JEE Advanced 2019 Question Paper with Solution

Joint Entrance Examination – Advanced

FINAL JEE(Advanced) EXAMINATION - 2019

(Held On Monday 27th MAY, 2019)

PAPER-1 | **TEST PAPER WITH ANSWER & SOLUTION**

PART-1 : PHYSICS

SECTION-1 : (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme :

  Full Marks : +3 If ONLY the correct option is chosen.

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered)

  Negative Marks : –1 In all other cases

1. A current carrying wire heats a metal rod. The wire provides a constant power (P) to the rod. The metal rod is enclosed in an insulated container. It is observed that the temperature (T) in the metal rod changes with time (t) as :

$$T(t) = T_0\,(1 + \beta t^{1/4})$$

where β is a constant with appropriate dimension while T_0 is a constant with dimension of temperature. The heat capacity of the metal is :

(1) $\dfrac{4P(T(t)-T_0)^3}{\beta^4 T_0^4}$

(2) $\dfrac{4P(T(t)-T_0)}{\beta^4 T_0^2}$

(3) $\dfrac{4P(T(t)-T_0)^4}{\beta^4 T_0^5}$

(4) $\dfrac{4P(T(t)-T_0)^2}{\beta^4 T_0^3}$

Ans. (1)

Sol. $P = \dfrac{dQ}{dt}$ $\qquad T_{(t)} = T_0\ (1+\beta t^{1/4})$

$$\frac{dQ}{dt} = \boxed{ms}\frac{dT}{dt} \Rightarrow S = \frac{P}{\left(\frac{dT}{dt}\right)}$$

$$\frac{dT}{dt} = T_0\left[0+\beta\frac{1}{4}\,.\,t^{-3/4}\right] = \frac{\beta T_0}{4}.t^{-3/4}$$

$$S = \frac{P}{(dT/dt)} = \frac{4P}{\beta T_0}.\ t^{3/4}$$

$$S = \frac{4P}{\beta}\left[\frac{t^{3/4}}{T_0}\right]$$

$$\frac{T(t)}{T_0} = (1+\beta t^{1/4})$$

$$\beta t^{1/4} = \frac{T(t)}{T_0} - 1 = \frac{T(t)-T_0}{T_0}$$

$$t^{3/4} = \left(\frac{T(t)-T_0}{\beta.\,T_0}\right)^3$$

$$\Rightarrow\ S = \frac{4P}{T_0\beta}\left[\frac{T(t)-T_0}{\beta.T_0}\right]^3 = \frac{4P}{\beta^4 T_0^4}[T(t)-T_0]^3$$

2. A thin spherical insulating shell of radius R carries a uniformly distributed charge such that the potential at its surface is V_0. A hole with a small area $\alpha 4\pi R^2$ (α<<1) is made on the shell without affecting the rest of the shell. Which one of the following statements is correct ?

(1) The ratio of the potential at the center of the shell to that of the point at $\frac{1}{2}R$ from center towards the hole will be $\dfrac{1-\alpha}{1-2\alpha}$

(2) The magnitude of electric field at the center of the shell is reduced by $\dfrac{\alpha V_0}{2R}$

(3) The magnitude of electric field at a point, located on a line passing through the hole and shell's center on a distance 2R from the center of the spherical shell will be reduced by $\dfrac{\alpha V_0}{2R}$

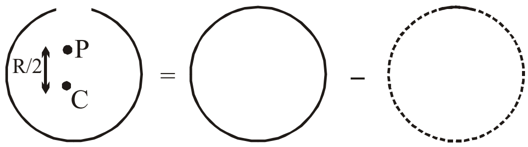
(4) The potential at the center of the shell is reduced by $2\alpha V_0$

Ans. **(1)**

Sol. Let charge on the sphere initially be Q.

$\therefore \ \dfrac{kQ}{R} = V_0$

and charge removed = αQ

(1) 

and $V_p = \dfrac{kQ}{R} - \dfrac{2K\alpha Q}{R} = \dfrac{kQ}{R}(1-2\alpha)$

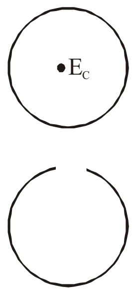
$V_C = \dfrac{kQ(1-\alpha)}{R}$

$\therefore \ \dfrac{V_C}{V_p} = \dfrac{1-\alpha}{1-2\alpha}$

(2) $(E_C)_{initial}$ = zero

$(E_C)_{final} = \dfrac{k\alpha Q}{R^2}$

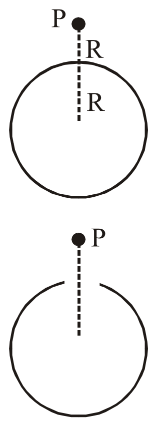
$\Rightarrow$ Electric field increases



(3) $(E_P)_{initial} = \dfrac{kQ}{4R^2}$

$(E_P)_{final} = \dfrac{kQ}{4R^2} - \dfrac{k\alpha Q}{R^2}$

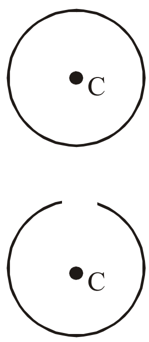
$\Delta E_P = \dfrac{kQ}{4R^2} - \dfrac{kQ}{4R^2} + \dfrac{k\alpha Q}{R^2} = \dfrac{k\alpha Q}{R^2} = \dfrac{V_0 \alpha}{R}$



(4) $(V_C)_{initial} = \dfrac{kQ}{R}$

$(V_C)_{final} = \dfrac{kQ(1-\alpha)}{R}$

$\Delta V_C = \dfrac{kQ}{R}(\alpha) = \alpha V_0$



3. Consider a spherical gaseous cloud of mass density $\rho(r)$ in free space where r is the radial distance from its center. The gaseous cloud is made of particles of equal mass m moving in circular orbits about the common center with the same kinetic energy K. The force acting on the particles is their mutual gravitational force. If $\rho(r)$ is constant in time, the particle number density $n(r) = \rho(r)/m$ is :

[G is universal gravitational constant]

(1) $\frac{K}{\pi r^2 m^2 G}$ (2) $\frac{K}{6\pi r^2 m^2 G}$

(3) $\frac{3K}{\pi r^2 m^2 G}$ (4) $\frac{K}{2\pi r^2 m^2 G}$

Ans. (4)

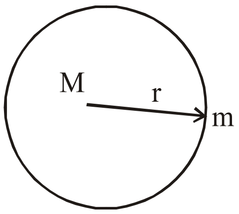
Sol. Let total mass included in a sphere of radius r be M.

For a particle of mass m,

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\Rightarrow \frac{GMm}{r} = 2K \quad \Rightarrow \quad M = \frac{2Kr}{Gm}$$

$$\therefore \ dM = \frac{2K dr}{Gm}$$



$$\Rightarrow (4\pi r^2 dr)\rho = \frac{2K\,dr}{Gm}$$

$$\Rightarrow \rho = \frac{K}{2\pi r^2 Gm}$$

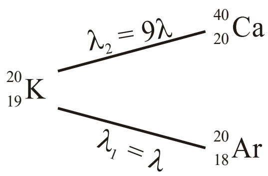
$$\therefore \ n = \frac{\rho}{m} = \frac{K}{2\pi r^2 m^2 G}$$

4. In a radioactive sample, ${}^{40}_{19}K$ nuclei either decay into stable ${}^{40}_{20}Ca$ nuclei with decay constant 4.5×10^{-10} per year or into stable ${}^{40}_{18}Ar$ nuclei with decay constant 0.5×10^{-10} per year. Given that in this sample all the stable ${}^{40}_{20}Ca$ and ${}^{40}_{18}Ar$ nuclei are produced by the ${}^{40}_{19}K$ nuclei only. In time $t \times 10^9$ years, if the ratio of the sum of stable ${}^{40}_{20}Ca$ and ${}^{40}_{18}Ar$ nuclei to the radioactive ${}^{40}_{19}K$ nuclei is 99, the value of t will be : [Given ln 10 = 2.3]

(1) 9.2 (2) 1.15 (3) 4.6 (4) 2.3

Ans. (1)

Sol. Parallel radioactive decay



$\lambda = \lambda_1 + \lambda_2 = 5 \times 10^{-10}$ per year

$N = N_0 e^{-\lambda t}$

$N_0 - N = N_{stable}$

$N = N_{radioactive}$

$$\frac{N_0}{N} - 1 = 99$$

$$\frac{N_0}{N} = 100$$

$$\frac{N}{N_0} = e^{-\lambda t} = \frac{1}{100}$$

$\Rightarrow \lambda t = 2\ \ell n 10$

$= 4.6$

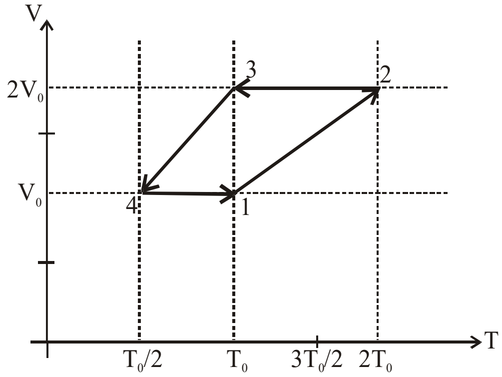
$t = 9.2 \times 10^9$ years

SECTION-2 : (Maximum Marks: 32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s)
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If only (all) the correct option(s) is (are) chosen.
  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen.
  Partial Marks : +2 If three or more options are correct but ONLY two options are chosen and both of which are correct.
  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).
  Negative Marks : –1 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 marks;
  choosing ONLY (B) will get +1 marks;
  choosing ONLY (D) will get +1 marks;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get –1 mark.

1. One mole of a monoatomic ideal gas goes through a thermodynamic cycle, as shown in the volume versus temperature (V-T) diagram. The correct statement(s) is/are :
[R is the gas constant]



(1) Work done in this thermodynamic cycle ($1\rightarrow2\rightarrow3\rightarrow4\rightarrow1$) is $|W| = \frac{1}{2}RT_0$

(2) The ratio of heat transfer during processes $1\rightarrow2$ and $2\rightarrow3$ is $\left|\frac{Q_{1\rightarrow2}}{Q_{2\rightarrow3}}\right| = \frac{5}{3}$

(3) The above thermodynamic cycle exhibits only isochoric and adiabatic processes.

(4) The ratio of heat transfer during processes $1\rightarrow2$ and $3\rightarrow4$ is $\left|\frac{Q_{1\rightarrow2}}{Q_{3\rightarrow4}}\right| = \frac{1}{2}$

Ans. (1,2)

Sol. From graph

Process $1 \rightarrow 2$ is isobaric with $P = \frac{RT_0}{V_0}$

Process $2 \rightarrow 3$ is isochoric with $V = 2V_0$

Process $3 \rightarrow 4$ is isobaric with $P = \frac{RT_0}{2V_0}$

Process $4 \rightarrow 1$ is isochoric with $V = V_0$

Work in cycle $= \frac{RT_0}{V_0}.V_0 - \frac{RT_0}{2V_0}.V_0 = \frac{RT_0}{2}$

$Q_{1-2} = nC_P\Delta T = n.\frac{5R}{2}.T_0$

$Q_{2-3} = nC_V\Delta T = n.\frac{3R}{2}.T_0$

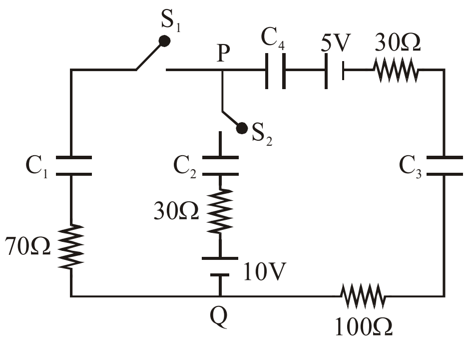
$$\therefore \left|\frac{Q_{1-2}}{Q_{2-3}}\right| = \frac{5}{3}$$

$$Q_{3-4} = nC_P\Delta T = n.\frac{5R}{2}.\frac{T_0}{2}$$

$$\therefore \left|\frac{Q_{1-2}}{Q_{3-4}}\right| = 2$$

Ans. 1, 2

2. In the circuit shown, initially there is no charge on capacitors and keys S_1 and S_2 are open. The values of the capacitors are $C_1 = 10\ \mu F$, $C_2 = 30\ \mu F$ and $C_3 = C_4 = 80\ \mu F$.



Which of the statement(s) is/are correct ?

(1) The keys S_1 is kept closed for long time such that capacitors are fully charged. Now key S_2 is closed, at this time, the instantaneous current across 30 Ω resistor (between points P and Q) will be 0.2 A (round off to 1st decimal place).

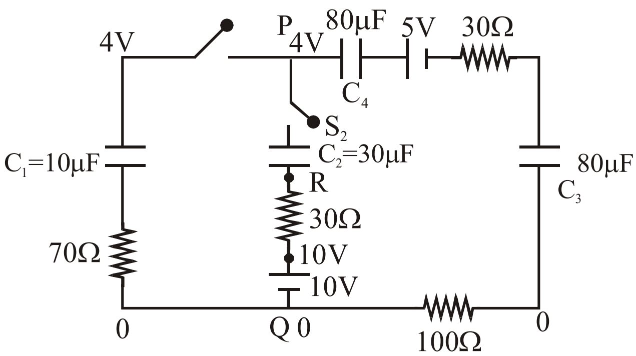
(2) If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage difference between points P and Q will be 10 V.

(3) At time t = 0, the key S_1 is closed, the instantaneous current in the closed circuit will be 25 mA.

(4) If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage across the capacitors C_1 will be 4V.

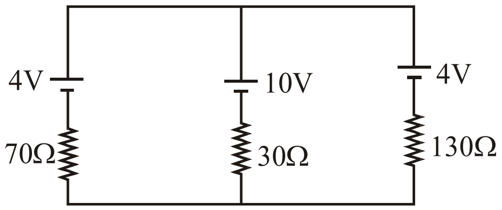
Ans. (3,4)

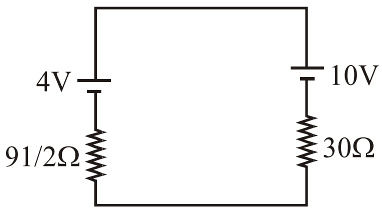
Sol.



(1) at t = 0, capacitor C_1 acts as a battery of 4V, C_4 & C_3 of $\frac{1}{2}V$ each, C_2 is shorted

Circuit is



⇒ 

$$\Rightarrow i = \frac{6}{30+\frac{91}{2}} = 0.079A$$

(2) and (4)

At steady state,

When capacitor is fully charged it behave as open circuit and current through it zero.

Hence, Charge on each capacitor is same.

$Q = C_{eq} V$

$= (8\ \mu F) \times 5$

$Q = 40\ \mu C$

Now,

$$V_P - \frac{40}{10} = V_Q$$

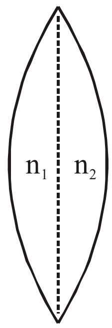
$$\boxed{V_P - V_Q = 4V}$$

(3) At t = 0, S_1 is closed, capacitor act as short circuit.

$$i = \frac{V}{R_{eq}} = \frac{5}{200} = 25mA$$

Ans. (3, 4)

3. A thin convex lens is made of two materials with refractive indices n_1 and n_2, as shown in figure. The radius of curvature of the left and right spherical surfaces are equal. f is the focal length of the lens when $n_1 = n_2 = n$. The focal length is $f + \Delta f$ when $n_1 = n$ and $n_2 = n + \Delta n$. Assuming $\Delta n << (n-1)$ and $1 < n < 2$, the correct statement(s) is/are :



(1) The relation between $\frac{\Delta f}{f}$ and $\frac{\Delta n}{n}$ remains unchanged if both the convex surfaces are replaced by concave surfaces of the same radius of curvature.

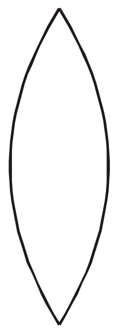
(2) $\left|\frac{\Delta f}{f}\right| < \left|\frac{\Delta n}{n}\right|$

(3) For $n = 1.5$, $\Delta n = 10^{-3}$ and $f = 20$ cm, the value of $|\Delta f|$ will be 0.02 cm (round off to 2nd decimal place)

(4) If $\frac{\Delta n}{n} < 0$ then $\frac{\Delta f}{f} > 0$

Ans. **(1,3,4)**

Sol. When $n_1 = n_2 = n$



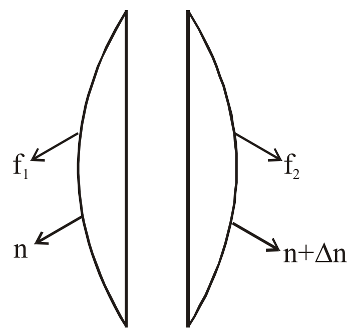
$$\frac{1}{f} = (n-1) \times \frac{2}{R}$$

$$\text{So } f = \frac{R}{2(n-1)} \qquad \text{........(1)}$$

2nd case :

$$\frac{1}{f_1} = \frac{n-1}{R}$$

$$\frac{1}{f_2} = \frac{(n+\Delta n)-1}{R}$$



$$\frac{1}{f_{eq}} = \frac{1}{f+\Delta f} = \left(\frac{n-1}{R}\right) + \frac{(n+\Delta n)-1}{R} = \frac{2(n-1)+\Delta n}{R}$$

$$\Delta f = \left(\frac{R}{2(n-1)+\Delta n}\right) - \left(\frac{R}{2(n-1)}\right)$$

$$= \frac{R}{2}\left[\frac{(n-1)-(n-1+\Delta n)}{(n-1+\Delta n)(n-1)}\right] = \frac{-\Delta n}{(n-1)^2} \times \frac{R}{2}$$

$$\frac{\Delta f}{f} = -\frac{\Delta n}{2(n-1)} \qquad ....(2)$$

(1) Relation between $\frac{\Delta f}{f}$ and $\frac{\Delta n}{n}$ is independent of R so (1) is correct.

(2) $2n - 2 < n$ because $n < 2$

$$\Rightarrow \frac{\Delta f}{f} = \frac{1}{2}\left|\frac{\Delta n}{n-1}\right| > \frac{\Delta n}{n}$$

So $\frac{\Delta f}{f} > \left|\frac{\Delta n}{n}\right|$ So (2) is wrong

(3) $|\Delta f| = \frac{f\Delta n}{(n-1)} = \frac{(20\times 10^{-3})}{1.5-1} = 40\times 10^{-3} = 0.04$

So (3) is wrong

(4) If $\frac{\Delta n}{n} < 0$ then $\frac{\Delta f}{f} > 0$ from equation (2)

4. Let us consider a system of units in which mass and angular momentum are dimensionless. If length has dimension of L, which of the following statement (s) is/are correct ?

(1) The dimension of force is L^{-3}

(2) The dimension of energy is L^{-2}

(3) The dimension of power is L^{-5}

(4) The dimension of linear momentum is L^{-1}

Ans. (1,2,4)

Sol. Mass = $M^0L^0T^0$

$MVr = M^0L^0T^0$

$$M^0\frac{L^1}{T^1}.L^1 = M^0L^0T^0$$

$L^2 = T^1$ ....(1)

Force = $M^1L^1T^{-2}$ (in SI)

$= M^0\,L^1\,L^{-4}$ (In new system from equation (1))

$= L^{-3}$

Energy = $M^1L^2T^{-2}$ (In SI)

$= M^0L^2L^{-4}$ (In new system from equation (1))

$= L^{-2}$

$$\text{Power} = \frac{\text{Energy}}{\text{Time}}$$

$= M^1L^2T^{-3}$ (in SI)

$= M^0L^2L^{-6}$ (In new system from equation (1))

$= L^{-4}$

Linear momentum $= M^1L^1T^{-1}$ (in SI)

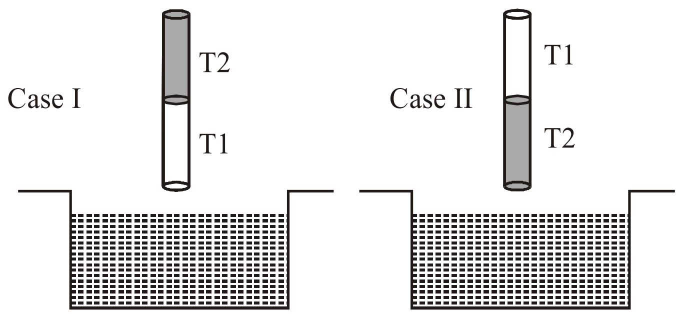
$= M^0L^1L^{-2}$ (In new system from equation (1))

$= L^{-1}$

Ans. (1, 2, 4)

5. A cylindrical capillary tube of 0.2 mm radius is made by joining two capillaries T1 and T2 of different materials having water contact angles of 0° and 60°, respectively. The capillary tube is dipped vertically in water in two different configurations, case I and II as shown in figure. Which of the following option(s) is(are) correct ?

(Surface tension of water = 0.075 N/m, density of water = 1000 kg/m^3, take g = 10 m/s^2)



(1) The correction in the height of water column raised in the tube, due to weight of water contained in the meniscus, will be different for both cases.

(2) For case I, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be more than 8.75 cm. (Neglect the weight of the water in the meniscus)

(3) For case I, if the joint is kept at 8 cm above the water surface, the height of water column in the tube will be 7.5 cm. (Neglect the weight of the water in the meniscus)

(4) For case II, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be 3.75 cm. (Neglect the weight of the water in the meniscus)

Ans. (1,3,4)

Sol. $h = \dfrac{2T\cos\theta}{\rho g R}$; $h_1 = \dfrac{2\times 0.075\times\cos 0°}{1000\times 10\times 0.2\times 10^{-3}}$

$\Rightarrow h_1 = 75$ mm (in T1) [If we assume entire tube of T1]

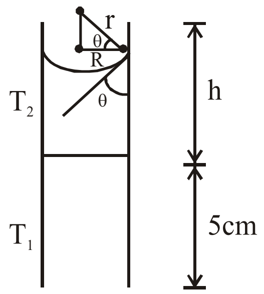
$\Rightarrow h_2 = \frac{2\times 0.075\times \cos 60°}{1000\times 10\times 0.2\times 10^{-3}} = 37.5$ mm (in T2) [If we assume entire tube of T2]

Option (1) : Since contact angles are different so correction in the height of water column raised in the tube will be different in both the cases, so option (1) is correct

Option (2) : If joint is 5 cm is above water surface, then lets say water crosses the joint by height h, then:

$\Rightarrow P_0 - \frac{2T}{r} + \rho gh + \rho g\times 5\times 10^{-2}$

$= P_0$



$\Rightarrow \cos\theta = \frac{R}{r}, r = \frac{R}{\cos\theta}$

$\Rightarrow \rho g(h + 5\times 10^{-2}) = \frac{2T\cos\theta}{R}$

$\Rightarrow h = \frac{2\times 0.075\times \cos 60}{0.2\times 10^{-3}\times 1000\times 10} - 5\times 10^{-2}$

$\Rightarrow$ h = –ve, not possible, so liquid will not cross the interface, but angle of contact at the interface will change, to balance the pressure,

So option (2) is wrong.

Option (3) : If interface is 8 cm above water then water will not even reach the interface, and water will rise till 7.5 cm only in T1, so option (3) is right.

Option (4) : If interface is 5 cm above the water in vessel, then water in capillary will not even reach the interface. Water will reach only till 3.75 cm, so option (4) is right.

6. Two identical moving coil galvanometer have 10 Ω resistance and full scale deflection at 2 μA current. One of them is converted into a voltmeter of 100 mV full scale reading and the other into an Ammeter of 1 mA full scale current using appropriate resistors. These are then used to measure the voltage and current in the Ohm's law experiment with R = 1000 Ω resistor by using an ideal cell. Which of the following statement(s) is/are correct ?

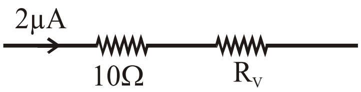
(1) The measured value of R will be 978 Ω < R < 982 Ω.

(2) The resistance of the Voltmeter will be 100 kΩ.

(3) The resistance of the Ammeter will be 0.02 Ω (round off to 2^{nd} decimal place)

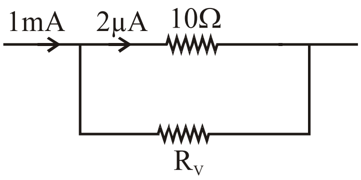
(4) If the ideal cell is replaced by a cell having internal resistance of 5 Ω then the measured value of R will be more than 1000 Ω.

Ans. (1,3)

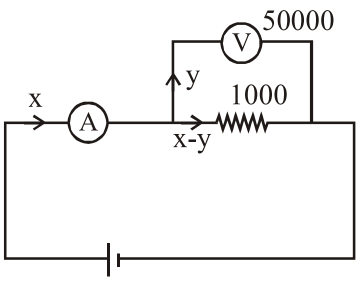
Sol. 

$0.1 = 2 \times 10^{-6} (10 + R_V)$

$\therefore\ R_V = 49990\ \Omega$



$2 \times 10^{-6} \times 10 = 10^{-3} R_A \ \therefore\ R_A = 0.02\ \Omega$



$y\ 50000 = (x - y)\ 1000$

$\therefore\ 51y = x$

$\text{Reading} = \dfrac{y\,50000}{x} \simeq 980$

7. A charged shell of radius R carries a total charge Q. Given Φ as the flux of electric field through a closed cylindrical surface of height h, radius r and with its center same as that of the shell. Here, center of the cylinder is a point on the axis of the cylinder which is equidistant from its top and bottom surfaces. Which of the following option(s) is/are correct ? [$\in_0$ is the permittivity of free space]

(1) If $h > 2R$ and $r > R$ then $\Phi = \dfrac{Q}{\in_0}$

(2) If $h < \dfrac{8R}{5}$ and $r = \dfrac{3R}{5}$ then $\Phi = 0$

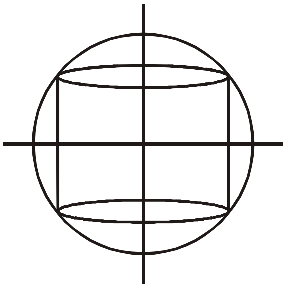
(3) If $h > 2R$ and $r = \dfrac{4R}{5}$ then $\Phi = \dfrac{Q}{5\in_0}$

(4) If $h > 2R$ and $r = \dfrac{3R}{5}$ then $\Phi = \dfrac{Q}{5\in_0}$

Ans. (1,2,4)

Sol. For option (1), cylinder encloses the shell, thus option is correct

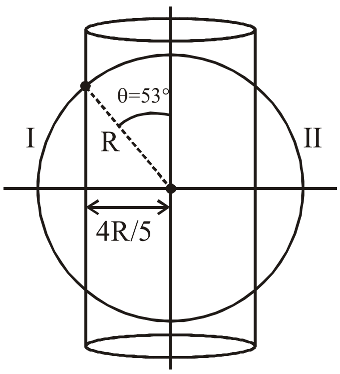
For option (2),



cylinder perfectly enclosed by shell,

thus $\phi = 0$, so option is correct.

For option (3)

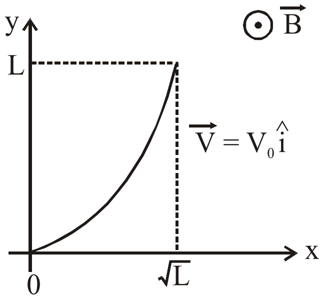


$$\phi = \frac{2 \times Q}{2 \epsilon_0}(1 - \cos 53°) = \frac{2Q}{5 \epsilon_0}$$

For option (4) :

Flux enclosed by cylinder = $\phi = \frac{2Q}{2 \epsilon_0}(1 - \cos 37°) = \frac{Q}{5 \epsilon_0}$

8. A conducting wire of parabolic shape, initially $y = x^2$, is moving with velocity $\vec{V} = V_0\hat{i}$ in a non-uniform magnetic field $\vec{B} = B_0\left(1+\left(\frac{y}{L}\right)^{\beta}\right)\hat{k}$, as shown in figure. If V_0, B_0, L and β are positive constants and Δϕ is the potential difference developed between the ends of the wire, then the correct statement(s) is/are:



(1) |Δϕ| remains the same if the parabolic wire is replaced by a straight wire, y = x initially, of length $\sqrt{2}L$

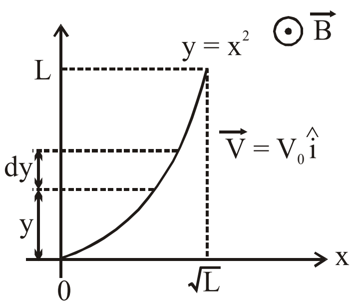
(2) |Δϕ| is proportional to the length of the wire projected on the y-axis.

(3) $|\Delta\phi| = \frac{1}{2}B_0V_0L$ for β = 0

(4) $|\Delta\phi| = \frac{4}{3}B_0V_0L$ for β = 2

Ans. (1,2,4)

Sol.



$y = x^2$

$$B = B_0\left[1+\left(\frac{y}{L}\right)^{\beta}\right]\hat{k}$$

$$\int d\phi = \int_0^L V_0 B_0\left(1+\frac{y^{\beta}}{L^{\beta}}\right)\cdot dy$$

$$\Delta\phi = V_0 B_0\left[L+\frac{L^{\beta+1}}{(\beta+1)L^{\beta}}\right]$$

$$\Delta\phi = V_0 B_0\left[L+\frac{L}{\beta+1}\right]$$

$$\because |\Delta\phi| = B_0 V_0\left(1+\frac{1}{\beta+1}\right)\cdot L$$

$|\Delta\phi| \propto L$ $\therefore$ option '2' is also correct

If $\beta = 0$

$\Delta\phi = V_0 B_0 [L + L]$

$\Delta\phi = 2V_0 B_0 L \Rightarrow$ option (3) is incorrect

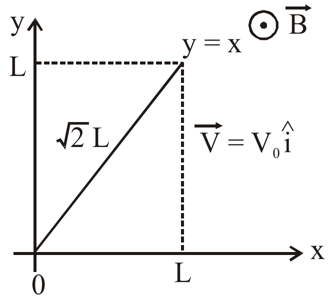
If $\beta = 2$

$$\Delta\phi = V_0 B_0\left[L+\frac{L}{3}\right]$$

$\Delta\phi = \frac{4}{3}V_0 B_0 L$ option (4) is correct

$\Delta\phi$ will be same if the wire is repalced by the straight wire of length $\sqrt{2}L$ and $y = x$

$\because$ range of y remains same



$\therefore$ option 1 is correct.

SECTION-3 : (Maximum Marks: 18)

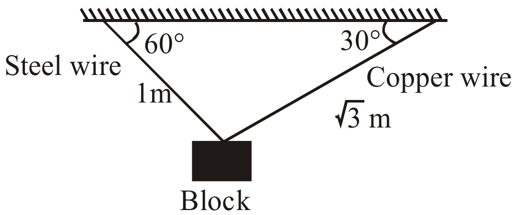
- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If ONLY the correct numerical value is entered.

  Zero Marks : 0 In all other cases.

1. A block of weight 100 N is suspended by copper and steel wires of same cross sectional area 0.5 cm^2 and, length $\sqrt{3}$ m and 1 m, respectively. Their other ends are fixed on a ceiling as shown in figure. The angles subtended by copper and steel wires with ceiling are 30° and 60°, respectively. If elongation in copper wire is $(\Delta\ell_C)$ and elongation in steel wire is $(\Delta\ell_S)$, then the ratio $\frac{\Delta\ell_C}{\Delta\ell_S}$ is ______.

[Young's modulus for copper and steel are 1×10^{11} N/m^2 and 2×10^{11} N/m^2 respectively]



Ans. (2.00)

Sol. Let T_S = tension in steel wire

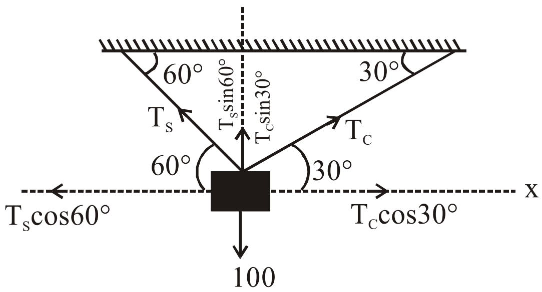
T_C = Tension in copper wire

in x direction

$T_C\cos30° = T_S\cos60°$

$$T_C \times \frac{\sqrt{3}}{2} = T_S \times \frac{1}{2}$$

$$\sqrt{3}T_C = T_S \text{ .... (i)}$$



in y direction

$T_C\sin30° + T_S\sin60° = 100$

$$\frac{T_C}{2} + \frac{T_S\sqrt{3}}{2} = 100 \text{ .... (ii)}$$

Solving equation (i) & (ii)

$T_C = 50 \text{ N}$

$T_S = 50\sqrt{3} \text{ N}$

We know

$$\Delta L = \frac{FL}{AY}$$

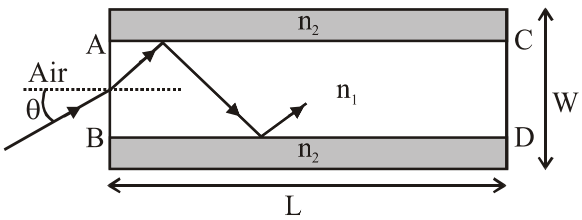
$$= \frac{\Delta L_C}{\Delta L_S} = \frac{T_C L_C}{A_C Y_C} \times \frac{A_S Y_S}{T_S L_S}$$

On solving above equation

$$\frac{\Delta L_C}{\Delta L_S} = 2$$

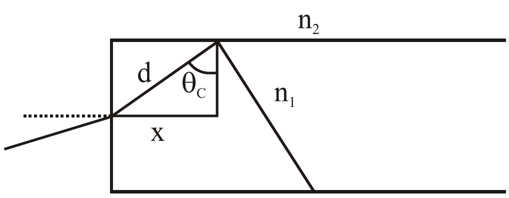
Ans. 2.00

2. A planar structure of length L and width W is made of two different optical media of refractive indices $n_1 = 1.5$ and $n_2 = 1.44$ as shown in figure. If $L >> W$, a ray entering from end AB will emerge from end CD only if the total internal reflection condition is met inside the structure. For $L = 9.6$ m, if the incident angle θ is varied, the maximum time taken by a ray to exit the plane CD is $t \times 10^{-9}$ s, where t is ____. [Speed of light $c = 3 \times 10^8$ m/s]



Ans. (50.00)

Sol. For maximum time the ray of light must undergo TIR at all surfaces at minimum angle i.e. θ_C



For TIR $n_1 \sin\theta_C = n_2$

$$\sin\theta_C = \frac{1.44}{1.5}$$

In above Δ $\sin\theta_C = \frac{x}{d}$

$$d = \frac{x}{\sin\theta_C}$$

Similarly $D = \frac{L}{\sin\theta_C}$

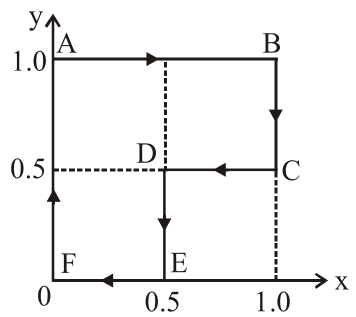
where L = length of tube, D = length of path of light

Time taken by light

$$t = \frac{D}{C} = \frac{L/\sin\theta_C}{2\times10^8}$$

$t = 50 \times 10^{-9}$s

3. A particle is moved along a path AB-BC-CD-DE-EF-FA, as shown in figure, in presence of a force $\vec{F} = \left(\alpha y\hat{i} + 2\alpha x\hat{j}\right)N$, where x and y are in meter and $\alpha = -1$ N/m^{-1}. The work done on the particle by this force $\vec{F}$ will be _____ Joule.



Ans. (0.75)

Sol. $F = \left(\alpha y\,\hat{i} + 2\alpha x\,\hat{j}\right)$

$$W_{AB} = \left(-1\hat{i}\right)\cdot\left(1\hat{i}\right) = -1\,J$$

$$\begin{bmatrix} \vec{F} = -1\hat{i} + 2\alpha\,x\,\hat{j} \\ \vec{S} = 1\hat{i} \end{bmatrix}$$

Similarly,

W_{BC} = 1J

$W_{CD} = 0.25J$

$W_{DE} = 0.5\ J$

$W_{EF} = W_{FA} = 0\ J$

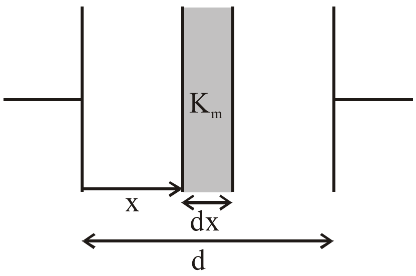
$\therefore$ New work in cycle = 0.75 J

4. A parallel plate capacitor of capacitance C has spacing d between two plates having area A. The region between the plates is filled with N dielectric layers, parallel to its plates, each with thickness $\delta = \frac{d}{N}$. The dielectric constant of the m^{th} layer is $K_m = K\left(1+\frac{m}{N}\right)$. For a very large N (> 10^3), the capacitance C is $\alpha\left(\frac{K \in_0 A}{d\ell n 2}\right)$. The value of α will be ______.

[$\in_0$ is the permittivity of free space]

Ans. (1.00)

Sol.



$$\delta = dx = \frac{d}{N} \quad \& \quad \frac{m}{N} = \frac{x}{d}$$

$$K_m = K\left(1+\frac{m}{N}\right)$$

$$\Rightarrow K_m = K\left(1+\frac{x}{d}\right)$$

$$C' = \frac{K_m A \in_0}{dx}$$

$$\frac{1}{C_{eq}} = \int_0^d \frac{dx}{K_m A \in_0} = \frac{1}{KA \in_0}\int_0^d \frac{dx}{\left(1+\frac{x}{d}\right)}$$

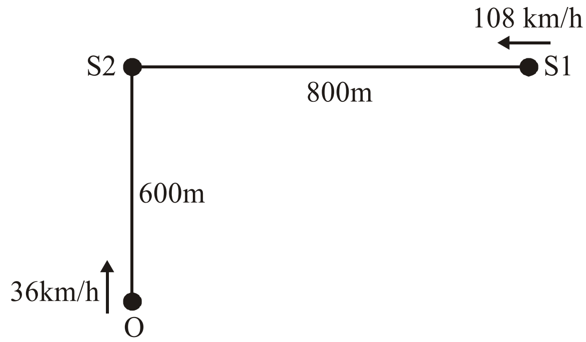
$$\Rightarrow \frac{1}{C_{eq}} = \frac{d}{KA \in_0}\left[\ell n\left(1+\frac{x}{d}\right)\right]_0^d$$

$$\Rightarrow \frac{1}{C_{eq}} = \frac{d}{KA \in_0}\left[\ell n2 - \ell n(1)\right]$$

$$\Rightarrow C_{eq} = \frac{KA \in_0}{d\ell n2} \Rightarrow \alpha = 1$$

5. A train S1, moving with a uniform velocity of 108 km/h, approaches another train S2 standing on a platform. An observer O moves with a uniform velocity of 36 km/h towards S2, as shown in figure. Both the trains are blowing whistles of same frequency 120 Hz. When O is 600 m away from S2 and distance between S1 and S2 is 800 m, the number of beats heard by O is ____.

[Speed of the sound = 330 m/s]



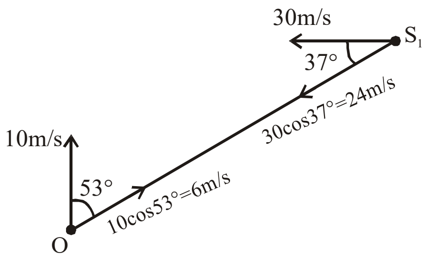
Ans. (8.12 to 8.13)

Sol. Frequency observed by O from S_2

$$f_2 = \frac{330+10}{330} \times 120 = \frac{340}{330} \times 120 = 123.63\,\text{Hz}$$

frequency observed by O from S_1

$$f_1 = \frac{330+6}{330-24} \times 120 = \frac{336}{306} \times 120 \approx 131.76\text{Hz}$$



beat frequency = 131.76 – 123.63 = 8.128 ≈ 8.12 to 8.13 Hz

6. A liquid at 30°C is poured very slowly into a Calorimeter that is at temperature of 110°C. The boiling temperature of the liquid is 80°C. It is found that the first 5 gm of the liquid completely evaporates. After pouring another 80 gm of the liquid the equilibrium temperature is found to be 50°C. The ratio of the Latent heat of the liquid to its specific heat will be _______ °C.

[Neglect the heat exchange with surrounding]

Ans. (270.00)

Sol. Let m = mass of calorimeter,

x = specific heat of calorimeter

s = specifc heat of liquid

L = latent heat of liquid

First 5 g of liquid at 30° is poured to calorimter at 110°C

$\therefore\ m \times x \times (110 - 80) = 5 \times s \times (80 \times 30) + 5\ L$

$\Rightarrow mx \times 30 = 250\ s + 5\ L$ ... (i)

Now, 80 g of liquid at 30° is poured into calorimeter at 80°C, the equilibrium temperature reaches to 50°C.

$\therefore\ m \times x \times (80 - 30) = 80 \times s \times (50 - 30)$

$\Rightarrow mx \times 30 = 1600\ s$ .... (ii)

From (i) & (ii)

$250\ s + 5\ L = 1600\ s \Rightarrow 5L = 1350\ s$

$\Rightarrow \frac{L}{s} = 270$

FINAL JEE(Advanced) EXAMINATION - 2019

(Held On Monday 27th MAY, 2019)

PAPER-1 | **TEST PAPER WITH ANSWER & SOLUTION**

PART-2 : CHEMISTRY

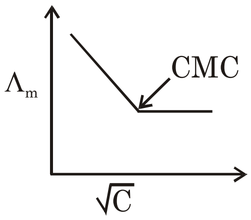
SECTION-1 : (Maximum Marks : 12)

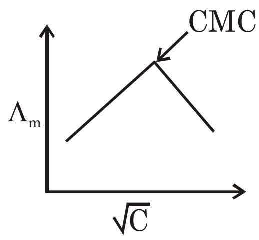
- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme :

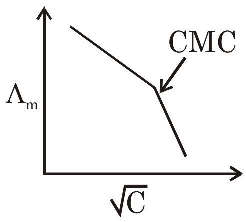
Full Marks : +3 If ONLY the correct option is chosen.
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered)
Negative Marks : –1 In all other cases

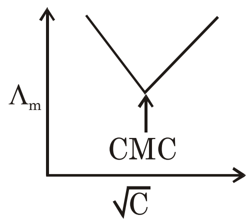
1. Molar conductivity (Λ_m) of aqueous solution of sodium stearate, which behaves as a strong electrolyte, is recorded at varying concentration(c) of sodium stearate. Which one of the following plots provides the correct representation of micelle formation in the solution ?

(Critical micelle concentration (CMC) is marked with an arrow in the figures.)

(1)


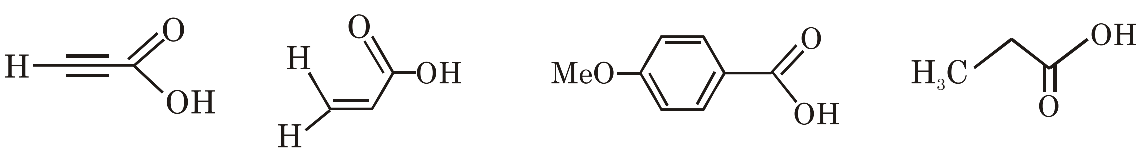
(2)


(3)


(4)


Ans. (3)

2. The correct order of acid strength of the following carboxylic acids is -

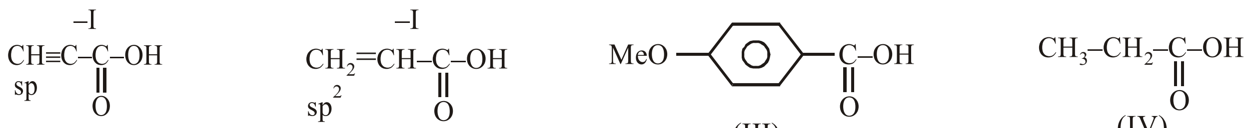


I II III IV

(1) I > III > II > IV
(2) III > II > I > IV
(3) II > I > IV > III
(4) I > II > III > IV

Ans. (4)

Sol. I > II > III > IV



(I) (pKa value 1.86)   (II) (pKa value 4.3)   (III) (pKa value 4.5)   (IV) (pKa value 4.88)

3. Calamine, malachite, magnetite and cryolite, respectively are

(1) $ZnSO_4$, $CuCO_3$, Fe_2O_3 , AlF_3 (2) $ZnCO_3$, $CuCO_3.Cu(OH)_2$, Fe_3O_4, Na_3AlF_6

(3) $ZnSO_4$, $Cu(OH)_2$, Fe_3O_4, Na_3AlF_6 (4) $ZnCO_3$, $CuCO_3$, Fe_2O_3, Na_3AlF_6

Ans. **(2)**

Sol.

Ore	Formula
Calamine	$ZnCO_3$
Malachite	$CuCO_3.Cu(OH)_2$
Magnetite	Fe_3O_4
Cryolite	Na_3AlF_6

So correct answer is option(2)

4. The green colour produced in the borax bead test of a chromium(III) salt is due to -

(1) $Cr(BO_2)_3$ (2) CrB   (3) $Cr_2(B_4O_7)_3$ (4) Cr_2O_3

Ans. **(1)**

Sol. Chromium (III) salt $\xrightarrow{\Delta}$ Cr_2O_3

Borax $\xrightarrow{\Delta}$ $B_2O_3 + NaBO_2$

$2Cr_2O_3 + 6B_2O_3 \longrightarrow 4\ Cr(BO_2)_3$

So correct answer is option(1)

SECTION-2 : (Maximum Marks: 32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s)
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If only (all) the correct option(s) is (are) chosen.
Partial Marks : +3 If all the four options are correct but ONLY three options are chosen.
Partial Marks : +2 If three or more options are correct but ONLY two options are chosen and both of which are correct.
Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).
Negative Marks : –1 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 marks;
  choosing ONLY (B) will get +1 marks;
  choosing ONLY (D) will get +1 marks;
  choosing no option (i.e. the question is unanswered) will get 0 marks, and
  choosing any other combination of options will get –1 mark.

---

1. Fusion of MnO_2 with KOH in presence of O_2 produces a salt **W**. Alkaline solution of **W** upon eletrolytic oxidation yields another salt **X**. The manganese containing ions present in **W** and **X**, respectively, are **Y** and **Z**. Correct statement(s) is (are)

(1) **Y** is diamagnetic in nature while **Z** is paramagnetic

(2) Both **Y** and **Z** are coloured and have tetrahedral shape

(3) In both **Y** and **Z** , π-bonding occurs between p-orbitals of oxygen and d-orbitals of manganese.

(4) In aqueous acidic solution, **Y** undergoes disproportionation reaction to give **Z** and MnO_2.

Ans. (2,3,4)

Sol. $$MnO_2 + 2KOH + \frac{1}{2}O_2 \xrightarrow{\Delta} \underset{(W)}{K_2MnO_4} + H_2O$$

$$\left[(W) = K_2MnO_{4(aq)} \rightleftharpoons 2K^{\oplus}_{(aq)} + \underset{(Y)}{MnO^{2-}_{4(aq)}}\right]$$

$$K_2MnO_4 + H_2O \xrightarrow[\text{Redox}]{\text{Electolytic}} H_2 + KOH + \underset{(X)}{KMnO_4}$$

[anion of X = $\underset{(Z)}{MnO_4^-}$]

$$\left[\because \underset{(Y)}{MnO_4^{2-}} \xrightarrow[\text{Oxidation}]{\text{Electrolytic}} \underset{(Z)}{MnO_4^-} + e^-\right]$$

∵ In acidic solution; Y undergoes disproportionation reaction

$$\left[3MnO^{2-}_{4(aq)} + 4H^{\oplus} \longrightarrow \underset{(Z)}{2MnO_4^-} + MnO_2 + 2H_2O\right]$$

2. Which of the following statement(s) is (are) correct regarding the root mean square speed (U_{rms}) and average translational kinetic energy (ε_{av}) of a molecule in a gas at equilibrium ?

(1) U_{rms} is doubled when its temperature is increased four times

(2) ε_{av} at a given temperature does not depend on its molecular mass

(3) U_{rms} is inversely proportional to the square root of its molecular mass

(4) ε_{av} is doubled when its temperature is increased four times

Ans. (1,2,3)

Sol. $U_{rms} = \sqrt{\frac{3RT}{M}}$

$$E_{avg} = \frac{3}{2}kT$$

3. In the decay sequence :

$$^{238}_{92}U \xrightarrow{-x_1} {}^{234}_{90}Th \xrightarrow{-x_2} {}^{234}_{91}Pa \xrightarrow{-x_3} {}^{234}Z \xrightarrow{-x_4} {}^{234}_{90}Th$$

x_1, x_2, x_3 and x_4 are particles/ radiation emitted by the respective isotopes. The correct option(s) is/are-

(1) Z is an isotope of uranium

(2) x_2 is β^-

(3) x_1 will deflect towards negatively charged plate

(4) x_3 is γ-ray

Ans. (1,2,3)

Sol. ${}_{92}U^{238} \longrightarrow {}_{90}Th^{234} + {}_{2}He^{4}(\text{or } \alpha)$ or x_1

$\downarrow$

${}_{91}Pa^{234} + \beta^{-}(\text{or } {}_{-1}e^{\circ})$ or x_2

$\downarrow$

${}_{92}Z^{234} + \beta^{-}(\text{or } {}_{-1}e^{\circ})$ or x_3

$\downarrow$

${}_{90}Th^{230} + {}_{2}He^{4}(\text{or } \alpha)$ or x_4

U and Z are isotopes

4. Which of the following statement(s) is(are) true ?

(1) Oxidation of glucose with bromine water gives glutamic acid

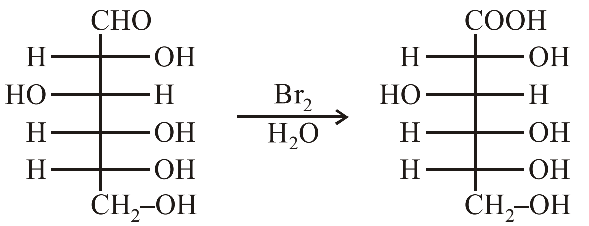
(2) The two six-membered cyclic hemiacetal forms of D-(+)-glucose ard called anomers

(3) Hydrolysis of sucrose gives dextrorotatory glucose and laevorotatory fructose

(4) Monosaccharides **cannot** be hydrolysed to give polyhydroxy aldehydes and ketones

Ans. (2,3,4)

Sol. (1) **FALSE :**



D(+) glucose → Gluconic acid

(2) **TRUE :** Six member hemiacetal on anomeric carbon gives α-D glucose & β-D glucose.

(3) **TRUE :** $C_{12}H_{22}O_{11} + H_2O \xrightarrow{\text{Invertase}} \underset{\substack{\text{Glucose} \\ (+)}}{C_6H_{12}O_6} + \underset{\substack{\text{Fructose} \\ (-)}}{C_6H_{12}O_6}$

(4) **TRUE :** Monosaccharide cannot be hydrolysed to give polyhydroxy aldehydes and ketones

5. A tin chloride **Q** undergoes the following reactions (not balanced)

Q + Cl^- → **X**

Q + Me_3N → **Y**

Q + $CuCl_2$ → **Z** + CuCl

X is a monoanion having pyramidal geometry. Both **Y** and **Z** are neutral compounds. Choose the correct option(s).

(1) The central atoms in **X** is sp^3 hybridized

(2) The oxidation state of the central atom in **Z** is +2

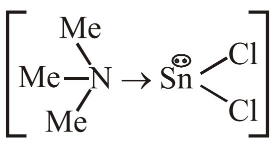
(3) The central atom in **Z** has one lone pair of electrons

(4) There is a coordinate bond in **Y**

Ans. (1,4)

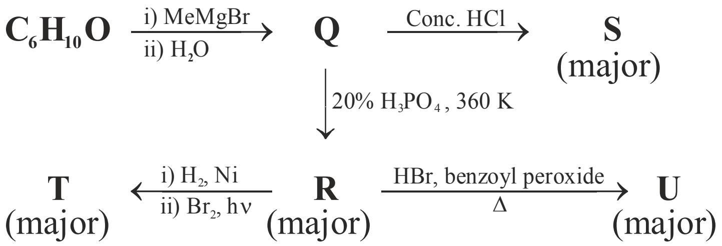
Sol. $\underset{(Q)}{SnCl_2} + Cl^- \longrightarrow \underset{(X)}{SnCl_3^-}$

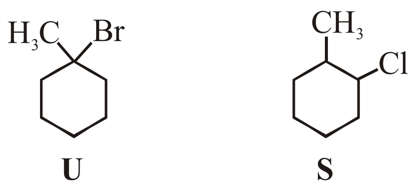
$\underset{(Q)}{SnCl_2} + Me_3N \longrightarrow \underset{(Y)}{SnCl_2.NMe_3}$

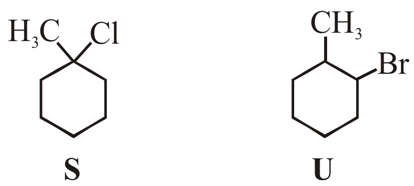


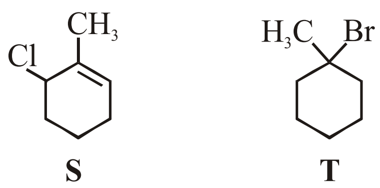
$\underset{(Q)}{SnCl_2} + 2CuCl_2 \longrightarrow \underset{(Z)}{SnCl_4} + 2CuCl$

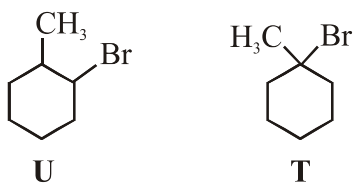
6. Choose the correct option(s) for the following set of reactions



(1) 

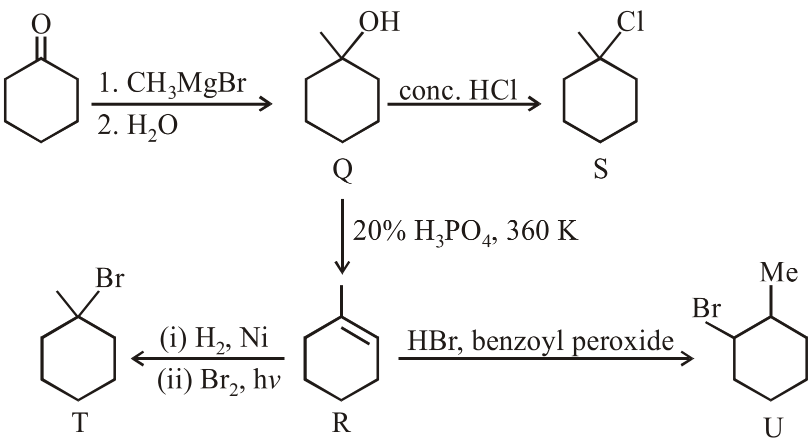
(2) 

(3) 

(4) 

Ans. **(2,4)**

Sol.



7. Each of the following options contains a set of four molecules. Identify the option(s) where all four molecules possess permanent dipole moment at room temperature.

(1) $BeCl_2$, CO_2, BCl_3, $CHCl_3$

(2) SO_2, C_6H_5Cl, H_2Se, BrF_5

(3) BF_3, O_3, SF_6, XeF_6

(4) NO_2, NH_3, $POCl_3$, CH_3Cl

Ans. **(2,4)**

Sol.

Polar molecule	Non-polar molecule
$CHCl_3$, SO_2, C_6H_5Cl,	$BeCl_2$, CO_2, BCl_3, SF_6
H_2Se, BrF_5, O_3, XeF_6,	
NO_2, NH_3, $POCl_3$, CH_3Cl	

So correct answer is option (2) and (4)

8. Choose the reaction(s) from the following options, for which the standard enthalpy of reaction is equal to the standard enthalpy of formation.

(1) $\frac{3}{2}O_2(g) \rightarrow O_3(g)$ (2) $\frac{1}{8}S_8(s) + O_2(g) \rightarrow SO_2(g)$

(3) $2H_2(g) + O_2(g) \rightarrow 2H_2O(l)$ (4) $2C(g) + 3H_2(g) \rightarrow C_2H_6(g)$

Ans. **(1,2)**

Sol. Enthalpy of formation is defined as enthalpy change for formation of 1 mole of substance from its elements, present in their natural most stable form.

SECTION-3 : (Maximum Marks: 18)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **Two** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If ONLY the correct numerical value is entered.

  Zero Marks : 0 In all other cases.

---

1. For the following reaction, the equilibrium constant K_c at 298 K is 1.6×10^{17}.

$Fe^{2+}(aq) + S^{2-}(aq) \rightleftharpoons FeS(s)$

When equal volumes of 0.06 M $Fe^{2+}(aq)$ and 0.2 M $S^{2-}(aq)$ solutions are mixed, the equilibrium concentration of $Fe^{2+}(aq)$ is found to be $\mathbf{Y \times 10^{-17}}$ **M**. The value of Y is ________

Ans. **(8.92 or 8.93)**

Sol. $Fe^{+2}_{(aq.)} + S^{-2}_{(aq.)} \rightleftharpoons FeS(s)$

0.03 M 0.1 M

(0.03–x) (0.1–x)

$\simeq y$ $\simeq 0.07$

$K_c >> 10^3 \Rightarrow 0.03 - x \simeq 0 \simeq y$

$\Rightarrow x = 0.03$

$$K_c = 1.6 \times 10^{17} = \frac{1}{y \times 0.07}$$

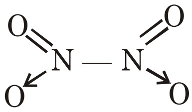
$$y = \frac{10^{-17}}{1.6 \times 0.07} = 8.928 \times 10^{-17} = Y \times 10^{-17}$$

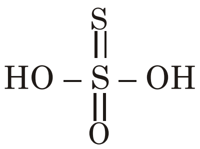
$\boxed{y \simeq 8.93}$

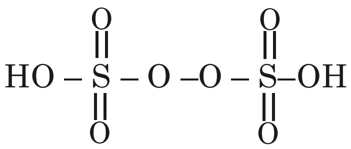
2. Among B_2H_6, $B_3N_3H_6$, N_2O, N_2O_4, $H_2S_2O_3$ and $H_2S_2O_8$, the total number of molecules containing covalent bond between two atoms of the same kind is ————

Ans. **(4.00)**

Sol. $N \equiv N \rightarrow O$







So correct answer is 4

3. Consider the kinetic data given in the following table for the reaction A + B + C → Product.

Experiment No.	[A] (mol dm^{-3})	[B] (mol dm^{-3})	[C] (mol dm^{-3})	Rate of reaction (mol dm^{-3} s^{-1})
1	0.2	0.1	0.1	6.0×10^{-5}
2	0.2	0.2	0.1	6.0×10^{-5}
3	0.2	0.1	0.2	1.2×10^{-4}
4	0.3	0.1	0.1	9.0×10^{-5}

The rate of the reaction for [A] = 0.15 mol dm^{-3}, [B] = 0.25 mol dm^{-3} and [C] = 0.15 mol dm^{-3} is found to be **Y** × 10^{-5} mol dm^{-3} s^{-1}. The value of **Y** is ————

Ans. **(6.75)**

Sol. $r = K[A]^{n_1} [B]^{n_2} [C]^{n_3}$

From table

$n_1 = 1$

$n_2 = 0$

$n_3 = 1$

$r = K[A]\,[C]$

From Exp-1

$6 \times 10^{-5} = K \times 0.2 \times 0.1$

$K = 3 \times 10^{-3}$

$r = (3 \times 10^{-3}) \times 0.15 \times 0.15$

$= 6.75 \times 10^{-5}$

$= Y \times 10^{-5}$

$Y = 6.75$

4. On dissolving 0.5 g of a non-volatile non-ionic solute to 39 g of benzene, its vapor pressure decreases from 650 mm Hg to 640 mm Hg. The depression of freezing point of benzene (in K) upon addition of the solute is ______

(Given data : Molar mass and the molal freezing point depression constant of benzene are 78 g mol^{-1} and 5.12 K kg mol^{-1}, respectively)

Ans. (1.02 or 1.03)

Sol. $$\frac{P^o - P_s}{P^o} = \frac{n_{solute}}{n_{solute} + n_{solvent}}$$

$$\frac{650-640}{650} = \frac{n_{solute}}{n_{solute} + 0.5}$$

$$n_{solute} = \left(\frac{5}{640}\right)$$

$$\text{Molality} = \frac{5\times1000}{640\times39}$$

$$\Delta T_f = m \times K_b$$

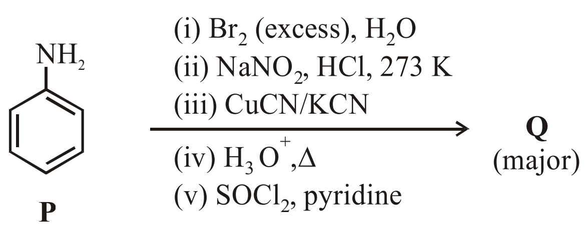
$$= \frac{5.12\times5\times1000}{640\times39}$$

$$= 1.0256$$

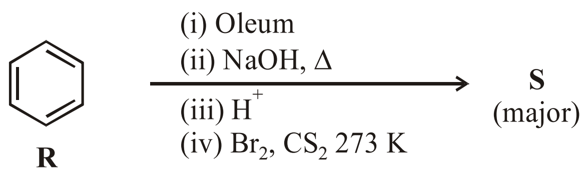
$$\boxed{\Delta T_f \approx 1.03}$$

5. Scheme 1 and 2 describe the conversion of **P** to **Q** and **R** to **S**, respectively. Scheme 3 describes the synthesis of **T** from **Q** and **S**. The total number of Br atoms in a molecule of **T** is ________

Scheme 1 :



Scheme 2 :

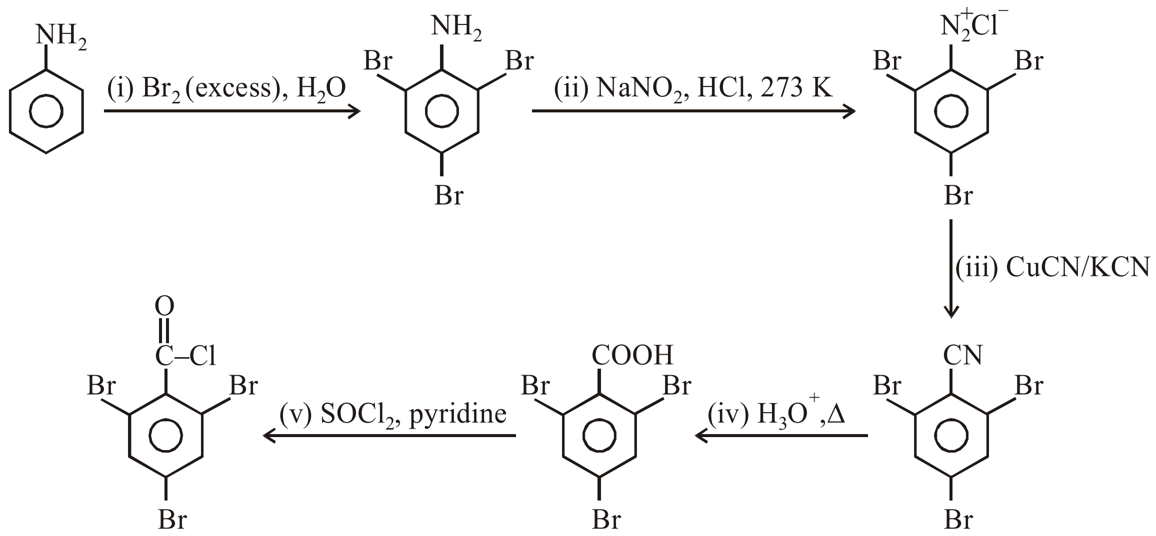


Scheme 3 :

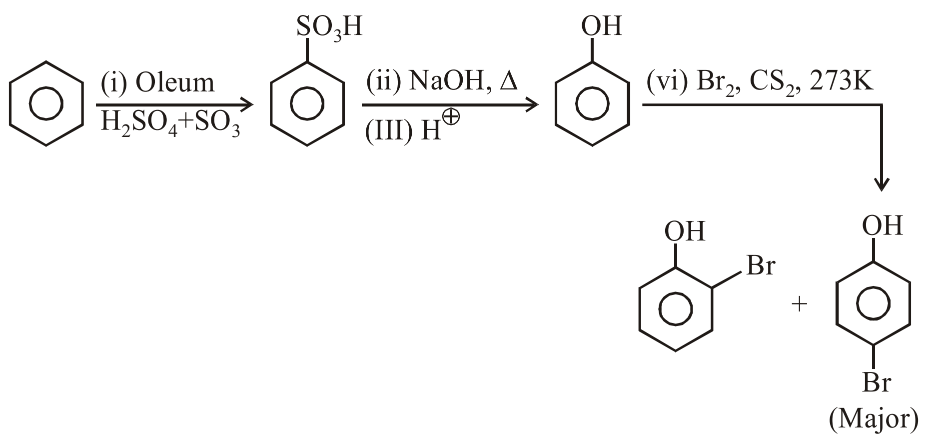
$$\mathbf{S} \xrightarrow[\text{(ii) } \mathbf{Q}]{\text{(i) NaOH}} \underset{\text{(major)}}{\mathbf{T}}$$

Ans. (4.00)

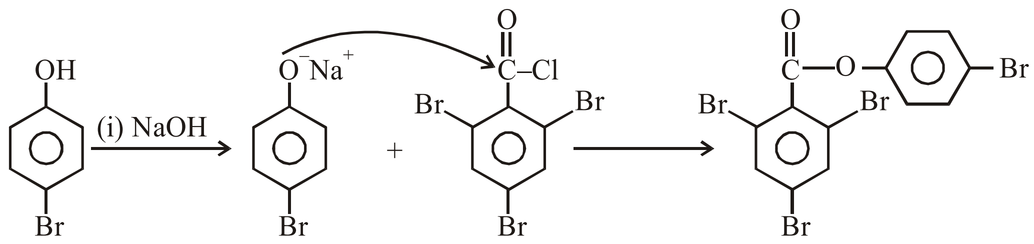
Sol. Scheme 1 :



Scheme 2 :



Scheme 3 :



(Q) (T)

6. At 143 K. the reaction of XeF_4 with O_2F_2 produces a xenon compound **Y**. The total number of lone pair(s) of electrons present on the whole molecule of **Y** is ______

Ans. (19.00)

Sol. $XeF_4 + O_2F_2 \rightarrow \underset{Y}{XeF_6} + O_2$

Y has 3 lone pair of electron in each fluorine and one lone pair of electron in xenon.

Hence total lone pair of electrons is 19.

Ans.(19)

FINAL JEE(Advanced) EXAMINATION - 2019

(Held On Monday 27th MAY, 2019)

PAPER-1	TEST PAPER WITH ANSWER & SOLUTION

PART-3 : MATHEMATICS

SECTION-1 : (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks : +3 If ONLY the correct option is chosen.
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered)
Negative Marks : –1 In all other cases

1. Let $M=\begin{bmatrix}\sin^4\theta & -1-\sin^2\theta\\ 1+\cos^2\theta & \cos^4\theta\end{bmatrix}=\alpha I+\beta M^{-1}$,

where $\alpha = \alpha(\theta)$ and $\beta = \beta(\theta)$ are real number, and I is the 2×2 identity matrix. If

α^* is the minimum of the set $\{\alpha(\theta) : \theta \in [0, 2\pi)\}$ and

β^* is the minimum of the set $\{\beta(\theta) : \theta \in [0, 2\pi)\}$,

then the value of $\alpha^* + \beta^*$ is

(1) $-\frac{37}{16}$ (2) $-\frac{29}{16}$ (3) $-\frac{31}{16}$ (4) $-\frac{17}{16}$

Ans. (2)

Sol. Given $M = \alpha I + \beta M^{-1}$

$\Rightarrow M^2 - \alpha M - \beta I = O$

By putting values of M and M^2, we get

$\alpha(\theta) = 1 - 2\sin^2\theta\cos^2\theta = 1-\frac{\sin^2 2\theta}{2}\geq\frac{1}{2}$

Also, $\beta(\theta) = -(\sin^4\theta\cos^4\theta + (1 + \cos^2\theta)(1 + \sin^2\theta))$

$= -(\sin^4\theta\cos^4\theta + 1 + \cos^2\theta + \sin^2\theta + \sin^2\theta\cos^2\theta)$

$= -(t^2 + t + 2),\ t=\frac{\sin^2 2\theta}{4}\in\left[0,\frac{1}{4}\right]$

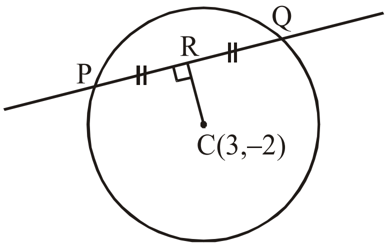
$\Rightarrow \beta(\theta) \geq -\frac{37}{16}$

2. A line $y = mx + 1$ intersects the circle $(x - 3)^2 + (y + 2)^2 = 25$ at the points P and Q. If the midpoint of the line segment PQ has x-coordinate $-\frac{3}{5}$, then which one of the following options is correct ?

(1) $6 \le m < 8$ (2) $2 \le m < 4$ (3) $4 \le m < 6$ (4) $-3 \le m < -1$

Ans. **(2)**

Sol.



$$R \equiv \left(-\frac{3}{5}, \frac{-3m}{5} + 1\right)$$

$$\text{So, } m\left(\frac{-\frac{3m}{5}+3}{-\frac{3}{5}-3}\right) = -1$$

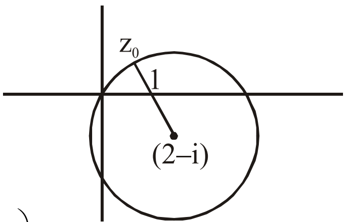
$\Rightarrow m^2 - 5m + 6 = 0$

$\Rightarrow m = 2, 3$

3. Let S be the set of all complex numbers z satisfying $|z - 2 + i| \ge \sqrt{5}$. If the complex number z_0 is such that $\frac{1}{|z_0 - 1|}$ is the maximum of the set $\left\{\frac{1}{|z-1|} : z \in S\right\}$, then the principal argument of $\frac{4 - z_0 - \bar{z}_0}{z_0 - \bar{z}_0 + 2i}$ is

(1) $\frac{\pi}{4}$ (2) $-\frac{\pi}{2}$ (3) $\frac{3\pi}{4}$ (4) $\frac{\pi}{2}$

Ans. **(2)**



Sol. $$\arg\left(\frac{4 - (z_0 - \bar{z}_0)}{(z_0 - \bar{z}_0) + zi}\right)$$

$$= \arg\left(\frac{4 - 2\operatorname{Re} z_0}{2i \operatorname{Im} z_0 + 2i}\right) = \arg\left(\frac{2 - \operatorname{Re} z_0}{(1 + \operatorname{Im} z_0)i}\right)$$

$$= \arg\left(-\left(\frac{2 - \operatorname{Re} z_0}{1 + \operatorname{Im} z_0}\right)i\right)$$

$= \arg(-ki)$; $k > 0$ (as $\operatorname{Re} z_0 < 2$ & $\operatorname{Im} z_0 > 0$)

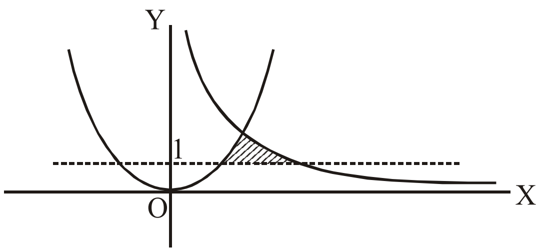
$$= -\frac{\pi}{2}$$

4. The area of the region $\{(x, y) : xy \le 8, 1 \le y \le x^2\}$ is

(1) $8\log_e 2 - \frac{14}{3}$ (2) $16\log_e 2 - \frac{14}{3}$ (3) $16\log_e 2 - 6$ (4) $8\log_e 2 - \frac{7}{3}$

Ans. (2)

Sol.



For intersection, $\frac{8}{y} = \sqrt{y} \Rightarrow y = 4$

Hence, required area = $\int_1^4 \left(\frac{8}{y} - \sqrt{y}\right) dy$

$$= \left[8\ell ny - \frac{2}{3}y^{3/2}\right]_1^4 = 16\ell n2 - \frac{14}{3}$$

Remark : The question should contain the phrase "area of the bounded region in the first quadrant". Because, in the 2^{nd} quadrant, the region above the line $y = 1$ and below $y = x^2$, satisfies the region, which is unbounded.

SECTION-2 : (Maximum Marks: 32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s)
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If only (all) the correct option(s) is (are) chosen.
  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen.
  Partial Marks : +2 If three or more options are correct but ONLY two options are chosen and both of which are correct.
  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).
  Negative Marks : –1 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 marks;
  choosing ONLY (B) will get +1 marks;
  choosing ONLY (D) will get +1 marks;
  choosing no option (i.e. the question is unanswered) will get 0 marks, and
  choosing any other combination of options will get –1 mark.

1. There are three bags B_1, B_2 and B_3. The bag B_1 contains 5 red and 5 green balls, B_2 contains 3 red and 5 green balls, and B_3 contains 5 red and 3 green balls, Bags B_1, B_2 and B_3 have probabilities $\frac{3}{10}$, $\frac{3}{10}$ and $\frac{4}{10}$ respectively of being chosen. A bag is selected at random and a ball is chosen at random from the bag. Then which of the following options is/are correct ?

(1) Probability that the selected bag is B_3 and the chosen ball is green equals $\frac{3}{10}$

(2) Probability that the chosen ball is green equals $\frac{39}{80}$

(3) Probability that the chosen ball is green, given that the selected bag is B_3, equals $\frac{3}{8}$

(4) Probability that the selected bag is B_3, given that the chosen balls is green, equals $\frac{5}{13}$

Ans. **(2,3)**

Sol.

Ball	Balls composition	$P(B_i)$
B_1	$5R+5G$	$\frac{3}{10}$
B_2	$3R+5G$	$\frac{3}{10}$
B_3	$5R+3G$	$\frac{4}{10}$

(1) $P(B_3 \cap G) = P\left(\frac{G_1}{B_3}\right)P(B_3)$

$$=\frac{3}{8}\times\frac{4}{10}=\frac{3}{20}$$

(2) $P(G) = P\left(\frac{G_1}{B_1}\right)P(B_1) + P\left(\frac{G}{B_2}\right)P(B_2) + P\left(\frac{G}{B_3}\right)P(B_3)$

$$=\frac{3}{20}+\frac{3}{16}+\frac{3}{20}=\frac{39}{80}$$

(3) $P\left(\frac{G}{B_3}\right)=\frac{3}{8}$

(4) $P\left(\frac{B_3}{G}\right)=\frac{P(G\cap B_3)}{P(G)}=\frac{3/20}{39/80}=\frac{4}{13}$

2. Define the collections $\{E_1, E_2, E_3, ...\}$ of ellipses and $\{R_1, R_2, R_3, ...\}$ of rectangles as follows :

$E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$;

R_1 : rectangle of largest area, with sides parallel to the axes, inscribed in E_1 ;

E_n : ellipse $\frac{x^2}{a_n^2} + \frac{y^2}{b_n^2} = 1$ of largest area inscribed in R_{n-1}, n > 1 ;

R_n : rectangle of largest area, with sides parallel to the axes, inscribed in E_n, n > 1.

Then which of the following options is/are correct ?

(1) The eccentricities of E_{18} and E_{19} are NOT equal

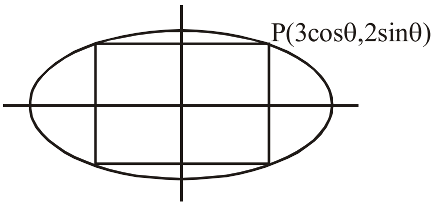
(2) The distance of a focus from the centre in E_9 is $\frac{\sqrt{5}}{32}$

(3) The length of latus rectum of E_9 is $\frac{1}{6}$

(4) $\sum_{n=1}^{N} (\text{area of } R_n) < 24$, for each positive integer N

Ans. (3,4)

Sol.



Area of R_1 = $3\sin 2\theta$; for this to be maximum

$\Rightarrow \theta = \frac{\pi}{4} \Rightarrow \left(\frac{3}{\sqrt{2}}, \frac{2}{\sqrt{2}}\right)$

Hence for subsequent areas of rectangles R_n to be maximum the coordinates will be in GP with common ratio $r = \frac{1}{\sqrt{2}} \Rightarrow a_n = \frac{3}{(\sqrt{2})^{n-1}}$; $b_n = \frac{3}{(\sqrt{2})^{n-1}}$

Eccentricity of all the ellipses will be same

Distance of a focus from the centre in $E_9 = a_9 e_9 = \sqrt{a_9^2 - b_9^2} = \frac{\sqrt{5}}{16}$

Length of latus rectum of $E_9 = \frac{2b_9^2}{a_9} = \frac{1}{6}$

$\because \sum_{n=1}^{\infty} \text{Area of } R_n = 12 + \frac{12}{2} + \frac{12}{4} + ...\infty = 24$

$\Rightarrow \sum_{n=1}^{N} (\text{area of } R_n) < 24$, for each positive integer N

3. Let $M=\begin{bmatrix}0 & 1 & a\\ 1 & 2 & 3\\ 3 & b & 1\end{bmatrix}$ and $\text{adj}M=\begin{bmatrix}-1 & 1 & -1\\ 8 & -6 & 2\\ -5 & 3 & -1\end{bmatrix}$ where a and b are real numbers. Which of the following options is/are correct ?

(1) $a + b = 3$

(2) $\det(\text{adj}M^2) = 81$

(3) $(\text{adj}M)^{-1} + \text{adj}M^{-1} = -M$

(4) If $M\begin{bmatrix}\alpha\\ \beta\\ \gamma\end{bmatrix}=\begin{bmatrix}1\\ 2\\ 3\end{bmatrix}$, then $\alpha - \beta + \gamma = 3$

Ans. **(1,3,4)**

Sol. $(\text{adj}M)_{11} = 2 - 3b = -1 \Rightarrow b = 1$

Also, $(\text{adj}M)_{22} = -3a = -6 \Rightarrow a = 2$

Now, $\det M=\begin{vmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{vmatrix}=-2$

$\Rightarrow \det(\text{adj}M^2) = (\det M^2)^2$

$= (\det M)^4 = 16$

Also $M^{-1}=\dfrac{\text{adj}M}{\det M}$

$\Rightarrow \text{adj}M = -2M^{-1}$

$\Rightarrow (\text{adj}M)^{-1} = \dfrac{-1}{2}M$

And, $\text{adj}(M^{-1}) = (M^{-1})^{-1} \det(M^{-1})$

$$=\frac{1}{\det M}M=\frac{-M}{2}$$

Hence, $(\text{adj}M)^{-1} + \text{adj}(M^{-1}) = -M$

Further, $MX = b$

$$\Rightarrow X = M^{-1}b = \frac{-\text{adj}M}{2}b$$

$$=\frac{-1}{2}\begin{bmatrix}-1 & 1 & -1\\ 8 & -6 & 2\\ -5 & 3 & -1\end{bmatrix}\begin{bmatrix}1\\ 2\\ 3\end{bmatrix}$$

$$=\frac{-1}{2}\begin{bmatrix}-2\\ 2\\ -2\end{bmatrix}=\begin{bmatrix}1\\ -1\\ 1\end{bmatrix}$$

$\Rightarrow (\alpha, \beta, \gamma) = (1, -1, 1)$

4. Let $f : \mathbb{R} \to \mathbb{R}$ be given by

$$f(x) = \begin{cases} x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1, & x < 0; \\ x^2 - x + 1, & 0 \le x < 1; \\ \frac{2}{3}x^3 - 4x^2 + 7x - \frac{8}{3}, & 1 \le x < 3; \\ (x-2)\log_e(x-2) - x + \frac{10}{3}, & x \ge 3 \end{cases}$$

Then which of the following options is/are correct ?

(1) f' has a local maximum at x = 1 (2) f is onto

(3) f is increasing on $(-\infty, 0)$ (4) f' is NOT differentiable at x = 1

Ans. **(1,2,4)**

Sol. $$f(x) = \begin{cases} (x+1)^5 - 2x, & x < 0; \\ x^2 - x + 1, & 0 \le x < 1; \\ \frac{2}{3}x^3 - 4x^2 + 7x - \frac{8}{3}, & 1 \le x < 3; \\ (x-2)\log_e(x-2) - x + \frac{10}{3}, & x \ge 3 \end{cases}$$

for $x < 0$, $f(x)$ is continuous

& $\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to 0^-} f(x) = 1$

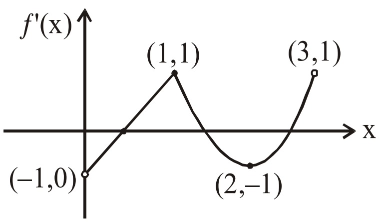
Hence, $(-\infty, 1) \subset$ Range of $f(x)$ in $(-\infty, 0)$

$f'(x) = 5(x + 1)^4 - 2$, which changes sign in $(-\infty, 0)$

$\Rightarrow$ $f(x)$ is non-monotonic in $(-\infty, 0)$

For $x \ge 3$, $f(x)$ is again continuous and $\lim_{x \to \infty} f(x) = \infty$ and $f(3) = \frac{1}{3}$

$\Rightarrow$ $\left[\frac{1}{3}, \infty\right) \subset$ Range of $f(x)$ in $[3, \infty)$



Hence, range of $f(x)$ is $\mathbb{R}$

$$f'(x) = \begin{cases} 2x - 1, & 0 \le x < 1 \\ 2x^2 - 8x + 7, & 1 \le x < 3 \end{cases}$$

Hence f' has a local maximum at x = 1 and f' is NOT differentiable at x = 1.

5. Let α and β be the roots of $x^2 - x - 1 = 0$, with $\alpha > \beta$. For all positive integers n, define

$$a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}, \quad n \geq 1$$

$b_1 = 1$ and $b_n = a_{n-1} + a_{n+1}$, $n \geq 2$.

Then which of the following options is/are correct ?

(1) $a_1 + a_2 + a_3 + ... + a_n = a_{n+2} - 1$ for all $n \geq 1$

(2) $\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \frac{10}{89}$

(3) $\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \frac{8}{89}$

(4) $b_n = \alpha^n + \beta^n$ for all $n \geq 1$

Ans. (1,2,4)

Sol. α, β are roots of $x^2 - x - 1$

$$a_{r+2} - a_r = \frac{(\alpha^{r+2} - \beta^{r+2}) - (\alpha^r - \beta^r)}{\alpha - \beta} = \frac{(\alpha^{r+2} - \alpha^r) - (\beta^{r+2} - \beta^r)}{\alpha - \beta}$$

$$= \frac{\alpha^r(\alpha^2 - 1) - \beta^r(\beta^2 - 1)}{\alpha - \beta} = \frac{\alpha^r\alpha - \beta^r\beta}{\alpha - \beta} = \frac{\alpha^{r+1} - \beta^{r+1}}{\alpha - \beta} = a_{r+1}$$

$\Rightarrow a_{r+2} - a_{r+1} = a_r$

$$\Rightarrow \sum_{r=1}^{n} a_r = a_{n+2} - a_2 = a_{n+2} - \frac{\alpha^2 - \beta^2}{\alpha - \beta} = a_{n+2} - (\alpha + \beta) = a_{n+2} - 1$$

Now $$\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \frac{\sum_{n=1}^{\infty}\left(\frac{\alpha}{10}\right)^n - \sum_{n=1}^{\infty}\left(\frac{\beta}{10}\right)^n}{\alpha - \beta}$$

$$= \frac{\frac{\frac{\alpha}{10}}{1 - \frac{\alpha}{10}} - \frac{\frac{\beta}{10}}{1 - \frac{\beta}{10}}}{\alpha - \beta} = \frac{\frac{\alpha}{10 - \alpha} - \frac{\beta}{10 - \beta}}{(\alpha - \beta)} = \frac{10}{(10 - \alpha)(10 - \beta)} = \frac{10}{89}$$

$$\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \sum_{n=1}^{\infty} \frac{a_{n-1} + a_{n+1}}{10^n} = \frac{\frac{\alpha}{10}}{1 - \frac{\alpha}{10}} + \frac{\frac{\beta}{10}}{1 - \frac{\beta}{10}} = \frac{12}{89}$$

Further, $b_n = a_{n-1} + a_{n+1}$

$$= \frac{(\alpha^{n-1} - \beta^{n-1}) + (\alpha^{n+1} - \beta^{n+1})}{\alpha - \beta}$$

(as $\alpha\beta = -1 \Rightarrow \alpha^{n-1} = -\alpha^n\beta$ & $\beta^{n-1} = -\alpha\beta^n$)

$$= \frac{\alpha^n(\alpha - \beta) + (\alpha - \beta)\beta^n}{\alpha - \beta} = \alpha^n + \beta^n$$

6. Let Γ denote a curve y = y(x) which is in the first quadrant and let the point (1, 0) lie on it. Let the tangent to Γ at a point P intersect the y-axis at Y_P. If PY_P has length 1 for each point P on Γ, then which of the following options is/are correct ?

(1) $y = \log_e\left(\frac{1+\sqrt{1-x^2}}{x}\right) - \sqrt{1-x^2}$

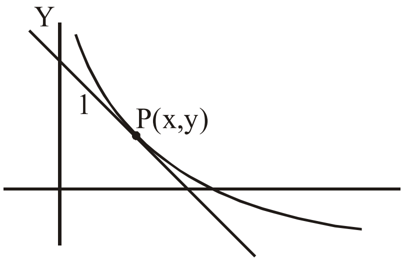
(2) $xy' - \sqrt{1-x^2} = 0$

(3) $y = -\log_e\left(\frac{1+\sqrt{1-x^2}}{x}\right) + \sqrt{1-x^2}$

(4) $xy' + \sqrt{1-x^2} = 0$

Ans. (1,4)

Sol.



$$Y - y = y'(X - x)$$

So, $Y_P = (0,\ y - xy')$

So, $x^2 + (xy')^2 = 1 \Rightarrow \frac{dy}{dx} = -\sqrt{\frac{1-x^2}{x^2}}$

[$\frac{dy}{dx}$ can not be positive i.e. $f(x)$ can not be increasing in first quadrant, for $x \in (0, 1)$]

Hence, $\int dy = -\int \frac{\sqrt{1-x^2}}{x}\,dx$

$\Rightarrow y = -\int \frac{\cos^2\theta\, d\theta}{\sin\theta}$; put $x = \sin\theta$

$\Rightarrow y = -\int \operatorname{cosec}\theta\, d\theta + \int \sin\theta\, d\theta$

$\Rightarrow y = \ln(\operatorname{cosec}\theta + \cot\theta) - \cos\theta + C$

$\Rightarrow y = \ell n\left(\frac{1+\sqrt{1-x^2}}{x}\right) - \sqrt{1-x^2} + C$

$\Rightarrow y = \ell n\left(\frac{1+\sqrt{1-x^2}}{x}\right) - \sqrt{1-x^2}$ (as $y(1) = 0$)

7. In a non-right-angled triangle ΔPQR, let p, q, r denote the lengths of the sides opposite to the angles at P, Q, R respectively. The median from R meets the side PQ at S, the perpendicular from P meets the side QR at E, and RS and PE intersect at O. If $p=\sqrt{3}$, $q=1$, and the radius of the circumcircle of the ΔPQR equals 1, then which of the following options is/are correct ?

(1) Area of $\Delta SOE = \frac{\sqrt{3}}{12}$

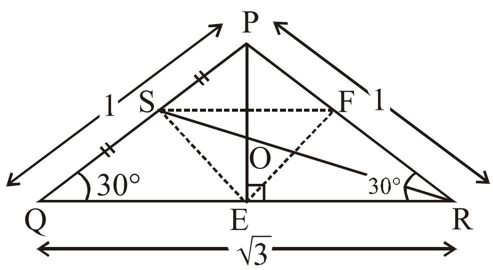
(2) Radius of incircle of $\Delta PQR = \frac{\sqrt{3}}{2}(2-\sqrt{3})$

(3) Length of $RS = \frac{\sqrt{7}}{2}$

(4) Length of $OE = \frac{1}{6}$

Ans. **(2,3,4)**

Sol. $\frac{\sin P}{\sqrt{3}} = \frac{\sin Q}{1} = \frac{1}{2R} = \frac{1}{2}$



$\Rightarrow \quad P = \frac{\pi}{3}$ or $\frac{2\pi}{3}$ and $Q = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

Since $p > q \Rightarrow P > Q$

So, if $P = \frac{\pi}{3}$ and $Q = \frac{\pi}{6} \Rightarrow R = \frac{\pi}{2}$ (not possible)

Hence, $P = \frac{2\pi}{3}$ and $Q = R = \frac{\pi}{6}$

$$r = \frac{\Delta}{s} = \frac{\frac{1}{2}(1)(\sqrt{3})\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}+2}{2}\right)} = \frac{\sqrt{3}}{2}(2-\sqrt{3})$$

Now, area of $\Delta SEF = \frac{1}{4}$ area of ΔPQR

$\Rightarrow$ area of $\Delta SOE = \frac{1}{3}$ area of $\Delta SEF = \frac{1}{12}$ area of $\Delta PQR = \frac{1}{12} \cdot \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{48}$

$$RS = \frac{1}{2}\sqrt{2(3)+2(1)-1} = \frac{\sqrt{7}}{2}$$

$$OE = \frac{1}{3}PE = \frac{1}{3} \cdot \frac{1}{2}\sqrt{2(1)^2+2(1)^2-3} = \frac{1}{6}$$

8. Let L_1 and L_2 denotes the lines

$$\vec{r} = \hat{i} + \lambda(-\hat{i} + 2\hat{j} + 2\hat{k}), \lambda \in \mathbb{R} \text{ and}$$

$$\vec{r} = \mu(2\hat{i} - \hat{j} + 2\hat{k}), \mu \in \mathbb{R}$$

respectively. If L_3 is a line which is perpendicular to both L_1 and L_2 and cuts both of them, then which of the following options describe(s) L_3?

(1) $\vec{r} = \frac{1}{3}(2\hat{i} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in \mathbb{R}$

(2) $\vec{r} = \frac{2}{9}(2\hat{i} - \hat{j} + 2\hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in \mathbb{R}$

(3) $\vec{r} = t(2\hat{i} + 2\hat{j} - \hat{k}), t \in \mathbb{R}$

(4) $\vec{r} = \frac{2}{9}(4\hat{i} + \hat{j} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in \mathbb{R}$

Ans. **(1,2,4)**

Sol. Points on L_1 and L_2 are respectively $A(1 - \lambda, 2\lambda, 2\lambda)$ and $B(2\mu, -\mu, 2\mu)$

So, $\overrightarrow{AB} = (2\mu + \lambda - 1)\hat{i} + (-\mu - 2\lambda)\hat{j} + (2\mu - 2\lambda)\hat{k}$

and vector along their shortest distance $= 2\hat{i} + 2\hat{j} - \hat{k}$.

Hence, $\frac{2\mu + \lambda - 1}{2} = \frac{-\mu - 2\lambda}{2} = \frac{2\mu - 2\lambda}{-1}$

$\Rightarrow \quad \lambda = \frac{1}{9}$ & $\mu = \frac{2}{9}$

Hence, $A \equiv \left(\frac{8}{9}, \frac{2}{9}, \frac{2}{9}\right)$ and $B \equiv \left(\frac{4}{9}, -\frac{2}{9}, \frac{4}{9}\right)$

$\Rightarrow$ Mid point of AB $\equiv \left(\frac{2}{3}, 0, \frac{1}{3}\right)$

SECTION-3 : (Maximum Marks: 18)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **Two** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct numerical value is entered.

Zero Marks : 0 In all other cases.

1. If

$$I = \frac{2}{\pi}\int_{-\pi/4}^{\pi/4} \frac{dx}{(1+e^{\sin x})(2-\cos 2x)}$$

then $27I^2$ equals ______

Ans. (4.00)

Sol. $2I = \frac{2}{\pi}\int_{-\pi/4}^{\pi/4}\left[\frac{1}{(1+e^{\sin x})(2-\cos 2x)} + \frac{1}{(1+e^{-\sin x})(2-\cos 2x)}\right]dx$ (using King's Rule)

$$\Rightarrow \quad I = \frac{1}{\pi}\int_{-\pi/4}^{\pi/4} \frac{dx}{2-\cos 2x}$$

$$\Rightarrow \quad I = \frac{2}{\pi}\int_{0}^{\pi/4} \frac{dx}{2-\cos 2x} = \frac{2}{\pi}\int_{0}^{\pi/4} \frac{\sec^2 dx dx}{1+3\tan^2 x}$$

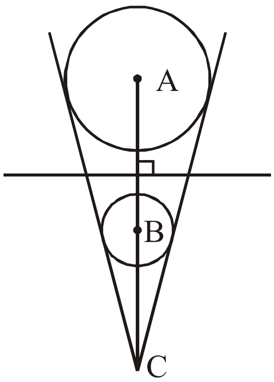
$$= \frac{2}{\sqrt{3}\pi}\left[\tan^{-1}\left(\sqrt{3}\tan x\right)\right]_0^{\pi/4} = \frac{2}{3\sqrt{3}}$$

$$\Rightarrow \quad 27I^2 = 27 \times \frac{4}{27} = 4$$

2. Let the point B be the reflection of the point A(2, 3) with respect to the line 8x – 6y – 23 = 0. Let Γ_A and Γ_B be circles of radii 2 and 1 with centres A and B respectively. Let T be a common tangent to the circles Γ_A and Γ_B such that both the circles are on the same side of T. If C is the point of intersection of T and the line passing through A and B, then the length of the line segment AC is ______

Ans. (10.00)

Sol. Distance of point A from given line $= \frac{5}{2}$



$$\frac{CA}{CB} = \frac{2}{1}$$

$$\Rightarrow \quad \frac{AC}{AB} = \frac{2}{1}$$

$$\Rightarrow \quad AC = 2 \times 5 = 10$$

3. Let AP (a; d) denote the set of all the terms of an infinite arithmetic progression with first term a and common difference $d > 0$. If $AP(1; 3) \cap AP(2; 5) \cap AP(3; 7) = AP(a; d)$ then $a + d$ equals ___

Ans. (157.00)

Sol. We equate the general terms of three respective

A.P.'s as $1 + 3a = 2 + 5b = 3 + 7c$

$\Rightarrow$ 3 divides $1 + 2b$ and 5 divides $1 + 2c$

$\Rightarrow$ $1 + 2c = 5, 15, 25$ etc.

So, first such terms are possible when $1 + 2c = 15$ i.e. $c = 7$

Hence, first term = a = 52

$d = \text{lcm}(3, 5, 7) = 105$

$\Rightarrow$ $a + d = 157$

4. Let S be the sample space of all 3×3 matrices with entries from the set {0, 1}. Let the events E_1 and E_2 be given by

$$E_1 = \{A \in S : \det A = 0\} \text{ and}$$

$$E_2 = \{A \in S : \text{sum of entries of A is } 7\}.$$

If a matrix is chosen at random from S, then the conditional probability $P(E_1|E_2)$ equals ____

Ans. (0.50)

Sol. $n(E_2) = {}^9C_2$ (as exactly two cyphers are there)

Now, det A = 0, when two cyphers are in the same column or same row

$\Rightarrow$ $n(E_1 \cap E_2) = 6 \times {}^3C_2$.

Hence, $P\left(\frac{E_1}{E_2}\right) = \frac{n(E_1 \cap E_2)}{n(E_2)} = \frac{18}{36} = \frac{1}{2} = 0.50$

5. Three lines are given by

$$\vec{r} = \lambda\hat{i},\ \lambda \in \mathbb{R}$$

$$\vec{r} = \mu(\hat{i} + \hat{j}), \mu \in \mathbb{R} \text{ and}$$

$$\vec{r} = v(\hat{i} + \hat{j} + \hat{k}), v \in \mathbb{R}.$$

Let the lines cut the plane $x + y + z = 1$ at the points A, B and C respectively. If the area of the triangle ABC is Δ then the value of $(6\Delta)^2$ equals ___

Ans. (0.75)

Sol. A(1, 0, 0), $B\left(\frac{1}{2},\frac{1}{2},0\right)$ & $C\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$

Hence, $\overrightarrow{AB}=-\frac{1}{2}\hat{i}+\frac{1}{2}\hat{j}$ & $\overrightarrow{AC}=-\frac{2}{3}\hat{i}+\frac{1}{3}\hat{j}+\frac{1}{3}\hat{k}$

So, $\Delta=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|=\frac{1}{2}\sqrt{\frac{1}{2}\times\frac{2}{3}-\frac{1}{4}}$

$=\frac{1}{2\times2\sqrt{3}}$

$\Rightarrow$ $(6\Delta)^2=\frac{3}{4}=0.75$

6. Let $\omega\neq1$ be a cube root of unity. Then the minimum of the set

$$\{|a+b\omega+c\omega^2|^2 : a, b, c \text{ distinct non-zero integers}\}$$

equals ____

Ans. **(3.00)**

Sol. $|a+b\omega+c\omega^2|^2=(a+b\omega+c\omega^2)\left(\overline{a+b\omega+c\omega^2}\right)$

$=(a+b\omega+c\omega^2)(a+b\omega^2+c\omega)$

$=a^2+b^2+c^2-ab-bc-ca$

$=\frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$

$\geq\frac{1+1+4}{2}=3$ (when a = 1, b = 2, c = 3)

FINAL JEE(Advanced) EXAMINATION - 2019

(Held On Monday 27th MAY, 2019)

PAPER-2 | **TEST PAPER WITH ANSWER**

PART-1 : PHYSICS

SECTION-1 : (Maximum Marks: 32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s)
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If only (all) the correct option(s) is (are) chosen.
Partial Marks : +3 If all the four options are correct but ONLY three options are chosen.
Partial Marks : +2 If three or more options are correct but ONLY two options are chosen and both of which are correct.
Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).
Negative Marks : –1 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2 marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 marks;
choosing ONLY (B) will get +1 marks;
choosing ONLY (D) will get +1 marks;
choosing no option (i.e. the question is unanswered) will get 0 marks, and
choosing any other combination of options will get –1 mark.

---

1. A mixture of ideal gas containing 5 moles of monatomic gas and 1 mole of rigid diatomic gas is initially at pressure P_0, volume V_0 and temperature T_0. If the gas mixture is adiabatically compressed to a volume $V_0/4$, then the correct statement(s) is/are,

(Give $2^{1.2} = 2.3$; $2^{3.2} = 9.2$; R is gas constant)

(1) The final pressure of the gas mixture after compression is in between $9P_0$ and $10P_0$

(2) The average kinetic energy of the gas mixture after compression is in between $18RT_0$ and $19RT_0$

(3) The work |W| done during the process is $13RT_0$

(4) Adiabatic constant of the gas mixture is 1.6

Ans. (1,3,4)

Sol. $n_1 = 5$ moles $C_{V_1} = \frac{3R}{2}$ $P_0V_0T_0$

$n_2 = 1$ mole $C_{V_2} = \frac{5R}{2}$

$$(C_V)_m = \frac{n_1C_{V_1} + n_2C_{V_2}}{n_1 + n_2} = \frac{5\times\frac{3R}{2} + 1\times\frac{5R}{2}}{6} = \frac{5R}{3}$$

$$\gamma_m = \frac{(c_P)_m}{(c_V)_m} = \frac{8}{5}$$

$\therefore$ Option 4 is correct

$$(C_P)_m = \frac{5R}{3} + R = \frac{8R}{3}$$

(1) $P_0V_0^{\gamma} = P\left(\frac{V_0}{4}\right)^{\gamma} \Rightarrow P = P_0(4)^{8/5} = 9.2\ P_0$ which is between $9P_0$ and $10P_0$

(2) Average K.E. $= 5 \times \frac{3}{2}RT + 1 \times \frac{5RT}{2}$

$= 10RT$

To calculate T

$$\frac{P_0V_0}{T_0} = 9.2P_0 \times \frac{V_0}{4\times T}$$

so $T = \frac{9.2}{4}T_0$

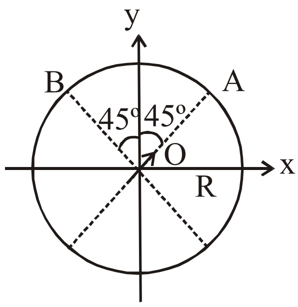
Now average KE $= 10\ R \times 9.2\ \frac{T_0}{4} = 23RT_0$

(3) $W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$

$$= \frac{P_0V_0 - 9.2P_0\times\frac{V_0}{4}}{3/5} = -13RT_0$$

2. An electric dipole with dipole moment $\frac{p_0}{\sqrt{2}}(\hat{i}+\hat{j})$ is held fixed at the origin O in the presence of an uniform electric field of magnitude E_0. If the potential is constant on a circle of radius R centered at the origin as shown in figure, then the correct statement(s) is/are:

(ε_0 is permittivity of free space, R >> dipole size)



(1) $R=\left(\frac{p_0}{4\pi\varepsilon_0 E_0}\right)^{1/3}$

(2) The magnitude of total electric field on any two points of the circle will be same

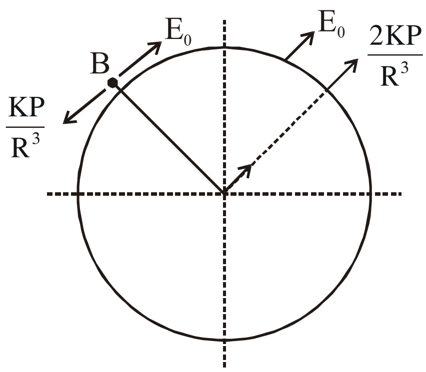
(3) Total electric field at point A is $\vec{E}_A=\sqrt{2}E_0(\hat{i}+\hat{j})$

(4) Total electric field at point B is $\vec{E}_B=0$

Ans. (1,4)

Sol. (1) $\vec{P}=\frac{P_0}{\sqrt{2}}\left(\hat{i}+\hat{j}\right)$

E.F. at B along tangent should be zero since circle is equipotential.



So, $E_0 = \frac{K|\vec{P}|}{R^3}$ & $E_B = 0$

So, $R^3 = \frac{KP_0}{E_0}=\left(\frac{P_0}{4\pi\in_0 E_0}\right)$

So $R = \left(\frac{P_0}{4\pi\in_0 E_0}\right)^{1/3}$

So, (1) is correct

(2) Because E_0 is uniform & due to dipole E.F. is different at different points, so magnitude of total E.F. will also be different at different points.

So, (2) is incorrect

(3) $E_A = \frac{2KP}{R^3} + \frac{KP}{R^3} = 3\frac{KP}{R^3}\frac{P_0}{\sqrt{2}}(\hat{i} + \hat{j})$

So, (3) is wrong

(4) $E_B = 0$

so, (4) is correct

3. A thin and uniform rod of mass M and length L is held vertical on a floor with large friction. The rod is released from rest so that it falls by rotating about its contact-point with the floor without slipping. Which of the following statement(s) is/are correct, when the rod makes an angle 60° with vertical ? [g is the acceleration due to gravity]

(1) The radial acceleration of the rod's center of mass will be $\frac{3g}{4}$

(2) The angular acceleration of the rod will be $\frac{2g}{L}$

(3) The angular speed of the rod will be $\sqrt{\frac{3g}{2L}}$

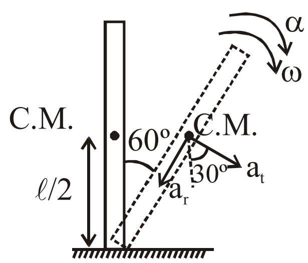
(4) The normal reaction force from the floor on the rod will be $\frac{Mg}{16}$

Ans. **(1,3,4)**

Sol. We can treat contact point as hinged.

Applying work energy theorem

$W_g = \Delta K.E.$



$$mg\frac{\ell}{4} = \frac{1}{2}\left(\frac{m\ell^2}{3}\right)\omega^2$$

$$\omega = \sqrt{\frac{3g}{2\ell}}$$

radial acceleration of C.M. of rod $= \left(\frac{\ell}{2}\right)\omega^2 = \frac{3g}{4}$

Using $\tau = I\,\alpha$ about contact point

$$\frac{mg\ell}{2}\sin 60° = \frac{m\ell^2}{3}\alpha$$

$\Rightarrow \quad \alpha = \frac{3\sqrt{3}}{4\ell}g$

Net vertical acceleration of C.M. of rod

$a_v = a_r \cos 60^\circ + a_t \cos 30^\circ$

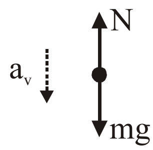
$= \left(\frac{3g}{4}\right)\left(\frac{1}{2}\right) + \left(\alpha\frac{\ell}{2}\right)\cos 30^\circ$

$= \frac{3g}{8} + \frac{3\sqrt{3}g}{4\ell}\left(\frac{\ell}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{3g}{8} + \frac{9g}{16} = \frac{15}{16}g$

Applying $F_{net} = ma$ in vertical direction on rod as system

$$mg - N = ma_v = m\left(\frac{15}{16}g\right)$$



$\Rightarrow \quad N = \frac{mg}{16}$

4. A small particle of mass m moving inside a heavy, hollow and straight tube along the tube axis undergoes elastic collision at two ends. The tube has no friction and it is closed at one end by a flat surface while the other end is fitted with a heavy movable flat piston as shown in figure. When the distance of the piston from closed end is $L = L_0$ the particle speed is $v = v_0$. The piston is moved inward at a very low speed V such that $V << \frac{dL}{L}v_0$, where dL is the infinitesimal displacement of the piston. Which of the following statement(s) is/are correct ?

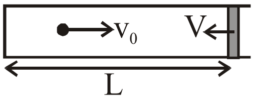
(1) The rate at which the particle strikes the piston is v/L

(2) After each collision with the piston, the particle speed increases by 2V

(3) The particle's kinetic energy increases by a factor of 4 when the piston is moved inward from L_0 to $\frac{1}{2}L_0$

(4) If the piston moves inward by dL, the particle speed increases by $2v\frac{dL}{L}$

Ans. (2,3)

Sol. 

(1) average rate of collision = $\frac{2L}{v}$

(2) speed of particle after collision = $2V + v_0$

change in speed = $(2V + v_0) - v_0$

after each collision = 2V

no. of collision per unit time (frequency) = $\frac{v}{2L}$

change in speed in dt time = 2V × number of collision in dt time

$$\Rightarrow dv = 2V\left(\frac{v}{2L}\right) \cdot \frac{dL}{V}$$

$$\boxed{dv = \frac{vdL}{L}}$$

Now, $dv = -\frac{vdL}{L}$ {as L decrease}

$$\int_{v_0}^{v} \frac{dv}{v} = -\int_{L_0}^{L_0/2} \frac{dL}{L}$$

$$\Rightarrow [\ln v]_{v_0}^{v} = -[\ln L]_{L}^{L_0/2}$$

$$\Rightarrow v = 2v_0$$

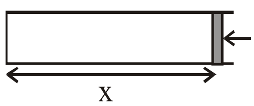
$$\Rightarrow KE_{L_0} = \frac{1}{2}mv_0^2$$

$$\boxed{\frac{KE_{L_0/2}}{KE_0} = 4}$$

$$KE_{L_0/2} = \frac{1}{2}m(2v_0)^2$$

or

$$(dt)\left(\frac{v}{2x}\right)\frac{2mv}{dt} = F$$



$$F = \frac{mv^2}{x}$$

$$-m\,v\frac{dv}{dx} = \frac{mv^2}{x}$$

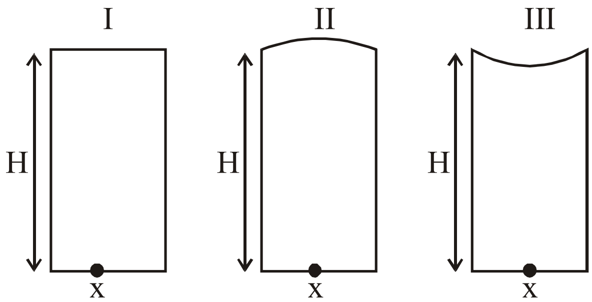
$$-\frac{dv}{v} = \frac{dx}{x}$$

$$\ln\frac{v_2}{v_1} = \ln\frac{x_1}{x_2}$$

vx = constant $\Rightarrow$ on decreasing length to half K.E. becomes 1/4

vdx + xdv = 0

5. Three glass cylinders of equal height H = 30 cm and same refractive index n = 1.5 are placed on a horizontal surfaces shown in figure. Cylinder I has a flat top, cylinder II has a convex top and cylinder III has a concave top. The radii of curvature of the two curved tops are same (R = 3m). If H_1, H_2 and H_3 are the apparent depths of a point X on the bottom of the three cylinders, respectively, the correct statement(s) is/are



(1) $H_3 > H_1$

(2) $0.8\text{ cm} < (H_2 - H_1) < 0.9\text{ cm}$

(3) $H_2 > H_3$

(4) $H_2 > H_1$

Ans. (3,4)

Sol. $H_1 = \frac{2H}{3} = \frac{2}{3}\times\frac{3}{10} = \frac{1}{5}\text{m}$

for 2nd

$$\frac{1}{v}+\frac{3}{2H}=\frac{-1}{2(-3)}$$

$$\frac{1}{v}=\frac{1}{6}-\frac{10}{2}=\frac{1}{6}-\frac{30}{6}=\frac{-29}{6}$$

$$H_2=\frac{6}{29}>H_1$$

For 3rd

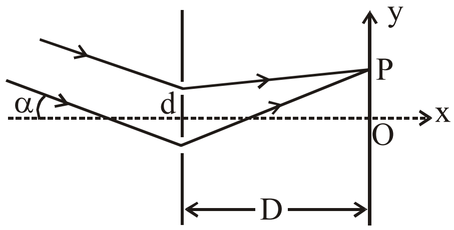
$$\frac{1}{v}+\frac{3}{2H}=\frac{-1}{2(3)}$$

$$\frac{1}{v}=\frac{-1}{6}-5=\frac{-31}{6}$$

$$H_3=\frac{6}{31}$$

so $\boxed{H_3 < H_1 < H_2}$ & $(H_2 - H_1) = \frac{6}{29}-\frac{6}{31}=0.68\,\text{cm}$

6. In a Young's double slit experiment, the slit separation d is 0.3 mm and the screen distance D is 1m. A parallel beam of light of wavelength 600nm is incident on the slits at angle α as shown in figure. On the screen, the point O is equidistant from the slits and distance PO is 11.0 mm. Which of the following statement(s) is/are correct ?



(1) For $\alpha = \frac{0.36}{\pi}$ degree, there will be destructive interference at point O.

(2) Fringe spacing depends on α

(3) For $\alpha = \frac{0.36}{\pi}$ degree, there will be destructive interference at point P

(4) For α = 0, there will be constructive interference at point P.

Ans. **(3)**

Sol. (1) $\Delta x = d\sin\alpha$

$= d\alpha$ (as α is very small) $\qquad \alpha = \frac{.36}{180} = (2\times10^{-3})\,\text{rad}$

$$\frac{\Delta x}{\lambda} = \frac{(3\times10^{-4})\,(2\times10^{-3})}{6\times10^{-7}} = 1$$

so constructive interference

(2) $\beta = \frac{D\lambda}{d}$

(3) $\Delta x_p = d\alpha + \frac{dy}{D}$

$= 3 \times 10^{-4} (2 \times 10^{-3} + 11 \times 10^{-3})$

$= 39 \times 10^{-7}$

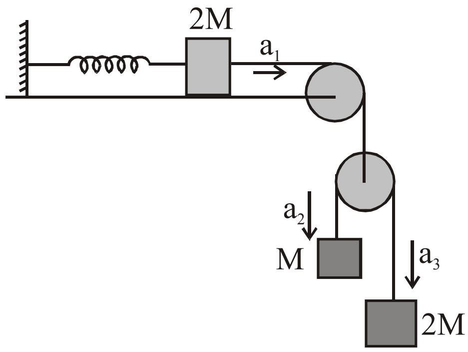
$$\frac{\Delta x_p}{\lambda} = \frac{39\times10^{-7}}{6\times10^{-7}} = 6.5 \text{ so destructive}$$

(4) $\Delta x_p = \frac{dy}{D} = (3\times10^{-4})\times11\times10^{-3}$

$= 33 \times 10^{-7}$

$$\frac{\Delta x_p}{\lambda} = \frac{33\times10^{-7}}{6\times10^{-7}} = 5.5 \Rightarrow \text{destructive}$$

7. A block of mass 2M is attached to a massless spring with spring-constant k. This block is connected to two other blocks of masses M and 2M using two massless pulleys and strings. The accelerations of the blocks are a_1, a_2 and a_3 as shown in figure. The system is released from rest with the spring in its unstretched state. The maximum extension of the spring is x_0. Which of the following option(s) is/are correct ? [g is the acceleration due to gravity. Neglect friction]



(1) $x_0 = \dfrac{4Mg}{k}$

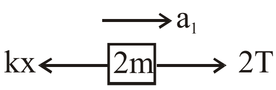
(2) When spring achieves an extension of $\dfrac{x_0}{2}$ for the first time, the speed of the block connected to the spring is $3g\sqrt{\dfrac{M}{5k}}$

(3) $a_2 - a_1 = a_1 - a_3$

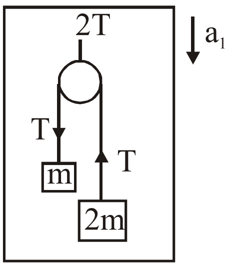
(4) At an extension of $\dfrac{x_0}{4}$ of the spring, the magnitude of acceleration of the block connected to the spring is $\dfrac{3g}{10}$

Ans. (3)

Sol.



$2T - kx = 2ma_1$



$$T = \frac{2(2m)(m)}{3m}(g - a_1)$$

$$= \frac{4m}{3}(g - a_1)$$

$$\frac{8m}{3}(g - a_1) - kx = 2ma_1$$

$$\frac{8Mg}{3}-\frac{8ma_1}{3}-kx=2ma_1$$

$$\frac{8Mg}{3}-kx=\frac{14ma_1}{3}$$

$$\frac{8Mg-3kx}{14m}=a_1$$

$$a_1 = \frac{8Mg-3kx}{14m}$$

$$\frac{vdv}{dx}=\left(\frac{8Mg}{14m}-\frac{3kx}{14m}\right)$$

$$\int vdv=\frac{1}{14m}\int(8Mg-3kx)dx$$

for max elongation

$$0 = \frac{1}{14m}\int_0^{x_0}(8Mg-3kx)dx$$

$$= \frac{1}{14m}\left(8Mgx_0-\frac{3kx_0^2}{2}\right)$$

$$8Mgx_0 = \frac{3kx_0^2}{2}$$

$$\boxed{x_0=\frac{16Mg}{3k}}$$

$$\text{at } x = \frac{x_0}{2}$$

$$\int_0^v vdv=\frac{1}{14m}\int_0^{x_0/2}(8Mg-3kx)dx$$

$$\frac{v^2}{2}=\frac{1}{14m}\left(\frac{8Mgx_0}{2}-\frac{3kx_0^2}{2\times 4}\right)$$

$$v^2 = \frac{1}{7m}\left(\frac{8Mg}{2}\times\frac{16Mg}{3x}-\frac{3x}{8}\times\frac{16M^2g^2}{3x\times 3x}\right)$$

$$= \frac{1}{7m}\left(\frac{64M^2g^2}{3x}-\frac{2M^2g^2}{3x}\right)$$

$$v^2 = \frac{62Mg^2}{21k}$$

For acc. $\boxed{2a_1=a_2+a_3}$ therefore

$a_2 - a_1 = a_1 - a_3$

$$a_1 = \frac{8Mg - 3k\,x_0/4}{14m}$$

$$= \frac{8g}{14} - \frac{3kx_0}{14m \times 4}$$

$$= \frac{8g}{14} - \frac{3x}{14m \times 4} \times \frac{16Mg}{3x}$$

$$= \frac{8g}{14} - \frac{4g}{14}$$

$$= \frac{4g}{14} = \frac{2g}{7}$$

OR

$$\frac{8mg}{3} - \frac{8m}{3}a_1 - kx = 2ma_1$$

$$\frac{14m}{3}a_1 = -k\left[x - \frac{8mg}{3k}\right]$$

$$a_1 = -\frac{3k}{14m}\left[x - \frac{8mg}{3k}\right] \text{ ....(i)}$$

that means, block 2m (connected with the spring) will perform SHM about $x_1 = \frac{8mg}{3k}$ therefore.

maximum elongation in the spring $x_0 = 2x_1 = \frac{16mg}{3k}$

on comparing equation (1) with
$a = -\omega^2 (x - x_0)$

$$\omega = \sqrt{\frac{3k}{14m}}$$

at $\left(\frac{x_0}{2}\right)$, block will be passing through its mean position therefore at mean position

$$v_0 = A\omega = \frac{8mg}{3k}.\sqrt{\frac{3k}{14m}}$$

At, $\frac{x_0}{4} \Rightarrow x = \frac{A}{2}$

$$\therefore\ a_{cc} = -\frac{A}{2}\omega^2$$

$$= -\frac{4mg}{3k}.\frac{3h}{14m} = -\frac{2g}{7}$$

8. A free hydrogen atom after absorbing a photon of wavelength λ_a gets excited from the state n = 1 to the state n = 4. Immediately after that the electron jumps to n = m state by emitting a photon of wavelength λ_e. Let the change in momentum of atom due to the absorption and the emission are Δp_a and Δp_e , respectively. If $\lambda_a/\lambda_e = \frac{1}{5}$. Which of the option(s) is/are correct ?

[Use hc = 1242 eV nm; 1 nm = 10^{-9} m, h and c are Planck's constant and speed of light, respectively]

(1) λ_e = 418 nm

(2) The ratio of kinetic energy of the electron in the state n = m to the state n = 1 is $\frac{1}{4}$

(3) m = 2

(4) $\Delta p_a/\Delta p_e = \frac{1}{2}$

Ans. **(2,3)**

Sol. $$\frac{hc}{\lambda_a} = 13.6\left[\frac{1}{1} - \frac{1}{4^2}\right] \quad \text{...(i)}$$

$$\frac{hc}{\lambda_e} = 13.6\left[\frac{1}{m^2} - \frac{1}{4^2}\right] \quad \text{...(ii)}$$

(ii) / (i) , we get

$$\frac{\lambda_a}{\lambda_e} = \frac{\left[\frac{1}{m^2} - \frac{1}{16}\right]}{\left[1 - \frac{1}{16}\right]} = \frac{1}{5}$$

$$\Rightarrow \frac{1}{m^2} - \frac{1}{16} = \frac{15}{16} \times \frac{1}{5}$$

$$\Rightarrow \frac{1}{m^2} - \frac{1}{16} = \frac{3}{16}$$

$$\Rightarrow \frac{1}{m^2} = \frac{3}{16} + \frac{1}{16}$$

$$\Rightarrow \boxed{m = 2}$$

from (ii)

$$\frac{hc}{\lambda_e} = 13.6\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = 13.6 \times \frac{3}{16} \text{ev}$$

$$\Rightarrow \lambda_e = \frac{12400 \times 16}{13.6 \times 3} \text{Å}$$

$\Rightarrow \quad \lambda_e \approx 4862 Å$

we have $KE_n \propto \frac{z^2}{n^2}$

$$\Rightarrow \frac{KE_2}{KE_1} = \frac{1}{4}$$

$$\Delta P_a = \frac{h}{\lambda_a}$$

$$\Delta P_e = \frac{h}{\lambda_e}$$

$$\Rightarrow \frac{\Delta P_a}{\Delta P_e} = \frac{\lambda_e}{\lambda_a}$$

SECTION-2 : (Maximum Marks: 18)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **Two** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct numerical value is entered.

Zero Marks : 0 In all other cases.

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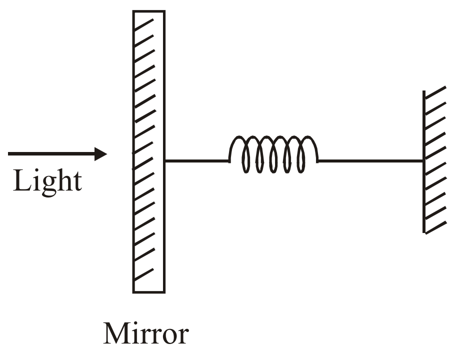
1. A perfectly reflecting mirror of mass M mounted on a spring constitutes a spring-mass system of angular frequency Ω such that $\frac{4\pi M\Omega}{h} = 10^{24} m^{-2}$ with h as Planck's constant. N photons of wavelength $\lambda = 8\pi \times 10^{-6}$m strike the mirror simultaneously at normal incidence such that the mirror gets displaced by 1μm. If the value of N is $x \times 10^{12}$, then the value of x is________.

[Consider the spring as massless]



Ans. (1.00)

Sol. Let momentum of one photon is p and after reflection velocity of the mirror is v.

conservation of linear momentum

$$Np\hat{i} = -Np\hat{i} + mv\hat{i}$$

$$mv\hat{i} = 2pN\hat{i}$$

mv = 2Np ....(1)

since v is velocity of mirror (spring mass system) at mean position,

$v = A\Omega$

Where A is maximum deflection of mirror from mean position. ($A = 1\ \mu m$) and Ω is angular frequency of mirror spring system,

momentum of 1 photon, $p = \frac{h}{\lambda}$

mv = 2Np .....(i)

$$mA\Omega = 2N\frac{h}{\lambda}$$

$$N = \frac{m\Omega}{h} \times \frac{\lambda A}{2}$$

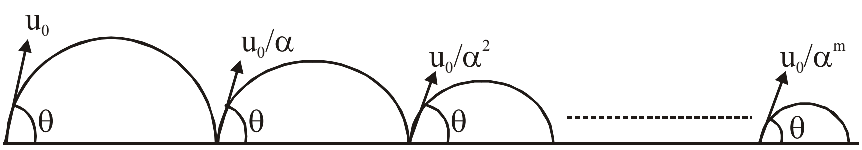
given, $\frac{m\Omega}{h} = \frac{10^{24}}{4\pi} m^{-2}$

$\lambda = 8\pi \times 10^{-6}$ m

$$N = \frac{10^{24}}{4\pi} \times \frac{8\pi \times 10^{-6} \times 10^{-6}}{2}$$

$N = 10^{12} = x \times 10^{12}$

2. A ball is thrown from ground at an angle θ with horizontal and with an initial speed u_0. For the resulting projectile motion, the magnitude of average velocity of the ball up to the point when it hits the ground for the first time is V_1. After hitting the ground, ball rebounds at the same angle θ but with a reduced speed of u_0/α. Its motion continues for a long time as shown in figure. If the magnitude of average velocity of the ball for entire duration of motion is 0.8 V_1, the value of α is______



Ans. (4.00)

Sol. $\underset{<v>}{\text{Average velocity}} = \frac{\text{Total displacement}}{\text{Total time}}$

Total time taken = $t_1 + t_2 + t_3 + ...$

$= t_1 + \frac{t_1}{\alpha} + \frac{t_1}{\alpha^2} + ...$

Total time = $\frac{t_1}{1-\frac{1}{\alpha}}$

Total displacement = $v_1t_1 + v_2t_2 + ...$

$= v_1t_1 + \frac{v_1}{\alpha}.\frac{t_1}{\alpha} + ...$

$= \frac{v_1t_1}{1-\frac{1}{\alpha^2}}$

On solving

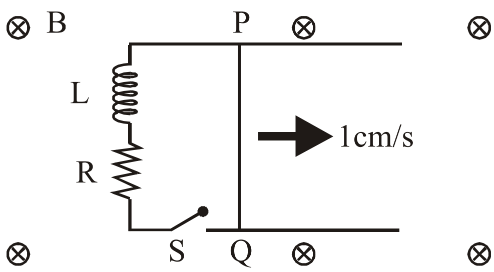
$<v> = \frac{v_1\alpha}{\alpha+1} = 0.8v_1$

$\boxed{\alpha = 4.00}$

3. A 10 cm long perfectly conducting wire PQ is moving, with a velocity 1cm/s on a pair of horizontal rails of zero resistance. One side of the rails is connected to an inductor L = 1 mH and a resistance R = 1Ω as shown in figure. The horizontal rails, L and R lie in the same plane with a uniform magnetic field B = 1 T perpendicular to the plane. If the key S is closed at certain instant, the current in the circuit after 1 millisecond is $x \times 10^{-3}$A, where the value of x is_______.

[Assume the velocity of wire PQ remains constant (1 cm/s) after key S is closed.

Given : $e^{-1} = 0.37$, where e is base of the natural logarithm]



Ans. (0.63)

Sol. Since velocity of PQ is constant. So emf developed across it remains constant.

$\varepsilon = Blv$ where ℓ = length of wire PQ

Current at any time t is given by

$$i = \frac{\varepsilon}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

$$i = \frac{B\ell v}{R}(1 - e^{-\frac{Rt}{L}})$$

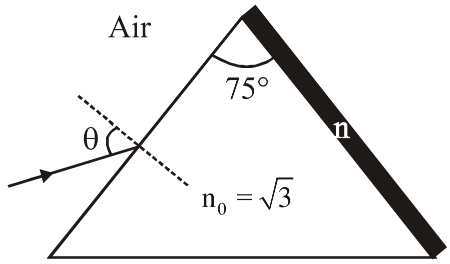
$= 1\times\left(\frac{10}{100}\right)\times\left(\frac{1}{100}\right)\times\frac{1}{1}\left(1-e^{\frac{-1\times10^{-3}}{1\times10^{-3}}}\right)$

$= \frac{1}{1000}\times(1-e^{-1})$

$= \frac{1}{1000}\times(1-0.37)$

$i = 0.63\times10^{-3}$ A $\Rightarrow x = 0.63$

4. A monochromatic light is incident from air on a refracting surface of a prism of angle 75° and refractive index $n_0=\sqrt{3}$. The other refracting surface of a prism is coated by a thin film of material of refractive index n as shown in figure. The light suffers total internal reflection at the coated prism surface for an incidence angle of $\theta \le 60°$. The value of n^2 is_______.



Ans. (1.50)

Sol. At θ = 60° ray incidents at critical angle at second surface

So,

$\sin\theta = \sqrt{3}\ \sin r_1$

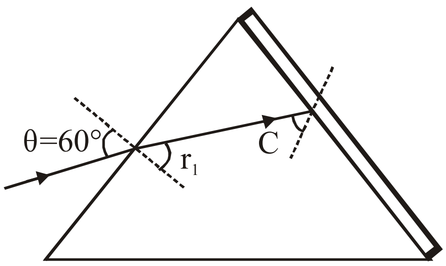
$\frac{\sqrt{3}}{2} = \sqrt{3}\sin r_1$

$r_1 = 30°$

$r_2 = 45° = C$

$\sqrt{3}\sin 45° = n\sin 90°$

$n = \sqrt{\frac{3}{2}} \Rightarrow n^2 = \frac{3}{2}$



5. Suppose a $^{226}_{88}Ra$ nucleus at rest and in ground state undergoes α-decay to a $^{222}_{86}Rn$ nucleus in its excited state. The kinetic energy of the emitted α particle is found to be 4.44 MeV. $^{222}_{86}Rn$ nucleus then goes to its ground state by γ-decay. The energy of the emitted γ-photon is _______ keV,

[Given: atomic mass of $^{226}_{88}Ra = 226.005u$, atomic mass of $^{222}_{86}Rn = 222.000u$, atomic mass of α particle = 4.000u, 1u = 931 MeV/c^2, c is speed of the light]

Ans. (135.00)

Sol. $Ra^{226} \longrightarrow Rn^{222} + \alpha$

Q = (226.005 – 222–4) 931 MeV

= 4.655 MeV

$K_\alpha = \frac{A-4}{A}\left(Q - E_\gamma\right)$

4.44 MeV = $\frac{222}{226}\left(Q - E_\gamma\right)$

$Q - E_\gamma = (4.44)\left(\frac{226}{222}\right)\text{MeV}$

E_γ = 4.655 – 4.520

= .135 MeV

= 135 KeV

6. An optical bench has 1.5 m long scale having four equal divisions in each cm. While measuring the focal length of a convex lens, the lens is kept at 75 cm mark of the scale and the object pin is kept at 45 cm mark. The image of the object pin on the other side of the lens overlaps with image pin that is kept at 135 cm mark. In this experiment, the percentage error in the measurement of the focal length of the lens is________.

Ans. **(0.69)**

Sol. For the given lens

u = –30 cm

v = 60 cm

& $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ on solving : f = 20 cm

also $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

on differentiation

$$\frac{df}{f^2} = \frac{dv}{v^2} + \frac{du}{u^2}$$

$$\frac{df}{f} = f\left[\frac{dv}{v^2} + \frac{du}{u^2}\right]$$

& $\frac{df}{f} \times 100 = f\left[\frac{dv}{v^2} + \frac{du}{u^2}\right] \times 100\%$

f = 20cm, du = dv = $\frac{1}{4}$cm

Since there are 4 divisions in 1 cm on scale

$$\therefore \frac{df}{f}\times 100 = 20\left[\frac{1/4}{(60)^2}+\frac{1/4}{(30)^2}\right]\times 100\%$$

$$= 5\left[\frac{1}{3600}+\frac{1}{900}\right]\times 100\%$$

$$= 5\left[\frac{5}{36}\right]\% = \frac{25}{36}\% \approx 0.69\%$$

SECTION–3 : (Maximum Marks : 12)

- This section contains **TWO (02)** List-Match sets.
- Each List-Match set has **Two (02)** Multiple Choice Questions.
- Each List-Match set has two lists : **List-I** and **List-II**
- **List-I** has **Four** entries (I), (II), (III) and (IV) and **List-II** has **Six** entries (P), (Q), (R), (S), (T) and (U)
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks : +3 If ONLY the option corresponding to the correct combination is chosen.

Zero Marks : 0 If none of the options is chosen (i.e., the question is unanswered);

Negative Marks : –1 In all other cases

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1. Answer the following by appropriately matching the lists based on the information given in the paragraph.

A musical instrument is made using four different metal strings, 1,2,3 and 4 with mass per unit length μ, 2μ, 3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0.

List-I gives the above four strings while list-II lists the magnitude of some quantity.

List-I	**List-II**
(I) String-1(μ)	(P) 1
(II) String-2 (2μ)	(Q) 1/2
(III) String-3 (3μ)	(R) $1/\sqrt{2}$
(IV) String-4 (4μ)	(S) $1/\sqrt{3}$
	(T) 3/16
	(U) 1/16

If the tension in each string is T_0, the correct match for the highest fundamental frequency in f_0 units will be,

(1) I→P, II→R, III→S, IV→Q

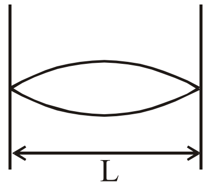
(2) I→P, II→Q, III→T, IV→S

(3) I→Q, II→S, III→R, IV→P

(4) I→Q, II→P, III→R, IV→T

Ans. (1)

Sol. For fundamental mode



$$\frac{\lambda}{2} = L \quad \Rightarrow \lambda = 2L$$

$$f = \frac{V}{\lambda} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

For string (1)

$$f_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \quad \Rightarrow (P)$$

For string (2)

$$f = \frac{1}{2L}\sqrt{\frac{T}{2\mu}} = \frac{f_0}{\sqrt{2}} \Rightarrow (R)$$

For string (3)

$$f = \frac{1}{2L}\sqrt{\frac{T}{3\mu}} = \frac{f_0}{\sqrt{3}} \Rightarrow (S)$$

For string (4)

$$f = \frac{1}{2L}\sqrt{\frac{T}{4\mu}} = \frac{f_0}{2} \Rightarrow (Q)$$

2. Answer the following by appropriately matching the lists based on the information given in the paragraph.

A musical instrument is made using four different metal strings, 1,2,3 and 4 with mass per unit length μ, 2μ, 3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0.

List-I gives the above four strings while list-II lists the magnitude of some quantity.

List-I	**List-II**
(I) String-1(μ)	(P) 1
(II) String-2 (2μ)	(Q) 1/2
(III) String-3 (3μ)	(R) $1/\sqrt{2}$
(IV) String-4 (4μ)	(S) $1/\sqrt{3}$
	(T) 3/16
	(U) 1/16

The length of the string 1,2,3 and 4 are kept fixed at $L_0, \frac{3L_0}{2}, \frac{5L_0}{4}$ and $\frac{7L_0}{4}$, respectively. Strings 1,2,3 and 4 are vibrated at their 1^{st}, 3^{rd}, 5^{th} and 14^{th} harmonics, respectively such that all the strings have same frequency. The correct match for the tension in the four strings in the units of T_0 will be.

(1) I→P, II→Q, III→T, IV→U

(2) I→T, II→Q, III→R, IV→U

(3) I→P, II→Q, III→R, IV→T

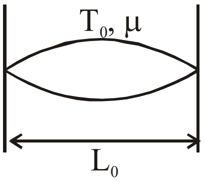
(4) I→P, II→R, III→T, IV→U

Ans. **(1)**

Sol. For string (1)

Length of string = L_0

It is vibrating in I^{st} harmonic i.e. fundamental mode.



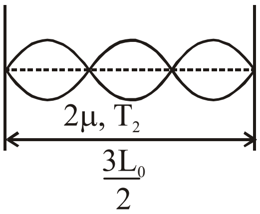
$$f_0 = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}} \Rightarrow (P)$$

For string (2)

Length of string = $\frac{3L_0}{2}$

It is vibrating in III^{rd} harmonic but frequency is still f_0.

$$f_0 = \frac{3v}{2L}$$



$$f_0 = \frac{3}{2\left(\frac{3L_0}{2}\right)}\sqrt{\frac{T_2}{2\mu}}$$

$$\Rightarrow f_0 = \frac{1}{L_0}\sqrt{\frac{T_2}{2\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$$

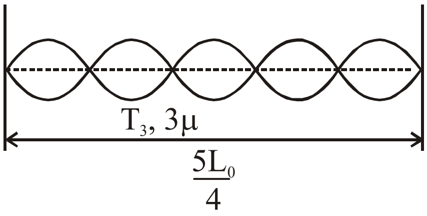
$$\Rightarrow \boxed{T_2 = \frac{T_0}{2}} \Rightarrow (Q)$$

For string (3)

Length of string = $\frac{5L_0}{4}$

It is vibrating in 5^{th} harmonic but frequency is still f_0.

$$f_0 = \frac{5V}{2L}$$



$$\Rightarrow f_0 = \frac{5}{2\left(\frac{5L_0}{4}\right)}\sqrt{\frac{T_3}{3\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$$

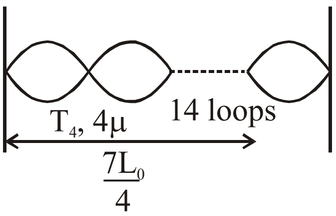
$$\Rightarrow \frac{2}{L_0}\sqrt{\frac{T_3}{3\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$$

$$T_3 = \frac{3T_0}{16} \Rightarrow (T)$$

For string (4)

Length of string = $\frac{7L_0}{4}$

It is vibrating in 14^{th} harmonic but frequency is still f_0.



$$f_0 = \frac{14v}{2L}$$

$$\Rightarrow \quad f_0 = \frac{14}{2\left(\frac{7L_0}{4}\right)}\sqrt{\frac{T_4}{4\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$$

$$\Rightarrow \frac{4}{L_0}\sqrt{\frac{T_4}{4\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}} \Rightarrow \boxed{T_4 = \frac{T_0}{16}} \Rightarrow (U)$$

3. **Answer the following by appropriately matching the lists based on the information given in the paragraph.**

In a thermodynamics process on an ideal monatomic gas, the infinitesimal heat absorbed by the gas is given by $T\Delta X$, where T is temperature of the system and ΔX is the infinitesimal change in a thermodynamic quantity X of the system. For a mole of monatomic ideal gas $X = \frac{3}{2}R\ln\left(\frac{T}{T_A}\right) + R\ln\left(\frac{V}{V_A}\right)$. Here, R is gas constant, V is volume of gas, T_A and V_A are constants.

The List-I below gives some quantities involved in a process and List-II gives some possible values of these quantities.

List-I

(I) Work done by the system in process $1 \rightarrow 2 \rightarrow 3$

(II) Change in internal energy in process $1 \rightarrow 2 \rightarrow 3$

(III) Heat absorbed by the system in process $1 \rightarrow 2 \rightarrow 3$

(IV) Heat absorbed by the system in process $1 \rightarrow 2$

List-II

(P) $\frac{1}{3}RT_0 \ln 2$

(Q) $\frac{1}{3}RT_0$

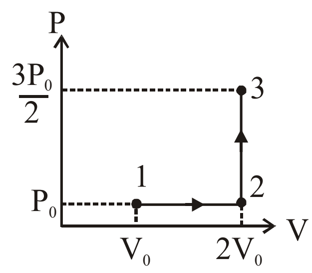
(R) RT_0

(S) $\frac{4}{3}RT_0$

(T) $\frac{1}{3}RT_0(3+\ln 2)$

(U) $\frac{5}{6}RT_0$

If the process carried out on one mole of monatomic ideal gas is as shown in figure in the PV-diagram with $P_0V_0 = \frac{1}{3}RT_0$, the correct match is,



(1) I → Q, II → R, III → P, IV → U

(2) I → S, II → R, III → Q, IV → T

(3) I → Q, II → R, III → S, IV → U

(4) I → Q, II → S, III → R, IV → U

Ans. **(3)**

Sol. (I) Degree of freedom f = 3

Work done in any process = Area under P–V graph

$\Rightarrow$ Work done in $1 \rightarrow 2 \rightarrow 3 = P_0V_0$

$$= \frac{RT_0}{3} \Rightarrow (Q)$$

(II) Change in internal energy $1 \rightarrow 2 \rightarrow 3$

$$\Delta U = nC_v\Delta T$$

$$= \frac{f}{2}nR\Delta T$$

$$= \frac{f}{2}\left(P_fV_f - P_iV_i\right)$$

$$= \frac{3}{2}\left(\frac{3P_0}{2}2V_0 - P_0V_0\right)$$

$$= 3P_0V_0$$

$$\Delta U = RT_0 \Rightarrow (R)$$

(III) Heat absorbed in $1 \rightarrow 2 \rightarrow 3$

for any process, I^{st} law of thermodynamics

$$\Delta Q = \Delta W + \omega$$

$$\Delta Q = RT_0 + \frac{RT_0}{3}$$

$$\Delta Q = \frac{4RT_0}{3} \Rightarrow (S)$$

(IV) Heat absorbed in process $1 \rightarrow 2$

$$\Delta Q = \Delta U + W$$

$$= \frac{f}{2}\left(P_fV_f - P_iV_i\right) + W$$

$$= \frac{3}{2}\left(P_0 2V_0 - P_0V_0\right) + P_0V_0$$

$$= \frac{5}{2}P_0V_0$$

$$= \frac{5}{2}\left(\frac{RT_0}{3}\right)$$

$$\boxed{\Delta Q = \frac{5RT_0}{6}} \Rightarrow (U)$$

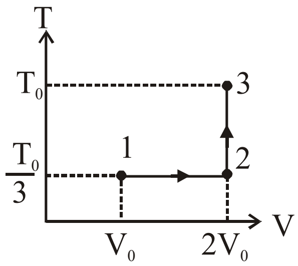
4. Answer the following by appropriately matching the lists based on the information given in the paragraph.

In a thermodynamic process on an ideal monatomic gas, the infinitesimal heat absorbed by the gas is given by TΔX, where T is temperature of the system and ΔX is the infinitesimal change in a thermodynamic quantity X of the system. For a mole of monatomic ideal gas $X = \frac{3}{2}R\ \ln\left(\frac{T}{T_A}\right) + R\ \ln\left(\frac{V}{V_A}\right)$. Here, R is gas constant, V is volume of gas, T_A and V_A are constants.

The List-I below gives some quantities involved in a process and List-II gives some possible values of these quantities.

List-I	List-II
(I) Work done by the system in process $1 \rightarrow 2 \rightarrow 3$	(P) $\frac{1}{3}RT_0 \ln 2$
(II) Change in internal energy in process $1 \rightarrow 2 \rightarrow 3$	(Q) $\frac{1}{3}RT_0$
(III) Heat absorbed by the system in process $1 \rightarrow 2 \rightarrow 3$	(R) RT_0
(IV) Heat absorbed by the system in process $1 \rightarrow 2$	(S) $\frac{4}{3}RT_0$
	(T) $\frac{1}{3}RT_0(3+\ln 2)$
	(U) $\frac{5}{6}RT_0$

If the process on one mole of monatomic ideal gas is an shown is as shown in the TV-diagram with $P_0V_0 = \frac{1}{3}RT_0$, the correct match is



(1) I $\rightarrow$ S, II $\rightarrow$ T, III $\rightarrow$ Q, IV $\rightarrow$ U

(2) I $\rightarrow$ P, II $\rightarrow$ R, III $\rightarrow$ T, IV $\rightarrow$ S

(3) I $\rightarrow$ P, II $\rightarrow$ T, III $\rightarrow$ Q, IV $\rightarrow$ T

(4) I $\rightarrow$ P, II $\rightarrow$ R, III $\rightarrow$ T, IV $\rightarrow$ P

Ans. (4)

Sol. Process $1 \rightarrow 2$ is isothermal (temperature constant)

Process $2 \rightarrow 3$ is isochoric (volume constant)

(I) Work done in $1 \rightarrow 2 \rightarrow 3$

$W = W_{1\rightarrow 2} + W_{2\rightarrow 3}$

$$= nRT \ln\left(\frac{V_f}{V_i}\right) + W_{2\rightarrow 3}$$

$$= \frac{RT_0}{3}\ln\left(\frac{2V_0}{V_0}\right) + 0$$

$$W = \frac{RT_0}{3}\ln 2 \quad \Rightarrow (P)$$

(II) ΔU in $1 \rightarrow 2 \rightarrow 3$

$$\Delta U = \frac{f}{2}nR(T_f - T_i)$$

$$= \frac{3}{2}R\left(T_0 - \frac{T_0}{3}\right)$$

$$= \frac{3}{2}R\left(\frac{2T_0}{3}\right)$$

$$\boxed{\Delta U = RT_0} \quad \Rightarrow (R)$$

(III) For any system, first law of thermodynamics

for $1 \rightarrow 2 \rightarrow 3$

$\Delta Q = \Delta U + W$

$$\Delta Q = RT_0 + \frac{RT_0}{3}\ln 2$$

$$\Delta Q = \frac{RT_0}{3}(3 + \ln 2) \Rightarrow (T)$$

(IV) For process $1 \rightarrow 2$ (isothermal)

$\Delta Q = \Delta U + W$

$$= \frac{f}{2}nR(T_f - T_i) + nRT\ln(V_f / V_i)$$

$$= 0 + R\left(\frac{T_0}{3}\right)\ln\left(\frac{2v_0}{v_0}\right)$$

$$\boxed{\Delta Q = \frac{RT_0}{3}\ln 2} \quad \Rightarrow (P)$$

FINAL JEE(Advanced) EXAMINATION - 2019

(Held On Monday 27th MAY, 2019)

PAPER-2 | **TEST PAPER WITH ANSWER & SOLUTION**

PART-2 : CHEMISTRY

SECTION-1 : (Maximum Marks: 32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s)
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If only (all) the correct option(s) is (are) chosen.
  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen.
  Partial Marks : +2 If three or more options are correct but ONLY two options are chosen and both of which are correct.
  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).
  Negative Marks : –1 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 marks;
  choosing ONLY (B) will get +1 marks;
  choosing ONLY (D) will get +1 marks;
  choosing no option (i.e. the question is unanswered) will get 0 marks, and
  choosing any other combination of options will get –1 mark.

---

1. The cyanide process of gold extraction involves leaching out gold from its ore with CN^- in the presence of **Q** in water to form **R**. Subsequently, **R** is treated with **T** to obtain Au and **Z**. Choose the correct option(s).

(1) **T** is Zn

(2) **R** is $[Au(CN)_4]^-$

(3) **Z** is $[Zn(CN)_4]^{2-}$

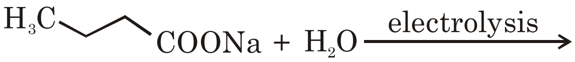
(4) **Q** is O_2

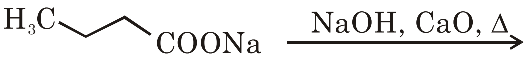
Ans. (1,3,4)

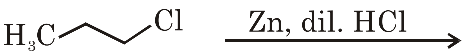
Sol. $4Au(s) + 8CN^-(aq) + 2H_2O(aq) + O_2(g) \rightarrow 4[Au(CN)_2]^-(aq) + 4OH^-(aq)$
(Q)

$2[Au(CN)_2]^-(aq) + Zn(s) \rightarrow [Zn(CN)_4]^{2-}(aq) + 2Au(s)$
(R) (T) (Z)

2. Which of the following reactions produce(s) propane as a major product?

(1) 

(2) 

(3) 

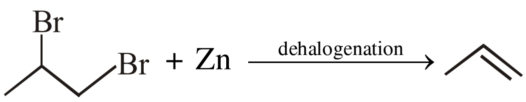
(4) 

Ans. **(2,3)**

Sol. $CH_3 - CH_2 - CH_2\text{–}CO_2Na + H_2O \xrightarrow{\text{electrolysis}}$ n-hexane

$CH_3 - CH_2 - CH_2 - CO_2Na \xrightarrow[\Delta]{NaOH + CaO} CH_3 - CH_2 - CH_3$

$CH_3 - CH_2 - CH_2\text{–}Cl + Zn \longrightarrow CH_3 - CH_2 - CH_2 - ZnCl \xrightarrow{\text{dil. HCl}} CH_3CH_2CH_3$



3. The ground state energy of hydrogen atom is –13.6 eV. Consider an electronic state Ψ of He^+ whose energy, azimuthal quantum number and magnetic quantum number are –3.4 eV, 2 and 0 respectively. Which of the following statement(s) is(are) true for the state Ψ?

(1) It has 2 angular nodes

(2) It has 3 radial nodes

(3) It is a 4d state

(4) The nuclear charge experienced by the electron in this state is less than 2e, where e is the magnitude of the electronic charge.

Ans. **(1,3)**

Sol. # $-3.4 = \dfrac{-13.6 \times 4}{n^2}$

$n = 4$

$\ell = 2$

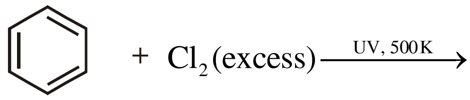
$m = 0$

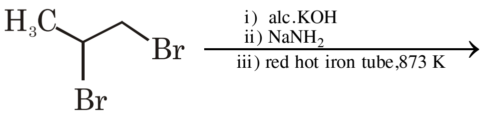
Angular nodes = ℓ = 2

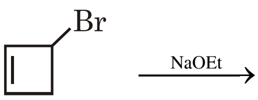
Radial nodes = $(n - \ell - 1) = 1$

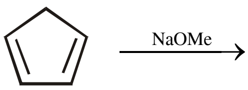
$n\ \ell$ = 4d state

4. Choose the correct option(s) that give(s) an aromatic compound as the major product.

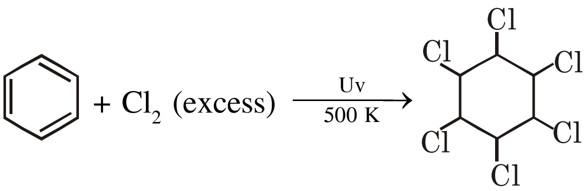
(1) 

(2) 

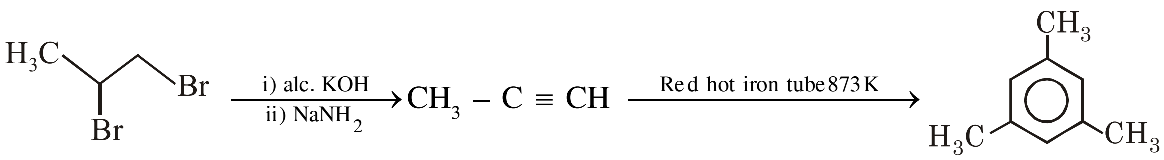
(3) 

(4) 

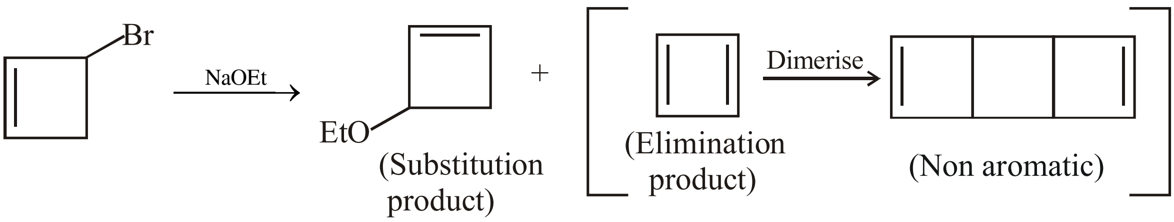
Ans. (2,4)

Sol. (1) 

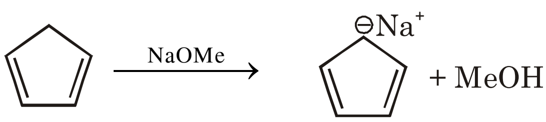
(Non aromatic)

(2) 

(Aromatic)

(3) 

(Non aromatic)

(4) 

(Aromatic ion)

5. Consider the following reactions (unbalanced)

Zn + hot conc. $H_2SO_4 \rightarrow G + R + X$

Zn + conc. $NaOH \rightarrow T + Q$

$G + H_2S + NH_4OH \rightarrow Z$ (a precipitate) $+ X + Y$

Choose the correct option(s).

(1) The oxidation state of Zn in T is +1

(2) Bond order of Q is 1 in its ground state

(3) Z is dirty white in colour

(4) R is a V-shaped molecule

Ans. **(2,3,4)**

Sol. $\underset{}{Zn} + 2H_2SO_4$ (Hot and conc.) $\rightarrow \underset{(G)}{ZnSO_4} + \underset{(R)}{SO_2\uparrow} + \underset{(X)}{2H_2O}$

$Zn + 2NaOH$ (conc.) $\rightarrow \underset{(T)}{Na_2ZnO_2} + \underset{(Q)}{H_2\uparrow}$

$ZnSO_4 + H_2S + 2NH_4OH \rightarrow \underset{(Z)}{ZnS\downarrow} + \underset{(X)}{2H_2O} + \underset{(Y)}{(NH_4)_2SO_4}$

6. With reference to *aqua regia*, choose the correct option(s).

(1) Reaction of gold with *aqua regia* produces NO_2 in the absence of air

(2) *Aqua regia* is prepared by mixing conc. HCl and conc. HNO_3 in 3 : 1 (v/v) ratio

(3) Reaction of gold with *aqua regia* produces an anion having Au in +3 oxidation state

(4) The yellow colour of *aqua regia* is due to the presence of NOCl and Cl_2

Ans. **(2,3,4)**

Sol. (1) $Au + HNO_3 + 4HCl \rightarrow AuCl_4^{\ominus} + H_3O^+ + NO + H_2O$

(2) Aqua regia = $3HCl$(conc.) + HNO_3(conc.)

(3) $AuCl_4^{\ominus}$ is produced

(4) Yellow colour of aqua regia is due to it's decomposition into NOCl(orange yellow) and Cl_2(greenish yellow).

7. Choose the correct option(s) from the following

(1) Natural rubber is polyisoprene containing *trans* alkene units

(2) Nylon-6 has amide linkages

(3) Cellulose has only α-D-glucose units that are joined by glycosidic linkages

(4) Teflon prepared by heating tetrafluoroethene in presence of a persulphate catalyst at high pressure

Ans. (2,4)

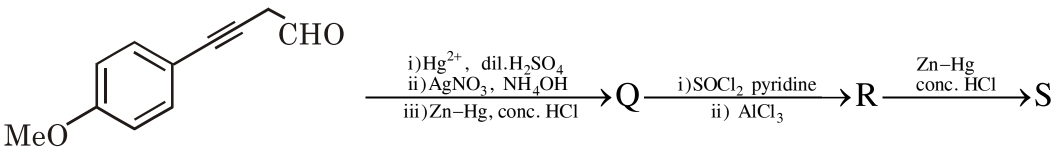
Sol. 1. Natural rubber is polyisoprene containing cis alkene units

2. Nylon-6 has amide linkage $\left[HN-(CH_2)_5-\underset{\underset{O}{\|}}{C} \right]_n$

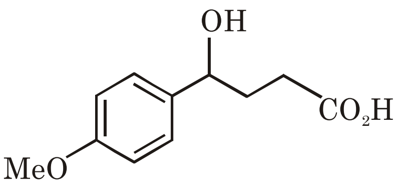
3. Cellulose has only β-D glucose units.

4. $F_2C=CF_2 \xrightarrow{\text{Per sulphate}} \left[CF_2-CF_2 \right]_n$

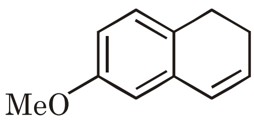
8. Choose the correct option(s) for the following reaction sequence



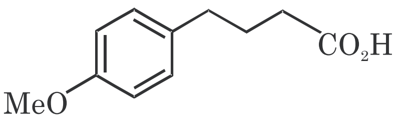
Consider Q, R and S as major products

(1) 

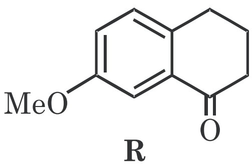
Q



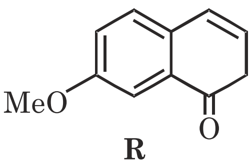
S

(2) 

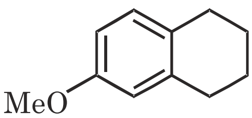
Q



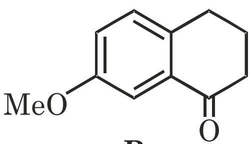
R

(3) 

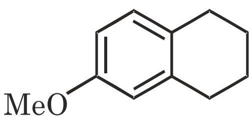
R



S

(4) 

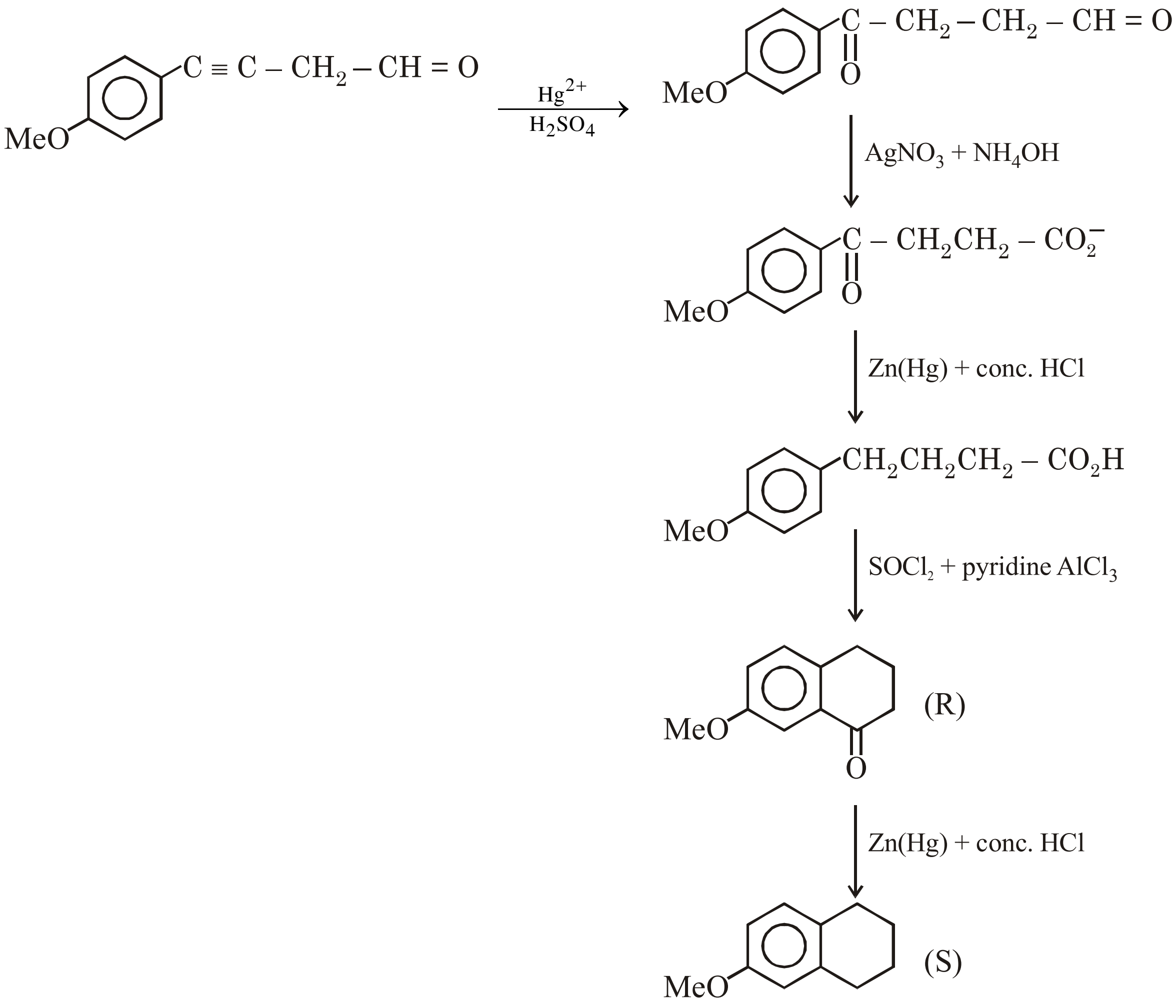
R



S

Ans. (2,4)

Sol.



SECTION-2 : (Maximum Marks: 18)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **Two** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If ONLY the correct numerical value is entered.

  Zero Marks : 0 In all other cases.

---

1. The decomposition reaction $2N_2O_5(g) \xrightarrow{\Delta} 2N_2O_4(g) + O_2(g)$ is started in a closed cylinder under isothermal isochoric condition at an initial pressure of 1 atm. After $Y \times 10^3$ s, the pressure inside the cylinder is found to be 1.45 atm. If the rate constant of the reaction is $5 \times 10^{-4}\ s^{-1}$, assuming ideal gas behavior, the value of Y is ___

Ans. (2.30)

Sol. $2N_2O_5(g) \xrightarrow{\Delta} 2N_2O_4(g) + O_2(g)$ at constant V, T

t = 0   1

t = $y \times 10^3$ sec $(1 - 2P)$ $2P$ P

$P_T = (1 + P) = 1.45$

$P = 0.45$ atm

$$(2K)t = 2.303\log\left(\frac{1}{1-2P}\right)$$

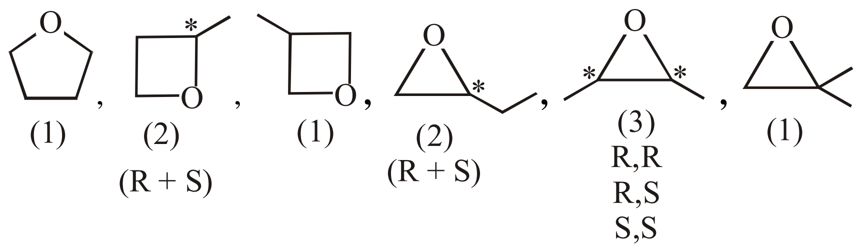
$$(2 \times 5 \times 10^{-4}) \times y \times 10^3 = 2.303\log\frac{1}{0.1}$$

$y = 2.303 = 2.30$

2. Total number of isomers, considering both structural and stereoisomers, of cyclic ethers with the molecular formula C_4H_8O is ___

Ans. (10.00)

Sol.



3. The amount of water produced (in g) in the oxidation of 1 mole of rhombic sulphur by conc.HNO_3 to a compound with the highest oxidation state of sulphur is ___

(Given data : Molar mass of water = 18 g mol^{-1})

Ans. (288.00)

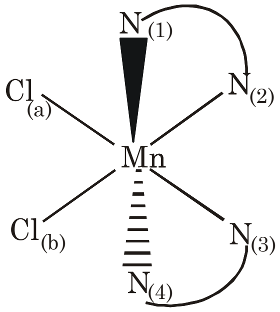
Sol. $S_8 + 48\ HNO_3 \longrightarrow 8H_2SO_4 + 48NO_2 + 16H_2O$

1 mole of rhombic sulphur produce 16 mole of H_2O i.e. 288 gm of H_2O

4. Total number of *cis* N–Mn–Cl bond angles (that is, Mn–N and Mn–Cl bonds in *cis* positions) present in a molecule of *cis*-$[Mn(en)_2Cl_2]$ complex is ____ (*en* = $NH_2CH_2CH_2NH_2$)

Ans. (6.00)

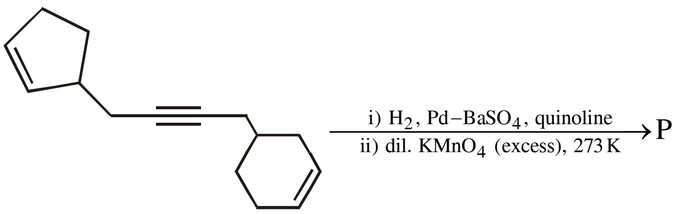
Sol. cis$[M(en)_2Cl_2]$



$Cl_{(a)}$ — Mn — $N_{(1)}$
$Cl_{(a)}$ — Mn — $N_{(2)}$
$Cl_{(a)}$ — Mn — $N_{(4)}$
$Cl_{(b)}$ — Mn — $N_{(1)}$
$Cl_{(b)}$ — Mn — $N_{(3)}$
$Cl_{(b)}$ — Mn — $N_{(4)}$

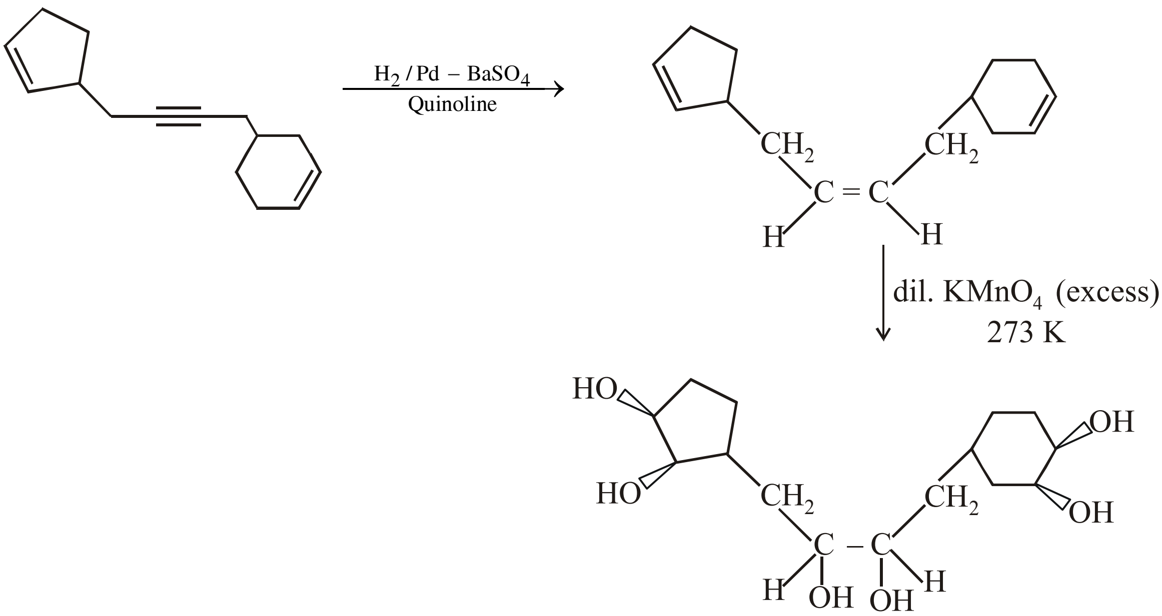
Number of cis (Cl—Mn—N) = 6

5. Total number of hydroxyl groups present in a molecule of the major product P is ___



Ans. (6.00)

Sol.



total 6 –OH group present in a molecule of the major product.

6. The mole fraction of urea in an aqueous urea solution containing 900 g of water is 0.05. If the density of the solution is 1.2 g cm^{-3}, the molarity of urea solution is ___

(Given data : Molar masses of urea and water are 60 g mol^{-1} and 18 g mol^{-1}, respectively)

Ans. (2.98 or 2.99)

Sol. $X_{urea} = 0.05 = \dfrac{n}{n+50}$

$19n = 50$

$n = 2.6315$

$V_{sol} = \dfrac{(2.6315 \times 60 + 900)}{1.2} = 881.5789 \text{ ml}$

$\text{Molarity} = \dfrac{2.6315 \times 1000}{881.5789} = 2.9849$

Molarity = 2.98M

SECTION–3 : (Maximum Marks : 12)

- This section contains **TWO (02)** List-Match sets.
- Each List-Match set has **Two (02)** Multiple Choice Questions.
- Each List-Match set has two lists : **List-I** and **List-II**
- **List-I** has **Four** entries (I), (II), (III) and (IV) and **List-II** has **Six** entries (P), (Q), (R), (S), (T) and (U)
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks : +3 If ONLY the option corresponding to the correct combination is chosen.

Zero Marks : 0 If none of the options is chosen (i.e., the question is unanswered);

Negative Marks : –1 In all other cases

---

1. Answer the following by appropriately matching the lists based on the information given in the paragraph

Consider the Bohr's model of a one-electron atom where the electron moves around the nucleus. In the following List-I contains some quantities for the n^{th} orbit of the atom and List-II contains options showing how they depend on n.

List-I		**List-II**
(I)	Radius of the n^{th} orbit	**(P)** $\propto n^{-2}$
(II)	Angular momentum of the electron in the n^{th} orbit	**(Q)** $\propto n^{-1}$
(III)	Kinetic energy of the electron in the n^{th} orbit	**(R)** $\propto n^{0}$
(IV)	Potential energy of the electron in the n^{th} orbit	**(S)** $\propto n^{1}$
		(T) $\propto n^{2}$
		(U) $\propto n^{1/2}$

Which of the following options has the correct combination considering List-I and List-II?

(1) (II), (R) (2) (I), (P) (3) (I), (T) (4) (II), (Q)

Ans. (3)

Sol. $r = 0.529 \times \frac{n^2}{z}$ $\Rightarrow$ $r \propto n^2$ $\Rightarrow$ (I) (T)

$mvr = \frac{nh}{2\pi}$ $\Rightarrow$ $(mvr) \propto n$ $\Rightarrow$ (II) (S)

$KE = +13.6 \times \frac{z^2}{n^2}$ $\Rightarrow$ $KE \propto n^{-2}$ $\Rightarrow$ (III) (P)

$PE = -2 \times 13.6 \times \frac{z^2}{n^2}$ $\Rightarrow$ $PE \propto n^{-2}$ $\Rightarrow$ (IV) (P)

2. **Answer the following by appropriately matching the lists based on the information given in the paragraph**

Consider the Bohr's model of a one-electron atom where the electron moves around the nucleus. In the following List-I contains some quantities for the n^{th} orbit of the atom and List-II contains options showing how they depend on n.

	List-I	List-II
(I)	Radius of the n^{th} orbit	**(P)** $\propto n^{-2}$
(II)	Angular momentum of the electron in the n^{th} orbit	**(Q)** $\propto n^{-1}$
(III)	Kinetic energy of the electron in the n^{th} orbit	**(R)** $\propto n^{0}$
(IV)	Potential energy of the electron in the n^{th} orbit	**(S)** $\propto n^{1}$
		(T) $\propto n^{2}$
		(U) $\propto n^{1/2}$

Which of the following options has the correct combination considering List-I and List-II?

(1) (III), (S) (2) (IV), (Q) (3) (IV), (U) (4) (III), (P)

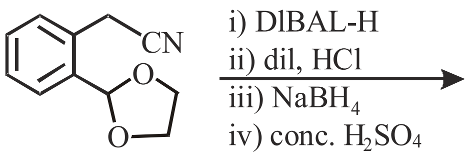
Ans. (4)

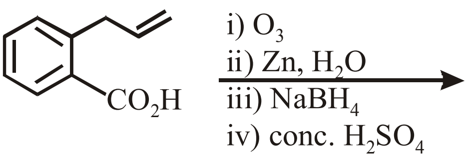
Sol. Same as 1 (Section-3)

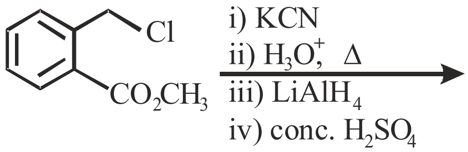
3. Answer the following by appropriately matching the lists based on the information given in the paragraph

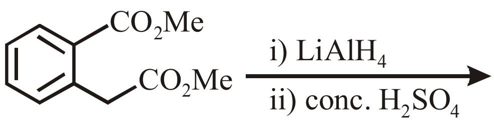
List-I includess starting materials and reagents of selected chemical reactions. List-II gives structures of compounds that may be formed as intermediate products and/or final products from the reactions of List-I

List-I

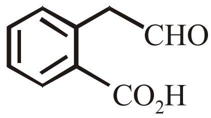
(I) 

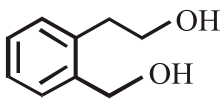
(II) 

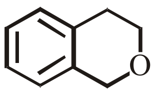
(III) 

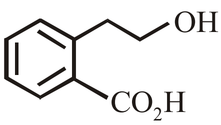
(IV) 

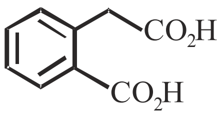
List-II

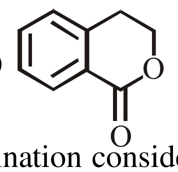
(P) 

(Q) 

(R) 

(S) 

(T) 

(U) 

Which of the following options has correct combination considering List-I and List-II?

(1) (III), (S), (R)

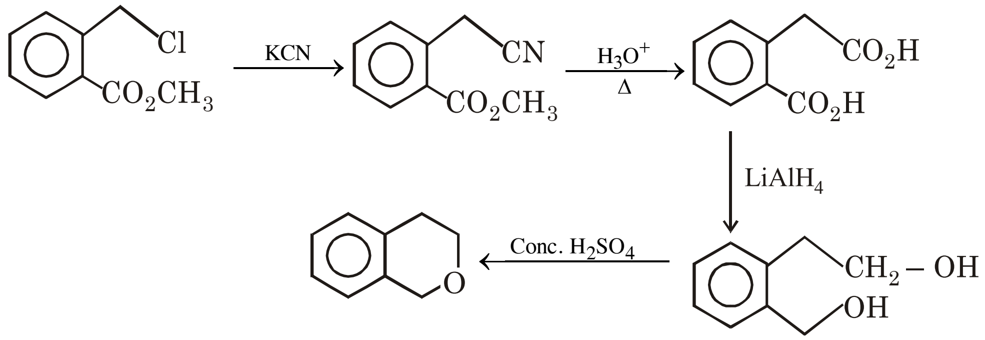
(2) (IV), (Q), (R)

(3) (III), (T), (U)

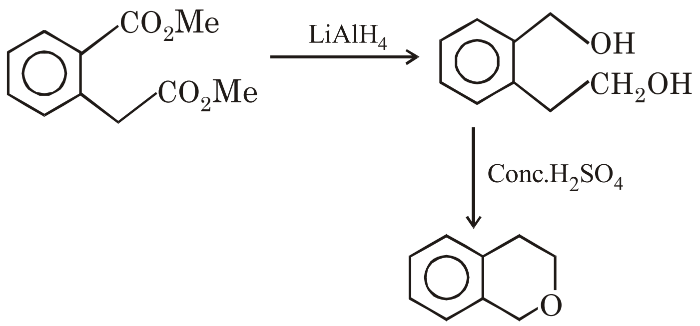
(4) (IV), (Q), (U)

Ans. (2)

Sol.



III, T, Q, R

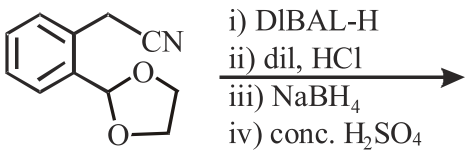


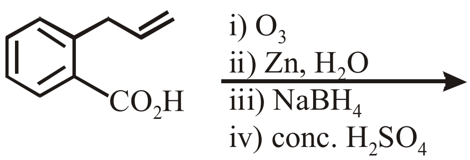
IV, Q, R

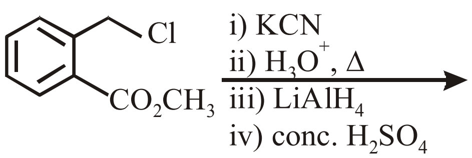
4. **Answer the following by appropriately matching the lists based on the information given in the paragraph**

List-I includess starting materials and reagents of selected chemical reactions. List-II gives structures of compounds that may be formed as intermediate products and/or final products from the reactions of List-I

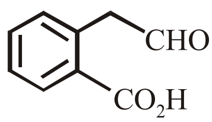
List-I

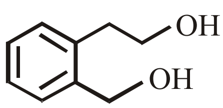
(I) 

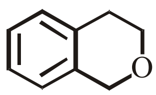
(II) 

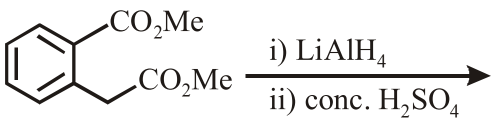
(III) 

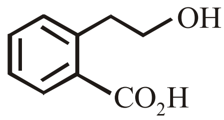
List-II

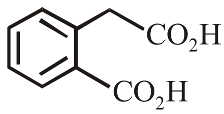
(P) 

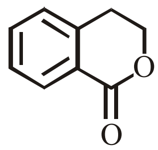
(Q) 

(R) 

(IV) 

(S) 

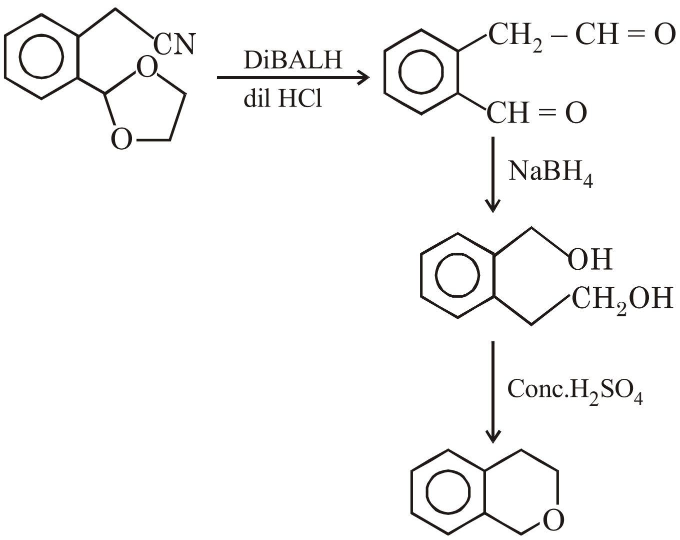
(T) 

(U) 

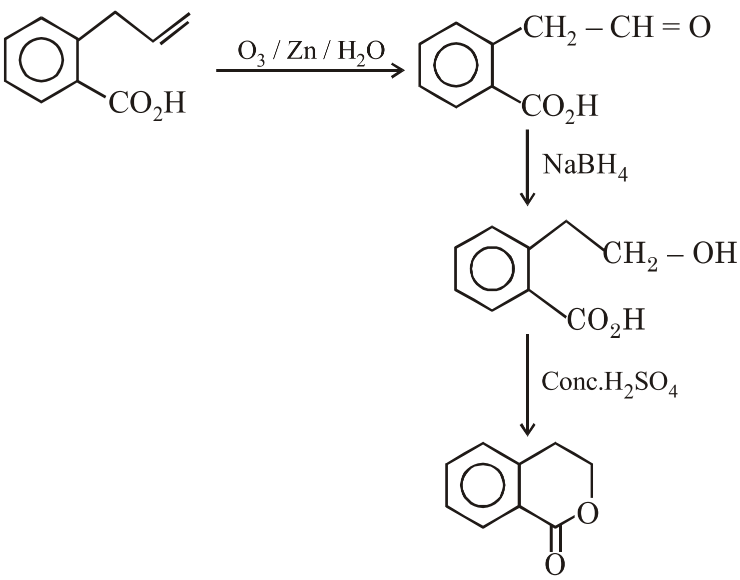
Which of the following options has correct combination considering List-I and List-II?

(1) (I), (Q), (T), (U)
(2) (II), (P), (S), (U)
(3) (II), (P), (S), (T)
(4) (I), (S), (Q), (R)

Ans. (2)

Sol. 

I, Q, R



II, P, S, U

FINAL JEE(Advanced) EXAMINATION - 2019

(Held On Monday 27th MAY, 2019)

PAPER-2	TEST PAPER WITH ANSWER & SOLUTION

PART-3 : MATHEMATICS

SECTION-1 : (Maximum Marks: 32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s)
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If only (all) the correct option(s) is (are) chosen.

  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen.

  Partial Marks : +2 If three or more options are correct but ONLY two options are chosen and both of which are correct.

  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).

  Negative Marks : –1 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 marks;
  choosing ONLY (B) will get +1 marks;
  choosing ONLY (D) will get +1 marks;
  choosing no option (i.e. the question is unanswered) will get 0 marks, and
  choosing any other combination of options will get –1 mark.

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1. Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = (x - 1)(x - 2)(x - 5)$. Define $F(x) = \int_0^x f(t)\,dt$, $x > 0$. Then which of the following options is/are correct ?

(1) F has a local minimum at x = 1

(2) F has a local maximum at x = 2

(3) $F(x) \neq 0$ for all $x \in (0, 5)$

(4) F has two local maxima and one local minimum in $(0, \infty)$

Ans. (1,2,3)

Sol. $f(x) = (x-1)(x-2)(x-5)$

$$F(x) = \int_0^x f(t)\,dt, x > 0$$

$$F'(x) = f(x) = (x-1)(x-2)(x-5), x > 0$$

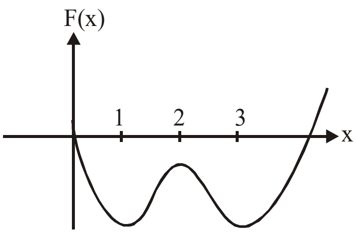
clearly F(x) has local minimum at x = 1,5

F(x) has local maximum at x = 2

$f(x) = x^3 - 8x^2 + 17x - 10$

$$\Rightarrow F(x) = \int_0^x (t^3 - 8t^2 + 17t - 10)\,dt$$

$$F(x) = \frac{x^4}{4} - \frac{8x^3}{3} + \frac{17x^2}{2} - 10x$$



from the graph of y = F(x), clearly $F(x) \neq 0 \ \forall \ x \in (0,5)$

2. For $a \in \mathbb{R}$, $|a| > 1$, let $\lim_{n\to\infty}\left(\frac{1+\sqrt[3]{2}+.....+\sqrt[3]{n}}{n^{7/3}\left(\frac{1}{(an+1)^2}+\frac{1}{(an+2)^2}+.......+\frac{1}{(an+n)^2}\right)}\right) = 54$. Then the possible value(s) of a is/are :

(1) 8 (2) –9 (3) –6 (4) 7

Ans. (1,2)

Sol. $$\lim_{n\to\infty}\frac{n^{1/3}\left(\sum_{r=1}^{n}\left(\frac{r}{n}\right)^{1/3}\right)}{n^{7/3}\left(\sum_{r=1}^{n}\frac{1}{(an+r)^2}\right)} = 54 \Rightarrow \lim_{n\to\infty}\left(\frac{\frac{1}{n}\sum_{r=1}^{n}\left(\frac{r}{n}\right)^{1/3}}{\frac{1}{n}\sum_{r=1}^{n}\frac{1}{(a+r/n)^2}}\right) = 54 \Rightarrow \frac{\int_0^1 x^{1/3}dx}{\int_0^1 \frac{1}{(a+x)^2}dx} = 54 \Rightarrow \frac{\frac{3}{4}}{\frac{1}{a(a+1)}} = 54$$

$$\Rightarrow a(a+1) = 72 \quad \Rightarrow \quad a^2 + a - 72 = 0 \quad \Rightarrow \quad a = -9, 8$$

3. Three lines

$$L_1 : \vec{r} = \lambda\hat{i},\ \lambda \in \mathbb{R},$$

$$L_2 : \vec{r} = \vec{k} + \mu\hat{j},\ \mu \in \mathbb{R} \text{ and}$$

$$L_3 : \vec{r} = \hat{i} + \hat{j} + v\hat{k},\ v \in \mathbb{R}$$

are given. For which point(s) Q on L_2 can we find a point P on L_1 and a point R on L_3 so that P, Q and R are collinear ?

(1) $\hat{k} + \hat{j}$ (2) $\hat{k}$ (3) $\hat{k} + \frac{1}{2}\hat{j}$ (4) $\hat{k} - \frac{1}{2}\hat{j}$

Ans. (3,4)

Sol. Let P(λ, 0, 0), Q(0, μ, 1), R(1, 1, ν) be points. L_1, L_2 and L_3 respectively

Since P, Q, R are collinear, $\overrightarrow{PQ}$ iscollinear with $\overrightarrow{QR}$

Hence $\frac{-\lambda}{1} = \frac{\mu}{1-\mu} = \frac{1}{\nu - 1}$

For every $\mu \in R - \{0, 1\}$ there exist unique $\lambda, \nu \in R$

Hence Q cannot have coordinates (0, 1, 1) and (0, 0, 1).

4. Let F : $\mathbb{R} \to \mathbb{R}$ be a function. We say that f has

PROPERTY 1 if $\lim\limits_{h\to 0} \frac{f(h) - f(0)}{\sqrt{|h|}}$ exists and is finite, and

PROPERTY 2 if $\lim\limits_{h\to 0} \frac{f(h) - f(0)}{h^2}$ exists and is finite.

Then which of the following options is/are correct ?

(1) f(x) = x|x| has PROPERTY 2
(2) $f(x) = x^{2/3}$ has PROPERTY 1
(3) f(x) = sin x has PROPERTY 2
(4) f(x) = |x| has PROPERTY 1

Ans. (2,4)

Sol. P -1 :

$$\lim_{h\to 0} \frac{f(h) - f(0)}{\sqrt{|h|}} = \text{exist and finite}$$

(B) $f(x) = x^{2/3}, \lim\limits_{h\to 0} \frac{h^{2/3} - 0}{\sqrt{|h|}} = \lim\limits_{h\to 0} \frac{|h|^{2/3}}{\sqrt{|h|}} = 0$

(D) $f(x)=|x|, \lim_{h\to 0}\frac{|h|-0}{\sqrt{|h|}} \Rightarrow \lim_{h\to 0}\sqrt{|h|}=0$

P-2 :

$$\lim_{h\to 0}\frac{f(h)-f(0)}{h^2} = \text{exist and finite}$$

(A) $f(x) = x|x|$, $\lim_{h\to 0}\frac{h|h|-0}{h^2} = \begin{bmatrix} RHL = \lim_{h\to 0}\frac{h^2}{h^2}=1 \\ LHL = \lim_{h\to 0}\frac{-h^2}{h^2}=-1 \end{bmatrix}$

(C) $f(x) = \sin x$ $\lim_{h\to 0}\frac{\sin h-0}{h^2} = \text{DNE}$

5. For non-negative integers n, let

$$f(n)=\frac{\sum_{k=0}^{n}\sin\left(\frac{k+1}{n+2}\pi\right)\sin\left(\frac{k+2}{n+2}\pi\right)}{\sum_{k=0}^{n}\sin^2\left(\frac{k+1}{n+2}\pi\right)}$$

Assuming $\cos^{-1}x$ takes values in $[0, \pi]$, which of the following options is/are correct ?

(1) $\sin(7\cos^{-1} f(5)) = 0$

(2) $f(4)=\frac{\sqrt{3}}{2}$

(3) $\lim_{n\to\infty} f(n)=\frac{1}{2}$

(4) If $\alpha = \tan(\cos^{-1} f(6))$, then $\alpha^2 + 2\alpha - 1 = 0$

Ans. (1,2,4)

Sol. $$f(n)=\frac{\sum_{k=0}^{n}\left(\cos\left(\frac{\pi}{n+2}\right)-\cos\left(\frac{2k+3}{n+2}\right)\pi\right)}{\sum_{k=0}^{n}\left(1-\cos\left(\frac{2k+2}{n+2}\right)\pi\right)}$$

$$f(n)=\frac{(n+1)\cos\left(\frac{\pi}{n+2}\right)-\left(\sum_{k=0}^{n}\cos\left(\frac{2k+3}{n+2}\right)\pi\right)}{(n+1)-\left(\sum_{k=0}^{n}\cos\left(\frac{2k+2}{n+2}\right)\pi\right)}$$

$$f(n)=\frac{(n+1)\cos\frac{\pi}{n+2}-\left(\frac{\sin\left(\frac{(n+1)\pi}{n+2}\right)}{\sin\left(\frac{\pi}{n+2}\right)}.\cos\left(\frac{n+3}{n+2}\right)\pi\right)}{(n+1)-\left(\frac{\sin\left(\frac{(n+1)\pi}{n+2}\right)}{\sin\left(\frac{\pi}{n+2}\right)}.\cos\left(\frac{2(n+2)\pi}{2(n+2)}\right)\right)}$$

$$f(n)=\frac{(n+1)\cos\left(\frac{\pi}{n+2}\right)+\cos\left(\frac{\pi}{n+2}\right)}{(n+1)+1}\Rightarrow g(x)=\cos\left(\frac{\pi}{n+2}\right)$$

(A) $\sin\left(7\cos^{-1}\cos\frac{\pi}{7}\right)=\sin\pi=0$

(B) $f(4)=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$

(C) $\lim\limits_{n\to\infty}\cos\left(\frac{\pi}{n+2}\right)=1$

(D) $\alpha=\tan\left(\cos^{-1}\cos\frac{\pi}{8}\right)=\sqrt{2}-1\Rightarrow\alpha+1=\sqrt{2}$

$\alpha^2+2\alpha-1=0$

6. Let $P_1=I=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$, $P_2=\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}$, $P_3=\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}$, $P_4=\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}$, $P_5=\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}$,

$P_6=\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}$ and $X=\sum\limits_{k=1}^{6}P_K\begin{bmatrix}2&1&3\\1&0&2\\3&2&1\end{bmatrix}P_K^T$

where P_K^T denotes the transpose of the matrix P_K. Then which of the following options is/are correct?

(1) $X-30I$ is an invertible matrix

(2) The sum of diagonal entries of X is 18

(3) If $X\begin{bmatrix}1\\1\\1\end{bmatrix}=\alpha\begin{bmatrix}1\\1\\1\end{bmatrix}$, then $\alpha=30$

(4) X is a symmetric matrix

Ans. (2,3,4)

Sol. Let $Q=\begin{bmatrix}2&1&3\\1&0&2\\3&2&1\end{bmatrix}$

$$X=\sum_{k=1}^{6}\left(P_kQP_K^T\right)$$

$$X^T=\sum_{k=1}^{6}\left(P_kQP_K^T\right)^T=X.$$

X is symmetric

Let $R=\begin{bmatrix}1\\1\\1\end{bmatrix}$

$$XR=\sum_{k=1}^{6}P_KQP_K^TR.\ [\because\ P_K{}^TR=R]$$

$$=\sum_{K=1}^{6}P_KQR.=\left(\sum_{K=1}^{6}P_K\right)QR$$

$$\sum_{K=1}^{6}P_K=\begin{bmatrix}2&2&2\\2&2&2\\2&2&2\end{bmatrix}\qquad QR=\begin{bmatrix}6\\3\\6\end{bmatrix}$$

$$\Rightarrow XR=\begin{bmatrix}2&2&2\\2&2&2\\2&2&2\end{bmatrix}\begin{bmatrix}6\\3\\6\end{bmatrix}=\begin{bmatrix}30\\30\\30\end{bmatrix}=30R$$

$\Rightarrow \alpha = 30.$

Trace X = Trace $\left(\sum_{K=1}^{6}P_KQP_K^T\right)$

$$=\sum_{K=1}^{6}\text{Trace}\left(P_K Q P_K^T\right)=6(\text{Trace } Q)=18$$

$$X\begin{bmatrix}1\\1\\1\end{bmatrix}=30\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$\Rightarrow (X-30I)\begin{bmatrix}1\\1\\1\end{bmatrix}=O\Rightarrow |X-30I|=0$$

$\Rightarrow X-30I$ is non-invertible

7. Let $x \in \mathbb{R}$ and let $P=\begin{bmatrix}1&1&1\\0&2&2\\0&0&3\end{bmatrix}$, $Q=\begin{bmatrix}2&x&x\\0&4&0\\x&x&6\end{bmatrix}$ and $R = PQP^{-1}$.

Then which of the following options is/are correct ?

(1) For x = 1, there exists a unit vector $\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ for which $R\begin{bmatrix}\alpha\\\beta\\\gamma\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

(2) There exists a real number x such that PQ = QP

(3) $\det R = \det\begin{bmatrix}2&x&x\\0&4&0\\x&x&5\end{bmatrix}+8$, for all $x \in \mathbb{R}$

(4) For x = 0, if $R\begin{bmatrix}1\\a\\b\end{bmatrix}=6\begin{bmatrix}1\\a\\b\end{bmatrix}$, then a + b = 5

Ans. (3,4)

Sol. $\det(R) = \det(PQP^{-1}) = (\det P)(\det Q)\left(\dfrac{1}{\det P}\right)$

$= \det Q$

$= 48 - 4x^2$

Option-1 :

for $x = 1$ $\det(R) = 44 \neq 0$

$\therefore$ for equation $R\begin{bmatrix}\alpha\\\beta\\\gamma\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

We will have trivial solution

$\alpha = \beta = \gamma = 0$

Option-2 :

$PQ = QP$

$PQP^{-1} = Q$

$R = Q$

No value of x.

Option-3 :

$$\det\begin{bmatrix}2 & x & x\\0 & 4 & 0\\x & x & 5\end{bmatrix}+8$$

$= (40 - 4x^2) + 8 = 48 - 4x^2 = \det R\ \forall\ x \in R$

Option-4 :

$$R=\begin{bmatrix}2 & 1 & 2/3\\0 & 4 & 4/3\\0 & 0 & 6\end{bmatrix}$$

$$(R-6I)\begin{bmatrix}1\\a\\b\end{bmatrix}=O$$

$$\Rightarrow -4+a+\frac{2b}{3}=0$$

$$-2a+\frac{4b}{3}=0$$

$\Rightarrow a=2 \qquad b=3$

$a + b = 5$

8. Let $f(x)=\frac{\sin \pi x}{x^2}, x>0$

Let $x_1 < x_2 < x_3 < ... < x_n < ...$ be all the points of local maximum of f and $y_1 < y_2 < y_3 < ... < y_n < ...$ be all the points of local minimum of f. Then which of the following options is/are correct ?

(1) $|x_n - y_n| > 1$ for every n

(2) $x_1 < y_1$

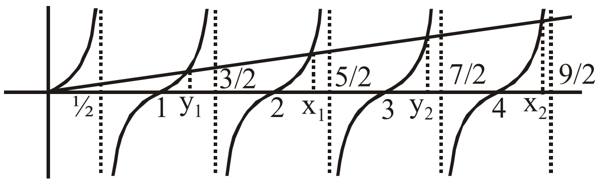
(3) $x_n \in \left(2n, 2n+\frac{1}{2}\right)$ for every n

(4) $x_{n+1} - x_n > 2$ for every n

Ans. (1,3,4)

Sol. $f(x)=\frac{\sin \pi x}{x^2}$

$$f'(x)=\frac{2x\cos \pi x\left(\frac{\pi x}{2}-\tan \pi x\right)}{x^4}$$



$\Rightarrow$ $|x_n - y_n| > 1$ for every n

$x_1 > y_1$

$x_n \in (2n, 2n + 1/2)$

$x_{n+1} - x_n > 2.$

SECTION-2 : (Maximum Marks: 18)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **Two** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct numerical value is entered.

Zero Marks : 0 In all other cases.

---

1. The value of $\sec^{-1}\left(\frac{1}{4}\sum_{k=0}^{10}\sec\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\sec\left(\frac{7\pi}{12}+\frac{(k+1)\pi}{2}\right)\right)$ in the interval $\left[-\frac{\pi}{4},\frac{3\pi}{4}\right]$ equals

Ans. (0.00)

Sol. $\sec^{-1}\left(\frac{1}{4}\sum_{k=0}^{10}\frac{1}{\cos\left(\frac{7\pi}{12}+\frac{k\pi}{12}\right)\cos\left(\frac{7\pi}{12}+\frac{(k+1)\pi}{2}\right)}\right)$

$$=\sec^{-1}\left(\frac{1}{4}\sum_{k=0}^{10}\frac{\sin\left(\frac{7\pi}{12}+\frac{(k+1)\pi}{2}-\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\right)}{\cos\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right).\cos\left(\frac{7\pi}{12}+\frac{(k+1)\pi}{2}\right)}\right)$$

$$=\sec^{-1}\left(\frac{1}{4}\left(\sum_{k=0}^{10}\tan\left(\frac{7\pi}{12}+(k+1)\frac{\pi}{2}\right)-\tan\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\right)\right)$$

$$=\sec^{-1}\left(\frac{1}{4}\left(\tan\left(\frac{11\pi}{2}+\frac{7\pi}{12}\right)-\tan\left(\frac{7\pi}{12}\right)\right)\right)$$

$$=\sec^{-1}\left(\frac{1}{4}\left(-\cot\frac{7\pi}{12}-\tan\frac{7\pi}{12}\right)\right)$$

$$=\sec^{-1}\left(\frac{1}{4}\left(-\frac{1}{\sin\frac{7\pi}{12}\cos\frac{7\pi}{12}}\right)\right)$$

$$=\sec^{-1}\left(-\frac{1}{2}\times\frac{1}{\sin\frac{7\pi}{6}}\right)=\sec^{-1}(1)=0.00$$

2. Let |X| denote the number of elements in set X. Let S = {1,2,3,4,5,6} be a sample space, where each element is equally likely to occur. If A and B are indepenent events associated with S, then the number of ordered pairs (A,B) such that $1 \leq |B| < |A|$, equals

Ans. (422.00)

Sol. $P\left(\frac{B}{A}\right)=P(B)$

$\Rightarrow$ $n\frac{(A\cap B)}{n(A)}=\frac{n(B)}{n(s)}$ .... (1)

$\Rightarrow$ n(A) should have 2 or 3 as prime factors

$\Rightarrow$ n(A) can be 2, 3, 4 or 6 as n(A) > 1 n(A) = 2 does not satisfy the constraint (1).

for n(A) = 3. n(B) = 2 and $n(A \cap B) = 1$

$\Rightarrow$ No. of ordered pair = ${}^6C_4 \times \frac{4!}{2!} = 180$

for n(A) = 4. n(B) = 3 and $n(A \cap B) = 2$

$\Rightarrow$ No. of ordered pairs = ${}^6C_5 \times \frac{5!}{2!2!} = 180$

for n(A) = 6. n(B) can be 1, 2, 3, 4, 5.

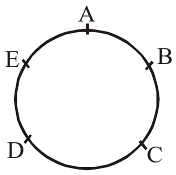
$\Rightarrow$ No. of ordered pairs = $2^6 - 2 = 62$

Total ordered pair = 180 + 180 + 62 = 422.

3. Five person A,B,C,D and E are seated in a ciruclar arrangement. If each of them is given a hat of one of the three colours red, blue and green ,then the number of ways of distributing the hats such that the persons seated in adjacent seats get different coloured hats is

Ans. (30.00)

Sol.



When 1R, 2B, 2G

$5C_1 \times 2 = 10$

Other possibilities

1B, 2R, 2G

or 1G, 2R, 2B

So total no. of ways = 3 × 10 = 30

4. Suppose

$$\det\begin{bmatrix} \sum_{k=0}^{n} k & \sum_{k=0}^{n} {}^nC_k k^2 \\ \sum_{k=0}^{n} {}^nC_k k & \sum_{k=0}^{n} {}^nC_k 3^k \end{bmatrix} = 0$$, holds for some positive integer n. Then $\sum_{k=0}^{n} \frac{{}^nC_k}{k+1}$ equals

Ans. (6.20)

Sol. Suppose

$$\begin{vmatrix} \frac{n(n+1)}{2} & n(n-1).2^{n-2}+n.2^{n-1} \\ n.2^{n-1} & 4^n \end{vmatrix} = 0$$

$$\frac{n(n+1)}{2}.4^n - n^2(n-1).2^{2n-3} - n^2 2^{2n-2} = 0$$

$$\frac{n(n+1)}{2} - \frac{n^2(n-1)}{8} - \frac{n^2}{4} = 0$$

$n^2 - 3n - 4 = 0$

$n = 4$

Now $\sum_{k=0}^{4} \frac{{}^4C_k}{k+1} = \sum_{k=0.}^{4} \frac{k+1}{5}.{}^5C_{k+1}\frac{1}{k+1}$

$$= \frac{1}{5}.\left[{}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5\right] = \frac{1}{5}\left[2^5 - 1\right] = \frac{31}{5} = 6.20$$

5. The value of the integral $\int_0^{\pi/2} \frac{3\sqrt{\cos\theta}}{\left(\sqrt{\cos\theta}+\sqrt{\sin\theta}\right)^5} d\theta$ equals

Ans. (0.50)

Sol. $I = \int_0^{\pi/2} \frac{3\sqrt{\cos\theta}}{\left(\sqrt{\cos\theta}+\sqrt{\sin\theta}\right)^5} d\theta$

$$= \int_0^{\pi/2} \frac{3\sqrt{\sin\theta}}{\left(\sqrt{\cos\theta}+\sqrt{\sec\theta}\right)} d\theta$$

$$2I = \int_0^{\pi/2} \frac{3d\theta}{\left(\sqrt{\cos\theta}+\sqrt{\sec\theta}\right)^4}$$

$$= 3\int_0^{\pi/2} \frac{\sec^2\theta d\theta}{\left(1+\sqrt{\tan\theta}\right)^4}$$

Let $1+\sqrt{\tan\theta} = t$

$$\frac{\sec^2\theta}{2\sqrt{\tan\theta}}d\theta = dt$$

$\sec^2\theta d\theta = 2(t-1)dt$

$$= 3\int_1^{\infty}\frac{2(t-1)dt}{t^4}$$

$$= 6\int_1^{\infty}\left(t^{-3}-t^{-4}\right)dt$$

$$2I = 6\left(\frac{t^{-2}}{-2}-\frac{t^{-3}}{-3}\right)_1^{\infty} = 6\left[0-0-\left\{-\frac{1}{2}+\frac{1}{3}\right\}\right]$$

$I = 0.50$

6. Let $\vec{a} = 2\hat{i}+\hat{j}-\hat{k}$ and $\vec{b} = \hat{i}+2\hat{j}+\hat{k}$ be two vectors. Consider a vector $\vec{c} = \alpha\vec{a}+\beta\vec{b}$, $\alpha,\beta \in \mathbb{R}$. If the projection of $\vec{c}$ on the vector $\left(\vec{a}+\vec{b}\right)$ is $3\sqrt{2}$, then the minimum value of $\left(\vec{c}-\left(\vec{a}\times\vec{b}\right)\right).\vec{c}$ equals

Ans. (18.00)

Sol. $\vec{c} = (2\alpha+\beta)\hat{i}+\hat{j}(\alpha+2\beta)+\hat{k}(\beta-\alpha)$

$$\frac{\vec{c}.(\vec{a}+\vec{b})}{|\vec{a}+\vec{b}|} = 3\sqrt{2}$$

$\Rightarrow \quad \alpha + \beta = 2 \quad$ .... (1)

$(\vec{c}-(\vec{a}\times\vec{b})).(\alpha\vec{a}+\beta\vec{b})$

$= |\vec{c}|^2 = \alpha^2|\vec{a}|^2 + \beta^2|b|^2 + 2\alpha\beta(\vec{a}.\vec{b})$

$= 6(\alpha^2 + \beta^2 + \alpha\beta)$

$= 6(\alpha^2 + (2-\alpha)^2 + \alpha(2-\alpha))$

$= 6((\alpha-1)^2 + 3)$

$\Rightarrow$ Min. value = 18

SECTION–3 : (Maximum Marks : 12)

- This section contains **TWO (02)** List-Match sets.
- Each List-Match set has **Two (02)** Multiple Choice Questions.
- Each List-Match set has two lists : **List-I** and **List-II**
- **List-I** has **Four** entries (I), (II), (III) and (IV) and **List-II** has **Six** entries (P), (Q), (R), (S), (T) and (U)
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks	:	+3	If ONLY the option corresponding to the correct combination is chosen.
Zero Marks	:	0	If none of the options is chosen (i.e., the question is unanswered);
Negative Marks	:	–1	In all other cases

1. **Answer the following by appropriately matching the lists based on the information given in the paragraph**

Let $f(x) = \sin(\pi \cos x)$ and $g(x) = \cos(2\pi \sin x)$ be two functions defined for $x > 0$. Define the following sets whose elements are written in the increasing order :

$$X = \{x : f(x) = 0\}, \quad Y = \{x : f'(x) = 0\}$$
$$Z = \{x : g(x) = 0\}, \quad W = \{x : g'(x) = 0\}.$$

List-I contains the sets X,Y,Z and W. List -II contains some information regarding these sets.

List-I		List-II	
(I)	X	(P)	$\supseteq \left\{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\right\}$
(II)	Y	(Q)	an arithmetic progression
(III)	Z	(R)	NOT an arithmetic progression
(IV)	W	(S)	$\supseteq \left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\right\}$
		(T)	$\supseteq \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$
		(U)	$\supseteq \left\{\frac{\pi}{6}, \frac{3\pi}{4}\right\}$

Which of the following is the only CORRECT combination ?

Options

(1) (II), (R), (S)
(2) (I), (P), (R)
(3) (II), (Q), (T)
(4) (I), (Q), (U)

Ans. (3)

2. **Answer the following by appropriately matching the lists based on the information given in the paragraph**

Let $f(x) = \sin(\pi \cos x)$ and $g(x) = \cos(2\pi \sin x)$ be two functions defined for $x > 0$. Define the following sets whose elements are written in the increasing order :

$$X = \{x : f(x) = 0\}, \quad Y = \{x : f'(x) = 0\}.$$
$$Z = \{x : g(x) = 0\}, \quad W = \{x : g'(x) = 0\}.$$

List-I contains the sets X,Y,Z and W. List -II contains some information regarding these sets.

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(III)	Z	(R)	NOT an arithmetic progression
(IV)	W	(S)	$\supseteq \left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\right\}$
		(T)	$\supseteq \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$
		(U)	$\supseteq \left\{\frac{\pi}{6}, \frac{3\pi}{4}\right\}$

Which of the following is the only CORRECT combination ?

Options

(1) (IV), (Q), (T)
(2) (IV), (P), (R), (S)
(3) (III), (R), (U)
(4) (III), (P), (Q), (U)

Ans. (2)

Solution Q.1 and Q.2

Q.1 Ans. (3)

Q.2 Ans. (2)

Sol. f(x) = sin (π cos x)

X : {x : f(x) = 0}

$f(x) = 0 \Rightarrow \sin(\pi \cos x) = 0 \Rightarrow \cos x = n \Rightarrow \cos x = 1, -1, 0 \Rightarrow x = \frac{n\pi}{2}$

$$X = \left\{\frac{n\pi}{2} : n \in N\right\} = \left\{\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, ...\right\}$$

g(x) = cos (2π sin x)

Z = {x : g(x) = 0}

$\cos(2\pi \sin x) = 0 \Rightarrow 2\pi \sin x = (2n+1)\frac{\pi}{2} \Rightarrow \sin x = \frac{(2n+1)}{4}$

$$\sin x = -\frac{1}{4}, \frac{1}{4}, \frac{-3}{4}, \frac{3}{4}$$

$$Z = \left\{n\pi \pm \sin^{-1}\left(\frac{1}{4}\right), n\pi \pm \sin^{-1}\left(\frac{3}{4}\right), n \in I\right\}$$

Y = {x : f'(x) = 0}

$f(x) = \sin(\pi \cos x) \Rightarrow f'(x) = \cos(\pi \cos x) . (-\pi \sin x) = 0$

$\sin x = 0 \Rightarrow x = n\pi.$

$\cos(\pi \cos x) = 0 \Rightarrow \pi \cos x = (2n+1)\frac{\pi}{2} \Rightarrow \cos x = \frac{(2n+1)}{2} \Rightarrow \cos x = -\frac{1}{2}, \frac{1}{2}$

$$Y = \left\{n\pi, n\pi \pm \frac{\pi}{3}\right\} = \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}, 2\pi, ...\right\}$$

W = {x : g'(x) = 0}

$g(x) = \cos(2\pi \sin x) \Rightarrow g'(x) = -\sin(2\pi \sin x).(2\pi \cos x) = 0$

$\cos x = 0 \Rightarrow x = (2n+1)\frac{\pi}{2}$

$\sin(2\pi \sin x) = 0 \Rightarrow 2\pi \sin x = n\pi \Rightarrow \sin x = \frac{n}{2} = -1, -\frac{1}{2}, 0, \frac{1}{2}, 1$

$$W = \left\{\frac{n\pi}{2}, n\pi \pm \frac{\pi}{6}, n \in I\right\} = \left\{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{3\pi}{2}, ...\right\}$$

Now check the options

3. Answer the following by appropriately matching the lists based on the information given in the paragraph

Let the circles $C_1 : x^2 + y^2 = 9$ and $C_2 : (x-3)^2 + (y-4)^2 = 16$, intersect at the points X and Y. Suppose that another circle $C_3 : (x-h)^2 + (y-k)^2 = r^2$ satisfies the following conditions :

(i) centre of C_3 is collinear with the centres of C_1 and C_2

(ii) C_1 and C_2 both lie inside C_3, and

(iii) C_3 touches C_1 at M and C_2 at N.

Let the line through X and Y intersect C_3 at Z and W, and let a common tangent of C_1 and C_3 be a tangent to the parabola $x^2 = 8\alpha y$.

There are some expression given in the List-I whose values are given in List-II below:

	List-I		**List-II**
(I)	$2h + k$	(P)	6
(II)	$\dfrac{\text{Length of ZW}}{\text{Length of XY}}$	(Q)	$\sqrt{6}$
(III)	$\dfrac{\text{Area of triangle MZN}}{\text{Area of triangle ZMW}}$	(R)	$\dfrac{5}{4}$
(IV)	α	(S)	$\dfrac{21}{5}$
		(T)	$2\sqrt{6}$
		(U)	$\dfrac{10}{3}$

Which of the following is the only INCORRECT combination ?

Options

(1) (IV), (S) (2) (IV), (U) (3) (III), (R) (4) (I), (P)

Ans. (1)

4. Answer the following by appropriately matching the lists based on the information given in the paragraph

Let the circles $C_1 : x^2 + y^2 = 9$ and $C_2 : (x-3)^2 + (y-4)^2 = 16$, intersect at the points X and Y. Suppose that another circle $C_3 : (x-h)^2 + (y-k)^2 = r^2$ satisfies the following conditions :

(i) centre of C_3 is collinear with the centres of C_1 and C_2

(ii) C_1 and C_2 both lie inside C_3, and

(iii) C_3 touches C_1 at M and C_2 at N.

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(III)	$\frac{\text{Area of triangle MZN}}{\text{Area of triangle ZMW}}$	(R)	$\frac{5}{4}$
(IV)	α	(S)	$\frac{21}{5}$
		(T)	$2\sqrt{6}$
		(U)	$\frac{10}{3}$

Which of the following is the only CORRECT combination ?

Options

(1) (II), (T)

(2) (I), (S)

(3) (I), (U)

(4) (II), (Q)

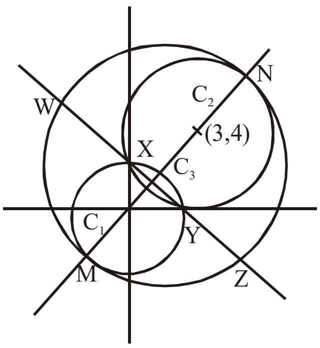
Ans. (4)

Solution Q.3 and Q.4

Q.3 Ans. (1)

Q.4 Ans. (4)

Sol.



$MC_1 + C_1C_2 + C_2N = 2r$

$\Rightarrow 3 + 5 + 4 = 2r \Rightarrow r = 6 \Rightarrow$ Radius of $C_3 = 6$

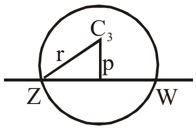
Suppose centre of C_3 be $(0 + r_4\cos\theta, 0 + r_4\sin\theta)$, $\left\{\begin{array}{l} r_4 = C_1C_3 = 3 \\ \tan\theta = \frac{4}{3} \end{array}\right\}$

$C_3 = \left(\frac{9}{5}, \frac{12}{5}\right) = (h,k) \Rightarrow 2h + k = 6$

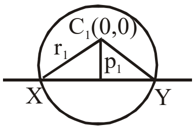
Equation of ZW and XY is $3x + 4y - 9 = 0$

(common chord of circle $C_1 = 0$ and $C_2 = 0$)

$ZW = 2\sqrt{r^2 - p^2} = \frac{24\sqrt{6}}{5}$ (where $r = 6$ and $p = \frac{6}{5}$)



$XY = 2\sqrt{r_1^2 - p_1^2} = \frac{24}{5}$ (where $r_1 = 3$ and $p_1 = \frac{9}{5}$)



$$\frac{\text{Length of ZW}}{\text{Length of XY}} = \sqrt{6}$$

Let length of perpendicular from M to ZW be λ, $\lambda = 3 + \frac{9}{5} = \frac{24}{5}$

$$\frac{\text{Area of } \Delta MZN}{\text{Area of } \Delta ZMW} = \frac{\frac{1}{2}(MN) \times \frac{1}{2}(ZW)}{\frac{1}{2} \times ZW \times \lambda} = \frac{1}{2}\frac{MN}{\lambda} = \frac{5}{4}$$

$$C_3 : \left(x - \frac{9}{5}\right)^2 + \left(y - \frac{12}{5}\right)^2 = 6^2$$

$C_1 : x^2 + y^2 - 9 = 0$

common tangent to C_1 and C_3 is common chord of C_1 and C_3 is $3x + 4y + 15 = 0$.

Now $3x + 4y + 15 = 0$ is tangent to parabola $x^2 = 8\alpha y$.

$$x^2 = 8\alpha\left(\frac{-3x - 15}{4}\right) \Rightarrow 4x^2 + 24\alpha x + 120\alpha = 0$$

$$D = 0 \Rightarrow \alpha = \frac{10}{3}$$