IIT JAM 2020 Question Paper (All Subjects)

IIT Joint Admission Test for Masters

Paper Specific Instructions

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, **A**, **B** and **C**. All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ).** Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ).** Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.

7. The Scribble Pad will be provided for rough work.

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 Deficiency of the enzyme phenylalanine hydroxylase causes Phenylketonuria. Phenylalanine hydroxylase converts phenylalanine to

(A) tryptophan (B) alanine (C) tyrosine (D) threonine

Q.2 T cells and B cells are

(A) lymphocytes (B) erythrocytes (C) epithelial cells (D) squamous cells

Q.3 In humans, the testis temperature is maintained below the core body temperature with the help of

(A) proximal tubule
(B) loop of Henle
(C) scrotum
(D) seminal vesicles

Q.4 Protein that helps other proteins to fold correctly is

(A) chaperone
(B) proteasome
(C) ubiquitin
(D) desmosome

Q.5 The CORRECT sequence of phases during mitosis is

(A) prophase, metaphase, anaphase, telophase
(B) prophase, anaphase, metaphase, telophase
(C) anaphase, prophase, metaphase, telophase
(D) anaphase, metaphase, prophase, telophase

Q.6 The curve $y = x^4 - 4x^3 + 4x^2 - 4$ has tangents parallel to the x-axis at the following points (x, y)

(A) $(1, 4),\ (-2, 2)\ and\ (0, -1)$
(B) $(0, -4), (2, -4)\ and\ (1, -3)$
(C) $(-1, 2), (-2, 1)\ and\ (1, -2)$
(D) $(1, -4), (1, -3)\ and\ (2, -4)$

Q.7 In viruses, capsids are made up of

(A) proteins (B) nucleic acids (C) lipids (D) sterols

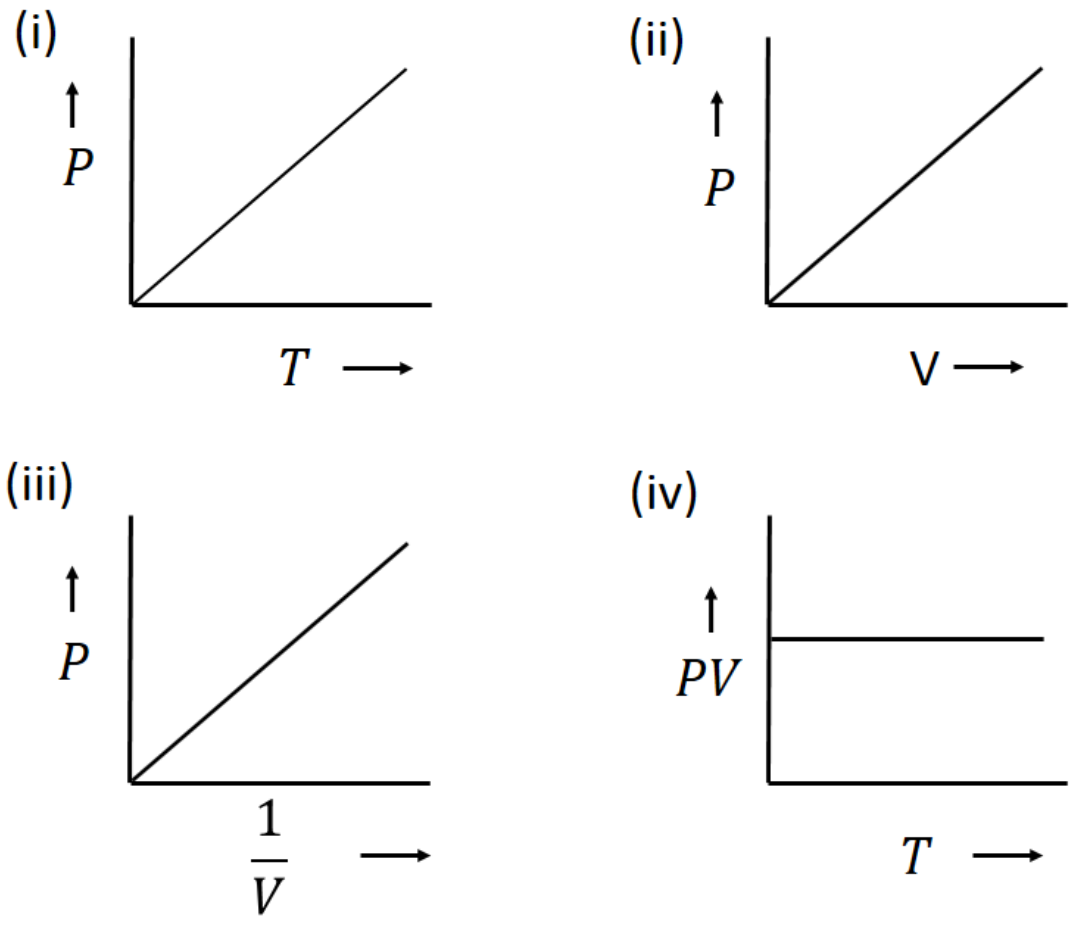
Q.8 The common component in crustacean exoskeleton and fungal cell wall is

(A) lignin (B) cellulose (C) chitin (D) peptidoglycan

Q.9 The CORRECT sequence of evolution (simplest to complex) is

(A) algae, bryophytes, ferns, angiosperms
(B) algae, ferns, bryophytes, angiosperms
(C) bryophytes, ferns, algae, angiosperms
(D) bryophytes, algae, ferns, angiosperms

Q.10 For an ideal gas at room temperature, choose the CORRECT representation(s) of Boyle's Law.
(P = Pressure, V = Volume, T = Temperature)



(A) only (i) (B) both (ii) and (iii) (C) only (iii) (D) both (iii) and (iv)

Q. 11 – Q. 30 carry two marks each.

Q.11 During El Niño,

(A) cold water of the north flowing Peru current is displaced by a low-nutrient warm southward current
(B) warm water of the north flowing Peru current is displaced by a low-nutrient cold southward current
(C) cold water of the south flowing Peru current is displaced by a warm northward current rich in nutrients
(D) warm water of the south flowing Peru current is displaced by a cold northward current rich in nutrients

Q.12 Match the deficiency conditions in Group I with the corresponding vitamin in Group II

Group I	Group II
(P) Beriberi	(1) Ascorbic acid
(Q) Scurvy	(2) Retinol
(R) Birth defects	(3) Thiamine
(S) Night blindness	(4) Folic acid

(A) P-3, Q-2, R-1, S-4
(B) P-2, Q-3, R-4, S-1
(C) P-3, Q-1, R-4, S-2
(D) P-1, Q-2, R-3, S-4

Q.13 Eutrophication refers to an aging process from a

(A) low production ecosystem to high production ecosystem due to availability of excess nutrients
(B) high production ecosystem to low production ecosystem due to nutrient deficiency
(C) high production ecosystem to low production ecosystem due to light scarcity
(D) low production ecosystem to high production ecosystem due to light scarcity

Q.14 Which one of the following ions has the maximum number of unpaired electrons?

(A) Cu^{2+} (B) Na^{+} (C) Cr^{3+} (D) Fe^{3+}

Q.15 Match the enzymes in Group I with the corresponding substrate in Group II

Group I	Group II
(P) Amylase	(1) Protein
(Q) Pepsin	(2) Fat
(R) Lipase	(3) RNA
(S) Ribozyme	(4) Starch

(A) P-2, Q-3, R-1, S-4
(B) P-4, Q-1, R-2, S-3
(C) P-3, Q-1, R-4, S-2
(D) P-4, Q-2, R-3, S-1

Q.16 In ABO blood group testing, which one of the following is INCORRECT

(A) A group – agglutination with anti-A antibodies
(B) B group – agglutination with anti-B antibodies
(C) AB group – no agglutination with either anti-A or anti-B antibodies
(D) O group – no agglutination with either anti-A or anti-B antibodies

Q.17 If a coin is tossed three times, what is the probability that NO two successive tosses show the same face?

(A) 0.25 (B) 0.33 (C) 0.20 (D) 0.125

Q.18 Match the RNAs in Group I with their corresponding function in Group II

Group I	Group II
(P) snRNA	(1) rRNA processing
(Q) piRNA	(2) RNA splicing
(R) snoRNA	(3) Selected mRNAs degradation
(S) siRNA	(4) Germ line protection from transposable elements

(A) P-4, Q-3, R-2, S-1
(B) P-4, Q-1, R-3, S-2
(C) P-3, Q-2, R-1, S-4
(D) P-2, Q-4, R-1, S-3

Q.19 In a typical mammalian cell, the protein content is 20 % of its net weight. If the density and volume of the cell are 1.2 g/mL and 4×10^{-9} mL, respectively, then the concentration (in mg/mL) of the protein is

(A) 60 (B) 600 (C) 166 (D) 240

Q.20 Consider a spherical particle of mass *m* and radius *r* moving in a medium. Its velocity at any time *t* is given by $v = v_0 \exp\left(\frac{-6\pi X r t}{m}\right)$, where v_0 is initial velocity of the particle. The dimensions of X are

(A) MLT^{-1} (B) $M^{-1}LT$ (C) $ML^{-1}T^{-1}$ (D) Dimensionless

Q.21 Match the type of bacterial flagella in Group I with their definitions in Group II

Group I	Group II
(P) Monotrichous	(1) Flagella only at both poles of the cell
(Q) Peritrichous	(2) Two or more flagella at one pole of the cell
(R) Lophotrichous	(3) Flagella distributed over the entire cell
(S) Amphitrichous	(4) A single polar flagellum

(A) P-4, Q-1, R-2, S-3
(B) P-4, Q-3, R-2, S-1
(C) P-4, Q-3, R-1, S-2
(D) P-3, Q-1, R-4, S-2

Q.22 Consider monoatomic ideal gas molecules of equal mass m in thermal equilibrium, at temperature T. Which one of the following equations is correct? (the angular brackets denote average, k_B is Boltzmann constant, v is velocity, and v_x is the x-component of velocity)

(A) $\langle\frac{3}{2}mv^2\rangle = \frac{1}{2}k_BT$ (B) $\langle\frac{1}{2}mv^2\rangle = \frac{3}{2}k_BT$

(C) $\frac{1}{2}mv^2 = \frac{3}{2}k_BT$ (D) $\langle\frac{1}{2}mv_x^2\rangle = \frac{3}{2}k_BT$

Q.23 Match the diseases in Group I with their causative organisms in Group II

Group I	Group II
(P) Syphilis	(1) *Shigella dysenteriae*
(Q) Bacillary dysentery	(2) *Bordetella pertusis*
(R) Gas gangrene	(3) *Treponema pallidum*
(S) Whooping cough	(4) *Clostridium perfringens*

(A) P-2, Q-1, R-4, S-3
(B) P-3, Q-4, R-2, S-1
(C) P-2, Q-3, R-4, S-1
(D) P-3, Q-1, R-4, S-2

Q.24 Determine the correctness or otherwise of the following Assertion [a] and the Reason [r].

Assertion [a]: The difference in the respective melting points of butter and coconut oil is caused by the degrees of saturation of the corresponding fatty acid chain.

Reason [r]: Unsaturated fatty acid chains in fats/lipids become solid more easily because they are relatively straight and thus able to pack together more closely than saturated chain.

(A) Both [a] and [r] are true and [r] is the correct reason for [a]
(B) Both [a] and [r] are true but [r] is not the correct reason for [a]
(C) Both [a] and [r] are false
(D) [a] is true but [r] is false

Q.25 Match the techniques in Group I with their applications in Group II for protein analysis

Group I	Group II
(P) Nuclear magnetic resonance spectroscopy	(1) Proximity between specific sites
(Q) Fluorescence resonance energy transfer	(2) Concentration
(R) Ultraviolet absorption spectroscopy	(3) Size
(S) Dynamic light scattering	(4) Structure

(A) P-3, Q-1, R-2, S-4
(B) P-4, Q-1, R-2, S-3
(C) P-2, Q-1, R-4, S-3
(D) P-3, Q-4, R-1, S-2

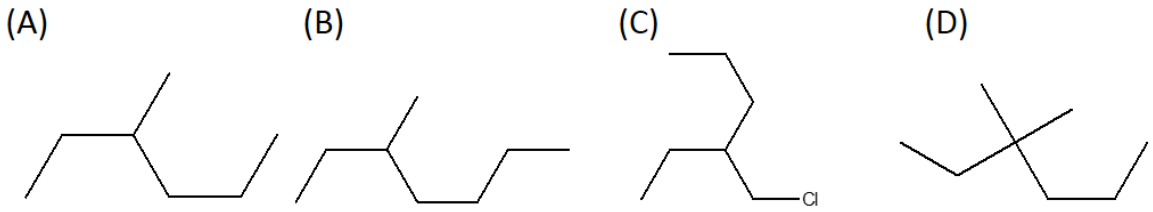
Q.26 The following carboxylic acids have a general formula, R-COOH.

(i) HCOOH
(ii) CH_3 - COOH
(iii) $ClCH_2$ - COOH
(iv) CF_3 - COOH

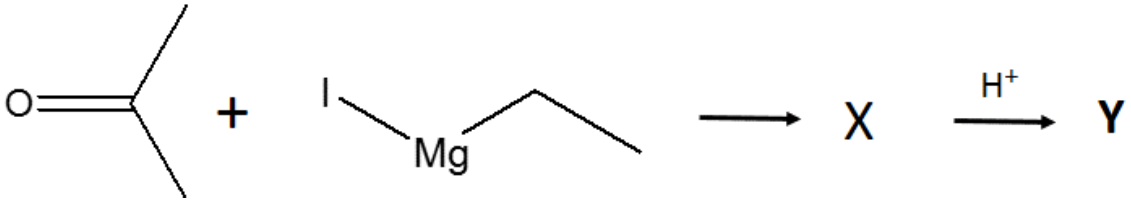
Which one of the following represents the decreasing order of their respective pK_a values?

(A) i > ii > iii > iv (B) ii > i > iii > iv (C) iv > iii > i > ii (D) iv > iii > ii > i

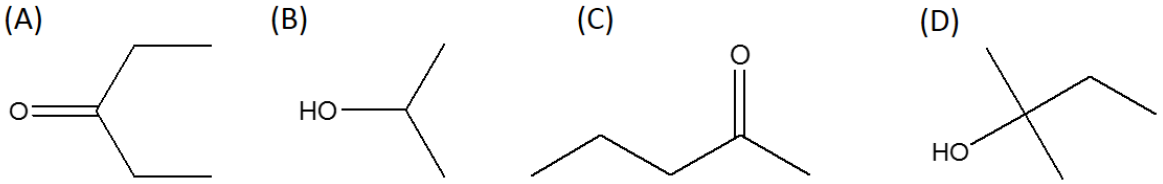
Q.27 Which one of the following compounds is the simplest alkane that is optically active?



Q.28 In the following reaction, **X** is an intermediate and **Y** is one of the end products.



Which one of the following compounds is the end product **Y**?



Q.29 Determine the correctness or otherwise of the following Assertion [a] and the Reason [r].

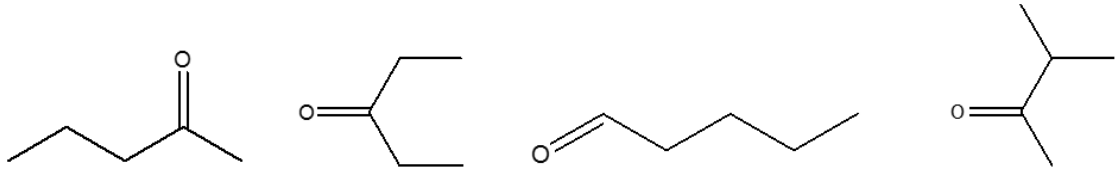
Assertion [a]: *lac* operon is an inducible operon.

Reason [r]: *lac* operon is not induced when the repressor protein remains bound to operator DNA sequence.

(A) Both [a] and [r] are true and [r] is the correct reason for [a]
(B) Both [a] and [r] are true but [r] is not the correct reason for [a]
(C) Both [a] and [r] are false
(D) [a] is true but [r] is false

Q.30 IR spectrum of a compound $C_5H_{10}O$ shows a band at 1715 cm^{-1}. The same compound showed two signals, a triplet and a quartet, in its NMR spectrum. Identify the compound from the following.

(A) (B) (C) (D)



SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 Which of the following is(are) auxins?

(A) 1-Naphthaleneacetic acid
(B) Indole-3-butyric acid
(C) 2,4-Dichlorophenoxyacetic acid
(D) Indole-3-acetic acid

Q.32 Which of the following is(are) TRUE about photosynthesis?

(A) In C3 plants the first organic product of carbon fixation is 3-phosphoglycerate
(B) In C4 plants the first organic product of carbon fixation is oxaloacetate
(C) Crassulacean acid metabolism occurs in succulent plants living in arid conditions
(D) Oxygen is generated from carbon dioxide

Q.33 Which of the following is(are) involved in the activation of cytotoxic T cells?

(A) MHC I
(B) FcR
(C) T cell receptor
(D) CTLA 4

Q.34 DNA and RNA are acidic in nature due to the presence of

(A) pentose sugar
(B) nitrogenous bases
(C) phosphate groups
(D) large number of hydrogen bonds

Q.35 Which of the following is(are) CORRECT?

(A) Light has wave nature only
(B) Light can have both wave and particle nature
(C) Photo electric effect shows that light can behave like particles
(D) Interference experiments show that light behaves like particles

Q.36 Protozoa are

(A) unicellular (B) multicellular (C) eukaryotic (D) prokaryotic

Q.37 Which of the following curve/straight line equations will pass through the origin when plotted on a graph?

(A) $\frac{-x}{2}+\frac{y}{2}=0$
(B) $1+y+x=1$
(C) $xy=1$
(D) $2y-2x+2=0$

Q.38 Consider two bodies with equal masses of 10^{12} kg each and R distance apart. Let G be the gravitational constant and V_0 be a constant with dimensions of energy. Which of the following represent(s) gravitational potential energy (V) between the bodies, such that Newton's law of gravitation is valid?

(A) $V=\frac{-G}{R}10^{24}$
(B) $V=\frac{-G}{R}10^{24}+1000\,V_0$
(C) $V=\frac{G}{R^2}10^{24}$
(D) $V=10^{12}GR$

Q.39 Which of the following is(are) CORRECT?

(A) Both glucose and fructose have the same molecular formula
(B) The positions of the oxygen and carbon differ in the structures of glucose and fructose
(C) Both glucose and fructose have the same physical properties
(D) Both glucose and fructose are monosaccharides

Q.40 Which of the following gas(es) function(s) as signaling molecule(s) in the human nervous system?

(A) Nitric oxide
(B) Carbon monoxide
(C) Helium
(D) Argon

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 The generation time of *E. coli* is 20 minutes. If there are 10^6 *E. coli* present in an exponentially growing synchronous culture, then the average time (in minutes) required to obtain a final population of 4×10^6 *E. coli* is _____.

Q.42 The solution to the integral $\int_0^1 2y\sqrt{1+y^2}\,dy$, rounded off to TWO decimal places, is _____.

Q.43 Let $A = \begin{pmatrix} \sin\theta & \cos\theta \\ \cos\theta & \sin\theta \end{pmatrix}$ and $A + A^T - 2I = 0$, where A^T is the transpose of A and I is the identity matrix. The value of θ (in degrees) is _____.

Q.44 Truth table of a logic gate is given below:

Input A	Input B	Output Y
0	0	1
0	1	1
1	1	0
1	0	X

The value of X in the above table is _____.

Q.45 Consider two particles, each of mass 20 g; the first particle is moving with a speed of 10 m/s along a one-dimensional track in the positive x-direction and collides with the second particle at rest. Assuming that the collision is elastic, the speed (in m/s) of the first particle after the collision is _____.

Q.46 In the following reaction, the values of ΔH and ΔS at temperature 25 °C are $-13.7\ kcal/mole$ and $-16.0\ cal/(K \cdot mole)$, respectively.

$$Cd^{2+} + 4CH_3NH_2 \rightarrow Cd(CH_3NH_2)_4^{2+}$$

The value of ΔG (in $kcal/mole$) of the reaction, rounded off to TWO decimal places, is _____.

Q.47 The volume (in mL) required to prepare 350 mL of 1X buffer solution from a fifty times (50X) concentrated buffer stock solution is _____.

Q.48 A compound microscope has its objective with linear magnification of 10. In order to achieve a final magnification of 100, the angular magnification of the eyepiece should be _____.

Q.49 The decimal reduction time (DRT or D value) of a bacterial culture is one minute. If a suspension of the bacterial culture contains an initial population of 10^6 cells, then the time (in minutes) required to reduce the number of bacteria to 10 by heat treatment is _____.

Q.50 The median of Y in the following data is _____.

Serial number	1	2	3	4	5	6
Y	22	12	10	14	16	20

Q. 51 – Q. 60 carry two marks each.

Q.51 A variable number of tandem repeats (VNTR) locus has 15 different alleles. The number of genotypes possible in a population for this VNTR is _____.

Q.52 The vibrational frequency (expressed in wavenumber) of $^1H^{35}Cl$ is 2990.6 cm^{-1}. Assuming that the force constant is same in both the cases, vibrational frequency (in cm^{-1}) of $^2D^{35}Cl$ is _____.

Q.53 The length of transverse and conjugate axis in a hyperbola are 6 and 8, respectively. The eccentricity of the hyperbola, rounded off to TWO decimal places, is _____.

Q.54 The solution to the limit $\lim_{x\to 0}\left(\frac{2-\sqrt{4-x}}{x}\right)$ is _____.

Q.55 The average value of function $f(x)=\sqrt{9-x^2}$ on $[-3,3]$, rounded off to TWO decimal places, is _____.

Q.56 A bouncing ball is dropped from an initial height of h meters above a flat surface. Each time the ball hits the surface, it rebounds a distance $r \times h$ meters and it bounces indefinitely. Consider the value of h = 5 meters and $r = 1/3$. The total vertical distance (up and down) travelled (in meters) by the ball is _____.

Q.57 One point charge (q) each, is placed along a line at 3 different points x = 0, x = 2 nm and x = 6 nm. The force between two charges separated by 2 nm is 2 piconewton (pN). The magnitude of force (in pN) on the charge in the middle due to the other two charges is _____.

Q.58 Energy of the electron in hydrogen atom in its ground state is 13.6 eV. The energy required (in eV) to move the electron from its ground state to the first excited state, rounded off to TWO decimal places, is _____.

Q.59 At a given time *t*, velocity (*v*) and acceleration (*a*) of a particle undergoing simple harmonic motion are given by

$$v(t) = -100\sin\left(20t+\frac{\pi}{3}\right),$$

$$a(t) = -2000\cos\left(20t+\frac{\pi}{3}\right).$$

Assuming all quantities are in SI units, the amplitude of the oscillation is _____.

Q.60 In an enzyme catalyzed first-order reaction, the substrate conversion follows an exponential pattern such that 80 % of the substrate is converted in 10 minutes. The first-order rate constant (in min^{-1}) of the reaction, rounded off to THREE decimal places, is _____.

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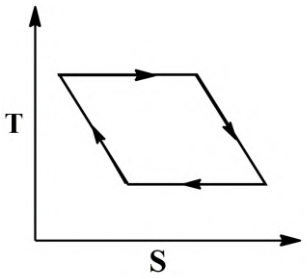
SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

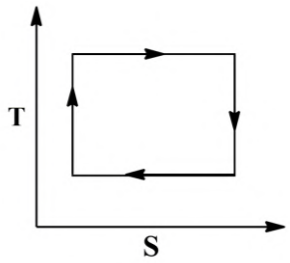
Q. 1 – Q.10 carry one mark each.

Q.1 The graph that represents the temperature (T) – entropy (S) variation of a Carnot cycle is

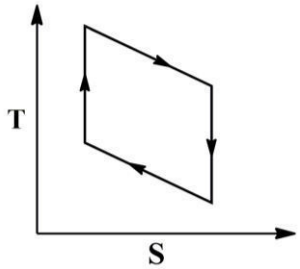
(A)



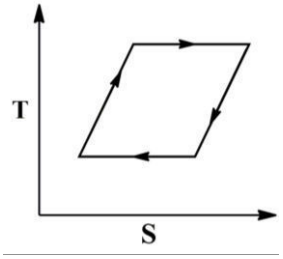
(B)



(C)



(D)



Q.2 For the radical chain reaction below, the correct classification for step 2 and step 3 is, respectively,

Step 1: $Br_2 + M \longrightarrow 2Br\bullet + M$

Step 2: $Br\bullet + H_2 \rightleftharpoons HBr + H\bullet$

Step 3: $H\bullet + Br_2 \longrightarrow HBr + Br\bullet$

(A) chain propagating, chain terminating

(B) chain branching, chain terminating

(C) chain propagating, chain propagating

(D) chain propagating, chain branching

Q.3 The salt bridge in a galvanic cell allows the flow of

(A) ions but NOT electrons

(B) BOTH ions and electrons

(C) electrons but NOT ions

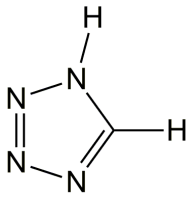
(D) NEITHER ions NOR electrons

Q.4 The nucleobase NOT found in DNA is

(A) Thymine (B) Uracil

(C) Guanine (D) Adenine

Q.5 The correct statement for the following structures is



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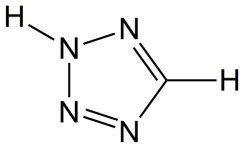

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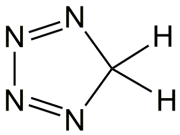


3

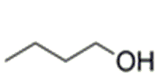
(A) **1**, **2** and **3** are resonance structures

(B) **1** and **2** are resonance structures, whereas **3** is an isomer of **1** and **2**

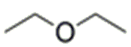
(C) **1** and **3** are resonance structures, whereas **2** is an isomer of **1** and **3**

(D) **1**, **2** and **3** are constitutional isomers

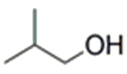
Q.6 The correct order of boiling points of compounds I–IV is



I



II



III

IV

(A) II > I > III > IV (B) II > III > I > IV

(C) I > III > IV > II (D) I > IV > III > II

Q.7 One of the products of the hydrolysis of calcium phosphide at 25° C is

(A) phosphine

(B) phosphoric acid

(C) phosphorus pentoxide

(D) white phosphorus

Q.8 Treatment of formic acid with concentrated sulfuric acid gives

(A) $CO + H_2O$

(B) $CO_2 + H_2$

(C) $HCHO + ½\ O_2$

(D) no product (no reaction)

Q.9 The *d*-orbitals involved in the hybridization to form square planar and trigonal bipyramidal geometries are, respectively,

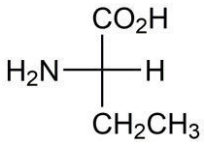
(A) d_{z^2} and d_{z^2}

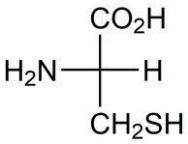
(B) d_{yz} and d_{z^2}

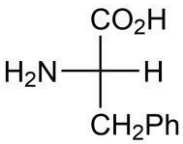
(C) $d_{x^2-y^2}$ and d_{z^2}

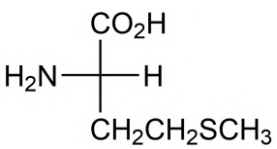
(D) $d_{x^2-y^2}$ and d_{yz}

Q.10 The amino acid with *R* configuration is

(A) 

(B) 

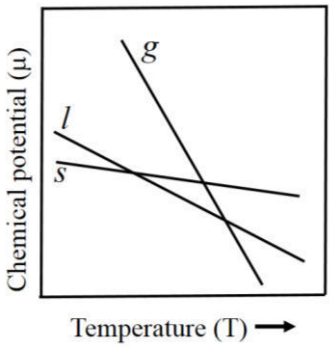
(C) 

(D) 

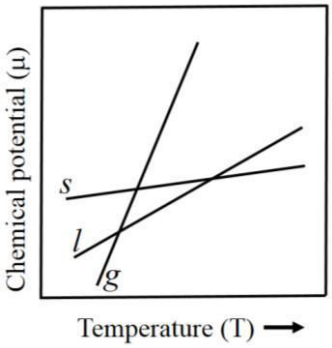
Q. 11 – Q. 30 carry two marks each.

Q.11 At constant pressure, the μ – T diagram for a pure substance that sublimes is (s = solid, l = liquid and g = gas)

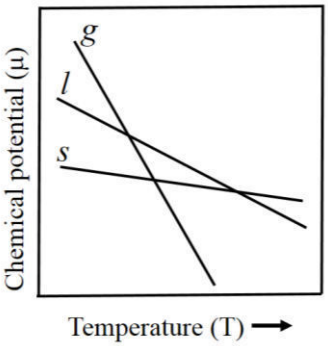
(A)



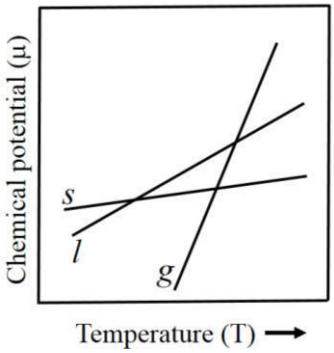
(B)



(C)



(D)



Q.12 The force constant for $H^{35}Cl$ and $D^{35}Cl$ are the same and both can be considered as harmonic oscillators. $H^{35}Cl$ has a fundamental vibrational transition at 2886 cm^{-1}. The ratio of the zero-point energy of $H^{35}Cl$ to that of $D^{35}Cl$ is

(A) 0.515 (B) 0.717 (C) 1.395 (D) 1.946

Q.13 The correct statement regarding the determinants (Det) of matrices *R*, *S* and *T* is

$$R = \begin{bmatrix} 3 & 2 & 4 \\ 4 & 5 & 7 \\ 1 & 3 & 8 \end{bmatrix} \qquad S = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 4 & 7 \\ 3 & 1 & 8 \end{bmatrix} \qquad T = \begin{bmatrix} 3 & 4 & 1 \\ 2 & 5 & 3 \\ 4 & 7 & 8 \end{bmatrix}$$

(A) Det(*R*) = Det(*S*) ≠ Det(*T*)

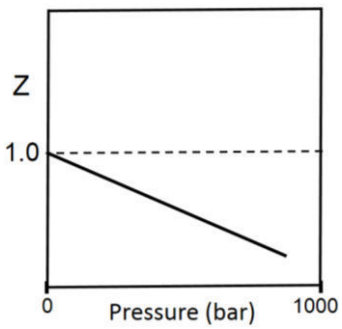
(B) Det(*R*) = Det(*T*) ≠ Det(*S*)

(C) Det(*R*) = Det(*S*) = Det(*T*)

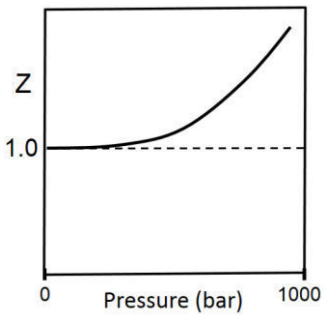
(D) Det(*R*), Det(*S*), Det(*T*) are all different

Q.14 The Boyle temperature (T_B) is defined as the temperature at which the properties of a real gas coincide with those of an ideal gas in the low pressure limit. The graph that shows the pressure dependence of the compression factor (Z) for a real gas at T_B is

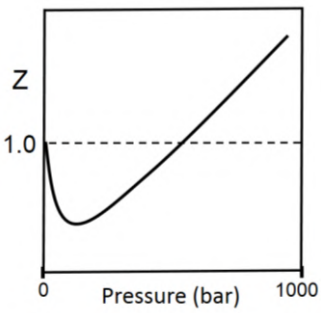
(A)



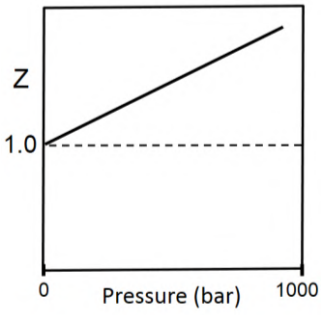
(B)



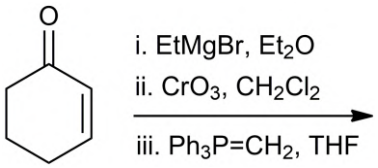
(C)

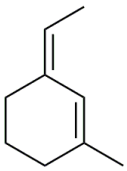


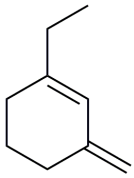
(D)

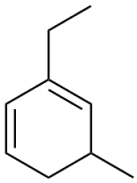


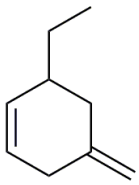
Q.15 The major product formed in the following reaction sequence is



(A) 

(B) 

(C) 

(D) 

Q.16 The geometries of the species $[Br_3]^+$, $[Br_3]^-$ and $[BrF_3]$ are, respectively,

(A) linear, trigonal bipyramidal and trigonal bipyramidal

(B) linear, linear and trigonal planar

(C) tetrahedral, trigonal bipyramidal and trigonal bipyramidal

(D) tetrahedral, trigonal pyramidal and trigonal planar

Q.17 The cage – type structure adopted by boron hydride, $[B_5H_{11}]$, is

(A) *closo* (B) *nido* (C) *hypo* (D) *arachno*

Q.18 The order of the M–C bond strength in the following species is
(Atomic number for Cr = 24, Mn = 25, Ti = 22, Co = 27)

$[Cr(CO)_6]$ $[Mn(CO)_6]^+$ $[Ti(CO)_6]^{2-}$ $[Co(CO)_4]^-$
I II III IV

(A) II > I > IV > III (B) I > III > II > IV

(C) III > IV > I > II (D) III > I > II > IV

Q.19 The number of non-bonding electrons present in the frontier molecular orbitals of HF is

(A) 10 (B) 4 (C) 6 (D) 8

Q.20 The coordination number of aluminum ion and the number of bridging hydrogen atoms in $[Al(BH_4)_4]^-$ are, respectively,

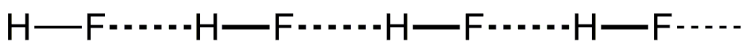
(A) 8 and 8 (B) 6 and 6 (C) 4 and 6 (D) 8 and 12

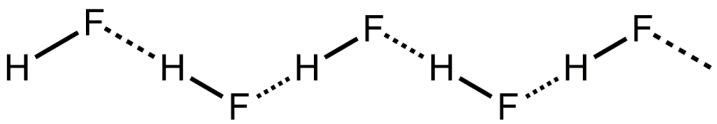
Q.21 The complex which does NOT obey 18-electron rule is
(atomic number for Mn = 25, Fe = 26, Co = 27, Ru = 44)

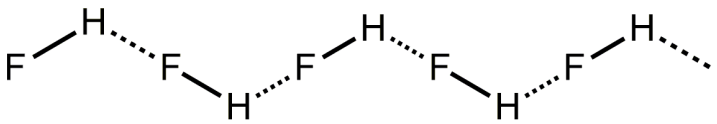
(A) $[Co_2(CO)_8]$ (B) $[Fe(CO)_4]^{2-}$

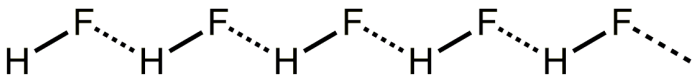
(C) $[HMn(CO)_5]$ (D) $[(\eta^5\text{-}C_5H_5)RuCl(CO)(PPh_2)]$

Q.22 The solid state structure of HF is

(A) 

(B) 

(C) 

(D) 

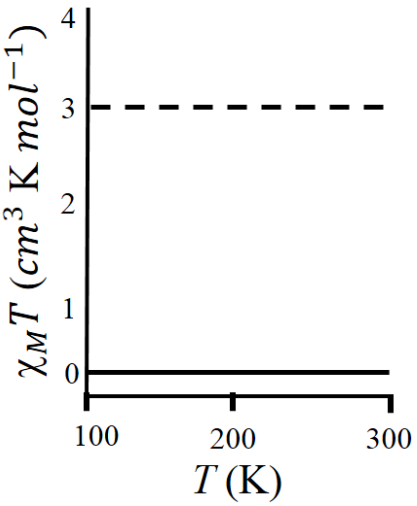
Q.23 The number of *d-d* transition(s) expected for the complex $[Cu(NH_3)_2(H_2O)_4]^{2+}$ is

(A) 1 (B) 2 (C) 3 (D) 4

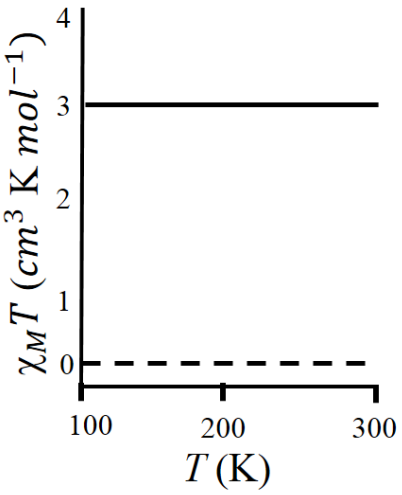
Q.24 The plot showing the magnetic behavior of oxy- (solid line) and deoxy-haemoglobin (dashed line) is

(χ_M = molar magnetic susceptibility, T = temperature)

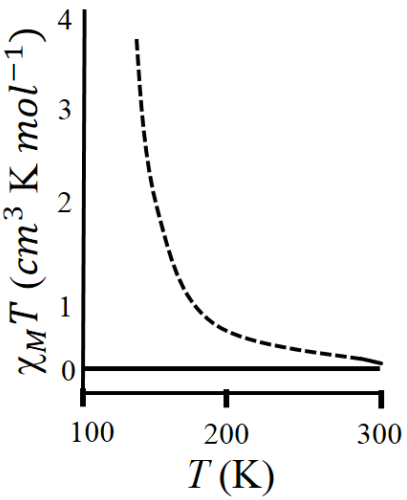
(A)



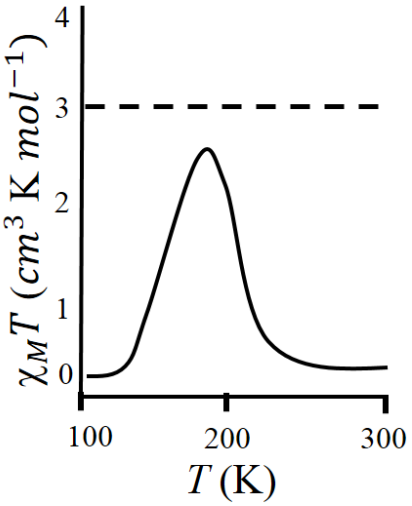
(B)



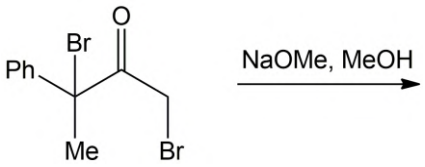
(C)

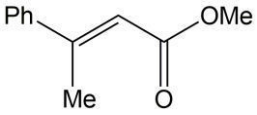


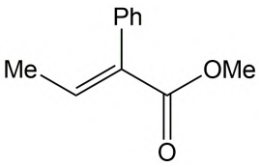
(D)

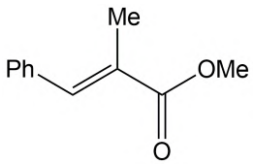


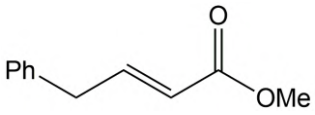
Q.25 The major product formed in the following reaction is



(A) 

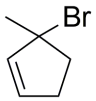
(B) 

(C) 

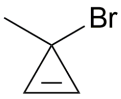
(D) 

Q.26 The rate of solvolysis of I–IV follows

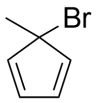
I



II



III



IV

(A) I > II > III > IV

(B) III > I > II > IV

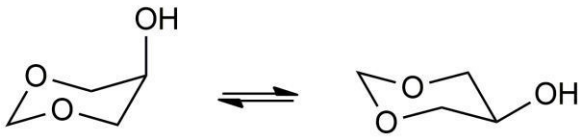
(C) III > II > I > IV

(D) IV > I > II > III

Q.27 The more stable species in each pair of conformers are

I II

III IV

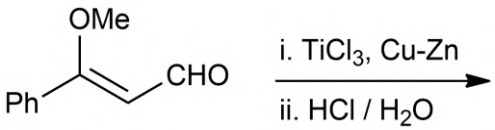


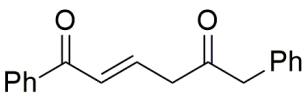
V VI

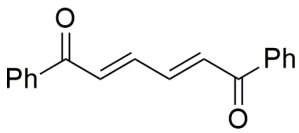
(A) II, IV and V (B) I, IV and V

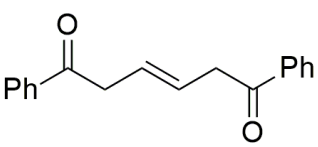
(C) II, III and V (D) I, IV and VI

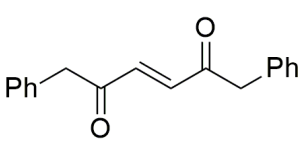
Q.28 The major product formed in the following reaction sequence is



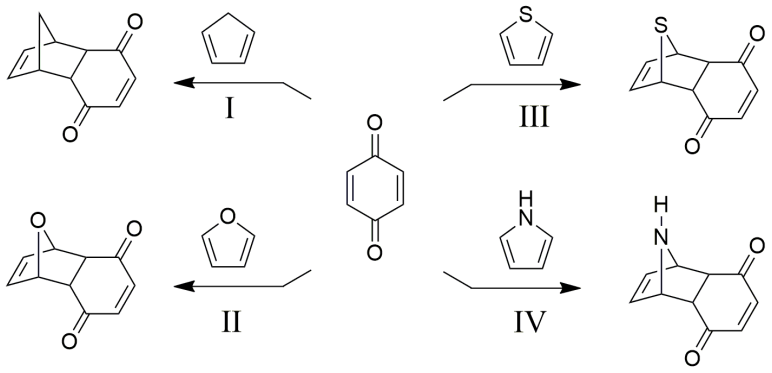
(A) 

(B) 

(C) 

(D) 

Q.29 For the Diels-Alder reactions I–IV, the activation barriers follow the order



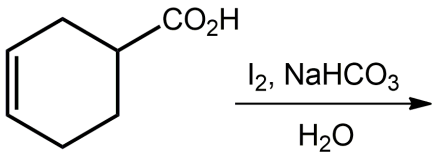
(A) II > I > III > IV

(B) I > III > IV > II

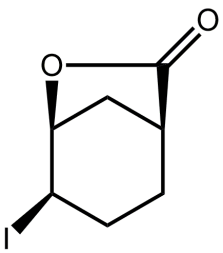
(C) III > IV > II > I

(D) IV > III > II > I

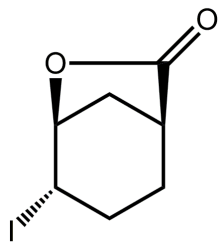
Q.30 The major product formed in the following reaction is



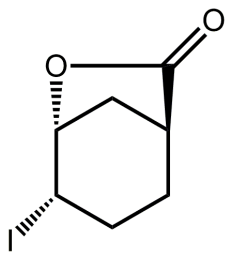
(A)



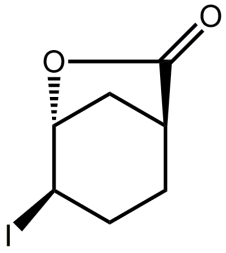
(B)



(C)



(D)

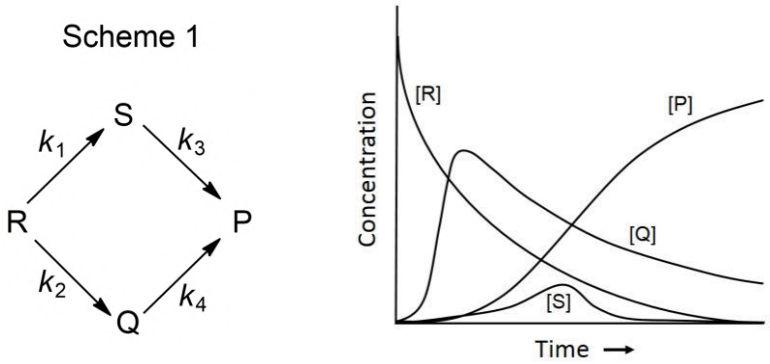


SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 For the reaction shown in Scheme 1, the concentration profiles of different species are provided.



Based on this graph, the correct condition(s) regarding the rate constants is(are)

(A) $k_2 > k_4$ (B) $k_3 > k_1$ (C) $k_2 > k_1$ (D) $k_1 = k_2$

Q.32 $\psi(x,y,z)$ describes the wavefunction of a particle. The probability of finding the particle between x and $x + dx$, y and $y + dy$, z and $z + dz$, can be expressed as

(A) $\psi^*(x,y,z)\ \psi(x,y,z)$

(B) $|\psi(x,y,z)|^2\ dx\ dy\ dz$

(C) $\psi^*(x,y,z)\ \psi(x,y,z) dx\ dy\ dz$

(D) $\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dz\ \psi^*(x,y,z)\ \psi(x,y,z)$

Q.33 In water, the enthalpy of a protein in its folded state (H_F) is lower than that in its unfolded state (H_{UF}). The entropies of the folded and unfolded states are S_F and S_{UF}, respectively. The condition(s) under which this protein spontaneously folds in water at a temperature T, is(are)

(A) $S_{UF} < S_F$ (B) $S_{UF} = 0$

(C) $S_{UF} = S_F$ (D) $(S_F - S_{UF}) > (H_F - H_{UF})/T$

Q.34 The soft Lewis base(s) is(are)

(A) I^- (B) CO (C) H^- (D) CH_3NC

Q.35 The boron adduct(s), which show(s) three signals in ^1H NMR spectrum with the intensity ratio 1:2:3 is(are)

(A) $(CH_3)_3B{:}N(CH_3)_3$ (B) $(CH_3CH_2)_3B{:}N(CH_2CH_3)_3$

(C) $H_3B{:}N(CH_2CH_3)_3$ (D) $(CH_3CH_2)_3B{:}NH_3$

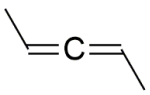
Q.36 The transition metal complex(es) with zero magnetic moment, zero dipole moment and CFSE of $-2.4\ \Delta_o$ is(are)

(A) $[Mn(CO)_5(CH_3)]$ (B) [*trans*-Ni(ethylene diamine)$_2Cl_2$]

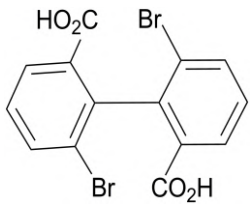
(C) $[trans\text{-}Co(CN)_4(H_2O)_2]^-$ (D) $[trans\text{-}Fe(CN)_4Cl_2]^{4-}$

Q.37 Achiral stereoisomer(s) is(are) possible for

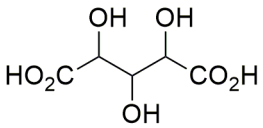
(A)



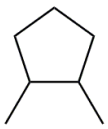
(B)



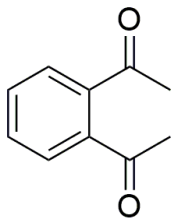
(C)

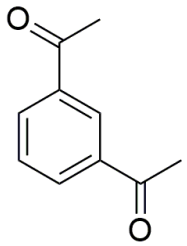


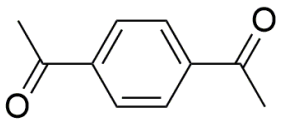
(D)

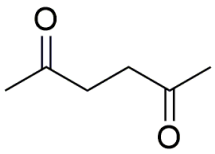


Q.38 The compound(s) which will have only two signals in the ^1H NMR spectrum in 3:2 ratio is(are)

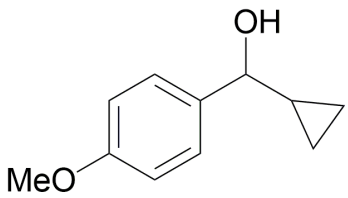
(A) 

(B) 

(C) 

(D) 

Q.39 The correct sequence of reactions for the synthesis of the following molecule is(are)



(A) (i) 4-Iodophenol, Mg, ether
(ii) Cyclopropane carboxaldehyde, THF
(iii) $CsCO_3$, MeI, THF

(B) (i) Cyclopropyl bromide, Mg, ether
(ii) 4-Hydroxybenzaldehyde, THF
(iii) $CsCO_3$, MeI, THF

(C) (i) 4-Iodophenol, $CsCO_3$, MeI, THF
(ii) Mg, ether
(iii) Cyclopropane carboxaldehyde, THF

(D) (i) Cyclopropyl bromide, Mg, ether
(ii) Methyl 4-methoxybenzoate, THF

Q.40 The organometallic reagent(s) among the following is(are)

(A) Lithium divinylcuprate

(B) Lithium diisopropylamide

(C) Potassium *tert*-butoxide

(D) Isopropyl magnesiumiodide

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 The function $x^4 e^{-2x/3}$ (for $x > 0$) has a maximum at a value of x equal to _____ (Round off to two decimal places)

Q.42 The longest wavelength of light absorbed by a hydrogen-like atom is 2.48 nm. The nuclear charge (Z) of the atom is _____ (Round off to nearest integer)
(Rydberg constant R_H = 109700 cm^{-1})

Q.43 Fullerene (C_{60}) crystallizes in an FCC unit cell (edge length = 14.14 Å) with one C_{60} centered at each lattice point. The smallest distance (in Å) between the centers of two C_{60} molecules is _____ (Round off to two decimal places)

Q.44 A film of stearic acid partially covers the water surface in a container. The work needed to decrease this coverage by 1 cm^2 is 25.0×10^{-7} J. The surface tension (in N/m) of the film is _____ (Round off to three decimal places)
(Surface tension of pure water is 0.072 N/m)

Q.45 The value of 'n' in $[P_nO_{18}]^{6-}$ is _____

Q.46 The total number of all possible isomers of $[Co(H_2NCH_2CH_2NH_2)_2Cl_2]^+$ and $[Co(H_2NCH_2CH_2NH_2)_3]^{3+}$ together is _____

Q.47 The number of lone pairs present in phosphonic acid (phosphorus acid) is _____

Q.48 Total number of constitutional isomers possible for trimethyl cyclohexane is _____

Q.49 The dihedral (torsional) angle (in degrees) between the two methyl groups in the most stable conformation of *n*-butane is _____ (Round off to nearest integer)

Q.50 The degree of unsaturation (double bond equivalent) for a compound with molecular formula $C_{14}H_{12}O_2$ is _____

Q. 51 – Q. 60 carry two marks each.

Q.51 The heat of formation of MgO at 300 K and 1 bar pressure is –600.60 kJ mol^{-1}. The free energy (in kJ mol^{-1}) of formation of MgO at 280 K is _____ (Round off to nearest integer)

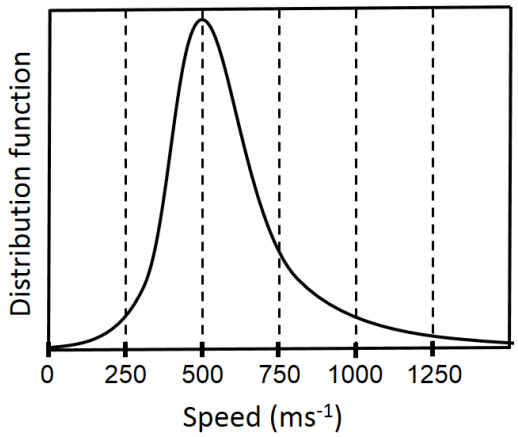
Given: In the range 280-300 K, the constant pressure heat capacities (C_P) and molar entropies (S_m) are:

	Mg	O_2	MgO
C_P (in J mol^{-1} K^{-1})	24.9	29.4	27.0
S_m (in J mol^{-1} K^{-1})	0	205.2	0

Q.52 Sea water containing 1 M NaCl has to be desalinated at 300 K using a membrane permeable only to water. The minimum pressure (in bars) required on the sea-water side of the membrane is _____ (Round off to one decimal place)
(R = 8.3 J mol^{-1} K^{-1}, 1 bar = 10^5 N/m^2)

Q.53 A bacterial colony grows via cell division where each mother bacterium independently produces two daughter cells in 20 minutes. If the concentration of bacteria is 10^4 cm^{-3}, the colony becomes harmful. Starting from a colony with an initial concentration of 5 cm^{-3}, the time taken (in minutes) for the colony to become harmful is _____ (Round off to nearest integer)

Q.54 The Maxwell distribution of speeds of a gas at 300 K is given below



The molar mass (in g mol^{-1}) of this gas is _____ (Round off to one decimal place)
($R = 8.3$ J mol^{-1} K^{-1})

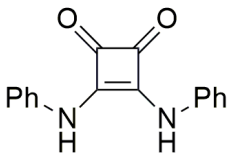
Q.55 At a certain wavelength, liquid P transmits 70%, whereas liquid Q transmits 30% of the incident light when separately placed in a spectrophotometric cell (path length = 1 cm). In a binary mixture of liquids P and Q (assume non-interacting liquids), the absorbance in the same cell is 0.25. The volume fraction of liquid P in the binary mixture is _____
(Round off to two decimal places)

Q.56 The mean ionic activity coefficient for a 0.01 M aqueous solution of $Ca_3(PO_4)_2$ is _____ (Round off to three decimal places)
(Given: $\log_{10} \gamma_{\pm} = -0.509\, z_+ |z_-| \sqrt{I}$)

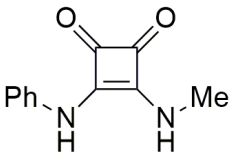
Q.57 For the reaction, $CuSO_4(aq) + Zn(s) \rightarrow ZnSO_4(aq) + Cu(s)$, the value of $\Delta G°$ (in kJ mol^{-1}) is _____ (Round off to the nearest integer)
(Reduction potential: $Cu^{2+}(aq)/Cu(s) = +0.34$ V; $Zn^{2+}(aq)/Zn(s) = -0.76$ V)
(Faraday constant = 96485 C mol^{-1})

Q.58 Titanium tetrachloride ($TiCl_4$) reacts with THF to form an octahedral complex **X** under inert atmosphere at 25 °C. If 5.0 g of $TiCl_4$ is used and the yield is 80%, the amount of **X** (in grams) formed is _____ (Round off to one decimal place)
(Use atomic weights: Ti = 48, Cl = 35.5, O = 16, C = 12, and H = 1)

Q.59 The total number of tautomers possible for I and II together is _____

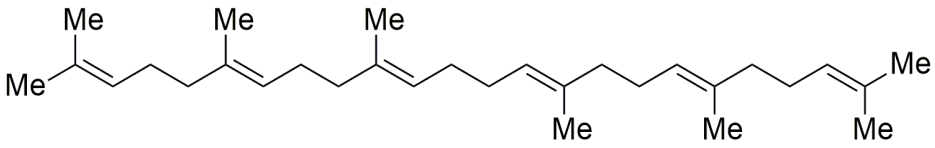


I



II

Q.60 The total number of head to tail isoprene linkages in the following molecule is _____



END OF THE QUESTION PAPER

Paper Specific Instructions

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, **A**, **B** and **C**. All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ).** Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ).** Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for this type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.

7. The Scribble Pad will be provided for rough work.

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 The Mesozoic Era approximately ranges from

(A) 1000-540 Ma (B) 250-65 Ma (C) 540-250 Ma (D) 65 Ma-Present

Q.2 A trench is found at a

(A) divergent plate boundary
(B) convergent plate boundary
(C) transform boundary
(D) passive margin

Q.3 The strike (in degree notation) of a bed dipping 30° towards N45W is

(A) 045-225 (B) 025-205 (C) 020-200 (D) 030-210

Q.4 Randomly oriented ore-bearing veinlets in a mass of rock are known as

(A) en echelon veins
(B) sheeted veins
(C) lode
(D) stockworks

Q.5 The diagnostic amphibole in a blueschist facies metabasalt is

(A) hornblende (B) anthophyllite (C) glaucophane (D) actinolite

Q.6 Atoll is a geomorphic feature formed by

(A) glacial erosion
(B) wind flow abrasion
(C) fluvial deposition
(D) coral reef accumulation

Q.7 Approximately 71% of the planetary mass in the solar system is concentrated in

(A) Uranus (B) Mercury (C) Saturn (D) Jupiter

Q.8 Which of the following oil field is NOT located in the western part of India?

(A) Bombay High
(B) Ankleshwar
(C) Gandhar
(D) Moran

Q.9 The most abundant element in the Earth's continental crust is

(A) silicon (B) aluminium (C) oxygen (D) iron

Q.10 Mammalian fossils are commonly found in

(A) Haimanta Group
(B) Jabalpur Group
(C) Siwalik Group
(D) Uttatur Group

Q. 11 – Q. 30 carry two marks each.

Q.11 Hardness of groundwater is determined by

(A) Mohs' scale of hardness
(B) concentrations of calcium and magnesium
(C) Bernoulli equation
(D) Darcy's law

Q.12 Which one of the following defines a hexagonal dipyramid?

(A) A vertical six fold axis of symmetry
(B) A horizontal mirror plane
(C) Six vertical mirror planes at an angle of 30° with each other
(D) A mirror plane that is perpendicular to the vertical six fold axis of symmetry

Q.13 Match the twinning in **Group I** with the corresponding mineral in **Group II**.

Group I	**Group II**
P. Cross-hatched	1. Plagioclase
Q. Carlsbad	2. Microcline
R. Polysynthetic	3. Sanidine
S. Brazil	4. Quartz

(A)	(B)	(C)	(D)
P-4	P-2	P-1	P-2
Q-2	Q-3	Q-2	Q-3
R-1	R-1	R-3	R-4
S-3	S-4	S-4	S-1

Q.14 Evidence of Late Paleozoic glaciation is recorded in

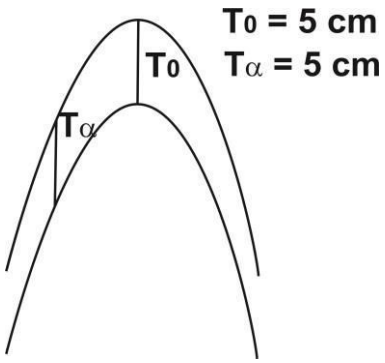
(A) Panchet Formation
(B) Talchir Formation
(C) Motur Formation
(D) Barakar Formation

Q.15 Match the landform in **Group I** with the corresponding geological process in **Group II**.

Group I	**Group II**
P. Cinder Cone	1. Eolian
Q. Ox-bow lake	2. Glacial
R. Draas	3. Volcanic
S. Drumlin	4. Fluvial

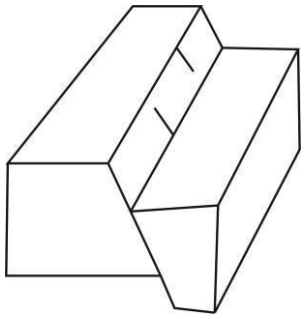
(A)	(B)	(C)	(D)
P-3	P-4	P-2	P-3
Q-4	Q-2	Q-4	Q-2
R-1	R-1	R-3	R-4
S-2	S-3	S-1	S-1

Q.16 Identify the fold in the given figure, where $\mathbf{T_0}$ and $\mathbf{T_\alpha}$ represent the axial plane thicknesses at the hinge and limb, respectively.



(A) Parallel fold
(B) Similar fold
(C) Supratenuous fold
(D) Flattened parallel fold

Q.17 Identify the type of fault in the given figure.



(A) Normal fault
(B) Strike-slip fault
(C) Reverse fault
(D) Thrust fault

Q.18 In clastic sediments, the correct order of decreasing grain size is

(A) Boulder > pebble > silt > sand
(B) Granule > pebble > clay > silt
(C) Cobble > granule > silt > clay
(D) Granule > pebble > sand > silt

Q.19 Match the primary sedimentary structure in **Group I** with the corresponding process of formation in **Group II**.

Group I	**Group II**
P. Asymmetric ripples	1. Water escape
Q. Dish and pillar	2. Bed load transportation of sediments
R. Flute cast	3. Deposition from alternate traction and suspension load
S. Wavy bedding	4. Scouring by turbulent eddy

(A)	(B)	(C)	(D)
P-1	P-2	P-2	P-3
Q-3	Q-1	Q-4	Q-1
R-2	R-4	R-1	R-4
S-4	S-3	S-3	S-2

Q.20 The correct pair of metal and the ore mineral is

(A) Nickel - *Sphalerite* (B) Tin - *Cassiterite*
(C) Zinc - *Pyrolusite* (D) Lead - *Bornite*

Q.21 Match the mineral/metal deposit in **Group I** with the corresponding Indian occurrence in **Group II**.

Group I	**Group II**
P. Diamond	1. Malanjkhand
Q. Iron	2. Wajrakarur
R. Fluorite	3. Dalli Rajhara
S. Copper	4. Ambadongar

(A)	(B)	(C)	(D)
P-4	P-3	P-2	P-2
Q-3	Q-4	Q-1	Q-3
R-2	R-1	R-4	R-4
S-1	S-2	S-3	S-1

Q.22 The mineral sequence arranged in order of increasing degree of sharing of $(SiO_4)^{4-}$ tetrahedra is

(A) Olivine→Tremolite→Enstatite→Quartz
(B) Olivine→Enstatite→Tremolite→Quartz
(C) Quartz→Tremolite→Enstatite→Olivine
(D) Olivine→Quartz→Enstatite→Tremolite

Q.23 The metamorphic facies series that explains oceanic subduction zone metamorphism is

(A) Prehnite-Pumpellyite→Blueschist→Eclogite
(B) Greenschist→Epidote amphibolite→Amphibolite→Granulite
(C) Albite-epidote hornfels→Hornblende hornfels→Pyroxene hornfels→Sanidinite
(D) Greenschist→Amphibolite→Granulite→Eclogite

Q.24 The correct combination of textural feature of magmatic rocks with corresponding petrological process is

(A) Exsolution lamellae – *Sub-solidus cooling*
(B) Reaction rim – *Eutectic crystallization*
(C) Graphic intergrowth – *Quenching of ultramafic lava*
(D) Spinifex texture – *Peritectic crystallization*

Q.25 A volcanic rock consisting of alkali feldspar (70%), sodic plagioclase (10%) and nepheline (20%) is named as

(A) Phonolite (B) Tephrite (C) Trachyte (D) Andesite

Q.26 Match the name of granitoid in **Group I** with the corresponding Craton in **Group II**.

Group I	**Group II**
P. Closepet	1. Singhbhum
Q. Berach	2. Bastar
R. Dongargarh	3. Dharwar
S. Mayurbhanj	4. Aravalli

(A)	(B)	(C)	(D)
P-4	P-3	P-3	P-4
Q-2	Q-4	Q-4	Q-2
R-1	R-2	R-1	R-3
S-3	S-1	S-2	S-1

Q.27 Match the seismic discontinuity in **Group I** with their occurrence in Earth's interior in **Group II**.

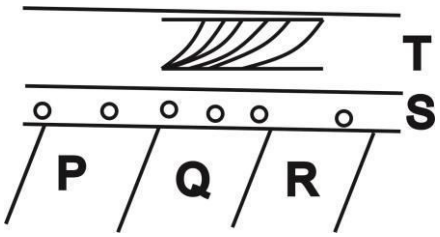
Group I	**Group II**
P. Conrad	1. Between lower mantle and outer core
Q. Mohorovičić	2. Between crust and upper mantle
R. Gutenberg	3. Between inner and outer core
S. Lehmann	4. Between lower and upper crust

(A)	(B)	(C)	(D)
P-4	P-4	P-3	P-2
Q-2	Q-2	Q-2	Q-4
R-1	R-3	R-4	R-1
S-3	S-1	S-1	S-3

Q.28 Choose the correct statement:

(A) Porosity of weathered granite is less than a crystalline granite.
(B) Coarse sands have high porosity and high permeability.
(C) Clays have high porosity and high permeability.
(D) Ground water table does not fluctuate with water recharge.

Q.29 In the given profile section, **P**, **Q**, **R**, **S** and **T** are sedimentary rocks. Identify the type of unconformity in the sedimentary sequence.



(A) Angular unconformity
(B) Disconformity
(C) Nonconformity
(D) Paraconformity

Q.30 Which of the following pairs is **NOT** correctly matched?

(A) *Productus* - Brachiopoda
(B) *Redlichia* - Arthropoda
(C) *Belemnites* - Cephalopoda
(D) *Gryphea* - Gastropoda

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 Which of the following statements in relation to the solar system is/are correct?

(A) The most abundant elements are H and He.
(B) The abundances of elements with atomic numbers 1-50 show an overall decreasing trend.
(C) The abundances of heavier elements (atomic number >50) are mostly higher than that of lighter elements (atomic number <50).
(D) Elements having odd atomic numbers are more abundant than their immediate neighbours.

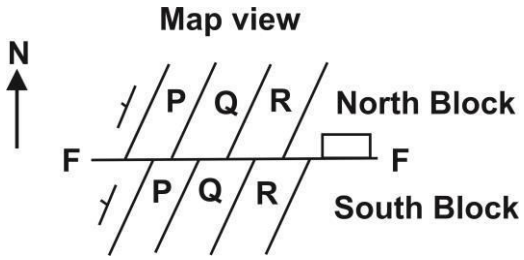
Q.32 Find out the correct statement(s).

(A) Authigenic minerals form during diagenesis of sandstone.
(B) Heavy minerals in sandstone are good indicators of provenance.
(C) An arkose is mineralogically more mature than a quartz arenite.
(D) Matrix in sandstone may form by post-depositional infiltration and/or authigenic filling.

Q.33 Which of the following is/are correctly matched?

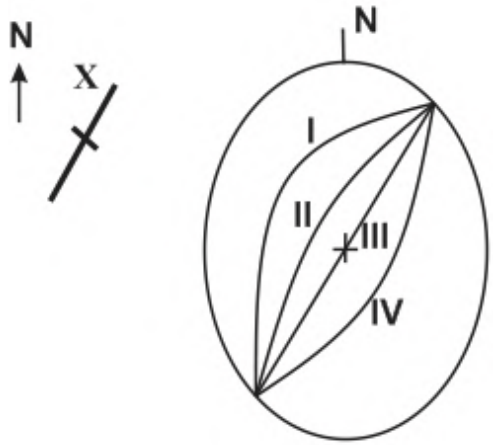
(A) Dendritic drainage pattern – *uniform substrate and gentle slope*
(B) Trellis drainage pattern – *parallel valleys and ridges*
(C) Radial drainage pattern – *uniform flat topography*
(D) Rectangular drainage pattern – *joints and faults*

Q.34 A northerly dipping fault (F-F) has displaced beds **P**, **Q** and **R**. The thickness of the beds across the fault is same. Identify the fault type(s).



(A) Dextral fault
(B) Reverse fault
(C) Normal fault
(D) Sinistral fault

Q.35 A bedding plane, pictorially represented at X, will be plotted in stereonet as



(A) I (B) II (C) III (D) IV

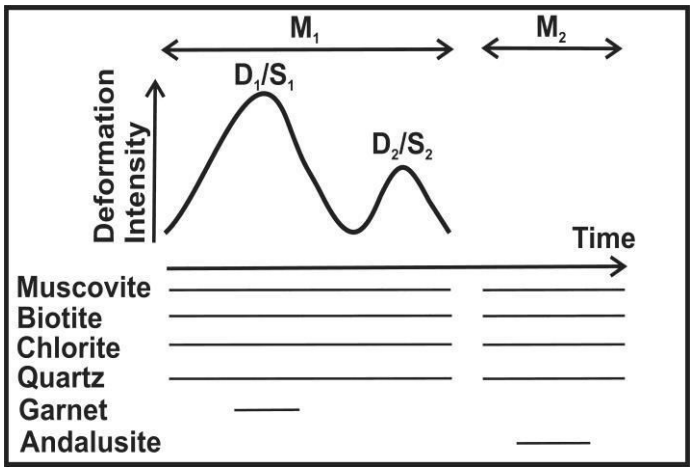
Q.36 Choose the correct statement(s):

(A) Stratiform chromite deposits form by weathering of ultramafic rocks.
(B) Skarn deposits mostly form in calcareous host rocks in contact with felsic intrusives.
(C) Nickeliferous laterite is a type of placer deposit.
(D) Gold-bearing vein deposits can form from hydrothermal solutions.

Q.37 Under crossed polars, a mineral remains dark at all stages of rotation of the microscope stage. Which of the following statement(s) is/are **NOT** correct?

(A) The mineral is isotropic.
(B) The mineral is biaxial with an optic orientation corresponding to the principal section of the indicatrix.
(C) The mineral is biaxial with an optic orientation containing the Y-vibration direction only.
(D) The mineral is uniaxial with an orientation perpendicular to the optic axis.

Q.38 A schematic deformation (D)-metamorphism (M)-time map of a metamorphic belt is shown below. The belt recorded two phases of deformation, D_1-D_2 and attendant schistosities, S_1-S_2 and metamorphism, M_1-M_2. The metamorphic minerals stable during these events are shown by solid lines.
On the basis of this information, which of the following statement(s) is/are correct?



(A) M_1 and M_2 events refer to regional and contact metamorphism, respectively.
(B) The S_1 schistosity developed as a crenulation cleavage.
(C) Garnet grew syntectonic with D_1.
(D) Andalusite grew during the waning phase of D_2.

Q.39 Choose the correct statement(s):

(A) Gravity dam is a rigid structure, straight or slightly curved in places.
(B) Arch dams are commonly built on very strong foundations.
(C) Buttress dam chiefly consists of reinforced concrete slab that slopes upstream.
(D) Earth dams have a concrete wall curved with convex face pointed towards the upstream.

Q.40 Choose the correct pair(s):

(A) Proterozoic – *Origin of flowering plants*
(B) Paleozoic – *Origin of amphibians*
(C) Mesozoic – *Acme of trilobites*
(D) Cenozoic – *Dominance of mammals*

SECTION – C

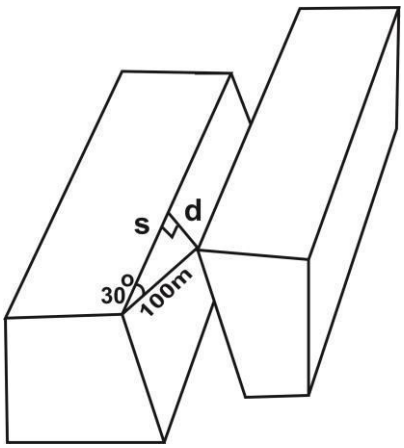
NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

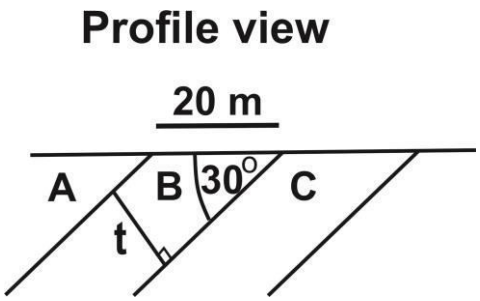
Q.41 The wt.% (correct to two decimal places) of Cu in chalcopyrite ($CuFeS_2$) (atomic weight of Cu=63.55, Fe=55.85, S=32.07) is ____

Q.42 The general formula of an amphibole mineral is $\mathbf{A_{0\text{-}1}B_2C_5T_8O_{22}(OH)_2}$, where A, B, C and T are cationic sites with different co-ordination numbers as stated below:
A=12, B=6-8, C=6, T=4.
The amount of octahedral Al in an amphibole of composition
$\mathbf{Na_{0.6}Ca_2Mg_{3.8}Al_{3.0}Si_{6.2}O_{22}(OH)_2}$ is ____

Q.43 In the block diagram, the net slip (=100 m) is resolved into strike slip (s) and dip slip (d) components. The value (in m, correct to two decimal places) of "**s**" is ____



Q.44 The true thickness (t, in m) of bed B in the given diagram is _____.



Q.45 If (326) is the Miller Index of a crystal face, then the value of **x** in the corresponding Weiss Parameter of the same face, **x**a: **y**b: **z**c is ____

Q.46 The value of h in the Miller-Bravais Index ($\bar{4}$1h0) is ____

Q.47 In an ocean basin, a 4 Ma old oceanic crust lies 40 km away from the ridge axis. The average velocity (in cm/yr) of the oceanic lithosphere is ____

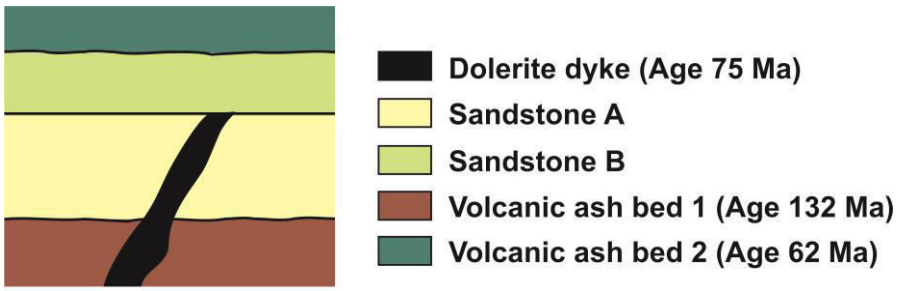
Q.48 An aquifer has a cross sectional area of 1000 m^2 and a hydraulic gradient of 0.01. If water is flowing from the aquifer at a rate of 10 m^3/sec, the hydraulic conductivity (in m/sec) of the aquifer is ____

Q.49 According to the mineralogical phase rule, the number of minerals that can coexist at equilibrium in a 8 component chemical system with 2 degrees of freedom is ____

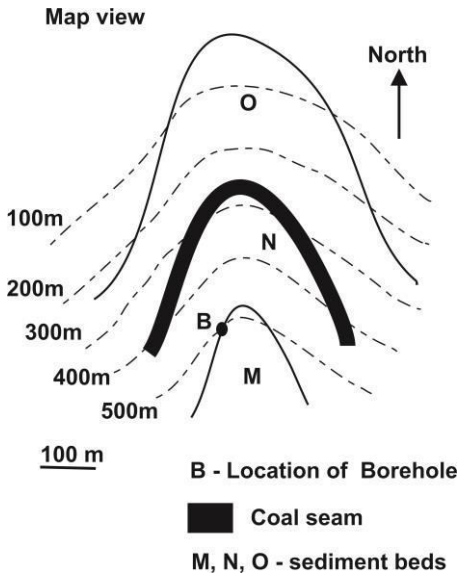
Q.50 The grain density (of solids only) and bulk density (solids + voids) of a sandstone sample are 2.7 gm/cm^3 and 2.3 gm/cm^3, respectively. The total porosity (in %, correct to two decimal places) of the sample is ____

Q. 51 – Q. 60 carry two marks each.

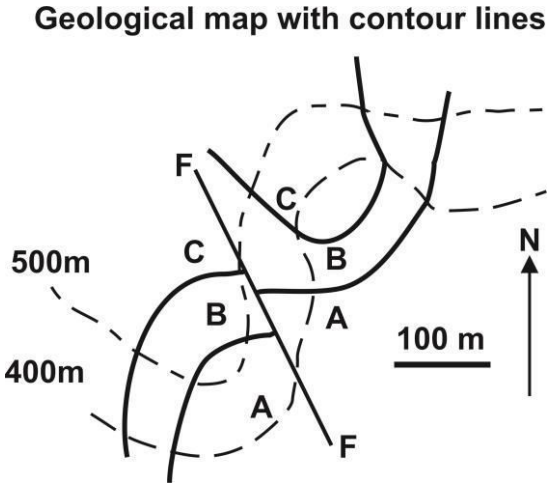
Q.51 In an undeformed and normal stratigraphic succession, a dolerite dyke was emplaced before deposition of sandstone B. The difference between the maximum ages (in Myr) of deposition of sandstone A and sandstone B is ____



Q.52 A coal seam occurs in a stratigraphic sequence as shown in the figure. If a vertical borehole is drilled at location B, the coal seam will be intersected at a depth (in m) of ____



Q.53 A set of sedimentary rocks **A**, **B** and **C** are affected by a fault F-F.
The amount of vertical throw (in m) along the fault is ____



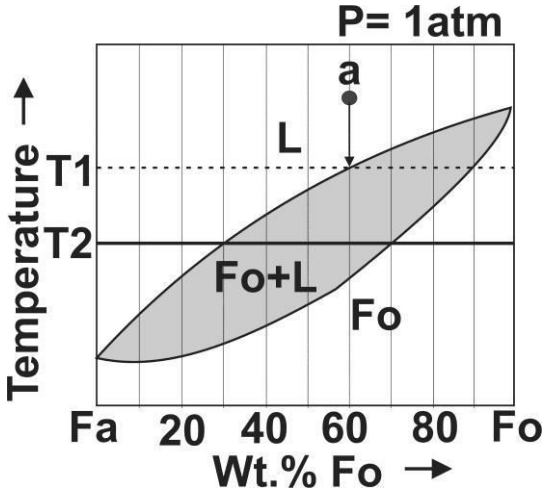
Q.54 The retardation of a uniaxial negative mineral of thickness 0.03 mm is 5160 nm in its principal section of indicatrix. If the refractive index corresponding to the E-ray is 1.486, the value of the refractive index (correct to three decimal places) of the O-ray is ____

Q.55 A spherical ore body (diameter=40m) has 7% metal content and density of 3300 kg/m^3.
The reserve (in tonne) of the ore body is ____

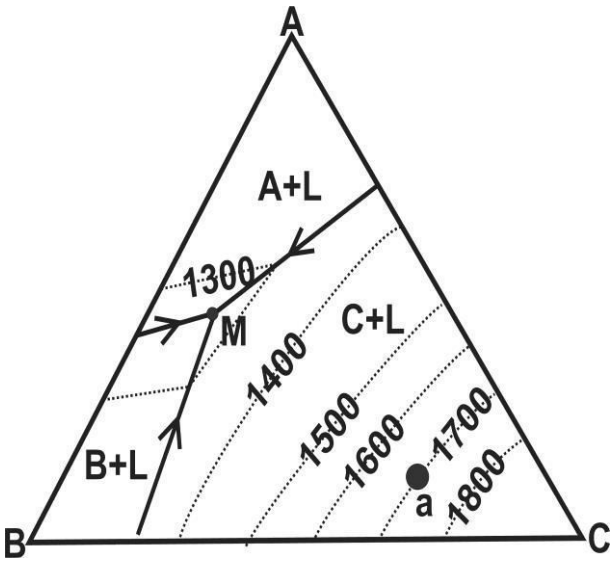
Q.56 From the data shown in the table, the weighted mean size (in micrometer, correct to two decimal places) of the sediment population is _____.

Grain size (micrometer)	Dry sediment weight (in gram)
4	50
20	75
40	125
60	50

Q.57 Consider the schematic isobaric T-X phase diagram in the binary forsterite (Fo)-fayalite (Fa) chemical system. If there is equilibrium crystallization of melt (L), the wt.% of olivine crystallized from a melt of composition "**a**" at a temperature T2 is ____



Q.58 Consider a schematic isobaric ternary phase diagram A-B-C, shown below. The diagram, which is contoured with isopleths of liquidus temperatures (in $^{\circ}$C), reveals crystallization behaviour of melt (L) of different compositions during cooling. When a melt of composition "**a**" lies at a temperature of 1800°C, the variance (or degree of freedom) of the magmatic system is ____



Q.59 An eclogite consists of garnet (60%) and omphacite (40%), where the mineral abundances are in mole %. X_{Mg} [=Mg/(Mg+Fe^{2+})] of garnet and omphacite is 0.50 and 0.75, respectively. The X_{Mg} of eclogite is ____

Q.60 A harzburgite contains pure forsterite and pure enstatite in a molecular ratio of 60:40. The mole % of MgO in the rock is ____

END OF THE QUESTION PAPER

Paper Specific Instructions

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, **A**, **B** and **C**. All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ).** Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ).** Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.

7. The Scribble Pad will be provided for rough work.

Special Instructions / Useful Data

$\mathbb{R}$	The set of all real numbers
$\mathbb{R}^n$	$\{(x_1, x_2, \dots, x_n): x_i \in \mathbb{R}, i = 1,2, \dots, n\}, n = 2, 3, \dots$
f'	First order derivative of the differentiable function f
M^{-1}	Inverse of the non-singular matrix M
$\det(M)$	Determinant of the square matrix M
$\text{trace}(M)$	Trace of the square matrix M
I_n	Identity matrix of order $n \times n$, $n = 2, 3, \dots$
$P(H)$	Probability of event H
H^c	Complement of an event H
$E(X)$	Expectation of the random variable X
$\text{Var}(X)$	Variance of the random variable X
i.i.d.	Independently and identically distributed
$\text{Bin}(n, p)$	Binomial distribution with parameters n and p, $n = 1, 2, \dots$ and $p \in (0, 1)$
$\text{Poisson}(\lambda)$	Poisson distribution with mean λ, $\lambda > 0$
$U(a, b)$	Continuous uniform distribution on the interval (a, b), $-\infty < a < b < \infty$
$\text{Exp}(\lambda)$	Exponential distribution with probability density function $f(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$, $\lambda > 0$
$N(\mu, \sigma^2)$	Normal distribution with mean μ and variance σ^2, $\mu \in \mathbb{R}$ and $\sigma > 0$
χ^2_n	Central Chi-squared distribution with n degrees of freedom, $n = 1, 2, \dots$
t_n	Central t-distribution with n degrees of freedom, $n = 1, 2, \dots$
$\chi^2_{n,\alpha}$	For $\alpha \in (0, 1)$ and positive integer n, $P(Y_n > \chi^2_{n,\alpha}) = \alpha$, where $Y_n \sim \chi^2_n$
$F_{m,n}$	Central F distribution with m and n degrees of freedom; $m, n = 1, 2, \dots$
$f_{m,n,\alpha}$	For $\alpha \in (0, 1)$ and positive integers m and n, $P(F_{m,n} > f_{m,n,\alpha}) = \alpha$
$n!$	$n \cdot (n-1) \cdots 3 \cdot 2 \cdot 1$, $n = 1, 2, \dots$ and $0! = 1$
$\binom{n}{k}$	$\frac{n!}{(n-k)!\,k!}$, $k = 0,1, \dots, n$ and $n = 1, 2, \dots$

$$\Phi(a) = \int_{-\infty}^{a} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2}\, dt, -\infty < a < \infty$$

$$\Phi(0.5) = 0.6915, \Phi(1) = 0.8413, \Phi(1.5) = 0.9332, \Phi(2) = 0.9772$$

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 If $\{x_n\}_{n\geq 1}$ is a sequence of real numbers such that $\lim_{n\to\infty} \frac{x_n}{n} = 0.001$, then

(A) $\{x_n\}_{n\geq 1}$ is a bounded sequence

(B) $\{x_n\}_{n\geq 1}$ is an unbounded sequence

(C) $\{x_n\}_{n\geq 1}$ is a convergent sequence

(D) $\{x_n\}_{n\geq 1}$ is a monotonically decreasing sequence

Q.2 For real constants a and b, let

$$f(x) = \begin{cases} \dfrac{a\sin x - 2x}{x}, & x < 0 \\ bx, & x \geq 0 \end{cases}$$

If f is a differentiable function then the value of $a + b$ is

(A) 0 (B) 1 (C) 2 (D) 3

Q.3 The area of the region bounded by the curves $y_1(x) = x^4 - 2x^2$ and $y_2(x) = 2x^2$, $x \in \mathbb{R}$, is

(A) $\frac{128}{15}$ (B) $\frac{129}{15}$ (C) $\frac{133}{15}$ (D) $\frac{134}{15}$

Q.4 Consider the following system of linear equations

$$\begin{aligned} ax + 2y + z &= 0 \\ y + 5z &= 1 \\ by - 5z &= -1 \end{aligned}$$

Which one of the following statements is TRUE?

(A) The system has unique solution for $a = 1, b = -1$

(B) The system has unique solution for $a = -1, b = 1$

(C) The system has no solution for $a = 1, b = 0$

(D) The system has infinitely many solutions for $a = 0, b = 0$

Q.5 Let E and F be two events. Then which one of the following statements is **NOT** always TRUE?

(A) $P(E \cap F) \leq \max\{1 - P(E^c) - P(F^c), 0\}$ (B) $P(E \cup F) \geq \max\{P(E), P(F)\}$

(C) $P(E \cup F) \leq \min\{P(E) + P(F), 1\}$ (D) $P(E \cap F) \leq \min\{P(E), P(F)\}$

Q.6 Let X be a random variable having Poisson(2) distribution. Then $E\left(\frac{1}{1+X}\right)$ equals

(A) $1-e^{-2}$ (B) e^{-2} (C) $\frac{1}{2}(1-e^{-2})$ (D) $\frac{1}{2}e^{-1}$

Q.7 The mean and the standard deviation of weights of ponies in a large animal shelter are 20 kg and 3 kg, respectively. A pony is selected at random from the shelter. Using Chebyshev's inequality, the value of the lower bound of the probability that the weight of the selected pony is between 14 kg and 26 kg is

(A) $\frac{3}{4}$ (B) $\frac{1}{4}$ (C) 0 (D) 1

Q.8 Let $X_1, X_2, \ldots, X_{10}$ be a random sample from $N(1,2)$ distribution. If

$$\bar{X} = \frac{1}{10}\sum_{i=1}^{10} X_i \quad \text{and} \quad S^2 = \frac{1}{9}\sum_{i=1}^{10}(X_i - \bar{X})^2,$$

then $\text{Var}(S^2)$ equals

(A) $\frac{2}{5}$ (B) $\frac{4}{9}$ (C) $\frac{11}{9}$ (D) $\frac{8}{9}$

Q.9 Let $\{X_n\}_{n\geq 1}$ be a sequence of i.i.d. random variables such that $E(X_i) = 1$ and $\text{Var}(X_i) = 1$, $i = 1, 2, \ldots$. Then the approximate distribution of $\frac{1}{\sqrt{n}}\sum_{i=1}^{n}(X_{2i} - X_{2i-1})$, for large n, is

(A) $N(0,1)$ (B) $N(0,2)$ (C) $N(0,0.5)$ (D) $N(0,0.25)$

Q.10 Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables having $N(\mu, \sigma^2)$ distribution, where $\mu \in \mathbb{R}$ and $\sigma > 0$. Define

$$W = \frac{1}{2n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}(X_i - X_j)^2$$

Then W, as an estimator of σ^2, is

(A) biased and consistent (B) unbiased and consistent

(C) biased and inconsistent (D) unbiased and inconsistent

Q. 11 – Q. 30 carry two marks each.

Q.11 Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers such that $a_1 = 1, a_2 = 7$ and $a_{n+1} = \frac{a_n + a_{n-1}}{2}, n \geq 2$. Assuming that $\lim_{n\to\infty} a_n$ exists, the value of $\lim_{n\to\infty} a_n$ is

(A) $\frac{19}{4}$ (B) $\frac{9}{2}$ (C) 5 (D) $\frac{21}{4}$

Q.12 Which one of the following series is convergent?

(A) $\sum_{n=1}^{\infty} \left(\frac{5n+1}{4n+1}\right)^n$ (B) $\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^n$

(C) $\sum_{n=1}^{\infty} \frac{\sin\left(\frac{1}{n}\right)}{n^{1/n}}$ (D) $\sum_{n=1}^{\infty} \sqrt{n}\left(1 - \cos\left(\frac{1}{n}\right)\right)$

Q.13 Let α and β be two real numbers. If

$$\lim_{x\to 0} \frac{\tan 2x - 2\sin \alpha x}{x(1-\cos 2x)} = \beta$$

then $\alpha + \beta$ equals

(A) $\frac{1}{2}$ (B) 1 (C) $\frac{3}{2}$ (D) $\frac{5}{2}$

Q.14 Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} \frac{2x^3 + 3y^3}{x^2 + y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

Let $f_x(0,0)$ and $f_y(0,0)$ denote the first order partial derivatives of $f(x,y)$ with respect to x and y, respectively, at the point $(0,0)$. Then which one of the following statements is TRUE?

(A) f is continuous at $(0,0)$ but $f_x(0,0)$ and $f_y(0,0)$ do not exist

(B) f is differentiable at $(0,0)$

(C) f is not differentiable at $(0,0)$

(D) f is not continuous at $(0,0)$ but $f_x(0,0)$ and $f_y(0,0)$ exist

Q.15 If the volume of the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 2y$ is given by

$$\int_0^{\alpha} \int_{\beta(y)}^{2y} \int_{-\sqrt{z-y^2}}^{\sqrt{z-y^2}} dx\, dz\, dy$$

then

(A) $\alpha = 2$ and $\beta(y) = y,\ y \in [0, 2]$ (B) $\alpha = 1$ and $\beta(y) = y^2,\ y \in [0, 1]$

(C) $\alpha = 2$ and $\beta(y) = y^2,\ y \in [0, 2]$ (D) $\alpha = 1$ and $\beta(y) = y,\ y \in [0, 1]$

Q.16 The value of the integral

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2 + y^2}\ dy\, dx$$

is

(A) $\frac{17}{9}$ (B) $\frac{16}{9}$ (C) $\frac{14}{9}$ (D) $\frac{13}{9}$

Q.17 Let $T: \mathbb{R}^3 \to \mathbb{R}^4$ be a linear transformation. If $T(1, 1, 0) = (2, 0, 0, 0)$, $T(1, 0, 1) = (2, 4, 0, 0)$ and $T(0, 1, 1) = (0, 0, 2, 0)$, then $T(1, 1, 1)$ equals

(A) $(1, 1, 1, 0)$ (B) $(0, 1, 1, 1)$ (C) $(2, 2, 1, 0)$ (D) $(0, 0, 0, 0)$

Q.18 Let M be an $n \times n$ non-zero skew symmetric matrix. Then the matrix $(I_n - M)(I_n + M)^{-1}$ is always

(A) singular (B) symmetric (C) orthogonal (D) idempotent

Q.19 A packet contains 10 distinguishable firecrackers out of which 4 are defective. If three firecrackers are drawn at random (without replacement) from the packet, then the probability that all three firecrackers are defective equals

(A) $\frac{1}{10}$ (B) $\frac{1}{20}$ (C) $\frac{1}{30}$ (D) $\frac{1}{40}$

Q.20 Let X_1, X_2, X_3, X_4 be i.i.d. random variables having a continuous distribution. Then $P(X_3 < X_2 < \max(X_1, X_4))$ equals

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{6}$

Q.21 Consider the simple linear regression model

$$y_i = \alpha + \beta x_i + \epsilon_i, \qquad i = 1, 2, \ldots, n,$$

where $\epsilon_i's$ are i.i.d. random variables with mean 0 and variance $\sigma^2 \in (0, \infty)$. Suppose that we have a data set $(x_1, y_1), \ldots, (x_n, y_n)$ with $n = 20$, $\sum_{i=1}^{n} x_i = 100$, $\sum_{i=1}^{n} y_i = 50$, $\sum_{i=1}^{n} x_i^2 = 600$, $\sum_{i=1}^{n} y_i^2 = 500$ and $\sum_{i=1}^{n} x_i y_i = 400$. Then the least square estimates of α and β are, respectively,

(A) 5 and $\frac{3}{2}$ (B) -5 and $\frac{3}{2}$ (C) 5 and $-\frac{3}{2}$ (D) -5 and $-\frac{3}{2}$

Q.22 Let Z_1 and Z_2 be i.i.d. $N(0, 1)$ random variables. If $Y = Z_1^2 + Z_2^2$, then $P(Y > 4)$ equals

(A) e^{-2} (B) $1 - e^{-2}$ (C) $\frac{1}{2}e^{-2}$ (D) e^{-4}

Q.23 Consider a sequence of independent Bernoulli trials with probability of success in each trial being $\frac{1}{3}$. Let X denote the number of trials required to get the second success. Then $P(X \geq 5)$ equals

(A) $\frac{3}{7}$ (B) $\frac{16}{27}$ (C) $\frac{16}{21}$ (D) $\frac{9}{13}$

Q.24 Let the joint probability density function of (X, Y) be

$$f(x, y) = \begin{cases} 2e^{-(x+y)}, & 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases}$$

Then $P\left(X < \frac{Y}{2}\right)$ equals

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{1}{2}$

Q.25 Let X_1, X_2, X_3, X_4, X_5 be a random sample from $N(0, 1)$ distribution and let $W = \frac{X_1^2}{X_2^2 + X_3^2 + X_4^2 + X_5^2}$. Then $E(W)$ equals

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{5}$

Q.26 Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu_1, \sigma^2)$ distribution and $Y_1, Y_2, \ldots, Y_m$ be a random sample from $N(\mu_2, \sigma^2)$ distribution, where $\mu_i \in \mathbb{R}, i = 1, 2$ and $\sigma > 0$. Suppose that the two random samples are independent. Define

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \quad \text{and} \quad W = \frac{\sqrt{mn}\ (\bar{X} - \mu_1)}{\sqrt{\sum_{i=1}^{m}(Y_i - \mu_2)^2}}$$

Then which one of the following statements is TRUE for all positive integers m and n?

(A) $W \sim t_m$ (B) $W \sim t_n$

(C) $W^2 \sim F_{m,1}$ (D) $W^2 \sim F_{m,n}$

Q.27 Let $X_1, X_2, \ldots, X_n$ be a random sample from $U(\theta - 0.5, \theta + 0.5)$ distribution, where $\theta \in \mathbb{R}$. If $X_{(1)} = \min\{X_1, X_2, \ldots, X_n\}$ and $X_{(n)} = \max\{X_1, X_2, \ldots, X_n\}$, then which one of the following estimators is **NOT** a maximum likelihood estimator of θ?

(A) $\frac{1}{2}\left(X_{(1)} + X_{(n)}\right)$ (B) $\frac{1}{4}\left(3X_{(1)} + X_{(n)} + 1\right)$

(C) $\frac{1}{4}\left(X_{(1)} + 3X_{(n)} - 1\right)$ (D) $\frac{1}{2}\left(3X_{(n)} - X_{(1)} - 2\right)$

Q.28 Let $X_1, X_2, \ldots, X_n$ be a random sample from $\text{Exp}(\theta)$ distribution, where $\theta \in (0, \infty)$. If $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$, then a 95% confidence interval for θ is

(A) $\left(0, \frac{\chi^2_{2n,0.95}}{n\bar{X}}\right]$ (B) $\left[\frac{\chi^2_{2n,0.95}}{n\bar{X}}, \infty\right)$

(C) $\left(0, \frac{\chi^2_{2n,0.05}}{2n\bar{X}}\right]$ (D) $\left[\frac{\chi^2_{2n,0.05}}{2n\bar{X}}, \infty\right)$

Q.29 Let $X_1, X_2, \ldots, X_n$ be a random sample from $U(1, 2)$ distribution and let $Y_1, Y_2, \ldots, Y_n$ be a random sample from $U(0, 1)$ distribution. Suppose that the two random samples are independent. Define

$$Z_i = \begin{cases} 1, & \text{if } X_i Y_i < 1 \\ 0, & \text{otherwise} \end{cases}, \quad i = 1, 2, \ldots n$$

If $\lim_{n\to\infty} P\left(\left|\frac{1}{n}\sum_{i=1}^{n} Z_i - \theta\right| < \epsilon\right) = 1$, for all $\epsilon > 0$, then θ equals

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\log_e \frac{3}{2}$ (D) $\log_e 2$

Q.30 Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with probability density function

$$f_\theta(x) = \begin{cases} \theta(1-x)^{\theta-1}, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}, \qquad \theta > 0$$

To test $H_0: \theta = 1$ against $H_1: \theta > 1$, the uniformly most powerful test of size α $(0 < \alpha < 1)$ would reject H_0 if

(A) $-\sum_{i=1}^{n} \log_e(1-X_i)^2 < \chi^2_{2n,1-\alpha}$ (B) $-\sum_{i=1}^{n} \log_e(1-X_i)^2 < \chi^2_{n,1-\alpha}$

(C) $-\sum_{i=1}^{n} \log_e(1-X_i)^2 < \chi^2_{2n,\alpha}$ (D) $-\sum_{i=1}^{n} \log_e(1-X_i)^2 < \chi^2_{n,\alpha}$

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 Let the sequence $\{x_n\}_{n\geq 1}$ be given by $x_n = \sin\frac{n\pi}{6}$, $n = 1, 2, \ldots$. Then which of the following statements is/are TRUE?

(A) The sequence $\{x_n\}_{n\geq 1}$ has a subsequence that converges to $\frac{1}{2}$

(B) $\limsup_{n\to\infty} x_n = 1$

(C) $\liminf_{n\to\infty} x_n = -1$

(D) The sequence $\{x_n\}_{n\geq 1}$ has a subsequence that converges to $\frac{1}{\sqrt{2}}$

Q.32 Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y) = x^2(2-y) - y^3 + 3y^2 + 9y$, where $(x,y) \in \mathbb{R}^2$. Which of the following is/are saddle point(s) of f?

(A) $(0,-1)$ (B) $(0,3)$ (C) $(3,2)$ (D) $(-3,2)$

Q.33 The arc length of the parabola $y^2 = 2x$ intercepted between the points of intersection of the parabola $y^2 = 2x$ and the straight line $y = 2x$ equals

(A) $\int_0^1 \sqrt{1+y^2}\, dy$ (B) $\int_0^1 \sqrt{1+4y^2}\, dy$

(C) $\int_0^{1/2} \frac{\sqrt{1+2x}}{\sqrt{2x}}\, dx$ (D) $\int_0^{1/2} \frac{\sqrt{1+4x}}{\sqrt{2x}}\, dx$

Q.34 For real constants a and b, let

$$M = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ a & b \end{bmatrix}$$

be an orthogonal matrix. Then which of the following statements is/are always TRUE?

(A) $a + b = 0$ (B) $b = \sqrt{1 - a^2}$ (C) $ab = -\frac{1}{2}$ (D) $M^2 = I_2$

Q.35 Consider a sequence of independent Bernoulli trials with probability of success in each trial being $\frac{1}{5}$. Then which of the following statements is/are TRUE?

(A) Expected number of trials required to get the first success is 5

(B) Expected number of successes in first 50 trials is 10

(C) Expected number of failures preceding the first success is 4

(D) Expected number of trials required to get the second success is 10

Q.36 Let (X, Y) have the joint probability mass function

$$f(x,y) = \begin{cases} \binom{x+1}{y}\binom{16}{x}\left(\frac{1}{6}\right)^{y}\left(\frac{5}{6}\right)^{x+1-y}\left(\frac{1}{2}\right)^{16}, & y = 0, 1, \ldots, x+1;\ x = 0, 1, \ldots, 16 \\ 0, & \text{otherwise} \end{cases}$$

Then which of the following statements is/are TRUE?

(A) $E(Y) = \frac{3}{2}$ (B) $\text{Var}(Y) = \frac{49}{36}$ (C) $E(XY) = \frac{37}{3}$ (D) $\text{Var}(X) = 3$

Q.37 Let X_1, X_2, X_3 be i.i.d. $N(0, 1)$ random variables. Then which of the following statements is/are TRUE?

(A) $\frac{\sqrt{2}\,X_1}{\sqrt{X_2^2 + X_3^2}} \sim t_2$ (B) $\frac{\sqrt{2}\,X_1}{|X_2 + X_3|} \sim t_1$

(C) $\frac{(X_1 - X_2)^2}{(X_1 + X_2)^2} \sim F_{1,1}$ (D) $\sum_{i=1}^{3} X_i^2 \sim \chi_2^2$

Q.38 Let $\{X_n\}_{n\geq 1}$ be a sequence of i.i.d. random variables such that

$$P(X_1 = 0) = \frac{1}{4} = 1 - P(X_1 = 1)$$

Define

$$U_n = \frac{1}{n}\sum_{i=1}^{n} X_i \quad \text{and} \quad V_n = \frac{1}{n}\sum_{i=1}^{n}(1 - X_i)^2 \ , \qquad n = 1, 2, \ldots$$

Then which of the following statements is/are TRUE?

(A) $\lim_{n\to\infty} P\left(\left|U_n - \frac{3}{4}\right| < \frac{1}{100}\right) = 1$

(B) $\lim_{n\to\infty} P\left(\left|U_n - \frac{3}{4}\right| > \frac{1}{100}\right) = 0$

(C) $\lim_{n\to\infty} P\left(\sqrt{n}\left(U_n - \frac{3}{4}\right) \leq 1\right) = \Phi(2)$

(D) $\lim_{n\to\infty} P\left(\sqrt{n}\left(V_n - \frac{1}{4}\right) \leq 1\right) = \Phi\left(\frac{4}{\sqrt{3}}\right)$

Q.39 Let $X_1, X_2, \ldots, X_n$ be i.i.d. Poisson(λ) random variables, where $\lambda > 0$. Define

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \quad \text{and} \quad S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$$

Then which of the following statements is/are TRUE?

(A) $\text{Var}(\bar{X}) < \text{Var}(S^2)$

(B) $\text{Var}(\bar{X}) = \text{Var}(S^2)$

(C) $\text{Var}(\bar{X})$ attains the Cramer-Rao lower bound

(D) $E(\bar{X}) = E(S^2)$

Q.40 Consider the following two probability density functions (pdfs)

$$f_0(x) = \begin{cases} 2x, & \text{if } 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad f_1(x) = \begin{cases} 1, & \text{if } 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Let X be a random variable having pdf $f \in \mathcal{P} = \{f_0, f_1\}$. Consider testing $H_0: f(x) = f_0(x), \forall\, x \in [0,1]$ against $H_1: f(x) = f_1(x), \forall\, x \in [0,1]$ at $\alpha = 0.05$ level of significance. For which of the following observed values of random observation X, the most powerful test would reject H_0?

(A) 0.19 (B) 0.22 (C) 0.25 (D) 0.28

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 $\lim_{n\to\infty} \dfrac{\left(\sqrt{n^2+1}+n\right)^2}{\sqrt[3]{n^6+1}}$ equals _____

Q.42 The maximum value of the function $y = \dfrac{x^2}{x^4+4}, x \in \mathbb{R}$, is _____

Q.43 The value of the integral

$$\int_0^1 \int_{y^2}^1 \frac{e^x}{\sqrt{x}}\, dx\, dy$$

equals _____ *(round off to two decimal places)*

Q.44 The rank of the matrix

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 2 \\ 2 & 5 & 6 & 4 \\ 2 & 6 & 8 & 5 \end{bmatrix}$$

is _____

Q.45 Let (X, Y) have the joint probability density function

$$f(x,y) = \begin{cases} \frac{3}{4}(y-x), & 0 < x < y < 2 \\ 0, & \text{otherwise} \end{cases}$$

Then the conditional expectation $E(X|Y=1)$ equals _____ *(round off to two decimal places)*

Q.46 Let X be a random variable having the Poisson(4) distribution and let E be an event such that $P(E|X=i) = 1-2^{-i}, i = 0, 1, 2, \ldots$. Then $P(E)$ equals _____ *(round off to two decimal places)*

Q.47 Let X_1, X_2 and X_3 be independent random variables such that $X_1 \sim N(47, 10)$, $X_2 \sim N(55,15)$ and $X_3 \sim N(60, 14)$. Then $P(X_1 + X_2 \geq 2X_3)$ equals _____ *(round off to two decimal places)*

Q.48 Let $U \sim F_{5,8}$ and $V \sim F_{8,5}$. If $P[U > 3.69] = 0.05$, then the value of c such that $P[V > c] = 0.95$ equals _____ *(round off to two decimal places)*

Q.49 Let the sample mean based on a random sample from Exp(λ) distribution be 3.7. Then the maximum likelihood estimate of $1 - e^{-\lambda}$ equals _____ *(round off to two decimal places)*

Q.50 Let X be a single observation drawn from $U(0, \theta)$ distribution, where $\theta \in \{1, 2\}$. To test $H_0: \theta = 1$ against $H_1: \theta = 2$ consider the test procedure that rejects H_0 if and only if $X > 0.75$. If the probabilities of Type-I and Type-II errors are α and β, respectively, then $\alpha + \beta$ equals _____ *(round off to two decimal places)*

Q. 51 – Q. 60 carry two marks each.

Q.51 Let $f: [-1, 3] \to \mathbb{R}$ be a continuous function such that f is differentiable on $(-1, 3)$, $|f'(x)| \leq \frac{3}{2}$, $\forall x \in (-1, 3)$, $f(-1) = 1$ and $f(3) = 7$. Then $f(1)$ equals _____

Q.52 Let α be the real number such that the coefficient of x^{125} in Maclaurin's series of $(x + \alpha^3)e^x$ is $\frac{28}{124!}$. Then α equals _____

Q.53 Consider the matrix

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 2 \\ 0 & 1 & 4 \end{bmatrix}$$

Let P be a non-singular matrix such that $P^{-1}MP$ is a diagonal matrix. Then the trace of the matrix $P^{-1}M^3\,P$ equals _____

Q.54 Let P be a 3×3 matrix having characteristic roots $\lambda_1 = -\frac{2}{3}$, $\lambda_2 = 0$ and $\lambda_3 = 1$. Define $Q = 3P^3 - P^2 - P + I_3$ and $R = 3P^3 - 2P$. If $\alpha = \det(Q)$ and $\beta = \text{trace}(R)$, then $\alpha + \beta$ equals _____ *(round off to two decimal places)*

Q.55 Let X and Y be independent random variables with respective moment generating functions

$$M_X(t) = \frac{(8 + e^t)^2}{81} \quad \text{and} \quad M_Y(t) = \frac{(1 + 3e^t)^3}{64}, -\infty < t < \infty$$

Then $P(X + Y = 1)$ equals _____ *(round off to two decimal places)*

Q.56 Let X be a random variable having $U(0,10)$ distribution and $Y = X - [X]$, where $[X]$ denotes the greatest integer less than or equal to X. Then $P(Y > 0.25)$ equals _____

Q.57 A computer lab has two printers handling certain types of printing jobs. Printer-I and Printer-II handle 40% and 60% of the jobs, respectively. For a typical printing job, printing time (in minutes) of Printer-I follows $N(10, 4)$ distribution and that of Printer-II follows $U(1, 21)$ distribution. If a randomly selected printing job is found to have been completed in less than 10 minutes, then the conditional probability that it was handled by the Printer-II equals _____ *(round off to two decimal places)*

Q.58 Let $x_1 = 1, x_2 = 0, x_3 = 0, x_4 = 1, x_5 = 0, x_6 = 1$ be the data on a random sample of size 6 from $\text{Bin}(1, \theta)$ distribution, where $\theta \in (0, 1)$. Then the uniformly minimum variance unbiased estimate of $\theta(1 + \theta)$ equals _____

Q.59 Let $x_1 = 1, x_2 = 4$ be the data on a random sample of size 2 from a $\text{Poisson}(\theta)$ distribution, where $\theta \in (0, \infty)$. Let $\hat{\psi}$ be the uniformly minimum variance unbiased estimate of $\psi(\theta) = \sum_{k=4}^{\infty} \frac{e^{-\theta}\,\theta^k}{k!}$ based on the given data. Then $\hat{\psi}$ equals _____ *(round off to two decimal places)*

Q.60 Let X be a random variable having $N(\theta, 1)$ distribution, where $\theta \in \mathbb{R}$. Consider testing $H_0: \theta = 0$ against $H_1: \theta \neq 0$ at $\alpha = 0.617$ level of significance. The power of the likelihood ratio test at $\theta = 1$ equals _____ *(round off to two decimal places)*

END OF THE QUESTION PAPER

Paper Specific Instructions

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, **A**, **B** and **C**. All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of **30 Multiple Choice Questions (MCQ)**. Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ)**. Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for this type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.

7. The Scribble Pad will be provided for rough work.

NOTATION

1. $\mathbb{N} = \{1, 2, 3, \cdots\}$
2. $\mathbb{R}$ - the set of all real numbers
3. $\mathbb{R} \setminus \{0\}$ - the set of all non-zero real numbers
4. $\mathbb{C}$ - the set of all complex numbers
5. $f \circ g$ - composition of the functions f and g
6. f' and f'' - first and second derivatives of the function f, respectively
7. $f^{(n)}$ - n^{th} derivative of f
8. $\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$
9. $\oint\limits_C$ - the line integral over an oriented closed curve C
10. $\hat{i}, \hat{j}, \hat{k}$ - unit vectors along the Cartisean right handed rectangular co-ordinate system
11. $\hat{n}$ - unit outward normal vector
12. I - identity matrix of appropriate order
13. $\det(M)$ - determinant of the matrix M
14. M^{-1} - inverse of the matrix M
15. M^T - transpose of the matrix M
16. id - identity map
17. $\langle a \rangle$ - cyclic subgroup generated by an element a of a group
18. S_n - permutation group on n symbols
19. $S^1 = \{z \in \mathbb{C} : |z| = 1\}$
20. $o(g)$ - order of the element g in a group

SECTION – A
MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q. 10 carry one mark each.

Q. 1 Let $s_n = 1 + \dfrac{(-1)^n}{n}, n \in \mathbb{N}$. Then the sequence $\{s_n\}$ is

(A) monotonically increasing and is convergent to 1

(B) monotonically decreasing and is convergent to 1

(C) neither monotonically increasing nor monotonically decreasing but is convergent to 1

(D) divergent

Q. 2 Let $f(x) = 2x^3 - 9x^2 + 7$. Which of the following is true?

(A) f is one-one in the interval $[-1, 1]$

(B) f is one-one in the interval $[2, 4]$

(C) f is NOT one-one in the interval $[-4, 0]$

(D) f is NOT one-one in the interval $[0, 4]$

Q. 3 Which of the following is FALSE?

(A) $\lim\limits_{x\to\infty} \dfrac{x}{e^x} = 0$

(B) $\lim\limits_{x\to 0^+} \dfrac{1}{xe^{1/x}} = 0$

(C) $\lim\limits_{x\to 0^+} \dfrac{\sin x}{1+2x} = 0$

(D) $\lim\limits_{x\to 0^+} \dfrac{\cos x}{1+2x} = 0$

Q. 4 Let $g : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function. If $f(x, y) = g(y) + xg'(y)$, then

(A) $\dfrac{\partial f}{\partial x} + y\dfrac{\partial^2 f}{\partial x \partial y} = \dfrac{\partial f}{\partial y}$

(B) $\dfrac{\partial f}{\partial y} + y\dfrac{\partial^2 f}{\partial x \partial y} = \dfrac{\partial f}{\partial x}$

(C) $\dfrac{\partial f}{\partial x} + x\dfrac{\partial^2 f}{\partial x \partial y} = \dfrac{\partial f}{\partial y}$

(D) $\dfrac{\partial f}{\partial y} + x\dfrac{\partial^2 f}{\partial x \partial y} = \dfrac{\partial f}{\partial x}$

Q. 5 If the equation of the tangent plane to the surface $z = 16 - x^2 - y^2$ at the point $P(1, 3, 6)$ is $ax + by + cz + d = 0$, then the value of $|d|$ is

(A) 16 (B) 26 (C) 36 (D) 46

Q. 6 If the directional derivative of the function $z = y^2e^{2x}$ at $(2, -1)$ along the unit vector $\vec{b} = \alpha\hat{i} + \beta\hat{j}$ is zero, then $|\alpha + \beta|$ equals

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\sqrt{2}$ (D) $2\sqrt{2}$

Q. 7 If $u = x^3$ and $v = y^2$ transform the differential equation $3x^5dx - y(y^2 - x^3)dy = 0$ to $\frac{dv}{du} = \frac{\alpha u}{2(u - v)}$, then α is

(A) 4 (B) 2 (C) -2 (D) -4

Q. 8 Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation given by $T(x, y) = (-x, y)$. Then

(A) $T^{2k} = T$ for all $k \geq 1$

(B) $T^{2k+1} = -T$ for all $k \geq 1$

(C) the range of T^2 is a proper subspace of the range of T

(D) the range of T^2 is equal to the range of T

Q. 9 The radius of convergence of the power series

$$\sum_{n=1}^{\infty} \left(\frac{n+2}{n}\right)^{n^2} x^n$$

is

(A) e^2 (B) $\frac{1}{\sqrt{e}}$ (C) $\frac{1}{e}$ (D) $\frac{1}{e^2}$

Q. 10 Consider the following group under matrix multiplication:

$$H = \left\{ \begin{bmatrix} 1 & p & q \\ 0 & 1 & r \\ 0 & 0 & 1 \end{bmatrix} : p, q, r \in \mathbb{R} \right\}.$$

Then the center of the group is isomorphic to

(A) $(\mathbb{R} \setminus \{0\}, \times)$ (B) $(\mathbb{R}, +)$

(C) $(\mathbb{R}^2, +)$ (D) $(\mathbb{R}, +) \times (\mathbb{R} \setminus \{0\}, \times)$

Q. 11 – Q. 30 carry two marks each.

Q. 11 Let $\{a_n\}$ be a sequence of positive real numbers. Suppose that $l = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$. Which of the following is true?

(A) If $l = 1$, then $\lim_{n\to\infty} a_n = 1$ (B) If $l = 1$, then $\lim_{n\to\infty} a_n = 0$

(C) If $l < 1$, then $\lim_{n\to\infty} a_n = 1$ (D) If $l < 1$, then $\lim_{n\to\infty} a_n = 0$

Q. 12 Define $s_1 = \alpha > 0$ and $s_{n+1} = \sqrt{\frac{1+s_n^2}{1+\alpha}}$, $n \geq 1$. Which of the following is true?

(A) If $s_n^2 < \frac{1}{\alpha}$, then $\{s_n\}$ is monotonically increasing and $\lim_{n\to\infty} s_n = \frac{1}{\sqrt{\alpha}}$

(B) If $s_n^2 < \frac{1}{\alpha}$, then $\{s_n\}$ is monotonically decreasing and $\lim_{n\to\infty} s_n = \frac{1}{\alpha}$

(C) If $s_n^2 > \frac{1}{\alpha}$, then $\{s_n\}$ is monotonically increasing and $\lim_{n\to\infty} s_n = \frac{1}{\sqrt{\alpha}}$

(D) If $s_n^2 > \frac{1}{\alpha}$, then $\{s_n\}$ is monotonically decreasing and $\lim_{n\to\infty} s_n = \frac{1}{\alpha}$

Q. 13 Suppose that S is the sum of a convergent series $\sum_{n=1}^{\infty} a_n$. Define $t_n = a_n + a_{n+1} + a_{n+2}$. Then the series $\sum_{n=1}^{\infty} t_n$

(A) diverges

(B) converges to $3S - a_1 - a_2$

(C) converges to $3S - a_1 - 2a_2$

(D) converges to $3S - 2a_1 - a_2$

Q. 14 Let $a \in \mathbb{R}$. If $f(x) = \begin{cases} (x+a)^2, & x \leq 0 \\ (x+a)^3, & x > 0, \end{cases}$

then

(A) $\frac{d^2 f}{dx^2}$ does not exist at $x = 0$ for any value of a

(B) $\frac{d^2 f}{dx^2}$ exists at $x = 0$ for exactly one value of a

(C) $\frac{d^2 f}{dx^2}$ exists at $x = 0$ for exactly two values of a

(D) $\frac{d^2 f}{dx^2}$ exists at $x = 0$ for infinitely many values of a

Q. 15 Let $f(x,y) = \begin{cases} x^2 \sin \frac{1}{x} + y^2 \sin \frac{1}{y}, & xy \neq 0 \\ x^2 \sin \frac{1}{x}, & x \neq 0, y = 0 \\ y^2 \sin \frac{1}{y}, & y \neq 0, x = 0 \\ 0, & x = y = 0. \end{cases}$

Which of the following is true at $(0,0)$?

(A) f is not continuous

(B) $\frac{\partial f}{\partial x}$ is continuous but $\frac{\partial f}{\partial y}$ is not continuous

(C) f is not differentiable

(D) f is differentiable but both $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are not continuous

Q. 16 Let S be the surface of the portion of the sphere with centre at the origin and radius 4, above the xy-plane. Let $\vec{F} = y\hat{i} - x\hat{j} + yx^3\hat{k}$. If $\hat{n}$ is the unit outward normal to S, then

$$\iint_S (\nabla \times \vec{F}) \cdot \hat{n}\, dS$$

equals

(A) -32π (B) -16π (C) 16π (D) 32π

Q. 17 Let $f(x, y, z) = x^3 + y^3 + z^3 - 3xyz$. A point at which the gradient of the function f is equal to zero is

(A) $(-1, 1, -1)$ (B) $(-1, -1, -1)$ (C) $(-1, 1, 1)$ (D) $(1, -1, 1)$

Q. 18 The area bounded by the curves $x^2 + y^2 = 2x$ and $x^2 + y^2 = 4x$, and the straight lines $y = x$ and $y = 0$ is

(A) $3\left(\frac{\pi}{2} + \frac{1}{4}\right)$ (B) $3\left(\frac{\pi}{4} + \frac{1}{2}\right)$ (C) $2\left(\frac{\pi}{4} + \frac{1}{3}\right)$ (D) $2\left(\frac{\pi}{3} + \frac{1}{4}\right)$

Q. 19 Let M be a real 6×6 matrix. Let 2 and -1 be two eigenvalues of M. If $M^5 = aI + bM$, where $a, b \in \mathbb{R}$, then

(A) $a = 10, b = 11$ (B) $a = -11, b = 10$

(C) $a = -10, b = 11$ (D) $a = 10, b = -11$

Q. 20 Let M be an $n \times n$ $(n \geq 2)$ non-zero real matrix with $M^2 = 0$ and let $\alpha \in \mathbb{R} \setminus \{0\}$. Then

(A) α is the only eigenvalue of $(M + \alpha I)$ and $(M - \alpha I)$

(B) α is the only eigenvalue of $(M + \alpha I)$ and $(\alpha I - M)$

(C) $-\alpha$ is the only eigenvalue of $(M + \alpha I)$ and $(M - \alpha I)$

(D) $-\alpha$ is the only eigenvalue of $(M + \alpha I)$ and $(\alpha I - M)$

Q. 21 Consider the differential equation $L[y] = (y - y^2)dx + xdy = 0$. The function $f(x, y)$ is said to be an integrating factor of the equation if $f(x, y)L[y] = 0$ becomes exact.

If $f(x, y) = \dfrac{1}{x^2y^2}$, then

(A) f is an integrating factor and $y = 1 - kxy,\ k \in \mathbb{R}$ is NOT its general solution

(B) f is an integrating factor and $y = -1 + kxy,\ k \in \mathbb{R}$ is its general solution

(C) f is an integrating factor and $y = -1 + kxy,\ k \in \mathbb{R}$ is NOT its general solution

(D) f is NOT an integrating factor and $y = 1 + kxy,\ k \in \mathbb{R}$ is its general solution

Q. 22 A solution of the differential equation $2x^2\dfrac{d^2y}{dx^2} + 3x\dfrac{dy}{dx} - y = 0, x > 0$ that passes through the point $(1, 1)$ is

(A) $y = \dfrac{1}{x}$ (B) $y = \dfrac{1}{x^2}$ (C) $y = \dfrac{1}{\sqrt{x}}$ (D) $y = \dfrac{1}{x^{3/2}}$

Q. 23 Let M be a 4×3 real matrix and let $\{e_1, e_2, e_3\}$ be the standard basis of $\mathbb{R}^3$. Which of the following is true?

(A) If rank$(M) = 1$, then $\{Me_1, Me_2\}$ is a linearly independent set

(B) If rank$(M) = 2$, then $\{Me_1, Me_2\}$ is a linearly independent set

(C) If rank$(M) = 2$, then $\{Me_1, Me_3\}$ is a linearly independent set

(D) If rank$(M) = 3$, then $\{Me_1, Me_3\}$ is a linearly independent set

Q. 24 The value of the triple integral $\iiint_V (x^2y+1)\, dxdydz$, where V is the region given by $x^2+y^2 \leq 1, 0 \leq z \leq 2$ is

(A) π (B) 2π (C) 3π (D) 4π

Q. 25 Let S be the part of the cone $z^2 = x^2 + y^2$ between the planes $z = 0$ and $z = 1$. Then the value of the surface integral $\iint_S (x^2 + y^2)\, dS$ is

(A) π (B) $\dfrac{\pi}{\sqrt{2}}$ (C) $\dfrac{\pi}{\sqrt{3}}$ (D) $\dfrac{\pi}{2}$

Q. 26 Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}, x, y, z \in \mathbb{R}$. Which of the following is FALSE?

(A) $\nabla(\vec{a} \cdot \vec{r}) = \vec{a}$

(B) $\nabla \cdot (\vec{a} \times \vec{r}) = 0$

(C) $\nabla \times (\vec{a} \times \vec{r}) = \vec{a}$

(D) $\nabla \cdot ((\vec{a} \cdot \vec{r})\vec{r}) = 4(\vec{a} \cdot \vec{r})$

Q. 27 Let $D = \{(x, y) \in \mathbb{R}^2 : |x| + |y| \leq 1\}$ and $f : D \to \mathbb{R}$ be a non-constant continuous function. Which of the following is TRUE?

(A) The range of f is unbounded

(B) The range of f is a union of open intervals

(C) The range of f is a closed interval

(D) The range of f is a union of at least two disjoint closed intervals

Q. 28 Let $f : [0, 1] \to \mathbb{R}$ be a continuous function such that $f\left(\frac{1}{2}\right) = -\frac{1}{2}$ and

$$|f(x) - f(y) - (x - y)| \leq \sin\left(|x - y|^2\right)$$

for all $x, y \in [0, 1]$. Then $\int_0^1 f(x)\, dx$ is

(A) $-\frac{1}{2}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$

Q. 29 Let $S^1 = \{z \in \mathbb{C} : |z| = 1\}$ be the circle group under multiplication and $i = \sqrt{-1}$. Then the set $\{\theta \in \mathbb{R} : \langle e^{i2\pi\theta} \rangle$ is infinite$\}$ is

(A) empty

(B) non-empty and finite

(C) countably infinite

(D) uncountable

Q. 30 Let $F = \{\omega \in \mathbb{C} : \omega^{2020} = 1\}$. Consider the groups

$$G = \left\{ \begin{pmatrix} \omega & z \\ 0 & 1 \end{pmatrix} : \omega \in F, z \in \mathbb{C} \right\}$$

and

$$H = \left\{ \begin{pmatrix} 1 & z \\ 0 & 1 \end{pmatrix} : z \in \mathbb{C} \right\}$$

under matrix multiplication. Then the number of cosets of H in G is

(A) 1010 (B) 2019 (C) 2020 (D) infinite

SECTION – B
MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q. 31 Let $a, b, c \in \mathbb{R}$ such that $a < b < c$. Which of the following is/are true for any continuous function $f : \mathbb{R} \to \mathbb{R}$ satisfying $f(a) = b, f(b) = c$ and $f(c) = a$?

(A) There exists $\alpha \in (a, c)$ such that $f(\alpha) = \alpha$

(B) There exists $\beta \in (a, b)$ such that $f(\beta) = \beta$

(C) There exists $\gamma \in (a, b)$ such that $(f \circ f)(\gamma) = \gamma$

(D) There exists $\delta \in (a, c)$ such that $(f \circ f \circ f)(\delta) = \delta$

Q. 32 If $s_n = \dfrac{(-1)^n}{2^n + 3}$ and $t_n = \dfrac{(-1)^n}{4n - 1}, n = 0, 1, 2, ...$, then

(A) $\sum_{n=0}^{\infty} s_n$ is absolutely convergent

(B) $\sum_{n=0}^{\infty} t_n$ is absolutely convergent

(C) $\sum_{n=0}^{\infty} s_n$ is conditionally convergent

(D) $\sum_{n=0}^{\infty} t_n$ is conditionally convergent

Q. 33 Let $a, b \in \mathbb{R}$ and $a < b$. Which of the following statement(s) is/are true?

(A) There exists a continuous function $f : [a, b] \to (a, b)$ such that f is one-one

(B) There exists a continuous function $f : [a, b] \to (a, b)$ such that f is onto

(C) There exists a continuous function $f : (a, b) \to [a, b]$ such that f is one-one

(D) There exists a continuous function $f : (a, b) \to [a, b]$ such that f is onto

Q. 34 Let V be a non-zero vector space over a field F. Let $S \subset V$ be a non-empty set. Consider the following properties of S:

(I) For any vector space W over F, any map $f : S \to W$ extends to a linear map from V to W.

(II) For any vector space W over F and any two linear maps $f, g : V \to W$ satisfying $f(s) = g(s)$ for all $s \in S$, we have $f(v) = g(v)$ for all $v \in V$.

(III) S is linearly independent.

(IV) The span of S is V.

Which of the following statement(s) is /are true?

(A) (I) implies (IV)

(B) (I) implies (III)

(C) (II) implies (III)

(D) (II) implies (IV)

Q. 35 Let $L[y] = x^2\dfrac{d^2y}{dx^2} + px\dfrac{dy}{dx} + qy$, where p, q are real constants. Let $y_1(x)$ and $y_2(x)$ be two solutions of $L[y] = 0, x > 0$, that satisfy $y_1(x_0) = 1, y_1'(x_0) = 0, y_2(x_0) = 0$ and $y_2'(x_0) = 1$ for some $x_0 > 0$. Then,

(A) $y_1(x)$ is not a constant multiple of $y_2(x)$

(B) $y_1(x)$ is a constant multiple of $y_2(x)$

(C) $1, \ln x$ are solutions of $L[y] = 0$ when $p = 1, q = 0$

(D) $x, \ln x$ are solutions of $L[y] = 0$ when $p + q \neq 0$

Q. 36 Consider the following system of linear equations

$$x + y + 5z = 3, \quad x + 2y + mz = 5 \quad \text{and} \quad x + 2y + 4z = k.$$

The system is consistent if

(A) $m \neq 4$ (B) $k \neq 5$ (C) $m = 4$ (D) $k = 5$

Q. 37 Let $a = \lim_{n\to\infty}\left(\frac{1}{n^2} + \frac{2}{n^2} + \cdots + \frac{(n-1)}{n^2}\right)$ and $b = \lim_{n\to\infty}\left(\frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n}\right)$. Which of the following is/are true?

(A) $a > b$ (B) $a < b$ (C) $ab = \ln\sqrt{2}$ (D) $\frac{a}{b} = \ln\sqrt{2}$

Q. 38 Let S be that part of the surface of the paraboloid $z = 16 - x^2 - y^2$ which is above the plane $z = 0$ and D be its projection on the xy-plane. Then the area of S equals

(A) $\iint_D \sqrt{1 + 4(x^2 + y^2)}\, dxdy$

(B) $\iint_D \sqrt{1 + 2(x^2 + y^2)}\, dxdy$

(C) $\int_0^{2\pi}\int_0^4 \sqrt{1 + 4r^2}\, drd\theta$

(D) $\int_0^{2\pi}\int_0^4 \sqrt{1 + 4r^2}\, rdrd\theta$

Q. 39 Let f be a real valued function of a real variable, such that $|f^{(n)}(0)| \leq K$ for all $n \in \mathbb{N}$, where $K > 0$. Which of the following is/are true?

(A) $\left|\frac{f^{(n)}(0)}{n!}\right|^{\frac{1}{n}} \to 0$ as $n \to \infty$

(B) $\left|\frac{f^{(n)}(0)}{n!}\right|^{\frac{1}{n}} \to \infty$ as $n \to \infty$

(C) $f^{(n)}(x)$ exists for all $x \in \mathbb{R}$ and for all $n \in \mathbb{N}$

(D) The series $\sum_{n=1}^{\infty} \frac{f^{(n)}(0)}{(n-1)!}$ is absolutely convergent

Q. 40 Let G be a group with identity e. Let H be an abelian non-trivial proper subgroup of G with the property that $H \cap gHg^{-1} = \{e\}$ for all $g \notin H$.

If $K = \{g \in G : gh = hg \text{ for all } h \in H\}$, then

(A) K is a proper subgroup of H

(B) H is a proper subgroup of K

(C) $K = H$

(D) there exists no abelian subgroup $L \subseteq G$ such that K is a proper subgroup of L

SECTION – C
NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q. 41 Let $x_n = n^{\frac{1}{n}}$ and $y_n = e^{1-x_n}$, $n \in \mathbb{N}$. Then the value of $\lim\limits_{n\to\infty} y_n$ is _____.

Q. 42 Let $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$ and S be the sphere given by $(x-2)^2 + (y-2)^2 + (z-2)^2 = 4$. If $\hat{n}$ is the unit outward normal to S, then

$$\frac{1}{\pi} \iint_S \vec{F} \cdot \hat{n}\, dS$$

is _____.

Q. 43 Let $f : \mathbb{R} \to \mathbb{R}$ be such that f, f', f'' are continuous functions with $f > 0, f' > 0$ and $f'' > 0$. Then

$$\lim_{x\to-\infty} \frac{f(x) + f'(x)}{2}$$

is _____.

Q. 44 Let $S = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$ and $f : S \to \mathbb{R}$ be defined by $f(x) = \frac{1}{x}$. Then

$$\max\left\{\delta : \left|x - \frac{1}{3}\right| < \delta \Longrightarrow \left|f(x) - f\left(\frac{1}{3}\right)\right| < 1\right\}$$

is _____. (rounded off to two decimal places)

Q. 45 Let $f(x, y) = e^x \sin y, x = t^3 + 1$ and $y = t^4 + t$. Then $\frac{df}{dt}$ at $t = 0$ is _____. (rounded off to two decimal places)

Q. 46 Consider the differential equation

$$\frac{dy}{dx} + 10y = f(x),\ x > 0,$$

where $f(x)$ is a continuous function such that $\lim_{x\to\infty} f(x) = 1$. Then the value of

$$\lim_{x\to\infty} y(x)$$

is _____.

Q. 47 If $\int_0^1 \int_{2y}^2 e^{x^2} dxdy = k(e^4 - 1)$, then k equals _____.

Q. 48 Let $f(x, y) = 0$ be a solution of the homogeneous differential equation

$$(2x + 5y)dx - (x + 3y)dy = 0.$$

If $f(x + \alpha, y - 3) = 0$ is a solution of the differential equation

$$(2x + 5y - 1)dx + (2 - x - 3y)dy = 0,$$

then the value of α is _____.

Q. 49 Consider the real vector space $P_{2020} = \{\sum_{i=0}^{n} a_i x^i : a_i \in \mathbb{R} \text{ and } 0 \leq n \leq 2020\}$. Let W be the subspace given by

$$W = \left\{ \sum_{i=0}^{n} a_i x^i \in P_{2020} : a_i = 0 \text{ for all odd } i \right\}.$$

Then, the dimension of W is _____.

Q. 50 Let $\phi : S_3 \to S^1$ be a non-trivial non-injective group homomorphism. Then, the number of elements in the kernel of ϕ is _____.

Q. 51 – Q. 60 carry two marks each.

Q. 51 The sum of the series $\frac{1}{2(2^2-1)}+\frac{1}{3(3^2-1)}+\frac{1}{4(4^2-1)}+\cdots$ is _____.

Q. 52 Consider the expansion of the function $f(x)=\frac{3}{(1-x)(1+2x)}$ in powers of x, that is valid in $|x|<\frac{1}{2}$. Then the coefficient of x^4 is _____.

Q. 53 The minimum value of the function $f(x,y)=x^2+xy+y^2-3x-6y+11$ is _____.

Q. 54 Let $f(x)=\sqrt{x}+\alpha x,\ x>0$ and

$$g(x)=a_0+a_1(x-1)+a_2(x-1)^2$$

be the sum of the first three terms of the Taylor series of $f(x)$ around $x=1$. If $g(3)=3$, then α is _____.

Q. 55 Let C be the boundary of the square with vertices $(0,0),(1,0),(1,1)$ and $(0,1)$ oriented in the counter clockwise sense. Then, the value of the line integral

$$\oint_C x^2y^2dx+(x^2-y^2)dy$$

is _____. (rounded off to two decimal places)

Q. 56 Let $f:\mathbb{R}\to\mathbb{R}$ be a differentiable function with $f'(x)=f(x)$ for all x. Suppose that $f(\alpha x)$ and $f(\beta x)$ are two non-zero solutions of the differential equation

$$4\frac{d^2y}{dx^2}-p\frac{dy}{dx}+3y=0$$

satisfying

$$f(\alpha x)f(\beta x)=f(2x)\ \text{ and }\ f(\alpha x)f(-\beta x)=f(x).$$

Then, the value of p is _____.

Q. 57 If $x^2 + xy^2 = c$, where $c \in \mathbb{R}$, is the general solution of the exact differential equation

$$M(x, y)\, dx + 2xy\, dy = 0,$$

then $M(1, 1)$ is _____.

Q. 58 Let $M = \begin{bmatrix} 9 & 2 & 7 & 1 \\ 0 & 7 & 2 & 1 \\ 0 & 0 & 11 & 6 \\ 0 & 0 & -5 & 0 \end{bmatrix}$. Then, the value of $\det\Big((8I - M)^3\Big)$ is _____.

Q. 59 Let $T : \mathbb{R}^7 \to \mathbb{R}^7$ be a linear transformation with Nullity$(T) = 2$. Then, the minimum possible value for Rank(T^2) is _____.

Q. 60 Suppose that G is a group of order 57 which is NOT cyclic. If G contains a unique subgroup H of order 19, then for any $g \notin H$, $o(g)$ is _____.

END OF THE QUESTION PAPER

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 Which one of the following functions has a discontinuity in the second derivative at $x = 0$, where x is a real variable?

(A) $f(x) = |x|^3$
(B) $f(x) = x|x|$
(C) $f(x) = \cos(|x|)$
(D) $f(x) = |x|^2$

Q.2 A collimated beam of laser light of wavelength 514 nm is normally incident on a smooth glass slab placed in air. Given the refractive indices of glass and air are 1.47 and 1.0, respectively, the percentage of light intensity reflected back is

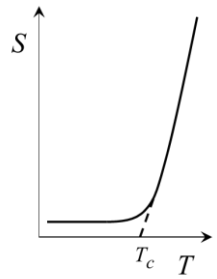
(A) 0 (B) 4.0 (C) 3.6 (D) 4.2

Q.3 Two stationary point particles with equal and opposite charges are at some fixed distance from each other. The points having zero electric potential lie on:

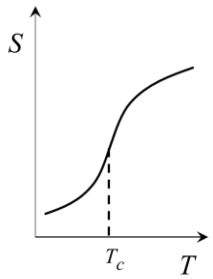
(A) A sphere (B) A plane (C) A cylinder (D) Two parallel planes

Q.4 For a system undergoing a first order phase transition at a temperature T_c, which one of the following graphs best describes the variation of entropy (S) as a function of temperature (T)?

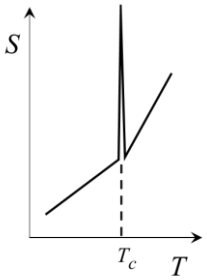
(A)



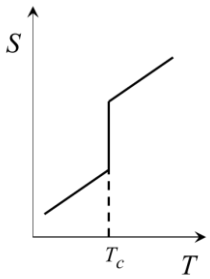
(B)



(C)



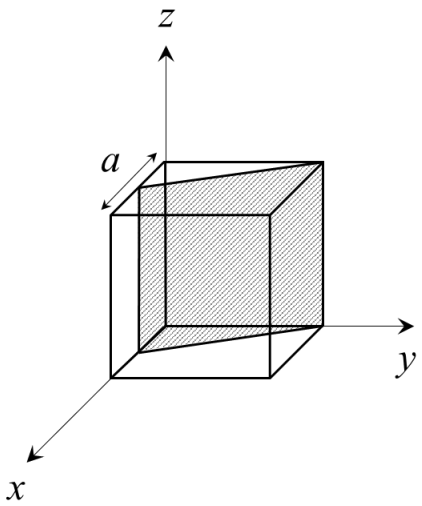
(D)



Q.5 In a photoelectric effect experiment, a monochromatic light source emitting photons with energy greater than the work function of the metal under test is used. If the power of the light source is doubled, which one of the following statements is correct?

(A) The number of emitted photoelectrons remains the same.
(B) The stopping potential remains the same.
(C) The number of emitted photoelectrons decreases.
(D) The stopping potential doubles.

Q.6 The figure below shows a cubic unit cell with lattice constant a. The shaded crystallographic plane intersects the x-axis at $0.5a$. The Miller indices of the shaded plane are

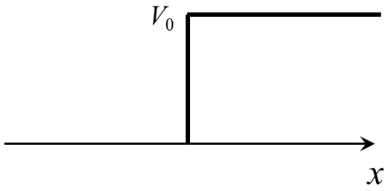


(A) (210) (B) $(\bar{2}10)$ (C) (110) (D) (102)

Q.7 For a particle moving in a central potential, which one of the following statements is correct?

(A) The motion is restricted to a plane due to the conservation of angular momentum.
(B) The motion is restricted to a plane due to the conservation of energy only.
(C) The motion is restricted to a plane due to the conservation of linear momentum.
(D) The motion is not restricted to a plane.

Q.8 Consider the motion of a quantum particle of mass m and energy E under the influence of a step potential of height V_0. If R denotes the reflection coefficient, which one of the following statements is true?



(A) If $E = \frac{4}{3}V_0$, $R = 1$ (B) If $E = \frac{4}{3}V_0$, $R = 0$

(C) If $E = \frac{1}{2}V_0$, $R = 1$ (D) If $E = \frac{1}{2}V_0$, $R = 0.5$

Q.9 The Boolean function $\overline{PQ}(\bar{P} + Q)(Q + \bar{Q})$ is equivalent to:

(A) P (B) $\bar{P}$ (C) $\bar{P}Q$ (D) $P\bar{Q}$

Q.10 Three point charges each carrying a charge q are placed on the vertices of an equilateral triangle of side L. The electrostatic potential energy of the configuration is:

(A) $\frac{1}{4\pi\epsilon_0}\frac{q^2}{L}$ (B) $\frac{2}{4\pi\epsilon_0}\frac{q^2}{L}$

(C) $\frac{3}{4\pi\epsilon_0}\frac{q^2}{L}$ (D) $\frac{1}{\pi\epsilon_0}\frac{q^2}{L}$

Q. 11 – Q. 30 carry two marks each.

Q.11 Which one of the following statements is correct?

Given, $\binom{n}{m} = \frac{n!}{m!(n-m)!}$ is the binomial coefficient.

(A) $\cos n\theta = \cos^n\theta - \binom{n}{2}\cos^{n-2}\theta\ \sin^2\theta + \binom{n}{4}\cos^{n-4}\theta\ \sin^4\theta - \cdots$

(B) $\sin n\theta = \binom{n}{1}\cos^{n-1}\theta\sin\theta + \binom{n}{3}\cos^{n-3}\theta\sin^3\theta + \cdots$

(C) $\cos n\theta = \cos^n\theta + \binom{n}{2}\cos^{n-2}\theta\ \sin^2\theta + \binom{n}{4}\cos^{n-4}\theta\ \sin^4\theta + \cdots$

(D) $\sin n\theta = \cos^n\theta - \binom{n}{2}\cos^{n-2}\theta\ \sin^2\theta + \binom{n}{4}\cos^{n-4}\theta\ \sin^4\theta - \cdots$

Q.12 The volume integral $\int_V e^{-\left(\frac{r}{R}\right)^2}\ \vec{\nabla}\cdot\left(\frac{\hat{r}}{r^2}\right)d^3r$, where V is the volume of a sphere of radius R centered at the origin, is equal to

(A) 4π (B) 0 (C) $\frac{4}{3}\pi R^3$ (D) 1

Q.13 $\lim_{x\to 0+} x^x$ is equal to

(A) 0 (B) ∞ (C) e (D) 1

Q.14 A wheel is rotating at a frequency f_0 Hz about a fixed vertical axis. The wheel stops in t_o seconds, with constant angular deceleration. The number of turns covered by the wheel before it comes to rest is given by:

(A) $f_o t_0$ (B) $2f_o t_0$ (C) $f_o t_0/2$ (D) $f_o t_0/\sqrt{2}$

Q.15 Two objects of masses m and $2m$ are moving at speeds of v and $v/2$, respectively. After undergoing a completely inelastic collision, they move together with a speed of $v/3$. The angle between the initial velocity vectors of the two objects is

(A) 60^0 (B) 120^0 (C) 45^0 (D) 90^0

Q.16 Two planets P_1 and P_2 having masses M_1 and M_2 revolve around the Sun in elliptical orbits, with time periods T_1 and T_2, respectively. The minimum and maximum distances of planet P_1 from the Sun are R and $3R$, respectively, whereas for planet P_2 these are $2R$ and $4R$, respectively, where R is a constant. Assuming M_1 and M_2 are much smaller than the mass of the Sun, the magnitude of $\frac{T_2}{T_1}$ is

(A) $\frac{2}{3}\sqrt{\frac{2\,M_1}{3\,M_2}}$ (B) $\frac{3}{2}\sqrt{\frac{3\,M_2}{2\,M_1}}$ (C) $\frac{3}{2}\sqrt{\frac{3}{2}}$ (D) $\frac{2}{3}\sqrt{\frac{2}{3}}$

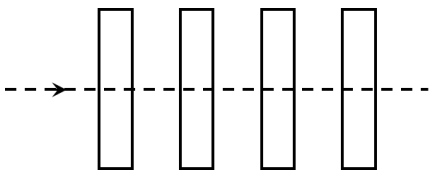
Q.17 The intensity of the primary maximum in a two-slit interference pattern is given by I_2 and the intensity of the primary maximum in a three-slit interference pattern is given by I_3. Assuming the far-field approximation, same slit parameters and intensity of the incident light in both the cases, I_2 and I_3 are related as

(A) $I_2 = \frac{3}{2}I_3$ (B) $I_2 = \frac{9}{4}I_3$ (C) $I_2 = \frac{2}{3}I_3$ (D) $I_2 = \frac{4}{9}I_3$

Q.18 A short rod of length L and negligible diameter lies along the optical axis of a concave mirror at a distance of 3 m. The focal length of the mirror is 1 m and L << 1 m. If L' is the length of image of the object in the mirror, then

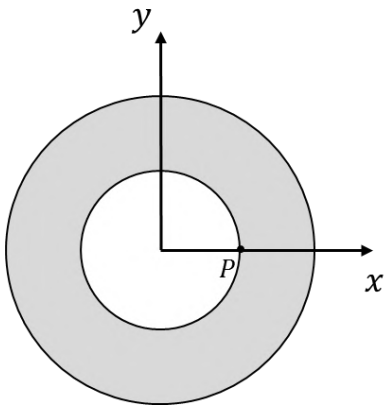
(A) $\frac{L'}{L} = 4$ (B) $\frac{L'}{L} = 2$ (C) $\frac{L'}{L} = \frac{1}{16}$ (D) $\frac{L'}{L} = \frac{1}{4}$

Q.19 A beam of unpolarized light of intensity I_0 falls on a system of four identical linear polarizers placed in a line as shown in the figure. The transmission axes of any two successive polarizers make an angle of 30^0 with each other. If the transmitted light has intensity I, the ratio $\frac{I}{I_0}$ is



(A) $\frac{81}{256}$ (B) $\frac{9}{16}$ (C) $\frac{27}{64}$ (D) $\frac{27}{128}$

Q.20 Consider an annular region in free space containing a uniform magnetic field in the *z*-direction, schematically represented by the shaded region in the figure. A particle having charge Q and mass M starts off from point $P\ (a, 0, 0)$ in the +*x*-direction with constant speed v. If the radii of inner and outer circles are a and b, respectively, the minimum magnetic field required so that the particle returns to the inner circle is



(A) $\frac{Mv}{Q}\left(\frac{b^2-a^2}{b}\right)^{-1}$ (B) $\frac{Mv}{Q}\left(\frac{b^2-a^2}{2b}\right)^{-1}$

(C) $\frac{Mv}{Q}\left(\frac{b^2-a^2}{3b}\right)^{-1}$ (D) $\frac{Mv}{Q}\left(\frac{b^2-a^2}{4b}\right)^{-1}$

Q.21 A thin conducting square loop of side L is placed in the first quadrant of the *xy*-plane with one of the vertices at the origin. If a changing magnetic field $\vec{B}(t) = \beta_0(5zyt\,\hat{x} + zxt\,\hat{y} + 3y^2t\hat{z})$ is applied, where β_0 is a constant, then the magnitude of the induced electromotive force in the loop is

(A) $4\beta_0 L^4$ (B) $3\beta_0 L^4$ (C) $2\beta_0 L^4$ (D) $\beta_0 L^4$

Q.22 In which one of the following limits the Fermi-Dirac distribution $n_F(\varepsilon, T) = \left(e^{\frac{\varepsilon-\mu}{k_B T}} + 1\right)^{-1}$ and Bose-Einstein distribution $n_B(\varepsilon, T) = \left(e^{\frac{\varepsilon-\mu}{k_B T}} - 1\right)^{-1}$ reduce to Maxwell-Boltzmann distribution?
(Here ε is the energy of the state, μ is the chemical potential, k_B is the Boltzmann constant and T is the temperature).

(A) $\mu = 0$ (B) $(\varepsilon - \mu) \ll k_B T$

(C) $(\varepsilon - \mu) \gg k_B T$ (D) $\mu \gg k_B T$

Q.23 Consider N classical particles at temperature T, each of which can have two possible energies 0 and ε. The number of particles in the lower energy level (N_0) and higher energy level (N_ε) levels are related by
(k_B is the Boltzmann constant)

(A) $\frac{N_o}{N_\varepsilon} = e^{\frac{-\varepsilon}{k_B T}}$ (B) $\frac{N_o}{N_\varepsilon} = e^{\frac{\varepsilon}{k_B T}}$

(C) $\frac{N_o}{N_\varepsilon} = 1 + e^{\frac{\varepsilon}{k_B T}}$ (D) $\frac{N_o}{N_\varepsilon} = 1 - e^{\frac{-\varepsilon}{k_B T}}$

Q.24 The root mean square (*rms*) speeds of Hydrogen atoms at 500 K, V_H, and Helium atoms at 2000 K, V_{He}, are related as

(A) $V_H > V_{He}$ (B) $V_H < V_{He}$ (C) $V_H = V_{He}$ (D) $V_H \gg V_{He}$

Q.25 The normalized ground-state wave function of a one-dimensional quantum harmonic oscillator with force constant K and mass m is $\psi_0(x) = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{-\alpha x^2/2}$, where $\alpha = m\omega_0/\hbar$ and $\omega_0^2 = K/m$. Which one of the following is the probability of finding the particle outside the classically allowed region?
(The classically allowed region is where the total energy is greater than the potential energy)

(A) $\frac{2}{\sqrt{\pi}}\int_1^\infty y^2 e^{-y^2} dy$ (B) $\frac{2}{\sqrt{\pi}}\int_1^\infty e^{-y^2} dy$

(C) 0.5 (D) 0

Q.26 A linear operator $\hat{O}$ acts on two orthonormal states of a system ψ_1 and ψ_2 as per following: $\hat{O}\psi_1 = \psi_2$, $\hat{O}\psi_2 = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2)$. The system is in a superposed state defined by $\psi = \frac{1}{\sqrt{2}}\psi_1 + \frac{i}{\sqrt{2}}\psi_2$. The expectation value of $\hat{O}$ in the state ψ is

(A) $\frac{1}{2\sqrt{2}}\left(1 + i(\sqrt{2}+1)\right)$ (B) $\frac{1}{2\sqrt{2}}\left(1 - i(\sqrt{2}+1)\right)$

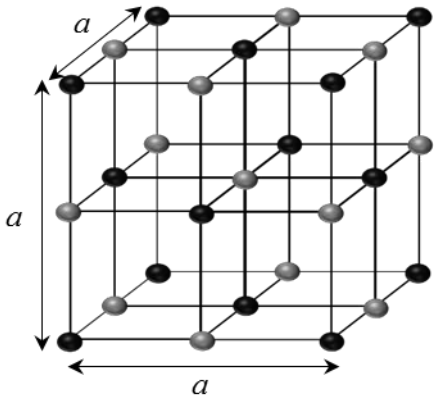
(C) $\frac{1}{2\sqrt{2}}\left(1 + i(\sqrt{2}-1)\right)$ (D) $\frac{1}{2\sqrt{2}}\left(1 - i(\sqrt{2}-1)\right)$

Q.27 Consider a one-dimensional infinite potential well of width *a*. This system contains five non-interacting electrons, each of mass *m*, at temperature $T = 0$ K. The energy of the highest occupied state is

(A) $\frac{25\pi^2\hbar^2}{2ma^2}$ (B) $\frac{10\pi^2\hbar^2}{2ma^2}$

(C) $\frac{5\pi^2\hbar^2}{2ma^2}$ (D) $\frac{9\pi^2\hbar^2}{2ma^2}$

Q.28 Consider the crystal structure shown in the figure, where black and grey spheres represent atoms of two different elements and a denotes the lattice constant. The Bravais lattice for this structure is



(A) Simple cubic

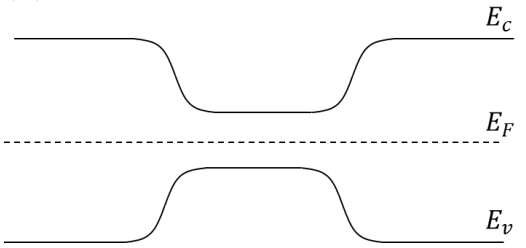
(B) Face-centered cubic

(C) Body-centered cubic

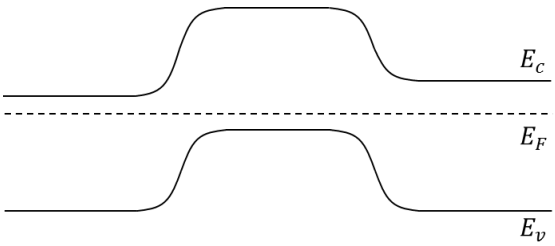
(D) Triclinic

Q.29 For an unbiased Silicon *n-p-n* transistor in thermal equilibrium, which one of the following electronic energy band diagrams is correct? (E_c= conduction band minimum, E_v = valence band maximum, E_F = Fermi level).

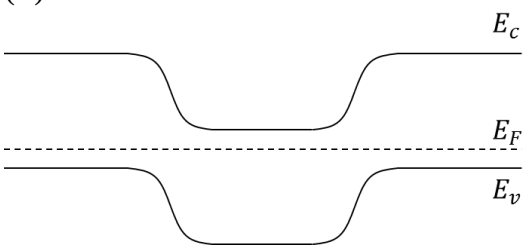
(A)



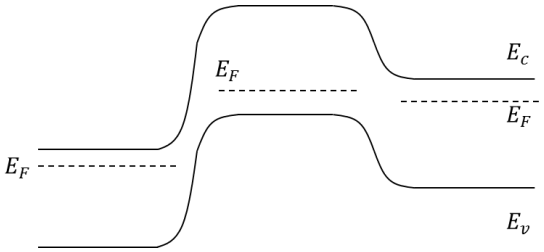
(B)



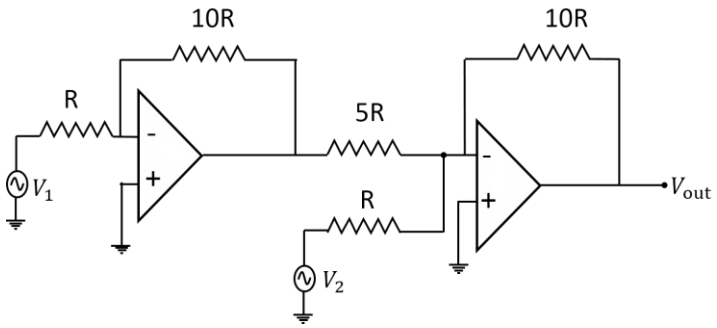
(C)



(D)



Q.30 In the circuit shown in the figure, both OPAMPs are ideal. The output for the circuit V_{out} is



(A) $20V_1 + 10V_2$

(B) $-20V_1 + 10V_2$

(C) $10V_1 - 20V_2$

(D) $20V_1 - 10V_2$

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 If P and Q are Hermitian matrices, which of the following is/are true?
(A matrix P is Hermitian if $P = P^{\dagger}$, where the elements $P_{ij}^{\dagger} = P_{ji}^{*}$)

(A) $PQ + QP$ is always Hermitian

(B) $i(PQ - QP)$ is always Hermitian

(C) PQ is always Hermitian

(D) $PQ - QP$ is always Hermitian

Q.32 Consider a vector function $\vec{u}(\vec{r})$ and two scalar functions $\psi(\vec{r})$ and $\phi(\vec{r})$.The unit vector $\hat{n}$ is normal to the elementary surface dS, dV is an infinitesimal volume, $\overrightarrow{dl}$ is an infinitesimal line element, and $\partial/\partial n$ denotes the partial derivative along $\hat{n}$. Which of the following identities is/are correct?

(A) $\int_V \vec{\nabla} \cdot \vec{u}\, dV = \oint_S \vec{u} \cdot \hat{n}\, dS$, where surface S bounds the volume V.

(B) $\int_V [\psi\, \nabla^2\phi - \phi\, \nabla^2\psi]\, dV = \oint_S \left[\psi \frac{\partial \phi}{\partial n} - \phi \frac{\partial \psi}{\partial n}\right] dS$, where surface S bounds the volume V.

(C) $\int_V [\psi\, \nabla^2\phi - \phi\, \nabla^2\psi]\, dV = \oint_S \left[\psi \frac{\partial \phi}{\partial n} + \phi \frac{\partial \psi}{\partial n}\right] dS$, where surface S bounds the volume V.

(D) $\oint_C \vec{u} \cdot \overrightarrow{dl} = \iint_S (\vec{\nabla} \times \vec{u}) \cdot \hat{n}\, dS$, where C is the boundary of surface S.

Q.33 A thin rod of uniform density and length $2\sqrt{3}$ m is undergoing small oscillations about a pivot point. The time period of oscillation (T_m) is minimum when the distance of the pivot point from the center-of-mass of the rod is x_m. Which of the following is/are correct?
(Assume acceleration due to gravity $g = 10$ m/s^2)

(A) x_m=1 m (B) $x_m = \frac{\sqrt{3}}{2}$ m (C) $T_m = \frac{2\pi}{\sqrt{3}}$ s (D) $T_m = \frac{2\pi}{\sqrt{5}}$ s

Q.34 Three sinusoidal waves of the same frequency travel with the same speed along the positive *x*-direction. The amplitudes of the waves are a, $a/2$, and $a/3$, and the phase constants of the waves are $\pi/2$, π, and $3\pi/2$, respectively. If A_m and φ_m are the amplitude and phase constant of the wave resulting from the superposition of the three waves, which of the following is/are correct?

(A) $A_m = \frac{5}{6}a$
(B) $\varphi_m = \frac{\pi}{2} + \tan^{-1}\left(\frac{3}{4}\right)$
(C) $A_m = \frac{7}{6}a$
(D) $\varphi_m = \tan^{-1}\left(\frac{2}{3}\right)$

Q.35 An object executes simple harmonic motion along the *x*-direction with angular frequency ω and amplitude a. The speed of the object is 4 cm/s and 2 cm/s, when it is at distances 2 cm and 6 cm, respectively from the equilibrium position. Which of the following is/are correct?

(A) $\omega = \sqrt{\frac{3}{8}}$ rad/s
(B) $\omega = \sqrt{\frac{5}{6}}$ rad/s
(C) $a = \sqrt{\frac{140}{3}}$ cm
(D) $a = \sqrt{\frac{175}{6}}$ cm

Q.36 For electric and magnetic fields, $\vec{E}$ and $\vec{B}$, due to a charge density $\rho(\vec{r}, t)$ and a current density $\vec{J}(\vec{r}, t)$, which of the following relations is/are always correct?

(A) $\vec{\nabla} \times \vec{E} = 0$

(B) $\vec{\nabla} \cdot \vec{B} = 0$

(C) $\vec{\nabla} \cdot \vec{J} - \frac{\partial \rho}{\partial t} = 0$

(D) $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$, where $\vec{F}$ is the force on a particle with charge q moving with velocity $\vec{v}$

Q.37 A spherical dielectric shell with inner radius a and outer radius b, has polarization $\vec{P} = \frac{k}{r^2}\hat{r}$, where k is a constant and $\hat{r}$ is the unit vector along the radial direction. Which of the following statements is/are correct?

(A) The surface density of bound charges on the inner and outer surfaces are $-k$ and $+k$, respectively. The volume density of bound charges inside the dielectric is zero.

(B) The surface density of bound charges is zero on both the inner and outer surfaces. The volume density of bound charges inside the dielectric is $+k$.

(C) The surface density of bound charges on the inner and outer surfaces are $\frac{-k}{a^2}$ and $\frac{k}{b^2}$, respectively. The volume density of bound charges inside the dielectric is zero.

(D) The surface density of bound charges is zero on both the inner and outer surfaces. The volume density of bound charges inside the dielectric is $\frac{3k}{4\pi(b^3-a^3)}$.

Q.38 One mole of an ideal gas having specific heat ratio (γ) of 1.6 is mixed with one mole of another ideal gas having specific heat ratio of 1.4. If C_V and C_P are the molar specific heat capacities of the gas mixture at constant volume and pressure, respectively, which of the following is/are correct? (R denotes the universal gas constant).

(A) $C_V = 2.08\,R$ (B) $C_P = 2.9\,R$ (C) $C_P = 1.48\,C_V$ (D) $C_P = 1.52\,C_V$

Q.39 Two relativistic particles with opposite velocities collide head-on and come to rest by sticking with each other. Which of the following quantities is/are conserved in the collision?

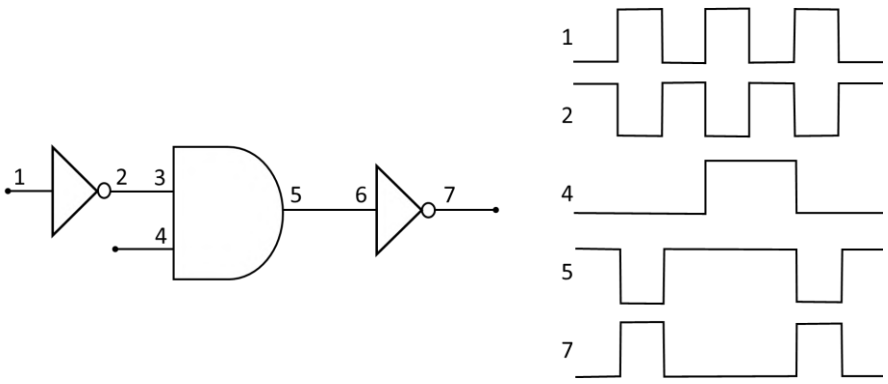
(A) Total momentum

(B) Total energy

(C) Total kinetic energy

(D) Total rest mass

Q.40 Figure shows a circuit diagram comprising Boolean logic gates and the corresponding timing diagrams show the digital signals at various points in the circuit. Which of the following is/are true?



(A) Points 3 and 7 are shorted.
(B) The NOT gate on the right is faulty.
(C) The AND gate is faulty and acts like a NOR gate.
(D) The AND gate is faulty and acts like an OR gate.

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 The line integral of the vector function $u(x, y) = 2y\,\hat{\imath} + x\,\hat{\jmath}$ along the straight line from (0, 0) to (2, 4) is _____

Q.42 Consider a thin bi-convex lens of relative refractive index n = 1.5. The radius of curvature of one surface of the lens is twice that of the other. The magnitude of larger radius of curvature in units of the focal length of the lens is _____ (Round off to 1 decimal place).

Q.43 Water flows in a horizontal pipe in a streamlined manner at an absolute pressure of 4×10^5 Pa and speed of 6 m/s. If it exits the pipe at a pressure of 10^5 Pa, the speed of water at the exit point is _____ m/s (Round off to 1 decimal place).

(The density of water is 1000 kg/m^3)

Q.44 Consider a retarder with refractive indices n_e=1.551 and n_o=1.542 along the extraordinary and ordinary axes, respectively. The thickness of this retarder for which a left circularly polarized light of wavelength 600 nm will be converted into a right circularly polarized light is _____ µm. (Round off to 2 decimal places)

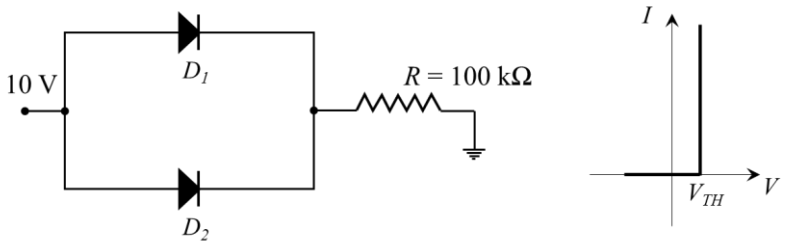
Q.45 Using a battery, a 10 pF capacitor is charged to 50 V and then the battery is removed. After that, a second uncharged capacitor is connected to the first capacitor in parallel. If the final voltage across the second capacitor is 20 V, its capacitance is _____pF.

Q.46 Consider two spherical perfect blackbodies with radii R_1 and R_2 at temperatures $T_1 = 1000$ K and $T_2 = 2000$ K, respectively. They both emit radiation of power 1 kW. The ratio of their radii, R_1/R_2 is given by _____.

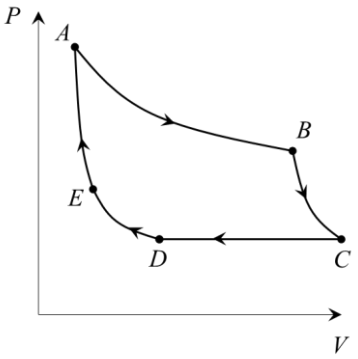
Q.47 In a Compton scattering experiment, the wavelength of incident X-rays is 0.500 Å. If the Compton wavelength λ_C is 0.024 Å, the value of the longest wavelength possible for the scattered X-ray is _____ Å. (Specify up to 3 decimal places)

Q.48 A solid with FCC crystal structure is probed using X-rays of wavelength 0.2 nm. For the crystallographic plane given by (2, 0, 0), a first order diffraction peak is observed for a Bragg angle of 21^0. The unit cell size is _____ nm. (Round off to 2 decimal places)

Q.49 The figure shows a circuit containing two diodes D_1 and D_2 with threshold voltages V_{TH} of 0.7 V and 0.3 V, respectively. Considering the simplified diode model, which assumes diode *I-V* characteristic as shown in the plot on the right, the current through the resistor *R* is _____μA.



Q.50 An ideal gas undergoes an isothermal expansion along a path AB, adiabatic expansion along BC, isobaric compression along CD, isothermal compression along DE, and adiabatic compression along EA, as shown in the figure. The work done by the gas along the process BC is 10 J. The change in the internal energy along process EA is 16 J. The absolute value of the change in the internal energy along the process CD is _____ J.



Q. 51 – Q. 60 carry two marks each.

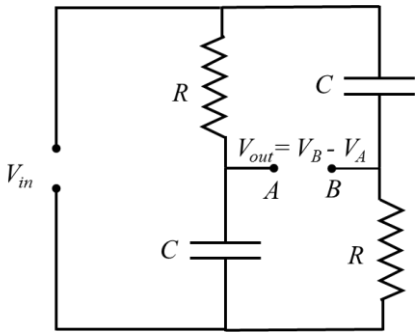
Q.51 If a function $y(x)$ is described by the initial-value problem, $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$, with initial conditions $y(0) = 2$, and $\left(\frac{dy}{dx}\right)_{x=0} = 0$, then the value of y at $x = 1$ is _____ .
(Round off to 2 decimal places)

Q.52 A vehicle of mass 600 kg with an engine operating at constant power *P* accelerates from rest on a straight horizontal road. The vehicle covers a distance of 600 m in 1 minute. Neglecting all losses, the magnitude of *P* is _____ kW. (Round off to 2 decimal places)

Q.53 The angular momentum of a particle relative to origin varies with time (*t*) as $\vec{L} = (4\,\hat{x} + \alpha\, t^2\, \hat{y})$ kg m^2/s, where $\alpha = 1$ kg m^2/s^3. The angle between $\vec{L}$ and the torque acting on the particle becomes 45^0 after a time of _____ s.

Q.54 Two transverse waves $y_1 = 5\cos(kx - \omega t)$ cm, and $y_2 = 5\cos(kx + \omega t)$ cm, travel on a string along *x*-axis. If the speed of a point at $x = 0$ is zero at t = 0 s, 0.25 s and 0.5 s, then the minimum frequency of the waves is _____ Hz.

Q.55 For the *ac* circuit shown in the figure, $R = 100\ k\Omega$ and $C = 10$ pF, the phase difference between V_{in} and V_{out} is 90^0 at the input signal frequency of _____ kHz. (Round off to 2 decimal places)

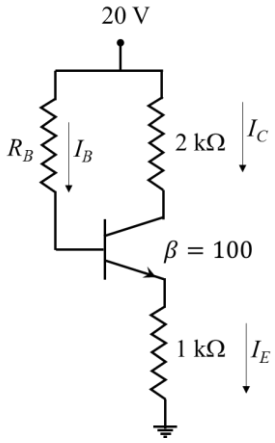


Q.56 The magnetic fields in tesla in the two regions separated by the $z = 0$ plane are given by $\overrightarrow{B_1} = 3\hat{x} + 5\hat{z}$ and $\overrightarrow{B_2} = \hat{x} + 3\hat{y} + 5\hat{z}$. The magnitude of the surface current density at the interface between the two regions is $\alpha \times 10^6$ A/m. Given the permeability of the free space $\mu_0 = 4\pi \times 10^{-7}$ N/A^2, the value of α is _____. (Round off to 2 decimal places)

Q.57 A body at a temperature *T* is brought into contact with a reservoir at temperature 2*T*. Thermal equilibrium is established at constant pressure. The heat capacity of the body at constant pressure is C_p. The total change in entropy of the body and the reservoir in units of C_p is _____. (Round off to 2 decimal places)

Q.58 One mole of an ideal monatomic gas at pressure *P*, volume *V* and temperature *T* is expanded isothermally to volume 4*V*. Thereafter, the gas is heated isochorically (at constant volume) till its pressure becomes *P*. If *R* is the universal gas constant, the total heat transfer in the process, in units of *RT* is _____. (Round off to 2 decimal places)

Q.59 In the transistor circuit given in the figure, the emitter-base junction has a voltage drop of 0.7 V. A collector-emitter voltage of 14 V reverse biases the collector. Assuming the collector current to be the same as the emitter current, the value of R_B is _____ kΩ.



Q.60 The radioactive nuclei ^{40}K decay to ^{40}Ar with a half-life of 1.25×10^9 years. The ^{40}K/^{40}Ar isotopic ratio for a particular rock is found to be 50. The age of the rock is $m \times 10^7$ years. The value of m is _____. (Round off to 2 decimal place)

END OF THE QUESTION PAPER