JEE Advanced 2015 Question Paper with Solution

Joint Entrance Examination – Advanced

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JEE (ADVANCED) 2015

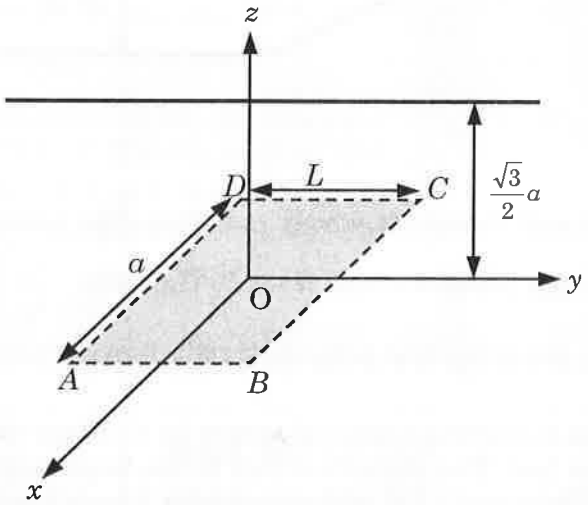
Answer Keys for Paper 1

PART I : PHYSICS

SECTION 1 (Maximum Marks: 32)

- This section contains **EIGHT** questions
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive
- For each question, darken the bubble corresponding to the correct integer in the ORS
- Marking scheme:
  - +4 If the bubble corresponding to the answer is darkened
  - 0 In all other cases

Q.1 An infinitely long uniform line charge distribution of charge per unit length λ lies parallel to the y-axis in the y-z plane at $z = \frac{\sqrt{3}}{2}a$ (see figure). If the magnitude of the flux of the electric field through the rectangular surface $ABCD$ lying in the x-y plane with its centre at the origin is $\frac{\lambda L}{n\varepsilon_0}$ (ε_0 = permittivity of free space), then the value of n is



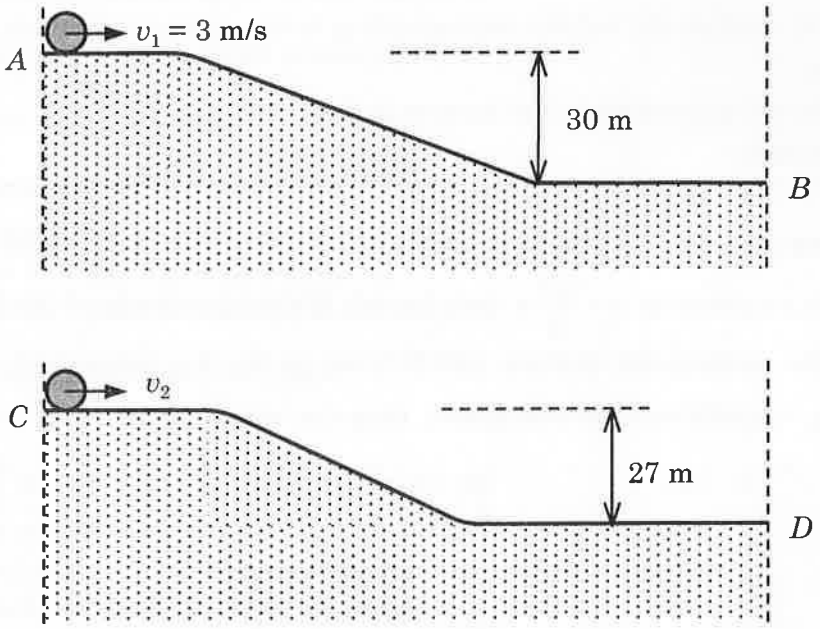
Q.2 Consider a hydrogen atom with its electron in the n^{th} orbital. An electromagnetic radiation of wavelength 90 nm is used to ionize the atom. If the kinetic energy of the ejected electron is 10.4 eV, then the value of n is (hc = 1242 eV nm)

Q.3 A bullet is fired vertically upwards with velocity v from the surface of a spherical planet. When it reaches its maximum height, its acceleration due to the planet's gravity is $1/4^{th}$ of its value at the surface of the planet. If the escape velocity from the planet is $v_{esc} = v\sqrt{N}$, then the value of N is (ignore energy loss due to atmosphere)

Answers for the above questions

Ans for Q.1: 6	**Ans for Q.2: 2**	**Ans for Q.3: 2**

Q.4 Two identical uniform discs roll without slipping on two different surfaces AB and CD (see figure) starting at A and C with linear speeds v_1 and v_2, respectively, and always remain in contact with the surfaces. If they reach B and D with the same linear speed and $v_1 = 3$ m/s, then v_2 in m/s is ($g = 10$ m/s^2)



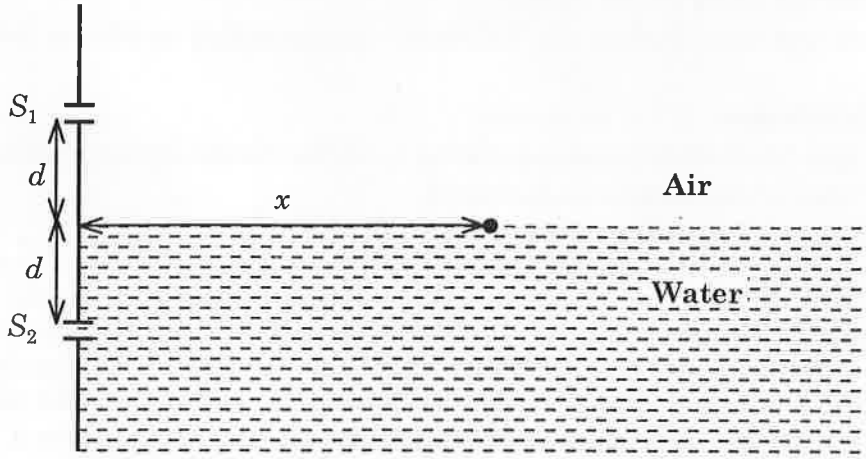
Q.5 Two spherical stars A and B emit blackbody radiation. The radius of A is 400 times that of B and A emits 10^4 times the power emitted from B. The ratio $\left(\frac{\lambda_A}{\lambda_B}\right)$ of their wavelengths λ_A and λ_B at which the peaks occur in their respective radiation curves is

Q.6 A nuclear power plant supplying electrical power to a village uses a radioactive material of half life T years as the fuel. The amount of fuel at the beginning is such that the total power requirement of the village is 12.5% of the electrical power available from the plant at that time. If the plant is able to meet the total power needs of the village for a maximum period of nT years, then the value of n is

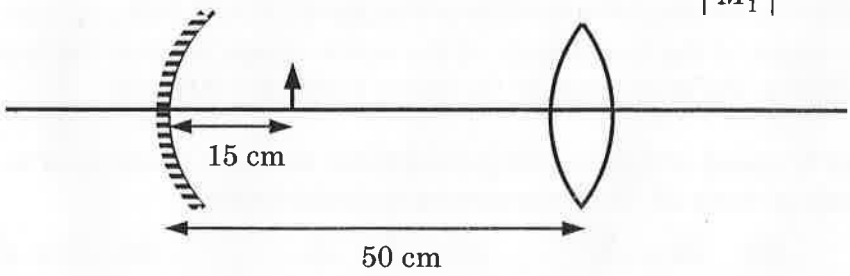
Answers for the above questions

Ans for Q.4: 7	Ans for Q.5: 2	Ans for Q.6: 3

Q.7 A Young's double slit interference arrangement with slits S_1 and S_2 is immersed in water (refractive index = 4/3) as shown in the figure. The positions of maxima on the surface of water are given by $x^2 = p^2m^2\lambda^2 - d^2$, where λ is the wavelength of light in air (refractive index = 1), $2d$ is the separation between the slits and m is an integer. The value of p is



Q.8 Consider a concave mirror and a convex lens (refractive index = 1.5) of focal length 10 cm each, separated by a distance of 50 cm in air (refractive index = 1) as shown in the figure. An object is placed at a distance of 15 cm from the mirror. Its erect image formed by this combination has magnification M_1. When the set-up is kept in a medium of refractive index 7/6, the magnification becomes M_2. The magnitude $\left|\frac{M_2}{M_1}\right|$ is



Answers for the above questions

Ans for Q.7: 3	Ans for Q.8: 7

SECTION 2 (Maximum Marks: 40)

- This section contains **TEN** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- Marking scheme:
  - +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened
  - 0 If none of the bubbles is darkened
  - –2 In all other cases

Q.9 Consider a Vernier callipers in which each 1 cm on the main scale is divided into 8 equal divisions and a screw gauge with 100 divisions on its circular scale. In the Vernier callipers, 5 divisions of the Vernier scale coincide with 4 divisions on the main scale and in the screw gauge, one complete rotation of the circular scale moves it by two divisions on the linear scale. Then:

(A) If the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.01 mm.

(B) If the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.005 mm.

(C) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.01 mm.

(D) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.005 mm.

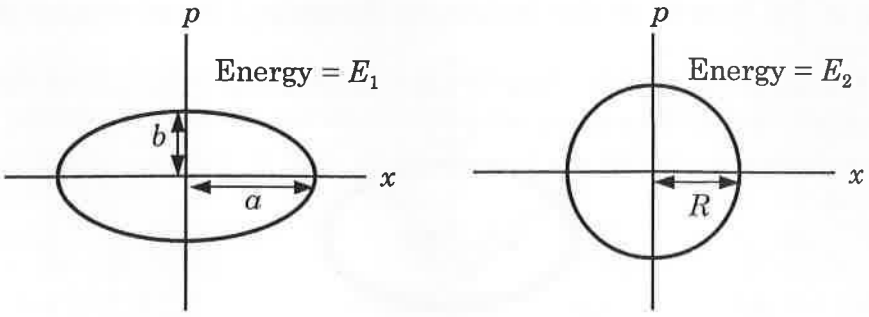
Q.10 Planck's constant h, speed of light c and gravitational constant G are used to form a unit of length L and a unit of mass M. Then the correct option(s) is(are)

(A) $M \propto \sqrt{c}$ (B) $M \propto \sqrt{G}$ (C) $L \propto \sqrt{h}$ (D) $L \propto \sqrt{G}$

Answers for the above questions

Ans for Q.9: (B) and (C)	Ans for Q.10: (A), (C) and (D)

Q.11 Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2, respectively. The variations of their momenta p with positions x are shown in the figures. If $\frac{a}{b} = n^2$ and $\frac{a}{R} = n$, then the correct equation(s) is(are)

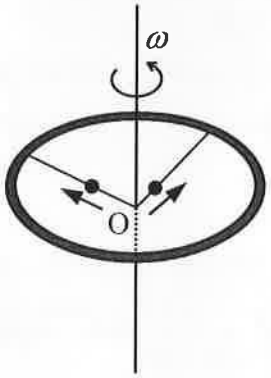


(A) $E_1\omega_1 = E_2\omega_2$ (B) $\frac{\omega_2}{\omega_1} = n^2$ (C) $\omega_1\omega_2 = n^2$ (D) $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

Answer for the above question

Ans for Q.11: (B) and (D)

Q.12 A ring of mass M and radius R is rotating with angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass $\frac{M}{8}$ at rest at O. These masses can move radially outwards along two massless rods fixed on the ring as shown in the figure. At some instant the angular speed of the system is $\frac{8}{9}\omega$ and one of the masses is at a distance of $\frac{3}{5}R$ from O. At this instant the distance of the other mass from O is

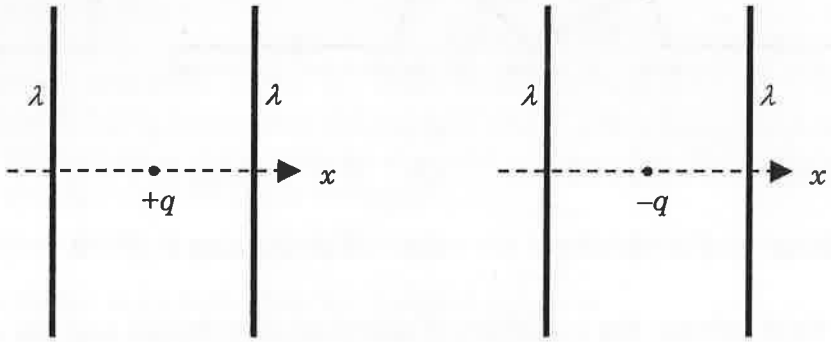


(A) $\frac{2}{3}R$ (B) $\frac{1}{3}R$ (C) $\frac{3}{5}R$ (D) $\frac{4}{5}R$

Answer for the above question

Ans for Q.12: (C) OR (D) OR (C) and (D)

Q.13 The figures below depict two situations in which two infinitely long static line charges of constant positive line charge density λ are kept parallel to each other. In their resulting electric field, point charges q and $-q$ are kept in equilibrium between them. The point charges are confined to move in the x direction only. If they are given a small displacement about their equilibrium positions, then the correct statement(s) is(are)



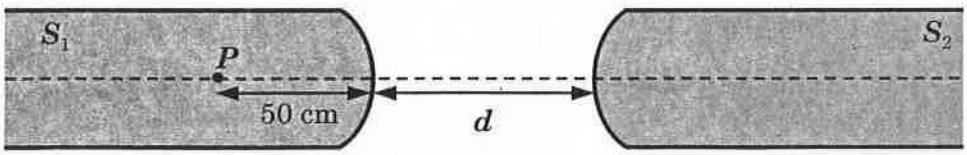
(A) Both charges execute simple harmonic motion.

(B) Both charges will continue moving in the direction of their displacement.

(C) Charge $+q$ executes simple harmonic motion while charge $-q$ continues moving in the direction of its displacement.

(D) Charge $-q$ executes simple harmonic motion while charge $+q$ continues moving in the direction of its displacement.

Q.14 Two identical glass rods S_1 and S_2 (refractive index = 1.5) have one convex end of radius of curvature 10 cm. They are placed with the curved surfaces at a distance d as shown in the figure, with their axes (shown by the dashed line) aligned. When a point source of light P is placed inside rod S_1 on its axis at a distance of 50 cm from the curved face, the light rays emanating from it are found to be parallel to the axis inside S_2. The distance d is

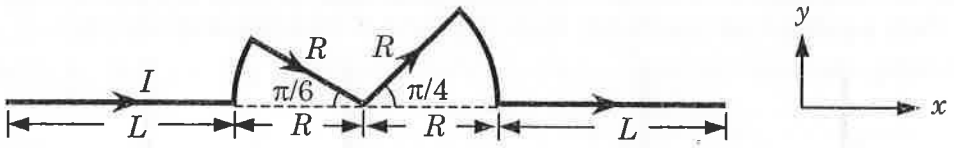


(A) 60 cm (B) 70 cm (C) 80 cm (D) 90 cm

Answers for the above questions

Ans for Q.13: (C)	Ans for Q.14: (B)

Q.15 A conductor (shown in the figure) carrying constant current I is kept in the x-y plane in a uniform magnetic field $\vec{B}$. If F is the magnitude of the total magnetic force acting on the conductor, then the correct statement(s) is(are)



(A) If $\vec{B}$ is along $\hat{z}$, $F \propto (L+R)$

(B) If $\vec{B}$ is along $\hat{x}$, $F=0$

(C) If $\vec{B}$ is along $\hat{y}$, $F \propto (L+R)$

(D) If $\vec{B}$ is along $\hat{z}$, $F=0$

Q.16 A container of fixed volume has a mixture of one mole of hydrogen and one mole of helium in equilibrium at temperature T. Assuming the gases are ideal, the correct statement(s) is(are)

(A) The average energy per mole of the gas mixture is $2RT$.

(B) The ratio of speed of sound in the gas mixture to that in helium gas is $\sqrt{6/5}$.

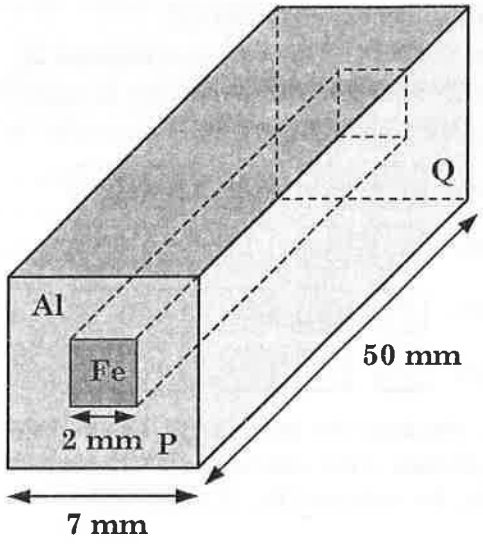
(C) The ratio of the rms speed of helium atoms to that of hydrogen molecules is $1/2$.

(D) The ratio of the rms speed of helium atoms to that of hydrogen molecules is $1/\sqrt{2}$.

Answers for the above questions

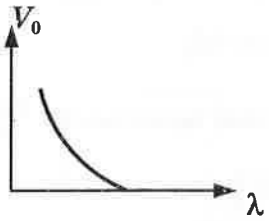
Ans for Q.15: (A), (B) and (C)	Ans for Q.16: (A), (B) and (D)

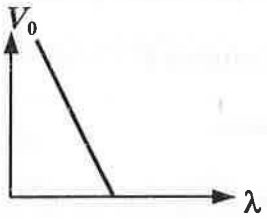
Q.17 In an aluminum (Al) bar of square cross section, a square hole is drilled and is filled with iron (Fe) as shown in the figure. The electrical resistivities of Al and Fe are 2.7×10^{-8} Ω m and 1.0×10^{-7} Ω m, respectively. The electrical resistance between the two faces P and Q of the composite bar is

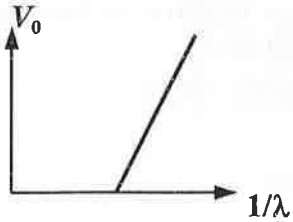


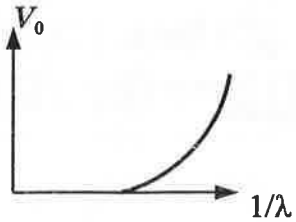
(A) $\frac{2475}{64}\mu\Omega$ (B) $\frac{1875}{64}\mu\Omega$ (C) $\frac{1875}{49}\mu\Omega$ (D) $\frac{2475}{132}\mu\Omega$

Q.18 For photo-electric effect with incident photon wavelength λ, the stopping potential is V_0. Identify the correct variation(s) of V_0 with λ and $1/\lambda$.

(A)


(B)


(C)


(D)


Answers for the above questions

Ans for Q.17: (B)	Ans for Q.18: (A) and (C)

SECTION 3 (Maximum Marks: 16)

- This section contains **TWO** questions
- Each question contains two columns, **Column I** and **Column II**
- **Column I** has **four** entries (A), (B), (C) and (D)
- **Column II** has **five** entries (P), (Q), (R), (S) and (T)
- Match the entries in **Column I** with the entries in **Column II**
- One or more entries in **Column I** may match with one or more entries in **Column II**
- The ORS contains a 4×5 matrix whose layout will be similar to the one shown below:

(A) (P) (Q) (R) (S) (T)
(B) (P) (Q) (R) (S) (T)
(C) (P) (Q) (R) (S) (T)
(D) (P) (Q) (R) (S) (T)

- For each entry in **Column I**, darken the bubbles of all the matching entries. For example, if entry (A) in **Column I** matches with entries (Q), (R) and (T), then darken these three bubbles in the ORS. Similarly, for entries (B), (C) and (D).
- Marking scheme:
  For each entry in **Column I**,
  +2 If only the bubble(s) corresponding to all the correct match(es) is(are) darkened
  0 If none of the bubbles is darkened
  –1 In all other cases

Q.19 Match the nuclear processes given in column I with the appropriate option(s) in column II.

	Column I		Column II
(A)	Nuclear fusion	(P)	Absorption of thermal neutrons by $^{235}_{92}U$
(B)	Fission in a nuclear reactor	(Q)	$^{60}_{27}Co$ nucleus
(C)	β-decay	(R)	Energy production in stars via hydrogen conversion to helium
(D)	γ-ray emission	(S)	Heavy water
		(T)	Neutrino emission

Answers for the above question

Ans for Q.19:
(A) Matches to (R) OR (R) and (T)
(B) Matches to (P) and (S)
(C) Matches to (Q) and (T)
(D) Matches to (R)

Q.20 A particle of unit mass is moving along the x-axis under the influence of a force and its total energy is conserved. Four possible forms of the potential energy of the particle are given in column I (a and U_0 are constants). Match the potential energies in column I to the corresponding statement(s) in column II.

Column I	Column II
(A) $U_1(x) = \frac{U_0}{2}\left[1-\left(\frac{x}{a}\right)^2\right]^2$	(P) The force acting on the particle is zero at $x = a$.
(B) $U_2(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2$	(Q) The force acting on the particle is zero at $x = 0$.
(C) $U_3(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2 \exp\left[-\left(\frac{x}{a}\right)^2\right]$	(R) The force acting on the particle is zero at $x = -a$.
(D) $U_4(x) = \frac{U_0}{2}\left[\frac{x}{a}-\frac{1}{3}\left(\frac{x}{a}\right)^3\right]$	(S) The particle experiences an attractive force towards $x = 0$ in the region $\lvert x\rvert < a$.
	(T) The particle with total energy $\frac{U_0}{4}$ can oscillate about the point $x = -a$.

END OF PART I : PHYSICS

Answers for the above question

Ans for Q.20:

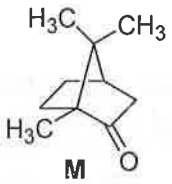
(A) Matches to (P), (Q), (R) and (T)
(B) Matches to (Q) and (S)
(C) Matches to (P), (Q), (R) and (S)
(D) Matches to (P), (R) and (T)

PART II : CHEMISTRY

SECTION 1 (Maximum Marks: 32)

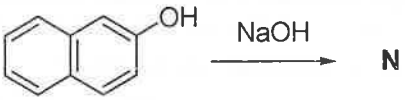
- This section contains **EIGHT** questions
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive
- For each question, darken the bubble corresponding to the correct integer in the ORS
- Marking scheme:
  +4 If the bubble corresponding to the answer is darkened
  0 In all other cases

Q.21 The total number of stereoisomers that can exist for **M** is



M

Q.22 The number of resonance structures for **N** is



N

Q.23 The total number of lone pairs of electrons in N_2O_3 is

Q.24 For the octahedral complexes of Fe^{3+} in SCN^- (thiocyanato-S) and in CN^- ligand environments, the difference between the spin-only magnetic moments in Bohr magnetons (when approximated to the nearest integer) is
[Atomic number of Fe = 26]

Answers for the above questions

Ans for Q.21: 2	**Ans for Q.22: 9**	**Ans for Q.23: 8**	**Ans for Q.24: 4**

Q.25 Among the triatomic molecules/ions, $BeCl_2$, N_3^-, N_2O, NO_2^+, O_3, SCl_2, ICl_2^-, I_3^- and XeF_2, the total number of linear molecule(s)/ion(s) where the hybridization of the central atom does not have contribution from the *d*-orbital(s) is
[Atomic number : S = 16, Cl = 17, I = 53 and Xe = 54]

Q.26 Not considering the electronic spin, the degeneracy of the second excited state ($n = 3$) of H atom is 9, while the degeneracy of the second excited state of H^- is

Q.27 All the energy released from the reaction $\mathbf{X} \rightarrow \mathbf{Y}$, $\Delta_r G^0 = -193\,\text{kJ mol}^{-1}$ is used for oxidizing $\mathbf{M}^+$ as $\mathbf{M}^+ \rightarrow \mathbf{M}^{3+} + 2e^-$, $E^0 = -0.25\,V$.
Under standard conditions, the number of moles of $\mathbf{M}^+$ oxidized when **one** mole of **X** is converted to **Y** is
[$F = 96500\ \text{C mol}^{-1}$]

Q.28 If the freezing point of a 0.01 molal aqueous solution of a cobalt(III) chloride-ammonia complex (which behaves as a strong electrolyte) is $-0.0558\,°\text{C}$, the number of chloride(s) in the coordination sphere of the complex is
[K_f of water = 1.86 K kg mol^{-1}]

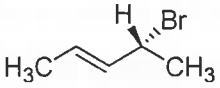
Answers for the above questions

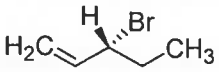
Ans for Q.25: 4	**Ans for Q.26: 3**	**Ans for Q.27: 4**	**Ans for Q.28: 1**

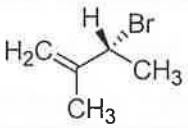
SECTION 2 (Maximum Marks: 40)

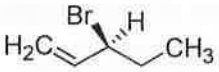
- This section contains **TEN** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- Marking scheme:
  - +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened
  - 0 If none of the bubbles is darkened
  - –2 In all other cases

Q.29 Compound(s) that on hydrogenation produce(s) optically inactive compound(s) is(are)

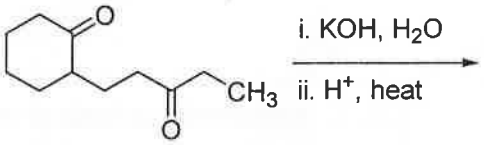
(A) 

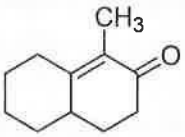
(B) 

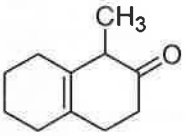
(C) 

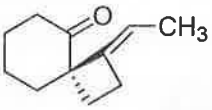
(D) 

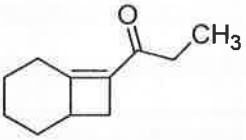
Q.30 The major product of the following reaction is



(A) 

(B) 

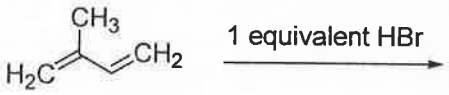
(C) 

(D) 

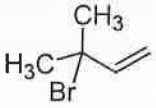
Answers for the above questions

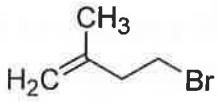
Ans for Q.29: (B) and (D)	Ans for Q.30: (A)

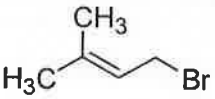
Q.31 In the following reaction, the major product is



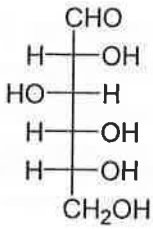
(A) 

(B) 

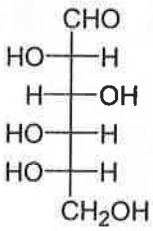
(C) 

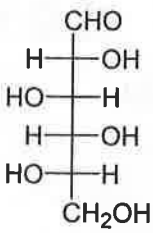
(D) 

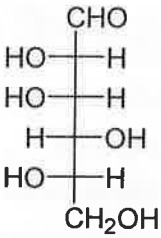
Q.32 The structure of D-(+)-glucose is

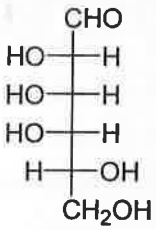


The structure of L-(–)-glucose is

(A) 

(B) 

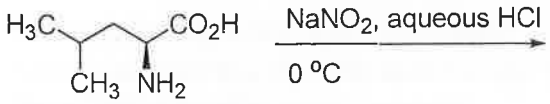
(C) 

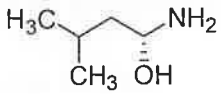
(D) 

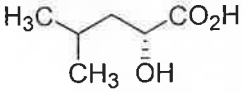
Answers for the above questions

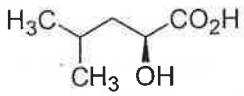
Ans for Q.31: (D)	Ans for Q.32: (A)

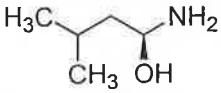
Q.33 The major product of the reaction is



(A) 

(B) 

(C) 

(D) 

Q.34 The correct statement(s) about Cr^{2+} and Mn^{3+} is(are)
[Atomic numbers of Cr = 24 and Mn = 25]

(A) Cr^{2+} is a reducing agent

(B) Mn^{3+} is an oxidizing agent

(C) Both Cr^{2+} and Mn^{3+} exhibit d^4 electronic configuration

(D) When Cr^{2+} is used as a reducing agent, the chromium ion attains d^5 electronic configuration

Q.35 Copper is purified by electrolytic refining of blister copper. The correct statement(s) about this process is (are)

(A) Impure Cu strip is used as cathode

(B) Acidified aqueous $CuSO_4$ is used as electrolyte

(C) Pure Cu deposits at cathode

(D) Impurities settle as anode-mud

Q.36 Fe^{3+} is reduced to Fe^{2+} by using

(A) H_2O_2 in presence of NaOH

(B) Na_2O_2 in water

(C) H_2O_2 in presence of H_2SO_4

(D) Na_2O_2 in presence of H_2SO_4

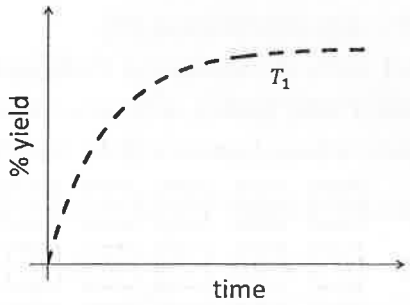
Answers for the above questions

Ans for Q.33: (C)	**Ans for Q.34: (A), (B) and (C)**
Ans for Q.35: (B), (C) and (D)	**Ans for Q.36: (A) and (B)**

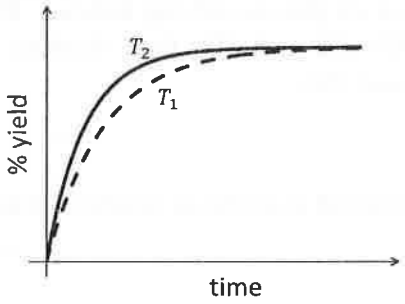
Q.37 The %yield of ammonia as a function of time in the reaction

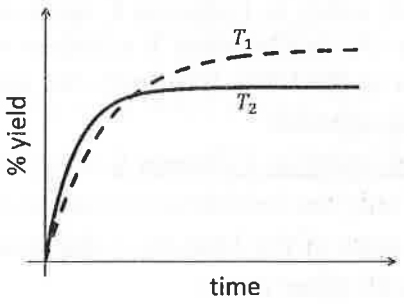
$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g), \Delta H < 0$

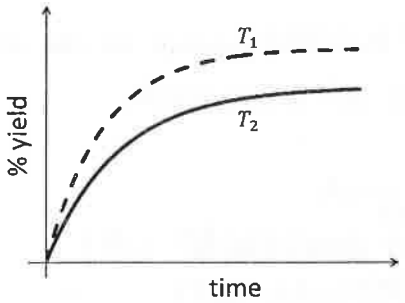
at (P, T_1) is given below.

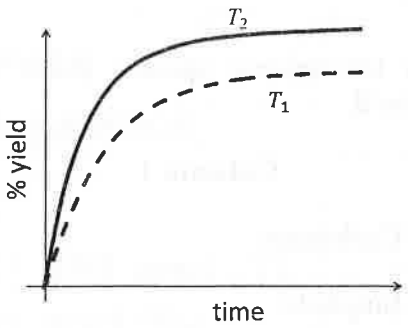


If this reaction is conducted at (P, T_2), with $T_2 > T_1$, the %yield of ammonia as a function of time is represented by

(A)


(B)


(C)


(D)


Q.38 If the unit cell of a mineral has cubic close packed (*ccp*) array of oxygen atoms with ***m*** fraction of octahedral holes occupied by aluminium ions and ***n*** fraction of tetrahedral holes occupied by magnesium ions, ***m*** and ***n***, respectively, are

(A) $\frac{1}{2}, \frac{1}{8}$ (B) $1, \frac{1}{4}$ (C) $\frac{1}{2}, \frac{1}{2}$ (D) $\frac{1}{4}, \frac{1}{8}$

Answers for the above questions

Ans for Q.37: (B)	Ans for Q.38: (A)

SECTION 3 (Maximum Marks: 16)

- This section contains **TWO** questions
- Each question contains two columns, **Column I** and **Column II**
- **Column I** has **four** entries (A), (B), (C) and (D)
- **Column II** has **five** entries (P), (Q), (R), (S) and (T)
- Match the entries in **Column I** with the entries in **Column II**
- One or more entries in **Column I** may match with one or more entries in **Column II**
- The ORS contains a 4×5 matrix whose layout will be similar to the one shown below:

(A)	(P)	(Q)	(R)	(S)	(T)
(B)	(P)	(Q)	(R)	(S)	(T)
(C)	(P)	(Q)	(R)	(S)	(T)
(D)	(P)	(Q)	(R)	(S)	(T)

- For each entry in **Column I**, darken the bubbles of all the matching entries. For example, if entry (A) in **Column I** matches with entries (Q), (R) and (T), then darken these three bubbles in the ORS. Similarly, for entries (B), (C) and (D).
- Marking scheme:

  For each entry in **Column I**,

  +2 If only the bubble(s) corresponding to all the correct match(es) is(are) darkened

  0 If none of the bubbles is darkened

  –1 In all other cases

Q.39 Match the anionic species given in Column I that are present in the ore(s) given in Column II.

Column I		Column II	
(A)	Carbonate	(P)	Siderite
(B)	Sulphide	(Q)	Malachite
(C)	Hydroxide	(R)	Bauxite
(D)	Oxide	(S)	Calamine
		(T)	Argentite

Answers for the above question

Ans for Q.39:

(A) Matches to (P), (Q) and (S)

(B) Matches to (T)

(C) Matches to (Q) and (R)

(D) Matches to (R)

Q.40 Match the thermodynamic processes given under Column I with the expressions given under Column II.

	Column I		Column II
(A)	Freezing of water at 273 K and 1 atm	(P)	$q = 0$
(B)	Expansion of 1 mol of an ideal gas into a vacuum under isolated conditions	(Q)	$w = 0$
(C)	Mixing of equal volumes of two ideal gases at constant temperature and pressure in an isolated container	(R)	$\Delta S_{sys} < 0$
(D)	Reversible heating of $H_2(g)$ at 1 atm from 300 K to 600 K, followed by reversible cooling to 300 K at 1 atm	(S)	$\Delta U = 0$
		(T)	$\Delta G = 0$

END OF PART II : CHEMISTRY

Answers for the above question

Ans for Q.40:
(A) Matches to (R) and (T)
(B) Matches to (P), (Q) and (S)
(C) Matches to (P), (Q) and (S)
(D) Matches to (P), (Q), (S) and (T)

PART III : MATHEMATICS

SECTION 1 (Maximum Marks: 32)

- This section contains **EIGHT** questions
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive
- For each question, darken the bubble corresponding to the correct integer in the ORS
- Marking scheme:
  +4 If the bubble corresponding to the answer is darkened
  0 In all other cases

Q.41 The number of distinct solutions of the equation

$$\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$$

in the interval $[0, 2\pi]$ is

Q.42 Let the curve C be the mirror image of the parabola $y^2 = 4x$ with respect to the line $x + y + 4 = 0$. If A and B are the points of intersection of C with the line $y = -5$, then the distance between A and B is

Q.43 The minimum number of times a fair coin needs to be tossed, so that the probability of getting at least two heads is at least 0.96, is

Q.44 Let n be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that all the girls stand consecutively in the queue. Let m be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that exactly four girls stand consecutively in the queue. Then the value of $\frac{m}{n}$ is

Q.45 If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x-3)^2 + (y+2)^2 = r^2$, then the value of r^2 is

Answers for the above questions

Ans for Q.41: 8	**Ans for Q.42: 4**	**Ans for Q.43: 8**
Ans for Q.44: 5	**Ans for Q.45: 2**	

Q.46 Let $f:\mathbb{R}\to\mathbb{R}$ be a function defined by $f(x)=\begin{cases}[x], & x\le 2\\ 0, & x>2\end{cases}$,

where $[x]$ is the greatest integer less than or equal to x. If $I=\int_{-1}^{2}\frac{xf(x^2)}{2+f(x+1)}dx$, then the value of $(4I-1)$ is

Q.47 A cylindrical container is to be made from certain solid material with the following constraints: It has a fixed inner volume of V mm^3, has a 2 mm thick solid wall and is open at the top. The bottom of the container is a solid circular disc of thickness 2 mm and is of radius equal to the outer radius of the container.

If the volume of the material used to make the container is minimum when the inner radius of the container is 10 mm, then the value of $\frac{V}{250\pi}$ is

Q.48 Let $F(x)=\int_{x}^{x^2+\frac{\pi}{6}}2\cos^2 t\,dt$ for all $x\in\mathbb{R}$ and $f:\left[0,\frac{1}{2}\right]\to[0,\infty)$ be a continuous function. For $a\in\left[0,\frac{1}{2}\right]$, if $F'(a)+2$ is the area of the region bounded by $x=0$, $y=0$, $y=f(x)$ and $x=a$, then $f(0)$ is

Answers for the above questions

Ans for Q.46: 0	**Ans for Q.47: 4**	**Ans for Q.48: 3**

SECTION 2 (Maximum Marks: 40)

- This section contains **TEN** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- Marking scheme:
  - +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened
  - 0 If none of the bubbles is darkened
  - –2 In all other cases

Q.49 Let X and Y be two arbitrary, 3×3, non-zero, skew-symmetric matrices and Z be an arbitrary 3×3, non-zero, symmetric matrix. Then which of the following matrices is (are) skew symmetric?

(A) $Y^3Z^4 - Z^4Y^3$ (B) $X^{44} + Y^{44}$

(C) $X^4Z^3 - Z^3X^4$ (D) $X^{23} + Y^{23}$

Q.50 Which of the following values of α satisfy the equation

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648\alpha \ ?$$

(A) –4 (B) 9 (C) –9 (D) 4

Q.51 In $\mathbb{R}^3$, consider the planes $P_1 : y = 0$ and $P_2 : x + z = 1$. Let P_3 be a plane, different from P_1 and P_2, which passes through the intersection of P_1 and P_2. If the distance of the point $(0, 1, 0)$ from P_3 is 1 and the distance of a point (α, β, γ) from P_3 is 2, then which of the following relations is (are) true?

(A) $2\alpha + \beta + 2\gamma + 2 = 0$ (B) $2\alpha - \beta + 2\gamma + 4 = 0$

(C) $2\alpha + \beta - 2\gamma - 10 = 0$ (D) $2\alpha - \beta + 2\gamma - 8 = 0$

Answers for the above questions

Ans for Q.49:	Ans for Q.50:	Ans for Q.51:
(C) and (D)	(B) and (C)	(B) and (D)

Q.52 In $\mathbb{R}^3$, let L be a straight line passing through the origin. Suppose that all the points on L are at a constant distance from the two planes $P_1 : x+2y-z+1=0$ and $P_2 : 2x-y+z-1=0$. Let M be the locus of the feet of the perpendiculars drawn from the points on L to the plane P_1. Which of the following points lie(s) on M ?

(A) $\left(0, -\frac{5}{6}, -\frac{2}{3}\right)$ (B) $\left(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{6}\right)$ (C) $\left(-\frac{5}{6}, 0, \frac{1}{6}\right)$ (D) $\left(-\frac{1}{3}, 0, \frac{2}{3}\right)$

Q.53 Let P and Q be distinct points on the parabola $y^2 = 2x$ such that a circle with PQ as diameter passes through the vertex O of the parabola. If P lies in the first quadrant and the area of the triangle ΔOPQ is $3\sqrt{2}$, then which of the following is (are) the coordinates of P ?

(A) $\left(4, 2\sqrt{2}\right)$ (B) $\left(9, 3\sqrt{2}\right)$ (C) $\left(\frac{1}{4}, \frac{1}{\sqrt{2}}\right)$ (D) $\left(1, \sqrt{2}\right)$

Q.54 Let $y(x)$ be a solution of the differential equation $(1+e^x)y' + ye^x = 1$. If $y(0) = 2$, then which of the following statements is (are) true?

(A) $y(-4) = 0$

(B) $y(-2) = 0$

(C) $y(x)$ has a critical point in the interval $(-1, 0)$

(D) $y(x)$ has no critical point in the interval $(-1, 0)$

Answers for the above questions

Ans for Q.52:	Ans for Q.53:	Ans for Q.54:
(A) and (B)	(A) and (D)	(A) and (C)

Q.55 Consider the family of all circles whose centers lie on the straight line $y = x$. If this family of circles is represented by the differential equation $Py'' + Qy' + 1 = 0$, where P, Q are functions of x, y and y' (here $y' = \frac{dy}{dx}$, $y'' = \frac{d^2y}{dx^2}$), then which of the following statements is (are) true?

(A) $P = y + x$ (B) $P = y - x$

(C) $P + Q = 1 - x + y + y' + (y')^2$ (D) $P - Q = x + y - y' - (y')^2$

Q.56 Let $g : \mathbb{R} \to \mathbb{R}$ be a differentiable function with $g(0) = 0$, $g'(0) = 0$ and $g'(1) \neq 0$. Let

$$f(x) = \begin{cases} \frac{x}{|x|} g(x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

and $h(x) = e^{|x|}$ for all $x \in \mathbb{R}$. Let $(f \circ h)(x)$ denote $f(h(x))$ and $(h \circ f)(x)$ denote $h(f(x))$. Then which of the following is (are) true?

(A) f is differentiable at $x = 0$ (B) h is differentiable at $x = 0$

(C) $f \circ h$ is differentiable at $x = 0$ (D) $h \circ f$ is differentiable at $x = 0$

Answers for the above questions

Ans for Q.55: (B) and (C)	Ans for Q.56: (A) and (D)

Q.57 Let $f(x)=\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$ for all $x\in\mathbb{R}$ and $g(x)=\frac{\pi}{2}\sin x$ for all $x\in\mathbb{R}$. Let $(f\circ g)(x)$ denote $f(g(x))$ and $(g\circ f)(x)$ denote $g(f(x))$. Then which of the following is (are) true?

(A) Range of f is $\left[-\frac{1}{2},\frac{1}{2}\right]$

(B) Range of $f\circ g$ is $\left[-\frac{1}{2},\frac{1}{2}\right]$

(C) $\lim\limits_{x\to 0}\frac{f(x)}{g(x)}=\frac{\pi}{6}$

(D) There is an $x\in\mathbb{R}$ such that $(g\circ f)(x)=1$

Q.58 Let ΔPQR be a triangle. Let $\vec{a}=\overrightarrow{QR}$, $\vec{b}=\overrightarrow{RP}$ and $\vec{c}=\overrightarrow{PQ}$. If $|\vec{a}|=12$, $|\vec{b}|=4\sqrt{3}$ and $\vec{b}.\vec{c}=24$, then which of the following is (are) true?

(A) $\frac{|\vec{c}|^2}{2}-|\vec{a}|=12$

(B) $\frac{|\vec{c}|^2}{2}+|\vec{a}|=30$

(C) $|\vec{a}\times\vec{b}+\vec{c}\times\vec{a}|=48\sqrt{3}$

(D) $\vec{a}.\vec{b}=-72$

Answers for the above questions

Ans for Q.57: (A), (B) and (C)	Ans for Q.58: (A), (C) and (D)

SECTION 3 (Maximum Marks: 16)

- This section contains **TWO** questions
- Each question contains two columns, **Column I** and **Column II**
- **Column I** has **four** entries (A), (B), (C) and (D)
- **Column II** has **five** entries (P), (Q), (R), (S) and (T)
- Match the entries in **Column I** with the entries in **Column II**
- One or more entries in **Column I** may match with one or more entries in **Column II**
- The ORS contains a 4×5 matrix whose layout will be similar to the one shown below:

(A)	(P)	(Q)	(R)	(S)	(T)
(B)	(P)	(Q)	(R)	(S)	(T)
(C)	(P)	(Q)	(R)	(S)	(T)
(D)	(P)	(Q)	(R)	(S)	(T)

- For each entry in **Column I**, darken the bubbles of all the matching entries. For example, if entry (A) in **Column I** matches with entries (Q), (R) and (T), then darken these three bubbles in the ORS. Similarly, for entries (B), (C) and (D).
- Marking scheme:

  For each entry in **Column I**,

  +2 If only the bubble(s) corresponding to all the correct match(es) is(are) darkened

  0 If none of the bubbles is darkened

  –1 In all other cases

Space for rough work

Q.59

Column I		Column II	
(A)	In $\mathbb{R}^2$, if the magnitude of the projection vector of the vector $\alpha\hat{i}+\beta\hat{j}$ on $\sqrt{3}\hat{i}+\hat{j}$ is $\sqrt{3}$ and if $\alpha = 2+\sqrt{3}\beta$, then possible value(s) of $\lvert\alpha\rvert$ is (are)	(P)	1
(B)	Let a and b be real numbers such that the function $f(x)=\begin{cases}-3ax^2-2, & x<1\\ bx+a^2, & x\geq 1\end{cases}$ is differentiable for all $x\in\mathbb{R}$. Then possible value(s) of a is (are)	(Q)	2
(C)	Let $\omega\neq 1$ be a complex cube root of unity. If $(3-3\omega+2\omega^2)^{4n+3}+(2+3\omega-3\omega^2)^{4n+3}+(-3+2\omega+3\omega^2)^{4n+3}=0$, then possible value(s) of n is (are)	(R)	3
(D)	Let the harmonic mean of two positive real numbers a and b be 4. If q is a positive real number such that $a, 5, q, b$ is an arithmetic progression, then the value(s) of $\lvert q-a\rvert$ is (are)	(S)	4
		(T)	5

Answers for the above question

Ans for Q.59:

(A) Matches to (P) and (Q)
(B) Matches to (P) and (Q)
(C) Matches to (P), (Q), (S) and (T)
(D) Matches to (Q) and (T)

Q.60

Column I		Column II	
(A)	In a triangle ΔXYZ, let a, b and c be the lengths of the sides opposite to the angles X, Y and Z, respectively. If $2(a^2-b^2)=c^2$ and $\lambda=\frac{\sin(X-Y)}{\sin Z}$, then possible values of n for which $\cos(n\pi\lambda)=0$ is (are)	(P)	1
(B)	In a triangle ΔXYZ, let a, b and c be the lengths of the sides opposite to the angles X, Y and Z, respectively. If $1+\cos 2X-2\cos 2Y=2\sin X\sin Y$, then possible value(s) of $\frac{a}{b}$ is (are)	(Q)	2
(C)	In $\mathbb{R}^2$, let $\sqrt{3}\hat{i}+\hat{j}$, $\hat{i}+\sqrt{3}\hat{j}$ and $\beta\hat{i}+(1-\beta)\hat{j}$ be the position vectors of X, Y and Z with respect to the origin O, respectively. If the distance of Z from the bisector of the acute angle of $\overrightarrow{OX}$ with $\overrightarrow{OY}$ is $\frac{3}{\sqrt{2}}$, then possible value(s) of $\lvert\beta\rvert$ is (are)	(R)	3
(D)	Suppose that $F(\alpha)$ denotes the area of the region bounded by $x=0$, $x=2$, $y^2=4x$ and $y=\lvert\alpha x-1\rvert+\lvert\alpha x-2\rvert+\alpha x$, where $\alpha\in\{0,1\}$. Then the value(s) of $F(\alpha)+\frac{8}{3}\sqrt{2}$, when $\alpha=0$ and $\alpha=1$, is (are)	(S)	5
		(T)	6

Answers for the above question

Ans for Q.60:

(A) Matches to (P), (R) and (S)

(B) Matches to (P)

(C) Matches to (P) and (Q)

(D) Matches to (S) and (T)

JEE (ADVANCED) 2015

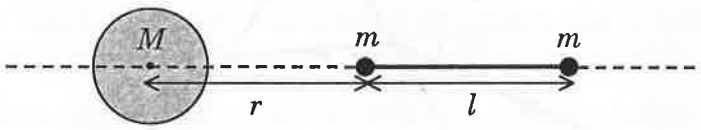
Answer Keys for Paper 2

PART I : PHYSICS

SECTION 1 (Maximum Marks: 32)

- This section contains **EIGHT** questions
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive
- For each question, darken the bubble corresponding to the correct integer in the ORS
- Marking scheme:
  - +4 If the bubble corresponding to the answer is darkened
  - 0 In all other cases

Q.1 A large spherical mass M is fixed at one position and two identical point masses m are kept on a line passing through the centre of M (see figure). The point masses are connected by a rigid massless rod of length l and this assembly is free to move along the line connecting them. All three masses interact only through their mutual gravitational interaction. When the point mass nearer to M is at a distance $r = 3l$ from M, the tension in the rod is zero for $m = k\left(\frac{M}{288}\right)$. The value of k is



Q.2 The energy of a system as a function of time t is given as $E(t) = A^2 \exp(-\alpha t)$, where $\alpha = 0.2\ \text{s}^{-1}$. The measurement of A has an error of 1.25%. If the error in the measurement of time is 1.50%, the percentage error in the value of $E(t)$ at $t = 5$ s is

Q.3 The densities of two solid spheres A and B of the same radii R vary with radial distance r as $\rho_A(r) = k\left(\frac{r}{R}\right)$ and $\rho_B(r) = k\left(\frac{r}{R}\right)^5$, respectively, where k is a constant. The moments of inertia of the individual spheres about axes passing through their centres are I_A and I_B, respectively. If $\frac{I_B}{I_A} = \frac{n}{10}$, the value of n is

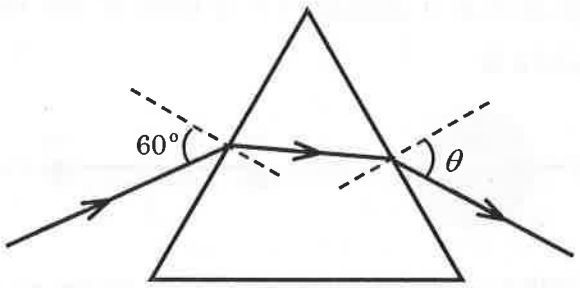
Answers for the above questions

Ans for Q.1: 7	**Ans for Q.2: 4**	**Ans for Q.3: 6**

Q.4 Four harmonic waves of equal frequencies and equal intensities I_0 have phase angles 0, $\pi/3$, $2\pi/3$ and π. When they are superposed, the intensity of the resulting wave is nI_0. The value of n is

Q.5 For a radioactive material, its activity A and rate of change of its activity R are defined as $A = -\frac{dN}{dt}$ and $R = -\frac{dA}{dt}$, where $N(t)$ is the number of nuclei at time t. Two radioactive sources P (mean life τ) and Q (mean life 2τ) have the same activity at $t = 0$. Their rates of change of activities at $t = 2\tau$ are R_P and R_Q, respectively. If $\frac{R_P}{R_Q} = \frac{n}{e}$, then the value of n is

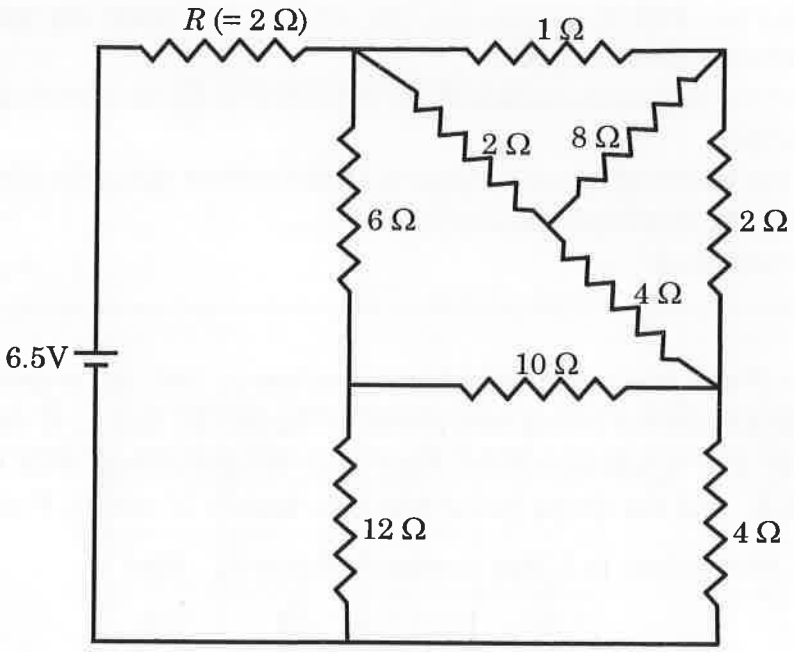
Q.6 A monochromatic beam of light is incident at 60° on one face of an equilateral prism of refractive index n and emerges from the opposite face making an angle $\theta(n)$ with the normal (see the figure). For $n = \sqrt{3}$ the value of θ is 60° and $\frac{d\theta}{dn} = m$. The value of m is



Answers for the above questions

Ans for Q.4: 3	Ans for Q.5: 2	Ans for Q.6: 2

Q.7 In the following circuit, the current through the resistor R (= 2Ω) is I Amperes. The value of I is



Q.8 An electron in an excited state of Li^{2+} ion has angular momentum $3h/2\pi$. The de Broglie wavelength of the electron in this state is $p\pi a_0$ (where a_0 is the Bohr radius). The value of p is

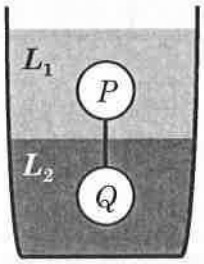
Answers for the above questions

Ans for Q.7: 1	Ans for Q.8: 2

SECTION 2 (Maximum Marks : 32)

- This section contains **EIGHT** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- Marking scheme :
  - +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened
  - 0 If none of the bubbles is darkened
  - –2 In all other cases

Q.9 Two spheres P and Q of equal radii have densities ρ_1 and ρ_2, respectively. The spheres are connected by a massless string and placed in liquids L_1 and L_2 of densities σ_1 and σ_2 and viscosities η_1 and η_2, respectively. They float in equilibrium with the sphere P in L_1 and sphere Q in L_2 and the string being taut (see figure). If sphere P alone in L_2 has terminal velocity $\vec{V}_P$ and Q alone in L_1 has terminal velocity $\vec{V}_Q$, then



(A) $\frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_1}{\eta_2}$ (B) $\frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_2}{\eta_1}$ (C) $\vec{V}_P \cdot \vec{V}_Q > 0$ (D) $\vec{V}_P \cdot \vec{V}_Q < 0$

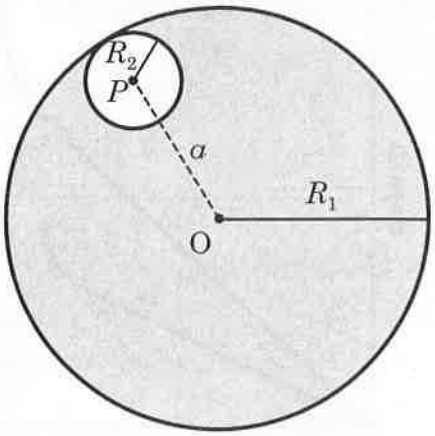
Q.10 In terms of potential difference V, electric current I, permittivity ε_0, permeability μ_0 and speed of light c, the dimensionally correct equation(s) is(are)

(A) $\mu_0 I^2 = \varepsilon_0 V^2$ (B) $\varepsilon_0 I = \mu_0 V$ (C) $I = \varepsilon_0 c V$ (D) $\mu_0 c\, I = \varepsilon_0 V$

Answers for the above questions

Ans for Q.9: (A) and (D)	Ans for Q.10: (A) and (C)

Q.11 Consider a uniform spherical charge distribution of radius R_1 centred at the origin O. In this distribution, a spherical cavity of radius R_2, centred at P with distance $OP = a = R_1 - R_2$ (see figure) is made. If the electric field inside the cavity at position $\vec{r}$ is $\vec{E}(\vec{r})$, then the correct statement(s) is(are)



(A) $\vec{E}$ is uniform, its magnitude is independent of R_2 but its direction depends on $\vec{r}$

(B) $\vec{E}$ is uniform, its magnitude depends on R_2 and its direction depends on $\vec{r}$

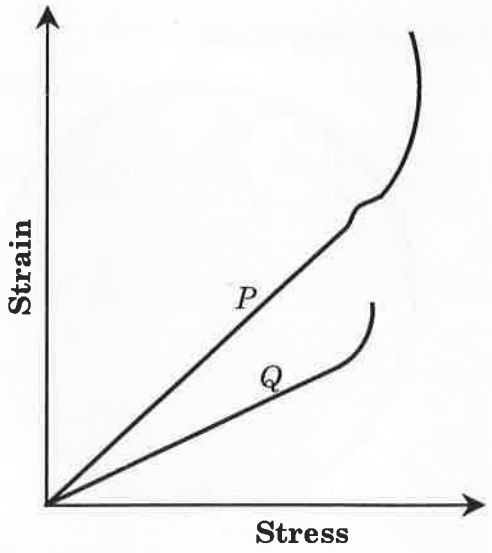
(C) $\vec{E}$ is uniform, its magnitude is independent of a but its direction depends on $\vec{a}$

(D) $\vec{E}$ is uniform and both its magnitude and direction depend on $\vec{a}$

Answer for the above question

Ans for Q.11: (D)

Q.12 In plotting stress versus strain curves for two materials P and Q, a student by mistake puts strain on the y-axis and stress on the x-axis as shown in the figure. Then the correct statement(s) is(are)



(A) P has more tensile strength than Q
(B) P is more ductile than Q
(C) P is more brittle than Q
(D) The Young's modulus of P is more than that of Q

Answer for the above question

Ans for Q.12: (A) and (B)

Q.13 A spherical body of radius R consists of a fluid of constant density and is in equilibrium under its own gravity. If $P(r)$ is the pressure at $r(r < R)$, then the correct option(s) is(are)

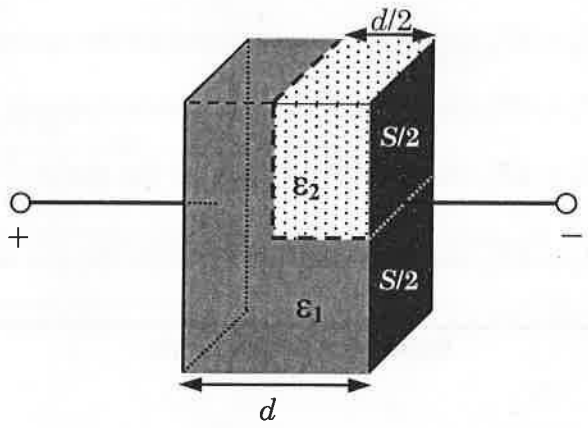
(A) $P(r=0)=0$

(B) $\dfrac{P(r=3R/4)}{P(r=2R/3)}=\dfrac{63}{80}$

(C) $\dfrac{P(r=3R/5)}{P(r=2R/5)}=\dfrac{16}{21}$

(D) $\dfrac{P(r=R/2)}{P(r=R/3)}=\dfrac{20}{27}$

Q.14 A parallel plate capacitor having plates of area S and plate separation d, has capacitance C_1 in air. When two dielectrics of different relative permittivities ($\varepsilon_1 = 2$ and $\varepsilon_2 = 4$) are introduced between the two plates as shown in the figure, the capacitance becomes C_2. The ratio $\dfrac{C_2}{C_1}$ is

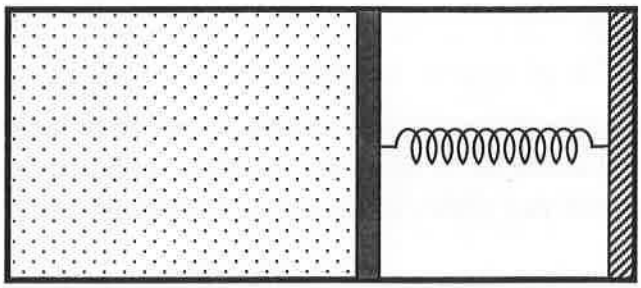


(A) 6/5 (B) 5/3 (C) 7/5 (D) 7/3

Answers for the above questions

Ans for Q.13: (B) and (C)	Ans for Q.14: (D)

Q.15 An ideal monoatomic gas is confined in a horizontal cylinder by a spring loaded piston (as shown in the figure). Initially the gas is at temperature T_1, pressure P_1 and volume V_1 and the spring is in its relaxed state. The gas is then heated very slowly to temperature T_2, pressure P_2 and volume V_2. During this process the piston moves out by a distance x. Ignoring the friction between the piston and the cylinder, the correct statement(s) is(are)



(A) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the energy stored in the spring is $\frac{1}{4}P_1V_1$

(B) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the change in internal energy is $3P_1V_1$

(C) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the work done by the gas is $\frac{7}{3}P_1V_1$

(D) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the heat supplied to the gas is $\frac{17}{6}P_1V_1$

Answer for the above question

Ans for Q.15: (A), (B) and (C)

Q.16 A fission reaction is given by ${}^{236}_{92}U \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + x + y$, where x and y are two particles. Considering ${}^{236}_{92}U$ to be at rest, the kinetic energies of the products are denoted by K_{Xe}, K_{Sr}, K_x (2 MeV) and K_y (2 MeV), respectively. Let the binding energies per nucleon of ${}^{236}_{92}U$, ${}^{140}_{54}Xe$ and ${}^{94}_{38}Sr$ be 7.5 MeV, 8.5 MeV and 8.5 MeV, respectively. Considering different conservation laws, the correct option(s) is(are)

(A) $x = n$, $y = n$, $K_{Sr} = 129$ MeV, $K_{Xe} = 86$ MeV

(B) $x = p$, $y = e^-$, $K_{Sr} = 129$ MeV, $K_{Xe} = 86$ MeV

(C) $x = p$, $y = n$, $K_{Sr} = 129$ MeV, $K_{Xe} = 86$ MeV

(D) $x = n$, $y = n$, $K_{Sr} = 86$ MeV, $K_{Xe} = 129$ MeV

Answer for the above question

Ans for Q.16: (A)

SECTION 3 (Maximum Marks : 16)

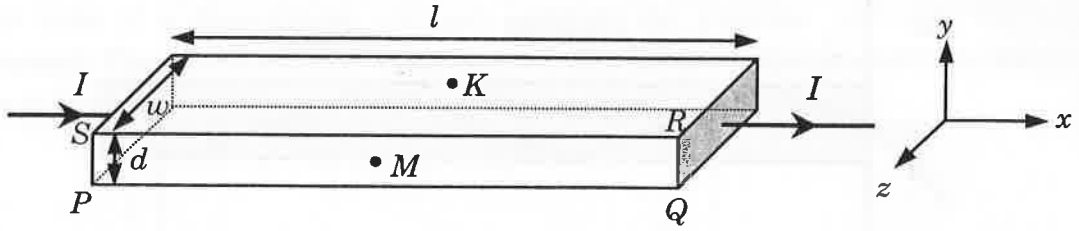
- This section contains **TWO** paragraphs
- Based on each paragraph, there will be **TWO** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- Marking scheme :
  - +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened
  - 0 If none of the bubbles is darkened
  - –2 In all other cases

Space for rough work

PARAGRAPH 1

In a thin rectangular metallic strip a constant current I flows along the positive x-direction, as shown in the figure. The length, width and thickness of the strip are l, w and d, respectively.

A uniform magnetic field $\vec{B}$ is applied on the strip along the positive y-direction. Due to this, the charge carriers experience a net deflection along the z-direction. This results in accumulation of charge carriers on the surface $PQRS$ and appearance of equal and opposite charges on the face opposite to $PQRS$. A potential difference along the z-direction is thus developed. Charge accumulation continues until the magnetic force is balanced by the electric force. The current is assumed to be uniformly distributed on the cross section of the strip and carried by electrons.



Q.17 Consider two different metallic strips (1 and 2) of the same material. Their lengths are the same, widths are w_1 and w_2 and thicknesses are d_1 and d_2, respectively. Two points K and M are symmetrically located on the opposite faces parallel to the x-y plane (see figure). V_1 and V_2 are the potential differences between K and M in strips 1 and 2, respectively. Then, for a given current I flowing through them in a given magnetic field strength B, the correct statement(s) is(are)

(A) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = 2V_1$

(B) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = V_1$

(C) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = 2V_1$

(D) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = V_1$

Q.18 Consider two different metallic strips (1 and 2) of same dimensions (length l, width w and thickness d) with carrier densities n_1 and n_2, respectively. Strip 1 is placed in magnetic field B_1 and strip 2 is placed in magnetic field B_2, both along positive y-directions. Then V_1 and V_2 are the potential differences developed between K and M in strips 1 and 2, respectively. Assuming that the current I is the same for both the strips, the correct option(s) is(are)

(A) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = 2V_1$

(B) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = V_1$

(C) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = 0.5V_1$

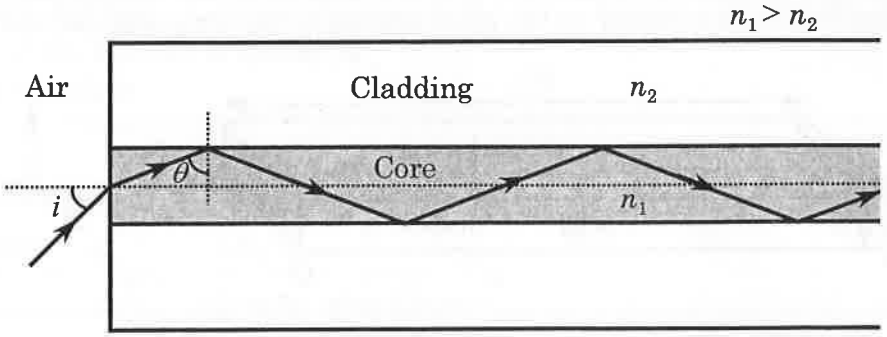
(D) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = V_1$

Answers for the above questions

Ans for Q.17: (A) and (D)	Ans for Q.18: (A) and (C)

PARAGRAPH 2

Light guidance in an optical fiber can be understood by considering a structure comprising of thin solid glass cylinder of refractive index n_1 surrounded by a medium of lower refractive index n_2. The light guidance in the structure takes place due to successive total internal reflections at the interface of the media n_1 and n_2 as shown in the figure. All rays with the angle of incidence i less than a particular value i_m are confined in the medium of refractive index n_1. The numerical aperture (NA) of the structure is defined as $\sin i_m$.



Q.19 For two structures namely S_1 with $n_1 = \sqrt{45}/4$ and $n_2 = 3/2$, and S_2 with $n_1 = 8/5$ and $n_2 = 7/5$ and taking the refractive index of water to be 4/3 and that of air to be 1, the correct option(s) is(are)

(A) NA of S_1 immersed in water is the same as that of S_2 immersed in a liquid of refractive index $\frac{16}{3\sqrt{15}}$

(B) NA of S_1 immersed in liquid of refractive index $\frac{6}{\sqrt{15}}$ is the same as that of S_2 immersed in water

(C) NA of S_1 placed in air is the same as that of S_2 immersed in liquid of refractive index $\frac{4}{\sqrt{15}}$.

(D) NA of S_1 placed in air is the same as that of S_2 placed in water

Q.20 If two structures of same cross-sectional area, but different numerical apertures NA_1 and NA_2 ($NA_2 < NA_1$) are joined longitudinally, the numerical aperture of the combined structure is

(A) $\frac{NA_1 NA_2}{NA_1 + NA_2}$ (B) $NA_1 + NA_2$ (C) NA_1 (D) NA_2

Answers for the above questions

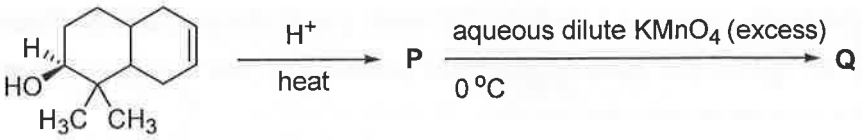
Ans for Q.19: (A) and (C)	Ans for Q.20: (D)

PART II : CHEMISTRY

SECTION 1 (Maximum Marks: 32)

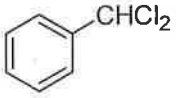
- This section contains **EIGHT** questions
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive
- For each question, darken the bubble corresponding to the correct integer in the ORS
- Marking scheme:
  - +4 If the bubble corresponding to the answer is darkened
  - 0 In all other cases

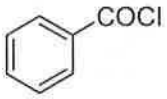
Q.21 The number of hydroxyl group(s) in **Q** is

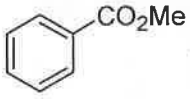


Q.22 Among the following, the number of reaction(s) that produce(s) benzaldehyde is

I. 

II. 

III. 

IV. 

Q.23 In the complex acetylbromidodicarbonylbis(triethylphosphine)iron(II), the number of Fe–C bond(s) is

Answers for the above questions

Ans for Q.21: 4	**Ans for Q.22: 4**	**Ans for Q.23: 3**

Q.24 Among the complex ions, $[Co(NH_2\text{-}CH_2\text{-}CH_2\text{-}NH_2)_2Cl_2]^+$, $[CrCl_2(C_2O_4)_2]^{3-}$, $[Fe(H_2O)_4(OH)_2]^+$, $[Fe(NH_3)_2(CN)_4]^-$, $[Co(NH_2\text{-}CH_2\text{-}CH_2\text{-}NH_2)_2(NH_3)Cl]^{2+}$ and $[Co(NH_3)_4(H_2O)Cl]^{2+}$, the number of complex ion(s) that show(s) *cis-trans* isomerism is

Q.25 Three moles of B_2H_6 are completely reacted with methanol. The number of moles of boron containing product formed is

Q.26 The molar conductivity of a solution of a weak acid HX (0.01 M) is 10 times smaller than the molar conductivity of a solution of a weak acid HY (0.10 M). If $\lambda^0_{X^-} \approx \lambda^0_{Y^-}$, the difference in their pK_a values, $pK_a(HX) - pK_a(HY)$, is (consider degree of ionization of both acids to be $\ll 1$)

Q.27 A closed vessel with rigid walls contains 1 mol of $^{238}_{92}U$ and 1 mol of air at 298 K. Considering complete decay of $^{238}_{92}U$ to $^{206}_{82}Pb$, the ratio of the final pressure to the initial pressure of the system at 298 K is

Q.28 In dilute aqueous H_2SO_4, the complex diaquodioxalatoferrate(II) is oxidized by MnO_4^-. For this reaction, the ratio of the rate of change of $[H^+]$ to the rate of change of $[MnO_4^-]$ is

Answers for the above questions

Ans for Q.24: 6	Ans for Q.25: 6	Ans for Q.26: 3
Ans for Q.27: 9	Ans for Q.28: 8	

SECTION 2 (Maximum Marks: 32)

- This section contains **EIGHT** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are)correct
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
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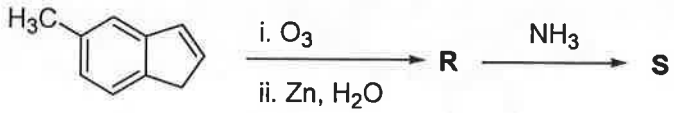
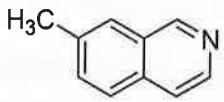
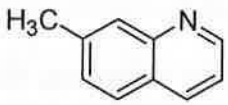
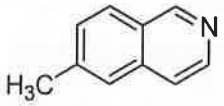
  - 0 If none of the bubbles is darkened
  - –2 In all other cases

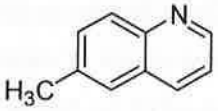
Q.29 In the following reactions, the product **S** is



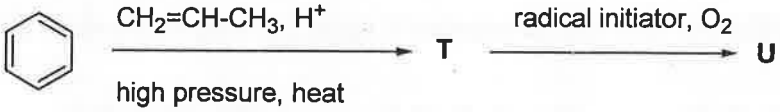
(A) 

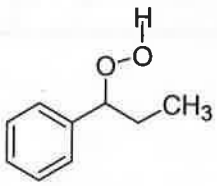
(B) 

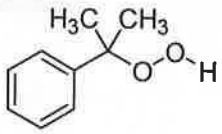
(C) 

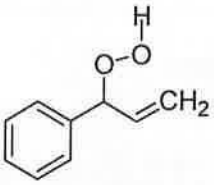
(D) 

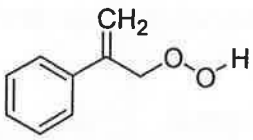
Q.30 The major product **U** in the following reactions is



(A) 

(B) 

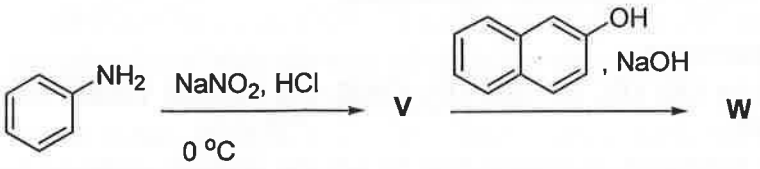
(C) 

(D) 

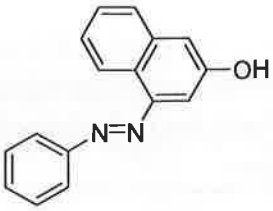
Answers for the above questions

Ans for Q.29: (A)	Ans for Q.30: (B)

Q.31 In the following reactions, the major product W is



(A) 

(B) 

(C) 

(D) 

Q.32 The correct statement(s) regarding, (i) $HClO$, (ii) $HClO_2$, (iii) $HClO_3$ and (iv) $HClO_4$, is(are)

(A) The number of Cl=O bonds in (ii) and (iii) together is two

(B) The number of lone pairs of electrons on Cl in (ii) and (iii) together is three

(C) The hybridization of Cl in (iv) is sp^3

(D) Amongst (i) to (iv), the strongest acid is (i)

Q.33 The pair(s) of ions where BOTH the ions are precipitated upon passing H_2S gas in presence of dilute HCl, is(are)

(A) Ba^{2+}, Zn^{2+} (B) Bi^{3+}, Fe^{3+} (C) Cu^{2+}, Pb^{2+} (D) Hg^{2+}, Bi^{3+}

Answers for the above questions

Ans for Q.31:	Ans for Q.32:	Ans for Q.33:
(A)	(B) and (C)	(C) and (D)

Q.34 Under hydrolytic conditions, the compounds used for preparation of linear polymer and for chain termination, respectively, are

(A) CH_3SiCl_3 and $Si(CH_3)_4$ (B) $(CH_3)_2SiCl_2$ and $(CH_3)_3SiCl$

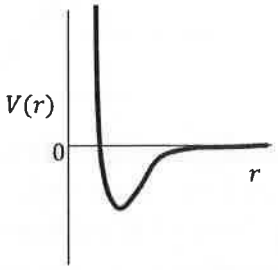
(C) $(CH_3)_2SiCl_2$ and CH_3SiCl_3 (D) $SiCl_4$ and $(CH_3)_3SiCl$

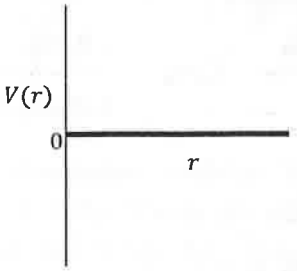
Q.35 When O_2 is adsorbed on a metallic surface, electron transfer occurs from the metal to O_2. The **TRUE** statement(s) regarding this adsorption is(are)

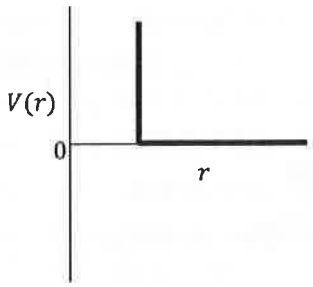
(A) O_2 is physisorbed (B) heat is released

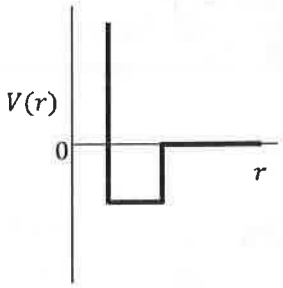
(C) occupancy of π^*_{2p} of O_2 is increased (D) bond length of O_2 is increased

Q.36 One mole of a monoatomic real gas satisfies the equation $p(V-b)=RT$ where b is a constant. The relationship of interatomic potential $V(r)$ and interatomic distance r for the gas is given by

(A)


(B)


(C)


(D)


Answers for the above questions

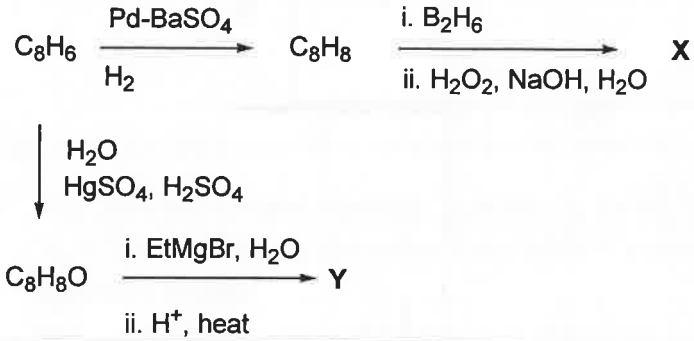
Ans for Q.34:	Ans for Q.35:	Ans for Q.36:
(B)	(B), (C) and (D)	(C)

SECTION 3 (Maximum Marks: 16)

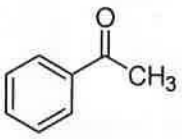
- This section contains **TWO** paragraphs
- Based on each paragraph, there will be **TWO** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- Marking scheme:
  - +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened
  - 0 If none of the bubbles is darkened
  - –2 In all other cases

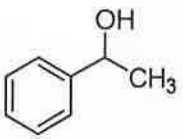
PARAGRAPH 1

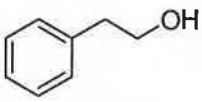
In the following reactions

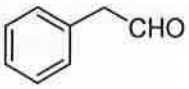


Q.37 Compound **X** is

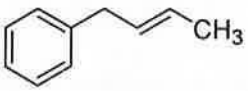
(A) 

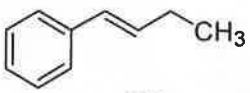
(B) 

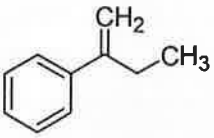
(C) 

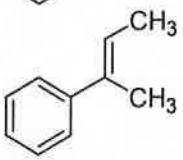
(D) 

Q.38 The major compound **Y** is

(A) 

(B) 

(C) 

(D) 

Answers for the above questions

Ans for Q.37: (C)	Ans for Q.38: (D)

PARAGRAPH 2

When 100 mL of 1.0 M HCl was mixed with 100 mL of 1.0 M NaOH in an insulated beaker at constant pressure, a temperature increase of 5.7 °C was measured for the beaker and its contents (**Expt. 1**). Because the enthalpy of neutralization of a strong acid with a strong base is a constant (–57.0 kJ mol^{-1}), this experiment could be used to measure the calorimeter constant.

In a second experiment (**Expt. 2**), 100 mL of 2.0 M acetic acid ($K_a = 2.0 \times 10^{-5}$) was mixed with 100 mL of 1.0 M NaOH (under identical conditions to **Expt. 1**) where a temperature rise of 5.6 °C was measured.

(Consider heat capacity of all solutions as 4.2 J g^{-1} K^{-1} and density of all solutions as 1.0 g mL^{-1})

Q.39 Enthalpy of dissociation (in kJ mol^{-1}) of acetic acid obtained from the **Expt. 2** is

(A) 1.0 (B) 10.0 (C) 24.5 (D) 51.4

Q.40 The pH of the solution after **Expt. 2** is

(A) 2.8 (B) 4.7 (C) 5.0 (D) 7.0

END OF PART II : CHEMISTRY

Answers for the above questions

Ans for Q.39: (A)	Ans for Q.40: (B)

PART III : MATHEMATICS

SECTION 1 (Maximum Marks: 32)

- This section contains **EIGHT** questions
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive
- For each question, darken the bubble corresponding to the correct integer in the ORS
- Marking scheme:
  - +4 If the bubble corresponding to the answer is darkened
  - 0 In all other cases

Q.41 For any integer k, let $\alpha_k = \cos\left(\frac{k\pi}{7}\right) + i\sin\left(\frac{k\pi}{7}\right)$, where $i = \sqrt{-1}$. The value of the expression

$$\frac{\sum_{k=1}^{12} |\alpha_{k+1} - \alpha_k|}{\sum_{k=1}^{3} |\alpha_{4k-1} - \alpha_{4k-2}|}$$ is

Q.42 Suppose that all the terms of an arithmetic progression (A.P.) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is 6 : 11 and the seventh term lies in between 130 and 140, then the common difference of this A.P. is

Q.43 The coefficient of x^9 in the expansion of $(1+x)(1+x^2)(1+x^3)\ldots(1+x^{100})$ is

Answers for the above questions

Ans for Q.41: 4	Ans for Q.42: 9	Ans for Q.43: 8

Q.44 Suppose that the foci of the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ are $(f_1, 0)$ and $(f_2, 0)$ where $f_1>0$ and $f_2<0$. Let P_1 and P_2 be two parabolas with a common vertex at (0, 0) and with foci at $(f_1, 0)$ and $(2f_2, 0)$, respectively. Let T_1 be a tangent to P_1 which passes through $(2f_2, 0)$ and T_2 be a tangent to P_2 which passes through $(f_1, 0)$. If m_1 is the slope of T_1 and m_2 is the slope of T_2, then the value of $\left(\frac{1}{m_1^2}+m_2^2\right)$ is

Q.45 Let m and n be two positive integers greater than 1. If

$$\lim_{\alpha\to 0}\left(\frac{e^{\cos(\alpha^n)}-e}{\alpha^m}\right)=-\left(\frac{e}{2}\right)$$

then the value of $\frac{m}{n}$ is

Q.46 If

$$\alpha=\int_0^1 (e^{9x+3\tan^{-1}x})\left(\frac{12+9x^2}{1+x^2}\right)dx$$

where $\tan^{-1}x$ takes only principal values, then the value of $\left(\log_e|1+\alpha|-\frac{3\pi}{4}\right)$ is

Answers for the above questions

Ans for Q.44: 4	Ans for Q.45: 2	Ans for Q.46: 9

Q.47 Let $f:\mathbb{R}\to\mathbb{R}$ be a continuous odd function, which vanishes exactly at one point and $f(1)=\frac{1}{2}$. Suppose that $F(x)=\int_{-1}^{x} f(t)\,dt$ for all $x\in[-1,2]$ and $G(x)=\int_{-1}^{x} t\,|f(f(t))|\,dt$ for all $x\in[-1,2]$. If $\lim_{x\to 1}\frac{F(x)}{G(x)}=\frac{1}{14}$, then the value of $f\left(\frac{1}{2}\right)$ is

Q.48 Suppose that $\vec{p}$, $\vec{q}$ and $\vec{r}$ are three non-coplanar vectors in $\mathbb{R}^3$. Let the components of a vector $\vec{s}$ along $\vec{p}$, $\vec{q}$ and $\vec{r}$ be 4, 3 and 5, respectively. If the components of this vector $\vec{s}$ along $(-\vec{p}+\vec{q}+\vec{r})$, $(\vec{p}-\vec{q}+\vec{r})$ and $(-\vec{p}-\vec{q}+\vec{r})$ are x, y and z, respectively, then the value of $2x+y+z$ is

Answers for the above questions

Ans for Q.47: 7	Ans for Q.48: 9 However, considering the level of the question, +4 marks will be awarded to ALL the candidates.

SECTION 2 (Maximum Marks: 32)

- This section contains **EIGHT** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- Marking scheme:
  - +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened
  - 0 If none of the bubbles is darkened
  - –2 In all other cases

Q.49 Let S be the set of all non-zero real numbers α such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots x_1 and x_2 satisfying the inequality $|x_1 - x_2| < 1$. Which of the following intervals is(are) a subset(s) of S?

(A) $\left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right)$ (B) $\left(-\frac{1}{\sqrt{5}}, 0\right)$ (C) $\left(0, \frac{1}{\sqrt{5}}\right)$ (D) $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

Q.50 If $\alpha = 3\sin^{-1}\left(\frac{6}{11}\right)$ and $\beta = 3\cos^{-1}\left(\frac{4}{9}\right)$, where the inverse trigonometric functions take only the principal values, then the correct option(s) is(are)

(A) $\cos\beta > 0$ (B) $\sin\beta < 0$ (C) $\cos(\alpha+\beta) > 0$ (D) $\cos\alpha < 0$

Answers for the above questions

Ans for Q.49: (A) and (D)	Ans for Q.50: (B), (C) and (D)

Q.51 Let E_1 and E_2 be two ellipses whose centers are at the origin. The major axes of E_1 and E_2 lie along the x-axis and the y-axis, respectively. Let S be the circle $x^2+(y-1)^2=2$. The straight line $x+y=3$ touches the curves S, E_1 and E_2 at P, Q and R, respectively. Suppose that $PQ=PR=\frac{2\sqrt{2}}{3}$. If e_1 and e_2 are the eccentricities of E_1 and E_2, respectively, then the correct expression(s) is(are)

(A) $e_1^2+e_2^2=\frac{43}{40}$ (B) $e_1 e_2=\frac{\sqrt{7}}{2\sqrt{10}}$ (C) $\left|e_1^2-e_2^2\right|=\frac{5}{8}$ (D) $e_1 e_2=\frac{\sqrt{3}}{4}$

Q.52 Consider the hyperbola $H: x^2-y^2=1$ and a circle S with center $N(x_2,0)$. Suppose that H and S touch each other at a point $P(x_1,y_1)$ with $x_1>1$ and $y_1>0$. The common tangent to H and S at P intersects the x-axis at point M. If (l, m) is the centroid of the triangle ΔPMN, then the correct expression(s) is(are)

(A) $\frac{dl}{dx_1}=1-\frac{1}{3x_1^2}$ for $x_1>1$ (B) $\frac{dm}{dx_1}=\frac{x_1}{3\left(\sqrt{x_1^2-1}\right)}$ for $x_1>1$

(C) $\frac{dl}{dx_1}=1+\frac{1}{3x_1^2}$ for $x_1>1$ (D) $\frac{dm}{dy_1}=\frac{1}{3}$ for $y_1>0$

Answers for the above questions

Ans for Q.51: (A) and (B)	Ans for Q.52: (A), (B) and (D)

Q.53 The option(s) with the values of a and L that satisfy the following equation is(are)

$$\frac{\int_0^{4\pi} e^t\,(\sin^6 at + \cos^4 at)\,dt}{\int_0^{\pi} e^t(\sin^6 at + \cos^4 at)\,dt} = L\,?$$

(A) $a = 2,\ L = \dfrac{e^{4\pi}-1}{e^{\pi}-1}$

(B) $a = 2,\ L = \dfrac{e^{4\pi}+1}{e^{\pi}+1}$

(C) $a = 4,\ L = \dfrac{e^{4\pi}-1}{e^{\pi}-1}$

(D) $a = 4,\ L = \dfrac{e^{4\pi}+1}{e^{\pi}+1}$

Q.54 Let $f, g : [-1, 2] \to \mathbb{R}$ be continuous functions which are twice differentiable on the interval $(-1, 2)$. Let the values of f and g at the points -1, 0 and 2 be as given in the following table:

	$x=-1$	$x=0$	$x=2$
$f(x)$	3	6	0
$g(x)$	0	1	-1

In each of the intervals $(-1, 0)$ and $(0, 2)$ the function $(f - 3g)''$ never vanishes. Then the correct statement(s) is(are)

(A) $f'(x) - 3g'(x) = 0$ has exactly three solutions in $(-1, 0) \cup (0, 2)$

(B) $f'(x) - 3g'(x) = 0$ has exactly one solution in $(-1, 0)$

(C) $f'(x) - 3g'(x) = 0$ has exactly one solution in $(0, 2)$

(D) $f'(x) - 3g'(x) = 0$ has exactly two solutions in $(-1, 0)$ and exactly two solutions in $(0, 2)$

Answers for the above questions

Ans for Q.53: (A) and (C)	Ans for Q.54: (B) and (C)

Q.55 Let $f(x)=7\tan^8 x+7\tan^6 x-3\tan^4 x-3\tan^2 x$ for all $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. Then the correct expression(s) is(are)

(A) $\int_0^{\pi/4} x f(x)\,dx=\frac{1}{12}$

(B) $\int_0^{\pi/4} f(x)dx=0$

(C) $\int_0^{\pi/4} x f(x)dx=\frac{1}{6}$

(D) $\int_0^{\pi/4} f(x)dx=1$

Q.56 Let $f'(x)=\frac{192x^3}{2+\sin^4\pi x}$ for all $x\in\mathbb{R}$ with $f\left(\frac{1}{2}\right)=0$. If $m\le\int_{1/2}^{1} f(x)dx\le M$, then the possible values of m and M are

(A) $m=13,\ M=24$

(B) $m=\frac{1}{4},\ M=\frac{1}{2}$

(C) $m=-11,\ M=0$

(D) $m=1,\ M=12$

Answers for the above questions

Ans for Q.55: (A) and (B)	Ans for Q.56: (D)

SECTION 3 (Maximum Marks : 16)

- This section contains **TWO** paragraphs
- Based on each paragraph, there will be **TWO** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- Marking scheme :
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  - 0 If none of the bubbles is darkened
  - –2 In all other cases

PARAGRAPH 1

Let n_1 and n_2 be the number of red and black balls, respectively, in box I. Let n_3 and n_4 be the number of red and black balls, respectively, in box II.

Q.57 One of the two boxes, box I and box II, was selected at random and a ball was drawn randomly out of this box. The ball was found to be red. If the probability that this red ball was drawn from box II is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1, n_2, n_3 and n_4 is(are)

(A) $n_1 = 3,\ n_2 = 3,\ n_3 = 5,\ n_4 = 15$ (B) $n_1 = 3,\ n_2 = 6,\ n_3 = 10,\ n_4 = 50$

(C) $n_1 = 8,\ n_2 = 6,\ n_3 = 5,\ n_4 = 20$ (D) $n_1 = 6,\ n_2 = 12,\ n_3 = 5,\ n_4 = 20$

Q.58 A ball is drawn at random from box I and transferred to box II. If the probability of drawing a red ball from box I, after this transfer, is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1 and n_2 is(are)

(A) $n_1 = 4$ and $n_2 = 6$ (B) $n_1 = 2$ and $n_2 = 3$

(C) $n_1 = 10$ and $n_2 = 20$ (D) $n_1 = 3$ and $n_2 = 6$

Answers for the above questions

Ans for Q.57: (A) and (B)	Ans for Q.58: (C) and (D)

PARAGRAPH 2

Let $F:\mathbb{R}\to\mathbb{R}$ be a thrice differentiable function. Suppose that $F(1)=0$, $F(3)=-4$ and $F'(x)<0$ for all $x\in(1/2\ ,3)$. Let $f(x)=xF(x)$ for all $x\in\mathbb{R}$.

Q.59 The correct statement(s) is(are)

(A) $f'(1)<0$ (B) $f(2)<0$

(C) $f'(x)\neq 0$ for any $x\in(1,3)$ (D) $f'(x)=0$ for some $x\in(1,3)$

Q.60 If $\int_1^3 x^2F'(x)dx=-12$ and $\int_1^3 x^3F''(x)dx=40$, then the correct expression(s) is(are)

(A) $9f'(3)+f'(1)-32=0$ (B) $\int_1^3 f(x)dx=12$

(C) $9f'(3)-f'(1)+32=0$ (D) $\int_1^3 f(x)dx=-12$

Answers for the above questions

Ans for Q.59: (A), (B) and (C)	Ans for Q.60: (C) and (D)

END OF THE QUESTION PAPER