IIT JAM Previous Year Question Papers - 2023

IIT Joint Admission Test for Masters

Section A: Q.1 – Q.10 Carry ONE mark each.	
Q.1	Which one of the following compounds inhibits the polymerization of tubulin to microtubules in animal cells?
(A)	ATP
(B)	Taxol
(C)	Thymosin
(D)	Vinblastine
Q.2	Arrange the following elements in increasing order of their electronegativity according to the Pauling scale C, Na, Be and Br
(A)	Be, Na, C, Br
(B)	Br, C, Na, Be
(C)	Na, Be, C, Br
(D)	Na, C, Be, Br

Q.3	Which one of the following is **NOT** a plant vascular tissue?
(A)	Phloem
(B)	Periderm
(C)	Stele
(D)	Xylem
Q.4	A growing shoot of a germinating seedling encounters an underground obstacle. Which one of the following hormones elicits 'triple response' to the underground obstacle?
(A)	Auxin
(B)	Cytokinin
(C)	Ethylene
(D)	Gibberellins

Q.5	Which one of the following pairs of antibodies contains ‘J-chain’ in their multimeric form?
(A)	IgA and IgE
(B)	IgA and IgM
(C)	IgD and IgE
(D)	IgD and IgG
Q.6	Restriction enzymes that recognize the same nucleotide sequence but cleave at different positions are called
(A)	heterohypekomers
(B)	isocaudomers
(C)	isoschizomers
(D)	neoschizomers

Q.7	Which one of the following does **NOT** belong to the freshwater ecosystem?
(A)	Estuary
(B)	Lentic
(C)	Lotic
(D)	Wetland
Q.8	Which one of the following is transcribed by RNA polymerase III in eukaryotes?
(A)	18S rRNA
(B)	28S rRNA
(C)	miRNA
(D)	tRNA

Q.9	Given the following sets: $A = \{2, 4, 6, 8, 10, 12\}$ $B = \{8, 10, 12, 14, 16, 18\}$ $C = \{7, 8, 9, 10\ 11, 12, 13\}$ $(A \cap B) \cup (B \cap C)$ is
(A)	$\{8, 10, 12, 14\}$
(B)	$\{8, 10, 12\}$
(C)	$\{7, 8, 10, 11, 12, 13, 14\}$
(D)	$\{4, 6, 7, 8\ 10, 11, 12, 13\}$

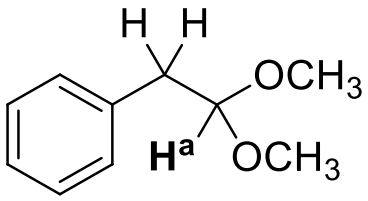
Q.10	Rain is falling vertically with a speed of 40 m s^{-1}. Wind starts blowing with a speed of 16 m s^{-1} in the west to east direction. How should a person, who is standing, hold his umbrella to avoid getting wet?
(A)	At an angle of about 22° with vertical towards east
(B)	At an angle of about 22° with vertical towards west
(C)	At an angle of about 66° with vertical towards east
(D)	At an angle of about 66° with vertical towards west

Section A: Q.11 – Q.30 Carry TWO marks each.	
Q.11	Which one of the following statements about the G1 checkpoint of eukaryotic cell division cycle is **INCORRECT**?
(A)	Cell assures the existence of favorable extracellular environment
(B)	Cell assures the DNA has no damage
(C)	Cell assures the damaged DNAs are directed for repair mechanism
(D)	Cell assures complete replication of DNA

Q.12	Determine the correctness or otherwise of the following Assertion [a] and the Reason [r]. Assertion [a]: Nitric oxide is involved in transient paracrine and autocrine signaling. Reason [r]: Nitric oxide is highly reactive, with a lifetime of few seconds, yet can diffuse freely across membranes
(A)	Both [a] and [r] are true and [r] is the correct reason for [a]
(B)	Both [a] and [r] are true but [r] is not the correct reason for [a]
(C)	Both [a] and [r] are false
(D)	Only [a] is true but [r] is false

Q.13	In mice, a trait is determined by a dominant allele *Y* and recessive allele *y*. What proportion of the offspring from a *YY* × *yy* cross is expected to be homozygous recessive in F1 generation?
(A)	0
(B)	0.25
(C)	0.5
(D)	1

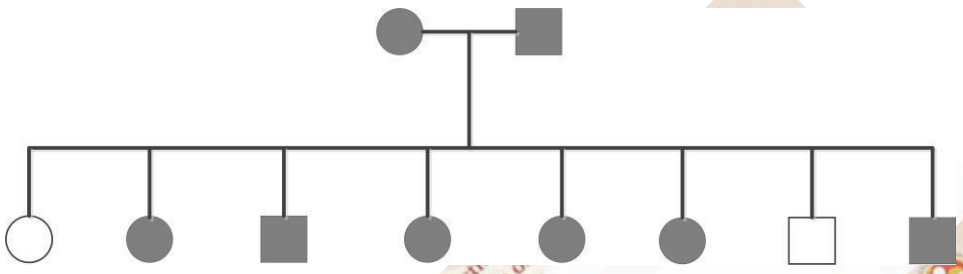
Q.14	Match the molecules in **Group I** with the type of bonds present in them, in **Group II** **Group I** — **Group II** P) NaCl — 1) Coordination bond Q) H_2 — 2) Polar covalent bond R) Pd-P bond in $Pd(PPh_3)_4$ — 3) Covalent bond S) C-Cl bond in CH_3Cl — 4) Ionic bond
(A)	P-4, Q-1, R-3, S-2
(B)	P-2, Q-3, R-1, S-4
(C)	P-4, Q-3, R-1, S-2
(D)	P-4, Q-3, R-2, S-1

Q.15	What is the splitting pattern of proton $\mathbf{H^a}$ of the following compound in its ^{1}H NMR spectrum? 
(A)	Doublet
(B)	Doublet of doublet
(C)	Multiplet
(D)	Triplet

Q.16	Which one of the following statements is correct about solute transport across membranes?
(A)	Passive transporters decrease the activation energy and does not facilitate the transport of polar compounds
(B)	The direction in which a charged solute tends to move spontaneously across a membrane does not depend on the electrical gradient across the membrane
(C)	All ABC transporters do not have nucleotide binding domain
(D)	P-type ATPases get reversibly phosphorylated as a part of transport cycle

Q.17	Match the type of DNA repair mechanism in **Group I** with the enzyme(s) involved in **Group II** **Group I** — **Group II** P) Mismatch repair — 1) DNA glycosylase Q) Base excision repair — 2) UvrA, UvrB, UvrC and UvrD R) Nucleotide excision repair — 3) RecA S) Double strand break repair — 4) MutL, MutS and MutH
(A)	P-4, Q-1, R-3, S-2
(B)	P-4, Q-2, R-1, S-3
(C)	P-4, Q-1, R-2, S-3
(D)	P-2, Q-1, R-4, S-3

Q.18	Tetracycline binds to the
(A)	30S subunit and inhibits aminoacyl-tRNA binding
(B)	50S subunit and inhibits aminoacyl-tRNA binding
(C)	30S subunit and prevents codon:anticodon interactions
(D)	50S subunit and blocks exit of growing polypeptide chain
Q.19	In the 'Southern blot' technique, which of the following reagents is used to detect the presence of a desired DNA fragment?
(A)	Ethidium bromide
(B)	DNA probe
(C)	Silver nitrate
(D)	DNase

Q.20	The pedigree given below shows individuals affected (shaded circles/rectangles) by chronic hypertension. Assuming 100% penetrance, the inheritance of this trait is 
(A)	autosomal dominant
(B)	autosomal recessive
(C)	sex-linked dominant
(D)	sex-linked recessive

Q.21	Which one of the following statements about photoproteins in plants is **INCORRECT**?
(A)	Phytochromes are activated by red light
(B)	Phytochromes are inactivated by far-red light
(C)	Cryptochromes are sensitive to blue light
(D)	Phototropins are insensitive to blue light

Q.22	Match the microorganisms in **Group I** with the human disease in **Group II** **Group I** — **Group II** P) *Treponema pallidum* — 1) Sleeping sickness Q) *Trypanosoma cruzi* — 2) Whooping cough R) *Trypanosoma gambiense* — 3) Chagas disease S) *Bordetella pertussis* — 4) Syphilis
(A)	P-4, Q-3, R-1, S-2
(B)	P-1, Q-2, R-4, S-3
(C)	P-3, Q-1, R-2, S-4
(D)	P-2, Q-1, R-3, S-4

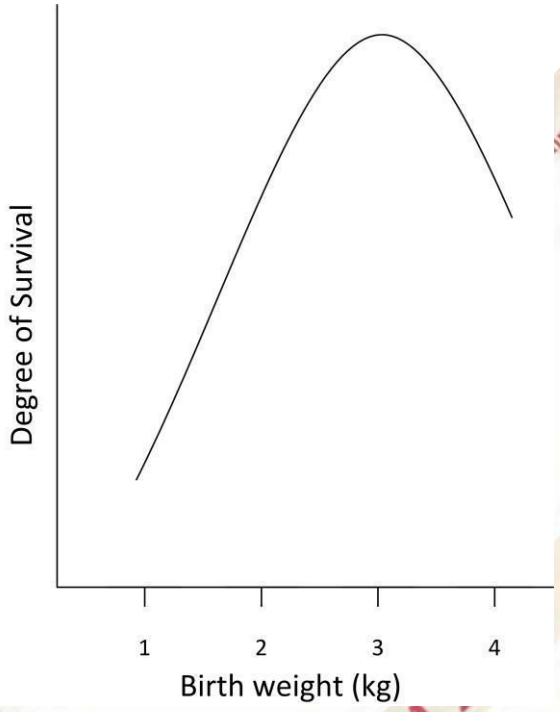
Q.23	Which one of the following is correct in the case of conjugation of a high frequency recombination (Hfr) strain with F^- strain of *E. coli*?
(A)	Recombination frequency is high, F factor transfer frequency is low
(B)	Recombination frequency is high, F factor transfer frequency is high
(C)	Recombination frequency is low, F factor transfer frequency is high
(D)	Recombination frequency is low, F factor transfer frequency is low

Q.24	Determine the correctness or otherwise of the following Assertion [a] and the Reason [r]. Assertion [a]: The cardiovascular organization called double circulation provides vigorous flow of blood to the brain, muscles, and other organs. Reason [r]: The blood is pumped a second time after it loses pressure in the capillary beds of the lungs or skin.
(A)	Both [a] and [r] are true and [r] is the correct reason for [a]
(B)	Both [a] and [r] are true but [r] is not the correct reason for [a]
(C)	Both [a] and [r] are false
(D)	[a] is true but [r] is false

Q.25	The inability in humans to taste capsaicin resides in a single gene difference between two alleles *P* and *p*. The allele *P* for tasting is dominant over the nontasting allele. In a population of 400 individuals in Hardy-Weinberg equilibrium, 64 are nontasters. How many individuals are heterozygous for the gene?
(A)	64
(B)	128
(C)	144
(D)	192

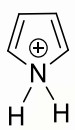
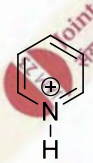
Q.26	A genetic linkage map represents the
(A)	relative locations of genes on a chromosome
(B)	distribution of the mutational hotspots
(C)	phylogenetic linkage among organisms
(D)	accurate physical distances among loci
Q.27	Class II MHC molecules are **NOT** expressed by
(A)	B-cells
(B)	dendritic cells
(C)	macrophages
(D)	T-cells

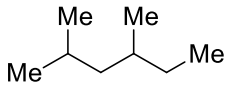
Q.28	Which one of the following enzymes is required to ensure the replication of a negative-sense or negative-strand RNA virus?
(A)	DNA-dependent RNA polymerase
(B)	DNA polymerase
(C)	RNA-dependent DNA polymerase
(D)	RNA-dependent RNA polymerase

Q.29	The relationship between birth weight and degree of survival (log $[\frac{survivors}{non-survivors}]$) in 6908 human births in an obstetric hospital is shown in the figure below. The mode of selection for birth weight is 
(A)	directional
(B)	disruptive
(C)	diversifying
(D)	stabilizing

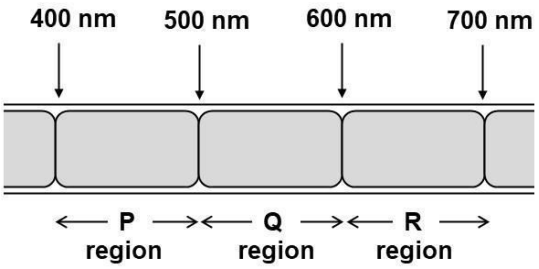
Q.30	Match the recombinant DNA products in **Group I** with its application in **Group II** **Group I** — **Group II** P) Tissue plasminogen activator — 1) Emergency treatment of heart attack Q) Erythropoietin — 2) Treatment of anemia R) Superoxide dismutase — 3) Prevents tissue damage S) Interferon — 4) Stimulates cells to inhibit viral replication
(A)	P-1, Q-2, R-3, S-4
(B)	P-1, Q-3, R-4, S-2
(C)	P-3, Q-1, R-4, S-2
(D)	P-4, Q-3, R-1, S-2

Section B: Q.31 – Q.40 Carry TWO marks each.	
Q.31	Which of the following statement(s) is/are correct about telophase?
(A)	Daughter chromosomes are yet to form
(B)	New nuclear envelop starts to reassemble
(C)	Division of cytoplasm begins
(D)	Nuclear membrane disappears
Q.32	The characteristic morphological change(s) in cells undergoing apoptosis is/are
(A)	formation of blebs on cell surface
(B)	swelling and bursting of cells
(C)	collapse of the cytoskeleton
(D)	condensation and fragmentation of nuclear chromatin

Q.33	A species of fish living in a lake are separated by drying up of the lake into two separate lakes. After several hundreds of years of separation, the two groups are unable to mate. These groups are now considered to be different_____.
(A)	communities
(B)	organisms
(C)	populations
(D)	species
Q.34	Which of the following compound(s) is/are aromatic?
(A)	
(B)	
(C)	
(D)	

Q.35	Which of the following statement(s) is/are correct for the following compound? 
(A)	It can have a maximum of four stereoisomers
(B)	It can have a maximum of two stereoisomers
(C)	It is a chiral compound
(D)	It is an achiral compound
Q.36	Which of the following is/are essential feature(s) of high-fidelity DNA polymerases used in polymerase chain reaction?
(A)	5′→3′ exonuclease activity
(B)	Endonuclease activity
(C)	3′→5′ exonuclease activity
(D)	Optimum temperature for activity ≥72 °C

Q.37	Which of the following option(s) represent(s) the evolutionary relationship between the bird and bat wings as structures for flying?
(A)	analogous
(B)	convergence
(C)	divergence
(D)	homologous

Q.38	Different segments of a photosynthetic filamentous alga are exposed to different wavelengths of light as shown below. After a period of time, bacteria known to migrate towards high oxygen concentration, is spread on the surface of the alga. Which region(s) of the alga will have maximum bacterial congregation? 
(A)	P and R
(B)	P and Q
(C)	Only P
(D)	Only Q

Q.39	Hyperventilation (breathing rapidly and deeply) causes which of the following event(s) in the arterial blood?
(A)	Decrease in CO_2 concentration
(B)	Decrease in proton concentration
(C)	Increase in pH
(D)	Increase in O_2 concentration
Q.40	Which of the given statement(s) about synthetic oligonucleotides is/are correct?
(A)	Chemical synthesis extends the DNA chain from 3′→5′end
(B)	They can be utilized for site-directed mutagenesis
(C)	Chemical synthesis extends the DNA chain from 5′→3′end
(D)	They can be utilized as radiolabeled probes

Section C: Q.41 – Q.50 Carry ONE mark each.	
Q.41	The net number of molecule(s) of NADH formed from one molecule of glucose in glycolysis under aerobic conditions is/are _____.
Q.42	The number of possible unique combination(s) of linear tetrapeptides that can be made from four different amino acids using each amino acid only once in the chain is/are _____.
Q.43	Among *i*-$BuNH_2$, NH_3, Me_2NH, $EtNH_2$, the number of compound(s) more basic than $MeNH_2$ is/are _____.
Q.44	Among K^+, Li^+, Rb^+, Cs^+, the number of cation(s) having ionic radii more than Na^+ is/are _____.

Q.45	Among the five fragments given below, $\overset{\bullet}{C}H_2\text{-}CH_3$, $CH_3\text{-}CH_3$, $CH_3\text{-}CH_2\text{-}\overset{\bullet}{C}H_2$, $\overset{\oplus}{C}H_2\text{-}CH{=}CH_2$, $[CH_3\text{-}CH_2\text{-}CH_3]^{\oplus\bullet}$ the number of fragment(s) accelerated to the analyzer tube in mass spectrometer with electron ionization is/are _____.
Q.46	A restriction endonuclease has a recognition site of 3 bases. Assuming random arrangement of nucleotides, the probability that this endonuclease will cut a piece of DNA is _____ (rounded off to three decimal places).
Q.47	A massless ideal spring is hanging vertically. A sphere of mass of 500 g, suspended from the spring, stretches the spring from its initial position by 50 cm when it reaches equilibrium. The force constant of the spring is _____ N m^{-1}. (Use g=10 m s^{-2})

Q.48	Whales can dive undersea to depths of 2 km. The pressure on the whale at this depth (ignoring atmospheric pressure) is _____ $\times 10^6$ Pa. (Density of sea water = 1 g cm^{-3} and g =10 m s^{-2})
Q.49	The order of differential equation $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 6x^4y = 0$ is _____.
Q.50	The value of $\lim_{x\to -3} \frac{(2x+6)}{(x+3)}$ is _____.

Section C: Q.51 – Q.60 Carry TWO marks each.	
Q.51	The $\Delta G'$ and K'_{eq} values of ATP hydrolysis are -32.34 kJ mol^{-1} and 4.6 $x10^5$, respectively. The $\Delta G'$ and K'_{eq} values of enzymatic hydrolysis of glucose-6-phosphate to glucose and phosphate are -13.18 kJ mol^{-1} and 203.8, respectively. The $\Delta G'$ value of reaction of glucose-6-phosphate formation from glucose and ATP by hexokinase is _____ kJ mol^{-1} (rounded off to 2 decimal places). [All reactions are carried out at pH 7.0 and 25 °C].
Q.52	K_m and V_{max} of an enzyme preparation are 5 µM and 30 µM min^{-1} respectively. Considering, K_i value of competitive inhibitor is 60 µM, the velocity (V_0) of this enzyme-catalyzed reaction in the presence of 200 µM of substrate and 600 µM of competitive inhibitor is _____ µM min^{-1} (rounded off to two decimal places).
Q.53	The heat required to convert 2 kg of water at 20 °C in a calorimeter to steam at 100 °C and at atmospheric pressure (1 atm) is _____ kJ. (Specific heat capacity of water is 4.2 kJ kg^{-1} K^{-1} and latent heat of steam is 2256 kJ kg^{-1})

Q.54	An electron is accelerated from rest through a potential difference of 200 V. The de Broglie wavelength associated with this electron is _____ nm. (Rounded off to 2 decimal places) (Planck's constant = 6.6×10^{-34} J s, 1eV=1.6×10^{-19} J, mass of an electron = 9.1×10^{-31} kg)
Q.55	Given data consists of distinct values of x_i occurring with frequencies f_i. The mean value for the data is _____. (rounded off to one decimal place)

x_i	5	6	8	10
f_i	8	10	10	12

Q.56

A random variable X and its probability distribution is given below. The value of P(X<5) is _____. (rounded off to one decimal place)

X	0	1	2	3	4	5
P(X)	0	k	2k	3k	6k	8k

Q.57

A protein solution of 1 μM has transmission of 40 % at 280 nm, when measured in a 1 cm cuvette using a UV-Visible spectrophotometer. The transmission of the same solution, when measured using a 2 cm cuvette is _____ %. (rounded off to the nearest integer)

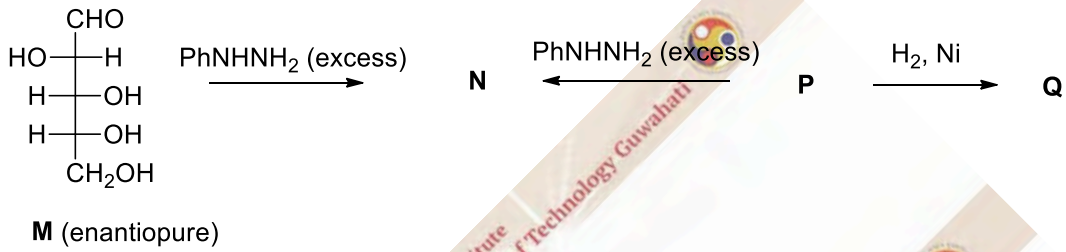
Q.58

If a bacterial culture with a doubling time of 30 minutes starts with two cells, then the number of cells after 4 hours are _____.

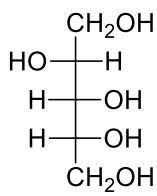
Q.59	The rate of transcription in a bacterium is 50 nucleotides/min and the average molecular weight of an amino acid is 110 Da. Time taken for synthesis of the mRNA of a protein with molecular weight of 110 kDa is _____ min. (rounded off to one decimal place) Assume no abortive transcriptions and no sequences upstream of the start codon.
Q.60	Consider a first order reaction $A \rightarrow B$. The initial concentration of A is 100 mol L^{-1} and the value of first order rate constant is 0.01 min^{-1}. The concentration of A after 10 min of reaction is _____ mol L^{-1} (rounded off to one decimal place).

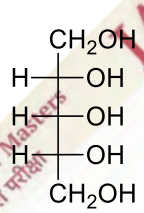
Section A: Q.1 – Q.10 Carry ONE mark each.

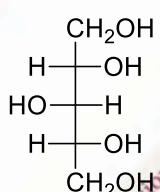
Q.1 The structure of **Q** in the following reaction scheme is

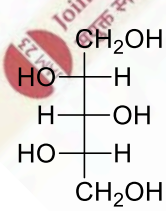


M (enantiopure)

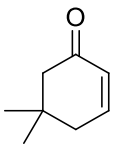
(A) 

(B) 

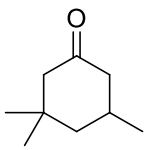
(C) 

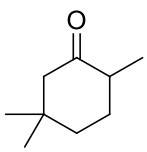
(D) 

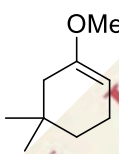
Q.2 The major product of the reaction is

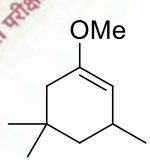


(i) CH_2I_2, Zn-Cu
(ii) Li, NH_3 (liq.)
(iii) H_3O^+

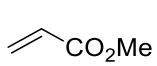
(A) 

(B) 

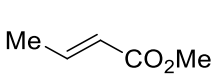
(C) 

(D) 

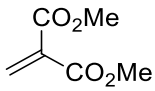
Q.3 The rate of addition of 1-hexyl radical to the given molecules follows the order



P



Q



R

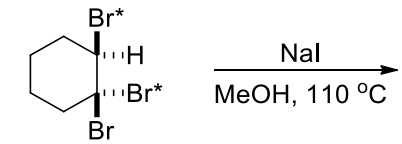
(A) **P > R > Q**

(B) **Q > P > R**

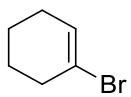
(C) **R > P > Q**

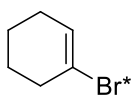
(D) **P > Q > R**

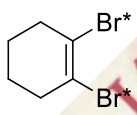
Q.4 The major product of the reaction is

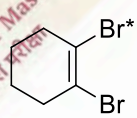


Br* = ^{82}Br

(A) 

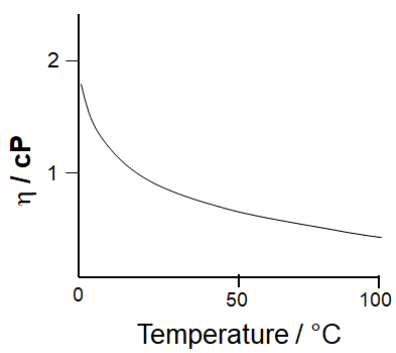
(B) 

(C) 

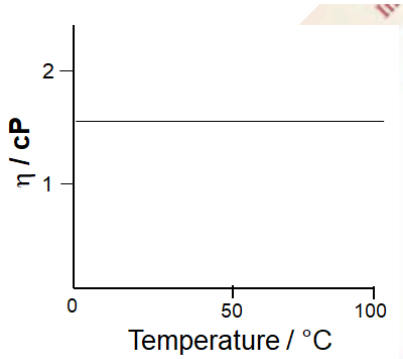
(D) 

Q.5 The diagram that best describes the variation of viscosity (η) of water with temperature at 1 atm is

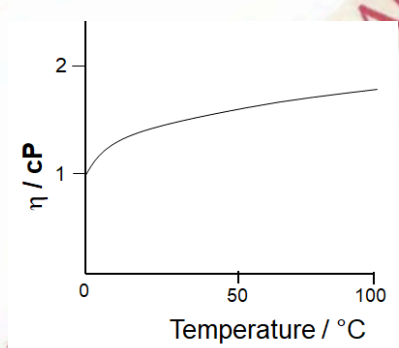
(A)



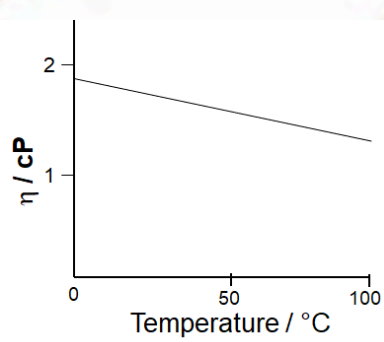
(B)



(C)



(D)



Q.6 The SI unit of the molar conductivity of an electrolyte solution is

(A) $S\ m^{-1}\ mol^{-1}$

(B) $S\ mol^{-1}$

(C) $S\ m\ mol^{-1}$

(D) $S\ m^{2}\ mol^{-1}$

Q.7 The system with the lowest zero-point energy when it is confined to a one-dimensional box of length L is

(A) an electron

(B) a hydrogen atom

(C) a helium atom

(D) a proton

Q.8 The metal ion present in human *carbonic anhydrase* is

(A) Fe^{3+}

(B) Cu^{2+}

(C) Zn^{2+}

(D) Ni^{2+}

Q.9 The oxoacid of sulfur that has S – O – S bond is

(A) Pyrosulfuric acid

(B) Pyrosulfurous acid

(C) Dithionous acid

(D) Dithionic acid

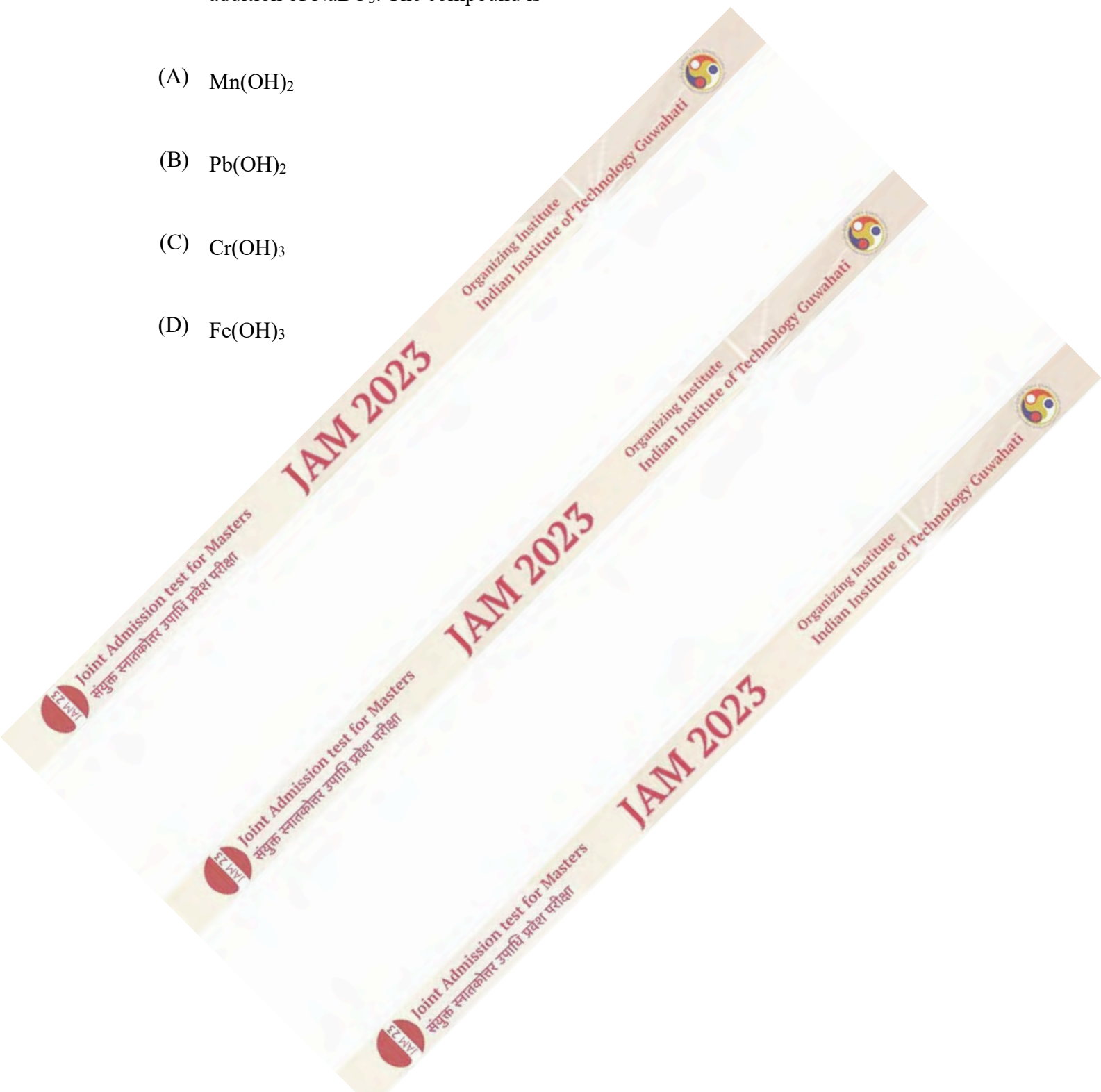
Q.10 An alkaline (NaOH) solution of a compound produces a yellow colored solution on addition of $NaBO_3$. The compound is

(A) $Mn(OH)_2$

(B) $Pb(OH)_2$

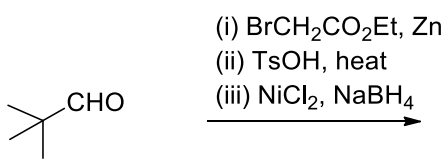
(C) $Cr(OH)_3$

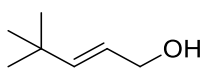
(D) $Fe(OH)_3$

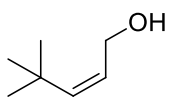


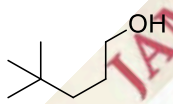
Section A: Q.11 – Q.30 Carry TWO marks each.

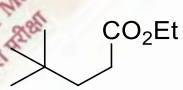
Q.11 The major product of the reaction is



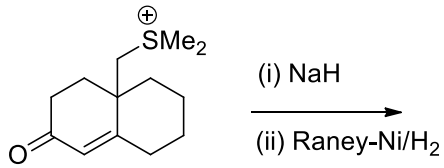
(A) 

(B) 

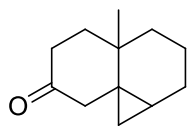
(C) 

(D) 

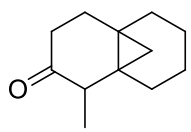
Q.12 The major product in the following reaction is



(A)



(B)



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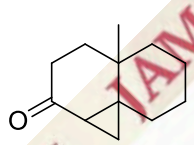
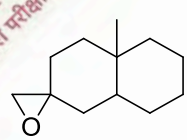
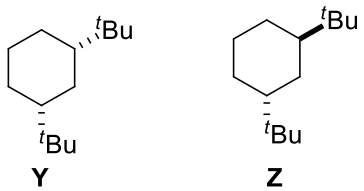


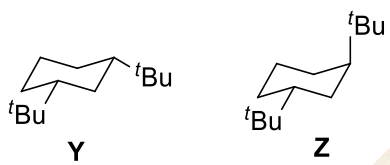
(D)



Q.13 The most stable conformation of **Y** and that of **Z** are

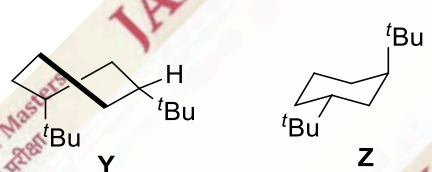


(A)

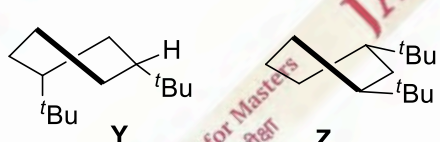


(B)

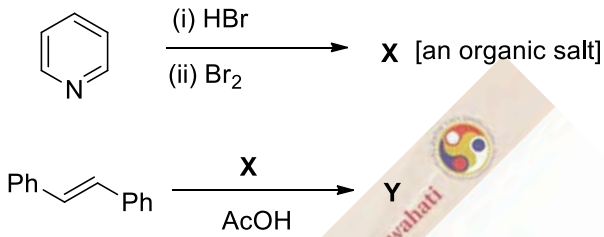
(C)

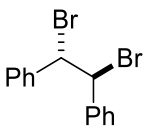


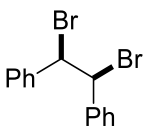
(D)

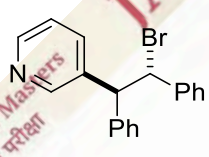


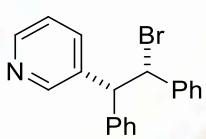
Q.14 The major product **Y** in the following reaction scheme is



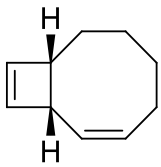
(A) 

(B) 

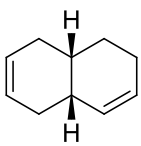
(C) 

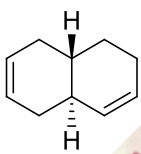
(D) 

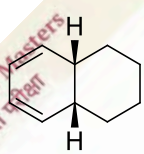
Q.15 The major product of the reaction is

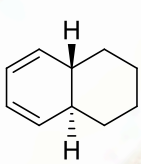


150 °C

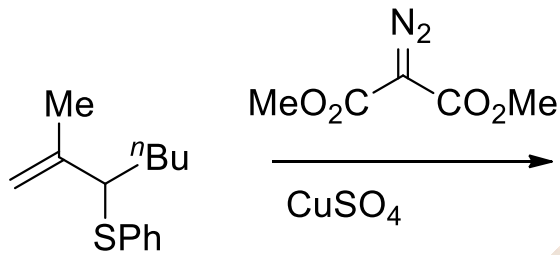
(A) 

(B) 

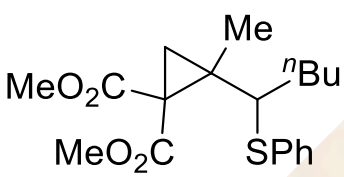
(C) 

(D) 

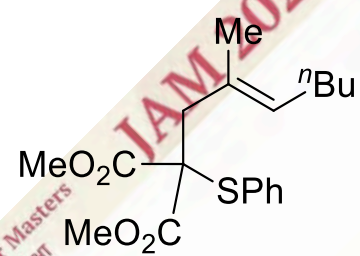
Q.16 The major product of the reaction is



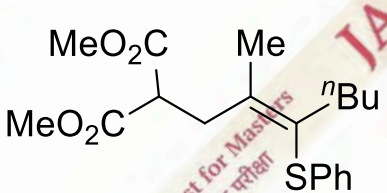
(A)



(B)



(C)



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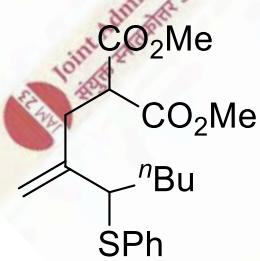


Q.17 Adsorption of a gas on a solid surface follows the Langmuir isotherm. If $k_a / k_d = 1.0$ bar^{-1}, the fraction of adsorption sites occupied by the gas at equilibrium under 2.0 bar pressure of the gas at 25 °C is

(k_a and k_d are the rate constants for adsorption and desorption processes, respectively, at 25 °C)

(A) 1/4

(B) 1/3

(C) 1/2

(D) 2/3

Q.18 The vapor pressure of a dilute solution of a non-volatile solute and the vapor pressure of the pure solvent at the same temperature are *P* and *P**, respectively.

$\frac{P^* - P}{P^*}$ is equal to

(Assume that the vapor phase behaves as an ideal gas)

(A) molality of the solution

(B) mole fraction of the solvent

(C) weight fraction of the solute

(D) mole fraction of the solute

Q.19 The volume of water (in mL) required to be added to a 100 mL solution (aq. 0.1 M) of a weak acid (HA) at 25 °C to double its degree of dissociation is

[Given: K_a of HA at 25 °C = 1.8×10^{-5}]

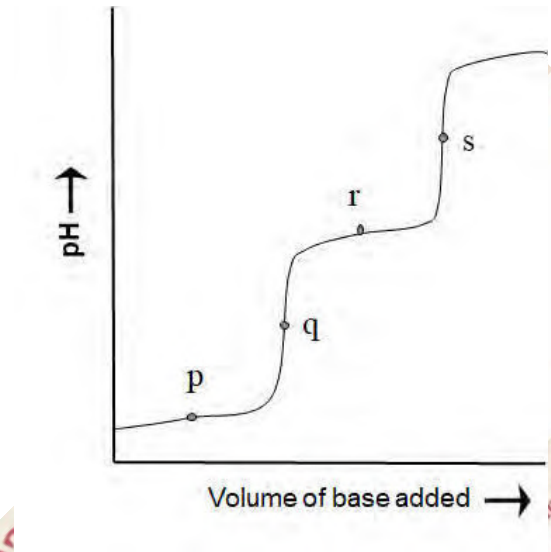
(A) 100

(B) 200

(C) 300

(D) 400

Q.20 The following diagram is obtained in a pH-metric titration of a weak dibasic acid (H_2A) with a strong base. The point that best represents $[HA^-] = [A^{-2}]$ is



(A) **p**

(B) **q**

(C) **r**

(D) **s**

Q.21 Equal number of gas molecules **A** (mass m and radius r) and **B** (mass $2m$ and radius $2r$) are placed in two separate containers of equal volume. At a given temperature, the ratio of the collision frequency of **B** to that of **A** is

(Assume the gas molecules as hard spheres)

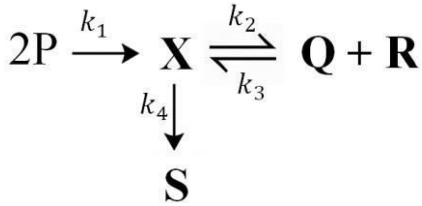
(A) $\sqrt{2} : 1$

(B) $2\sqrt{2} : 1$

(C) $1 : \sqrt{2}$

(D) $1 : 2\sqrt{2}$

Q.22 For the given elementary reactions, the steady-state concentration of **X** is



(A) $\dfrac{k_1[\mathbf{P}]^2 + k_3[\mathbf{Q}][\mathbf{R}]}{k_2 + k_4}$

(B) $\dfrac{\frac{1}{2}k_1[\mathbf{P}]^2 + k_3[\mathbf{Q}][\mathbf{R}]}{k_2 + k_4}$

(C) $\dfrac{k_1[\mathbf{P}]^2 + k_3[\mathbf{Q}][\mathbf{R}]}{-k_1 + k_2 - k_3 + k_4}$

(D) $\dfrac{k_1[\mathbf{P}] + k_3[\mathbf{Q}][\mathbf{R}]}{k_2 + k_4}$

Q.23 The separation (in nm) of {134} planes of an orthorhombic unit cell (with cell parameters $a = 0.5$ nm, $b = 0.6$ nm, and $c = 0.8$ nm) is

(A) 0.036

(B) 0.136

(C) 0.236

(D) 0.336

Q.24 The transition metal (**M**) complex that can have all isomers (geometric, linkage, and ionization) is

(A) $[\mathbf{M}(NH_3)_4Br_2]SCN$

(B) $[\mathbf{M}(NH_3)_4Cl_2]Br$

(C) $[\mathbf{M}(NH_3)_4(H_2O)_2]Cl_3$

(D) $[\mathbf{M}(NH_3)_4\,(H_2O)_2](SCN)_3$

Q.25 The geometry of $[VO(acac)_2]$ is

(A) square pyramidal

(B) trigonal bipyramidal

(C) pentagonal planar

(D) distorted trigonal bipyramidal

Q.26 The products **X** and **Y** in the following reaction sequence, respectively, are

$$BCl_3 \xrightarrow[150\ ^oC]{NH_4Cl \text{ in } C_6H_5Cl} \mathbf{X} \xrightarrow{NaBH_4} \mathbf{Y}$$

(A) $B_3N_3Cl_6$ and $B_3N_3H_6$

(B) $B_3N_3H_3Cl_3$ and $B_3N_3H_6$

(C) $B_3N_3H_3Cl_3$ and $B_3N_3H_{12}$

(D) $B_3N_3H_9Cl_3$ and $B_3N_3H_{12}$

Q.27 The correct order of the energy of the d orbitals of a square planar complex is

(A) $d_{xz} = d_{yz} < d_{xy} < d_{z^2} < d_{x^2-y^2}$

(B) $d_{xz} = d_{yz} < d_{z^2} < d_{xy} < d_{x^2-y^2}$

(C) $d_{yz} < d_{xz} < d_{z^2} < d_{xy} < d_{x^2-y^2}$

(D) $d_{xy} < d_{xz} < d_{yz} < d_{x^2-y^2} < d_{z^2}$

Q.28 **X** and **Y** in the following reactions, respectively, are

$EtOH + 2H_2SO_4 \longrightarrow \mathbf{X} + H_3O^+ + HSO_4^-$

$HNO_3 + 2H_2SO_4 \longrightarrow \mathbf{Y} + H_3O^+ + 2HSO_4^-$

(A) CH_3COOH and NO^+

(B) CH_3CHO and NO_2^+

(C) $EtOSO_3H$ and NO_2^+

(D) $EtOSO_3H$ and NO^+

Q.29 The correct order of energy levels of the molecular orbitals of N_2 is

(A) $1\sigma_g < 1\sigma_u < 2\sigma_g < 2\sigma_u < 1\pi_u < 3\sigma_g < 1\pi_g < 3\sigma_u$

(B) $1\sigma_g < 1\sigma_u < 2\sigma_g < 2\sigma_u < 3\sigma_g < 3\sigma_u < 1\pi_u < 1\pi_g$

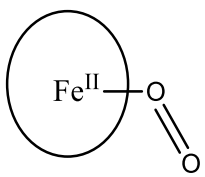
(C) $1\sigma_g < 1\sigma_u < 2\sigma_g < 2\sigma_u < 1\pi_g < 3\sigma_g < 1\pi_u < 3\sigma_u$

(D) $1\sigma_g < 1\sigma_u < 2\sigma_g < 2\sigma_u < 3\sigma_g < 1\pi_u < 1\pi_g < 3\sigma_u$

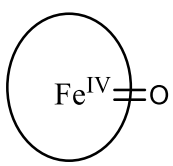
Q.30 Free heme in aqueous solution when exposed to dioxygen is finally converted to

(circle around iron in the given choices represents the protoporphyrin IX)

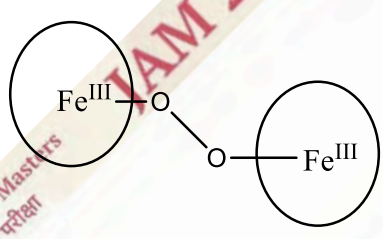
(A)



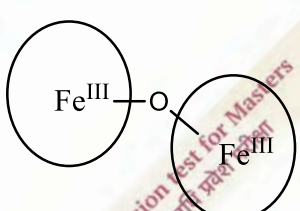
(B)



(C)

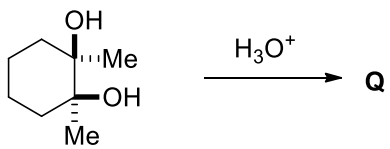


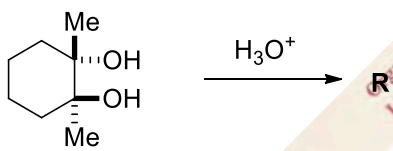
(D)



Section B: Q.31 – Q.40 Carry TWO marks each.

Q.31 Correct statement(s) about **Q** and **R** is/are





(A) Both **Q** and **R** give positive Fehling's test

(B) **Q** gives positive iodoform test and its ^{1}H NMR spectrum shows singlets at 1.0 ppm (3H) and at 1.3 ppm (3H)

(C) **R** gives positive iodoform test and its ^{1}H NMR spectrum shows singlets at 1.0 ppm (3H) and at 2.2 ppm (3H)

(D) A bright yellow precipitate is formed when **Q** and **R** treated separately with 2,4-dinitrophenyl hydrazine

Q.32 The correct statement(s) is/are

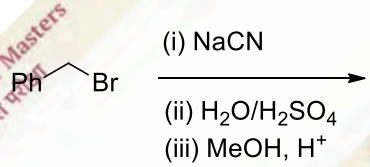
(A) The pK_{a1} of *cis*-cyclohexane 1,3-diol is greater than that of the *trans* isomer.

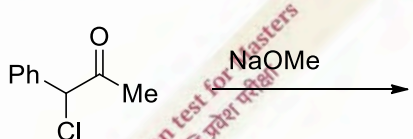
(B) The *trans*-4-(*tert*-butyl)cyclohexanamine is more basic than its *cis* isomer.

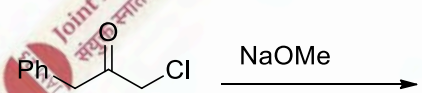
(C) 2,6-Dihydroxybenzoic acid is more acidic than salicylic acid.

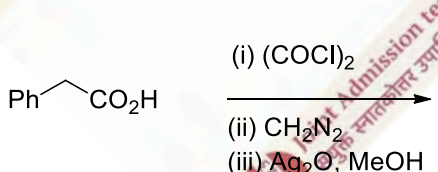
(D) 2,4,6-Trinitrophenol is more acidic than 2,4,6-trinitrobenzoic acid.

Q.33 The reaction(s) that yield(s) Ph-CH_2-CH_2-CO_2Me as the major product is/are

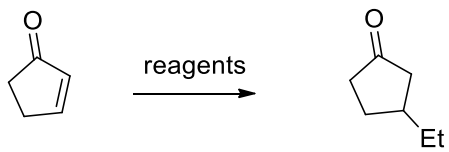
(A) 

(B) 

(C) 

(D) 

Q.34 The correct option(s) of the reagents required for the following reaction is/are



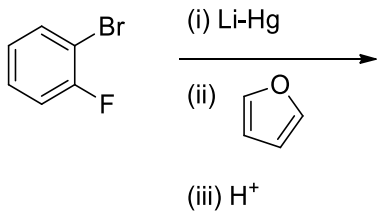
(A) (i) Et_3B, O_2 (cat), THF; (ii) H_2O

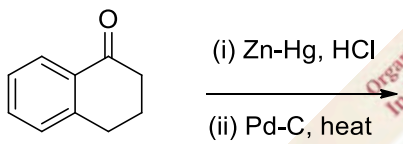
(B) (i) Et_2CuLi, Me_3SiCl; (ii) H_3O^+

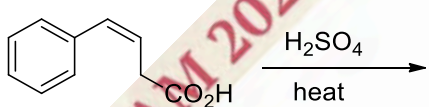
(C) (i) EtMgBr, Et_2O; (ii) H_2O

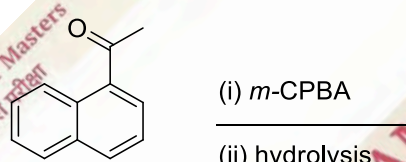
(D) (i) ^{n}BuLi, THF; (ii) EtI

Q.35 The reaction(s) that yield(s) 1-naphthol as the major product is/are

(A) 

(B) 

(C) 

(D) 

Q.36 The correct relation(s) for an ideal gas in a closed system is/are

(A) $\left(\frac{\partial H}{\partial V}\right)_T = 0$

(B) $\left(\frac{\partial T}{\partial P}\right)_H = 0$

(C) $\left(\frac{\partial H}{\partial P}\right)_T = 0$

(D) $\left(\frac{\partial H}{\partial T}\right)_P = 0$

Q.37 The molecule(s) that follow(s) $I_a < I_b = I_c$ (I_a, I_b, and I_c are the principal moments of inertia) is/are

(A) HCN

(B) CH_3Cl

(C) $CH_3C{\equiv}CH$

(D) C_6H_6

Q.38 The role(s) of fluorspar in the electrolytic reduction of Al_2O_3 is/are to

(A) decrease the melting point of Al_2O_3

(B) improve the electrical conductivity of the melt

(C) prevent the corrosion of anode

(D) prevent the radiation loss of heat

Q.39 The correct statement(s) about the complexes **I** ($K_3[CoF_6]$) and **II** ($K_3[RhF_6]$) is/are

(A) Both complexes are high spin.

(B) Complex **I** is paramagnetic.

(C) Complex **II** is diamagnetic.

(D) The crystal field stabilization energy of complex **II** is more than that of complex **I**.

Q.40 The diatomic molecule(s) that has/have bond order of one is/are

(A) B_2

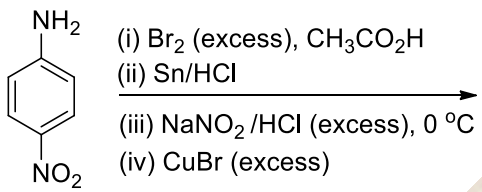
(B) N_2^{2-}

(C) Li_2

(D) O_2^{2-}

Section C: Q.41 – Q.50 Carry ONE mark each.

Q.41 The molecular weight of the major product of the reaction is ____ (*in integer*).

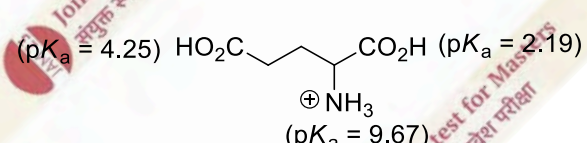


[Given: atomic weight of H = 1, C = 12, N = 14, and Br = 80]

Q.42 A 0.06 g/mL solution of (*S*)-1-phenylethanol placed in a 5 cm long polarimeter tube shows an optical rotation of 1.2°. The specific rotation is _____°.

(*round off to the nearest integer*)

Q.43 The isoelectric point of glutamic acid is _____.



(*round off to two decimal places*)

Q.44 Consider the following reaction:

$$2\,C_6H_6 + 15\,O_2 \rightarrow 12\,CO_2 + 6\,H_2O \qquad \Delta_r H^0_{298} = -3120\ \text{kJ mol}^{-1}.$$

A closed system initially contains 5 moles of benzene and 25 moles of oxygen under standard conditions at 298 K. The reaction was stopped when 17.5 moles of oxygen is left. The amount of heat evolved during the reaction is _____ kJ.

(*round off to the nearest integer*)

Q.45 For the elementary reaction $\mathbf{C} \overset{k_2}{\leftarrow} \mathbf{A} \overset{k_1}{\rightarrow} \mathbf{B}$, $k_1 = 2k_2$. At time $t = 0$, $[\mathbf{A}] = A_0$ and $[\mathbf{B}] = [\mathbf{C}] = 0$. At a later time t, the value of $[\mathbf{B}]/[\mathbf{C}]$ is _____.

(*round off to the nearest integer*)

Q.46 The highest possible energy of a photon in the emission spectrum of hydrogen atom is _____ eV.

[Given: Rydberg constant = 13.61 eV]

(*round off to two decimal places*)

Q.47 The standard reduction potential (E^0) of $Fe^{3+} \rightarrow Fe$ is _____V.

[Given: $Fe^{3+} \rightarrow Fe^{2+}$ $E^0 = 0.77$ V and

$Fe^{2+} \rightarrow Fe$ $E^0 = -0.44$ V]

(*round off to three decimal places*)

Q.48 The number of valence electrons in $Na_2[Fe(CO)_4]$ (the Colman's reagent) is _____.

Q.49 In the Born-Haber cycle, the heat of formation of CuCl is _____ kJ/mol.

[Given: Heat of atomization of Cu = +338 kJ/mol,
Ionization energy of Cu = +746 kJ/mol,
Heat of atomization of Cl_2 = +121 kJ/mol,
Electron affinity of Cl = −349 kJ/mol, and
Lattice energy of CuCl = −973 kJ/mol]

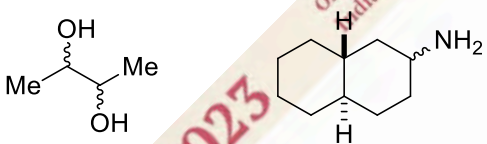
(*round off to the nearest integer*)

Q.50 The spin-only magnetic moment of B_2 molecule is_____ μ_B.

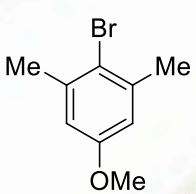
(*round off to two decimal places*)

Section C: Q.51 – Q.60 Carry TWO marks each.

Q.51 The sum of the total number of stereoisomers (including enantiomers) present in the following molecules is _____.



Q.52 The number of singlets observed in the ^{1}H NMR spectrum of **P** is _____.



P

Q.53 When a glass capillary tube is dipped in water, a 1.0 cm rise in the water level is observed at 18 °C. The internal radius of the capillary is _____ cm.

[Given: Surface tension of water at 18 °C = 73.2 dyne cm^{-1}; difference in the densities of water and air at 18 °C = 0.996 g cm^{-3}; gravitational acceleration constant, g = 980 cm s^{-2}.

Assume that water completely wets the glass capillary and the interface between the water and the air phase inside the capillary is a hemisphere.]

(*round off to two decimal places*)

Q.54 The volume of 2.0 mol of an ideal gas is reduced to half isothermally at 300 K in a closed system. The value of ΔG is_____ kJ.

[Given: R = 8.314 J mol^{-1} K^{-1}]

(*round off to two decimal places*)

Q.55 The harmonic vibrational frequency of a diatomic molecule is 2000 cm^{-1}. Its zero-point energy is ____ eV.

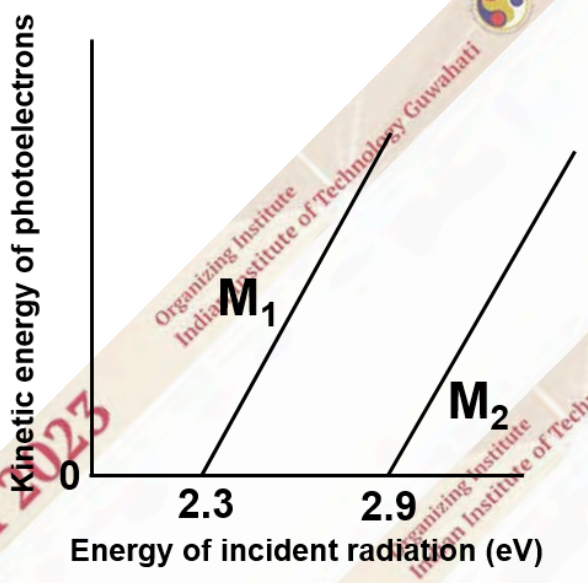
[Given: Planck's constant = 6.62 × 10^{-34} J s; 1 eV = 1.6 × 10^{-19} J]

(*round off to two decimal places*)

Q.56 An elementary reaction 2**A** → **P** follows a second order rate law with rate constant 2.5 × 10^{-3} dm^3 mol^{-1} s^{-1}. The time required for the concentration of **A** to change from 0.4 mol dm^{-3} to 0.2 mol dm^{-3} is_____ s.

(*round off to the nearest integer*)

Q.57 The following diagram shows the kinetic energy of the ejected photoelectrons against the energy of incident radiation for two metal surfaces **M_1** and **M_2**. If the energy of the incident radiation on **M_1** is equal to the work function of **M_2**, the de Broglie wavelength of the ejected photoelectron is _____ nm.



[Given: Mass of electron = 9.11×10^{-31} kg; Planck's constant = 6.62×10^{-34} J s; 1 eV = 1.6×10^{-19} J.]

(*round off to two decimal places*)

Q.58 The spin-only magnetic moment of $[Fe(acac)_3]$ is _____ μ_B.

(*round off to two decimal places*)

Q.59 The amount of ethane produced in the following reaction is _____ kg.

$$C_2H_4(2\text{ kg}) + H_2(2\text{ kg}) \xrightarrow{\text{Wilkinson's Catalyst}} C_2H_6\ (90\%\text{ catalytic conversion})$$

(*round off to two decimal places*)

Q.60 In a gravimetric estimation of Al, a sample of 0.1000 g $AlCl_3$ is precipitated with 8-hydroxyquinoline. The weight of the precipitate is _____ g.

[Given: atomic weight of Al is 26.98; molecular weight of $AlCl_3$ is 133.34; and molecular weight of 8-hydroxyquinoline is 145.16]

(*round off to 4 decimal places*)

Section A: Q.1 – Q.10 Carry ONE mark each.

Q.1 A competitive firm can sell any output at price $P = 1$. Production depends on capital alone, and the production function $y = f(K)$ is twice continuously differentiable, with

$$f(0) = 0,\ \ f' > 0,\ \ f'' < 0,\ \ \lim_{K\to 0} f'(K) = \infty,\ \ \lim_{K\to\infty} f'(K) = 0.$$

The firm has positive capital stock, $\bar{K}$ to start with, and can buy and sell capital at price r per unit of capital. If the firm is maximizing profit then which of the following statements is NOT CORRECT?

(A) If $\bar{K}$ is large enough, profit maximizing $y = 0$ and the profit is $r\bar{K}$

(B) If $f'(\bar{K}) > r$, the firm will buy additional capital

(C) If $f'(\bar{K}) < r$, the firm will sell some of its capital

(D) If $f'(\bar{K}) = r$, the firm will neither buy nor sell any capital

Q.2 Let $f, g : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x+2, & x \leq 1 \\ 2x+1, & x > 1 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} 2x, & x \leq 2 \\ x+2, & x > 2. \end{cases}$$

Then

(A) f is convex and g is concave

(B) f is concave and g is convex

(C) both f and g are concave

(D) both f and g are convex

Q.3 Let S be a feasible set of a linear programming problem (P). If the dual problem of (P) is unbounded then

(A) (P) is unbounded

(B) S is empty

(C) S is unbounded

(D) (P) has multiple optimal solutions

Q.4 Which of the following is NOT CORRECT?

(A) A quasiconcave function is necessarily a concave function

(B) A concave function is necessarily a quasiconcave function

(C) A quasiconcave function can also be a quasiconvex function

(D) A quasiconcave function can also be a convex function

Q.5 Among the following statements which one is CORRECT?

S1: $x^2 + y^2 = 6$ is a level curve of

$$f(x, y) = \sqrt{x^2 + y^2} - x^2 - y^2 + 2$$

S2: $x^2 - y^2 = -3$ is a level curve of

$$g(x, y) = e^{-x^2} e^{y^2} + x^4 - 2 - 2x^2 y^2 + y^4$$

(A) both S1 and S2

(B) only S1

(C) only S2

(D) neither S1 nor S2

Q.6 Which of the following is NOT a component of Gross Domestic Product?

(A) Investment

(B) Rental Income

(C) Transfer Payments

(D) Wages and Salaries

Q.7 Which of the following are the direct instruments exercised by the Reserve Bank of India to control the money supply?

(i) Cash Reserve Ratio
(ii) Open Market Operations
(iii) Foreign Exchange Rate
(iv) Statutory Liquidity Ratio

(A) (i, ii, iii)

(B) (i, ii, iv)

(C) (ii, iii, iv)

(D) (i, iii, iv)

Q.8 Which of the following committees for the first time recommended for India

(i) use of implicit prices derived from quantity and value data collected in household consumer expenditure surveys for computing and updating the poverty lines

(ii) Mixed Reference Period (MRP) in estimating poverty lines

(A) Y K Alagh Committee

(B) D T Lakdawala Committee

(C) S D Tendulkar Committee

(D) C Rangarajan Committee

Q.9 Which of the following Five Year Plans focused on rapid industrialization-heavy and basic industries, and advocated for a socialistic pattern of society as the goal of economic policy?

(A) 1st Five Year Plan (1951-56)

(B) 2nd Five Year Plan (1956-61)

(C) 3rd Five Year Plan (1961-66)

(D) 4th Five Year Plan (1969-74)

Q.10 Let M and N be events defined on the sample space S. If $P(M) = \frac{1}{3}$ and $P(N^c) = \frac{1}{4}$ then which one of the following is necessarily CORRECT?

(A) M and N are disjoint

(B) M and N are not disjoint

(C) M and N are independent

(D) M and N are not independent

Section A: Q.11 – Q.30 Carry TWO marks each.

Q.11 Consider a 2-agent, 2-good exchange economy where agent i has utility function $u_i(x_i, y_i) = \max\{x_i, y_i\}, i = 1,2$. The initial endowments of goods X and Y that the agents have are $(\overline{x_1}, \overline{y_1}, \overline{x_2}, \overline{y_2}) = (25, 5, 5, 5)$. Then select the CORRECT choice below where the price vector (p_x, p_y) specified is part of a competitive equilibrium.

(A) $(p_x, p_y) = (2,1)$

(B) $(p_x, p_y) = (2,2)$

(C) $(p_x, p_y) = (1,2)$

(D) $(p_x, p_y) = (4,2)$

Q.12 For a firm operating in a perfectly competitive market which of the following statements is CORRECT?

(A) Profit function is convex and homogeneous of degree 1 in prices

(B) Profit function is concave and homogeneous of degree 1 in prices

(C) Profit function is convex but not homogeneous in prices

(D) Profit function is neither concave nor convex in prices

Q.13 A firm is operating in a perfectly competitive environment. A change in the market condition leads to an increase in the firm's profit by an amount K. Which of the following describes the change in the Producer's Surplus due to the above change in the market condition?

(A) The Producer's Surplus increases by K

(B) The Producer's Surplus increases by less than K but greater than 0

(C) The Producer's Surplus changes but it is not possible to know the direction of the change

(D) The Producer's Surplus doesn't change

Q.14 Two people, 1 and 2, are engaged in a joint project. Person $i \in \{1,2\}$ puts in effort x_i $(0 \leq x_i \leq 1)$, and incurs cost $C_i(x_i) = x_i$. The monetary outcome of the project is $4x_1x_2$ which is split equally between them. Considering the situation as a strategic game, the set of all Nash Equilibria in pure strategies is

(A) $\{(0,0),(1,1)\}$

(B) $\left\{(0,0),\left(\frac{1}{4},\frac{1}{4}\right),\left(\frac{1}{2},\frac{1}{2}\right),\left(\frac{3}{4},\frac{3}{4}\right),(1,1)\right\}$

(C) $\left\{(0,0),\left(\frac{1}{2},\frac{1}{2}\right),(1,1)\right\}$

(D) a null set

Q.15 Two firms, *X* and *Y*, are operating in a perfectly competitive market. The price elasticity of supply of *X* and *Y* are respectively 0.5 and 1.5. Then

(A) if the market price increases by 1 %, *X* supplies 0.5 % less quantity

(B) *Y* experiences a slower increase in marginal cost in comparison to *X*

(C) if market price increases by 0.5 %, *X* supplies 1 % more quantity

(D) *Y* experiences a rapid increase in marginal cost in comparison to *X*

Q.16 Let $y = y(x)$ be a solution curve of the differential equation

$$x\frac{dy}{dx} = y \ln\left(\frac{y}{x}\right), \qquad y > x > 0.$$

If $y(1) = e^2$ and $y(2) = \alpha$, then the value of $\frac{dy}{dx}$ at $(2, \alpha)$ is equal to

(A) α

(B) $\frac{\alpha}{2}$

(C) 2α

(D) $\frac{3\alpha}{2}$

Q.17 Let $2z = -3 + \sqrt{3}\,i$, $i = \sqrt{-1}$. Then $2z^8$ is equal to

(A) $-81\,(1 + \sqrt{3}\,i)$

(B) $81\,(-1 + \sqrt{3}\,i)$

(C) $81\,(\sqrt{3} + i)$

(D) $9\,(-\sqrt{3} + i)$

Q.18 Let $a_n = \left(1 + \frac{1}{n}\right)^{\frac{n}{2}}$ be the n^{th} term of the sequence $\langle a_n \rangle, n = 1, 2, 3, \ldots$. Then which one of the following is NOT CORRECT?

(A) $\langle a_n \rangle$ is bounded

(B) $\langle a_n \rangle$ is increasing

(C) $\sum_{n=1}^{\infty} \ln(a_n)$ is a convergent series

(D) $\lim_{n\to\infty} \left(\frac{1}{n} \sum_{k=1}^{n} a_k\right) = \sqrt{e}$

Q.19 Consider a linear programming problem (P)

$$\min\ z = 4x_1 + 6x_2 + 6x_3$$
$$\text{subject to}$$
$$x_1 + 3x_2 \geq 3$$
$$x_1 + 2x_3 \geq 5$$
$$x_1,\ x_2,\ x_3 \geq 0$$

If $x^* = (x_1^*, x_2^*, x_3^*)$ is an optimal solution and z^* is an optimal value of (P) and $w^* = (w_1^*, w_2^*)$ is an optimal solution of the dual of (P) then

(A) $x_2^* + x_3^* = w_1^* + w_2^*$

(B) $z^* = 4(x_1^* + w_2^*)$

(C) $z^* = 6(w_1^* + x_3^*)$

(D) $x_1^* + x_3^* = w_1^* + w_2^*$

Q.20 For $\alpha, \beta \in \mathbb{R}$, consider the system of linear equations

$$x + y + z = 1$$
$$3x + y + 2z = 2$$
$$5x + \alpha y + \beta z = 3$$

Then

(A) for every (α, β), $\alpha = \beta$, the system is consistent

(B) there exists (α, β), satisfying $\alpha - 2\beta + 5 = 0$, for which the system has a unique solution

(C) there exists a unique pair (α, β) for which the system has infinitely many solutions

(D) for every (α, β), $\alpha \neq \beta$, satisfying $\alpha - 2\beta + 5 = 0$, the system has infinitely many solutions

Q.21 For a positively sloped LM curve, which of the following statements is CORRECT?

(A) A decrease in the price level will shift the LM curve to the left

(B) A lower nominal money supply will shift the LM curve to the right

(C) An increase in the price level will shift the LM curve to the right

(D) A higher nominal money supply will shift the LM curve to the right

Q.22 Consider an Economy that produces only Apples and Bananas. The following Table contains per unit price (in INR) and quantity (in kg) of these goods. Assuming 2010 as the Base Year and using GDP deflator to calculate the annual inflation rate, which of the following options is CORRECT?

Year	Price of Apple	Quantity of Apple	Price of Banana	Quantity of Banana
2010	1	100	2	50
2011	1	200	2	100
2012	2	200	4	100

(A) GDP deflator for the year 2011 is 100 and the inflation rate for the year 2011 is 0 %

(B) GDP deflator for the year 2012 is 50 and the inflation rate for the year 2012 is 100 %

(C) GDP deflator for the year 2011 is 50 and the inflation rate for the year 2011 is 0 %

(D) GDP deflator for the year 2012 is 100 and the inflation rate for the year 2012 is 100 %

Q.23 Which of the following statements is NOT CORRECT in the context of an Open Economy IS-LM Model under Floating Exchange Rate (with fixed price) and Perfect Capital Mobility?

(A) An expansionary fiscal policy would appreciate the domestic currency value

(B) An expansionary monetary policy would depreciate the domestic currency value

(C) Exchange rate has significant impact on determining the equilibrium level of income and employment

(D) Monetary policy is fully effective in determining income and employment whereas fiscal policy is ineffective

Q.24 Among the following statements which one is CORRECT?

S1: Structural unemployment arises in between two jobs, the first job which an individual has quit in order to find the second job

S2: Frictional unemployment arises due to the mismatch of vacancies and skills of the individual

(A) only S1

(B) only S2

(C) both S1 and S2

(D) neither S1 nor S2

Q.25 Matching List-I and List-II, choose the CORRECT option.

List-I	List-II
(a) Fiscal Deficit	(i) Difference between Government revenue expenditure and Government revenue receipts
(b) Revenue Deficit	(ii) Difference between Government total expenditure and Government total non-debt receipts minus interest payments
(c) Primary Deficit	(iii) Difference between Government total expenditure and Government total non-debt receipts

(A) (a, iii), (b, ii), (c, i)

(B) (a, iii), (b, i), (c, ii)

(C) (a, i), (b, iii), (c, ii)

(D) (a, ii), (b, i), (c, iii)

Q.26 A production function at time t is given by

$$Y_t = A_t\, K_t^{\alpha}\, L_t^{1-\alpha}, \quad \alpha \in (0,1), \quad \alpha \neq 0.5,$$

where Y is output, K is capital, L is labour and A is the level of Total Factor Productivity. Define per capita output as $y_t \equiv \frac{Y_t}{L_t}$ and capital-output ratio as $k_t \equiv \frac{K_t}{Y_t}$. For any variable x_t, denote $\frac{dx_t}{dt}$ by $\dot{x}$. The per capita output growth rate is

(A) $\frac{\dot{y}}{y} = \frac{1}{(1-\alpha)}\frac{\dot{A}}{A} + \frac{\alpha}{(1-\alpha)}\frac{\dot{k}}{k}$

(B) $\frac{\dot{y}}{y} = \frac{\alpha}{(1-\alpha)}\frac{\dot{A}}{A} + \frac{1}{(1-\alpha)}\frac{\dot{k}}{k}$

(C) $\frac{\dot{y}}{y} = (1-\alpha)\frac{\dot{A}}{A} + \alpha\frac{\dot{k}}{k}$

(D) $\frac{\dot{y}}{y} = \alpha\frac{\dot{A}}{A} + (1-\alpha)\frac{\dot{k}}{k}$

Q.27 Matching List-I and List-II, choose the CORRECT option.

List-I (Regulatory and Supervisory Financial Institutions)	List-II (Established as statutory bodies via Parliamentary Acts in year)
(a) Reserve Bank of India	(i) 2016
(b) Security and Exchange Board of India	(ii) 1934
(c) Insurance Regulatory Development Authority of India	(iii) 1992
(d) Insolvency and Bankruptcy Board of India	(iv) 1999

(A) (a, ii), (b, iv), (c, iii), (d, i)

(B) (a, iii), (b, ii), (c, iv), (d, i)

(C) (a, ii), (b, iii), (c, i), (d, iv)

(D) (a, ii), (b, iii), (c, iv), (d, i)

Q.28 Let $X \sim Normal(0,1)$ and $Y = |X|$. If the probability density function of Y is $f_Y(y)$ then for $y > 0$, $\sqrt{\frac{\pi}{2}}\, f_Y(y)$ is

(A) $e^{-\frac{y^2}{2}}$

(B) $e^{\frac{y^2}{2}}$

(C) e^{-y^2}

(D) $e^{-\frac{y}{2}}$

Q.29 Let the probability density function of the continuous random variable X be

$$f_X(x,\lambda) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases},$$

where $\lambda > 0$ is a parameter. If the observed sample values of X are

$$x_1 = 1.75,\ x_2 = 2.25,\ x_3 = 2.50,\ x_4 = 2.75,\ x_5 = 3.25,$$

then the Maximum Likelihood Estimator of λ is

(A) $\frac{5}{2}$

(B) $\frac{1}{5}$

(C) $\frac{5}{12}$

(D) $\frac{2}{5}$

Q.30 From a set comprising of 10 students, four girls $G_i, i = 1, \ldots, 4,$ and six boys $B_j, j = 1, \ldots, 6,$ a team of five students is to be formed. The probability that a randomly selected team comprises of 2 girls and 3 boys, with at least one of them to be B_1 or B_2, is equal to

(A) $\frac{3}{7}$

(B) $\frac{6}{7}$

(C) $\frac{8}{21}$

(D) $\frac{5}{21}$

Section B: Q.31 – Q.40 Carry TWO marks each.

Q.31 Suppose that the utility function $u: \mathbb{R}_+^n \rightarrow \mathbb{R}_+$ represents a complete, transitive and continuous preference relation over all bundles of n goods. Then select the choices below in which the function also represents the same preference relation.

(A) $f(x_1, x_2, \ldots, x_n) = u(x_1, x_2, \ldots, x_n) + (u(x_1, x_2, \ldots, x_n))^3$

(B) $g(x_1, x_2, \ldots, x_n) = u(x_1, x_2, \ldots, x_n) + \sum_{i=1}^{n} x_i$

(C) $h(x_1, x_2, \ldots, x_n) = (u(x_1, x_2, \ldots, x_n))^{\frac{1}{n}}$

(D) $m(x_1, x_2, \ldots, x_n) = u(x_1, x_2, \ldots, x_n) + (x_1^2 + x_2^2 + \cdots + x_n^2)^{0.5}$

Q.32 Consider a 2-agent, 2-good economy with an aggregate endowment of 30 units of good X and 10 units of good Y. Agent i has utility function

$$u_i(x_i, y_i) = \max\{x_i, y_i\}, i = 1, 2.$$

Select the choices below in which the specified allocation of the goods to the agents is Pareto optimal for this economy

(A) $(x_1, y_1, x_2, y_2) = (5, 5, 25, 5)$

(B) $(x_1, y_1, x_2, y_2) = (10, 10, 20, 0)$

(C) $(x_1, y_1, x_2, y_2) = (30, 0, 0, 10)$

(D) $(x_1, y_1, x_2, y_2) = (0, 10, 30, 0)$

Q.33 In a 3-player game, player 1 can choose either Up or Down as strategies. Player 2 can chose either Left or Right as strategies. Player 3 can choose either Table 1 or Table 2 as strategies.

Player 1		Player 2			Player 1		Player 2	
		Left	Right				Left	Right
	Up	3, 2, 5	4, 1, 3			Up	2, 3, 4	4, 5, 7
	Down	2, 6, 1	5, 4, 6			Down	6, 4, 0	3, 3, 3
	Table 1					Table 2		
Player 3								

Which of the following strategy profile(s) is/are Nash Equilibrium?

(A) (Up, Left, Table 1)

(B) (Down, Right, Table 1)

(C) (Down, Left, Table 2)

(D) (Up, Right, Table 2)

Q.34 Let $f: \mathbb{R}^2 \to \mathbb{R}$ be the function defined by

$$f(x,y) = \begin{cases} \dfrac{x^2 - y^3}{x^2 + y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

Then

(A) f is not continuous at $(0,0)$

(B) $f_x(0,0) = 0$

(C) $f_y(0,0) = -1$

(D) $f_x(0,0)$ does not exists

Q.35 For $\alpha, \beta \in \mathbb{R}$, $\alpha \neq \beta$, if -2 and 5 are the eigenvalues of the matrix

$$M = \begin{bmatrix} 1-\alpha & 1+\beta \\ \beta & \alpha+\beta \end{bmatrix}$$

and $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ is an eigenvector of M associated to -2, then

(A) $2x_1 + x_2 = 0$

(B) $\beta - \alpha = 5$

(C) $\alpha^2 - \beta^2 = 5$

(D) $x_1 + 3x_2 = 0$

Q.36 Which of the following statements is/are CORRECT in the context of the Absolute Income Hypothesis?

(A) The marginal propensity to consume (MPC) is a constant

(B) As income increases, the average propensity to consume (APC) tends to approach the marginal propensity to consume (MPC)

(C) Average propensity to consume (APC) increases as income increases

(D) Current saving/dis-saving has no bearing on future consumption

Q.37 GDP_F = Gross Domestic Product at Factor Cost; GDP_M = Gross Domestic Product at Market Price; NNP_F = Net National Product at Factor Cost; C = Consumption; I = Investment; G = Government Expenditure; X = Export; M = Import; T = Tax; S = Saving; D = Depreciation; NIA = Net Income from Abroad

Which of the following expressions is/are CORRECT?

(A) $GDP_F = C + I + G + X - M$

(B) $GDP_M = C + I + G + X - M$

(C) $NNP_F = C + I + G + X - M - T + S - D + NIA$

(D) $NNP_F = C + I + G + X - M - T + S - D$

Q.38 Which of the following major developments have been undertaken after the initiation of structural reforms in 1991 of the Indian Economy?

(A) A general deregulation of interest rates and a greater role for market forces in the determination of both interest and exchange rates

(B) The phase out of ad hoc Treasury Bill, which puts a check on the automatic monetization of the fiscal deficit

(C) An exchange rate anchor under a Proportional Reserve System

(D) A commitment to the Fiscal Responsibility and Budget Management (FRBM) which sought to put ceiling on the overall fiscal deficit

Q.39 Which of the following functions qualify to be a cumulative density function of a random variable X ?

(A) $F(x) = \begin{cases} 1 - e^{-x}, & x \in (0, \infty) \\ 0, & \text{otherwise} \end{cases}$

(B) $F(x) = (1 + e^{-x})^{-1}, \quad x \in (-\infty, \infty)$

(C) $F(x) = \begin{cases} 1 - x^{-1}\ln(x), & x \in (e, \infty) \\ 0, & \text{otherwise} \end{cases}$

(D) $F(x) = \begin{cases} 1 - (\ln(x))^{-1}, & x \in (e, \infty) \\ 0, & \text{otherwise} \end{cases}$

Q.40 Let the joint probability density function of the random variables X and Y be

$$f(x,y) = \begin{cases} 1, & 0 < x < 1, \quad x < y < x+1 \\ 0, & \text{otherwise.} \end{cases}$$

Let the marginal density of X and Y be $f_X(x)$ and $f_Y(y)$, respectively. Which of the following is/are CORRECT?

(A) $f_X(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$ and $f_Y(y) = \begin{cases} 2-y, & 0 < y < 2 \\ 0, & \text{otherwise} \end{cases}$

(B) $f_X(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$ and $f_Y(y) = \begin{cases} y, & 0 < y < 1 \\ 2-y, & 1 \le y < 2 \\ 0, & \text{otherwise} \end{cases}$

(C) $E(X) = \frac{1}{2}, \quad Var(X) = \frac{1}{12}$

(D) $E(Y) = 1, \quad Var(Y) = \frac{1}{6}$

Section C: Q.41 – Q.50 Carry ONE mark each.

Q.41 Let $X \sim Uniform(8, 20)$ and $Z \sim Uniform(0, 6)$ be independent random variables. Let $Y = X + Z$ and $W = X - Z$. Then $Cov(Y, W)$ is _____ (*in integer*).

Q.42 Let $Y \sim Normal(3, 1)$, $W \sim Normal(1, 2)$ and $X \sim Bernoulli\ (p = 0.9)$ where $X = 1$ is success and $X = 0$ is failure. Let $S = XY + (1 - X)W$. Then $E(S) =$ _____ (*round off to 1 decimal place*).

Q.43 If X denotes the sum of the numbers appearing on a throw of two fair six-faced dice then the probability

$P(7 < X < 10) =$ _____ (*round off to 2 decimal places*).

Q.44 Using the following table,

Year	Population of the Economy	GDP of the Economy (in crore)
2010	20,000	25,000
2020	25,000	40,000

the average growth rate (compounded annually) of per capita GDP in an economy during the period 2010-2020 is _____ (*in percent, round off to 2 decimal places*).

Q.45 Consider a Keynesian Cross Model with following features,

Consumption Function: $C = C_0 + b(Y-T)$
Tax Function: $T = T_0 + tY$
Income Identity: $Y = C + I_0 + G_0$

Where, C = Consumption; Y = Real Income; T = Tax; I = Investment; G = Government Expenditure; b = Parameter; t = Tax Rate
(The subscript 0 (zero) indicates that the concerned variable is autonomous)

If $b = 0.7$ and $t = 0.2$, value of the Keynesian multiplier is _____
(*round off to 2 decimal places*).

Q.46 Let $[t]$ denote the greatest integer $\leq t$. The number of points of discontinuity of the function $f(x) = [\, x^2 - 3x + 2 \,]$ for $x \in [0, 4]$ is _____ (*in integer*).

Q.47 Let E be the area of the region bounded by the curves $y = x^2$ and $y = 8\sqrt{x}$, $x \geq 0$. Then $30E$ is equal to _____ (*round off to 1 decimal place*).

Q.48 A firm has production function $y = K^{0.5}L^{0.5}$ and faces wage rate $w = 4$ and rental rate of capital $r = 4$. The firm's marginal cost is equal to _____ (*in integer*).

Q.49 Let $\hat{y} = 5.5 + 3.2\,x$ be an estimated regression equation using a large sample. The 95% confidence interval of the coefficient of x is $[0.26,\ 6.14]$ and $R^2 = 0.26$. The standard error of the estimated coefficient is _____ (*round off to 1 decimal place*).

Q.50 Let π be the proportion of a population vaccinated against a disease. An estimate $\hat{\pi} = 0.64$ is found using a sample of 100 individuals from the population. The z test statistic for the null hypothesis $H_0 : \pi = 0.58$ is _____ (*round off to 2 decimal places*).

Section C: Q.51 – Q.60 Carry TWO marks each.

Q.51 An industry has 3 firms (1, 2 and 3) in Cournot competition. They have no fixed costs, and their constant marginal costs are respectively

$$c_1 = \frac{9}{30}, c_2 = \frac{10}{30}, \qquad c_3 = \frac{11}{30}.$$

They face an industry inverse demand function $P = 1 - Q$, where P is the market price and Q is the industry output (sum of outputs of the 3 firms). Suppose that Q^c is the industry output under Cournot-Nash equilibrium. Then $(Q^c)^{-1}$ is equal to _____ (*in integer*).

Q.52 A consumer has utility function

$$u(x_1, x_2) = \max\{0.5\,x_1,\ 0.5\,x_2\} + \min\{x_1, x_2\}.$$

She has some positive income y, and faces positive prices p_1, p_2 for goods 1 and 2 respectively. Suppose $p_2 = 1$. There exists a lowest price $\overline{p_1}$ such that if $p_1 > \overline{p_1}$ then the unique utility maximizing choice is to buy ONLY good 2. Then $\overline{p_1}$ is _____ (*in integer*).

Q.53 An economy has three firms: X, Y and Z. Every unit of output that X produces creates a benefit of INR 700 for Y and a cost of INR 300 for Z. Firm X's cost curve is

$$C(Q_X) = 2Q_X^2 + 10$$

where C represents cost and Q_X is the output. The market price for the output of X is INR 1600 per unit. The difference between the socially optimal output and private profit maximizing output of firm X (in INR) is _____ (*in integer*).

Q.54 Let $\int \sin^9 x \cos(11x)\, dx = \cos(10x)\, f(x) + c$, where c is a constant. If $f''\left(\frac{\pi}{4}\right) - k\, f'\left(\frac{\pi}{4}\right) = 0$, then k is equal to _____ (*in integer*).

Q.55 Let $M = \begin{bmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{bmatrix}$ and I_3 be the identity matrix of order 3. If the rank of the matrix $10\, I_3 - M$ is 2 then k is equal to _____ (*in integer*).

Q.56 In a two period model, a consumer is maximizing the present discounted utility

$$W_t = \ln(c_t) + \frac{1}{1+\theta}\ln(c_{t+1})$$

with respect to c_t and c_{t+1} and subject to the following budget constraint

$$c_t + \frac{c_{t+1}}{1+r} \leq y_t + \frac{y_{t+1}}{1+r}$$

where c_i and y_i are the consumption and income in period i $(i = t, t+1)$ respectively, $\theta \in [0, \infty)$ is the time discount rate and $r \in [0, \infty)$ is the rate of interest. Suppose, consumer is in the interior equilibrium and $\theta = 0.05$ and $r = 0.08$. In equilibrium, the ratio $\frac{c_{t+1}}{c_t}$ is equal to _____ (*round off to 2 decimal places*).

Q.57 The portfolio of an investment firm comprises of two risky assets, S and T, whose returns are denoted by random variables R_s and R_T respectively. The mean, the variance and the covariance of the returns are

$$E(R_s) = 0.08,\ Var(R_s) = 0.07,$$

$$E(R_T) = 0.05,\ Var(R_T) = 0.05,\ Cov(R_s, R_T) = 0.04.$$

Let w be the proportion of assets allotted to S so that the return from the portfolio is $R = wR_s + (1-w)R_T$. The value of w which minimizes $Var(R)$ is _____ (*round off to 2 decimal places*).

Q.58 A number x is randomly chosen from the set of the first 100 natural numbers. The probability that x satisfies the condition $x + \frac{300}{x} > 65$ is _____ (*round off to 2 decimal places*).

Q.59 For $k \in \mathbb{R}$, let $f(x) = x^4 + 2x^3 + kx^2 - k, \ x \in \mathbb{R}$. If $x = \frac{3}{2}$ is a point of local minima of f and m is the global minimum value of f then $f(0) - m$ is equal to _____ (*in integer*).

Q.60 If (x^*, y^*) is the optimal solution of the problem

$$\begin{aligned} &\text{maximize} && f(x,y) = 100 - e^{-x} - e^{-y} \\ &\text{subject to} && ex + y = \frac{e}{e-1}, \quad x \geq 0, \quad y \geq 0. \end{aligned}$$

Then $\sqrt{\frac{y^*}{x^*}}$ is equal to _____ (*round off to 2 decimal places*).

Section A: Q.1 – Q.10 Carry ONE mark each.

Q.1 Hollandite is an ore mineral of which one of the following elements?

(A) Fe

(B) Mn

(C) Pt

(D) Cr

Q.2 The transition from spinel to perovskite structure occurs between _____.

(A) lower mantle and outer core

(B) outer core and inner core

(C) upper mantle and lower mantle

(D) lower crust and upper mantle

Q.3 Which one of the following textures shows cuneiform-shape intergrowth between alkali feldspar and quartz?

(A) Spherulite texture

(B) Graphic texture

(C) Porphyritic texture

(D) Spinifex texture

Q.4 A pelitic rock consisting of cordierite + garnet + K-feldspar + sillimanite belongs to which one of the following metamorphic facies?

(A) Granulite

(B) Eclogite

(C) Greenschist

(D) Blueschist

Q.5 Which one of the following dams resists external forces by its own weight?

(A) Earthen dam

(B) Gravity dam

(C) Storage dam

(D) Detention dam

Q.6 Which one of the following minerals is NOT a framework silicate?

(A) Feldspar

(B) Zeolite

(C) Chlorite

(D) Quartz

Q.7 Crustal thickness is maximum at the _____.

(A) ocean-ocean convergent plate boundary

(B) ocean-continent convergent plate boundary

(C) continent-continent convergent plate boundary

(D) continent-continent divergent plate boundary

Q.8 Which one of the following causes sediment movement parallel to shoreline in the coastal area?

(A) Longshore current

(B) Rip current

(C) Backwash

(D) Edge wave

Q.9 Which one of the following dinosaur fossils is a theropod?

(A) *Kotasaurus*

(B) *Titanosaurus*

(C) *Rajasaurus*

(D) *Barapasaurus*

Q.10 Spiti Shale was deposited during the _____ time.

(A) Palaeozoic

(B) Mesozoic

(C) Cenozoic

(D) Proterozoic

Section A: Q.11 – Q.30 Carry TWO marks each.

Q.11 Which one of the following is a gently sloping (< 10°) volcanic landform resulting from eruption of basaltic lava?

(A) Shield volcano

(B) Composite volcano

(C) Lava dome

(D) Caldera

Q.12 On the magnetic polarity time scale, the present day epoch/chron is called _____.

(A) Matuyama

(B) Gilbert

(C) Gauss

(D) Bruhnes

Q.13 Which one of the following options is the CORRECT sequence of seismic waves in order of arrival time recorded on a seismogram after an earthquake?

(A) P-waves, S-waves, Rayleigh waves, Love waves

(B) P-waves, Rayleigh waves, S-waves, Love waves

(C) S-waves, P-waves, Love waves, Rayleigh waves

(D) P-waves, S-waves, Love waves, Rayleigh waves

Q.14 Match the geomorphic agents in **Column-I** with their corresponding landforms in **Column-II**.

Column-I	**Column-II**
P. Wind	1. Backswamp
Q. Groundwater	2. Yardang
R. Glacier	3. Doline
S. River	4. Drumlin

(A) P – 2, Q – 3, R – 4, S – 1

(B) P – 3, Q – 1, R – 2, S – 4

(C) P – 2, Q – 1, R – 4, S – 3

(D) P – 4, Q – 3, R – 2, S – 1

Q.15 Which one of the following processes is NOT a mechanism for bedload sediment transport in a river channel?

(A) Cavitation

(B) Sliding

(C) Rolling

(D) Saltation

Q.16 Which one of the following relationships between the topographic contour value (t) and the stratum contour value (x) of a bed must be TRUE for an outcrop of the bed to occur on the topographic surface?

(A) $t = x$

(B) $t = 2x$

(C) $t = 3x$

(D) $t = 4x$

Q.17 As per Ramsay's classification of folds, the maximum thickening of fold hinge and the maximum thinning of the fold limbs are observed in _____.

(A) Class 1A

(B) Class 1B

(C) Class 2

(D) Class 3

Q.18 The number of hinge(s) in a monocline is _____.

(A) 0

(B) 1

(C) 2

(D) 3

Q.19 Which one of the following Gondwana flora is a seed?

(A) *Dadoxylon*

(B) *Cordaicarpus*

(C) *Taeniopteris*

(D) *Palaeovittaria*

Q.20 Which one of the following gastropod genera displays sinistral coiling?

(A) *Physa*

(B) *Cypraea*

(C) *Murex*

(D) *Conus*

Q.21 Which one of the following was emplaced in the Neoproterozoic time?

(A) Singhbhum Granite

(B) Dongargarh Granite

(C) Closepet Granite

(D) Erinpura Granite

Q.22 Match the lithostratigraphic groups in **Column-I** with their corresponding formations in **Column-II**.

Column-I	**Column-II**
P. Papaghni Group	1. Kajrahat Formation
Q. Uttatur Group	2. Nagri Formation
R. Siwalik Group	3. Vempalle Formation
S. Semri Group	4. Karai Formation

(A) P – 4, Q – 3, R – 1, S – 2

(B) P – 3, Q – 4, R – 2, S – 1

(C) P – 3, Q – 1, R – 2, S – 4

(D) P – 2, Q – 4, R – 3, S – 1

Q.23 Which one of the following symmetry elements is an INCORRECT representation of rotoinversion operation?

(A) $1A_3$ + inversion centre = $\bar{3}$

(B) $1A_2 = \bar{4}$

(C) Mirror plane = $\bar{2}$

(D) $1A_3/m = \bar{6}$

Q.24 A plutonic igneous rock is composed of 50% orthopyroxene, 45% olivine and 5% clinopyroxene. What is the appropriate name of the rock according to the IUGS classification?

(A) Norite

(B) Wehrlite

(C) Troctolite

(D) Harzburgite

Q.25 Which one of the following is NOT a sediment-gravity flow?

(A) Hypopycnal flow

(B) Cohesive debris flow

(C) Turbidity flow

(D) Mud flow

Q.26 Which one among the following mineral pairs crystallise early during the cooling of a basaltic melt?

(A) Forsterite and albite

(B) Biotite and anorthite

(C) Enstatite and bytownite

(D) Forsterite and quartz

Q.27 Match the ore deposits in **Column-I** with their corresponding ores in **Column-II**.

Column-I	**Column-II**
P. Malanjkhand	1. Uranium ore
Q. Tummalapalle	2. Gold ore
R. Bhukia	3. Tin ore
S. Tosham	4. Copper ore

(A) P – 4, Q – 3, R – 2, S – 1

(B) P – 3, Q – 1, R – 4, S – 2

(C) P – 4, Q – 1, R – 2, S – 3

(D) P – 2, Q – 4, R – 1, S – 3

Q.28 Which one of the following statements is CORRECT?

(A) Banded Iron Formations are of chemogenic origin

(B) Porphyry-type deposits are formed purely by sedimentary processes

(C) Quartz-Pebble Conglomerate hosted gold deposits are formed by supergene enrichment

(D) Fullerene is formed by residual concentration process

Q.29 Which one of the following statements about the hydrological cycle is CORRECT?

(A) Groundwater represents the largest share of fresh water on Earth

(B) 'Precipitation rate greater than infiltration rate' is a necessary condition to generate surface runoff

(C) All precipitation falling on the land finally ends up as groundwater

(D) Groundwater flows in curved and concave-upward path

Q.30 Which one of the following mineral deposits is NOT related to the mining for energy production?

(A) Narwapahar

(B) Rampura-Agucha

(C) Jaduguda

(D) Turamdih

Section B: Q.31 – Q.40 Carry TWO marks each.

Q.31 At which of the following locations do lignite deposits occur in India?

(A) Raniganj

(B) Singrauli

(C) Barmer

(D) Neyveli

Q.32 Which of the following types of dunes form(s) primarily by uni-directional wind?

(A) Linear dunes

(B) Parabolic dunes

(C) Barchan dunes

(D) Star dunes

Q.33 The attitude of a fault plane was measured to be 350°, 75°E. The rake of the slickenline on the fault plane was found to be 90°. Which of the faults listed below satisfy(ies) these observations?

(A) Dip-slip fault

(B) Normal fault

(C) Reverse fault

(D) Strike-slip fault

Q.34 What type(s) of fossil remains is/are studied in ichnology?

(A) Fishes and amphibians

(B) Spores and pollens

(C) Tracks and trails

(D) Burrows and bioturbation

Q.35 Which of the following combinations of Basin and Formation is/are CORRECT?

(A) Cauvery Basin – Niniyur Formation

(B) Assam Basin – Tipam Formation

(C) Bengal Basin – Jalangi Formation

(D) Kutch Basin – Dhok Pathan Formation

Q.36 Which of the following optical properties CORRECTLY indentify(ies) the apatite {0001} section?

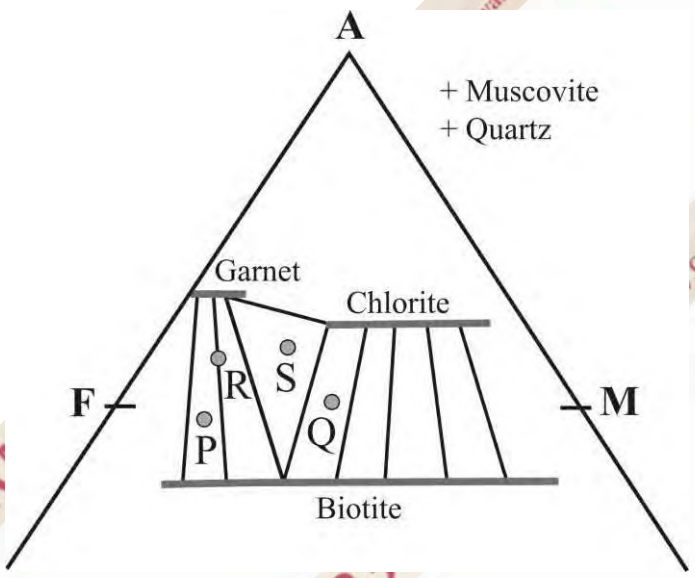
(A) Isotropic under crossed nicols

(B) Second-order interference colour

(C) Centered uniaxial interference figure

(D) High birefringence

Q.37 The AFM diagram given below shows stability of minerals in the garnet zone. If P, Q, R and S represent the compositions of different pelitic rocks, which of the following is/are characterised by the equilibrium assemblage of muscovite + garnet + biotite + quartz?



(A) P

(B) Q

(C) R

(D) S

Q.38 Which of the following sedimentary structures is/are tool marks?

(A) Bounce marks

(B) Wrinkle marks

(C) Prod marks

(D) Skip marks

Q.39 Which of the following is/are NOT copper-bearing mineral(s)?

(A) Bornite

(B) Chalcocite

(C) Braunite

(D) Chrysocolla

Q.40 Which of the following is/are used to estimate the strength of a rock mass?

(A) API gravity

(B) Resistivity

(C) Kriging

(D) RQD

Section C: Q.41 – Q.50 Carry ONE mark each.

Q.41 The amplitude recorded at a station for a magnitude 5 earthquake is x. If another earthquake recorded at the same station has an amplitude of $15x$, then the magnitude of this earthquake is _____. (*Round off to two decimal places*)

Q.42 If the intercepts of crystallographic axes are 0.5a : 1b : 0.75c on a crystallographic plane {*h k l*}, the value of '*l*' is _____. (*In integer*)

Q.43 An ocean wave with a wavelength of 200 m approaches the coast. If water depth at the observation point is 75 m, the wave velocity is _____ m/s. (*Round off to two decimal places*) (Use g = 10 m/s^2)

Q.44 A bed with an attitude 045°, 20°SE is rotated 60° clockwise (looking down) about a vertical axis. The strike value (in the azimuthal convention following right hand rule) of the rotated bed is _____ degrees. (*In integer*)

Q.45 A one-meter deep and sheet-like waterflow on a sandy beach developed antidunes. The minimum velocity of the waterflow was _____ m/s. (*Round off to two decimal places*) (Use g = 10 m/s^2)

Q.46 If the angular aperture of a 20X objective is 46°, the numerical aperture of the water immersion objective is _____. (*Round off to two decimal places*) (Use RI of water = 1.33)

Q.47 A metamorphic rock is composed of grossular garnet ($Ca_3Al_2Si_3O_{12}$), kyanite (Al_2SiO_5), anorthite ($CaAl_2Si_2O_8$) and quartz (SiO_2). If these minerals show an univariant reaction relationship, the number of components in this assemblage is _____. (*In integer*)

Q.48 If the dip separation vector on a normal fault plane has an attitude 60° → 040° and a magnitude of 6 m, the heave on the fault is _____ m. (*In integer*)

Q.49 A hillslope with an angle of 40° consists of soil having an internal friction angle of 30°. The factor of safety of the hillslope is _____. (*Round off to two decimal places*)

Q.50 The water table over an area of 1 km^2 was lowered by 4 m. If the porosity of rock is 30% and the specific retention is 10%, the change in the groundwater storage is _____ $\times 10^3$ m^3. (*In integer*)

Section C: Q.51 – Q.60 Carry TWO marks each.

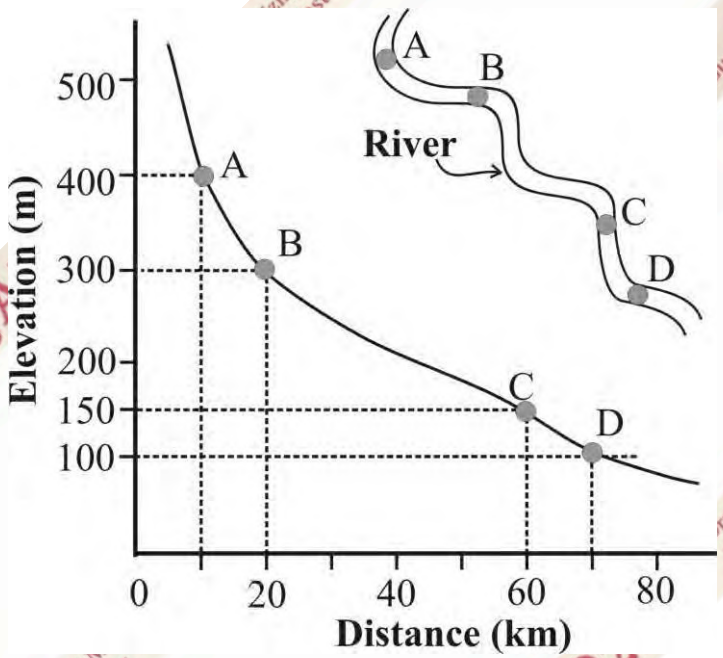
Q.51 The $\frac{^{143}Nd}{^{144}Nd}$ and $\frac{^{147}Sm}{^{144}Nd}$ ratios of a rock are 0.516 and 0.389, respectively. The rock evolved as a closed system. As per the exact parent-daughter relationship equation, the $\frac{^{143}Nd}{^{144}Nd}$ ratio of the rock 4.6×10^9 years ago was _____.

(*Round off to three decimal places*)

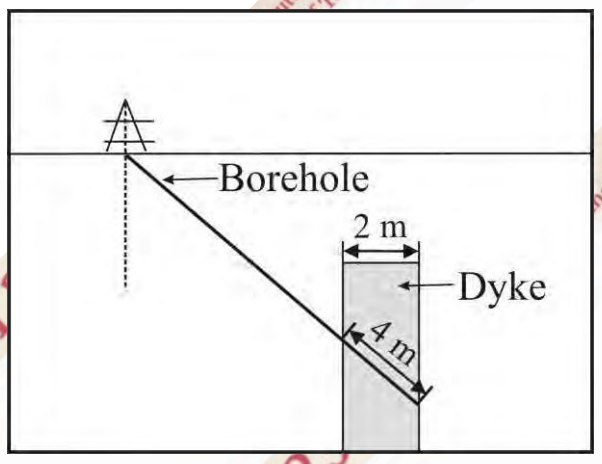
(Use decay constant for $^{147}Sm = 6.54 \times 10^{-12}$ y^{-1})

Q.52 A longitudinal profile of a river is shown in the figure below. If the average discharge of the river at reach AB is 200 m^3/s and increases to 300 m^3/s at reach CD, then the stream power from the reach AB to CD will change by a factor of _____. (*Round off to two decimal places*)

(Use g = 10 m/s^2, ρ_{water} = 1000 kg/m^3)



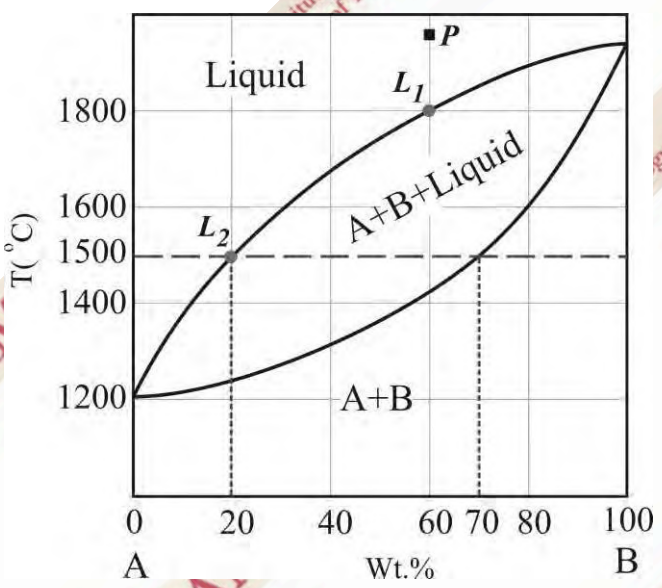
Q.53 An underground vertical dyke is intercepted by an inclined borehole as shown in the figure below. The length of the dyke core intercepted by the borehole is 4 m. If the true thickness of the dyke is 2 m, the inclination of the borehole from the vertical is _____ degrees. (*In integer*)



Q.54 A cylindrical copper ore body has a vertical thickness of 45 m and a diameter of 14 m with a density of 2.9 g/cm^3. The reserve of the copper ore body is _____ tons. (*In integer*)

Q.55 The density of a FCC unit cell is 6.5 g/cm^3. If the mass of a single atom is 60 g/mol, the diagonal length of the face {100} is _____ Å. (*Round off to two decimal places*) (Use $N_A = 6.022 \times 10^{23}$)

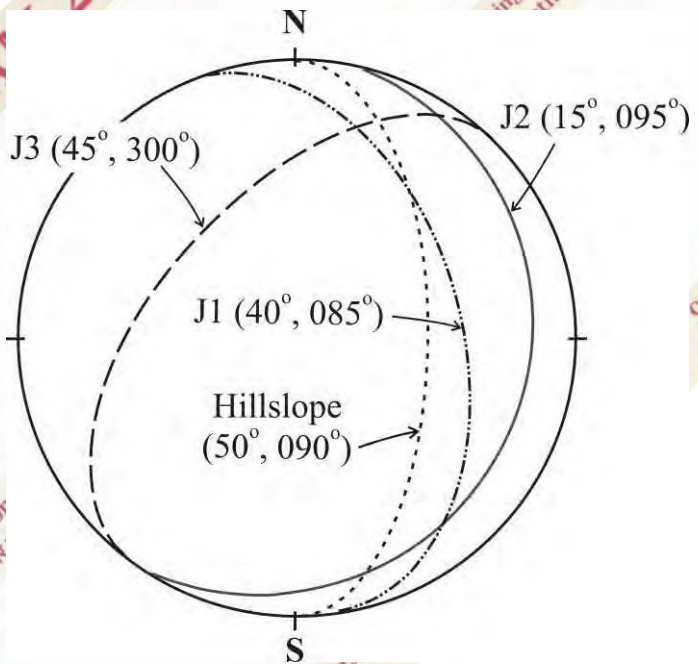
Q.56 The following figure shows an isobaric temperature-composition (T-X) phase diagram for the binary system A-B. If '***P***' is the initial composition of liquid, the amount of liquid that remains in the system when the liquid cools from 1800 °C (point L_1) to 1500 °C (point L_2) is _____ %. (*In integer*)



Q.57 A water flow transports spherical particles (diameter = 2 mm; density = 3 g/cm^3) in suspension mode. If additional particles of density 2 g/cm^3 are added into the flow, then the diameter of the particles that can be transported without a change in terminal fall velocity, using Stokes law, is _____ mm. (*Round off to two decimal places*) (Use density of water = 1 g/cm^3)

Q.58 If an iron ore body contains 50% hematite (Fe_2O_3) and 50% magnetite (Fe_3O_4), then the grade of the iron ore body is _____%. (*Round off to two decimal places*) (Use atomic weight of Fe = 55.85 amu and O = 16 amu).

Q.59 The schematic stereographic projection below shows dip amount and dip direction of three sets of joints (J1, J2 and J3) on a hillslope. If the internal friction angle of the hillslope material is 30°, the strike of the potential failure joint plane (in azimuthal convention following right hand rule) is _____ degrees. (*In integer*)



Q.60 The hydraulic conductivity of a 100 cm long cylindrical core is estimated as 1.2 cm/min when hydraulic head difference is 20 cm in an experimental setup. If the effective porosity of the core is 20%, then, assuming steady state Darcy flow, the average interstitial velocity of groundwater through the core is _____ m/day. (*Round off to two decimal places*)

Section A: Q.1 – Q.10 Carry ONE mark each.

Q.1 Let $M = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}$. If a non-zero vector $X = (x, y, z)^T \in \mathbb{R}^3$ satisfies $M^6X = X$, then a subspace of $\mathbb{R}^3$ that contains the vector X is

(A) $\{(x, y, z)^T \in \mathbb{R}^3 : x = 0,\ y + z = 0\}$

(B) $\{(x, y, z)^T \in \mathbb{R}^3 : y = 0,\ x + z = 0\}$

(C) $\{(x, y, z)^T \in \mathbb{R}^3 : z = 0,\ x + y = 0\}$

(D) $\{(x, y, z)^T \in \mathbb{R}^3 : x = 0,\ y - z = 0\}$

Q.2 Let $M = M_1M_2$, where M_1 and M_2 are two 3×3 distinct matrices. Consider the following two statements:

(I) The rows of M are linear combinations of rows of M_2.

(II) The columns of M are linear combinations of columns of M_1.

Then,

(A) only (I) is TRUE

(B) only (II) is TRUE

(C) both (I) and (II) are TRUE

(D) neither (I) nor (II) is TRUE

Q.3 Let $X \sim F_{6,2}$ and $Y \sim F_{2,6}$. If $P(X \leq 2) = \frac{216}{343}$ and $P\left(Y \leq \frac{1}{2}\right) = \alpha$, then 686α equals

(A) 246

(B) 254

(C) 260

(D) 264

Q.4 Let $Y \sim F_{4,2}$. Then, $P(Y \leq 2)$ equals

(A) 0.60

(B) 0.62

(C) 0.64

(D) 0.66

Q.5 Let $X_1, X_2, \ldots$ be a sequence of $i.i.d.$ random variables each having $U(0,1)$ distribution. Let Y be a random variable having the distribution function G. Suppose that

$$\lim_{n\to\infty} P\left(\frac{X_1 + X_2 + X_n}{4} \leq x\right) = G(x), \quad \text{for all } x \in \mathbb{R},$$

Then, $Var(Y)$ equals

(A) $\frac{1}{12}$

(B) $\frac{1}{32}$

(C) $\frac{1}{48}$

(D) $\frac{1}{64}$

Q.6 Let X_1, X_2, X_3 be a random sample from an $N(\theta, 1)$ distribution, where $\theta \in \mathbb{R}$ is an unknown parameter. Then, which one of the following conditional expectations does NOT depend on θ ?

(A) $E(X_1 + X_2 - X_3 \mid X_1 + X_2)$

(B) $E(X_1 + X_2 - X_3 \mid X_2 + X_3)$

(C) $E(X_1 + X_2 - X_3 \mid X_1 - X_3)$

(D) $E(X_1 + X_2 - X_3 \mid X_1 + X_2 + X_3)$

Q.7 For the function $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ defined by

$$f(x, y) = 2x^2 - xy - 3y^2 - 3x + 7y\,,$$

the point $(1, 1)$ is

(A) a point of local maximum

(B) a point of local minimum

(C) a saddle point

(D) NOT a critical point

Q.8 Let E_1, E_2 and E_3 be three events such that

$$P(E_1 \cap E_2) = \frac{1}{4}\ , \quad P(E_1 \cap E_3) = P(E_2 \cap E_3) = \frac{1}{5} \quad \text{and} \quad P(E_1 \cap E_2 \cap E_3) = \frac{1}{6}\ .$$

Then, among the events E_1, E_2 and E_3, the probability that at least two events occur, equals

(A) $\frac{17}{60}$

(B) $\frac{23}{60}$

(C) $\frac{19}{60}$

(D) $\frac{29}{60}$

Q.9 Let X be a continuous random variable such that $P(X \geq 0) = 1$ and $Var(X) < \infty$. Then, $E(X^2)$ is

(A) $2\int_0^\infty x^2\, P(X > x)\, dx$

(B) $\int_0^\infty x^2\, P(X > x)\, dx$

(C) $2\int_0^\infty x\, P(X > x)\, dx$

(D) $\int_0^\infty x\, P(X > x)\, dx$

Q.10 Let X be a random variable having a probability density function

$$f(x;\theta) = \begin{cases} (3-\theta)\, x^{2-\theta}, & \text{if } 0 < x < 1, \\ 0, & \text{otherwise,} \end{cases}$$

where $\theta \in \{0, 1\}$. For testing the null hypothesis $H_0 : \theta = 0$ against $H_1 : \theta = 1$, the power of the most powerful test, at the level of significance $\alpha = 0.125$, equals

(A) 0.15

(B) 0.25

(C) 0.35

(D) 0.45

Section A: Q.11 – Q.30 Carry TWO marks each.

Q.11 Let X_1 and X_2 be $i.i.d.$ random variables having the common probability density function

$$f(x) = \begin{cases} e^{-x}, & x \geq 0, \\ 0, & \text{otherwise}. \end{cases}$$

Define $X_{(1)} = \min\{X_1, X_2\}$ and $X_{(2)} = \max\{X_1, X_2\}$. Then, which one of the following statements is FALSE?

(A) $\frac{2X_{(1)}}{X_{(2)} - X_{(1)}} \sim F_{2,2}$

(B) $2\left(X_{(2)} - X_{(1)}\right) \sim \chi^2_2$

(C) $E\left(X_{(1)}\right) = \frac{1}{2}$

(D) $P\left(3X_{(1)} < X_{(2)}\right) = \frac{1}{3}$

Q.12 Let X and Y be random variables such that $X \sim N(1,2)$ and $P\left(Y = \frac{X}{2} + 1\right) = 1$. Let $\alpha = Cov(X,Y)$, $\beta = E(Y)$ and $\gamma = Var(Y)$. Then, the value of $\alpha + 2\beta + 4\gamma$ equals

(A) 5

(B) 6

(C) 7

(D) 8

Q.13 A point (a,b) is chosen at random from the rectangular region $[0,2] \times [0,4]$. Then, the probability that the area of the region

$$R = \{(x,y) \in \mathbb{R} \times \mathbb{R} : \ bx + ay \leq ab, \qquad x,y \geq 0\}$$

will be less than 2, equals

(A) $\frac{1+\ln 2}{4}$

(B) $\frac{1+\ln 2}{2}$

(C) $\frac{2+\ln 2}{4}$

(D) $\frac{1+2\ln 2}{4}$

Q.14 Let $X_1, X_2, \ldots$ be a sequence of independent random variables such that

$$P(X_i = i) = \frac{1}{4} \quad \text{and} \quad P(X_i = 2i) = \frac{3}{4}, \quad i = 1, 2, \ldots .$$

For some real constants c_1 and c_2, suppose that

$$\frac{c_1}{\sqrt{n}} \sum_{i=1}^{n} \frac{X_i}{i} + c_2\sqrt{n} \xrightarrow{d} Z \sim N(0,1), \quad \text{as } n \to \infty .$$

Then, the value of $\sqrt{3}\,(3c_1 + c_2)$ equals

(A) 2

(B) 3

(C) 4

(D) 5

Q.15 Let $X_1, X_2, \ldots$ be a sequence of $i.i.d.$ random variables such that

$$P(X_1 = 0) = P(X_1 = 1) = P(X_1 = 2) = \frac{1}{3}.$$

Let $S_n = \frac{1}{n}\sum_{i=1}^{n} X_i$ and $T_n = \frac{1}{n}\sum_{i=1}^{n} X_i^2$, $n = 1, 2, \ldots$. Suppose that

$$\alpha_1 = \lim_{n\to\infty} P\left(\left|S_n - \frac{1}{2}\right| < \frac{3}{4}\right), \qquad \alpha_2 = \lim_{n\to\infty} P\left(\left|S_n - \frac{1}{3}\right| < 1\right),$$

$$\alpha_3 = \lim_{n\to\infty} P\left(\left|T_n - \frac{1}{3}\right| < \frac{3}{2}\right) \quad \text{and} \quad \alpha_4 = \lim_{n\to\infty} P\left(\left|T_n - \frac{2}{3}\right| < \frac{1}{2}\right).$$

Then, the value of $\alpha_1 + 2\alpha_2 + 3\alpha_3 + 4\alpha_4$ equals

(A) 6

(B) 5

(C) 4

(D) 3

Q.16 For $x \in \mathbb{R}$, the curve $y = x^2$ intersects the curve $y = x\sin x + \cos x$ at exactly n points. Then, n equals

(A) 1

(B) 2

(C) 4

(D) 8

Q.17 Let (X, Y) be a random vector having the joint probability density function

$$f(x,y) = \begin{cases} \alpha\,|x|\,, & \text{if } x^2 \le y \le 2x^2, \quad -1 \le x \le 1\,, \\ 0\,, & \text{otherwise}\,, \end{cases}$$

where α is a positive constant. Then, $P(X > Y)$ equals

(A) $\frac{5}{48}$

(B) $\frac{7}{48}$

(C) $\frac{5}{24}$

(D) $\frac{7}{24}$

Q.18 Let X_1, X_2, X_3, X_4 be a random sample of size 4 from an $N(\theta, 1)$ distribution, where $\theta \in \mathbb{R}$ is an unknown parameter. Let $\bar{X} = \frac{1}{4}\sum_{i=1}^{4} X_i$, $g(\theta) = \theta^2 + 2\theta$ and $L(\theta)$ be the Cramer-Rao lower bound on variance of unbiased estimators of $g(\theta)$. Then, which one of the following statements is FALSE?

(A) $L(\theta) = (1+\theta)^2$

(B) $\bar{X} + e^{\bar{X}}$ is a sufficient statistic for θ

(C) $(1+\bar{X})^2$ is the uniformly minimum variance unbiased estimator of $g(\theta)$

(D) $Var((1+\bar{X})^2) \geq \frac{(1+\theta)^2}{2}$

Q.19 Let $X_1, X_2, \ldots, X_n$ be a random sample from a population having the probability density function

$$f(x;\mu) = \begin{cases} \frac{1}{2} e^{-\left(\frac{x-2\mu}{2}\right)}, & \text{if } x > 2\mu, \\ 0, & \text{otherwise}, \end{cases}$$

where $-\infty < \mu < \infty$. For estimating μ, consider estimators

$$T_1 = \frac{\bar{X} - 2}{2} \quad \text{and} \quad T_2 = \frac{nX_{(1)} - 2}{2n},$$

where $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ and $X_{(1)} = \min\{X_1, X_2, \ldots, X_n\}$. Then, which one of the following statements is TRUE?

(A) T_1 is consistent but T_2 is NOT consistent

(B) T_2 is consistent but T_1 is NOT consistent

(C) Both T_1 and T_2 are consistent

(D) Neither T_1 nor T_2 is consistent

Q.20 Let $X_1, X_2, \ldots, X_n$ be a random sample from a $U\left(\theta + \frac{\sigma}{\sqrt{3}}, \theta + \sqrt{3}\,\sigma\right)$ distribution, where $\theta \in \mathbb{R}$ and $\sigma > 0$ are unknown parameters. Let $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ and $S = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2}$. Let $\hat{\theta}$ and $\hat{\sigma}$ be the method of moment estimators of θ and σ, respectively. Then, which one of the following statements is FALSE?

(A) $\hat{\sigma} + \sqrt{3}\,\hat{\theta} = \sqrt{3}\,\bar{X} - 3S$

(B) $2\sqrt{3}\,\hat{\sigma} + \hat{\theta} = \bar{X} - 4\sqrt{3}\,S$

(C) $\sqrt{3}\,\hat{\sigma} + \hat{\theta} = \bar{X} + \sqrt{3}\,S$

(D) $\hat{\sigma} - \sqrt{3}\,\hat{\theta} = 9S - \sqrt{3}\,\bar{X}$

Q.21

Let (X, Y, Z) be a random vector having the joint probability density function

$$f(x,y,z) = \begin{cases} \frac{1}{2xy}, & \text{if } 0 < z < y < x < 1, \\ \frac{1}{2x^2}, & \text{if } 0 < z < x < y < 2x < 2, \\ 0, & \text{otherwise.} \end{cases}$$

Then, which one of the following statements is FALSE?

(A) $P(Z < Y < X) = \frac{1}{2}$

(B) $P(X < Y < Z) = 0$

(C) $E(\min\{X, Y\}) = \frac{1}{4}$

(D) $Var\left(Y \,\middle|\, X = \frac{1}{2}\right) = \frac{1}{12}$

Q.22 Let X be a random variable such that the moment generating function of X exists in a neighborhood of zero and

$$E(X^n) = (-1)^n \frac{2}{5} + \frac{2^{n+1}}{5} + \frac{1}{5}, \quad n = 1, 2, 3, \ldots .$$

Then, $P\left(\left|X - \frac{1}{2}\right| > 1\right)$ equals

(A) $\frac{1}{5}$

(B) $\frac{2}{5}$

(C) $\frac{3}{5}$

(D) $\frac{4}{5}$

Q.23 Let X be a random variable having a probability mass function $p(x)$ which is positive only for non-negative integers. If

$$p(x+1) = \left(\frac{\ln 3}{x+1}\right) p(x), \qquad x = 0, 1, 2, \ldots,$$

then $Var(X)$ equals

(A) $\ln 3$

(B) $\ln 6$

(C) $\ln 9$

(D) $\ln 18$

Q.24 Let $\{a_n\}_{n\geq 1}$ be a sequence such that $a_1 = 1$ and $4a_{n+1} = \sqrt{45 + 16a_n}$, $n = 1, 2, 3, \ldots$. Then, which one of the following statements is TRUE?

(A) $\{a_n\}_{n\geq 1}$ is monotonically increasing and converges to $\frac{17}{8}$

(B) $\{a_n\}_{n\geq 1}$ is monotonically increasing and converges to $\frac{9}{4}$

(C) $\{a_n\}_{n\geq 1}$ is bounded above by $\frac{17}{8}$

(D) $\sum_{n=1}^{\infty} a_n$ is convergent

Q.25 Let the series S and T be defined by

$$\sum_{n=0}^{\infty} \frac{2 \cdot 5 \cdot 8 \cdots (3n+2)}{1 \cdot 5 \cdot 9 \cdots (4n+1)} \quad \text{and} \quad \sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^{-n^2},$$

respectively. Then, which one of the following statements is TRUE?

(A) S is convergent and T is divergent

(B) S is divergent and T is convergent

(C) Both S and T are convergent

(D) Both S and T are divergent

Q.26 The volume of the region

$$R = \{(x, y, z) \in \mathbb{R} \times \mathbb{R} \times \mathbb{R} : x^2 + y^2 \leq 4, \quad 0 \leq z \leq 4 - y\}$$

is

(A) $16\pi - 16$

(B) 16π

(C) 8π

(D) $16\pi + 4$

Q.27 For real constants α and β, suppose that the system of linear equations

$$x + 2y + 3z = 6; \quad x + y + \alpha z = 3; \quad 2y + z = \beta,$$

has infinitely many solutions. Then, the value of $4\alpha + 3\beta$ equals

(A) 18

(B) 23

(C) 28

(D) 32

Q.28 Let x_1, x_2, x_3 and x_4 be observed values of a random sample from an $N(\theta, \sigma^2)$ distribution, where $\theta \in \mathbb{R}$ and $\sigma > 0$ are unknown parameters. Suppose that $\bar{x} = \frac{1}{4}\sum_{i=1}^{4} x_i = 3.6$ and $\frac{1}{3}\sum_{i=1}^{4}(x_i - \bar{x})^2 = 20.25$. For testing the null hypothesis $H_0 : \theta = 0$ against $H_1 : \theta \neq 0$, the p-value of the likelihood ratio test equals

(A) 0.712

(B) 0.208

(C) 0.104

(D) 0.052

Q.29 Let X and Y be jointly distributed random variables such that, for every fixed $\lambda > 0$, the conditional distribution of X given $Y = \lambda$ is the Poisson distribution with mean λ. If the distribution of Y is $Gamma\left(2, \frac{1}{2}\right)$, then the value of $P(X = 0) + P(X = 1)$ equals

(A) $\frac{7}{27}$

(B) $\frac{20}{27}$

(C) $\frac{8}{27}$

(D) $\frac{16}{27}$

Q.30 Among all the points on the sphere $x^2 + y^2 + z^2 = 24$, the point (α, β, γ) is closest to the point $(1, 2, -1)$. Then, the value of $\alpha + \beta + \gamma$ equals

(A) 4

(B) -4

(C) 2

(D) -2

Section B: Q.31 – Q.40 Carry TWO marks each.

Q.31 Let M be a 3×3 real matrix. If $P = M + M^T$ and $Q = M - M^T$, then which of the following statements is/are always TRUE?

(A) $\det(P^2Q^3) = 0$

(B) $\text{trace}(Q + Q^2) = 0$

(C) $X^TQ^2X = 0$, for all $X \in \mathbb{R}^3$

(D) $X^TPX = 2X^TMX$, for all $X \in \mathbb{R}^3$

Q.32 Let X_1, X_2, X_3 be $i.i.d.$ random variables, each having the $N(0,1)$ distribution. Then, which of the following statements is/are TRUE?

(A) $\dfrac{\sqrt{2}\ (X_1 - X_2)}{\sqrt{(X_1+X_2)^2 + 2X_3^2}} \sim t_1$

(B) $\dfrac{(X_1+X_2)^2}{(X_1-X_2)^2 + 2X_3^2} \sim F_{1,2}$

(C) $E\left(\dfrac{X_1}{X_2^2 + X_3^2}\right) = 0$

(D) $P(X_1 < X_2 + X_3) = \dfrac{1}{3}$

Q.33 Let $x_1, x_2, \ldots, x_{10}$ be the observed values of a random sample of size 10 from an $N(\theta, \sigma^2)$ distribution, where $\theta \in \mathbb{R}$ and $\sigma > 0$ are unknown parameters. If

$$\bar{x} = \frac{1}{10}\sum_{i=1}^{10} x_i = 0 \quad \text{and} \quad s = \sqrt{\frac{1}{9}\sum_{i=1}^{10}(x_i - \bar{x})^2} = 2,$$

then based on the values of $\bar{x}$ and s and using Student's t-distribution with 9 degrees of freedom, 90% confidence interval(s) for θ is/are

(A) $(-0.8746, \infty)$

(B) $(-0.8746, 0.8746)$

(C) $(-1.1587, 1.1587)$

(D) $(-\infty, 0.8746)$

Q.34 Let $(X_1,\ X_2)$ be a random vector having the probability mass function

$$f(x_1, x_2) = \begin{cases} \dfrac{c}{x_1!\, x_2!\, (12 - x_1 - x_2)!}, & \text{if } x_1, x_2 \in \{0, 1, \ldots, 12\},\ x_1 + x_2 \leq 12, \\ 0, & \text{otherwise}, \end{cases}$$

where c is a real constant. Then, which of the following statements is/are TRUE?

(A) $E(X_1 + X_2) = 8$

(B) $Var(X_1 + X_2) = \frac{8}{3}$

(C) $Cov(X_1, X_2) = -\frac{5}{3}$

(D) $Var(X_1 + 2X_2) = 8$

Q.35 Let P be a 3×3 matrix having the eigenvalues $1, 1$ and 2. Let $(1, -1, 2)^T$ be the only linearly independent eigenvector corresponding to the eigenvalue 1. If the adjoint of the matrix $2P$ is denoted by Q, then which of the following statements is/are TRUE?

(A) $\text{trace}(Q) = 20$

(B) $\det(Q) = 64$

(C) $(2, -2, 4)^T$ is an eigenvector of the matrix Q

(D) $Q^3 = 20Q^2 - 124Q + 256I_3$

Q.36 Let $f: \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be a function defined by

$$f(x,y) = \begin{cases} \dfrac{xy(x+y)}{x^2+y^2}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

Then, which of the following statements is/are TRUE?

(A) f is continuous on $\mathbb{R} \times \mathbb{R}$

(B) The partial derivative of f with respect to y exists at $(0,0)$, and is 0

(C) The partial derivative of f with respect to x is continuous on $\mathbb{R} \times \mathbb{R}$

(D) f is NOT differentiable at $(0,0)$

Q.37 Let X and Y be *i.i.d.* random variables each having the $N(0,1)$ distribution. Let $U = \frac{X}{Y}$ and $Z = |U|$. Then, which of the following statements is/are TRUE?

(A) U has a Cauchy distribution

(B) $E(Z^p) < \infty$, for some $p \geq 1$

(C) $E(e^{tZ})$ does not exist for all $t \in (-\infty, 0)$

(D) $Z^2 \sim F_{1,1}$

Q.38 Which of the following is/are TRUE?

(A) $\int_0^1 \int_0^1 e^{\max\{x^2, y^2\}}\, dx\, dy = e - 1$

(B) $\int_0^1 \int_0^1 e^{\min\{x^2, y^2\}}\, dx\, dy = \int_0^1 e^{t^2} dt - (e - 1)$

(C) $\int_0^1 \int_0^1 e^{\max\{x^2, y^2\}}\, dx\, dy = 2\int_0^1 \int_y^1 e^{x^2}\, dx\, dy$

(D) $\int_0^1 \int_0^1 e^{\min\{x^2, y^2\}}\, dx\, dy = 2\int_0^1 \int_1^y e^{y^2}\, dx\, dy$

Q.39 Let X be a random variable having the probability density function

$$f(x) = \begin{cases} \frac{5}{x^6}, & \text{if } x > 1, \\ 0, & \text{otherwise.} \end{cases}$$

Then, which of the following statements is/are TRUE for the distribution of X ?

(A) The coefficient of variation is $\frac{4}{\sqrt{15}}$

(B) The first quartile is $\left(\frac{3}{4}\right)^{\frac{1}{5}}$

(C) The median is $(2)^{\frac{1}{5}}$

(D) The upper bound obtained by Chebyshev's inequality for $P\left(X \geq \frac{5}{2}\right)$ is $\frac{1}{15}$

Q.40 Based on 10 data points $(x_1, y_1), (x_2, y_2), \ldots, (x_{10}, y_{10})$ on a variable (X, Y), the simple regression lines of Y on X and X on Y are obtained as $2y - x = 8$ and $y - x = -3$, respectively. Let $\bar{x} = \frac{1}{10}\sum_{i=1}^{10} x_i$ and $\bar{y} = \frac{1}{10}\sum_{i=1}^{10} y_i$. Then, which of the following statements is/are TRUE?

(A) $\sum_{i=1}^{10} x_i = 140$

(B) $\sum_{i=1}^{10} y_i = 110$

(C) $$\frac{\sum_{i=1}^{10}(x_i - \bar{x})y_i}{\sqrt{\left(\sum_{i=1}^{10}(x_i - \bar{x})^2\right)\left(\sum_{i=1}^{10}(y_i - \bar{y})^2\right)}} = -\frac{1}{\sqrt{2}}$$

(D) $$\frac{\sum_{i=1}^{10}(x_i - \bar{x})^2}{\sum_{i=1}^{10}(y_i - \bar{y})^2} = 2$$

Section C: Q.41 – Q.50 Carry ONE mark each.

Q.41 Let $f: \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = x^2 - x,\ x \in \mathbb{R}$. Let $g: \mathbb{R} \to \mathbb{R}$ be a twice differentiable function such that $g(x) = 0$ has exactly three distinct roots in the open interval $(0, 1)$. Let $h(x) = f(x)g(x)$, $x \in \mathbb{R}$, and h'' be the second order derivative of the function h. If n is the number of roots of $h''(x) = 0$ in $(0, 1)$, then the minimum possible value of n equals _____

Q.42 Let $X_1,\ X_2, \ldots$ be a sequence of $i.i.d.$ random variables, each having the probability density function

$$f(x) = \begin{cases} \dfrac{x^2 e^{-x}}{2}, & \text{if } x \geq 0\,, \\ 0\,, & \text{otherwise}\,. \end{cases}$$

For some real constants $\beta,\ \gamma$ and k, suppose that

$$\lim_{n\to\infty} P\left(\frac{1}{n}\sum_{i=1}^{n} X_i \leq x\right) = \begin{cases} 0\,, & \text{if } x < \beta, \\ kx\,, & \text{if } \beta \leq x \leq \gamma, \\ k\gamma, & \text{if } x > \gamma. \end{cases}$$

Then, the value of $2\beta + 3\gamma + 6k$ equals _____

Q.43 Let α and β be real constants such that

$$\lim_{x \to 0^+} \frac{\int_0^x \left(\frac{\alpha t^2}{1+t^4}\right) dt}{\beta x - \sin x} = 1 .$$

Then, the value of $\alpha + \beta$ equals _____

Q.44 Let $X_1, X_2, \ldots, X_{10}$ be a random sample from an $N(0, \sigma^2)$ distribution, where $\sigma > 0$ is an unknown parameter. For some real constant c, let $Y = \frac{c}{10} \sum_{i=1}^{10} |X_i|$ be an unbiased estimator of σ. Then, the value of c equals _____ *(round off to two decimal places)*

Q.45 Let X be a random variable having the probability density function

$$f(x) = \begin{cases} \frac{x}{2}, & \text{if } 0 < x < 2, \\ 0, & \text{otherwise.} \end{cases}$$

Then, $Var\left(\ln\left(\frac{2}{X}\right)\right)$ equals _____

Q.46 Let X_1, X_2, X_3 be $i.i.d.$ random variables, each having the $N(2,4)$ distribution. If

$$P(2X_1 - 3X_2 + 6X_3 > 17) = 1 - \Phi(\beta),$$

then β equals _____

Q.47 Let the probability mass function of a random variable X be given by

$$P(X = n) = \frac{k}{(n-1)n}, \quad n = 2, 3, \ldots,$$

where k is a positive constant. Then, $P(X \geq 17 \mid X \geq 5)$ equals _____

Q.48 Let

$$S_n = \sum_{k=1}^{n} \frac{1 + k\,2^k}{4^{k-1}}, \quad n = 1, 2, \ldots.$$

Then, $\lim_{n\to\infty} S_n$ equals _____ *(round off to two decimal places)*

Q.49 A box contains a certain number of balls out of which 80% are white, 15% are blue and 5% are red. All the balls of the same color are indistinguishable. Among all the white balls, α% are marked defective, among all the blue balls, 6% are marked defective and among all the red balls, 9% are marked defective. A ball is chosen at random from the box. If the conditional probability that the chosen ball is white, given that it is defective, is 0.4, then α equals _____

Q.50 Let X_1, X_2 be a random sample from a distribution having a probability density function

$$f(x;\theta) = \begin{cases} \frac{1}{\theta}\, e^{-\frac{x}{\theta}}, & \text{if } x > 0, \\ 0, & \text{otherwise,} \end{cases}$$

where $\theta \in (0, \infty)$ is an unknown parameter. For testing the null hypothesis $H_0 : \theta = 1$ against $H_1 : \theta \neq 1$, consider a test that rejects H_0 for small observed values of the statistic $W = \frac{X_1 + X_2}{2}$. If the observed values of X_1 and X_2 are 0.25 and 0.75, respectively, then the p-value equals _____ *(round off to two decimal places)*

Section C: Q.51 – Q.60 Carry TWO marks each.

Q.51 Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by

$$f(x) = x^2 \sin(x-1) + x\, e^{(x-1)}, \; x \in \mathbb{R}.$$

Then,

$$\lim_{n\to\infty} n\left(f\left(1+\frac{1}{n}\right) + f\left(1+\frac{2}{n}\right) + \cdots + f\left(1+\frac{10}{n}\right) - 10\right)$$

equals _____

Q.52 Let the random vector (X_1, X_2) follow the bivariate normal distribution with $E(X_1) = E(X_2) = 1$, $Var(X_2) = 4\,Var(X_1) = 4$ and $Cov(X_1, X_2) = 1$.

Then, $Var\left(X_1 + X_2 \,\middle|\, X_1 = \frac{1}{2}\right)$ equals _____

Q.53 If, for some $\alpha \in (0, \infty)$,

$$\int_0^\infty 2^{-x^2}\, dx = \alpha\sqrt{\pi},$$

then α equals _____ *(round off to two decimal places)*

Q.54 Let $x_1 = 2.1,\ x_2 = 4.2, x_3 = 5.8$ and $x_4 = 3.9$ be the observed values of a random sample X_1, X_2, X_3 and X_4 from a population having a probability density function

$$f(x;\theta) = \begin{cases} \frac{x}{\theta^2}\, e^{-\frac{x}{\theta}}, & \text{if } x > 0, \\ 0, & \text{otherwise}, \end{cases}$$

where $\theta \in (0, \infty)$ is an unknown parameter. Then, the maximum likelihood estimate of $Var(X_1)$ equals _____

Q.55 Let $x_1 = 2, x_2 = 5$ and $x_3 = 4$ be the observed values of a random sample from a population having a probability mass function

$$f(x;\theta) = \begin{cases} \theta(1-\theta)^x, & \text{if } x = 0, 1, 2, \ldots, \\ 0, & \text{otherwise}, \end{cases}$$

where $\theta \in (0, 1)$ is an unknown parameter. If $\hat{\tau}$ is the uniformly minimum variance unbiased estimate of θ^2, then $156\,\hat{\tau}$ equals _____

Q.56 Let $X_1, X_2, \ldots, X_5$ be $i.i.d.$ random variables, each having the $Bin\left(1, \frac{1}{2}\right)$ distribution. Let $K = X_1 + X_2 + \cdots + X_5$ and

$$U = \begin{cases} 0\,, & \text{if } K = 0\,, \\ X_1 + X_2 + \cdots + X_K\,, & \text{if } K = 1, 2, \ldots, 5\,. \end{cases}$$

Then, $E(U)$ equals _____

Q.57 Let $X_1 \sim Gamma(1,4)$, $X_2 \sim Gamma(2,2)$ and $X_3 \sim Gamma(3,4)$ be three independent random variables. If $Y = X_1 + 2X_2 + X_3$, then $E\left(\left(\frac{Y}{4}\right)^4\right)$ equals _____

Q.58 Let X_1, X_2 be a random sample from a $U(0,\theta)$ distribution, where $\theta > 0$ is an unknown parameter. For testing the null hypothesis $H_0 : \theta \in (0,1] \cup [2,\infty)$ against $H_1 : \theta \in (1,2)$, consider the critical region

$$R = \left\{(x_1, x_2) \in \mathbb{R} \times \mathbb{R} \,:\, \frac{5}{4} < \max\{x_1, x_2\} < \frac{7}{4}\right\}.$$

Then, the size of the critical region equals _____

Q.59 Let $X_1, X_2, \ldots, X_5$ be a random sample from a $Bin(1, \theta)$ distribution, where $\theta \in (0, 1)$ is an unknown parameter. For testing the null hypothesis $H_0 : \theta \leq 0.5$ against $H_1 : \theta > 0.5$, consider the two tests T_1 and T_2 defined as:

T_1: Reject H_0 if, and only if, $\sum_{i=1}^{5} X_i = 5$.

T_2: Reject H_0 if, and only if, $\sum_{i=1}^{5} X_i \geq 3$.

Let β_i be the probability of making Type-II error, at $\theta = \frac{2}{3}$, when the test T_i, $i = 1, 2$, is used. Then, the value of $\beta_1 + \beta_2$ equals _____ *(round off to two decimal places)*

Q.60 Let $X_1 \sim N(2, 1)$, $X_2 \sim N(-1, 4)$ and $X_3 \sim N(0, 1)$ be mutually independent random variables. Then, the probability that exactly two of these three random variables are less than 1, equals _____ *(round off to two decimal places)*

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q. 10 carry one mark each.

Q. 1 Let G be a finite group. Then G is necessarily a cyclic group if the order of G is

(A) 4

(B) 7

(C) 6

(D) 10

Q. 2 Let $\mathbf{v}_1, \ldots, \mathbf{v}_9$ be the column vectors of a non-zero 9×9 real matrix A. Let $a_1, \ldots, a_9 \in \mathbb{R}$, not all zero, be such that $\sum_{i=1}^{9} a_i \mathbf{v}_i = \mathbf{0}$. Then the system $A\mathbf{x} = \sum_{i=1}^{9} \mathbf{v}_i$ has

(A) no solution

(B) a unique solution

(C) more than one but only finitely many solutions

(D) infinitely many solutions

Q. 3 Which of the following is a subspace of the real vector space $\mathbb{R}^3$?

(A) $\{(x, y, z) \in \mathbb{R}^3 : (y + z)^2 + (2x - 3y)^2 = 0\}$

(B) $\{(x, y, z) \in \mathbb{R}^3 : y \in \mathbb{Q}\}$

(C) $\{(x, y, z) \in \mathbb{R}^3 : yz = 0\}$

(D) $\{(x, y, z) \in \mathbb{R}^3 : x + 2y - 3z + 1 = 0\}$

Q. 4 Consider the initial value problem

$$\frac{dy}{dx} + \alpha y = 0,$$
$$y(0) = 1,$$

where $\alpha \in \mathbb{R}$. Then

(A) there is an α such that $y(1) = 0$

(B) there is a unique α such that $\lim_{x \to \infty} y(x) = 0$

(C) there is NO α such that $y(2) = 1$

(D) there is a unique α such that $y(1) = 2$

Q. 5 Let $p(x) = x^{57} + 3x^{10} - 21x^3 + x^2 + 21$ and

$$q(x) = p(x) + \sum_{j=1}^{57} p^{(j)}(x) \quad \text{for all } x \in \mathbb{R},$$

where $p^{(j)}(x)$ denotes the j^{th} derivative of $p(x)$. Then the function q admits

(A) NEITHER a global maximum NOR a global minimum on $\mathbb{R}$

(B) a global maximum but NOT a global minimum on $\mathbb{R}$

(C) a global minimum but NOT a global maximum on $\mathbb{R}$

(D) a global minimum and a global maximum on $\mathbb{R}$

Q. 6 The limit

$$\lim_{a \to 0} \left(\frac{\int_0^a \sin(x^2)\, dx}{\int_0^a (\ln(x+1))^2\, dx} \right)$$

is

(A) 0

(B) 1

(C) $\frac{\pi}{e}$

(D) non-existent

Q. 7 The value of

$$\int_0^1 \int_0^{1-x} \cos(x^3 + y^2)\, dy\, dx - \int_0^1 \int_0^{1-y} \cos(x^3 + y^2)\, dx\, dy$$

is

(A) 0

(B) $\dfrac{\cos(1)}{2}$

(C) $\dfrac{\sin(1)}{2}$

(D) $\cos\left(\dfrac{1}{2}\right) - \sin\left(\dfrac{1}{2}\right)$

Q. 8 Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $f(x, y) = (e^x \cos(y), e^x \sin(y))$. Then the number of points in $\mathbb{R}^2$ that do NOT lie in the range of f is

(A) 0

(B) 1

(C) 2

(D) infinite

Q. 9 Let $a_n = \left(1 + \frac{1}{n}\right)^n$ and $b_n = n \cos\left(\frac{n!\pi}{2^{10}}\right)$ for $n \in \mathbb{N}$. Then

(A) (a_n) is convergent and (b_n) is bounded

(B) (a_n) is NOT convergent and (b_n) is bounded

(C) (a_n) is convergent and (b_n) is unbounded

(D) (a_n) is NOT convergent and (b_n) is unbounded

Q. 10 Let (a_n) be a sequence of real numbers defined by

$$a_n = \begin{cases} 1 & \text{if } n \text{ is prime} \\ -1 & \text{if } n \text{ is not prime.} \end{cases}$$

Let $b_n = \frac{a_n}{n}$ for $n \in \mathbb{N}$. Then

(A) both (a_n) and (b_n) are convergent

(B) (a_n) is convergent but (b_n) is NOT convergent

(C) (a_n) is NOT convergent but (b_n) is convergent

(D) both (a_n) and (b_n) are NOT convergent

Q. 11 Let $a_n = \sin\left(\frac{1}{n^3}\right)$ and $b_n = \sin\left(\frac{1}{n}\right)$ for $n \in \mathbb{N}$. Then

(A) both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are convergent

(B) $\sum_{n=1}^{\infty} a_n$ is convergent but $\sum_{n=1}^{\infty} b_n$ is NOT convergent

(C) $\sum_{n=1}^{\infty} a_n$ is NOT convergent but $\sum_{n=1}^{\infty} b_n$ is convergent

(D) both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are NOT convergent

Q. 12 Consider the following statements:

I. There exists a linear transformation from $\mathbb{R}^3$ to itself such that its range space and null space are the same.

II. There exists a linear transformation from $\mathbb{R}^2$ to itself such that its range space and null space are the same.

Then

(A) both I and II are TRUE

(B) I is TRUE but II is FALSE

(C) II is TRUE but I is FALSE

(D) both I and II are FALSE

Q. 13 Let

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ -2 & 2 & 2 \end{pmatrix}$$

and $B = A^5 + A^4 + I_3$. Which of the following is NOT an eigenvalue of B?

(A) 1

(B) 2

(C) 49

(D) 3

Q. 14 The system of linear equations in x_1, x_2, x_3

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 2 & 3 & \alpha \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ \beta \end{pmatrix}$$

where $\alpha, \beta \in \mathbb{R}$, has

(A) at least one solution for any α and β

(B) a unique solution for any β when $\alpha \neq 1$

(C) NO solution for any α when $\beta \neq 5$

(D) infinitely many solutions for any α when $\beta = 5$

Q. 15 Let S and T be non-empty subsets of $\mathbb{R}^2$, and W be a non-zero proper subspace of $\mathbb{R}^2$. Consider the following statements:

I. If $\text{span}(S) = \mathbb{R}^2$, then $\text{span}(S \cap W) = W$.

II. $\text{span}(S \cup T) = \text{span}(S) \cup \text{span}(T)$.

Then

(A) both I and II are TRUE

(B) I is TRUE but II is FALSE

(C) II is TRUE but I is FALSE

(D) both I and II are FALSE

Q. 16 Let $f(x, y) = e^{x^2+y^2}$ for $(x, y) \in \mathbb{R}^2$, and a_n be the determinant of the matrix

$$\begin{pmatrix} \dfrac{\partial^2 f}{\partial x^2} & \dfrac{\partial^2 f}{\partial x \partial y} \\ \dfrac{\partial^2 f}{\partial y \partial x} & \dfrac{\partial^2 f}{\partial y^2} \end{pmatrix}$$

evaluated at the point $(\cos(n), \sin(n))$. Then the limit $\lim_{n\to\infty} a_n$ is

(A) non-existent

(B) 0

(C) $6e^2$

(D) $12e^2$

Q. 17 Let $f(x,y) = \ln(1 + x^2 + y^2)$ for $(x,y) \in \mathbb{R}^2$. Define

$$P = \left.\frac{\partial^2 f}{\partial x^2}\right|_{(0,0)} \qquad Q = \left.\frac{\partial^2 f}{\partial x \partial y}\right|_{(0,0)}$$

$$R = \left.\frac{\partial^2 f}{\partial y \partial x}\right|_{(0,0)} \qquad S = \left.\frac{\partial^2 f}{\partial y^2}\right|_{(0,0)}$$

Then

(A) $PS - QR > 0$ and $P < 0$

(B) $PS - QR > 0$ and $P > 0$

(C) $PS - QR < 0$ and $P > 0$

(D) $PS - QR < 0$ and $P < 0$

Q. 18 The area of the curved surface

$$S = \left\{(x,y,z) \in \mathbb{R}^3 : z^2 = (x-1)^2 + (y-2)^2\right\}$$

lying between the planes $z = 2$ and $z = 3$ is

(A) $4\pi\sqrt{2}$

(B) $5\pi\sqrt{2}$

(C) 9π

(D) $9\pi\sqrt{2}$

Q. 19 Let $a_n = \dfrac{1 + 2^{-2} + \cdots + n^{-2}}{n}$ for $n \in \mathbb{N}$. Then

(A) both the sequence (a_n) and the series $\sum_{n=1}^{\infty} a_n$ are convergent

(B) the sequence (a_n) is convergent but the series $\sum_{n=1}^{\infty} a_n$ is NOT convergent

(C) both the sequence (a_n) and the series $\sum_{n=1}^{\infty} a_n$ are NOT convergent

(D) the sequence (a_n) is NOT convergent but the series $\sum_{n=1}^{\infty} a_n$ is convergent

Q. 20 Let (a_n) be a sequence of real numbers such that the series $\sum_{n=0}^{\infty} a_n(x-2)^n$ converges at $x = -5$. Then this series also converges at

(A) $x = 9$

(B) $x = 12$

(C) $x = 5$

(D) $x = -6$

Q. 21 Let (a_n) and (b_n) be sequences of real numbers such that

$$|a_n - a_{n+1}| = \frac{1}{2^n} \quad \text{and} \quad |b_n - b_{n+1}| = \frac{1}{\sqrt{n}} \quad \text{for } n \in \mathbb{N}.$$

Then

(A) both (a_n) and (b_n) are Cauchy sequences

(B) (a_n) is a Cauchy sequence but (b_n) need NOT be a Cauchy sequence

(C) (a_n) need NOT be a Cauchy sequence but (b_n) is a Cauchy sequence

(D) both (a_n) and (b_n) need NOT be Cauchy sequences

Q. 22 Consider the family of curves $x^2 + y^2 = 2x + 4y + k$ with a real parameter $k > -5$. Then the orthogonal trajectory to this family of curves passing through $(2, 3)$ also passes through

(A) $(3, 4)$

(B) $(-1, 1)$

(C) $(1, 0)$

(D) $(3, 5)$

Q. 23 Consider the following statements:

I. Every infinite group has infinitely many subgroups.

II. There are only finitely many non-isomorphic groups of a given finite order.

Then

(A) both I and II are TRUE

(B) I is TRUE but II is FALSE

(C) I is FALSE but II is TRUE

(D) both I and II are FALSE

Q. 24 Suppose $f : (-1, 1) \to \mathbb{R}$ is an infinitely differentiable function such that the series $\sum_{j=0}^{\infty} a_j \frac{x^j}{j!}$ converges to $f(x)$ for each $x \in (-1, 1)$, where,

$$a_j = \int_0^{\pi/2} \theta^j \cos^j(\tan\theta) d\theta + \int_{\pi/2}^{\pi} (\theta - \pi)^j \cos^j(\tan\theta) d\theta$$

for $j \geq 0$. Then

(A) $f(x) = 0$ for all $x \in (-1, 1)$

(B) f is a non-constant even function on $(-1, 1)$

(C) f is a non-constant odd function on $(-1, 1)$

(D) f is NEITHER an odd function NOR an even function on $(-1, 1)$

Q. 25 Let $f(x) = \cos(x)$ and $g(x) = 1 - \frac{x^2}{2}$ for $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then

(A) $f(x) \geq g(x)$ for all $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(B) $f(x) \leq g(x)$ for all $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(C) $f(x) - g(x)$ changes sign exactly once on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(D) $f(x) - g(x)$ changes sign more than once on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Q. 26 Let

$$f(x, y) = \iint_{(u-x)^2+(v-y)^2 \leq 1} e^{-\sqrt{(u-x)^2+(v-y)^2}} \, du \, dv.$$

Then $\lim_{n\to\infty} f(n, n^2)$ is

(A) non-existent

(B) 0

(C) $\pi(1 - e^{-1})$

(D) $2\pi(1 - 2e^{-1})$

Q. 27 How many group homomorphisms are there from $\mathbb{Z}_2$ to S_5?

(A) 40

(B) 41

(C) 26

(D) 25

Q. 28 Let $y : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function such that y'' is continuous on $[0, 1]$ and $y(0) = y(1) = 0$. Suppose $y''(x) + x^2 < 0$ for all $x \in [0, 1]$. Then

(A) $y(x) > 0$ for all $x \in (0, 1)$

(B) $y(x) < 0$ for all $x \in (0, 1)$

(C) $y(x) = 0$ has exactly one solution in $(0, 1)$

(D) $y(x) = 0$ has more than one solution in $(0, 1)$

Q. 29 From the additive group $\mathbb{Q}$ to which one of the following groups does there exist a non-trivial group homomorphism?

(A) $\mathbb{R}^\times$, the multiplicative group of non-zero real numbers

(B) $\mathbb{Z}$, the additive group of integers

(C) $\mathbb{Z}_2$, the additive group of integers modulo 2

(D) $\mathbb{Q}^\times$, the multiplicative group of non-zero rational numbers

Q. 30 Let $f : \mathbb{R} \to \mathbb{R}$ be an infinitely differentiable function such that f'' has exactly two distinct zeroes. Then

(A) f' has at most 3 distinct zeroes

(B) f' has at least 1 zero

(C) f has at most 3 distinct zeroes

(D) f has at least 2 distinct zeroes

SECTION – B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q. 31 For each $t \in (0,1)$, the surface P_t in $\mathbb{R}^3$ is defined by

$$P_t = \{(x,y,z) : (x^2+y^2)z = 1,\ t^2 \le x^2+y^2 \le 1\}.$$

Let $a_t \in \mathbb{R}$ be the surface area of P_t. Then

(A) $a_t = \displaystyle\iint_{t^2 \le x^2+y^2 \le 1} \sqrt{1 + \frac{4x^2}{(x^2+y^2)^4} + \frac{4y^2}{(x^2+y^2)^4}}\, dx\, dy$

(B) $a_t = \displaystyle\iint_{t^2 \le x^2+y^2 \le 1} \sqrt{1 + \frac{4x^2}{(x^2+y^2)^2} + \frac{4y^2}{(x^2+y^2)^2}}\, dx\, dy$

(C) the limit $\lim\limits_{t \to 0^+} a_t$ does NOT exist

(D) the limit $\lim\limits_{t \to 0^+} a_t$ exists

Q. 32 Let $A \subseteq \mathbb{Z}$ with $0 \in A$. For $r, s \in \mathbb{Z}$, define

$$rA = \{ra : a \in A\}, \qquad rA + sA = \{ra + sb :\ a, b \in A\}.$$

Which of the following conditions imply that A is a subgroup of the additive group $\mathbb{Z}$?

(A) $-2A \subseteq A,\ \ A + A = A$

(B) $A = -A,\ \ A + 2A = A$

(C) $A = -A,\ \ A + A = A$

(D) $2A \subseteq A,\ \ A + A = A$

Q. 33 Let $y : (\sqrt{2/3}, \infty) \to \mathbb{R}$ be the solution of

$$(2x - y)y' + (2y - x) = 0,$$
$$y(1) = 3.$$

Then

(A) $y(3) = 1$

(B) $y(2) = 4 + \sqrt{10}$

(C) y' is bounded on $(\sqrt{2/3}, 1)$

(D) y' is bounded on $(1, \infty)$

Q. 34 Let $f : (-1, 1) \to \mathbb{R}$ be a differentiable function satisfying $f(0) = 0$. Suppose there exists an $M > 0$ such that $|f'(x)| \leq M|x|$ for all $x \in (-1, 1)$. Then

(A) f' is continuous at $x = 0$

(B) f' is differentiable at $x = 0$

(C) ff' is differentiable at $x = 0$

(D) $(f')^2$ is differentiable at $x = 0$

Q. 35 Which of the following functions is/are Riemann integrable on $[0, 1]$?

(A) $f(x) = \int_0^x \left| \frac{1}{2} - t \right| dt$

(B) $f(x) = \begin{cases} x \sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

(C) $f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \cap [0, 1] \\ -1 & \text{otherwise} \end{cases}$

(D) $f(x) = \begin{cases} x & \text{if } x \in [0, 1) \\ 0 & \text{if } x = 1 \end{cases}$

Q. 36 A subset $S \subseteq \mathbb{R}^2$ is said to be *bounded* if there is an $M > 0$ such that $|x| \leq M$ and $|y| \leq M$ for all $(x, y) \in S$. Which of the following subsets of $\mathbb{R}^2$ is/are bounded?

(A) $\{(x, y) \in \mathbb{R}^2 : e^{x^2} + y^2 \leq 4\}$

(B) $\{(x, y) \in \mathbb{R}^2 : x^4 + y^2 \leq 4\}$

(C) $\{(x, y) \in \mathbb{R}^2 : |x| + |y| \leq 4\}$

(D) $\{(x, y) \in \mathbb{R}^2 : e^{x^3} + y^2 \leq 4\}$

Q. 37 Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined as follows:

$$f(x,y) = \begin{cases} \dfrac{x^4y^3}{x^6+y^6} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

Then

(A) $\lim\limits_{t\to 0} \dfrac{f(t,t)-f(0,0)}{t}$ exists and equals $\dfrac{1}{2}$

(B) $\left.\dfrac{\partial f}{\partial x}\right|_{(0,0)}$ exists and equals 0

(C) $\left.\dfrac{\partial f}{\partial y}\right|_{(0,0)}$ exists and equals 0

(D) $\lim\limits_{t\to 0} \dfrac{f(t,2t)-f(0,0)}{t}$ exists and equals $\dfrac{1}{3}$

Q. 38 Which of the following is/are true?

(A) Every linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ maps lines onto points or lines

(B) Every surjective linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ maps lines onto lines

(C) Every bijective linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ maps pairs of parallel lines to pairs of parallel lines

(D) Every bijective linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ maps pairs of perpendicular lines to pairs of perpendicular lines

Q. 39 Which of the following is/are linear transformations?

(A) $T : \mathbb{R} \to \mathbb{R}$ given by $T(x) = \sin(x)$

(B) $T : M_2(\mathbb{R}) \to \mathbb{R}$ given by $T(A) = \text{trace}(A)$

(C) $T : \mathbb{R}^2 \to \mathbb{R}$ given by $T(x, y) = x + y + 1$

(D) $T : P_2(\mathbb{R}) \to \mathbb{R}$ given by $T(p(x)) = p(1)$

Q. 40 Let R_1 and R_2 be the radii of convergence of the power series $\sum_{n=1}^{\infty}(-1)^n x^{n-1}$ and $\sum_{n=1}^{\infty}(-1)^n \frac{x^{n+1}}{n(n+1)}$, respectively. Then

(A) $R_1 = R_2$

(B) $R_2 > 1$

(C) $\sum_{n=1}^{\infty}(-1)^n x^{n-1}$ converges for all $x \in [-1, 1]$

(D) $\sum_{n=1}^{\infty}(-1)^n \frac{x^{n+1}}{n(n+1)}$ converges for all $x \in [-1, 1]$

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q. 41 Let $f : \mathbb{R}^2 \to \mathbb{R}$ be the function defined as follows:

$$f(x,y) = \begin{cases} (x^2-1)^2 \cos^2\left(\dfrac{y^2}{(x^2-1)^2}\right) & \text{if } x \neq \pm 1 \\ 0 & \text{if } x = \pm 1. \end{cases}$$

The number of points of discontinuity of $f(x,y)$ is equal to _____.

Q. 42 Let $T : P_2(\mathbb{R}) \to P_4(\mathbb{R})$ be the linear transformation given by $T(p(x)) = p(x^2)$. Then the rank of T is equal to _____.

Q. 43 If y is the solution of

$$y'' - 2y' + y = e^x,$$
$$y(0) = 0, \quad y'(0) = -1/2,$$

then $y(1)$ is equal to _____. (rounded off to two decimal places)

Q. 44 The value of

$$\lim_{n\to\infty} \left(n \int_0^1 \frac{x^n}{x+1}\, dx \right)$$

is equal to _____. (rounded off to two decimal places)

Q. 45 For $\sigma \in S_8$, let $o(\sigma)$ denote the order of σ. Then $\max\{o(\sigma) : \sigma \in S_8\}$ is equal to _____.

Q. 46 For $g \in \mathbb{Z}$, let $\bar{g} \in \mathbb{Z}_8$ denote the residue class of g modulo 8. Consider the group $\mathbb{Z}_8^\times = \{\bar{x} \in \mathbb{Z}_8 : 1 \leq x \leq 7, \gcd(x,8) = 1\}$ with respect to multiplication modulo 8. The number of group isomorphisms from $\mathbb{Z}_8^\times$ onto itself is equal to _____.

Q. 47 Let $f(x) = \sqrt[3]{x}$ for $x \in (0, \infty)$, and $\theta(h)$ be a function such that

$$f(3+h) - f(3) = hf'(3+\theta(h)h)$$

for all $h \in (-1, 1)$. Then $\lim_{h \to 0} \theta(h)$ is equal to _____.
(rounded off to two decimal places)

Q. 48 Let V be the volume of the region $S \subseteq \mathbb{R}^3$ defined by

$$S = \{(x, y, z) \in \mathbb{R}^3 : \ xy \leq z \leq 4, \ 0 \leq x^2 + y^2 \leq 1\}.$$

Then $\dfrac{V}{\pi}$ is equal to _____. (rounded off to two decimal places)

Q. 49 The sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{2n+1}{(n^2+1)(n^2+2n+2)}$ is equal to _____.
(rounded off to two decimal places)

Q. 50 The value of $\lim_{n \to \infty} \left(1 + \frac{1}{2^n} + \frac{1}{3^n} + \cdots + \frac{1}{(2023)^n}\right)^{\frac{1}{n}}$ is equal to _____.
(rounded off to two decimal places)

Q. 51 – Q. 60 carry two marks each.

Q. 51 Let $f : \mathbb{R}^3 \to \mathbb{R}$ be defined as $f(x, y, z) = x^3 + y^3 + z^3$, and let $L : \mathbb{R}^3 \to \mathbb{R}$ be the linear map satisfying

$$\lim_{(x,y,z)\to(0,0,0)} \frac{f(1+x, 1+y, 1+z) - f(1,1,1) - L(x,y,z)}{\sqrt{x^2+y^2+z^2}} = 0.$$

Then $L(1, 2, 4)$ is equal to _____. (rounded off to two decimal places)

Q. 52 The global minimum value of

$$f(x) = |x-1| + |x-2|^2$$

on $\mathbb{R}$ is equal to _____. (rounded off to two decimal places)

Q. 53 Let $y : (1, \infty) \to \mathbb{R}$ be the solution of the differential equation

$$y'' - \frac{2y}{(1-x)^2} = 0$$

satisfying $y(2) = 1$ and $\lim_{x\to\infty} y(x) = 0$. Then $y(3)$ is equal to _____.
(rounded off to two decimal places)

Q. 54 The number of permutations in S_4 that have exactly two cycles in their cycle decompositions is equal to _____.

Q. 55 Let S be the triangular region whose vertices are $(0, 0)$, $\left(0, \frac{\pi}{2}\right)$, and $\left(\frac{\pi}{2}, 0\right)$. The value of $\iint_S \sin(x)\cos(y)\, dx\, dy$ is equal to _____.
(rounded off to two decimal places)

Q. 56 Let

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 3 \\ 1 & 1 & 4 & 4 & 4 \end{pmatrix}$$

and B be a 5×5 real matrix such that AB is the zero matrix. Then the maximum possible rank of B is equal to _____.

Q. 57 Let W be the subspace of $M_3(\mathbb{R})$ consisting of all matrices with the property that the sum of the entries in each row is zero and the sum of the entries in each column is zero. Then the dimension of W is equal to _____.

Q. 58 The maximum number of linearly independent eigenvectors of the matrix

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

is equal to _____.

Q. 59 Let S be the set of all real numbers α such that the solution y of the initial value problem

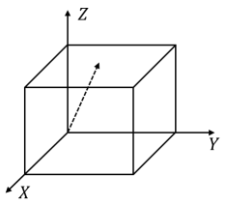
$$\frac{dy}{dx} = y(2 - y),$$
$$y(0) = \alpha,$$

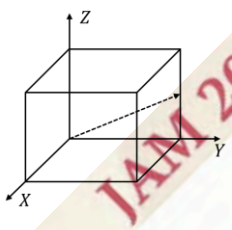
exists on $[0, \infty)$. Then the minimum of the set S is equal to _____.
(rounded off to two decimal places)

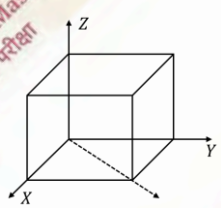
Q. 60 Let $f : \mathbb{R} \to \mathbb{R}$ be a bijective function such that for all $x \in \mathbb{R}$, $f(x) = \sum_{n=1}^{\infty} a_n x^n$ and $f^{-1}(x) = \sum_{n=1}^{\infty} b_n x^n$, where f^{-1} is the inverse function of f. If $a_1 = 2$ and $a_2 = 4$, then b_1 is equal to _____.

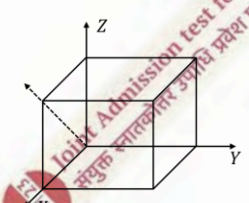
Section A: Q.1 – Q.10 Carry ONE mark each.

Q.1 For a cubic unit cell, the dashed arrow in which of the following figures represents the direction [220]?

(A) 

(B) 

(C) 

(D) 

Q.2 Which of the following fields has non-zero curl?

(A) $x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

(B) $(y+z)\hat{\imath} + (x+z)\hat{\jmath} + (x+y)\hat{k}$

(C) $y^2\hat{\imath} + (2xy + z^2)\hat{\jmath} + 2yz\hat{k}$

(D) $xy\hat{\imath} + 2yz\hat{\jmath} + 3xz\hat{k}$

Q.3 Which of the following statements about the viscosity of a dilute ideal gas is correct?

(A) It is independent of pressure at fixed temperature

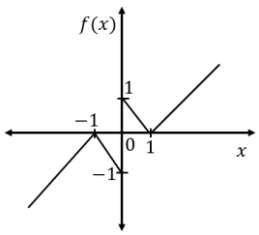
(B) It increases with increasing pressure at fixed temperature

(C) It is independent of temperature

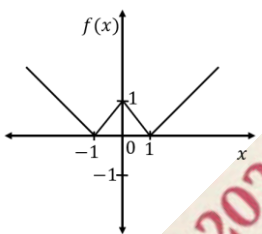
(D) It decreases with increasing temperature

Q.4 The plot of the function $f(x) = ||x| - 1|$ is

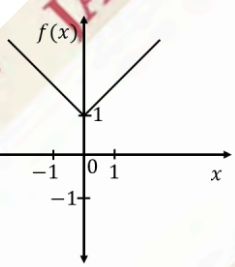
(A)



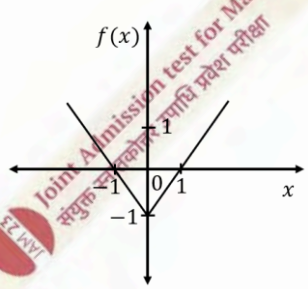
(B)



(C)



(D)



Q.5 A system has N spins, where each spin is capable of existing in 4 possible states. The difference in entropy of disordered states (where all possible spin configurations are equally probable) and ordered states is

(A) $2(N-1)k_{\mathrm{B}}\ln 2$

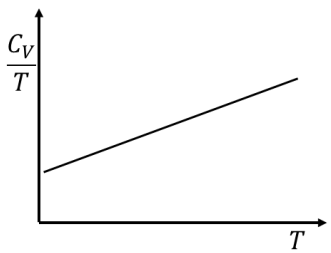
(B) $(N-1)k_{\mathrm{B}}\ln 2$

(C) $4k_{\mathrm{B}}\ln N$

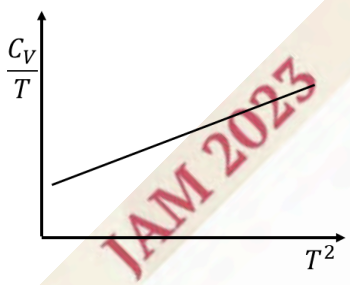
(D) $Nk_{\mathrm{B}}\ln 2$

Q.6 Temperature (T) dependence of the total specific heat (C_v) for a two dimensional metallic solid at low temperatures is

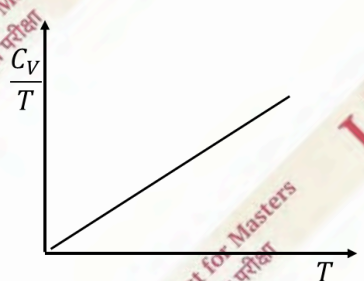
(A)



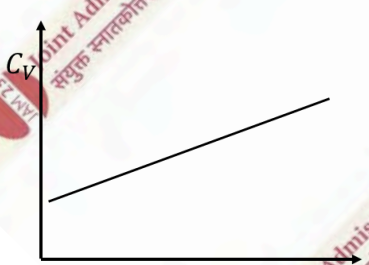
(B)



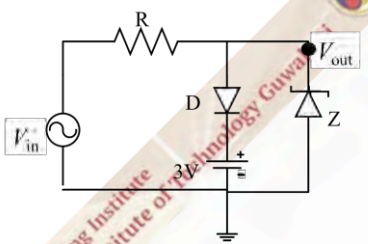
(C)



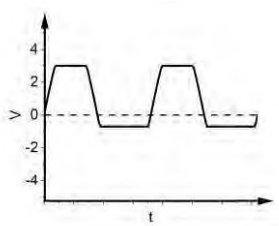
(D)



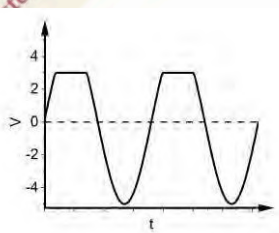
Q.7 For the following circuit, choose the correct waveform corresponding to the output signal (V_{out}). Given $V_{\text{in}} = 5\sin(200\pi t)$ V, forward bias voltage of the diodes (D and Z) = 0.7 V and reverse Zener voltage = 3 V.



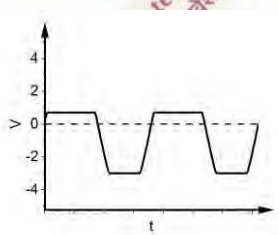
(A)



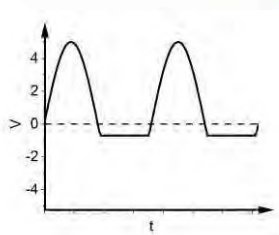
(B)



(C)



(D)



Q.8 If the ground state energy of a particle in an infinite potential well of width L_1 is equal to the energy of the second excited state in another infinite potential well of width L_2, then the ratio $\frac{L_1}{L_2}$ is equal to

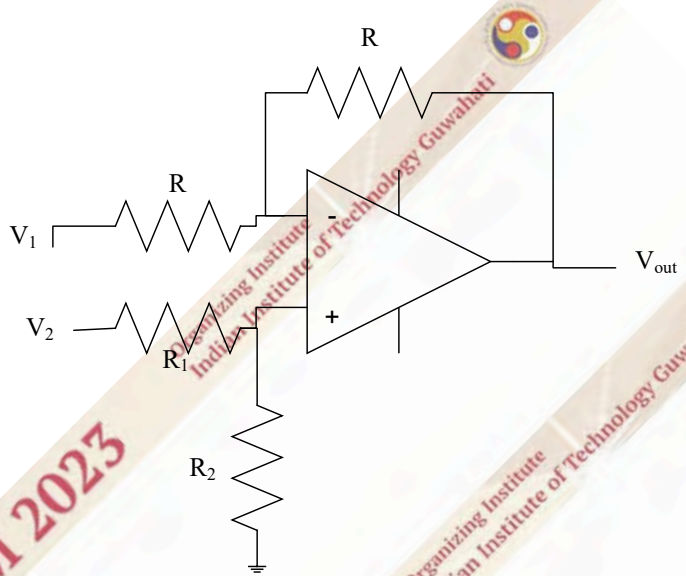
(A) 1

(B) $1/3$

(C) $1/\sqrt{3}$

(D) $1/9$

Q.9 In the given circuit, with an ideal op-amp for what value of $\frac{R_1}{R_2}$ the output of the amplifier $V_{out} = V_2 - V_1$?



(A) 1

(B) 1/2

(C) 2

(D) 3/2

Q.10 A projectile of mass m is moving in the vertical x-y plane with the origin on the ground and y-axis pointing vertically up. Taking the gravitational potential energy to be zero on the ground, the total energy of the particle written in planar polar coordinates (r, θ) is (here g is the acceleration due to gravity)

(A) $\frac{m}{2}\dot{r}^2 + mgr\sin\theta$

(B) $\frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + mgr\cos\theta$

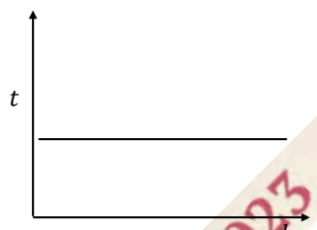
(C) $\frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + mgr\sin\theta$

(D) $\frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - mgr\cos\theta$

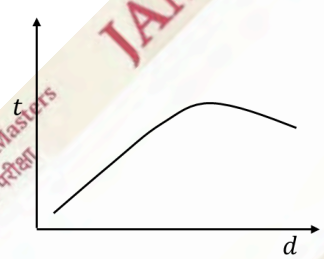
Section A: Q.11 – Q.30 Carry TWO marks each.

Q.11 A small bar magnet is dropped through different hollow copper tubes with same length and inner diameter but with different outer diameter. The variation in the time (t) taken for the magnet to reach the bottom of the tube depends on its wall thickness (d) as

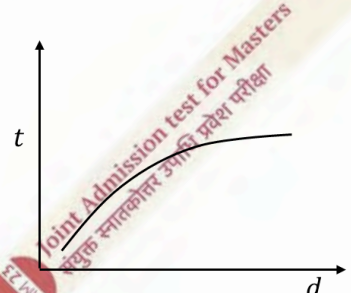
(A)



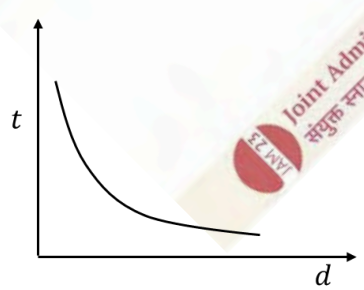
(B)



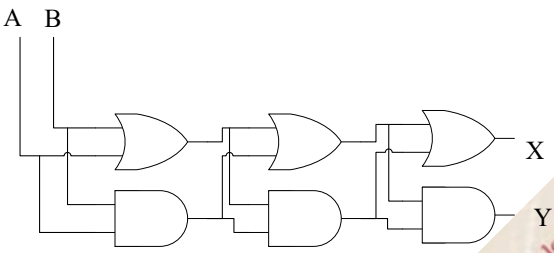
(C)



(D)



Q.12 Two digital inputs A and B are given to the following circuit. For $A = 1, B = 0$, the values of X and Y are:



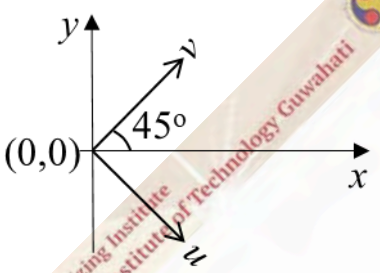
(A) $X = 0, Y = 0$

(B) $X = 1, Y = 0$

(C) $X = 0, Y = 1$

(D) $X = 1, Y = 1$

Q.13 The Jacobian matrix for transforming from (x, y) to another orthogonal coordinates system (u, v) as shown in the figure is



(A) $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

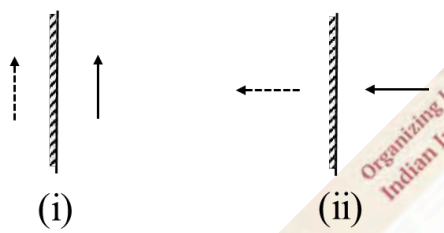
(B) $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

(C) $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

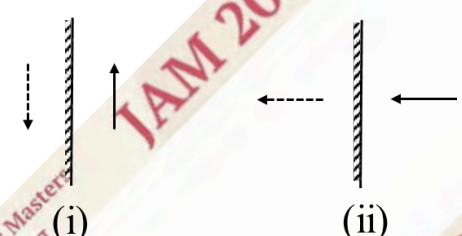
(D) $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$

Q.14 A rotating disc is held in front of a plane mirror in two different orientations which are (i) angular momentum parallel to the mirror and (ii) angular momentum perpendicular to the mirror. Which of the following schematic figures correctly describes the angular momentum (solid arrow) and its mirror image (shown by dashed arrows) in the two orientations?

(A)



(B)



(C)

(D)

Q.15 Inverse of the matrix $\begin{bmatrix} 1 & 1 & 0 \\ 2 & 3 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ is

(A) $\begin{bmatrix} 1 & -2 & 1 \\ -1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 3 & -1 & 0 \\ -2 & 1 & 0 \\ -3 & 1 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & -1 & 0 \\ 2 & 3 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 3 & -2 & -3 \\ -2 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

Q.16 Suppose the divergence of magnetic field $\vec{B}$ is nonzero and is given as $\vec{\nabla} \cdot \vec{B} = \mu_0 \rho_m$, where μ_0 is the permeability of vacuum and ρ_m is the magnetic charge density. If the corresponding magnetic current density is $\vec{J}_m$, then the curl $\vec{\nabla} \times \vec{E}$ of the electric field $\vec{E}$ is

(A) $\vec{J}_m - \frac{\partial \vec{B}}{\partial t}$

(B) $\mu_0 \vec{J}_m - \frac{\partial \vec{B}}{\partial t}$

(C) $-\vec{J}_m - \frac{\partial \vec{B}}{\partial t}$

(D) $-\mu_0 \vec{J}_m - \frac{\partial \vec{B}}{\partial t}$

Q.17 For a thermodynamic system, the coefficient of volume expansion $\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$ and compressibility $\kappa = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$, where V, T, and P are respectively the volume, temperature, and pressure. Considering that $\frac{dV}{V}$ is a perfect differential, we get

(A) $\left(\frac{\partial \beta}{\partial P}\right)_T = \left(\frac{\partial \kappa}{\partial T}\right)_P$

(B) $\left(\frac{\partial \beta}{\partial T}\right)_P = -\left(\frac{\partial \kappa}{\partial P}\right)_T$

(C) $\left(\frac{\partial \beta}{\partial P}\right)_T = -\left(\frac{\partial \kappa}{\partial T}\right)_P$

(D) $\left(\frac{\partial \beta}{\partial T}\right)_P = \left(\frac{\partial \kappa}{\partial P}\right)_T$

Q.18 A linearly polarized light of wavelength 590 nm is incident normally on the surface of a 20 μm thick quartz film. The plane of polarization makes an angle 30° with the optic axis. Refractive indices of ordinary and extraordinary waves differ by 0.0091, resulting in a phase difference of $f\pi$ between them after transmission. The value of f (rounded off to two decimal places) and the state of polarization of the transmitted light is

(A) 0.62 and linear

(B) 0.62 and elliptical

(C) -0.38 and elliptical

(D) 0.5 and circular

Q.19 The phase velocity v_p of transverse waves on a one-dimensional crystal of atomic separation d is related to the wavevector k as

$$v_p = C\frac{\sin(kd/2)}{(kd/2)}$$

The group velocity of these waves is

(A) $C\left[\cos(kd/2) - \frac{\sin(kd/2)}{(kd/2)}\right]$

(B) $C\cos(kd/2)$

(C) $C\left[\cos(kd/2) + \frac{\sin(kd/2)}{(kd/2)}\right]$

(D) $C\frac{\sin(kd/2)}{(kd/2)}$

Q.20 In a dielectric medium of relative permittivity 5, the amplitudes of the displacement current and conduction current are equal for an applied sinusoidal voltage of frequency $f = 1$ MHz. The value of conductivity (in $\Omega^{-1}m^{-1}$) of the medium at this frequency is

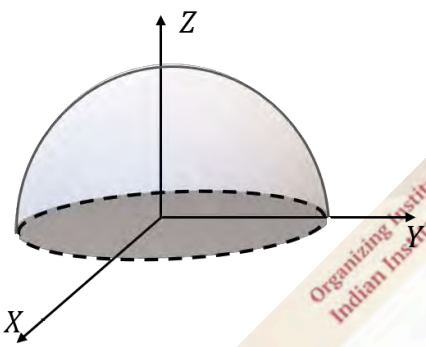
(A) 2.78×10^{-4}

(B) 2.44×10^{-4}

(C) 2.78×10^{-3}

(D) 2.44×10^{-3}

Q.21 For a given vector $\vec{F} = -y\hat{\imath} + z\hat{\jmath} + x^2\hat{k}$, the surface integral $\int_S (\vec{\nabla} \times \vec{F}) \cdot \hat{r} dS$ over the surface S of a hemisphere of radius R with the centre of the base at the origin is



(A) πR^2

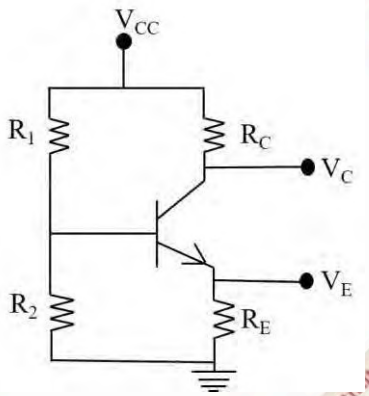
(B) $\frac{2\pi R^2}{3}$

(C) $-\pi R^2$

(D) $-\frac{2\pi R^2}{3}$

Q.22 In the circuit shown, assuming the current gain $\beta = 100$ and $V_{BE} = 0.7$ V, what will be the collector voltage V_C in V?

Given: V_{CC} = 15 V, R_1 = 100 kΩ, R_2 = 50 kΩ, R_C = 4.7 kΩ, and R_E = 3.3 kΩ



(A) 8.9

(B) 5.1

(C) 4.3

(D) 3.2

Q.23 A uniform stick of length l and mass m pivoted at its top end is oscillating with an angular frequency ω_r. Assuming small oscillations, the ratio ω_r/ω_s, where ω_s is the angular frequency of a simple pendulum of the same length, will be

(A) $\sqrt{3}$

(B) $\sqrt{\frac{3}{2}}$

(C) $\sqrt{2}$

(D) $\frac{1}{\sqrt{3}}$

Q.24 An oil film in air of thickness 255 nm is illuminated by white light at normal incidence. As a consequence of interference, which colour will be predominantly visible in the reflected light?

Given the refractive index of oil = 1.47

(A) Red (~ 650 nm)

(B) Blue (~ 450 nm)

(C) Green (~ 500 nm)

(D) Yellow (~560 nm)

Q.25 Water from a tank is flowing down through a hole at its bottom with velocity 5 ms^{-1}. If this water falls on a flat surface kept below the hole at a distance of 0.1 m and spreads horizontally, the pressure (in kNm^{-2}) exerted on the flat surface is closest to

Given: acceleration due to gravity = 9.8 ms^{-2} and density of water = 1000 kgm^{-3}

(A) 13.5

(B) 27.0

(C) 17.6

(D) 6.8

Q.26 At the planar interface of two dielectrics, which of the following statements related to the electric field ($\vec{E}$), electric displacement ($\vec{D}$) and polarization ($\vec{P}$) is true?

(A) Normal component of both $\vec{D}$ and $\vec{P}$ are continuous

(B) Normal component of both $\vec{D}$ and $\vec{E}$ are discontinuous

(C) Normal component of $\vec{D}$ is continuous and that of $\vec{P}$ is discontinuous

(D) Normal component of both $\vec{E}$ and $\vec{P}$ are continuous

Q.27 Consider a system of large number of particles that can be in three energy states with energies 0 meV, 1 meV, and 2 meV. At temperature $T = 300$ K, the mean energy of the system (in meV) is closest to

Given: Boltzmann constant $k_B = 0.086$ meVK^{-1}

(A) 0.12

(B) 0.97

(C) 1.32

(D) 1.82

Q.28 For the Maxwell-Boltzmann speed distribution, the ratio of the root-mean-square speed (v_{rms}) and the most probable speed (v_{max}) is

Given: Maxwell-Boltzmann speed distribution function for a collection of particles of mass m is

$$f(v) = \left(\frac{m}{2\pi k_{\text{B}} T}\right)^{3/2} 4\pi v^2 \exp\left(-\frac{mv^2}{2k_{\text{B}} T}\right)$$

where, v is the speed and $k_{\text{B}}T$ is the thermal energy.

(A) $\sqrt{\frac{3}{2}}$

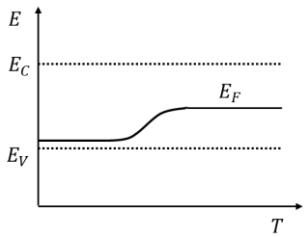
(B) $\sqrt{\frac{2}{3}}$

(C) $\frac{3}{2}$

(D) $\frac{2}{3}$

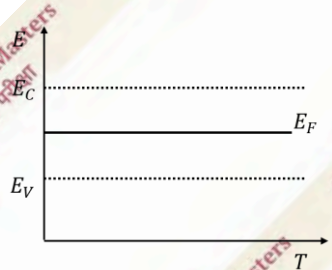
Q.29 In an extrinsic p-type semiconductor, which of the following schematic diagram depicts the variation of the Fermi energy level (E_F) with temperature (T)?

(A)

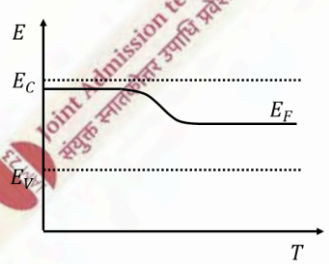


(B)

(C)



(D)



Q.30 A container is occupied by a fixed number of non-interacting particles. If they are obeying Fermi-Dirac, Bose-Einstein, and Maxwell-Boltzmann statistics, the pressure in the container is P_{FD}, P_{BE} and P_{MB}, respectively. Then

(A) $P_{FD} > P_{MB} > P_{BE}$

(B) $P_{FD} > P_{MB} = P_{BE}$

(C) $P_{FD} > P_{BE} > P_{MB}$

(D) $P_{FD} = P_{MB} = P_{BE}$

Section B: Q.31 – Q.40 Carry TWO marks each.

Q.31 The spectral energy density $u_T(\lambda)$ vs wavelength (λ) curve of a black body shows a peak at $\lambda = \lambda_{\max}$. If the temperature of the black body is doubled, then

(A) the maximum of $u_T(\lambda)$ shifts to $\lambda_{\max}/2$

(B) the maximum of $u_T(\lambda)$ shifts to $2\lambda_{\max}$

(C) the area under the curve becomes 16 times the original area

(D) the area under the curve becomes 8 times the original area

Q.32 A periodic function $f(x) = x^2$ for $-\pi < x < \pi$ is expanded in a Fourier series. Which of the following statement(s) is/are correct?

(A) Coefficients of all the sine terms are zero

(B) The first term in the series is $\frac{\pi^2}{3}$

(C) The second term in the series is $-4\cos x$

(D) Coefficients of all the cosine terms are zero

Q.33 The state of a harmonic oscillator is given as $\Psi = \frac{1}{\sqrt{3}}\psi_0 - \frac{1}{\sqrt{6}}\psi_1 + \frac{1}{\sqrt{2}}\psi_2$, where ψ_0, ψ_1 and ψ_2 are the normalized wave functions of ground, first excited, and second excited states, respectively. Which of the following statement(s) is/are true?

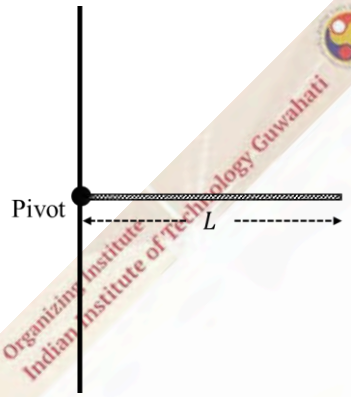
(A) A measurement of the energy of the system yields $E = \frac{1}{2}\hbar\omega$ with non-zero probability

(B) A measurement of the energy of the system yields $E = \frac{5}{3}\hbar\omega$ with non-zero probability

(C) Expectation value of the energy of the system $\langle E \rangle = \frac{5}{3}\hbar\omega$

(D) Expectation value of the energy of the system $\langle E \rangle = \frac{7}{6}\hbar\omega$

Q.34 A rod of mass M, length L and non-uniform mass per unit length $\lambda(x) = \frac{3Mx^2}{L^3}$, is held horizontally by a pivot, as shown in the figure, and is free to move in the plane of the figure. For this rod, which of the following statements are true?



(A) Moment of inertia of the rod about an axis passing through the pivot is $\frac{3}{5}ML^2$

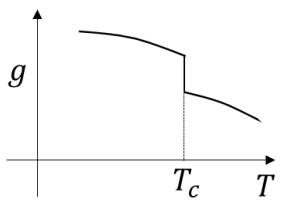
(B) Moment of inertia of the rod about an axis passing through the pivot is $\frac{1}{3}ML^2$

(C) Torque on the rod about the pivot is $\frac{3}{4}MgL$

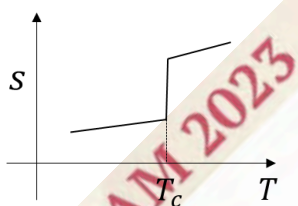
(D) If the rod is released, the point at a distance $\frac{2L}{3}$ from the pivot will fall with acceleration g

Q.35 Which of the following schematic plots correctly represent(s) a first order phase transition occurring at temperature $T = T_c$? Here g, s, v are specific Gibbs free energy, entropy and volume, respectively.

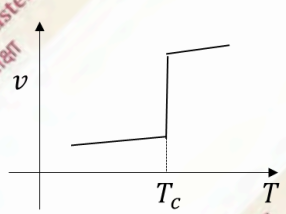
(A)



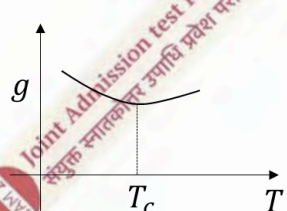
(B)



(C)



(D)



Q.36 A particle (p_1) of mass m moving with speed v collides with a stationary identical particle (p_2). The particles bounce off each other elastically with p_1 getting deflected by an angle $\theta = 30°$ from its original direction. Then, which of the following statement(s) is/are true after the collision?

(A) Speed of p_1 is $\frac{\sqrt{3}}{2}v$

(B) Kinetic energy of p_2 is 25% of the total energy

(C) Angle between the directions of motion of the two particles is 90°

(D) The kinetic energy of the centre of mass of p_1 and p_2 decreases

Q.37 A wave travelling along the x-axis with y representing its displacement is described by (v is the speed of the wave)

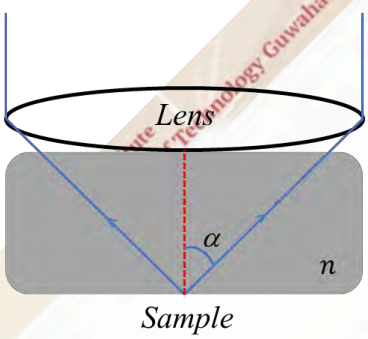
(A) $\frac{\partial y}{\partial x} + \frac{1}{v}\frac{\partial y}{\partial t} = 0$

(B) $\frac{\partial y}{\partial x} - \frac{1}{v}\frac{\partial y}{\partial t} = 0$

(C) $\frac{\partial^2 y}{\partial x^2} + \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = 0$

(D) $\frac{\partial^2 y}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = 0$

Q.38 An objective lens with half angular aperture α is illuminated with light of wavelength λ. The refractive index of the medium between the sample and the objective is n. The lateral resolving power of the optical system can be increased by



(A) decreasing both λ and α

(B) decreasing λ and increasing α

(C) increasing both α and n

(D) decreasing λ and increasing n

Q.39 Which of the following statement(s) is/are true for a LC circuit with L = 25 mH and C = 4 μF?

(A) Resonance frequency is close to 503 Hz

(B) The impedance at 1 kHz is 15 Ω

(C) At a frequency of 200 Hz, the voltage lags the current in the circuit

(D) At a frequency of 700 Hz, the voltage lags the current in the circuit

Q.40 For a particle moving in a general central force field, which of the following statement(s) is/are true?

(A) The angular momentum is a constant of motion

(B) Kepler's second law is valid

(C) The motion is confined to a plane

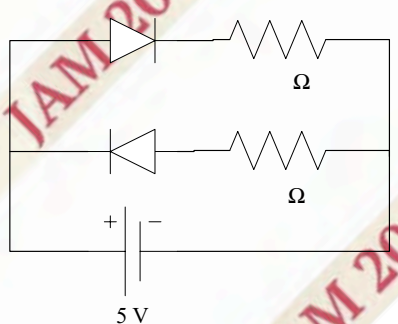
(D) Kepler's third law is valid

Section C: Q.41 – Q.50 Carry ONE mark each.

Q.41 The lattice constant (in Å) of copper, which has FCC structure, is _____ (rounded off to two decimal places).

Given: density of copper is 8.91 $g\,cm^{-3}$ and its atomic mass is 63.55 $g\,mol^{-1}$; Avogadro's number $= 6.023 \times 10^{23}\ mol^{-1}$

Q.42 Two silicon diodes are connected to a battery and two resistors as shown in the figure. The current through the battery is _____ A (rounded off to two decimal places).



Given: The forward voltage drop across each diode = 0.7 V

Q.43 The absolute error in the value of $\sin\theta$ if approximated up to two terms in the Taylor's series for $\theta = 60°$ is _____ (rounded off to three decimal places).

Q.44 A single pendulum hanging vertically in an elevator has a time period T_0 when the elevator is stationary. If the elevator moves upward with an acceleration of $a = 0.2g$, the time period of oscillations is T_1. Here g is the acceleration due to gravity. The ratio $\frac{T_0}{T_1}$ is _____ (rounded off to two decimal places).

Q.45 A spacecraft has speed $v_s = fc$ with respect to the earth, where c is the speed of light in vacuum. An observer in the spacecraft measures the time of one complete rotation of the earth to be 48 hours. The value of f is _____ (rounded off to two decimal places).

Q.46 The sum of the x-components of unit vectors $\hat{r}$ and $\hat{\theta}$ for a particle moving with angular speed 2 rad s^{-1} at angle $\theta = 215°$ is_____ (rounded off to two decimal places)

Q.47 Consider a spring mass system with mass 0.5 kg and spring constant $k = 2\ \text{Nm}^{-1}$ in a viscous medium with drag coefficient $b = 3\ \text{kg s}^{-1}$. The additional mass required so that the motion becomes critically damped is _____ kg (rounded off to three decimal places).

Q.48 Unit vector normal to the equipotential surface of $V(x, y, z) = 4x^2 + y^2 + z$ at (1,2,1) is given by $(a\hat{\imath} + b\hat{\jmath} + c\hat{k})$. The value of $|b|$ is_____ (rounded off to two decimal places).

Q.49 A rectangular pulse of width 0.5 cm is travelling to the right on a taut string (shown by full line in the figure) that has mass per unit length μ_1. The string is attached to another taut string (shown by dashed line) of mass per unit length μ_2. If the tension in both the strings is the same, and the transmitted pulse has width 0.7 cm, the ratio μ_1/μ_2 is_____(rounded off to two decimal places).

Q.50 An α particle with energy of 3 MeV is moving towards a nucleus of ^{50}Sn. Its minimum distance of approach to the nucleus is $f \times 10^{-14}$ m. The value of f is _____ (rounded off to one decimal place).

Section C: Q.51 – Q.60 Carry TWO marks each.

Q.51 In a X-Ray tube operating at 20 kV, the ratio of the de-Broglie wavelength of the incident electrons to the shortest wavelength of the generated X-rays is _____ (rounded off to two decimal places).

Given: e/m ratio for an electron = 1.76×10^{11} Ckg^{-1} and the speed of light in vacuum is 3×10^{8} ms^{-1}

Q.52 A point source emitting photons of 2 eV energy and 1 W of power is kept at a distance of 1m from a small piece of a photoelectric material of area 10^{-4} m^2. If the efficiency of generation of photoelectrons is 10%, then the number of photoelectrons generated are $f \times 10^{12}$ per second. The value of f is _____ (rounded off to two decimal places).

Given: $1eV = 1.6 \times 10^{-19}$ J

Q.53 Consider the α-decay $^{90}Th^{232} \rightarrow {}^{88}Ra^{228}$. In an experiment with one gram of $^{90}Th^{232}$, the average count rate (integrated over the entire volume) measured by the α-detector is 3000 counts s^{-1}. If the half life of $^{90}Th^{232}$ is given as 4.4×10^{17} s, then the efficiency of the α-detector is _____ (rounded off to two decimal places).

Given: Avogadro's number $= 6.023 \times 10^{23}\ mol^{-1}$

Q.54 In the Thomson model of hydrogen atom, the nuclear charge is distributed uniformly over a sphere of radius R. The average potential energy of an electron confined within this atom can be taken as $V = -\frac{e^2}{4\pi\epsilon_0 R}$. Taking the uncertainty in position to be the radius of the atom, the minimum value of R for which an electron will be confined within the atom is estimated to be $f \times 10^{-11}$ m. The value of f is _____ (rounded off to one decimal place).

Given: The uncertainty product of momentum and position is $\hbar = 1 \times 10^{-34}\ Js^{-1}$, $e = 1.6 \times 10^{-19} C$, and $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9\ Nm^2C^{-2}$

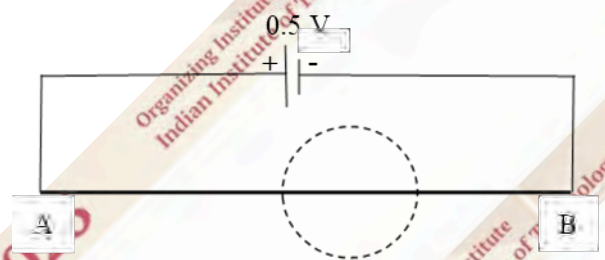
Q.55 The sum of the eigenvalues λ_1 and λ_2 of matrix $B = I + A + A^2$, where $A = \begin{bmatrix} 2 & 1 \\ -0.5 & 0.5 \end{bmatrix}$ is _____ (rounded off to two decimal places).

Q.56 A container of volume V has helium gas in it with N number of He atoms. The mean free path of these atoms is λ_{He}. Another container has argon gas with the same number of Ar atoms in volume $2V$ with their mean free path being λ_{Ar}. Taking the radius of Ar atoms to be 1.5 times the radius of He atoms, the ratio $\lambda_{Ar}/\lambda_{He}$ is_____ (rounded off to two decimal places).

Q.57 Three frames F_0, F_1 and F_2 are in relative motion. The frame F_0 is at rest, F_1 is moving with velocity $v_1\hat{\imath}$ with respect to F_0 and F_2 is moving with velocity $v_2\hat{\imath}$ with respect to F_1. A particle is moving with velocity $v_3\hat{\imath}$ with respect to F_2. If $v_1 = v_2 = v_3 = c/2$, where c is the speed of light, the speed of the particle with respect to F_0 is fc. The value of f is _____ (rounded off to two decimal places).

Q.58 A fission device explodes into two pieces of rest masses m and $0.5m$ with no loss of energy into any other form. These masses move apart respectively with speeds $\frac{c}{\sqrt{13}}$ and $\frac{c}{2}$, with respect to the stationary frame. If the rest mass of the device is fm then f is_____ (rounded off to two decimal places).

Q.59 A conducting wire AB of length m has resistance of .6 Ω. It is connected to a voltage source of 0.5 V with negligible resistance as shown in the figure. The corresponding electric and magnetic fields give Poynting vectors $\vec{S}(\vec{r})$ all around the wire. Surface integral $\int \vec{S}.d\vec{a}$ is calculated over a virtual sphere of diameter 0.2 m with its centre on the wire, as shown. The value of the integral is _____W (rounded off to three decimal places).



Q.60 A metallic sphere of radius R is held at electrostatic potential V. It is enclosed in a concentric thin metallic shell of radius $2R$ at potential $2V$. If the potential at the distance $\frac{3}{2}R$ from the centre of the sphere is fV, then the value of f is _____ (rounded off to two decimal places).