JEE Advanced 2022 Question Paper with Solution

Joint Entrance Examination – Advanced

JEE(Advanced) EXAMINATION - 2022

(Held On Sunday 28th AUGUST, 2022)

PAPER-1

PHYSICS

SECTION-1 : (Maximum Marks : 24)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 **ONLY** if the correct numerical value is entered;
Zero Marks : 0 In all other cases.

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1. Two spherical stars A and B have densities ρ_A and ρ_B, respectively. A and B have the same radius, and their masses M_A and M_B are related by $M_B = 2M_A$. Due to an interaction process, star A loses some of its mass, so that its radius is halved, while its spherical shape is retained, and its density remains ρ_A. The entire mass lost by A is deposited as a thick spherical shell on B with the density of the shell being ρ_A. If v_A and v_B are the escape velocities from A and B after the interaction process, the ratio $\frac{v_B}{v_A} = \sqrt{\frac{10n}{15^{1/3}}}$. The value of n is _______

Ans. **2.30**

Sol. Given $R_A = R_B = R$

$M_B = 2M_A$

Calculation of escape velocity for A:

Radius of remaining star $= \frac{R_A}{2}$.

Mass of remaining star $= \rho_A \frac{4}{3}\pi \frac{R_A^3}{8} = \frac{M_A}{8}$

$$\frac{-GM_{A/B}}{R_{A/2}} + \frac{1}{2}mv_A^2 = 0 \Rightarrow v_A = \sqrt{\frac{2GM_{A/B}}{R_{A/2}}} = \sqrt{\frac{GM_A}{2R}}$$

Calculation of escape velocity for B

Mass collected over $B = \frac{7}{8}M_A$

Let the radius of B becomes r.

$$\therefore \frac{4}{3}\pi(r^3 - R_B^3)\rho_A = \frac{7}{8}\rho_A \frac{4}{3}\pi R_A^3 \Rightarrow \pi^3 = \frac{7}{8}R_A^3 + R_B^3 = \frac{(15)^{1/3}R}{2}$$

$$\therefore \frac{V_B^2}{2} = \frac{23GM_A}{8\times15^{1/3}\frac{R}{2}} = \frac{23GM_A}{4\times15^{1/3}R}$$

$$\therefore V_B = \sqrt{\frac{23GM_A}{2\times15^{1/3}R}}$$

$$\therefore \frac{V_B}{V_A} = \sqrt{\frac{23}{15^{1/3}}} = \sqrt{\frac{10\times2.30}{15^{1/3}}}$$

n = 2.30

2. The minimum kinetic energy needed by an alpha particle to cause the nuclear reaction ${}^{16}_{7}N + {}^{4}_{2}He \rightarrow {}^{1}_{1}H + {}^{19}_{8}O$ in a laboratory frame is n (in *MeV*). Assume that ${}^{16}_{7}N$ is at rest in the laboratory frame. The masses of ${}^{16}_{7}N$, ${}^{4}_{2}He$, ${}^{1}_{1}H$ and ${}^{19}_{8}O$ can be taken to be 16.006 u, 4.003 u, 1.008 u and 19.003 u, respectively, where $1\ u = 930\ MeVc^{-2}$. The value of n is_________.

Ans. **2.32 to 2.33**

Sol. ${}^{16}_{7}N + {}^{4}_{2}He \rightarrow {}^{1}_{1}He + {}^{19}_{8}O$

${}^{16}_{7}N$	+	${}^{4}_{2}He$	$\rightarrow$	${}^{1}_{1}He$	+	${}^{19}_{8}O$
16.006		4.003		1.008		19.003

$4v_0 = 1v_1 + 19v_2 = 20v_2$ (For max loss of KE)

$$v_0 = \frac{v_2}{5}$$

E required = (1.008 + 19.003 – 16.006 – 4.003) × 930 = 1.86

$$\frac{1}{2}4v_0^2 - \frac{1}{2}20v^2 = 1.86$$

$$\frac{1}{2}4v_0^2 - 10\frac{v_0^2}{25}20v^2 = 1.86$$

$$2v_0^2 - \frac{2}{5}v_0^2 = 1.86$$

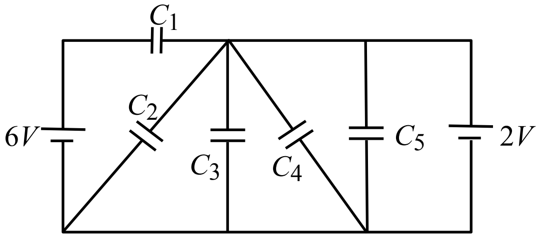
$$\frac{8}{5}v_0^2 = 1.86$$

$$v_0^2 = \frac{1.86\times5}{8}$$

$$KE = \frac{1}{2}4v_0^2 = 2v_0^2 = \frac{18.6\times5}{4}$$

= 2.325

3. In the following circuit $C_1 = 12\ \mu F$, $C_2 = C_3 = 4\ \mu F$ and $C_4 = C_5 = 2\ \mu F$. The Charge stored in C_3 is _______ μC.

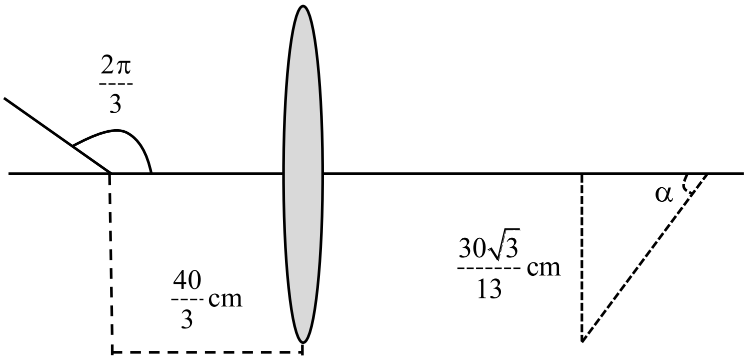


Ans. **8**

Sol. Potential difference across the terminals of C_3 is 2V.

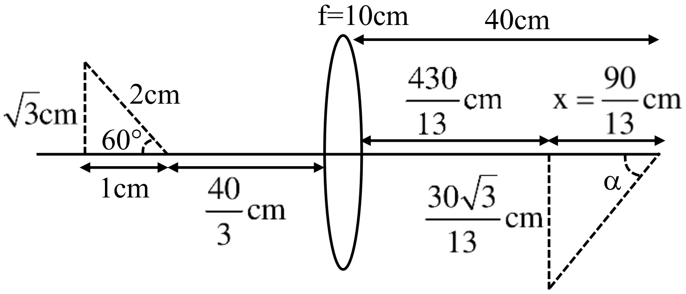
$\therefore\ Q_3 = CV = (4\mu)\,(2) = 8\mu C$

4. A rod of length 2 *cm* makes an angle $\frac{2\pi}{3}$ rad with the principal axis of a thin convex lens. The lens has a focal length of 10 *cm* and is placed at a distance of $\frac{40}{3}$ *cm* from the object as shown in the figure. The height of the image is $\frac{30\sqrt{3}}{13}$ *cm* and the angle made by it with respect to the principal axis is α rad. The value of α is $\frac{\pi}{n}$ *rad*, where *n* is _________.



Ans. **6**

Sol.



$$\frac{h_i}{h_0}=\frac{v}{u}\Rightarrow\frac{-\frac{30\sqrt{3}}{13}}{\sqrt{3}}=\frac{v}{-\frac{43}{3}}\Rightarrow v_1=\frac{430}{13}\text{cm}$$

$$*\ \frac{1}{v}-\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{v}=\frac{1}{10}-\frac{3}{40}\Rightarrow v=40\text{cm}$$

$$*\ x=40-\frac{430}{13}=\frac{90}{13}\text{cm}$$

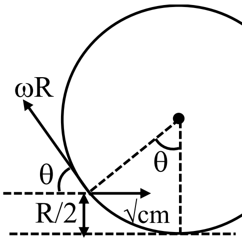
$$\tan\alpha=\frac{\frac{30\sqrt{3}}{13}}{\frac{90}{13}}=\frac{1}{\sqrt{3}}\Rightarrow\alpha=30^\circ=\frac{\pi}{6}$$

N = 6 Ans.

5. At time t = 0, a disk of radius 1 m starts to roll without slipping on a horizontal plane with an angular acceleration of $\alpha=\frac{2}{3}$ $rad\ s^{-2}$. A small stone is stuck to the disk. At t = 0, it is at the contact point of the disk and the plane. Later, at time $t=\sqrt{\pi}\ s$, the stone detaches itself and flies off tangentially from the disk. The maximum height (in m) reached by the stone measured from the plane is $\frac{1}{2}+\frac{x}{10}$. The value of x is______. [Take g = 10 $m\ s^{-2}$.]

Ans. 0.52

Sol.



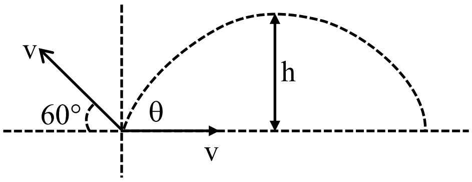
At t = 0, ω = 0

$$\text{at } t=\sqrt{\pi},\ \omega=\alpha t=\frac{2}{3}\sqrt{\pi}, v=\omega r=\frac{2}{3}\sqrt{\pi}$$

$$\theta=\frac{1}{2}\alpha t^2$$

$$\theta=\frac{1}{2}\times\frac{2}{3}\times\pi=\frac{\pi}{3}$$

$$\theta=60^\circ$$



$$v_y = v\sin 60 = \frac{\sqrt{3}}{2}V$$

$$h = \frac{u_y^2}{2g} = \frac{\frac{3}{4}v^2}{2g}$$

$$h = \frac{\frac{3}{4}\times\frac{4}{9}\pi}{2g}$$

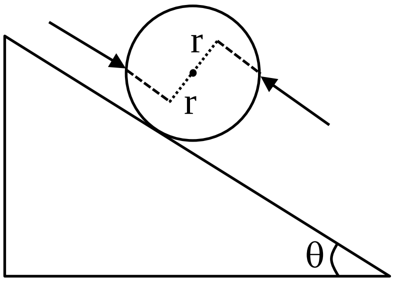
$$h = \frac{3\pi}{9\times 2g} = \frac{\pi}{6g}$$

Maximum height from plane, $H = \frac{R}{2} + h$

$$H = \frac{1}{2} + \frac{\pi}{6\times 10}$$

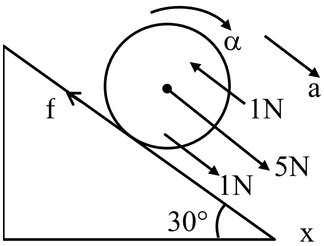
$$x = \frac{\pi}{6}\text{ ; } x = 0.52$$

6. A solid sphere of mass 1 *kg* and radius 1 *m* rolls without slipping on a fixed inclined plane with an angle of inclination $\theta = 30°$ from the horizontal. Two forces of magnitude 1 *N* each, parallel to the incline, act on the sphere, both at distance $r = 0.5$ *m* from the center of the sphere, as shown in the figure. The acceleration of the sphere down the plane is______ms^{-2}. (Take $g = 10\ m\ s^{-2}$.)



Ans. 2.85 to 2.86

Sol. Solid sphere 1kg, 1m



$5 + 1 - 1 - f = 1a$

$5 - f = a$

About COM

$$f\,1 - 2(1(0.5)) = \frac{2}{5}Mr^2\alpha$$

$$\Rightarrow f - 1 = \frac{2}{5}a \Rightarrow f = 1 + \frac{2}{5}a$$

$$5 - a = 1 + \frac{2}{5}a$$

$$\Rightarrow 4 = \frac{7a}{5} \;\Rightarrow a = \frac{20}{7} = 2.86\text{ m/s}^2$$

7. Consider an LC circuit, with inductance L = 0.1 *H* and capacitance $C = 10^{-3}\,F$, kept on a plane. The area of the circuit is 1 m^2. It is placed in a constant magnetic field of strength B_0 which is perpendicular to the plane of the circuit. At time $t = 0$, the magnetic field strength starts increasing linearly as $B = B_0 + \beta t$ with $\beta = 0.04\ Ts^{-1}$. The maximum magnitude of the current in the circuit is____*mA*.

Ans. 4

Sol. Maximum energy will be

$$\frac{q_0^2}{2C} = \frac{1}{2}LI_0^2$$

$$\frac{q_0^2}{CL} = I_0^2$$

$$I_0 = \frac{q_0}{\sqrt{LC}}$$

$$I_0 = \frac{CV}{\sqrt{LC}}$$

$$I_0 = \sqrt{\frac{C}{L}} \times V \qquad V = \text{emf} = \left|\frac{AdB}{dt}\right|$$

$I_0 = \sqrt{\frac{10^{-3}}{0.1}} \times 0.04 \qquad V = (1 \times 0.04)$

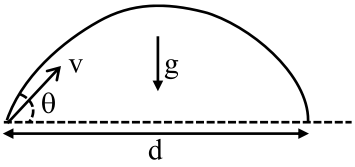
Maximum current $I_0 = 0.004 = 4mA$

Ans. (4)

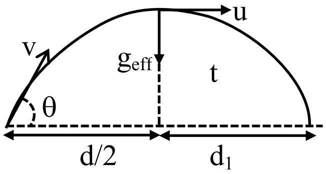
8. A projectile is fired from horizontal ground with speed *v* and projection angle *θ.* When the acceleration due to gravity is *g*, the range of the projectile is *d*. If at the highest point in its trajectory, the projectile enters a different region where the effective acceleration due to gravity is $g' = \frac{g}{0.81}$, then the new range is $d' = nd$. The value of *n* is_______.

Ans. 0.95

Sol.



$$d = \frac{v^2 \sin 2\theta}{g}$$



$$H_{max} = \frac{v^2 \sin^2 \theta}{2g}; \ \frac{1}{2} g_{eff} t^2 = H_{max} \Rightarrow t^2 = \frac{2H_{max}}{g_{eff}}; \ t = \sqrt{\frac{v^2 \sin^2 \theta \times 0.81}{g^2}}; \ t = \frac{0.9 v \sin \theta}{g}$$

$$t^2 = \frac{2 \times v^2 \sin^2 \theta}{2g\left(\frac{g}{0.81}\right)}$$

$$d' = \text{New range} = \frac{d}{2} + d_1$$

$$d_1 = v\cos\theta^\circ t$$

$$= \frac{v^2 \sin^2 \theta \cos\theta \times 0.9}{g}; \ d' = \frac{v^2 \sin 2\theta}{2g} + \frac{v^2 \sin 2\theta \times 0.9}{2g}$$

$$= \frac{v^2 \sin 2\theta}{g}\left(\frac{1.0}{2}\right) = 0.95d$$

$n = 0.95$

SECTION-2 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	: +3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	: +2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	: +1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: −2	In all other cases.

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9. A medium having dielectric constant $K > 1$ fills the space between the plates of a parallel plate capacitor. The plates have large area, and the distance between them is d. The capacitor is connected to a battery of voltage V. as shown in Figure (a). Now, both the plates are moved by a distance of $\frac{d}{2}$ from their original positions, as shown in Figure (b).

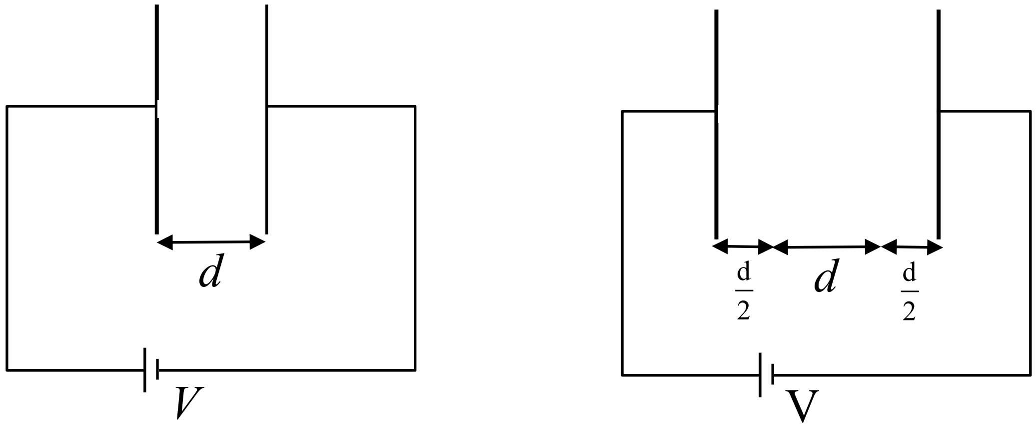


Figure (a) Figure (b)

In the process of going from the configuration depicted in Figure (a) to that in Figure (b), which of the following statement(s) is(are) correct?

(A) The electric field inside the dielectric material is reduced by a factor of $2K$.

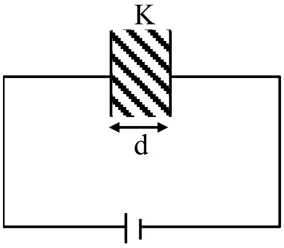
(B) The capacitance is decreased by a factor of $\frac{1}{K+1}$.

(C) The voltage between the capacitor plates is increased by a factor of $(K+1)$.

(D) The work done in the process **DOES NOT** depend on the presence of the dielectric material.

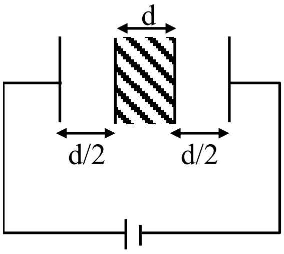
Ans. (B)

Sol. For figure(a)



$$E_0 = \frac{V}{d};\ C = \frac{K\varepsilon_0 A}{d}$$

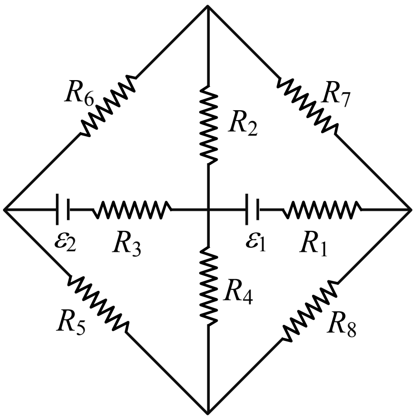
For figure(b)



$$C' = \frac{\varepsilon_0 A}{2d - d + d/k};$$

$$C' = \frac{K\varepsilon_0 A}{(K+1)d};\ C' = \frac{C}{K+1}$$

10. The figure shows a circuit having eight resistances of 1 Ω each, labelled R_1 to R_8, and two ideal batteries with voltages $\varepsilon_1 = 12\ V$ and $\varepsilon_2 = 6\ V$.



Which of the following statement(s) is(are) correct?

(A) The magnitude of current flowing through R_1 is 7.2 A.

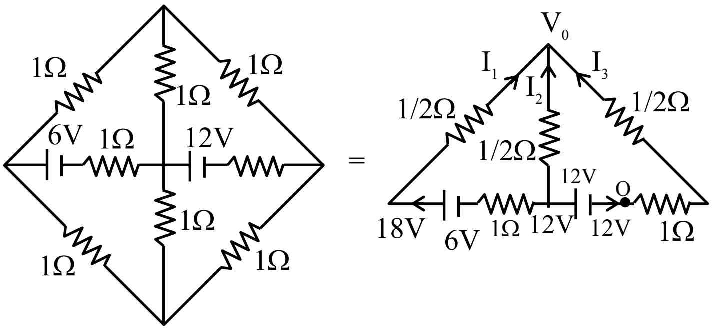
(B) The magnitude of current flowing through R_2 is 1.2 A.

(C) The magnitude of current flowing through R_3 is 4.8 A.

(D) The magnitude of current flowing through R_5 is 2.4 A.

Ans. (A,B,C,D)

Sol.



From KCL

$i_1 + i_2 + i_3 = 0$

$$\Rightarrow \frac{18 - V_0}{3/2} + \frac{12 - V_0}{1/2} + \frac{0 - V_0}{3/2} = 0$$

$$\Rightarrow 18 - V_0 + 36 - 3V_0 - V_0 = 0$$

$$\Rightarrow 54 = 5V_0$$

$$\frac{2\left(\frac{54}{5} - v'\right)}{1} + \frac{18 - v'}{1} = 0$$

$$\Rightarrow \frac{108}{5} + 18 = 3V'$$

$$\Rightarrow v' = \frac{198}{5 \times 3} = \frac{66}{5} V$$

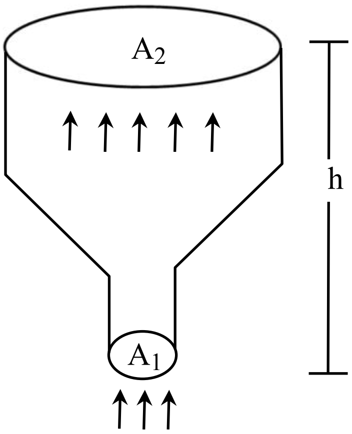
$$I_{R_1} = \frac{36}{5} = 7.2A$$

$$I_{R_2} = \frac{6}{5} = 1.2A$$

$$I_{R_3} = \frac{24}{5} = 4.8A$$

$$I_{R_5} = \frac{12}{5} = 2.4A$$

11. An ideal gas of density $\rho = 0.2\ kg\ m^{-3}$ enters a chimney of height h at the rate of $\alpha = 0.8\ kg\ s^{-1}$ from its lower end, and escapes through the upper end as shown in the figure. The cross-sectional area of the lower end is $A_1 = 0.1\ m^2$ and the upper end is $A_2 = 0.4\ m^2$. The pressure and the temperature of the gas at the lower end are 600 *Pa* and 300 *K*, respectively, while its temperature at the upper end is 150 *K*. The chimney is heat insulated so that the gas undergoes adiabatic expansion. Take $g = 10\ ms^{-2}$ and the ratio of specific heats of the gas $\gamma = 2$. Ignore atmospheric pressure.



Which of the following statement(s) is(are) correct?

(A) The pressure of the gas at the upper end of the chimney is 300 *Pa*.

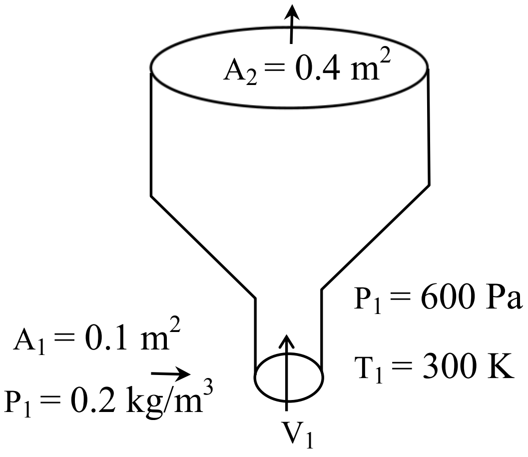
(B) The velocity of the gas at the lower end of the chimney is 40 ms^{-1} and at the upper end is 20 ms^{-1}.

(C) The height of the chimney is 590 *m*.

(D) The density of the gas at the upper end is 0.05 $kg\ m^{-3}$.

Ans. (B)

Sol.



$$\frac{dm}{dt} = \rho_1 A_1 v_1 = 0.8\ kg/s\ A$$

$$v_1 = \frac{0.8}{0.2 \times 0.1} = 40 m/s$$

$g = 10 \text{ m/s}^2$

$\gamma = 2$

Gas undergoes adiabatic expansion,

$p^{1-\gamma} T^{\gamma} = \text{Constant}$

$$\frac{P_2}{P_1} = \left(\frac{T_1}{T_2}\right)^{\frac{r}{1-\gamma}}$$

$$P_2 = \left(\frac{300}{150}\right)^{\frac{2}{-1}} \times 600$$

$$P_2 = \frac{600}{4} = 150\,\text{Pa}$$

Now $\rho = \frac{PM}{RT} \Rightarrow \rho \propto \frac{P}{T}$

$$\frac{\rho_1}{\rho_2} = \left(\frac{P_1}{P_2}\right)\left(\frac{T_1}{T_2}\right) = \left(\frac{150}{600}\right)\left(\frac{300}{150}\right) = \frac{1}{2}$$

$$\rho_2 = \frac{\rho_1}{2} = 0.1 \text{ kg/m}^3$$

Now $\rho_2 A_2 v_2 = 0.8 \Rightarrow v_2 = \frac{0.8}{0.1 \times 0.4} = 20\text{m / s}$

Now $W_{\text{on gas}} = \Delta K + \Delta U + (\text{Internal energy})$

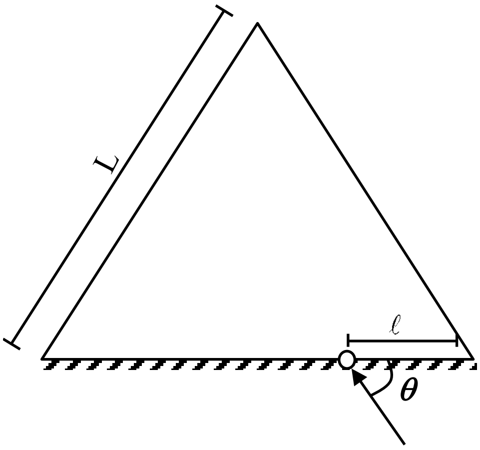
$$P_1A_1\Delta x_1 - P_2A_2\Delta x_2 = \frac{1}{2}\Delta m V_2^2 - \frac{1}{2}\Delta m V_1^2 + \Delta mgh \;\; + \frac{f}{2}(P_2\Delta V_2 - P_1\Delta V_1)$$

$$\Rightarrow 2P_1\frac{\Delta V_1}{\Delta m} - 2P_2\frac{\Delta V_2}{\Delta m} = \frac{V_2^2 - V_1^2}{2} + gh$$

$$\Rightarrow \frac{2\times 600}{0.2} - \frac{2\times 150}{0.1} = \frac{20^2 - 40^2}{2} + 10h$$

$h = 360$ m

12. Three plane mirrors form an equilateral triangle with each side of length L. There is a small hole at a distance $l > 0$ from one of the corners as shown in the figure. A ray of light is passed through the hole at an angle θ and can only come out through the same hole. The cross section of the mirror configuration and the ray of light lie on the same plane.



Which of the following statement(s) is(are) correct?

(A) The ray of light will come out for $\theta = 30°$, for $0 < l < L$.

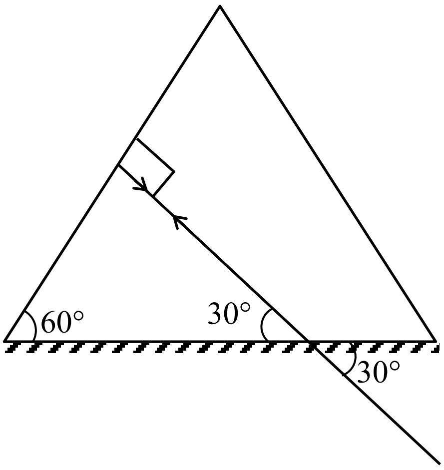
(B) There is an angle for $l = \frac{L}{2}$ at which the ray of light will come out after two reflections.

(C) The ray of light will **NEVER** come out for $\theta = 60°$, and $l = \frac{L}{3}$.

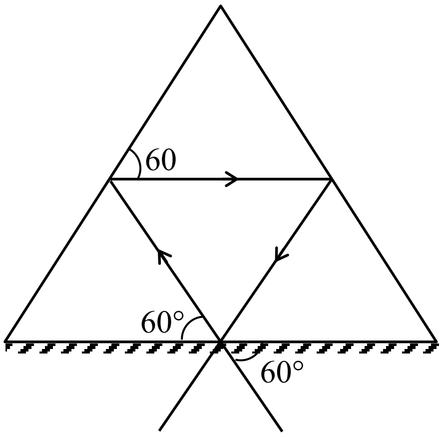
(D) The ray of light will come out for $\theta = 60°$, and $0 < l < \frac{L}{2}$ after six reflections.

Ans. **(A,B)**

Sol. (A) Ray will come out after one reflection for $\theta = 30°$ & $0 < \ell < L$

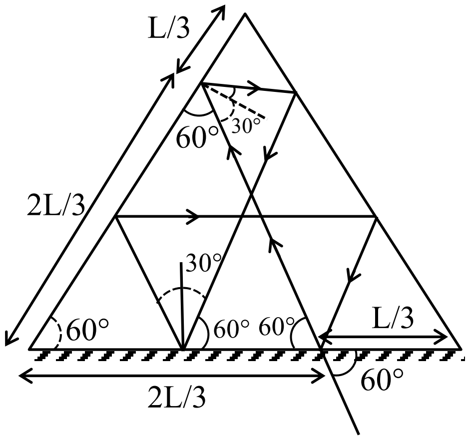


(B)

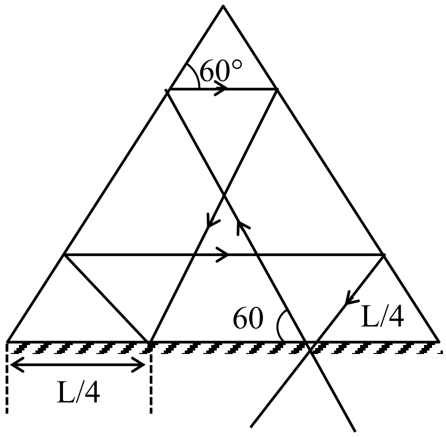


for θ = 60° & $\ell = \frac{L}{2}$, ray will come out after two reflections.

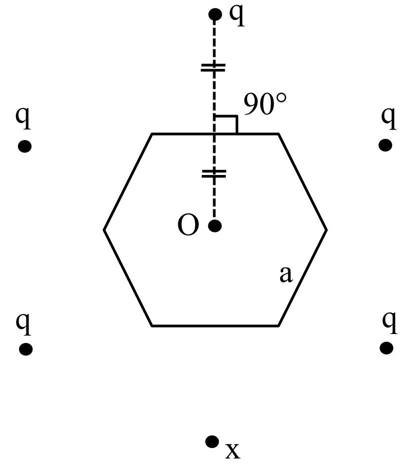
(C) For $\ell = \frac{L}{3}$ & θ = 60° ray will come out after five reflections.



(D) For θ = 60° & $0 < \ell < \frac{L}{2}$, ray will come out after five reflections



13. Six charges are placed around a regular hexagon of side length a as shown in the figure. Five of them have charge q, and the remaining one has charge x. The perpendicular from each charge to the nearest hexagon side passes through the center O of the hexagon and is bisected by the side.



Which of the following statement(s) is(are) correct in SI units?

(A) When $x = q$, the magnitude of the electric field at O is zero.

(B) When $x = -q$, the magnitude of the electric field at O is $\frac{q}{6\pi \in_0 a^2}$.

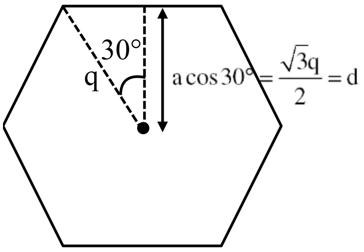
(C) When $x = 2q$, the potential at O is $\frac{7q}{4\sqrt{3}\pi \in_0 a}$.

(D) When $x = -3q$, the potential at O is $\frac{3q}{4\sqrt{3}\pi \in_0 a}$.

Ans. (A,B,C)

Sol. (A) Due to symmetry $\vec{E}_0 = 0$

(B)



$$E_{net} = \frac{kq}{(2d)^2} \times 2 = \frac{2q \times 4}{4\pi\varepsilon_0 \cdot 4 \cdot 3a^2}$$

$$= \frac{q}{6\pi\varepsilon_0 a^2}$$

(C) $v = \frac{7kq}{2d} = \frac{7q}{4\pi\varepsilon_0 \cdot \sqrt{3}a} = \frac{7q}{4\sqrt{3}\pi\varepsilon_0 q}$

(D) $v = \frac{2kq}{2d} = \frac{2q}{4\pi\varepsilon_0 \cdot \sqrt{3}a} = \frac{q}{2\sqrt{3}\pi\varepsilon_0 q}$

Ans. (A,B,C)

14. The binding energy of nucleons in a nucleus can be affected by the pairwise Coulomb repulsion. Assume that all nucleons are uniformly distributed inside the nucleus. Let the binding energy of a proton be E_b^p and the binding energy of a neutron be E_b^n in the nucleus.

Which of the following statement(s) is(are) correct?

(A) $E_b^p - E_b^n$ is proportional to $Z(Z-1)$ where Z is the atomic number of the nucleus.

(B) $E_b^p - E_b^n$ is proportional to $A^{-\frac{1}{3}}$ where A is the mass number of the nucleus.

(C) $E_b^p - E_b^n$ is positive.

(D) E_b^p increases if the nucleus undergoes a beta decay emitting a positron.

Ans. **(A,B,D)**

Sol. Binding energy of proton & neutron due to nuclear force is same. So difference in binding energy is only due to electrostatic P.E. and it is positive

$E_0^P - E_0^n$ = electrostatic P.E.

= Z × P.E. of one proton

$$= Z \times \frac{1}{4\pi\varepsilon_0} \frac{(Z-1)e^2}{R}$$

Where $R = R_0 A^{1/3}$

$$= \frac{1}{4\pi\varepsilon_0} \frac{Z(Z-1)e^2}{R_0 A^{\frac{1}{3}}}$$

Ans. (A,B,D)

SECTION-3 : (Maximum Marks : 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists : **List-I** and **List-II**.
- **List-I** has **Four** entries (I), (II), (III) and (IV) and **List-II** has **Five** entries (P), (Q), (R), (S) and (T).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 **ONLY** if the option corresponding to the correct combination is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : –1 In all other cases.

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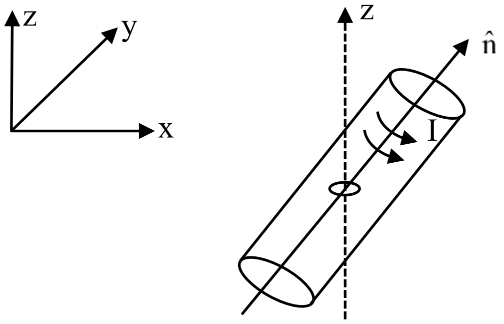
15. A small circular loop of area A and resistance R is fixed on a horizontal xy-plane with the center of the loop always on the axis $\hat{n}$ of a long solenoid. The solenoid has m turns per unit length and carries current I counterclockwise as shown in the figure. The magnetic field due to the solenoid is in $\hat{n}$ direction. List-I gives time dependences of $\hat{n}$ in terms of a constant angular frequency ω. List-II gives the torques experienced by the circular loop at time $t = \frac{\pi}{6\omega}$, Let $\alpha = \frac{A^2\mu_0^2 m^2 I^2 \omega}{2R}$.



List-I		List-II	
(I)	$\frac{1}{\sqrt{2}}\left(\sin\omega t\,\hat{j} + \cos\omega t\,\hat{k}\right)$	(P)	0
(II)	$\frac{1}{\sqrt{2}}\left(\sin\omega t\,\hat{i} + \cos\omega t\,\hat{j}\right)$	(Q)	$-\frac{\alpha}{4}\hat{i}$
(III)	$\frac{1}{\sqrt{2}}\left(\sin\omega t\,\hat{i} + \cos\omega t\,\hat{k}\right)$	(R)	$\frac{3\alpha}{4}\hat{i}$
(IV)	$\frac{1}{\sqrt{2}}\left(\cos\omega t\,\hat{i} + \sin\omega t\,\hat{k}\right)$	(S)	$\frac{\alpha}{4}\hat{j}$
		(T)	$-\frac{3\alpha}{4}\hat{i}$

Which one of the following options is correct?

(A) I→Q, II→P, III→S, IV→T
(B) I→S, II→T, III→Q, IV→P
(C) I→Q, II→P, III→S, IV→R
(D) I→T, II→Q, III→P, IV→R

Ans. (C)

Sol. (I) $\vec{B}=\frac{\mu_0 mI}{\sqrt{2}}\left(\sin\omega t\,\hat{j}+\cos\omega t\,\hat{k}\right)$

$$\phi=\vec{B}\cdot\vec{A}=\frac{\mu_0 mI}{\sqrt{2}}\cos\left(\omega t\right)\cdot A$$

$$\varepsilon=\frac{d\phi}{dt}=\frac{\mu_0 mI\omega A}{\sqrt{2}}\sin\left(\omega t\right)$$

$$i=\frac{\varepsilon}{R}=\frac{\mu_0 mI\omega A}{\sqrt{2}R}\sin\left(\omega t\right)$$

$$\vec{M}=i\vec{A}=iA\left(\hat{k}\right)=\frac{\mu_0 mI\omega A^2}{\sqrt{2}R}\sin\left(\omega t\right)\left(\hat{k}\right)$$

$$\vec{\tau}=\vec{M}\times\vec{B}=\frac{\mu_0 m^2I^2\omega A^2}{\sqrt{2}R}\sin^2\left(\omega t\right)\left(-\hat{i}\right)$$

$$=-\left(\frac{\alpha}{4}\right)\hat{i}$$

(II) $\vec{B}=\frac{\mu_0 mI}{\sqrt{2}}\left(\sin\omega t\,\hat{i}+\cos\omega t\,\hat{j}\right)$

$$\phi=0,\ \varepsilon=0,\ i=0,\ t=0$$

(III) $\vec{B}=\frac{\mu_0 mI}{\sqrt{2}}\left(\sin\omega t\,\hat{i}+\cos\omega t\,\hat{k}\right)$

$$\phi=\vec{B}\cdot\vec{A}=\frac{\mu_0 mI}{\sqrt{2}}\cdot\cos\left(\omega t\right)\cdot A$$

$$\varepsilon=-\frac{d\phi}{dt}=\frac{\mu_0 mI\omega A}{\sqrt{2}}\sin\left(\omega t\right)$$

$$i = \frac{\varepsilon}{R} = \frac{\mu_0 mI\omega A}{\sqrt{2}R}\sin(\omega t)$$

$$\vec{M} = i\vec{A} = iA(\hat{k}) = \frac{\mu_0 mI\omega A^2}{\sqrt{2}R}\sin(\omega t)(\hat{k})$$

$$\vec{\tau} = \vec{M}\times\vec{B} = \frac{\mu_0 m^2 I^2\omega A^2}{2R}\sin^2(\omega t)(+\hat{j})$$

$$= \frac{\alpha}{4}\hat{j}$$

(IV) $\vec{B} = \frac{\mu_0 mI}{\sqrt{2}}\left(\cos\omega t\,\hat{j} + \sin\omega t\,\hat{k}\right)$

$$\phi = \vec{B}\cdot\vec{A} = \frac{\mu_0 mI}{\sqrt{2}}\cdot\sin(\omega t)\cdot A$$

$$\varepsilon = -\frac{d\phi}{dt} = \frac{\mu_0 mI\omega A}{\sqrt{2}}\cos(\omega t)$$

$$i = \frac{\varepsilon}{R} = -\frac{\mu_0 mI\omega A}{\sqrt{2}R}\cos(\omega t)$$

$$\vec{M} = i\vec{A} = iA(\hat{k}) = -\frac{\mu_0 mI\omega A^2}{\sqrt{2}R}\cos(\omega t)(\hat{k})$$

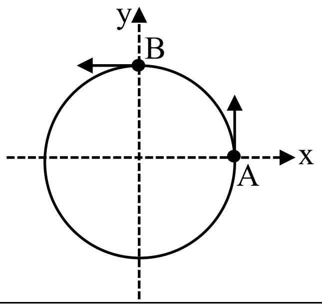
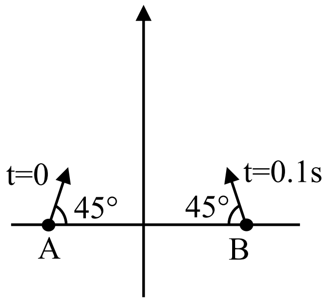
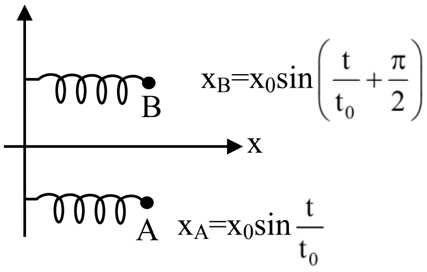
$$\vec{\tau} = \vec{M}\times\vec{B} = -\frac{\mu_0 m^2 I^2\omega A^2}{2R}\cos^2(\omega t)(-\hat{i})$$

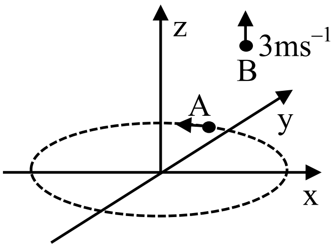
$$= \alpha\cdot\cos^2\left(\frac{\pi}{6}\right)\hat{i}$$

$$= \frac{3\alpha}{4}\hat{i}$$

Ans. (C) I –Q, II-P, III-S, IV-R

16. List I describes four systems, each with two particles A and B in relative motion as shown in figure. List II gives possible magnitudes of then relative velocities (in ms^{-1}) at time $t = \frac{\pi}{3}s$.

List-I		List-II	
(I)	A and B are moving on a horizontal circle of radius 1 m with uniform angular speed ω = 1 $rad\ s^{-1}$. The initial angular positions of A and B at time t = 0 are θ = 0 and $\theta = \frac{\pi}{2}$ respectively.  	(P)	$\frac{\sqrt{3}+1}{2}$
(II)	Projectiles A and B are fired (in the same vertical plane) at t = 0 and t = 0.1 s respectively, with the same speed $v = \frac{5\pi}{\sqrt{2}}\ m\ s^{-1}$ and at 45° from the horizontal plane. The initial separation between A and B is large enough so that they do not collide, ($g = 10\ m\ s^{-2}$).  	(Q)	$\frac{(\sqrt{3}-1)}{\sqrt{2}}$
(III)	Two harmonic oscillators A and B moving in the x direction according to $x_A = x_0 \sin\frac{t}{t_0}$ and $x_B = x_0 \sin\left(\frac{t}{t_0}+\frac{\pi}{2}\right)$ respectively, starting from t = 0. Take x_0 = 1 m, t_0 = 1 s.  	(R)	$\sqrt{10}$

(IV)	Particle *A* is rotating in a horizontal circular path of radius 1 *m* on the *xy* plane, with constant angular speed $\omega = 1\ rad\ s^{-1}$. Particle *B* is moving up at a constant speed 3 $m\ s^{-1}$ in the vertical direction as shown in the figure. (Ignore gravity.)  	(S)	$\sqrt{2}$
		(T)	$\sqrt{25\pi^2+1}$

Which one of the following options is correct?

(A) I → R, II → T, III → P, IV → S

(B) I → S, II → P, III → Q, IV → R

(C) I → S, II → T, III → P, IV → R

(D) I → T, II → P, III → R, IV → S

Ans. **(C)**

Sol. (I) $v_{BA}^2 = v_A^2 + v_B^2 - 2v_{AB}\cos\theta$

As $\omega_A = \omega_B, \theta = 90°$ remains constant.

Also, $v_A = v_B = 1$ m/s

So, $v_{BA} = \sqrt{2}\text{m/s}$

(II) $\vec{u}_A = \frac{5\pi}{2}\hat{i} + \frac{5\pi}{2}\hat{j}$

$$\vec{v}_A = \frac{5\pi}{2}\hat{i} + \left(\frac{5\pi}{2} - 10\cdot\frac{\pi}{3}\right)\hat{j}$$

$$= \frac{5\pi}{2}\hat{i} - \frac{5\pi}{6}\hat{j}$$

$$\vec{u}_B = -\frac{5\pi}{2}\hat{i} + \frac{5\pi}{2}\hat{j}$$

$$\vec{u}_B = -\frac{5\pi}{2}\hat{i} - \left(\frac{5\pi}{6}+1\right)\hat{j}$$

$$\vec{v}_{B,A} = -5\pi\hat{i} - \hat{j}$$

$$v_{BA} = \sqrt{25\pi^2+1}$$

(III) $x_A = \sin t$

$$v_A = \cos t = \frac{1}{2}\,m/s$$

$x_B = \cos t$

$$v_B = -\sin t = -\frac{\sqrt{3}}{2}\,m/s$$

$$v_{BA} = -\frac{\sqrt{3}}{2} - \frac{1}{2}$$

(IV) $\vec{v}_A$ & $\vec{v}_B$ are always perpendicular

So, $|\vec{v}_{BA}| = \sqrt{v_A^2 + v_B^2} = \sqrt{10}\,m/s$

Ans. (C), I-S, II-T, III-P, IV-R

17. List I describes thermodynamic processes in four different systems. List II gives the magnitudes (either exactly or as a close approximation) of possible changes in the internal energy of the system due to the process.

List-I		**List-II**	
(I)	10^{-3} kg of water at 100°C is converted to steam at the same temperature, at a pressure of 10^5 *Pa*. The volume of the system changes from $10^{-6}\ m^3$ to $10^{-3}\ m^3$ in the process. Latent heat of water = 2250 *kJ/kg*.	(P)	2 *kJ*
(II)	0.2 moles of a rigid diatomic ideal gas with volume *V* at temperature 500 *K* undergoes an isobaric expansion to volume 3 *V*. Assume $R = 8.0\ J\,mol^{-1}K^{-1}$.	(Q)	7 *kJ*
(III)	On mole of a monatomic ideal gas is compressed adiabatically from volume $V = \frac{1}{3}m^3$ and pressure 2 *kPa* to volume $\frac{v}{8}$	(R)	4 *kJ*
(IV)	Three moles of a diatomic ideal gas whose molecules can vibrate, is given 9 *kJ* of heat and undergoes isobaric expansion.	(S)	5 *kJ*
		(T)	3 *kJ*

Which one of the following options is correct ?

(A) I → T, II → R, III → S, IV → Q
(B) I → S, II → P, III → T, IV → P
(C) I → P, II → R, III → T, IV → Q
(D) I → Q, II → R, III → S, IV → T

Ans. (C)

Sol. (I) $\Delta U = \Delta Q - \Delta W$

$$= \left\{\left(10^{-3} \times 2250\right) - \frac{10^{5}\left(10^{-3} - 10^{-6}\right)}{10^{3}}\right\} \text{kJ}$$

$= (2.25 - 0.0999)$ kJ

$= (2.1501)$ kJ

(II) $\Delta U = nC_V\Delta T$

$$= \frac{5}{2} nR\Delta T$$

$$= \frac{5}{2} \cdot (0.2)(8)(1500 - 500)\,\text{J}$$

$= 4$ kJ

(III) $P_1V_2^{\gamma} = P_2V_2^{\gamma}$

$$\Rightarrow 2\left(\frac{1}{3}\right)^{5/3} = P_2\left(\frac{1}{24}\right)^{5/3}$$

$\Rightarrow P_2 = 64$ kPa

$$\Delta U = nC_V\Delta T = \frac{3}{2} \cdot \left(P_2V_2 - P_1V_1\right)$$

$$= \frac{3}{2}\left(64 \times \frac{1}{24} - 2 \times \frac{1}{3}\right) \text{kJ}$$

$= 3$ kJ

(IV) $\Delta U = nC_V\Delta T$

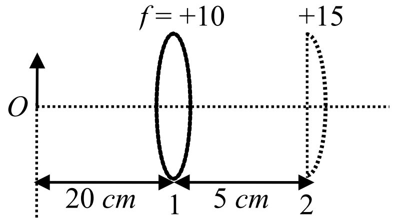
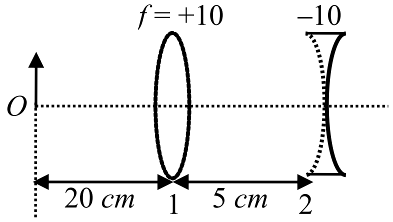
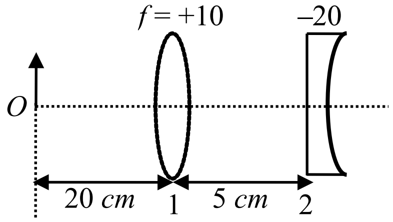
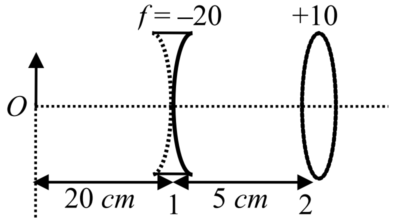
$$= n \cdot \frac{7}{2} R\Delta T$$

$$= \frac{7}{9} \Delta Q$$

$= 7$ kJ

Ans. (C); I-P, II-R, III-T, IV-Q

18. List I contains four combinations of two lenses (1 and 2) whose focal lengths (in cm) are indicated in the figures. In all cases, the object is placed 20 *cm* from the first lens on the left, and the distance between the two lenses is 5 *cm*. List II contains the positions of the final images.

List-I		**List-II**	
(I)		(P)	Final image is farmed at 7.5 *cm* on the right side of lens 2.
(II)		(Q)	Final image is formed at 60.0 *cm* on the right side of lens 2.
(III)		(R)	Final image is formed at 30.0 *cm* on the left side of lens 2.
(IV)		(S)	Final image is formed at 6.0 *cm* on the right side of lens 2.
		(T)	Final image is formed at 30.0 *cm* on the right side of lens 2.

Which one of the following options is correct ?

(A) I → P, II → R, III → Q, IV → T

(B) I → Q, II → P, III → T, IV → S

(C) I → P, II → T, III → R, IV → Q

(D) I → T, II → S, III → Q, IV → R

Ans. (A)

Sol. (I) $v_1 = \frac{uf}{u+f}$

$$= \frac{(-20)(10)}{(-20)+(10)} = +20$$

$u_2 = +15$

$$v_2 = \frac{(15)(15)}{(15)+(15)} = +7.5$$

(II) $v_1 = +20$

$u_2 = +15$

$$v_2 = \frac{(15)(-10)}{(15)+(-10)} = -30$$

(III) $v_1 = +20$

$u_2 = +15$

$$v_2 = \frac{(15)(-20)}{(15)+(-20)} = 60$$

(IV) $v_1 = \frac{(-20)(-20)}{(-20)+(-20)} = -10$

$u_2 = -15$

$$v_2 = \frac{(-15)(10)}{(-15)+(10)} = 30$$

Ans. (A), I-P, II-R, III-Q, IV-T

CHEMISTRY

SECTION-1 : (Maximum Marks : 24)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +3	**ONLY** if the correct numerical value is entered;
Zero Marks	: 0	In all other cases.

---

1. 2 mol of Hg(g) is combusted in a fixed volume bomb calorimeter with excess of O_2 at 298 K and 1 atm into HgO(s). During the reaction, temperature increases from 298.0 K to 312.8 K. If heat capacity of the bomb calorimeter and enthalpy of formation of Hg(g) are 20.00 kJ K^{-1} and 61.32 kJ mol^{-1} at 298 K, respectively, the calculated standard molar enthalpy of formation of HgO(s) at 298 K is X kJ mol^{-1}. The value of |X| is ______.

[Given : Gas constant R = 8.3 J K^{-1} mol^{-1}]

Ans. **(90.39)**

Sol. $Q_{rxn} = C\Delta T$

$|\Delta U| \times 2 = 20 \times 14.8$

$|\Delta U| = 148$ kJ/mol

$\Delta U = -148$ kJ/mol

$Hg(g) + \frac{1}{2}O_2(g) \longrightarrow HgO(s) : \Delta U = -148$ kJ/mol

$\Delta H = \Delta U + \Delta n_g RT$

$= -148 - \frac{3}{2}\times\frac{8.3}{1000} \times 298 = -151.7101$

$Hg(l) + \frac{1}{2}O_2(g) \longrightarrow HgO(s)$

$\Delta H = -151.7101 + 61.32 = -90.39$ kJ/mol

Ans. 90.39

2. The reduction potential (E^0, in V) of $\mathrm{MnO_4^-(aq)/Mn(s)}$ is ______.

[Given : $E^0_{\left(\mathrm{MnO_4^-(aq)/MnO_2(s)}\right)} = 1.68\,\mathrm{V}$; $E^0_{\left(\mathrm{MnO_2(s)/Mn^{2+}(aq)}\right)} = 1.21\,\mathrm{V}$; $E^0_{\left(\mathrm{Mn^{2+}(aq)/Mn(s)}\right)} = -1.03\,\mathrm{V}$]

Ans. (0.77)

Sol.

$$\begin{tikzcd} \overset{+7}{\mathrm{MnO_4^-}} \arrow[r, "(3)"] \arrow[rrr, bend right, "7"'] & \overset{+4}{\mathrm{MnO_2}} \arrow[r, "(2)"] & \mathrm{Mn^{+2}} \arrow[r, "(2)"] & \mathrm{Mn} \end{tikzcd}$$

For the required reaction $\Delta G^\circ = \Delta G^\circ_1 + \Delta G^\circ_2 + \Delta G^\circ_3$

$\Rightarrow 7 \times E = 1.68 \times 3 + 1.21 \times 2 + (-1.03) \times 2$

$$E = \frac{5.4}{7} = 0.7714$$

Ans. = 0.77

3. A solution is prepared by mixing 0.01 mol each of H_2CO_3, $NaHCO_3$, Na_2CO_3, and NaOH in 100 mL of water. pH of the resulting solution is ______.

[Given : pK_{a1} and pK_{a2} of H_2CO_3 are 6.37 and 10.32, respectively ; log 2 = 0.30]

Ans. (10.02)

Sol.

$$H_2CO_3 + NaOH \longrightarrow NaHCO_3 + H_2O$$

	H_2CO_3	NaOH	$NaHCO_3$
Milli moles	10	10	–
At end	0	0	10 + 10 = 20

Final mixture has 20 milli moles $NaHCO_3$ and 10 milli moles Na_2CO_3

$$\mathrm{pH} = \mathrm{pKa_2} + \log \frac{\mathrm{Salt}}{\mathrm{Acid}}$$

$$\mathrm{pH} = \mathrm{pKa_2} + \log \left(\frac{10}{20}\right) \qquad [\text{Buffer : } Na_2CO_3 + NaHCO_3]$$

$= 10.32 - \log 2 = 10.02$

4. The treatment of an aqueous solution of 3.74 g of $Cu(NO_3)_2$ with excess KI results in a brown solution along with the formation of a precipitate. Passing H_2S through this brown solution gives another precipitate X. The amount of X (in g) is ______.

[Given : Atomic mass of H = 1, N = 14, O = 16, S = 32, K = 39, Cu = 63, I = 127]

Ans. (0.32)

Sol. $2Cu(NO_3)_2 + 5KI \longrightarrow Cu_2I_2 + KI_3 + 4KNO_3$

0.02   0.01

$KI_3 + H_2S \longrightarrow S\downarrow + KI + 2HI$

0.01   0.01

$n_S = 0.01$ mole

weight of sulphur = 32 × 0.01 = 0.32 gm

5. Dissolving 1.24 g of white phosphorous in boiling NaOH solution in an inert atmosphere gives a gas **Q**. The amount of $CuSO_4$ (in g) required to completely consume the gas **Q** is ______.

[Given : Atomic mass of H = 1, O = 16, Na = 23, P = 31, S = 32, Cu = 63]

Ans. (2.38 / 2.39)

Sol. Mole of $P_4 = \frac{1.24}{31 \times 4} = 0.01$

$$P_4 + 3NaOH + 3H_2O \longrightarrow PH_3 + 3NaH_2PO_2$$

0.01 mole 0.01 mole

$$2PH_3 + 3CuSO_4 \rightarrow Cu_3P_2 + 3H_2SO_4$$

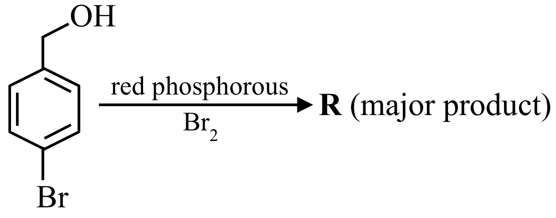
$0.01 \quad \frac{3}{2} \times 0.01$

$= \frac{0.03}{2}$ moles

$W_{CuSO_4} = \frac{0.03}{2} \times 159 = 2.385$ gm

Ans. = 2.38 or 2.39

6. Consider the following reaction.

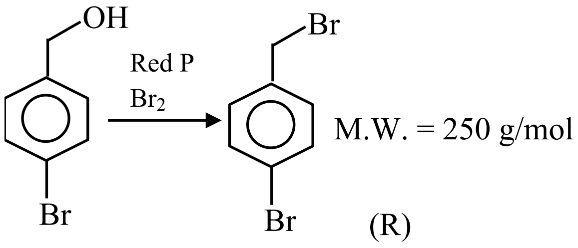


On estimation of bromine in 1.00 g of **R** using Carius method, the amount of AgBr formed (in g) is ______.

[Given : Atomic mass of H = 1, C = 12, O = 16, P = 31, Br = 80, Ag = 108]

Ans. (1.50)

Sol.



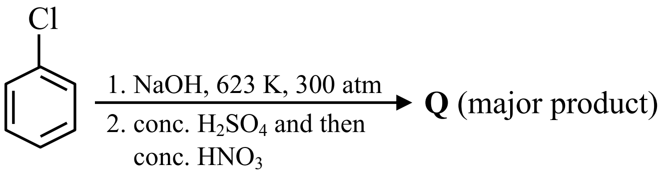
$1g\ R \rightarrow \frac{1}{250}$ moles

No. of Br Atoms $\rightarrow \frac{2}{250}$ moles

Moles of AgBr $\rightarrow \frac{2}{250}$ moles

Mass of AgBr = $\frac{2}{250} \times (108+80) = 1.504$

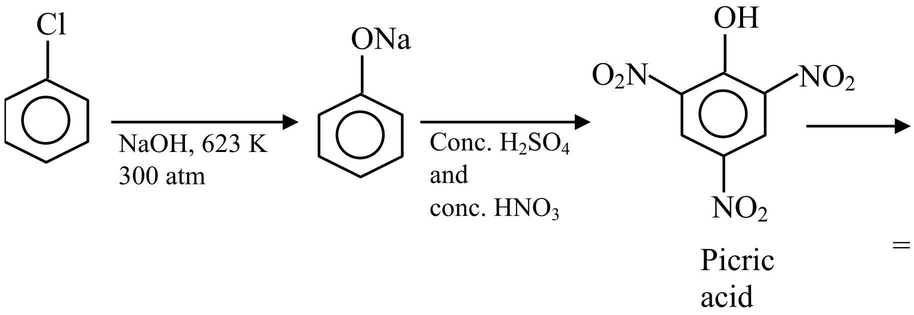
7. The weight percentage of hydrogen in **Q**, formed in the following reaction sequence, is ______.



[Given : Atomic mass of H = 1, C = 12, N = 14, O = 16, S = 32, Cl = 35]

Ans. (1.31)

Sol.

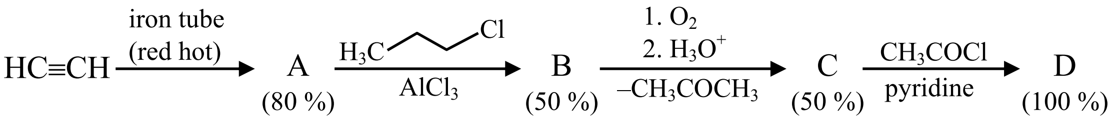


$C_6H_3N_3O_7$

Mass % of H

$$=\frac{3}{229}\times 100 = 1.31\%$$

8. If the reaction sequence given below is carried out with 15 moles of acetylene, the amount of the product **D** formed (in g) is ______.

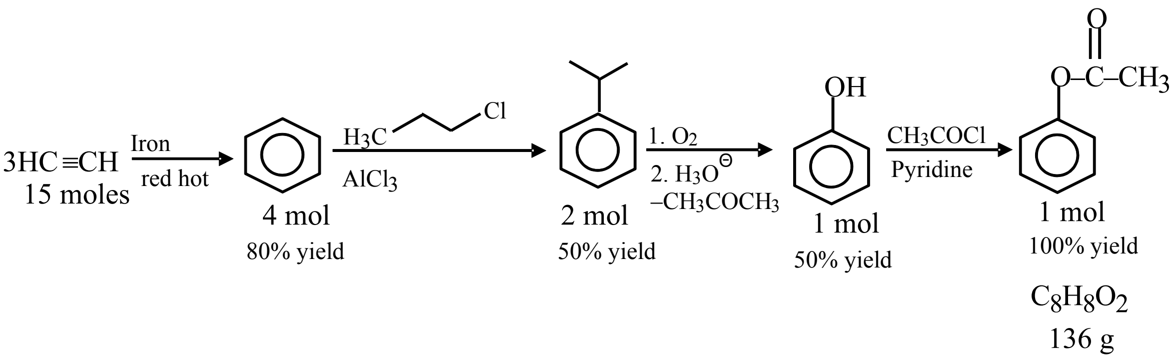


The yields of **A, B, C** and **D** are given in parentheses.

[Given : Atomic mass of H = 1, C = 12, O = 16, Cl = 35]

Ans. (136)

Sol.



SECTION-2 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	: +3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	: +2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	: +1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: −2	In all other cases.

---

9. For diatomic molecules, the correct statement(s) about the molecular orbitals formed by the overlap to two $2p_z$ orbitals is(are)

(A) σ orbital has a total of two nodal planes.

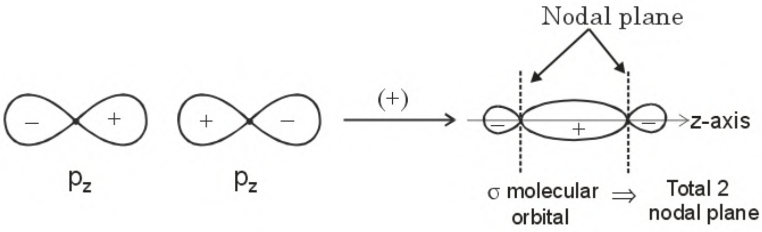
(B) σ^* orbital has one node in the *xz*-plane containing the molecular axis.

(C) π orbital has one node in the plane which is perpendicular to the molecular axis and goes through the center of the molecule.

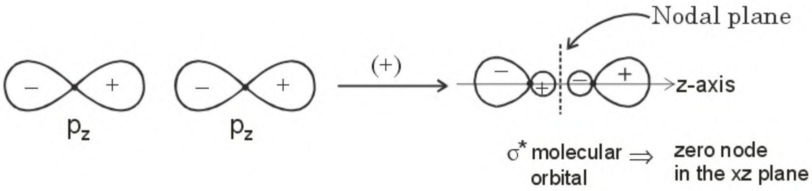
(D) π^* orbital has one node in the *xy*-plane containing the molecular axis.

Ans. (A,D)

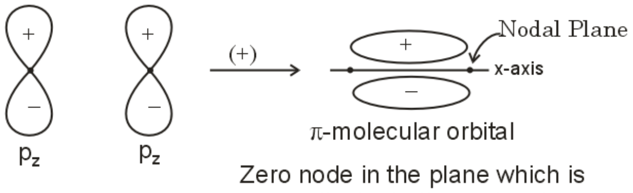
Sol. (A)



(B)

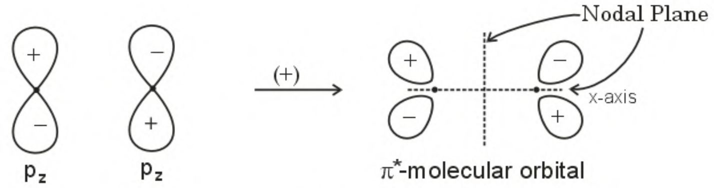


(C)



Zero node in the plane which is perpendicular to the molecular axis and goes through the center of the molecule

(D)



One node in xy plane containing
the molecular axis

10. The correct option(s) related to adsorption processes is(are)

(A) Chemisorption results in a unimolecular layer.

(B) The enthalpy change during physisorption is in the range of 100 to 140 kJ mol^{-1}.

(C) Chemisorption is an endothermic process.

(D) Lowering the temperature favors physisorption processes.

Ans. (A,D)

Sol. (A) Chemisorption is unimolecular layered.

(B) Enthalpy of physisorption is much less in magnitude.

(C) Chemisorption of gases on solids is exothermic.

(D) As physisorption is exothermic so lowering temperature favours it.

11. The electrochemical extraction of aluminum from bauxite ore involves.

(A) the reaction of Al_2O_3 with coke (C) at a temperature > 2500°C.

(B) the neutralization of aluminate solution by passing CO_2 gas to precipitate hydrated alumina ($Al_2O_3.3H_2O$)

(C) the dissolution of Al_2O_3 in hot aqueous NaOH.

(D) the electrolysis of Al_2O_3 mixed with Na_3AlF_6 to give Al and CO_2.

Ans. (B,C,D)

Sol. (A) Electrochemical extraction of Aluminum from bauxite done below 2500°C

(B) $2Na[Al(OH)_4]_{aq.} + 2CO_{2(g)} \rightarrow Al_2O_3.3H_2O_{(s)} \downarrow + 2NaHCO_{3(aq.)}$

The sodium aluminate present in solution is neutralised by passing CO_2 gas and hydrated Al_2O_3 is precipitated.

(C) $Al_2O_{3(s)} + 2NaOH_{(aq.)} + 3H_2O_{(l)} \rightarrow 2Na[Al(OH)_4]_{aq.}$

Concentration of bauxite is carried out by heating the powdered ore with hot concentrated solution of NaOH

(D) In metallurgy of aluminum, Al_2O_3 is mixed with Na_3AlF_6

12. The treatment of galena with HNO_3 produces a gas that is

(A) paramagnetic

(B) bent in geometry

(C) an acidic oxide

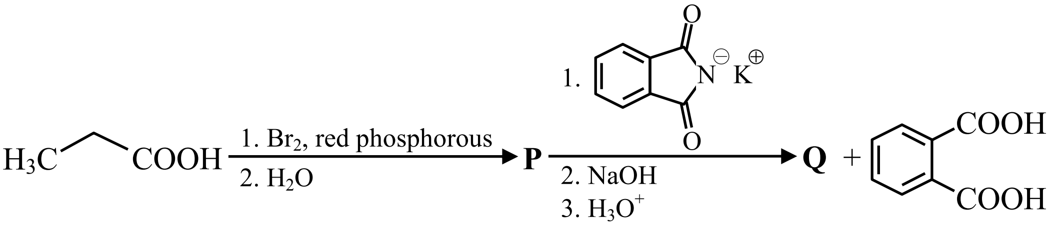
(D) colorless

Ans. (A,D)

Sol. $3PbS + 8HNO_3 \rightarrow 3Pb(NO_3)_2 + 2NO + 4H_2O + S$

$NO \Rightarrow$ Neutral oxide, Paramagnetic, Linear geometry, Colourless gas

13. Considering the reaction sequence given below, the correct statement(s) is(are)



(A) **P** can be reduced to a primary alcohol using $NaBH_4$.

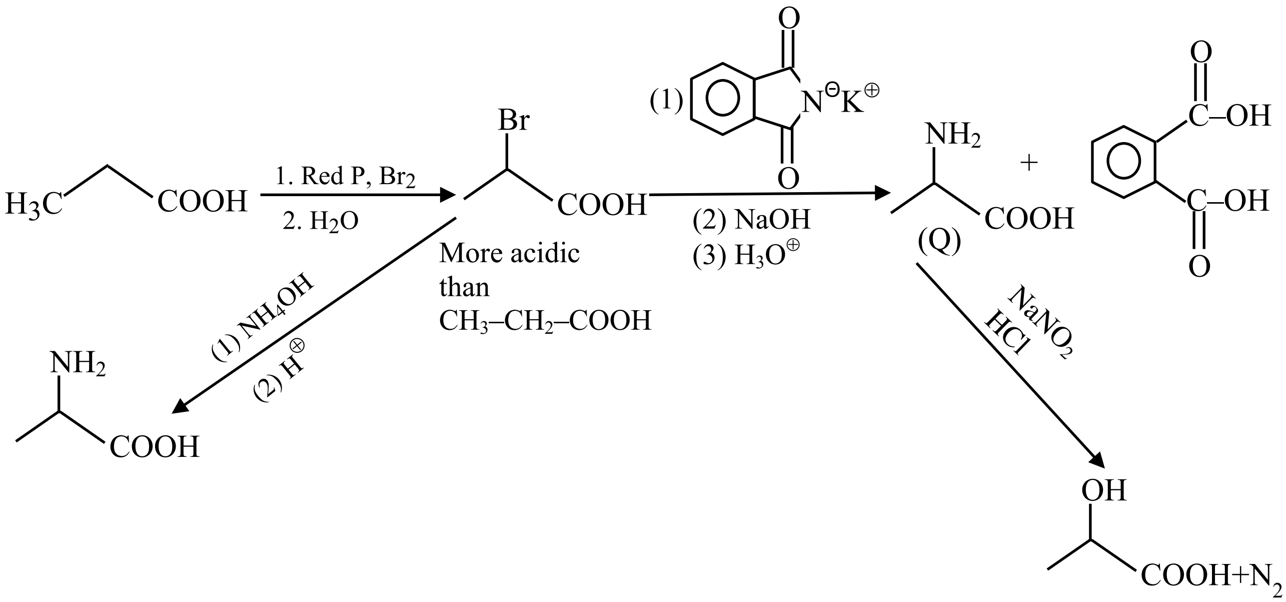
(B) Treating **P** with conc. NH_4OH solution followed by acidification gives **Q**.

(C) Treating **Q** with a solution of $NaNO_2$ in aq. HCl liberates N_2.

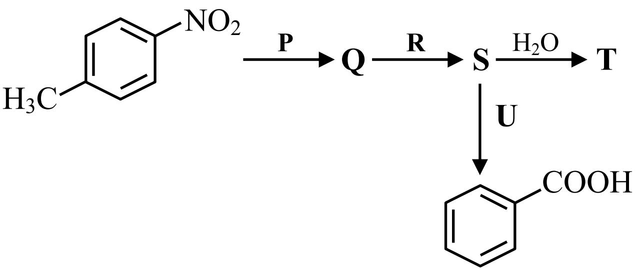
(D) **P** is more acidic than CH_3CH_2COOH.

Ans. (B,C,D)

Sol.



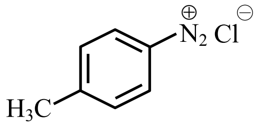
14. Consider the following reaction sequence,



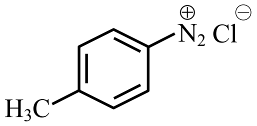
the correct option(s) is(are)

(A) **P** = H_2/Pd, ethanol **R** = $NaNO_2$/HCl **U** = 1. H_3PO_2
2. $KMnO_4$ - KOH, heat

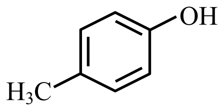
(B) **P** = Sn/HCl **R** = HNO_2 **S** =



(C) **S** =

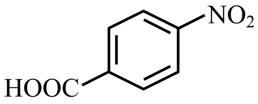


T =

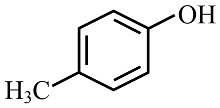


U = 1. CH_3CH_2OH
2. $KMnO_4$ - KOH, heat

(D) **Q** =

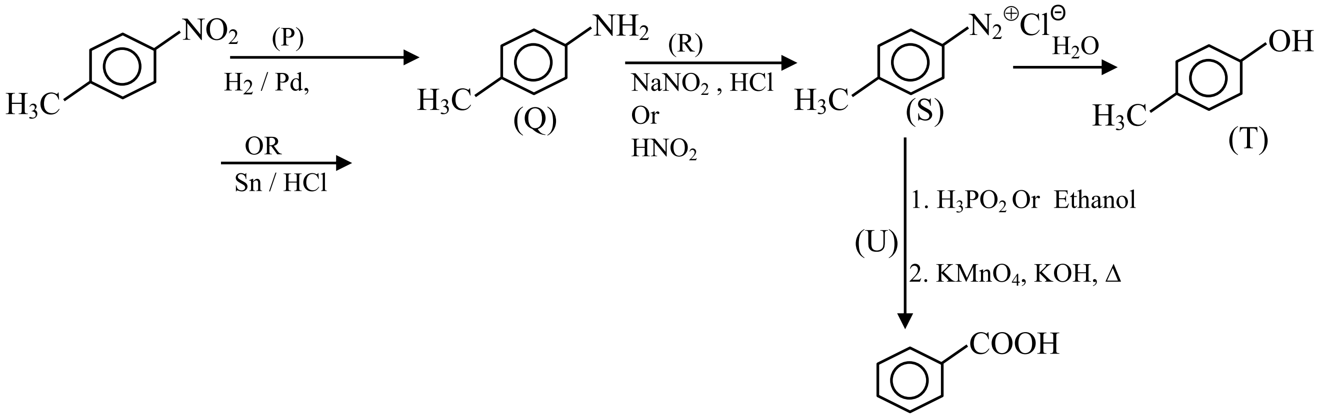


R = H_2/Pd, ethanol **T** =



Ans. (A,B,C)

Sol.



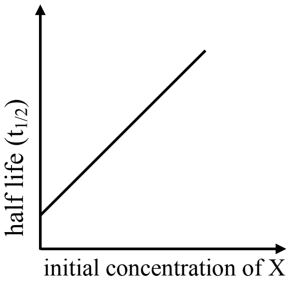
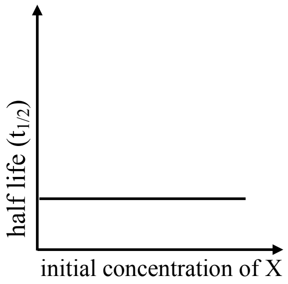
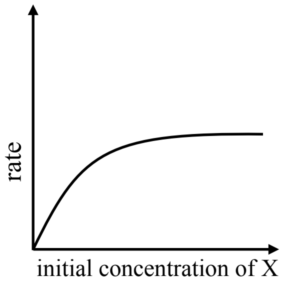
SECTION-3 : (Maximum Marks : 12)

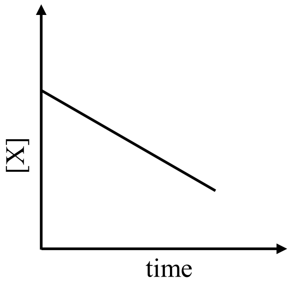
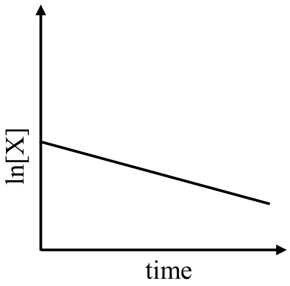
- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists : **List-I** and **List-II**.
- **List-I** has **Four** entries (I), (II), (III) and (IV) and **List-II** has **Five** entries (P), (Q), (R), (S) and (T).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 **ONLY** if the option corresponding to the correct combination is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : –1 In all other cases.

---

15. Match the rate expressions in LIST-I for the decomposition of X with the corresponding profiles provided in LIST-II. X_s and k constants having appropriate units.

LIST-I	**LIST-II**
(I) $\text{rate} = \frac{k[X]}{X_s + [X]}$ under all possible initial concentration of X	(P) 
(II) $\text{rate} = \frac{k[X]}{X_s + [X]}$ where initial concentration of X are much less than X_s	(Q) 
(III) $\text{rate} = \frac{k[X]}{X_s + [X]}$ where initial concentration of X are much higher than X_s	(R) 

(IV) rate $= \frac{k[X]^2}{X_s + [X]}$ where initial concentration of X is much higher than X_s	(S) 
	(T) 

(A) I → P; II → Q; III → S; IV → T

(B) I → R; II → S; III → S; IV → T

(C) I → P; II → Q; III → Q; IV → R

(D) I → R; II → S; III → Q; IV → R

Ans. **(A)**

Sol. (I) rate $= \frac{k[x]}{x_s + [x]} = \frac{k}{\frac{x_s}{[x]} + 1}$

If $[x] \to \infty \Rightarrow$ rate $\to k \Rightarrow$ order = 0

$\Rightarrow$ (I) – (R), (P)

(II) $[x] << x_s \Rightarrow$ rate $= \frac{k[x]}{x_s} \Rightarrow$ order = 1

$\Rightarrow$ (II) – (Q), (T)

(III) $[x] >> x_s \Rightarrow$ rate $= k \Rightarrow$ order = 0

$\Rightarrow$ (III) – (P), (S)

(IV) rate $= \frac{k[x]^2}{x_s + [x]}$

$[x] >> x_s \Rightarrow$ rate $= k[x]$

$\Rightarrow$ (IV) – (Q), (T)

Ans. (A)

16. LIST-I contains compounds and LIST-II contains reaction

LIST-I	LIST-II
(I) H_2O_2	(P) $Mg(HCO_3)_2 + Ca(OH)_2 \rightarrow$
(II) $Mg(OH)_2$	(Q) $BaO_2 + H_2SO_4 \rightarrow$
(III) $BaCl_2$	(R) $Ca(OH)_2 + MgCl_2$
(IV) $CaCO_3$	(S) $BaO_2 + HCl \rightarrow$
	(T) $Ca(HCO_3)_2 + Ca(OH)_2 \rightarrow$

Match each compound in LIST – I with its formation reaction(s) in LIST-II, and choose the correct option

(A) I → Q; II → P; III → S; IV → R

(B) I → T; II → P; III → Q; IV → R

(C) I → T; II → R; III → Q; IV → P

(D) I → Q; II → R; III → S; IV → P

Ans. (D)

Sol. (P) $Mg(HCO_3)_2 + 2Ca(OH)_2 \rightarrow Mg(OH)_2 + 2CaCO_3 + 2H_2O$

(Q) $BaO_2 + H_2SO_4 \rightarrow H_2O_2 + BaSO_4$

(R) $Ca(OH)_2 + MgCl_2 \rightarrow Mg(OH)_2 + CaCl_2$

(S) $BaO_2 + 2HCl \rightarrow BaCl_2 + H_2O_2$

(T) $Ca(HCO_3)_2 + Ca(OH)_2 \rightarrow 2CaCO_3 + 2H_2O$

17. LIST-I contains metal species and LIST-II contains their properties.

LIST-I	LIST-II
(I) $[Cr(CN)_6]^{4-}$	(P) t_{2g} orbitals contain 4 electrons
(II) $[RuCl_6]^{2-}$	(Q) μ(spin-only) = 4.9 BM
(III) $[Cr(H_2O)_6]^{2+}$	(R) low spin complex ion
(IV) $[Fe(H_2O)_6]^{2+}$	(S) metal ion in 4+ oxidation state
	(T) d^4 species

[Given : Atomic number of Cr = 24, Ru = 44, Fe = 26]

Metal each metal species in LIST-I with their properties in LIST-II, and choose the correct option

(A) I → R, T; II → P, S; III → Q, T; IV → P, Q

(B) I → R, S; II → P, T; III → P, Q; IV → Q, T

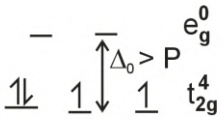
(C) I → P, R; II → R, S; III → R, T; IV → P, T

(D) I → Q, T; II → S, T; III → P, T; IV → Q, R

Ans. (A)

Sol. (1) $[Cr(CN)_6]^{4-}$

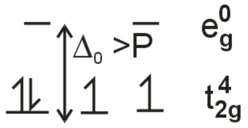
$Cr^{+2} = [Ar]_{18}\ 3d^4 4s^0$; low spin complex



P,R,T

(2) $[RuCl_6]^{2-}$

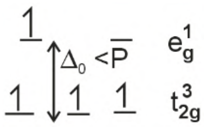
$Ru^{+4} = [Kr]_{36} 4d^4 5s^0$; low spin complex



P,R,S,T

(3) $[Cr(H_2O)_6]^{2+}$

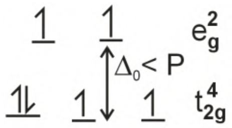
$Cr^{+2} = [Ar]_{18} 3d^4 4s^0$; high spin complex



Q,T

(4) $[Fe(H_2O)_6]^{2+}$

$Fe^{+2} = [Ar]_{18} 3d^6$; High spin complex



P,Q

18. Match the compounds in LIST-I with the observation in LIST-II, and choose the correct option.

LIST-I	LIST-II
(I) Aniline	(P) Sodium fusion extract of the compound on boiling with $FeSO_4$, followed by acidification with conc. H_2SO_4, gives Prussian blue color.
(II) o-Cresol	(Q) Sodium fusion extract of the compound on treatment with sodium nitroprusside gives blood red color.
(III) Cysteine	(R) Addition of the compound to a saturated solution of $NaHCO_3$ results in effervescence.

(IV) Coprolactam	(S) The compound reacts with bromine water to give a white precipitate.
	(T) Treating the compound with neutral $FeCl_3$ solution produces violet color.

(A) I → P, Q; II → S; III → Q, R; IV → P

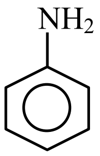
(B) I → P ; II → R, S; III → R; IV → Q, S

(C) I → Q, S; II → P, T; III → P; IV → S

(D) I → P, S; II → T; III → Q, R; IV → P

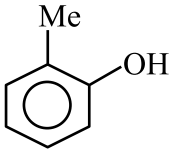
Ans. (D)

Sol.



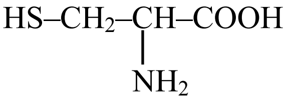
Aniline

: Blue colour in Lassign test due to presence of N



o-Cresol

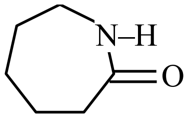
:Violet colour with $FeCl_3$ due to presence of phenolic OH



$HS{-}CH_2{-}\underset{\substack{|\\ NH_2}}{CH}{-}COOH$

Cystein

: It gives blod red colour with NaSCN



Caprolactam

: Blue colour in Lassign test due to presence of N

MATHEMATICS

SECTION-1 : (Maximum Marks : 24)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 **ONLY** if the correct numerical value is entered;
  Zero Marks : 0 In all other cases.

---

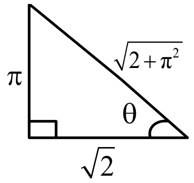
1. Considering only the principal values of the inverse trigonometric functions, the value of

$$\frac{3}{2}\cos^{-1}\sqrt{\frac{2}{2+\pi^2}}+\frac{1}{4}\sin^{-1}\frac{2\sqrt{2}\pi}{2+\pi^2}+\tan^{-1}\frac{\sqrt{2}}{\pi}$$

is ________.

Ans. **(2.35 or 2.36)**

Sol. $\cos^{-1}\sqrt{\frac{2}{2+\pi^2}}=\tan^{-1}\frac{\pi}{\sqrt{2}}$



$$\sin^{-1}\left(\frac{2\sqrt{2}\pi}{2+\pi^2}\right)=\sin^{-1}\left(\frac{2\times\frac{\pi}{\sqrt{2}}}{1+\left(\frac{\pi}{\sqrt{2}}\right)^2}\right)$$

$$=\pi-2\tan^{-1}\left(\frac{\pi}{\sqrt{2}}\right)$$

$$\left(\text{As, }\sin^{-1}\left(\frac{2x}{1+x^2}\right)=\pi-2\tan^{-1}x, x\geq 1\right)$$

and $\tan^{-1}\frac{\sqrt{2}}{\pi}=\cot^{-1}\left(\frac{\pi}{\sqrt{2}}\right)$

$$\therefore \text{ Expression } = \frac{3}{2}\left(\tan^{-1}\frac{\pi}{\sqrt{2}}\right)+\frac{1}{4}\left(\pi-2\tan^{-1}\frac{\pi}{\sqrt{2}}\right)+\cot^{-1}\left(\frac{\pi}{\sqrt{2}}\right)$$

$$=\left(\frac{3}{2}-\frac{2}{4}\right)\tan^{-1}\frac{\pi}{\sqrt{2}}+\frac{\pi}{4}+\cot^{-1}\frac{\pi}{\sqrt{2}}$$

$$=\left(\tan^{-1}\frac{\pi}{\sqrt{2}}+\cot^{-1}\frac{\pi}{\sqrt{2}}\right)+\frac{\pi}{4}$$

$$=\frac{\pi}{2}+\frac{\pi}{4}=\frac{3\pi}{4}$$

= 2.35 or 2.36

2. Let α be a positive real number. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g : (\alpha, \infty) \rightarrow \mathbb{R}$ be the functions defined by

$$f(x)=\sin\left(\frac{\pi x}{12}\right) \text{ and } g(x)=\frac{2\log_e\left(\sqrt{x}-\sqrt{\alpha}\right)}{\log_e\left(e^{\sqrt{x}}-e^{\sqrt{\alpha}}\right)}.$$

Then the value of $\lim\limits_{x\to\alpha^+} f(g(x))$ is _______.

Ans. (0.50)

Sol. $$\lim_{x\to\alpha^+}\frac{2\ell n\left(\sqrt{x}-\sqrt{\alpha}\right)}{\ell n\left(e^{\sqrt{x}}-e^{\sqrt{\alpha}}\right)} \quad \left(\frac{0}{0}\text{form}\right)$$

∴ Using Lopital rule,

$$=2\lim_{x\to\alpha^+}\frac{\left(\frac{1}{\sqrt{x}-\sqrt{\alpha}}\right).\frac{1}{2\sqrt{x}}}{\left(\frac{1}{e^{\sqrt{x}}-e^{\sqrt{\alpha}}}\right).e^{\sqrt{x}}.\frac{1}{2\sqrt{x}}}$$

$$=\frac{2}{e^{\sqrt{\alpha}}}\lim_{x\to\alpha^+}\frac{\left(e^{\sqrt{x}}-e^{\sqrt{\alpha}}\right)}{\left(\sqrt{x}-\sqrt{\alpha}\right)} \quad \left(\frac{0}{0}\right)$$

$$=\frac{2}{e^{\sqrt{\alpha}}}\lim_{x\to\alpha^+}\frac{\left(e^{\sqrt{x}}.\frac{1}{2\sqrt{x}}-0\right)}{\left(\frac{1}{2\sqrt{x}}-0\right)}=2$$

so, $\lim\limits_{x\to\alpha^+} f\left(g\left(x\right)\right)=\lim\limits_{x\to\alpha^+} f\left(2\right)$

$$=f\left(2\right)=\sin\frac{\pi}{6}=\frac{1}{2}$$

= 0.50

3. In a study about a pandemic, data of 900 persons was collected. It was found that

190 persons had symptom of fever,

220 persons had symptom of cough,

220 persons had symptom of breathing problem,

330 persons had symptom of fever or cough or both,

350 persons had symptom of cough or breathing problem or both,

340 persons had symptom of fever or breathing problem or both,

30 persons had all three symptoms (fever, cough and breathing problem).

If a person is chosen randomly from these 900 persons, then the probability that the person has at most one symptom is ______________.

Ans. **(0.80)**

Sol. $n(U) = 900$

Let $A \equiv$ Fever, $B \equiv$ Cough

$C \equiv$ Breathing problem

$\therefore\ n(A) = 190,\ n(B) = 220,\ n(C) = 220$

$n(A \cup B) = 330,\ n(B \cup C) = 350,$

$n(A \cup C) = 340,\ n(A \cap B \cap C) = 30$

Now $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow 330 = 190 + 220 - n(A \cap B)$

$\Rightarrow n(A \cap B) = 80$

Similarly,

$350 = 220 + 220 - n(B \cap C)$

$\Rightarrow n(B \cap C) = 90$

and $340 = 190 + 220 - n(A \cap C)$

$\Rightarrow n(A \cap C) = 70$

$\therefore\ n(A \cup B \cup C) = (190 + 220 + 220) - (80 + 90 + 70) + 30$

$= 660 - 240 = 420$

$\Rightarrow$ Number of person without any symptom

$= n(\cup) - n(A \cup B \cup C)$

$= 900 - 420 = 480$

Now, number of person suffering from exactly one symptom

$= (n(A) + n(B) + n(C)) - 2(n(A \cap B) + n(B \cap C) + n(C \cap A)) + 3n(A \cap B \cap C)$

$= (190 + 220 + 220) - 2(80 + 90 + 70) + 3(30)$

$= 630 - 480 + 90 = 240$

$\therefore$ Number of person suffering from atmost one symotom

$= 480 + 240 = 720$

$\Rightarrow$ Probability $= \frac{720}{900} = \frac{8}{10} = \frac{4}{5} = 0.80$

4. Let z be a complex number with non-zero imaginary part. If

$$\frac{2+3z+4z^2}{2-3z+4z^2}$$

is a real number, then the value of $|z|^2$ is ______________.

Ans. **(0.50)**

Sol. Given that

$z \neq \overline{z}$

Let $\alpha = \frac{2+3z+4z^2}{2-3z+4z^2} = \frac{\left(2-3z+4z^2\right)+6z}{2-3z+4z^2}$

$\therefore \ \alpha = 1 + \frac{6z}{2-3z+4z^2}$

If α is a real number, then

$\alpha = \overline{\alpha}$

$\Rightarrow \frac{z}{2-3z+4z^2} = \frac{\overline{z}}{2-3\overline{z}+4\overline{z}^2}$

$\therefore \ 2\left(z-\overline{z}\right) = 4z\overline{z}\left(z-\overline{z}\right)$

$\Rightarrow \left(z-\overline{z}\right)\left(2-4z\overline{z}\right) = 0$

As $z \neq \overline{z}$ (Given)

$\Rightarrow z\overline{z} = \frac{2}{4} = \frac{1}{2}$

$\Rightarrow |z|^2 = 0.50$

5. Let $\bar{z}$ denote the complex conjugate of a complex number z and let $i = \sqrt{-1}$. In the set of complex numbers, the number of distinct roots of the equation

$$\bar{z} - z^2 = i(\bar{z} + z^2)$$

is ______________.

Ans. **(4.00)**

Sol. Given ,

$$\bar{z}-z^2=i\left(\bar{z}+z^2\right)$$

$$\Rightarrow(1-i)\bar{z}=(1+i)z^2$$

$$\Rightarrow\frac{(1-i)}{(1+i)}\bar{z}=z^2$$

$$\Rightarrow\left(-\frac{2i}{2}\right)\bar{z}=z^2$$

$$\therefore\ z^2=-i\,\bar{z}$$

Let z = x + *i*y,

$\therefore\ (x^2-y^2)+i\,(2xy)=-i\,(x-iy)$

so, $x^2-y^2+y=0$ ...(1)

and $(2y+1)x=0$ ...(2)

$\Rightarrow x=0$ or $y=-\frac{1}{2}$

Case I : When x = 0

$\therefore\ (1)\Rightarrow y(1-y)=0\Rightarrow y=0,1$

$\therefore\ (0,0),(0,1)$

Case II : When $y=-\frac{1}{2}$

$$\therefore\ (1)\Rightarrow x^2-\frac{1}{4}-\frac{1}{2}=0\Rightarrow x^2=\frac{3}{4}\Rightarrow x=\pm\frac{\sqrt{3}}{2}$$

$$\therefore\ \left(\frac{\sqrt{3}}{2},-\frac{1}{2}\right),\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$$

$\Rightarrow$ Number of distinct 'z' is equal to 4.

6. Let $l_1, l_2,\ldots, l_{100}$ be consecutive terms of an arithmetic progression with common difference d_1, and let $w_1, w_2,\ldots, w_{100}$ be consecutive terms of another arithmetic progression with common difference d_2, where $d_1d_2 = 10$. For each $i = 1, 2,\ldots,100$, let R_i be a rectangle with length l_i, width w_i and area A_i. If $A_{51} - A_{50} = 1000$, then the value of $A_{100} - A_{90}$ is _____________.

Ans. (18900.00)

Sol. Given

$$A_{51}-A_{50}=1000 \Rightarrow \ell_{51}w_{51}-\ell_{50}w_{50}=1000$$

$$\Rightarrow(\ell_1+50d_1)(w_1+50d_2)-(\ell_1+49d_1)(w_1+49d_2)=1000$$

$$\Rightarrow(\ell_1d_2+w_1d_1)=10 \qquad \ldots.(1)$$

(As $d_1d_2 = 10$)

$$\therefore\ A_{100}-A_{90}=\ell_{100}w_{100}-\ell_{90}w_{90}$$

$$=(\ell_1+99d_1)(w_1+99d_2)-(\ell_1+89d_1)(w_1+89d_2)$$

$$=10(\ell_1d_2+w_1d_1)+(99^2-89^2)d_1d_2$$

$$=10(10)+\underbrace{(99-89)}_{=10}(99+89)(10)$$

(As, $d_1d_2 = 10$)

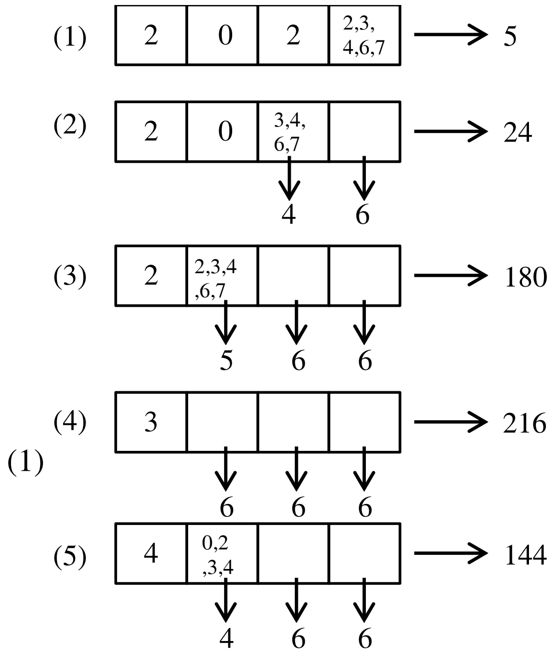
= 100 (1 + 188) = 100 (189)

= 18900

7. The number of 4-digit integers in the closed interval [2022, 4482] formed by using the digits 0, 2, 3, 4, 6, 7 is ____________.

Ans. (569.00)

Sol. Ans. 569



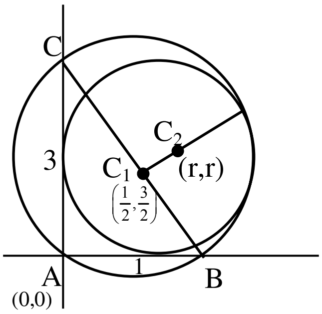
Number of 4 digit integers in [2022,4482]

= 5 + 24 + 180 + 216 + 144 = 569

8. Let ABC be the triangle with $AB = 1$, $AC = 3$ and $\angle BAC = \frac{\pi}{2}$. If a circle of radius $r > 0$ touches the sides AB, AC and also touches internally the circumcircle of the triangle ABC, then the value of r is ______________.

Ans. **(0.83 or 0.84)**

Sol. $4-\sqrt{10} = 0.83$ or 0.84



$C_1\left(\frac{1}{2},\frac{3}{2}\right)$ and $r_1 = \frac{\sqrt{10}}{2}$

$C_2 = (r,r)$

$\therefore$ circle C_2 touches C_1 internally

$$\Rightarrow C_1C_2 = \left|r-\frac{\sqrt{10}}{2}\right|$$

$$\Rightarrow \left(r-\frac{1}{2}\right)^2+\left(r-\frac{3}{2}\right)^2=\left(r-\frac{\sqrt{10}}{2}\right)^2$$

$r^2-4r+\sqrt{10}r=0$

$r = 0$ (reject) or $r = 4-\sqrt{10}$

SECTION-2 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	: +3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	: +2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	: +1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: –2	In all other cases.

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9. Consider the equation

$$\int_1^e \frac{(\log_e x)^{1/2}}{x\left(a-(\log_e x)^{3/2}\right)^2}dx=1, \qquad a\in(-\infty,0)\cup(1,\infty).$$

Which of the following statements is/are TRUE ?

(A) **No** *a* satisfies the above equation

(B) An integer *a* satisfies the above equation

(C) An irrational number *a* satisfies the above equation

(D) More than one *a* satisfy the above equation

Ans. (C, D)

Sol. $\displaystyle\int_1^e \frac{(\log_e x)^{1/2}}{x\left(a-(\log_e x)^{3/2}\right)^2}=1$

Let $a-(\log_e x)^{3/2}=t$

$$\frac{(\log_e x)^{1/2}}{x}dx=-\frac{2}{3}dt$$

$$=\frac{2}{3}\int_a^{a-1}\frac{-dt}{t^2}=\frac{2}{3}\left(\frac{1}{t}\right)_a^{a-1}=1$$

$$\frac{2}{3a(a-1)}=1$$

$3a^2-3a-2=0$

$$a=\frac{3\pm\sqrt{33}}{6}$$

10. Let $a_1, a_2, a_3, \ldots$ be an arithmetic progression with $a_1 = 7$ and common difference 8. Let $T_1, T_2, T_3, \ldots$ be such that $T_1 = 3$ and $T_{n+1} - T_n = a_n$ for $n \geq 1$. Then, which of the following is/are TRUE ?

(A) $T_{20} = 1604$

(B) $\sum_{k=1}^{20} T_k = 10510$

(C) $T_{30} = 3454$

(D) $\sum_{k=1}^{30} T_k = 35610$

Ans. **(B,C)**

Sol. $a_1 = 7, d = 8$

$T_{n+1} - T_n = a_n \forall n \geq 1$

$S_n = T_1 + T_2 + T_3 + \ldots + T_{n-1} + T_n$

$S_n = \quad T_1 + T_2 + T_3 + \ldots. + T_{n-1} + T_n$

on subtraction

$T_n = T_1 + a_1 + a_2 + \ldots. + a_{n-1}$

$T_n = 3 + (n - 1)(4n - 1)$

$T_n = 4n^2 - 5n + 4$

$$\sum_{k=1}^{n} T_k = 4\sum n^2 - 5\sum n + 4n$$

$T_{20} = 1504$

$T_{30} = 3454$

$$\sum_{k=1}^{30} T_k = 35615$$

$$\sum_{k=1}^{20} T_k = 10510$$

11. Let P_1 and P_2 be two planes given by

$$P_1: 10x + 15y + 12z - 60 = 0,$$

$$P_2: -2x + 5y + 4z - 20 = 0.$$

Which of the following straight lines can be an edge of some tetrahedron whose two faces lie on P_1 and P_2 ?

(A) $\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{5}$

(B) $\frac{x-6}{-5} = \frac{y}{2} = \frac{z}{3}$

(C) $\frac{x}{-2} = \frac{y-4}{5} = \frac{z}{4}$

(D) $\frac{x}{1} = \frac{y-4}{-2} = \frac{z}{3}$

Ans. **(A,B,D)**

Sol. line of intersection is $\frac{x}{0} = \frac{y-4}{-4} = \frac{z}{5}$

(1) Any skew line with the line of intersection of given planes can be edge of tetrahedron.

(2) any intersecting line with line of intersection of given planes must lie either in plane P_1 or P_2 can be edge of tetrahedron.

12. Let S be the reflection of a point Q with respect to the plane given by

$$\vec{r} = -(t+p)\hat{i} + t\hat{j} + (1+p)\hat{k}$$

where t, p are real parameters and $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors along the three positive coordinate axes. If the position vectors of Q and S are $10\hat{i} + 15\hat{j} + 20\hat{k}$ and $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ respectively, then which of the following is/are TRUE ?

(A) $3(\alpha + \beta) = -101$

(B) $3(\beta + \gamma) = -71$

(C) $3(\gamma + \alpha) = -86$

(D) $3(\alpha + \beta + \gamma) = -121$

Ans. **(A,B,C)**

Sol. $\vec{r} = \hat{k} + t\left(-\hat{i} + \hat{j}\right) + p\left(-\hat{i} + \hat{k}\right)$

$\vec{n} = \hat{i} + \hat{j} + \hat{k}$

$\Rightarrow x + y + z = 1$

Q(10,15,20) and S(α,β,γ)

$$\frac{\alpha - 10}{1} = \frac{\beta - 15}{1} = \frac{\gamma - 20}{1} = -2\left(\frac{10+15+20-1}{1+1+1}\right)$$

$$= -\frac{88}{3}$$

$$\Rightarrow (\alpha, \beta, \gamma) \equiv \left(-\frac{58}{3}, -\frac{43}{3}, -\frac{28}{3}\right)$$

$\Rightarrow$ A,B,C are correct options

13. Consider the parabola $y^2 = 4x$. Let S be the focus of the parabola. A pair of tangents drawn to the parabola from the point $P = (-2, 1)$ meet the parabola at P_1 and P_2. Let Q_1 and Q_2 be points on the lines SP_1 and SP_2 respectively such that PQ_1 is perpendicular to SP_1 and PQ_2 is perpendicular to SP_2. Then, which of the following is/are TRUE ?

(A) $SQ_1 = 2$

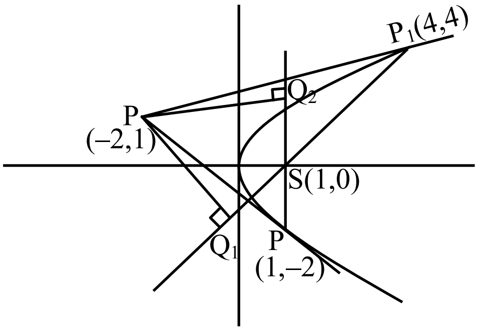
(B) $Q_1Q_2 = \frac{3\sqrt{10}}{5}$

(C) $PQ_1 = 3$

(D) $SQ_2 = 1$

Ans. **(B,C,D)**

Sol. Let equation of tangent with slope 'm' be



$$T : y = mx + \frac{1}{m}$$

T : passes through (–2, 1) so

$$1 = -2m + \frac{1}{m}$$

$\Rightarrow m = -1 \text{ or } m = \frac{1}{2}$

Points are given by $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

So, one point will be (1, –2) & (4, 4)

Let $P_1(4, 4)$ & $P_2(1, -2)$

$P_1S : 4x - 3y - 4 = 0$

$P_2S : x - 1 = 0$

$$PQ_1 = \left|\frac{4(-2) - 3(1) - 4}{5}\right| = 3$$

$SP = \sqrt{10}$; $PQ_2 = 3$; $SQ_1 = 1 = SQ_2$

$$\frac{1}{2}\left(\frac{Q_1Q_2}{2}\right) \times \sqrt{10} = \frac{1}{2} \times 3 \times 1 \quad \text{(comparing Areas)}$$

$$\Rightarrow Q_1Q_2 = \frac{2 \times 3}{\sqrt{10}} = \frac{3\sqrt{10}}{5}$$

14. Let $|M|$ denote the determinant of a square matrix M. Let $g : \left[0, \frac{\pi}{2}\right] \to \mathbb{R}$ be the function defined by

$$g(\theta) = \sqrt{f(\theta) - 1} + \sqrt{f\left(\frac{\pi}{2} - \theta\right) - 1}$$

where

$$f(\theta) = \frac{1}{2}\begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix} + \begin{vmatrix} \sin\pi & \cos\left(\theta + \frac{\pi}{4}\right) & \tan\left(\theta - \frac{\pi}{4}\right) \\ \sin\left(\theta - \frac{\pi}{4}\right) & -\cos\frac{\pi}{2} & \log_e\left(\frac{4}{\pi}\right) \\ \cot\left(\theta + \frac{\pi}{4}\right) & \log_e\left(\frac{\pi}{4}\right) & \tan\pi \end{vmatrix}.$$

Let $p(x)$ be a quadratic polynomial whose roots are the maximum and minimum values of the function $g(\theta)$, and $p(2) = 2 - \sqrt{2}$. Then, which of the following is/are TRUE ?

(A) $p\left(\frac{3+\sqrt{2}}{4}\right) < 0$

(B) $p\left(\frac{1+3\sqrt{2}}{4}\right) > 0$

(C) $p\left(\frac{5\sqrt{2}-1}{4}\right) > 0$

(D) $p\left(\frac{5-\sqrt{2}}{4}\right) < 0$

Ans. (A,C)

Sol. $$f(\theta)=\frac{1}{2}\begin{vmatrix}1 & \sin\theta & 1\\ -\sin\theta & 1 & \sin\theta\\ -1 & -\sin\theta & 1\end{vmatrix}+\begin{vmatrix}\sin\pi & \cos\left(\theta+\frac{\pi}{4}\right) & \tan\left(\theta-\frac{\pi}{4}\right)\\ \sin\left(\theta-\frac{\pi}{4}\right) & -\cos\frac{\pi}{2} & \log_e\left(\frac{4}{\pi}\right)\\ \cot\left(\theta+\frac{\pi}{4}\right) & \log_e\frac{\pi}{4} & \tan\pi\end{vmatrix}$$

$$f(\theta)=\frac{1}{2}\begin{vmatrix}2 & \sin\theta & 1\\ 0 & 1 & \sin\theta\\ 0 & -\sin\theta & 1\end{vmatrix}+\begin{vmatrix}0 & -\sin\left(\theta-\frac{\pi}{4}\right) & \tan\left(\theta-\frac{\pi}{4}\right)\\ \sin\left(\theta-\frac{\pi}{4}\right) & 0 & \log_e\left(\frac{4}{\pi}\right)\\ -\tan\left(\theta-\frac{\pi}{4}\right) & -\log_e\left(\frac{4}{\pi}\right) & 0\end{vmatrix}$$

$f(\theta) = (1 + \sin^2\theta) + 0$ (skew symmetric)

$$g(\theta)=\sqrt{f(\theta)-1}+\sqrt{f\left(\frac{\pi}{2}-\theta\right)-1}$$

$= |\sin\theta| + |\cos\theta| \qquad$ for $\theta\in\left[0,\frac{\pi}{2}\right]$

$g(\theta)\in\left[1,\sqrt{2}\right]$

Again let $P(x)=k\left(x-\sqrt{2}\right)(x-1)$

$2-\sqrt{2}=k\left(2-\sqrt{2}\right)(2-1)$

$\Rightarrow k=1 \qquad (P(2)=2-\sqrt{2}$ given)

$\therefore\ P(x)=\left(x-\sqrt{2}\right)(x-1)$

for option (A) $P\left(\frac{3+\sqrt{2}}{4}\right)<0$ correct

option (B) $P\left(\frac{1+3\sqrt{2}}{4}\right)<0$ incorrect

option (C) $P\left(\frac{5\sqrt{2}-1}{4}\right)>0$ correct

option (D) $P\left(\frac{5-\sqrt{2}}{4}\right)>0$ incorrect

SECTION-3 : (Maximum Marks : 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists : **List-I** and **List-II**.
- **List-I** has **Four** entries (I), (II), (III) and (IV) and **List-II** has **Five** entries (P), (Q), (R), (S) and (T).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 **ONLY** if the option corresponding to the correct combination is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : –1 In all other cases.

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15. Consider the following lists:

List-I		List-II	
(I)	$\left\{x \in \left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right] : \cos x + \sin x = 1\right\}$	(P)	has two elements
(II)	$\left\{x \in \left[-\frac{5\pi}{18}, \frac{5\pi}{18}\right] : \sqrt{3}\tan 3x = 1\right\}$	(Q)	has three elements
(III)	$\left\{x \in \left[-\frac{6\pi}{5}, \frac{6\pi}{5}\right] : 2\cos(2x) = \sqrt{3}\right\}$	(R)	has four elements
(IV)	$\left\{x \in \left[-\frac{7\pi}{4}, \frac{7\pi}{4}\right] : \sin x - \cos x = 1\right\}$	(S)	has five elements
		(T)	has six elements

The correct option is:

(A) (I) → (P); (II) → (S); (III) → (P); (IV) → (S)

(B) (I) → (P); (II) → (P); (III) → (T); (IV) → (R)

(C) (I) → (Q); (II) → (P); (III) → (T); (IV) → (S)

(D) (I) → (Q); (II) → (S); (III) → (P); (IV) → (R)

Ans. (B)

Sol. (I) $\left\{x \in \left[\frac{-2\pi}{3}, \frac{2\pi}{3}\right] : \cos x + \sin x = 1\right\}$

$\cos x + \sin x = 1$

$$\Rightarrow \frac{1}{\sqrt{2}}\cos x+\frac{1}{\sqrt{2}}\sin x=\frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos\left(x-\frac{\pi}{4}\right)=\cos\frac{\pi}{4}$$

$$\Rightarrow x-\frac{\pi}{4}=2n\pi\pm\frac{\pi}{4}\ ;\ n\in Z$$

$$\Rightarrow x=2n\pi\ ;\ x=2n\pi+\frac{\pi}{2}\ ;\ n\in Z$$

$\Rightarrow x\in\left\{0,\frac{\pi}{2}\right\}$ in given range has two solutions

(II) $\left\{x\in\left[\frac{-5\pi}{18},\frac{5\pi}{18}\right]:\sqrt{3}\tan 3x=1\right\}$

$$\sqrt{3}\tan 3x=1\ \Rightarrow\ \tan 3x=\frac{1}{\sqrt{3}}\ \Rightarrow\ 3x=n\pi+\frac{\pi}{6}$$

$$\Rightarrow x=(6n+1)\frac{\pi}{18}\ ;\ n\in Z$$

$\Rightarrow x\in\left\{\frac{\pi}{18},\frac{-5\pi}{18}\right\}$ in given range has two solutions

(III) $\left\{x\in\left[-\frac{6\pi}{5},\frac{6\pi}{5}\right]:2\cos(2x)=\sqrt{3}\right\}$

$2\cos 2x=\sqrt{3}$

$$\Rightarrow \cos 2x=\frac{\sqrt{3}}{2}=\cos\frac{\pi}{6}$$

$$\Rightarrow 2x=2n\pi\pm\frac{\pi}{6}\ ;\ n\in Z$$

$$\Rightarrow x=n\pi\pm\frac{\pi}{12}\ ;\ n\in Z$$

$$x\in\left\{\pm\frac{\pi}{12},\ \pi\pm\frac{\pi}{12},\ -\pi\pm\frac{\pi}{12}\right\}$$

Six solutions in given range

(IV) $\left\{x\in\left[-\frac{7\pi}{4},\frac{7\pi}{4}\right]:\sin x-\cos x=1\right\}$

$\cos x-\sin x=-1$

$$\Rightarrow \cos\left(x+\frac{\pi}{4}\right)=\frac{-1}{\sqrt{2}}=\cos\frac{3\pi}{4}$$

$$\Rightarrow x+\frac{\pi}{4}=2n\pi\pm\frac{3\pi}{4}\ ;\ n\in Z$$

$\Rightarrow x=2n\pi+\frac{\pi}{2}$ or $x=2n\pi-\pi\ ;\ n\in Z$

$\Rightarrow x\in\left\{\frac{\pi}{2},\frac{-3\pi}{2},\pi,-\pi\right\}$ four solutions in given range

16. Two players, P_1 and P_2, play a game against each other. In every round of the game, each player rolls a fair die once, where the six faces of the die have six distinct numbers. Let x and y denote the readings on the die rolled by P_1 and P_2, respectively. If $x > y$, then P_1 scores 5 points and P_2 scores 0 point. If $x = y$, then each player scores 2 points. If $x < y$, then P_1 scores 0 point and P_2 scores 5 points. Let X_i and Y_i be the total scores of P_1 and P_2, respectively, after playing the i^{th} round.

List-I		**List-II**	
(I)	Probability of $(X_2 \geq Y_2)$ is	(P)	$\frac{3}{8}$
(II)	Probability of $(X_2 > Y_2)$ is	(Q)	$\frac{11}{16}$
(III)	Probability of $(X_3 = Y_3)$ is	(R)	$\frac{5}{16}$
(IV)	Probability of $(X_3 > Y_3)$ is	(S)	$\frac{355}{864}$
		(T)	$\frac{77}{432}$

The correct option is:

(A) (I) → (Q); (II) → (R); (III) → (T); (IV) → (S)

(B) (I) → (Q); (II) → (R); (III) → (T); (IV) → (T)

(C) (I) → (P); (II) → (R); (III) → (Q); (IV) → (S)

(D) (I) → (P); (II) → (R); (III) → (Q); (IV) → (T)

Ans. (A)

Sol. $\text{P(draw in 1 round)} = \frac{6}{36} = \frac{1}{6}$

$\text{P(win in 1 round)} = \frac{1}{2}\left(1 - \frac{1}{6}\right) = \frac{5}{12}$

$\text{P(loss in 1 round)} = \frac{5}{12}$

$$P(X_2 > Y_2) = P(10,0) + P(7,2) = \frac{5}{12}\times\frac{5}{12} + \frac{5}{12}\times\frac{1}{6}\times 2 = \frac{45}{144} = \frac{5}{16}$$

$$P(X_2 = Y_2) = P(5,5) + P(4,4) = \frac{5}{12}\times\frac{5}{12}\times 2 + \frac{1}{6}\times\frac{1}{6} = \frac{25+2}{72} = \frac{3}{8}$$

$$P(X_3 = Y_3) = P(6,6) + P(7,7) = \frac{1}{6\times 6\times 6} + \frac{5}{12}\times\frac{1}{6}\times\frac{5}{12}\times 6 = \frac{2}{432} + \frac{75}{432} = \frac{77}{432}$$

$$P(X_3 > Y_3) = \frac{1}{2}\left(1 - \frac{77}{432}\right) = \frac{355}{864}$$

17. Let p, q, r be nonzero real numbers that are, respectively, the 10^{th}, 100^{th} and 1000^{th} terms of a harmonic progression. Consider the system of linear equations

$$x + y + z = 1$$

$$10x + 100y + 1000z = 0$$

$$qr\,x + pr\,y + pq\,z = 0.$$

List-I		**List-II**	
(I)	If $\frac{q}{r} = 10$, then the system of linear equations has	(P)	$x = 0,\ y = \frac{10}{9}, z = -\frac{1}{9}$ as a solution
(II)	If $\frac{p}{r} \neq 100$, then the system of linear equations has	(Q)	$x = \frac{10}{9},\ y = -\frac{1}{9}, z = 0$ as a solution
(III)	If $\frac{p}{q} \neq 10$, then the system of linear equations has	(R)	infinitely many solutions
(IV)	If $\frac{p}{q} = 10$, then the system of linear equations has	(S)	no solution
		(T)	at least one solution

The correct option is:

(A) (I) → (T); (II) → (R); (III) → (S); (IV) → (T)

(B) (I) → (Q); (II) → (S); (III) → (S); (IV) → (R)

(C) (I) → (Q); (II) → (R); (III) → (P); (IV) → (R)

(D) (I) → (T); (II) → (S); (III) → (P); (IV) → (T)

Ans. **(B)**

Sol. If $\frac{q}{r} = 10 \Rightarrow A = D \Rightarrow D_x = D_y = D_z = 0$

So, there are infinitely many solutions

Look of infinitely many solutions can be given as

$x + y + z = 1$

& $10x + 100y + 1000z = 0 \quad \Rightarrow \quad x + 10y + 100z = 0$

Let $z = \lambda$

then $x + y = 1 - \lambda$

and $x + 10y = -100\lambda$

$$\Rightarrow x = \frac{10}{9} + 10\lambda \ ; \ y = \frac{-1}{9} - 11\lambda$$

$$\text{i.e., } (x, y, z) \equiv \left(\frac{10}{9} + 10\lambda, \frac{-1}{9} - 11\lambda, \lambda\right)$$

$Q\left(\frac{10}{9}, \frac{-1}{9}, 0\right)$ valid for $\lambda = 0$

$P\left(0, \frac{10}{9}, \frac{-1}{9}\right)$ not valid for any λ.

(I) $\rightarrow$ Q,R,T

(II) If $\frac{p}{r} \neq 100$, then $D_y \neq 0$

So no solution

(II) $\rightarrow$ (S)

(III) If $\frac{p}{q} \neq 10$, then $D_z \neq 0$ so, no solution

(III) $\rightarrow$ (S)

(IV) If $\frac{p}{q} = 10 \Rightarrow D_z = 0 \Rightarrow D_x = D_y = 0$

so infinitely many solution

(IV) $\rightarrow$ Q,R,T

18. Consider the ellipse

$$\frac{x^2}{4}+\frac{y^2}{3}=1.$$

Let H(α, 0), $0 < \alpha < 2$, be a point. A straight line drawn through H parallel to the y-axis crosses the ellipse and its auxiliary circle at points E and F respectively, in the first quadrant. The tangent to the ellipse at the point E intersects the positive x-axis at a point G. Suppose the straight line joining F and the origin makes an angle ϕ with the positive x-axis.

List-I		List-II	
(I)	If $\phi=\frac{\pi}{4}$, then the area of the triangle FGH is	(P)	$\frac{\left(\sqrt{3}-1\right)^4}{8}$
(II)	If $\phi=\frac{\pi}{3}$, then the area of the triangle FGH is	(Q)	1
(III)	If $\phi=\frac{\pi}{6}$, then the area of the triangle FGH is	(R)	$\frac{3}{4}$
(IV)	If $\phi=\frac{\pi}{12}$, then the area of the triangle FGH is	(S)	$\frac{1}{2\sqrt{3}}$
		(T)	$\frac{3\sqrt{3}}{2}$

The correct option is:

(A) (I) → (R); (II) → (S); (III) → (Q); (IV) → (P)

(B) (I) → (R); (II) → (T); (III) → (S); (IV) → (P)

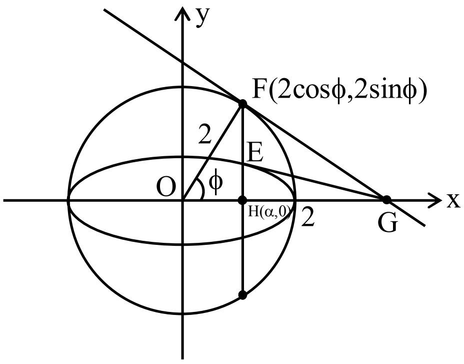
(C) (I) → (Q); (II) → (T); (III) → (S); (IV) → (P)

(D) (I) → (Q); (II) → (S); (III) → (Q); (IV) → (P)

Ans. **(C)**

Sol. Let F(2cosϕ, 2sinϕ)

& E(2cosϕ, $\sqrt{3}$ sinϕ)



EG : $\frac{x}{2}\cos\phi+\frac{y}{\sqrt{3}}\sin\phi=1$

$\therefore$ G$\left(\frac{2}{\cos\phi},0\right)$ and α = 2cosϕ

ar(ΔFGH) = $\frac{1}{2}$ HG × FH

$$=\frac{1}{2}\left(\frac{2}{\cos\phi}-2\cos\phi\right)\times 2\sin\phi$$

f(ϕ) = 2tanϕ$\sin^2$ϕ

$\therefore$ (I) $f\left(\frac{\pi}{4}\right)=1$ (II) $f\left(\frac{\pi}{3}\right)=\frac{3\sqrt{3}}{2}$ (III) $f\left(\frac{\pi}{6}\right)=\frac{1}{2\sqrt{3}}$

(IV) $f\left(\frac{\pi}{12}\right)=2\left(2-\sqrt{3}\right)\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2=\left(4-2\sqrt{3}\right)\frac{\left(\sqrt{3}-1\right)^2}{8}=\frac{\left(\sqrt{3}-1\right)^4}{8}$

$\therefore$ (I) → (Q) ; (II) → (T) ; (III) → (S) ; (IV) → (P)

JEE(Advanced) EXAMINATION - 2022

(Held On Sunday 28th AUGUST, 2022)

PAPER-2

PHYSICS

SECTION-1 : (Maximum Marks : 24)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER ranging from 0 TO 9, BOTH INCLUSIVE.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +3	If **ONLY** the correct integer is entered;
Zero Marks	: 0	If the question is unanswered;
Negative Marks	: −1	In all other cases.

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1. A particle of mass 1 *kg* is subjected to a force which depends on the position as $\vec{F} = -k(x\hat{i} + y\hat{j})kgms^{-2}$ with k = 1 kgs^{-2}. At time t = 0, the particle's position $\vec{r} = \left(\frac{1}{\sqrt{2}}\hat{i} + \sqrt{2}\hat{j}\right)m$ and its velocity $\vec{v} = \left(-\sqrt{2}\hat{i} + \sqrt{2}\hat{j} + \frac{2}{\pi}\hat{k}\right)ms^{-1}$. Let v_x and v_y denote the x and the y components of the particle's velocity, respectively. **Ignore gravity.** When z = 0.5 m, the value of $(x\ v_y — y\ v_x)$ is_________ m^2s^{-1}.

Ans. (3)

Sol. Torque about origin is zero

So angular momentum about origin remains conserved.

$$\begin{vmatrix} i & j & k \\ \frac{1}{\sqrt{2}} & \sqrt{2} & 0 \\ -\sqrt{2} & \sqrt{2} & \frac{2}{\pi} \end{vmatrix} = \begin{vmatrix} i & j & k \\ x & y & 0.5 \\ v_x & v_y & \frac{2}{\pi} \end{vmatrix}$$

$$\hat{i}\left[\sqrt{2}\times\frac{2}{\pi}\right] - \hat{j}\left[\frac{\sqrt{2}}{\pi}\right] + \hat{k}[1+2] = i\left[\frac{y\times 2}{\pi} - 0.5v_y\right] - \hat{j}\left[\frac{x\times 2}{\pi} - 0.5v_x\right] + k\left[xv_y - yv_x\right]$$

$xv_y - yv_x = 3$

2. In a radioactive decay chain reaction, ${}^{230}_{90}\text{Th}$ nucleus decays into ${}^{214}_{84}\text{Po}$ nucleus. The ratio of the number of α to number of β^- particles emitted in this process is ________.

Ans. (2)

Sol. $\text{Th}_{90}^{230} \rightarrow \text{Po}_{84}^{214} + n\alpha_2^4 + m\beta_{-1}^0$

$$230 = 214 + 4n$$

$$n = \frac{16}{4} = 4$$

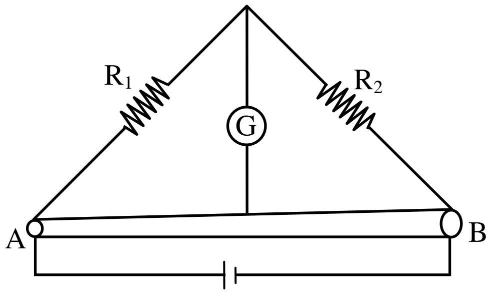
$$90 = 84 + n \times 2 - m \times 1$$

$$90 = 84 + 4 \times 2 - m \times 1$$

$$m = 92 - 90 = 2$$

Hence $\frac{n}{m} = \frac{4}{2} = 2$ **Ans.**

3. Two resistances $R_1 = X\ \Omega$ and $R_2 = 1\ \Omega$ are connected to a wire AB of uniform resistivity, as shown in the figure. The radius of the wire varies linearly along its axis from 0.2 *mm* at *A* to 1 *mm* at *B*. A galvanometer (G) connected to the center of the wire, 50 *cm* from each end along its axis, shows zero deflection when *A* and *B* are connected to a battery. The value of *X* is __________.



Ans. (5)

Sol. For the balanced Wheatstone bridge

$$\frac{R_1}{R_2} = \frac{\int_0^{0.5} \frac{\rho dx}{\pi r_x^2}}{\int_{0.5}^{1} \frac{\rho dx}{\pi r_x^2}}$$

$$\frac{R_1}{R_2} = \frac{+\left[\frac{1}{r_x}\right]_0^{0.5}}{+\left[\frac{1}{r_x}\right]_{0.5}^{1}}$$

$\therefore R_1 = 5R_2 = 5\Omega$

4. In a particular system of units, a physical quantity can be expressed in terms of the electric charge e, electron mass m_e, Planck's constant h, and Coulomb's constant $k = \frac{1}{4\pi \epsilon_0}$, where ϵ_0 is the permittivity of vacuum. In terms of these physical constants, the dimension of the magnetic field is $[B] = [e]^{\alpha} [m_e]^{\beta} [h]^{\gamma} [k]^{\delta}$. The value of $\alpha + \beta + \gamma + \delta$ is ________.

Ans. **(4)**

Sol. $B = e^{\alpha}\left(m_e\right)^{\beta} h^{\gamma} k^{\delta}$

$$[B] = \left[e^{\alpha}\right][m_e]^{\beta}[h]^{\gamma}\left[k^{\delta}\right]$$

$$[M^1T^{-2}A^{-1}] = [AT]^{\alpha}[m]^{\beta}\left[ML^2T^{-1}\right]^{\gamma}\left[ML^3A^{-2}T^{-4}\right]^{\delta}$$

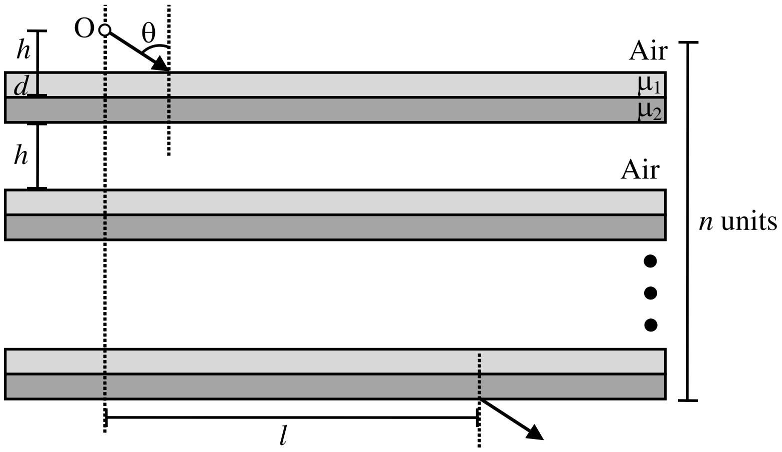
$$M^1T^{-2}A^{-1} = m^{\beta+\gamma+\delta}L^{2r+3\delta}T^{\alpha-\gamma-4\delta}A^{\alpha-2\delta}$$

Compare : $\beta + \gamma + \delta = 1;\ 2\gamma + 3\delta = 0,\ \alpha - \gamma - 4\delta = -2,\ \alpha - 2\delta = -1$

On solving $\alpha = 3, \beta = 2, \gamma = -3, \delta = 2$

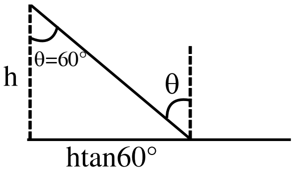
$\alpha + \beta + \gamma + \delta = 4$

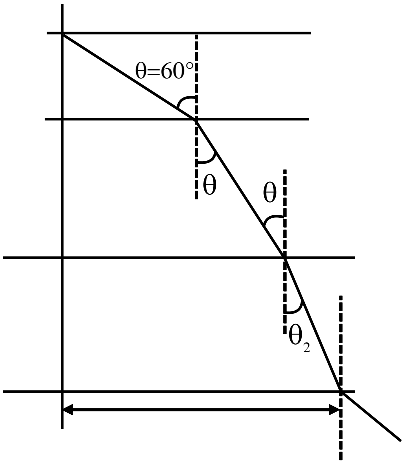
5. Consider a configuration of n identical units, each consisting of three layers. The first layer is a column of air of height $h = \frac{1}{3}cm$, and the second and third layers are of equal thickness $d = \frac{\sqrt{3}-1}{2}cm$, and refractive indices $\mu_1 = \sqrt{\frac{3}{2}}$ and $\mu_2 = \sqrt{3}$, respectively. A light source O is placed on the top of the first unit, as shown in the figure. A ray of light from O is incident on the second layer of the first unit at an angle of $\theta = 60°$ to the normal. For a specific value of n, the ray of light emerges from the bottom of the configuration at a distance $l = \frac{8}{\sqrt{3}}cm$, as shown in the figure. The value of n is ________.



Ans. **(4)**

Sol.





$$1\sin 60° = \sqrt{\frac{3}{2}}\sin\theta$$

$\Rightarrow \theta_1 = 45°$

$$\sqrt{\frac{3}{2}}\sin 45° = \sqrt{3}\sin\theta_2$$

$$= \sqrt{\frac{3}{2}}\frac{1}{\sqrt{2}} = \sqrt{3}\sin\theta_2$$

$= \theta_2 = 30°$

h tan60° + d tan45° + d tan 30°

$$\frac{1}{3}\sqrt{3} + \left(\frac{\sqrt{3}-1}{2}\right) + \left(\frac{\sqrt{3}-1}{2}\right)\frac{1}{\sqrt{3}}$$

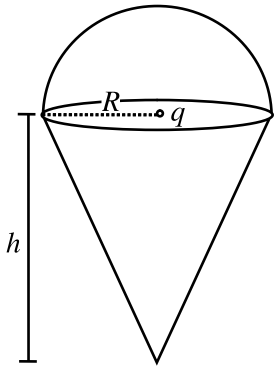
$$\frac{2\sqrt{3}+3\sqrt{3}-3+3-\sqrt{3}}{6}$$

$$\frac{4\sqrt{3}}{6}$$

$$\therefore n\frac{4\sqrt{3}}{6} = \frac{8}{\sqrt{3}}$$

n = 4

6. A charge q is surrounded by a closed surface consisting of an inverted cone of height h and base radius R, and a hemisphere of radius R as shown in the figure. The electric flux through the conical surface is $\frac{nq}{6\in_0}$ (in SI units). The value of n is ________.



Ans. **(3)**

Sol. From Gauss law,

$$\phi_{\text{hemisphere}}+\phi_{\text{Cone}}=\frac{q}{\varepsilon_0} \quad \ldots..$$

Total flux produced from q in α angle

$$\phi=\frac{q}{2\varepsilon_0}[1-\cos\alpha]$$

For hemisphere, $\alpha=\frac{\pi}{2}$

$$\phi_{\text{hemisphere}}=\frac{q}{2\varepsilon_0}$$

From equation (i)

$$=\frac{q}{2\varepsilon_0}+\phi_{\text{cone}}=\frac{q}{\varepsilon_0}$$

$$\phi_{\text{cone}}=\frac{q}{2\varepsilon_0}$$

$$\frac{4q}{6\varepsilon_0}=\frac{q}{2\varepsilon_0}$$

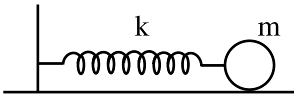
n = 3

Alternatively, $\phi \propto$ no of electric field lines passing through surface q is point charge which has uniformly distributed electric field lines thus half of electric field lines will pass through hemisphere & other half will pass through conical surface.

7. On a frictionless horizontal plane, a bob of mass m = 0.1 kg is attached to a spring with natural length $l_0 = 0.1$ m. The spring constant is $k_1 = 0.009$ Nm^{-1} when the length of the spring $l > l_0$ and is $k_2 = 0.016$ Nm^{-1} when $l < l_0$. Initially the bob is released from $l = 0.15$ m. Assume that Hooke's law remains valid throughout the motion. If the time period of the full oscillation is $\mathrm{T} = (n\pi)$ s, then the integer closest to n is ________.

Ans. (6)

Sol.



$\ell > \ell_0 \rightarrow \mathrm{k} = \mathrm{k}_1$

$\ell < \ell_0 \rightarrow \mathrm{k} = \mathrm{k}_2$

Time period of oscillation,

$$\mathrm{T} = \pi\sqrt{\frac{\mathrm{m}}{\mathrm{k}_1}} + \pi\sqrt{\frac{\mathrm{m}}{\mathrm{k}_2}}$$

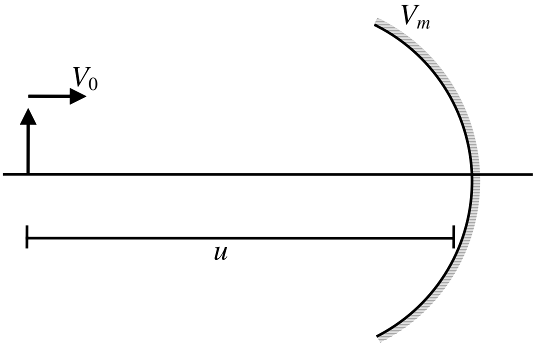
$$\mathrm{T} = \pi\sqrt{\frac{0.1}{0.009}} + \pi\sqrt{\frac{0.1}{0.016}}$$

$$\mathrm{T} = \frac{\pi}{0.3} + \frac{\pi}{0.4} \Rightarrow \mathrm{T} = \frac{0.7}{0.12}\pi \Rightarrow \mathrm{T} = 5.83\pi$$

$\mathrm{T} \approx 6\pi$

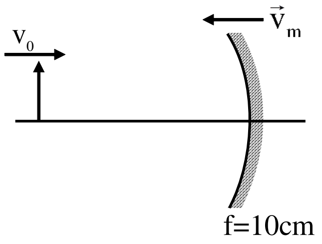
So, n = 6

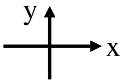
8. An object and a concave mirror of focal length f = 10 cm both move along the principal axis of the mirror with constant speeds. The object moves with speed V_0 = 15 cm s^{-1} towards the mirror with respect to a laboratory frame. The distance between the object and the mirror at a given moment is denoted by u. When u = 30 cm, the speed of the mirror V_m is such that the image is instantaneously at rest with respect to the laboratory frame, and the object forms a real image. The magnitude of V_m is ________ cm s^{-1}.



Ans. (3)

Sol.



Let 

$u = -30$ cm

$f = -\ 10$ cm

$$v = \frac{f_0}{u-f} = -15\text{cm}$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{du}{dt} = -\frac{v^2}{u^2}\frac{du}{dt}$$

$$\vec{v}_{Im} = -\left(\frac{v}{u}\right)^2 \vec{v}_{om}$$

Given $\vec{v}_I = \vec{0}$

$$\vec{v}_I - \vec{v}_m = -\left(\frac{-15}{-30}\right)^2 (\vec{v}_{O/m})$$

$$\vec{v}_I - \vec{v}_m = -\frac{1}{4}\vec{v}_0 + \frac{1}{4}\vec{v}_m$$

$$\vec{v}_0 = 15\text{cm}/\text{s}\ \hat{i}$$

$$\vec{v}_I = \vec{0}\text{cm}/\text{s}$$

$$\frac{5}{4}\vec{v}_m = \frac{\vec{v}_0}{4}$$

$$\vec{v}_m = \frac{\vec{v}_0}{4} = \frac{15\text{cm}/\text{s}\,\hat{i}}{5} = 3\text{m}/\text{s}\ \hat{i}$$

$$\left|\vec{v}_m\right|_m \text{cm}/\text{s} = 3$$

SECTION-2 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	: +3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	: +2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	: +1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	: 0	If unanswered;
Negative Marks	: −2	In all other cases.

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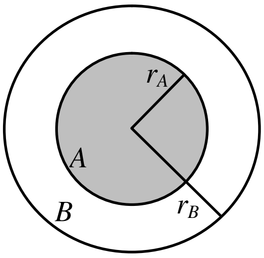
9. In the figure, the inner (shaded) region A represents a sphere of radius $r_A = 1$, within which the electrostatic charge density varies with the radial distance r from the center as $\rho_A = kr$, where k is positive. In the spherical shell B of outer radius r_B, the electrostatic charge density varies as $\rho_B = \frac{2k}{r}$. Assume that dimensions are taken care of. All physical quantities are in their SI units.



Which of the following statement(s) is(are) correct?

(A) If $r_B = \sqrt{\frac{3}{2}}$, then the electric field is zero everywhere outside B.

(B) If $r_B = \frac{3}{2}$, then the electric potential just outside B is $\frac{k}{\epsilon_0}$.

(C) If $r_B = 2$, then the total charge of the configuration is $15\pi k$.

(D) If $r_B = \frac{5}{2}$, then the magnitude of the electric field just outside B is $\frac{13\pi k}{\epsilon_0}$.

Ans. (B)

Sol. $q_1 = \int_0^1 kr4\pi r^2 dr = \frac{4\pi k}{4} = \pi k$

$$q_2 = \int_1^r \frac{2k}{r} 4\pi r^2 dr = \frac{8\pi k\left(r^2 - 1^2\right)}{2}$$

$q_2 = 4\pi k[r^2 - 1] = 4\pi kr^2 - 4\pi k$

$q_{net} = q_1 + q_2$

$= 4\pi kr^2 - 3\pi k$

$q_{net} = \pi k\ [4r^2 - 3]$

(A) $E_{net} = 0 \Rightarrow q_{net} = 0 \Rightarrow r = \frac{\sqrt{3}}{2}$

(B) $V = \frac{kQ_{net}}{r} = \frac{1}{4\pi\varepsilon_0} \frac{\pi k\left(4r^2 - 3\right)}{r}$

$$V = \frac{k}{4\varepsilon_0}\left[4r - \frac{3}{r}\right]$$

$$= \frac{k}{4\varepsilon_0}\left[4 \times \frac{3}{2} - \frac{3 \times 2}{3}\right] = \frac{k}{\varepsilon_0}$$

(C) $q_{net} = \pi k\left[4(2)^2 - 3\right]$

$= 13\pi k$

(D) $E_2 = \frac{kQ}{r^2}$

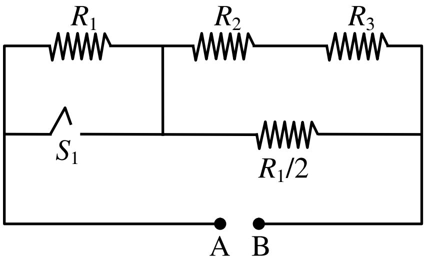
$$= \frac{1}{4\pi\varepsilon_0} \frac{\pi k\left(4r^2 - 3\right)}{r^2}$$

$$= \frac{k}{4\varepsilon_0}\left[\frac{4\left(\frac{5}{2}\right)^2 - 3}{(5/2)^2}\right]$$

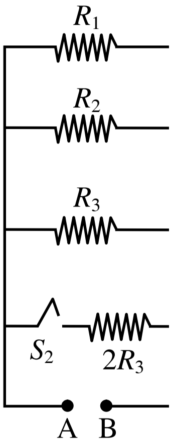
$$= \frac{k}{25\varepsilon_0}[25 - 3] = \frac{22}{25}\frac{k}{\varepsilon_0}$$

10. In Circuit-1 and Circuit-2 shown in the figures, $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$ and $R_3 = 3\ \Omega$. P_1 and P_2 are the power dissipations in Circuit-1 and Circuit-2 when the switches S_1 and S_2 are in open conditions, respectively.

Q_1 and Q_2 are the power dissipations in Circuit-1 and Circuit-2 when the switches S_1 and S_2 are in closed conditions, respectively.



Circuit-1



Circuit-2

Which of the following statement(s) is(are) correct?

(A) When a voltage source of 6 *V* is connected across A and B in both circuits, $P_1 < P_2$.

(B) When a constant current source of 2 *Amp* is connected across A and B in both circuits, $P_1 > P_2$.

(C) When a voltage source of 6 *V* is connected across A and B in Circuit-1, $Q_1 > P_1$.

(D) When a constant current source of 2 *Amp* is connected across A and B in both circuits, $Q_2 < Q_1$

Ans. **(A,B,C)**

Sol. Case (i)

When both switches are open equivalent resistance in circuit 1

$$R_{C_1} = \frac{16}{11}\Omega$$

Equivalent resistance in circuit 2

$$R_{C_2} = \frac{6}{11}\Omega$$

For voltage source

$$P = \frac{V^2}{R}$$

$$P \propto \frac{1}{R}$$

$$R_{C_1} > R_{C_2}$$

$\Rightarrow P_2 > P_1$ (Option (A) correct)

For constant current source

$$P = i^2R$$

$$P \propto R$$

$\Rightarrow \quad P_1 > P_2$ (Option (B) correct)

Case-II

When switch is closed

$$R'_{C_1} = \frac{5}{11}\Omega$$

$$R'_{C_2} = \frac{1}{2}\Omega$$

$$R'_{C_1} < R_{C_1}$$

For voltage source

$P \propto \frac{1}{R} \Rightarrow Q_1 > P_1$ (Option (C) correct)

& $R'_{C_1} > R'_{C_2}$

For current source $P \propto R$

$Q_1 > Q_2$ (Option (D) also correct)

11. A bubble has surface tension S. The ideal gas inside the bubble has ratio of specific heats $\gamma = \frac{5}{3}$. The bubble is exposed to the atmosphere and it always retains its spherical shape. When the atmospheric pressure is P_{a1}, the radius of the bubble is found to be r_1 and the temperature of the enclosed gas is T_1. When the atmospheric pressure is P_{a2}, the radius of the bubble and the temperature of the enclosed gas are r_2 and T_2, respectively.

Which of the following statement(s) is(are) correct?

(A) If the surface of the bubble is a perfect heat insulator, then $\left(\frac{r_1}{r_2}\right)^5 = \frac{P_{a2} + \frac{2S}{r_2}}{P_{a1} + \frac{2S}{r_1}}$

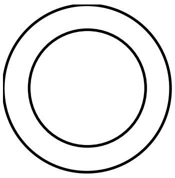
(B) If the surface of the bubble is a perfect heat insulator, then the total internal energy of the bubble including its surface energy does not change with the external atmospheric pressure.

(C) If the surface of the bubble is a perfect heat conductor and the change in atmospheric temperature is negligible, then $\left(\frac{r_1}{r_2}\right)^3 = \frac{P_{a2} + \frac{4S}{r_2}}{P_{a1} + \frac{4S}{r_1}}$.

(D) If the surface of the bubble is a perfect heat insulator, then $\left(\frac{T_2}{T_1}\right)^{\frac{5}{2}} = \frac{P_{a2} + \frac{4S}{r_2}}{P_{a1} + \frac{4S}{r_1}}$.

Ans. (C,D)

Sol.



$$P_{gas} = P_a + \frac{4S}{r}$$

PV^{γ} = constant [adiabatic process]

$$\left(Pa_1 + \frac{4S}{r_1}\right)\left(\frac{4}{3}\pi r_1^3\right)^{5/3} = \left(P_{a_2} + \frac{4S}{r_2}\right)\left(\frac{4}{3}\pi r_2^3\right)^{5/3}$$

$$\frac{r_1^3}{r_2^3}=\left(\frac{P_{a_2}+\frac{4S}{r_2}}{P_{a_1}+\frac{4S}{r_1}}\right)$$

$P^{1-y}T^{y}$ = constant

$$\left(P_{a_2}+\frac{4S}{r_2}\right)^{1-5/3}T_2^{5/3}=\left(P_{a_1}+\frac{4S}{r_1}\right)^{1-5/3}T_1^{5/3}$$

$$\left(\frac{T_2}{T_1}\right)^{5/3}=\left(\frac{P_{a_1}+\frac{4S}{r_1}}{P_{a_2}+\frac{4S}{r_2}}\right)^{-2/3}$$

$$\left(\frac{T_2}{T_1}\right)^{5/2}=\left(\frac{P_{a_2}+\frac{4S}{r_2}}{P_{a_1}+\frac{4S}{r_1}}\right)$$

(D) is correct

12. A disk of radius R with uniform positive charge density σ is placed on the xy plane with its center at the origin. The Coulomb potential along the z-axis is

$$V(z)=\frac{\sigma}{2\in_0}\left(\sqrt{R^2+z^2}-z\right)$$

A particle of positive charge q is placed initially at rest at a point on the z axis with $z = z_0$ and $z_0 > 0$. In addition to the Coulomb force, the particle experiences a vertical force $\vec{F}=-c\hat{k}$ with $c > 0$. Let $\beta=\frac{2c\in_0}{q\sigma}$. Which of the following statement(s) is(are) correct?

(A) For $\beta=\frac{1}{4}$ and $z_0=\frac{25}{7}R$, the particle reaches the origin.

(B) For $\beta=\frac{1}{4}$ and $z_0=\frac{3}{7}R$, the particle reaches the origin.

(C) For $\beta=\frac{1}{4}$ and $z_0=\frac{R}{\sqrt{3}}$, the particle returns back to $z = z_0$.

(D) For $\beta > 1$ and $z_0 > 0$, the particle always reaches the origin.

Ans. (A,C,D)

Sol. $W_{el} + W_{ext} = k_f - k_i$

$qv_i - qv_f + W_{ext} = k_f - k_i$

$\frac{q\sigma}{2\in_0}\left[\sqrt{R^2+Z^2} - Z\right] - \frac{q\sigma R}{2\in_0} + CZ = k_f - 0$

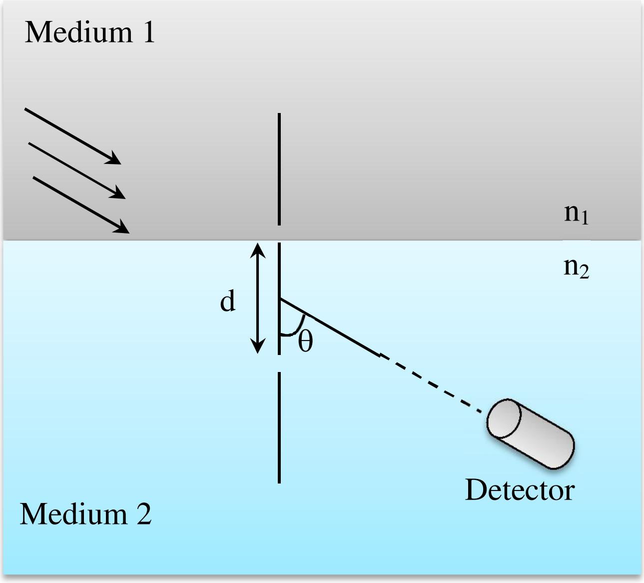
$C = \frac{q\sigma B}{2\in_0}$

Substitute β & Z, calculate kinetic energy at z = 0

If kinetic energy is positive, then particle will reach at origin

If kinetic energy is negative, then particle will not reach at origin.

13. A double slit setup is shown in the figure. One of the slits is in medium 2 of refractive index n_2. The other slit is at the interface of this medium with another medium 1 of refractive index $n_1 (\neq n_2)$. The line joining the slits is perpendicular to the interface and the distance between the slits is *d*. The slit widths are much smaller than *d*. A monochromatic parallel beam of light is incident on the slits from medium 1. A detector is placed in medium 2 at a large distance from the slits, and at an angle θ from the line joining them, so that θ equals the angle of refraction of the beam. Consider two approximately parallel rays from the slits received by the detector.



Which of the following statement(s) is (are) correct?

(A) The phase difference between the two rays is independent of *d*.

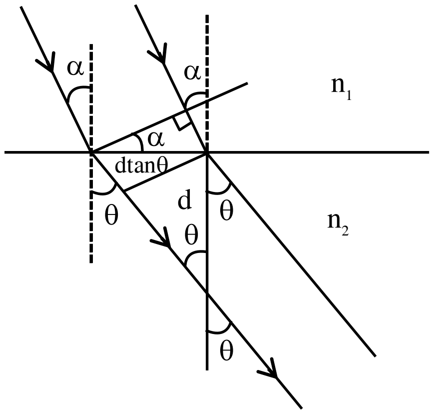
(B) The two rays interfere constructively at the detector.

(C) The phase difference between the two rays depends on n_1 but is independent of n_2.

(D) The phase difference between the two rays vanishes only for certain values of *d* and the angle of incidence of the beam, with θ being the corresponding angle of refraction.

Ans. **(A,B)**

Sol.



Optical path difference →

$\Delta x = n_1(d\tan\theta)\sin\alpha - n_2(d\tan\theta)\sin\theta$

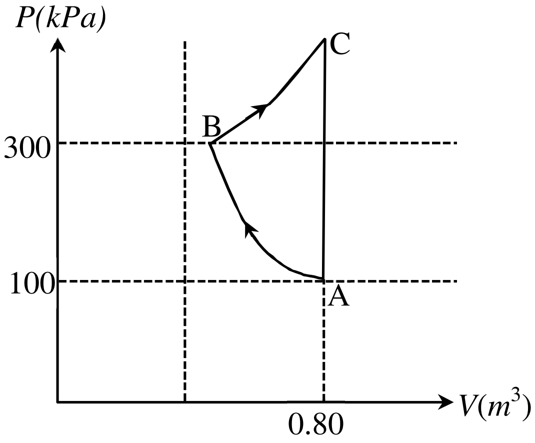
$= (n_1\sin\alpha - n_2\sin\theta)\, d\tan\theta$

$= 0$

$\Rightarrow \Delta\phi = 0$

Ans. (A,B)

14. In the given P-V diagram, a monoatomic gas $\left(\gamma = \frac{5}{3}\right)$ is first compressed adiabatically from state A to state B. Then it expands isothermally from state B to state C. [Given: $\left(\frac{1}{3}\right)^{0.6} \simeq 0.5$, $\ln 2 \simeq 0.7$].



Which of the following statement(s) is(are) correct?

(A) The magnitude of the total work done in the process $A \to B \to C$ is 144 kJ.

(B) The magnitude of the work done in the process $B \to C$ is 84 kJ.

(C) The magnitude of the work done in the process $A \to B$ is 60 kJ.

(D) The magnitude of the work done in the process $C \to A$ is zero.

Ans. (B,C,D)

Sol. For adiabatic process (A → B)

$$P_A V_A^{\gamma} = P_B V_B^{\gamma}$$

$$10^5 \times (0.8)^{\frac{5}{3}} = 3 \times 10^5 (V_B)^{\frac{5}{3}}$$

$$\Rightarrow V_B = 0.8 \times \left(\frac{1}{3}\right)^{0.6} = 0.4$$

Work done in process A → B

$$W_{AB} = \frac{P_A V_A - P_B V_B}{\gamma - 1}$$

$$\Rightarrow W_{AB} = \frac{10^5 \times 0.8 - 3 \times 10^5 \times 0.4}{\frac{5}{3} - 1}$$

$\Rightarrow W_{AB} = -60$ kJ $= \Rightarrow |W_{AB}| = 60$ kJ

Work done in process B → C (Isothermal process)

$$W_{BC} = nRT \ell n \frac{V_C}{V_B} = P_B V_B \ell n \frac{V_C}{V_B}$$

$$\Rightarrow W_{BC} = 3 \times 10^5 \times 0.4 \ell n \frac{0.8}{0.4}$$

$\Rightarrow W_{BC} = 84$ kJ

Work done in process C → A

$W_{CA} = P\Delta V = 0 \quad (\because \Delta V = 0)$

So total work done in the process A→B→C

$W_{ABC} = W_{AB} + W_{BC} + W_{CA} = -60 + 84 + 0$

$W_{ABC} = 24$ kJ

So correct options are (B,C,D)

SECTION-3 : (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +3	If **ONLY** the correct option is chosen;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: −1	In all other cases.

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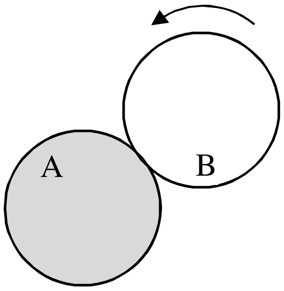
15. A flat surface of a thin uniform disk *A* of radius *R* is glued to a horizontal table. Another thin uniform disk *B* of mass *M* and with the same radius *R* rolls without slipping on the circumference of *A*, as shown in the figure. A flat surface of *B* also lies on the plane of the table. The center of mass of *B* has fixed angular speed ω about the vertical axis passing through the center of *A*. The angular momentum of *B* is $nM\omega R^2$ with respect to the center of *A*. Which of the following is the value of *n*?



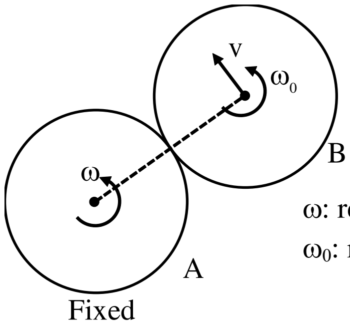
(A) 2 (B) 5 (C) $\frac{7}{2}$ (D) $\frac{9}{2}$

Ans. **(B)**

Sol. v = ω (2R)

v = ω_0R : no slipping

$\therefore \omega_0 = 2\omega$



ω: revolution

ω_0: rotation

$$\vec{L} = m\vec{r} \times \vec{v}_c + I_c\omega_0$$

$$= M2Rv + \frac{1}{2}MR^2\omega_0$$

$$= 4MR^2\omega + \frac{1}{2}MR^2(2\omega) = 5MR^2\omega$$

$\therefore$ n = 5

16. When light of a given wavelength is incident on a metallic surface, the minimum potential needed to stop the emitted photoelectrons is 6.0 V. This potential drops to 0.6 V if another source with wavelength four times that of the first one and intensity half of the first one is used. What are the wavelength of the first source and the work function of the metal, respectively? [Take = $\frac{hc}{e} = 1.24\times10^{-6}\,Jm\,C^{-1}$.]

(A) 1.72×10^{-7} m, 1.20 eV (B) 1.72×10^{-7} m, 5.60 eV

(C) 3.78×10^{-7} m, 5.60 eV (D) 3.78×10^{-7} m, 1.20 eV

Ans. **(A)**

Sol. $\frac{\text{hc}}{\lambda} = \phi + 6$ … (i)

$\frac{\text{hc}}{4\lambda} = \phi + 0.6$ … (ii)

$\frac{3\text{hc}}{4\lambda} = 5.4\text{eV}$ $\therefore\ \phi = 1.2$ eV

$$\Rightarrow \frac{3}{4}\times\frac{6.63\times10^{-24}\times3\times10^{8}}{5.4\times1.6\times10^{-19}} = \lambda = 1.72\times10^{-7}\text{m}$$

17. Area of the cross-section of a wire is measured using a screw gauge. The pitch of the main scale is 0.5 mm. The circular scale has 100 divisions and for one full rotation of the circular scale, the main scale shifts by two divisions. The measured readings are listed below.

Measurement condition	**Main scale reading**	**Circular scale reading**
Two arms of gauge touching each other without wire	0 division	4 division
Attempt-1: With wire	4 divisions	20 divisions
Attempt-2: With wire	4 divisions	16 divisions

What are the diameter and cross-sectional area of the wire measured using the screw gauge?

(A) 2.22 ± 0.02 mm, $\pi(1.23 \pm 0.02)\ mm^2$

(B) 2.22 ± 0.01 mm, $\pi(1.23 \pm 0.01)\ mm^2$

(C) 2.14 ± 0.02 mm, $\pi(1.14 \pm 0.02)\ mm^2$

(D) 2.14 ± 0.01 mm, $\pi(1.14 \pm 0.01)\ mm^2$

Ans. (C)

Sol. $LC = \frac{0.1}{100} = 0.001mm$

Zero error = $4 \times 0.001 = 0.004$ mm

Reading 1 = $0.5 \times 4 + 20 \times 0.001 - 0.004 = 2.16$ mm

Reading 2 = $0.5 \times 4 + 16 \times 0.001 - 0.004 = 2.12$ mm

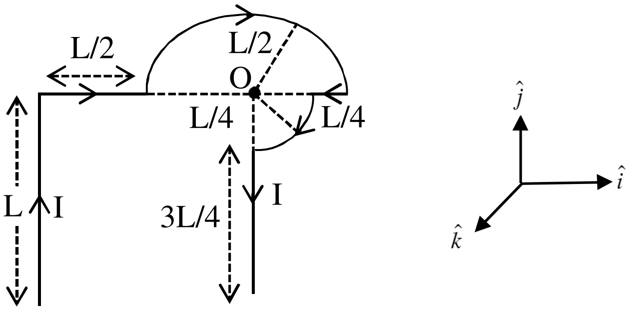
Mean value = 2.14 mm

Mean absolute error = $\frac{0.02 + 0.02}{2} = 0.02$

Diameter = 2.14 ± 0.02

Area = $\frac{\pi}{4} d^2$

18. Which one of the following options represents the magnetic field $\vec{B}$ at O due to the current flowing in the given wire segments lying on the *xy* plane?



(A) $\vec{B} = \frac{-\mu_0 I}{L}\left(\frac{3}{2} + \frac{1}{4\sqrt{2}\pi}\right)\hat{k}$

(B) $\vec{B} = -\frac{\mu_0 I}{L}\left(\frac{3}{2} + \frac{1}{2\sqrt{2}\pi}\right)\hat{k}$

(C) $\vec{B} = \frac{-\mu_0 I}{L}\left(1 + \frac{1}{4\sqrt{2}\pi}\right)\hat{k}$

(D) $\vec{B} = \frac{-\mu_0 I}{L}\left(1 + \frac{1}{4\pi}\right)\hat{k}$

Ans. (C)

Sol. $\vec{B} = \frac{\mu_0 I}{4\pi L}\sin 45°\left(-\hat{k}\right) + \frac{\mu_0 I \pi}{4\pi \frac{L}{2}}\left(-\hat{k}\right) + \frac{\mu_0 I}{4\pi \frac{L}{4}} \times \frac{\pi}{2}\left(-\hat{k}\right)$

CHEMISTRY

SECTION-1 : (Maximum Marks : 24)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER ranging from 0 TO 9, BOTH INCLUSIVE.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If **ONLY** the correct integer is entered;
Zero Marks : 0 If the question is unanswered;
Negative Marks : −1 In all other cases.

---

1. Concentration of H_2SO_4 and Na_2SO_4 in a solution is 1 M and 1.8×10^{-2} M, respectively. Molar solubility of $PbSO_4$ in the same solution is $X \times 10^{-Y}$ M (expressed in scientific notation). The value of Y is __________.
[Given: Solubility product of $PbSO_4$ (K_{sp}) = 1.6×10^{-8}. For H_2SO_4, K_{a1} is very large and $K_{a2} = 1.2 \times 10^{-2}$]

Ans. (6)

Sol.

$$\begin{array}{llll} H_2SO_4 & \rightleftharpoons & HSO_4^- & +H^+ \\ 1M & & - & - \\ - & & 1M & 1M \end{array}$$

$$\begin{array}{llllll} Na_2SO_4 & \longrightarrow & 2Na^+ & + & SO_4^{2-} \\ 1.8 \times 10^{-2}\ M & & - & & - \\ & & 3.6\times10^{-2}\ M & & 1.8\times 10^{-2}\ M \end{array}$$

$$\begin{array}{llllll} HSO_4^- & \rightleftharpoons & H^+ & + & SO_4^{2-} & ;\ K_{a_2} = 1.2 \times 10^{-2}\ M \\ 1M & & 1M & & 1.8\times 10^{-2}\ M & \end{array}$$

Since $Q_C > K_C$ it will move in backward direction.

$$\begin{array}{lllll} 1+x & & 1-x & & 1.8 \times 10^{-2} - x \end{array}$$

$$K_{a_2} = 1.2 \times 10^{-2} = \frac{(1-x)(1.8\times10^{-2}-x)}{(1+x)}$$

Since x is very small (1 + x) ≃ **1 and (1 – x)** ≃ 1

$x = \left(1.8\times10^{-2} - 1.2\times10^{-2}\right)\text{M}$

$\left[SO_4^{2-}\right] = \left(1.8\times10^{-2} - 0.6\times10^{-2}\right)\text{M}$

$= 1.2 \times 10^{-2}$ M

$PbSO_4$	$\longrightarrow$	Pb^{2+} +	SO_4^{2-}
s		-	1.2×10^{-2} M
-		s	$(s + 1.2 \times 10^{-2})$

$K_{sp} = s\,(s + 1.2 \times 10^{-2}) = 1.6 \times 10^{-8}$
$(PbSO_4)$

Here, $(s + 1.2 \times 10^{-2}) \simeq 1.2 \times 10^{-2}$ (since 's' is very small)

$s(1.2\times10^{-2}) = 1.6 \times 10^{-8}$

$\Rightarrow s = \dfrac{1.6}{1.2}\times10^{-6}\text{M} = X\times10^{-Y}\text{M}$

$\Rightarrow Y = 6$

2. An aqueous solution is prepared by dissolving 0.1 mol of an ionic salt in 1.8 kg of water at 35 ºC. The salt remains 90% dissociated in the solution. The vapour pressure of the solution is 59.724 mm of Hg. Vapor pressure of water at 35 ºC is 60.000 mm of Hg. The number of ions present per formula unit of the ionic salt is _______.

Ans. (5)

Sol. 0.1 mole ionic salt in 1.8 kg water at 35° C

Vapour pressure of solution = 59.724 mm of Hg

Vapour pressure of pure H_2O = 60.000 mm of Hg

Let the number of ions present per formula unit of the ionic salt be 'x'

A_x	$\longrightarrow$	xA
(Salt)		(Ions)
0.1		-
0.1 (1 – 0.9)		(0.1 × 0.9) x

Total moles of non-volatile particles = 0.01 + 0.09 x
in 1.8 kg water

Moles of water $= \dfrac{1.8\times10^3}{18} = 100$ moles

Relative lowering of vapour pressure $\dfrac{P^\circ - P_s}{P^\circ}$ = Mole fraction of non – volatile particles

$$\frac{P^\circ - P_s}{P_s} = \frac{\text{moles of non} - \text{volatile particles}}{\text{moles of water}}$$

$$\frac{60.000 - 59.724}{59.724} = \frac{0.01 + 0.09x}{100}$$

$$(0.276) \times 100 = 0.59274 + (0.59274 \times 9)x$$

$$27.6 - 0.59274 = (0.59274 \times 9)x$$

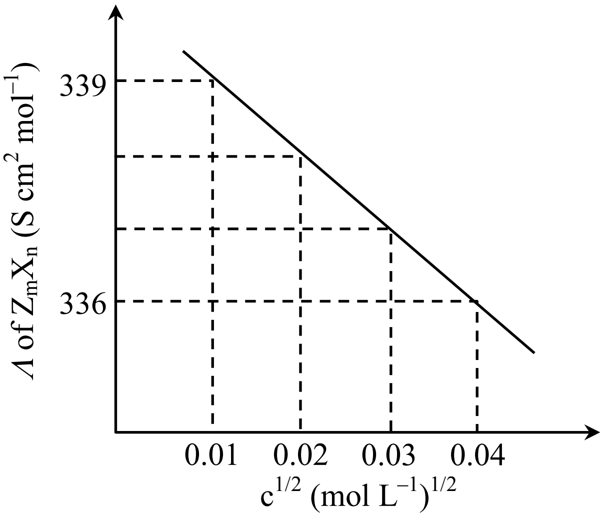
$$\Rightarrow x \simeq \frac{27}{0.6 \times 9} = 5$$

3. Consider the strong electrolytes Z_mX_n, U_mY_p and V_mX_n . Limiting molar conductivity (Λ^0) of U_mY_p and V_mX_n are 250 and 440 S cm^2 mol^{-1}, respectively. The value of (m + n + p) is _______.
Given:

Ion	Z^{n+}	U^{p+}	V^{n+}	X^{m-}	Y^{m-}
λ^0(S cm^2 mol^{-1})	50.0	25.0	100.0	80.0	100.0

λ^0 is the limiting molar conductivity of ions

The plot of molar conductivity (Λ) of Z_mX_n *vs* $c^{1/2}$ is given below.



Ans. (7)

Sol. $\Lambda^\circ(U_mY_p) = m \times \lambda^\circ_{U^{p+}} + p \times \lambda^\circ_{Y^{m-}} = 250$

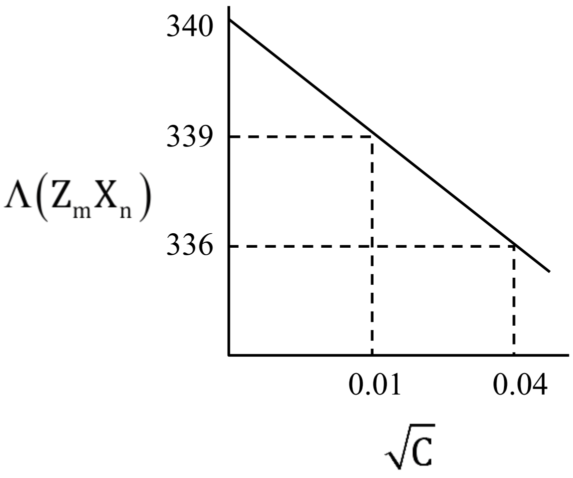
$25m + 100p = 250$

$m + 4p = 10$ ......(1)

$\Lambda^\circ(V_mX_n) = m \times \lambda_{V^{n+}} + n \times \lambda^\circ_{X^{m-}} = 440$

$100m + 80n = 440$

$5m + 4n = 22$ ......(2)



From the extrapolation of curve

$$\Lambda^\circ(Z_mX_n)=340$$

$$m\times\lambda^\circ_{Z^{n+}}+n\lambda^\circ_{X^{m-}}=340$$

$$50m + 80n = 340$$

$$5m + 8n = 34 \quad ....(3)$$

(3) — (2) ⇒ $4n = 12 \Rightarrow n = 3$

Putting in (2) we get $m = 2$

Putting in (1) we get $p = 2$

$m + n + p = 2 + 3 + 2 = 7$

4. The reaction of Xe and O_2F_2 gives a Xe compound **P**. The number of moles of HF produced by the complete hydrolysis of 1 mol of **P** is _______.

Ans. **(4)**

Sol. $Xe + 2O_2F_2 \rightarrow XeF_4 + 2O_2$

$$3XeF_4 + 6H_2O \rightarrow 2Xe + XeO_3 + \frac{3}{2}O_2 + 12HF$$

∴ One mole of XeF_4 gives 4 moles of HF on hydrolysis.

5. Thermal decomposition of $AgNO_3$ produces two paramagnetic gases. The total number of electrons present in the antibonding molecular orbitals of the gas that has the higher number of unpaired electrons is _______.

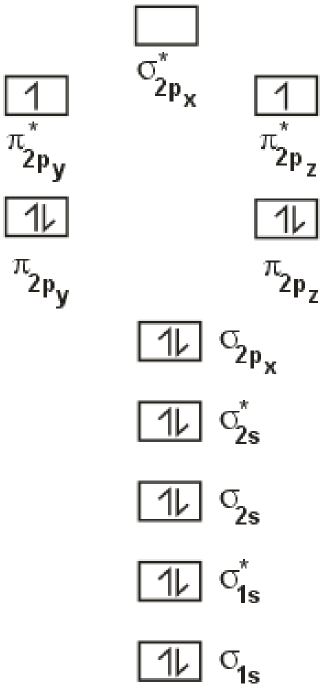
Ans. **(6)**

Sol. $AgNO_3 \rightarrow 2Ag + 2NO_2 + ½O_2$

– Both NO_2 & O_2 are paramagnetic

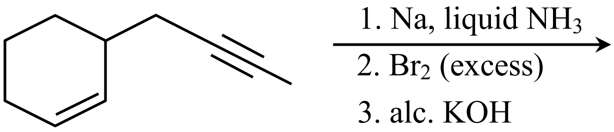
– NO_2 is odd electron molecule with one unpaired electron

–O_2 has two unpaired electrons



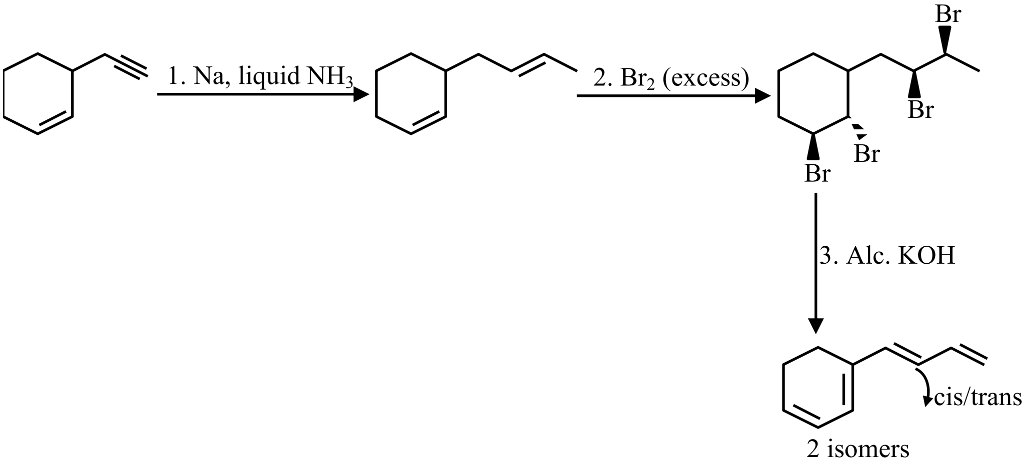
Total number of antibonding electrons = 6

6. The number of isomeric tetraenes (**NOT** containing *sp*-hybridized carbon atoms) that can be formed from the following reaction sequence is ________.

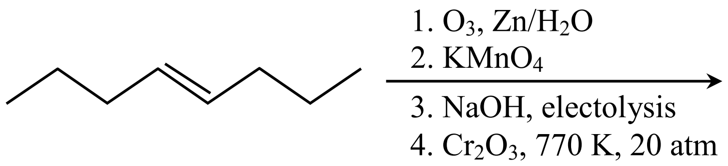


Ans. (2)

Sol.

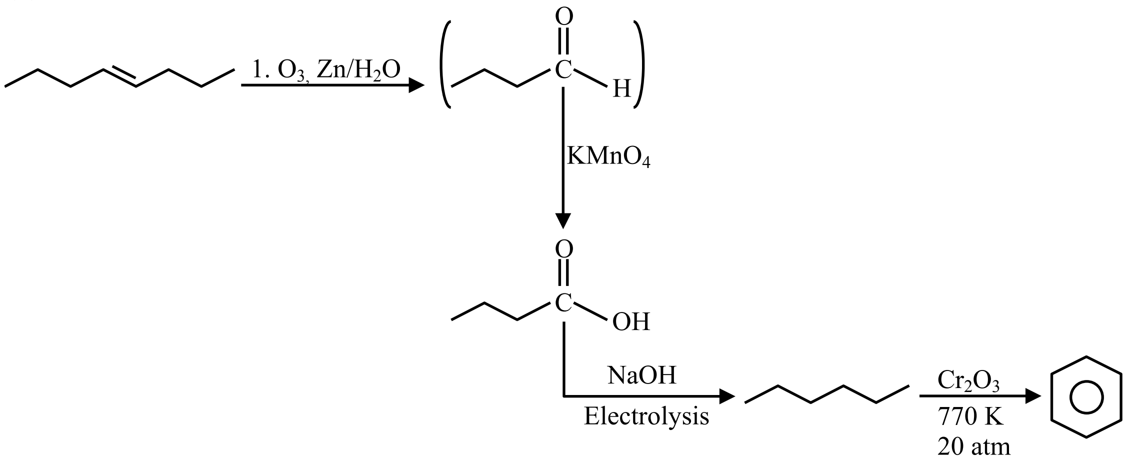


7. The number of -*CH*$_2$- (methylene) groups in the product formed from the following reaction sequence is ________.

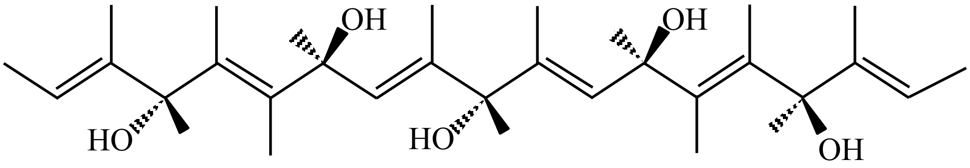


Ans. (0)

Sol.



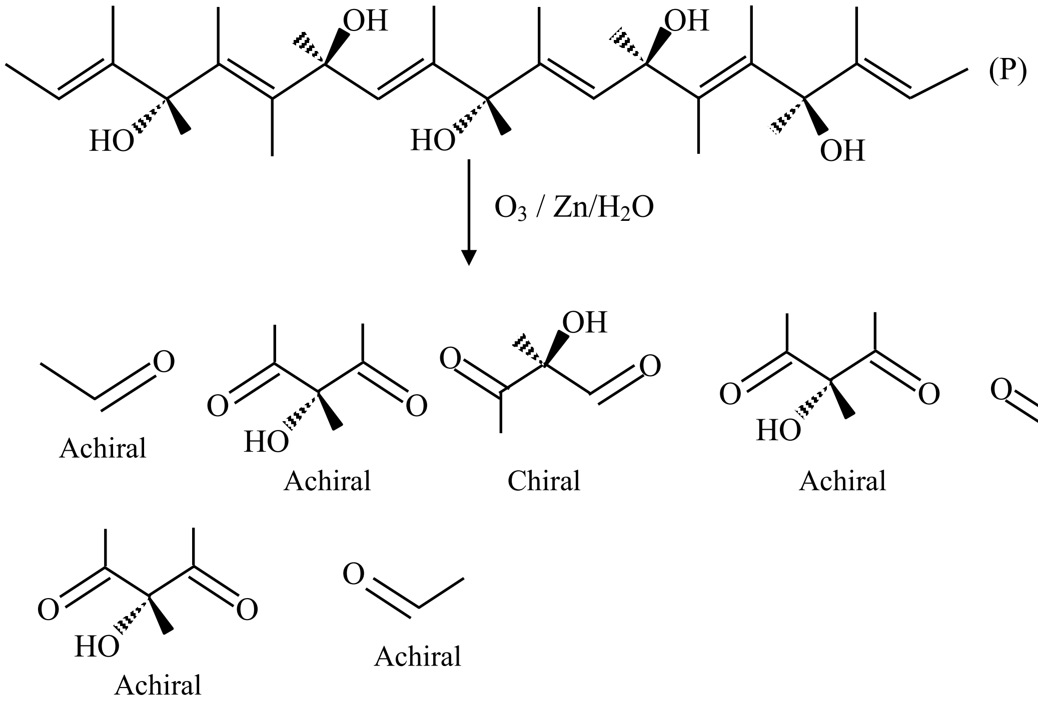
8. The total number of chiral molecules formed from one molecule of **P** on complete ozonolysis (O_3, Zn/H_2O) is ________.



P

Ans. (2)

Sol.



SECTION-2 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks : 0 If unanswered;
Negative Marks : −2 In all other cases.

9. To check the principle of multiple proportions, a series of pure binary compounds (P_mQ_n) were analyzed and their composition is tabulated below. The correct option(s) is(are)

Compound	Weight % of P	Weight % of Q
1	50	50
2	44.4	55.6
3	40	60

(A) If empirical formula of compound **3** is P_3Q_4, then the empirical formula of compound **2** is P_3Q_5.

(B) If empirical formula of compound **3** is P_3Q_2 and atomic weight of element P is 20, then the atomic weight of Q is 45.

(C) If empirical formula of compound **2** is PQ, then the empirical formula of the compound **1** is P_5Q_4.

(D) If atomic weight of P and Q are 70 and 35, respectively, then the empirical formula of compound **1** is P_2Q.

Ans. **(B,C)**

Sol.

Compound	Weight % of P	Weight % of Q
1	50	50
2	44.4	55.6
3	40	60

For option (A)

Let atomic mass of P be M_P and atomic mass of Q be M_Q

Molar ratio of atoms P : Q in compound 3 is

$$\frac{40}{M_P}:\frac{60}{M_Q}=3:4$$

$$\frac{2M_Q}{3M_p}=\frac{3}{4}\Rightarrow 9M_P=8M_Q$$

Molar ratio of atoms P : Q in compound 2 is

$$\frac{44.4}{M_P}:\frac{55.6}{M_Q}$$

$= 44.4\ M_Q : 55.6\ M_P$

$= 44.4\ M_Q : 55.6 \times \frac{8M_Q}{9}$

$= 44.4 : 55.6 \times \frac{8}{9}$

$= 9 : 10$

⇒ Empirical formula of compound 2 is therefore P_9Q_{10}

Option (A) in incorrect

For option (B)

Molar Ratio of atoms P : Q in compound 3 is $\frac{40}{M_P}:\frac{60}{M_Q}=3:2$

$$\frac{2M_Q}{3M_P}=\frac{3}{2}\Rightarrow 9M_P=4M_Q$$

If $M_P = 20$ $\quad\Rightarrow M_Q=\frac{9\times 20}{4}=45$

Option (B) is correct

For option (C)

Molar ratio of atoms P : Q in compound 2 is

$$\frac{44.4}{M_P}:\frac{55.6}{M_Q}=44.4M_Q:55.6\,M_P=1:1$$

$$\Rightarrow\frac{M_P}{M_Q}=\frac{44.4}{55.6}$$

Molar ratio of atoms P : Q in compound 1 is

$$\frac{50}{M_P}:\frac{50}{M_Q}=M_Q:M_P$$

$$= 55.6 : 44.4$$

$$\simeq 5 : 4$$

Hence, empirical formula of compound 1 is P_5Q_4

Hence, option (C) is correct

For option (D)

Molar ratio of atoms P : Q in compound 1 is

$$\frac{50}{M_P}:\frac{50}{M_Q}=M_Q:M_P$$

$$= 35 : 70 = 1 : 2$$

Hence, empirical formula of compound 1 is PQ_2

Hence, option (D) is incorrect

10. The correct option(s) about entropy (S) is(are)

[R = gas constant, F = Faraday constant, T = Temperature]

(A) For the reaction, $M(s) + 2H^+(aq) \rightarrow H_2(g) + M^{2+}(aq)$, if $\frac{dE_{cell}}{dT}=\frac{R}{F}$, then the entropy change of the reaction is R (assume that entropy and internal energy changes are temperature independent).

(B) The cell reaction, $Pt(s) \mid H_2(g, 1bar) \mid H^+(aq, 0.01M) \parallel H^+(aq, 0.1M) \mid H_2(g, 1bar) \mid Pt(s)$, is an entropy driven process.

(C) For racemization of an optically active compound, $\Delta S > 0$.

(D) $\Delta S > 0$, for $[Ni(H_2O)_6]^{2+} + 3\ en \rightarrow [Ni(en)_3]^{2+} + 6H_2O$ (where en = ethylenediamine).

Ans. (B,C,D)

Sol. $\Delta G = \Delta H - T\Delta S$

$$\Delta G = \Delta H + T\left(\frac{d\Delta G}{dT}\right)_p$$

$$-nF\left(\frac{dE_{cell}}{dT}\right) = -\Delta S$$

$$\frac{dE_{cell}}{dT} = \frac{\Delta S}{nF} = \frac{R}{F}(\text{given})$$

$\Rightarrow \Delta S = nR$

For the reaction, $M(g) + 2H^{\oplus}(aq) \longrightarrow H_2(g) + M^{2\oplus}(aq)$

$n = 2$

$\Rightarrow \quad \Delta S = 2R$

Hence, option (A) is incorrect

For the reaction, $Pt_{(s)}\,|H_{2(g)}, 1\text{ bar}|\, H^{\oplus}_{aq}(0.01M)||\, H^{\oplus}(aq, 0.1M)\,|\, H_2(g, 1\text{ bar})|\, Pt_{(s)}$

$$E_{cell} = E^{\circ}_{cell} - \frac{0.0591}{1}\log\frac{0.01}{0.1} = 0.0591V$$

E_{cell} is positive $\Rightarrow \Delta G < 0$ and $\Delta S > 0$ ($\Delta H = 0$ for concentration cells)

Hence, option (B) is correct

Racemization of an optically active compound is a spontaneous process.

Here, $\Delta H = 0$ (similar type of bonds are present in enantiomers)

$\Rightarrow \Delta S > 0$

Hence, option (C) is correct.

$\left[Ni(H_2O)_6\right]^{2+} + 3\,en \rightarrow \left[Ni(en)_3\right]^{2+} + 6H_2O$ is a spontaneous process

more stable complex is formed

$\Rightarrow \Delta S > 0$

Hence, option (D) is correct.

11. The compound(s) which react(s) with NH_3 to give boron nitride (BN) is(are)

(A) B (B) B_2H_6 (C) B_2O_3 (D) HBF_4

Ans. (B,C)

Sol. (A) $2B + 2NH_3 \rightarrow 2BN + 3H_2$

Boron produced BN with ammonia but **Boron is element not compound.** So that this option not involve in answer.

(B) $3B_2H_6 + 6NH_3 \rightarrow 3[BH_2(NH_3)_2]^+[BH_4^-] \xrightarrow{T = 200°C} 2B_3N_3H_6 + 12H_2$

$B_3N_3H_6 \xrightarrow{T > 200°C} (BN)_x$

(C) $B_2O_3(\ell) + 2NH_3 \xrightarrow{1200°C} 2BN_{(s)} + 3H_2O_{(g)}$

(D) $HBF_4 + NH_3 \rightarrow NH_4[BF_4]$

12. The correct option(s) related to the extraction of iron from its ore in the blast furnace operating in the temperature range 900 – 1500 K is(are)

(A) Limestone is used to remove silicate impurity.

(B) Pig iron obtained from blast furnace contains about 4% carbon.

(C) Coke (C) converts CO_2 to CO.

(D) Exhaust gases consist of NO_2 and CO.

Ans. (A,B,C)

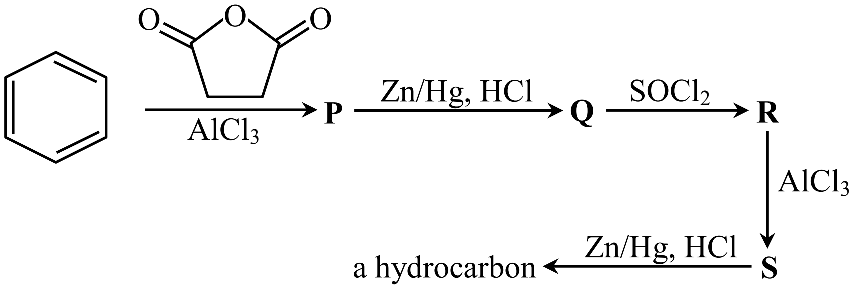
Sol. (A) $CaO + SiO_2 \rightarrow CaSiO_3$ (in the temperature range 900 – 1500 K)

(B) In fusion zone molten iron becomes heavy by absorbing elemental impurities and produces Pig iron. (in the temperature range 900 – 1500 K)

(C) $C + CO_2 \rightarrow 2CO$ (in the temperature range 900 – 1500 K)

(D) Exhaust gases does not contain NO_2.

13. Considering the following reaction sequence, the correct statement(s) is(are)



(A) Compounds **P** and **Q** are carboxylic acids.

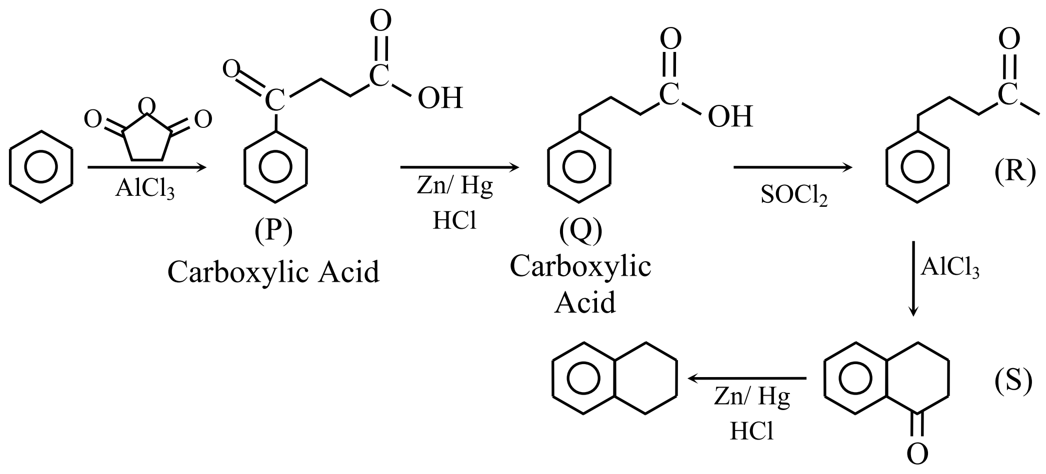
(B) Compound **S** decolorizes bromine water.

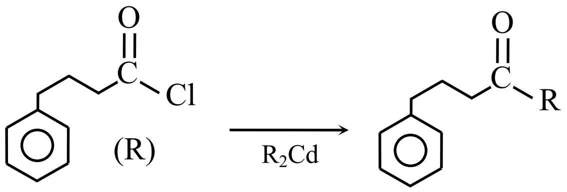
(C) Compounds **P** and **S** react with hydroxylamine to give the corresponding oximes.

(D) Compound **R** reacts with dialkylcadmium to give the corresponding tertiary alcohol.

Ans. (A,C)

Sol.





14. Among the following, the correct statement(s) about polymers is(are)

(A) The polymerization of chloroprene gives natural rubber.

(B) Teflon is prepared from tetrafluoroethene by heating it with persulphate catalyst at high pressures.

(C) PVC are thermoplastic polymers.

(D) Ethene at 350-570 K temperature and 1000-2000 atm pressure in the presence of a peroxide initiator yields high density polythene.

Ans. **(B,C)**

Sol. (a) The polymerisation of neoprene gives natural rubber.

(b) is correct statement

(c) is correct statement

(d) Ethene at 350-570 K temperature and 1000-2000 atm pressure in the pressure of a peroxide initiator yields low density polythene.

SECTION-3 : (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +3	If **ONLY** the correct option is chosen;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: −1	In all other cases.

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15. Atom X occupies the fcc lattice sites as well as alternate tetrahedral voids of the same lattice. The packing efficiency (in %) of the resultant solid is closest to

(A) 25 (B) 35 (C) 55 (D) 75

Ans. (B)

Atom 'X' occupies FCC lattice points as well as alternate tetrahedral voids of the same lattice

$\Rightarrow \frac{1}{4}$th distance of body diagonal

$$= \frac{\sqrt{3}a}{4} = 2r_X$$

$$\Rightarrow a = \frac{8r_X}{\sqrt{3}}$$

Number of atoms of X per unit cell

= 4	+	4	= 8
(FCC lattice points)		(Alternate tetrahedral voids)	

$$\% \text{ packing efficiency} = \frac{\text{Volume occupied by X}}{\text{Volume of cubic unit cell}} \times 100$$

$$= \frac{8 \times \frac{4}{3}\pi(r_X)^3}{a^3} \times 100$$

$$= \frac{8 \times \frac{4}{3}\pi(r_X)^3}{\left(\frac{8r_X}{\sqrt{3}}\right)^3} \times 100$$

$$= \left(8 \times \frac{4}{3} \times \pi \times \frac{1}{8^3} \times 3\sqrt{3}\right) \times 100$$

$$= \frac{\sqrt{3}\pi}{16} \times 100$$

$$= 34\%$$

Hence, option (B) is the most appropriate option

16. The reaction of $HClO_3$ with HCl gives a paramagnetic gas, which upon reaction with O_3 produces

(A) Cl_2O (B) ClO_2 (C) Cl_2O_6 (D) Cl_2O_7

Ans. (C)

Sol. $HClO_3 + HCl \rightarrow \underset{\text{(Paramagnetic)}}{ClO_2} + \frac{1}{2}Cl_2 + H_2O$

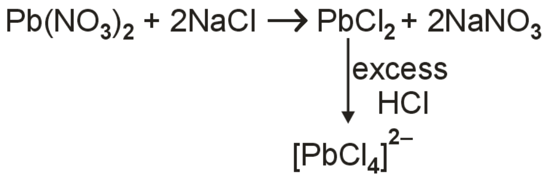
$2ClO_2 + 2O_3 \rightarrow Cl_2O_6 + 2O_2$

17. The reaction $Pb(NO_3)_2$ and NaCl in water produces a precipitate that dissolves upon the addition of HCl of appropriate concentration. The dissolution of the precipitate is due to the formation of

(A) $PbCl_2$ (B) $PbCl_4$ (C) $[PbCl_4]^{2-}$ (D) $[PbCl_6]^{2-}$

Ans. (C)

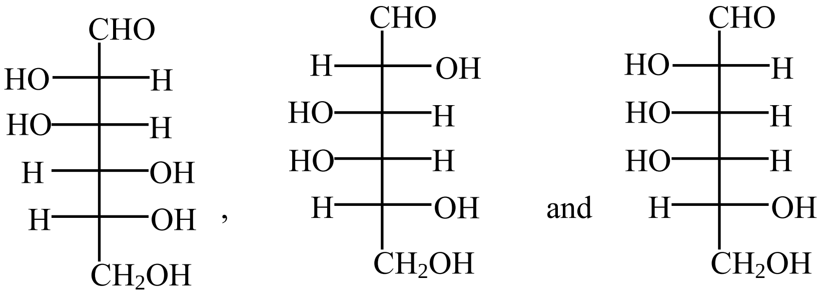
Sol.



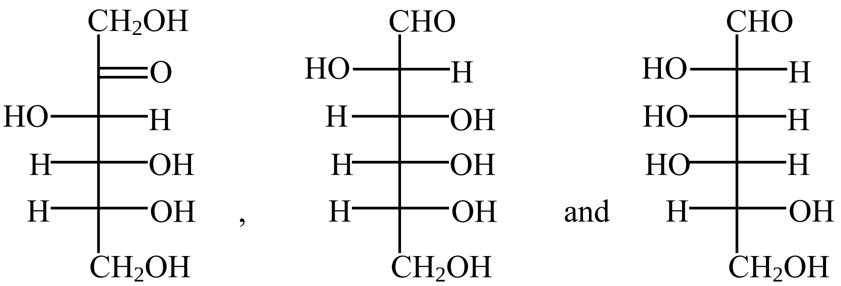
$$\begin{tikzcd} Pb(NO_3)_2 + 2NaCl \arrow[r] & PbCl_2 + 2NaNO_3 \arrow[d, "\text{excess HCl}"] \\ & {[PbCl_4]^{2-}} \end{tikzcd}$$

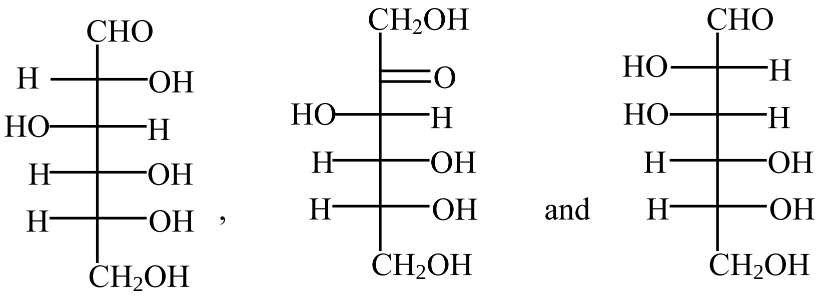
18. Treatment of D- glucose with aqueous NaOH results in a mixture of monosaccharides, which are

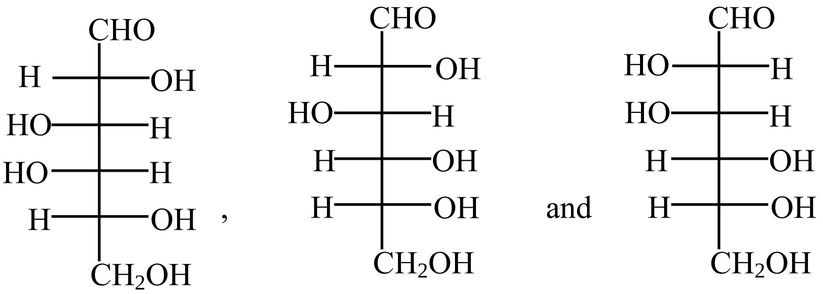
(A)



(B)

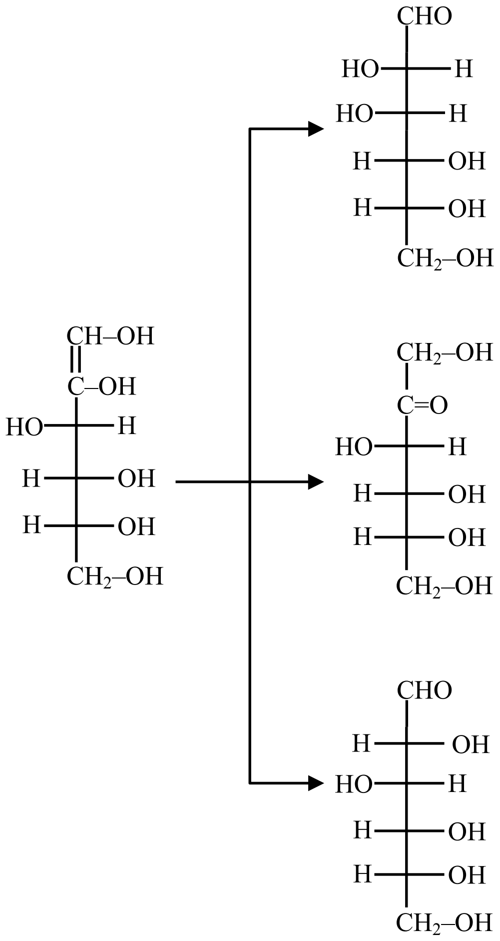


(C) 

(D) 

Ans. (C)

Sol. Basic catalyse tautomerism through enediol intermediate



MATHEMATICS

SECTION-1 : (Maximum Marks : 24)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER ranging from 0 TO 9, BOTH INCLUSIVE.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +3	If **ONLY** the correct integer is entered;
Zero Marks	: 0	If the question is unanswered;
Negative Marks	: −1	In all other cases.

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1. Let α and β be real numbers such that $-\frac{\pi}{4} < \beta < 0 < \alpha < \frac{\pi}{4}$. If $\sin(\alpha+\beta) = \frac{1}{3}$ and $\cos(\alpha-\beta) = \frac{2}{3}$,

then the greatest integer less than or equal to

$$\left(\frac{\sin\alpha}{\cos\beta} + \frac{\cos\beta}{\sin\alpha} + \frac{\cos\alpha}{\sin\beta} + \frac{\sin\beta}{\cos\alpha}\right)^2$$

is _______.

Ans. 1

Sol. $\alpha \in \left(0, \frac{\pi}{4}\right), \beta \in \left(-\frac{\pi}{4}, 0\right) \Rightarrow \alpha + \beta \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

$\sin(\alpha+\beta) = \frac{1}{3}, \cos(\alpha-\beta) = \frac{2}{3}$

$$\left(\frac{\sin\alpha}{\cos\beta} + \frac{\cos\alpha}{\sin\beta} + \frac{\cos\beta}{\sin\alpha} + \frac{\sin\beta}{\cos\alpha}\right)^2$$

$$\left(\frac{\cos(\alpha-\beta)}{\cos\beta\sin\beta} + \frac{\cos(\beta-\alpha)}{\sin\alpha\cos\alpha}\right)^2$$

$$= 4\cos^2(\alpha-\beta)\left(\frac{1}{\sin 2\beta} + \frac{1}{\sin 2\alpha}\right)^2$$

$$= 4\cos^2(\alpha-\beta)\left(\frac{2\sin(\alpha+\beta)\cos(\alpha-\beta)}{\sin 2\alpha \sin 2\beta}\right) \qquad \ldots(1)$$

$$=\frac{16\cos^4(\alpha-\beta)\sin^2(\alpha+\beta)\times 4}{\left(\cos 2(\alpha-\beta)-\cos 2(\alpha+\beta)\right)^2}$$

$$=\frac{64\cos^4(\alpha-\beta)\sin^2(\alpha+\beta)}{\left(2\cos^2(\alpha-\beta)-1-1+2\sin^2(\alpha+\beta)\right)^2}$$

$$=64\times\frac{16}{81}\times\frac{1}{9}\frac{1}{\left(2\times\frac{4}{9}-1-1+\frac{2}{9}\right)^2}$$

$$=\frac{64\times 16}{81\times 9}\cdot\frac{81}{64}=\frac{16}{9}$$

$$\left[\frac{16}{9}\right]=1\ \text{Ans.}$$

2. If y(x) is the solution of the differential equation

$$xdy-(y^2-4y)dx=0 \text{ for } x>0,\ y(1)=2,$$

and the slope of the curve y = y(x) is never zero, then the value of $10y\left(\sqrt{2}\right)$ is ______.

Ans. 8

Sol. $xdy-\left(y^2-4y\right)dx=0, x>0$

$$\int\frac{dy}{y^2-4y}=\int\frac{dx}{x}$$

$$\int\left(\frac{1}{y-4}-\frac{1}{y}\right)dy=4\int\frac{dx}{x}$$

$$\log_e|y-4|-\log_e|y|=4\log_e x+\log_e c$$

$$\frac{|y-4|}{|y|}=cx^4\xrightarrow{(1,2)}c=1$$

$$|y-4|=|y|x^4$$

C-1	and	C-2
$y-4=yx^4$		$y-4=-yx^4$
$y=\frac{4}{1-x^4}$		$y=\frac{4}{1+x^4}$
y(1) = ND (rejected)		y(1) = 2

$$y\left(\sqrt{2}\right)=\frac{4}{5}\Rightarrow 10y\left(\sqrt{2}\right)=8$$

3. The greatest integer less than or equal to

$$\int_1^2 \log_2(x^3+1)dx + \int_1^{\log_2 9} \left(2^x-1\right)^{\frac{1}{3}} dx$$

is______.

Ans. 5

Sol. $f\left(x\right)=\log_2\left(x^3+1\right)=y$

$x^3+1=2^y \Rightarrow x=\left(2^y-1\right)^{1/3}=f^{-1}\left(y\right)$

$f^{-1}\left(x\right)=\left(2^x-1\right)^{1/3}$

$=\int_1^2 \log_2\left(x^3+1\right)dx+\int_1^{\log_2 9}\left(2^x-1\right)^{1/3}dx$

$=\int_1^2 f\left(x\right)dx+\int_1^{\log_2 9} f^{-1}\left(x\right)dx=2\log_2 9-1$

$=8<9<2^{7/2}\Rightarrow 3<\log_2 9<\frac{7}{2}$

$=5<2\log_2 9-1<6$

$\left[2\log_2 9-1\right]=5$

4. The product of all positive real values of x satisfying the equation

$$x^{(16(\log_5 x)^3-68\log_5 x)} = 5^{-16}$$

is______.

Ans. 1

Sol. $x^{16\left(\log_5 x\right)^3-68\log_5 x}=5^{-16}$

Take log to the base 5 on both sides and put $\log_5 x=t$

$16t^4 - 68t^2 + 16 = 0$

$\Rightarrow 4t^4-17t^2+4=0\left\{\begin{array}{l}t_1\\t_2\\t_3\\t_4\end{array}\right.$

$t_1 + t_2 + t_3 + t_4 = 0$

$\log_5 x_1+\log_5 x_2+\log_5 x_3+\log_5 x_4=0$

$x_1x_2x_3x_4=1$

5. If

$$\beta = \lim_{x\to 0}\frac{e^{x^3}-\left(1-x^3\right)^{\frac{1}{3}}+\left(\left(1-x^2\right)^{\frac{1}{2}}-1\right)\sin x}{x\sin^2 x}$$

then the value of 6β is ______.

Ans. 5

Sol. $$\beta = \lim_{x\to 0}\frac{e^{x^3}-\left(1-x^3\right)^{1/3}}{\frac{x\sin^2 x}{x^2}x^2}+\frac{\left(\left(1-x^2\right)^{1/2}-1\right)\sin x}{x\frac{\sin^2 x}{x^2}x^2}$$

use expansion

$$\beta = \lim_{x\to 0}\frac{\left(1+x^3\right)-\left(1-\frac{x^3}{3}\right)}{x^3}+\lim_{x\to 0}\frac{\left(\left(1-\frac{x^2}{2}\right)-1\right)}{x^2}\frac{\sin x}{x}$$

$$\beta = \lim_{x\to 0}\frac{4x^3}{3x^3}+\lim_{x\to 0}\frac{-x^2}{2x^2}$$

$$\beta = \frac{4}{3}-\frac{1}{2}=\frac{5}{6}$$

$6\beta = 5$

6. Let β be a real number. Consider the matrix

$$A = \begin{pmatrix}\beta & 0 & 1\\ 2 & 1 & -2\\ 3 & 1 & -2\end{pmatrix}$$

If $A^7 - (\beta - 1)A^6 - \beta A^5$ is a singular matrix, then the value of 9β is ______.

Ans. 3

Sol. $A = \begin{pmatrix}\beta & 0 & 1\\ 2 & 1 & -2\\ 3 & 1 & -2\end{pmatrix}$ $|A| = -1$

$\Rightarrow |A^7 - (\beta - 1)A^6 - \beta A^5| = 0$

$\Rightarrow |A|^5\, |A^2 - (\beta - 1)A - \beta I| = 0$

$\Rightarrow |A|^5\, |(A^2 - \beta A) + A - \beta I| = 0$

$\Rightarrow |A|^5\, |A(A - \beta I) + I(A - \beta I)| = 0$

$|A|^5\, |(A + I)\,(A - \beta I)| = 0$

$$A+I=\begin{pmatrix}\beta+1 & 0 & 1\\ 2 & 2 & -2\\ 3 & 1 & -1\end{pmatrix}\Rightarrow |A+I|=-4\text{, Here } |A|\neq 0\ \&\ |A+I|\neq 0$$

$$A-\beta I=\begin{pmatrix}0 & 0 & 1\\ 2 & 1-\beta & -2\\ 3 & 1 & -2-\beta\end{pmatrix}$$

$$|A-\beta I|=2-3(1-\beta)=3\beta-1=0\Rightarrow \beta=\frac{1}{3}$$

$9\beta = 3$

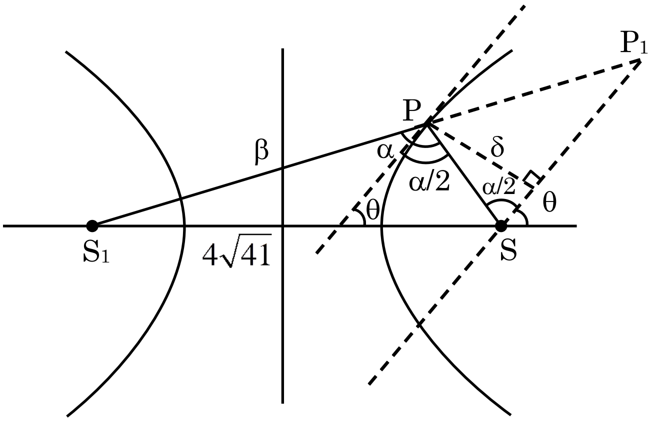
7. Consider the hyperbola

$$\frac{x^2}{100}-\frac{y^2}{64}=1$$

with foci at S and S_1, where S lies on the positive x-axis. Let P be a point on the hyperbola, in the first quadrant. Let $\angle SPS_1 = \alpha$, with $\alpha < \frac{\pi}{2}$. The straight line passing through the point S and having the same slope as that of the tangent at P to the hyperbola, intersects the straight line S_1P at P_1. Let δ be the distance of P from the straight line SP_1, and $\beta = S_1P$. Then the greatest integer less than or equal to $\frac{\beta\delta}{9}\sin\frac{\alpha}{2}$ is______.

Ans. 7

Sol.



$S_1P - SP = 20$

$$\beta - \frac{\delta}{\sin\frac{\alpha}{2}} = 20$$

$$\beta^2 + \frac{\delta^2}{\sin^2\frac{\alpha}{2}} - 400 = \frac{2\beta\delta}{\sin\frac{\alpha}{2}}$$

$$\frac{1}{SP} = \frac{\sin\frac{\alpha}{2}}{\delta}$$

$$\cos\alpha = \frac{SP^2 + \beta^2 - 656}{2\beta\frac{\delta}{\sin\frac{\alpha}{2}}}$$

$$= \frac{\frac{2\beta\delta}{\sin\frac{\alpha}{2}} - 256}{\frac{2\beta S}{\sin\frac{\alpha}{2}}} = \cos\alpha$$

$$\frac{\lambda - 128}{\lambda} = \cos\alpha$$

$\lambda(1 - \cos\alpha) = 128$

$$\frac{\beta\delta}{\sin\frac{\alpha}{2}}.2\sin^2\frac{\alpha}{2} = 128$$

$\frac{\beta\delta}{9}\sin\frac{\alpha}{2} = \frac{64}{9} \Rightarrow \left[\frac{\beta\delta}{9}\sin\frac{\alpha}{2}\right] = 7$ where [.] denotes greatest integer function

8. Consider the functions $f, g : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = x^2 + \frac{5}{12} \text{ and } g(x) = \begin{cases} 2\left(1 - \frac{4|x|}{3}\right), & |x| \le \frac{3}{4}, \\ 0, & |x| > \frac{3}{4}. \end{cases}$$

If α is the area of the region

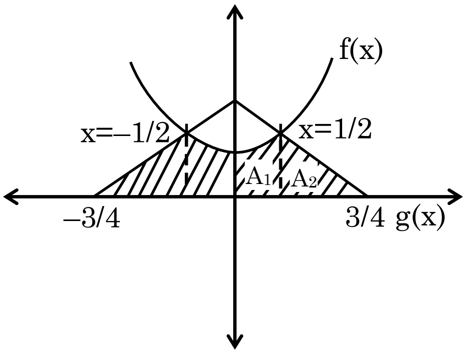
$$\left\{(x,y) \in \mathbb{R} \times \mathbb{R} : |x| \le \frac{3}{4},\ 0 \le y \le \min\{f(x), g(x)\}\right\},$$

then the value of 9α is ______.

Ans. 6

Sol. $x^2+\frac{5}{12}=\frac{2-8x}{3}$

$x^2+\frac{8x}{3}+\frac{5}{12}-2=0$



$12x^2 + 32x - 19 = 0$

$12x^2 + 38x - 6x - 19 = 0$

$2x(6x + 19) - 1(6x + 19) = 0$

$(6x + 19)(2x - 1) = 0$

$\boxed{x=\frac{1}{2}}$

$\alpha = 2A_1 + A_2$

$$\alpha=2\left(\int_0^{1/2} x^2+\frac{5}{12}dx+\frac{1}{2}\times\frac{1}{4}\times\frac{2}{3}\right)$$

$$\Rightarrow \alpha=2\left[\left(\frac{x^3}{3}+\frac{5x}{12}\right)_0^{1/2}+\frac{1}{12}\right]$$

$$\Rightarrow \alpha=2\left[\frac{1}{24}+\frac{5}{24}+\frac{1}{12}\right]$$

$$\Rightarrow \alpha=2\left[\frac{1+5+2}{24}\right]\Rightarrow \alpha=2\times\frac{8}{24}\Rightarrow 9\alpha=9\times\frac{8}{12}$$

$$\Rightarrow 9\alpha=6$$

SECTION-2 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	: +3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	: +2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	: +1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	: 0	If unanswered;
Negative Marks	: –2	In all other cases.

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9. Let PQRS be a quadrilateral in a plane, where QR = 1, $\angle PQR = \angle QRS = 70^\circ$, $\angle PQS = 15^\circ$ and $\angle PRS = 40^\circ$. If $\angle RPS = \theta^\circ$, PQ = α and PS = β, then the interval(s) that contain(s) the value of $4\alpha\beta\sin\theta^\circ$ is/are

(A) $\left(0,\sqrt{2}\right)$ (B) $(1, 2)$

(C) $\left(\sqrt{2},3\right)$ (D) $\left(2\sqrt{2},3\sqrt{2}\right)$

Ans. **(A,B)**

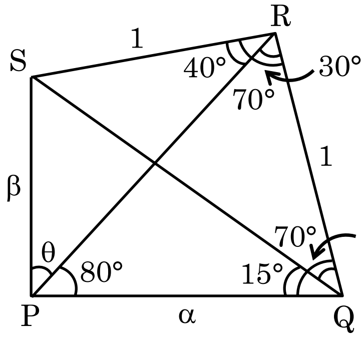
Sol. $\angle PRQ = 70^\circ - 40^\circ = 30^\circ$

$\angle RQS = 70^\circ - 15^\circ = 55^\circ$

$\angle QSR = 180^\circ - 55^\circ - 70^\circ = 55$

$\therefore$ QR = RS = 1

$\angle QPR = 180^\circ - 70^\circ - 30^\circ = 80^\circ$



Apply sine-rule in ΔPRQ :

$$\frac{\alpha}{\sin 30^\circ} = \frac{1}{\sin 80^\circ} \Rightarrow \alpha = \frac{1}{2\sin 80^\circ} \qquad \ldots.(1)$$

Apply sine-rule in ΔPRS

$$\frac{\beta}{\sin 40^\circ} = \frac{1}{\sin\theta} \Rightarrow \beta\sin\theta = \sin 40^\circ \qquad \ldots.(2)$$

$$4\alpha\beta\sin\theta = \frac{4\sin 40^\circ}{2\sin 80^\circ} = \frac{4\sin 40^\circ}{2(2\sin 40^\circ\cos 40^\circ)}$$

$= \sec 40^\circ$

Now $\sec 30^\circ < \sec 40^\circ < \sec 45^\circ$

$$\Rightarrow \frac{2}{\sqrt{3}} < \sec 40^\circ < \sqrt{2}$$

10. Let

$$\alpha = \sum_{k=1}^{\infty} \sin^{2k}\left(\frac{\pi}{6}\right).$$

Let $g : [0, 1] \rightarrow \mathbb{R}$ be the function defined by

$$g(x) = 2^{\alpha x} + 2^{\alpha(1-x)}$$

Then, which of the following statements is/are TRUE?

(A) The minimum value of $g(x)$ is $2^{\frac{7}{6}}$

(B) The maximum value of $g(x)$ is $1 + 2^{\frac{1}{3}}$

(C) The function $g(x)$ attains its maximum at more than one point

(D) The function $g(x)$ attains its minimum at more than one point

Ans. (A,B,C)

Sol. $\alpha = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^6 + \ldots$

$$\alpha = \frac{\frac{1}{4}}{1-\frac{1}{4}} = \frac{1}{3}$$

$\therefore$ g(x) = $2^{x/3} + 2^{1/3(1-x)}$

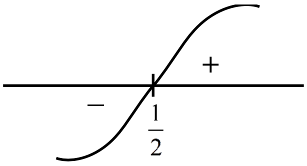
$\therefore$ g(x) = $2^{x/3} + \frac{2^{1/3}}{2^{x/3}}$

where g(0) = $1 + 2^{1/3}$ & g(1) = $1 + 2^{1/3}$

$\therefore\ g'(x) = \frac{1}{3}\left(2^{x/3} - \frac{2^{1/3}}{2^{x/3}}\right) = 0$

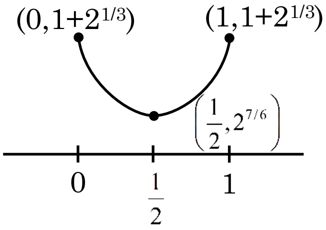
$\Rightarrow 2^{2x/3} = 2^{1/3} \Rightarrow x = \frac{1}{2}$ = critical point

$\therefore$ graph of g'(x) =



& $g\left(\frac{1}{2}\right) = 2^{\frac{7}{6}}$

$\therefore$ graph of g(x) in [0, 1]



11. Let $\bar{z}$ denote the complex conjugate of a complex number z. If z is a non-zero complex number for which both real and imaginary parts of

$$(\bar{z})^2 + \frac{1}{z^2}$$

are integers, then which of the following is/are possible value(s) of |z| ?

(A) $\left(\frac{43+3\sqrt{205}}{2}\right)^{\frac{1}{4}}$

(B) $\left(\frac{7+\sqrt{33}}{4}\right)^{\frac{1}{4}}$

(C) $\left(\frac{9+\sqrt{65}}{4}\right)^{\frac{1}{4}}$

(D) $\left(\frac{7+\sqrt{13}}{6}\right)^{\frac{1}{4}}$

Ans. (A)

Sol. Let $(\overline{z})^2+\frac{1}{z^2}=m+in\ ,\ m,n\in\mathbb{Z}$

$$(\overline{z})^2+\frac{\overline{z}^2}{|z|^4}=m+in$$

$$\Rightarrow (x^2-y^2)\left(1+\frac{1}{|z|^4}\right)=m \qquad \ldots(1)$$

$$\& \ -2xy\left(1+\frac{1}{|z|^4}\right)=n \qquad \ldots(2)$$

Equation $(1)^2+(2)^2$

$$\left(1+\frac{1}{|z|^4}\right)^2\left[\left(x^2+y^2\right)^2\right]=m^2+n^2$$

$$\left(1+\frac{1}{|z|^4}\right)^2(|z|)^4=m^2+n^2$$

$$\Rightarrow |z|^4+\frac{1}{|z|^4}+2=m^2+n^2$$

Now for option (A)

$$|z|^4=\frac{43+3\sqrt{205}}{2}$$

$$\Rightarrow m^2+n^2=45$$

$$\Rightarrow m=\pm 6,\ \ n=\pm 3$$

Option (B)

$$|z|^4+\frac{1}{|z|^4}+2=\frac{7+\sqrt{33}}{4}+\frac{7-\sqrt{33}}{4}+2=\frac{7}{2}+2=\frac{11}{2}$$

Option (C)

$$|z|^4+\frac{1}{|z|^4}+2=\frac{9+\sqrt{65}}{4}+\frac{9-\sqrt{65}}{4}+2=\frac{18}{4}+2=\frac{9}{2}+2=\frac{13}{2}$$

Option (D)

$$|z|^4+\frac{1}{|z|^4}+2=\frac{7+\sqrt{13}}{6}+\frac{7-\sqrt{13}}{6}+2=\frac{14}{6}+2=\frac{7}{3}+2=\frac{13}{2}$$

12. Let G be a circle of radius $R > 0$. Let $G_1, G_2, \ldots, G_n$ be n circles of equal radius $r > 0$. Suppose each of the n circles $G_1, G_2, \ldots, G_n$ touches the circle G externally. Also, for $i = 1, 2, \ldots, n-1$, the circle G_i touches G_{i+1} externally, and G_n touches G_1 externally. Then, which of the following statements is/are TRUE ?

(A) If $n = 4$, then $(\sqrt{2} - 1)r < R$

(B) If $n = 5$, then $r < R$

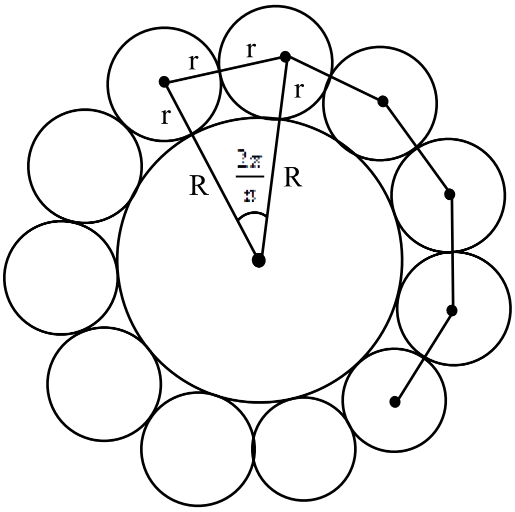
(C) If $n = 8$, then $(\sqrt{2} - 1)r < R$

(D) If $n = 12$, then $\sqrt{2}(\sqrt{3} + 1)r > R$

Ans. **(C,D)**

Sol. $2(R+r)\sin\frac{\pi}{n} = 2r$

$$\frac{R+r}{r} = \operatorname{cosec}\frac{\pi}{n}$$



(A) $n = 4,\ R + r = \sqrt{2}\,r$

(B) $n = 5,\ \frac{R+r}{r} = \operatorname{cosec}\frac{\pi}{5} < \operatorname{cosec}\frac{\pi}{6}$

$R + r < 2r \Rightarrow r > R$

(C) $n = 8,\ \frac{R+r}{r} = \operatorname{cosec}\frac{\pi}{8} > \operatorname{cosec}\frac{\pi}{4}$

$R + r > \sqrt{2}\,r$

(D) $n = 12,\ \frac{R+r}{r} = \operatorname{cosec}\frac{\pi}{12} = \sqrt{2}\left(\sqrt{3}+1\right)$

$R + r = \sqrt{2}\left(\sqrt{3}+1\right)r$

$\sqrt{2}\left(\sqrt{3}+1\right)r > R$

13. Let $\hat{i}, \hat{j}$ and $\hat{k}$ be the unit vectors along the three positive coordinate axes. Let

$$\vec{a} = 3\hat{i} + \hat{j} - \hat{k},$$

$$\vec{b} = \hat{i} + b_2\hat{j} + b_3\hat{k}, \qquad b_2, b_3 \in \mathbb{R},$$

$$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}, \qquad c_1, c_2, c_3 \in \mathbb{R}$$

be three vectors such that $b_2b_3 > 0$, $\vec{a} \cdot \vec{b} = 0$ and

$$\begin{pmatrix} 0 & -c_3 & c_2 \\ c_3 & 0 & -c_1 \\ -c_2 & c_1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 3-c_1 \\ 1-c_2 \\ -1-c_3 \end{pmatrix}.$$

Then, which of the following is/are TRUE ?

(A) $\vec{a} \cdot \vec{c} = 0$ (B) $\vec{b} \cdot \vec{c} = 0$

(C) $|\vec{b}| > \sqrt{10}$ (D) $|\vec{c}| \le \sqrt{11}$

Ans. **(B,C,D)**

Sol. $\vec{a} = 3\hat{i} + \hat{j} - \hat{k}$

$\vec{b} = \hat{i} + b_2\hat{j} + b_3\hat{k}$

$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

$$\begin{pmatrix} 0 & -c_3 & c_2 \\ c_3 & 0 & -c_1 \\ -c_2 & c_1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 3-c_1 \\ 1-c_2 \\ -1-c_3 \end{pmatrix}$$

multiply & compare

$b_2c_3 - b_3c_2 = c_1 - 3$ ...(1)

$c_3 - b_3c_1 = 1 - c_2$ ...(2)

$c_2 - b_2c_1 = 1 + c_3$ ...(3)

$(1)\hat{i} - (2)\hat{j} + (3)\hat{k}$

$\hat{i}(b_2c_3 - b_3c_2) - \hat{j}(c_3 - b_3c_1) + \hat{k}(c_2 - b_2c_1)$

$= c_1\hat{i} + c_2\hat{j} + c_2\hat{k} - 3\hat{i} - \hat{j} + \hat{k}$

$\vec{b} \times \vec{c} = \vec{c} - \vec{a}$

Take dot product with $\vec{b}$

$$0 = \vec{c}.\vec{b} - \vec{a}.\vec{b}$$

$$\vec{b}.\vec{c} = 0$$

$$\vec{b} \perp \vec{c}$$

$$\vec{b}\,\hat{}\,\vec{c} = 90^\circ$$

Take dot product with $\vec{c}$

$$0 = |\vec{c}|^2 - \vec{a}.\vec{c}$$

$$\vec{a}.\vec{c} = |\vec{c}|^2$$

$$\vec{a}.\vec{c} \neq 0$$

$$\vec{b} \times \vec{c} = \vec{c} - \vec{a}$$

Squaring

$$|\vec{b}|^2\,|\vec{c}|^2 = |\vec{c}|^2 + |\vec{a}|^2 - 2\vec{c}.\vec{a}$$

$$|\vec{b}|^2\,|\vec{c}|^2 = |\vec{c}|^2 + 11 - 2|\vec{c}|^2$$

$$|\vec{b}|^2\,|\vec{c}|^2 = 11 - |\vec{c}|^2$$

$$|\vec{c}|^2\left(|\vec{b}|^2 + 1\right) = 11$$

$$|\vec{c}|^2 = \frac{11}{|\vec{b}|^2 + 1}$$

$$|\vec{c}| \leq \sqrt{11}$$

given $\vec{a}.\vec{b} = 0$

$b_2 - b_3 = -3$ also

$b_2^2 + b_3^2 - 2b_2b_3 = 9$ $b_2b_3 > 0$

$b_2^2 + b_3^2 = 9 + 2b_2b_3$

$b_2^2 + b_3^2 = 9 + 2b_2b_3 > 9$

$b_2^2 + b_3^2 > 9$

$$|\vec{b}| = \sqrt{1 + b_2^2 + b_3^2}$$

$$|\vec{b}| > \sqrt{10}$$

14. For $x \in \mathbb{R}$, let the function y(x) be the solution of the differential equation

$$\frac{dy}{dx}+12y=\cos\left(\frac{\pi}{12}x\right),\ y(0)=0.$$

Then, which of the following statements is/are TRUE?

(A) y(x) is an increasing function

(B) y(x) is a decreasing function

(C) There exists a real number β such that the line $y = \beta$ intersects the curve $y = y(x)$ at infinitely many points

(D) y(x) is a periodic function

Ans. **(C)**

Sol. $\dfrac{dy}{dx}+12y=\cos\left(\dfrac{\pi}{12}x\right)$

Linear D.E.

$\text{I.F.} = e^{\int 12.dx}=e^{12x}$

Solution of DE

$$y.e^{12x}=\int e^{12x}.\cos\left(\frac{\pi}{12}x\right)dx$$

$$y.e^{12x}=\frac{e^{12x}}{(12)^2+\left(\frac{\pi}{12}\right)^2}\left(12\cos\frac{\pi}{12}x+\frac{\pi}{12}\sin\frac{\pi}{12}x\right)+C$$

$$\Rightarrow\ y=\frac{(12)}{(12)^4+\pi^2}\left((12)^2\cos\left(\frac{\pi x}{12}\right)+\pi\sin\left(\frac{\pi x}{12}\right)\right)+\frac{C}{e^{12x}}$$

Given y(0) = 0

$$\Rightarrow 0=\frac{12}{12^4+\pi^2}(12^2+0)+C \Rightarrow C=\frac{-12^3}{12^4+\pi^2}$$

$$\therefore\ y=\frac{12}{12^4+\pi^2}\left[(12)^2\cos\left(\frac{\pi x}{12}\right)+\pi\sin\left(\frac{\pi x}{12}\right)-12^2.e^{-12x}\right]$$

$$\text{Now }\frac{dy}{dx}=\frac{12}{12^4+\pi^2}\left[\underbrace{-12\pi\sin\left(\frac{\pi x}{12}\right)+\frac{\pi^2}{12}\cos\left(\frac{\pi x}{12}\right)}_{\text{min. value}}+12^3e^{-12x}\right]$$

$$\left(-\sqrt{144\pi^2+\frac{\pi^4}{144}}=-12\pi\sqrt{1+\frac{\pi^2}{12^4}}\right)$$

$$\Rightarrow\frac{dy}{dx}>0\ \ \forall\ x\le 0\ \&\ \text{may be negative/positive for } x>0$$

So, f(x) is neither increasing nor decreasing

For some $\beta \in R$, $y = \beta$ intersects $y = f(x)$ at infinitely many points

So option C is correct

SECTION-3 : (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If **ONLY** the correct option is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : −1 In all other cases.

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15. Consider 4 boxes, where each box contains 3 red balls and 2 blue balls. Assume that all 20 balls are distinct. In how many different ways can 10 balls be chosen from these 4 boxes so that from each box at least one red ball and one blue ball are chosen?

(A) 21816 (B) 85536

(C) 12096 (D) 156816

Ans. **(A)**

Sol.

$\boxed{\begin{matrix}3R\\2B\end{matrix}}$	$\boxed{\begin{matrix}3R\\2B\end{matrix}}$	$\boxed{\begin{matrix}3R\\2B\end{matrix}}$	$\boxed{\begin{matrix}3R\\2B\end{matrix}}$
B-1	B-2	B-3	B-4

Case-I : when exactly one box provides four balls (3R 1B or 2R 2B)

Number of ways in this case ${}^5C_4\,({}^3C_1 \times {}^2C_1)^3 \times 4$

Case-II : when exactly two boxes provide three balls (2R 1B or 1R 2B) each

Number of ways in this case $({}^5C_3 - 1)^2\,({}^3C_1 \times {}^2C_1)^2 \times 6$

Required number of ways = 21816

Language ambiguity : If we consider at least one red ball and exactly one blue ball, then required number of ways is 9504. None of the option is correct.

16. If $M = \begin{pmatrix} \frac{5}{2} & \frac{3}{2} \\ -\frac{3}{2} & -\frac{1}{2} \end{pmatrix}$, then which of the following matrices is equal to M^{2022} ?

(A) $\begin{pmatrix} 3034 & 3033 \\ -3033 & -3032 \end{pmatrix}$ (B) $\begin{pmatrix} 3034 & -3033 \\ 3033 & -3032 \end{pmatrix}$

(C) $\begin{pmatrix} 3033 & 3032 \\ -3032 & -3031 \end{pmatrix}$ (D) $\begin{pmatrix} 3032 & 3031 \\ -3031 & -3030 \end{pmatrix}$

Ans. **(A)**

Sol. $M=\begin{bmatrix}\frac{5}{2} & \frac{3}{2}\\ \frac{-3}{2} & \frac{-1}{2}\end{bmatrix}$

$$M=\begin{bmatrix}\frac{3}{2}+1 & \frac{3}{2}\\ \frac{-3}{2} & \frac{-3}{2}+1\end{bmatrix}$$

$$M=I+\frac{3}{2}\begin{bmatrix}1 & 1\\ -1 & -1\end{bmatrix}$$

Let $A=\begin{bmatrix}1 & 1\\ -1 & -1\end{bmatrix}$

$$A^2=\begin{bmatrix}1 & 1\\ -1 & -1\end{bmatrix}\begin{bmatrix}1 & 1\\ -1 & -1\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$$M^{2022}=\left(I+\frac{3}{2}A\right)^{2022}$$

$$= I + 3033A$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+3033\begin{bmatrix}1 & 1\\ -1 & -1\end{bmatrix}$$

$$=\begin{bmatrix}3034 & 3033\\ -3033 & -3032\end{bmatrix}$$

17. Suppose that

Box-I contains 8 red, 3 blue and 5 green balls,
Box-II contains 24 red, 9 blue and 15 green balls,
Box-III contains 1 blue, 12 green and 3 yellow balls,
Box-IV contains 10 green, 16 orange and 6 white balls.

A ball is chosen randomly from Box-I ; call this ball *b*. If *b* is red then a ball is chosen randomly from Box-II, if *b* is blue then a ball is chosen randomly from Box-III, and if *b* is green then a ball is chosen randomly from Box-IV. The conditional probability of the event 'one of the chosen balls is white' given that the event 'at least one of the chosen balls is green' has happened, is equal to

(A) $\frac{15}{256}$ (B) $\frac{3}{16}$ (C) $\frac{5}{52}$ (D) $\frac{1}{8}$

Ans. (C)

Sol. Box I 8(R) 3(B) 5(G)

Box II 24(R) 9(B) 15(G)

Box III 1(B) 12(G) 3(y)

Box IV 10(G) 16(o) 6(w)

A (one of the chosen balls is white)

B (at least one of the chosen ball is green)

$$P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}$$

$A \cap B \rightarrow (wG)$

$$=\frac{\frac{5}{16}\times\frac{6}{32}}{\frac{5}{16}\times 1+\frac{8}{16}\times\frac{15}{48}+\frac{3}{16}\times\frac{12}{16}}$$

$$=\frac{15}{156}=\frac{5}{52}$$

18. For positive integer n, define

$$f(n)=n+\frac{16+5n-3n^2}{4n+3n^2}+\frac{32+n-3n^2}{8n+3n^2}+\frac{48-3n-3n^2}{12n+3n^2}+\ldots+\frac{25n-7n^2}{7n^2}.$$

Then, the value of $\lim_{n\to\infty} f(n)$ is equal to

(A) $3+\frac{4}{3}\log_e 7$

(B) $4-\frac{3}{4}\log_e\left(\frac{7}{3}\right)$

(C) $4-\frac{4}{3}\log_e\left(\frac{7}{3}\right)$

(D) $3+\frac{3}{4}\log_e 7$

Ans. (B)

18. $f(n) = n + \sum_{r=1}^{n} \frac{16r + (9-4r)n - 3n^2}{4rn + 3n^2}$

$$f(n) = n + \sum_{r=1}^{n} \frac{(16r+9n) - (4rn+3n^2)}{4rn+3n^2}$$

$$f(n) = n + \left(\sum_{r=1}^{n} \frac{16r+9n}{4rn+3n^2}\right) - n$$

$$\lim_{n\to\infty} f(n) = \lim_{n\to\infty} \sum \frac{16r+9n}{4rn+3n^2}$$

$$= \lim_{n\to\infty} \sum_{r=1}^{n} \frac{\left(16\left(\frac{r}{n}\right)+9\right)\frac{1}{n}}{4\left(\frac{r}{n}\right)+3}$$

$$= \int_0^1 \frac{16x+9}{4x+3}dx = \int_0^1 4\,dx - \int_0^1 \frac{3\,dx}{4x+3}$$

$$= 4 - \frac{3}{4}\left(\ell n|4x+3|\right)_0^1$$

$$= 4 - \frac{3}{4}\ell n\frac{7}{3}$$