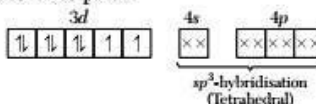
103. (b) $_{28}$Ni $= 3d^8 4s^2$
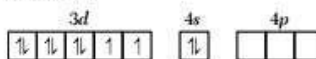
In $[Ni(CO)_4]$, CO is a strong field ligand which causes pairing of electrons.

sp^3-hybridisation
(Tetrahedral)
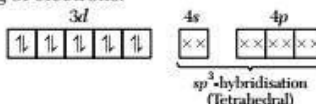
In $[Ni(CN)_4]^{2-}$, CN^- is a strong field ligand which causes pairing of electrons.


dsp^2-hybridisation
(Square planar)
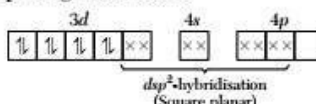
In $[NiCl_4]^{2-}$, Cl^- is a weak field ligand, so no pairing of electrons takes place.
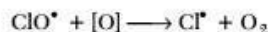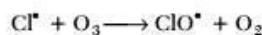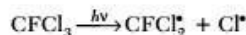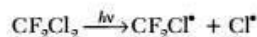

sp^3-hybridisation
(Tetrahedral)

104. (c) The formula for sodium trioxalatoaluminate (II) is $Na_3[Al(C_2O_4)_3]$.

105. (c) Octahedral complex of general formula $[M(AA)_2 a_2]^{n\pm}$ show optical isomerism. *Trans* form of $[M(AA)_2 a_2]^{n\pm}$ does not show optical isomerism due to symmetry.
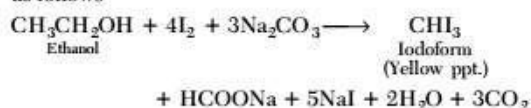

cis-d-isomer   Mirror   *cis*-l-isomer
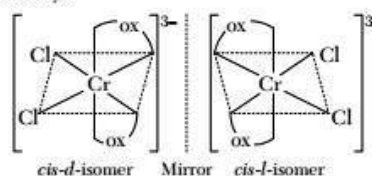
cis-$[Co(NH_3)_4Cl_2]^+$ and *trans*-$[Co(NH_3)_4Cl_2]^+$ do not show optical isomerism due to symmetry.

106. (a) Freons are chlorofluorocarbons introduced into the atmosphere from aerosol sprays and refrigerating equipments. They have a very long life time and when they reach stratosphere, they undergo photochemical decomposition and destroy ozone by the following sequence of reactions.

$$CF_2Cl_2 \xrightarrow{h\nu} CF_2Cl^\bullet + Cl^\bullet$$
$$CFCl_3 \xrightarrow{h\nu} CFCl_2^\bullet + Cl^\bullet$$
$$Cl^\bullet + O_3 \longrightarrow ClO^\bullet + O_2$$
$$ClO^\bullet + [O] \longrightarrow Cl^\bullet + O_2$$

107. (b) Ethanol gives iodoform test to produce yellow ppt as follows

$$CH_3CH_2OH + 4I_2 + 3Na_2CO_3 \longrightarrow CHI_3$$
Ethanol   Iodoform
                                 (Yellow ppt.)

$$+ HCOONa + 5NaI + 2H_2O + 3CO_2$$

108. (c) Due to $-I$ and $-R$-effects of $-NO_2$ group, nitrobenzoic acids are stronger acids, than benzoic acid Due to *ortho*-effect, *o*-nitrobenzoic acid is the strongest acid.

In *p*-nitrobenzoic acid both $-R$-effect and $-I$-effect of the $-NO_2$ group increase the acidity, while in *m*-nitrobenzoic acid only the weaker $-I$-effect increases the acidity.

Hence, the order of acidic strength is

o-nitrobenzoic acid > *p*-nitrobenzoic acid
                    > *m*-nitrobenzoic acid > benzoic acid.

109. (a) Acid strength increases as the electronegativity of halogen increases because the anion formed after H^+ ion release, is stabilised by $-I$-effect.

The order of $-I$-effect is F > Cl > Br > I.

∴ Order of acidic strength is as follows :

$FCH_2COOH > ClCH_2COOH > BrCH_2COOH$

110. (b) A mixture of methylamine (1° amine) and dimethylamine (2° amine) can KOH be separated with the help of $C_6H_5SO_2Cl$ and (Hinsberg's method). Methylamine will form N-methylbenzene - sulphonamide, which is soluble in KOH.

Dimethylamine will form N, N-dimethylbenzenesulphonamide which is insoluble in KOH.

111. (c) Gabriel phthalimide synthesis is used for the preparation of pure aliphatic and alkyl primary amines.

112. (c) Aniline is less basic than ammonia and aliphatic amines due to delocalisation of lone pair of electrons on the benzene ring.

Aliphatic amines are stronger bases, than ammonia due to $+I$-effect of alkyl group. More the number of alkyl groups stronger is the base.

Hence, basic strength will follow the following order

$(C_2H_5)_2NH > C_2H_5NH_2 > NH_3 > C_6H_5NH_2$

113. (a) In RNA molecule, the base uracil (U) appears opposite to base adenine (A), cytosine (C) appears opposite to guanine (G), adenine (A) appears opposite to thymine (T) and guanine (G) appears opposite to cytosine (C).

The sequence of RNA is CGUAU.

114. (b) α-helix and β-pleated structures of proteins are classified as secondary structure.

115. (b) Glycosidic linkages are present in carbohydrates.

116. (c) Glucose and ribose are monosaccharides.

117. (d) Nylon-6, 6 is an example of step growth polymerisation as monomers adipic acid and hexamethylenediamine are bifunctional and form bond with the loss of simple molecules. The dimer formed also contains two functional groups, thus undergoes a series of condensations in a stepwise manner.

118. (a) Glycine and aminocaproic acid forms nylon-2-nylon-6, which is a biodegradable polymer.

$$nH_2N-CH_2-COOH$$
Glycine
$$+nH_2N-(CH_2)_5-COOH$$
Aminocaproic acid
$$\downarrow -(n-1)H_2O$$

Nylon-2-nylon-6
(Biodegradable)
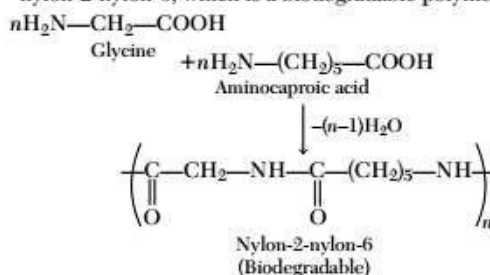
119. (c) Detergents can be made biodegradable and pollution free by using unbranched hydrocarbon chain.

120. (b) Sodium rosinate is generally added in soap because it produces rich lather.

Mathematics

121. (b) Since, $n(A \cap B) = 99$

$$n((A \times B) \cap (B \times A))$$
$$= n((A \cap B) \times (B \cap A))$$
$$= n(A \cap B) \cdot n(B \cap A)$$
$$= n(A \cap B) \cdot n(A \cap B) = 99 \cdot 99 = (99)^2$$

122. (b) We have, $f(x) = e^x$ and $g(x) = \log e^x$

Now, $f(g(x)) = f(\log e^x) = e^{\log e^x} = e^x$

and $g(f(x)) = g(e^x) = \log e^{e^x} = e^x \log e = e^x$

Hence, $f(g(x)) = g(f(x))$.

123. (b) Since, number of one-one onto functions from a set A having n elements to itself is $n!$.

124. (c) Let M, P and C be the set of students who opted Mathematics, Physics and Chemistry respectively.

Then, $n(M) = 100$, $n(P) = 70$, $n(C) = 40$

$n(M \cap P) = 30$, $n(M \cap C) = 28$, $n(P \cap C) = 23$, $n(M \cap P \cap C) = 18$

Number of students who opted Mathematics alone

$$= n(M \cap P' \cap C')$$
$$= n(M \cap (P \cup C)')$$
$$= n(M) - n(M \cap (P \cup C))$$
$$= n(M) - n((M \cap P) \cup (M \cap C))$$
$$= n(M) - n(M \cap P) - n(M \cap C) + n(M \cap P \cap C)$$
$$= 100 - 30 - 28 + 18 = 60$$

125. (c) $n(A) = 4$ (given)

∴ Total number of subsets of $A = 2^4 = 16$

∴ Number of proper subsets $= 16 - 1 = 15$

126. (d) We have, $\tan(x + y) = 33$

$\Rightarrow \dfrac{\tan x + \tan y}{1 - \tan x \tan y} = 33$

$\Rightarrow \dfrac{3 + \tan y}{1 - 3 \tan y} = 33 \qquad [\because x = \tan^{-1} 3 \Rightarrow \tan x = 3]$

$\Rightarrow 3 + \tan y = 33 - 99 \tan y$

$\Rightarrow 100 \tan y = 30$

$\Rightarrow \tan y = \dfrac{3}{10}$

$\Rightarrow y = \tan^{-1} \dfrac{3}{10}$

127. (c) We have, $\cot^{-1}(21) + \cot^{-1}(13) + \cot^{-1}(-8)$

$= \tan^{-1}\left(\dfrac{1}{21}\right) + \tan^{-1}\left(\dfrac{1}{13}\right) + \cot^{-1}(-8)$

$= \tan^{-1}\left(\dfrac{\frac{1}{21} + \frac{1}{13}}{1 - \frac{1}{21} \times \frac{1}{13}}\right) + \cot^{-1}(-8)$

$= \tan^{-1}\left(\dfrac{34}{272}\right) + (\pi - \cot^{-1} 8)$

$\qquad\qquad [\because \cot^{-1}(-x) = \pi - \cot^{-1} x]$

$= \tan^{-1}\left(\dfrac{1}{8}\right) - \cot^{-1} 8 + \pi$

$= \tan^{-1}\left(\dfrac{1}{8}\right) - \tan^{-1}\left(\dfrac{1}{8}\right) + \pi = \pi$

128. (d) $\tan\left(\cos^{-1}\left(\dfrac{1}{5\sqrt{2}}\right) - \sin^{-1}\left(\dfrac{4}{\sqrt{17}}\right)\right)$

$= \tan\left(\dfrac{\pi}{2} - \sin^{-1}\left(\dfrac{1}{5\sqrt{2}}\right) - \sin^{-1}\left(\dfrac{4}{\sqrt{17}}\right)\right)$

$\qquad\qquad \left[\because \cos^{-1} x = \dfrac{\pi}{2} - \sin^{-1} x\right]$

$= \tan\left(\dfrac{\pi}{2} - \left(\sin^{-1}\dfrac{4}{\sqrt{17}} + \sin^{-1}\dfrac{1}{5\sqrt{2}}\right)\right)$

$= \tan\left(\dfrac{\pi}{2} - \sin^{-1}\left(\dfrac{4}{\sqrt{17}} \cdot \sqrt{1 - \dfrac{1}{50}} + \dfrac{1}{5\sqrt{2}} \cdot \sqrt{1 - \dfrac{16}{17}}\right)\right)$

$= \tan\left(\dfrac{\pi}{2} - \sin^{-1}\left(\dfrac{4}{\sqrt{17}} \cdot \dfrac{7}{5\sqrt{2}} + \dfrac{1}{5\sqrt{2}} \cdot \dfrac{1}{\sqrt{17}}\right)\right)$

$= \cot\left(\sin^{-1}\dfrac{29}{5\sqrt{2} \cdot \sqrt{17}}\right) \quad \left[\because \tan\left(\dfrac{\pi}{2} - x\right) = \cot x\right]$

$= \cot\left(\cot^{-1}\dfrac{3}{29}\right) = \dfrac{3}{29}$

129. (b) With the help of group of $\cot x$, we know that

When $x \in \left(0, \dfrac{\pi}{2}\right] \cup \left(\pi, \dfrac{3\pi}{2}\right]$, then $\cot x \geq 0$

∴ $\qquad |\cot x| = \cot x + \dfrac{1}{\sin x}$

$\Rightarrow \cot x = \cot x + \dfrac{1}{\sin x} \Rightarrow \dfrac{1}{\sin x} = 0$

$\Rightarrow$ No solution exist. $\qquad\qquad\qquad$ ...(i)

When $x \in \left(\dfrac{\pi}{2}, \pi\right) \cup \left(\dfrac{3\pi}{2}, 2\pi\right)$, then $\cot x < 0$

$\qquad |\cot x| = \cot x + \dfrac{1}{\sin x}$

∴ $\qquad -\cot x = \cot x + \dfrac{1}{\sin x}$

$\Rightarrow \qquad -2\cot x = \dfrac{1}{\sin x}$

$\Rightarrow \qquad \cos x = \dfrac{-1}{2} \Rightarrow x = \dfrac{2\pi}{3} \qquad$ ...(ii)

∴ From Eqs. (i) and (ii), only one solution exists.

130. (d) $\dfrac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x}$

$= \dfrac{2 \sin\left(\dfrac{x - 3x}{2}\right) \cos\left(\dfrac{x + 3x}{2}\right)}{-(\cos^2 x - \sin^2 x)}$

$= \dfrac{2 \cos 2x \sin(-x)}{-\cos 2x} = \dfrac{-2 \sin x \cos 2x}{-\cos 2x} = 2\sin x$

131. (b) $\operatorname{cosec} A(\sin B \cos C + \cos B \sin C)$

$= \operatorname{cosec} A \sin(B + C)$

$= \operatorname{cosec} A \sin(180° - A)$

$\qquad [\because A, B \text{ and } C \text{ are angles of a triangle}]$

$= \dfrac{1}{\sin A} \sin A = 1$

132. (c) Let $z = i \sin \dfrac{\pi}{19}$

or $\qquad z = 0 + i \sin \dfrac{\pi}{19}$

Let amplitude of z be θ, then

$\sin \theta = \dfrac{\sin \dfrac{\pi}{19}}{\sqrt{0^2 + \left(\sin \dfrac{\pi}{19}\right)^2}} \qquad \left[\because \sin \theta = \dfrac{b}{\sqrt{a^2 + b^2}}\right]$

or $\sin \theta = \dfrac{\sin \dfrac{\pi}{19}}{\sin \dfrac{\pi}{19}} = 1 \Rightarrow \theta = \dfrac{\pi}{2}$

133. (a) To form a triangle, 3 line segments are needed. We can choose 3 line segment from 5 line segments in $^5C_3$ ways. We know that in a triangle, sum of two sides is always greater than third side.

So, line segments of length (2, 3, 5), (2, 3, 6) and (2, 4, 6) do not form any triangle.

∴ Total number of triangles formed = $^5C_3 - 3$.

134. *(b)* Since, the number is to be greater than 1000000 it must be of seven digits and should begin with either 2 or 3 or 4.

Number of numbers formed $= \dfrac{6 \times 6!}{2! \times 3!} = 360$

135. *(d)* We have,

$$\dfrac{1}{\sqrt{(3x+1)}} \left\{ \left(\dfrac{1+\sqrt{3x+1}}{2} \right)^7 - \left(\dfrac{1-\sqrt{3x+1}}{2} \right)^7 \right\}$$

$$= \dfrac{1}{2^7 \sqrt{(3x+1)}} \{ (1+\sqrt{(3x+1)})^7 - (1-\sqrt{(3x+1)})^7 \}$$

... (i)

Now, $(1+\sqrt{3x+1})^7 - (1-\sqrt{3x+1})^7$

$[^7C_0 + {}^7C_1(\sqrt{3x+1}) + {}^7C_2(\sqrt{3x+1})^2 +$

$..... + {}^7C_7(\sqrt{3x+1})^7]$

$-[^7C_0 - {}^7C_1\sqrt{3x+1} + {}^7C_2(\sqrt{3x+1}))^2$

$.... - {}^7C_7 (\sqrt{3x+1})^7]$

$= 2[{}^7C_1(\sqrt{3x+1}) + {}^7C_3(\sqrt{3x+1})^3$

$+ {}^7C_5(\sqrt{3x+1})^5 + {}^7C_7(\sqrt{3x+1})^7]$

$= 2\sqrt{3x+1} \times [7 + 35(3x+1)$

$+ 21(3x+1)^2 + (3x+1)^3]$

Now, putting above value in Eq. (i), so the given expression becomes

$$\dfrac{1}{2^6}[42 + 105x + 21(3x+1)^2 + (3x+1)^3]$$

So, degree of x in given expression is 3.

136. *(d)* We have, $(1+px)^n$

∴ $T_{r+1} = {}^nC_r(1)^{n-r}(px)^r \Rightarrow T_{r+1} = {}^nC_r p^r x^r$

Coefficient of $x^r = {}^nC_r p^r$

Now, coefficient of $x = {}^nC_1 p = 8$ (put $r=1$)

$\Rightarrow \qquad np = 8$ ... (i)

Also, coefficient of $x^2 = {}^nC_2 p^2 = 24$ (put $r=2$)

$\dfrac{n(n-1)}{2} p^2 = 24$

$\Rightarrow \quad \dfrac{n(n-1)}{2} \left(\dfrac{8}{n} \right)^2 = 24$ (from Eq. (i))

$\Rightarrow \quad \dfrac{64(n-1)}{2n} = 24 \Rightarrow 4(n-1) = 3n \Rightarrow n = 4$

From Eq. (i), we get $p = 2$

137. *(b)* We have,

$11^3 + 12^3 + 13^3 + ... + 20^3$

$= (1^3 + 2^3 + ... + 10^3 + 11^3 + 12^3 + ... + 20^3)$
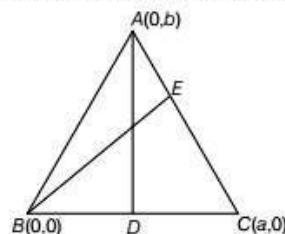
$- (1^3 + 2^3 + ... + 10^3)$

$= \Sigma(20)^3 - \Sigma(10)^3$

$= \left(\dfrac{20(20+1)}{2} \right)^2 - \left(\dfrac{10(10+1)}{2} \right)^2$

$= (10 \times 21)^2 - (5 \times 11)^2 = (210-55)(210+55)$

$= 155 \times 265 = 41075 = 5 \times 8215$

138. *(d)* We have, BE and AD are the medians. So, E and D are the mid-points of AC and BC respectively.

∴ Coordinates of $E = \left(\dfrac{a}{2}, \dfrac{b}{2} \right)$

and coordinates of $D = \left(\dfrac{a}{2}, 0 \right)$

Now, slope of median $BE = m_1 = \dfrac{b}{a}$

Also, slope of median $AD = m_2 = \dfrac{-2b}{a}$

Now, m_1 and m_2 are perpendicular if $m_1 m_2 = -1$

$\Rightarrow \dfrac{b}{a} \times \dfrac{(-2b)}{a} = -1 \Rightarrow 2b^2 = a^2 \Rightarrow a = \pm\sqrt{2}b$

139. *(c)* Given, equation of lines are

$$\dfrac{x}{a} + \dfrac{y}{b} = 1 \qquad ... (i)$$

and $\qquad \dfrac{x}{b} + \dfrac{y}{a} = 1 \qquad ... (ii)$

$\Rightarrow \qquad bx + ay = ab \qquad ... (iii)$

and $\qquad ax + by = ab \qquad ... (iv)$

Solving Eqs. (iii) and (iv), we get

$(a^2 - b^2)y = a^2 b - ab^2 = ab(a-b) \Rightarrow y = \dfrac{ab}{a+b}$

Substituting the value of y in Eq. (iii), we get

$bx + a\left(\dfrac{ab}{a+b} \right) = ab \Rightarrow bx = ab - \dfrac{a^2 b}{a+b}$

$\Rightarrow bx = \dfrac{ab^2}{a+b} \Rightarrow x = \dfrac{ab}{a+b}$

∴ Point of intersection is $\left(\dfrac{ab}{a+b}, \dfrac{ab}{a+b} \right)$.

Since, equation of the line passing through origin is $y = mx$
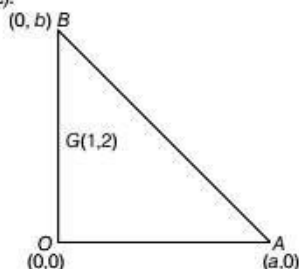
∴ When it passes through $\left(\dfrac{ab}{a+b}, \dfrac{ab}{a+b} \right)$, then we get

$m = 1$

Hence, required equation of line is $y - x = 0$.

140. (b) Since, straight line meets the coordinate axes at A and B, so equation of line intercept form is $\dfrac{x}{a} + \dfrac{y}{b} = 1$.

Let G represents the centroid of ΔOAB. Given that $G = (1, 2)$.

$\therefore$ Centroid $G\left(\dfrac{0 + a + 0}{3}, \dfrac{0 + 0 + b}{3}\right) = (1, 2)$ (given)

$\Rightarrow \dfrac{a}{3} = 1 \Rightarrow a = 3, \dfrac{b}{3} = 2 \Rightarrow b = 6$

Hence, required equation of line is

$\dfrac{x}{3} + \dfrac{y}{6} = 1 \Rightarrow 2x + y = 6$

141. (a) Let DR's of line are $< a, b, c >$, then

$$2a - 3b + c = 0 \quad \text{and} \quad a + 2b - 2c = 0$$

$\Rightarrow \quad \dfrac{a}{6 - 2} = \dfrac{b}{1 + 4} = \dfrac{c}{4 + 3} \Rightarrow \dfrac{a}{4} = \dfrac{b}{5} = \dfrac{c}{7}$

DR's of line are $<4, 5, 7>$.

142. (a) Let the line makes an angle x with positive direction of Z-axis. Then, we have

$$\cos^2 45^\circ + \cos^2 60^\circ + \cos^2 x = 1$$

$\Rightarrow \quad \left(\dfrac{1}{\sqrt{2}}\right)^2 + \left(\dfrac{1}{2}\right)^2 + \cos^2 x = 1$

$\Rightarrow \quad \cos^2 x = 1 - \dfrac{1}{2} - \dfrac{1}{4}$

$\Rightarrow \quad \cos^2 x = \dfrac{1}{4} \Rightarrow \cos x = \dfrac{1}{2}$

Neglecting $\cos x = \dfrac{-1}{2}$, because line is in I-octant)

$\Rightarrow \quad x = 60^\circ$

143. (d) $L_1 : \dfrac{x - 3}{3} = \dfrac{y - 8}{-1} = \dfrac{z - 3}{1}$

$L_2 : \dfrac{x + 3}{-3} = \dfrac{y + 7}{2} = \dfrac{z - 6}{4}$

Shortest distance between two lines

$$= \left| \dfrac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1 c_2 - b_2 c_1)^2 + (c_1 a_2 - c_2 a_1)^2 + (a_1 b_2 - a_2 b_1)^2}} \right|$$

$$= \dfrac{\begin{vmatrix} -3 - 3 & -7 - 8 & 6 - 3 \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix}}{\sqrt{(-4 - 2)^2 + (-3 - 12)^2 + (6 - 3)^2}}$$

$$= \dfrac{\begin{vmatrix} -6 & -15 & 3 \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix}}{\sqrt{(-6)^2 + (-15)^2 + (3)^2}}$$ On expanding the determinant along C_3.

$= \dfrac{3(6 - 3) - 1(-12 - 45) + 4(6 + 45)}{\sqrt{36 + 225 + 9}}$

$= \dfrac{9 + 57 + 204}{\sqrt{270}} = \dfrac{270}{\sqrt{270}} = \sqrt{270} = 3\sqrt{30}$

144. (a) We have, $\lim\limits_{x \to \infty} (\sqrt{a^2 x^2 + bx + x} - ax)$

$= \lim\limits_{x \to \infty} \left((\sqrt{a^2 x^2 + bx + x} - ax) \times \dfrac{(\sqrt{a^2 x^2 + bx + x} + ax)}{(\sqrt{a^2 x^2 + bx + x} + ax)} \right)$

$= \lim\limits_{x \to \infty} \left(\dfrac{(a^2 x^2 + bx + x) - a^2 x^2}{x\left\{\sqrt{a^2 + \dfrac{b}{x} + \dfrac{1}{x}} + a\right\}} \right)$

$= \lim\limits_{x \to \infty} \left(\dfrac{(b + 1)x}{x\left\{\sqrt{a^2 + \dfrac{b}{x} + \dfrac{1}{x}} + a\right\}} \right) = \dfrac{b + 1}{2a}$

145. (c) We have, $y = |\cos x| + |\sin x|$

At $\quad x = \dfrac{2\pi}{3}$, $\cos x$ is negative and $\sin x$ is positive.

$\therefore \quad y = -\cos x + \sin x$

$\Rightarrow \dfrac{dy}{dx} = \sin x + \cos x$

$\therefore \left.\dfrac{dy}{dx}\right|_{\text{at } x = 2\pi/3} = \sin\left(\dfrac{2\pi}{3}\right) + \cos\left(\dfrac{2\pi}{3}\right)$

$= \dfrac{\sqrt{3}}{2} - \dfrac{1}{2} = \dfrac{\sqrt{3} - 1}{2}$

146. (d) We have, slant height of cone $(l) = 7$ cm

$\Rightarrow \quad l^2 = h^2 + r^2$ ... (i)

$\Rightarrow \quad r^2 = 7^2 - 4^2$ [when $h = 4$ cm]

$\Rightarrow \quad r^2 = 33$

$\Rightarrow \quad r = \sqrt{33}$ cm

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Now, differentiating Eq. (i) w.r.t. l, we get

$$0 = 2h\frac{dh}{dt} + 2r\frac{dr}{dt}$$

$$\Rightarrow \quad \frac{dr}{dt} = -\frac{h}{r}\frac{dh}{dt} \qquad \dots \text{(ii)}$$

Volume of the cone, $V = \frac{1}{3}\pi r^2 h$

$$\frac{dV}{dt} = \frac{1}{3}\pi\left[2rh\frac{dr}{dt} + r^2\frac{dh}{dt}\right]$$

$$\Rightarrow \frac{dV}{dt} = \frac{1}{3}\pi\left[2rh\left(\frac{-h}{r}\frac{dh}{dt}\right) + r^2\frac{dh}{dt}\right] \text{ (from Eq. (ii))}$$

$$\Rightarrow \frac{dV}{dt} = \frac{1}{3}\pi\left[-2h^2\frac{dh}{dt} + r^2\frac{dh}{dt}\right]$$

$$\Rightarrow \frac{dV}{dt} = \frac{1}{3}\pi\frac{dh}{dt}[-2h^2 + r^2]$$

$$\Rightarrow \left[\frac{dV}{dt}\right]_{h=4} = \frac{1}{3}\pi(0.3)[-2\times 4^2 + (\sqrt{33})^2]$$

$$= \frac{\pi}{10}[1] = \frac{\pi}{10} \text{ cc/sec}$$

147. *(b)* Let AB be the ladder of length 5 m.

We are given, $\dfrac{dx}{dt} = 2\,\text{m/sec}$
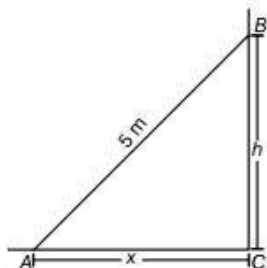
In $\triangle ABC$,

$$AB^2 = AC^2 + BC^2$$

$$\Rightarrow \quad (5)^2 = x^2 + h^2 \qquad \dots \text{(i)}$$

Differentiating Eq. (i) w.r.t. t, we get

$$0 = 2x\frac{dx}{dt} + 2h\frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = \frac{-x}{h}\frac{dx}{dt} \qquad \dots \text{(ii)}$$

Now, from Eq. (i), when $x = 4$

$$h^2 = 25 - 16$$

$$\Rightarrow h^2 = 9 \Rightarrow h = 3 \text{ m}$$

From Eq. (ii), we get $\left[\dfrac{dh}{dt}\right] = \dfrac{-4}{3}\times 2 = \dfrac{-8}{3}$

$\because$ The negative sign shows the height decreases and decreasing rate is $\dfrac{8}{3}$ m/sec.

148. *(b)* We have, $\qquad y^2 = 4ax \qquad \dots \text{(i)}$

and $\qquad\qquad\qquad ay = 2x^2 \qquad \dots \text{(ii)}$

Solving Eqs. (i) and (ii), we get

$$x = a \text{ and } y = 2a$$

Differentiating Eq. (i) w.r.t. x, we get

$$2y\frac{dy}{dx} = 4a$$

Now, $\qquad \left[\dfrac{dy}{dx}\right]_{(a,\,2a)} = \dfrac{4a}{2(2a)} = 1$

$$\Rightarrow \qquad m_1 = 1$$

Now, differentiating Eq. (ii) w.r.t. x, we get

$$a\frac{dy}{dx} = 4x \Rightarrow \left[\frac{dy}{dx}\right]_{(a,\,2a)} = \frac{4a}{a} = 4$$

$$\Rightarrow \qquad m_2 = 4$$

$\because$ Angle between curves is equal to angle between their tangents.

$$\tan\theta \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| \Rightarrow \tan\theta = \left|\frac{4 - 1}{1 + 4\times(1)}\right|$$

$$\Rightarrow \quad \tan\theta = \frac{3}{5} \Rightarrow \theta = \tan^{-1}\left(\frac{3}{5}\right)$$
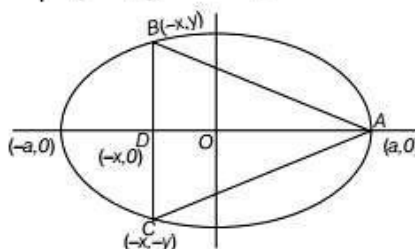
149. *(b)* Let $\triangle ABC$ be an isosceles triangle inscribed in the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

Let coordinate of A be $(a, 0)$.

Area of $\triangle ABC$, $A = \dfrac{1}{2}\times AD \times BC$

$$= \frac{1}{2}\times(a + x)\times 2y = y(a + x)$$

$$= \sqrt{b^2\left(1 - \frac{x^2}{a^2}\right)}(a + x) = \frac{b}{a}\sqrt{a^2 - x^2}(a + x)$$

$$\Rightarrow \frac{dA}{dx} = \frac{b}{a}\left[(a + x)\frac{1}{2}\times\frac{-2x}{\sqrt{a^2 - x^2}} + \sqrt{a^2 - x^2}\right]$$

$$= \frac{b}{a}\left[\frac{-x(a + x) + (a^2 - x^2)}{\sqrt{a^2 - x^2}}\right]$$

Put $\dfrac{dA}{dx} = 0$

$$\Rightarrow \qquad 2x^2 + ax - a^2 = 0$$

$$\Rightarrow \quad 2x^2 + 2ax - ax - a^2 = 0$$

$$\Rightarrow \qquad (2x - a)(x + a) = 0$$

$$\Rightarrow \qquad x = \frac{a}{2}, -a$$

Since, $\dfrac{d^2A}{dx^2} < 0$, for $x = \dfrac{a}{2}$

$\therefore$ Maximum area $= \dfrac{b}{a} \cdot \left(a + \dfrac{a}{2}\right)\sqrt{a^2 - \dfrac{a^2}{4}}$

$= \dfrac{b}{a} \cdot \dfrac{3}{2}(a)\dfrac{\sqrt{3}\,a}{2} = \dfrac{3\sqrt{3}}{4}\,ab$ sq units

150. (b) We have, $f(x + y) = f(x)\,f(y)$

Putting $x = y = 0$

$\Rightarrow \qquad f(0 + 0) = f(0)\,f(0)$

$\Rightarrow \qquad\qquad f(0) = [f(0)]^2$

$\Rightarrow \qquad\qquad f(0) = 1$

Now, $f'(5) = \lim\limits_{h \to 0} \dfrac{f(5 + h) - f(5)}{h}$

$= \lim\limits_{h \to 0} \dfrac{f(5)\,f(h) - f(5)}{h} = \lim\limits_{h \to 0} f(5)\left[\dfrac{f(h) - 1}{h}\right]$

$= f(5) \lim\limits_{h \to 0}\left[\dfrac{f(h) - f(0)}{h}\right] \qquad [\because f(0) = 1]$

$= 2 \times f'(0) = 2 \times 3 = 6$

151. (d) We have, $f(x) = \log_{x^2}(\log x)$

$\Rightarrow \qquad f(x) = \dfrac{1}{2}[\log_x(\log x)] \qquad \left[\because \log_{a^n} b = \dfrac{1}{n}\log_a b\right]$

$\Rightarrow \qquad f(x) = \dfrac{1}{2}\left[\dfrac{\log(\log x)}{\log x}\right] \qquad \left(\because \log_a b = \dfrac{\log b}{\log a}\right)$

$\Rightarrow$

$f'(x) = \dfrac{1}{2}\left[\dfrac{\log(x)\dfrac{d}{dx}[\log(\log(x))] - \log(\log x)\dfrac{d}{dx}\log x}{(\log x)^2}\right]$

$= \dfrac{1}{2}\left[\dfrac{\log(x)\dfrac{1}{\log x}\times\dfrac{1}{x} - \left(\log(\log x)\dfrac{1}{x}\right)}{(\log x)^2}\right]$

$= \dfrac{1}{2}\left[\dfrac{\dfrac{1}{x} - \dfrac{\log(\log(x))}{x}}{(\log x)^2}\right]$

$\Rightarrow \qquad f'(x) = \dfrac{1}{2}\left[\dfrac{1 - \log\log(x)}{x(\log x)^2}\right]$

$\Rightarrow \qquad f'(e) = \dfrac{1}{2}\left[\dfrac{1 - \log\log(e)}{e(\log e)^2}\right] = \dfrac{1}{2}\left[\dfrac{1 - 0}{e}\right] = \dfrac{1}{2e}$

152. (b) $f(x) = \log(\sin(x))$ is continuous in $\left[\dfrac{\pi}{6}, \dfrac{5\pi}{6}\right]$ and

differentiable in $\left(\dfrac{\pi}{6}, \dfrac{5\pi}{6}\right)$ as its derivative

$f'(x) = \dfrac{1}{\sin x}\cos x = \cot x$

Now, $f\left(\dfrac{\pi}{6}\right) = \log\left(\sin\left(\dfrac{\pi}{6}\right)\right) = \log\left(\dfrac{1}{2}\right)$

$f\left(\dfrac{5\pi}{6}\right) = \log\sin\left(\dfrac{5\pi}{6}\right) = \log\sin\left(\pi - \dfrac{\pi}{6}\right) = \log\left(\dfrac{1}{2}\right)$

Now, according to Lagrange's mean value theorem, there exists a point $c \in \left(\dfrac{\pi}{6}, \dfrac{5\pi}{6}\right)$ such that

$f'(c) = \dfrac{f\left(\dfrac{\pi}{6}\right) - f\left(\dfrac{5\pi}{6}\right)}{\left(\dfrac{\pi}{6} - \dfrac{5\pi}{6}\right)} \Rightarrow \cot c = 0 \quad \Rightarrow c = \dfrac{\pi}{2}$

153. (c) Let $I = \int\limits_{-1}^{1}(x^{27}\cos x + e^x)\,dx$

$\Rightarrow \qquad I = \int\limits_{-1}^{1}x^{27}\cos x\,dx + \int\limits_{-1}^{1}e^x\,dx$

Since, $f(x) = x^{27}\cos x$ is an odd function.

$\left\{\because \int\limits_{-a}^{a}f(x)\,dx = 0 \text{ if } f(x) \text{ is odd function}\right\}$

So, $\qquad I = 0 + [e^x]_{-1}^{1} = e - \dfrac{1}{e}$

154. (c) Let $I = \int\limits_{0}^{\pi}\dfrac{x\sin^{2n}x}{\sin^{2n}x + \cos^{2n}x}\,dx$ ... (i)

$\Rightarrow \qquad I = \int\limits_{0}^{\pi}\dfrac{(\pi - x)\sin^{2n}(\pi - x)}{\sin^{2n}(\pi - x) + \cos^{2n}(\pi - x)}\,dx$

$= \int\limits_{0}^{\pi}\dfrac{(\pi - x)\sin^{2n}x}{\sin^{2n}x + \cos^{2n}x}\,dx$ ... (ii)

Adding Eqs. (i) and (ii), we get

$2I = \pi\int\limits_{0}^{\pi}\dfrac{\sin^{2n}x}{\sin^{2n}x + \cos^{2n}x}\,dx$ ... (iii)

Let $I_1 = \int\limits_{0}^{\pi}\dfrac{\sin^{2n}x}{\sin^{2n}x + \cos^{2n}x}\,dx$

$\Rightarrow \quad I_1 = 2\int\limits_{0}^{\pi/2}\dfrac{\sin^{2n}x}{\sin^{2n}x + \cos^{2n}x}\,dx$ ... (iv)

$\Rightarrow \quad I_1 = 2\int\limits_{0}^{\pi/2}\dfrac{\sin^{2n}\left(\dfrac{\pi}{2} - x\right)}{\sin^{2n}\left(\dfrac{\pi}{2} - x\right) + \cos^{2n}\left(\dfrac{\pi}{2} - x\right)}\,dx$

$= 2\int\limits_{0}^{\pi/2}\dfrac{\cos^{2n}x}{\cos^{2n}x + \sin^{2n}x}\,dx$ ... (v)

Adding Eqs. (iv) and (v), we get

$2I_1 = 2\int\limits_{0}^{\pi/2}1\,dx = 2\cdot\dfrac{\pi}{2} = \pi$

$\Rightarrow \qquad I_1 = \dfrac{\pi}{2}$

Substituting the value of I_1 in Eq. (iii), we get

$2I = (\pi)\left(\dfrac{\pi}{2}\right) \Rightarrow I = \dfrac{\pi^2}{4}$

155. (b) Let

$$I = \int \frac{x^3 - 1}{x^3 + x} dx = \int \left(1 - \frac{x+1}{x^3 + x}\right) dx$$

$$= \int 1 dx - \int \frac{x+1}{x(x^2+1)} dx = x - \int \frac{x+1}{x(x^2+1)} dx \ \dots \text{(i)}$$

Now, $\dfrac{x+1}{x(x^2+1)} = \dfrac{A}{x} + \dfrac{Bx+C}{x^2+1}$

(By using partial fractions)

$$\Rightarrow x + 1 = A(x^2+1) + (Bx+C)x$$
$$\Rightarrow x + 1 = (A+B)x^2 + Cx + A$$

Comparing coefficients of x^2, x and constant, we get
$$A + B = 0, C = 1, A = 1 \Rightarrow B = -1$$

$\therefore$ From Eq. (i), we get

$$I = x - \int \frac{1}{x} dx - \int \frac{1-x}{x^2+1} dx$$

$$= x - \log x - \int \frac{1}{x^2+1} dx + \frac{1}{2}\int \frac{2x}{x^2+1} dx$$

$$= x - \log x - \tan^{-1}x + \frac{1}{2}\log(x^2+1) + c$$

156. (c)   LHS $= \int \dfrac{\cos 8x + 1}{\tan 2x - \cot 2x} dx$

$$= \int \frac{2\cos^2 4x}{\left(\dfrac{\sin^2 2x - \cos^2 2x}{\sin 2x \cos 2x}\right)} dx$$

$$[\because \cos 2x = 2\cos^2 x - 1]$$

$$= -\int \frac{\cos^2 4x (2\sin 2x \cos 2x)}{(\cos^2 2x - \sin^2 2x)} dx$$

$$= -\int \frac{\cos^2 4x \times \sin 4x}{\cos 4x} dx$$

$$[\because \sin 2x = 2\sin x \cos x]$$

$$= -\frac{1}{2}\int 2\sin 4x \cos 4x \, dx$$

$$= -\frac{1}{2}\int \sin 8x \, dx = \frac{1}{2} \times \frac{\cos 8x}{8} + c$$

Now, $\dfrac{1}{2} \dfrac{\cos 8x}{8} + c = a\cos 8x + c$

$\therefore \quad a = \dfrac{1}{16}$

157. (d) We have,

$$\int_0^\pi x f(\sin x) \, dx = A \int_0^{\pi/2} f(\sin x) dx$$

Let $\quad I = \int_0^\pi x \cdot f(\sin x) \, dx \qquad \dots \text{(i)}$

$$\Rightarrow \quad I = \int_0^\pi (\pi - x) f(\sin(\pi - x)) dx$$

$$\Rightarrow \quad I = \int_0^\pi (\pi - x) f(\sin x) dx \qquad \dots \text{(ii)}$$
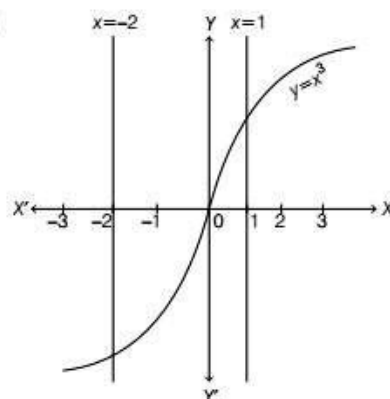
Adding Eqs. (i) and (ii), we get

$$2I = \pi\int_0^\pi f(\sin x)dx \Rightarrow I = \frac{\pi}{2}\int_0^\pi f(\sin x)dx$$

$$\Rightarrow \quad I = \frac{\pi}{2} \times 2\int_0^{\pi/2} f(\sin x)dx$$

$$\Rightarrow \quad I = \pi\int_0^{\pi/2} f(\sin x)dx$$

$\therefore$ On comparing, we get $A = \pi$

158. (d)

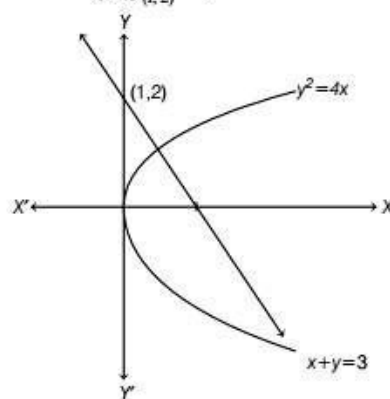Required area $= \left|\int_{-2}^0 x^3 \, dx\right| + \int_0^1 x^3 \, dx$

$$= \left|\left[\frac{x^4}{4}\right]_{-2}^0\right| + \left[\frac{x^4}{4}\right]_0^1 = \frac{16}{4} + \frac{1}{4} = \frac{17}{4} \text{ sq units}$$

159. (a) We have, $y^2 = 4x$

Differentiating w.r.t. x, we get

$$2y\frac{dy}{dx} = 4 \Rightarrow \frac{dy}{dx} = \frac{2}{y}$$

$$\left(\frac{dy}{dx}\right)_{(1,2)} = \frac{2}{2} = 1$$

Equation of normal to the curve at $(1, 2)$ is

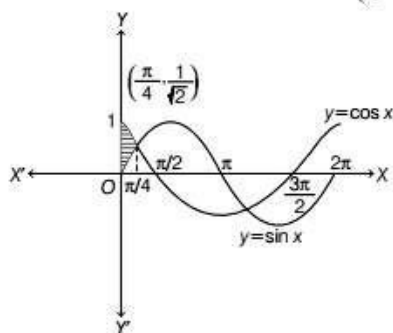$$y - y_1 = \frac{1}{\left(-\dfrac{dy}{dx}\right)}(x - x_1) \quad \Rightarrow \quad (y - 2) = -\frac{1}{1}(x - 1)$$

$\Rightarrow \ y - 2 = -x + 1 \Rightarrow x + y = 3$

The line $x + y = 3$ meets the X-axis at $x = 3$

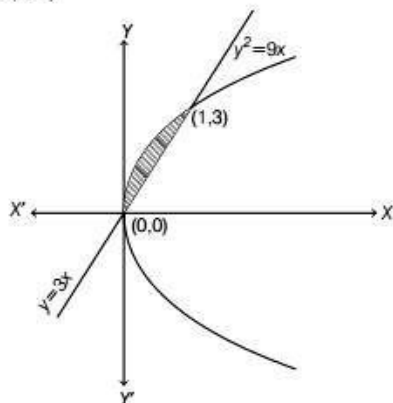$\therefore$ Required area $= \int\limits_0^1 \sqrt{4x}\, dx + \int\limits_1^3 (3 - x)\, dx$

$$= 2\left[\frac{x^{3/2}}{3/2}\right]_0^1 + \left[3x - \frac{x^2}{2}\right]_1^3$$

$$= \frac{4}{3}(1) + \left[9 - \frac{9}{2} - 3 + \frac{1}{2}\right] = \frac{4}{3} + \left(\frac{9}{2} - \frac{5}{2}\right)$$

$$= \frac{4}{3} + \frac{4}{2} = \frac{4}{3} + 2 = \frac{10}{3} \text{ sq units}$$

160. (a) $\sin x$ and $\cos x$ intersects each other at $\left(\dfrac{\pi}{4}, \dfrac{1}{\sqrt{2}}\right)$

$\therefore$ Required area $= \int\limits_0^{\pi/4} \cos x\, dx - \int\limits_0^{\pi/4} \sin x\, dx$

$$= [\sin x]_0^{\pi/4} - [-\cos x]_0^{\pi/4}$$

$$= \frac{1}{\sqrt{2}} - \left[\frac{-1}{\sqrt{2}} + 1\right] = \frac{2}{\sqrt{2}} - 1 = \sqrt{2} - 1$$

161. (c) Intersecting points of $y^2 = 9x$ and $y = 3x$ is $(0, 0)$ and $(1, 3)$.

$\therefore$ Required area $= \int\limits_0^1 (\sqrt{9x} - 3x)\, dx$

$$= 3\int\limits_0^1 (\sqrt{x} - x)\, dx = 3\left[\frac{x^{3/2}}{3/2} - \frac{x^2}{2}\right]_0^1$$

$$= 3\left(\frac{2}{3} - \frac{1}{2}\right) = 3\left(\frac{1}{6}\right) = \frac{1}{2} \text{ sq unit}$$

162. (a) We have, $\int\limits_1^e \log x\, dx$

$$= \left[\log x \int 1\, dx\right]_1^e - \int\limits_1^e \left(\frac{d}{dx}\log x \int 1\, dx\right) dx$$

$$= [\log x \cdot x]_1^e - \int\limits_1^e \frac{1}{x} \cdot x\, dx$$

$$= [e \log e - 1 \log(1)] - \int\limits_1^e 1\, dx$$

$$= e - [x]_1^e = e - [e - 1] = 1$$

163. (a) Given, equation of parabola is $y^2 = 4ax$ ... (i)

Differentiating Eq. (i) w.r.t. x, we get

$$2y\frac{dy}{dx} = 4a$$

$\Rightarrow \qquad \dfrac{dy}{dx} = \dfrac{2a}{y} \Rightarrow a = \dfrac{y}{2}\dfrac{dy}{dx}$

Substituting the value of a in Eq. (i), we get

$$y^2 = 4 \cdot \frac{y}{2}\frac{dy}{dx}x$$

$\Rightarrow \qquad y^2 = 2xy\dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx} = \dfrac{y}{2x}$

164. (b) We have, $\dfrac{dy}{dx} = \dfrac{y + x \tan \dfrac{y}{x}}{x}$ ... (i)

Given, differential equation is in homogeneous form.

$\therefore$ Put $y = vx$ in Eq. (i), we get

$$v + x\frac{dv}{dx} = v + \tan v \Rightarrow \frac{1}{\tan v}\, dv = \frac{dx}{x}$$

Taking integration on both sides, we get

$$\log(\sin v) = \log x + \log c$$

$\Rightarrow \log\dfrac{\sin v}{x} = \log c \Rightarrow \sin\left(\dfrac{y}{x}\right) = xc$

165. (b) Given, differential equation is

$$\frac{dy}{dx} + y \cdot g'(x) = g(x) \cdot g'(x)$$

This is in the form of linear differential equation.

So, $\qquad$ IF $= e^{\int g'(x)\, dx} = e^{g(x)}$

Required solution is

$$y \cdot e^{g(x)} = \int g(x) \cdot g'(x) \cdot e^{g(x)}\, dx + c'$$

Put $g(x) = t \Rightarrow g'(x)\, dx = dt$

$\therefore \quad y \cdot e^{g(x)} = \int t e^t\, dt + c'$

$\Rightarrow y \cdot e^{g(x)} = te^t - e^t + c'$

$\Rightarrow y \cdot e^{g(x)} = (g(x) - 1)e^{g(x)} + c'$

or $(y - g(x) + 1)e^{g(x)} = c'$

Taking log on both sides, we get

$g(x) + \log(1 + y - g(x)) = c$

Where, $c = \log c'$

166. (a) Given, differential equation is $\left(\dfrac{d^2y}{dx^2}\right)^2 - \left(\dfrac{dy}{dx}\right)^3 = y^3$

So, Order $= 2$ and Degree $= 2$

$\therefore$ Order $\times$ Degree $= 2 \times 2 = 4$

167. (c) Let $a = \hat{i} - \hat{j}$ and $b = \hat{i} - 2\hat{j}$

The required vector is along the vector

$a \times (a \times b) = (a \cdot b)a - (a \cdot a)b$

$= 3(\hat{i} - \hat{j}) - 2(\hat{i} - 2\hat{j})$

$= 3\hat{i} - 3\hat{j} - 2\hat{i} + 4\hat{j} = \hat{i} + \hat{j}$

Hence, required vectors are given by

$\pm \dfrac{(\hat{i} + \hat{j})}{\sqrt{1 + 1}} = \pm \dfrac{1}{\sqrt{2}}(\hat{i} + \hat{j})$

168. (a) According to the question,

$a \cdot (b + c) = 0, b \cdot (c + a) = 0, c \cdot (a + b) = 0$

$\Rightarrow \quad a \cdot b + a \cdot c = 0 \qquad \ldots (i)$

$\qquad b \cdot c + b \cdot a = 0 \qquad \ldots (ii)$

$\qquad c \cdot a + c \cdot b = 0 \qquad \ldots (ii)$

Adding Eqs. (i), (ii) and (iii), we get

$2(a \cdot b + b \cdot c + c \cdot a) = 0 \qquad \ldots (iv)$

Now, $|a + b + c|^2$

$= |a|^2 + |b|^2 + |c|^2 + 2(a \cdot b + b \cdot c + c \cdot a)$

$= |a|^2 + |b|^2 + |c|^2$ [using Eq. (iv)]

169. (a) We have, $BA = B$

$\Rightarrow (BA)B = BB \Rightarrow B(AB) = B^2$

$\Rightarrow BA = B^2 \Rightarrow B = B^2$

170. (a) Let $\Delta = \begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$

Applying $R_2 \to 2R_2$, we get

$\Delta = \dfrac{1}{2}\begin{vmatrix} x+2 & x+3 & x+2a \\ 2x+6 & 2x+8 & 2x+4b \\ x+4 & x+5 & x+2c \end{vmatrix}$

Applying $R_2 \to R_2 - (R_1 + R_3)$, we get

$\Delta = \dfrac{1}{2}\begin{vmatrix} x+2 & x+3 & x+2a \\ 0 & 0 & 4b - (2a + 2c) \\ x+4 & x+5 & x+2c \end{vmatrix}$

$\Rightarrow \Delta = \dfrac{1}{2}\begin{vmatrix} x+2 & x+3 & x+2a \\ 0 & 0 & 0 \\ x+4 & x+5 & x+2c \end{vmatrix}$ $\begin{bmatrix} \because a, b, c \text{ are in AP} \\ \therefore 2b = a + c \end{bmatrix}$

$\Rightarrow \Delta = 0$

171. (c) Since, $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$

$\Rightarrow |A| = a \cdot a \cdot a = a^3$

Using formula $|\text{adj } A| = |A|^{m-1}$, we get

$\det(\text{adj } A) = (a^3)^{3-1} = (a^3)^2 = a^6$

172. (c) We have, $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$A^2 = A \cdot A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$A^3 = A^2 \cdot A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

Similarly, $A^8 = \begin{bmatrix} 1 & 0 \\ 8 & 1 \end{bmatrix}$

Now, $A^8 = aA + bI$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 8 & 1 \end{bmatrix} = a\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} + b\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 8 & 1 \end{bmatrix} = \begin{bmatrix} a+b & 0 \\ a & a+b \end{bmatrix}$

$\therefore a = 8$ and $a + b = 1$

$\Rightarrow b = 1 - 8 = -7$

173. (b) Let E_1, E_2 and E_3 be the the events such that drawn ball is white, blue and black respectively.

The required probability

$= {}^3P_1 P(E_1)P(E_1)P(\overline{E})_1 + {}^3P_1 P(E_2) P(E_2)P(\overline{E}_2)$

$\qquad\qquad + {}^3P_1 P(E_3)P(E_3)P(\overline{E}_3)$

$= 3 \times \dfrac{2}{9} \times \dfrac{1}{8} \times \dfrac{7}{7} + 3 \times \dfrac{3}{9} \times \dfrac{2}{8} \times \dfrac{6}{7} + 3 \times \dfrac{4}{9} \times \dfrac{3}{8} \times \dfrac{5}{7}$

$= 3\left[\dfrac{14 + 36 + 60}{9 \times 8 \times 7}\right] = \dfrac{3 \times 110}{9 \times 8 \times 7} = \dfrac{55}{84}$

174. (c) 6 boys and 6 girls sit in a row in 12! ways. Number of ways when all six girls sit together $= 7! \times 6!$

Required probability $= \dfrac{7! \times 6!}{12!} = \dfrac{1}{132}$

175. (c) Total number of outcomes $= 6 \times 6 \times 6 = 216$

When sum of numbers is 5, then outcomes are (1, 1, 3), (1, 2, 2), (1, 3, 1), (2, 1, 2), (2, 2, 1) and (3, 1, 1).

Required probability $= \dfrac{6}{216} = \dfrac{1}{36}$

176. *(c)* Total number of balls = 9

Probability of drawn two white balls

$$\frac{3}{9} \times \frac{2}{8} = \frac{1}{12}$$

177. *(b)* Given, inequality is

$$\frac{6x}{4x-1} < \frac{1}{2} \Rightarrow \frac{12x}{4x-1} < 1$$

$$\Rightarrow \quad \frac{12x}{4x-1} - 1 < 0 \Rightarrow \frac{8x+1}{4x-1} < 0 \qquad \dots \text{(i)}$$

For above inequality Eq. (i), we have two cases either

Case I $8x + 1 > 0, 4x - 1 < 0$

Case II $8x + 1 < 0, 4x - 1 > 0$

From Case (I), we get $x > \dfrac{-1}{8}, x < \dfrac{1}{4}$

$$\therefore \quad x \in \left(-\frac{1}{8}, \frac{1}{4} \right) \qquad \dots \text{(ii)}$$

From Case (II), $x < \dfrac{-1}{8}, x > \dfrac{1}{4}$, both of which can not be true.

$\therefore$ Eq. (ii) represent the solution set of given inequality.

178. *(d)* We have, $S^2 = at^2 + 2bt + c$ $\qquad \dots \text{(i)}$

Differentiating Eq. (i) w.r.t. t, we get

$$2S \frac{dS}{dt} = 2at + 2b$$

$$\Rightarrow \frac{dS}{dt} = \left(\frac{at+b}{S} \right) \qquad \dots \text{(ii)}$$

Differentiating Eq. (ii) w.r.t. t, we get

$$\frac{d^2S}{dt^2} = \frac{S \dfrac{d}{dt}(at+b) - (at+b)\dfrac{dS}{dt}}{S^2}$$

$$\Rightarrow \frac{d^2S}{dt^2} = \frac{aS - (at+b)\dfrac{(at+b)}{S}}{S^2} \qquad \text{[using Eq. (ii)]}$$

$$\Rightarrow \frac{d^2S}{dt^2} = \frac{aS^2 - (at+b)^2}{S^3}$$

$$\Rightarrow \frac{d^2S}{dt^2} = \frac{a(at^2 + 2bt + c) - [(at)^2 + b^2 + 2atb]}{S^3}$$

$$\text{[using Eq. (i)]}$$

$$\Rightarrow \frac{d^2S}{dt^2} = \frac{a^2t^2 + 2abt + ac - a^2t^2 - b^2 - 2atb}{S^3}$$

$$\Rightarrow \frac{d^2S}{dt^2} = \frac{ac - b^2}{S^3}$$

$$\Rightarrow \text{Acceleration} = \left(\frac{d^2S}{dt^2} \right) \propto \frac{1}{S^3}$$

179. *(d)* Data is 6, 7, 10, 12, 13, 4, 8, 12

Here, $n = 8$

$\therefore$ Mean

$$= \frac{6 + 7 + 10 + 12 + 13 + 4 + 8 + 12}{8}$$

$$= \frac{72}{8} = 9$$

Now, variance $= \dfrac{1}{n} \Sigma x_i^2 - (\bar{x})^2$

$$= \frac{1}{8}[(6)^2 + (7)^2 + (10)^2 + (12)^2 + (13)^2$$
$$+ (4)^2 + (8)^2 + (12)^2] - 81$$

$$= \frac{1}{8}[722] - 81 = 90.25 - 81 = 9.25$$

180. *(b)* We have, $\overline{(x + iy)(1 - 2i)} = 1 + i$

$$\Rightarrow \quad \overline{(x + iy)}\overline{(1 - 2i)} = 1 + i$$

$$\Rightarrow \quad (x - iy)(1 + 2i) = 1 + i$$

$$\Rightarrow \quad x - iy = \frac{1+i}{1+2i}$$

$$\Rightarrow \quad \overline{x - iy} = \overline{\left(\frac{1+i}{1+2i} \right)} = \frac{1-i}{1-2i}$$

$$\Rightarrow \quad x + iy = \frac{1-i}{1-2i}$$