IIT JAM 2016 Question Paper with Answer Key (All Subjects)

IIT Joint Admission Test for Masters

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 Pure IgG antibody was run on an SDS-PAGE under reducing condition. How many bands would you see after staining with Coomassie blue?

(A) 4 (B) 2 (C) 1 (D) 6

Q.2 During mitosis, disappearance of the nucleolus is a hallmark of

(A) metaphase (B) prophase (C) anaphase (D) telophase

Q.3 Eukaryotic cell containing flagella is

(A) cell lining the fallopian tube
(B) sperm
(C) Paramecium
(D) cell lining the respiratory tract

Q.4 The circulatory levels of estrogen is derived mainly from

(A) thecal and granulosa cells
(B) gonadotrophs
(C) endometrial epithelia
(D) Leydig cells

Q.5 Radial symmetry is the characteristic feature of which of the following phyla?

(A) Arthropoda (B) Mollusca (C) Cnidaria (D) Chordata

Q.6 In ecotone, some species become abundant and they are called

(A) sibling species
(B) endemic species
(C) rare species
(D) edge species

Q.7 From the set of 10 numbers {1, 2,...,10} three numbers are selected at random without replacement. The probability that the sum of these selected numbers is 9, is

(A) 1/40 (B) 1/20 (C) 3/10 (D) 3/80

Q.8 Consider two nuclei with the same mass number A. For which of the following values of A, the fusion reaction is **NOT** possible?

(A) 15 (B) 22 (C) 29 (D) 36

Q.9 The time period and the amplitude of an object executing simple harmonic motion, under the restoring force of a spring, are 3.14 seconds and 0.2 m, respectively. If the mass of the object is 2 kg, the maximum force (in Newton) exerted by the spring on the object is

(A) 1.6
(B) 3.2
(C) 4.6
(D) 5.2

Q.10 The migratory aptitude of (alkyl or aryl) substituents in Baeyer-Villiger oxidation is

(A) methyl < primary < secondary < tertiary
(B) tertiary < secondary < primary < methyl
(C) phenyl < methyl < primary < tertiary
(D) tertiary < primary < methyl < phenyl

Q. 11 – Q. 30 carry two marks each.

Q.11 Match the entries in **Group I** with the entries in **Group II**

Group I	Group II
P) Epilepsy	1) Degeneration of neurons in cerebral cortex
Q) Alzheimer's disease	2) Degeneration of dopamine releasing neurons
R) Parkinson's disease	3) Decreased production of acetylcholine
S) Huntington's disease	4) Defect in electric discharge in the neurons

(A) P-3, Q-2, R-4, S-1
(B) P-4, Q-3, R-2, S-1
(C) P-4, Q-1, R-2, S-3
(D) P-1, Q-3, R-4, S-2

Q.12 Determine the correctness or otherwise of the following Assertion [a] and Reason [r].

Assertion: In the process of ATP synthesis in oxidative phosphorylation, ATP synthase is not a part of electron transport chain on inner mitochondrial membrane.

Reason: ATP synthase is coupled to electron transport chain through proton motive force.

(A) [a] and [r] are true and [r] is the correct reason for [a]
(B) [a] and [r] are true but [r] is not the correct reason for [a]
(C) both [a] and [r] are false
(D) [a] is false but [r] is true

Q.13 Higher levels of glycosylated hemoglobin (HbA1c) indicate

(A) high hemoglobin level (B) anaemic condition
(C) diabetes (D) favism

Q.14 Match the entries in **Group I** with the entries in **Group II**

Group I	Group II
P) DNA replication	1) Hershey and Chase
Q) Genetic Code	2) Miller and Urey
R) Life on Earth	3) Messelson and Stahl
S) DNA as Genetic material	4) Nirenberg and Khorana

(A) P-2, Q-1, R-3, S-4 (B) P-4, Q-3, R-2, S-1
(C) P-3, Q-4, R-2, S-1 (D) P-3, Q-4, R-1, S-2

Q.15 Match the entries in **Group I** with the entries in **Group II**

Group I	Group II
P) Bacteria	1) Leishmaniasis
Q) Virus	2) Anthrax
R) Fungus	3) Rubella
S) Protozoa	4) Athletes Foot

(A) P-2, Q-3, R-4, S-1
(B) P-2, Q-3, R-1, S-4
(C) P-2, Q-4, R-3, S-1
(D) P-1, Q-2, R-4, S-3

Q.16 In an experiment conducted in the dark, isolated chloroplasts are kept in buffer (pH 4.0) at 4 °C until their internal pH is equal to 4.0. Then, they are transferred to a buffer of pH 8.0, and ADP and P_i are added at the same time. Which of the following will happen?

(A) Chloroplasts will be destroyed
(B) Chlorophyll in the chloroplast will release bound Magnesium
(C) Chloroplasts will be intact but no ATP will be produced
(D) Chloroplasts will be intact and ATP will be produced

Q.17 Two mammalian cell lines with doubling times of 12 h and 36 h were cultured with radioactive thymidine for 8 h. The cells were further cultured without the radioactive thymidine for 72 h. Incorporated radioactivity was measured in equal number of cells in each culture, which revealed that

(A) both the cell lines had the same amount of radioactivity
(B) the fast growing cells had more radioactivity
(C) the slow growing cells had more radioactivity
(D) neither of the cells had any radioactivity

Q.18 Match the entries in **Group I** with the entries in **Group II**

Group I	Group II
P) Yeast 2 Hybrid System	1) *in vivo* protein-DNA interaction
Q) Electrophoretic Mobility Shift Assay	2) protein structure determination
R) Chromatin Immunoprecipitation	3) *in vitro* protein-DNA interaction
S) Nuclear Magnetic Resonance	4) protein-protein interaction

(A) P-3, Q-2, R-1, S-4
(B) P-4, Q-3, R-1, S-2
(C) P-4, Q-1, R-3, S-2
(D) P-1, Q-3, R-4, S-2

Q.19 A line *L* parallel to the vector $\hat{i}+\hat{j}+\hat{k}$ passes through the point (1,2,4) and meets the *xy*-plane at a point *P*. The distance between the origin and *P* is

(A) $\sqrt{10}$ (B) $\sqrt{11}$ (C) $\sqrt{12}$ (D) $\sqrt{13}$

Q.20 Let $P(t)$ denote the population of a species at time t. If $P(t)$ is given by the equation $\frac{dP}{dt} = P(1-P)$ and if the initial population $P(0)$ = 0.1 million, then the population at t = 1 is

(A) $\frac{e}{9+e}$ (B) $\frac{e}{9-e}$ (C) $\frac{9e}{e-1}$ (D) $\frac{9e}{9+e}$

Q.21 Let Z be the set of all integers and f and g are one-one mappings from Z into itself.

If $\begin{cases} f(g(n)) = g(n+1)+1 \ \ for \ \ even \ \ n \\ g(f(n)) = f(n-1)-1 \ \ for \ odd \ \ n \end{cases}$ and $f(1)=3$ then

(A) $g(2)=0$ (B) $f(3)=2$
(C) $g(2)=1$ (D) $f(3)=1$

Q.22 In a p-n junction, the depletion region has a width of 3×10^{-7} m and the intensity of electric field in the depletion region is 10^6 V/m. An electron approaches the junction from the n-side with velocity v_1 and enters the p-side with velocity v_2. If $v_2 = 4 \times 10^5$ m/s, the value of v_1 is

Given data: Charge of electron = 1.6×10^{-19} C; Mass of electron = 9.1×10^{-31} kg

(A) 3.2×10^5 m/s (B) 4.2×10^5 m/s (C) 5.2×10^5 m/s (D) 6.2×10^5 m/s

Q.23 An alpha particle and a proton have the same de Broglie wavelength. Which of the following is also the same for the two particles if they are moving at non-relativistic speeds?

(A) Frequency (B) Kinetic energy (C) Momentum (D) Speed

Q.24 An electron is accelerated from rest through a potential difference of 400 V. The electron then enters a uniform magnetic field that is perpendicular to the direction of electrons. The radius of the circular path experienced by the electron is 10 cm. The angular speed of electrons, in radians/sec, is

Given data: Charge of electron = 1.6×10^{-19} C; Mass of electron = 9.1×10^{-31} Kg

(A) 1.18×10^7 (B) 1.18×10^8 (C) 2.18×10^7 (D) 2.18×10^8

Q.25 Consider two vectors **P** and **Q** of equal magnitude. If the magnitude of **P** + **Q** is two-times larger than that of **P** – **Q**, then the angle between them is

(A) 107° (B) 117° (C) 127° (D) 137°

Q.26 Match the equations in the left column with their names in the right column

(i) $\ln k = \ln A - \frac{E_a}{RT}$ (p) Kirchhoff's law

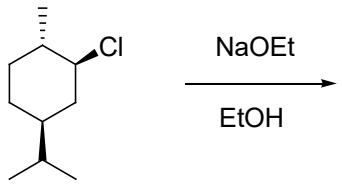
(ii) $\ln K = -\frac{\Delta_r H^0}{RT} + \frac{\Delta_r S^0}{R}$ (q) *van't* Hoff equation

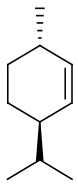
(iii) $\Delta_r H_2 - \Delta_r H_1 = \Delta C_p (T_2 - T_1)$ (r) Clausius-Clapeyron equation

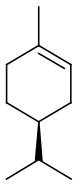
(iv) $\ln P = -\frac{\Delta \overline{H}}{RT}$ + constant (s) Arrhenius equation

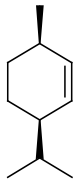
(A) (i)-(s), (ii)-(r), (iii)-(p), (iv)-(q)
(B) (i)-(p), (ii)-(q), (iii)-(r), (iv)-(s)
(C) (i)-(p), (ii)-(q), (iii)-(s), (iv)-(r)
(D) (i)-(s), (ii)-(q), (iii)-(p), (iv)-(r)

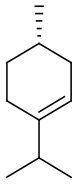
Q.27 The major product in the following reaction is



(A) 

(B) 

(C) 

(D) 

Q.28 Of the given isomers of molecules **I** and **II**, the *meso*-form is

$CH_3CH(Br)CH(Cl)CH_3$ $CH_3CH(Cl)CH(Cl)CH_3$

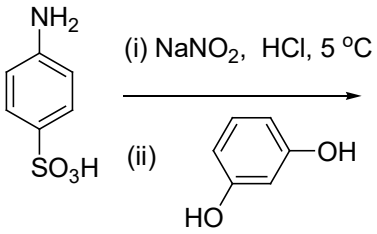
I **II**

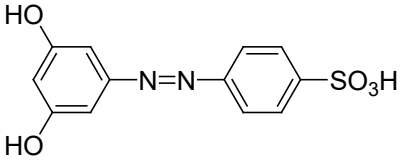
(A) (*R*,*R*)-isomer of **I** (B) (*R*,*S*)-isomer of **II**
(C) (*R*,*S*)-isomer of **I** (D) (*S*,*S*)-isomer of **II**

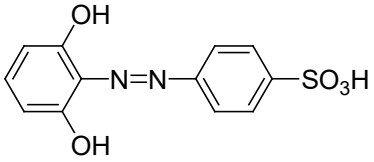
Q.29 Considering the periodic trends of elements, which of the following is **NOT** correct?

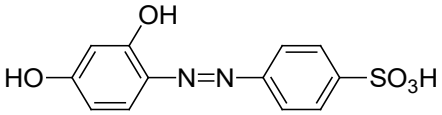
(A) MgO and Na_2O are basic, and SiO_2 is acidic
(B) Atomic radius decreases in a period from left to right
(C) Order of first ionization energies: K < Mg < Ca
(D) Order of bond energies: C–C < Si–O < N≡N

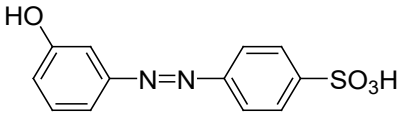
Q.30 Which one is the major product of the following reaction?



(A) 

(B) 

(C) 

(D) 

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 Identify the autoimmune diseases among the following

(A) Type II Diabetes Mellitus
(B) Type I Diabetes Mellitus
(C) Gestational Diabetes
(D) Pernicious Anaemia

Q.32 Which of the following statements are **TRUE** for hydrogen bonds?

Strength of hydrogen bond is

(A) low in a solvent of high dielectric constant
(B) low in a solvent of low dielectric constant
(C) lower in water as compared to organic solvents
(D) higher in water as compared to organic solvents

Q.33 Which of the following statements are **TRUE** for cellulose?

(A) Cellulose serves a structural role
(B) Cellulose is a branched polysaccharide
(C) Cellulose is a homopolysaccharide composed of ($\alpha 1 \rightarrow 4$) linked D-glucose units
(D) Cellulose is a homopolysaccharide composed of ($\beta 1 \rightarrow 4$) linked D-glucose units

Q.34 Which of the following are **NOT** true for photosynthesis?

(A) Reduction of CO_2 and H_2O
(B) Oxidation of CO_2 and H_2O
(C) Reduction of CO_2 and oxidation of H_2O
(D) Oxidation of CO_2 and reduction of H_2O

Q.35 Apoptosis is a controlled process of cell death. The process involves

(A) exposure of phosphatidyl serine on the outer surface of the cell membrane
(B) decreased permeability of the outer mitochondrial membrane
(C) increased lysosomal activity
(D) inter-nucleosomal cleavage of genomic DNA

Q.36 If $\begin{cases} f(x) = ax^2 + b \text{ for } 0 \le x \le 1 \\ \quad\quad = cx + \sin(\frac{\pi}{2}x) \text{ for } 1 \le x \le 2 \end{cases}$ is continuous and differentiable at all points in the interval [0,2] and $f(2) = \frac{\pi}{4}$, then

(A) $a = \frac{\pi}{16}$ and $b = \frac{\pi}{16} + 1$
(B) $b = \frac{\pi}{16} + 1$ and $c = \frac{\pi}{8}$
(C) $a = \frac{\pi}{8}$ and $c = \frac{\pi}{8}$
(D) $a = \frac{\pi}{8}$ and $b = -\frac{3\pi}{16}$

Q.37 In ΔPQR, $\angle Q = 60°$ and S is the mid-point of QR. If $QS = PS$ and $PR = 5$, then

(A) $PQ = 5/\sqrt{3}$
(B) $PS = 5/\sqrt{2}$
(C) Area of the triangle $PSR = (25 - 5\sqrt{3})/2\sqrt{3}$
(D) S is the circumcenter of ΔPQR

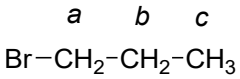
Q.38 The magnetic field in the interior of a long current-carrying solenoid can be increased by

(A) increasing the length of solenoid while keeping the number of turns per unit length as constant
(B) increasing the number of turns per unit length
(C) increasing the current in the solenoid
(D) decreasing the length of solenoid while keeping the number of turns per unit length as constant

Q.39 Which of the following statements are **CORRECT**?

(A) Fluorescence has a much longer decay period than that of phosphorescence
(B) Radiative transition from T_1 to S_0 is phosphorescence
(C) Radiative transition from S_1 to S_0 is fluorescence
(D) Enhancing the life time of the excited state is quenching

Q.40 In the ^1H NMR spectrum of 1-bromopropane (structure shown below), which of the following statements are **CORRECT**?



(A) Protons *'a'* resonate upfield to protons *'c'*
(B) Protons *'b'* resonate downfield to protons *'c'*
(C) There are two triplets and one quartet in the spectrum
(D) Protons *'a'* appear as triplet with high chemical shift in comparison to protons *'c'*

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 The net charge on the following peptide at pH 7.0 is _____ .

Val-Asp-Asn-Lys-Ser-Ile

Q.42 A 152 nm long Watson-Crick double helical DNA (B-DNA) will contain _____ turns.

Q.43 A population is in Hardy-Weinberg equilibrium for a gene with only two alleles ("A" and "a"). If the gene frequency of the allele "A" is 0.7, genotype frequency of heterozygous "Aa" is _____ .

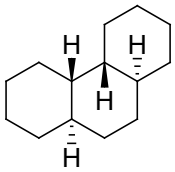
Q.44 A receptor binds to its ligand with a dissociation constant $K_d = 10^{-8}$ M. The concentration of the ligand required to occupy 10% of the receptors would be 10^{-x} M. The value of x is _____ .

Q.45 The plane $x + y + z = 0$ intersects the sphere $x^2 + y^2 + z^2 = 9$ along a circle. If $(2, y, z)$ is a point on the circle, then the value of $|y + z|$ is _____ .

Q.46 In Young's double slit experiment, the slits are separated by a distance of 0.05 mm and the source emits light of two wavelengths 450 and 520 nm. If the distance between the slit and viewing screen is 2 m, the separation between 2nd order bright fringes for the two wavelengths is _____ cm.

Q.47 A jet plane lands on an aircraft carrier at 70 m/s and stops in 3 seconds. Assuming that the acceleration is constant, the jet plane travels a distance of _____ m before it stops.

Q.48 The number of signals in ^{13}C NMR for the following structure is _____ .



Q.49 From the database of a clinic it was found that out of 2000 patients who had visited the clinic in a year, 900 had high BP, 900 had high Sugar and 400 had neither high BP nor high Sugar. On a given day, if 20 patients visit the clinic, the expected number of patients who have both high BP and high Sugar is _____ .

Q.50 An enzyme catalyzes the conversion of 4×10^{-4} M substrate into product at a rate of 20 μM/min. If the K_m value for the enzyme is 2×10^{-4} M, the value of V_{max} is _____ μM/min.

Q. 51 – Q. 60 carry two marks each.

Q.51 The following polypeptide chain was sequentially treated with dithiothreitol, cyanogen bromide, and trypsin.

Phe-Trp-Lys-Tyr-Met-Gly-Ala-Cys-Cys-Pro-Met-Asp-Gly-Arg-Phe-Ala-Gly-Trp

The total number of fragments expected at the end of complete digestion of the polypeptide are _____ .

(consider that none of the reagents interfere with each other's activities)

Q.52 In maize, the genes for colored seed and round seed are dominant over the genes for colorless seed and shrunken seed. Pure breeding strains of the double dominant variety were crossed with the double recessive variety and a test cross of the F1 generation produced the following:

Phenotypes	**Number of seeds**
Colored, round seed	380
Colorless, shrunken seed	396
Colored, shrunken seed	14
Colorless, round seed	10

For the above, the distance between the genes for seed color and seed shape on the chromosomes would be _____ centimorgan units.

Q.53 A culture of 10^6 bacteria, with doubling time of 60 min, is grown in a nutrient medium at 37 °C. Considering that the nutrients are unlimited, the number of bacteria at the end of 10 h would be _____ × 10^6.

Q.54 A 50-amino acid residue stretch of a globular protein adopts an extended structure containing a true α-helix of 24 residues and β-strand of 26 residues. The total length of the stretch will be _____ nm.

Q.55 The right limit $\lim\limits_{x \to 3^+} (x-3)^2 \left(\log(x-3) + \operatorname{cosec}(x-3)^2\right)$ is _____ .

Q.56 If [x] denotes the greatest integer valued function (e.g., [1.16] = 1 & [1.8] = 1) and $\int_1^{\sqrt{3}} \frac{1+[x]}{1+x^2}\,dx = L$, then $L =$ ____ degrees.

Q.57 A copper wire having a cross sectional area of 6.62×10^{-6} m^2 carries a current of 20 A. Assuming that each atom contributes one free electron to the current, the time required by electrons to travel a distance of 1 m is _____ min.

Given data: Density of copper = 8.92 g/cm^3 and molar mass = 63.5 g/mol,
Avogadro number = 6.02×10^{23}

Q.58 A piece of charcoal, containing 36 grams of Carbon, found in ancient ruins shows a ^{14}C activity of 300 decays/min. The tree, from which this charcoal came, has been dead for _____ years.

Given data: The ratio of ^{14}C to ^{12}C is 1.3×10^{-12} in the CO_2 molecules of atmosphere and the half life of ^{14}C is 5730 years.

Q.59 At constant pressure, 200 g of water was heated from 10 °C to 22 °C. The molar heat capacity of H_2O at constant pressure is 75.3 $J\ K^{-1}\ mol^{-1}$. The increase in entropy for this process is _____ $J\ K^{-1}$.

(Consider that molar heat capacity of water is independent of temperature and that water does not expand when heated)

Q.60 The number of **optically inactive** geometrical isomers of $[Pt(NH_3)_2(py)_2Cl_2]^{2+}$ is _____ .

(where, 'py' is pyridine)

END OF THE QUESTION PAPER

JAM 2016: Biotechnology			
Qn. No.	Qn. Type	Key(s)	Mark(s)
1	MCQ	B	1
2	MCQ	B	1
3	MCQ	B	1
4	MCQ	A	1
5	MCQ	C	1
6	MCQ	D	1
7	MCQ	A	1
8	MCQ	D	1
9	MCQ	A	1
10	MCQ	A	1
11	MCQ	B	2
12	MCQ	A	2
13	MCQ	C	2
14	MCQ	C	2
15	MCQ	A	2
16	MCQ	D	2
17	MCQ	C	2
18	MCQ	B	2
19	MCQ	D	2
20	MCQ	A	2
21	MCQ	A	2
22	MCQ	C	2
23	MCQ	C	2
24	MCQ	B	2
25	MCQ	C	2
26	MCQ	D	2
27	MCQ	A	2
28	MCQ	B	2
29	MCQ	C	2
30	MCQ	C	2

JAM 2016: Biotechnology			
Qn. No.	Qn. Type	Key(s)	Mark(s)
31	MSQ	B;D	2
32	MSQ	A;C	2
33	MSQ	A;D	2
34	MSQ	A;B;D	2
35	MSQ	A;D	2
36	MSQ	A;B	2
37	MSQ	A;D	2
38	MSQ	B;C	2
39	MSQ	B;C	2
40	MSQ	B;D	2

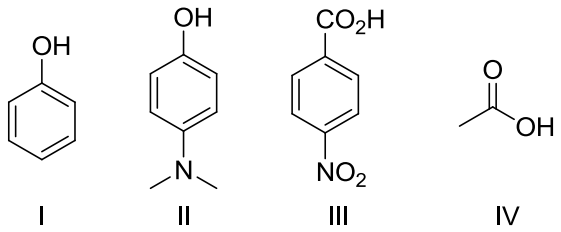
JAM 2016: Biotechnology			
Qn. No.	Qn. Type	Key(s)	Mark(s)
41	NAT	0.0 to 0.0	1
42	NAT	41.0 to 44.0	1
43	NAT	0.41 to 0.43	1
44	NAT	9.0 to 9.0	1
45	NAT	2.0 to 2.0	1
46	NAT	0.53 to 0.60	1
47	NAT	105 to 105	1
48	NAT	7.0 to 7.0	1
49	NAT	2.0 to 2.0	1
50	NAT	29.0 to 31.0	1
51	NAT	5.0 to 5.0	2
52	NAT	2.9 to 3.1	2
53	NAT	1020 to 1026	2
54	NAT	11.0 to 14.0	2
55	NAT	1.0 to 1.0	2
56	NAT	29.0 to 30.0	2
57	NAT	73.0 to 78.0	2
58	NAT	4800 to 4900	2
59	NAT	34.0 to 35.0	2
60	NAT	4.0 to 4.0	2

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

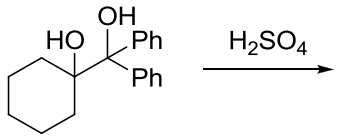
Q.1 The correct order of pKa for the following compounds is



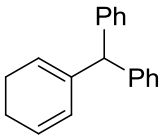
I II III IV

(A) II > I > III > IV
(B) II > I > IV > III
(C) III > IV > I > II
(D) IV > II > I > III

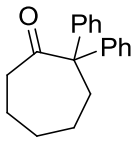
Q.2 The major product formed in the following reaction is



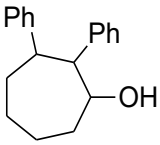
(A)



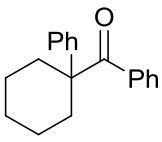
(B)



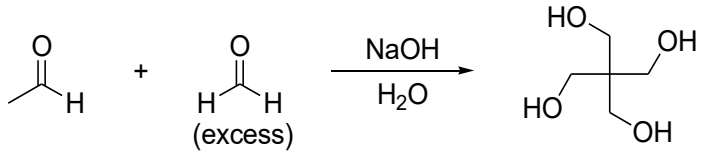
(C)



(D)



Q.3 The mechanism of the following transformation involves



(A) Aldol reaction and Cannizzaro reaction
(B) Aldol reaction and Claisen-Schmidt reaction
(C) Knoevenagel condensation and Cannizzaro reaction
(D) Stobbe condensation and Cannizzaro reaction

Q.4 The most basic amino acid among the following is

(A) tyrosine
(B) methionine
(C) arginine
(D) glutamine

Q.5 The crystal field stabilization energy (CFSE) in $[Mn(H_2O)_6]^{2+}$ is

(A) 0 Δ_O (B) 2.0 Δ_O – 2P (C) 0.4 Δ_O – 2P (D) 2.0 Δ_O

Q.6 Indicator used in redox titration is

(A) Eriochrome black T
(B) Methyl orange
(C) Phenolphthalein
(D) Methylene blue

Q.7 Among the following, the compound that has the lowest degree of ionic character is

(A) NaCl (B) $MgCl_2$ (C) $AlCl_3$ (D) $CaCl_2$

Q.8 The correct order of entropy for various states of CO_2 is

(A) $CO_2(s) > CO_2(l) > CO_2(g)$
(B) $CO_2(l) > CO_2(s) > CO_2(g)$
(C) $CO_2(g) > CO_2(l) > CO_2(s)$
(D) $CO_2(g) > CO_2(s) > CO_2(l)$

Q.9 The coordination numbers of Cs^+ and Cl^- ions in the CsCl structure, respectively, are

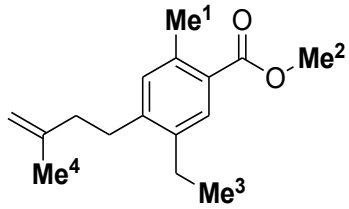
(A) 4,4 (B) 4,8 (C) 6,6 (D) 8,8

Q.10 Determinant of a square matrix is always

(A) a square matrix
(B) a column matrix
(C) a row matrix
(D) a number

Q. 11 – Q. 30 carry two marks each.

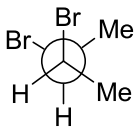
Q.11 The correct order of ^{1}H NMR chemical shift (δ) values for the labeled methyl groups in the following compound is



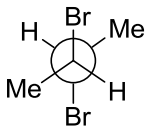
(A) $Me^1 < Me^2 < Me^3 < Me^4$
(B) $Me^3 < Me^4 < Me^1 < Me^2$
(C) $Me^3 < Me^1 < Me^4 < Me^2$
(D) $Me^2 < Me^4 < Me^3 < Me^1$

Q.12 Among the following, the most stable conformation of *meso*-2,3-dibromobutane is

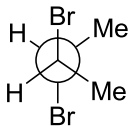
(A)



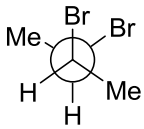
(B)



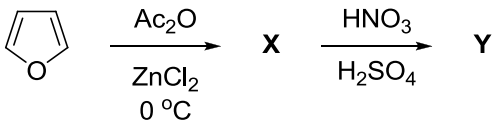
(C)



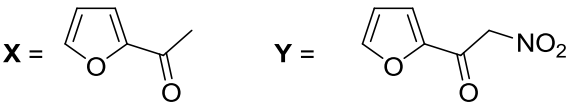
(D)



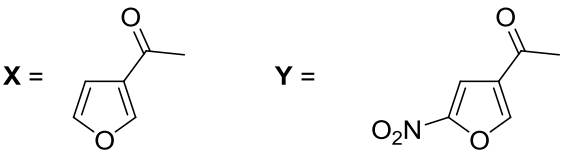
Q.13 The major products **X** and **Y** in the following reaction sequence are



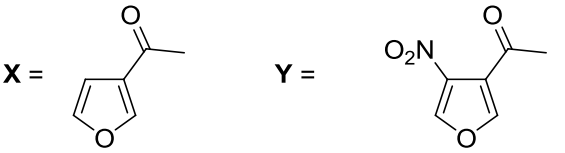
(A)



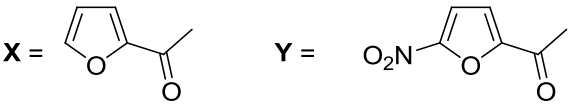
(B)



(C)

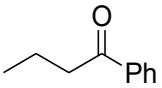


(D)

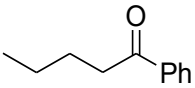


Q.14 The major product formed in the reaction of butanenitrile with phenylmagnesium bromide followed by acidification is

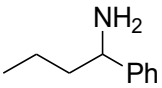
(A)



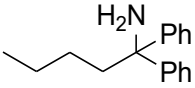
(B)



(C)

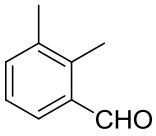


(D)

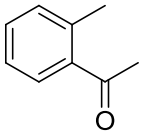


Q.15 An organic compound on reaction with 2,4-dinitrophenylhydrazine (2,4-DNP) gives a yellow precipitate. It also gives silver mirror on reaction with ammoniacal $AgNO_3$. It gives an alcohol and sodium salt of a carboxylic acid on reaction with concentrated NaOH. It yields benzene-1,2-dicarboxylic acid on heating with alkaline $KMnO_4$. The structure of the compound among the following is

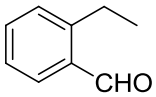
(A)



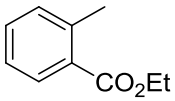
(B)



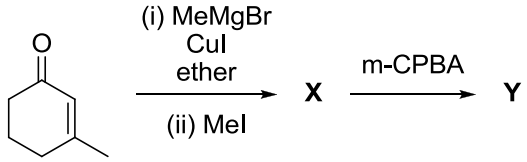
(C)



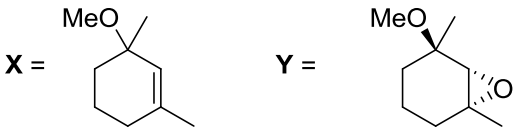
(D)



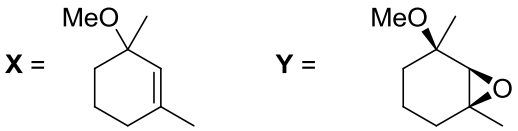
Q.16 The major products **X** and **Y** in the following reaction sequence are



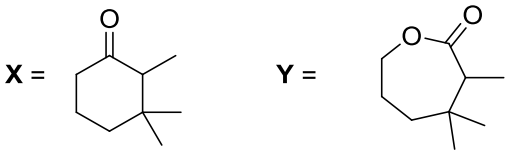
(A)



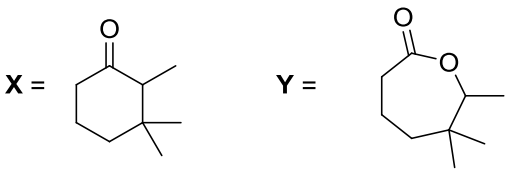
(B)



(C)



(D)



Q.17 The TRUE statement about $[Cu(H_2O)_6]^{2+}$ is

(A) All Cu–O bond lengths are equal
(B) One Cu–O bond length is shorter than the remaining five
(C) Three Cu–O bond lengths are shorter than the remaining three
(D) Four Cu–O bond lengths are shorter than the remaining two

Q.18 The complexes $[Pt(CN)_4]^{2-}$ and $[NiCl_4]^{2-}$, respectively, are

(A) paramagnetic, paramagnetic
(B) diamagnetic, diamagnetic
(C) paramagnetic, diamagnetic
(D) diamagnetic, paramagnetic

Q.19 The value of 'x' in $[Cu(CO)_x]^+$ such that it obeys the 18 electron rule is

(A) 6 (B) 5 (C) 4 (D) 3

Q.20 The correct order of ν_{NO} (cm^{-1}) in the following compounds is

(A) $NO^+ > NO > [NiCp(NO)] > [Cr(Cp)_2(NO)_4]$
(B) $[Cr(Cp)_2(NO)_4] > [NiCp(NO)] > NO^+ > NO$
(C) $NO^+ > [Cr(Cp)_2(NO)_4] > NO > [NiCp(NO)]$
(D) $[NiCp(NO)] > NO > [Cr(Cp)_2(NO)_4] > NO^+$

Q.21 The red color of ruby is due to

(A) d-d transition of Cr^{3+} ion in Cr_2O_3 lattice
(B) d-d transition of Cr^{3+} ion in Al_2O_3 lattice
(C) ligand to metal charge transfer transition
(D) metal to metal charge transfer transition

Q.22 The final products in the reaction of BF_3 with water are

(A) $B(OH)_3$ and OF_2
(B) H_3BO_3 and HBF_4
(C) B_2O_3 and HBF_4
(D) B_2H_6 and HF

Q.23 The correct order of bond angles in BF_3, NH_3, NF_3 and PH_3 is

(A) $BF_3 > NH_3 > NF_3 > PH_3$
(B) $PH_3 > BF_3 > NF_3 > NH_3$
(C) $BF_3 > PH_3 > NH_3 > NF_3$
(D) $NH_3 > NF_3 > BF_3 > PH_3$

Q.24 The maximum of a function Ae^{-ax^2} (A > 0; a > 0) is at x =

(A) 0 (B) $+\infty$
(C) $-\infty$ (D) $1/\sqrt{a}$

Q.25 At 298 K, 0.1 mol of ammonium acetate and 0.14 mol of acetic acid are dissolved in 1 L of water. The *p*H of the resulting solution is
[Given: pK_a of acetic acid is 4.75]

(A) 4.9 (B) 4.6 (C) 4.3 (D) 2.3

Q.26 An electrochemical cell consists of two half-cell reactions
$AgCl(s) + e^- \rightarrow Ag(s) + Cl^-(aq)$
$Cu(s) \rightarrow Cu^{2+}(aq) + 2e^-$
The mass of copper (in grams) dissolved on passing 0.5 A current for 1 hour is
[Given: atomic mass of Cu is 63.6; F = 96500 C mol^{-1}]

(A) 0.88 (B) 1.18 (C) 0.29 (D) 0.59

Q.27 For a zero order reaction, the half-life depends on the initial concentration $[C_0]$ of the reactant as

(A) $[C_0]$ (B) $[C_0]^0$ (C) $[C_0]^{-1}$ (D) $[C_0]^{1/2}$

Q.28 The effective nuclear charge of helium atom is 1.7. The first ionization energy of helium atom in eV is

(A) 13.6 (B) 23.1 (C) 39.3 (D) 27.2

Q.29 The relationship between the van der Waals 'b' coefficient of N_2 and O_2 is

(A) $b(N_2) = b(O_2) = 0$
(B) $b(N_2) = b(O_2) \neq 0$
(C) $b(N_2) > b(O_2)$
(D) $b(N_2) < b(O_2)$

Q.30 From the kinetic theory of gases, the ratio of most probable speed (C_{mp}) to root mean square speed (C_{rms}) is

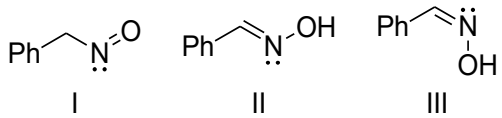
(A) $\sqrt{3}$ (B) $\sqrt{2}/\sqrt{3}$ (C) $\sqrt{3}/\sqrt{2}$ (D) $3/\sqrt{2}$

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

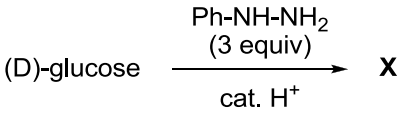
Q.31 The correct statement(s) about the following species is(are)



I II III

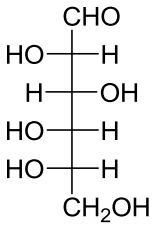
(A) I and II are resonance structures
(B) II and III are resonance structures
(C) II and III are diastereomers
(D) III is a tautomer of I

Q.32 Consider the following reaction:

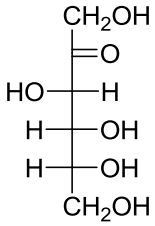


Among the following, the compound(s) whose osazone derivative(s) will have the same melting point as that of **X** is(are)

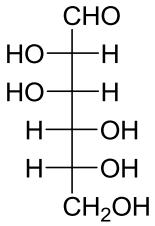
(A)



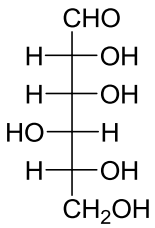
(B)



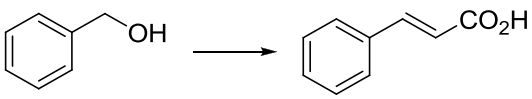
(C)



(D)

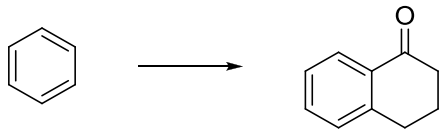


Q.33 The appropriate reagents required for carrying out the following transformation are



(A) (i) PCC, CH_2Cl_2; (ii) $Ph_3P{=}CHCO_2Et$; (iii) aq. NaOH, heat, then acidify
(B) (i) CrO_3, H_2SO_4, aq. acetone (ii) Ac_2O, NaOAc
(C) (i) MnO_2; (ii) $CH_2(CO_2H)_2$, piperidine, pyridine
(D) (i) PCC, CH_2Cl_2; (ii) $BrCH_2CO_2C(CH_3)_3$, Zn (iii) H_3O^+, heat

Q.34 The appropriate reagents required for carrying out the following transformation are



(A) (i) succinic anhydride, $AlCl_3$; (ii) Zn/Hg, HCl; (iii) polyphosphoric acid
(B) (i) maleic anhydride, $AlCl_3$; (ii) $H_2N\text{-}NH_2$, KOH; (iii) H_2SO_4
(C) (i) succinic anhydride, $FeCl_3$; (ii) $LiAlH_4$; (iii) H_2SO_4
(D) (i) phthalic anhydride, $F_3B\cdot OEt_2$; (ii) $HS(CH_2)_2SH$, H^+; (iii) Raney Ni; (iv) polyphosphoric acid

Q.35 The protein(s) that belong to the class of blue copper proteins is(are)

(A) ceruloplasmin
(B) superoxide dismutase
(C) hemocyanin
(D) azurin

Q.36 The ion(s) that exhibit only charge transfer bands in the absorption spectra (UV-visible region) is(are)

(A) $[Cr(C_2O_4)_3]^{3-}$ (B) $[CrO_4]^{2-}$
(C) $[ReO_4]^-$ (D) $[NiO_2]^{2-}$

Q.37 The type(s) of interaction(s) that hold layers of graphite together is(are)

(A) π-π stacking (B) van der Waals
(C) hydrogen bonding (D) Coulombic

Q.38 TRUE statement(s) about Langmuir isotherm is(are)

(A) valid for monolayer coverage
(B) all adsorption sites are equivalent
(C) there is dynamic equilibrium between free gas and adsorbed gas
(D) adsorption probability is independent of occupancy at the neighboring sites

Q.39 The $3p_z$ orbital has

(A) one radial node (B) two radial nodes
(C) one angular node (D) two angular nodes

Q.40 The diatomic molecule(s) that has(have) two π-type bonds is(are)

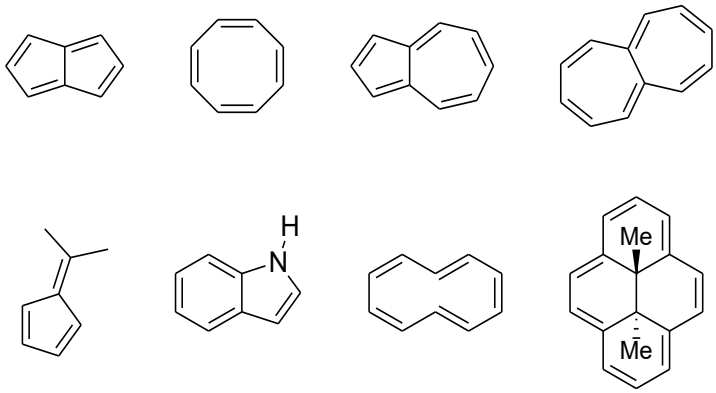
(A) B_2 (B) C_2 (C) N_2 (D) O_2

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 Among the following, the number of molecules that are aromatic is __.



Q.42 The number of all possible isomers for the molecular formula C_6H_{14} is __.

Q.43 Hydrolysis of 15.45 g of benzonitrile produced 10.98 g of benzoic acid. The percentage yield of acid formed is __.

Q.44 Acetic acid content in commercial vinegar was analyzed by titrating against 1.5 M NaOH solution. A 20 mL vinegar sample required 18 mL of titrant to give endpoint. The concentration of acetic acid in the vinegar (in mol L^{-1}) is __.

Q.45 The bond order of Be_2 molecule is __.

Q.46 The number of P–H bonds in hypophosphorus acid is __.

Q.47 The isotope $^{214}_{84}Po$ undergoes one alpha and one beta particle emission sequentially to form an isotope "X". The number of neutrons in "X" is __.

Q.48 In a diffraction experiment with X-rays of wavelength 1.54 Å, a diffraction line corresponding to $2\theta = 20.8°$ is observed. The inter-planar separation in Å is __.

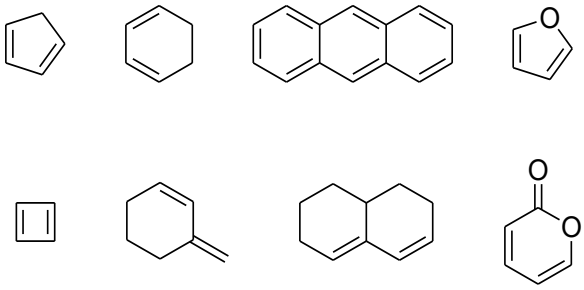
Q.49 The potential energy of interaction between two ions in an ionic compound is given by $U = 1389.4\left[\frac{Z_1Z_2}{r/\text{Å}}\right]$ kJ mol^{-1}. Assuming that $CaCl_2$ is linear molecule of length 5.6 Å, the potential energy for $CaCl_2$ molecule in kJ mol^{-1} is __.

Q.50 The enthalpy of formation for $CH_4(g)$, $C(g)$ and $H(g)$ are –75, 717 and 218 kJ mol^{-1}, respectively. The enthalpy of the C–H bond in kJ mol^{-1} is __.

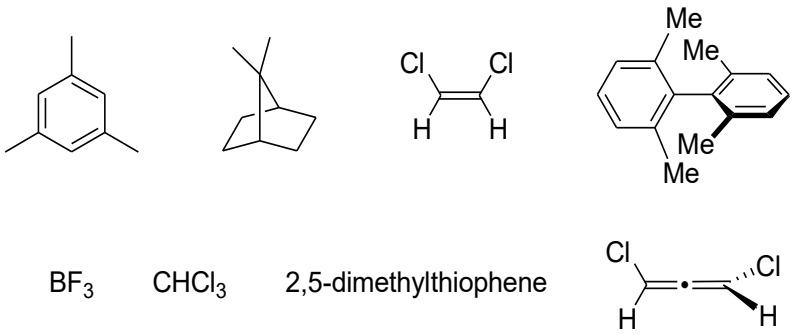
Q. 51 – Q. 60 carry two marks each.

Q.51 Specific rotation of the (*R*)-enantiomer of a chiral compound is 48. The specific rotation of a sample of this compound which contains 25% of (*S*)-enantiomer is ___.

Q.52 Among the following, the number of compounds, which can participate as '**diene**' component in a Diels-Alder reaction is __.



Q.53 Among the following, the number of molecules that possess C_2 axis of symmetry is __.



BF_3 $CHCl_3$ 2,5-dimethylthiophene

Q.54 Effective nuclear charge for 3d electron in vanadium (atomic number = 23) according to Slater's rule is __.

Q.55 The total number of isomers possible for the molecule $[Co(NH_3)_4Cl(NO_2)]^+$ is __.

Q.56 The bond angle in PBr_3 is 101°. The percent 's' character of the central atom is __.

Q.57 $Cu(s) + 4\ H^+(aq) + 2NO_3^-(aq) \rightarrow 2NO_2(g) + Cu^{2+}(aq) + 2H_2O(l)$
In the above reaction at 1 atm and 298 K, if 6.36 g of copper is used. Assuming ideal gas behavior, the volume of NO_2 produced in liters is __.
[Given: atomic mass of Cu is 63.6; R = 0.0821 L atm K^{-1} mol^{-1}]

Q.58 The ΔH^o for the reaction $CO(g) + \frac{1}{2}O_2(g) \rightarrow CO_2(g)$ at 400 K in kJ mol^{-1} is __.
Given at 298 K:

	ΔH_f^o kJ mol^{-1}	C_p^o J mol^{-1} K^{-1}
O_2	0	29.4
CO	–110	29.1
CO_2	–394	37.1

Q.59 The rate constants for a reaction at 300 and 350 K are 8 and 160 L mol^{-1} s^{-1}, respectively. The activation energy of the reaction in kJ mol^{-1} is __.
[Given R = 8.314 J K^{-1} mol^{-1}]

Q.60 A 10 L flask containing 10.8 g of N_2O_5 is heated to 373 K, which leads to its decomposition according to the equation $2\ N_2O_5(g) \rightarrow 4\ NO_2(g) + O_2(g)$. If the final pressure in the flask is 0.5 atm, then the partial pressure of $O_2(g)$ in atm is __.
[Given R = 0.0821 L atm K^{-1} mol^{-1}]

END OF THE QUESTION PAPER

JAM 2016: Chemistry			
Qn. No.	Qn. Type	Key(s)	Mark(s)
1	MCQ	B	1
2	MCQ	B	1
3	MCQ	A	1
4	MCQ	C	1
5	MCQ	A	1
6	MCQ	D	1
7	MCQ	C	1
8	MCQ	C	1
9	MCQ	D	1
10	MCQ	D	1
11	MCQ	B	2
12	MCQ	B	2
13	MCQ	D	2
14	MCQ	A	2
15	MCQ	C	2
16	MCQ	D	2
17	MCQ	D	2
18	MCQ	D	2
19	MCQ	C	2
20	MCQ	MTA	2
21	MCQ	B	2
22	MCQ	B	2
23	MCQ	A	2
24	MCQ	A	2
25	MCQ	B	2
26	MCQ	D	2
27	MCQ	A	2
28	MCQ	C	2
29	MCQ	C	2
30	MCQ	B	2

JAM 2016: Chemistry			
Qn. No.	Qn. Type	Key(s)	Mark(s)
31	MSQ	C;D	2
32	MSQ	A;B;C	2
33	MSQ	A;C;D	2
34	MSQ	A	2
35	MSQ	A;D	2
36	MSQ	B;C	2
37	MSQ	A;B	2
38	MSQ	A;B;C;D	2
39	MSQ	A;C	2
40	MSQ	B;C	2

JAM 2016: Chemistry			
Qn. No.	Qn. Type	Key(s)	Mark(s)
41	NAT	3.0 to 3.0	1
42	NAT	5.0 to 5.0	1
43	NAT	60.0 to 60.0	1
44	NAT	1.3 to 1.4	1
45	NAT	0.0 to 0.0	1
46	NAT	2.0 to 2.0	1
47	NAT	127.0 to 127.0	1
48	NAT	4.2 to 4.3	1
49	NAT	-1738 to -1734	1
50	NAT	-417.0 to -415.0	1
51	NAT	24.0 to 24.0	2
52	NAT	6.0 to 6.0	2
53	NAT	7.0 to 7.0	2
54	NAT	4.2 to 4.4	2
55	NAT	4.0 to 4.0	2
56	NAT	MTA	2
57	NAT	4.8 to 5.0	2
58	NAT	-284.70 to -284.65	2
59	NAT	52.0 to 53.0	2
60	NAT	0.06 to 0.07	2

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

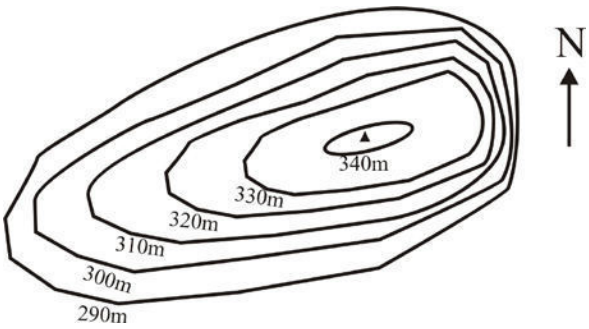
Q.1 The most abundant metal (by weight %) in the Earth's crust is

(A) Al (B) Fe (C) Na (D) Mg

Q.2 The correct order of increasing stability of minerals during chemical weathering is

(A) olivine, pyroxene, biotite, quartz
(B) olivine, biotite, pyroxene, quartz
(C) quartz, biotite, pyroxene, olivine
(D) pyroxene, olivine, biotite, quartz

Q.3 In the topographic map the steepest slope is



(A) northerly (B) southerly (C) easterly (D) westerly

Q.4 Gabbro contains much more alumina than peridotite because it is richer in

(A) orthopyroxene
(B) orthoamphibole
(C) olivine
(D) plagioclase

Q.5 Coking coal in India is found in

(A) Neyvelli, Tamil Nadu
(B) Jharia, Jharkhand
(C) Palana, Rajasthan
(D) Garampani, Meghalya

Q.6 The amplitude of ground motion during an earthquake of magnitude 7 in Richter scale is how many times more than that of a magnitude 5?

(A) 10 (B) 100 (C) 1000 (D) 10,000

Q.7 The failed arm of a continental rift is called

(A) hot spot (B) horst (C) decollement (D) aulacogen

Q.8 The hardest mineral (with the exception of diamond) in the Moh's scale of hardness is

(A) an oxide
(B) a silicate
(C) a phosphate
(D) a carbonate

Q.9 Which one of the following is capable of transporting sediments against the slope?

(A) river current
(B) turbidity current
(C) tidal current
(D) rip current

Q.10 Which is the most abundant ion in the normal seawater?

(A) Cl^- (B) SO_4^- (C) Na^+ (D) K^+

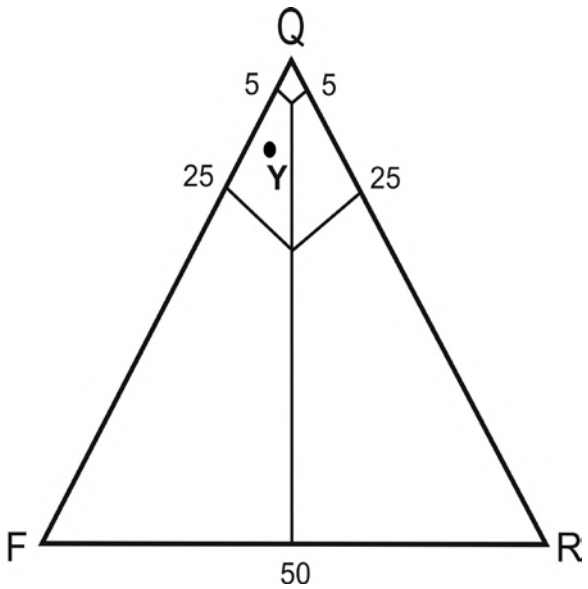
Q. 11 – Q. 30 carry two marks each.

Q.11 Match minerals in Group I with the corresponding silicate structure in Group II.

Group I	**Group II**
i) Anthophyllite	P) Cyclosilicate
ii) Hedenbergite	Q) Tectosilicate
iii) Cordierite	R) Double chain silicate
iv) Zeolite	S) Single chain silicate

(A) i – R; ii – S; iii – Q; iv – P
(B) i – S; ii – R; iii – Q; iv – P
(C) i – R; ii – S; iii – P; iv – Q
(D) i – P; ii – Q; iii – R; iv – S

Q.12 The correct name for the well sorted sandstone, whose framework composition plots at Y in the following QFR diagram, is



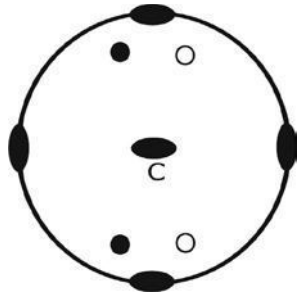
(A) arkose
(B) subarkose
(C) lithic arkose
(D) quartz arenite

Q.13 Which one of the following optical properties of minerals is NOT observed under crossed nicols?

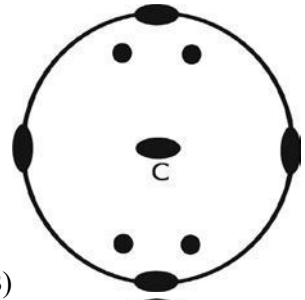
(A) Extinction
(B) Interference colour
(C) Interference figure
(D) Pleochroism

Q.14 Choose the correct stereographic projection among the following that represents 222 crystal symmetry.

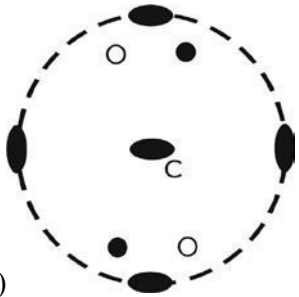
(A)



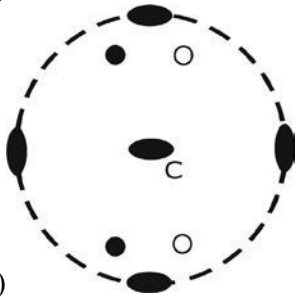
(B)



(C)



(D)

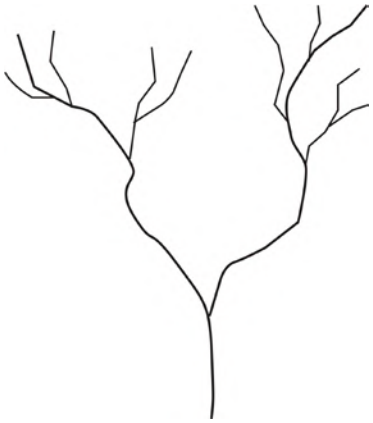


Q.15 Match the folds listed in Group I with corresponding geometric characteristics in Group II.

Group I	**Group II**
i) Parallel	P) Angular hinge
ii) Chevron	Q) Minimum thickness parallel to the axial surface is at the hinge
iii) Similar	R) Negative inter-limb angle
iv) Mushroom	S) Constant thickness parallel to the axial surface

(A) i – Q; ii – R; iii – S; iv – P
(B) i – S; ii – P; iii – Q; iv – R
(C) i – Q; ii – P; iii – S; iv – R
(D) i – S; ii – R; iii – Q; iv – P

Q.16 The number of 4^{th} order stream(s) present in the drainage network shown below, as per hierarchical classification, is



(A) one (B) two (C) three (D) four

Q.17 During metamorphism, temperature can increase at constant pressure in the case of

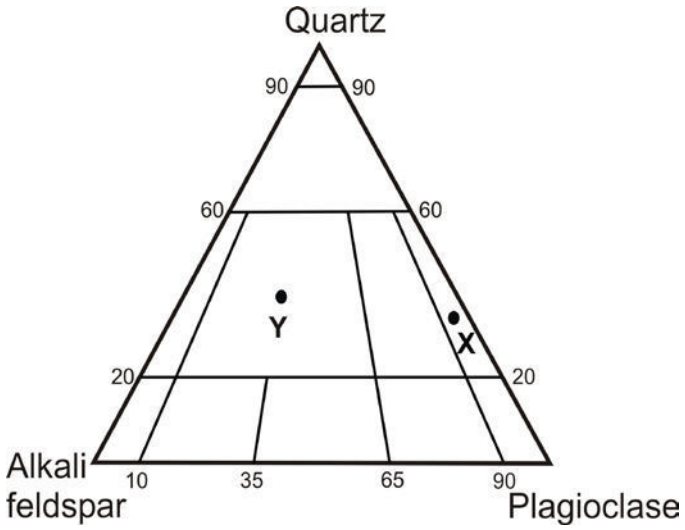
(A) exhumation by erosion
(B) burial by subduction
(C) burial by underthrusting
(D) intrusion of batholith

Q.18 Choose the correct chronological order from oldest to youngest for the following stratigraphic units

P - Chitradurga Group; Q - Sargur Group; R - Ajabgarh Group; S - Udaipur Group

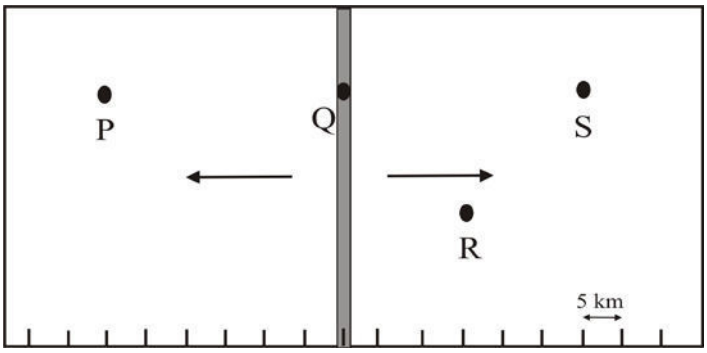
(A) Q, P, S, R
(B) P, Q, S, R
(C) Q, R, P, S
(D) Q, R, S, P

Q.19 Identify the rock types at X and Y in the following QAP diagram of IUGS.



(A)X is granodiorite and Y is granite
(B) X is granodiorite and Y is alkali feldspar granite
(C) X is tonalite and Y is granite
(D) X is tonalite and Y is alkali feldspar granite

Q.20 A schematic diagram of a divergent plate boundary, with arrows indicating directions of plate movement, is given below. Which one of the following statements is NOT true for points P, Q, R and S, if the spreading rate for both the plates is uniform and same through time and space?



(A) The rocks at P and S have the same age
(B) The rocks at S are twice as old as those at R
(C) The age of rocks at Q is 0 Ma
(D) The age of the rocks decreases progressively from P to S

Q.21 Mississippian and Pennsylvanian belong to which period?

(A) Permian
(B) Devonian
(C) Carboniferous
(D) Silurian

Q.22 A gravity dam with E–W axis is to be constructed in a narrow river valley between two N–S trending parallel ridges. The river is flowing from south to north. The lithology of the area is represented by 2 to 5 m thick metasedimentary rocks – quartzite, phyllite and schist. Which of the following geological conditions will be most suitable?

(A) Beds in both the ridges strike N–S but dip towards each other
(B) Beds in both the ridges strike N–S but dip in opposite direction to each other
(C) Beds in both the ridges strike E–W and dip towards N (downstream)
(D) Beds in both the ridges strike E–W and dip towards S (upstream)

Q.23 Match the ore deposits in Group I with the localities in Group II

Group I	**Group II**
i) Copper	P) Balaghat, M.P.
ii) Lead-Zinc	Q) Panchpatmali, Odisha
iii) Manganese	R) Rampura-Agucha, Rajasthan
iv) Bauxite	S) Khetri, Rajasthan

(A) i - S, ii - R, iii - Q, iv - P
(B) i - S, ii - R, iii - P, iv - Q
(C) i - P, ii - R, iii - Q, iv - S
(D) i - S, ii - Q, iii - R, iv - P

Q.24 Match the following stratigraphic units (Group-I) with their ages (Group-II)

Group I	**Group II**
i) Cumbum Formation	P) Cenozoic
ii) Baisakhi Formation	Q) Mesozoic
iii) Kopili Formation	R) Palaeozoic
iv) Barakar Formation	S) Proterozoic

(A) i - R, ii - P, iii - Q, iv - S
(B) i - S, ii - Q, iii - P, iv - R
(C) i - S, ii - R, iii - P, iv - Q
(D) i - S, ii - Q, iii - R, iv - P

Q.25 Which one of the following sedimentary structures is related to gravitational instability?

(A) groove cast (B) load cast (C) gutter cast (D) flute cast

Q.26 Match features mentioned in Group-I with the fossil types in Group-II.

Group I	**Group II**
i) Escutcheon	P) Brachiopoda
ii) Plastron	Q) Nautiloidea
iii) Delthyrium	R) Echinoidea
iv) Siphuncle	S) Bivalvia

(A) i - S, ii - R, iii - Q, iv - P
(B) i - S, ii - R, iii - P, iv - Q
(C) i - S, ii - P, iii - Q, iv - R
(D) i - R, ii - Q, iii - P, iv - S

Q.27 Which one of the following is an upper Gondwana flora?

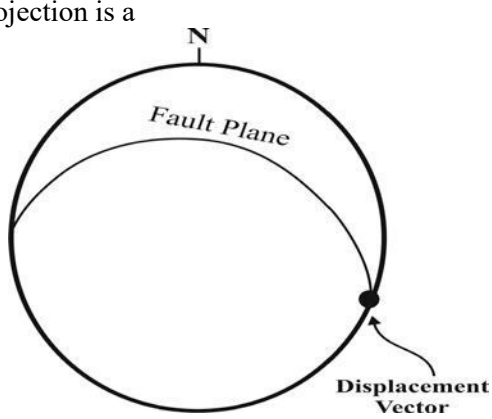
(A) *Ptillophyllum*
(B) *Phyllotheca*
(C) *Glossopteris*
(D) *Schizoneura*

Q.28 Choose the correct match of items in Group I with the items in Group II.

Group I	**Group II**
i) Eutectic crystallisation	P) Perthitic texture
ii) Fractional crystallisation	Q) Graphic texture
iii) Exsolution	R) Pillow structure
iv) Submarine eruption	S) Crystal zoning

(A) i - Q, ii - S, iii - R, iv - P
(B) i - Q, ii - S, iii - P, iv - R
(C) i - S, ii - Q, iii - P, iv - R
(D) i - S, ii-Q, iii - R, iv - P

Q.29 The fault for which the orientation and displacement vector are given in the following lower hemisphere stereographic projection is a



(A) vertical fault (B) thrust fault (C) normal fault (D) strike-slip fault

Q.30 The change in coordination number of silicon (Si) when α-quartz transforms to stishovite, which has octahedral coordination, is

(A) 2
(B) 4
(C) 6
(D) 8

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 Which of the following is/are found only in the brittle shear zones?

(A) Cataclasite
(B) Mylonite
(C) Psuedotachylite
(D) Gouge

Q.32 Which among the following statements related to headward erosion by river is/are correct?

(A) Length of tributaries increases
(B) Length of the main channel increases
(C) Streams get captured
(D) Channel gradient increases upstream

Q.33 In a thin section of 30 μm thickness, the R.I. of a mineral are: ε = 1.565 and ω = 1.468. Choose the correct statement(s) about its optical properties.

(A) Uniaxial +ve
(B) Uniaxial –ve
(C) 1st order interference colour
(D) low to moderate relief

Q.34 Which of the following statements is/are NOT true for equilibrium crystallization of plagioclase from a melt in the binary system $NaAlSi_3O_8$–$CaAlSi_2O_8$ at constant pressure?

(A) Composition of the first-formed solid depends on the initial composition of the melt.
(B) Solid composition is always more anorthitic than the coexisting melt.
(C) Composition of the final solid is the same as the initial composition of the melt.
(D) Equilibrium crystallization leads to the formation of zoned crystals.

Q.35 The value of gravity at the Earth's surface is dependent on

(A) latitude
(B) altitude
(C) composition of underlying material
(D) relative position of Sun–Earth

Q.36 Choose the burrowing form(s) of bivalve.

(A) *Mya* (B) *Teredo* (C) *Pecten* (D) *Venus*

Q.37 Which of the following environments is/are characterized by predominant deposit of mud?

(A) Barrier bar
(B) Lagoon
(C) Fluvial flood plain
(D) Fluvial channel

Q.38 Which of the following stratigraphic units belong(s) to Cretaceous?

(A) Bhuj Formation
(B) Ariyalur Group
(C) Patcham Formation
(D) Katrol Formation

Q.39 Which of the following ore deposit(s) is/are formed only by hydrothermal process?

(A) 'Sn-W' ore associated with greissenised rock
(B) Layered type chromite ore associated with dunite-peridotite-pyroxenite
(C) Vein type gold ore associated with greenstone belt
(D) Ni-Cu sulphide ore associated with gabbroic rocks.

Q.40 Migmatite is a rock

(A) in which mafic-rich parts are intermixed with pods or layers of granitic composition
(B) with melanosome and leucosome
(C) with a solid residue and partial melt
(D) which forms at high grade metamorphic conditions

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

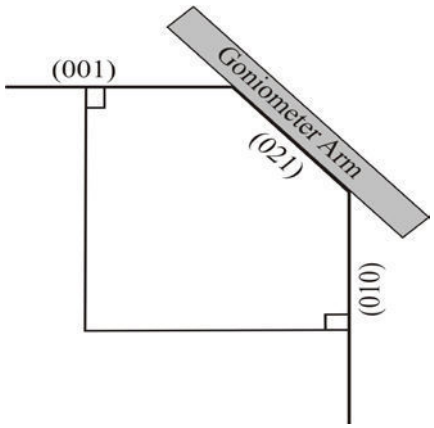
Q.41 Based on the ideal end member formula of diopside, the mole proportion (%) of CaO for plotting the mineral on a CaO-MgO-SiO_2 triangular diagram is_____.

Q.42 The phi (ϕ) value of a sediment particle having 4 mm diameter is _____.

Q.43 Calcite, quartz, wollastonite and CO_2 fluid were present in equilibrium during the formation of a calc-silicate rock. In the chemical system CaO–SiO_2–CO_2, the degree of freedom of this assemblage is _____.

Q.44 Weight of a10 cm^3 medium grained sandstone block with 20% (v/v) porosity, in dry state is 26g. The density of the block when fully saturated with water is _____g/cm^3.

Q.45 In the following figure, the exterior angle measured between (001)∧(021) with a goniometer in a crystal is 40°. The interior angle between (010)∧(021) in degrees is _____.



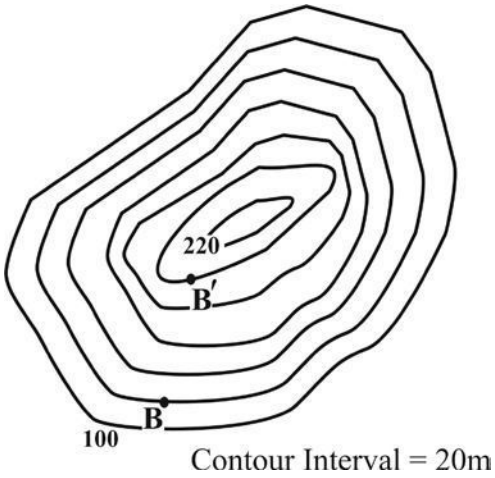
Q.46 If the elevation of a wave cut platform is 55 m above the sea level and the age of the erosional surface is 120 kilo years, the rate of rock uplift at this coastal location is _____m/kilo years (give answer in two decimal places).

Q.47 A foliation plane has strike 025° and 60° easterly dip. A mineral lineation on this foliation plane has a rake/pitch of 90°. The plunge direction of the mineral lineation in whole circle bearing is _____degrees.

Q.48 Two outcrops on a 1 : 25000 map are 12 cm apart. The ground distance between the two outcrops is _____ km.

Q.49 Fine muds are deposited at a rate of 1 cm per 1000 y. Assuming constant sedimentation rate and absence of compaction, a 1 km thick sequence would be deposited in _____million years.

Q.50 B and B′are two points on the topographic map shown below. The distance between B and B′ along the linear traverse BB′ is 220m. The angle of the slope along this traverse is _____ degree (give answer in two decimal places).



Q. 51 – Q. 60 carry two marks each.

Q.51 The half life of a radionuclide A is double that of a radionuclide B. The fraction of A remaining when B is reduced to 1/64 is _____. Give answer in three decimal places.

Q.52 The total metal content of a mineable 40m × 40m ×3m ore block having bulk density 2.75 g/cm^3 and assay value 1.5 wt % Cu is _____metric tonnes.

Q.53 The temperature at the Earth's surface is 25^{o}C. The temperature at the base of the Earth's crust (30 km thick), if the geothermal gradients are 25^{o}C/km up to 15km depth and 15^{o}C/km further down, is _____^{o}C.

Q.54 A melt in the binary system MgO–SiO_2 contains 89.92 wt% SiO_2. If all the magnesium is consumed to form enstatite ($MgSiO_3$), how many moles of this mineral will crystallize from 100 grams of the melt? Give answer in two decimal places.

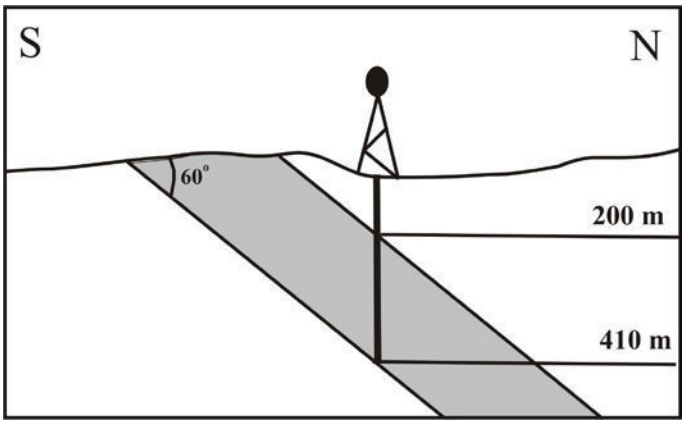
Molecular weight : MgO = 40.3, SiO_2 = 60.1

Q.55 A lherzolite xenolith from the mantle contains 50 volume % olivine, the rest being equal proportions of orthopyroxene and clinopyroxene. If the densities of the minerals are (in g/cc) olivine = 3.42, orthopyroxene = 3.28 and clinopyroxene = 3.46, the bulk density of the xenoliths in g/cc is_____ (give answer in two decimal places).

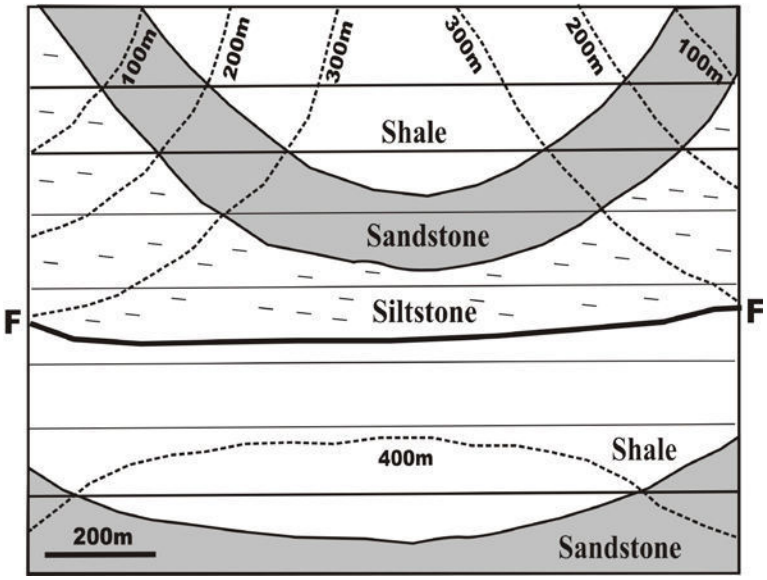
Q.56 The SiO_2 value, recalculated on volatile free basis, of the rock whose major oxide (wt%) composition given below is _____. Give answer in two decimal places.

SiO_2	45.58
Al_2O_3	12.49
FeO (T)	12.44
MgO	11.36
CaO	09.93
Na_2O	02.25
K_2O	01.67
Total	**95.72**

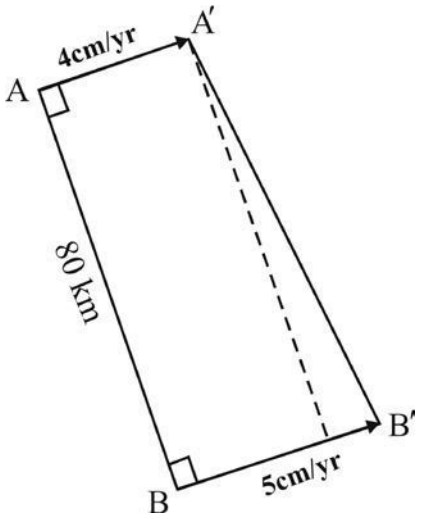
Q.57 As shown in the following figure, a vertical well intersects the top and bottom of an inclined bed at 200 m and 410 m depths, respectively. If the true dip of the bed is 60^o to the north, the true thickness of the bed is _____metres.



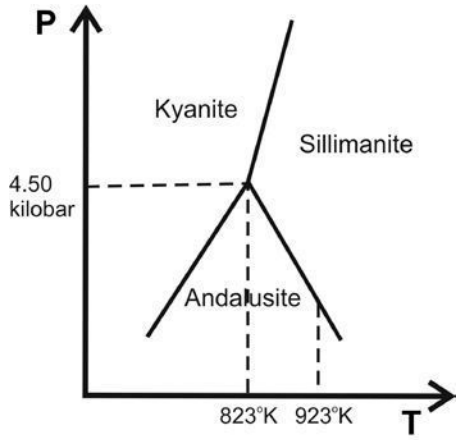
Q.58 In the given geologic map, the horizontal lines are stratum contours. The throw on the fault (F-F) is _____meters.



Q.59 Two localities A and B on a continental plate as shown in the figure below, are separated by a distance of 80 km. The plate velocities measured at A and B are 4 cm/yr and 5 cm/yr, respectively. Assuming no faulting in the area, the new distance between A and B will be_____ km in one million years. Give answer in two decimal places.



Q.60 In the following schematic diagram, the aluminosilicate triple point is located at the pressure of 4.50 kbar and temperature of 823°K. If the reaction andalusite = sillimanite has negative slope with a value of –18.22 bar/°K, the pressure of the reaction at 923°K is _____kbar (give answer in two decimal places).



(Figure not to scale)

END OF THE QUESTION PAPER

JAM 2016: Geology			
Qn. No.	Qn. Type	Key(s)	Mark(s)
1	MCQ	A	1
2	MCQ	A	1
3	MCQ	C	1
4	MCQ	D	1
5	MCQ	B	1
6	MCQ	B	1
7	MCQ	D	1
8	MCQ	A	1
9	MCQ	C	1
10	MCQ	A	1
11	MCQ	C	2
12	MCQ	B	2
13	MCQ	D	2
14	MCQ	C	2
15	MCQ	C	2
16	MCQ	A	2
17	MCQ	D	2
18	MCQ	A	2
19	MCQ	C	2
20	MCQ	D	2
21	MCQ	C	2
22	MCQ	D	2
23	MCQ	B	2
24	MCQ	B	2
25	MCQ	B	2
26	MCQ	B	2
27	MCQ	A	2
28	MCQ	B	2
29	MCQ	D	2
30	MCQ	A	2

JAM 2016: Geology			
Qn. No.	Qn. Type	Key(s)	Mark(s)
31	MSQ	A;C;D	2
32	MSQ	B;C;D	2
33	MSQ	A;D	2
34	MSQ	D	2
35	MSQ	A;B;C;D	2
36	MSQ	A;D	2
37	MSQ	B;C	2
38	MSQ	A;B	2
39	MSQ	A;C	2
40	MSQ	A;B;C;D	2

JAM 2016: Geology			
Qn. No.	Qn. Type	Key(s)	Mark(s)
41	NAT	25.0 to 25.0	1
42	NAT	-2.0 to -2.0	1
43	NAT	1.0 to 1.0	1
44	NAT	2.8 to 2.8	1
45	NAT	140 to 140	1
46	NAT	0.45 to 0.46	1
47	NAT	115 to 115	1
48	NAT	3.0 to 3.0	1
49	NAT	100 to 100	1
50	NAT	21.32 to 21.33	1
51	NAT	0.125 to 0.125	2
52	NAT	198 to 198	2
53	NAT	625 to 625	2
54	NAT	0.24 to 0.26	2
55	NAT	3.39 to 3.40	2
56	NAT	47.61 to 47.63	2
57	NAT	105 to 105	2
58	NAT	400 to 400	2
59	NAT	80.62 to 80.63	2
60	NAT	2.67 to 2.69	2

Special Instructions / Useful Data	
$\mathbb{R}$	Set of all real numbers
$\mathbb{R}^n$	$\{(x_1,\ldots,x_n): x_i \in \mathbb{R}, i=1,\ldots,n\}$
$P(A)$	Probability of an event A
i.i.d.	Independently and identically distributed
$Bin(n,p)$	Binomial distribution with parameters n and p
$Poisson(\theta)$	Poisson distribution with mean θ
$N(\mu,\sigma^2)$	Normal distribution with mean μ and variance σ^2
$Exp(\lambda)$	The exponential distribution with probability density function $f(x\mid\lambda)=\begin{cases}\lambda e^{-\lambda x}, & x>0,\\ 0, & \text{otherwise}\end{cases}$, $\lambda>0$
t_n	Student's t distribution with n degrees of freedom
χ^2_n	Chi-square distribution with n degrees of freedom
$\chi^2_{n,\alpha}$	A constant such that $P(W>\chi^2_{n,\alpha})=\alpha$, where W has χ^2_n distribution
$\Phi(x)$	Cumulative distribution function of $N(0,1)$
$\phi(x)$	Probability density function of $N(0,1)$
A^C	Complement of an event A
$E(X)$	Expectation of a random variable X
$Var(X)$	Variance of a random variable X
$B(m,n)$	$\int_0^1 x^{m-1}(1-x)^{n-1}dx$, $m>0$, $n>0$
$[x]$	The greatest integer less than or equal to real number x
f'	Derivative of function f
$\Phi(0.25)=0.5987$, $\Phi(0.5)=0.6915$, $\Phi(0.625)=0.7341$, $\Phi(0.71)=0.7612$, $\Phi(1)=0.8413$, $\Phi(1.125)=0.8697$, $\Phi(2)=0.9772$	

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 Let

$$P=\begin{bmatrix} 1 & 2 & 0 & 2 \\ -1 & -2 & 1 & 1 \\ 1 & 2 & -3 & -7 \\ 1 & 2 & -2 & -4 \end{bmatrix}.$$

Then rank of P equals

(A) 4
(B) 3
(C) 2
(D) 1

Q.2 Let α, β, γ be real numbers such that $\beta \neq 0$ and $\gamma \neq 0$. Suppose

$$P=\begin{bmatrix} \alpha & \beta \\ \gamma & 0 \end{bmatrix},$$

and $P^{-1} = P$. Then

(A) $\alpha = 0$ and $\beta\gamma = 1$
(B) $\alpha \neq 0$ and $\beta\gamma = 1$
(C) $\alpha = 0$ and $\beta\gamma = 2$
(D) $\alpha = 0$ and $\beta\gamma = -1$

Q.3 Let $m > 1$. The volume of the solid generated by revolving the region between the y-axis and the curve $xy = 4, 1 \le y \le m,$ about the y-axis is 15π. The value of m is

(A) 14 (B) 15 (C) 16 (D) 17

Q.4 Consider the region S enclosed by the surface $z = y^2$ and the planes $z = 1, x = 0, x = 1, y = -1$ and $y = 1$. The volume of S is

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) 1 (D) $\frac{4}{3}$

Q.5 Let X be a discrete random variable with the moment generating function

$$M_X(t) = e^{0.5(e^t - 1)}, \; t \in \mathbb{R}.$$

Then $P(X \le 1)$ equals

(A) $e^{-1/2}$ (B) $\frac{3}{2}e^{-1/2}$ (C) $\frac{1}{2}e^{-1/2}$ (D) $e^{-(e-1)/2}$

Q.6 Let E and F be two independent events with

$$P(E \mid F) + P(F \mid E) = 1, \; P(E \cap F) = \frac{2}{9} \text{ and } P(F) < P(E).$$

Then $P(E)$ equals

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

Q.7 Let X be a continuous random variable with the probability density function

$$f(x) = \frac{1}{(2+x^2)^{3/2}}, \quad x \in \mathbb{R}.$$

Then $E(X^2)$

(A) equals 0
(B) equals 1
(C) equals 2
(D) does not exist

Q.8 The probability density function of a random variable X is given by

$$f(x) = \begin{cases} \alpha x^{\alpha-1}, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}, \; \alpha > 0.$$

Then the distribution of the random variable $Y = \log_e X^{-2\alpha}$ is

(A) χ^2_2 (B) $\frac{1}{2}\chi^2_2$ (C) $2\chi^2_2$ (D) χ^2_1

Q.9 Let $X_1, X_2, \ldots$ be a sequence of i.i.d. $N(0,1)$ random variables. Then, as $n \to \infty$, $\frac{1}{n}\sum_{i=1}^{n} X_i^2$ converges in probability to

(A) 0 (B) 0.5 (C) 1 (D) 2

Q.10 Consider the simple linear regression model with n random observations $Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$, $i = 1,\ldots,n, (n > 2)$. β_0 and β_1 are unknown parameters, $x_1,\ldots,x_n$ are observed values of the regressor variable and $\varepsilon_1,\ldots,\varepsilon_n$ are error random variables with $E(\varepsilon_i) = 0$, $i = 1,\ldots,n$, and for $i, j = 1,\ldots,n$, $Cov(\varepsilon_i, \varepsilon_j) = \begin{cases} 0, & \text{if } i \neq j, \\ \sigma^2, & \text{if } i = j \end{cases}$. For real constants $a_1,\ldots,a_n$, if $\sum_{i=1}^{n} a_i Y_i$ is an unbiased estimator of β_1, then

(A) $\sum_{i=1}^{n} a_i = 0$ and $\sum_{i=1}^{n} a_i x_i = 0$ (B) $\sum_{i=1}^{n} a_i = 0$ and $\sum_{i=1}^{n} a_i x_i = 1$

(C) $\sum_{i=1}^{n} a_i = 1$ and $\sum_{i=1}^{n} a_i x_i = 0$ (D) $\sum_{i=1}^{n} a_i = 1$ and $\sum_{i=1}^{n} a_i x_i = 1$

Q. 11 – Q. 30 carry two marks each.

Q.11 Let (X, Y) have the joint probability density function

$$f(x,y) = \begin{cases} \frac{1}{2} y^2 e^{-x}, & \text{if } 0 < y < x < \infty, \\ 0, & \text{otherwise.} \end{cases}$$

Then $P(Y < 1 \mid X = 3)$ equals

(A) $\frac{1}{81}$ (B) $\frac{1}{27}$ (C) $\frac{1}{9}$ (D) $\frac{1}{3}$

Q.12 Let $X_1, X_2, \ldots$ be a sequence of i.i.d. random variables having the probability density function

$$f(x) = \begin{cases} \frac{1}{B(6,4)} x^5 (1-x)^3, & 0 < x < 1, \\ 0, & \text{otherwise.} \end{cases}$$

Let $Y_i = \frac{X_i}{1 - X_i}$ and $U_n = \frac{1}{n} \sum_{i=1}^{n} Y_i$. If the distribution of $\frac{\sqrt{n}(U_n - 2)}{\alpha}$ converges to $N(0,1)$ as $n \to \infty$, then a possible value of α is

(A) $\sqrt{7}$ (B) $\sqrt{5}$ (C) $\sqrt{3}$ (D) 1

Q.13 Let $X_1,\ldots,X_n$ be a random sample from a population with the probability density function

$$f(x\,|\,\theta)=\begin{cases}4e^{-4(x-\theta)}, & x>\theta,\\ 0, & \text{otherwise}\end{cases},\quad \theta\in\mathbb{R}.$$

If $T_n=\min\{X_1,\ldots,X_n\}$, then

(A) T_n is unbiased and consistent estimator of θ

(B) T_n is biased and consistent estimator of θ

(C) T_n is unbiased but NOT consistent estimator of θ

(D) T_n is NEITHER unbiased NOR consistent estimator of θ

Q.14 Let $X_1,\ldots,X_n$ be i.i.d. random variables with the probability density function

$$f(x)=\begin{cases}e^{-x}, & x>0,\\ 0, & \text{otherwise.}\end{cases}$$

If $X_{(n)}=\max\{X_1,\ldots,X_n\}$, then $\lim\limits_{n\to\infty}P\left(X_{(n)}-\log_e n\le 2\right)$ equals

(A) $1-e^{-2}$

(B) $e^{-e^{-0.5}}$

(C) $e^{-e^{-2}}$

(D) $e^{-e^{2}}$

Q.15 Let X and Y be two independent $N(0,1)$ random variables. Then $P\left(0<X^2+Y^2<4\right)$ equals

(A) $1-e^{-2}$ (B) $1-e^{-4}$ (C) $1-e^{-1}$ (D) e^{-2}

Q.16 Let X be a random variable with the cumulative distribution function

$$F(x)=\begin{cases}0, & x<0,\\ \dfrac{x}{8}, & 0\le x<2,\\ \dfrac{x^2}{16}, & 2\le x<4,\\ 1, & x\ge 4.\end{cases}$$

Then $E(X)$ equals

(A) $\dfrac{12}{31}$ (B) $\dfrac{13}{12}$ (C) $\dfrac{31}{21}$ (D) $\dfrac{31}{12}$

Q.17 Let $X_1,\ldots,X_n$ be a random sample from a population with the probability density function

$$f(x)=\frac{1}{2\theta}e^{-|x|/\theta},\ x\in\mathbb{R},\ \theta>0.$$

For a suitable constant K, the critical region of the most powerful test for testing $H_0:\theta=1$ against $H_1:\theta=2$ is of the form

(A) $\sum_{i=1}^{n}|X_i|>K$ (B) $\sum_{i=1}^{n}|X_i|<K$

(C) $\sum_{i=1}^{n}\frac{1}{|X_i|}<K$ (D) $\sum_{i=1}^{n}\frac{1}{|X_i|}>K$

Q.18 Let $X_1,\ldots,X_n,X_{n+1},X_{n+2},\ldots,X_{n+m}$ $(n>4,\ m>4)$ be a random sample from $N(\mu,\sigma^2)$; $\mu\in\mathbb{R},\ \sigma>0$. If $\bar{X}_1=\frac{1}{n}\sum_{i=1}^{n}X_i$ and $\bar{X}_2=\frac{1}{m-2}\sum_{i=n+1}^{n+m-2}X_i$, then the distribution of the random variable

$$T=\frac{X_{n+m}-X_{n+m-1}}{\sqrt{\sum_{i=1}^{n}(X_i-\bar{X}_1)^2+\sum_{i=n+1}^{n+m-2}(X_i-\bar{X}_2)^2}}$$

is

(A) t_{n+m-2}

(B) $\sqrt{\frac{2}{n+m-1}}\,t_{n+m-1}$

(C) $\sqrt{\frac{2}{n+m-4}}\,t_{n+m-4}$

(D) t_{n+m-4}

Q.19 Let $X_1,\ldots,X_n$ $(n>1)$ be a random sample from a $Poisson(\theta)$ population, $\theta>0$, and $T=\sum_{i=1}^{n}X_i$. Then the uniformly minimum variance unbiased estimator of θ^2 is

(A) $\frac{T(T-1)}{n^2}$ (B) $\frac{T(T-1)}{n(n-1)}$

(C) $\frac{T(T-1)}{n(n+1)}$ (D) $\frac{T^2}{n^2}$

Q.20 Let X be a random variable whose probability mass functions $f(x \mid H_0)$ (under the null hypothesis H_0) and $f(x \mid H_1)$ (under the alternative hypothesis H_1) are given by

$X = x$	0	1	2	3
$f(x \mid H_0)$	0.4	0.3	0.2	0.1
$f(x \mid H_1)$	0.1	0.2	0.3	0.4

For testing the null hypothesis $H_0 : X \sim f(x \mid H_0)$ against the alternative hypothesis $H_1 : X \sim f(x \mid H_1)$, consider the test given by: Reject H_0 if $X > \frac{3}{2}$.

If $\alpha =$ size of the test and $\beta =$ power of the test, then

(A) $\alpha = 0.3$ and $\beta = 0.3$
(B) $\alpha = 0.3$ and $\beta = 0.7$
(C) $\alpha = 0.7$ and $\beta = 0.3$
(D) $\alpha = 0.7$ and $\beta = 0.7$

Q.21 Let $X_1, \ldots, X_n$ be a random sample from a $N(2\theta, \theta^2)$ population, $\theta > 0$. A consistent estimator for θ is

(A) $\frac{1}{n}\sum_{i=1}^{n} X_i$ (B) $\left(\frac{5}{n}\sum_{i=1}^{n} X_i^2\right)^{1/2}$

(C) $\frac{1}{5n}\sum_{i=1}^{n} X_i^2$ (D) $\left(\frac{1}{5n}\sum_{i=1}^{n} X_i^2\right)^{1/2}$

Q.22 An institute purchases laptops from either vendor V_1 or vendor V_2 with equal probability. The lifetimes (in years) of laptops from vendor V_1 have a $U(0,4)$ distribution, and the lifetimes (in years) of laptops from vendor V_2 have an $Exp(1/2)$ distribution. If a randomly selected laptop in the institute has lifetime more than two years, then the probability that it was supplied by vendor V_2 is

(A) $\frac{2}{2+e}$ (B) $\frac{1}{1+e}$ (C) $\frac{1}{1+e^{-1}}$ (D) $\frac{2}{2+e^{-1}}$

Q.23 Let $y(x)$ be the solution to the differential equation

$$x^4 \frac{dy}{dx} + 4x^3 y + \sin x = 0; \quad y(\pi) = 1, \quad x > 0.$$

Then $y\left(\frac{\pi}{2}\right)$ is

(A) $\frac{10\left(1+\pi^4\right)}{\pi^4}$ (B) $\frac{12\left(1+\pi^4\right)}{\pi^4}$

(C) $\frac{14\left(1+\pi^4\right)}{\pi^4}$ (D) $\frac{16\left(1+\pi^4\right)}{\pi^4}$

Q.24 Let $a_n = e^{-2n}\sin n$ and $b_n = e^{-n} n^2 (\sin n)^2$ for $n \geq 1$. Then

(A) $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty} b_n$ does NOT converge

(B) $\sum_{n=1}^{\infty} b_n$ converges but $\sum_{n=1}^{\infty} a_n$ does NOT converge

(C) both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge

(D) NEITHER $\sum_{n=1}^{\infty} a_n$ NOR $\sum_{n=1}^{\infty} b_n$ converges

Q.25 Let

$$f(x) = \begin{cases} x \sin^2(1/x), & x \neq 0, \\ 0, & x = 0, \end{cases} \quad \text{and} \quad g(x) = \begin{cases} x(\sin x)\sin(1/x), & x \neq 0, \\ 0, & x = 0. \end{cases}$$

Then

(A) f is differentiable at 0 but g is NOT differentiable at 0

(B) g is differentiable at 0 but f is NOT differentiable at 0

(C) f and g are both differentiable at 0

(D) NEITHER f NOR g is differentiable at 0

Q.26 Let $f:[0,4]\to\mathbb{R}$ be a twice differentiable function. Further, let $f(0)=1$, $f(2)=2$ and $f(4)=3$. Then

(A) there does NOT exist any $x_1\in(0,2)$ such that $f'(x_1)=\frac{1}{2}$

(B) there exist $x_2\in(0,2)$ and $x_3\in(2,4)$ such that $f'(x_2)=f'(x_3)$

(C) $f''(x)>0$ for all $x\in(0,4)$

(D) $f''(x)<0$ for all $x\in(0,4)$

Q.27 Let $f(x,y)=x^2-400\,x\,y^2$ for all $(x,y)\in\mathbb{R}^2$. Then f attains its

(A) local minimum at $(0,0)$ but NOT at $(1,1)$

(B) local minimum at $(1,1)$ but NOT at $(0,0)$

(C) local minimum both at $(0,0)$ and $(1,1)$

(D) local minimum NEITHER at $(0,0)$ NOR at $(1,1)$

Q.28 Let $y(x)$ be the solution to the differential equation

$$4\frac{d^2y}{dx^2}+12\frac{dy}{dx}+9y=0,\quad y(0)=1,\quad y'(0)=-4.$$

Then $y(1)$ equals

(A) $-\frac{1}{2}e^{-3/2}$ (B) $-\frac{3}{2}e^{-3/2}$

(C) $-\frac{5}{2}e^{-3/2}$ (D) $-\frac{7}{2}e^{-3/2}$

Q.29 Let $g:[0,2]\to\mathbb{R}$ be defined by

$$g(x)=\int_0^x(x-t)e^t\,dt.$$

The area between the curve $y=g''(x)$ and the x-axis over the interval $[0,2]$ is

(A) e^2-1 (B) $2(e^2-1)$

(C) $4(e^2-1)$ (D) $8(e^2-1)$

Q.30 Let P be a 3×3 singular matrix such that $P\vec{v}=\vec{v}$ for a nonzero vector $\vec{v}$ and

$$P\begin{bmatrix}1\\0\\-1\end{bmatrix}=\begin{bmatrix}2/5\\0\\-2/5\end{bmatrix}.$$

Then

(A) $P^3=\frac{1}{5}\left(7P^2-2P\right)$

(B) $P^3=\frac{1}{4}\left(7P^2-2P\right)$

(C) $P^3=\frac{1}{3}\left(7P^2-2P\right)$

(D) $P^3=\frac{1}{2}\left(7P^2-2P\right)$

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 For two nonzero real numbers a and b, consider the system of linear equations

$$\begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} b/2 \\ a/2 \end{bmatrix}.$$

Which of the following statements is (are) TRUE?

(A) If $a=b$, the solutions of the system lie on the line $x+y=1/2$

(B) If $a=-b$, the solutions of the system lie on the line $y-x=1/2$

(C) If $a\neq\pm b$, the system has no solution

(D) If $a\neq\pm b$, the system has a unique solution

Q.32 For $n\geq 1$, let

$$a_n=\begin{cases} n\ 2^{-n}, & \text{if } n \text{ is odd,} \\ -3^{-n}, & \text{if } n \text{ is even.} \end{cases}$$

Which of the following statements is (are) TRUE?

(A) The sequence $\{a_n\}$ converges

(B) The sequence $\{|a_n|^{1/n}\}$ converges

(C) The series $\sum_{n=1}^{\infty} a_n$ converges

(D) The series $\sum_{n=1}^{\infty} |a_n|$ converges

Q.33 Let $f:(0,\infty)\to\mathbb{R}$ be defined by

$$f(x)=x\left(e^{1/x^3}-1+\frac{1}{x^3}\right).$$

Which of the following statements is (are) TRUE?

(A) $\lim_{x\to\infty} f(x)$ exists

(B) $\lim_{x\to\infty} x f(x)$ exists

(C) $\lim_{x\to\infty} x^2 f(x)$ exists

(D) There exists $m>0$ such that $\lim_{x\to\infty} x^m f(x)$ does NOT exist.

Q.34 For $x \in \mathbb{R}$, define $f(x) = \cos(\pi x) + [x^2]$ and $g(x) = \sin(\pi x)$. Which of the following statements is (are) TRUE?

(A) $f(x)$ is continuous at $x = 2$

(B) $g(x)$ is continuous at $x = 2$

(C) $f(x) + g(x)$ is continuous at $x = 2$

(D) $f(x)\, g(x)$ is continuous at $x = 2$

Q.35 Let E and F be two events with $0 < P(E) < 1$, $0 < P(F) < 1$ and $P(E \mid F) > P(E)$. Which of the following statements is (are) TRUE?

(A) $P(F \mid E) > P(F)$

(B) $P(E \mid F^C) > P(E)$

(C) $P(F \mid E^C) < P(F)$

(D) E and F are independent

Q.36 Let $X_1, \ldots, X_n$ $(n > 1)$ be a random sample from a $U(2\theta - 1, 2\theta + 1)$ population, $\theta \in \mathbb{R}$, and $Y_1 = \min\{X_1, \ldots, X_n\}$, $Y_n = \max\{X_1, \ldots, X_n\}$. Which of the following statistics is (are) maximum likelihood estimator (s) of θ?

(A) $\frac{1}{4}(Y_1 + Y_n)$

(B) $\frac{1}{6}(2Y_1 + Y_n + 1)$

(C) $\frac{1}{8}(Y_1 + 3Y_n - 2)$

(D) Every statistic $T(X_1, \ldots, X_n)$ satisfying $\frac{(Y_n - 1)}{2} < T(X_1, \ldots, X_n) < \frac{(Y_1 + 1)}{2}$

Q.37 Let $X_1, \ldots, X_n$ be a random sample from a $N(0, \sigma^2)$ population, $\sigma > 0$. Which of the following testing problems has (have) the region $\left\{(x_1, \ldots, x_n) \in \mathbb{R}^n : \sum_{i=1}^{n} x_i^2 \geq \chi^2_{n,\alpha}\right\}$ as the most powerful critical region of level α?

(A) $H_0 : \sigma^2 = 1$ against $H_1 : \sigma^2 = 2$

(B) $H_0 : \sigma^2 = 1$ against $H_1 : \sigma^2 = 4$

(C) $H_0 : \sigma^2 = 2$ against $H_1 : \sigma^2 = 1$

(D) $H_0 : \sigma^2 = 1$ against $H_1 : \sigma^2 = 0.5$

Q.38 Let $X_1,\ldots,X_n$ be a random sample from a $N\left(0,\,2\theta^2\right)$ population, $\theta>0$. Which of the following statements is (are) TRUE?

(A) $\left(X_1,\ldots,X_n\right)$ is sufficient and complete

(B) $\left(X_1,\ldots,X_n\right)$ is sufficient but NOT complete

(C) $\sum_{i=1}^{n} X_i^2$ is sufficient and complete

(D) $\frac{1}{2n}\sum_{i=1}^{n} X_i^2$ is the uniformly minimum variance unbiased estimator for θ^2

Q.39 Let $X_1,\ldots,X_n$ be a random sample from a population with the probability density function

$$f\left(x\,|\,\theta\right)=\begin{cases}\theta e^{-\theta x}, & x>0,\\ 0, & \text{otherwise}\end{cases},\ \theta>0.$$

Which of the following is (are) $100\left(1-\alpha\right)\%$ confidence interval(s) for θ?

(A) $\left(\frac{\chi^2_{2n,1-\alpha/2}}{2\sum_{i=1}^{n} X_i},\ \frac{\chi^2_{2n,\alpha/2}}{2\sum_{i=1}^{n} X_i}\right)$

(B) $\left(0,\ \frac{\chi^2_{2n,\alpha}}{2\sum_{i=1}^{n} X_i}\right)$

(C) $\left(\frac{\chi^2_{2n,1-\alpha/2}}{\sum_{i=1}^{n} X_i},\ \frac{\chi^2_{2n,\alpha/2}}{\sum_{i=1}^{n} X_i}\right)$

(D) $\left(\frac{2\sum_{i=1}^{n} X_i}{\chi^2_{2n,\alpha/2}},\ \frac{2\sum_{i=1}^{n} X_i}{\chi^2_{2n,1-\alpha/2}}\right)$

Q.40 The cumulative distribution function of a random variable X is given by

$$F\left(x\right)=\begin{cases}0, & x<2,\\ \frac{1}{10}\left(x^2-\frac{7}{3}\right), & 2\le x<3,\\ 1, & x\ge 3.\end{cases}$$

Which of the following statements is (are) TRUE?

(A) $F\left(x\right)$ is continuous everywhere

(B) $F\left(x\right)$ increases only by jumps

(C) $P\left(X=2\right)=\frac{1}{6}$

(D) $P\left(X=\frac{5}{2}\,|\,2\le X\le 3\right)=0$

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 Let $X_1,\ldots,X_{10}$ be a random sample from a $N(3,12)$ population. Suppose $Y_1=\frac{1}{6}\sum_{i=1}^{6}X_i$ and $Y_2=\frac{1}{4}\sum_{i=7}^{10}X_i$. If $\frac{(Y_1-Y_2)^2}{\alpha}$ has a χ_1^2 distribution, then the value of α is _____

Q.42 Let X be a continuous random variable with the probability density function

$$f(x)=\begin{cases}\frac{2x}{9}, & 0<x<3,\\ 0, & \text{otherwise.}\end{cases}$$

Then the upper bound of $P(|X-2|>1)$ using Chebyshev's inequality is _____

Q.43 Let X and Y be continuous random variables with the joint probability density function

$$f(x,y)=\begin{cases}e^{(x+y)}, & -\infty<x,\ y<0,\\ 0, & \text{otherwise.}\end{cases}$$

Then $P(X<Y)=$_____

Q.44 Let X and Y be continuous random variables with the joint probability density function

$$f(x,y)=\frac{1}{2\pi}e^{-(x^2+y^2)/2},\ (x,\ y)\in\mathbb{R}^2.$$

Then $P(X>0,\ Y<0)=$ _____

Q.45 Let Y be a $Bin\left(72,\frac{1}{3}\right)$ random variable. Using normal approximation to binomial distribution, an approximate value of $P(22\le Y\le 28)$ is _____

Q.46 Let X be a $Bin(2, p)$ random variable and Y be a $Bin(4, p)$ random variable, $0 < p < 1$. If $P(X \geq 1) = \frac{5}{9}$, then $P(Y \geq 1) =$ _____

Q.47 Consider the linear transformation

$$T(x, y, z) = (2x + y + z,\ x + z,\ 3x + 2y + z).$$

The rank of T is _____

Q.48 The value of $\lim_{n \to \infty} n\left[e^{-n}\cos(4n) + \sin\left(\frac{1}{4n}\right)\right]$ is _____

Q.49 Let $f : [0, 13] \to \mathbb{R}$ be defined by $f(x) = x^{13} - e^{-x} + 5x + 6$. The minimum value of the function f on $[0,13]$ is_____

Q.50 Consider a differentiable function f on $[0,1]$ with the derivative $f'(x) = 2\sqrt{2x}$. The arc length of the curve $y = f(x)$, $0 \leq x \leq 1$, is _____

Q. 51 – Q. 60 carry two marks each.

Q.51 Let m be a real number such that $m > 1$. If

$$\int_1^m \int_0^1 \int_{\sqrt{x}}^1 e^{y^3}\, dy\, dx\, dz = e - 1,$$

then $m =$ _____

Q.52 Let

$$P = \begin{bmatrix} 1 & -3 & 3 \\ 0 & -5 & 6 \\ 0 & -3 & 4 \end{bmatrix}.$$

The product of the eigen values of P^{-1} is _____

Q.53 The value of the real number m in the following equation

$$\int_0^1 \int_x^{\sqrt{2-x^2}} \left(x^2+y^2\right) dy\, dx = \int_{m\pi}^{\pi/2} \int_0^{\sqrt{2}} r^3\, dr\, d\theta$$

is _____

Q.54 Let $a_1 = 1$ and $a_n = 2 - \frac{1}{n}$ for $n \geq 2$. Then

$$\sum_{n=1}^{\infty} \left(\frac{1}{a_n^2} - \frac{1}{a_{n+1}^2} \right)$$

converges to _____

Q.55 Let $X_1, X_2, \ldots$ be a sequence of i.i.d. random variables with the probability density function

$$f(x) = \begin{cases} 4x^2 e^{-2x}, & x > 0, \\ 0, & \text{otherwise} \end{cases}$$

and let $S_n = \sum_{i=1}^{n} X_i$. Then $\lim_{n \to \infty} P\left(S_n \leq \frac{3n}{2} + \sqrt{3n} \right)$ is _____

Q.56 Let X and Y be continuous random variables with the joint probability density function

$$f(x,y) = \begin{cases} \frac{c\, x^2}{y^3}, & 0 < x < 1,\ y > 1, \\ 0, & \text{otherwise} \end{cases},$$

where c is a suitable constant. Then $E(X) =$ _____

Q.57 Two points are chosen at random on a line segment of length 9 cm. The probability that the distance between these two points is less than 3 cm is _____

Q.58 Let X be a continuous random variable with the probability density function

$$f(x)=\begin{cases}\dfrac{x+1}{2}, & -1<x<1,\\ 0, & \text{otherwise.}\end{cases}$$

Then $P\left(\frac{1}{4}<X^2<\frac{1}{2}\right)=$ _____

Q.59 If X is a $U(0,1)$ random variable, then $P\left(\min(X,1-X)\le\frac{1}{4}\right)=$ _____

Q.60 In a colony all families have at least one child. The probability that a randomly chosen family from this colony has exactly k children is $(0.5)^k$; $k=1,2,\ldots$. A child is either a male or a female with equal probability. The probability that such a family consists of at least one male child and at least one female child is _____

END OF THE QUESTION PAPER

JAM 2016: Mathematical Statistics			
Qn. No.	Qn. Type	Key(s)	Mark(s)
1	MCQ	C	1
2	MCQ	A	1
3	MCQ	C	1
4	MCQ	D	1
5	MCQ	B	1
6	MCQ	C	1
7	MCQ	D	1
8	MCQ	A	1
9	MCQ	C	1
10	MCQ	B	1
11	MCQ	B	2
12	MCQ	C	2
13	MCQ	B	2
14	MCQ	C	2
15	MCQ	A	2
16	MCQ	D	2
17	MCQ	A	2
18	MCQ	C	2
19	MCQ	A	2
20	MCQ	B	2
21	MCQ	D	2
22	MCQ	A	2
23	MCQ	D	2
24	MCQ	C	2
25	MCQ	B	2
26	MCQ	B	2
27	MCQ	D	2
28	MCQ	B	2
29	MCQ	A	2
30	MCQ	A	2

JAM 2016: Mathematical Statistics			
Qn. No.	Qn. Type	Key(s)	Mark(s)
31	MSQ	A;B;D	2
32	MSQ	A;C;D	2
33	MSQ	A;B;C;D	2
34	MSQ	B;D	2
35	MSQ	A;C	2
36	MSQ	A;B;C;D	2
37	MSQ	A;B	2
38	MSQ	B;C;D	2
39	MSQ	A;B	2
40	MSQ	C;D	2

JAM 2016: Mathematical Statistics			
Qn. No.	Qn. Type	Key(s)	Mark(s)
41	NAT	4.9 to 5.1	1
42	NAT	0.49 to 0.51	1
43	NAT	0.49 to 0.51	1
44	NAT	0.24 to 0.26	1
45	NAT	0.52 to 0.62	1
46	NAT	0.80 to 0.81	1
47	NAT	1.9 to 2.1	1
48	NAT	0.24 to 0.26	1
49	NAT	4.9 to 5.1	1
50	NAT	2.1 to 2.2	1
51	NAT	3.9 to 4.1	2
52	NAT	-0.51 to -0.49	2
53	NAT	0.24 to 0.26	2
54	NAT	0.74 to 0.76	2
55	NAT	0.97 to 0.98	2
56	NAT	0.74 to 0.76	2
57	NAT	0.5 to 0.6	2
58	NAT	0.20 to 0.21	2
59	NAT	0.49 to 0.51	2
60	NAT	0.3 to 0.4	2

Notation

$\mathbb{N}$	The set of all natural numbers $\{1,2,3, \ldots\}$
$\mathbb{Z}$	The set of all integers
$\mathbb{Q}$	The set of all rational numbers
$\mathbb{R}$	The set of all real numbers
S_n	The group of permutations of n distinct symbols
$\mathbb{Z}_n$	$\{0, 1, 2, \ldots, n-1\}$ with addition and multiplication modulo n
ϕ	empty set
A^T	Transpose of A
i	$\sqrt{-1}$
$\hat{\imath}, \hat{\jmath}, \hat{k}$	unit vectors having the directions of the positive x, y and z axes of a three dimensional rectangular coordinate system
∇	$\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$
I_n	Identity matrix of order n
ln	logarithm with base e

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 The sequence $\{s_n\}$ of real numbers given by

$$s_n = \frac{\sin\frac{\pi}{2}}{1\cdot 2} + \frac{\sin\frac{\pi}{2^2}}{2\cdot 3} + \cdots + \frac{\sin\frac{\pi}{2^n}}{n\cdot(n+1)}$$

is

(A) a divergent sequence
(B) an oscillatory sequence
(C) not a Cauchy sequence
(D) a Cauchy sequence

Q.2 Let P be the vector space (over $\mathbb{R}$) of all polynomials of degree ≤ 3 with real coefficients. Consider the linear transformation $T: P \to P$ defined by

$$T(a_0 + a_1x + a_2x^2 + a_3x^3) = a_3 + a_2x + a_1x^2 + a_0x^3.$$

Then the matrix representation M of T with respect to the ordered basis $\{1, x, x^2, x^3\}$ satisfies

(A) $M^2 + I_4 = 0$ (B) $M^2 - I_4 = 0$
(C) $M - I_4 = 0$ (D) $M + I_4 = 0$

Q.3 Let $f: [-1, 1] \to \mathbb{R}$ be a continuous function. Then the integral

$$\int_0^{\pi} x\, f(\sin x)\, dx$$

is equivalent to

(A) $\frac{\pi}{2}\int_0^{\pi} f(\sin x)\, dx$ (B) $\frac{\pi}{2}\int_0^{\pi} f(\cos x)\, dx$

(C) $\pi\int_0^{\pi} f(\cos x)\, dx$ (D) $\pi\int_0^{\pi} f(\sin x)\, dx$

Q.4 Let σ be an element of the permutation group S_5. Then the maximum possible order of σ is

(A) 5 (B) 6 (C) 10 (D) 15

Q.5 Let f be a strictly monotonic continuous real valued function defined on $[a, b]$ such that $f(a) < a$ and $f(b) > b$. Then which one of the following is TRUE?

(A) There exists exactly one $c \in (a, b)$ such that $f(c) = c$
(B) There exist exactly two points $c_1, c_2 \in (a, b)$ such that $f(c_i) = c_i,\ i = 1,2$
(C) There exists no $c \in (a, b)$ such that $f(c) = c$
(D) There exist infinitely many points $c \in (a, b)$ such that $f(c) = c$

Q.6 The value of $\lim_{(x,\ y)\to(2,-2)} \frac{\sqrt{(x-y)}-2}{x-y-4}$ is

(A) 0 (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$

Q.7 Let $\vec{r} = \left(x\,\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k}\right)$ and $r = |\vec{r}|$. If $f(r) = \ln r$ and $g(r) = \frac{1}{r}$, $r \neq 0$, satisfy $2\,\nabla f + h(r)\nabla g = \vec{0}$, then $h(r)$ is

(A) r (B) $\frac{1}{r}$ (C) $2r$ (D) $\frac{2}{r}$

Q.8 The nonzero value of n for which the differential equation

$$(3xy^2 + n^2x^2y)dx + (nx^3 + 3x^2y)\,dy = 0, \quad x \neq 0,$$

becomes exact is

(A) -3 (B) -2 (C) 2 (D) 3

Q.9 One of the points which lies on the solution curve of the differential equation

$$(y - x)dx + (x + y)dy = 0,$$

with the given condition $y(0) = 1$, is

(A) $(1, -2)$ (B) $(2, -1)$ (C) $(2, 1)$ (D) $(-1, 2)$

Q.10 Let S be a closed subset of $\mathbb{R}$, T a compact subset of $\mathbb{R}$ such that $S \cap T \neq \phi$. Then $S \cap T$ is

(A) closed but not compact
(B) not closed
(C) compact
(D) neither closed nor compact

Q. 11 – Q. 30 carry two marks each.

Q.11 Let S be the series

$$\sum_{k=1}^{\infty} \frac{1}{(2k-1)\,2^{(2k-1)}}$$

and T be the series

$$\sum_{k=2}^{\infty} \left(\frac{3k-4}{3k+2}\right)^{\frac{(k+1)}{3}}$$

of real numbers. Then which one of the following is TRUE?

(A) Both the series S and T are convergent
(B) S is convergent and T is divergent
(C) S is divergent and T is convergent
(D) Both the series S and T are divergent

Q.12 Let $\{a_n\}$ be a sequence of positive real numbers satisfying

$$\frac{4}{a_{n+1}} = \frac{3}{a_n} + \frac{a_n^3}{81}, \qquad n \geq 1, \ a_1 = 1.$$

Then all the terms of the sequence lie in

(A) $\left[\frac{1}{2}, \frac{3}{2}\right]$ (B) $[0, 1]$ (C) $[1, 2]$ (D) $[1, 3]$

Q.13 The largest eigenvalue of the matrix $\begin{bmatrix} 1 & 4 & 16 \\ 4 & 16 & 1 \\ 16 & 1 & 4 \end{bmatrix}$ is

(A) 16 (B) 21

(C) 48 (D) 64

Q.14 The value of the integral

$$\frac{(2n)!}{2^{2n}\,(n!)} \int_{-1}^{1} (1-x^2)^n \, dx, \qquad n \in \mathbb{N}$$

is

(A) $\frac{2}{(2n+1)!}$ (B) $\frac{2n}{(2n+1)!}$

(C) $\frac{2\,(n!)}{2n+1}$ (D) $\frac{(n+1)!}{2n+1}$

Q.15 If the triple integral over the region bounded by the planes

$$2x + y + z = 4, \qquad x = 0, \qquad y = 0, \qquad z = 0$$

is given by

$$\int_0^2 \int_0^{\lambda(x)} \int_0^{\mu(x,y)} dz\, dy\, dx\,,$$

then the function $\lambda(x) - \mu(x, y)$ is

(A) $x + y$ (B) $x - y$ (C) x (D) y

Q.16 The surface area of the portion of the plane $y + 2z = 2$ within the cylinder $x^2 + y^2 = 3$ is

(A) $\frac{3\sqrt{5}}{2}\pi$ (B) $\frac{5\sqrt{5}}{2}\pi$ (C) $\frac{7\sqrt{5}}{2}\pi$ (D) $\frac{9\sqrt{5}}{2}\pi$

Q.17 Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} \dfrac{xy^2}{x+y} & \text{if } x+y \neq 0 \\ 0 & \text{if } x+y = 0 \end{cases}.$$

Then the value of $\left(\frac{\partial^2 f}{\partial x\,\partial y} + \frac{\partial^2 f}{\partial y\,\partial x}\right)$ at the point $(0,0)$ is

(A) 0 (B) 1 (C) 2 (D) 4

Q.18 The function $f(x,y) = 3x^2y + 4y^3 - 3x^2 - 12y^2 + 1$ has a saddle point at

(A) $(0,\ 0)$ (B) $(0,\ 2)$ (C) $(1,\ 1)$ (D) $(-2,\ 1)$

Q.19 Consider the vector field $\vec{F} = r^{\beta}(y\,\hat{\imath} - x\hat{\jmath})$, where $\beta \in \mathbb{R}$, $\vec{r} = x\hat{\imath} + y\hat{\jmath}$ and $r = |\vec{r}|$. If the absolute value of the line integral $\oint_C \vec{F} \cdot d\vec{r}$ along the closed curve $C: x^2 + y^2 = a^2$ (oriented counter clockwise) is 2π, then β is

(A) -2 (B) -1 (C) 1 (D) 2

Q.20 Let S be the surface of the cone $z = \sqrt{x^2 + y^2}$ bounded by the planes $z = 0$ and $z = 3$. Further, let C be the closed curve forming the boundary of the surface S. A vector field $\vec{F}$ is such that $\nabla \times \vec{F} = -x\hat{\imath} - y\hat{\jmath}$. The absolute value of the line integral $\oint_C \vec{F} \cdot d\vec{r}$, where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $r = |\vec{r}|$, is

(A) 0 (B) 9π (C) 15π (D) 18π

Q.21 Let $y(x)$ be the solution of the differential equation

$$\frac{d}{dx}\left(x\frac{dy}{dx}\right) = x;\quad y(1) = 0,\quad \left.\frac{dy}{dx}\right|_{x=1} = 0.$$

Then $y(2)$ is

(A) $\frac{3}{4} + \frac{1}{2}\ln 2$ (B) $\frac{3}{4} - \frac{1}{2}\ln 2$

(C) $\frac{3}{4} + \ln 2$ (D) $\frac{3}{4} - \ln 2$

Q.22 The general solution of the differential equation with constant coefficients

$$\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$

approaches zero as $x \to \infty$, if

(A) b is negative and c is positive
(B) b is positive and c is negative
(C) both b and c are positive
(D) both b and c are negative

Q.23 Let $S \subset \mathbb{R}$ and ∂S denote the set of points x in $\mathbb{R}$ such that every neighbourhood of x contains some points of S as well as some points of complement of S. Further, let $\overline{S}$ denote the closure of S. Then which one of the following is FALSE?

(A) $\partial\mathbb{Q} = \mathbb{R}$
(B) $\partial(\mathbb{R} \setminus T) = \partial T, \quad T \subset \mathbb{R}$
(C) $\partial(T \cup V) = \partial T \cup \partial V, \quad T, V \subset \mathbb{R},\ T \cap V \neq \phi$
(D) $\partial T = \overline{T} \cap (\overline{\mathbb{R} \setminus T}), \quad T \subset \mathbb{R}$

Q.24 The sum of the series

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n^2 + n - 2}$$

is

(A) $\frac{1}{3}\ln 2 - \frac{5}{18}$ (B) $\frac{1}{3}\ln 2 - \frac{5}{6}$ (C) $\frac{2}{3}\ln 2 - \frac{5}{18}$ (D) $\frac{2}{3}\ln 2 - \frac{5}{6}$

Q.25 Let $f(x) = \frac{1}{1+|x|} + \frac{1}{1+|x-1|}$ for all $x \in [-1, 1]$. Then which one of the following is TRUE?

(A) Maximum value of $f(x)$ is $\frac{3}{2}$
(B) Minimum value of $f(x)$ is $\frac{1}{3}$
(C) Maximum of $f(x)$ occurs at $x = \frac{1}{2}$
(D) Minimum of $f(x)$ occurs at $x = 1$

Q.26 The matrix $M = \begin{bmatrix} \cos\alpha & \sin\alpha \\ i\sin\alpha & i\cos\alpha \end{bmatrix}$ is a unitary matrix when α is

(A) $(2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$ (B) $(3n+1)\frac{\pi}{3}, n \in \mathbb{Z}$

(C) $(4n+1)\frac{\pi}{4}, n \in \mathbb{Z}$ (D) $(5n+1)\frac{\pi}{5}, n \in \mathbb{Z}$

Q.27 Let $M = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & \alpha \\ 2 & -\alpha & 0 \end{bmatrix}$, $\alpha \in \mathbb{R} \setminus \{0\}$ and $\boldsymbol{b}$ a non-zero vector such that $M\boldsymbol{x} = \boldsymbol{b}$ for some $\boldsymbol{x} \in \mathbb{R}^3$. Then the value of $\boldsymbol{x}^T\boldsymbol{b}$ is

(A) $-\alpha$ (B) α (C) 0 (D) 1

Q.28 The number of group homomorphisms from the cyclic group $\mathbb{Z}_4$ to the cyclic group $\mathbb{Z}_7$ is

(A) 7 (B) 3 (C) 2 (D) 1

Q.29 In the permutation group S_n $(n \geq 5)$, if H is the smallest subgroup containing all the 3-cycles, then which one of the following is TRUE?

(A) Order of H is 2
(B) Index of H in S_n is 2
(C) H is abelian
(D) $H = S_n$

Q.30 Let $f: \mathbb{R} \to \mathbb{R}$ be defined as

$$f(x) = \begin{cases} x(1 + x^{\alpha} \sin(\ln x^2)) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}.$$

Then, at $x = 0$, the function f is

(A) continuous and differentiable when $\alpha = 0$
(B) continuous and differentiable when $\alpha > 0$
(C) continuous and differentiable when $-1 < \alpha < 0$
(D) continuous and differentiable when $\alpha < -1$

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 Let $\{s_n\}$ be a sequence of positive real numbers satisfying

$$2\, s_{n+1} = s_n^2 + \frac{3}{4}, \qquad n \geq 1.$$

If α and β are the roots of the equation $x^2 - 2x + \frac{3}{4} = 0$ and $\alpha < s_1 < \beta$, then which of the following statement(s) is(are) TRUE ?

(A) $\{s_n\}$ is monotonically decreasing
(B) $\{s_n\}$ is monotonically increasing
(C) $\lim_{n\to\infty} s_n = \alpha$
(D) $\lim_{n\to\infty} s_n = \beta$

Q.32 The value(s) of the integral

$$\int_{-\pi}^{\pi} |x| \cos nx \, dx \ , \quad n \geq 1$$

is (are)

(A) 0 when n is even
(B) 0 when n is odd
(C) $-\frac{4}{n^2}$ when n is even
(D) $-\frac{4}{n^2}$ when n is odd

Q.33 Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} \dfrac{xy}{|x|} & \text{if } x \neq 0 \\ 0 & \text{elsewhere} \end{cases}.$$

Then at the point $(0,0)$, which of the following statement(s) is(are) TRUE ?

(A) f is not continuous
(B) f is continuous
(C) f is differentiable
(D) Both first order partial derivatives of f exist

Q.34 Consider the vector field $\vec{F} = x\hat{\imath} + y\hat{\jmath}$ on an open connected set $S \subset \mathbb{R}^2$. Then which of the following statement(s) is(are) TRUE ?

(A) Divergence of $\vec{F}$ is zero on S
(B) The line integral of $\vec{F}$ is independent of path in S
(C) $\vec{F}$ can be expressed as a gradient of a scalar function on S
(D) The line integral of $\vec{F}$ is zero around any piecewise smooth closed path in S

Q.35 Consider the differential equation

$$\sin 2x \ \frac{dy}{dx} = 2y + 2\cos x \ , \ y\left(\frac{\pi}{4}\right) = 1 - \sqrt{2}.$$

Then which of the following statement(s) is(are) TRUE?

(A) The solution is unbounded when $x \to 0$
(B) The solution is unbounded when $x \to \frac{\pi}{2}$
(C) The solution is bounded when $x \to 0$
(D) The solution is bounded when $x \to \frac{\pi}{2}$

Q.36 Which of the following statement(s) is(are) TRUE?

(A) There exists a connected set in $\mathbb{R}$ which is not compact
(B) Arbitrary union of closed intervals in $\mathbb{R}$ need not be compact
(C) Arbitrary union of closed intervals in $\mathbb{R}$ is always closed
(D) Every bounded infinite subset V of $\mathbb{R}$ has a limit point in V itself

Q.37 Let $P(x) = \left(\frac{5}{13}\right)^x + \left(\frac{12}{13}\right)^x - 1$ for all $x \in \mathbb{R}$. Then which of the following statement(s) is(are) TRUE?

(A) The equation $P(x) = 0$ has exactly one solution in $\mathbb{R}$
(B) $P(x)$ is strictly increasing for all $x \in \mathbb{R}$
(C) The equation $P(x) = 0$ has exactly two solutions in $\mathbb{R}$
(D) $P(x)$ is strictly decreasing for all $x \in \mathbb{R}$

Q.38 Let G be a finite group and $o(G)$ denotes its order. Then which of the following statement(s) is(are) TRUE?

(A) G is abelian if $o(G) = pq$ where p and q are distinct primes
(B) G is abelian if every non identity element of G is of order 2
(C) G is abelian if the quotient group $\frac{G}{Z(G)}$ is cyclic, where $Z(G)$ is the center of G
(D) G is abelian if $o(G) = p^3$, where p is prime

Q.39 Consider the set $V = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \;\middle|\; \alpha x + \beta y + z = \gamma,\ \alpha, \beta, \gamma \in \mathbb{R} \right\}$. For which of the following choice(s) the set V becomes a two dimensional subspace of $\mathbb{R}^3$ over $\mathbb{R}$?

(A) $\alpha = 0,\ \beta = 1,\ \gamma = 0$
(B) $\alpha = 0,\ \beta = 1,\ \gamma = 1$
(C) $\alpha = 1,\ \beta = 0,\ \gamma = 0$
(D) $\alpha = 1,\ \beta = 1,\ \gamma = 0$

Q.40 Let $S = \left\{ \frac{1}{3^n} + \frac{1}{7^m} \;\middle|\; n, m \in \mathbb{N} \right\}$. Then which of the following statement(s) is(are) TRUE?

(A) S is closed
(B) S is not open
(C) S is connected
(D) 0 is a limit point of S

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 Let $\{s_n\}$ be a sequence of real numbers given by

$$s_n = 2^{(-1)^n}\left(1 - \frac{1}{n}\right)\sin\frac{n\pi}{2}, \qquad n \in \mathbb{N}.$$

Then the least upper bound of the sequence $\{s_n\}$ is _____

Q.42 Let $\{s_k\}$ be a sequence of real numbers, where

$$s_k = k^{\alpha/k}, \qquad k \geq 1, \qquad \alpha > 0.$$

Then

$$\lim_{n\to\infty} \left(s_1\ s_2\ \ldots\ s_n\right)^{1/n}$$

is _____

Q.43 Let $\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3$ be a non-zero vector and $A = \frac{\boldsymbol{x}\,\boldsymbol{x}^T}{\boldsymbol{x}^T\boldsymbol{x}}$. Then the dimension of the vector space $\left\{ \boldsymbol{y} \in \mathbb{R}^3 \;\middle|\; A\boldsymbol{y} = \boldsymbol{0} \right\}$ over $\mathbb{R}$ is _____

Q.44 Let f be a real valued function defined by

$$f(x,y) = 2\ln\left(x^2y^2e^{\frac{y}{x}}\right), \qquad x > 0, y > 0.$$

Then the value of $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ at any point (x,y), where $x > 0$, $y > 0$, is _____

Q.45 Let $\overrightarrow{F} = \sqrt{x}\,\hat{\imath} + (x + y^3)\hat{\jmath}$ be a vector field for all (x,y) with $x \geq 0$ and $\vec{r} = x\hat{\imath} + y\hat{\jmath}$. Then the value of the line integral $\int_C \overrightarrow{F} \cdot d\vec{r}$ from $(0,0)$ to $(1,1)$ along the path C: $x = t^2$, $y = t^3$, $0 \leq t \leq 1$ is _____

Q.46 If $f: (-1,\infty) \to \mathbb{R}$ defined by $f(x) = \frac{x}{1+x}$ is expressed as

$$f(x) = \frac{2}{3} + \frac{1}{9}(x-2) + \frac{c(x-2)^2}{(1+\xi)^3},$$

where ξ lies between 2 and x, then the value of c is _____

Q.47 Let $y_1(x)$, $y_2(x)$ and $y_3(x)$ be linearly independent solutions of the differential equation

$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0.$$

If the Wronskian $W(y_1, y_2, y_3)$ is of the form ke^{bx} for some constant k, then the value of b is_____

Q.48 The radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-4)^n}{n(n+1)}(x+2)^{2n} \quad \text{is} \_\_\_\_\_$$

Q.49 Let $f:(0,\infty)\to\mathbb{R}$ be a continuous function such that

$$\int_0^x f(t)dt = -2+\frac{x^2}{2}+4x\sin 2x+2\cos 2x.$$

Then the value of $\frac{1}{\pi}f\left(\frac{\pi}{4}\right)$ is _____

Q.50 Let G be a cyclic group of order 12. Then the number of non-isomorphic subgroups of G is _____

Q. 51 – Q. 60 carry two marks each.

Q.51 The value of $\lim_{n\to\infty}\left(8n-\frac{1}{n}\right)^{\frac{(-1)^n}{n^2}}$ is equal to _____

Q.52 Let R be the region enclosed by $x^2+4y^2\geq 1$ and $x^2+y^2\leq 1$. Then the value of

$$\iint_R |xy|\,dx\,dy$$ is _____

Q.53 Let

$$M=\begin{bmatrix}\alpha & 1 & 1\\ 1 & \beta & 1\\ 1 & 1 & \gamma\end{bmatrix},\ \alpha\beta\gamma=1,\ \alpha,\beta,\gamma\in\mathbb{R}\ \text{ and }\ \boldsymbol{x}=\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}\in\mathbb{R}^3.$$

Then $M\boldsymbol{x}=\boldsymbol{0}$ has infinitely many solutions if $trace(M)$ is _____

Q.54 Let C be the boundary of the region enclosed by $y=x^2$, $y=x+2$, and $x=0$. Then the value of the line integral

$$\oint_C (xy-y^2)dx-x^3dy,$$

where C is traversed in the counter clockwise direction, is _____

Q.55 Let S be the closed surface forming the boundary of the region V bounded by $x^2 + y^2 = 3$, $z = 0$, $z = 6$. A vector field $\vec{F}$ is defined over V with $\nabla \cdot \vec{F} = 2y + z + 1$. Then the value of

$$\frac{1}{\pi} \iint_S \vec{F} \cdot \hat{n}\, ds,$$

where $\hat{n}$ is the unit outward drawn normal to the surface S, is _____,

Q.56 Let $y(x)$ be the solution of the differential equation

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0, \qquad y(0) = 1, \qquad \left.\frac{dy}{dx}\right|_{x=0} = -1.$$

Then $y(x)$ attains its maximum value at $x =$ _____

Q.57 The value of the double integral

$$\int_0^{\pi} \int_0^{x} \frac{\sin y}{\pi - y}\, dy\, dx$$

is _____

Q.58 Let H denote the group of all 2×2 invertible matrices over $\mathbb{Z}_5$ under usual matrix multiplication. Then the order of the matrix $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ in H is _____

Q.59 Let $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 5 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 3 & 1 \end{bmatrix}$, $N(A)$ the null space of A and $R(B)$ the range space of B. Then the dimension of $N(A) \cap R(B)$ over $\mathbb{R}$ is _____

Q.60 The maximum value of $f(x, y) = x^2 + 2y^2$ subject to the constraint $y - x^2 + 1 = 0$ is _____

END OF THE QUESTION PAPER

JAM 2016: Mathematics			
Qn. No.	Qn. Type	Key(s)	Mark(s)
1	MCQ	D	1
2	MCQ	B	1
3	MCQ	A	1
4	MCQ	B	1
5	MCQ	A	1
6	MCQ	B	1
7	MCQ	C	1
8	MCQ	D	1
9	MCQ	C	1
10	MCQ	C	1
11	MCQ	B	2
12	MCQ	D	2
13	MCQ	B	2
14	MCQ	C	2
15	MCQ	D	2
16	MCQ	A	2
17	MCQ	B	2
18	MCQ	D	2
19	MCQ	A	2
20	MCQ	MTA	2
21	MCQ	B	2
22	MCQ	C	2
23	MCQ	C	2
24	MCQ	C	2
25	MCQ	A	2
26	MCQ	A	2
27	MCQ	C	2
28	MCQ	D	2
29	MCQ	B	2
30	MCQ	MTA	2

JAM 2016: Mathematics			
Qn. No.	Qn. Type	Key(s)	Mark(s)
31	MSQ	A;C	2
32	MSQ	A;D	2
33	MSQ	B;D	2
34	MSQ	B;C;D	2
35	MSQ	C;D	2
36	MSQ	A;B	2
37	MSQ	A;D	2
38	MSQ	B;C	2
39	MSQ	A;C;D	2
40	MSQ	B;D	2

JAM 2016: Mathematics			
Qn. No.	Qn. Type	Key(s)	Mark(s)
41	NAT	0.5:0.5	1
42	NAT	1.0:1.0	1
43	NAT	2.0:2.0	1
44	NAT	8.0:8.0	1
45	NAT	1.49:1.55	1
46	NAT	-1:-1	1
47	NAT	6.0:6.0	1
48	NAT	0.5:0.5	1
49	NAT	0.25:0.25	1
50	NAT	6.0:6.0	1
51	NAT	1.0:1.0	2
52	NAT	0.35:0.4	2
53	NAT	3.0:3.0	2
54	NAT	0.8:1.9	2
55	NAT	72.0:72.0	2
56	NAT	-0.3:-0.25	2
57	NAT	2.0:2.0	2
58	NAT	3.0:3.0	2
59	NAT	1.0:1.0	2
60	NAT	2.0:2.0	2

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 For an infinitely long wire with uniform line-charge density, λ, along the z-axis, the electric field at a point (a,b,0) away from the origin is
($\hat{e}_x, \hat{e}_y,$ and $\hat{e}_z$ are unit vectors in Cartesian − coordinate system.)

(A) $\frac{\lambda}{2\pi\varepsilon_0\sqrt{a^2+b^2}}\left(\hat{e}_x + \hat{e}_y\right)$.

(B) $\frac{\lambda}{2\pi\varepsilon_0(a^2+b^2)}\left(a\hat{e}_x + b\hat{e}_y\right)$.

(C) $\frac{\lambda}{2\pi\varepsilon_0\sqrt{a^2+b^2}}\hat{e}_x$.

(D) $\frac{\lambda}{2\pi\varepsilon_0\sqrt{a^2+b^2}}\hat{e}_z$.

Q.2 A 1 W point source at origin emits light uniformly in all the directions. If the units for both the axes are measured in centimetre, then the Poynting vector at the point (1,1,0) in $\mathrm{W/cm^2}$ is
($\hat{e}_x, \hat{e}_y,$ and $\hat{e}_z$ are unit vectors in Cartesian − coordinate system.)

(A) $\frac{1}{8\pi\sqrt{2}}\left(\hat{e}_x + \hat{e}_y\right)$.

(B) $\frac{1}{16\pi}\left(\hat{e}_x + \hat{e}_y\right)$.

(C) $\frac{1}{16\pi\sqrt{2}}\left(\hat{e}_x + \hat{e}_y\right)$.

(D) $\frac{1}{4\pi\sqrt{2}}\left(\hat{e}_x + \hat{e}_y\right)$.

Q.3 A charged particle in a uniform magnetic field $\vec{B} = B_0\hat{e}_z$ starts moving from the origin with velocity $\vec{v} = (3\hat{e}_x + 2\hat{e}_z)\ m/s$. The trajectory of the particle and the time t at which it reaches 2 meters above the xy- plane are
($\hat{e}_x, \hat{e}_y,$ and $\hat{e}_z$ are unit vectors in Cartesian − coordinate system.)

(A) Helical path ; $t = 1$ s.

(B) Helical path ; $t = 2/3$ s.

(C) Circular path ; $t = 1$ s.

(D) Circular path ; $t = 2/3$ s.

Q.4 Consider a particle of mass m following a trajectory given by $x = x_0 \cos\omega_1 t$ and $y = y_0 \sin\omega_2 t$, where $x_0, y_0, \omega_1,$ and ω_2 are constants of appropriate dimensions. The force on the particle is

(A) central only if $\omega_1 = \omega_2$.

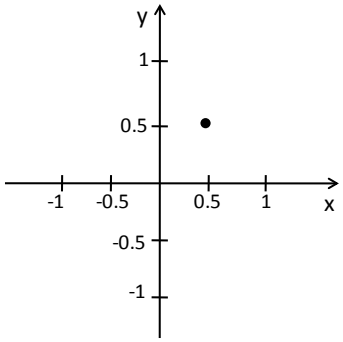
(B) central only if $x_0 = y_0$ and $\omega_1 = \omega_2$.

(C) always central.

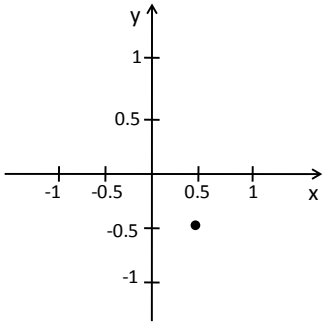
(D) central only if $x_0 = y_0$ and $\omega_1 \neq \omega_2$.

Q.5 Which one of the following points represent the complex number $= \frac{1}{1-i}$?

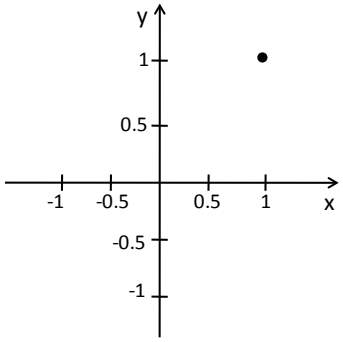
(A)



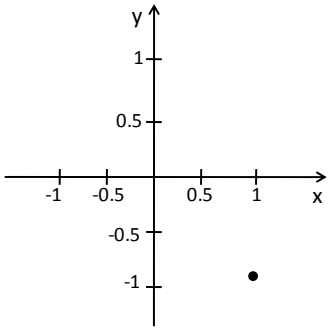
(B)



(C)



(D)



Q.6 The eigenvalues of the matrix representing the following pair of linear equations

$$x + iy = 0$$
$$ix + y = 0$$

are

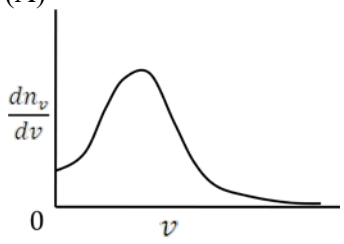
(A) $1+i, 1+i$ (B) $1-i, 1-i$ (C) $1, i$ (D) $1+i, 1-i$

Q.7 The solution of the Boolean equation $Y = \overline{A+B} + \overline{AB}$ is

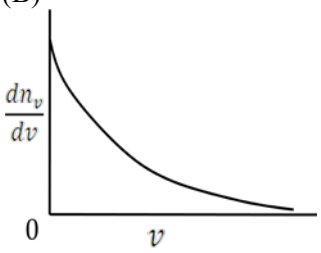
(A) 1 (B) $\overline{AB}$ (C) $\overline{A}\,\overline{B}$ (D) $\overline{A} + \overline{B}$

Q.8 A spherical closed container with smooth inner wall contains a monoatomic ideal gas. If the collisions between the wall and the atoms are elastic, then the Maxwell speed-distribution function $\left(\frac{dn_v}{dv}\right)$ for the atoms is best represented by:

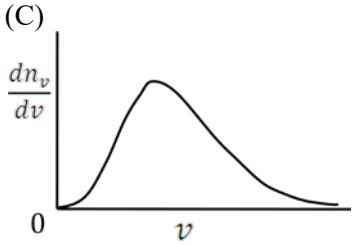
(A)



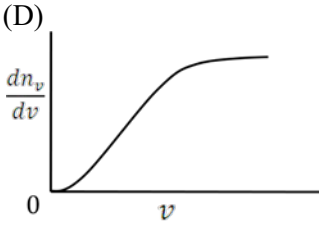
(B)



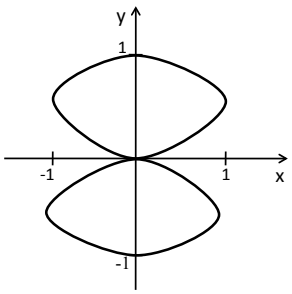
(C)



(D)



Q.9 Two sinusoidal signals of frequencies ω_x and ω_y having same amplitude are applied to x- and y-channels of a cathode ray oscilloscope (CRO), respectively. The following stationary figure will be observed when



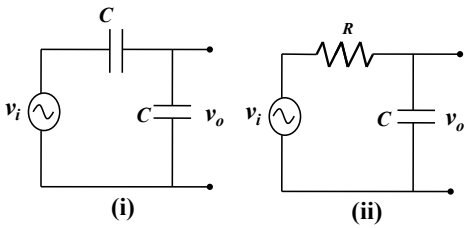
(A) $\omega_y = \omega_x$.

(B) phase difference is 0.

(C) $\omega_y = 2\omega_x$.

(D) phase difference is π/2.

Q.10 The phase difference (δ) between input and output voltage for the following circuits (i) and (ii)



will be

(A) 0 and 0.

(B) π/2 and $0 < \delta \leq \pi/2$ respectively.

(C) π/2 and π/2.

(D) 0 and $0 < \delta \leq \pi/2$ respectively.

Q. 11 – Q. 30 carry two marks each.

Q.11 For the given set of equations:

$$x + y = 1$$
$$y + z = 1$$
$$x + z = 1,$$

which one of the following statements is correct?

(A) Equations are inconsistent.

(B) Equations are consistent and a single non-trivial solution exists.

(C) Equations are consistent and many solutions exist.

(D) Equations are consistent and only a trivial solution exists.

Q.12 The tangent line to the curve $x^2 + xy + 5 = 0$ at (1, 1) is represented by

(A) $y = 3x - 2$. (B) $y = -3x + 4$.

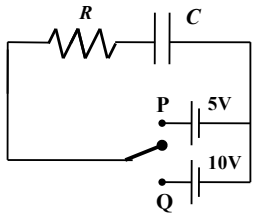
(C) $x = 3y - 2$. (D) $x = -3y + 4$.

Q.13 In the following RC circuit, the capacitor was charged in two different ways.

(i) The capacitor was first charged to 5V by moving the toggle switch to position P and then it was charged to 10V by moving the toggle switch to position Q.

(ii) The capacitor was directly charged to 10V, by keeping the toggle switch at position Q.

Assuming the capacitor to be ideal, which one of the following statements is correct?



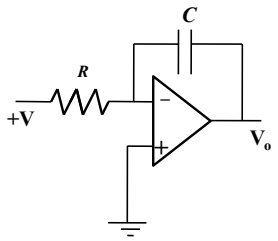
(A) The energy dissipation in cases (i) and (ii) will be equal and non-zero.

(B) The energy dissipation for case (i) will be more than that for case (ii).

(C) The energy dissipation for case (i) will be less than that for case (ii).

(D) The energy will not be dissipated in either case.

Q.14 If a constant voltage +V is applied to the input of the following OPAMP circuit for a time t, then the output voltage V_o will approach



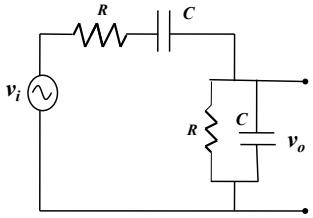
(A) +V exponentially.

(B) −V exponentially.

(C) +V linearly.

(D) −V linearly.

Q.15 In the following RC network, for an input signal frequency $f = \frac{1}{2\pi RC}$, the voltage gain $\left|\frac{v_o}{v_i}\right|$ and the phase angle φ between v_o and v_i respectively are



(A) $\frac{1}{2}$ and 0. (B) $\frac{1}{3}$ and 0. (C) $\frac{1}{2}$ and $\frac{\pi}{2}$. (D) $\frac{1}{3}$ and $\frac{\pi}{2}$.

Q.16 Light travelling between two points takes a path for which

(A) time of flight is always minimum.

(B) distance is always minimum.

(C) time of flight is extremum.

(D) distance is extremum.

Q.17 Consider a free electron (e) and a photon (ph) both having 10 eV of energy. If λ and P represent wavelength and momentum respectively, then
(mass of electron = 9.1 x 10^{-31} kg; speed of light = 3 x 10^8 m/s)

(A) $\lambda_e = \lambda_{ph}$ and $P_e = P_{ph}$.

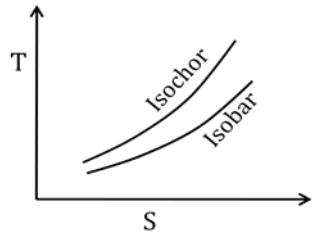
(B) $\lambda_e < \lambda_{ph}$ and $P_e > P_{ph}$.

(C) $\lambda_e > \lambda_{ph}$ and $P_e < P_{ph}$.

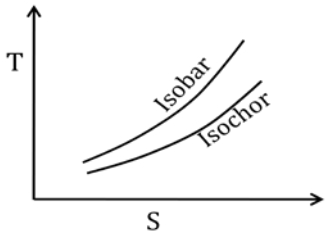
(D) $\lambda_e < \lambda_{ph}$ and $P_e < P_{ph}$.

Q.18 For an ideal gas, which one of the following T-S diagrams is valid?

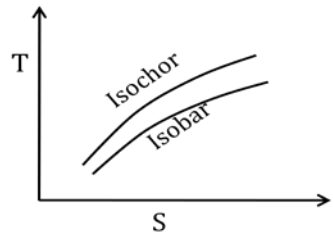
(A)



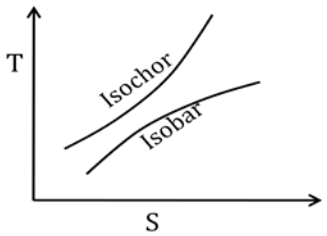
(B)



(C)



(D)



Q.19 If U, F, H, and G represent internal energy, Helmholtz free energy, enthalpy, and Gibbs free energy respectively, then which one of the following is a correct thermodynamic relation?

(A) $dU = PdV - TdS$ (B) $dH = VdP + TdS$

(C) $dF = -PdV + SdT$ (D) $dG = VdP + SdT$

Q.20 A train passes through a station with a constant speed. A stationary observer at the station platform measures the tone of the train whistle as 484 Hz when it approaches the station and 442 Hz when it leaves the station. If the sound velocity in air is 330 m/s, then the tone of the whistle and the speed of the train are

(A) 462 Hz, 54 km/h. (B) 463 Hz, 52 km/h.

(C) 463 Hz, 56 km/h. (D) 464 Hz, 52 km/h.

Q.21 The minimum length of a plane mirror to see the entire full-length image of an object is half of the object's height. Suppose δ is the distance between eye and top of the head of a person of height h. The person will be able to see his entire full-length image with a mirror of height $h/2$ fixed on the wall

(A) when the bottom edge of mirror is kept $h/2$ above the floor.

(B) when the bottom edge of mirror is kept $(h+\delta)/2$ above the floor.

(C) when the bottom edge of mirror is kept $(h-\delta)/2$ above the floor.

(D) when the centre of the mirror is at the same height as centre of the person.

Q.22 A particle is moving in a plane with a constant radial velocity of 12 m/s and constant angular velocity of 2 rad/s. When the particle is at a distance r = 8 m from the origin, the magnitude of the instantaneous velocity of the particle in m/s is

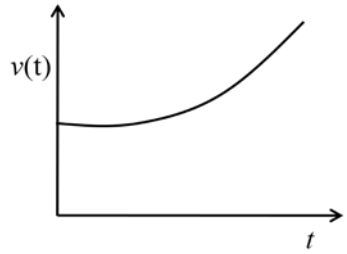
(A) $8\sqrt{15}$. (B) 20. (C) $2\sqrt{37}$. (D) 10.

Q.23 A cylindrical rod of length L has a mass density distribution given by $\rho(x) = \rho_0\left(1+\frac{x}{L}\right)$, where x is measured from one end of the rod and ρ_0 is a constant of appropriate dimensions. The center of mass of the rod is

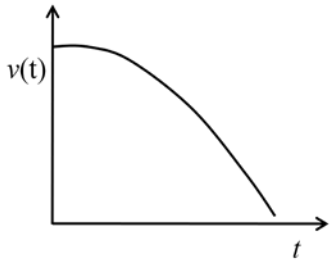
(A) $\frac{5}{9}L$. (B) $\frac{4}{9}L$. (C) $\frac{1}{9}L$. (D) $\frac{1}{2}L$.

Q.24 A particle travels in a medium along a horizontal linear path. The initial velocity of the particle is v_0 and the viscous force acting on it is proportional to its instantaneous velocity. In the absence of any other forces, which one of the following figures correctly represents the velocity of the particle as a function of time?

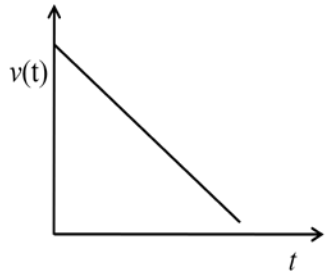
(A)



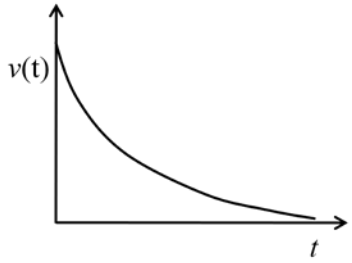
(B)



(C)



(D)

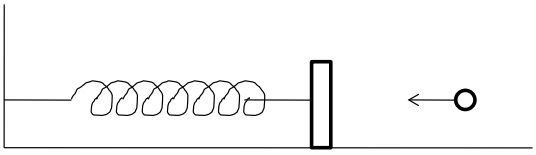


Q.25 A lightly damped harmonic oscillator with natural frequency ω_0 is driven by a periodic force of frequency ω. The amplitude of oscillation is maximum when

(A) ω is slightly lower than ω_0. (B) $\omega = \omega_0$.

(C) ω is slightly higher than ω_0. (D) the force is in phase with the displacement.

Q.26 A block of mass 0.38 kg is kept at rest on a frictionless surface and attached to a wall with a spring of negligible mass. A bullet weighing 0.02 kg moving with a speed of 200 m/s hits the block at time t = 0 and gets stuck to it. The displacement of the block (in metre) with respect to the equilibrium position is given by



(Spring constant = 640 N/m.)

(A) 2 sin5t. (B) cos10t. (C) 0.4 cos25t. (D) 0.25 sin40t.

Q.27 One mole of an ideal gas with average molecular speed v_0 is kept in a container of fixed volume. If the temperature of the gas is increased such that the average speed gets doubled, then

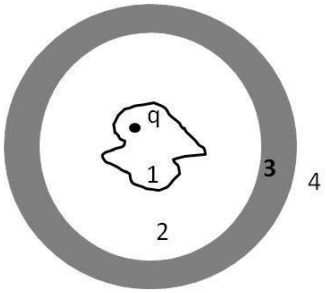
(A) the mean free path of the gas molecule will increase.

(B) the mean free path of the gas molecule will not change.

(C) the mean free path of the gas molecule will decrease.

(D) the collision frequency of the gas molecule with wall of the container remains unchanged.

Q.28 An arbitrarily shaped conductor encloses a charge q and is surrounded by a conducting hollow sphere as shown in the figure. Four different regions of space, 1, 2, 3, and 4, are indicated in the figure. Which one of the following statements is correct?



(A) The electric field lines in region 2 are not affected by the position of the charge q.

(B) The surface charge density on the inner wall of the hollow sphere is uniform.

(C) The surface charge density on the outer surface of the sphere is always uniform irrespective of the position of charge q in region 1.

(D) The electric field in region 2 has a radial symmetry.

Q.29 Consider a small bar magnet undergoing simple harmonic motion (SHM) along the *x*-axis. A coil whose plane is perpendicular to the *x*-axis is placed such that the magnet passes in and out of it during its motion. Which one of the following statements is correct? Neglect damping effects.

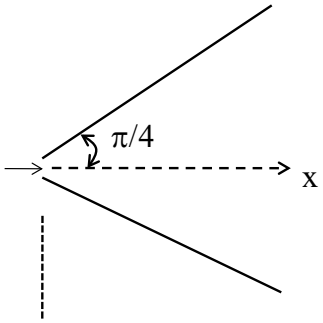
(A) Induced e.m.f. is minimum when the center of the bar magnet crosses the coil.

(B) The frequency of the induced current in the coil is half of the frequency of the SHM.

(C) Induced e.m.f. in the coil will not change with the velocity of the magnet.

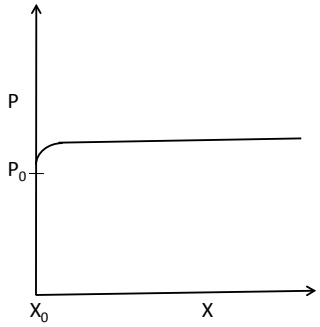
(D) The sign of the e.m.f. depends on the pole (N or S) face of the magnet which enters into the coil.

Q.30 An incompressible, non-viscous fluid is injected into a conical pipe at its orifice as schematically shown in the figure. The pressure at the orifice of area A_0 is P_0. Neglecting the effect of gravity and assuming streamline flow, which one of the following plots correctly predicts the pressure along axis of the cone?

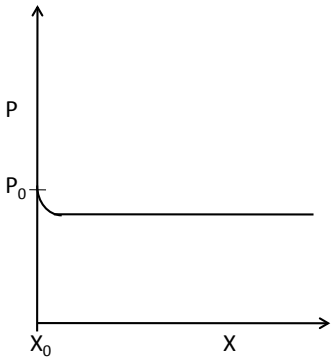


$X_0(P_0; v_0)$

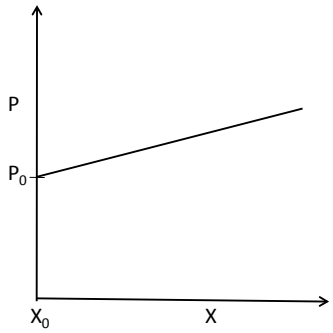
(A)



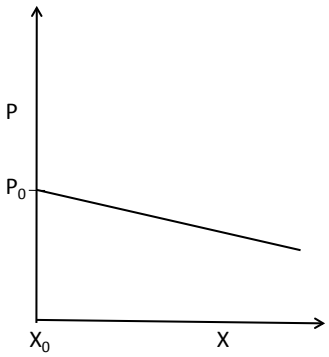
(B)



(C)



(D)



SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 Consider a spherical dielectric material of radius 'a', centered at origin. If the polarization vector $\vec{P} = P_0\hat{e}_x$, where P_0 is a constant of appropriate dimensions, then
($\hat{e}_x, \hat{e}_y,$ and $\hat{e}_z$ are unit vectors in Cartesian − coordinate system.)

(A) the bound volume charge density is zero.

(B) the bound surface charge density is zero at (0,0,a).

(C) the electric field is zero inside the dielectric.

(D) the sign of the surface charge density changes over the surface.

Q.32 For an electric dipole with moment $\vec{p} = p_0\hat{e}_z$ placed at the origin,
(p_0 is a constant of appropriate dimensions and $\hat{e}_x, \hat{e}_y,$ and $\hat{e}_z$ are unit vectors in Cartesian-coordinate system.)

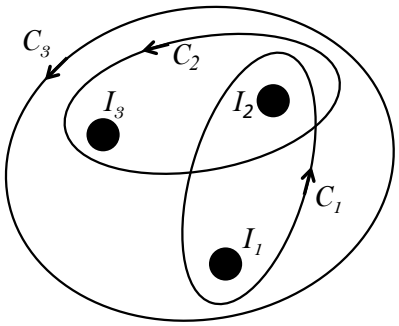
(A) potential falls as $\frac{1}{r^2}$, where r is the distance from origin.

(B) a spherical surface centered at origin is an equipotential surface.

(C) electric flux through a spherical surface enclosing the origin is zero.

(D) radial component of $\vec{E}$ is zero on the xy -plane.

Q.33 Three infinitely-long conductors carrying currents $I_1, I_2,$ and I_3 lie perpendicular to the plane of the paper as shown below.



If the value of the integral $\oint_C \vec{B}.\vec{dl}$ for the loops $C_1, C_2,$ and C_3 are $2\mu_0, 4\mu_0,$ and μ_0 in the units of N/A, respectively, then

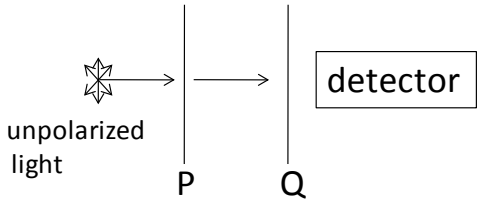
(A) $I_1 = 3$A into the paper.

(B) $I_2 = 5$A out of the paper.

(C) $I_3 = 0$.

(D) $I_3 = 1$A out of the paper.

Q.34 In the optical arrangement as shown below, the axes of two polarizing sheets P and Q are oriented such that no light is detected. Now when a third polarizing sheet (R) is placed in between P and Q, then light is detected. Which of the following statement(s) is(are) true?



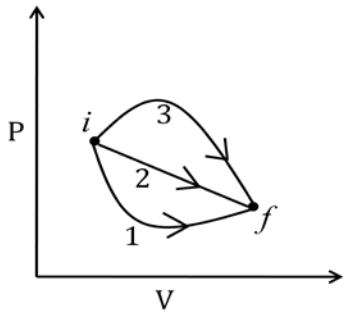
(A) Polarization axes of P and Q are perpendicular to each other.

(B) Polarization axis of R is not parallel to P.

(C) Polarization axis of R is not parallel to Q.

(D) Polarization axes of P and Q are parallel to each other.

Q.35 The P-V diagram below shows three possible paths for an ideal gas to reach the final state *f* from an initial state *i*. Which among the following statement(s) is(are) correct?



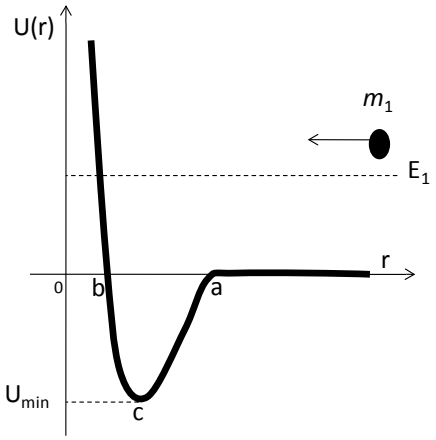
(A) The work done by the gas is maximum along path-3.

(B) Minimum change in the internal energy occurs along path-2.

(C) Maximum heat transfer is for path-1.

(D) Heat transfer is path independent.

Q.36 Potential energy U as a function of r is shown below. If a particle of mass m_1 and energy E_1, starting from $r \gg a$, moves towards the origin, then



(A) there is only one turning point for the particle.

(B) velocity of the particle starts to increase at $r = a$ and reaches its maximum at $r = c$.

(C) velocities of the particle at $r = a$ and $r = b$ are equal.

(D) the particle gets bounded if it elastically collides with a stationary particle of mass m_2 at $r = c$, imparting a momentum greater than $\sqrt{2m_2E_1}$.

Q.37 A particle moves in a circular path in the *xy* -plane centered at the origin. If the speed of the particle is constant, then its angular momentum

(A) about the origin is constant both in magnitude and direction.

(B) about (0, 0, 1) is constant in magnitude but not in direction.

(C) about (0, 0, 1) varies both in magnitude and direction.

(D) about (0, 0, 1) is constant in direction but not in magnitude.

Q.38 A *pn* junction was formed with a heavily doped (10^{18} cm^{-3}) *p*-region and lightly doped (10^{14} cm^{-3}) *n*-region. Which of the following statement(s) is(are) correct?

(A) The width of the depletion layer will be more in the *n*-side of the junction.

(B) The width of the depletion layer will be more in the *p*-side of the junction.

(C) The width of the depletion layer will be same on both sides of the junction.

(D) If the *pn* junction is reverse biased, then the width of the depletion region increases.

Q.39 A slit has width 'd' along the x-direction. If a beam of electrons, accelerated in y-direction to a particular velocity by applying a potential difference of 100 ± 0.1 kV passes through the slit, then, which of the following statement(s) is(are) correct?

(A) The uncertainty in the position of electrons in x-direction before passing the slit is zero.

(B) The momentum of electrons in x-direction is $\sim \hbar/d$ immediately after passing the slit.

(C) The uncertainty in the position of electrons in y-direction before passing the slit is zero.

(D) The presence of the slit does not affect the uncertainty in momentum of electrons in y-direction.

Q.40 A free particle of energy E collides with a one-dimensional square potential barrier of height V and width W. Which one of the following statement(s) is(are) correct?

(A) For $E > V$, the transmission coefficient for the particle across the barrier will always be unity.

(B) For $E < V$, the transmission coefficient changes more rapidly with W than with V.

(C) For $E < V$, if V is doubled, the transmission coefficient will also be doubled.

(D) Sum of the reflection and the transmission coefficients is always one.

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

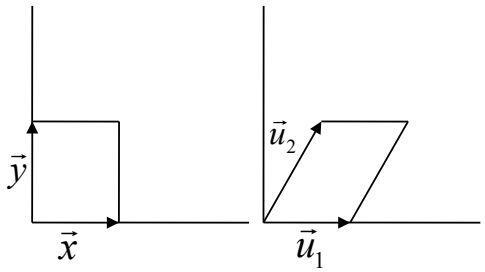
Q. 41 – Q. 50 carry one mark each.

Q.41 Fourier series of a given function $f(x)$ in the interval 0 to L is
$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(\frac{2\pi nx}{L}) + \sum_{n=1}^{\infty} b_n \sin(\frac{2\pi nx}{L})$.

If $f(x) = x$ in the region $(0, \pi)$, b_2 = _____.

Q.42 Consider a function $f(x, y) = x^3 + y^3$, where y represents a parabolic curve $x^2 + 1$. The total derivative of f with respect to x, at x =1 is _____.

Q.43 A rectangular area (A_1) is formed by two vectors $\vec{x}$ and $\vec{y}$ as shown in figure (i). A new set of vectors, representing the area (A_2) as shown in figure (ii), are given as: $\vec{u}_1 = \vec{x}$; $\vec{u}_2 = k\vec{x} + \vec{y}$, where k is a dimensionless constant.



The Jacobian of the frame $(\vec{u}_1, \vec{u}_2)$ with respect to $(\vec{x}, \vec{y})$ is _____.

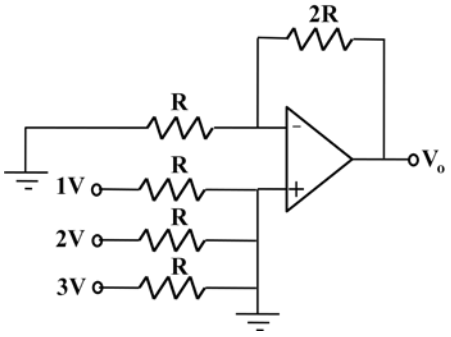
Q.44 A particular radioisotope has a half-life of 5 days. In 15 days the probability of decay in percentage will be _____.

Q.45 In a photoelectric experiment both sodium (work function = 2.3 eV) and tungsten (work function = 4.5 eV) metals were illuminated by an ultraviolet light of same wavelength. If the stopping potential for tungsten is measured to be 1.8 V, the value of the stopping potential for sodium will be _____ V.

Q.46 The addition of two binary numbers 1000.01 and 0001.11 in binary representation is _____.

Q.47 The number of second-nearest neighbor ions to a Na^+ ion in NaCl crystal is_____.

Q.48 The output voltage V_O of the OPAMP circuit given below is _____V.

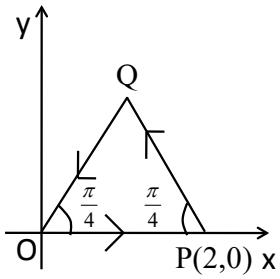


Q.49 A cylinder contains 16 g of O_2. The work done when the gas is compressed to 75% of the original volume at constant temperature of 27°C is _____ J.
[Universal gas constant R = 8.31 J/(mole K)]

Q.50 When sunlight is focused on a paper using a bi-convex lens, it starts to burn in the shortest time if the lens is kept 0.5 m above it. If the radius of curvature of the lens is 0.75 m then, the refractive index of the material is _____.

Q. 51 – Q. 60 carry two marks each.

Q.51 Consider a closed triangular contour traversed in counter-clockwise direction, as shown in the figure below.



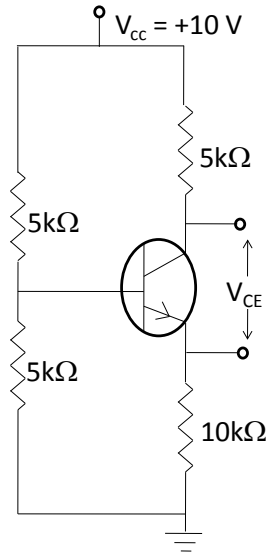
The value of the integral, $\oint \vec{F} \cdot \overrightarrow{dl}$ evaluated along this contour, for a vector field, $\vec{F} = y\hat{e}_x - x\hat{e}_y$, is _____.
($\hat{e}_x, \hat{e}_y,$ and $\hat{e}_z$ are unit vectors in Cartesian – coordinate system.)

Q.52 A hemispherical shell is placed on the *xy*– plane centered at the origin. For a vector field $\vec{E} = (-y\hat{e}_x + x\hat{e}_y)/(x^2 + y^2)$, the value of the integral $\int_S (\vec{\nabla} \times \vec{E}) \cdot d\vec{a}$ over the hemispherical surface is _____ π.

($d\vec{a}$ is the elemental surface area. $\hat{e}_x, \hat{e}_y,$ and $\hat{e}_z$ are unit vectors in Cartesian – coordinate system.)

Q.53 The shape of a dielectric lamina is defined by the two curves $y = 0$ and $y = 1 - x^2$. If the charge density of the lamina $\sigma = 15y$ C/m^2, then the total charge on the lamina is _____C.

Q.54 In the circuit given below, the collector to emitter voltage V_{CE} is _____V.
(Neglect V_{BE}, take $\beta = 100$)



Q.55 The *de* Broglie wavelength of a relativistic electron having 1 MeV of energy is _____ $\times 10^{-12}$ m. (Take the rest mass energy of the electron to be 0.5 MeV. Planck constant = 6.63×10^{-34} Js, Speed of light= 3×10^{8} m/s, Electronic charge = 1.6×10^{-19}C)

Q.56 X-ray diffraction of a cubic crystal gives an intensity maximum for Bragg angle of 20° corresponding to the (110) plane. The lattice parameter of the crystal is _____nm. (Consider wavelength of X-ray = 0.15 nm)

Q.57 X-rays of 20 keV energy is scattered inelastically from a carbon target. The kinetic energy transferred to the recoiling electron by photons scattered at 90° with respect to the incident beam is _____keV. (Planck constant = 6.6×10^{-34} Js, Speed of light = 3×10^{8} m/s, electron mass = 9.1×10^{-31} kg, Electronic charge = 1.6×10^{-19}C)

Q.58 An aluminum plate of mass 0.1 kg at 95°C is immersed in 0.5 litre of water at 20°C kept inside an insulating container and is then removed. If the temperature of the water is found to be 23°C, then the temperature of the aluminum plate is _____°C. (The specific heat of water and aluminum are 4200 J/kg-K and 900 J/kg-K respectively, the density of water is 1000 kg/m^3)

Q.59 The maximum and minimum speeds of a comet that orbits the Sun are 80 and 10 km/s respectively. The ratio of the aphelion distance of the comet to the radius of the Earth's orbit is _____. (Assume that Earth moves in a circular orbit of radius 1.5×10^{8} km with a speed of 30 km/s.)

Q.60 If there is a 10% decrease in the atmospheric pressure at a hill compared to the pressure at sea level, then the change in the boiling point of water is_____°C. (Take latent heat of vaporisation of water as 2270 kJ/kg and the change in the specific volume at the boiling point to be 1.2 m^3/kg.)

END OF THE QUESTION PAPER

JAM 2016: Physics			
Qn. No.	Qn. Type	Key(s)	Mark(s)
1	MCQ	B	1
2	MCQ	A	1
3	MCQ	A	1
4	MCQ	A	1
5	MCQ	A	1
6	MCQ	D	1
7	MCQ	D	1
8	MCQ	C	1
9	MCQ	B	1
10	MCQ	D	1
11	MCQ	B	2
12	MCQ	MTA	2
13	MCQ	C	2
14	MCQ	D	2
15	MCQ	B	2
16	MCQ	C	2
17	MCQ	B	2
18	MCQ	A	2
19	MCQ	B	2
20	MCQ	A	2
21	MCQ	C	2
22	MCQ	B	2
23	MCQ	A	2
24	MCQ	D	2
25	MCQ	A	2
26	MCQ	D	2
27	MCQ	B	2
28	MCQ	C	2
29	MCQ	A	2
30	MCQ	A	2

JAM 2016: Physics			
Qn. No.	Qn. Type	Key(s)	Mark(s)
31	MSQ	A;B;D	2
32	MSQ	A;C;D	2
33	MSQ	A;B	2
34	MSQ	A;B;C	2
35	MSQ	A	2
36	MSQ	A;B;C or A;B;C;D	2
37	MSQ	A;B	2
38	MSQ	A;D	2
39	MSQ	B;D	2
40	MSQ	B;D	2

JAM 2016: Physics			
Qn. No.	Qn. Type	Key(s)	Mark(s)
41	NAT	-0.5 to -0.5	1
42	NAT	27.0 to 27.0	1
43	NAT	1.0 to 1.0	1
44	NAT	87 to 88	1
45	NAT	4.0 to 4.0	1
46	NAT	1010 to 1010	1
47	NAT	12.0 to 12.0	1
48	NAT	6.0 to 6.0	1
49	NAT	358 to 359	1
50	NAT	1.73 to 1.77	1
51	NAT	-2 to -2	2
52	NAT	2.0 to 2.0 or -2.0 to -2.0	2
53	NAT	8.0 to 8.0	2
54	NAT	2.0 to 3.0	2
55	NAT	1.3 to 1.6	2
56	NAT	0.28 to 0.35	2
57	NAT	0.68 to 0.82	2
58	NAT	24.0 to 26.0	2
59	NAT	1.9 to 2.1	2
60	NAT	1.9 to 2.1	2