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## JEE Main 2020 Question Paper with Answer

January & September (Shift 1 & Shift 2)

JEE Main 2020 – 7th Jan Shift 1 Question Paper	Page No. 2 to 42
JEE Main 2020 – 7th Jan Shift 2 Question Paper	Page No. 43 to 86
JEE Main 2020 – 8th Jan Shift 1 Question Paper	Page No. 118 to 129
JEE Main 2020 – 8th Jan Shift 2 Question Paper	Page No. 130 to 176
JEE Main 2020 – 9th Jan Shift 1 Question Paper	Page No. 177 to 226
JEE Main 2020 – 9th Jan Shift 2 Question Paper	Page No. 227 to 274
JEE Main 2020 – 2nd Sep Shift 1 Question Paper	Page No. 275 to 314
JEE Main 2020 – 2nd Sep Shift 2 Question Paper	Page No. 333 to 354
JEE Main 2020 – 3rd Sep Shift 1 Question Paper	Page No. 355 to 393
JEE Main 2020 – 3rd Sep Shift 2 Question Paper	Page No. 394 to 430
JEE Main 2020 – 4th Sep Shift 1 Question Paper	Page No. 431 to 467
JEE Main 2020 – 4th Sep Shift 2 Question Paper	Page No. 468 to 504
JEE Main 2020 – 5th Sep Shift 1 Question Paper	Page No. 505 to 541
JEE Main 2020 – 5th Sep Shift 2 Question Paper	Page No. 542 to 578
JEE Main 2020 – 6th Sep Shift 1 Question Paper	Page No. 579 to 617

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$$\begin{vmatrix} 2 & 2a & a \\ 2 & 3b & b \\ 2 & 4c & c \end{vmatrix} = 0$$

$$R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$$

$$\Rightarrow \begin{vmatrix} 2 & 2a & a \\ 0 & 3b - 2a & b - a \\ 0 & 4c - 2a & c - a \end{vmatrix} = 0$$

$$\Rightarrow (3b - 2a)(c - a) - (4c - 2a)(b - a) = 0$$

$$\Rightarrow 3bc - 2ac - 3ab + 2a^2 - [4bc - 4ac - 2ab + 2a^2] = 0$$

$$\Rightarrow -bc + 2ac - ab = 0$$

$$\Rightarrow ab + bc = 2ac$$

$$\Rightarrow \frac{1}{c} + \frac{1}{a} = \frac{2}{b}$$

9. 5 numbers are in A.P whose sum is 25 and product is 2520. If one of these 5 numbers is  $-\frac{1}{2}$ , then the greatest number amongst them is

a.  $\frac{21}{2}$   
c. 27

b. 16

d. 7

**Answer:** (b)

**Solution:**

Let 5 numbers be  $a - 2d, a - d, a, a + d, a + 2d$

$$5a = 25$$

$$a = 5$$

$$(a - 2d)(a - d)a(a + d)(a + 2d) = 2520$$

$$(25 - 4d^2)(25 - d^2) = 504$$

$$4d^4 - 125d^2 + 121 = 0$$

$$4d^4 - 4d^2 - 121d^2 + 121 = 0$$

$$d^2 = 1 \text{ or } d^2 = \frac{121}{4}$$

$$d = \pm \frac{11}{2}$$

$$\text{For } d = \frac{11}{2}, a + 2d \text{ is the greatest term, } a + 2d = 5 + 11 = 16$$

10. If  $\alpha$  is a root of the equation  $x^2 + x + 1 = 0$  and  $A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}$  then  $A^{31}$  equal to

a.  $A$

c.  $A^3$

b.  $A^2$

d.  $A^4$

**Answer:** (c)

**Solution:**

The roots of equation  $x^2 + x + 1 = 0$  are complex cube roots of unity.

$$A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}$$

$$A^2 = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}$$

$$A^2 = \frac{1}{3} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & 3 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$$A^4 = I$$

$$A^{28} = I$$

Therefore, we get

$$A^{31} = A^{28} A^3$$

$$A^{31} = I A^3$$

$$A^{31} = A^3$$

11. Let  $x^k + y^k = a^k$  where  $a, k > 0$  and  $\frac{dy}{dx} + \left(\frac{y}{x}\right)^{\frac{1}{3}} = 0$ , then  $k$  is

a.  $\frac{1}{3}$   
c.  $\frac{4}{3}$

b.  $\frac{2}{3}$   
d. 2

**Answer:** (b)

**Solution:**

$$x^k + y^k = a^k$$

$$kx^{k-1} + ky^{k-1} \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\left(\frac{x}{y}\right)^{k-1}$$

$$\Rightarrow \frac{dy}{dx} + \left(\frac{y}{x}\right)^{1-k} = 0$$

$$\Rightarrow 1 - k = \frac{1}{3}$$

$$\Rightarrow k = \frac{2}{3}$$

12. If real part of  $\left(\frac{z-1}{2z+i}\right) = 1$  where  $z = x + iy$ , then the point  $(x, y)$  lies on

a. straight line with slope 2

b. straight line with slope  $\frac{1}{2}$

c. circle with diameter  $\frac{\sqrt{5}}{2}$

d. circle with diameter  $\frac{1}{2}$

**Answer:** (c)

**Solution:**

$$z = x + iy$$

$$\frac{x + iy - 1}{2x + 2iy + i} = \frac{(x-1) + iy}{2x + i(2y+1)} \left( \frac{2x - i(2y+1)}{2x - i(2y+1)} \right) = 1$$

$$\frac{2x(x-1) + y(2y+1)}{4x^2 + (2y+1)^2} = 1$$

$$2x^2 + 2y^2 - 2x + y = 4x^2 + 4y^2 + 4y + 1$$

$$x^2 + y^2 + x + \frac{3}{2}y + \frac{1}{2} = 0$$

Circle's centre will be  $\left(-\frac{1}{2}, -\frac{3}{4}\right)$

$$\text{Radius} = \sqrt{\frac{1}{4} + \frac{9}{16} - \frac{1}{2}} = \frac{\sqrt{5}}{4}$$

$$\text{Diameter} = \frac{\sqrt{5}}{2}$$

13. If  $y(\alpha) = \sqrt{\frac{2(\tan \alpha + \cot \alpha)}{1 + \tan^2 \alpha} + \frac{1}{\sin^2 \alpha}}$  where  $\alpha \in \left(\frac{3\pi}{4}, \pi\right)$  then find  $\frac{dy}{d\alpha}$  at  $\alpha = \frac{5\pi}{6}$

a. 4

b. 2

c. 3

d. -4

**Answer:** (a)

**Solution:**

$$y(\alpha) = \sqrt{2 \frac{\tan \alpha + \cot \alpha}{1 + \tan^2 \alpha} + \frac{1}{\sin^2 \alpha}}$$

$$y(\alpha) = \sqrt{2 \frac{1}{\sin \alpha \cos \alpha \times \frac{1}{\cos^2 \alpha}} + \frac{1}{\sin^2 \alpha}}$$

$$y(\alpha) = \sqrt{2 \cot \alpha + \operatorname{cosec}^2 \alpha}$$

$$y(\alpha) = \sqrt{(1 + \cot \alpha)^2}$$

$$y(\alpha) = -1 - \cot \alpha$$

$$\frac{dy}{d\alpha} = 0 + \operatorname{cosec}^2 \alpha \Big|_{\alpha = \frac{5\pi}{6}}$$

$$\frac{dy}{d\alpha} = \operatorname{cosec}^2 \frac{5\pi}{6}$$

$$\frac{dy}{d\alpha} = 4$$

14. Find the greatest integer  $k$  for which  $49^k + 1$  is a factor of the given sum  
 $49^{125} + 49^{124} + \dots + 49^2 + 49 + 1$

a. 63

b. 65

c. 32

d. 60

**Answer:** (a)

**Solution:**

$$\begin{aligned} 1 + 49 + 49^2 + \dots + 49^{125} &= \frac{49^{126} - 1}{49 - 1} \\ &= \frac{(49^{63} + 1)(49^{63} - 1)}{48} \\ &= \frac{(49^{63} + 1)((1 + 48)^{63} - 1)}{48} \\ &= \frac{(49^{63} + 1)(1 + 48I - 1)}{48}; \text{ Where } I \text{ is an integer} \\ &= (49^{63} + 1)I \end{aligned}$$

Greatest positive integer is  $k = 63$

15. If  $A(1,1)$ ,  $B(6,5)$ ,  $C\left(\frac{3}{2}, 2\right)$  are the vertices of  $\triangle ABC$ . A point  $P$  is such that area of  $\triangle PAB$ ,  $\triangle PAC$  and  $\triangle PBC$  are equal, then find the length of the line the segment  $PQ$ , where  $Q$  is the point  $\left(-\frac{7}{6}, -\frac{1}{3}\right)$

a. 2

b. 3

c. 4

d. 5

**Answer:** (d)

**Solution:**

$P$  is the centroid which is  $\equiv \left( \frac{1+6+\frac{3}{2}}{3}, \frac{1+5+2}{3} \right)$

$$P = \left( \frac{17}{6}, \frac{8}{3} \right)$$

$$Q = \left( -\frac{7}{6}, -\frac{1}{3} \right)$$

$$PQ = \sqrt{(4)^2 + (3)^2} = 5$$

16. If  $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$  lies in plane of  $\vec{b}$  and  $\vec{c}$  where  $\vec{b} = \hat{i} + \hat{j}$  and  $\vec{c} = \hat{i} - \hat{j} + 4\hat{k}$  and  $\vec{a}$  bisects the angle between  $\vec{b}$  and  $\vec{c}$ , then

a.  $\vec{a} \cdot \hat{k} + 2 = 0$

b.  $\vec{a} \cdot \hat{k} + 4 = 0$

c.  $\vec{a} \cdot \hat{k} - 2 = 0$

d.  $\vec{a} \cdot \hat{k} + 5 = 0$

**Answer:**

**Solution:**

*More data needed to solve the question.*

17. If  $f(x)$  is continuous and differentiable in  $x \in [-7, 0]$  and  $f'(x) \leq 2 \forall x \in [-7, 0]$ , also  $f(-7) = -3$  then the range of  $f(-1) + f(0)$  is

a.  $[-5, -7]$

b.  $(-\infty, 6]$

c.  $(-\infty, 20]$

d.  $[-5, 3]$

**Answer:** (c)

**Solution:**

$$f(-7) = -3 \text{ and } f'(x) \leq 2$$

Applying LMVT in  $[-7, 0]$ , we get

$$\frac{f(-7) - f(0)}{-7} = f'(c) \leq 2$$

$$\frac{-3 - f(0)}{-7} \leq 2$$

$$f(0) + 3 \leq 14$$

$$f(0) \leq 11$$

Applying LMVT in  $[-7, -1]$ , we get

$$\frac{f(-7) - f(-1)}{-7 + 1} = f'(c) \leq 2$$

$$\frac{-3 - f(-1)}{-6} \leq 2$$

$$f(-1) + 3 \leq 12$$

$$f(-1) \leq 9$$

$$\text{Therefore, } f(-1) + f(0) \leq 20$$

18. Find the image of the point (2,1,6) in the plane containing the points (2,1,0), (6,3,3) and (5,2,2)

a. (6, 5, -2)

b. (6, -5, 2)

c. (2, -3, 4)

d. (2, -5, 6)

**Answer:** (a)

**Solution:**

Points  $A(2,1,0)$ ,  $B(6,3,3)$   $C(5,2,2)$

$$\overrightarrow{AB} = (4, 2, 3)$$

$$\overrightarrow{AC} = (3, 1, 2)$$

$$\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC} = (1, 1, -2)$$

$$\text{Equation of the plane is } x + y - 2z = 3 \dots (1)$$

Let the image of point (2,1,6) is  $(l, m, n)$

$$\frac{l-2}{1} = \frac{m-1}{1} = \frac{n-6}{-2} = \frac{-2(-12)}{6} = 4$$

$$\Rightarrow l = 6, m = 5, n = -2$$

Hence the image of  $R$  in the plane  $P$  is (6, 5, -2)

19. If sum of all the coefficients of even powers in

$(1 - x + x^2 - x^3 \dots + x^{2n})(1 + x + x^2 + x^3 \dots + x^{2n})$  is 61 then  $n$  is equal to

a. 30

b. 32

c. 28

d. 36

**Answer:** (a)

**Solution:**

$$\text{Let } (1 + x + x^2 + \dots + x^{2n})(1 - x + x^2 + \dots + x^{2n}) = a_0 + a_1x + a_2x^2 + \dots$$

$$\text{Put } x = 1$$

$$2n + 1 = a_0 + a_1 + a_2 + a_3 + \dots \dots \dots (1)$$

$$\text{Put } x = -1$$

$$2n + 1 = a_0 - a_1 + a_2 - a_3 + \dots \dots \dots (2)$$

Add (1) and (2)

$$2(2n + 1) = 2(a_0 + a_2 + a_4 + \dots \dots \dots$$

$$2n + 1 = 61$$

$$n = 30$$



$$2I = \int_a^b f(a+b-x+1)dx + \int_a^b f(x)dx$$

$$2I = 2 \int_a^b f(x)dx$$

$$I = \int_a^b f(x)dx \quad ; \quad x = t+1, dx = dt$$

$$I = \int_{a-1}^{b-1} f(t+1)dt$$

$$I = \int_{a-1}^{b-1} f(x+1)dx$$

22. Total number of six-digit numbers in which only and all the five digits 1,3,5,7 and 9 appear, is \_\_\_\_\_.

**Answer:** (1800)

**Solution:**

Selecting all 5 digits =  ${}^5C_5 = 1$  way

Now, we need to select one more digit to make it a 6 digit number =  ${}^5C_1 = 5$  ways

Total number of permutations =  $\frac{6!}{2!}$

Total numbers =  ${}^5C_5 \times {}^5C_1 \times \frac{6!}{2!} = 1800$

23. Evaluate  $\lim_{x \rightarrow 2} \frac{3^x + 3^{x-1} - 12}{\frac{-x}{3^{\frac{x}{2}} - 3^{1-x}}}$

**Answer:** (72)

**Solution:**

$$\lim_{x \rightarrow 2} \frac{3^x + \frac{3^x}{3} - 12}{\frac{\frac{1}{x} + \frac{3}{3^x}}{3^{\frac{x}{2}}}}$$

$$\lim_{x \rightarrow 2} \frac{\frac{4}{3}3^x - 12}{\frac{\frac{1}{x} + \frac{3}{3^x}}{3^{\frac{x}{2}}}}$$

Put  $3^{\frac{x}{2}} = t$

$$\lim_{t \rightarrow 3} \frac{\frac{4t^2}{3} - 12}{-\frac{3}{t^2} + \frac{1}{t}} = \lim_{t \rightarrow 3} \frac{4(t^2-9)t^2}{3(-3+t)} = \lim_{t \rightarrow 3} \frac{4t^2(3+t)}{3} = \frac{4 \times 9 \times 6}{3} = 72$$

24. If variance of first  $N$  natural numbers is 10 and variance of first  $M$  even natural numbers is 16 then the value of  $M + N$  is \_\_\_\_\_.

**Answer:** (18)

**Solution:**

For  $N$  Natural number variance

$$\sigma^2 = \frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2$$

$$\frac{\sum x_i^2}{N} = \frac{1^2 + 2^2 + 3^2 + \dots + N^2}{N} = \frac{N(N+1)(2N+1)}{6N}$$

$$\frac{\sum x_i}{N} = \frac{1+2+3+\dots+N}{N} = \frac{N(N+1)}{2N}$$

$$\sigma^2 = \frac{N^2-1}{12} = 10 \text{ (given)}$$

$$\Rightarrow N = 11$$

$$\text{Variance of } (2, 4, 6, \dots) = 4 \times \text{variance of } (1, 2, 3, 4, \dots) = 4 \times \frac{M^2-1}{12} = \frac{M^2-1}{3} = 16 \text{ (given)}$$

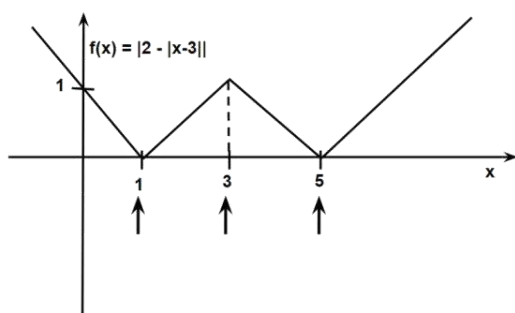
$$\Rightarrow M = 7$$

$$\text{Therefore, } N + M = 11 + 7 = 18$$

25. If  $f(x) = |2 - |x - 3||$  is non-differentiable in  $x \in S$ . Then, the value of  $\sum_{x \in S} (f(f(x)))$  is \_\_\_\_\_.

**Answer:** (3)

**Solution:**



There will be three points  $x = 1, 3, 5$  at which  $f(x)$  is non-differentiable.

$$\text{So } f(f(1)) + f(f(3)) + f(f(5))$$

$$= f(0) + f(2) + f(0)$$

$$= 1 + 1 + 1$$

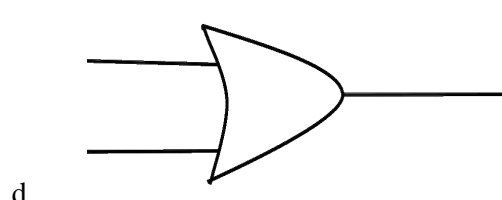
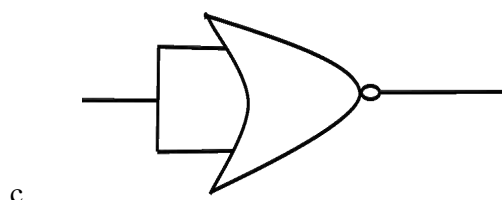
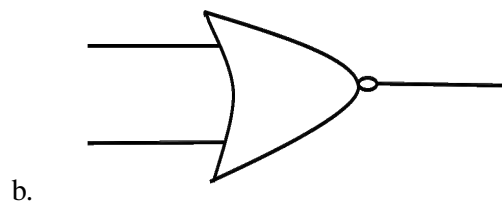
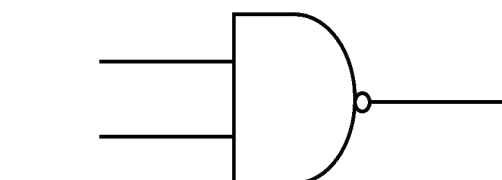
$$= 3$$

Date of Exam: 7<sup>th</sup> January 2020 (Shift 1)

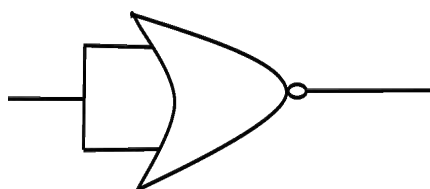
Time: 9:30 am- 12:30 pm

Subject: Physics

1. Which of the following gives reversible operation?



Solution: (c)



Since, there is only one input hence the operation is reversible.

2. A 60 HP electric motor lifts an elevator with a maximum total load capacity of 2000 kg. If the frictional force on the elevator is 4000 N, the speed of the elevator at full load is close to (Given 1 HP = 746 W,  $g = 10 \text{ m/s}^2$ )

a.  $1.9 \text{ m/s}$

b.  $1.7 \text{ m/s}$

c.  $2 \text{ m/s}$

d.  $1.5 \text{ m/s}$

Solution:(a)

Friction will oppose the motion

$$\text{Net force} = 2000g + 4000 = 24000 \text{ N}$$

$$\text{Power of lift} = 60 \text{ HP}$$

$$\text{Power} = \text{Force} \times \text{Velocity}$$

$$v = \frac{P}{F} = \frac{60 \times 746}{24000}$$

$$v = 1.86 \text{ m/s}$$

3. A LCR circuit behaves like a damped harmonic oscillator. Comparing it with a physical spring-mass damped oscillator having damping constant  $b$ , mass  $m$  and oscillating with a force constant  $k$ , the correct equivalence will be

a.  $L \leftrightarrow m, C \leftrightarrow \frac{1}{k}, R \leftrightarrow b$

b.  $L \leftrightarrow \frac{1}{b}, C \leftrightarrow \frac{1}{m}, R \leftrightarrow \frac{1}{k}$

c.  $L \leftrightarrow k, C \leftrightarrow b, R \leftrightarrow m$

d.  $L \leftrightarrow m, C \leftrightarrow k, R \leftrightarrow b$

Solution:(a)

For damped oscillator by Newton's second law

$$-kx - bv = ma$$

$$kx + bv + ma = 0$$

$$kx + b \frac{dx}{dt} + m \frac{d^2x}{dt^2} = 0$$

For LCR circuit by KVL

$$-IR - L \frac{dI}{dt} - \frac{q}{c} = 0$$

$$\Rightarrow IR + L \frac{dI}{dt} + \frac{q}{c} = 0$$

$$\Rightarrow \frac{q}{c} + R \frac{dq}{dt} + L \frac{d^2q}{dt^2} = 0$$

By comparing

$$R \Rightarrow b$$

$$c \Rightarrow \frac{1}{k}$$

$$m \Rightarrow L$$

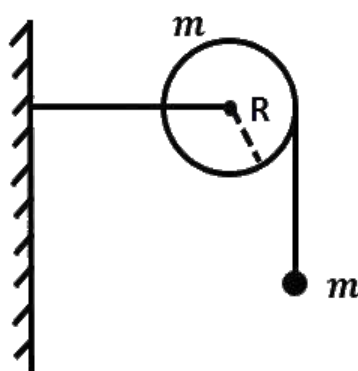
4. As shown in the figure, a bob of mass  $m$  is tied by a massless string whose other end portion is wound on a flywheel (disc) of radius  $R$  and mass  $m$ . When released from the rest, the bob starts falling vertically. When it has covered a distance  $h$ , the angular speed of the wheel will be (there is no slipping between string and wheel)

a.  $\frac{1}{R} \sqrt{\frac{4gh}{3}}$

b.  $\frac{1}{R} \sqrt{\frac{2gh}{3}}$

c.  $R \sqrt{\frac{2gh}{3}}$

d.  $R \sqrt{\frac{4gh}{3}}$



Solution:(a)

By energy conservation,

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$\Rightarrow gh = \frac{v^2}{2} + \frac{\omega^2 R^2}{4} \quad (1)$$

Since the rope is inextensible and also it is not slipping,

$$\therefore v = R\omega \quad (2)$$

from eq. (1) and (2)

$$gh = \frac{\omega^2 R^2}{2} + \frac{\omega^2 R^2}{4}$$

$$\Rightarrow gh = \frac{3}{4}R^2\omega^2$$

$$\Rightarrow \omega^2 = \frac{4gh}{3R^2}$$

$$\Rightarrow \omega = \frac{1}{R}\sqrt{\frac{4gh}{3}}$$

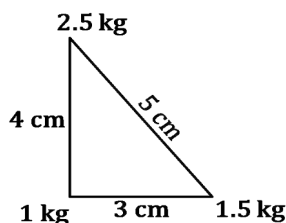
5. Three point particles of mass 1 kg, 1.5 kg and 2.5 kg are placed at corners of a right triangle of sides 4 cm, 3 cm and 5 cm as shown. The centre of mass of the system with respect to 1 kg mass is at the point

- 0.6 cm to the right of 1 kg and 2 cm above 1 kg mass
- 0.9 cm to the right of 1 kg and 2 cm above 1 kg mass
- 0.9 cm to the right of 1 kg and 2 cm below 1 kg mass
- 0.9 cm to the right of 1 kg and 1.5 cm above 1 kg mass

Solution: (b)

Taking 1 kg as the origin

$$x_{com} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$



$$x_{com} = \frac{1 \times 0 + 1.5 \times 3 + 2.5 \times 0}{5}$$

$$x_{com} = 0.9$$

$$y_{com} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

$$y_{com} = \frac{1 \times 0 + 1.5 \times 0 + 2.5 \times 4}{5}$$

$$y_{com} = 2$$

Centre of mass is at (0.9, 2)

6. A parallel plate capacitor has plates of area  $A$  separated by distance ' $d$ '. It is filled with a dielectric which has a dielectric constant varies as  $k(x) = k(1 + \alpha x)$ , where ' $x$ ' is the distance measured from one of the plates. If ( $\alpha d \ll 1$ ), the capacitance of the system is

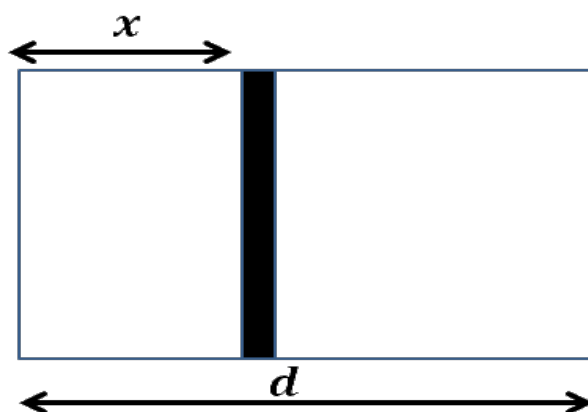
a.  $\frac{A\epsilon_0 k}{d} \left[ 1 + \left( \frac{\alpha d}{2} \right)^2 \right]$

b.  $\frac{A\epsilon_0 k}{d} \left[ 1 + \left( \frac{\alpha^2 d^2}{2} \right) \right]$

c.  $\frac{A\epsilon_0 k}{d} [1 + \alpha d]$

d.  $\frac{A\epsilon_0 k}{d} \left[ 1 + \left( \frac{\alpha d}{2} \right) \right]$

Solution:(d)



Given,  $k(x) = k(1 + \alpha x)$

$$dC = \frac{A\epsilon_0 k}{dx}$$

Since all capacitance are in series, we can apply

$$\frac{1}{C_{eq}} = \int \frac{1}{dC} = \int_0^d \frac{dx}{k(1 + \alpha x)\epsilon_0 A}$$

$$\frac{1}{Ceq} = \left[ \frac{\ln(1 + \alpha x)}{k\epsilon_0 A \alpha} \right]_0^d$$

On putting the limits from 0 to d

$$= \frac{\ln(1 + \alpha d)}{k\epsilon_0 A \alpha}$$

Using expression  $\ln(1 + x) = x - \frac{x^2}{2} + \dots$

And putting  $x = \alpha d$  where,  $x$  approaches to 0.

$$\frac{1}{C} = \frac{d}{k\epsilon_0 A d \alpha} \left[ \alpha d - \frac{(\alpha d)^2}{2} \right]$$

$$\frac{1}{C} = \frac{d}{k\epsilon_0 A} \left[ 1 - \frac{\alpha d}{2} \right]$$

$$C = \frac{k\epsilon_0 A}{d} \left[ 1 + \frac{\alpha d}{2} \right]$$

7. The time period of revolution of an electron in its ground state orbit in a hydrogen atom is  $1.6 \times 10^{-16} s$ . The frequency of the electron in its first excited state (in  $s^{-1}$ ) is :

a.  $7.8 \times 10^{14}$

b.  $7.8 \times 10^{16}$

c.  $3.7 \times 10^{14}$

d.  $3.7 \times 10^{16}$

Solution:(a)

Time period is proportional to  $\frac{n^3}{Z^2}$ .

Let  $T_1$  be the time period in ground state and  $T_2$  be the time period in its first excited state.

$$T_1 = \frac{n^3}{2^2}$$

(Where,  $n$  = excitation level and 2 is atomic no.)

$$\frac{T_1}{T_2} = \left( \frac{n_1}{n_2} \right)^3$$

Given,

$$T_1 = 1.6 \times 10^{-16} s$$

So,

$$\frac{1.6 \times 10^{-16}}{T_2} = \left( \frac{1}{2} \right)^3$$

$$T_2 = 12.8 \times 10^{-16} s$$

Frequency is given by  $f = \frac{1}{T}$

$$f = \frac{1}{12.8} \times 10^{16} Hz$$



Solution:(c)

As we know,

$$\begin{aligned}
 T.E_{ground} &= T.E_R \\
 \frac{1}{2}mu^2 + \left(\frac{-GMm}{R}\right) &= \frac{1}{2}mv^2 + \left(\frac{-GMm}{2R}\right) \\
 \frac{1}{2}mv^2 &= \frac{1}{2}mu^2 + \left(\frac{-GMm}{2R}\right) \\
 v^2 &= u^2 + \left(\frac{-GMm}{R}\right) \\
 \Rightarrow v &= \sqrt{u^2 + \left(\frac{-GMm}{R}\right)}
 \end{aligned} \tag{1}$$

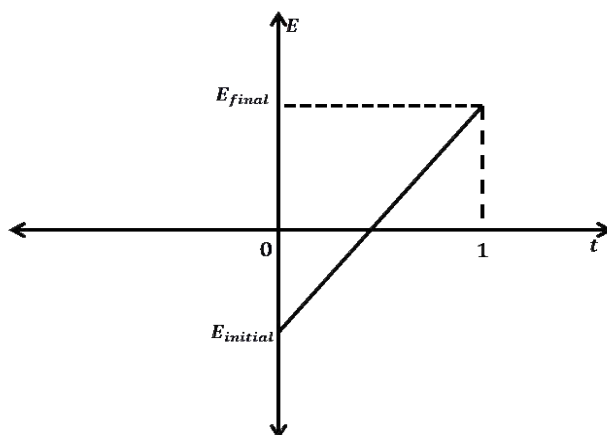
The rocket splits at height  $R$ . Since, separation of rocket is impulsive therefore conservation of momentum in both radial and tangential direction can be applied.

$$\begin{aligned}
 \frac{m}{10}V_T &= \frac{9m}{10}\sqrt{\frac{GM}{2R}} \\
 \frac{m}{10}V_r &= m\sqrt{u^2 - \frac{GM}{R}}
 \end{aligned}$$

$$\begin{aligned}
 \text{Kinetic energy of satellite} &= \frac{1}{2} \times \frac{m}{10}(V_T^2 + V_r^2) = \frac{m}{20} \left( 81\frac{GM}{2R} + 100u^2 - 100\frac{GM}{R} \right) \\
 &= \frac{m}{20} \left( 100u^2 - \frac{119GM}{2R} \right) \\
 &= 5m \left( u^2 - \frac{119GM}{200R} \right)
 \end{aligned}$$

10. A long solenoid of radius  $R$  carries a time ( $t$ ) dependent current  $I(t) = I_0 t(1 - t)$ . A ring of radius  $2R$  is placed coaxially to the middle. During the time instant  $0 \leq t \leq 1$ , the induced current ( $I_R$ ) and the induced EMF ( $V_R$ ) in the ring changes as
- Current will change its direction and its emf will be zero at  $t = 0.25 \text{ sec}$ .
  - Current will not change its direction and its emf will be maximum at  $t = 0.5 \text{ sec}$ .
  - Current will not change direction and emf will be zero at  $t = 0.25 \text{ sec}$ .
  - Current will change its direction and its emf will be zero at  $t = 0.5 \text{ sec}$ .

Solution:(d)



Field due to solenoid near the middle  $= \mu_o N I$

$$\text{Flux, } \phi = BA \quad \text{where } (A = \pi(R)^2)$$

$$= \mu_o N I_o t (1 - t) \pi R^2$$

$$E = -\frac{d\phi}{dt} \quad [\text{By Lenz's law}]$$

$$E = -\frac{d}{dt} (\mu_o N I_o t (1 - t)^2)$$

$$E = -\mu_o N I_o \pi R^2 \frac{d}{dt} [t(1 - t)]$$

$$E = -\pi \mu_o I_o N R^2 (1 - 2t)$$

Current will change its direction when EMF will be zero

$$\Rightarrow (1 - 2t) = 0$$

$$\text{So, } t = 0.5 \text{ sec}$$

11. The radius of gyration of a uniform rod of length  $l$  about an axis passing through a point  $l/4$  away from the center of the rod and perpendicular to its axis is

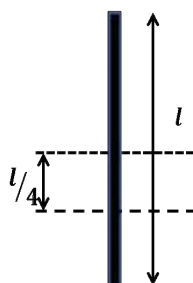
a.  $\sqrt{\frac{7}{48}} l$

b.  $\sqrt{\frac{5}{48}} l$

c.  $\sqrt{\frac{7}{24}} l$

d.  $\sqrt{\frac{19}{24}} l$

Solution:(a)



Moment of inertia of rod about axis perpendicular to it passing through its centre is given by

$$\begin{aligned} I &= \frac{Ml^2}{12} + M \left( \frac{l}{4} \right)^2 \\ &= \frac{3Ml^2 + 4Ml^2}{48} \\ &= \frac{7Ml^2}{48} \end{aligned}$$

Now, comparing with  $I = Mk^2$  where  $k$  is the radius of gyration

$$\begin{aligned} k &= \sqrt{\frac{7l^2}{48}} \\ k &= l\sqrt{\frac{7}{48}} \end{aligned}$$

12. Two moles of an ideal gas with  $\frac{C_p}{C_v} = 5/3$  are mixed with 3 moles of another ideal gas with  $\frac{C_p}{C_v} = 4/3$ . The value of  $\frac{C_p}{C_v}$  for the mixture is

- |         |         |
|---------|---------|
| a. 1.38 | b. 1.42 |
| c. 1.50 | d. 1.70 |

Solution:(b)

For first gas having  $\gamma = \frac{C_p}{C_v} = \frac{5}{3}$

Using formula  $C_p = \frac{R\gamma}{\gamma - 1}$

$$C_v = \frac{R}{\gamma - 1}$$

$$C_p = \frac{5R}{2} \quad C_v = \frac{3R}{2}$$

Similarly for 2<sup>nd</sup> gas having  $\gamma = \frac{C_p}{C_v} = \frac{4}{3}$

$$C_p = 4R \quad C_v = 3R$$



We know that,

$$\left| \frac{E_0}{B_0} \right| = c$$

$$B_0 = 3 \times 10^{-8}$$

$$\Rightarrow E_0 = B_0 \times c = 3 \times 10^{-8} \times 3 \times 10^8$$

$$= 9 \text{ N/C}$$

$$\therefore E = E_0 \sin(\omega t - kx + \phi) \hat{k} = 9 \sin(\omega t - kx + \phi) \hat{k}$$

15. A polarizer analyzer set is adjusted such that the intensity of light coming out of the analyzer is just 10 % of the original intensity. Assuming that the polarizer analyzer set does not absorb any light, the angle by which the analyzer needs to be rotated further to reduce the output intensity to be zero is

- |               |                 |
|---------------|-----------------|
| a. $45^\circ$ | b. $71.6^\circ$ |
| c. $90^\circ$ | d. $18.4^\circ$ |

Solution:(d)

$$\text{Intensity after polarisation through polaroid} = I_o \cos^2 \phi$$

$$\text{So, } 0.1 I_o = I_o \cos^2 \phi$$

$$\Rightarrow \cos \phi = \sqrt{0.1}$$

$$\Rightarrow \cos \phi = 0.316$$

Since,  $\cos \phi < \cos 45^\circ$  therefore,  $\phi > 45^\circ$  If the light is passing at  $90^\circ$  from the plane of polaroid, than its intensity will be zero.

Then,  $\theta = 90^\circ - \phi$  therefore,  $\theta$  will be less than  $45^\circ$ . So, the only option matching is option d which is  $18.4^\circ$

16. Speed of transverse wave of a straight wire having mass 6.0 g, length 60 cm and area of cross-section  $1.0 \text{ mm}^2$  is 90 m/s. If the Young's modulus of wire is  $1.6 \times 10^{11} \text{ Nm}^{-2}$ , the extension of wire over its natural length is
- |           |           |
|-----------|-----------|
| a. 0.3 mm | b. 0.2 mm |
| c. 0.1 mm | d. 0.4 mm |

Solution:(a)

Given,  $M = 6 \text{ grams} = 6 \times 10^{-3} \text{ kg}$

$$L = 60 \text{ cm} = 0.6 \text{ m}$$

$$A = 1 \text{ mm}^2 = 1 \times 10^{-6} \text{ m}^2$$

$$\text{Using the relation, } v^2 = \frac{T}{\mu}$$

$$\Rightarrow T = \mu v^2 = V^2 \times \frac{M}{L}$$

As Young's modulus,  $Y = \frac{\text{stress}}{\text{strain}}$

$$\text{Strain} = \frac{\text{Stress}}{Y} = \frac{T}{AY}$$

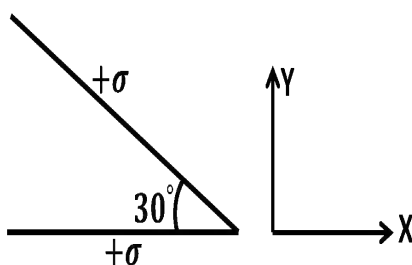
$$\text{Strain} = \frac{\Delta L}{L} = \frac{V^2 \frac{M}{L}}{AY} = V^2 \frac{M}{AYL}$$

$$\Rightarrow \Delta L = \frac{V^2 M}{AY}$$

$$\Delta L = \frac{8100 \times 6 \times 10^{-3}}{1 \times 10^{-6} \times 1.6 \times 10^{11}}$$

$$\Delta L = 0.3 \text{ mm}$$

17. Two infinite planes each with uniform surface charge density  $+\sigma \text{ C/m}^2$  are kept in such a way that the angle between them is  $30^\circ$ . The electric field in the region shown between them is given by:



- $\frac{\sigma}{2\epsilon_0} \left[ \left(1 - \frac{\sqrt{3}}{2}\right) \hat{y} - \frac{1}{2} \hat{x} \right]$
- $\frac{\sigma}{2\epsilon_0} \left[ \left(1 + \frac{\sqrt{3}}{2}\right) \hat{y} - \frac{1}{2} \hat{x} \right]$
- $\frac{\sigma}{2\epsilon_0} \left[ \left(1 - \frac{\sqrt{3}}{2}\right) \hat{y} + \frac{1}{2} \hat{x} \right]$
- $\frac{\sigma}{2\epsilon_0} \left[ \left(1 + \frac{\sqrt{3}}{2}\right) \hat{y} + \frac{1}{2} \hat{x} \right]$

Solution:(a)

$$\text{Field due to single plate} = \frac{\sigma}{2\epsilon_0} = [\vec{E}_1] = [\vec{E}_2]$$

$$\text{Net electric field } \vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= \frac{\sigma}{2\epsilon_0} \cos 30^\circ (-\hat{j}) + \frac{\sigma}{2\epsilon_0} \sin 30^\circ (-\hat{i}) + \frac{\sigma}{2\epsilon_0} (\hat{j})$$

$$= \frac{\sigma}{2\epsilon_0} \left( 1 - \frac{\sqrt{3}}{2} \right) (\hat{j}) - \frac{\sigma}{4\epsilon_0} (\hat{i})$$

$$= \frac{\sigma}{2\epsilon_0} \left[ \left( 1 - \frac{\sqrt{3}}{2} \right) \hat{y} - \frac{1}{2} \hat{x} \right]$$



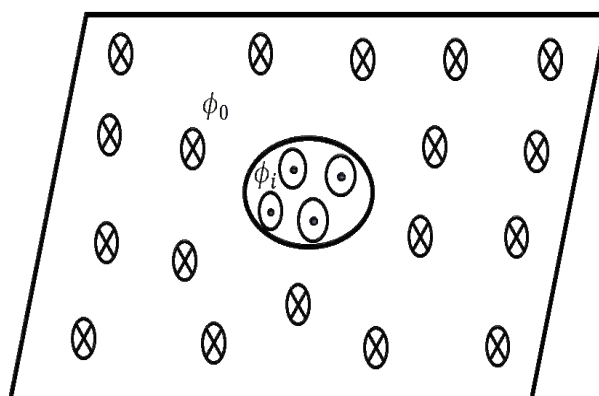
$$\Rightarrow \frac{\sqrt{3}}{2\sin\theta_1} = 2 \Rightarrow \sin\theta_1 = \frac{\sqrt{3}}{4} = 0.43$$

As, the value is coming less than  $30^\circ$  the only available option are  $20^\circ$  and  $25^\circ$  but by using approximation we get  $\theta_1 = 25^\circ$

20. Consider a coil of wire carrying current  $I$ , forming a magnetic dipole placed in an infinite plane. If  $\phi_i$  is the magnitude of magnetic flux through the inner region and  $\phi_o$  is magnitude of magnetic flux through outer region then which of the following is correct?

- |                       |                      |
|-----------------------|----------------------|
| a. $\phi_i = -\phi_o$ | b. $\phi_i > \phi_o$ |
| c. $\phi_i < \phi_o$  | d. $\phi_i = \phi_o$ |

Solution:(a)

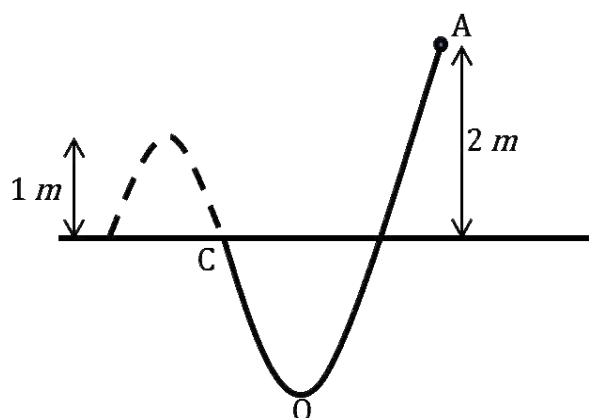


As magnetic field line of ring will form close loop therefore all outgoing from circular hole passing through the infinite plate.

$\therefore \phi_i = -\phi_o$  (because the magnetic field lines going inside is equal to the magnetic field lines coming out.)

21. A particle of mass  $1\text{ kg}$  slides down a frictionless track AOC starting from rest at A (height  $2\text{ m}$ ). After reaching C particle continues to move freely in air as a projectile. When it reaches its highest point P ( $h=1\text{ m}$ ) the kinetic energy of the particle (in  $J$ ) is..... (take  $g=10\text{ m/s}^2$ )

Solution:(10)



As the particle starts from rest the total energy at point A =  $mgh = T.E_A$  (where  $h = 2\text{ m}$ )

After reaching point P

$$T.E_c = K.E. + mgh$$

By conservation of energy

$$T.E_A = T.E_p$$

$$\implies K.E. = mgh = 10\text{ J}$$

22. A carnot engine operates between two reservoirs of temperature  $900\text{ K}$  and  $300\text{ K}$ . The engine performs  $1200\text{ J}$  of work per cycle. The heat energy in( $J$ ) delivered by the engine to the low temperature reservoir in a cycle is

-----

Solution:( $600\text{ J}$ )

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{900} = \frac{2}{3}$$

$$\text{Given, } W = 1200\text{ J}$$

From conservation of energy

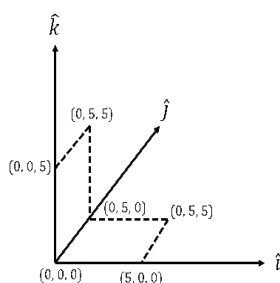
$$Q_1 - Q_2 = W$$

$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{W}{Q_1} \implies Q_1 = 1800\text{ J}$$

$$\implies Q_2 = Q_1 - W = 600\text{ J}$$

23. A loop  $ABCDEF A$  of straight edges has a six corner points  $A(0,0,0)$ ,  $B(5,0,0)$ ,  $C(5,5,0)$ ,  $D(0,5,0)$ ,  $E(0,5,5)$ ,  $F(0,0,5)$ . The magnetic field in this region is  $\vec{B} = (3\hat{i} + 4\hat{k})\text{ T}$ . The quantity of the flux through the loop  $ABCDEF A$  (in  $Wb$ ) is -----

Solution:( $175$ )



As we know, magnetic flux =  $\vec{B} \cdot \vec{A}$

$$\Rightarrow (B_x + B_z) \cdot (A_x + A_z)$$

$$\Rightarrow (3\hat{i} + 4\hat{k}) \cdot (25\hat{i} + 25\hat{k})$$

$$\Rightarrow (75 + 100) \text{ Wb}$$

$$\Rightarrow 175 \text{ Wb}$$

24. A beam of electromagnetic radiation of intensity  $6.4 \times 10^{-5} \text{ W/cm}^2$  is comprised of wavelength,  $\lambda = 310 \text{ nm}$ . It falls normally on a metal (work function  $2 \text{ eV}$ ) of surface area  $1 \text{ cm}^2$ . If one in  $10^3$  photons ejects an electron, total number of electrons ejected in  $1 \text{ s}$  is  $10^x$  ( $hc = 1240 \text{ eV} - \text{nm}$ ,  $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$ ), then the value of  $x$  is -----

Solution:(11)

$$\begin{aligned} P &= \text{Intensity} \times \text{Area} \\ &= 6.4 \times 10^{-5} \text{ W} - \text{cm}^{-2} \times 1 \text{ cm}^2 \\ &= 6.4 \times 10^{-5} \text{ W} \end{aligned}$$

For photoelectric effect to take place, energy should be greater than work function

Now,

$$E = \frac{1240}{310} = 4 \text{ eV} > 2 \text{ eV}$$

Therefore, photoelectric effect takes place

Here  $n$  is the number of photons emitted.

$$\begin{aligned} n \times E &= I \times A \\ \Rightarrow n &= \frac{IA}{E} = \frac{6.4 \times 10^{-5}}{6.4 \times 10^{-19}} = 10^{14} \end{aligned}$$

Where,  $n$  is number of incident photon

Since, 1 out of every 1000 photons are successful in ejecting 1 photoelectron

Therefore, the number of photoelectrons emitted is

$$= \frac{10^{14}}{10^3}$$

$$\therefore x = 11$$

25. A non- isotropic solid metal cube has coefficient of linear expansion as  $5 \times 10^{-5}/^{\circ}C$  along the x-axis and  $5 \times 10^{-6}/^{\circ}C$  along y-axis and z-axis. If the coefficient of volumetric expansion of the solid is  $n \times 10^{-6}/^{\circ}C$  then the value of  $n$  is -----

Solution:(60)

We know that,  $V = xyz$

$$\frac{\Delta v}{v} = \frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta z}{z}$$

$$\frac{1}{T} \frac{\Delta v}{v} = \frac{1}{T} \frac{\Delta x}{x} + \frac{1}{T} \frac{\Delta y}{y} + \frac{1}{T} \frac{\Delta z}{z}$$

$$\gamma = \alpha_x + \alpha_y + \alpha_z$$

$$\gamma = 50 \times 10^{-6}/^{\circ}C + 5 \times 10^{-6}/^{\circ}C + 5 \times 10^{-6}/^{\circ}C$$

$$\gamma = 60 \times 10^{-6}/^{\circ}C$$

$$\therefore n = 60$$

Date: 7<sup>th</sup> January 2020

Time: 09:30 am – 12:30 pm

## JEE Main 2020 Paper

Subject: Chemistry

1. The relative strength of inter-ionic/ intermolecular forces in the decreasing order is:

- |   |   |
|---|---|
| a) ion-dipole > dipole-dipole > ion-ion | b) dipole-dipole > ion-dipole > ion-ion |
| c) ion-ion > ion-dipole > dipole-dipole | d) ion-dipole > ion-ion > dipole-dipole |

Answer: c

Solution:

Ion-ion interactions are stronger because they have stronger electrostatic forces of attraction whereas dipoles have partial charges and hence the electrostatic forces in their case would be relatively weak.

2. The oxidation number of K in  $K_2O$ ,  $K_2O_2$  and  $KO_2$  respectively is:

- |                 |                 |
|-----------------|-----------------|
| a) +0.5, +4, +1 | b) +2, +1, +0.5 |
| c) +1, +1, +1   | d) +0.5, +1, +2 |

Answer: c

Solution:

Alkali metals always possess a +1 oxidation state, whereas oxygen present in  $K_2O$  (oxide) is -2, and in  $K_2O_2$  (peroxide) is -1 and in  $KO_2$  (superoxide) is  $-\frac{1}{2}$ .

3. At 35 °C the vapour pressure of  $CS_2$  is 512 mm of Hg and that of acetone is 344 mm of Hg. A solution of  $CS_2$  in acetone has a total vapour pressure of 600 mm of Hg. The false statement among the following is:

- a)  $CS_2$  and acetone are less attracted to each other than themselves.
- b) Heat must be absorbed in order to produce the solution at 35 °C
- c) Raoult's law is not obeyed by this system
- d) A mixture of 100 mL  $CS_2$  and 100 mL acetone has a volume less than 200 mL

Answer: d

Solution:

$$P_{\text{Total}} = P_T = P_A^{\circ}X_A + P_B^{\circ}X_B$$

The maximum value  $X_A$  can hold is 1, and hence the maximum value of  $P_T$  should come out to be 512 mm of Hg, which is less than the value of  $P_T$  observed (600 mm of Hg). Therefore, positive deviation from Raoult's law is observed. This implies that A-A interactions and B-B interactions are stronger than A-B interactions.

As we know, for a system not obeying Raoult's law and showing positive deviation,

$$\Delta V_{\text{mix}} > 0, \Delta H_{\text{mix}} > 0$$

4. The atomic radius of Ag is closest to:

- |       |       |
|-------|-------|
| a) Ni | b) Cu |
| c) Au | d) Hg |

**Answer:** c

**Solution:**

Because of Lanthanide contraction, an increase in  $Z_{\text{eff}}$  is observed and so, the size of Au instead of being greater, as is expected, turns out to be similar to that of Ag.

5. The dipole moments of  $\text{CCl}_4$ ,  $\text{CHCl}_3$  and  $\text{CH}_4$  are in the order:

- |   |   |
|---|---|
| a) $\text{CH}_4 > \text{CCl}_4 > \text{CHCl}_3$ | b) $\text{CHCl}_3 > \text{CCl}_4 > \text{CH}_4$ |
| c) $\text{CHCl}_3 > \text{CCl}_4 = \text{CH}_4$ | d) $\text{CCl}_4 = \text{CH}_4 > \text{CHCl}_3$ |

**Answer:** c

**Solution:**

All the three compounds possess a tetrahedral geometry. In both  $\text{CCl}_4$  and  $\text{CH}_4$ ,  $\mu_{\text{net}} = 0$ , whereas in  $\text{CHCl}_3$ ,  $\mu_{\text{net}} > 0$ .

6. In comparison to the zeolite process for the removal of permanent hardness, the synthetic resins method is:

- a) Less efficient as it exchanges only anions
- b) More efficient as it can exchange only cations
- c) Less efficient as the resins cannot be generated
- d) More efficient as it can exchange both cations and anions

**Answer:** d

7. Amongst the following statements, which was not proposed by Dalton:

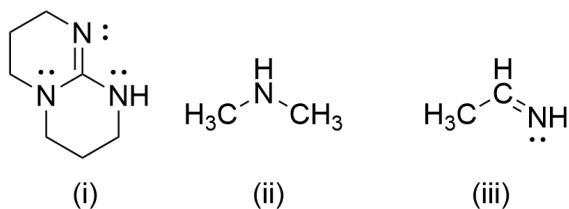
- a) Matter consists of indivisible atoms
- b) When gases combine or react in a chemical reaction, they do so in a simple ratio by volume provided all gases are maintained at the same temperature and pressure
- c) Chemical reactions involve reorganisation of atoms
- d) Atoms are neither created nor destroyed in a chemical reaction

**Answer:** b

**Solution:**

When gases combine or react in a chemical reaction they do so in a simple ratio by volume provided all gases are maintained at the same temperature and pressure - Gay-Lussac's law.

8. The increasing order of  $pK_b$  for the following compounds will be:



- a)  $i > ii > iii$   
 c)  $ii > i > iii$

- b)  $iii > ii > i$   
 d)  $i < iii < ii$

**Answer:** b

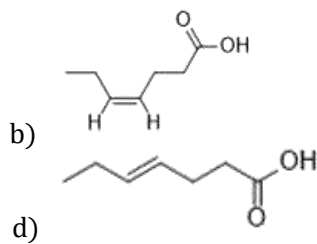
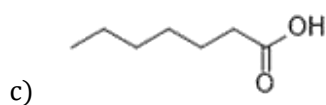
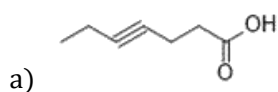
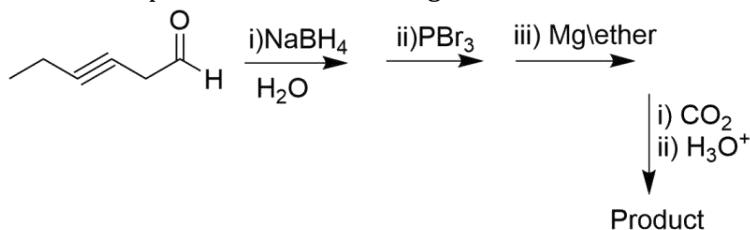
**Solution:**

Weaker the conjugate acid, stronger the base. (i) is the most basic as it has a guanidine like structure. It has a high tendency of accepting a proton, giving rise to a very stable conjugate acid and hence, is a very strong base.

In compound (iii), the N is  $sp^2$  hybridised and its electronegativity is higher as compared to the compound (ii) which is a  $2^0$  amine ( $sp^3$  hybridised). So compound (ii) is more basic compared to compound (iii).

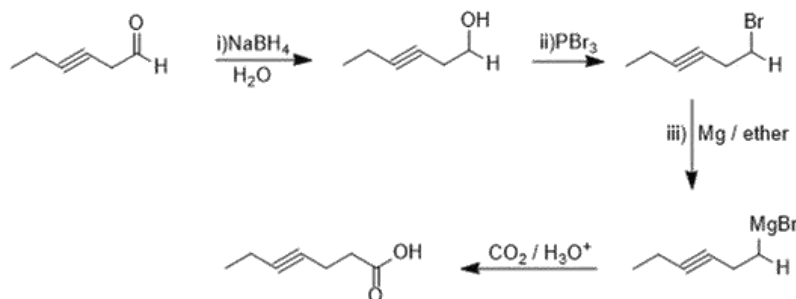
So the order of basicity is  $i > ii > iii$  and thus the order of  $pK_b$  value will be  $iii > ii > i$

9. What is the product of the following reaction?



**Answer:** a

**Solution:**



10. The number of orbitals associated with quantum number  $n=5$ ,  $m_s=+\frac{1}{2}$  is:

- a) 50
- b) 16
- c) 25
- d) 30

**Answer:** c

**Solution:**

$n = 5$ ;  $l = (n - 1) = 4$ ; hence the possible sub-shells for  $n=5$  are: 5s, 5p, 5d, 5f and 5g.

The number of orbitals in each would be 1,3,5,7 and 9, respectively and summing them up gives the answer as 25.

11. The purest form of commercial iron is:

- a) Cast iron
- b) Wrought iron
- c) Pig iron
- d) None of these

**Answer:** b

12. The theory that can completely/ properly explain the nature of bonding in  $[\text{Ni}(\text{CO})_4]$  is:

- a) Werner's theory
- b) Crystal Field Theory
- c) Molecular Orbital Theory
- d) Valence Bond Theory

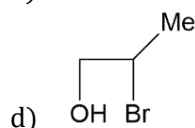
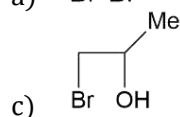
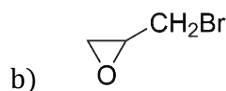
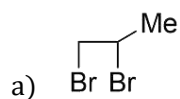
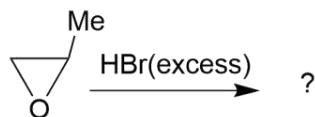
**Answer:** c

13. The IUPAC name of the complex  $[\text{Pt}(\text{NH}_3)_2\text{Cl}(\text{NH}_2\text{CH}_3)]\text{Cl}$  is:

- a) Diamminechloridomethylamineplatinum(II) chloride
- b) Chloridomethanaminodiammineplatinum(II) chloride
- c) Diamminechloridomethylamineplatinate(II) chloride
- d) None of these

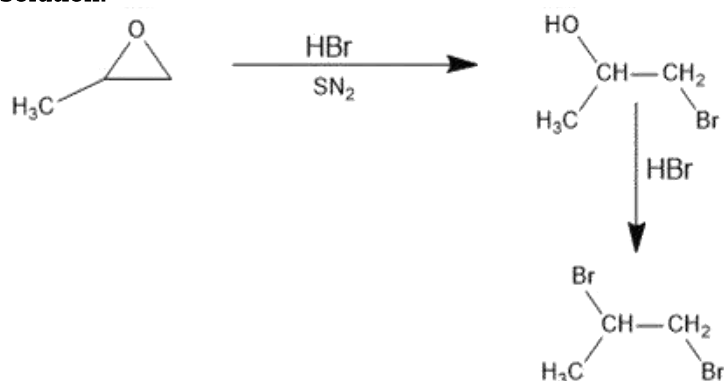
**Answer:** a

14.

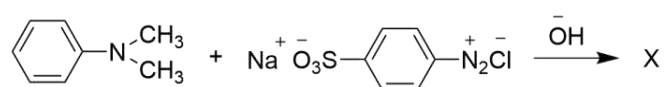


**Answer:** a

**Solution:**



15. Consider the following reaction:

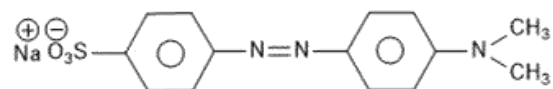


The product X is used:

- a) In protein estimation as an alternative to Ninhydrin
- b) As a food grade colourant
- c) In laboratory test for phenols
- d) In acid-base titration as an indicator

**Answer:** d

**Solution:**



X formed is methyl orange.

16. Match:

List I	List II
i) Riboflavin	p) Beri beri
ii) Thiamine	q) Scurvy
iii) Ascorbic acid	r) Cheilosis
iv) Pyridoxine	s) Convulsions

	i	ii	iii	iv
a)	s	q	p	r
b)	r	p	q	s
c)	p	r	q	s
d)	s	r	q	p

**Answer:** b

**Solution:**

Vitamins	Deficiency diseases
i) Riboflavin (Vitamin B <sub>2</sub> )	Cheilosis
ii) Thiamine (Vitamin B <sub>1</sub> )	Beri beri
iii) Ascorbic acid (Vitamin C)	Scurvy
iv) Pyridoxine (Vitamin B <sub>6</sub> )	Convulsions

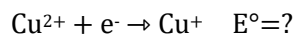
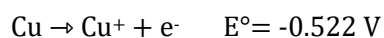
17. Given that the standard potential; E° of Cu<sup>2+</sup>|Cu and Cu<sup>+</sup>|Cu are 0.340 V and 0.522 V respectively. The E° of Cu<sup>2+</sup>|Cu<sup>+</sup> is:

- a) 0.158 V  
c) 0.182 V

- b) -0.158 V  
d) -0.182 V

**Answer:** a

**Solution:**



Applying  $\Delta G = -nFE^\circ$

We get,

$$(-1 \times F \times E^\circ) = (-2 \times F \times 0.340) + (-1 \times F \times -0.522)$$

Solving, we get,  $E^\circ = 0.158 \text{ V}$

18. A solution of m-chloroaniline, m-chlorophenol, m-chlorobenzoic acid in ethyl acetate was extracted initially with a saturated solution of  $\text{NaHCO}_3$  to give fraction A, the leftover organic phase was extracted with dil.  $\text{NaOH}$  to give fraction B. The final organic layer was labelled as fraction C. Fractions A, B and C contains respectively:

- a) m-chlorobenzoic acid, m-chlorophenol and m-chloroaniline
- b) m-chlorophenol, m-chlorobenzoic acid and m-chloroaniline
- c) m-chloroaniline, m-chlorophenol and m-chlorobenzoic acid
- d) m-chlorobenzoic acid, m-chloroaniline and m-chlorophenol

**Answer:** a

**Solution:**

m-chlorobenzoic acid being the most acidic can be separated by a weak base like  $\text{NaHCO}_3$  and hence will be labelled fraction A.

m-chlorophenol is not as acidic as m-chlorobenzoic acid, and can be separated by a stronger base like  $\text{NaOH}$ , and hence can be labelled as fraction B.

m-chloroaniline being a base, does not react with either of the bases and hence would be labelled as fraction C.

19. The electron gain enthalpy in kJ/mol of F, Cl, Br, and I respectively are:

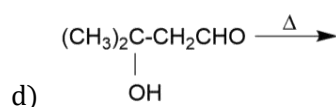
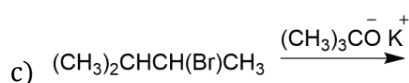
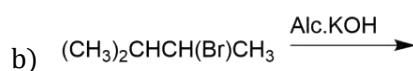
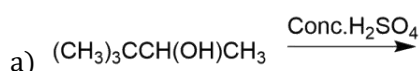
- a) -295, -324, -348, -333
- b) -348, -324, -333, -295
- c) -333, -348, -324, -295
- d) -348, -333, -295, -324

**Answer:** c

**Solution:**

$\text{Cl} > \text{F} > \text{Br} > \text{I}$

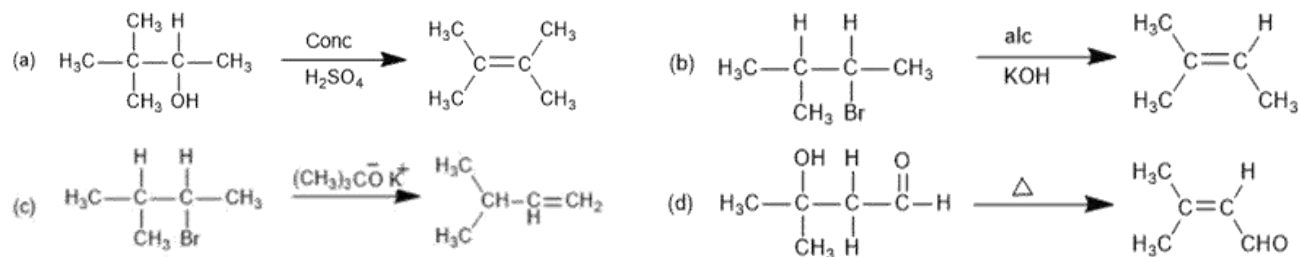
20. Consider the following reactions:



Which of these reactions will not produce Saytzeff product?

**Answer:** c

**Solution:**



21. Two solutions A and B each of 100 L was made by dissolving 4 g of NaOH and 9.8 g of  $\text{H}_2\text{SO}_4$  in water respectively. The pH of the resulting solution obtained by mixing 40 L of Sol A and 10 L of Sol B is:

**Answer:** 10.6

**Solution:**

Molarity of NaOH (4 g in 100 L) =  $10^{-3}$  M

Molarity of  $\text{H}_2\text{SO}_4$  (9.8 g in 100 L) =  $10^{-3}$  M

Equivalents of NaOH =  $M \times V \times n_f = 10^{-3} \times 40 \times 1 = 0.04$

Equivalents of  $\text{H}_2\text{SO}_4$  =  $M \times V \times n_f = 10^{-3} \times 10 \times 2 = 0.02$

$M_{\text{NaOH}} \cdot V_{\text{NaOH}} \cdot (n_f)_{\text{NaOH}} - M_{\text{H}_2\text{SO}_4} \cdot V_{\text{H}_2\text{SO}_4} \cdot (n_f)_{\text{H}_2\text{SO}_4} = M \cdot V_{\text{total}}$

$10^{-3} \times 40 \times 1 - 10^{-3} \times 10 \times 2 = M \cdot 50$

$M = 4 \times 10^{-4}$

$\text{pOH} = -\log M$

$= 4 - 2\log 2$

$= 3.4$

$\text{pH} = 14 - 3.4 = 10.6$

22. During the nuclear explosion, one of the products  $^{90}\text{Sr}$  was absorbed in the bones of a newly born baby in place of Ca. How much time in years is required to reduce it by 90% if it is not lost metabolically? ( $t_{1/2} = 6.93$  years)

**Answer:** 23.03

**Solution:**

All nuclear processes follow first order kinetics, and hence

$$t_{1/2} = \frac{0.693}{\lambda}$$

$$\lambda = 0.1 \text{ (year)}^{-1}$$

$$t = \frac{2.303}{\lambda} \log \left( \frac{a_0}{a_t} \right)$$

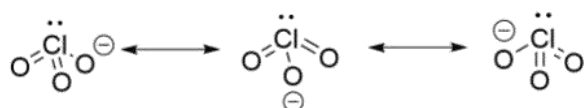
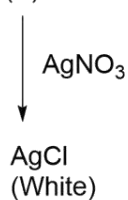
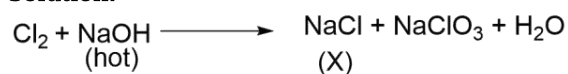
$$t = \frac{2.303}{0.1} \log \left( \frac{a_0}{0.1a_0} \right)$$

On solving,  $t = 23.03$  years

23. Chlorine reacts with hot and conc. NaOH and produces compounds X and Y. Compound X gives a white precipitate with  $\text{AgNO}_3$  soln. The average bond order between Cl and O atoms in Y is?

**Answer:** 1.67

**Solution:**

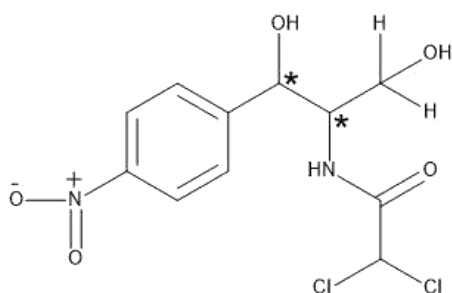


$$\text{Bond order} = \frac{\text{Total no of bonds}}{\text{Total resonating structures}} = \frac{5}{3} = 1.67$$

24. The number of chiral carbons in chloramphenicol is:

**Answer:** 2

**Solution:**



25. The reaction  $A_{(l)} \rightarrow 2B_{(g)}$

$\Delta U = 2.1 \text{ kcal}$ ,  $\Delta S = 20 \text{ cal/K}$  at 300 K, find  $\Delta G$  in kcal.

**Answer:** -2.7

**Solution:**

$$\Delta H = \Delta U + \Delta n_g RT$$

$$\Delta H = 2100 + (2 \times 2 \times 300) \quad (R = 2 \text{ calK}^{-1} \text{mol}^{-1})$$

$$= 3300 \text{ cal}$$

$$\Delta G = \Delta H - T\Delta S$$

$$\Delta G = 3300 - (300 \times 20) = -2700 \text{ cal} = -2.7 \text{ kcal}$$

Date: 7<sup>th</sup> January 2020 (Shift 2), JEE Main 2020 Paper

Time: 2:30 P.M. to 5:30 P.M.

Subject: Mathematics

1. From any point  $P$  on the line  $x = 2y$ , a perpendicular is drawn on  $y = x$ . Let the foot of perpendicular be  $Q$ . Find the locus of mid point of  $PQ$ .

a.  $5x = 7y$

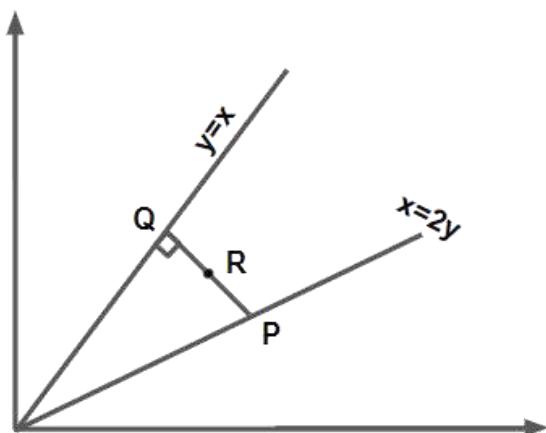
b.  $2x = 3y$

c.  $7x = 5y$

d.  $3x = 2y$

**Answer:** (a)

**Solution:**



Let  $R$  be the midpoint of  $PQ$

$PQ$  is perpendicular on line  $y = x$

$\therefore$  Equation of the line  $PQ$  can be written as  $y = -x + c$

$y = -x + c$  intersects  $y = x$  at  $Q: \left(\frac{c}{2}, \frac{c}{2}\right)$

$y = -x + c$  intersects  $x = 2y$  at  $P: \left(\frac{2c}{3}, \frac{c}{3}\right)$

$\therefore$  Midpoint  $R: \left(\frac{7c}{12}, \frac{5c}{12}\right)$

Locus of  $R : x = \frac{7c}{12}$

$$y = \frac{5c}{12}$$

$$\Rightarrow 5x = 7y$$

2. Let  $\theta_1$  and  $\theta_2$  (where  $\theta_1 < \theta_2$ ) are two solutions of  $2 \cot^2 \theta - \frac{5}{\sin \theta} + 4 = 0, \theta \in [0, 2\pi)$  then  $\int_{\theta_1}^{\theta_2} \cos^2 3\theta d\theta$  is equal to

a.  $\frac{\pi}{9}$

b.  $\frac{2\pi}{3}$

c.  $\frac{\pi}{3} + \frac{1}{6}$

d.  $\frac{\pi}{3}$

**Answer:** (d)

**Solution:**

$$2 \cot^2 \theta - \frac{5}{\sin \theta} + 4 = 0, \theta \in [0, 2\pi)$$

$$\Rightarrow 2 \operatorname{cosec}^2 \theta - 2 - 5 \operatorname{cosec} \theta + 4 = 0$$

$$\Rightarrow 2 \operatorname{cosec}^2 \theta - 4 \operatorname{cosec} \theta - \operatorname{cosec} \theta + 2 = 0$$

$$\Rightarrow \operatorname{cosec} \theta = 2 \text{ or } \frac{1}{2} \text{ (Not possible)}$$

As  $\theta \in [0, 2\pi)$ ,

$$\theta_1 = \frac{\pi}{6}, \theta_2 = \frac{5\pi}{6}$$

$$\Rightarrow \int_{\theta_1}^{\theta_2} \cos^2 3\theta d\theta = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{(1 + \cos 6\theta)}{2} d\theta$$

$$= \frac{1}{2} \left( \frac{5\pi}{6} - \frac{\pi}{6} \right) + \frac{\sin 6\theta}{12} \Big|_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$$

$$= \frac{\pi}{3}$$



$$P_5 = P_4 + P_3 = 11$$

$$\therefore P_5 \neq P_2 P_3 \text{ \& } P_1 + P_2 + P_3 + P_4 + P_5 = 26$$

$$\& P_3 = P_5 - P_4$$

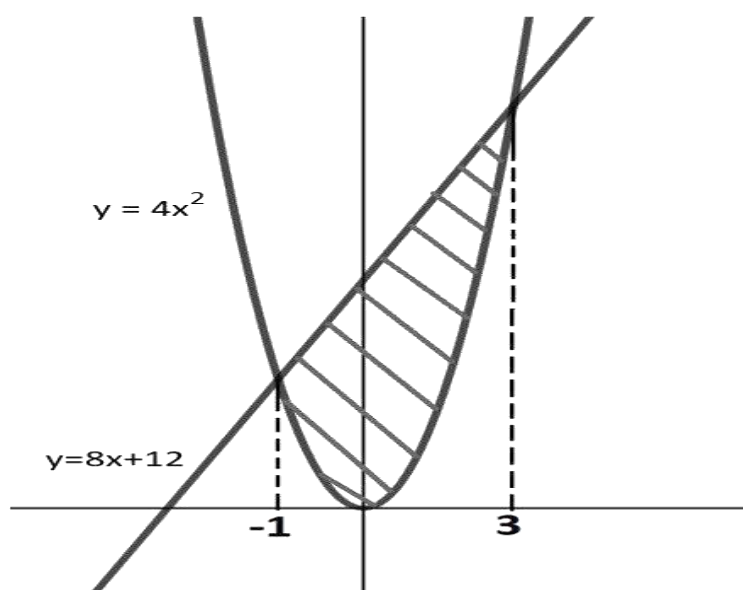
5. The area bounded by  $4x^2 \leq y \leq 8x + 12$  is

a.  $\frac{127}{3}$   
c.  $\frac{128}{3}$

b.  $\frac{125}{3}$   
d.  $\frac{124}{3}$

**Answer:** (c)

**Solution:**



For point of intersection,

$$4x^2 = 8x + 12$$

$$\Rightarrow x^2 - 2x - 3 = 0$$

$$\Rightarrow x = 3, -1$$

Area bounded is given by

$$A = \int_{-1}^3 (8x + 12 - 4x^2) dx$$

$$A = \left[ \frac{8x^2}{2} + 12x - \frac{4x^3}{3} \right]_{-1}^3$$

$$A = (36 + 36 - 36) - \left( 4 - 12 + \frac{4}{3} \right)$$

$$A = 44 - \frac{4}{3} = \frac{128}{3}$$

6. Contrapositive of  $A \subset B$  and  $B \subset C$  then  $C \subset D$

- a.  $C \not\subset D$  or  $A \not\subset B$  or  $B \not\subset C$   
 c.  $C \subset D$  and  $A \not\subset B$  or  $B \not\subset C$

- b.  $C \subset D$  or  $A \not\subset B$  and  $B \not\subset C$   
 d.  $C \subset D$  or  $A \not\subset B$  or  $B \not\subset C$

**Answer:** (d)

**Solution:**

Given statements:  $A \subset B$  and  $B \subset C$

Let  $A \subset B$  be  $p$

$B \subset C$  be  $q$

$C \subset D$  be  $r$

Modified statement:  $(p \wedge q) \Rightarrow r$

Contrapositive:  $\sim r \Rightarrow \sim (p \wedge q)$

$\therefore r \vee (\sim p \vee \sim q)$

$\Rightarrow C \subset D$  or  $A \not\subset B$  or  $B \not\subset C$

7. Let  $3 + 4 + 8 + 9 + 13 + 14 + 18 + \dots \dots \dots 40$  terms =  $S$ . If  $S = (102)m$  then  $m =$

- a. 5  
 c. 25  
 b. 10  
 d. 20

**Answer:** (d)

**Solution:**

$S = \underline{3+4} + \underline{8+9} + 13 + 14 + \dots \dots 40$  terms

$S = 7 + 17 + 27 + 37 + \dots \dots \dots 20$  terms

$$S = \frac{20}{2} [14 + (19)10] = 20 \times 102$$

$$\therefore m = 20$$

8.  $({}^{36}C_{r+1}) \times (k^2 - 3) = {}^{35}C_r \times 6$ , then the number of ordered pairs  $(r, k)$ , where  $k \in \mathbf{I}$ , are

- a. 2  
b. 6  
c. 3  
d. 4

**Answer:** (d)

**Solution:**

$$\text{using } {}^{36}C_{r+1} = \frac{36}{r+1} \times {}^{35}C_r$$

$$\frac{36}{r+1} \times {}^{35}C_r \times (k^2 - 3) = {}^{35}C_r \times 6$$

$$k^2 - 3 = \frac{r+1}{6}$$

$$k^2 = \frac{r+1}{6} + 3$$

$$k \in \mathbf{I}$$

$$r \rightarrow \text{Non-negative integer } 0 \leq r \leq 35$$

$$r = 5 \Rightarrow k = \pm 2$$

$$r = 35 \Rightarrow k = \pm 3$$

$$\text{No. of ordered pairs } (r, k) = 4$$

9. Let  $f(x)$  be a five-degree polynomial which has critical points  $x = \pm 1$  and  $\lim_{x \rightarrow 0} \left( 2 + \frac{f(x)}{x^3} \right) = 4$  then which one is incorrect.

- a.  $f(x)$  has minima at  $x = 1$  and maxima at  $x = -1$   
b.  $f(1) - 4f(-1) = 4$   
c.  $f(x)$  has maxima at  $x = 1$  and minima at  $x = -1$   
d.  $f(x)$  is odd

**Answer:** (a)

**Solution:**

Given  $\lim_{x \rightarrow 0} \left( 2 + \frac{f(x)}{x^3} \right) = 4$

$$\lim_{x \rightarrow 0} \frac{f(x)}{x^3} = 2$$

$\lim_{x \rightarrow 0} \frac{f(x)}{x^3}$  Limit exists and it is finite

$$\therefore f(x) = ax^5 + bx^4 + cx^3$$

$$\Rightarrow \lim_{x \rightarrow 0} (ax^2 + bx + c) = 2$$

$$c = 2$$

Also  $f'(x) = 5ax^4 + 4bx^3 + 6x^2$

$$f'(1) = 5a + 4b + 6 = 0$$

$$f'(-1) = 5a - 4b + 6 = 0$$

$$b = 0, \quad a = -\frac{6}{5}$$

$$f(x) = -\frac{6}{5}x^5 + 2x^3 \Rightarrow f(x) \text{ is odd}$$

$$f'(x) = -6x^4 + 6x^2$$

$$f''(x) = -24x^3 + 12x \quad (f''(1) < 0, \quad f''(-1) > 0)$$

At  $x = -1$  local minima      at  $x = 1$  local maxima

$$\text{And } f(1) - 4f(-1) = 4$$

10. If LMVT is applicable on  $f(x) = x^3 - 4x^2 + 8x + 11$  in  $[0,1]$ , the value of  $c$  is

a.  $\frac{4+\sqrt{5}}{3}$

b.  $\frac{4+\sqrt{7}}{3}$

c.  $\frac{4-\sqrt{7}}{3}$

d.  $\frac{4-\sqrt{5}}{3}$

**Answer:** (c)

**Solution:**

LMVT is applicable on  $f(x)$  in  $[0,1]$ , therefore it is continuous and differentiable in  $[0,1]$

$$\text{Now, } f(0) = 11, f(1) = 16$$

$$f'(x) = 3x^2 - 8x + 8$$

$$\therefore f'(c) = \frac{f(1)-f(0)}{1-0} = \frac{16-11}{1}$$

$$\Rightarrow 3c^2 - 8c + 8 = 5$$

$$\Rightarrow 3c^2 - 8c + 3 = 0$$

$$\Rightarrow c = \frac{8 \pm 2\sqrt{7}}{6} = \frac{4 \pm \sqrt{7}}{3}$$

$$\text{As } c \in (0,1)$$

$$\text{We get, } c = \frac{4-\sqrt{7}}{3}$$

11. Consider there are 5 machines. Probability of a machine being faulty is  $\frac{1}{4}$ . Probability of at most two machines being faulty is  $\left(\frac{3}{4}\right)^3 k$ , then the value of  $k$  is

a.  $\frac{17}{4}$

b.  $\frac{17}{8}$

c.  $\frac{17}{2}$

d. 4

**Answer:** (b)

**Solution:**

$$P(\text{machine being faulty}) = p = \frac{1}{4}$$

$$\therefore q = \frac{3}{4}$$

$$P(\text{at most two machines being faulty}) = P(\text{zero machine being faulty})$$

$$+ P(\text{one machine being faulty}) + P(\text{two machines being faulty})$$

$$= {}^5C_0 p^0 q^5 + {}^5C_1 p^1 q^4 + {}^5C_2 p^2 q^3$$

$$= q^5 + 5pq^4 + 10p^2q^3$$

$$= \left(\frac{3}{4}\right)^5 + 5 \times \frac{1}{4} \left(\frac{3}{4}\right)^4 + 10 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^3$$



$$y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$$

Differentiating w.r.t.  $x$  on both the sides, we get:

$$y'\sqrt{1-x^2} + y \times \frac{1}{2\sqrt{1-x^2}} \times (-2x) = -\sqrt{1-y^2} - x \times \frac{1}{2\sqrt{1-y^2}} \times (-2y)y'$$

$$\Rightarrow y'\sqrt{1-x^2} - \frac{xy}{\sqrt{1-x^2}} + \sqrt{1-y^2} - \frac{xy}{\sqrt{1-y^2}}y' = 0$$

$$\Rightarrow y' \left[ \sqrt{1-x^2} - \frac{xy}{\sqrt{1-y^2}} \right] = \frac{xy}{\sqrt{1-x^2}} - \sqrt{1-y^2}$$

Putting  $x = \frac{1}{2}, y = -\frac{1}{4}$

$$\Rightarrow y' \left[ \frac{\sqrt{3}}{2} + \frac{\frac{1}{8}}{\frac{\sqrt{15}}{4}} \right] = -\frac{\frac{1}{8}}{\frac{\sqrt{3}}{2}} - \frac{\sqrt{15}}{4}$$

$$\Rightarrow y' \left[ \frac{\sqrt{3}}{2} + \frac{1}{2\sqrt{15}} \right] = -\frac{1}{4\sqrt{3}} - \frac{\sqrt{15}}{4}$$

$$\Rightarrow y' \left[ \frac{\sqrt{45}+1}{2\sqrt{15}} \right] = -\frac{1+\sqrt{45}}{4\sqrt{3}}$$

$$\Rightarrow y' = -\frac{\sqrt{5}}{2}$$

14. Let  $A = [a_{ij}]$ ,  $B = [b_{ij}]$  are two  $3 \times 3$  matrices such that  $b_{ij} = \lambda^{i+j-2}a_{ij}$  and  $|B| = 81$ . Find  $|A|$  if  $\lambda = 3$

a.  $\frac{1}{81}$

b.  $\frac{1}{27}$

c.  $\frac{1}{9}$

d. 3

**Answer:** (c)

**Solution:**

$$b_{ij} = \lambda^{i+j-2}a_{ij}, \lambda = 3$$

$$B = \begin{bmatrix} 3^0 a_{11} & 3a_{12} & 3^2 a_{13} \\ 3a_{21} & 3^2 a_{22} & 3^3 a_{23} \\ 3^2 a_{31} & 3^3 a_{32} & 3^4 a_{33} \end{bmatrix}$$

$$|B| = \begin{vmatrix} 3^0 a_{11} & 3a_{12} & 3^2 a_{13} \\ 3a_{21} & 3^2 a_{22} & 3^3 a_{23} \\ 3^2 a_{31} & 3^3 a_{32} & 3^4 a_{33} \end{vmatrix}$$

Taking  $3^2$  Common each from  $C_3$  &  $R_3$

$$|B| = 81 \begin{vmatrix} a_{11} & 3a_{12} & a_{13} \\ 3a_{21} & 3^2 a_{22} & 3a_{23} \\ a_{31} & 3a_{32} & a_{33} \end{vmatrix}$$

Taking 3 common each from  $C_2$  &  $R_2$

$$|B| = 81(9) \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Given  $|B| = 81$

$$\Rightarrow 81 = 81(9)|A|$$

$$\Rightarrow |A| = \frac{1}{9}$$

15. Pair of tangents are drawn from the origin to the circle  $x^2 + y^2 - 8x - 4y + 16 = 0$ , then the square of length of chord of contact is

a.  $\frac{8}{5}$

b.  $\frac{8}{13}$

c.  $\frac{24}{5}$

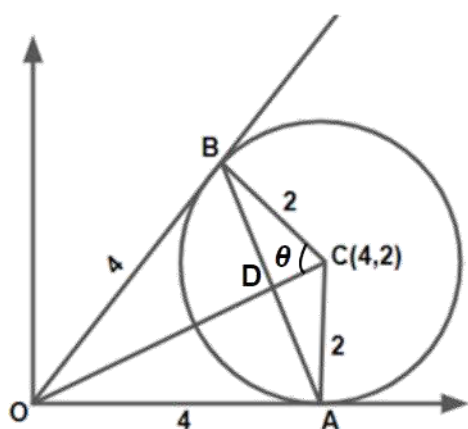
d.  $\frac{64}{5}$

**Answer:** (d)

**Solution:**

$$x^2 + y^2 - 8x - 4y + 16 = 0$$

$$(x - 4)^2 + (y - 2)^2 = 4 \Rightarrow \text{Centre } (4, 2), \text{ radius } (2)$$



$$OA = 4 = OB$$

In  $\triangle OBC$

$$\tan \theta = \frac{4}{2} = 2 \Rightarrow \sin \theta = \frac{2}{\sqrt{5}}$$

In  $\triangle BDC$

$$\sin \theta = \frac{BD}{2} \Rightarrow BD = \frac{4}{\sqrt{5}}$$

$$\text{Length of chord of contact } (AB) = \frac{8}{\sqrt{5}}$$

Alternative

(l) length of tangent = 4

(r) radius = 2

$$\Rightarrow \text{Length of chord of contact} = \frac{2lr}{\sqrt{l^2 + r^2}}$$

$$\text{Square of length of chord of contact} = \frac{64}{5}$$

16. Let  $y(x)$  is the solution of differential equation  $(y^2 - x) \frac{dy}{dx} = 1$  and  $y(0) = 1$ , then find the value of  $x$  where the curve cuts the  $x$ -axis.

- |            |        |
|------------|--------|
| a. $2 - e$ | b. $2$ |
| c. $2 + e$ | d. $e$ |

**Answer:** (a)

**Solution:**

$$(y^2 - x) \frac{dy}{dx} = 1$$

$$\frac{dx}{dy} + x = y^2$$

$$xe^y = \int y^2 e^y dy$$

$$x = y^2 - 2y + 2 + ce^{-y}$$

$$\text{Given } y(0) = 1$$

$$\Rightarrow c = -e$$

$$\therefore \text{Solution is } x = y^2 - 2y + 2 - e^{-y+1}$$

$$\therefore \text{The value of } x \text{ where the curve cuts the } x\text{-axis will be at } x = 2 - e$$

17. Let  $4\alpha \int_{-1}^2 e^{-\alpha|x|} dx = 5$  then  $\alpha =$

a.  $\ln \sqrt{2}$

b.  $\ln \frac{3}{4}$

c.  $\ln 2$

d.  $\ln \frac{4}{3}$

**Answer:** (c)

**Solution:**

$$4\alpha \int_{-1}^2 e^{-\alpha|x|} dx = 5$$

$$4\alpha \left[ \int_{-1}^0 e^{-\alpha|x|} dx + \int_0^2 e^{-\alpha|x|} dx \right] = 5$$

$$= 4\alpha \left[ \int_{-1}^0 e^{\alpha x} dx + \int_0^2 e^{-\alpha x} dx \right] = 5$$

$$= 4\alpha \left[ \left( \frac{1 - e^{-\alpha}}{\alpha} \right) + \left( \frac{e^{-2\alpha} - 1}{-\alpha} \right) \right] = 5$$

$$= 4[1 - e^{-2\alpha} - e^{-\alpha} + 1] = 5$$

$$\text{Let } e^{-\alpha} = t$$

$$\Rightarrow -4t^2 - 4t + 3 = 0$$

$$\Rightarrow t = \frac{1}{2} = e^{-\alpha} \Rightarrow \alpha = \ln 2$$

18. If  $\vec{a}, \vec{b}, \vec{c}$  are unit vectors such that  $\vec{a} + \vec{b} + \vec{c} = \vec{0}$  and  $\lambda = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$  and  $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$  then  $(\lambda, d) =$

a.  $\left(\frac{3}{2}, 3\vec{a} \times \vec{b}\right)$

b.  $\left(-\frac{3}{2}, 3\vec{a} \times \vec{c}\right)$

c.  $\left(-\frac{3}{2}, 3\vec{a} \times \vec{b}\right)$

d.  $\left(\frac{3}{2}, 3\vec{a} \times \vec{c}\right)$

**Answer:** (c)

**Solution:**

$$\text{Given } \vec{a} + \vec{b} + \vec{c} = \vec{0} \quad \lambda = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$$

$$\Rightarrow |\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{0}|^2$$

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$\lambda = -\frac{3}{2}$$

$$\text{Also } \vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$$

$$\Rightarrow \vec{d} = \vec{a} \times \vec{b} + \vec{b} \times (-\vec{a} - \vec{b}) + (-\vec{a} - \vec{b}) \times \vec{a}$$

$$\Rightarrow \vec{d} = \vec{a} \times \vec{b} + \vec{a} \times \vec{b} + \vec{a} \times \vec{b}$$

$$\Rightarrow \vec{d} = 3(\vec{a} \times \vec{b}) = 3\vec{a} \times \vec{b}$$



$$\Rightarrow x + y = 19$$

$$\text{Variance} = \frac{\sum x_i^2}{n} - (\bar{x})^2$$

$$\Rightarrow 25 = \frac{2^2 + 3^2 + 16^2 + 20^2 + 13^2 + 7^2 + x^2 + y^2}{8} - 100$$

$$\Rightarrow 1000 = 887 + x^2 + y^2$$

$$\Rightarrow x^2 + y^2 = 113$$

$$\Rightarrow (x + y)^2 - 2xy = 113$$

$$\Rightarrow 361 - 2xy = 113$$

$$\text{So, } xy = 124$$

21. If  $Q \equiv \left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$  is foot of perpendicular drawn from  $P(1, 0, 3)$  onto a line  $L$  and line  $L$  is passing through  $(\alpha, 7, 1)$ , then value of  $\alpha$  is \_\_\_\_\_.

**Answer:** (4)

**Solution:**

$$\text{Direction ratios of line } L: \left(\alpha - \frac{5}{3}, 7 - \frac{7}{3}, 1 - \frac{17}{3}\right)$$

$$= \left(\frac{3\alpha - 5}{3}, \frac{14}{3}, -\frac{14}{3}\right)$$

$$\text{Direction ratios of } PQ: \left(-\frac{2}{3}, -\frac{7}{3}, -\frac{8}{3}\right)$$

As line  $L$  is perpendicular to  $PQ$

$$\text{So, } \left(\frac{3\alpha - 5}{3}\right)\left(-\frac{2}{3}\right) + \left(\frac{14}{3}\right)\left(-\frac{7}{3}\right) + \left(-\frac{14}{3}\right)\left(-\frac{8}{3}\right) = 0$$

$$\Rightarrow -6\alpha + 10 - 98 + 112 = 0$$

$$\Rightarrow 6\alpha = 24 \Rightarrow \alpha = 4$$

22. If system of equations  $x + y + z = 6$ ,  $x + 2y + 3z = 10$ ,  $3x + 2y + \lambda z = \mu$  has more than 2 solutions, then  $(\mu - \lambda^2)$  is \_\_\_\_\_.

**Answer:** (13)

**Solution:**

The system of equations has more than 2 solutions

$$\therefore D = D_3 = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 2 & \lambda \end{vmatrix} = 0 \Rightarrow 2\lambda - 6 - \lambda + 9 + 2 - 6 = 0$$

$$\Rightarrow \lambda = 1$$

$$\begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 3 & 2 & \mu \end{vmatrix} = 0 \Rightarrow 2\mu - 20 - \mu + 30 - 24 = 0$$

$$\Rightarrow \mu = 14$$

$$\text{So, } \mu - \lambda^2 = 13$$

23. If  $f(x)$  is defined in  $x \in \left(-\frac{1}{3}, \frac{1}{3}\right)$  &

$$f(x) = \begin{cases} \left(\frac{1}{x}\right) \log_e \left(\frac{1+3x}{1-2x}\right) & x \neq 0 \\ k & x = 0 \end{cases}$$

The value of  $k$  such that  $f(x)$  is continuous is \_\_\_\_\_.

**Answer:** (5)

**Solution:**

As  $f(x)$  is continuous

$$\Rightarrow \lim_{x \rightarrow 0} f(x) = f(0) = k$$

$$\Rightarrow \lim_{x \rightarrow 0} \left(\frac{1}{x}\right) \log_e \left(\frac{1+3x}{1-2x}\right) = k$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{3 \log(1+3x)}{3x} - \lim_{x \rightarrow 0} \frac{(-2) \log(1-2x)}{(-2x)} = k$$

$$\Rightarrow 3 + 2 = k \Rightarrow k = 5$$

24. Let  $X = \{x: 1 \leq x \leq 50, x \in \mathbf{N}\}$ ,  $A = \{x: x \text{ is a multiple of } 2\}$ ,  $B = \{x: x \text{ is a multiple of } 7\}$ . Then the number of elements in the smallest subset of  $X$  which contain elements of both  $A$  and  $B$  is \_\_\_\_\_.

**Answer:** (29)

**Solution:**

$$A = \{x: x \text{ is multiple of } 2\} = \{2, 4, 6, 8, \dots\}$$

$$B = \{x: x \text{ is multiple of } 7\} = \{7, 14, 21, \dots\}$$

$$X = \{x: 1 \leq x \leq 50, x \in \mathbf{N}\}$$

Smallest subset of  $X$  which contains elements of both  $A$  and  $B$  is a set with multiples of 2 or 7 less than 50.

$$P = \{x: x \text{ is a multiple of } 2 \text{ less than or equal to } 50\}$$

$$Q = \{x: x \text{ is a multiple of } 7 \text{ less than or equal to } 50\}$$

$$n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$$

$$= 25 + 7 - 3$$

$$= 29$$







$$2 = \frac{T_1}{T} + \frac{T_2}{T}$$

$$T = \frac{T_1 + T_2}{2}$$

7. Activity of a substance changes from  $700 \text{ s}^{-1}$  to  $900 \text{ s}^{-1}$  in 30 minutes. Find its half-life in minutes.

- a. 66  
b. 62  
c. 56  
d. 50

Solution:

(b)

Using the half-life equation,

$$\ln \frac{A_0}{A_t} = \lambda t$$

At half-life,  $t = t_{\frac{1}{2}}$  and  $A_t = \frac{A_0}{2}$

$$\Rightarrow \ln 2 = \lambda t_{\frac{1}{2}} \text{ ----- (1)}$$

Also given

$$\ln \frac{500}{700} = \lambda (30) \text{ ----- (2)}$$

Dividing the equations,

$$\frac{\ln 2}{\ln \left( \frac{7}{5} \right)} = \frac{t_{\frac{1}{2}}}{30}$$

$$\Rightarrow t_{\frac{1}{2}} = 61.8 \text{ minutes}$$

8. In YDSE, separation between slits is  $0.15 \text{ mm}$ , distance between slits and screen is  $1.5 \text{ m}$  and wavelength of light is  $589 \text{ nm}$ . Then, fringe width is

- a.  $5.9 \text{ mm}$   
b.  $3.9 \text{ mm}$   
c.  $1.9 \text{ mm}$   
d.  $2.3 \text{ mm}$

Solution:

(a)

Given,

Maximum diameter of pipe =  $6.4 \text{ cm}$

Minimum diameter of pipe =  $4.8 \text{ cm}$

$$\beta = \lambda \frac{D}{d} = \frac{589 \times 10^{-9} \times 1.5}{0.5 \times 10^{-3}} = 5.9 \text{ mm}$$

9. An ideal fluid is flowing in a pipe in streamline flow. Pipe has maximum and minimum diameter of  $6.4 \text{ cm}$  and  $4.8 \text{ cm}$  respectively. Find out the ratio of minimum to maximum velocity.

a.  $\frac{81}{256}$   
c.  $\frac{3}{4}$

b.  $\frac{9}{16}$   
d.  $\frac{3}{16}$

Solution:

(b)

Using equation of continuity

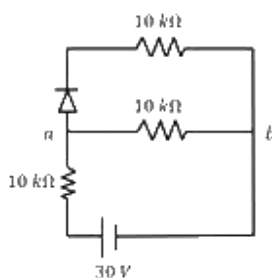
$$A_1 V_1 = A_2 V_2$$

$$\frac{V_1}{V_2} = \frac{A_2}{A_1} = \left(\frac{4.8}{6.4}\right)^2 = \frac{9}{16}$$

10. There is an electric circuit as shown in the figure. Find potential difference between points  $a$  and  $b$

a.  $0 \text{ V}$   
c.  $10 \text{ V}$

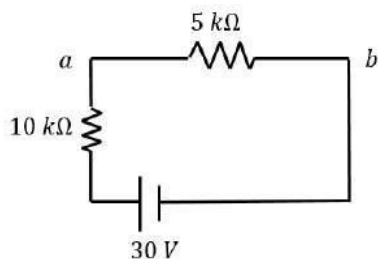
b.  $15 \text{ V}$   
d.  $5 \text{ V}$



Solution:

(c)

Diode is in forward bias, so it will behave as simple wire. So, the circuit effectively becomes



$$V_{ab} = \frac{30}{5+10} \times 5 = 10 \text{ V}$$

11. A particle of mass  $m$  and positive charge  $q$  is projected with a speed of  $V_0$  in  $y$ -direction in the presence of electric and magnetic field and both of them are in  $x$ -direction. Find the instant of time at which the speed of particle becomes double the initial speed.

a.  $t = \frac{mV_0\sqrt{3}}{qE}$   
 c.  $t = \frac{mV_0}{qE}$

b.  $t = \frac{mV_0\sqrt{2}}{qE}$   
 d.  $t = \frac{mV_0}{2qE}$

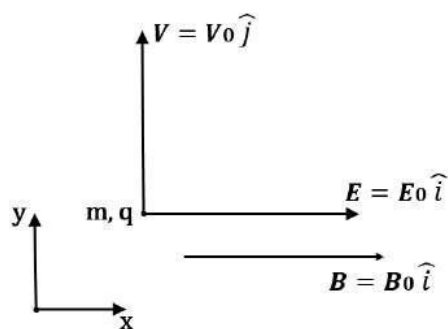
Solution:

(a)

As  $\vec{V} = V_0\hat{j}$  (magnitude of velocity does not change in  $y$ - $z$  plane)

$$(2V_0)^2 = V_0^2 + V_x^2$$

$$V_x = \sqrt{3}V_0$$



$$\therefore \sqrt{3}V_x = 0 + \frac{qE}{m}t \Rightarrow t = \frac{mV_0\sqrt{3}}{qE}$$

12. Two sources of sound moving with the same speed  $V$  and emitting frequencies of  $1400 \text{ Hz}$  are moving such that one source  $s_1$  is moving towards the observer and  $s_2$  is moving away from the observer. If observer hears a beat frequency of  $2 \text{ Hz}$ , then find the speed of the source (Given  $V_{\text{sound}} \gg V_{\text{source}}$  and  $V_{\text{sound}} = 350 \text{ m/s}$ .)

- a.  $\frac{1}{4}$   
c. 2

- b. 4  
d.  $\frac{1}{2}$

Solution:

(a)

$$f_0 \left( \frac{C}{C - V} \right) - f_0 \left( \frac{C}{C + V} \right) = 2$$

$$V = \frac{1}{4} m/s$$

13. An electron and a photon have same energy  $E$ . Find the de Broglie wavelength of electron to wavelength of photon. (Given mass of electron is  $m$  and speed of light is  $c$ )

a.  $\frac{2}{c} \left( \frac{E}{2m} \right)^{\frac{1}{2}}$

b.  $\frac{1}{c} \left( \frac{E}{2m} \right)^{\frac{1}{3}}$

c.  $\frac{1}{c} \left( \frac{E}{m} \right)^{\frac{1}{2}}$

d.  $\frac{1}{c} \left( \frac{E}{2m} \right)^{\frac{1}{2}}$

Solution:

(d)

$$\lambda_d \text{ for electron} = \frac{h}{\sqrt{2mE}}$$

$$\lambda \text{ for photon} = \frac{hc}{E}$$

$$\text{Ratio} = \frac{h}{\sqrt{2mE}} \frac{E}{hc} = \frac{1}{c} \sqrt{\frac{E}{2m}}$$

14. A ring is rotated about diametric axis in a uniform magnetic field perpendicular to the plane of the ring. If initially the plane of the ring is perpendicular to the magnetic field. Find the instant of time at which EMF will be maximum and minimum respectively.

a. 2.5 sec, 5 sec

b. 5 sec, 7.5 sec

c. 2.5 sec, 7.5 sec

d. 10 sec, 5 sec

Solution:

(a)

$$\omega = \frac{2\pi}{T} = \frac{\pi}{5}$$

$$\text{When } \omega t = \frac{\pi}{2}$$

Then  $\phi_{flux}$  will be minimum

$\therefore e$  will be maximum

$$t = \frac{\frac{\pi}{2}}{\frac{\pi}{5}} = 2.5 \text{ sec}$$

When  $\omega t = \pi$

Then  $\phi_{flux}$  will be maximum

$\therefore e$  will be minimum

$$t = \frac{\pi}{\frac{\pi}{5}} = 5 \text{ sec}$$

15. Electric field in space is given by  $\vec{E}(t) = \frac{E_0(\hat{i} + \hat{j})}{\sqrt{2}} \cos(\omega t + kz)$ . A positively charged particle at  $(0, 0, \pi/k)$  is given velocity  $v_0 \hat{k}$  at  $t = 0$ . Direction of force acting on particle is

a.  $f = 0$

b. Antiparallel to  $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$

c. Parallel to  $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$

d.  $\hat{k}$

Solution:

(b)

Force due to electric field is in direction  $-\frac{\hat{i} + \hat{j}}{\sqrt{2}}$

Because at  $t = 0, E = -\frac{(\hat{i} + \hat{j})}{\sqrt{2}} E_0$

Force due to magnetic field is in direction  $q(\vec{v} \times \vec{B})$  and  $\vec{v} \parallel \hat{k}$

$\therefore$  It is parallel to  $\vec{E}$

$\therefore$  Net force is antiparallel to  $\frac{(\hat{i} + \hat{j})}{\sqrt{2}}$ .

16. Focal length of convex lens in air is  $16 \text{ cm}$  ( $\mu_{glass} = 1.5$ ). Now the lens is submerged in liquid of refractive index 1.42. Find the ratio of focal length in medium to focal length in air.

a. 9

b. 17

c. 1

d. 5

Solution:

(a)

$$\frac{1}{f_a} = \left( \frac{\mu_g}{\mu_l} - 1 \right) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$





Solution:

(a)

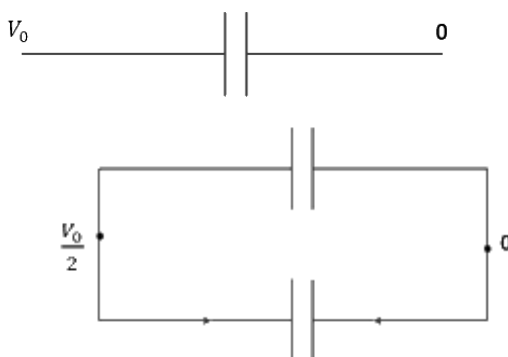
Energy density in magnetic field  $= \frac{B^2}{2\mu_0}$

$$= \frac{\text{Force} \times \text{displacement}}{(\text{displacement})^3} = \frac{MLT^{-2} \cdot L}{(L)^3} = ML^{-1}T^{-2}$$

21. A capacitor of  $60 \text{ pF}$  charged to  $20 \text{ V}$ . Now, the battery is removed, and this capacitor is connected to another identical uncharged capacitor. Find heat loss in  $\text{J}$ .

Solution:

(6)



$$V_0 = 20 \text{ V}$$

$$\text{Initial potential energy } U_i = \frac{1}{2} CV_0^2$$

After connecting identical capacitor in parallel, voltage across each capacitor will be  $\frac{V_0}{2}$ . Then, final potential energy  $U_f = 2 \left[ \frac{1}{2} C \left( \frac{V_0}{2} \right)^2 \right]$

$$\text{Heat loss} = U_i - U_f$$

$$= \frac{CV_0^2}{2} - \frac{CV_0^2}{4} = \frac{CV_0^2}{4} = \frac{60 \times 10^{-12} \times 20^2}{4} = 6 \times 10^{-9} = 6 \text{ nJ}$$

22. When  $m$  grams of steam at  $100^\circ \text{C}$  is mixed with  $200$  grams of ice at  $0^\circ \text{C}$ , it results in water at  $40^\circ \text{C}$ . Find the value of  $m$  in grams

(Given, Latent heat of fusion ( $L_f$ ) =  $80 \text{ cal/g}$ , Latent heat of vaporization ( $L_v$ ) =  $540 \text{ cal/g}$ , specific heat of water ( $C_w$ ) =  $1 \text{ cal/g}^\circ \text{C}$ )

Solution:

(40)

Here, heat absorbed by ice =  $m_{ice} L_f + m_{ice} C_w (40 - 0)$

Heat released by steam =  $m_{steam} L_v + m_{steam} C_w (100 - 40)$

Heat absorbed = heat released

$$m_{ice} L_f + m_{ice} C_w (40 - 0) = m_{steam} L_v + m_{steam} C_w (100 - 40)$$

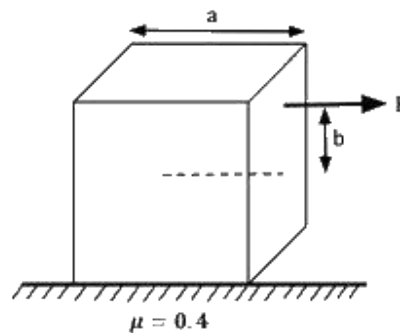
$$\Rightarrow 200 \times 80 \text{ cal/g} + 200 \times 1 \text{ cal/g/}^\circ\text{C} \times (40 - 0)$$

$$= m \times 540 \text{ cal/g} + 540 \times 1 \text{ cal/g/}^\circ\text{C} \times (100 - 40)$$

$$\Rightarrow 200 [80 + (40)1] = m[540 + (60)1]$$

$$m = 40 \text{ g}$$

23. A solid cube of side ' $a$ ' is shown in the figure. Find the maximum value of  $c \frac{100b}{a}$  for which the block does not topple before sliding.



Solution:

(50)

$F$  balances kinetic friction so that the block can move

$$\text{So, } F = \mu mg$$

For no toppling, the net torque about bottom right edge should be zero

i.e

$$F \left( \frac{a}{2} + b \right) \leq mg \frac{a}{2}$$

$$\mu mg \left( \frac{a}{2} + b \right) \leq mg \frac{a}{2}$$

$$F \mu \frac{a}{2} + \mu b \leq \frac{a}{2}$$

$$0.2a + 0.4b \leq 0.5a$$

7<sup>th</sup> Jan (Shift 2, Physics)

Page | 12

$$0.4b \leq 0.3a$$

$$b \leq \frac{3}{4} a$$

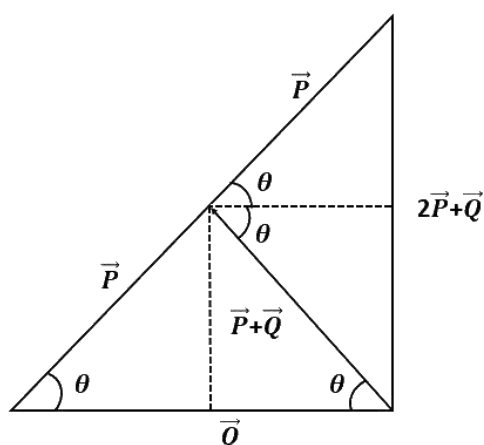
But, maximum value of  $b$  can only be  $0.5a$

$\therefore$  Maximum value of  $100 \frac{b}{a}$  is 50.

24. Magnitude of resultant of two vectors  $\vec{P}$  and  $\vec{Q}$  is equal to magnitude of  $\vec{P}$ . Find the angle between  $\vec{Q}$  and resultant of  $2\vec{P}$  and  $\vec{Q}$ .

Solution:

(90°)



25. A battery of unknown emf connected to a potentiometer has balancing length 560 cm. If a resistor of resistance  $10 \Omega$  is connected in parallel with the cell the balancing length change by 60 cm. If the internal resistance of the cell is  $\frac{n}{10} \Omega$ , the value of 'n' is

Solution:

(12)

Let the emf of cell is  $\varepsilon$  internal resistance is ' $r$ ' and potential gradient is  $x$ .

$$\varepsilon = 560 x \quad (1)$$

After connecting the resistor

$$\frac{\varepsilon \times 10}{10 + r} = 500x \quad (2)$$

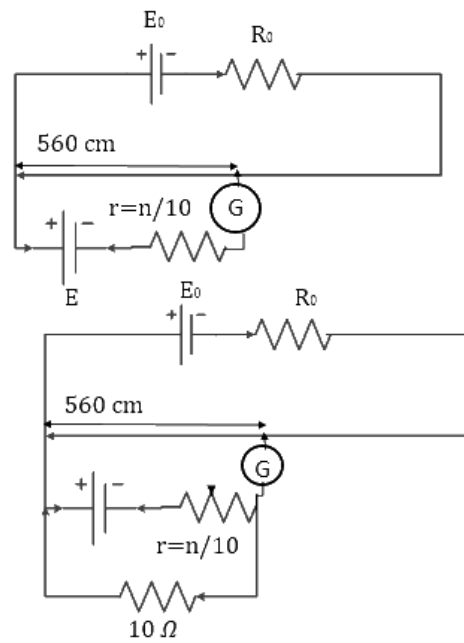
From (1) and (2)

$$\frac{560 \times 10}{10 + r} = 500x$$

$$56 = 540 + 5r$$

$$r = \frac{6}{5} = 1.2 \Omega$$

$$n = 12$$



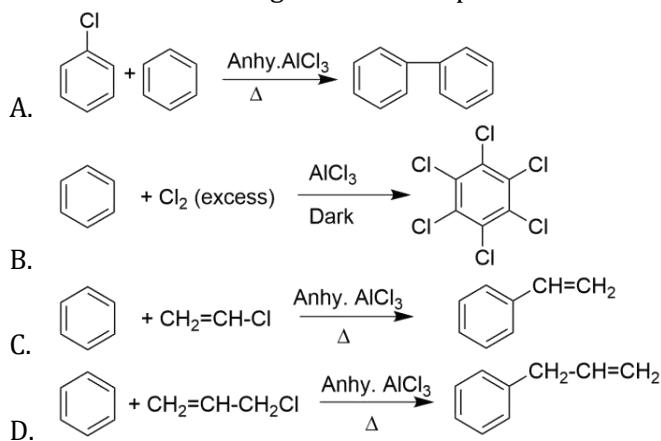
Date: 7<sup>th</sup> January 2020

# JEE Main 2020 Paper

Time: 02.30 PM – 05.30 PM

Subject: Chemistry

1. Which of the following reactions are possible?



- a. A, B, C  
c. A, C, D

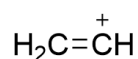
- b. B, D  
d. A, C

**Answer: b**

**Solution:**

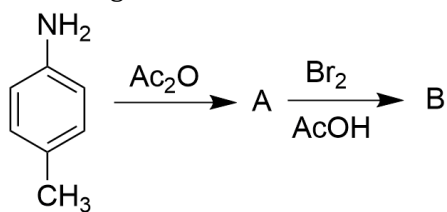
In aryl halides, due to the partial double bond character generated by chlorine, the aryl cation is not formed.

Vinyl halides do not give Friedel-Crafts reaction, because the intermediate that is generated (vinyl cation) is not stable.

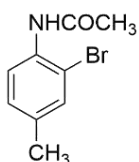


vinyl  
cation

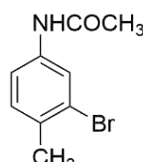
2. B in the given reaction is?



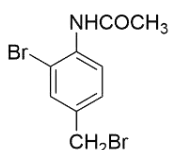
a.



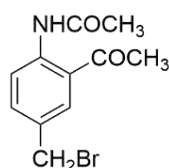
b.



c.



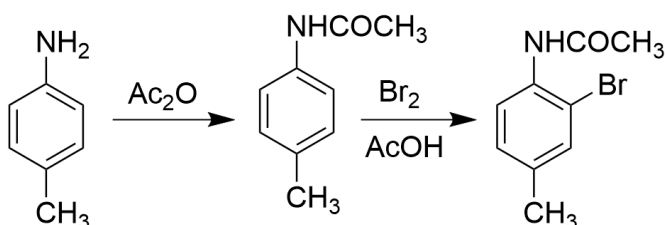
d.



**Answer: a**

**Solution:**

During trisubstitution, the acetanilide group attached to the benzene ring is more electron donating than the methyl group attached, owing to +M effect, and therefore, the incoming electrophile would prefer ortho w.r.t the acetanilide group.



3. The correct statement about gluconic acid is:

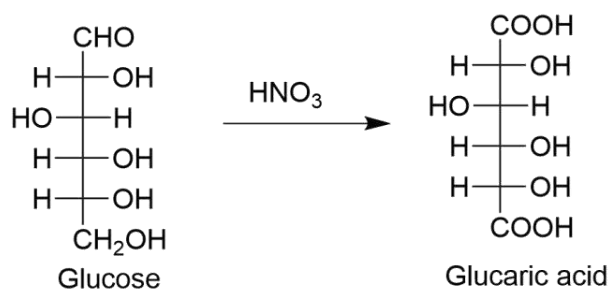
- It is prepared by oxidation of glucose with  $\text{HNO}_3$
- It is obtained by partial oxidation of glucose
- It is a dicarboxylic acid
- It forms hemiacetal or acetal

**Answer: b**

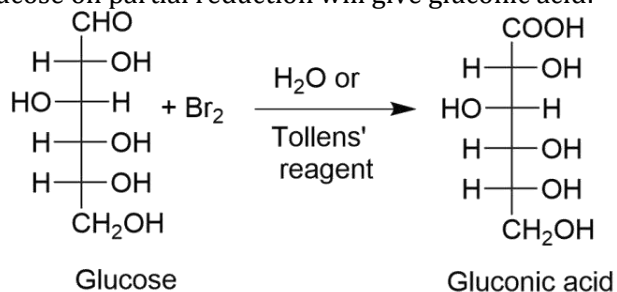
**Solution:**

The gluconic acid formed is a monocarboxylic acid which is formed during the partial oxidation of glucose

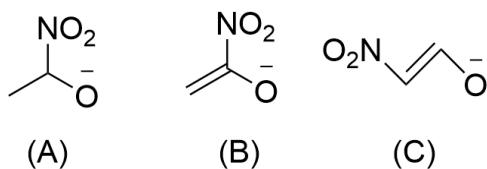
(a) Glucose on reaction with  $\text{HNO}_3$  will give glucaric acid:



(b) Glucose on partial reduction will give gluconic acid:



4. The stability order of the following alkoxide ions are:



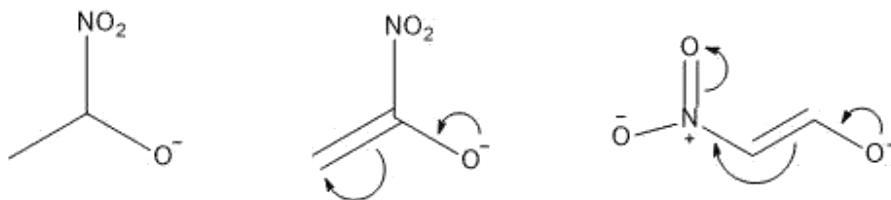
- a.  $\text{C} > \text{B} > \text{A}$   
b.  $\text{A} > \text{C} > \text{B}$

- c.  $\text{B} > \text{A} > \text{C}$   
d.  $\text{C} > \text{A} > \text{B}$

**Answer: a**

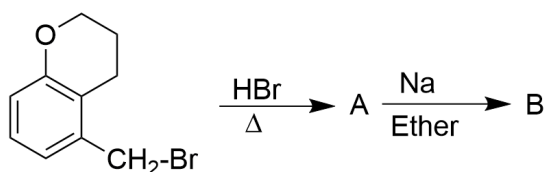
**Solution:**

Higher the delocalization of the negative charge, more will be the stability of the anion.



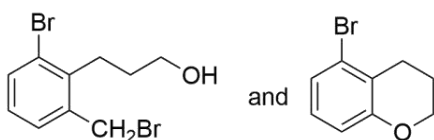
- (A) The negative charge is stabilized only through  $-I$  effect exhibited by the  $-\text{NO}_2$  group.  
 (B) The negative charge is stabilized by the delocalization of the double bond and the  $-I$  effect exhibited by the  $-\text{NO}_2$  group.  
 (C) The negative charge is stabilized by extended conjugation.

5.

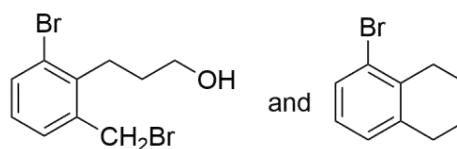


A and B are:

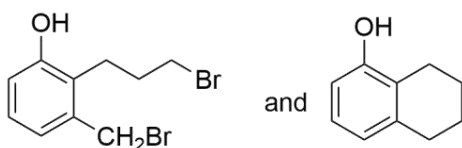
a.



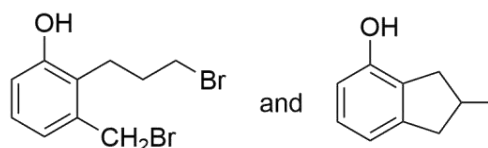
b.



c.

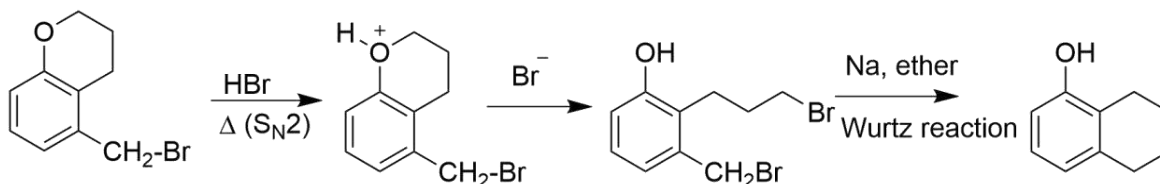


d.



**Answer: c**

**Solution:**





**Solution:**

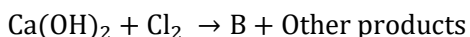
$$\Lambda_m^0 \text{NaI} - \Lambda_m^0 \text{NaBr} = \Lambda_m^0 \text{NaBr} - \Lambda_m^0 \text{KBr}$$

$$[\lambda_m^0 \text{Na}^+ + \lambda_m^0 \text{I}^-] - [\lambda_m^0 \text{Na}^+ + \lambda_m^0 \text{Br}^-] = [\lambda_m^0 \text{Na}^+ + \lambda_m^0 \text{Br}^-] - [\lambda_m^0 \text{K}^+ + \lambda_m^0 \text{Br}^-]$$

$$\lambda_m^0 \text{I}^- - \lambda_m^0 \text{Br}^- \neq \lambda_m^0 \text{Na}^+ - \lambda_m^0 \text{K}^+$$

9.  $\text{NaOH} + \text{Cl}_2 \rightarrow \text{A} + \text{Other products}$

Hot & conc.



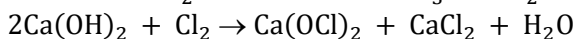
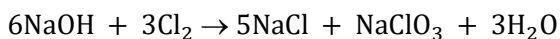
Cold & dil.

A and B respectively are:

- |   |                                       |
|---|---------------------------------------|
| a. $\text{NaClO}_3, \text{Ca(OCl)}_2$   | c. $\text{NaCl}, \text{Ca(ClO}_3)_2$  |
| b. $\text{NaClO}_3, \text{Ca(ClO}_3)_2$ | d. $\text{NaClO}, \text{Ca(ClO}_3)_2$ |

**Answer:** a

**Solution:**



10. There are two beakers (I) having pure volatile solvent and (II) having a volatile solvent and a non-volatile solute. If both the beakers are placed together in a closed container then:
- Volume of solvent beaker will decrease and solution beaker will increase
  - Volume of solvent beaker will increase and solution beaker will also increase
  - Volume of solvent beaker will decrease and solution beaker will also decrease
  - Volume of solvent beaker will increase and solution beaker will decrease

**Answer:** a

**Solution:**

Consider beaker I contains the solvent and beaker 2 contains the solution. Let the vapour pressure of the beaker I be  $P^o$  and the vapour pressure of beaker II be  $P^s$ . According to Raoult's law, the vapour pressure of the solvent ( $P^o$ ) is greater than the vapour pressure of the solution ( $P^s$ )

$$(P^o > P^s)$$

Due to a higher vapour pressure, the solvent flows into the solution. So volume of beaker II would increase.

In a closed beaker, both the liquids on attaining equilibrium with the vapour phase will end up having the same vapour pressure. Beaker II attains equilibrium at a lower vapour pressure and so in its case, condensation will occur so as to negate the increased vapour pressure from beaker I, which results in an increase in its volume.

On the contrary, since particles are condensing from the vapour phase in beaker II, the vapour pressure will decrease. Since beaker I at equilibrium attains a higher vapour pressure, there, evaporation will be favoured more so as to compensate for the decreased vapour pressure, as mentioned in the previous statement.

11. Metal with low melting point containing impurities of high melting point can be purified by
- Zone refining
  - Vapor phase refining
  - Distillation
  - Liquation

**Answer:** d

**Solution:**

Liquation is the process of refining a metal with a low melting point containing impurities of high melting point

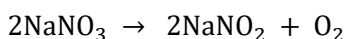
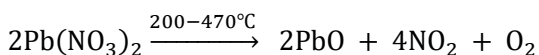
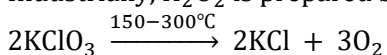
12. Which of the following statements are correct?
- On decomposition of  $\text{H}_2\text{O}_2$ ,  $\text{O}_2$  gas is released.
  - 2-ethylanthraquinol is used in the preparation of  $\text{H}_2\text{O}_2$
  - On heating  $\text{KClO}_3$ ,  $\text{Pb}(\text{NO}_3)_2$  and  $\text{NaNO}_3$ ,  $\text{O}_2$  gas is released.
  - In the preparation of sodium peroxoborate,  $\text{H}_2\text{O}_2$  is treated with sodium metaborate.
- I, II, IV
  - II, III, IV
  - I, II, III, IV
  - I, II, III

**Answer:** c

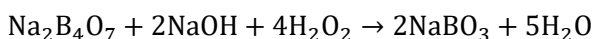
**Solution:**

Decomposition of  $\text{H}_2\text{O}_2$  :  $2\text{H}_2\text{O}_2(\text{l}) \rightarrow \text{O}_2(\text{g}) + 2\text{H}_2\text{O}(\text{l})$

Industrially,  $\text{H}_2\text{O}_2$  is prepared by the auto-oxidation of 2-alkylanthraquinols.



Synthesis of sodium perborate:



13. Among the following, which is a redox reaction?

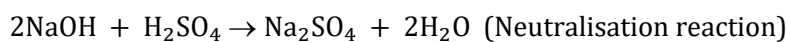
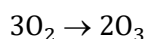
- $\text{N}_2 + \text{O}_2 \xrightarrow{2000 \text{ K}}$
- Formation of  $\text{O}_3$  from  $\text{O}_2$
- Reaction between  $\text{NaOH}$  and  $\text{H}_2\text{SO}_4$
- Reaction between  $\text{AgNO}_3$  and  $\text{NaCl}$

**Answer:** a

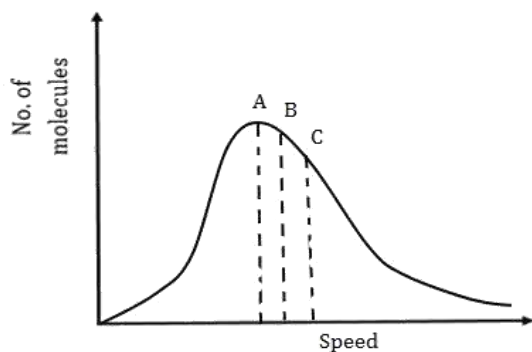
**Solution:**

$\text{N}_2 + \text{O}_2 \xrightarrow{2000 \text{ K}} 2\text{NO}$  : The oxidation state of N changes from 0 to +2, and the oxidation state of O changes from 0 to -2

In all the remaining reactions, there is no change in oxidation states of the elements participating in the reaction.



14.



Select the correct options:

- |   |   |
|---|---|
| a. $A = C_{\text{MPS}}, B = C_{\text{Average}}, C = C_{\text{RMS}}$ | b. $A = C_{\text{Average}}, B = C_{\text{MPS}}, C = C_{\text{RMS}}$ |
| c. $A = C_{\text{RMS}}, B = C_{\text{Average}}, C = C_{\text{MPS}}$ | d. $A = C_{\text{Average}}, B = C_{\text{MPS}}, C = C_{\text{RMS}}$ |

**Answer:** a

**Solution:**



$\text{Co}^{3+}$  has  $d^6$  electronic configuration. In the presence of strong field ligand,  $\Delta_o > P$ . Thus the splitting occurs as:  $t_{2g}^6 e_g^0$ ; so the magnetic moment is zero.

According to the spectrochemical series, en is a stronger ligand than F and therefore promotes pairing. This implies that the  $\Delta_o$  of en is more than the  $\Delta_o$  of F.

$$\Delta_o = \frac{hc}{\lambda_{\text{abs}}}$$

$$\Delta_t = \frac{4}{9} \Delta_o = 8000 \text{ cm}^{-1}$$

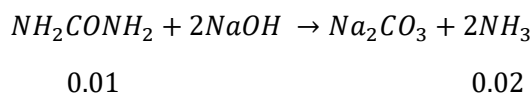
17. 0.6 g of urea on strong heating with NaOH evolves  $\text{NH}_3$ . The liberated  $\text{NH}_3$  will react completely with which of the following HCl solutions?

- |                        |                        |
|------------------------|------------------------|
| a. 100 mL of 0.2 N HCl | c. 100 mL of 0.1 N HCl |
| b. 400 mL of 0.2 N HCl | d. 200 mL of 0.2 N HCl |

**Answer:** a

**Solution:**

$$\text{Moles of urea} = \left( \frac{0.6}{60} \right) = 0.01$$



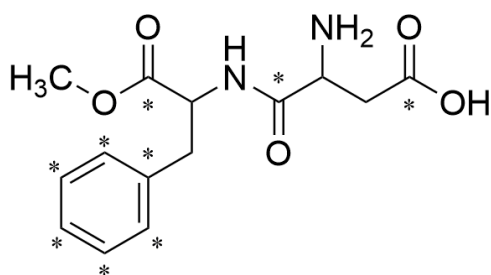
0.02 moles of  $\text{NH}_3$  reacts with 0.02 moles of HCl.

$$\text{Moles of HCl in option a} = 0.2 \times \frac{100}{1000} = 0.02$$

21. Number of  $sp^2$  hybrid carbon atoms in aspartame is \_\_\_\_.

**Answer:** 9

**Solution:**



The marked carbons are  $sp^2$  hybridised.

22. 3 grams of acetic acid is mixed in 250 mL of 0.1 M HCl. This mixture is now diluted to 500 mL. 20 mL of this solution is now taken in another container.  $\frac{1}{2}$  mL of 5 M NaOH is added to this. Find the pH of this solution. ( $\log 3 = 0.4771$ ,  $pK_a = 4.74$ ).

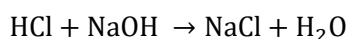
**Answer:** 5.22

**Solution:**

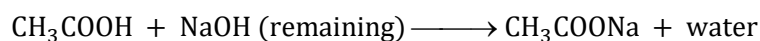
mmole of acetic acid in 20 mL = 2

mmole of HCl in 20 mL = 1

mmole of NaOH = 2.5



1	2.5	-	-
-	1.5	1	1



2	1.5	-	-
0.5	0	1.5	

$$\text{pH} = \text{pK}_a + \log \frac{1.5}{0.5} = 4.74 + \log 3 = 4.74 + 0.48 = 5.22$$

23. The flocculation value for  $\text{As}_2\text{S}_3$  sol by HCl is  $30 \text{ mmolL}^{-1}$ . Calculate mass of  $\text{H}_2\text{SO}_4$  required in grams for 250 mL sol is \_\_\_\_.

**Answer:** 0.3675 g

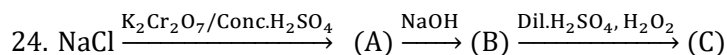
**Solution:**

For 1L sol 30 mmol of HCl is required

$\therefore$  For 1L sol 15 mmol of  $\text{H}_2\text{SO}_4$  is required

For 250 mL of sol,

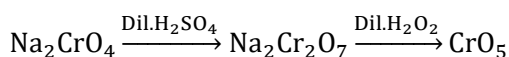
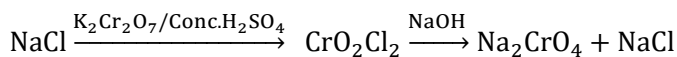
$$\frac{15}{4} \times 98 \times 10^{-3} \text{ g of } \text{H}_2\text{SO}_4 = 0.3675 \text{ g}$$



Determine the total number of atoms in per unit formula of (A), (B) & (C).

**Answer:** 18

**Solution:**

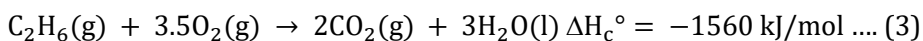
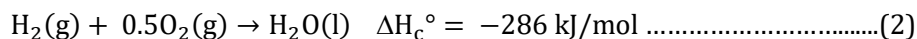
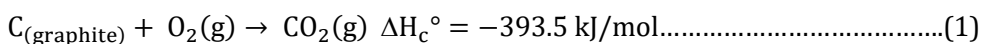


(A) =  $\text{CrO}_2\text{Cl}_2$ , (B) =  $\text{Na}_2\text{CrO}_4$  and (C) =  $\text{CrO}_5$

25. Calculate the  $\Delta H_f^\circ$  (in kJ/mol) for  $\text{C}_2\text{H}_6(\text{g})$ , if  $\Delta H_c^\circ [\text{C}_{(\text{graphite})}] = -393.5 \text{ kJ/mol}$ ,  $\Delta H_c^\circ [\text{H}_2(\text{g})] = -286 \text{ kJ/mol}$  and  $\Delta H_c^\circ [\text{C}_2\text{H}_6(\text{g})] = -1560 \text{ kJ/mol}$ .

**Answer:** -85 kJ/mol

**Solution:**



$$2 \times (-393.5) + 3 \times (-286) - (-1560) = -85 \text{ kJ/mol}$$

By inverting (3) and multiplying (1) by 2 and (2) by 3 and adding, we get,

$$2 \times (-393.5) + 3 \times (-286) - (-1560) = -85 \text{ kJ/mol}$$

Date: 8<sup>th</sup> January 2020 (Shift 1)

Time: 9:30 A.M. to 12:30 P.M.

Subject: Mathematics

1. The maximum values of  ${}^{19}C_p$ ,  ${}^{20}C_q$ ,  ${}^{21}C_r$  are  $a, b, c$  respectively. Then, the relation between  $a, b, c$  is

a.  $\frac{a}{22} = \frac{b}{42} = \frac{c}{11}$   
 c.  $\frac{a}{22} = \frac{b}{11} = \frac{c}{42}$

b.  $\frac{a}{11} = \frac{b}{22} = \frac{c}{42}$   
 d.  $\frac{a}{21} = \frac{b}{11} = \frac{c}{22}$

**Answer:** (b)

**Solution:**

We know that,  ${}^nC_r$  is maximum when  $r = \begin{cases} \frac{n}{2}, & n \text{ is even} \\ \frac{n+1}{2} \text{ or } \frac{n-1}{2}, & n \text{ is odd} \end{cases}$

$$\text{Therefore, } \max({}^{19}C_p) = {}^{19}C_9 = a$$

$$\max({}^{20}C_q) = {}^{20}C_{10} = b$$

$$\max({}^{21}C_r) = {}^{21}C_{11} = c$$

$$\therefore \frac{a}{{}^{19}C_9} = \frac{b}{{}^{20}C_{10}} = \frac{c}{{}^{21}C_{11}}$$

$$\frac{a}{1} = \frac{b}{2} = \frac{c}{11}$$

$$\Rightarrow \frac{a}{11} = \frac{b}{22} = \frac{c}{42}$$

2. Let  $P(A) = \frac{1}{3}$ ,  $P(B) = \frac{1}{6}$  where  $A$  and  $B$  are independent events, then

a.  $P\left(\frac{A}{B}\right) = \frac{2}{3}$

b.  $P\left(\frac{A}{B'}\right) = \frac{5}{6}$

c.  $P\left(\frac{A}{B'}\right) = \frac{1}{3}$

d.  $P\left(\frac{A}{B}\right) = \frac{1}{6}$

**Answer:** (c)

**Solution:**

If  $X$  and  $Y$  are independent events, then

$$P\left(\frac{X}{Y}\right) = \frac{P(X \cap Y)}{P(Y)} = \frac{P(X)P(Y)}{P(Y)} = P(X)$$

Therefore,  $P\left(\frac{A}{B}\right) = P(A) = \frac{1}{3} \Rightarrow P\left(\frac{A}{B'}\right) = P(A) = \frac{1}{3}$ .

3. If  $f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}$ , then inverse of  $f(x)$  is

a.  $\frac{1}{2} \log_8 \left( \frac{1+x}{1-x} \right)$

b.  $\frac{1}{2} \log_8 \left( \frac{1-x}{1+x} \right)$

c.  $\frac{1}{4} \log_8 \left( \frac{1-x}{1+x} \right)$

d.  $\frac{1}{4} \log_8 \left( \frac{1+x}{1-x} \right)$

**Answer: (d)**

**Solution:**

$$f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}} = \frac{8^{4x} - 1}{8^{4x} + 1}$$

$$\text{Put } y = \frac{8^{4x} - 1}{8^{4x} + 1}$$

Applying componendo-dividendo on both sides

$$\frac{y+1}{y-1} = \frac{2 \times 8^{4x}}{-2}$$

$$\frac{y+1}{y-1} = -8^{4x} \Rightarrow 8^{4x} = \frac{1+y}{1-y}$$

$$\Rightarrow x = \frac{1}{4} \log_8 \left( \frac{1+y}{1-y} \right)$$

$$f^{-1}(x) = \frac{1}{4} \log_8 \left( \frac{1+x}{1-x} \right).$$

4. Roots of the equation  $x^2 + bx + 45 = 0$ ,  $b \in \mathbf{R}$  lies on the curve  $|z + 1| = 2\sqrt{10}$ , where  $z$  is a complex number, then

a.  $b^2 + b = 12$

b.  $b^2 - b = 36$

c.  $b^2 - b = 30$

d.  $b^2 + b = 30$

**Answer: (c)**

**Solution:**

Given  $x^2 + bx + 45 = 0$ ,  $b \in \mathbf{R}$ , let roots of the equation be  $p \pm iq$

Then, sum of roots  $= 2p = -b$

Product of roots  $= p^2 + q^2 = 45$

As  $p \pm iq$  lies on  $|z + 1| = 2\sqrt{10}$ , we get

$$(p+1)^2 + q^2 = 40$$

$$\Rightarrow p^2 + q^2 + 2p + 1 = 40$$

$$\Rightarrow 45 - b + 1 = 40$$

$$\Rightarrow b = 6$$

$$\Rightarrow b^2 - b = 30.$$

5. Rolle's theorem is applicable on  $f(x) = \ln\left(\frac{x^2 + \alpha}{7x}\right)$  in  $[3, 4]$ . The value of  $f''(c)$  is equal to
- |                   |                    |
|-------------------|--------------------|
| a. $\frac{1}{12}$ | b. $\frac{-1}{12}$ |
| c. $\frac{-1}{6}$ | d. $\frac{1}{6}$   |

**Answer:** (a)

**Solution:**

Rolle's theorem is applicable on  $f(x)$  in  $[3, 4]$

$$\Rightarrow f(3) = f(4)$$

$$\Rightarrow \ln\left(\frac{9 + \alpha}{21}\right) = \ln\left(\frac{16 + \alpha}{28}\right)$$

$$\Rightarrow \frac{9 + \alpha}{21} = \frac{16 + \alpha}{28}$$

$$\Rightarrow 36 + 4\alpha = 48 + 3\alpha$$

$$\Rightarrow \alpha = 12$$

Now,

$$f(x) = \ln\left(\frac{x^2 + 12}{7x}\right) \Rightarrow f'(x) = \frac{7x}{x^2 + 12} \times \frac{7x \times 2x - (x^2 + 12) \times 7}{(7x)^2}$$

$$f'(x) = \frac{x^2 - 12}{x(x^2 + 12)}$$

$$f'(c) = 0 \Rightarrow c = 2\sqrt{3}$$

$$f''(x) = \frac{-x^4 + 48x^2 + 144}{x^2(x^2 + 12)^2}$$

$$f''(c) = \frac{1}{12}.$$

6. Let  $f(x) = x \cos^{-1}(\sin(-|x|))$ ,  $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ , then
- $f'(0) = -\frac{\pi}{2}$
  - $f'(x)$  is not defined at  $x = 0$
  - $f'(x)$  is decreasing in  $\left(-\frac{\pi}{2}, 0\right)$  and  $f'(x)$  is decreasing in  $\left(0, \frac{\pi}{2}\right)$
  - $f'(x)$  is increasing in  $\left(-\frac{\pi}{2}, 0\right)$  and  $f'(x)$  is increasing in  $\left(0, \frac{\pi}{2}\right)$

**Answer:** (c)

**Solution:**

$$f(x) = x \cos^{-1}(\sin(-|x|))$$

$$\Rightarrow f(x) = x \cos^{-1}(-\sin |x|)$$

$$\Rightarrow f(x) = x[\pi - \cos^{-1}(\sin |x|)]$$

$$\Rightarrow f(x) = x \left[ \pi - \left( \frac{\pi}{2} - \sin^{-1}(\sin |x|) \right) \right]$$

$$\Rightarrow f(x) = x \left( \frac{\pi}{2} + |x| \right)$$

$$\Rightarrow f(x) = \begin{cases} x \left( \frac{\pi}{2} + x \right), & x \geq 0 \\ x \left( \frac{\pi}{2} - x \right), & x < 0 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} \left( \frac{\pi}{2} + 2x \right), & x \geq 0 \\ \left( \frac{\pi}{2} - 2x \right), & x < 0 \end{cases}$$

Therefore,  $f'(x)$  is decreasing  $\left( -\frac{\pi}{2}, 0 \right)$  and increasing in  $\left( 0, \frac{\pi}{2} \right)$ .

7. Ellipse  $2x^2 + y^2 = 1$  and  $y = mx$  meet at a point  $P$  in the first quadrant. Normal to the ellipse at  $P$  meets  $x$ -axis at  $\left( -\frac{1}{3\sqrt{2}}, 0 \right)$  and  $y$ -axis at  $(0, \beta)$ , then  $|\beta|$  is

a.  $\frac{2}{3}$

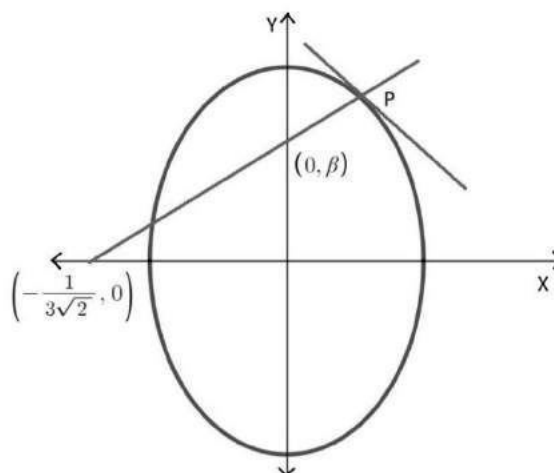
b.  $\frac{2\sqrt{2}}{3}$

c.  $\frac{\sqrt{2}}{3}$

d.  $\frac{2}{\sqrt{3}}$

**Answer:** (c)

**Solution:**



Let  $P \equiv (x_1, y_1)$

$2x^2 + y^2 = 1$  is given equation of ellipse.

$$\Rightarrow 4x + 2yy' = 0$$

$$\Rightarrow y'|_{(x_1, y_1)} = -\frac{2x_1}{y_1}$$

Therefore, slope of normal at  $P(x_1, y_1)$  is  $\frac{y_1}{2x_1}$

Equation of normal at  $P(x_1, y_1)$  is

$$(y - y_1) = \frac{y_1}{2x_1}(x - x_1)$$

It passes through  $(-\frac{1}{3\sqrt{2}}, 0)$

$$\Rightarrow -y_1 = \frac{y_1}{2x_1}\left(-\frac{1}{3\sqrt{2}} - x_1\right)$$

$$\Rightarrow x_1 = \frac{1}{3\sqrt{2}}$$

$$\Rightarrow y_1 = \frac{2\sqrt{2}}{3} \text{ as } P \text{ lies in first quadrant}$$

Since  $(0, \beta)$  lies on the normal of the ellipse at point  $P$ , hence we get

$$\beta = \frac{y_1}{2} = \frac{\sqrt{2}}{3}$$

8. If  $ABC$  is a triangle whose vertices are  $A(1, -1)$ ,  $B(0, 2)$ ,  $C(x', y')$  and area of  $\triangle ABC$  is 5, and  $C(x', y')$  lies on  $3x + y - 4\lambda = 0$ , then
- |                   |                  |
|-------------------|------------------|
| a. $\lambda = 3$  | b. $\lambda = 4$ |
| c. $\lambda = -3$ | d. $\lambda = 2$ |

**Answer:** (a)

**Solution:**

Area of triangle is

$$A = \frac{1}{2} \begin{vmatrix} 0 & 2 & 1 \\ 1 & -1 & 1 \\ x' & y' & 1 \end{vmatrix} = \pm 5$$

$$-2(1 - x') + (y' + x') = \pm 10$$

$$-2 + 2x' + y' + x' = \pm 10$$

$$3x' + y' = 12 \text{ or } 3x' + y' = -8$$

$$\Rightarrow \lambda = 3 \text{ or } -2$$

9. Shortest distance between the lines  $\frac{x-3}{1} = \frac{y-8}{4} = \frac{z-3}{22}$ ,  $\frac{x+3}{1} = \frac{y+7}{1} = \frac{z-6}{7}$  is
- $3\sqrt{30}$
  - $\sqrt{30}$
  - $2\sqrt{30}$
  - $4\sqrt{30}$

**Answer:** (a)

**Solution:**

$$\overrightarrow{AB} = -3\hat{i} - 7\hat{j} + 6\hat{k} - (3\hat{i} + 8\hat{j} + 3\hat{k}) = -6\hat{i} - 15\hat{j} + 3\hat{k}$$

$$\vec{p} = \hat{i} + 4\hat{j} + 22\hat{k}$$

$$\vec{q} = \hat{i} + \hat{j} + 7\hat{k}$$

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 22 \\ 1 & 1 & 7 \end{vmatrix} = 6\hat{i} + 15\hat{j} - 3\hat{k}$$

$$\text{Shortest distance} = \frac{|\overrightarrow{AB} \cdot (\vec{p} \times \vec{q})|}{|\vec{p} \times \vec{q}|} = \frac{|36 + 225 + 9|}{\sqrt{36 + 225 + 9}} = 3\sqrt{30}.$$

10. Let  $\int \frac{\cos x}{\sin^3 x (1 + \sin^6 x)^{\frac{2}{3}}} dx = f(x)(1 + \sin^6 x)^{\frac{1}{3}} + c$ , then the value of  $\lambda f\left(\frac{\pi}{3}\right)$  is
- 4
  - 2
  - 8
  - 4

**Answer:** (b)

**Solution:**

$$\text{Let } \sin x = t \Rightarrow \cos x dx = dt$$

$$\therefore \int \frac{dt}{t^3 (1+t^6)^{\frac{2}{3}}} = \int \frac{dt}{t^7 \left(1 + \frac{1}{t^6}\right)^{\frac{2}{3}}}$$

$$\text{Let } 1 + \frac{1}{t^6} = u \Rightarrow -6t^{-7} dt = du$$

$$\Rightarrow \int \frac{dt}{t^7 \left(1 + \frac{1}{t^6}\right)^{\frac{2}{3}}} = -\frac{1}{6} \int \frac{du}{u^{\frac{2}{3}}} = -\frac{3}{6} u^{\frac{1}{3}} + c = -\frac{1}{2} \left(1 + \frac{1}{t^6}\right)^{\frac{1}{3}} + c$$

$$= -\frac{(1 + \sin^6 x)^{\frac{1}{3}}}{2 \sin^2 x} + c = f(x)(1 + \sin^6 x)^{\frac{1}{3}}$$

$$\therefore \lambda = 3 \text{ and } f(x) = -\frac{1}{2 \sin^2 x}$$

$$\Rightarrow \lambda f\left(\frac{\pi}{3}\right) = -2.$$

11. If  $y(x)$  is a solution of the differential equation  $\sqrt{1-x^2} \frac{dy}{dx} + \sqrt{1-y^2} = 0$ , such that

$$y\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}, \text{ then}$$

a.  $y\left(\frac{1}{\sqrt{2}}\right) = -\frac{1}{\sqrt{2}}$

b.  $y\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2}$

c.  $y\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}$

d.  $y\left(\frac{1}{\sqrt{2}}\right) = \frac{\sqrt{3}}{2}$

**Answer:** (c)

**Solution:**

$$\sqrt{1-x^2} \frac{dy}{dx} + \sqrt{1-y^2} = 0$$

$$\Rightarrow \frac{dy}{\sqrt{1-y^2}} + \frac{dx}{\sqrt{1-x^2}} = 0$$

$$\Rightarrow \sin^{-1} y + \sin^{-1} x = c$$

If  $x = \frac{1}{2}, y = \frac{\sqrt{3}}{2}$  then,

$$\sin^{-1} \frac{\sqrt{3}}{2} + \sin^{-1} \frac{1}{2} = c$$

$$\therefore \frac{\pi}{3} + \frac{\pi}{6} = c \Rightarrow c = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1} y = \frac{\pi}{2} - \sin^{-1} x = \cos^{-1} x$$

$$\therefore \sin^{-1} y = \cos^{-1} \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin^{-1} y = \frac{\pi}{4}$$

$$\Rightarrow y\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}$$

12.  $\lim_{x \rightarrow 0} \left( \frac{3x^2+2}{7x^2+2} \right)^{\frac{1}{x^2}}$  is equal to

a.  $e^{-2}$

b.  $e^2$

c.  $e^{\frac{3}{7}}$

d.  $e^{\frac{2}{7}}$

**Answer:** (a)

**Solution:**

$$\text{Let } L = \lim_{x \rightarrow 0} \left( \frac{3x^2+2}{7x^2+2} \right)^{\frac{1}{x^2}} \quad [\text{Intermediate form } 1^\infty]$$



$$\Rightarrow \frac{\pi}{3} = \sin^{-1} \left( \sin \left( \frac{2\pi}{3} \right) \right) + c$$

$$\Rightarrow \frac{\pi}{3} = \frac{\pi}{3} + c \Rightarrow c = 0$$

$$\Rightarrow 2y = \sin^{-1} \sin(2 \tan^{-1} x)$$

$$\text{When } x = -\sqrt{3}$$

$$2y = \sin^{-1} \left( \sin \left( 2 \tan^{-1} (-\sqrt{3}) \right) \right) = \sin^{-1} \left( \sin \left( -\frac{2\pi}{3} \right) \right) = -\frac{\pi}{3}$$

$$\Rightarrow y = -\frac{\pi}{6}$$

15. The system of equation  $3x + 4y + 5z = \mu$

$$x + 2y + 3z = 1$$

$$4x + 4y + 4z = \delta$$

is inconsistent, then  $(\mu, \delta)$  can be

a. (4, 6)

b. (1, 0)

c. (4, 3)

d. (3, 4)

**Answer:** (c)

**Solution:**

$$D = \begin{vmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 4 & 4 & 4 \end{vmatrix}$$

$$R_3 \rightarrow R_3 - 2R_1 + 2R_2$$

$$D = \begin{vmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

For inconsistent system, one of  $D_x, D_y, D_z$  should not be equal to 0

$$D_x = \begin{vmatrix} \mu & 4 & 5 \\ 1 & 2 & 3 \\ \delta & 4 & 4 \end{vmatrix} \quad D_y = \begin{vmatrix} 3 & \mu & 5 \\ 1 & 1 & 3 \\ 4 & \delta & 4 \end{vmatrix}$$

$$D_z = \begin{vmatrix} 3 & 4 & \mu \\ 1 & 2 & 1 \\ 4 & 4 & \delta \end{vmatrix}$$

For inconsistent system,  $2\mu \neq \delta + 2$

$\therefore$  The system will be inconsistent for  $\mu = 4, \delta = 3$ .

16. If volume of parallelepiped whose conterminous edges are  $\vec{u} = \hat{i} + \hat{j} + \lambda \hat{k}$ ,  $\vec{v} = 2\hat{i} + \hat{j} + \hat{k}$  and  $\vec{w} = \hat{i} + \hat{j} + 3\hat{k}$  is 1 cubic units. Then, the cosine of angle between  $\vec{u}$  and  $\vec{v}$  is

a.  $\frac{5}{3\sqrt{10}}$   
c.  $\frac{7}{6\sqrt{3}}$

b.  $\frac{5}{7}$   
d.  $\frac{7}{3\sqrt{3}}$

**Answer:** (c)

**Solution:**

Volume of parallelepiped =  $[\vec{u} \ \vec{v} \ \vec{w}]$

$$\Rightarrow \begin{vmatrix} 1 & 1 & \lambda \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{vmatrix} = \pm 1$$

$$\Rightarrow \lambda = 2 \text{ or } 4$$

For  $\lambda = 4$ ,

$$\cos \theta = \frac{2+1+4}{\sqrt{6}\sqrt{18}} = \frac{7}{6\sqrt{3}}$$

17. If  $2^{1-x} + 2^{1+x}$ ,  $f(x)$ ,  $3^x + 3^{-x}$  are in A.P. then the minimum value of  $f(x)$  is

a. 4  
c. 3

b. 2  
d. 1

**Answer:** (c)

**Solution:**

$2^{1-x} + 2^{1+x}$ ,  $f(x)$ ,  $3^x + 3^{-x}$  are in A.P.

$$\therefore f(x) = \frac{3^x + 3^{-x} + 2^{1+x} + 2^{1-x}}{2} = \frac{(3^x + 3^{-x})}{2} + \frac{2^{1+x} + 2^{1-x}}{2}$$

Applying A.M.  $\geq$  G.M. inequality, we get

$$\frac{(3^x + 3^{-x})}{2} \geq \sqrt{3^x \cdot 3^{-x}}$$

$$\Rightarrow \frac{(3^x + 3^{-x})}{2} \geq 1 \quad \dots (1)$$

Also, Applying A.M.  $\geq$  G.M. inequality, we get

$$\frac{2^{1+x} + 2^{1-x}}{2} \geq \sqrt{2^{1+x} \cdot 2^{1-x}}$$

$$\Rightarrow \frac{2^{1+x} + 2^{1-x}}{2} \geq 2 \quad \dots (2)$$

Adding (1) and (2), we get

$$f(x) \geq 1 + 2 = 3$$

Thus, minimum value of  $f(x)$  is 3.

18. Which of the following is tautology?

a.  $(p \wedge (p \rightarrow q)) \rightarrow q$

b.  $q \rightarrow p \wedge (p \rightarrow q)$

c.  $(p \wedge (p \vee q))$

d.  $(p \vee (p \wedge q))$

**Answer:** (a)

**Solution:**

$$\begin{aligned} & (p \wedge (p \rightarrow q)) \rightarrow q \\ &= (p \wedge (\sim p \vee q)) \rightarrow q \\ &= [(p \wedge \sim p) \vee (p \wedge q)] \rightarrow q \\ &= (p \wedge q) \rightarrow q \\ &= \sim (p \wedge q) \vee q \\ &= \sim p \vee \sim q \vee q \\ &= T \end{aligned}$$

19.  $A$  is a  $3 \times 3$  matrix whose elements are from the set  $\{-1, 0, 1\}$ . Find the number of matrices  $A$  such that  $tr(AA^T) = 3$  where  $tr(A)$  is sum of diagonal elements of matrix  $A$

a. 612

b. 572

c. 672

d. 682

**Answer:** (c)

**Solution:**

$$tr(AA^T) = 3$$

$$\text{Let } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

$$tr(AA^T) = a_{11}^2 + a_{12}^2 + a_{13}^2 + a_{21}^2 + a_{22}^2 + a_{23}^2 + a_{31}^2 + a_{32}^2 + a_{33}^2 = 3$$

So out of 9 elements  $(a_{ij})$ 's, 3 elements must be equal to 1 or  $-1$  and rest elements must be 0.

So, the total possible cases will be

When there is  $6(0's)$  and  $3(1's)$  then the total possibilities is  ${}^9C_6$

For  $6(0's)$  and  $3(-1's)$  total possibilities is  ${}^9C_6$

For  $6(0's), 2(1's)$  and  $1(-1's)$  total possibilities is  ${}^9C_6 \times 3$

For  $6(0's), 1(1's)$  and  $2(-1's)$  total possibilities is  ${}^9C_6 \times 3$

$\therefore$  Total number of cases =  ${}^9C_6 \times 8 = 672$ .

20. Mean and standard deviation of 10 observations are 20 and 2 respectively. If  $p$  ( $p \neq 0$ ) is multiplied to each observation and then  $q$  ( $q \neq 0$ ) is subtracted from each of them, then new mean and standard deviation becomes half of it's original value. Then find  $q$
- |       |        |
|-------|--------|
| a. 10 | b. -20 |
| c. -5 | d. -10 |

**Answer:** (b)

**Solution:**

If mean  $\bar{x}$  is multiplied by  $p$  and then  $q$  is subtracted from it,

then new mean  $\bar{x}' = p\bar{x} - q$

$$\therefore \bar{x}' = \frac{1}{2}\bar{x} \text{ and } \bar{x} = 10$$

$$\Rightarrow 10 = 20p - q \quad \dots (1)$$

If standard deviation is multiplied by  $p$ , new standard deviation ( $\sigma'$ ) is  $p$  times of the initial standard deviation ( $\sigma$ ).

$$\sigma' = |p|\sigma$$

$$\Rightarrow \frac{1}{2}\sigma = |p|\sigma \Rightarrow |p| = \frac{1}{2}$$

$$\text{If } p = \frac{1}{2}, q = 0$$

$$\text{If } p = -\frac{1}{2}, q = -20.$$

21. Let  $P$  be a point on  $x^2 = 4y$ . The segment joining  $A(0, -1)$  and  $P$  is divided by a point  $Q$  in the ratio 1: 2, then locus of point  $Q$  is
- |                    |                     |
|--------------------|---------------------|
| a. $9x^2 = 3y + 2$ | b. $9x^2 = 12y + 8$ |
| c. $9y^2 = 3x + 2$ | d. $9y^2 = 12x + 8$ |

**Answer:** (b)

**Solution:**

Let point  $P$  be  $(2t, t^2)$  and  $Q$  be  $(h, k)$ .

$$h = \frac{2t}{3}, k = \frac{-2 + t^2}{3}$$

Now, eliminating  $t$  from the above equations we get:

$$3k + 2 = \left(\frac{3h}{2}\right)^2$$

Replacing  $h$  and  $k$  by  $x$  and  $y$ , we get the locus of the curve as  $9x^2 = 12y + 8$ .

22. If the curves  $y^2 = ax$  and  $x^2 = ay$  intersect each other at  $A$  and  $B$  such that the area bounded by the curves is bisected by the line  $x = b$  (given  $a > b > 0$ ) and the area of triangle formed by the lines  $AB$ ,  $x = b$  and the  $x$ -axis is  $\frac{1}{2}$ . Then

a.  $a^6 + 12a^3 + 4 = 0$

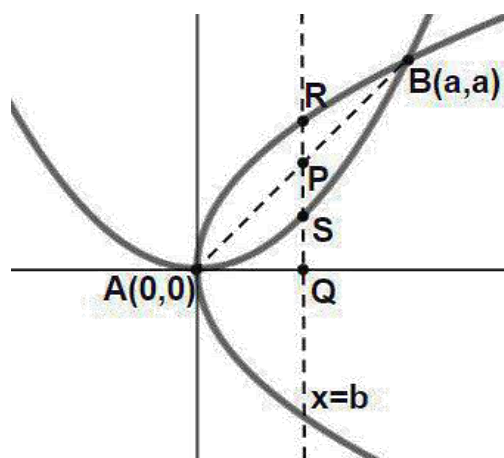
b.  $a^6 + 12a^3 - 4 = 0$

c.  $a^6 - 12a^3 + 4 = 0$

d.  $a^6 - 12a^3 - 4 = 0$

**Answer:** (c)

**Solution:**



Given,  $ar(\Delta APQ) = \frac{1}{2}$

$$\Rightarrow \frac{1}{2} \times b \times b = \frac{1}{2}$$

$$\Rightarrow b = 1$$

As per the question

$$\Rightarrow \int_0^1 \left( \sqrt{ax} - \frac{x^2}{a} \right) dx = \frac{1}{2} \int_0^a \left( \sqrt{ax} - \frac{x^2}{a} \right) dx$$

$$\Rightarrow \frac{2}{3} \sqrt{a} - \frac{1}{3a} = \frac{a^2}{6}$$

$$\Rightarrow 2a\sqrt{a} - 1 = \frac{a^3}{2}$$

$$\Rightarrow 4a\sqrt{a} = 2 + a^3$$

$$\Rightarrow 16a^3 = 4 + a^6 + 4a^3$$

$$\Rightarrow a^6 - 12a^3 + 4 = 0.$$

23. The sum  $\sum_{k=1}^{20} (1 + 2 + 3 + \dots + k)$  is \_\_\_\_\_.

**Answer:** (1540)

**Solution:**

$$\begin{aligned} &= \sum_{k=1}^{20} \frac{k(k+1)}{2} \\ &= \frac{1}{2} \sum_{k=1}^{20} k^2 + k \\ &= \frac{1}{2} \left[ \frac{20(21)(41)}{6} + \frac{20(21)}{2} \right] \\ &= \frac{1}{2} [2870 + 210] \\ &= 1540. \end{aligned}$$

24. If  $2x^2 + (a - 10)x + \frac{33}{2} = 2a, a \in \mathbf{Z}^+$  has real roots, then minimum value of ' $a$ ' is equal to \_\_\_\_\_.

**Answer:** (8)

**Solution:**

$$\because 2x^2 + (a - 10)x + \frac{33}{2} = 2a, a \in \mathbf{Z}^+ \text{ has real roots}$$

$$\Rightarrow D \geq 0$$

$$\Rightarrow (a - 10)^2 - 4 \times 2 \times \left( \frac{33}{2} - 2a \right) \geq 0$$

$$\Rightarrow (a - 10)^2 - 4(33 - 4a) \geq 0$$

$$\Rightarrow a^2 - 4a - 32 \geq 0 \Rightarrow a \in (-\infty, 4] \cup [8, \infty)$$

Thus, minimum value of ' $a$ '  $\forall a \in \mathbf{Z}^+$  is 8.

25. If normal at  $P$  on the curve  $y^2 - 3x^2 + y + 10 = 0$  passes through the point  $\left(0, \frac{3}{2}\right)$  and the slope of tangent at  $P$  is  $n$ . The value of  $|n|$  is equal to\_\_\_\_\_.

**Answer:** (4)

**Solution:**

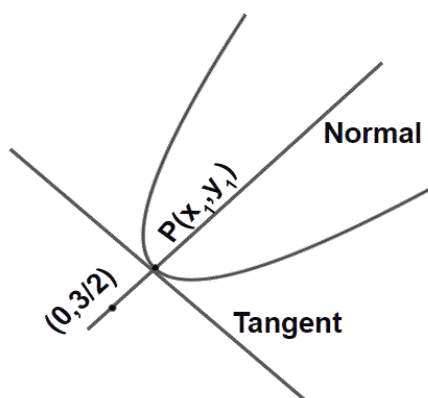
Let co-ordinate of  $P$  be  $(x_1, y_1)$

Differentiating the curve w.r.t  $x$

$$2yy' - 6x + y' = 0$$

Slope of tangent at  $P$

$$\Rightarrow y' = \frac{6x_1}{1 + 2y_1}$$



$$\therefore m_{\text{normal}} = \left( \frac{y_1 - \frac{3}{2}}{x_1 - 0} \right)$$

$$\therefore m_{\text{normal}} \times m_{\text{tangent}} = -1$$

$$\Rightarrow \frac{\frac{3}{2} - y_1}{-x_1} \times \frac{6x_1}{1 + 2y_1} = -1$$

$$\Rightarrow y_1 = 1$$

$$\Rightarrow x_1 = \pm 2$$

$$\text{Slope of tangent} = \pm \frac{12}{3} = \pm 4$$

$$\Rightarrow |n| = 4$$

Date of Exam: 8<sup>th</sup> January (Shift 1) **JEE Main 2020 Paper**

Time: 9:30 am – 12:30 pm

Subject: **Physics**

1. A block of mass  $m$  is connected at one end of natural length  $l_0$  and spring constant  $k$ . The spring is fixed at its other end. The block is rotated with constant angular speed ( $\omega$ ) in gravity free space. The elongation in spring is

- a.  $\frac{l_0 m \omega^2}{k - m \omega^2}$                       b.  $\frac{l_0 m \omega^2}{k + m \omega^2}$   
 c.  $\frac{l_0 m \omega^2}{k - m \omega}$                       d.  $\frac{l_0 m \omega^2}{k + m \omega}$

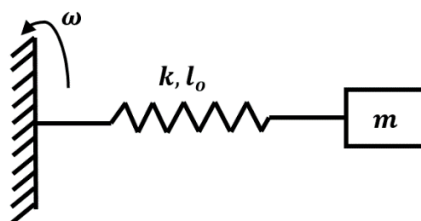
Solution: (a)

The centripetal force is provided by the spring force.

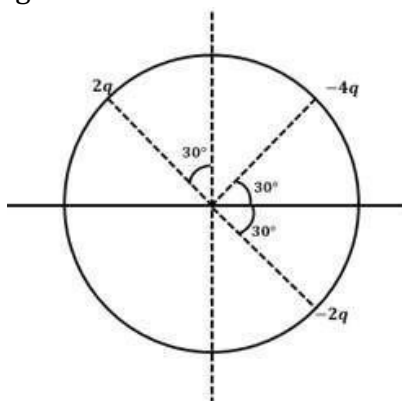
$$m \omega^2 (l_0 + x) = kx$$

$$\left(\frac{l_0}{x} + 1\right) = \frac{k}{m \omega^2}$$

$$x = \frac{l_0 m \omega^2}{k - m \omega^2}$$



2. Three charges are placed on the circumference of a circle of radius  $d$  as shown in the figure. The electric field along  $x$  – axis at the centre of the circle is



a.  $\frac{q}{4\pi\epsilon_0 d^2}$

b.  $\frac{q\sqrt{3}}{4\pi\epsilon_0 d^2}$

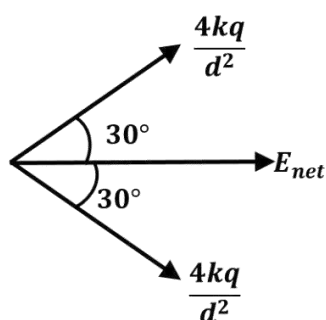
c.  $\frac{q\sqrt{3}}{\pi\epsilon_0 d^2}$

d.  $\frac{q\sqrt{3}}{2\pi\epsilon_0 d^2}$

Solution: (c)

Applying superposition principle,

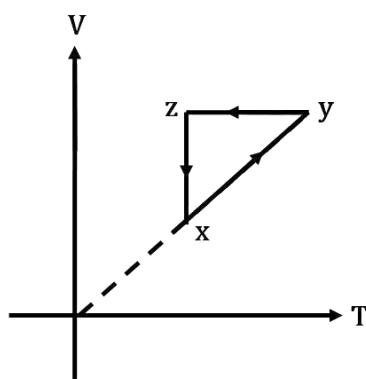
$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$



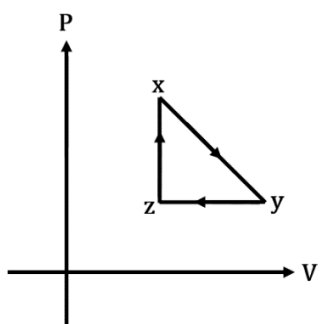
By symmetry, net electric field along the x-axis.

$$|\vec{E}_{net}| = \frac{4kq}{d^2} \times 2 \cos 30^\circ = \frac{q\sqrt{3}}{\pi\epsilon_0 d^2}$$

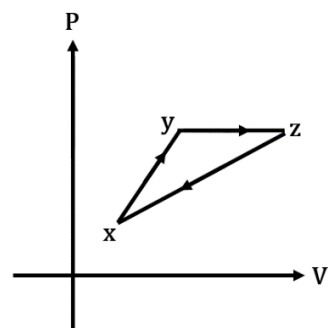
3. Choose the correct P – V graph of an ideal gas for the given V – T graph.



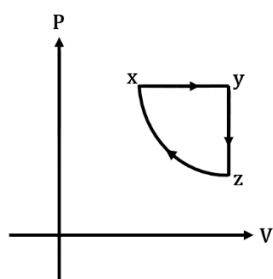
a.



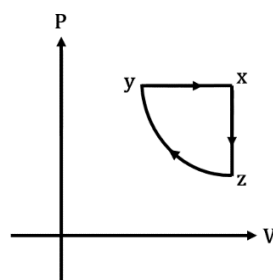
b.



c.



d.



Solution: (a)

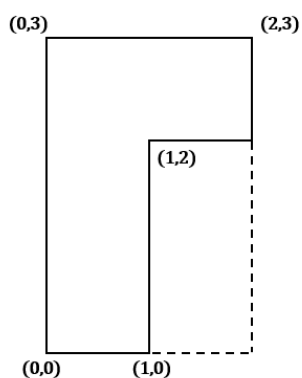
For the given  $V - T$  graph

For the process  $x \rightarrow y$ ;  $V \propto T$ ;  $P = \text{constant}$

For the process  $y \rightarrow z$ ;  $V = \text{constant}$

Only 'a' satisfies these two conditions.

4. Find the co-ordinates of center of mass of the lamina shown in the figure below.

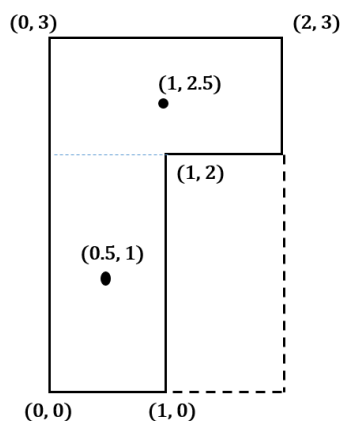


- a. (0.75, 1.75)  
c. (0.5, 1.75)

- b. (0.75, 1.5)  
d. (0.5, 1.5)

Solution: (a)

The Lamina can be divided into two parts having equal mass  $m$  each.

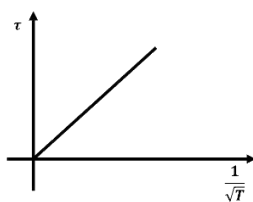


$$\vec{r}_{\text{cm}} = \frac{m \times \left(\frac{\hat{i}}{2} + \hat{j}\right) + m \times \left(\hat{i} + \frac{5\hat{j}}{2}\right)}{2m}$$

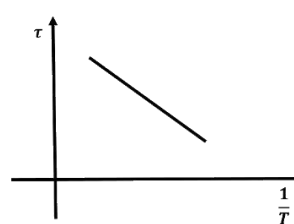
$$\vec{r}_{\text{cm}} = \frac{3}{4}\hat{i} + \frac{7}{4}\hat{j}$$

5. Which graph correctly represents the variation between relaxation time ( $\tau$ ) of gas molecules with absolute temperature ( $T$ ) of the gas?

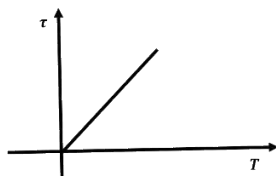
a.



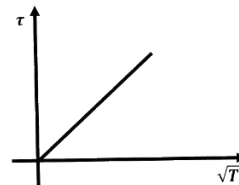
b.



c.



d.





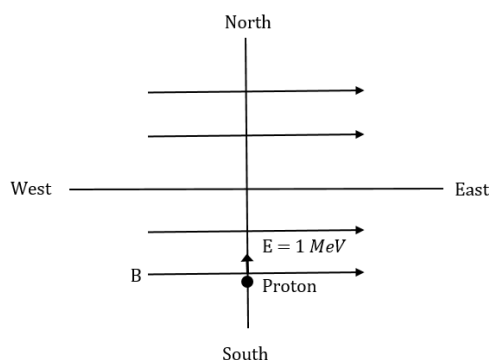






$$\Rightarrow \frac{m_1}{m_2} = \frac{1}{6}$$

12. When a proton of  $KE = 1.0 \text{ MeV}$  moving towards North enters a magnetic field (directed along East), it accelerates with an acceleration,  $a = 10^{12} \text{ m/s}^2$ . The magnitude of the magnetic field is



a. 0.71 mT

b. 7.1 mT

c. 71 mT

d. 710 mT

Solution: (a)

$$K.E = 1 \times 10^6 \text{ eV} = 1.6 \times 10^{-13} \text{ J}$$

$$= \frac{1}{2} m_e v^2$$

Where  $m_e$  is the mass of the electron  $= 1.6 \times 10^{-27} \text{ kg}$

$$\Rightarrow 1.6 \times 10^{-13} = \frac{1}{2} \times 1.6 \times 10^{-27} \times v^2$$

$$\therefore v = \sqrt{2} \times 10^7 \text{ m/s}$$

$$Bqv = m_e a$$

$$\therefore B = \frac{1.6 \times 10^{-27} \times 10^{12}}{1.6 \times 10^{-19} \times \sqrt{2} \times 10^7}$$

$$= 0.71 \times 10^{-3} \text{ T} = 0.71 \text{ mT}$$

13. If the electric field around a surface is given by  $|\vec{E}| = \frac{Q_{in}}{\epsilon_0 A}$  where A is the normal area of surface and  $Q_{in}$  is the charge enclosed by the surface. This relation of Gauss' law is valid when
- the surface is equipotential.
  - the magnitude of the electric field is constant.
  - the magnitude of the electric field is constant and the surface is equipotential.
  - for all the Gaussian surfaces.

Solution: (c)

The magnitude of the electric field is constant and the electric field must be along the area vector i.e. the surface is equipotential.

14. The stopping potential depends on the Planks constant(h), the current (I), the universal gravitational constant (G) and the speed of light (C). Choose the correct option for the dimension of the stopping potential (V)
- $h^1 I^{-1} G^1 C^5$
  - $h^{-1} I^1 G^6$
  - $h^0 I^1 G^1 C^6$
  - $h^0 I^{-1} G^{-1} C^5$

Solution: (d)

$$V = K(h)^a(I)^b(G)^c(C)^d \quad \text{Unit of stopping potential is (V) Volt.}$$

$$\text{We know } [h] = ML^2T^{-1}$$

$$[I] = A$$

$$[G] = M^{-1}L^3T^{-2}$$

$$[C] = LT^{-1}$$

$$[V] = ML^2T^{-3}A^{-1}$$

$$ML^2T^{-3}A^{-1} = (ML^2T^{-1})^a(A)^b(M^{-1}L^3T^{-2})^c(LT^{-1})^d$$

$$ML^2T^{-3}A^{-1} = M^{a-c}L^{2a+3c+d}T^{-a-2c-d}A^b$$

$$a - c = 1$$

$$2a + 3c + d = 2$$

$$-a - 2c - d = -3$$

$$b = -1$$

On solving,

$$c = -1$$

$$a = 0$$

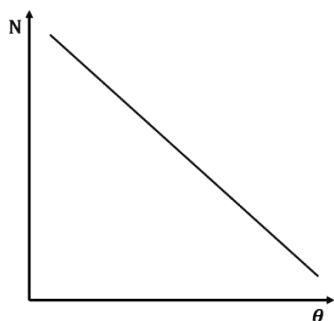
$$d = 5$$

$$b = -1$$

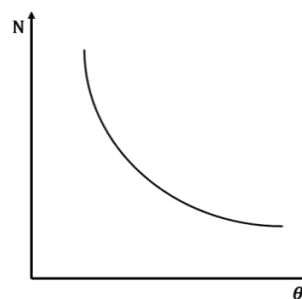
$$V = K(h)^0(I)^{-1}(G)^{-1}(C)^5$$



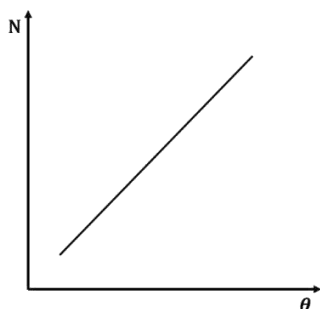
a.



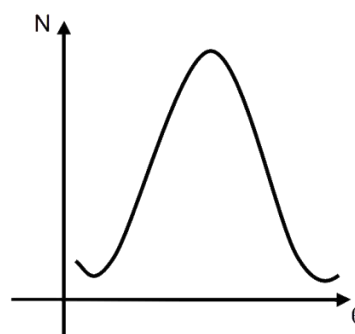
b.



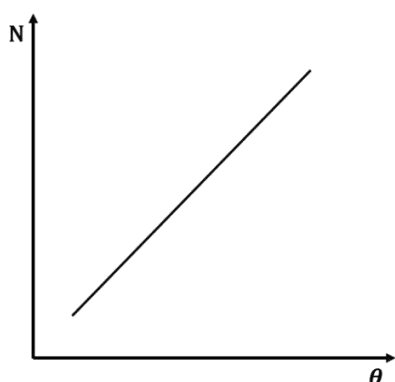
c.



d.



Solution: (c)



$$N \propto \frac{1}{\sin^4(\theta/2)}$$

17. If relative permittivity and relative permeability of a medium are 3 and  $\frac{4}{3}$  respectively, the critical angle for this medium is

- a.  $45^\circ$   
c.  $30^\circ$

- b.  $60^\circ$   
d.  $15^\circ$

Solution: (c)



- 115

$$\rho_L = \frac{2}{5} \rho_o$$

21. Two masses each of mass  $0.10 \text{ kg}$  are moving with velocities  $3 \text{ m/s}$  along  $x$  -axis and  $5 \text{ m/s}$  along  $y$  -axis respectively. After an elastic collision one of the mass moves with velocity  $4\hat{i} + 4\hat{j} \text{ m/s}$ . If the energy of the other mass after the collision is  $\frac{x}{10}$ , then  $x$  is

Solution: (1)

Mass of each object,  $m_1 = m_2 = 0.1 \text{ kg}$

Initial velocity of 1<sup>st</sup> object,  $u_1 = 5 \text{ m/s}$

Initial velocity of 2<sup>nd</sup> object,  $u_2 = 3 \text{ m/s}$

Final velocity of 1<sup>st</sup> object,  $V_1 = 4\hat{i} + 4\hat{j} \text{ m/s} = \sqrt{4^2 + 4^2} = 16\sqrt{2} \text{ m/s}$

For elastic collision, kinetic energy remains conserved

*Initial kinetic energy ( $K_i$ ) = Final kinetic energy ( $K_f$ )*

$$\frac{1}{2}mu_1^2 + \frac{1}{2}mu_2^2 = \frac{1}{2}mV_1^2 + \frac{1}{2}mV_2^2$$

$$\frac{1}{2}m(5)^2 + \frac{1}{2}m(3)^2 = \frac{1}{2}m(16\sqrt{2})^2 + \frac{1}{2}mV_2^2$$

$$V_2 = \sqrt{2} \text{ m/s}$$

$$\text{Kinetic energy of second object} = \frac{1}{2}mV_2^2 = \frac{1}{2} \times 0.1 \times \sqrt{2}^2 = \frac{1}{10}$$

$$\Rightarrow x = 1$$

22. A plano-convex lens of radius of curvature  $30 \text{ cm}$  and refractive index  $1.5$  is kept in air. Find its focal length (in  $\text{cm}$ ).

Solution: (60 cm)

Applying Lens makers' formula,

$$\frac{1}{f} = (\mu - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$R_1 = \infty$$

$$R_2 = -30 \text{ cm}$$

$$\frac{1}{f} = (1.5 - 1) \left( \frac{1}{\infty} - \frac{1}{-30} \right)$$

$$\frac{1}{f} = \frac{0.5}{30}$$

$$f = 60 \text{ cm}$$

23. The position of two particles  $A$  and  $B$  as a function of time are given by  $X_A = -3t^2 + 8t + c$  and  $Y_B = 10 - 8t^3$ . The velocity of  $B$  with respect to  $A$  at  $t = 1$  is  $\sqrt{v}$ . Find  $v$ .

Solution: (580 m/s)

$$X_A = -3t^2 + 8t + c$$

$$\vec{v}_A = (-6t + 8)\hat{i}$$

$$= 2\hat{i}$$

$$Y_B = 10 - 8t^3$$

$$\vec{v}_B = -24t^2\hat{j}$$

$$|\vec{v}_{B/A}| = |\vec{v}_B - \vec{v}_A| = |-24\hat{j} - 2\hat{i}|$$

$$v = \sqrt{24^2 + 2^2}$$

$$v = 580 \text{ m/s}$$

24. An open organ pipe of length 1 m contains a gas whose density is twice the density of the atmosphere at STP. Find the difference between its fundamental and second harmonic frequencies if the speed of sound in atmosphere is 300 m/s.

Solution: (105.75 Hz)

$$V = \sqrt{\frac{B}{\rho}}$$

$$\frac{V_{\text{pipe}}}{V_{\text{air}}} = \frac{\sqrt{\frac{B}{2\rho}}}{\sqrt{\frac{B}{\rho}}} = \frac{1}{\sqrt{2}}$$

$$V_{\text{pipe}} = \frac{V_{\text{air}}}{\sqrt{2}}$$

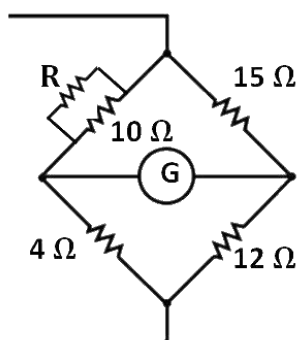
$$f_n = \frac{(n+1)}{2l} V_{\text{pipe}}$$

$$f_1 - f_0 = \frac{V_{\text{pipe}}}{2l} = \frac{300}{2\sqrt{2}} = 105.75 \text{ Hz (if } \sqrt{2} = 1.41)$$

$$= 106.05 \text{ Hz (if } \sqrt{2} = 1.414)$$

25. Four resistors of resistance  $15 \Omega$ ,  $12 \Omega$ ,  $4 \Omega$  and  $10 \Omega$  are connected in cyclic order to form a wheat stone bridge. The resistance (in  $\Omega$ ) that should be connected in parallel across the  $10 \Omega$  resistor to balance the wheat stone bridge is

Solution: ( $10 \Omega$ )



$$\frac{10 R}{10+R} \times 12 = 15 \times 4 \Rightarrow R = 10 \Omega$$

Date: 8<sup>th</sup> January 2020

# JEE Main 2020 Paper

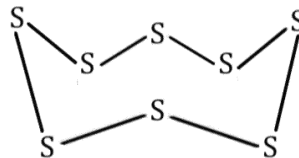
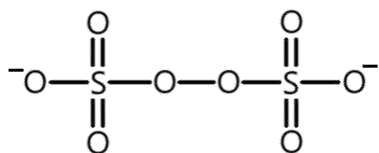
Time: 09:30 AM – 12:30 PM

Subject: Chemistry

1. The number of S–O bonds in  $S_2O_8^{2-}$  and number of S–S bonds in Rhombic sulphur, respectively, are:
- 8, 8
  - 6, 8
  - 2, 4
  - 4, 2

**Answer:** a

**Solution:** Here, we have to count S – O single bonds as well as S = O in  $S_2O_8^{2-}$ , as each double bond also has one sigma bond. The structure of  $S_2O_8^{2-}$  and  $S_8$  is shown below:



2. Which of the following van der Waals forces are present in ethyl acetate liquid?
- H-bond, London forces
  - Dipole-dipole interaction, H-bond
  - Dipole-dipole interaction, London forces
  - H-bond, dipole-dipole interaction, London forces

**Answer:** c

**Solution:** London dispersion forces (also called as induced dipole - induced dipole interactions), exist because of the generation of temporary polarity due to collision of particles and for this very reason, they are present in all molecules and inert gases as well.

Because of the presence of a permanent dipole, there will be dipole-dipole interactions present here.

There is no H that is directly attached to an oxygen atom, so H-bonding cannot be present.

3. Given, for H-atom

$$\bar{\nu} = R_H \left[ \frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

Select the correct options regarding this formula for Balmer series:

- A)  $n_1 = 2$ .
  - B) Ionization energy of H atom can be calculated from above formula.
  - C)  $\lambda_{\text{maximum}}$  is for  $n_2 = 3$ .
  - D) If  $\lambda$  decreases then spectrum lines will converge.
- a. A, B
  - b. C, D
  - c. A, C, D
  - d. A, B, C, D

**Answer:** c

**Solution:**

(A) is correct since the series studied in H-spectrum, including Balmer series, are de-excitation series or emission series. So, electrons get de-excited to  $n = 2$  which means that  $n_{\text{lower}} = 2$ .

(B) It is possible to obtain I.E. from the formula above, but since the question has stated the formula for the Balmer series,  $n_{\text{lower}}$  has been fixed as 2. So, it is not possible to calculate I.E. from it. To calculate I.E., we'll have to put  $n_{\text{lower}} = 1$ , which isn't possible here.

(C)  $\Delta E = hc/\lambda$

With  $n_{\text{lower}}$  fixed as 2,  $\Delta E$  increases as  $n_{\text{higher}}$  is increased. So, the last line of the Balmer series, i.e. from infinity to  $n = 2$ , will have the maximum energy in the series and thus, the lowest wavelength. Similarly, the first line in the series, i.e. from  $n = 3$  to  $n = 2$  will have the lowest energy in the series and thus, the highest wavelength. Which makes this statement correct.

(D) As orbits with higher orbit number or those that are further away from the nucleus are considered, the energy gap in-between subsequent orbits decreases. Now, consider the following for example and with  $n_{\text{lower}}$  fixed as 2.

Energy of a photon released on transition from  $n = 100$  to  $n = 2$  will have similar energy to that of the photon that gets released on transition from  $n = 101$  to  $n = 2$ , because energy of the 100<sup>th</sup> and the 101<sup>th</sup> orbit will be very close in value. That means they will also have very close values of wavelengths, which further implies that these two lines will be situated quite close to each other on the photographic plate.

In a similar fashion, we can see that as the  $n_{\text{higher}}$  increases, the lines start to converge together. And since, increasing the  $n_{\text{higher}}$  will indeed lead to an increase in the energy of the photon released, it will end up releasing photons of shorter wavelengths. Combining these two statements we can easily see that as the wavelength decreases, the spectral lines start to converge.

4. The correct order of the first ionization energies of the following metals; Na, Mg, Al, Si in  $\text{kJ mol}^{-1}$ , respectively is:
- 497, 737, 577, 786
  - 497, 577, 737, 786
  - 786, 739, 577, 497
  - 739, 577, 786, 487

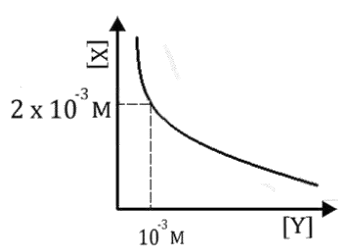
**Answer:** a

**Solution:**

The expected order is  $\text{Na} < \text{Mg} < \text{Al} < \text{Si}$ .

But the actual/experimental order turns out to be  $\text{Na} < \text{Al} < \text{Mg} < \text{Si}$ , because of the fully filled s-subshell of magnesium and the  $s^2p^1$  configuration of Al which makes it relatively easy for Al to lose its outermost electron.

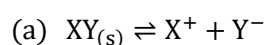
5. Select the correct stoichiometry and its  $K_{sp}$  value according to the given graph:



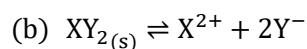
- $\text{XY}, K_{sp} = 2 \times 10^{-6}$
- $\text{XY}_2, K_{sp} = 4 \times 10^{-9}$
- $\text{X}_2\text{Y}, K_{sp} = 9 \times 10^{-9}$
- $\text{XY}_2, K_{sp} = 1 \times 10^{-9}$

**Answer:** a

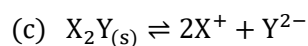
**Solution:**



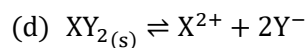
$$K_{sp} = [\text{X}^+][\text{Y}^-] = 2 \times 10^{-3} \times 10^{-3} = 2 \times 10^{-6}$$



$$K_{sp} = [\text{X}^{2+}][\text{Y}^-]^2 = 2 \times 10^{-3} \times 10^{-6} = 2 \times 10^{-9}$$



$$K_{sp} = [\text{X}^+]^2[\text{Y}^{2-}] = 4 \times 10^{-6} \times 10^{-3} = 4 \times 10^{-9}$$



$$K_{sp} = [\text{X}^{2+}][\text{Y}^-]^2 = 2 \times 10^{-3} \times 10^{-6} = 2 \times 10^{-9}$$

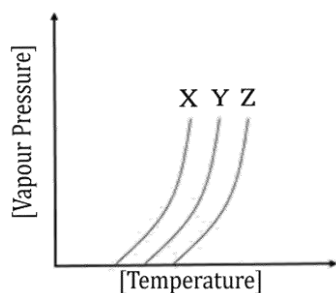
6. Which of the following complex exhibit facial-meridional geometrical isomerism?

- $[\text{Pt}(\text{NH}_3)\text{Cl}_3]^-$
- $[\text{PtCl}_2(\text{NH}_3)_2]$
- $[\text{Ni}(\text{CO})_4]$
- $[\text{Co}(\text{NO}_2)_3(\text{NH}_3)_3]$

**Answer:** d

**Solution:** Facial and meridional geometrical isomerism is observed only in  $[\text{MA}_3\text{B}_3]$  type complexes which is given in option d.

7.



- Intermolecular force of attraction of  $X > Y$
- Intermolecular force of attraction of  $X < Y$ .
- Intermolecular force of attraction of  $Z < X$ .

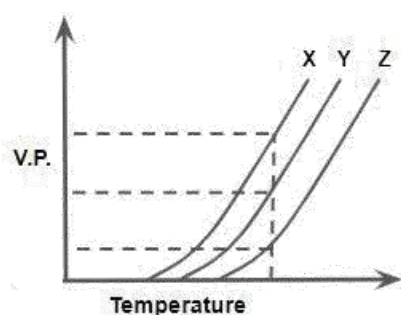
Select the correct option(s):

- A and C
- A and B
- B only
- B and C

**Answer:** c

**Solution:**

As shown in the plot below, for the same T, the vapour pressure of X is the highest and of Z is the lowest. Now, that means with the same average K.E. of X, Y and Z molecules, the X molecules are able to compensate their respective intermolecular forces better. So, X molecules have the highest vapour pressure. Which implies that the intermolecular forces in X are the weakest among the three. The opposite could be said for Z as well.



8. Rate of a reaction increases by  $10^6$  times when a reaction is carried out in presence of enzyme catalyst at the same temperature. Determine the change in activation energy.
- $-6 \times 2.303 RT$
  - $+6 \times 2.303 RT$
  - $+6RT$
  - $-6RT$

**Answer:** a

**Solution:**

$$K_1 = Ae^{-E_{a1}/RT} \text{----(1)}$$

$$K_2 = Ae^{-E_{a2}/RT} \text{----(2)}$$

Dividing equation 1 with equation 2, we get

$$\frac{K_1}{K_2} = e^{(E_{a2}-E_{a1})/RT}$$

$$10^{-6} = e^{(E_{a2}-E_{a1})/RT}$$

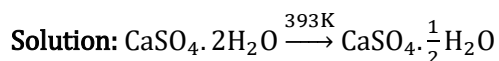
Taking  $\log_e$  on both sides, we get

$$\Delta E = E_{a2} - E_{a1} = -6 \times 2.303 RT$$

9. Gypsum on heating at 393K produces:

- Dead burnt plaster
- Anhydrous  $\text{CaSO}_4$
- $\text{CaSO}_4 \cdot \frac{1}{2} \text{H}_2\text{O}$
- $\text{CaSO}_4 \cdot 5\text{H}_2\text{O}$

**Answer:** c



10. Among the following, the least 3<sup>rd</sup> ionization energy is for:

- a. Mn
- b. Co
- c. Fe
- d. Ni

**Answer:** c

**Solution:**

Consider an element E

$E^{2+} \rightarrow E^{3+}$  would be the 3<sup>rd</sup> I.E. of the element E.

Electronic configuration of Mn is  $[Ar]4s^23d^5$

Electronic configuration of Co is  $[Ar]4s^23d^7$

Electronic configuration of Fe is  $[Ar]4s^23d^6$

Electronic configuration of Ni is  $[Ar]4s^23d^8$

Electronic configuration of  $Mn^{2+}$  is  $[Ar]3d^5$

Electronic configuration of  $Co^{2+}$  is  $[Ar]3d^7$

Electronic configuration of  $Fe^{2+}$  is  $[Ar]3d^6$

Electronic configuration of  $Ni^{2+}$  is  $[Ar]3d^8$

As it is evident from the above configurations of the  $E^{2+}$  for the given elements,  $Fe^{2+}$  would require the least amount of energy for removal of electron as it has the configuration  $3d^6 4s^0$ . That means that its  $E^{3+}$  form is the most stable among the four elements provided in their respective  $E^{3+}$  states, i.e., when compared, the next electron removal will require least amount of energy.

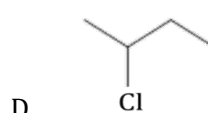
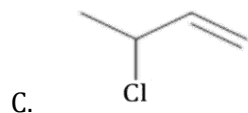
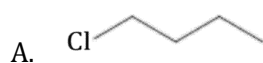
11. Accurate measurement of concentration of NaOH can be performed by which of the following titration?

- a. NaOH in burette and oxalic acid in conical flask
- b. NaOH in burette and concentrated  $H_2SO_4$  in conical flask
- c. NaOH in volumetric flask and concentrated  $H_2SO_4$  in conical flask
- d. Oxalic acid in burette and NaOH in conical flask

**Answer:** d

**Solution:** The standard solution is always kept in burette. The oxalic acid is a primary standard solution while  $H_2SO_4$  is a secondary standard solution.

12. Arrange the following compounds in order of dehydrohalogenation ( $E_1$ ) reaction:



- $C > B > D > A$
- $C > D > B > A$
- $B > C > D > A$
- $A > B > C > D$

**Answer:** b

**Solution:** In  $E_1$  mechanism, the rate determining step is formation of carbocation. So, stability of carbocation formed decides the rate.

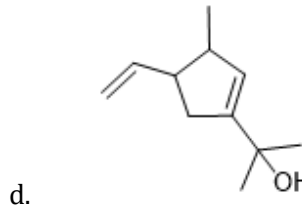
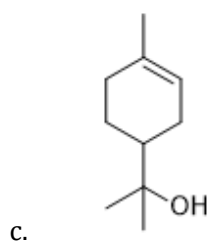
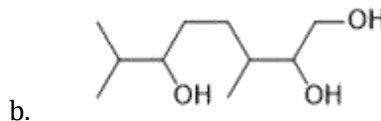
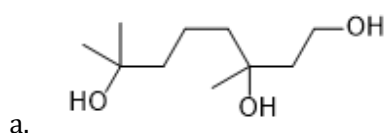
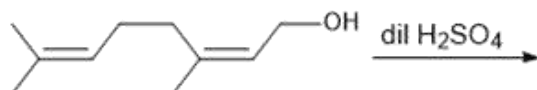
In option c, the cation formed is resonance stabilised.

In option d, the cation formed is a  $2^\circ$  carbocation.

In option a and b, the carbocations formed are  $1^\circ$  but there is a chance of rearrangement in option b and after the rearrangement, the carbocation formed in option b will be allylic. So, the order of reaction is as follows:

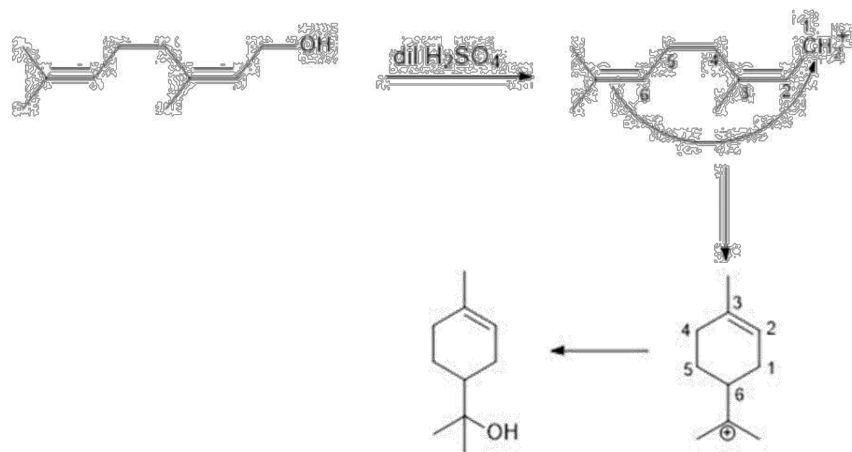
$C > D > B > A$ .

13. Major product in the following reaction is:



**Answer:** c

**Solution:**



14. Arrange the order of C—OH bond length in the following compounds:

- |          |        |                |
|----------|--------|----------------|
| Methanol | Phenol | p-Ethoxyphenol |
| (A)      | (B)    | (C)            |
- $A > B > C$
  - $A > C > B$
  - $C > B > A$
  - $B > C > A$

**Answer: b**

**Solution:** In methanol, there is no resonance. In phenol, there is resonance. In p-Ethoxyphenol, there is resonance involved but the involvement of lone pair of oxygen in OH group is poor as compared with phenol due to the presence of lone pair oxygen in OCH<sub>3</sub> group which are also involved in resonance.

So, partial double bond character develops in C—OH bond of phenol and p-Ethoxyphenol but in case of p-Ethoxyphenol, resonance is poor as compared to phenol. So, bond length follows the order:  $A > C > B$

15. Which of the following are "greenhouse gases"?

- CO<sub>2</sub>
  - O<sub>2</sub>
  - O<sub>3</sub>
  - CFC
  - H<sub>2</sub>O vapours
- i, ii and iv
  - i, ii, iii and iv
  - i, iii and iv
  - i, iii, iv and v

**Answer: d**

**Solution:**  $\text{CO}_2$ ,  $\text{O}_3$ ,  $\text{H}_2\text{O}$  vapours and CFC's are green house gases.

16. Two liquids, isohexane and 3-methylpentane have boiling points  $60^\circ\text{C}$  and  $63^\circ\text{C}$ , respectively. They can be separated by:
- Simple distillation and isohexane comes out first
  - Fractional distillation and isohexane comes out first.
  - Simple distillation and 3-Methylpentane comes out first.
  - Fractional distillation and 3-Methylpentane comes out first

**Answer:** b

**Solution:** When the difference between the B.P. of the two liquids is less than around  $40^\circ\text{C}$ , fractional distillation is more efficient. The difference between the boiling points of isohexane and 3-methylpentane is only 3 degrees. So, fractional distillation is the best suitable method. Since, isohexane has a lower boiling point, it comes out first.

17. Which of the given statement is incorrect about glucose?
- Glucose exists in two crystalline forms  $\alpha$  and  $\beta$ .
  - Glucose gives Schiff's test.
  - Penta acetate of glucose does not form oxime.
  - Glucose forms oxime with hydroxylamine.

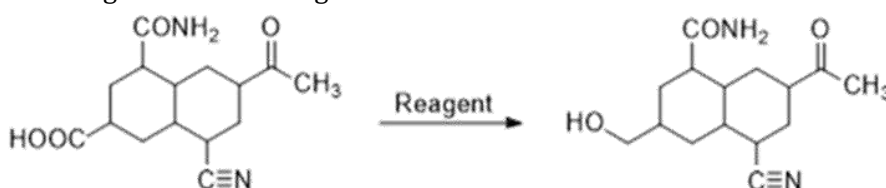
**Answer:** b

**Solution:** Glucose exists in two crystalline forms  $\alpha$  and  $\beta$  which are anomers of each other.

Glucose does not react with Schiff's reagent because after the internal cyclisation, it forms either  $\alpha$ -anomer or  $\beta$ -anomer. In these forms, free aldehydic group is not present.

Glucose forms open chain structure in aqueous solution which contains aldehyde at chain end. This aldehydic group reacts with  $\text{NH}_4\text{OH}$  to form oxime. On the other hand, glucose penta acetate being a cyclic structure even in aqueous form does not have terminal carbonyl group. Therefore it will not react with  $\text{NH}_4\text{OH}$ .

18. The reagent used for the given conversion is:



- $\text{H}_2$ , Pd
- $\text{B}_2\text{H}_6$
- $\text{NaBH}_4$
- $\text{LiAlH}_4$

**Answer:** b

**Solution:**  $\text{B}_2\text{H}_6$  does not reduce amide, carbonyl group and cyanide. It selectively reduces carboxylic acid to alcohol. So, for this conversion, it is the best suitable reagent.

19. 0.3 g  $[\text{ML}_6]\text{Cl}_3$  of molar mass 267.46 g/mol is reacted with 0.125 M  $\text{AgNO}_3(\text{aq})$  solution, calculate volume of  $\text{AgNO}_3$  required in mL.

**Answer:** 26.92

**Solution:** To react completely with one mole of  $[\text{ML}_6]\text{Cl}_3$ , 3 moles of  $\text{AgNO}_3$  is required.

0.3 g  $[\text{ML}_6]\text{Cl}_3$  means  $\frac{0.3}{267.46}$  moles of  $[\text{ML}_6]\text{Cl}_3$ .

So, moles of  $\text{AgNO}_3$  required will be  $\frac{0.3 \times 3}{267.46}$  moles

To find the volume,

$$\frac{0.3 \times 3}{267.46} = 0.125 \times V(\text{L})$$

$$V(\text{L}) = 0.02692$$

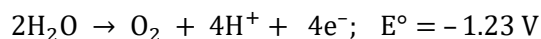
$$V(\text{mL}) = 26.92$$

20. Given :  $2\text{H}_2\text{O} \rightarrow \text{O}_2 + 4\text{H}^+ + 4\text{e}^-$ ;  $E^\circ = -1.23 \text{ V}$

Calculate the electrode potential at pH= 5.

**Answer:** -0.93

**Solution:**



$$E = -1.23 - \frac{0.0591}{4} \log [\text{H}^+]^4$$

$$= -1.23 + (0.0591 \times \text{pH}) = -1.23 + 0.0591 \times 5$$

$$= -1.23 + 0.2955 = -0.9345 \text{ V} = -0.93 \text{ V}$$

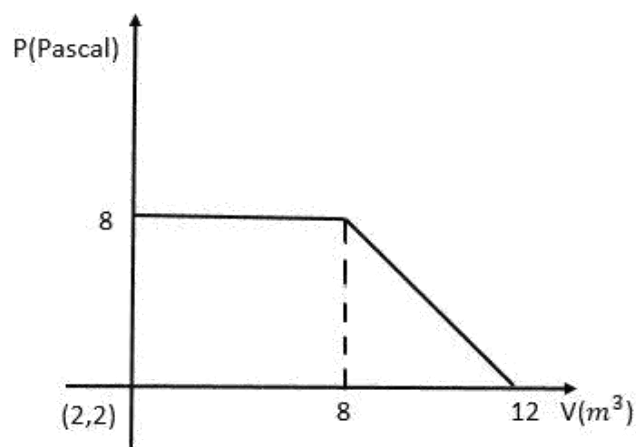
21. Calculate the mass of  $\text{FeSO}_4 \cdot 7\text{H}_2\text{O}$ , which must be added in 100 kg of wheat to get 10 ppm of Fe.

**Answer:** 4.96

**Solution:** 10 ppm of Fe means 10 g of Fe in  $10^6$  g of wheat. So, for 100 kg i.e.,  $10^5$  g

of wheat. Fe needed is 1 g. So, for 1 g of Fe, the mass of  $\text{FeSO}_4 \cdot 7\text{H}_2\text{O}$  required is  $\frac{278}{56} = 4.96$  g.

22. A gas undergoes expansion according to the following graph. Calculate work done by the gas (in Joules).



**Answer:** 48

**Solution:**

Work done by the gas

= The area under the curve

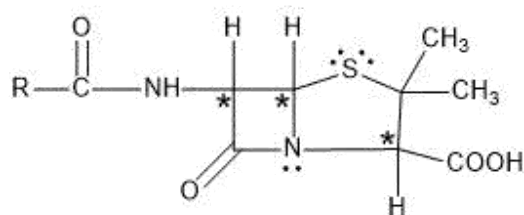
= (Area of the square) + (Area of the triangle)

= 48 J

23. The number of chiral centres in Penicillin is \_\_\_\_.

**Answer:** 3

**Solution:** The structure of penicillin is shown below:



So, the number of chiral centres= 3

**Date: 8<sup>th</sup> January 2020 (Shift 2)**

**Time: 2:30 P.M. to 5:30 P.M.**

**Subject: Mathematics**

1. Solution set of  $3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$  contains
- |                        |                           |
|------------------------|---------------------------|
| a. exactly one element | b. at least four elements |
| c. two elements        | d. infinite elements      |

**Answer: (a)**

**Solution:**

$$3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$$

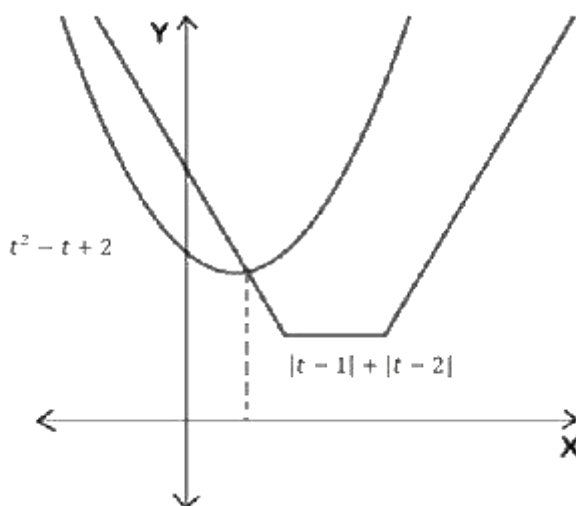
Let  $3^x = t$

$$t(t - 1) + 2 = |t - 1| + |t - 2|$$

$$\Rightarrow t^2 - t + 2 = |t - 1| + |t - 2|$$

We plot  $t^2 - t + 2$  and  $|t - 1| + |t - 2|$

As  $3^x$  is always positive, therefore only positive values of  $t$  will be the solution.



Therefore, we have only one solution.

2. Which of the following is a tautology?
- |   |   |
|---|---|
| a. $\sim(p \wedge \sim q) \rightarrow (p \vee q)$ | b. $(\sim p \vee q) \rightarrow (p \vee q)$       |
| c. $\sim(p \vee \sim q) \rightarrow (p \vee q)$   | d. $\sim(p \vee \sim q) \rightarrow (p \wedge q)$ |

**Answer: (c)**

**Solution:**

$$\sim(p \vee \sim q) \rightarrow (p \vee q)$$

$$= (p \vee \sim q) \vee (p \vee q)$$

$$= (p \vee p) \vee (q \vee \sim q)$$

$$= p \vee T$$

$$= T$$

3. If a hyperbola has vertices  $(\pm 6, 0)$  and  $P(10, 16)$  lies on it, then the equation of normal at  $P$  is
- a.  $2x + 5y = 10$
- b.  $2x + 5y = 100$
- c.  $2x - 5y = 100$
- d.  $5x + 2y = 100$

**Answer: (b)**

**Solution:**

Vertex of hyperbola is  $(\pm a, 0) \equiv (\pm 6, 0) \Rightarrow a = 6$

Let the equation of hyperbola be  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\Rightarrow \frac{x^2}{36} - \frac{y^2}{b^2} = 1$$

As  $P(10, 16)$  lies on the hyperbola.

$$\frac{100}{36} - \frac{256}{b^2} = 1$$

$$\Rightarrow \frac{64}{36} = \frac{256}{b^2} \Rightarrow b^2 = 144$$

Equation of hyperbola becomes  $\frac{x^2}{36} - \frac{y^2}{144} = 1$

Equation of normal is  $\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 + b^2$

$$\Rightarrow \frac{36x}{10} + \frac{144y}{16} = 180$$

$$\Rightarrow \frac{x}{50} + \frac{y}{20} = 1$$

$$\Rightarrow 2x + 5y = 100$$

4. If  $y = mx + c$  is a tangent to the circle  $(x - 3)^2 + y^2 = 1$  and also perpendicular to the tangent to the circle  $x^2 + y^2 = 1$  at  $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ , then
- a.  $c^2 - 6c - 7 = 0$
- b.  $c^2 - 6c + 7 = 0$
- c.  $c^2 + 6c - 7 = 0$
- d.  $c^2 + 6c + 7 = 0$

**Answer:** (d)

**Solution:**

For circle,  $x^2 + y^2 = 1$

$$2x + 2yy' = 0 \Rightarrow y' = -\frac{x}{y}$$

Slope of tangent to  $x^2 + y^2 = 1$  at  $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = -1$

$\Rightarrow$  Slope of tangent to  $(x - 3)^2 + y^2 = 1$  is 1  $\Rightarrow m = 1$

Tangent to  $(x - 3)^2 + y^2 = 1$  is  $y = x + c$

Perpendicular distance of tangent  $y = x + c$  from centre  $(3, 0)$  is equal to radius = 1

$$\left| \frac{3 + c}{\sqrt{2}} \right| = 1$$

$$\Rightarrow c + 3 = \pm\sqrt{2}$$

$$\Rightarrow c^2 + 6c + 9 = 2$$

$$\Rightarrow c^2 + 6c + 7 = 0$$

5. If  $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ ,  $\vec{b} = \hat{i} - \hat{j} + \hat{k}$  and  $\vec{c}$  is non-zero vector and  $\vec{b} \times \vec{c} = \vec{b} \times \vec{a}$ ,  $\vec{a} \cdot \vec{c} = 0$  then  $\vec{b} \cdot \vec{c}$  is equal to

a.  $\frac{1}{2}$

b.  $-\frac{1}{3}$

c.  $-\frac{1}{2}$

d.  $\frac{1}{3}$

**Answer:** (c)

**Solution:**

$$\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$$

$$\vec{b} = \hat{i} - \hat{j} + \hat{k}$$

$$\vec{b} \times \vec{c} = \vec{b} \times \vec{a}$$

$$\Rightarrow \vec{a} \times (\vec{b} \times \vec{c}) = \vec{a} \times (\vec{b} \times \vec{a})$$

$$\Rightarrow (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = (\vec{a} \cdot \vec{a})\vec{b} - (\vec{a} \cdot \vec{b})\vec{a}$$

$$\Rightarrow -(\vec{a} \cdot \vec{b})\vec{c} = (\vec{a} \cdot \vec{a})\vec{b} - (\vec{a} \cdot \vec{b})\vec{a}$$

$$\Rightarrow -4\vec{c} = 6(\hat{i} - \hat{j} + \hat{k}) - 4(\hat{i} - 2\hat{j} + \hat{k})$$

$$\Rightarrow \vec{c} = -\frac{1}{2}(\hat{i} + \hat{j} + \hat{k})$$

$$\therefore \vec{b} \cdot \vec{c} = -\frac{1}{2}$$



$$\Rightarrow x \left( \frac{dy}{dx} \right)^2 = 2y \left( \frac{dy}{dx} \right) + x$$

8. Image of point  $(1, 2, 3)$  w.r.t a plane is  $\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$  then which of the following points lie on this plane
- |                  |                   |
|------------------|-------------------|
| a. $(1, 1, -1)$  | b. $(-1, -1, 1)$  |
| c. $(-1, 1, -1)$ | d. $(-1, -1, -1)$ |

**Answer:** (a)

**Solution:**

Image of point  $P(1, 2, 3)$  w.r.t. a plane  $ax + by + cz + d = 0$  is  $Q\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$

Direction ratios of  $PQ$ :  $-\frac{10}{3}, -\frac{10}{3}, -\frac{10}{3} = 1, 1, 1$

Direction ratios of normal to plane is  $1, 1, 1$

Mid-point of  $PQ$  lies on the plane

$\therefore$  The mid-point of  $PQ = \left(-\frac{2}{3}, \frac{1}{3}, \frac{4}{3}\right)$

$\therefore$  Equation of plane is  $x + \frac{2}{3} + y - \frac{1}{3} + z - \frac{4}{3} = 0$

$$\Rightarrow x + y + z = 1$$

$(1, 1, -1)$  satisfies the equation of the plane.

9.  $\lim_{x \rightarrow 0} \frac{\int_0^x t \sin 10t \, dt}{x}$  is equal to

- |       |      |
|-------|------|
| a. 10 | b. 0 |
| c. 1  | d. 5 |

**Answer:** (b)

**Solution:**

$$\lim_{x \rightarrow 0} \frac{\int_0^x t \sin 10t \, dt}{x}$$

Applying L'Hospital's Rule:

$$= \lim_{x \rightarrow 0} \frac{x \sin 10x}{1} = 0$$

10. Let  $P$  be the set of points  $(x, y)$  such that  $(x^2 \leq y \leq -2x + 3)$ . Then area bounded by points in  $P$  is

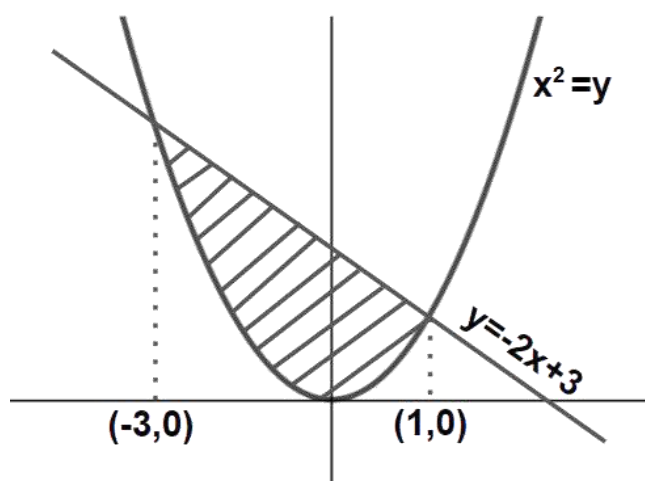
a.  $\frac{16}{3}$   
c.  $\frac{20}{3}$

b.  $\frac{29}{3}$   
d.  $\frac{32}{3}$

**Answer:** (d)

**Solution:**

We have  $x^2 \leq y \leq -2x + 3$



For point of intersection of two curves -

$$x^2 + 2x - 3 = 0$$

$$\Rightarrow x = -3, 1$$

$$\Rightarrow \text{Area} = \int_{-3}^1 ((-2x + 3) - x^2) dx$$

$$= \left[ -x^2 + 3x - \frac{x^3}{3} \right]_{-3}^1 = \frac{32}{3} \text{ sq. units.}$$

11. If  $f(x) = \frac{x[x]}{x^2+1} : (1, 3) \rightarrow \mathbf{R}$ , then the range of  $f(x)$  is (where  $[.]$  denotes greatest integer function)

a.  $\left(0, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{7}{5}\right]$   
c.  $\left(0, \frac{1}{3}\right) \cup \left(\frac{2}{5}, \frac{4}{5}\right]$

b.  $\left(\frac{2}{5}, 1\right) \cup \left(1, \frac{4}{5}\right]$   
d.  $\left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$

**Answer:** (d)

**Solution:**

$$f(x) = \frac{x[x]}{x^2+1}$$

$$\Rightarrow f(x) = \begin{cases} \frac{x}{x^2+1} : 1 < x < 2 \\ \frac{2x}{x^2+1} : 2 \leq x < 3 \end{cases}$$







$$\Rightarrow c = -9a$$

$$f(-1) = 10 \Rightarrow -a + b - c + d = 10$$

$$\Rightarrow -a - 3a + 9a + d = 10$$

$$d = -5a + 10$$

$$f(1) = 6 \Rightarrow a + b + c + d = 6$$

$$\Rightarrow a - 3a - 9a - 5a + 10 = 6$$

$$\Rightarrow a = \frac{1}{4}$$

$$\therefore f'(x) = \frac{3}{4}x^2 - \frac{6}{4}x - \frac{9}{4} = \frac{3}{4}(x^2 - 2x - 3)$$

$$\text{For } f'(x) = 0 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow x = 3, -1$$

Minima exists at  $x = 3$

18. Let  $I = \int_1^2 \frac{dx}{\sqrt{2x^3 - 9x^2 + 12x + 4}}$  then

a.  $\frac{1}{9} < I^2 < \frac{1}{8}$

b.  $\frac{1}{9} < I < \frac{1}{8}$

c.  $\frac{1}{3} < I^2 < \frac{1}{2}$

d.  $\frac{1}{3} < I < \frac{1}{2}$

**Answer:** (a)

**Solution:**

$$f(x) = \frac{1}{\sqrt{2x^3 - 9x^2 + 12x + 4}}$$

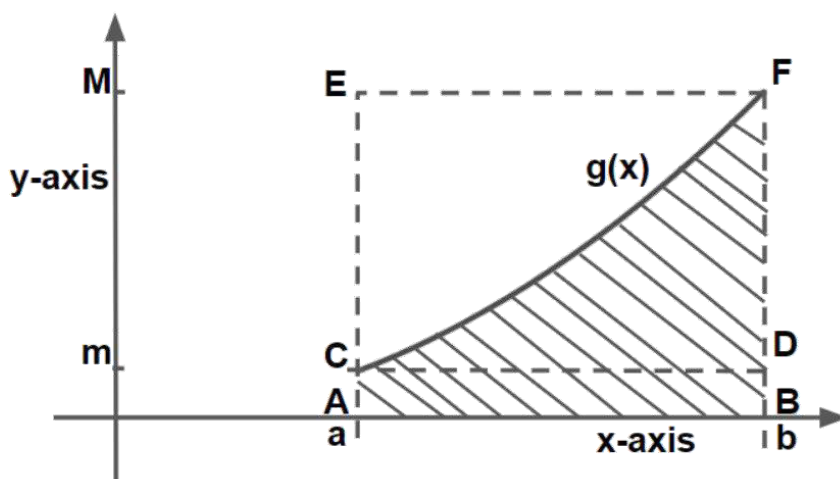
Differentiating w.r.t  $x$

$$f'(x) = -\frac{1}{2} \times \frac{(6x^2 - 18x + 12)}{(2x^3 - 9x^2 + 12x + 4)^{3/2}}$$

$$= \frac{-6(x-1)(x-2)}{2(2x^3 - 9x^2 + 12x + 4)^{3/2}}$$

Here  $f(x)$  is increasing in (1,2)

$$\text{At } x = 1, f(1) = \frac{1}{3} \quad \text{and } x = 2, f(2) = \frac{1}{\sqrt{8}}$$



Let  $g(x)$  be a function such that it is increasing in  $(a, b)$  and  $m \leq g(x) \leq M$ , then

$$\text{ar}(ABCD) < \int_a^b g(x) dx < \text{ar}(ABEF)$$

$$m(b-a) < \int_a^b g(x) dx < M(b-a)$$

$$\text{Thus, } \frac{1}{3} < \int_1^2 f(x) dx < \frac{1}{\sqrt{8}}$$

$$\frac{1}{3} < I < \frac{1}{\sqrt{8}}$$

$$\text{or } \frac{1}{9} < I^2 < \frac{1}{8}$$

19. Normal at  $(2, 2)$  to curve  $x^2 + 2xy - 3y^2 = 0$  is  $L$ . Then perpendicular distance from origin to line  $L$  is

- |                |      |
|----------------|------|
| a. $2\sqrt{2}$ | b. 4 |
| c. $4\sqrt{2}$ | d. 2 |

**Answer:** (a)

**Solution:**

$$\text{Given curve: } x^2 + 2xy - 3y^2 = 0$$

$$\Rightarrow x^2 + 3xy - xy - 3y^2 = 0$$

$$\Rightarrow (x + 3y)(x - y) = 0$$

Equating we get,

$$x + 3y = 0 \text{ or } x - y = 0$$

$$(2, 2) \text{ lies on } x - y = 0$$

$\therefore$  Equation of normal will be  $x + y = \lambda$

It passes through  $(2, 2)$

$$\therefore \lambda = 4$$

$$L : x + y = 4$$

$$\text{Distance of } L \text{ from the origin} = \left| \frac{-4}{\sqrt{2}} \right| = 2\sqrt{2}$$

20. If  $A$  and  $B$  are two events such that  $P(\text{exactly one}) = \frac{2}{5}$ ,  $P(A \cup B) = \frac{1}{2}$  then  $P(A \cap B)$  is

a.  $\frac{1}{8}$

b.  $\frac{1}{10}$

c.  $\frac{1}{12}$

d.  $\frac{2}{9}$

**Answer:** (b)

**Solution:**

$$P(\text{exactly one of } A \text{ or } B) = \frac{2}{5}$$

$$\Rightarrow P(A) - P(A \cap B) + P(B) - P(A \cap B) = \frac{2}{5}$$

$$\Rightarrow P(A \cup B) - P(A \cap B) = \frac{2}{5}$$

$$\Rightarrow P(A \cap B) = \frac{1}{10}$$

21. The number of four-letter words that can be made from the letters of word "EXAMINATION" is

**Answer:** (2454)

**Solution:**

Word "EXAMINATION" consists of 2A, 2I, 2N, E, X, M, T, O

Case I: All different letters are selected

$$\text{Number of words formed} = {}^8C_4 \times 4! = 1680$$

Case II: 2 letters are same and 2 are different

$$\text{Number of words formed} = {}^3C_1 \times {}^7C_2 \times \frac{4!}{2!} = 756$$

Case III: 2 pair of letters are same

$$\text{Number of words formed} = {}^3C_2 \times \frac{4!}{2! \times 2!} = 18$$

Total number of words formed =  $1680 + 756 + 18 = 2454$

22. Line  $y = mx$  intersects the curve  $y^2 = x$  at point  $P$ . The tangent to  $y^2 = x$  at  $P$  intersects  $x$ -axis at  $Q$ . If area  $\Delta OPQ = 4$ , find  $m$ , ( $m > 0$ )

**Answer:** (0.5)

**Solution:**

Let the co-ordinates of  $P$  be  $(t^2, t)$

Equation of tangent at  $P(t^2, t)$  is  $y - t = \frac{1}{2t}(x - t^2)$

Therefore, co-ordinates of  $Q$  will be  $(-t^2, 0)$

Area of  $\Delta OPQ = 4$

$$\Rightarrow \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ t^2 & t & 1 \\ -t^2 & 0 & 1 \end{vmatrix} = \pm 4$$

$$\Rightarrow t^3 = 8 \Rightarrow t = \pm 2 \Rightarrow t = 2 \text{ as } t > 0$$

$$m = \frac{1}{t} = \frac{1}{2}$$

23.  $\sum_{n=1}^7 \frac{n(n+1)(2n+1)}{4}$  is equal to

**Answer:** (504)

**Solution:**

$$\begin{aligned} & \sum_{n=1}^7 \frac{n(n+1)(2n+1)}{4} \\ &= \frac{1}{4} \sum_{n=1}^7 (2n^3 + 3n^2 + n) \\ &= \frac{1}{4} \left[ 2 \sum_{n=1}^7 n^3 + 3 \sum_{n=1}^7 n^2 + \sum_{n=1}^7 n \right] \\ &= \frac{1}{4} \left[ 2 \times \left( \frac{7 \times 8}{2} \right)^2 + 3 \times \frac{7 \times 8 \times 15}{6} + \frac{7 \times 8}{2} \right] \\ &= \frac{1}{4} [2 \times 784 + 420 + 28] = 504 \end{aligned}$$

24. Let  $\frac{\sqrt{2} \sin \alpha}{\sqrt{1+\cos 2\alpha}} = \frac{1}{7}$  and  $\sqrt{\frac{1-\cos 2\beta}{2}} = \frac{1}{\sqrt{10}}$ , where  $\alpha, \beta \in \left(0, \frac{\pi}{2}\right)$ . Then  $\tan(\alpha + 2\beta)$  is equal to

**Answer:** (1)

**Solution:**

$$\frac{\sqrt{2} \sin \alpha}{\sqrt{1+\cos 2\alpha}} = \frac{1}{7} \Rightarrow \frac{\sqrt{2} \sin \alpha}{\sqrt{2} \cos \alpha} = \frac{1}{7} \Rightarrow \tan \alpha = \frac{1}{7}$$

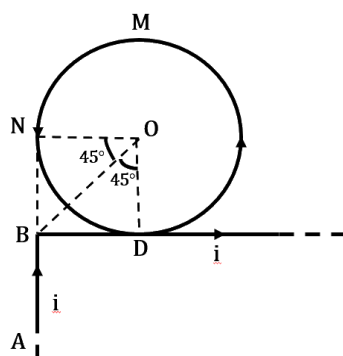
$$\sqrt{\frac{1-\cos 2\beta}{2}} = \frac{1}{\sqrt{10}} \Rightarrow \frac{\sqrt{2} \sin \beta}{\sqrt{2}} = \frac{1}{\sqrt{10}} \Rightarrow \sin \beta = \frac{1}{\sqrt{10}} \Rightarrow \tan \beta = \frac{1}{3}$$

$$\tan 2\beta = \frac{2 \tan \beta}{1 - \tan^2 \beta} = \frac{\frac{2}{3}}{1 - \frac{1}{9}} = \frac{3}{4}$$

$$\tan(\alpha + 2\beta) = \frac{\tan \alpha + \tan 2\beta}{1 - \tan \alpha \tan 2\beta} = \frac{\frac{1}{7} + \frac{3}{4}}{1 - \frac{1}{7} \times \frac{3}{4}} = 1$$

Date of Exam: 8<sup>th</sup> January (Shift II) **JEE Main 2020 Paper**  
 Time: 2:30 pm – 5:30 pm  
 Subject: **Physics**

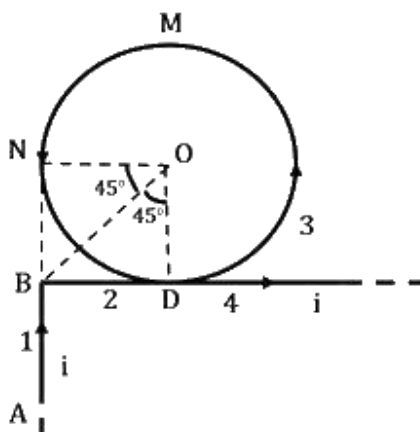
1. Find magnetic field at O. Where R is the radius of the loop



a.  $\frac{\mu_0 i}{2\pi R} \left[ \frac{-1}{\sqrt{2}} + \pi \right]$   
 c.  $\frac{\mu_0 i}{2R}$

b.  $\frac{\mu_0 i}{2\pi R} [\pi - 1]$   
 d.  $\frac{\mu_0 i}{2\pi R} \left[ \frac{1}{\sqrt{2}} + \pi \right]$

Solution: (d)



To get magnetic field at O, we need to find magnetic field due to each current carrying part 1, 2, 3 and 4 individually.

Let's take total magnetic field as  $B_T$ , then

$$\vec{B}_T = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4$$

Since 2 and 4 are parts of same wire, hence

$$\begin{aligned}\vec{B}_T &= \frac{\mu_0 i}{4\pi R} (\sin 90^\circ - \sin 45^\circ)(-\hat{k}) + \frac{\mu_0 i}{2R} \hat{k} + \frac{\mu_0 i}{4\pi R} (\sin 90^\circ + \sin 45^\circ)\hat{k} \\ &= \frac{-\mu_0 i}{4\pi R} \left[1 - \frac{1}{\sqrt{2}}\right] + \frac{\mu_0 i}{2R} + \frac{\mu_0 i}{4\pi R} \left[1 + \frac{1}{\sqrt{2}}\right] \hat{k} \\ \vec{B}_T &= \frac{\mu_0 i}{4\pi R} [\sqrt{2} + 2\pi] \hat{k} \\ \vec{B}_T &= \frac{\mu_0 i}{2\pi R} \left[\frac{1}{\sqrt{2}} + \pi\right] \hat{k}\end{aligned}$$

$\hat{k}$  denotes that direction of magnetic field is in the plane coming out of the plane of current.

2. Position of particle as a function of time is given as  $\vec{r} = \cos \omega t \hat{i} + \sin \omega t \hat{j}$ , where  $\omega$  is constant. Choose correct statement about  $\vec{r}$ ,  $\vec{v}$  and  $\vec{a}$  where  $\vec{v}$  and  $\vec{a}$  are the velocity and acceleration of the particle at time  $t$ .
- $\vec{v}$  and  $\vec{a}$  are perpendicular to  $\vec{r}$
  - $\vec{v}$  is parallel to  $\vec{r}$  and  $\vec{a}$  parallel to  $\vec{r}$
  - $\vec{v}$  is perpendicular to  $\vec{r}$  and  $\vec{a}$  is away from the origin
  - $\vec{v}$  is perpendicular to  $\vec{r}$  and  $\vec{a}$  is towards the origin

Solution:(d)

$$\begin{aligned}\vec{r} &= \cos \omega t \hat{i} + \sin \omega t \hat{j} \\ \vec{v} &= \frac{d\vec{r}}{dt} = \omega[-\sin \omega t \hat{i} + \cos \omega t \hat{j}] \\ \vec{a} &= \frac{d\vec{v}}{dt} = -\omega^2[\cos \omega t \hat{i} + \sin \omega t \hat{j}] \\ \vec{a} &= -\omega^2 \vec{r}\end{aligned}$$

Since there is negative sign in acceleration, this means that acceleration is in opposite direction of  $\vec{r}$

For velocity direction we can take dot product of  $\vec{v}$  and  $\vec{r}$ .

$$\begin{aligned}\vec{v} \cdot \vec{r} &= \omega(-\sin \omega t \hat{i} + \cos \omega t \hat{j}) \cdot (\cos \omega t \hat{i} + \sin \omega t \hat{j}) \\ &= \omega[-\sin \omega t \cos \omega t + \cos \omega t \sin \omega t] = 0\end{aligned}$$

which implies that  $\vec{v}$  is perpendicular to  $\vec{r}$ .

3. Two uniformly charged solid spheres are such that  $E_1$  is electric field at surface of 1<sup>st</sup> sphere due to itself.  $E_2$  is electric field at surface of 2<sup>nd</sup> sphere due to itself.  $r_1, r_2$  are radius of 1<sup>st</sup> and 2<sup>nd</sup> sphere respectively. If  $\frac{E_1}{E_2} = \frac{r_1}{r_2}$  then ratio of potential at the surface of spheres 1<sup>st</sup> and 2<sup>nd</sup> due to their self charges is :

- |                      |                                     |
|----------------------|-------------------------------------|
| a. $\frac{r_1}{r_2}$ | b. $\left(\frac{r_1}{r_2}\right)^2$ |
| c. $\frac{r_2}{r_1}$ | d. $\left(\frac{r_2}{r_1}\right)^2$ |

Solution: (b)

$$\begin{aligned}\frac{E_1}{E_2} &= \frac{r_1}{r_2} \\ \frac{V_1}{V_2} &= \frac{E_1 r_1}{E_2 r_2} = \frac{r_1}{r_2} \times \frac{r_1}{r_2} = \left(\frac{r_1}{r_2}\right)^2\end{aligned}$$

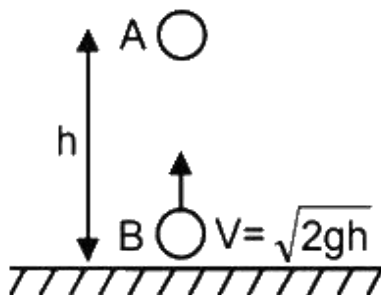
4. Velocity of a wave in a wire is  $v$  when tension in it is  $2.06 \times 10^4 \text{ N}$ . Find value of tension in wire when velocity of wave become  $\frac{v}{2}$ .

- |                                 |                                 |
|---------------------------------|---------------------------------|
| a. $5.15 \times 10^2 \text{ N}$ | b. $9.12 \times 10^4 \text{ N}$ |
| c. $9 \times 10^4 \text{ N}$    | d. $5.15 \times 10^3 \text{ N}$ |

Solution: (d)

$$\begin{aligned}V &\propto \sqrt{T} \\ \frac{V_1}{V_2} &= \sqrt{\frac{T_1}{T_2}} \\ \Rightarrow \frac{2V}{V} &= \sqrt{\frac{2.06 \times 10^4}{T}} \\ \Rightarrow T &= \frac{2.06 \times 10^4}{4} \text{ N} \\ &= 5.15 \times 10^3 \text{ N}\end{aligned}$$

5. There are two identical particles A and B. One is projected vertically upward with speed  $\sqrt{2gh}$  from ground and other is dropped from height  $h$  along the same vertical line. Collision between them is perfectly inelastic. Find time taken by them to reach the ground after collision in terms of  $\sqrt{\frac{h}{g}}$  is.



a.  $\sqrt{\frac{3}{2}}$   
c.  $\sqrt{3}$

b.  $\sqrt{\frac{1}{2}}$   
d.  $\sqrt{\frac{1}{5}}$

Solution:(a)

Time taken for the collision  $t_1 = \frac{h}{\sqrt{2gh}}$

After  $t_1$

$$V_A = 0 - gt_1 = -\frac{\sqrt{gh}}{2}$$

And  $V_B = \sqrt{2gh} - gt_1 = \sqrt{gh} \left[ \sqrt{2} - \frac{1}{\sqrt{2}} \right]$

At the time of collision

$$\vec{P}_i = \vec{P}_f$$

$$\Rightarrow m \vec{V}_A + m \vec{V}_B = 2 m \vec{V}_f$$

$$\Rightarrow -\sqrt{\frac{gh}{2}} + \sqrt{gh} \left[ \sqrt{2} - \frac{1}{\sqrt{2}} \right] = 2 \vec{V}_f$$

$$V_f = 0$$

And height from the ground =  $h - \frac{1}{2} g t_1^2 = h - \frac{h}{4} = \frac{3h}{4}$ .

So, time taken to reach ground after collision =  $\sqrt{2 \times \frac{(3h)}{4} / g} = \sqrt{\frac{3h}{2g}}$

6. A Carnot engine, having an efficiency of  $\eta = 1/10$  as heat engine, is used as a refrigerator. If the work done on the system is 10 J, the amount of energy absorbed from the reservoir at lower temperature is
- |         |          |
|---------|----------|
| a. 99 J | b. 90 J  |
| c. 1 J  | d. 100 J |

Solution:(b )

For Carnot engine using as refrigerator

Work done on engine is given by

$$W = Q_1 - Q_2 \dots (1)$$

where  $Q_1$  is heat rejected to the reservoir at higher temperature and  $Q_2$  is the heat absorbed from the reservoir at lower temperature.

It is given  $\eta = 1/10$

$$\eta = 1 - \frac{Q_2}{Q_1}$$

$$\Rightarrow \frac{Q_2}{Q_1} = \frac{9}{10} \dots (2)$$

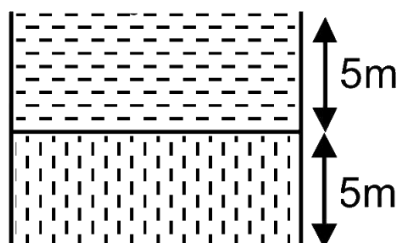
We are given,  $W = 10$  J

Therefore, from equations (1) and (2),

$$Q_2 = \frac{10}{\frac{10}{9} - 1}$$

$$\Rightarrow Q_2 = 90 \text{ J}$$

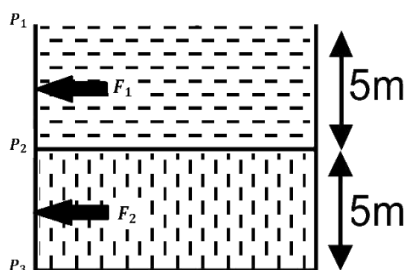
7. Two liquid columns of same height  $5\text{ m}$  and densities  $\rho$  and  $2\rho$  are filled in a container of uniform cross-sectional area. Then ratio of force exerted by the liquid on upper half of the wall to lower half of the wall is



a.  $\frac{2}{3}$   
c.  $\frac{1}{4}$

b.  $\frac{1}{2}$   
d.  $\frac{2}{5}$

Solution:(c)



The net force exerted on the wall by one type of liquid will be average value of pressure due to that liquid multiplied by the area of the wall.

Here, since the pressure due a liquid of uniform density varies linearly with depth, its average will be just the mean value of pressure at the top and pressure at the bottom.

So,

$$P_1 = 0$$

$$P_2 = \rho g \times 5$$

$$P_3 = 5\rho g + 2\rho g \times 5$$

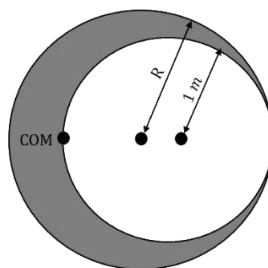
$$F_1 = \left( \frac{P_1 + P_2}{2} \right) A$$

$$F_2 = \left( \frac{P_2 + P_3}{2} \right) A$$

So,

$$\frac{F_1}{F_2} = \frac{1}{4}$$

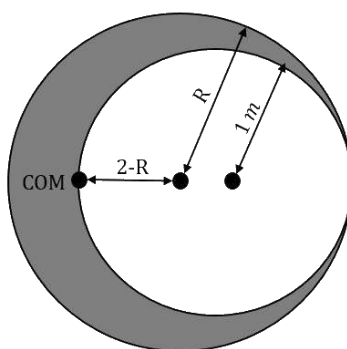
8. A uniform solid sphere of radius  $R$  has a cavity of radius  $1\text{ m}$  cut from it. If the center of mass of the system lies at the periphery of the cavity then



- a.  $(R^2 + R + 1)(2 - R) = 1$   
c.  $(R^2 - R + 1)(2 - R) = 1$

- b.  $(R^2 - R - 1)(2 - R) = 1$   
d.  $(R^2 + R - 1)(2 - R) = 1$

Solution (a)



Let  $M$  be the mass of the sphere and  $M'$  be the mass of the cavity.

Mass of the remaining part of the sphere =  $M - M'$

Mass moments of the cavity and the remaining part of sphere about the original CoM should add up to zero.

$$(M - M')(2 - R) - M'(R - 1) = 0$$

(Mass of the cavity to be taken negative)

$$\Rightarrow \frac{4}{3}\pi(R^3 - 1^3)\rho g(2 - R) = \frac{4}{3}\pi(1)^3\rho g(R - 1)$$

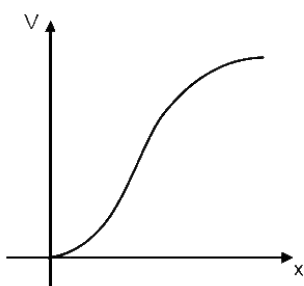
$$\Rightarrow (R^3 - 1^3)(2 - R) = (1^3)(R - 1)$$

$$\Rightarrow (R^2 + R + 1)(R - 1)(2 - R) = (R - 1)$$

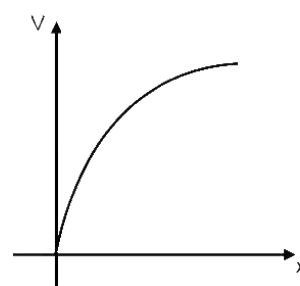
(using identity)

$$\Rightarrow (R^2 + R + 1)(2 - R) = 1$$

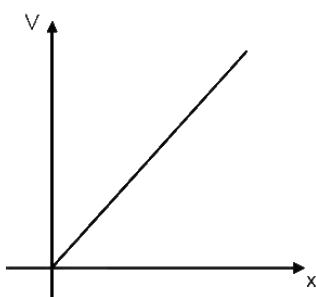
9. A charge particle of mass  $m$  and charge  $q$  is released from rest in uniform electric field. Its graph between velocity ( $v$ ) and distance ( $x$ ) will be :



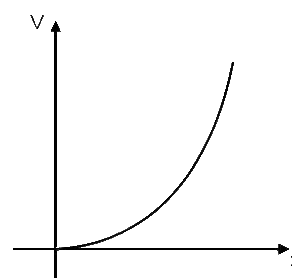
a.



b.



c.



d.



$$C_{p_{mix}} = C_{v_{mix}} + R = \frac{19R}{6}$$

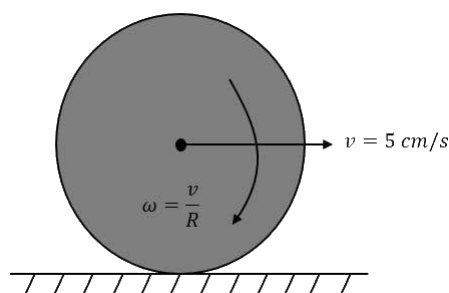
$$\therefore \gamma_{mix} = \frac{C_{p_{mix}}}{C_{v_{mix}}} = \frac{19}{13}$$

12. A solid sphere of mass  $m = 500 \text{ gm}$  is rolling without slipping on a horizontal surface. Find kinetic energy of the sphere if velocity of center of mass is  $5 \text{ cm/sec}$ .

a.  $\frac{35}{4} \times 10^{-4} \text{ J}$   
c.  $21 \times 10^{-4} \text{ J}$

b.  $\frac{35}{2} \times 10^{-4} \text{ J}$   
d.  $70 \times 10^{-3} \text{ J}$

Solution:(a)



Total K.E. = Translational K.E + Rotational K.E.

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 \left(1 + \frac{k^2}{R^2}\right)$$

$k$  is radius of gyration.

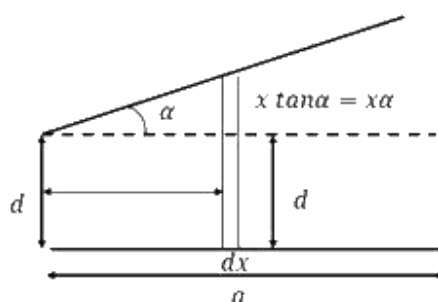
$$= \frac{1}{2} \times \frac{1}{2} \times \left(\frac{5}{100}\right)^2 \left(1 + \frac{2}{5}\right)$$

$$= \frac{35}{4} \times 10^{-4} \text{ J}$$

13. Two square plates of side 'a' are arranged as shown in the figure. The minimum separation between plates is 'd' and one of the plates is inclined at small angle  $\alpha$  with plane parallel to another plate. The capacitance of capacitor is (given  $\alpha$  is very small)

- a.  $\frac{\epsilon_0 a^2}{d} \left(1 - \frac{\alpha a}{d}\right)$       b.  $\frac{\epsilon_0 a^2}{d} \left(1 - \frac{\alpha a}{2d}\right)$   
 c.  $\frac{\epsilon_0 a^2}{d} \left(1 + \frac{\alpha a}{2d}\right)$       d.  $\frac{\epsilon_0 a^2}{d} \left(1 - \frac{\alpha a}{4d}\right)$

Solution:(b)



Let  $dC$  be the capacitance of the element of thickness  $dx$

$$dC = \frac{\epsilon_0 a dx}{d + \alpha x}$$

These are effectively in parallel combination

So,

$$C = \int dC$$

$$C = \int_0^a \frac{\epsilon_0 a dx}{d + \alpha x}$$

$$\Rightarrow C = \frac{\epsilon_0 a}{\alpha} [\ln(d + \alpha x)]_0^a$$

$$= \frac{\epsilon_0 a}{\alpha} \left[ \ln \left( 1 + \frac{\alpha a}{d} \right) \right]$$

$$\approx \frac{\epsilon_0 a^2}{d} \left( 1 - \frac{\alpha a}{2d} \right)$$





So, Trapezium's area

$$s = \frac{g \left[ t - \frac{1}{2} + t \right]}{2} \times \frac{1}{2} = 19 \quad (\text{i})$$

$$\frac{1}{2} g t^2 = 100 \quad (\text{ii})$$

Solving equations (i) and (ii), we get

$$g = 8 \text{ m/s}^2$$

17. Coming Soon

18. A simple pendulum of length 25.0 cm makes 40 oscillation in 50 sec. If resolution of stopwatch is 1 sec, then accuracy of  $g$  is (in %)

- |        |        |
|--------|--------|
| a. 1.2 | b. 3.2 |
| c. 4.4 | d. 5.4 |

Solution:(c)

$$T = 2\pi \sqrt{\frac{L}{g}}$$

$$g = L \cdot \left( \frac{2\pi}{T} \right)^2$$

$$\frac{\Delta g}{g} = 2 \frac{\Delta T}{T} + \frac{\Delta L}{L}$$

$$2 \left( \frac{1}{50} \right) + \frac{0.1}{25} = 4.4\%$$

19. An electron is moving initially with velocity  $v_0\hat{i} + v_0\hat{j}$  in uniform electric field  $\vec{E} = -E_0\hat{k}$ . If initial wavelength of electron is  $\lambda_0$  and mass of electron is  $m$ , find wavelength of electron as a function of time.

a.  $\frac{\lambda_0}{\sqrt{1 + \frac{e^2 E_0^2}{m^2 v_0^2} t^2}}$

b.  $\frac{2\lambda_0}{\sqrt{1 + \frac{e^2 E_0^2}{m^2 v_0^2} t^2}}$

c.  $\frac{\lambda_0 m v_0}{e E_0 t}$

d.  $\frac{2\lambda_0 m v_0}{e E_0 t}$

Solution:(b)

Momentum of an electron

$$p = mv = \frac{h}{\lambda}$$

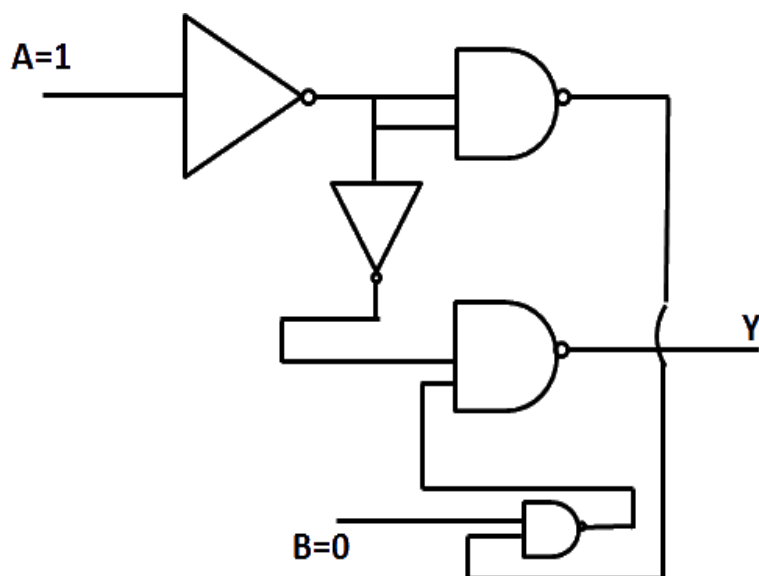
$$\text{Initially } m(\sqrt{2}v_0) = \frac{h}{\lambda_0}$$

$$\text{Velocity as a function of time} = v_0\hat{i} + v_0\hat{j} + \frac{eE_0}{m}t\hat{k}$$

$$\text{So, wavelength } \lambda = \frac{h}{m\sqrt{2v_0^2 + \frac{e^2 E_0^2}{m^2} t^2}}$$

$$\Rightarrow \lambda = \frac{\lambda_0}{\sqrt{1 + \frac{e^2 E_0^2}{m^2 v_0^2} t^2}}$$

20. Output at terminal Y of given logic circuit.



- a. 1
- b. 0
- c. Can't determine
- d. Oscillating between 0 and 1

Solution: b.

$$\begin{aligned}
 Y &= \overline{\overline{AB}.A} \\
 &= \overline{\overline{AB}} + \bar{A} \\
 &= AB + \bar{A} \\
 Y &= 0 + 0 = 0
 \end{aligned}$$

21. In  $H$ -spectrum wavelength of 1<sup>st</sup> line of Balmer series is  $\lambda = 6561\text{\AA}$ . Find out wavelength of 2<sup>nd</sup> line of same series in  $nm$ .

Solution: (486)

$$\begin{aligned}
 \frac{1}{\lambda} &= RZ^2 \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \\
 \frac{1}{\lambda_1} &= R(1)^2 \left( \frac{1}{2^2} - \frac{1}{3^2} \right) = \frac{5R}{36}
 \end{aligned}$$

$$\frac{1}{\lambda_2} = R(1)^2 \left( \frac{1}{2^2} - \frac{1}{4^2} \right) = \frac{3R}{36}$$

$$\frac{\lambda_2}{\lambda_1} = \frac{20}{27}$$

$$\lambda_2 = \frac{20}{27} \times 6561 \text{ Å} = 4860 \text{ Å}$$

22. An EMW is travelling along z-axis is  $\vec{B} = 5 \times 10^{-8} \hat{j} \text{ T}$ ,  $c = 3 \times 10^8 \text{ m/s}$  and frequency of wave is  $25 \text{ Hz}$ , then electric field in  $\frac{\text{volt}}{\text{m}}$ .

Solution: (15)

$$\frac{E}{B} = c$$

$$E = B \times c$$

Given,

$$\vec{B} = 5 \times 10^{-8} \hat{j} \text{ T and } C = 3 \times 10^8 \text{ m/s}$$

$$E = 15 \frac{\text{volt}}{\text{m}}.$$

23. There are three containers  $C_1$ ,  $C_2$  and  $C_3$  filled with same material at different constant temperature. When we mix them in different quantity (volume) then we get some final temperature as shown in the table. Then find the value of final temperature  $\theta$  as shown in the table.

$C_1$	$C_2$	$C_3$	$t(^{\circ}\text{C})$
1 l	2 l	0	60
0	1 l	2 l	30
2 l	0	1 l	60
1 l	1 l	1 l	$\theta$

Solution: (50)

Since, all the containers have same material, specific heat capacity is the same for all.

$$V_1\theta_1 + 2\theta_2 = (V_1 + V_2)\theta_f$$

$$1\theta_1 + 1\theta_2 = (1 + 2)60$$

$$\theta_1 + 2\theta_2 = 180$$

$$0 \times \theta_1 + 1 \times \theta_2 + 2 \times \theta_3 = (1 + 2)30$$

$$\begin{aligned}\theta_2 + 2\theta_3 &= 180 \\ \theta_1 + 2\theta_2 + \theta_3 &= (1 + 1 + 1)\theta\end{aligned}$$

From equation. (1)+(2)+(3)

$$3\theta_1 + 3\theta_2 + 3\theta_3 = 450$$

Where,  $\theta_1 + \theta_2 + \theta_3 = 150$

From (4) equation  $150=30$

So,  $\theta = 50^\circ C$

24. An asteroid of mass  $m$  ( $m \ll m_E$ ) is approaching with a velocity  $12 \text{ km/s}$  when it is at distance of  $9R$  from the surface of earth (where  $R$  is radius of earth). When it reaches at the surface of Earth, its velocity is (Nearest Integer) in  $\text{km/s}$ .

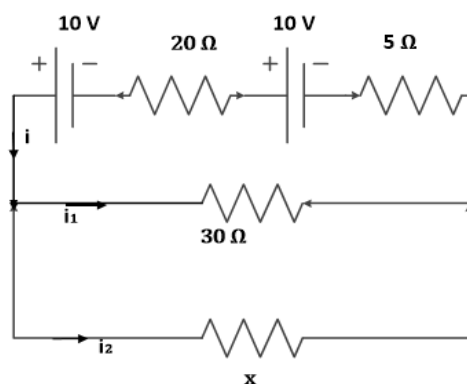
Solution: (16)

Taking, asteroid and earth as an isolated system conserving total energy.

$$\begin{aligned}KE_i + PE_i &= KE_f + PE_f \\ \frac{1}{2}mu_0^2 + \left(-\frac{GMm}{10R}\right) &= \frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right) \\ v^2 &= u_0^2 + \frac{2GM}{R}\left[1 - \frac{1}{10}\right] \\ v &= \sqrt{u_0^2 + \frac{9}{5}\frac{GM}{R}}\end{aligned}$$

$$\begin{aligned}\text{Since, escape velocity from surface of earth is } 11.2 \frac{\text{km}}{\text{sec}^2} &= \sqrt{\frac{2GM}{R}} \\ &= \sqrt{12^2 + \frac{9}{5}\frac{(11.2)^2}{2}} \\ &= \sqrt{256.9} \approx 16 \text{ km/s}.\end{aligned}$$

25. Two batteries (connected in series) of same emf  $10 \text{ V}$  of internal resistances  $20 \Omega$  and  $5 \Omega$  are connected to a load resistance of  $30 \Omega$ . Now an unknown resistance  $x$  is connected in parallel to the load resistance. Find value of  $x$  so that potential drop of battery having internal resistance  $20 \Omega$  becomes zero.



Solution: (30)

If  $V_1$  and  $V_2$  are terminal voltage across the two batteries.

$$V_1 = 0$$

$$V_1 = \varepsilon_1 - i \cdot r_1$$

$$0 = 10 - i \times 20$$

$$i = 0.5 \text{ A}$$

$$V_2 = 10 - 0.5 \times 5$$

$$V_2 = 7.5 \text{ V}$$

$$0.5 = \frac{7.5}{30} + \frac{7.5}{x}$$

$$\frac{7.5}{x} = 0.25$$

$$x = 30 \Omega$$

Date: 8<sup>th</sup> January 2020

Time: 02:30 PM – 05:30 PM

## JEE Main 2020 Paper

Subject: Chemistry

1. Correct bond energy order of the following is:

a.  $\text{C-Cl} > \text{C-Br} > \text{C-I} > \text{C-F}$

b.  $\text{C-F} < \text{C-Cl} < \text{C-Br} < \text{C-I}$

c.  $\text{C-F} > \text{C-Cl} > \text{C-Br} > \text{C-I}$

d.  $\text{C-I} < \text{C-Br} < \text{C-F} < \text{C-Cl}$

**Answer:** c

**Solution:** In C – F there is 2p-2p overlapping involved, in C – Cl the overlapping involved is 2p-3p whereas for C – Br and C – I the overlappings involved are 2p-4p and 2p-5p, respectively. The bond length for the various type of overlappings can be given as:

$$2p-2p < 2p-3p < 2p-4p < 2p-5p.$$

As we know that Bond energy  $\propto \frac{1}{\text{Bond length}}$

The order of bond energy comes out:  $\text{C-F} > \text{C-Cl} > \text{C-Br} > \text{C-I}$

2. Determine Bohr's radius of  $\text{Li}^{2+}$  ion for  $n = 2$ . Given (Bohr's radius of H-atom =  $a_0$ )

a.  $\frac{3a_0}{4}$

b.  $\frac{4a_0}{3}$

c.  $\frac{a_0}{3}$

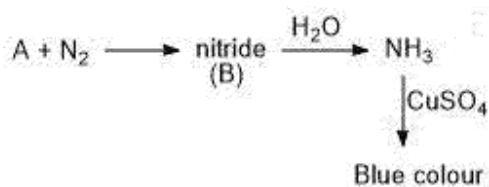
d.  $\frac{16a_0}{9}$

**Answer:** b

**Solution:** The formula for Bohr's radius for any unielelectronic species is:  $r = \frac{a_0 n^2}{z}$

$$\text{for } \text{Li}^{2+} : r = \frac{a_0 2^2}{3} = \frac{4a_0}{3}$$

3. Given the following reaction sequence:

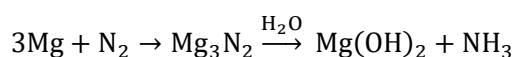


A and B are respectively;

- |                     |                 |
|---------------------|-----------------|
| a. Mg, $Mg_3N_2$    | b. Na, $Na_3N$  |
| c. Mg, $Mg(NO_3)_2$ | d. Na, $NaNO_3$ |

**Answer:** a

**Solution:** As it is provided in the question that nitride is being formed so the option c and d can be eliminated. Amongst Mg and Na we already know that Mg can only form nitride so the correct choice is option a.



4. Correct order of the magnetic moment (spin only) for the following complexes is:

- |                        |                        |
|------------------------|------------------------|
| A. $[Pd(PPh_3)_2Cl_2]$ | B. $[Ni(CO)_4]$        |
| C. $[Ni(CN)_4]^{2-}$   | D. $[Ni(H_2O)_6]^{2+}$ |

- |              |              |
|--------------|--------------|
| a. $A=B<C<D$ | b. $A<B<C<D$ |
| c. $A>B>C>D$ | d. $A=B>C>D$ |

**Answer:** a

**Solution:**  $[Pd(PPh_3)_2Cl_2]$  : Here Pd is in +2 oxidation state and configuration of  $Pd^{2+}$  is  $[Kr]4d^8$ . As the CFSE value for Pd is very high so all the electrons will be paired and hence magnetic moment for this complex will be zero.

$[Ni(CO)_4]$  : Here Ni is in 0 oxidation state and configuration of Ni is  $[Ar]3d^84s^2$ . As here the ligand is carbonyl which is a strong field ligand, all the electrons will be paired and hence magnetic moment for this complex will be zero.

$[Ni(CN)_4]^{2-}$  : Here Ni is in +2 oxidation state and configuration of  $Ni^{2+}$  is  $[Ar]3d^8$ . As here the ligand is cyanide which is a strong field ligand, all the electrons will be paired and hence magnetic moment for this complex will be zero.

$[Ni(H_2O)_6]^{2+}$  : Here Ni is in +2 oxidation state and configuration of  $Ni^{2+}$  is  $[Ar]3d^8$ . As here the ligand is water which is a weak field ligand, the electrons will not be paired and there are two unpaired electrons in this complex hence magnetic moment for this complex will be  $\sqrt{8}$  BM.

So the order of magnetic moment is  $A=B<C<D$ .

5. Determine the total number of neutrons in three isotopes of hydrogen.

- |      |      |
|------|------|
| a. 1 | b. 2 |
| c. 3 | d. 4 |

**Answer:** c

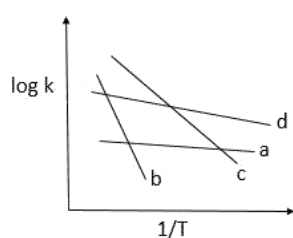
**Solution:** Number of neutrons in protium = 0

Number of neutrons in deuterium = 1

Number of neutrons in tritium = 2

So, total number of neutrons = 3

6.



Compare  $E_a$  (activation energy) for a, b, c and d.

- |                            |                            |
|----------------------------|----------------------------|
| a. $E_b > E_c > E_d > E_a$ | b. $E_a > E_d > E_c > E_b$ |
| c. $E_c > E_b > E_a > E_d$ | d. $E_d > E_a > E_b > E_c$ |

**Answer:** a

**Solution:**

To avoid confusion, in this question we'll be denoting activation energy by  $E_x$

$$K = Ae^{-E_x/RT}$$

$$\log K = \log A - \frac{E_x}{2.303RT} \quad \text{-----(1)}$$

Here, the graph given in the question is of a straight line and we know that the equation of straight line is

$$y = mx + c \quad \text{-----(2)}$$

Comparing equation 1 with 2 we get,

$$\text{Slope} = \frac{-E_x}{2.303R}$$

So, from the graph we can conclude that the line with the most negative slope will have the maximum activation energy value.

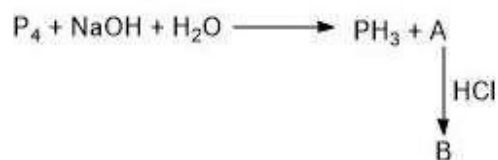
$$E_b > E_c > E_d > E_a$$

7. Which of the following exhibit both Frenkel and Schottky defect?
- AgBr
  - KCl
  - CsCl
  - ZnS

**Answer:** a

**Solution:** The radius ratio for AgBr is intermediate. Thus, it shows both Frenkel and Schottky defects.

8. Given:



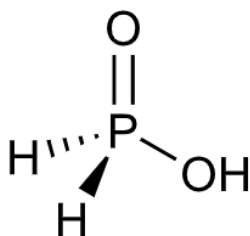
Basicity of B is:

- 1
- 2
- 3
- 4

**Answer:** a

**Solution:**  $\text{P}_4 + \text{NaOH} + \text{H}_2\text{O} \rightarrow \text{PH}_3 + \text{NaH}_2\text{PO}_2 \xrightarrow{\text{HCl}} \text{H}_3\text{PO}_2 + \text{NaCl}$

Here the product B which is mentioned in the question is  $\text{H}_3\text{PO}_2$ . The structure of  $\text{H}_3\text{PO}_2$  can be given as:



As only 1 Hydrogen atom is attached to the oxygen, its basicity is one.



12. **Assertion:** pH of water increases on increasing temperature.

**Reason:**  $\text{H}_2\text{O} \rightarrow \text{H}^+ + \text{OH}^-$  is an exothermic process

- a. Both assertion and reason are correct and reason is correct explanation of assertion.
- b. Both assertion and reason are correct and reason is not correct explanation of assertion.
- c. Assertion is true and reason is false.
- d. Both assertion and reason are incorrect.

**Answer:** d

**Solution:**  $\text{H}_2\text{O} \rightarrow \text{H}^+ + \text{OH}^-$  is an endothermic process. On increasing the temperature the value of  $K_w$  increases which will result in decrease in  $\text{p}K_w$ . So we can say that pH of water will decrease on increasing temperature because  $\text{pH for water} = \frac{1}{2} \text{p}K_w$ .

13. **Assertion:** It has been found that for hydrogenation reaction the catalytic activity increases from group-5 to group-11 metals with maximum activity being shown by group 7-9 elements of the periodic table.

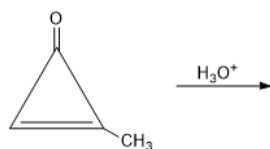
**Reason:** For 7-9 group elements adsorption rate is maximum.

- a. Both assertion and reason are correct and reason is correct explanation of assertion.
- b. Both assertion and reason are correct and reason is not correct explanation of assertion.
- c. Assertion is true & reason is false.
- d. Both are incorrect

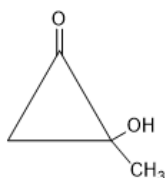
**Answer:** a

**Solution:** Group 7-9 elements of the periodic table show variable valencies so they have maximum activity because of the increase in adsorption rate.

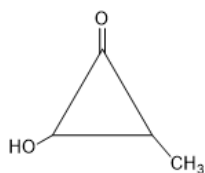
14. The major product of the following reactions is



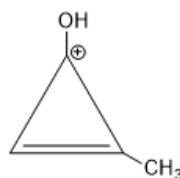
a.



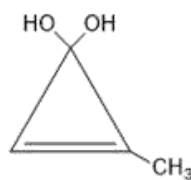
c.



b.

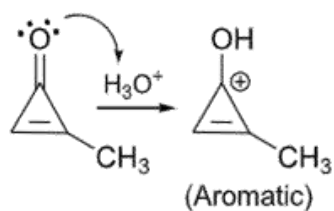


d.

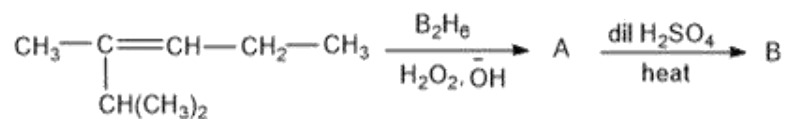


**Answer:** b

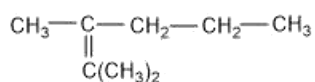
**Solution:**



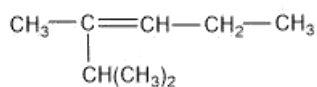
15. Find the final major product of the following reactions?



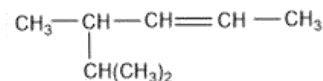
a.



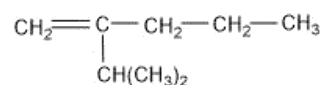
c.



b.

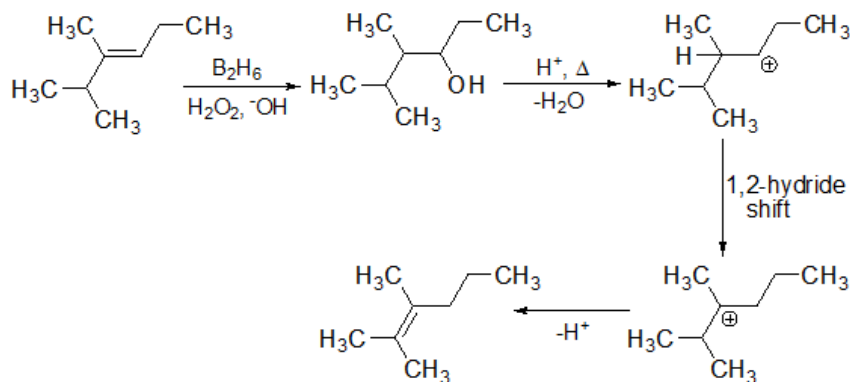


d.



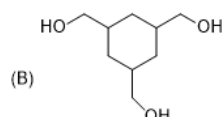
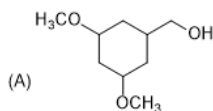
**Answer: a**

**Solution:**

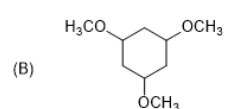
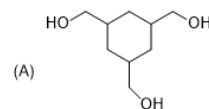


16. There are two compounds A and B of molecular formula  $C_9H_{18}O_3$ . A has higher boiling point than B. What are the possible structures of A and B?

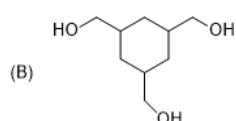
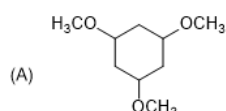
a.



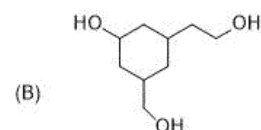
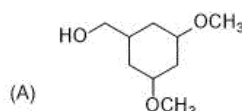
b.



c.



d.

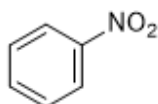


**Answer:** b

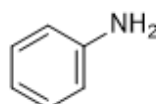
**Solution:** In option b compound A has extensive inter-molecular hydrogen bonding because of the 3  $-OH$  groups while in compound B there are  $-OCH_3$  groups present and no inter-molecular hydrogen bonding is possible.

17. Kjeldahl method cannot be used for :

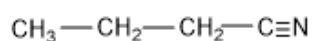
a.



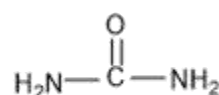
b.



c.



d.

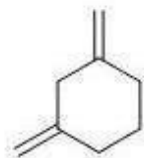


**Answer:** a

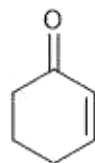
**Solution:** Kjeldahl method cannot be used for the estimation of nitrogen in the compounds in which nitrogen is involved in nitro, diazo groups or is present in the ring, as nitrogen atom can't be converted to ammonium sulphate under the reaction conditions.

18. A compound X adds 2 hydrogen molecules on hydrogenation. The compound X also gives 3-oxohexanedioic acid on oxidative ozonolysis. The compound 'X' is:

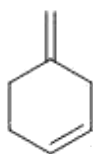
a.



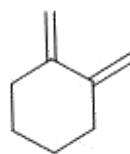
b.



c.

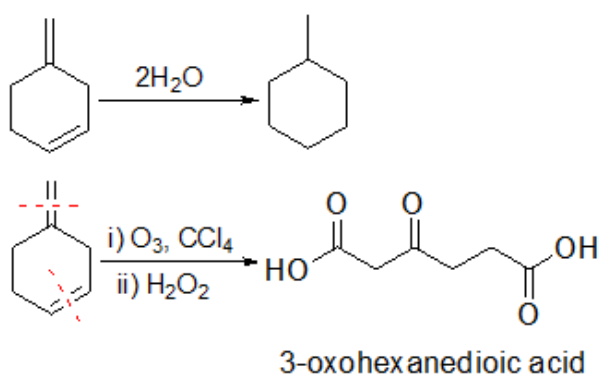


d.



**Answer: c**

**Solution:**

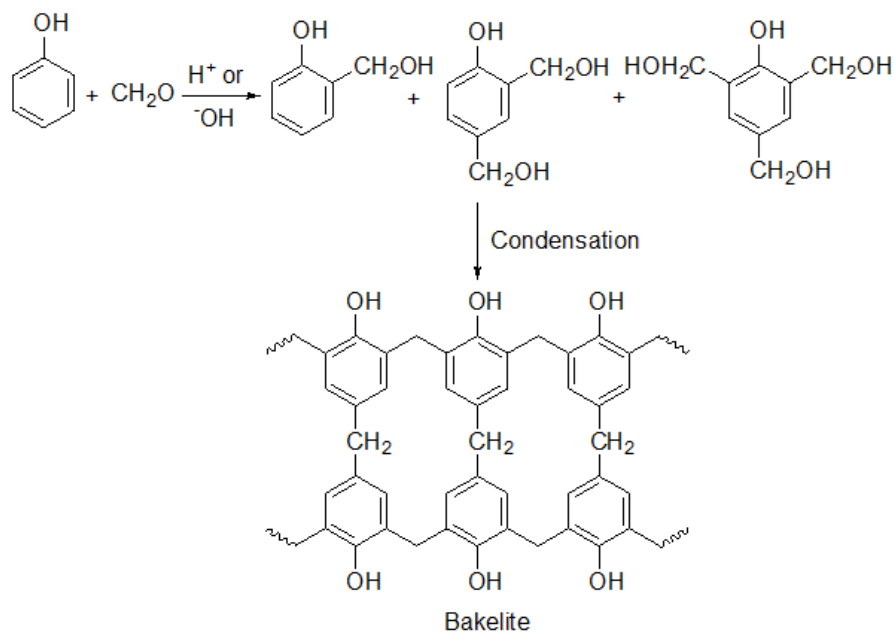


19. Formation of Bakelite follows :

- Electrophilic substitution followed by condensation.
- Nucleophilic addition followed by dehydration.
- Electrophilic addition followed by dehydration.
- Hydration followed by condensation.

**Answer:** a

**Solution:** Bakelite is a condensation polymer of phenol and formaldehyde.

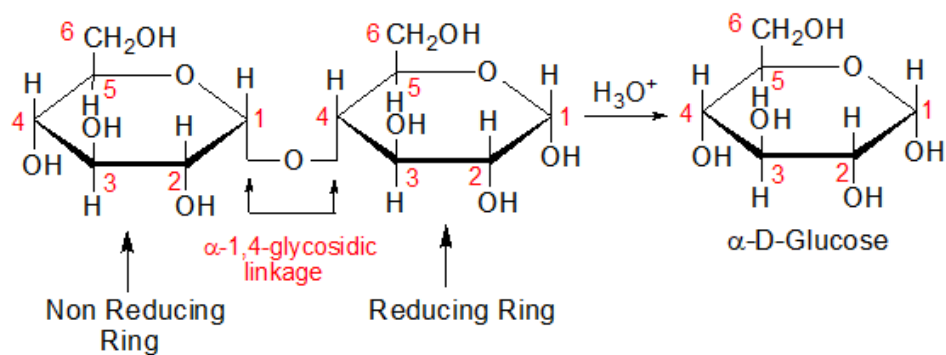


20. Products formed by hydrolysis of maltose are:

- a.  $\alpha$ -D-Glucose,  $\alpha$ -D-Glucose
- b.  $\alpha$ -D-Glucose,  $\beta$ -D-Glucose
- c.  $\alpha$ -D-Galactose,  $\beta$ -D-Glucose
- d.  $\beta$ -D-Galactose,  $\alpha$ -D-Glucose

**Answer:** a

**Solution:**



Maltose is formed by the glycosidic linkage between C-1 of one  $\alpha$ -D-Glucose unit to the C-4 of another  $\alpha$ -D-Glucose.

21. Temperature of 4 moles of gas increases from 300 K to 500 K find ' $C_v$ ', if  $\Delta U = 5000$  J.

**Answer:** 6.25

**Solution:**

$$\Delta U = nC_v\Delta T$$

$$5000 = 4 \times C_v (500 - 300)$$

$$C_v = 6.25 \text{ J K}^{-1} \text{ mol}^{-1}$$

22. Given :  $E_{\text{Sn}^{2+}/\text{Sn}}^0 = -0.14 \text{ V}$  ;  $E_{\text{Pb}^{2+}/\text{Pb}}^0 = -0.13 \text{ V}$ . Determine  $\frac{[\text{Sn}^{2+}]}{[\text{Pb}^{2+}]}$  at equilibrium

For the cell reaction  $\text{Sn}|\text{Sn}^{2+}||\text{Pb}^{2+}|\text{Pb}$

Take  $\frac{2.303RT}{F} = 0.06 \text{ V}$ , and  $\sqrt[3]{10} = 2.154$

**Answer:** 2.15

**Solution:**

Anodic half:  $\text{Sn} \rightarrow \text{Sn}^{2+} + 2e^-$

Cathodic half:  $\text{Pb}^{2+} + 2e^- \rightarrow \text{Pb}$

Net reaction:  $\text{Sn} + \text{Pb}^{2+} \rightarrow \text{Pb} + \text{Sn}^{2+}$

$$E_{\text{cell}}^0 = E_{\text{cathode}}^0 - E_{\text{anode}}^0$$

$$E_{\text{cell}}^0 = 0.01 \text{ V}$$

$$E_{\text{cell}} = E_{\text{cell}}^0 - \frac{0.06}{2} \log Q$$

At equilibrium state  $E_{\text{cell}} = 0$

So,

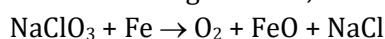
$$0 = 0.01 - \frac{0.06}{2} \log \frac{[\text{Sn}^{2+}]}{[\text{Pb}^{2+}]}$$

$$0.01 = \frac{0.06}{2} \log \frac{[\text{Sn}^{2+}]}{[\text{Pb}^{2+}]}$$

$$\log \frac{[\text{Sn}^{2+}]}{[\text{Pb}^{2+}]} = \frac{1}{3}$$

$$\frac{[\text{Sn}^{2+}]}{[\text{Pb}^{2+}]} = 10^{\frac{1}{3}} = 2.154$$

23. Given following reaction,



In the above reaction 492 L of  $\text{O}_2$  is obtained at 1 atm & 300 K temperature. Determine mass of  $\text{NaClO}_3$  required (in kg). ( $R = 0.082 \text{ L atm mol}^{-1} \text{ K}^{-1}$ )

**Answer:** 2.13

**Solution:**

Mol of  $\text{NaClO}_3 = \text{mol of } \text{O}_2$

$$\text{Mol of } \text{O}_2 = \frac{PV}{RT} = \frac{1 \times 492}{0.082 \times 300} = 20 \text{ mol}$$

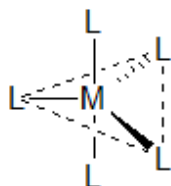
Molar mass of  $\text{NaClO}_3$  is 106.5

So, mass =  $20 \times 106.5 = 2130 \text{ g} = 2.13 \text{ Kg}$

24. Complex  $[ML_5]$  can exhibit trigonal bipyramidal and square pyramidal geometry. Determine total number of  $180^\circ$ ,  $90^\circ$  &  $120^\circ$  L-M-L bond angles

**Answer:** 20

**Solution:**



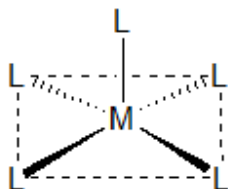
For trigonal bipyramidal geometry

Total number of  $180^\circ$  L-M-L bond angles = 1

Total number of  $90^\circ$  L-M-L bond angles = 6

Total number of  $120^\circ$  L-M-L bond angles = 3

Total = 10



For square pyramidal geometry

Total number of  $180^\circ$  L-M-L bond angles = 2

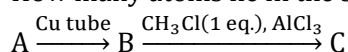
Total number of  $90^\circ$  L-M-L bond angles = 8

Total number of  $120^\circ$  L-M-L bond angles = 0

Total = 10

Total for both the structures = 20

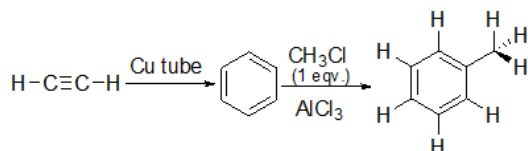
25. How many atoms lie in the same plane in the major product (C)?



(Where A is the alkyne of lowest molecular mass).

**Answer:** 13

**Solution:**



Date of Exam: **9<sup>th</sup> January 2020 (Shift 1)**

Time: **9:30 A.M. to 12:30 P.M.**

Subject: **Mathematics**

1. A sphere of 10 cm radius has a uniform thickness of ice around it. If the ice is melting at the rate of  $50 \text{ cm}^3/\text{min}$  when thickness is 5 cm, then the rate of change of thickness is

a.  $\frac{1}{12\pi}$   
c.  $\frac{1}{9\pi}$

b.  $\frac{1}{18\pi}$   
d.  $\frac{1}{36\pi}$

**Answer:** (b)

**Solution:**

Let thickness of ice be  $x$  cm.

Therefore, net radius of sphere =  $(10 + x)$  cm

$$\text{Volume of sphere } V = \frac{4}{3}\pi(10 + x)^3$$

$$\Rightarrow \frac{dV}{dt} = 4\pi(10 + x)^2 \frac{dx}{dt}$$

$$\text{At } x = 5, \frac{dV}{dt} = 50 \text{ cm}^3/\text{min}$$

$$\Rightarrow 50 = 4\pi \times 225 \times \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{1}{18\pi} \text{ cm/min}$$

2. The number of real roots of  $e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$  is

a. 1  
c. 3

b. 2  
d. 4

**Answer:** (a)

**Solution:**

$$e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$$

$$\Rightarrow e^{2x} + e^x - 4 + \frac{1}{e^x} + \frac{1}{e^{2x}} = 0$$

$$\Rightarrow \left(e^{2x} + \frac{1}{e^{2x}}\right) + \left(e^x + \frac{1}{e^x}\right) - 4 = 0$$

$$\Rightarrow \left(e^x + \frac{1}{e^x}\right)^2 - 2 + \left(e^x + \frac{1}{e^x}\right) - 4 = 0$$

$$\text{Let } e^x + \frac{1}{e^x} = u$$

$$\text{Then, } u^2 + u - 6 = 0$$

$$\Rightarrow u = 2, -3$$

$$u \neq -3 \text{ as } u > 0 (\because e^x > 0)$$

$$\Rightarrow e^x + \frac{1}{e^x} = 2 \Rightarrow (e^x - 1)^2 = 0 \Rightarrow e^x = 1 \Rightarrow x = 0$$

Hence, only one real solution is possible.

3. If  $f'(x) = \tan^{-1}(\sec x + \tan x)$ ,  $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$  and  $f(0) = 0$ , then the value of  $f(1)$  is

a.  $\frac{\pi-1}{4}$

b.  $\frac{\pi+1}{4}$

c.  $\frac{\pi+1}{2}$

d. 0

**Answer:** (b)

**Solution:**

$$f'(x) = \tan^{-1}(\sec x + \tan x) = \tan^{-1}\left(\frac{1 + \sin x}{\cos x}\right)$$

$$f'(x) = \tan^{-1}\left(\frac{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2}}{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}\right)$$

$$f'(x) = \tan^{-1}\left[\frac{\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)^2}{\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)\left(\cos \frac{x}{2} - \sin \frac{x}{2}\right)}\right]$$

$$f'(x) = \tan^{-1}\left[\frac{1 + \tan \frac{x}{2}}{1 - \tan \frac{x}{2}}\right]$$

$$f'(x) = \tan^{-1}\left[\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right]$$

$$f'(x) = \frac{\pi}{4} + \frac{x}{2}$$

$$\Rightarrow f(x) = \frac{\pi}{4}x + \frac{x^2}{4} + c$$

$$f(0) = 0 \Rightarrow c = 0$$

$$f(1) = \frac{\pi}{4} + \frac{1}{4} = \frac{\pi+1}{4}$$

4. The number of solutions of  $\log_{\frac{1}{2}} |\sin x| = 2 - \log_{\frac{1}{2}} |\cos x|$ ,  $x \in [0, 2\pi]$  is

a. 2

b. 4

c. 8

d. 6

**Answer:** (c)



6. The value of  $\int \frac{dx}{(x-3)^{\frac{6}{7}} \times (x+4)^{\frac{8}{7}}}$  is -

a.  $7 \left( \frac{x-3}{x+4} \right)^{\frac{1}{7}} + c$

b.  $7 \left( \frac{x-3}{x+4} \right)^{\frac{6}{7}} + c$

c.  $\left( \frac{x-3}{x+4} \right)^{\frac{1}{7}} + c$

d.  $7 \left( \frac{x+4}{x-3} \right)^{\frac{6}{7}} + c$

**Answer:** (c)

**Solution:**

$$I = \int \frac{dx}{(x-3)^{\frac{6}{7}} \times (x+4)^{\frac{8}{7}}}$$

$$\Rightarrow I = \int \frac{(x+4)^{\frac{6}{7}} dx}{(x-3)^{\frac{6}{7}} \times (x+4)^2} = \int \left( \frac{x-3}{x+4} \right)^{-\frac{6}{7}} \times \frac{dx}{(x+4)^2}$$

$$\text{Put } \frac{x-3}{x+4} = t \Rightarrow dt = 7 \left( \frac{1}{(x+4)^2} \right) dx$$

$$\Rightarrow I = \frac{\int t^{-\frac{6}{7}} dt}{7} = t^{\frac{1}{7}} + c = \left( \frac{x-3}{x+4} \right)^{\frac{1}{7}} + c$$

7. If  $\left| \frac{z-i}{z+2i} \right| = 1$ ,  $|z| = \frac{5}{2}$  then the value of  $|z + 3i|$  is

a.  $\sqrt{10}$

b.  $\sqrt{5}$

c.  $\frac{7}{2}$

d.  $\sqrt{3}$

**Answer:** (c)

**Solution:**

$$\text{If } \left| \frac{z-i}{z+2i} \right| = 1 \text{ \& } |z| = \frac{5}{2}$$

$$\Rightarrow |z - i| = |z + 2i|$$

$$\Rightarrow x^2 + (y - 1)^2 = x^2 + (y + 2)^2$$

$$\Rightarrow y - 1 = \pm(y + 2)$$

$$\Rightarrow y - 1 = -y - 2$$

$$\Rightarrow y = -\frac{1}{2}$$

$$\Rightarrow |z| = \frac{5}{2} \Rightarrow x^2 + y^2 = \frac{25}{4}$$

$$\Rightarrow x^2 + \frac{1}{4} = \frac{25}{4}$$

$$\Rightarrow x = \pm\sqrt{6}$$

$$|z + 3i| = \sqrt{x^2 + (y + 3)^2}$$

$$\Rightarrow |z + 3i| = \frac{7}{2}$$

8. The value of  $2^{\frac{1}{4}} \times 4^{\frac{1}{16}} \times 8^{\frac{1}{48}} \dots \infty$  is

a. 2

b. 1

c.  $\sqrt{2}$

d.  $2^{\frac{1}{4}}$

**Answer:** (c)

**Solution:**

$$2^{\frac{1}{4}} \times 4^{\frac{1}{16}} \times 8^{\frac{1}{48}} \dots \infty = 2^{\frac{1}{4}} \times 2^{\frac{2}{16}} \times 2^{\frac{4}{48}} \dots \infty$$

$$\Rightarrow 2^{\frac{1}{4}} \times 2^{\frac{1}{8}} \times 2^{\frac{1}{16}} \dots \infty = 2^{\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots \infty}$$

$$\Rightarrow 2^{\left(\frac{\frac{1}{4}}{1 - \frac{1}{2}}\right)} = \sqrt{2}$$

9. The value of  $\cos^3 \frac{\pi}{8} \cos \frac{3\pi}{8} + \sin^3 \frac{\pi}{8} \sin \frac{3\pi}{8}$  is -

a.  $\frac{1}{2}$

b.  $-\frac{1}{2}$

c.  $\frac{1}{\sqrt{2}}$

d.  $\frac{1}{2\sqrt{2}}$

**Answer:** (d)

**Solution:**

$$\cos^3 \frac{\pi}{8} \cos \frac{3\pi}{8} + \sin^3 \frac{\pi}{8} \sin \frac{3\pi}{8} = \cos^3 \frac{\pi}{8} \left[ 4 \cos^3 \frac{\pi}{8} - 3 \cos \frac{\pi}{8} \right] + \sin^3 \frac{\pi}{8} \left[ 3 \sin \frac{\pi}{8} - 4 \sin^3 \frac{\pi}{8} \right]$$

$$= 4 \left[ \cos^6 \frac{\pi}{8} - \sin^6 \frac{\pi}{8} \right] + 3 \left[ \sin^4 \frac{\pi}{8} - \cos^4 \frac{\pi}{8} \right]$$

$$= 4 \left[ \cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8} \right] \left[ \cos^4 \frac{\pi}{8} + \sin^4 \frac{\pi}{8} + \cos^2 \frac{\pi}{8} \sin^2 \frac{\pi}{8} \right] - 3 \left[ \cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8} \right]$$

$$= \left[ \cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8} \right] \left[ 4 \left( 1 - \cos^2 \frac{\pi}{8} \sin^2 \frac{\pi}{8} \right) - 3 \right]$$

$$= \cos \frac{\pi}{4} \left[ 1 - \sin^2 \frac{\pi}{4} \right] = \frac{1}{2\sqrt{2}}$$

10. The value of  $\int_0^{2\pi} \frac{x \sin^8 x}{\sin^8 x + \cos^8 x} dx$  is

a.  $4\pi^2$

b.  $2\pi^2$

c.  $\pi^2$

d.  $3\pi^2$

**Answer:** (c)

**Solution:**

$$\text{Let } I = \int_0^{2\pi} \frac{x \sin^8 x}{\sin^8 x + \cos^8 x} dx \quad \dots (1)$$

$$\begin{aligned} I &= \int_0^{2\pi} \frac{(2\pi-x) \sin^8(2\pi-x)}{\sin^8(2\pi-x) + \cos^8(2\pi-x)} dx \\ &= \int_0^{2\pi} \frac{(2\pi-x) \sin^8 x}{\sin^8 x + \cos^8 x} dx \quad \dots (2) \end{aligned}$$

Adding (1) & (2), we get:

$$\begin{aligned} \Rightarrow 2I &= 2\pi \int_0^{2\pi} \frac{\sin^8 x}{\sin^8 x + \cos^8 x} dx \\ I &= \pi \int_0^{2\pi} \frac{\sin^8 x}{\sin^8 x + \cos^8 x} dx \\ I &= 4\pi \int_0^{\frac{\pi}{2}} \frac{\sin^8 x}{\sin^8 x + \cos^8 x} dx \quad \dots (3) \end{aligned}$$

$$I = 4\pi \int_0^{\frac{\pi}{2}} \frac{\sin^8(\frac{\pi}{2}-x)}{\sin^8(\frac{\pi}{2}-x) + \cos^8(\frac{\pi}{2}-x)} dx = 4\pi \int_0^{\frac{\pi}{2}} \frac{\cos^8 x}{\sin^8 x + \cos^8 x} dx \quad \dots (4)$$

Adding (3) & (4), we get -

$$I = 2\pi \int_0^{\frac{\pi}{2}} 1 dx = 2\pi \times \frac{\pi}{2} = \pi^2$$

11. If  $f(x) = a + bx + cx^2$  where  $a, b, c \in \mathbf{R}$  then the value of  $\int_0^1 f(x) dx$  is -

- |  |  |
|--|--|
| a. $\frac{1}{6} \left( f(1) + f(0) - 4f\left(\frac{1}{2}\right) \right)$ | b. $\frac{1}{3} \left( f(1) + f(0) + 2f\left(\frac{1}{2}\right) \right)$ |
| c. $\frac{1}{6} \left( f(1) + f(0) + 4f\left(\frac{1}{2}\right) \right)$ | d. $\frac{1}{6} \left( f(1) - f(0) - 4f\left(\frac{1}{2}\right) \right)$ |

**Answer: (c)**

**Solution:**

$$f(x) = a + bx + cx^2$$

$$f(0) = a, f(1) = a + b + c$$

$$f\left(\frac{1}{2}\right) = \frac{c}{4} + \frac{b}{2} + a$$

$$\int_0^1 f(x) dx = \int_0^1 (a + bx + cx^2) dx = a + \frac{b}{2} + \frac{c}{3}$$

$$= \frac{1}{6} (6a + 3b + 2c) = \frac{1}{6} (a + (a + b + c) + (4a + 2b + c))$$



14. If  $f(x)$  is twice differentiable and continuous function in  $x \in [a, b]$ . Also  $f'(x) > 0$  and  $f''(x) < 0$  and  $c \in (a, b)$ , then  $\frac{f(c)-f(a)}{f(b)-f(c)}$  is greater than

- a. 1  
b.  $\frac{a+b}{b-c}$   
c.  $\frac{b-c}{c-a}$   
d.  $\frac{c-a}{b-c}$

**Answer:** (d)

**Solution:**

$\therefore c \in (a, b)$  and  $f$  is twice differentiable and continuous function  $(a, b)$

$\therefore$  LMVT is applicable

$$\text{For } p \in (a, c), \quad f'(p) = \frac{f(c)-f(a)}{c-a}$$

$$\text{For } q \in (c, b), \quad f'(q) = \frac{f(b)-f(c)}{b-c}$$

$\therefore f''(x) < 0 \Rightarrow f'(x)$  is decreasing

$$f'(p) > f'(q)$$

$$\Rightarrow \frac{f(c)-f(a)}{c-a} > \frac{f(b)-f(c)}{b-c}$$

$$\Rightarrow \frac{f(c)-f(a)}{f(b)-f(c)} > \frac{c-a}{b-c} \quad (\text{as } f'(x) > 0 \Rightarrow f(x) \text{ is increasing})$$

15. If three planes

$$x + 4y - 2z = 1$$

$$x + 7y - 5z = \beta$$

$$x + 5y + \alpha z = 5$$

intersect in a line, then  $\alpha + \beta =$

- a. -10  
b. 0  
c. 2  
d. 10

**Answer:** (d)

**Solution:**

The given planes intersect in a line

$$\therefore D = D_x = D_y = D_z = 0$$

$$D = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 4 & -2 \\ 1 & 7 & -5 \\ 1 & 5 & \alpha \end{vmatrix} = 0 \Rightarrow 7\alpha + 25 - 4\alpha - 20 + 4 = 0$$

$$\Rightarrow \alpha = -3$$

$$D_z = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 4 & 1 \\ 1 & 7 & \beta \\ 1 & 5 & 5 \end{vmatrix} = 0 \Rightarrow 35 - 5\beta - 20 + 4\beta - 2 = 0$$

$$\Rightarrow \beta = 13$$

$$\therefore \alpha + \beta = 10$$

16.  $\sum_{i=1}^{10}(x_i - 5) = 10$  and  $\sum_{i=1}^{10}(x_i - 5)^2 = 40$ . If mean and variance of observations  $(x_1 - 3), (x_2 - 3) \dots (x_{10} - 3)$  is  $\lambda$  and  $\mu$  respectively, then ordered pair  $(\lambda, \mu)$  is

- a. (1,1) b. (1,3)  
c. (3,1) d. (3,3)

**Answer:** (d)

**Solution:**

$$\sum_{i=1}^{10}(x_i - 5) = 10 \Rightarrow \sum_{i=1}^{10} x_i - 50 = 10$$

$$\Rightarrow \sum_{i=1}^{10} x_i = 60$$

$$\lambda = \frac{\sum_{i=1}^{10}(x_i - 3)}{10} = \frac{\sum_{i=1}^{10} x_i - 30}{10} = 3$$

Variance is unchanged, if a constant is added or subtracted from each observation

$$\begin{aligned} \mu &= Var(x_i - 3) = Var(x_i - 5) = \frac{\sum_{i=1}^{10}(x_i - 5)^2}{10} - \left( \frac{\sum (x_i - 5)}{10} \right)^2 \\ &= \frac{40}{10} - \left( \frac{10}{10} \right)^2 = 3 \end{aligned}$$

17. 20 cards are placed in a bag with 10 named as A and another 10 named as B. If cards are drawn one by one (with replacement), then the probability that second A comes before third B is

- a.  $\frac{11}{16}$  b.  $\frac{7}{16}$   
c.  $\frac{9}{16}$  d.  $\frac{13}{16}$

**Answer:** (a)

**Solution:**

Here  $P(A) = P(B) = \frac{1}{2}$

Then, these following cases are possible  $\rightarrow AA, BAA, ABA, ABBA, BBAA, BABA$

So, the required probability is  $= \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{11}{16}$

18. The negation of ' $\sqrt{5}$  is an integer or 5 is an irrational number' is

- $\sqrt{5}$  is an integer and 5 is not an irrational number.
- $\sqrt{5}$  is not an integer and 5 is not an irrational number.
- $\sqrt{5}$  is not an integer or 5 is not an irrational number.
- $\sqrt{5}$  is not an integer and 5 is an irrational number.

**Answer:** (b)

**Solution:**

$p$ :  $\sqrt{5}$  is an integer

$q$ : 5 is an irrational number

Given statement :  $p \vee q$

Required negation statement:  $\sim(p \vee q) = \sim p \wedge \sim q$

' $\sqrt{5}$  is not an integer and 5 is not an irrational number'

19. If  $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$ ,  $B = \text{adj}(A)$  and  $C = 3A$ , then  $\frac{|\text{adj } B|}{|C|}$  is
- 2
  - 4
  - 8
  - 16

**Answer:** (c)

**Solution:**

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{vmatrix} = 13 + 1 - 8 = 6$$

$$B = \text{adj}(A) \Rightarrow |\text{adj } B| = |\text{adj}(\text{adj } A)| = |A|^4 = 6^4$$

$$|C| = |3A| = 3^3 |A| = 3^3 \times 6$$

$$\frac{|\text{adj } B|}{|C|} = \frac{6^4}{3^3 \times 6} = \frac{2^3 \times 3^3}{3^3} = 8$$

20. If a circle touches y-axis at (0,4) and passes through (2,0), then which of the following can be the tangent to the circle?

a.  $3x + 4y - 24 = 0$

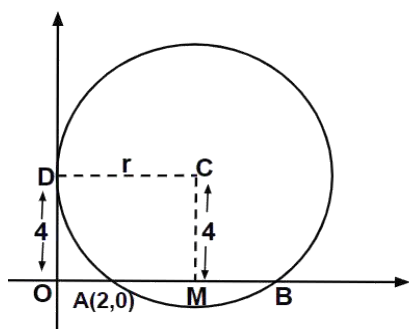
c.  $4x + 3y - 6 = 0$

b.  $4x - 3y - 17 = 0$

d.  $3x + 4y - 6 = 0$

**Answer:** (d)

**Solution:**



$$OD^2 = OA \times OB \Rightarrow 16 = 2 \times OB \Rightarrow OB = 8$$

$$\therefore AB = 6$$

$$\therefore AM = 3, CM = 4 \Rightarrow CA = 5$$

$$\therefore OM = 5$$

Centre will be (5,4) and radius is 5

Now checking option (d)

$$3x + 4y - 6 = 0$$

$$\frac{15 + 16 - 6}{\sqrt{3^2 + 4^2}} = 5 \quad (p = r)$$

21.  $(1+x) \frac{dy}{dx} = [(1+x)^2 + (y-3)]$ . If  $y(2) = 0$ , then the value of  $y(3)$  is

**Answer:** (3)

**Solution:**

$$(1+x) \frac{dy}{dx} = [(1+x)^2 + (y-3)]$$

$$\Rightarrow (1+x) \frac{dy}{dx} - y = (1+x)^2 - 3$$

$$\Rightarrow \frac{dy}{dx} - \frac{1}{(1+x)} y = 1+x - \frac{3}{1+x}$$

$$\text{I. F.} = e^{-\int \frac{1}{1+x} dx} = \frac{1}{1+x}$$

$$y \times \frac{1}{1+x} = \int 1 - \frac{3}{(1+x)^2} dx$$

$$\frac{y}{1+x} = x + \frac{3}{1+x} + c$$

$$\Rightarrow y = x(1+x) + 3 + c(1+x)$$

At  $x = 2, y = 0$ , we get

$$0 = 6 + 3 + 3c$$

$$\Rightarrow c = -3$$

$$\Rightarrow \text{At } x = 3,$$

$$y = x^2 - 2x = 9 - 6 = 3$$

$$\Rightarrow y(3) = 3$$

22. Function  $f(x) = \begin{cases} \frac{\sin(a+2)x + \sin x}{x}, & x < 0 \\ b, & x = 0 \\ \frac{(x+3x^2)^{\frac{1}{3}} - x^{\frac{1}{3}}}{x^{\frac{4}{3}}}, & x > 0 \end{cases}$  is continuous at  $x = 0$ . The value of  $a + 2b$  is

**Answer:** (0)

**Solution:**

$f(x)$  is continuous at  $x = 0$

$$\therefore \lim_{x \rightarrow 0^-} f(x) = b = \lim_{x \rightarrow 0^+} f(x)$$

$$b = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} \frac{(h+3h^2)^{\frac{1}{3}} - h^{\frac{1}{3}}}{h^{\frac{4}{3}}}$$

$$\Rightarrow b = \lim_{h \rightarrow 0} \frac{h^{\frac{1}{3}} \left[ (1+3h)^{\frac{1}{3}} - 1 \right]}{h^{\frac{4}{3}}}$$

$$\Rightarrow b = \lim_{h \rightarrow 0} \frac{(1+3h)^{\frac{1}{3}} - 1}{h}$$

$$\Rightarrow b = \lim_{h \rightarrow 0} \frac{1}{3} (1+3h)^{-\frac{2}{3}} \times 3$$

$$\text{or, } b = 1$$

$$\lim_{x \rightarrow 0^-} f(x) = 1 \Rightarrow \lim_{h \rightarrow 0} \frac{\sin(a+2)(-h) + \sin(-h)}{-h} = 1$$

$$\Rightarrow a + 3 = 1 \Rightarrow a = -2$$

$$\Rightarrow a + 2b = 0$$

23. The coefficient of  $x^4$  in  $(1 + x + x^2)^{10}$  is

**Answer:** (615)

**Solution:**

General term of the given expression is given by  $\frac{10!}{p!q!r!} x^{q+2r}$

Here,  $q + 2r = 4$

$$\text{For } p = 6, q = 4, r = 0, \text{ coefficient} = \frac{10!}{6! \times 4!} = 210$$

$$\text{For } p = 7, q = 2, r = 1, \text{ coefficient} = \frac{10!}{7! \times 2! \times 1!} = 360$$

$$\text{For } p = 8, q = 0, r = 2, \text{ coefficient} = \frac{10!}{8! \times 2!} = 45$$

Therefore, sum = 615

24. If  $\vec{P} = (a+1)\hat{i} + a\hat{j} + a\hat{k}$

$$\vec{Q} = a\hat{i} + (a+1)\hat{j} + a\hat{k}$$

$$\vec{R} = a\hat{i} + a\hat{j} + (a+1)\hat{k}$$

and  $\vec{P}, \vec{Q}, \vec{R}$  are coplanar vectors and  $3(\vec{P} \cdot \vec{Q})^2 - \lambda |\vec{R} \times \vec{Q}|^2 = 0$ , then value of  $\lambda$  is

**Answer:** (1)

**Solution:**

As  $\vec{P}, \vec{Q}, \vec{R}$  are coplanar,

$$\begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$$R_1 \rightarrow R_1 + R_2 + R_3$$

$$\begin{vmatrix} 3a+1 & 3a+1 & 3a+1 \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$$(3a + 1) \begin{vmatrix} 1 & 1 & 1 \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$$C_2 \rightarrow C_2 - C_1 \text{ and } C_3 \rightarrow C_3 - C_1$$

$$(3a + 1) \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ a & 0 & 1 \end{vmatrix} = 0$$

$$3a + 1 = 0$$

$$\Rightarrow a = -\frac{1}{3}$$

$$\vec{P} = \frac{1}{3}(2\hat{i} - \hat{j} - \hat{k}), \quad \vec{Q} = \frac{1}{3}(-\hat{i} + 2\hat{j} - \hat{k}), \quad \vec{R} = \frac{1}{3}(-\hat{i} - \hat{j} + 2\hat{k})$$

$$\vec{R} \times \vec{Q} = \frac{1}{9} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -1 & 2 \\ -1 & 2 & -1 \end{vmatrix}$$

$$\vec{R} \times \vec{Q} = \frac{1}{9}(-3\hat{i} - 3\hat{j} - 3\hat{k}) = -\frac{1}{3}(\hat{i} + \hat{j} + \hat{k})$$

$$|\vec{R} \times \vec{Q}|^2 = \frac{1}{3}$$

$$\vec{P} \cdot \vec{Q} = \frac{1}{9}(-2 - 2 + 1) = -\frac{1}{3}$$

$$3(\vec{P} \cdot \vec{Q})^2 - \lambda |\vec{R} \times \vec{Q}|^2 = 0$$

$$\Rightarrow \frac{1}{3} - \lambda \times \frac{1}{3} = 0 \Rightarrow \lambda = 1$$

25. Points  $A(2, 4, 0)$ ,  $B(3, 1, 8)$ ,  $C(3, 1, -3)$ ,  $D(7, -3, 4)$  are four points. The projection of line segment  $AB$  on line  $CD$  is

**Answer:** (8)

**Solution:**

$$\vec{AB} = \hat{i} - 3\hat{j} + 8\hat{k}$$

$$\vec{CD} = 4\hat{i} - 4\hat{j} + 7\hat{k}$$

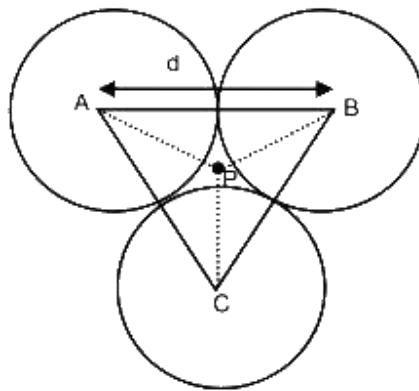
$$\text{Projection of } \vec{AB} \text{ on } \vec{CD} \text{ is } = \frac{\vec{AB} \cdot \vec{CD}}{|\vec{CD}|} = \frac{4+12+56}{\sqrt{4^2+4^2+7^2}} = \frac{72}{9} = 8$$

Date of Exam: 9<sup>th</sup> January (Shift I)

Time: 9:30 am – 12:30 pm

Subject: Physics

1. Three identical solid spheres each have mass 'm' and diameter 'd' are touching each as shown in the figure. Calculate ratio of moment of inertia about the axis perpendicular to plane of paper and passing through point P and B as shown in the figure. Given P is centroid of the triangle



a.  $\frac{13}{23}$   
c.  $\frac{7}{9}$

b.  $\frac{11}{19}$   
d.  $\frac{13}{11}$

Solution: (a)

$$\text{Moment of Inertia of solid sphere} = \frac{2}{5} M \left( \frac{d}{2} \right)^2$$

$$\text{Distance of centroid (Point P) from centre of sphere} = \left( \frac{2}{3} \times \frac{\sqrt{3}d}{2} \right) = \frac{d}{\sqrt{3}}$$

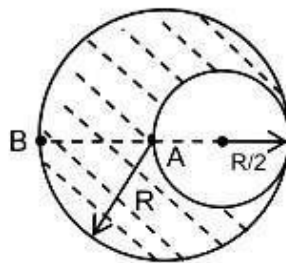
By Parallel axis theorem,

$$\text{Moment of Inertia about P} = 3 \left[ \frac{2}{5} M \left( \frac{d}{2} \right)^2 + M \left( \frac{d}{\sqrt{3}} \right)^2 \right] = \frac{13}{10} M d^2$$

$$\text{Moment of Inertia about B} = 2 \left[ \frac{2}{5} M \left( \frac{d}{2} \right)^2 + M(d)^2 \right] + \frac{2}{5} M \left( \frac{d}{2} \right)^2 = \frac{23}{10} M d^2$$

$$\text{Now ratio} = \frac{13}{23}$$

2. A solid sphere having a radius  $R$  and uniform charge density  $\rho$  has a radius  $R/2$  as shown in the figure. Find the ratio of the magnitude of electric field at point A and B



a.  $\frac{18}{19}$   
c.  $\frac{9}{17}$

b.  $\frac{11}{17}$   
d.  $\frac{9}{91}$

Solution: (c)

For solid sphere,

Field inside sphere,  $E = \frac{\rho r}{3\epsilon_0}$  & Field outside sphere,  $E = \frac{\rho R^3}{3r^2\epsilon_0}$  where,  $r$  is distance from centre and  $R$  is radius of sphere

Electric field at A due to sphere of radius  $R$  (sphere 1) is zero and therefore, net electric field will be because of sphere of radius  $\frac{R}{2}$  (sphere 2) having charge density  $(-\rho)$

$$E_A = \frac{-\rho R}{2(3\epsilon_0)}$$

$$|E_A| = \frac{\rho R}{6\epsilon_0}$$

Similarly, Electric field at point B =  $E_B = E_{1B} + E_{2B}$

$E_{1B}$  = Electric Field Due to solid sphere of radius  $R = \frac{\rho R}{3\epsilon_0}$

$E_{2B}$  = Electric Field Due to solid sphere of radius  $\frac{R}{2}$  which having charge density  $(-\rho)$

$$= -\frac{\rho \left(\frac{R}{2}\right)^3}{3 \left(\frac{3R}{2}\right)^2 \epsilon_0} = -\frac{\rho R}{54\epsilon_0}$$

$$E_B = E_{1A} + E_{2A} = \frac{\rho R}{3\epsilon_0} - \frac{\rho R}{54\epsilon_0} = \frac{17\rho R}{54\epsilon_0}$$

$$\frac{|E_A|}{|E_B|} = \frac{9}{17}$$

3. Consider an infinitely long current carrying cylindrical straight wire having radius 'a'. Then the ratio of magnetic field at distance  $a/3$  and  $2a$  from axis of wire is.

a.  $3/5$   
c.  $1/2$

b.  $2/3$   
d.  $4/3$

Solution: (b)

$$B_A = \frac{\mu_0 i r}{2\pi a^2} = \frac{\frac{\mu_0 i a}{3}}{2\pi a^2} = \frac{\mu_0 i}{\pi a^2} \frac{a}{6} = \frac{\mu_0 i}{6\pi a}$$

$$B_B = \frac{\mu_0 i (2a)^2}{2\pi (2a)} = \frac{\mu_0 i}{4\pi a}$$

$$\frac{B_A}{B_B} = \frac{4}{6} = \frac{2}{3}$$

- 

- $$\begin{aligned} d\vec{s} &= (dx\hat{i} + dy\hat{j}) \\ &= (-x\hat{i} + y\hat{j}) \cdot (dx\hat{i} + dy\hat{j}) \\ &= \int_1^0 -x\,dx + \int_0^1 y\,dy \end{aligned}$$



Solution: (a)

Since, we have to find vector parallel to electric field at position  $\vec{r}$

We have to find  $\vec{p} \cdot \vec{r} = 0$

Since already in question,  $\vec{p} \cdot \vec{r} = 0$  is given we need to find E such that

$$\vec{E} = \lambda (\vec{p})$$

where  $\lambda$  is a arbitrary positive constant

On putting,  $\lambda = -1$ , we get,  $\vec{E} = \hat{i} + 3\hat{j} - 2\hat{k}$

7. A particle of mass  $m$  is revolving around a planet in a circular orbit of radius  $R$ . At the instant the particle has velocity  $\vec{V}$ , another particle of mass  $\frac{m}{2}$  moving at velocity of  $\frac{V}{2}$  in same direction collides perfectly in-elastically with the first particle. The new path of the combined body will take is

- a. Elliptical  
b. Circular  
c. Straight Line  
d. Spiral

Solution: (a)

By conservation of linear momentum

$$\frac{m}{2} \frac{V}{2} + mV = (m + \frac{m}{2})V_f$$

$$V_f = \frac{5V}{6}$$

Escape velocity will be at  $\sqrt{2}V$  and at velocity less than escape velocity path will be elliptical or part of ellipse except for velocity  $V$  where path will be circular.

Hence the resultant mass will go on to an elliptical path

8. Two particles of same mass  $m$  moving with velocities  $\vec{v}_1 = v\hat{i}$  and  $\vec{v}_2 = \frac{v}{2}\hat{i} + \frac{v}{2}\hat{j}$  collide in - elastically. Find the loss in kinetic energy.

- a.  $\frac{mv^2}{8}$   
b.  $\frac{1mv^2}{8}$   
c.  $\frac{9mv^2}{8}$   
d.  $\frac{3mv^2}{8}$

Solution: (a)

Conserving linear momentum

$$mv\hat{i} + m(\frac{v}{2}\hat{i} + \frac{v}{2}\hat{j}) = 2m(v_1\hat{i} + v_2\hat{j})$$

By equating  $\hat{i}$  and  $\hat{j}$

$$v_1 = \frac{3v}{4} \text{ and } v_2 = \frac{v}{4}$$

$$\text{Initial K.E} = \frac{mv^2}{2} + \frac{m}{2} \times (\frac{v}{\sqrt{2}})^2 = \frac{3mv^2}{4}$$

$$\text{Final K.E} = \frac{2m}{2} \times (\frac{v\sqrt{10}}{4})^2 = \frac{mv^2}{8}$$

Change in KE =

$$\frac{3mv^2}{4} - \frac{5mv^2}{8} = \frac{mv^2}{8}$$

9. Three waves of same intensity ( $I_0$ ) having initial phases  $0, \frac{\pi}{4}, -\frac{\pi}{4}$  rad respectively interfere at a point. Find the resultant intensity.

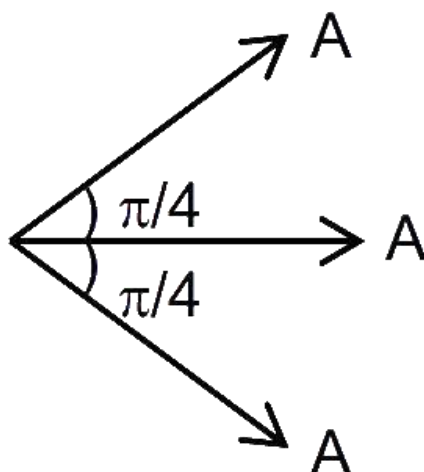
a.  $5.8 I_0$

b.  $I_0$

c.  $0.4 I_0$

d.  $0.3 I_0$

Solution: (a)



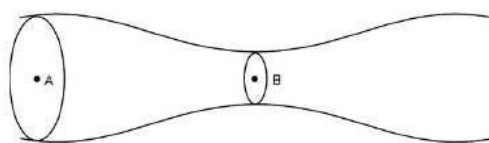
Amplitudes can be vectorially added

$$A_{\text{resultant}} = (\sqrt{2} + 1)A$$

Since,  $I \propto A^2$

$$\text{Therefore, } I_{\text{res}} = (\sqrt{2} + 1)^2 I_0 = 5.8 I_0$$

10. An ideal liquid (water) flowing through a tube of non-uniform cross section area at A and B are  $40 \text{ cm}^2$  and  $20 \text{ cm}^2$  respectively. If pressure difference between A & B is  $700 \text{ N/m}^2$  then volume flow rate is



a.  $2732 \text{ cm}^3/\text{s}$

b.  $2142 \text{ cm}^3/\text{s}$

c.  $1832 \text{ cm}^3/\text{s}$

d.  $3218 \text{ cm}^3/\text{s}$

Solution: (a)

Using equation of continuity

$$V_A \times \text{Area}_A = V_B \times \text{Area}_B$$

$$40V_A = 20V_B$$

$$2V_A = V_B$$

Using Bernoulli's equation

$$P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2$$

$$P_A - P_B = \frac{1}{2}\rho(V_B^2 - V_A^2)$$

$$\Delta P = \frac{1}{2}1000\left(V_B^2 - \frac{V_B^2}{4}\right)$$

$$\Delta P = 500 \times \frac{3V_B^2}{4}$$

$$V_B = \sqrt{\frac{(\Delta P) \times 4}{1500}} = \sqrt{\frac{(700) \times 4}{1500}} = \sqrt{\frac{28}{15}} \text{ m/s}$$

$$\text{Volume flow rate} = V_B \times \text{Area}_B = 20 \times 100 \times \sqrt{\frac{28}{15}} \text{ cm}^3/\text{s} = 2732 \text{ cm}^3/\text{s}$$

11. A screw gauge advances by 3mm in 6 rotations. There are 50 divisions on circular scale. Find least count of screw gauge?

- a. 0.002 cm  
c. 0.01 cm

- b. 0.001 cm  
d. 0.02 cm

Solution: (b)

$$\text{Pitch} = \frac{3}{6} = 0.5 \text{ mm}$$

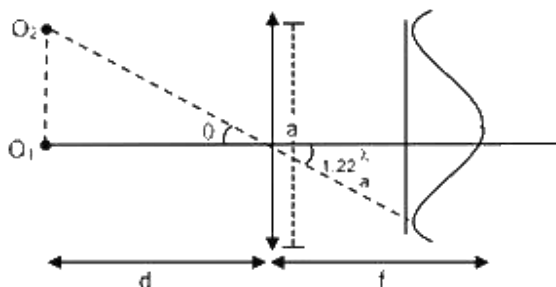
$$\text{Least count} = \frac{\text{Pitch}}{\text{Number of division}} = \frac{0.5\text{mm}}{50} = \frac{1}{100} \text{ mm} = 0.01 \text{ mm} = 0.001 \text{ cm}$$

12. A telescope of aperture diameter 5m is used to observe the moon from the earth. Distance between the moon and earth is  $4 \times 10^5$  km. Determine the minimum distance between two points on the moon's surface which can be resolved using this telescope. (Wave length of light is  $5893 \text{ \AA}$ )

- a. 60 m  
c. 600 m

- b. 20 m  
d. 200 m

Solution: (a)



Minimum angle for clear resolution,

$$\theta = 1.22 \frac{\lambda}{a}$$

$$\text{distance} = O_1 O_2 = d\theta$$

$$= 1.22 \frac{\lambda}{a} d$$

$$\text{distance} = O_1 O_2 = \frac{1.22 \times 5893 \times 10^{-10} \times 4 \times 10^8}{5} \approx 57.52 \text{ m}$$

$\therefore$  Nearest option is 60 m

13. Photons of wavelength  $6556 \text{ \AA}$  falls on a metal surface. If ejected electrons with maximum K.E moves in magnetic field of  $3 \times 10^{-4} \text{ T}$  in circular orbit of radius  $10^{-2} \text{ m}$ , then work function of metal surface is

- a. 1.8 eV  
c. 1.1 eV

- b. 0.8 eV  
d. 1.4 eV

Solution: (c)

From photoelectric equation,

$$\frac{hc}{\lambda} = W + K.E_{\max}$$

Where,  $hc = 12400 \text{ eV \AA}$

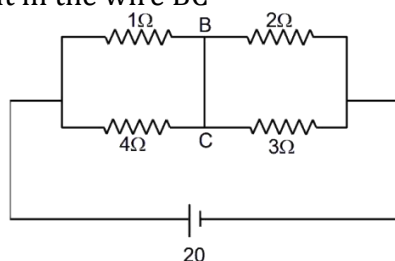
$$\Rightarrow \frac{12400}{6556} = W + K.E_{\max}$$

$$\Rightarrow 1.9 \text{ eV} = W + K.E_{\max} \text{ --- (1)}$$





17. Find the current in the wire BC



- a. 1.6 A  
c. 2.4 A

- b. 2 A  
d. 3 A

Solution: (b)

Since resistance  $1\ \Omega$  and  $4\ \Omega$  are in parallel

$$\therefore R' = \frac{4 \times 1}{4 + 1} = \frac{4}{5}$$

Similarly we can find equivalent resistance ( $R''$ ) for resistances  $2\ \Omega$  and  $3\ \Omega$

$$\Rightarrow R'' = \frac{6}{5}$$

And  $R'$  and  $R''$  are in series

$$\therefore R_{eff} = \frac{4}{5} + \frac{6}{5} = 2\ \Omega$$

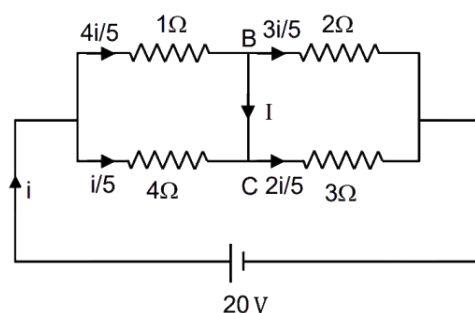
So total current flowing in the circuit ' $i$ ' can be given as

$$i = \frac{V}{R_{eff}} = \frac{20}{2} = 10\ A$$

Current will distribute in ratio opposite to resistance.

So, distribution will be as

So current in the branch BC will be



$$I = \frac{4i}{5} - \frac{3i}{5} = \frac{i}{5} = \frac{10}{5} = 2\ A$$



Solution: (d)

We know that,

Molar heat capacity at constant volume,  $C_V = \frac{fR}{2}$  (Where  $f$  is degree of freedom)

Since, A is diatomic and rigid, degree of freedom for A is 5

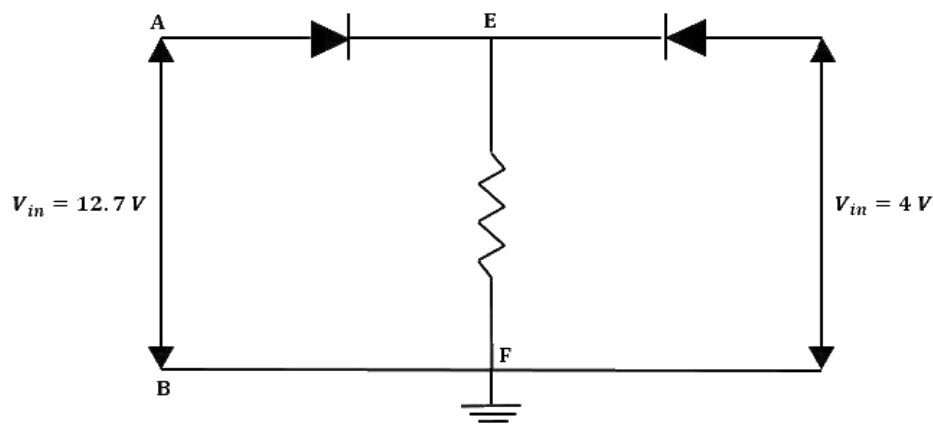
Therefore, Molar heat capacity of A at constant volume  $(C_V)_A = \frac{5R}{2}$

Since, B is diatomic and have extra degree of freedom because of vibration, degree of freedom for B is  $5 + 2 \times 1 = 7$  (1 vibration for each atom).

Therefore, Molar heat capacity of B at constant volume  $(C_V)_B = \frac{7R}{2}$

Ratio of molar specific heat of A and B =  $\frac{(C_V)_A}{(C_V)_B} = \frac{5}{7}$

20. In the given circuit both diodes are ideal having zero forward resistance and built-in potential of 0.7 V. Find the potential of point E in volts



Solution: (12)

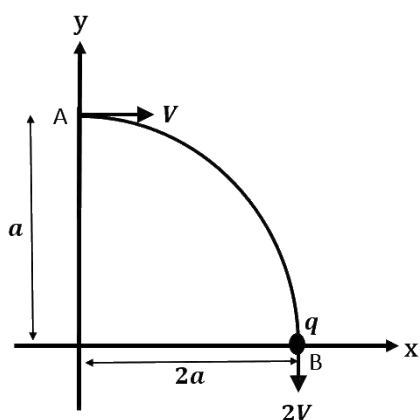
By applying Kirchhoff's Voltage Law in the loop ACBFA

$$12.7 - 0.7 - V_{EF} = 0$$

$$\Rightarrow V_{EF} = 12 \text{ V}$$

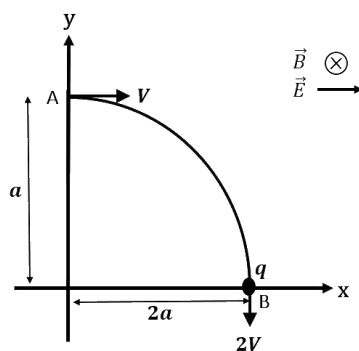
$$\Rightarrow V_E = 12 \text{ V}$$

21. A particle having mass  $m$  and charge  $q$  is moving in a region as shown in figure. This region contains a uniform magnetic field directed into the plane of the figure, and a uniform electric field directed along positive  $x$  - axis. Which of the following statements are correct for moving charge as shown in figure?



- A. Magnitude of electric field  $\vec{E} = \frac{3}{4} \left( \frac{mv^2}{qa} \right)$
- B. Rate of change of work done at a point A is  $\frac{3}{4} \left( \frac{mv^3}{a} \right)$
- C. Rate of change of work done by both fields at point B is zero
- D. Change in angular momentum about the origin is  $2mva$
- a. A, B and C are correct
- b. A, B, C and D are correct
- c. A and B are correct
- d. B, C and D are correct

Solution: (a)



Considering statement A  
By Work-Energy theorem

$$W_{mag} + W_{ele} = \frac{1}{2} m(2v)^2 - \frac{1}{2} mv^2$$

$$\Rightarrow 0 + qE_o 2a = \frac{3}{2} mv^2$$

$$E_o = \frac{3}{4} \frac{mv^2}{qa}$$

So statement A is correct

Now considering statement B

Rate of change of work done at A = Power of electric force

$$= 9E_0 v$$

$$= \frac{3mv^3}{4a}$$

So statement B is correct

Coming to statement C

At B,

$$\vec{E} \perp \vec{v}$$

So,  $\frac{dw}{dt} = 0$  for both forces

Coming to statement D.

Change in angular momentum about the origin is

$$\Delta \vec{L} = \Delta \vec{L}_B - \Delta \vec{L}_A$$

$$\vec{L}_B = m(2v)(2a)$$

$$\vec{L}_A = m(v)(a)$$

$$\text{Hence, } \Delta L = 3mva$$

22. If reversible voltage of 200 V is applied across an inductor, current in it reduces from 0.25A to 0A in 0.025ms. Find inductance of inductor (in mH).

Solution: (20)

By using KVL,

$$V - L \frac{di}{dt} = 0$$

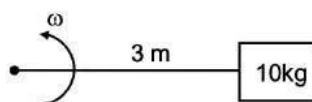
$$\Rightarrow 200 = \frac{L(0.25)}{0.025} \times 10^3$$

$$L = 200 \times 10^{-4} \text{ H}$$

$$= 20 \text{ mH}$$

23. A wire of length  $l = 3 \text{ m}$  and area of cross section  $10^{-2} \text{ cm}^2$  and breaking stress  $4.8 \times 10^8 \text{ N/m}^2$  is attached with block of mass 10 kg. Find the maximum possible value of angular velocity ( $\text{rad/s}$ ) with which block can be moved in circle with string fixed at one end.

Solution: (4)



Breaking stress

$$\sigma = \frac{T}{A}$$

$$T = m\omega^2 l$$

$$\Rightarrow \sigma = \frac{m\omega^2 l}{A}$$

$$\Rightarrow \omega^2 = \frac{\sigma A}{ml} = \frac{4.8 \times 10^8 \times 10^{-6}}{10 \times 3} = 16$$

$$\Rightarrow \omega = 4 \text{ rad/s}$$

24. Position of a particle as a function of time is given as  $x^2 = at^2 + 2bt + c$ , where  $a, b, c$  are constants. Acceleration of particle varies with  $x^{-n}$  then value of  $n$  is

Solution: (3)

Let,  $v$  be velocity,  $a$  be the acceleration then,

$$x^2 = at^2 + 2bt + c$$

$$2xv = 2at + 2b$$

$$xv = at + b \quad \text{---(1)}$$

$$\Rightarrow v = \frac{at + b}{x}$$

Now, differentiating equation (1),

$$v^2 + ax = a$$

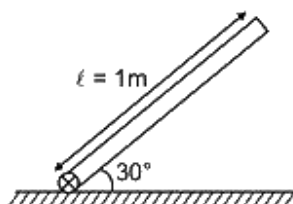
$$ax = a - \left( \frac{at + b}{x} \right)^2$$

$$\alpha = \frac{a(at^2 + 2bt + c) - (at + b)^2}{x^3}$$

$$\alpha = \frac{ac - b^2}{x^3}$$

$$\alpha \propto x^{-3}$$

25. A rod of length 1 m is released from rest as shown in the figure below.



If  $\omega$  of rod is  $\sqrt{n}$  at the moment it hits the ground, then find n

Solution: (15)

: By using conservation of energy,

$$mg \frac{l}{2} \sin 30^\circ = \frac{1}{2} \frac{ml^2}{3} \omega^2$$

On solving

$$\omega^2 = 15$$

$$\omega = \sqrt{15}$$

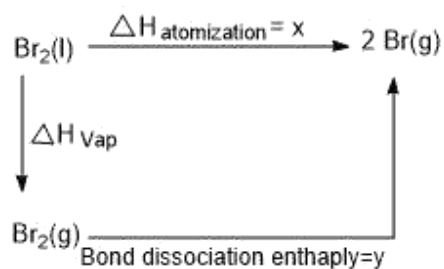
Therefore, n = 15



3. For  $\text{Br}_2(\text{l})$ , the enthalpy of atomisation =  $x$  kJ/mol and the bond dissociation enthalpy of bromine =  $y$  kJ/mol. Then,
- $x > y$
  - $x < y$
  - $x = y$
  - Relation does not exist.

Answer: a

Solution:



$$\Delta H_{\text{atomisation}} = \Delta H_{\text{vap}} + y$$

$$x - y = \Delta H_{\text{vap}}$$

4. Which of the following oxides are acidic, basic and amphoteric, respectively?
- $\text{MgO}$ ,  $\text{P}_4\text{O}_{10}$ ,  $\text{Al}_2\text{O}_3$
  - $\text{N}_2\text{O}_3$ ,  $\text{Li}_2\text{O}$ ,  $\text{Al}_2\text{O}_3$
  - $\text{SO}_3$ ,  $\text{Al}_2\text{O}_3$ ,  $\text{Na}_2\text{O}$
  - $\text{P}_4\text{O}_{10}$ ,  $\text{Al}_2\text{O}_3$ ,  $\text{MgO}$

Answer: b

Solution:

Non-metallic oxides are acidic in nature, metallic oxides are basic in nature and  $\text{Al}_2\text{O}_3$  is amphoteric in nature

5. The complex  $\text{Cr}(\text{H}_2\text{O})_6\text{Cl}_n$ , shows geometrical isomerism and also reacts with  $\text{AgNO}_3$  solution. Given: The spin only magnetic moment = 3.8 B. M. What is the IUPAC name of the complex?
- Hexaaquachromium(III) chloride
  - Tetraaquadichloridochromium(III) chloride dihydrate
  - Hexaaquachromium(IV) chloride
  - Tetraaquadichloridochromium(IV) chloride dehydrate

Answer: b

Solution:

Spin only magnetic moment = 3.8 B. M. This implies,  $\mu = \sqrt{n(n+2)}$  B.M.

$(\sqrt{16} = 4)$  implies that  $\sqrt{15}$  should be less than four.

This means,  $n=3$  as  $\sqrt{15} = \sqrt{3(3+2)}$

$\text{Cr} (24) = [\text{Ar}]4s^1 3d^5$

(g.s)

For 3 unpaired electrons, the oxidation state of Cr should be +3

$\text{Cr}^{3+}$  can be attained if the complex has a structure that looks like:  $[\text{Cr}(\text{H}_2\text{O})_4\text{Cl}_2]\text{Cl} \cdot 2\text{H}_2\text{O}$

$[\text{Cr}(\text{H}_2\text{O})_4\text{Cl}_2]\text{Cl} \cdot 2\text{H}_2\text{O}$  has the IUPAC name : Tetraaquadichloridochromium(III) chloride dihydrate

6. The electronic configuration of bivalent Europium and trivalent Cerium, respectively is:  
(Atomic Number : Xe = 54, Ce = 58, Eu = 63)
- |  |   |
|--|---|
| a. $[\text{Xe}]4f^7, [\text{Xe}]4f^1$                | b. $[\text{Xe}]4f^7 6s^2, [\text{Xe}]4f^1$      |
| c. $[\text{Xe}]4f^7 6s^2, [\text{Xe}]4f^1 5d^1 6s^2$ | d. $[\text{Xe}]4f^7, [\text{Xe}]4f^1 5d^1 6s^2$ |

Answer: a

Solution:

Ce (58):  $[\text{Xe}] 6s^2 4f^2$

(g.s)

$\text{Ce}^{3+}$ :  $[\text{Xe}]4f^1$

Eu(63) :  $[\text{Xe}]6s^2 4f^7$

(g.s)

$\text{Eu}^{2+}$  :  $[\text{Xe}]4f^7$



9. The first Ionisation energy of Be is higher than that of Boron. Select the correct statements regarding this:

- (i) It is easier to extract electron from 2p orbital than 2s orbital
- (ii) Penetration power of 2s orbital is greater than 2p orbital
- (iii) Shielding of 2p electron by 2s electron
- (iv) Radius of Boron atom is larger than that of Be

- a. (i), (ii), (iii), (iv)
- c. (ii), (iii), (iv)

- b. (i), (iii), (iv)
- d. (i), (ii), (iii)

Answer: d

Solution:

Be (4):  $1s^2 2s^2$

B (5):  $1s^2 2s^2 2p^1$

The electron in  $2p^1$  can easily be extracted.

The penetrating power is of the order:  $s > p > d > f$

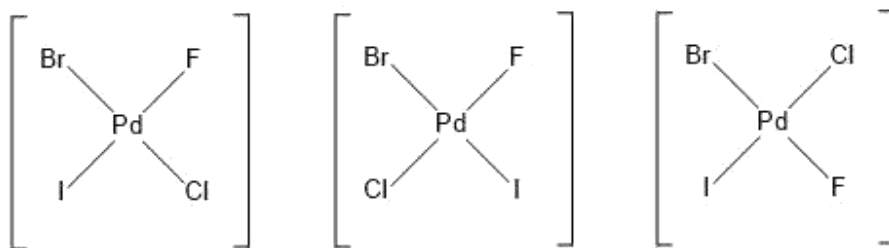
The shielding power order:  $s > p > d > f$

As we move along the period, the size decreases, as  $Z_{\text{eff}}$  increases. Hence the radius of B is smaller than the radius of Be.

10. For  $[\text{PdFClBrI}]^{2-}$ , the number of geometrical Isomers = n. Determine the spin only magnetic moment and CFSE for  $[\text{Fe}(\text{CN})_6]^{n-6}$  (Ignore pairing energy).
- 1.73 B. M.,  $-2\Delta_0$
  - 2.84 B. M.,  $-1.6\Delta_0$
  - 0,  $-1.6\Delta_0$
  - 5.92 B. M.,  $-2.4\Delta_0$

Answer: a

Solution:



Number of geometrical isomers (n) = 3

$$[\text{Fe}(\text{CN})_6]^{n-6} = [\text{Fe}(\text{CN})_6]^{3-6} = [\text{Fe}(\text{CN})_6]^{-3}$$

This implies, that Iron is in its +3 oxidation state.

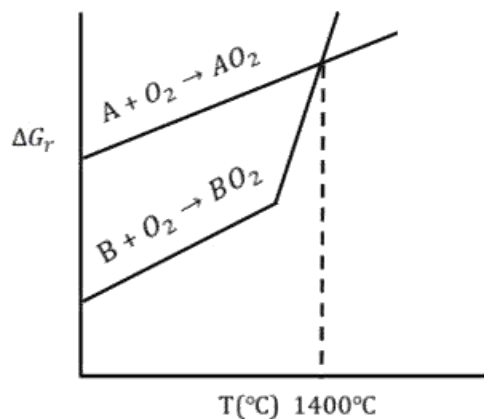
$$\text{Fe}^{3+} (26): [\text{Ar}]3d^5$$

$\text{CN}^-$  is a strong ligand in  $[\text{Fe}(\text{CN})_6]^{-3}$  and causes pairing. Hence, according to CFT, the configuration will be  $t_{2g}^5 e_g^0$ .

Hence, there is only 1 unpaired electron, i.e,  $n=1$  in  $\sqrt{n(n+2)} = \sqrt{3} = 1.73 \text{ B.M}$

$$\begin{aligned} \text{CFSE} &= (-0.4 \times n_{t_{2g}} + 0.6 \times n_{e_g})\Delta_0 \\ &= (-0.4 \times 5 + 0.6 \times 0)\Delta_0 \\ &= -2\Delta_0 \end{aligned}$$

11. A can reduce  $\text{BO}_2$  under which conditions?



- |                           |                           |
|---------------------------|---------------------------|
| a. $> 1400^\circ\text{C}$ | b. $< 1400^\circ\text{C}$ |
| c. $> 1200^\circ\text{C}$ | d. $< 1200^\circ\text{C}$ |

Answer: a

Solution: In Ellingham's diagram, the line of the element that lies below can reduce the oxide of the element which lies above it. Therefore, for A to reduce  $\text{BO}_2$ , the temperature when the line for element A is below that of  $\text{BO}_2$ , according to the graph when  $T > 1400^\circ\text{C}$ .

For  $T > 1400^\circ\text{C}$ ,  $\Delta G_r < 0$  for  $\text{A} + \text{BO}_2 \rightarrow \text{B} + \text{AO}_2$

12.  $A \rightarrow B$ ; 700 K

$A \xrightarrow{C} B$ ; 500 K

Rate of the reaction in absence of catalyst at 700 K is same as in presence of catalyst at 500 K. If catalyst decreases the activation energy barrier by 30 kJ/mol, determine the activation energy in presence of catalyst. (Assume 'A' factor to be same in both cases)

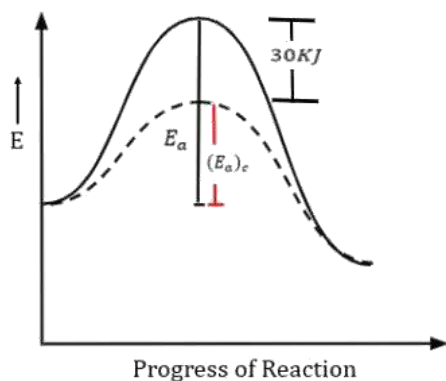
- a. 75 kJ
- b. 135 kJ
- c. 105 kJ
- d. 125 kJ

Answer: c

Solution:

$$K = Ae^{\left(\frac{-E_a}{RT}\right)}$$

$$K_{\text{catalyst}} = K_{\text{without catalyst}}$$



$$Ae^{\left(\frac{-(E_a)_c}{RT_{500k}}\right)} = Ae^{\left(\frac{-(E_a)}{RT_{700k}}\right)}$$

$$e^{\left(\frac{-(E_a)_c}{RT_{500k}}\right)} = e^{\left(\frac{-(E_a)}{RT_{700k}}\right)}$$

$$-\frac{(E_a)_c}{RT_{500k}} = -\frac{(E_a)}{RT_{700k}}$$

$$(E_a)_c = E_a - 30$$

$$-\frac{(E_a - 30)}{T_{500k}} = -\frac{(E_a)}{T_{700k}}$$

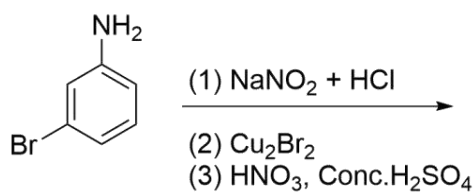
On Solving,  $E_a = 105 \text{ kJmol}^{-1}$

13. A substance 'X' having low melting point, does not conduct electricity in both solid and liquid state. 'X' can be :
- |        |                   |
|--------|-------------------|
| a. Hg  | b. SiC            |
| c. ZnS | d. $\text{CCl}_4$ |

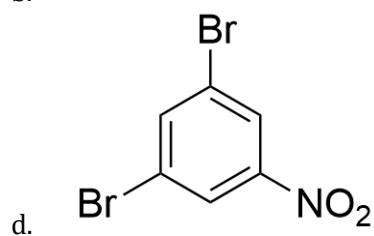
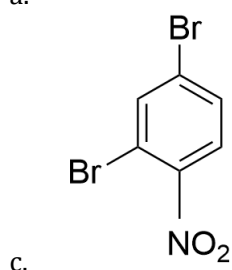
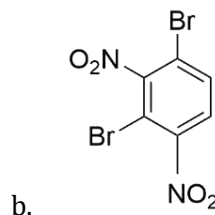
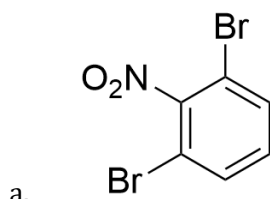
Answer: d

Solution:  $\text{CCl}_4$  is non polar and does not conduct in either solid or liquid state.

14.

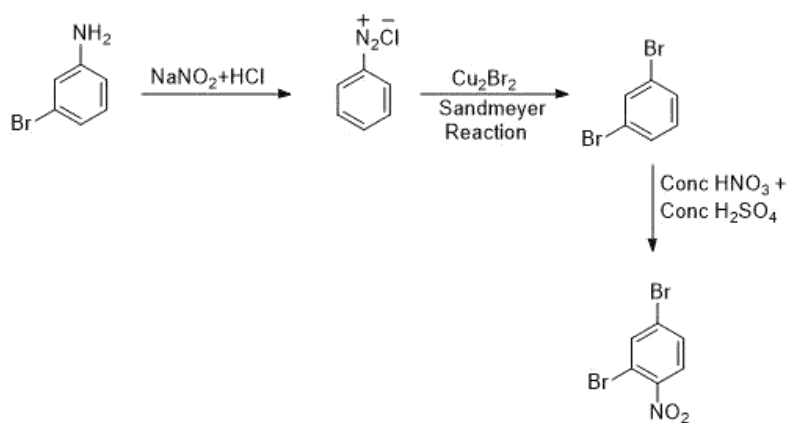


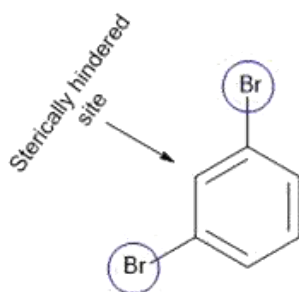
The major product for above sequence of reaction is:



Answer: c

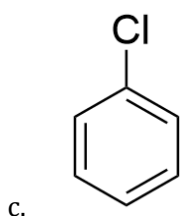
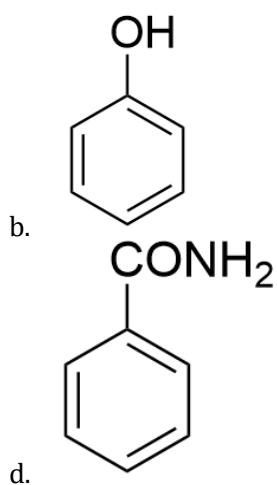
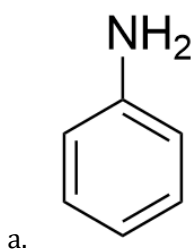
Solution:





Hence, major product formed is that of option b.

15. Which of the following can give the highest yield in Friedel-Craft's reaction?

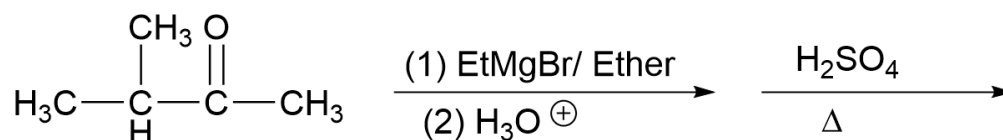


d.

Answer: b

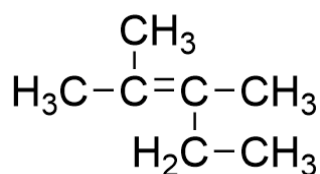
Solution: Out of the four options given, only aniline and phenol show strong +R effects, but as we know, aniline is a Lewis base and can react with a Lewis acid that is added during the reaction. Hence, Phenol gives the highest yield in Friedel-Craft's reaction.

16.

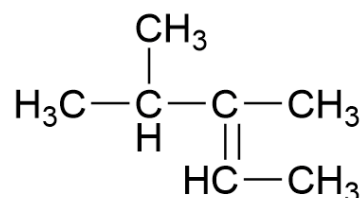


What will be the major product?

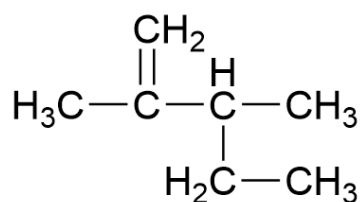
a.



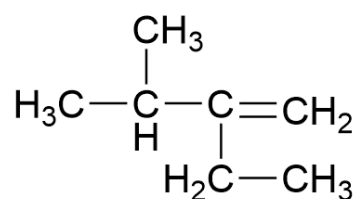
c.



b.

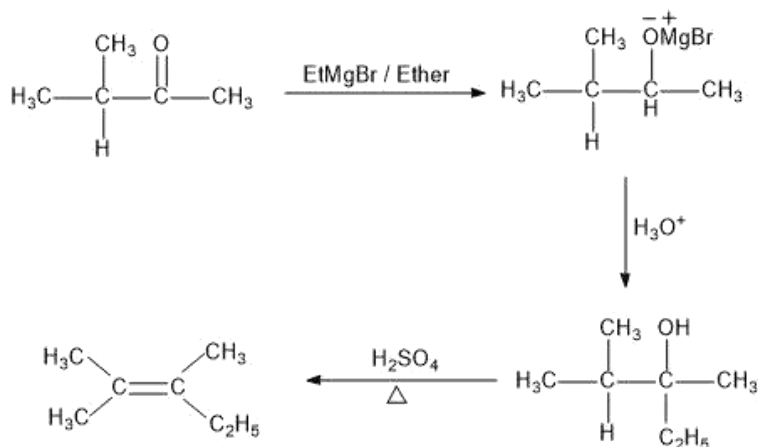


d.

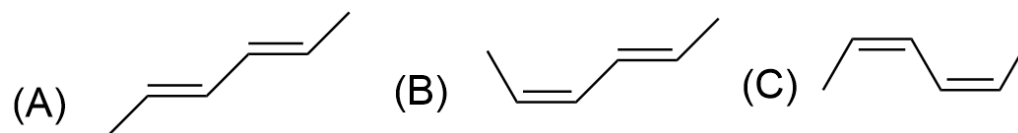


Answer: a

Solution:



17. Which of the following is the correct order for heat of combustion?



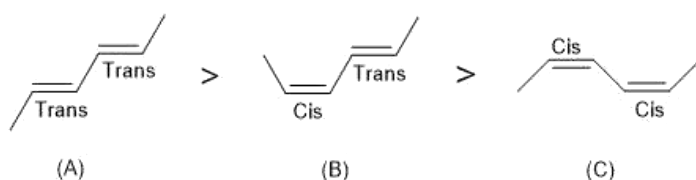
- a.  $C > B > A$   
c.  $A > B > C$

- b.  $B > A > C$   
d.  $C > A > B$

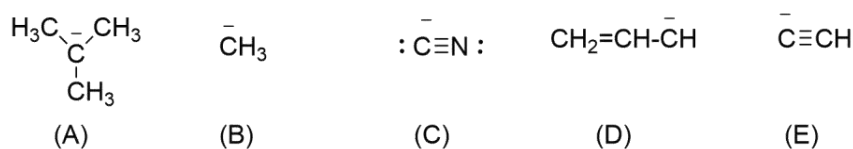
Answer: c

Solution: Heat of combustion  $\propto \frac{1}{\text{stability}}$

The trans-isomer is more stable than the cis-isomer. More the number of trans forms in a structure, higher the stability.



18. Write the correct order of basicity:



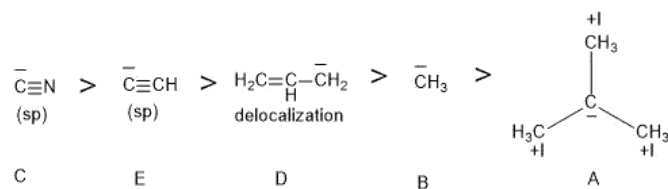
- a.  $A > B > D > E > C$   
c.  $A > B > E > D > C$

- b.  $B > A > D > C > E$   
c.  $C > E > D > B > A$

Answer: a

Solution: As we know weaker the conjugate base, stronger the acid.

The order of stability of conjugate base:



Hence, the order of basicity or acidic strength is:

$A > B > D > E > C$

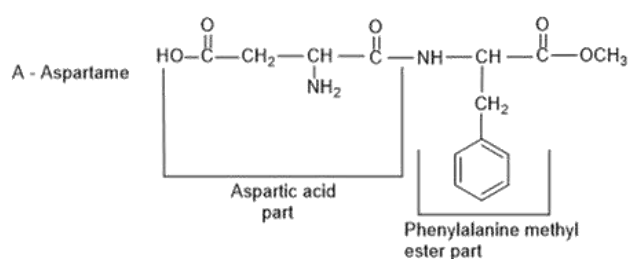
19. A, B, C, and D are four artificial sweetners.
- (i) A & D give positive test with ninhydrin.
  - (ii) C form precipitate with  $\text{AgNO}_3$  in the lassaigne extract of the sugar.
  - (iii) B & D give positive test with sodium nitroprusside.

Correct option is :

- a. A – Saccharine, B – Aspartame, C – Sucralose, D – Alitame
- b. A – Aspartame, B – Saccharine, C – Sucralose, D – Alitame
- c. A – Saccharine, B – Aspartame, C – Alitame , D – Sucralose
- d. A – Aspartame, B – Sucralose, C – Saccharine, D – Alitame

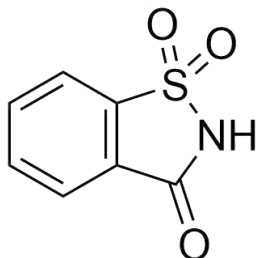
Answer: b

Solution:

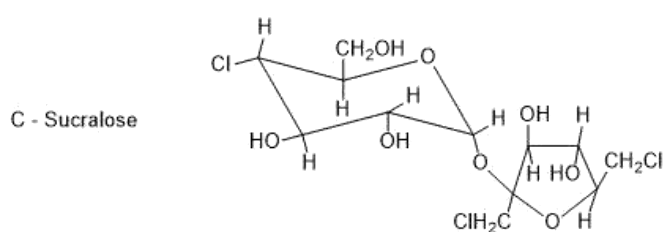


It has a free amine group and hence reacts with ninhydrin to give a purple colour known as Ruhemann's purple.

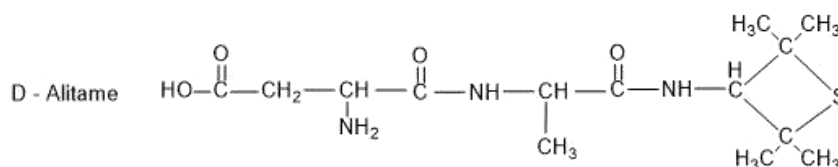
B- Saccharine



It has Sulphur, therefore, it will give a positive test with sodium nitroprusside.



It has chlorine and hence it forms a precipitate with  $\text{AgNO}_3$  in the Lassaigne's extract of the sugar.

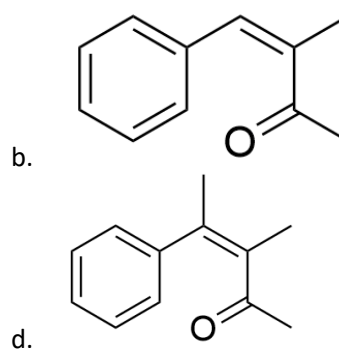
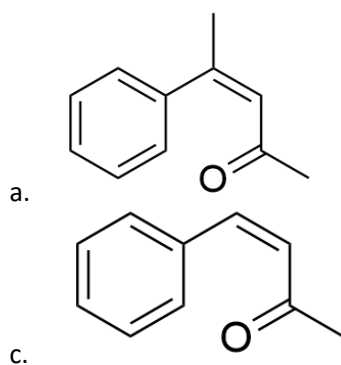


It has a free amine group and hence reacts with ninhydrin to give purple colour known as Ruhemann's purple. Also, it has Sulphur, therefore, it will give positive test with sodium nitroprusside.

20.

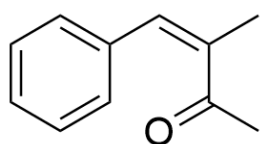


Predict the compound (P) on the basis of above sequence of the reactions, where compound (P) gives positive Iodoform test:

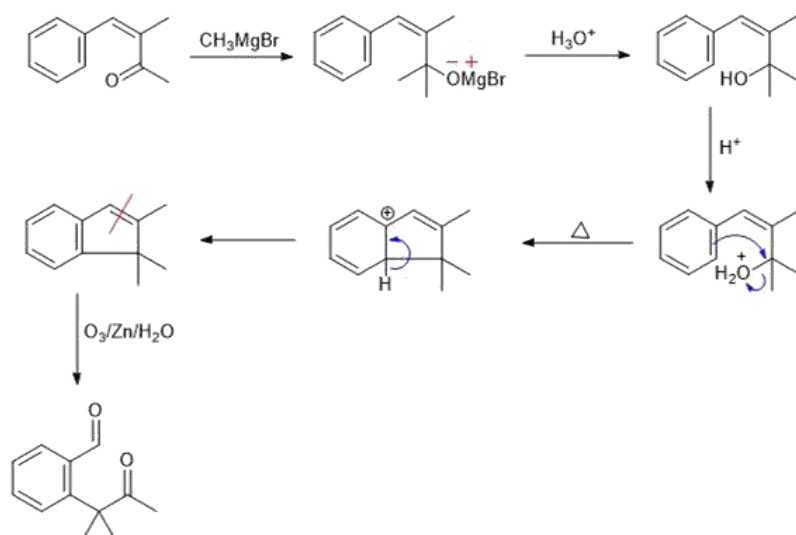


Answer: b

Solution:



is a methyl ketone, which gives positive Iodoform test.



21. Given a solution of  $\text{HNO}_3$  of density  $1.4 \text{ g/mL}$  and  $63\% \frac{w}{w}$ . Determine molarity of  $\text{HNO}_3$  solution.

Answer: 14.00

Solution:  $\% \frac{w}{w} = 63\%$

$$\rho = 1.4 \text{ g/mL}$$

$$M = \frac{\left(\% \frac{w}{w} \times \rho \times 10\right)}{\text{MM}}$$

$$M = \frac{(63 \times 1.4 \times 10)}{63}$$

$$M = 14 \text{ mol/L}$$

22. Determine the degree of hardness in terms of ppm of  $\text{CaCO}_3$  of  $10^{-3}$  molar  $\text{MgSO}_4$  (aq).

Answer: 100.00

Solution:

Hardness of water is measured in ppm in terms  $\text{CaCO}_3$ .

$$n_{\text{CaCO}_3} = n_{\text{MgSO}_4}$$

ppm is the parts (in grams) present per million i.e,  $10^6$

1000 mL has  $10^{-3}$  moles of  $\text{MgSO}_4$ .

Grams of  $\text{CaCO}_3$  in 1000 mL =  $10^{-3} \times 100$  grams

$$\text{Grams of } \text{CaCO}_3 \text{ in 1 mL} = \frac{10^{-3} \times 100}{1000 \text{ mL}} \text{ grams}$$

$$\text{Hardness} = \frac{10^{-3} \times 100}{1000 \text{ mL}} \times 10^6 = 100$$

23. Determine the amount of NaCl to be dissolved in 600 g of  $\text{H}_2\text{O}$  to decrease the freezing point by  $0.2^\circ\text{C}$ . Given :  $k_f$  of  $\text{H}_2\text{O}$  =  $2 \text{ K m}^{-1}$

Answer: 1.76

Solution: NaCl is strong electrolyte and gives 2 ions in the solution. This implies,  $i=2$ .

$$\text{Molality} = \frac{w \times 1000}{58.5 \times 600}$$

$$\Delta T_f = 0.2^\circ\text{C}$$

$$\Delta T_f = i \times k_f \times m$$

On solving we get,

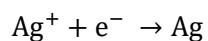
$$w = 1.76 \text{ grams}$$

24. On passing a particular amount of electricity in  $\text{AgNO}_3$  solution, 108 g of Ag is deposited. What will be the volume of  $\text{O}_2(\text{g})$  in litres liberated at 1 bar, 273 K by the same quantity of electricity?

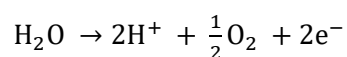
Answer: 5.68

Solution: On applying Faraday's 1<sup>st</sup> law,

Moles of Ag deposited =  $108/108 = 1$  mol.



1 Faraday is required to deposit 1 mole of Ag.



$\frac{1}{2}$  moles of  $\text{O}_2$  are deposited by 2 F of charge.

This implies, 1 F will deposit  $\frac{1}{4}$  moles of  $\text{O}_2$

Using  $PV = nRT$

$P = 1$  bar

$T = 273$  K

$R = 0.0823$  Lbar  $\text{mol}^{-1}\text{K}^{-1}$

On solving we get,

$V = 5.68$  L

25. Find percentage nitrogen by mass in Histamine?

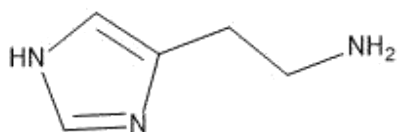
Answer: 37.84

Solution:

Molecular mass of Histamine = 111

In Histamine, 3 Nitrogens are present (42g)

The percentage of Nitrogen by mass in Histamine =  $\frac{42}{111} \times 100 = 37.84\%$



**JEEMain 2020 Paper**  
**Date: 9<sup>th</sup> January 2020 (Shift 2)**  
**Time: 2:30 P.M. to 5:30 P.M.**  
**Subject: Mathematics**

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1. If  $f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$  and  $a - 2b + c = 1$  then

a.  $f(-50) = -1$

b.  $f(50) = 1$

c.  $f(50) = -501$

d.  $f(50) = 501$

**Answer: (a)**

**Solution:**

Given  $f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$

$a - 2b + c = 1$

Applying  $R_1 \rightarrow R_1 - 2R_2 + R_3$

$f(x) = \begin{vmatrix} a-2b+c & 0 & 0 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$

Using  $a - 2b + c = 1$

$\therefore f(x) = (x+3)^2 - (x+2)(x+4)$

$\Rightarrow f(x) = 1$

$\Rightarrow f(50) = 1$

$\Rightarrow f(-50) = 1$

2. If  $f(x) = \begin{cases} x & 0 < x < \frac{1}{2} \\ \frac{1}{2} & x = \frac{1}{2} \\ 1-x & \frac{1}{2} < x < 1 \end{cases}$

$g(x) = \left(x - \frac{1}{2}\right)^2$  then find the area bounded by  $f(x)$  and  $g(x)$  from  $x = \frac{1}{2}$  to  $x = \frac{\sqrt{3}}{2}$ .

a.  $\frac{\sqrt{3}}{2} - \frac{1}{3}$

b.  $\frac{\sqrt{3}}{4} + \frac{1}{3}$

c.  $2\sqrt{3}$

d.  $3\sqrt{3}$

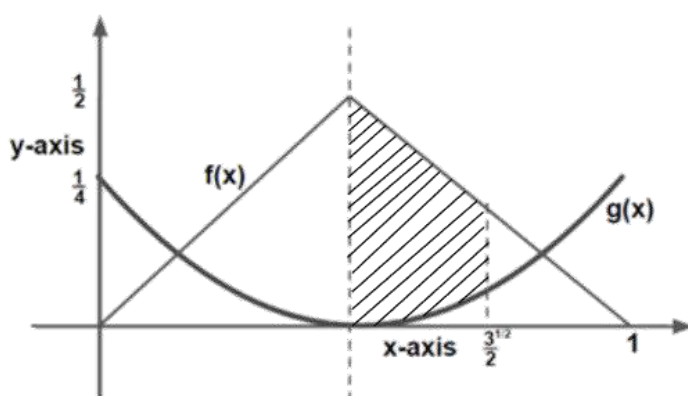
**Answer:** (a)

**Solution:**

$$\text{Given } f(x) = \begin{cases} x & 0 < x < \frac{1}{2} \\ \frac{1}{2} & x = \frac{1}{2} \\ 1 - x & \frac{1}{2} < x < 1 \end{cases}$$

$$g(x) = \left(x - \frac{1}{2}\right)^2$$

The area between  $f(x)$  and  $g(x)$  from  $x = \frac{1}{2}$  to  $x = \frac{\sqrt{3}}{2}$  :



Points of intersection of  $f(x)$  and  $g(x)$  :

$$1 - x = \left(x - \frac{1}{2}\right)^2$$

$$\Rightarrow x = \frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$$

$$\text{Required area} = \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} (f(x) - g(x)) dx$$

$$= \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \left(1 - x - \left(x - \frac{1}{2}\right)^2\right) dx$$

$$= x - \frac{x^2}{2} - \frac{1}{3} \left(x - \frac{1}{2}\right)^3 \Big|_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}$$

$$= \frac{\sqrt{3}}{4} - \frac{1}{3}$$





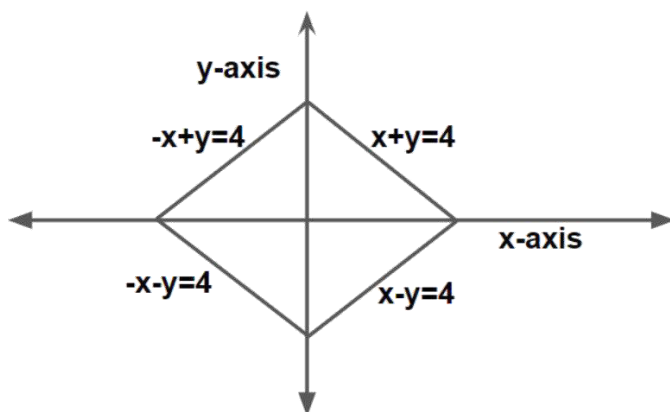
**Answer:** (b)

**Solution:**

$$|\operatorname{Re}(z)| + |\operatorname{Im}(z)| = 4$$

$$\text{Let } z = x + iy$$

$$\Rightarrow |x| + |y| = 4$$



$\therefore z$  lies on the rhombus.

Maximum value of  $|z| = 4$  when  $z = 4, -4, 4i, -4i$

Minimum value of  $|z| = 2\sqrt{2}$  when  $z = 2 \pm 2i, \pm 2 + 2i$

$$|z| \in [2\sqrt{2}, 4]$$

$$|z| \in [\sqrt{8}, \sqrt{16}]$$

$$|z| \neq \sqrt{7}$$

7.  $f(x) : [0,5] \rightarrow \mathbb{R}, F(x) = \int_0^x x^2 g(x) dx, f(1) = 3, g(x) = \int_1^x f(t) dt$  then correct choice is

- $F(x)$  has no critical point
- $F(x)$  has local minimum at  $x = 1$
- $F(x)$  has local maximum at  $x = 1$
- $F(x)$  has point of inflection at  $x = 1$

**Answer:** (b)

**Solution:**

$$F(x) = x^2 g(x)$$

$$\text{Put } x = 1$$

$$\Rightarrow F(1) = g(1) = 0 \quad \dots (1)$$

$$\text{Now } F''(x) = 2xg(x) + g'(x)x^2$$

$$F''(1) = 2g(1) + g'(1) \quad \{\because g'(x) = f(x)\}$$

$$F''(1) = f(1) = 3 \quad \dots (2)$$

From (1) and (2),  $F(x)$  has local minimum at  $x = 1$

8. Let  $x = 2 \sin \theta - \sin 2\theta$  and  $y = 2 \cos \theta - \cos 2\theta$ , then the value of  $\frac{d^2y}{dx^2}$  at  $\theta = \pi$  is

a.  $\frac{3}{8}$

b.  $\frac{5}{8}$

c.  $\frac{7}{8}$

d.  $\frac{3}{2}$

**Answer:** (a)

**Solution:**

$$\frac{dx}{d\theta} = 2 \cos \theta - 2 \cos 2\theta$$

$$\frac{dy}{d\theta} = -2 \sin \theta + 2 \sin 2\theta$$

$$\frac{dy}{dx} = \frac{2 \cos \frac{3\theta}{2} \sin \frac{\theta}{2}}{2 \sin \frac{3\theta}{2} \sin \frac{\theta}{2}}$$

$$\frac{dy}{dx} = \cot \frac{3\theta}{2}$$

$$\frac{d^2y}{dx^2} = -\frac{3}{2} \operatorname{cosec}^2 \frac{3\theta}{2} \frac{d\theta}{dx}$$

$$\frac{d^2y}{dx^2} = \left( -\frac{3}{2} \operatorname{cosec}^2 \frac{3\theta}{2} \right) \frac{1}{(2 \cos \theta - 2 \cos 2\theta)}$$

$$\left. \frac{d^2y}{dx^2} \right|_{\theta=\pi} = \frac{3}{8}$$

9. If  $f(x)$  and  $g(x)$  are continuous functions,  $f \circ g$  is identity function,  $g'(b) = 5$  and  $g(b) = a$ , then  $f'(a)$  is

a.  $\frac{3}{5}$

b. 5

c.  $\frac{2}{5}$

d.  $\frac{1}{5}$

**Answer:** (d)

**Solution:**

$$f(g(x)) = x$$

$$f'(g(x))g'(x) = 1$$

Put  $x = b$

$$f'(g(b))g'(b) = 1$$

$$f'(a) \times 5 = 1$$

$$f'(a) = \frac{1}{5}$$

10. Let  $x + 6y = 8$  is tangent to standard ellipse where minor axis is  $\frac{4}{\sqrt{3}}$ , then eccentricity of ellipse is

a.  $\frac{1}{4}\sqrt{\frac{11}{12}}$

b.  $\frac{1}{4}\sqrt{\frac{11}{3}}$

c.  $\sqrt{\frac{5}{6}}$

d.  $\sqrt{\frac{11}{12}}$

**Answer:** (d)

**Solution:**

$$\text{If } 2b = \frac{4}{\sqrt{3}}$$

$$b = \frac{2}{\sqrt{3}}$$

$$\text{Comparing } y = -\frac{x}{6} + \frac{8}{6} \text{ with } y = mx \pm \sqrt{a^2m^2 + b^2}$$

$$m = -\frac{1}{6} \text{ and } a^2m^2 + b^2 = \frac{16}{9}$$

$$\frac{a^2}{36} + \frac{4}{3} = \frac{16}{9}$$

$$\Rightarrow \frac{a^2}{36} = \frac{16}{9} - \frac{4}{3}$$

$$\Rightarrow a^2 = 16$$

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$\Rightarrow e = \sqrt{\frac{11}{12}}$$

11. If one end of focal chord of parabola  $y^2 = 8x$  is  $\left(\frac{1}{2}, -2\right)$ , then the equation of tangent at the other end of this focal chord is

a.  $x + 2y + 8 = 0$

b.  $x - 2y = 8$

c.  $x - 2y + 8 = 0$

d.  $x + 2y = 8$

**Answer:** (c)

**Solution:**

Let  $PQ$  be the focal chord of the parabola  $y^2 = 8x$

$$\Rightarrow P(t_1) = (2t_1^2, 4t_1) \text{ \& } Q(t_2) = (2t_2^2, 4t_2)$$

$$\Rightarrow t_1 t_2 = -1$$

$\therefore \left(\frac{1}{2}, -2\right)$  is one of the ends of the focal chord of the parabola

$$\text{Let } \left(\frac{1}{2}, -2\right) = (2t_2^2, 4t_2)$$

$$\Rightarrow t_2 = -\frac{1}{2}$$

$\Rightarrow$  Other end of focal chord will have parameter  $t_1 = 2$

$\Rightarrow$  The co-ordinate of the other end of the focal chord will be  $(8, 8)$

$\therefore$  The equation of the tangent will be given as  $\rightarrow 8y = 4(x + 8)$

$$\Rightarrow 2y - x = 8$$

12. If  $7x + 6y - 2z = 0$ ,  $3x + 4y + 2z = 0$  &  $x - 2y - 6z = 0$ , then the system of equations has

a. No solution

b. Infinite non-trivial solution for  $(x = 2z)$

c. Infinite non-trivial solution for  $(y = 2z)$

d. Only trivial solution

**Answer:** (b)

**Solution:**

$$7x + 6y - 2z = 0$$

$$3x + 4y + 2z = 0$$

$$x - 2y - 6z = 0$$

As the system of equations are Homogeneous  $\Rightarrow$  the system is consistent.

$$\Rightarrow \begin{vmatrix} 7 & 6 & -2 \\ 3 & 4 & 2 \\ 1 & -2 & -6 \end{vmatrix} = 0$$

$\Rightarrow$  Infinite solutions exist (both trivial and non-trivial solutions)

When  $y = 2z$

Let's take  $y = 2, z = 1$

When  $(x, 2, 1)$  is substituted in the system of equations

$$\Rightarrow 7x + 10 = 0$$

$$3x + 10 = 0$$

$$x - 10 = 0 \text{ (which is not possible)}$$

$\therefore y = 2z \Rightarrow$  Infinite non-trivial solutions does not exist.

For  $x = 2z$ , let's take  $x = 2, z = 1, y = y$

Substitute  $(2, y, 1)$  in system of equations

$$\Rightarrow y = -2$$

$\therefore$  For each pair of  $(x, z)$ , we get a value of  $y$ .

Therefore, for  $x = 2z$  infinite non-trivial solution exists.

13. If both the roots of the equation  $ax^2 - 2bx + 5 = 0$  are  $\alpha$  and of the equation  $x^2 - 2bx - 10 = 0$  are  $\alpha$  and  $\beta$ . Then the value of  $\alpha^2 + \beta^2$

- |       |       |
|-------|-------|
| a. 15 | b. 20 |
| c. 25 | d. 30 |

**Answer:** (c)

**Solution:**

$ax^2 - 2bx + 5 = 0$  has both roots as  $\alpha$

$$\Rightarrow 2\alpha = \frac{2b}{a} \Rightarrow \alpha = \frac{b}{a}$$

$$\text{And } \alpha^2 = \frac{5}{a}$$

$$\Rightarrow b^2 = 5a(a \neq 0) \quad \dots (1)$$

$$\Rightarrow \alpha + \beta = 2b \text{ \& } \alpha\beta = -10$$

$\alpha = \frac{b}{a}$  is also a root of  $x^2 - 2bx - 10 = 0$

$$\Rightarrow b^2 - 2ab^2 - 10a^2 = 0$$

$$\because b^2 = 5a \Rightarrow 5a - 10a^2 - 10a^2 = 0$$

$$\Rightarrow a = \frac{1}{4} \Rightarrow b^2 = \frac{5}{4}$$

$$\Rightarrow \alpha^2 = 20, \beta^2 = 5$$

$$\Rightarrow \alpha^2 + \beta^2 = 25$$

14. If  $A = \{x: |x| < 2$  and  $B = \{x: |x - 2| \geq 3\}$  then

a.  $A \cap B = [-2, -1]$

c.  $A - B = [-1, 2)$

b.  $B - A = \mathbf{R} - (-2, 5)$

d.  $A \cup B = \mathbf{R} - (2, 5)$

**Answer:** (b)

**Solution:**

$$A = \{x: x \in (-2, 2)\}$$

$$B = \{x: x \in (-\infty, -1] \cup [5, \infty)\}$$

$$A \cap B = \{x: x \in (-2, -1]\}$$

$$B - A = \{x: x \in (-\infty, -2] \cup [5, \infty)\}$$

$$A - B = \{x: x \in (-1, 2)\}$$

$$A \cup B = \{x: x \in (-\infty, 2) \cup [5, \infty)\}$$





$$\sqrt{A+5} = 3$$

$$\sqrt{A+1} = \sqrt{5}$$

$$\sqrt{A+21} = 5$$

$$\sqrt{A} = 2$$

$\therefore$  Points of discontinuity for  $f(x)$  is  $x = \sqrt{5}$

19. Circles  $(x-0)^2 + (y-4)^2 = k$  and  $(x-3)^2 + (y-0)^2 = 1^2$  touch each other. The maximum value of  $k$  is \_\_\_\_\_.

**Answer:** (36)

**Solution:**

Two circles touch each other if  $C_1 C_2 = |r_1 \pm r_2|$

$$\sqrt{k} + 1 = 5 \text{ or } |\sqrt{k} - 1| = 5$$

$$\Rightarrow k = 16 \text{ or } 36$$

Maximum value of  $k$  is 36

20. If  ${}^{25}C_0 + 5{}^{25}C_1 + 9{}^{25}C_2 + \dots + 101{}^{25}C_{25} = 2^{25}k$ , then the value of  $k$  is \_\_\_\_\_.

**Answer:** (51)

**Solution:**

$$S = {}^{25}C_0 + 5{}^{25}C_1 + 9{}^{25}C_2 + \dots + 97{}^{25}C_{24} + 101{}^{25}C_{25} = 2^{25}k \quad (1)$$

Reverse and apply property  ${}^nC_r = {}^nC_{n-r}$  in all coefficients

$$S = 101{}^{25}C_0 + 97{}^{25}C_1 + \dots + 5{}^{25}C_{24} + {}^{25}C_{25} \quad (2)$$

Adding (1) and (2), we get

$$2S = 102[{}^{25}C_0 + {}^{25}C_1 + \dots + {}^{25}C_{25}]$$

$$S = 51 \times 2^{25}$$

$$\Rightarrow k = 51$$

21. Number of common terms in both the sequences 3, 7, 11, ... 407 and 2, 9, 16, ... 905 is \_\_\_\_\_.

**Answer:** (14)

**Solution:**

First common term is 23

Common difference =  $\text{LCM}(7, 4) = 28$

$$23 + (n - 1)28 \leq 407$$

$$n - 1 \leq 13.71$$

$$n = 14$$

22. Let  $|\vec{a}| = \sqrt{3}$ ,  $|\vec{b}| = 5$ ,  $\vec{b} \cdot \vec{c} = 10$  and angle between  $\vec{b}$  and  $\vec{c}$  is equal to  $\frac{\pi}{3}$ . If  $\vec{a}$  is perpendicular to  $\vec{b} \times \vec{c}$ , then the value of  $|\vec{a} \times (\vec{b} \times \vec{c})|$  is

**Answer:** (30)

**Solution:**

$$|\vec{a} \times (\vec{b} \times \vec{c})| = |\vec{a}| |\vec{b} \times \vec{c}| \sin \theta \quad \text{where } \theta \text{ is the angle between } \vec{a} \text{ and } \vec{b} \times \vec{c}$$

$$\theta = \frac{\pi}{2} \text{ given}$$

$$\Rightarrow |\vec{a} \times (\vec{b} \times \vec{c})| = \sqrt{3} |\vec{b} \times \vec{c}| = \sqrt{3} |\vec{b}| |\vec{c}| \sin \frac{\pi}{3}$$

$$\Rightarrow |\vec{a} \times (\vec{b} \times \vec{c})| = \sqrt{3} \times 5 \times |\vec{c}| \times \frac{\sqrt{3}}{2}$$

$$\Rightarrow |\vec{a} \times (\vec{b} \times \vec{c})| = \frac{15}{2} |\vec{c}|$$

$$\text{Now, } |\vec{b}| |\vec{c}| \cos \theta = 10$$

$$5 |\vec{c}| \frac{1}{2} = 10$$

$$|\vec{c}| = 4$$

23. If minimum value of term free from  $x$  for  $\left(\frac{x}{\sin \theta} + \frac{1}{x \cos \theta}\right)^{16}$  is  $L_1$  in  $\theta \in \left[\frac{\pi}{8}, \frac{\pi}{4}\right]$  and  $L_2$  in  $\theta \in \left[\frac{\pi}{16}, \frac{\pi}{8}\right]$ , the value of  $\frac{L_2}{L_1}$  is

**Answer:** (16)

**Solution:**

$$T_{r+1} = {}^{16}C_r \left( \frac{x}{\sin \theta} \right)^{16-r} \left( \frac{1}{x \cos \theta} \right)^r$$

For term independent of  $x$ ,

$$16 - 2r = 0 \Rightarrow r = 8$$

$$T_9 = {}^{16}C_8 \left( \frac{1}{\sin \theta \cos \theta} \right)^8 = {}^{16}C_8 2^8 \left( \frac{1}{\sin 2\theta} \right)^8$$

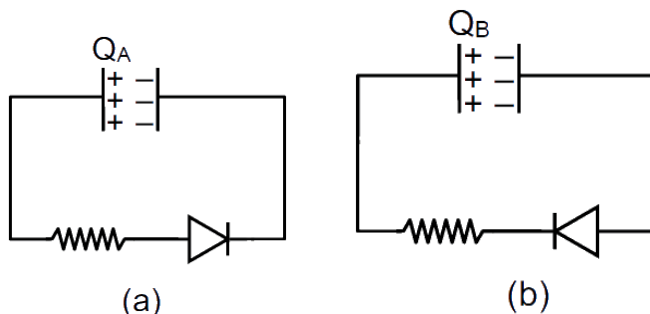
$$L_1 = {}^{16}C_8 2^8 \quad \text{at } \theta = \frac{\pi}{4}$$

$$L_2 = {}^{16}C_8 \frac{2^8}{\left( \frac{1}{\sqrt{2}} \right)^8} = {}^{16}C_8 2^{12} \quad \text{at } \theta = \frac{\pi}{8}$$

$$\frac{L_2}{L_1} = 16$$



The charge on the capacitor after time  $RC$  in (a) and (b) respectively is  $Q_A$  and  $Q_B$ . Their values are



- a.  $\frac{5CV}{e}, 5CV$   
c.  $5CV, 5CV$

- b.  $\frac{5CV}{e}, \frac{5CV}{2e}$   
d.  $5CV, \frac{5CV}{e}$

Solution:(a)

Maximum charge on capacitor =  $5CV$

is forward biased and (b) is reverse biased

For case (a)

$$q = q_{max}(1 - e^{\frac{-t}{RC}}) = 5CV$$

$$Q_A = 5CVe^{-1}$$

For case (b)

$$Q_B = 5CV$$

3. Different values of  $a, b$  and  $c$  are given and their sum is  $d$ . Arrange the value of  $d$  in increasing order

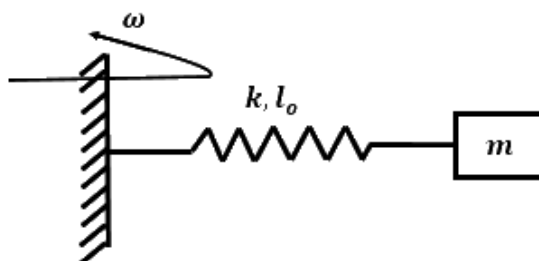
	$a$	$b$	$c$
1	220.1	20.4567	40.118
2	218.2	22.3625	40.372
3	221.2	20.2435	39.432
4	221.4	18.3625	40.281

- e.  $d_1 = d_2 = d_3 = d_4$   
g.  $d_1 > d_2 > d_3 > d_4$

- f.  $d_1 < d_2 < d_3 < d_4$   
h.  $d_4 < d_1 < d_3 = d_2$

Solution:(d)





Using Newton's second law of dynamics,

$$m\omega^2(l_0 + x) = kx$$

$$\left(\frac{l_0}{x} + 1\right) = \frac{k}{m\omega^2}$$

$$x = \frac{l_0 m \omega^2}{k - m \omega^2}$$

$$k \gg m \omega^2$$

So,  $\frac{x}{l_0}$  is equal to  $\frac{m \omega^2}{k}$

6. A loop of radius  $R$  and mass  $m$  is placed in a uniform magnetic field  $B$  with its plane perpendicular to the field. A current  $i$  is flowing in it. Now the loop is slightly rotated about its diameter and released. Find the time period of oscillations.

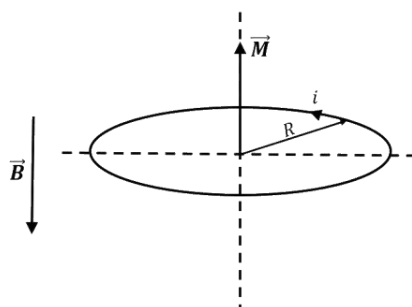
a.  $2\sqrt{\frac{\pi M}{iB}}$

b.  $\sqrt{\frac{2\pi M}{iB}}$

c.  $2\sqrt{\frac{M}{\pi iB}}$

d.  $\sqrt{\frac{M}{\pi iB}}$

Solution: (b)



Considering the torque situation on the loop,

$$\tau = MB \sin \theta = -i\alpha$$

$$\pi R^2 i B \theta = -\frac{mR^2}{2} \alpha$$

The above equation is analogous to  $\theta = -C\alpha$ , where  $C = \omega^2 = \frac{2\pi i B}{M}$

$$\omega = \sqrt{\frac{2\pi i B}{M}} = \frac{2\pi}{T}$$

$$T = \sqrt{\frac{2\pi M}{iB}}$$

7. A sphere of density  $\rho$  is half submerged in a liquid of density  $\sigma$  and surface tension  $T$ . The sphere remains in equilibrium. Find the radius of this sphere. (Assume the force due to surface tension acts tangentially to surface of sphere.)

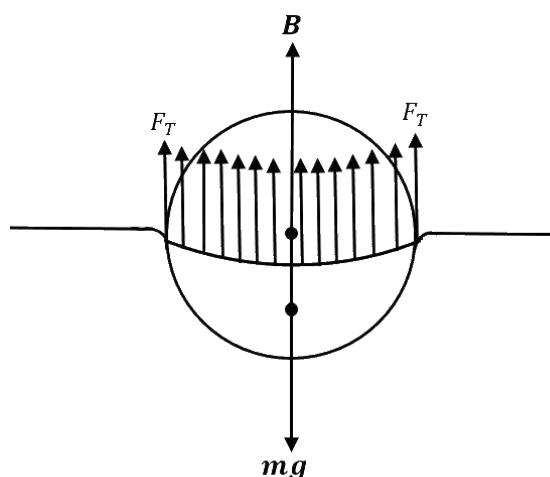
a.  $\sqrt{\frac{3T}{2(\rho+\sigma)g}}$

b.  $\sqrt{\frac{3T}{4(\rho-\sigma)g}}$

c.  $\sqrt{\frac{3T}{2(\rho-\frac{\sigma}{2})g}}$

d.  $\sqrt{\frac{T}{(\rho+\sigma)g}}$

Solution:(c)



In equilibrium, net external force acting on the sphere is zero.

$$mg = F_T + B$$



EM wave is in direction  $\frac{\hat{i} + \hat{j}}{2}$

Electric field is in direction  $\hat{k}$

Direction of propagation of EM wave is given by  $\vec{E} \times \vec{B}$

10. Two gases Ar (40) and Xe (131) at equal temperature have the same number density. Their diameters are 0.07 nm and 0.10 nm respectively. Find the ratio of their mean free time
- |         |         |
|---------|---------|
| a. 1.03 | b. 2.04 |
| c. 2.09 | d. 2.49 |

Solution: (a)

$$\text{Mean free time} = \frac{1}{\sqrt{2} n \pi d^2}$$

$$\frac{t_{Ar}}{t_{Xe}} = \frac{d_{Xe}^2}{d_{Ar}^2} = \left( \frac{0.1}{0.07} \right)^2 = \left( \frac{10}{7} \right)^2 = 2.04$$

11. When the same mass is suspended from two steel rods, the ratio of their energy densities is 1: 4. If the lengths of both the rods are equal, then the ratio of their diameters will be
- |                  |                  |
|------------------|------------------|
| a. $\sqrt{2}: 1$ | b. $1: \sqrt{2}$ |
| c. $1: 2$        | d. $2: 1$        |

Solution: (a)

$$\begin{aligned} \frac{dU}{dV} &= \frac{1}{2} \times \text{stress} \times \frac{\text{stress}}{Y} \\ &= \frac{1}{2} \times \frac{F^2}{A^2 Y} \\ \frac{dU}{dV} &\propto \frac{1}{D^4} \\ \frac{\left( \frac{dU}{dV} \right)_1}{\left( \frac{dU}{dV} \right)_2} &= \frac{D_2^4}{D_1^4} = \frac{1}{4} \end{aligned}$$



c.  $\frac{l}{4} \left( \frac{a+b}{3a+b} \right)$

d.  $l \left( \frac{a+b}{3a+b} \right)$

Solution:(b)

Here we take a small element along the length as  $dx$  at a distance  $x$  from the left end as shown.

$$\begin{aligned} x_{cm} &= \frac{1}{M} \int_0^l x \cdot dm \\ \Rightarrow dM &= \lambda \cdot dx = \left( a + b \left( \frac{x}{l} \right)^2 \right) \cdot dx \\ x_{cm} &= \frac{\int x \lambda dx}{\int \lambda dx} = \frac{\int_0^l x \left( a + \frac{bx^2}{l^2} \right) dx}{\int_0^l \left( a + \frac{bx^2}{l^2} \right) dx} \\ &= \frac{a \left( \frac{x^2}{2} \right)_0^l + \frac{b}{l^2} \left( \frac{x^4}{4} \right)_0^l}{a(x)_0^l + \frac{b}{l^2} \left( \frac{x^3}{3} \right)_0^l} \\ &= \frac{\frac{al^2}{2} + \frac{bl^2}{4}}{al + \frac{bl}{3}} \\ &= \frac{3l}{4} \left( \frac{2a+b}{3a+b} \right) \end{aligned}$$

14. A particle is projected from the ground with a speed  $u$  at an angle of  $60^\circ$  from horizontal. It collides with a second particle of equal mass moving with a horizontal speed  $u$  in the same direction at the highest point of its trajectory. If the collision is perfectly inelastic then find the horizontal distance travelled by them after this collision when they reached the ground.

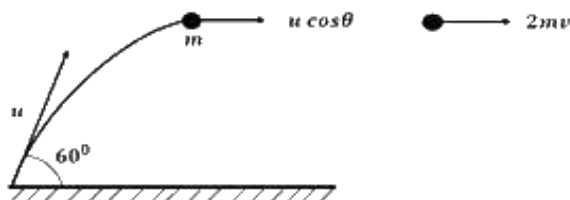
a.  $\frac{3\sqrt{6}u^2}{8g}$

b.  $\frac{3\sqrt{3}u^2}{8g}$

c.  $\frac{u^2}{8g}$

d.  $\frac{\sqrt{3}u^2}{g}$

Solution: (b)



The only external force acting on the colliding system during the collision is the gravitational force. Since gravitational force is non-impulsive, the linear momentum of the system is conserved just before and just after the collision.

$$p_i = p_f$$

$$mu + mu \cos \theta = 2mv$$

$$\Rightarrow v = \frac{u(1 + \cos 60^\circ)}{2} = \frac{3}{4}u$$

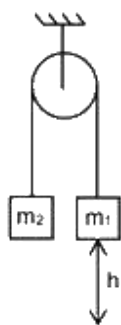
So the horizontal range after the collision  $= vt$

$$= v \sqrt{\frac{2H_{\max}}{g}}$$

$$= \frac{3}{4}u \sqrt{\frac{2u^2 \sin^2 60^\circ}{2g^2}}$$

$$= \frac{3}{4}u^2 \frac{\sqrt{3}}{g} = \frac{3\sqrt{3}u^2}{8g}$$

15. System is released from rest. Moment of inertia of pulley is  $I$ . Find angular speed of pulley when  $m_1$  block falls by  $h$ . (Given  $m_1 > m_2$  and assume no slipping between string and pulley)



a.  $\frac{1}{R} \sqrt{\frac{2(m_1 - m_2)gh}{m_1 + m_2 + \frac{I}{R^2}}}$

b.  $\frac{1}{R} \sqrt{\frac{4(m_2 + m_1)gh}{m_1 + m_2 + \frac{I}{R^2}}}$

$$c. \frac{1}{R} \sqrt{\frac{(m_1 - m_2)gh}{m_1 + m_2 + \frac{1}{R^2}}}$$

$$d. \frac{1}{R} \sqrt{\frac{2(m_2 + m_1)gh}{m_1 + m_2 + \frac{1}{R^2}}}$$

Solution: (a)

Assume initial potential energy of the blocks to be zero. Initial kinetic energy is also zero since the blocks are at rest.

When block  $m_1$  falls by  $h$ ,  $m_2$  goes up by  $h$  (because of length constraint)

Final P.E =  $m_2gh - m_1gh$

Let the final speed of the blocks be  $v$  and angular velocity of the pulley be  $\omega$

Final K.E =  $\frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}I\omega^2$

Total energy is conserved. Hence,

$$0 = m_2gh - m_1gh + \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}I\omega^2$$

$v = \omega r$  (due to no slip condition)

$$\Rightarrow \frac{1}{2}(m_1 + m_2)\omega^2 R^2 + \frac{1}{2}I\omega^2 = (m_1 - m_2)gh$$

$$\Rightarrow \omega^2 \left[ \frac{1}{2}(m_1 + m_2)R^2 + \frac{1}{2}I \right] = (m_1 - m_2)gh$$

$$\Rightarrow \omega^2 = \frac{2(m_1 - m_2)gh}{R^2 \left[ (m_1 + m_2) + \frac{I}{R^2} \right]}$$

$$\Rightarrow \omega = \frac{1}{R} \sqrt{\frac{2(m_1 - m_2)gh}{\left[ (m_1 + m_2) + \frac{I}{R^2} \right]}}$$

16. An H-like atom has its ionization energy equal to  $9R$ . Find the wavelength of light emitted (in  $nm$ ) when an electron jumps from the second excited state to the ground state. ( $R$  is Rydberg constant)

- a. 12.40  
c. 5.80

- b. 11.39  
d. 22.76

Solution:(b)

$$\frac{hc}{\lambda} = (13.6 \text{ eV})Z^2 \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$n_1 = 1$$

$$n_2 = 3$$

For an H-like atom, ionization energy is  $(R)Z^2$ .

This gives  $Z = 3$

$$\frac{hc}{\lambda} = (13.6 \text{ eV})(3^2) \left( \frac{1}{1^2} - \frac{1}{3^2} \right)$$

$$\Rightarrow \frac{hc}{\lambda} = (13.6 \text{ eV})(9) \times \frac{8}{9}$$

$$\text{wavelength} = \frac{1240}{8 \times 13.6} \text{ nm}$$

$$\lambda = 11.39 \text{ nm}$$

17. A point source is placed at a depth  $h$  in a liquid of refractive index is  $\frac{4}{3}$ . Find the percentage of energy of light that escapes from the liquid. (Assuming 100 % transmission of emerging light)

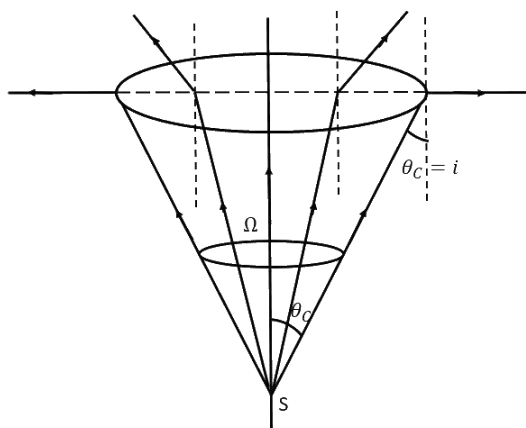
a. 17 %

b. 25 %

c. 10 %

d. 8 %

Solution:(a)



The portion of light escaping into the air from the liquid will form a cone. As long as the angle of incidence on the liquid – air interface is less than the critical angle, i.e.  $i < \theta_c$ , the light rays will undergo refraction and emerge into the air.

For  $i > \theta_c$ , the light rays will suffer TIR. So, these rays will not emerge into the air.

The portion of light rays emerging into the air from the liquid will form a cone of half angle  $= \theta_c$

$$\sin \theta_c = \frac{1}{\mu_{Liq}} = \frac{3}{4}, \quad \cos \theta_c = \frac{\sqrt{7}}{4}$$

Solid angle contained in this cone is

$$\Omega = 2\pi(1 - \cos \theta_c)$$

$$\text{Percentage of light that escapes from liquid} = \frac{\Omega}{4\pi} \times 100$$

Putting values we get

$$\text{Percentage} = \frac{4-\sqrt{7}}{8} \times 100 \approx 17\%$$

18. An electron  $(-|e|, m)$  is released in Electric field  $E$  from rest. Rate of change of de-Broglie wavelength with time will be.

a.  $-\frac{h}{2|e|}$

b.  $-\frac{h}{2|e|t}$

c.  $-\frac{h}{|e|Et^2}$

d.  $-\frac{2ht^2}{|e|E}$

Solution:(c)

$$\lambda_D = \frac{h}{mv}$$

where,  $v = at$

$$v = \frac{eE}{m} t \quad (a = \frac{eE}{m})$$

$$\lambda_D = \frac{h}{m \frac{eE}{m} t}$$

$$\lambda_D = \frac{h}{eEt}$$

$$\frac{d\lambda_D}{dt} = \frac{h}{|e|Et^2}$$

19. An AC source is connected to the LC series circuit with  $V = 10 \sin(314t)$ . Find the current in the circuit as function of time ? ( $L = 40 \text{ mH}$ ,  $C = 100 \mu\text{F}$ )

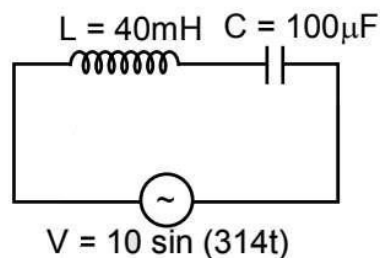
a.  $10.4 \sin(314t)$

b.  $0.52 \cos(314t)$

c.  $0.52 \sin(314t)$

d.  $5.2 \cos(314t)$

Solution:(b)



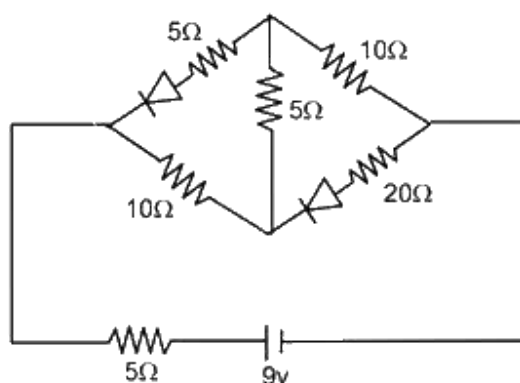
$$\begin{aligned}
 \text{Impedance } Z &= \sqrt{R^2 + (X_C - X_L)^2} \\
 &= \sqrt{(X_C - X_L)^2} \\
 &= X_C - X_L \\
 &= \frac{1}{\omega C} - \omega L \\
 &= \frac{1}{314 \times 100 \times 10^{-6}} - 314 \times 40 \times 10^{-3} \\
 &= 31.84 - 12.56 = 19.28 \Omega
 \end{aligned}$$

For  $X_C > X_L$ , current leads voltage by  $\frac{\pi}{2}$

$$\begin{aligned}
 \therefore i &= \frac{V}{Z} = \frac{10 \sin(314t + \frac{\pi}{2})}{19.28} \\
 &= 0.52 \cos(314t)
 \end{aligned}$$

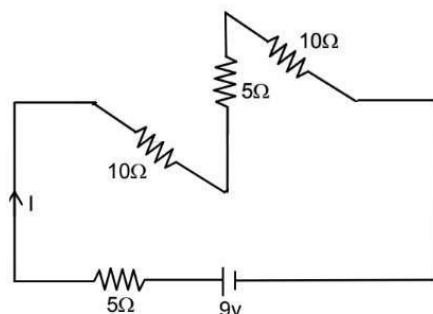
20. Find the current supplied by the battery.

- |          |          |
|----------|----------|
| a. 0.1 A | b. 0.3 A |
| c. 0.4 A | d. 0.5 A |



Solution:(b)

Since the diodes are reverse biased, they will not conduct.  
Hence, the circuit will look like

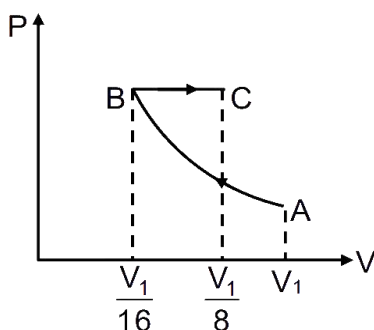


$$R_{eff} = 5 + 10 + 5 + 10 = 30 \Omega$$

$$I = \frac{9}{30} = 0.3 A$$

21. An ideal gas at an initial temperature  $300 K$  is compressed adiabatically ( $\gamma = 1.4$ ) to its initial volume. The gas is then expanded isobarically to double its volume. Then the final temperature of the gas rounded off to nearest integer is.

Solution: (1819 K)



$$PV^\gamma = \text{Constant}$$

$$TV^{(\gamma-1)} = \text{constant}$$

$$300(V_1)^{(1.4-1)} = T_B \left(\frac{V_1}{16}\right)^{\frac{2}{5}}$$

$$T_B = 300 \times 2^{\left(\frac{8}{5}\right)}$$

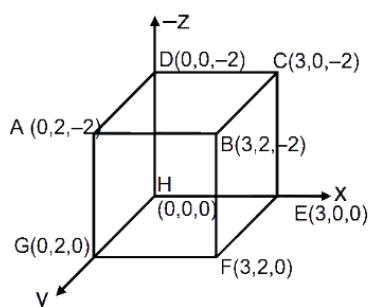
Now for BC process

$$\frac{V_B}{T_B} = \frac{V_C}{T_C}$$

$$T_C = \frac{V_C T_B}{T_B} = 2 \times 300 \times 2^{\left(\frac{8}{5}\right)}$$

$$T_C = 1819 \text{ K}$$

22. If electric field in the space is given by  $\vec{E} = 4x\hat{i} - (y^2 + 1)\hat{j}$ , and electric flux through ABCD is  $\phi_1$  and electric flux through BCEF is  $\phi_2$ , then find  $(\phi_1 - \phi_2)$ .



Solution: (-40)

Electric flux through a surface is  $\phi = \int \vec{E} \cdot d\vec{A}$

For surface ABCD,

$d\vec{A}$  is along  $(-\hat{k})$

So, at all the points of this surface,

$$\vec{E} \cdot d\vec{A} = 0$$

Because,  $\phi_{ABCD} = \phi_1 = 0$

For surface BCEF,

$d\vec{A}$  is along  $(\hat{i})$

So,

$$\vec{E} \cdot d\vec{A} = E_x dA$$

$$\phi_{BCEF} = \phi_2 = 4x(2 \times 2)$$

If  $x = 3$

$$\phi_2 = 48 \frac{N-m^2}{C}$$

Hence,  $\phi_1 - \phi_2 = -48 \frac{N-m^2}{C}$

23. In a YDSE interference pattern obtained with light of wavelength  $\lambda_1 = 500 \text{ nm}$ , 15 fringes are obtained on a certain segment of screen. If number of fringes for light of wavelength  $\lambda_2$  on same segment of screen is 10, then the value of  $\lambda_2$  (in nm) is

Solution:(750)

If the length of the segment is  $y$ ,

Then  $y = n \beta$

$n$  = no. of fringes,

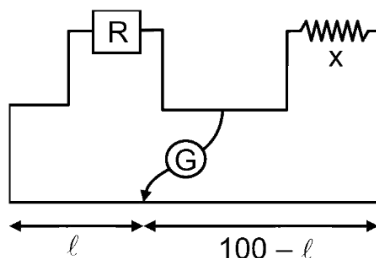
$\beta$  = fringe width

$$15 \times 500 \times \frac{D}{d} = 10 \times \lambda_2 \times \frac{D}{d}$$

$$\lambda_2 = 15 \times 50 \text{ nm}$$

$$\lambda_2 = 750 \text{ nm}$$

24. In a meter bridge experiment, the balancing length was 25 cm for the situation shown in the figure. If the length and diameter of the wire of resistance  $R$  is halved, then the new balancing length in centimetre is



Solution:(40)

$$\frac{X}{R} = \frac{75}{25} = 3$$

$$R = \frac{\rho l}{A} = \frac{4\rho l}{\pi d^2}$$

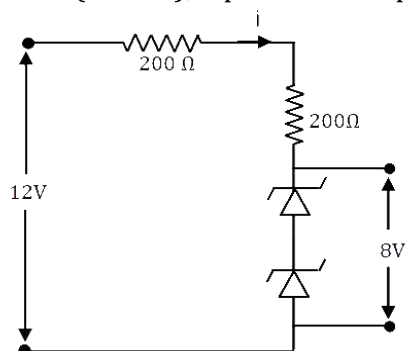
$$R' = \frac{4\rho \frac{l}{2}}{\pi \left(\frac{d}{2}\right)^2} = 2R$$

$$\text{Then } \frac{X}{R'} = \left(\frac{100-l}{l}\right)$$

$$\frac{100-l}{l} = \frac{X}{2R} = \frac{3}{2}$$

$$l = 40.00 \text{ cm}$$

25. Find the power loss in each diode (in  $mW$ ), if potential drop across the Zener diode is  $8V$ .



Solution:(40)

$$i = \left( \frac{12-8}{200+200} \right) A = \frac{4}{400} = 10^{-2} A$$

$$\text{Power loss in each diode} = (4)(10^{-2}) W = 40 mW$$







Complex (III) has the central metal ion as  $\text{Co}^{2+}$  with weak field ligands.

Configuration of  $\text{Co}^{2+} = [\text{Ar}] 3d^7$

As weak field ligands are present no pairing can occur. There will be 3 unpaired electrons and hence the magnetic moment  $= \sqrt{15}$  B.M.

Complex (IV) has the central metal ion as  $\text{Fe}^{3+}$  with strong field ligands.

Configuration of  $\text{Fe}^{3+} = [\text{Ar}] 3d^5$

Strong field ligands will pair up the electrons but as we have a  $[\text{Ar}] 3d^5$  configuration, one electron will remain unpaired and hence the magnetic moment will be  $\sqrt{3}$  B.M.

7. Select the correct option:
- Entropy is a function of temperature and also entropy change is a function of temperature.
  - Entropy is a function of temperature & entropy change is not a function of temperature.
  - Entropy is not a function of temperature & entropy change is a function of temperature.
  - Both entropy & entropy change are not a function of temperature.

**Answer:** a

**Solution:**

Entropy is a function of temperature, at any temperature, the entropy can be given as:

$$S_T = \int_0^T \frac{nCdT}{T}$$

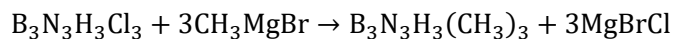
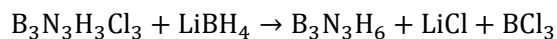
Change in entropy is also a function of temperature, at any temperature, the entropy change can be given as:

$$\Delta S = \int \frac{dq}{T}$$

8. A compound (A:  $\text{B}_3\text{N}_3\text{H}_3\text{Cl}_3$ ) reacts with  $\text{LiBH}_4$  to form inorganic benzene (B). (A) reacts with (C) to form  $\text{B}_3\text{N}_3\text{H}_3(\text{CH}_3)_3$ . (B) and (C) respectively are:
- Boron nitride,  $\text{MeMgBr}$
  - Boron nitride,  $\text{MeBr}$
  - Borazine,  $\text{MeBr}$
  - Borazine,  $\text{MeMgBr}$

**Answer:** d

**Solution:**



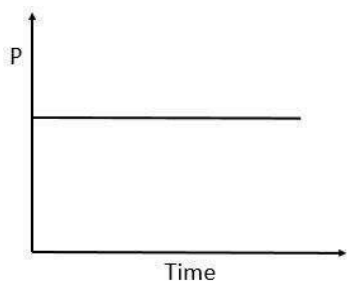
So, we can say that,

B is  $\text{B}_3\text{N}_3\text{H}_6$

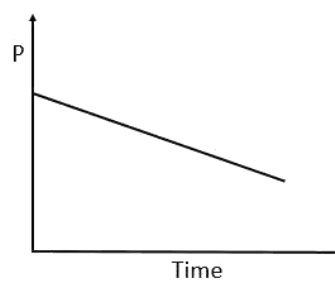
C is  $\text{CH}_3\text{MgBr}$

9. In a box, a mixture containing  $H_2$ ,  $O_2$  and  $CO$  along with charcoal is present. Then, the variation of pressure with time will be:

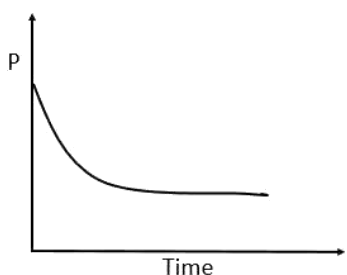
a.



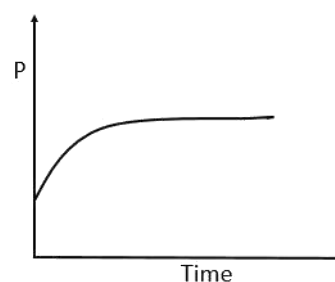
b.



c.



d.



**Answer: c**

**Solution:**

As  $H_2$ ,  $O_2$  and  $CO$  gets adsorbed on the surface of charcoal, the pressure decreases. So, option (a) and (d) can be eliminated. After some time, as almost all the surface sites are occupied, the pressure becomes constant.

10. Given the complex:  $[Co(NH_3)_4Cl_2]$ . If in this complex, the  $Cl-Co-Cl$  bond angle is  $90^\circ$ , then it is a:
- Cis-isomer
  - Trans-isomers
  - Meridional and Trans
  - Cis and trans both

**Answer: a**

**Solution:**

In cis-isomer, similar ligands are at an angle of  $90^\circ$ .

11. Amongst the following, which has the least conductivity?
- |   |               |
|---|---------------|
| a. Distilled water                              | b. Sea water  |
| c. Saline water used for intra venous injection | d. Well water |

**Answer:** a

**Solution:**

In distilled water there are no ions present except  $H^+$  and  $OH^-$  ions, both of which are immensely minute in concentration, that renders their collective conductivity negligible.

12. Number of  $sp^2$  hybrid orbitals in Benzene is:
- |       |       |
|-------|-------|
| a. 18 | b. 24 |
| c. 6  | d. 12 |

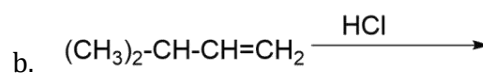
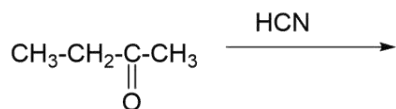
**Answer:** a

**Solution:**

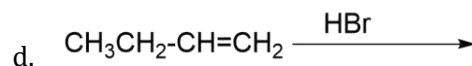
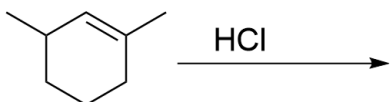
Benzene ( $C_6H_6$ ) has 6  $sp^2$  hybridized carbons. Each carbon has 3  $\sigma$ -bonds and 1  $\pi$ -bond. 3  $\sigma$ -bonds means that there are 3  $sp^2$  hybrid orbitals for each carbon. Hence, the total number of  $sp^2$  hybrid orbitals is 18.

13. Which of the following reaction will not give a racemic mixture as the product?

a.

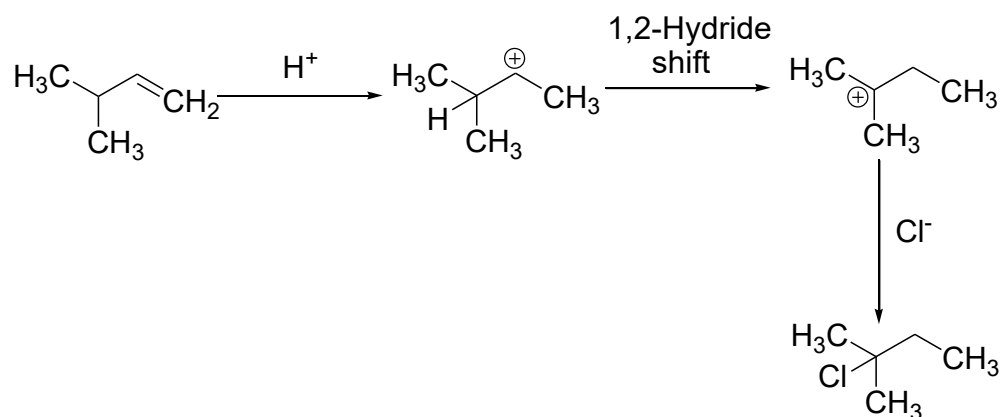


c.



**Answer: b**

**Solution:**

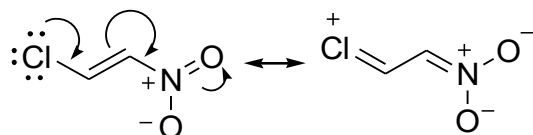


14. In which compound is the C-Cl bond length the shortest?
- $\text{Cl} - \text{CH} = \text{CH}_2$
  - $\text{Cl} - \text{CH} = \text{CH} - \text{CH}_3$
  - $\text{Cl} - \text{CH} = \text{CH} - \text{OCH}_3$
  - $\text{Cl} - \text{CH} = \text{CH} - \text{NO}_2$

**Answer:** d

**Solution:**

There is extended conjugation present in option (d), which will reduce the length of C-Cl bond to the greatest extent which can be represented as follows:



15. Biochemical oxygen demand (BOD) is defined as ..... in ppm of  $\text{O}_2$ .
- Required to sustain life.
  - The amount of oxygen required by bacteria to break down the organic matter present in a certain volume of a sample of water.
  - The amount of oxygen required by anaerobic bacteria to break down the inorganic matter present in a certain volume of a sample of water.
  - Required photochemical reaction to degrade waste.

**Answer:** b

**Solution:**

Biochemical oxygen demand (BOD) is the amount of dissolved oxygen used by microorganisms in the biological process of metabolizing organic matter in water.

16. Monomer(s) of which of the given polymer is chiral?
- Buna-S
  - Neoprene
  - Nylon-6,6
  - PHBV

**Answer:** d

**Solution:**

	Polymers	Monomers
Buna-S	$\left( \text{H}_2\text{C}-\underset{\text{Ph}}{\text{CH}}-\text{CH}=\text{CH}_2 \right)_n$	$\text{Ph}-\text{CH}=\text{CH}_2$ & $\text{H}_2\text{C}=\text{CH}-\text{CH}=\text{CH}_2$
Neoprene	$\left( \text{H}_2\text{C}-\underset{\text{Cl}}{\text{CH}}=\text{CH}_2 \right)_n$	$\text{H}_2\text{C}=\underset{\text{Cl}}{\text{CH}}-\text{CH}_2$
Nylon-6,6	$\left( \text{C}_6\text{H}_4-\text{C}(=\text{O})-\text{NH}-(\text{CH}_2)_4-\text{NH}-\text{C}(=\text{O})-\text{C}_6\text{H}_4 \right)_n$	$\text{Cl}-\text{C}(=\text{O})-\text{C}_6\text{H}_4-\text{C}(=\text{O})-\text{Cl}$ & $\text{H}_2\text{N}-(\text{CH}_2)_6-\text{NH}_2$
PHBV	$\left( \text{O}-\underset{\text{CH}_3}{\text{CH}}-\text{C}(=\text{O})-\text{O}-\underset{\text{C}_2\text{H}_5}{\text{CH}}-\text{C}(=\text{O}) \right)_n$	$\text{H}_3\text{C}-\underset{\text{OH}}{\text{CH}}-\text{C}(=\text{O})-\text{OH}$ & $\text{C}_2\text{H}_5-\underset{\text{OH}}{\text{CH}}-\text{C}(=\text{O})-\text{OH}$

17.

Lab tests			
Compound	Molisch's test	Barfoed's test	Biuret test
A	✓	x	x
B	✓	✓	x
C	x	x	✓

Which of the following options is correct?

- |    |         |          |         |
|----|---------|----------|---------|
|    | A       | B        | C       |
| a. | Lactose | Glucose  | Albumin |
| b. | Lactose | Glucose  | Alanine |
| c. | Lactose | Fructose | Alanine |
| d. | Glucose | Sucrose  | Albumin |

**Answer:** a

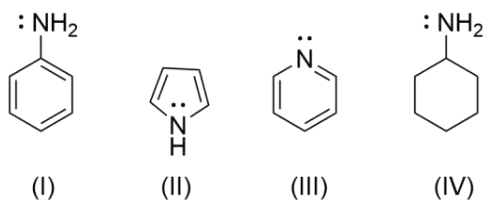
**Solution:**

Lactose, glucose and fructose gives positive Molisch's test.

Glucose gives positive Barfoed's test whereas sucrose gives a negative for Barfoed's test.

Albumin gives positive for Biuret test whereas alanine gives a negative Biuret test.

18. The order of basic character is:



a. I > II > III > IV

b. IV > III > I > II

c. II > I > III > IV

d. IV > I > II > III

**Answer:** b

**Solution:**

The basicity of the compound depends on the availability of the lone pairs.

In compound IV, Nitrogen is  $sp^3$  hybridized.

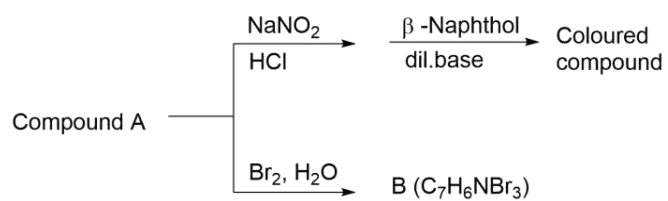
In compound III, Nitrogen is  $sp^2$  hybridized and the lone pairs are not involved in resonance.

In compound I, Nitrogen is  $sp^2$  hybridized and the lone pairs are involved in resonance.

In compound II, Nitrogen is  $sp^2$  hybridized and the lone pairs are involved in resonance such that, they are contributing to the aromaticity of the ring.

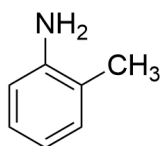
From the above points we can conclude that the basicity order should be IV > III > I > II.

19.

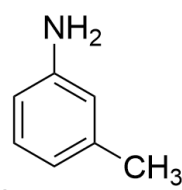


Compound A will be:

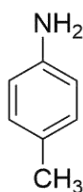
a.



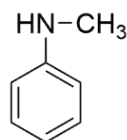
b.



c.

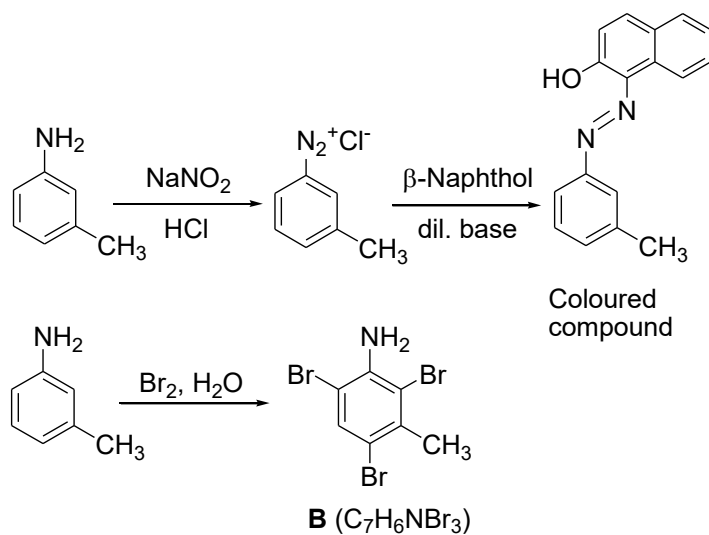


d.

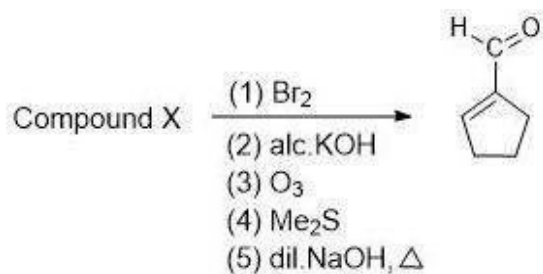


**Answer: b**

**Solution:**

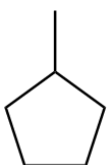


20.

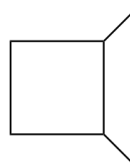


Compound X will be:

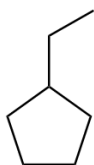
a.



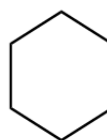
b.



c.

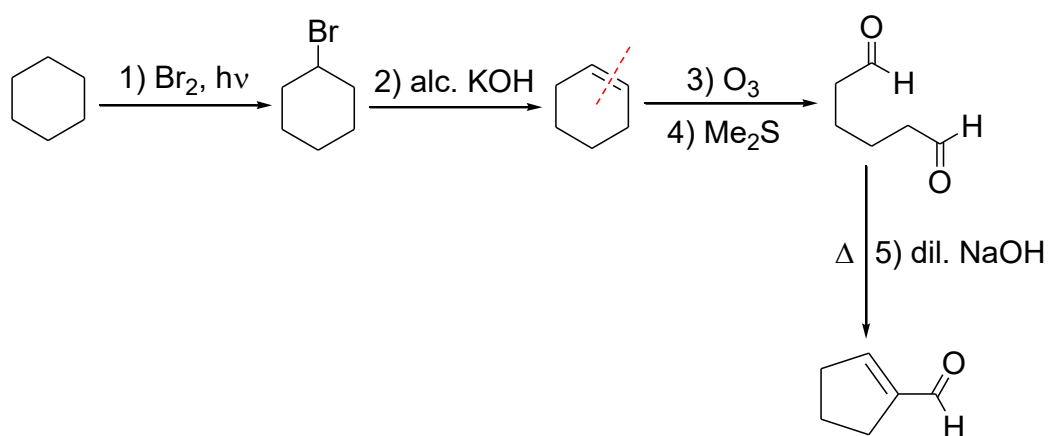


d.



**Answer: d**

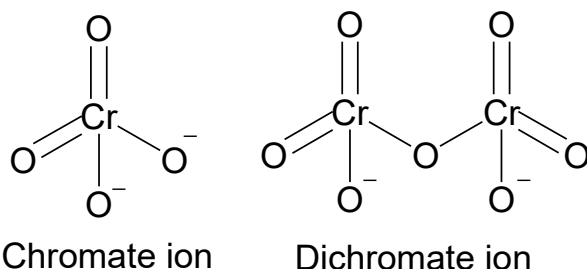
**Solution:**



21. Total number of Cr-O bonds in Chromate ion and Dichromate ion is:

**Answer: 12**

**Solution:**



22. Lacto bacillus has a generation time of 60 minutes at 300K and 40 minutes at 400K. Determine the activation energy in  $\frac{\text{kJ}}{\text{mol}}$ . ( $R = 8.3 \text{ J K}^{-1} \text{ mol}^{-1}$ ) [ $\ln\left(\frac{2}{3}\right) = -0.4$ ]

**Answer: 3.98**

**Solution:**

The generation time can be utilized to get an indication of the rate ratio. Let the amount generated be (x).

$$\text{Rate} = \frac{\text{Amount generated}}{\text{Time taken}}$$

$$\text{Rate}_{300 \text{ K}} = \frac{(x)}{60} \qquad \text{Rate}_{400 \text{ K}} = \frac{(x)}{40}$$

$$\frac{\text{Rate}_{300 \text{ K}}}{\text{Rate}_{400 \text{ K}}} = \frac{40}{60}$$

For the same concentration (which is applicable here), the rate ratio can also be equaled to the ratio of rate constants.

$$\ln \left[ \frac{K_{\text{at } 400 \text{ K}}}{K_{\text{at } 300 \text{ K}}} \right] = \frac{E_a}{R} \left[ \frac{1}{T_1} - \frac{1}{T_2} \right]$$

$$\ln \frac{60}{40} = \frac{E_a}{8.3} \left[ \frac{1}{300} - \frac{1}{400} \right]$$

$$E_a = 0.4 \times 8.3 \times 1200 = 3984 \text{ J/mol} = 3.98 \text{ kJ/mol}$$

23. One litre of sea water ( $d = 1.03 \frac{\text{g}}{\text{cm}^3}$ ) contains 10.3 mg of  $\text{O}_2$  gas. Determine the concentration of  $\text{O}_2$  in ppm:

**Answer:** 10.00

**Solution:**

$$\text{Ppm} = \frac{W_{\text{Solute}}}{W_{\text{Solution}}} \times 100$$

Using the density of the solution and its volume ( $1\text{L} = 1000\text{ mL} = 1000\text{ cm}^3$ ), the weight of the solution can be calculated.

$$W_{\text{solution}} = 1.03 \times 1000 = 1030\text{ g}$$

$$\text{Thus, ppm} = \frac{10.3 \times 10^{-3}\text{g}}{1030\text{ g}} \times 100$$

24. 0.1 mole of an ideal gas has volume  $1\text{ dm}^3$  in a locked box with a frictionless piston. The gas is in thermal equilibrium with an excess of 0.5 m aqueous ethylene glycol at its freezing point. If the piston is released all of a sudden at 1 atm, then determine the final volume of gas in  $\text{dm}^3$  ( $R = 0.08\text{ atm L mol}^{-1}\text{ K}^{-1}$ ;  $K_f = 2.0\text{ K molal}^{-1}$ )

**Answer:** 2.18

**Solution:**

$$K_f = 2$$

$$\text{Molality, 'm'} = 0.5$$

$$\Delta T_f = K_f \cdot m$$

$$= (0.5 \times 2) = 1$$

So, the initial temperature now becomes 272 K. Further using the given value of moles and initial volume of the gas and the calculated initial temperature value, we can find out the initial pressure of the ideal gas contained inside the piston.

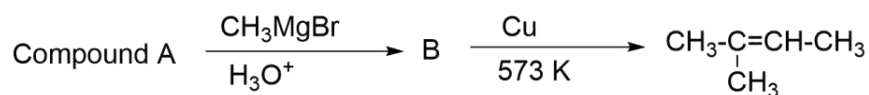
$$\begin{aligned} P_{\text{gas}} &= \frac{nRT}{V_1} \\ &= (0.1)(0.08)(272) = 2.176\text{ atm} \end{aligned}$$

Now, on releasing the piston against an external pressure of 1 atm, the gas will expand until the final pressure of the gas, i.e.  $P_2$  becomes equal to 1 atm. During this expansion, since no reaction is happening and the temperature of the gas is not changing as well, the boyle's law relation can be applied.

$$P_1V_1 = P_2V_2$$

$$2.176 \times 1 = 1 \times V_2$$

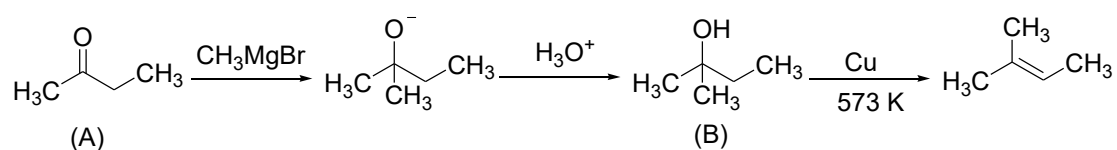
25.



The percentage of carbon in compound A is:

**Answer:** 66.67

**Solution:**



Compound A is  $\text{CH}_3(\text{CO})\text{CH}_2\text{CH}_3$  ( $\text{C}_4\text{H}_8\text{O}$ )

The percentage of carbon in compound A by weight is  $\frac{w_{\text{Carbon}}}{w_{\text{Compound}}} = \frac{12 \times 4}{72} = 66.67$

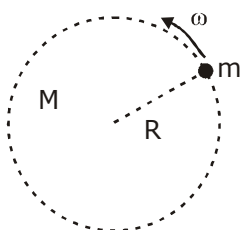
# QUESTION PAPER WITH SOLUTION

## PHYSICS - 2 Sep 2020. - SHIFT - 1

**Q.1** The mass density of a spherical galaxy K varies as  $\frac{K}{r}$  over a large distance 'r' from its centre. In that region, a small star is in a circular orbit of radius R. Then the period of revolution, T depends on R as:

- (1)  $T^2 \propto R$                       (2)  $T^2 \propto R^3$                       (3)  $T^2 \propto \frac{1}{R^3}$                       (4)  $T \propto R$

**Sol. (1)**



$$\text{Mass of galaxy} = \int_0^R \rho dv$$

$$= \int_0^R \frac{k}{r} 4\pi r^2 dr$$

$$= 4\pi k \int_0^R r dr$$

$$M = \frac{4\pi k R^2}{2} = k_1 R^2$$

$$F = m\omega^2 R$$

$$\frac{GMm}{R^2} = m\omega^2 R$$

$$\frac{Gk_1 R^2}{R^2} = \omega^2 R$$

$$\therefore \omega^2 = \frac{k_2}{R}$$

$$\omega = \sqrt{\frac{k_2}{R}}$$

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{R}{k_2}}$$

$$T = k_3 \sqrt{R}$$

$$T^2 \propto R$$

**Q.2** An amplitude modulated wave is represented by the expression  $v_m = 5(1 + 0.6 \cos 6280t) \sin (211 \times 10^4 t)$  volts. The minimum and maximum amplitudes of the amplitude modulated wave are, respectively :

- (1)  $\frac{3}{2}$  V, 5V                      (2) 5V, 8V                      (3) 3V, 5V                      (4)  $\frac{5}{2}$  V, 8V

**Sol. (4)**

$$\frac{A_m}{A_c} = 0.6$$

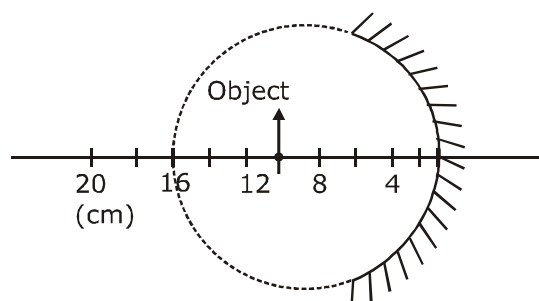
$$= (5 + 3 \cos 6280t) \sin (211 \times 10^4 t)$$

$$\text{maximum Amp.} = 5 + 3 = 8 \text{ V}$$

$$\text{minimum Amp.} = 5 - 3 = 2 \text{ V}$$

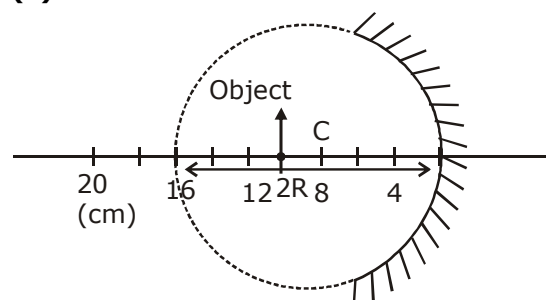
from the given option nearest value of minimum Amplitude =  $\frac{5}{2}$  V

**Q.3** A spherical mirror is obtained as shown in the figure from a hollow glass sphere. If an object is positioned in front of the mirror, what will be the nature and magnification of the image of the object ? (Figure drawn as schematic and not to scale)



- (1) Erect, virtual and unmagnified                      (2) Inverted, real and magnified  
(3) Erect, virtual and magnified                      (4) Inverted, real and unmagnified

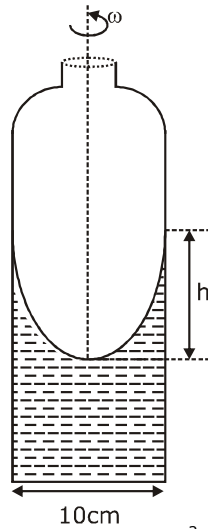
**Sol. (4)**



$\therefore$  beyond C i.e.  $-\infty < u < C$

$\therefore$  real, inverted  
and unmagnified

- Q.4** A cylindrical vessel containing a liquid is rotated about its axis so that the liquid rises at its sides as shown in the figure. The radius of vessel is 5 cm and the angular speed of rotation is  $\omega$  rad s<sup>-1</sup>. The difference in the height,  $h$  (in cm) of liquid at the centre of vessel and at the side will be :



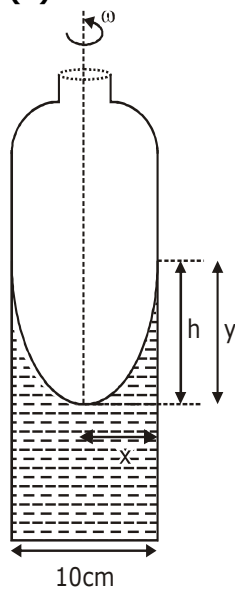
(1)  $\frac{5\omega^2}{2g}$

(2)  $\frac{2\omega^2}{25g}$

(3)  $\frac{25\omega^2}{2g}$

(4)  $\frac{2\omega^2}{5g}$

**Sol.**



$$y = \frac{\omega^2 x^2}{2g}$$

at  $x = 5\text{ cm}$ ,  $y = h$

$$h = \frac{\omega^2 (5)^2}{2g} = \frac{25\omega^2}{2g}$$

**Q.5** If speed  $V$ , area  $A$  and force  $F$  are chosen as fundamental units, then the dimension of Young's modulus will be

- (1)  $FA^2V^{-3}$  (2)  $FA^2V^{-2}$  (3)  $FA^{-1}V^0$  (4)  $FA^2V^{-1}$

**Sol. (3)**

$$Y = k [F]^x [A]^y [V]^z$$

$$[ML^{-1}T^{-2}] = [MLT^{-2}]^x [L^2]^y [LT^{-1}]^z$$

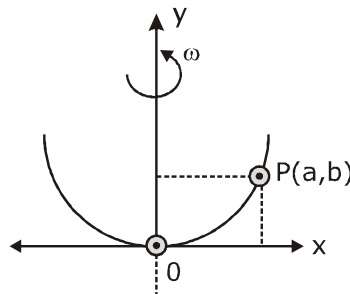
$$[ML^{-1}T^{-2}] = [M^x L^{x+2y+z} T^{-2x-z}]$$

$$x = 1, -2x - z = -2, x + 2y + z = -1$$

$$\Rightarrow z = 0$$

$$\Rightarrow y = -1$$

**Q.6** A bead of mass  $m$  stays at point  $P(a, b)$  on a wire bent in the shape of a parabola  $y = 4Cx^2$  and rotating with angular speed  $\omega$  (see figure). The value of  $\omega$  is (neglect friction):



- (1)  $\sqrt{\frac{2g}{C}}$  (2)  $2\sqrt{gC}$  (3)  $\sqrt{\frac{2gC}{ab}}$  (4)  $2\sqrt{2gC}$

**Sol. (4)**

$$y = 4cx^2$$

$$\frac{dy}{dx} = 8cx$$

$$\tan \theta = \frac{m\omega^2 a}{mg}$$

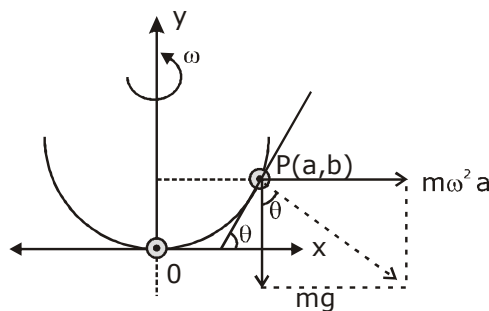
$$\tan \theta = \frac{dy}{dx} = 8cx$$

$$8cx = \frac{\omega^2 a}{g}$$

$$(x = a), 8ca = \frac{\omega^2 a}{g}$$

$$\sqrt{8cg} = \omega$$

$$2\sqrt{2gc} = \omega$$



- Q.7** Magnetic materials used for making permanent magnets (P) and magnets in a transformer (T) have different properties of the following, which property best matches for the type of magnet required ?  
 (1) P : Small retentivity, large coercivity    (2) P : Large retentivity, large coercivity  
 (3) T : Large retentivity, large coercivity    (4) T : Large retentivity, small coercivity

**Sol. (2)**

Permanent magnet must retain for long use and should not be easily demagnetised.

- Q. 8** Interference fringes are observed on a screen by illuminating two thin slits 1 mm apart with a light source ( $\lambda = 632.8 \text{ nm}$ ). The distance between the screen and the slits is 100 cm. If a bright fringe is observed on screen at a distance of 1.27 mm from the central bright fringe, then the path difference between the waves, which are reaching this point from the slits is close to :  
 (1)  $2.05 \mu\text{m}$                       (2)  $2.87 \text{ nm}$                       (3)  $2 \text{ nm}$                       (4)  $1.27 \mu\text{m}$

**Sol. (4)**

given,  $d = 1\text{mm}$

$\lambda = 632.8 \text{ nm}$

$D = 100\text{cm}$

$y = 1.27 \text{ mm}$

$$\Delta x = d \sin \theta$$

$$\because (\theta = \text{small})$$

$$\Delta x = d \tan \theta$$

$$\Delta x = \frac{dy}{D} = \frac{1 \times 10^{-3} \times 1.27 \times 10^{-3}}{100 \times 10^{-2}}$$

$$= 1.27 \times 10^{-6} \text{ m}$$

$$= 1.27 \mu\text{m}$$

- Q.9** A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T. Assuming the gases to be ideal and the oxygen bond to be rigid, the total internal energy (in units of RT) of the mixture is:

- (1) 11                      (2) 13                      (3) 15                      (4) 20

**Sol. (3)**

$$U = n_1 C_{v_1} T + n_2 C_{v_2} T$$

$$= 3 \times \frac{5}{2} RT + 5 \times \frac{3}{2} RT$$

$$= \frac{30}{2} RT = 15RT$$

**Q. 10** A plane electromagnetic wave, has frequency of  $2.0 \times 10^{10}$  Hz and its energy density is  $1.02 \times 10^{-8} \text{ J/m}^3$  in vacuum. The amplitude of the magnetic field of the wave is close

$$\left( \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \text{ and speed of light} = 3 \times 10^8 \text{ ms}^{-1} \right);$$

- (1) 160 nT                      (2) 150 nT                      (3) 180 nT                      (4) 190 nT

**Sol. (1)**

$$\text{energy density} = \frac{B_0^2}{2\mu_0} \quad \dots(1)$$

$$\& C = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \quad \dots(2)$$

$$\mu_0 = \frac{1}{C^2 \epsilon_0}$$

$$B = \sqrt{U \times 2\mu_0}$$

$$= \sqrt{1.02 \times 10^{-8} \times 2 \times \frac{1}{9 \times 10^{16}} 4\pi \times 9 \times 10^9}$$

$$= \sqrt{25.62 \times 10^{-15}}$$

$$\cong \sqrt{25600 \times 10^{-18}}$$

$$\cong 160 \times 10^{-9}$$

$$= 160 \text{ nT}$$

**Q. 11** Consider four conducting materials copper, tungsten, mercury and aluminium with resistivity  $\rho_C$ ,  $\rho_T$ ,  $\rho_M$  and  $\rho_A$  respectively. Then :

- (1)  $\rho_C > \rho_A > \rho_T$                       (2)  $\rho_A > \rho_M > \rho_C$                       (3)  $\rho_A > \rho_T > \rho_C$                       (4)  $\rho_M > \rho_A > \rho_C$

**Sol. (4)**

(Theoretical concept)

**Q.12** A beam of protons with speed  $4 \times 10^5 \text{ ms}^{-1}$  enters a uniform magnetic field of 0.3 T at an angle of  $60^\circ$  to the magnetic field. The pitch of the resulting helical path of protons is close to : (Mass of the proton =  $1.67 \times 10^{-27} \text{ kg}$ , charge of the proton =  $1.69 \times 10^{-19} \text{ C}$ )

- (1) 4 cm                      (2) 2 cm                      (3) 12 cm                      (4) 5 cm

**Sol. (1)**

$$\text{pitch} = V \cos 60^\circ, T = \frac{V}{2} \frac{2\pi m}{eB}$$

$$= 4 \times 10^5 \times \frac{1}{2} \times \frac{2\pi}{0.3} \left( \frac{m}{e} \right)$$

$$= \frac{4\pi \times 10^5 \times 10^{-8}}{0.3}$$

$$= \frac{4 \times 3.14 \times 10^{-3}}{3 \times 10^{-1}}$$

$$\approx 4 \times 10^{-2} \text{ m}$$

$$\approx 4 \text{ cm}$$

**Q.13** Two identical strings X and Z made of same material have tension  $T_x$  and  $T_z$  in them. If their fundamental frequencies are 450 Hz and 300 Hz, respectively, then the ratio  $T_x/T_z$  is :

(1) 2.25

(2) 1.25

(3) 0.44

(4) 1.5

**Sol. (1)**

$$f = \frac{1}{2L} \sqrt{\frac{T}{\mu}}$$

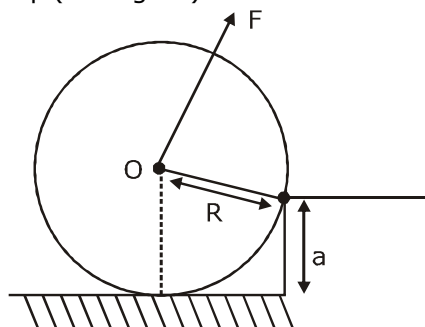
given,  $\mu_x = \mu_z$  &  $L_x = L_z$  as identical

$$\therefore f \propto \sqrt{T}$$

$$\Rightarrow \frac{T_x}{T_z} = \frac{f_x^2}{f_z^2} = \left( \frac{450}{300} \right)^2 = \left( \frac{3}{2} \right)^2 = \frac{9}{4}$$

$$\frac{T_x}{T_y} = 2.25$$

**Q.14** A uniform cylinder of mass  $M$  and radius  $R$  is to be pulled over a step of height  $a$  ( $a < R$ ) by applying a force  $F$  at its centre 'O' perpendicular to the plane through the axes of the cylinder on the edge of the step (see figure). The minimum value of  $F$  required is



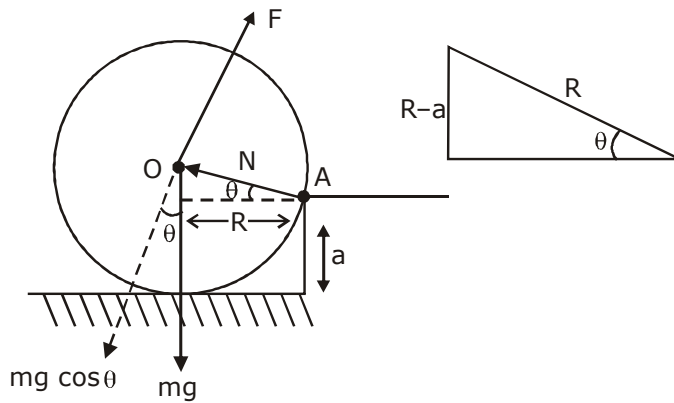
(1)  $Mg \sqrt{\left( \frac{R}{R-a} \right)^2 - 1}$

(2)  $Mg \sqrt{1 - \frac{a^2}{R^2}}$

(3)  $Mg \frac{a}{R}$

(4)  $Mg \sqrt{1 - \left( \frac{R-a}{R} \right)^2}$

**Sol. (4)**



$$\cos \theta = \frac{\sqrt{R^2 - (R - a)^2}}{R}$$

$$= \sqrt{\frac{R^2}{R^2} - \left(\frac{R - a}{R}\right)^2}$$

$$= \sqrt{1 - \left(\frac{R - a}{R}\right)^2}$$

to pull up,  $\tau_F \geq \tau_{mg}$

$$FR \geq mg \cos \theta R$$

for min F,  $F_{\min} = mg \cos \theta$

$$F_{\min} = mg \sqrt{1 - \left(\frac{R - a}{R}\right)^2}$$

- Q.15** In a reactor, 2 kg of  ${}_{92}\text{U}^{235}$  fuel is fully used up in 30 days. The energy released per fission is 200 MeV. Given that the Avogadro number,  $N = 6.023 \times 10^{26}$  per kilo mole and  $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$ . The power output of the reactor is close to  
 (1) 60 MW (2) 54 MW (3) 125 MW (4) 35 MW

**Sol. (1)**

$$n(\text{moles}) = \frac{2\text{kg}}{235\text{gm}} = \frac{2000}{235}$$

$$\text{no. of nucleus} = N_A \times n$$

$$= 6.022 \times 10^{23} \times \frac{2000}{235}$$

$$= 51.25 \times 10^{23}$$

$$\text{total energy released} = 200 \times 51.25 \times 10^{23} \text{ MeV}$$

$$= 102.5 \times 10^{25} \text{ MeV}$$

$$= 102.5 \times 10^{25} \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$$

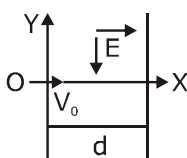
$$= 164 \times 10^6 \text{ MJ}$$

$$\text{power} = \frac{164 \times 10^6 \text{ MJ}}{30 \times 24 \times 60 \times 60 \text{ S}}$$

$$= 0.063 \times 10^3 \text{ MW}$$

$$\approx 60 \text{ MW}$$

- Q.16** A charged particle (mass  $m$  and charge  $q$ ) moves along X axis with velocity  $V_0$ . When it passes through the origin it enters a region having uniform electric field  $\vec{E} = -E\hat{j}$  which extends upto  $x = d$ . Equation of path of electron in the region  $x > d$  is :



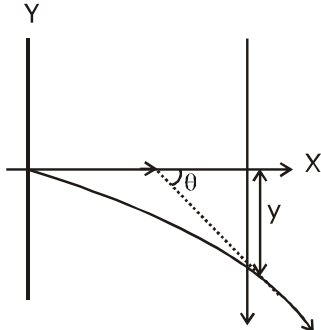
$$(1) y = \frac{qEd^2}{mV_0^2} x$$

$$(2) y = \frac{qEd}{mV_0^2} \left( \frac{d}{2} - x \right)$$

$$(3) y = \frac{qEd}{mV_0^2} (x - d)$$

$$(4) y = \frac{qEd}{mV_0^2} x$$

**Sol. (2)**



$$-y = \frac{1}{2} at^2$$

$$-y = \frac{1}{2} \frac{qE}{m} t^2 \quad \dots(1)$$

$$X = V_0 t$$

$$\Rightarrow t = \frac{x}{V_0} \quad \dots(2)$$

for  $x \leq d$ ,

$$y = -\frac{1}{2} \frac{qE}{m} \frac{x^2}{V_0^2} \quad \dots(3)$$

$$\left. \frac{dy}{dx} \right|_{x=d} = -\frac{1}{2} \frac{qE}{m} \times \frac{2x}{V_0^2} \bigg|_{x=d}$$

$$\text{Slope} = m = \tan \theta = -\frac{qEd}{mV_0^2}$$

equation of straight line,

$$y = (\tan \theta) x + c \quad \dots(4)$$

$$= -\left( \frac{qEd}{mV_0^2} \right) x + c$$

$$(\text{now for } C, \text{ at } x = d, y = -\frac{qEd^2}{2mV_0^2} \text{ put in (4)})$$

$$-\frac{qEd^2}{2mV_0^2} = -\frac{qEd^2}{mV_0^2} + c$$

$$\Rightarrow c = \frac{qEd^2}{2mv_0^2}$$

for  $x > d$ , as no  $\vec{E}$

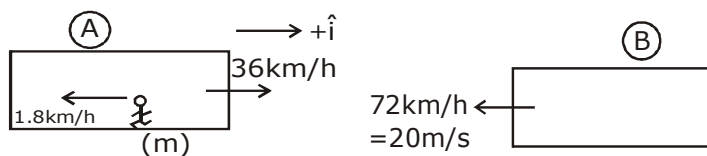
$$y = -\left(\frac{qEd}{mv_0^2}\right)x + \frac{qEd^2}{2mv_0^2}$$

$$y = \frac{qEd}{mv_0^2}\left(\frac{d}{2} - x\right)$$

- Q.17** Train A and train B are running on parallel tracks in the opposite directions with speeds of 36 km/hour and 72 km/hour, respectively. A person is walking in train A in the direction opposite to its motion with a speed of 1.8 km/hour. Speed (in  $\text{ms}^{-1}$ ) of this person as observed from train B will be close to : (take the distance between the tracks as negligible)

(1)  $29.5 \text{ ms}^{-1}$  (2)  $30.5 \text{ ms}^{-1}$  (3)  $31.5 \text{ ms}^{-1}$  (4)  $28.5 \text{ ms}^{-1}$

**Sol. (1)**



$$\vec{V}_m = \vec{V}_{m/A} + \vec{V}_A$$

$$= (-1.8\hat{i} + 36\hat{i}) \text{ km/h}$$

$$= \left(-1.8 \times \frac{5}{18} + 36 \times \frac{5}{18}\right) \text{ m/s}$$

$$= (-0.5\hat{i} + 10\hat{i}) \text{ m/s}$$

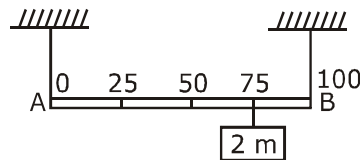
$$= 9.5\hat{i} \text{ m/s}$$

$$\vec{V}_{m/B} = \vec{V}_m - \vec{V}_B$$

$$= 9.5\hat{i} - (-20\hat{i}) \text{ m/s}$$

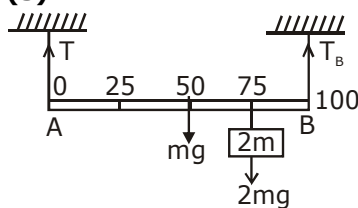
$$= 29.5 \text{ m/s } \hat{i}$$

- Q.18** Shown in the figure is rigid and uniform one meter long rod AB held in horizontal position by two strings tied to its ends and attached to the ceiling. The rod is of mass 'm' and has another weight of mass 2 m hung at a distance of 75 cm from A. The tension in the string at A is:



- (1) 0.75 mg      (2) 0.5 mg      (3) 1 mg      (4) 2 mg

**Sol. (3)**



$\tau_{\text{net}}$  about B = 0

$$T \times 100\text{cm} = mg \times 50\text{cm} + 2mg \times 25\text{cm}$$

$$T = 1 \text{ mg}$$

- Q.19** The least count of the main scale of a vernier callipers is 1 mm. Its vernier scale is divided into 10 divisions and coincide with 9 divisions of the main scale. When jaws are touching each other, the 7<sup>th</sup> division of vernier scale coincides with a division of main scale and the zero of vernier scale is lying right side of the zero of main scale. When this vernier is used to measure length of a cylinder the zero of the vernier scale between 3.1 cm and 3.2 cm and 4<sup>th</sup> VSD coincides with a main scale division. The length of the cylinder is: (VSD is vernier scale division)

- (1) 3.21 cm      (2) 3.07 cm      (3) 2.99 cm      (4) 3.2 cm

**Sol. (2)**

$$\text{L.C.} = 1\text{MSD} - 1\text{VSD}$$

$$\text{L.C.} = 0.1\text{MSD}$$

$$1 \text{ MSD} = 1\text{mm}$$

$$\text{L. C.} = 0.1\text{mm}$$

$$\begin{aligned} \text{+ve zero error} &= +7 \times \text{L.C.} \\ &= 0.7\text{mm} \end{aligned}$$

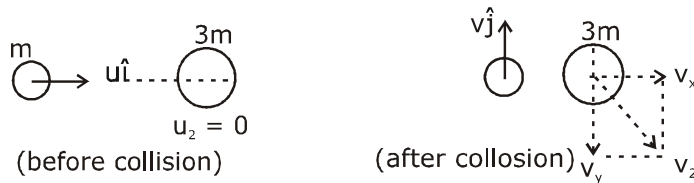
$$\begin{aligned} \text{Reading} &= (3.1\text{cm} + 4 \times \text{L.C.}) - \text{zero error} \\ &= 3.1\text{cm} + 0.4\text{mm} - 0.7\text{mm} \\ &= 3.1\text{cm} - 0.03\text{cm} \text{ (as given } 1 \text{ MSD} = 1\text{mm)} \\ &= 3.07 \text{ cm} \end{aligned}$$

**Q. 20** A particle of mass  $m$  with an initial velocity  $u\hat{i}$  collides perfectly elastically with a mass  $3m$  at rest.

It moves with a velocity  $v\hat{j}$  after collision, then  $v$  is given by:

- (1)  $v = \frac{1}{\sqrt{6}}u$       (2)  $v = \sqrt{\frac{2}{3}}u$       (3)  $v = \frac{u}{\sqrt{3}}$       (4)  $v = \frac{u}{\sqrt{2}}$

**Sol. (4)**



$$\vec{p}_i = \vec{p}_f$$

$$mu\hat{i} + 0 = mv\hat{j} + 3m\vec{V}_2$$

$$\frac{mu\hat{i}}{3m} - \frac{mv\hat{j}}{3m} = \vec{V}_2$$

$$\vec{V}_2 = \frac{u}{3}\hat{i} - \frac{v}{3}\hat{j}$$

now, K.E conserved as elastic collision

$$\Sigma KE_i = \Sigma KE_f$$

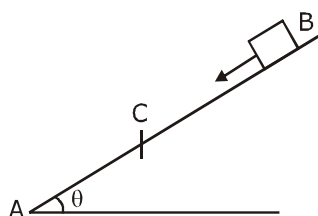
$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}3m\left(\frac{u^2}{9} + \frac{v^2}{9}\right)$$

$$\Rightarrow u^2 = v^2 + \frac{u^2}{3} + \frac{v^2}{3}$$

$$\frac{2}{3}u^2 = \frac{4}{3}v^2$$

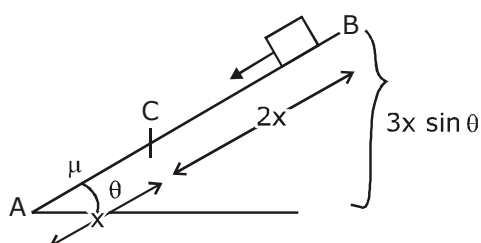
$$\Rightarrow v = \frac{u}{\sqrt{2}}$$

**Q.21**



A small block starts slipping down from a point B on an inclined plane AB, which is making an angle  $\theta$  with the horizontal. Section BC is smooth and the remaining section CA is rough with a coefficient of friction  $\mu$ . It is found that the block comes to rest as it reaches the bottom (point A) of the inclined plane. If  $BC = 2AC$ , the coefficient of friction is given by  $\mu = k \tan \theta$ . The value of  $k$  is \_\_\_\_\_.

**Sol. (3)**



from work energy theorem

$$W_g + W_f = \Delta K E$$

$$mg \cdot 3x \sin \theta - \mu mg \cos \theta \cdot x = 0 - 0$$

$$\Rightarrow mg 3x \sin \theta = \mu mg \cos \theta x$$

$$3 \tan \theta = \mu$$

$$k = 3$$

**Q.22** An engine takes in 5 moles of air at  $20^\circ\text{C}$  and  $1\text{atm}$ , and compresses it adiabatically to  $1/10^{\text{th}}$  of the original volume. Assuming air to be a diatomic ideal gas made up of rigid molecules, the change in its internal energy during this process comes out to be  $X$  kJ. The value of  $X$  to the nearest integer is \_\_\_\_\_.

**Sol. (46)**

$$T_2 V_2^{\gamma-1} = T_1 V_1^{\gamma-1}$$

$$T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{\gamma-1}$$

$$= 293 \left( \frac{V}{V/10} \right)^{\frac{7}{5}-1}$$

$$T_2 = 293 \times (10)^{2/5}$$

$$\Delta U = n C_v \Delta T = 5 \times \frac{5}{2} R \left( 293 \times 10^{\frac{2}{5}} - 293 \right)$$

$$= \frac{25}{2} R \times 293 \left( 10^{\frac{2}{5}} - 1 \right) = \frac{25R}{2} \times 293(2.5 - 1)$$

$$= \frac{25 \times 8.314 \times 293 \times 1.5}{2}$$

$$= 45675 \text{ J} = 46 \text{ kJ}$$

**Q.23** When radiation of wavelength  $\lambda$  is used to illuminate a metallic surface, the stopping potential is  $V$ .

When the same surface is illuminated with radiation of wavelength  $3\lambda$ , the stopping potential is  $\frac{V}{4}$ .

If the threshold wavelength for the metallic surface is  $n\lambda$  then value of  $n$  will be \_\_\_\_\_.

**Sol. (9)**

$$\frac{hc}{\lambda} = \phi + eV \quad \dots(1)$$

$$\frac{hc}{3\lambda} = \phi + \frac{eV}{4} \quad \dots(2)$$

$$\frac{\text{eq.(1)}}{\text{eq.(2)}} \quad 3 = \frac{\phi + eV}{\phi + \frac{eV}{4}}$$

$$3\phi + \frac{3eV}{4} = \phi + eV$$

$$2\phi = \frac{eV}{4}$$

$$\phi = \frac{eV}{8}$$

$$\frac{hc}{\lambda} = \frac{eV}{8} + eV$$

$$= \frac{9}{8} eV$$

$$\therefore eV = \frac{8}{9} \frac{hc}{\lambda}$$

$$\text{so } \phi = \frac{hc}{\lambda} - \frac{8}{9} \frac{hc}{\lambda}$$

$$\phi = \frac{1}{9} \frac{hc}{\lambda}$$

$$\frac{hc}{\lambda_{th}} = \frac{hc}{9\lambda}$$

$$\therefore \lambda_{th} = 9\lambda$$

- Q.24** A circular coil of radius 10 cm is placed in uniform magnetic field of  $3.0 \times 10^{-5}$  T with its plane perpendicular to the field initially. It is rotated at constant angular speed about an axis along the diameter of coil and perpendicular to magnetic field so that it undergoes half of rotation in 0.2s. The maximum value of EMF induced (in  $\mu\text{V}$ ) in the coil will be close to the integer \_\_\_\_\_.

**Sol. (15)**

$$\phi = BA \cos \omega t$$

$$E = \frac{-d\phi}{dt} = BA\omega \sin \omega t$$

$$E_{\max} = BA\omega \quad \left( \omega = \frac{\pi}{0.2} \right)$$

$$= 3 \times 10^{-5} \times \pi R^2 \times \frac{\pi}{0.2}$$

$$= 15 \times 10^{-6} \text{ V}$$

$$= 15 \mu\text{V}$$

- Q.25** A  $5\mu\text{F}$  capacitor is charged fully by a 220V supply. It is then disconnected from the supply and is connected in series to another uncharged  $2.5 \mu\text{F}$  capacitor. If the energy change during the charge redistribution is  $\frac{X}{100}$  J then value of X to the nearest integer is \_\_\_\_\_.

**Sol. (40) Our Answer**

**NTA Answer 36**

$$\text{heat} = U_i - U_f$$

$$= \frac{1}{2} \frac{C_1 C_2}{C_1 + C_2} (V_1 - V_2)^2$$

$$= \frac{1}{2} \frac{5 \times 2.5}{7.5} (220 - 0)^2$$

$$= \frac{5}{6} \times 220 \times 220 \times 10^{-6} \text{ J}$$

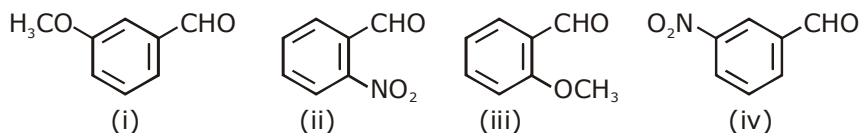
$$= 40,333.33 \times 10^{-6} \text{ J}$$

$$\frac{X}{100} = 0.4 \text{ J}$$

$$X = 40$$

## CHEMISTRY - 2 Sep 2020. - SHIFT - 1

1. The increasing order of the following compounds towards HCN addition is:



- (1) (iii) < (i) < (iv) < (ii)

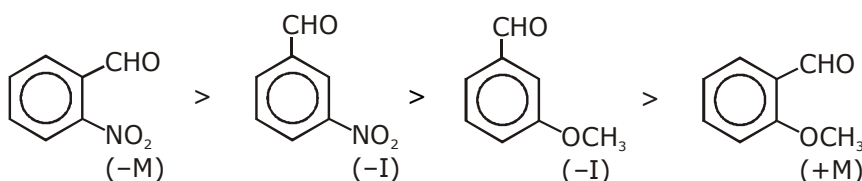
(3) (i) < (iii) < (iv) < (ii)

(2) (iii) < (iv) < (i) < (ii)

(4) (iii) < (iv) < (ii) < (i)

Sol.

**1**  
In HCN,  $\text{CN}^-$  acts as nucleophile, attack first that  $-\text{CHO}$  group which has maximum positive charge. The magnitude of the (+ve) charge increases by  $-\text{M}$  and  $-\text{I}$  group. So reactivity order will be



So, option (1) is correct answer.

2. Which of the following is used for the preparation of colloids?

- (1) Van Arkel Method

(3) Mond Process

(2) Ostwald Process

(4) Bredig's Arc Method

Sol.

**4**  
Bredig's Arc method  
Chapter name surface chemistry

3. An open beaker of water in equilibrium with water vapour is in a sealed container. When a few grams of glucose are added to the beaker of water, the rate at which water molecules:

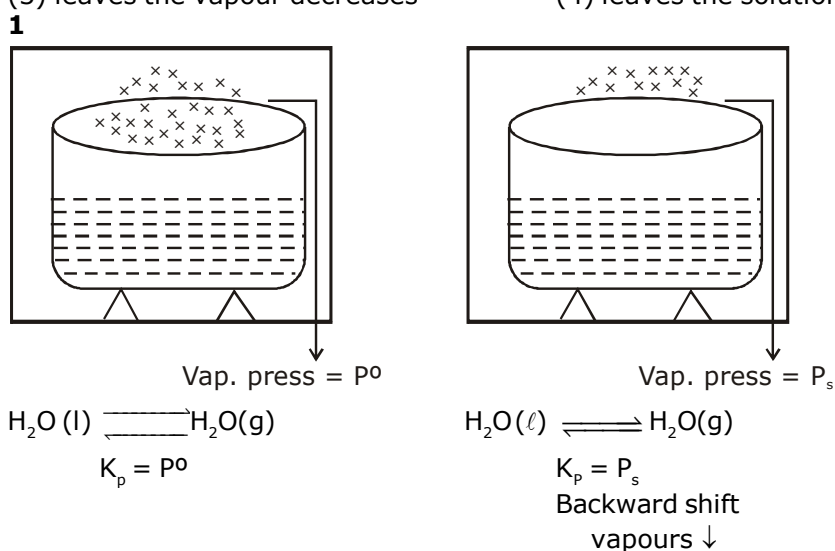
- (1) leaves the vapour increases

(3) leaves the vapour decreases

(2) leaves the solution increases

(4) leaves the solution decreases

Sol.



Hence Rate at which water molecules leaves the vap. increases.

4. For octahedral Mn(II) and tetrahedral Ni(II) complexes, consider the following statements:

- (I) both the complexes can be high spin.
- (II) Ni(II) complex can very rarely be low spin.
- (III) with strong field ligands, Mn(II) complexes can be low spin.
- (IV) aqueous solution of Mn(II) ions is yellow in colour.

The correct statements are:

- (1) (I), (III) and (IV) only
- (2) (I), (II) and (III) only
- (3) (II), (III) and (IV) only
- (4) (I) and (II) only

**Sol. 2**

Mn<sup>2+</sup> [Ar]3d<sup>5</sup> it can form low spin as well as high spin complex depending upon nature of ligand same of Ni<sup>2+</sup> ion with coordination no 4. It can be dsp<sup>2</sup> or sp<sup>3</sup> i.e low spin or high spin depending open nature of ligand.

5. The statement that is not true about ozone is:

- (1) in the stratosphere, it forms a protective shield against UV radiation.
- (2) in the atmosphere, it is depleted by CFCs.
- (3) in the stratosphere, CFCs release chlorine free radicals (Cl) which reacts with O<sub>3</sub> to give chlorine dioxide radicals.
- (4) it is a toxic gas and its reaction with NO gives NO<sub>2</sub>.

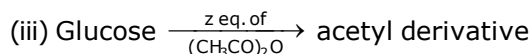
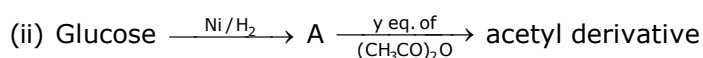
**Sol. 3**



Chlorine monoxide

Hence option (3)

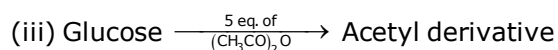
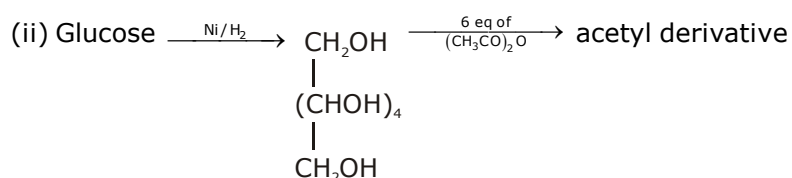
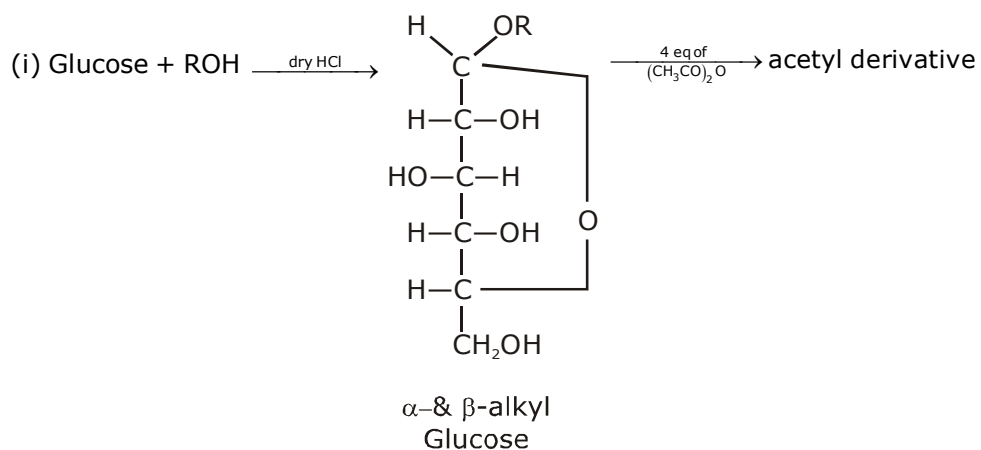
6. Consider the following reactions:



'x', 'y' and 'z' in these reactions are respectively.

- (1) 4, 5 & 5
- (2) 5, 4 & 5
- (3) 5, 6 & 5
- (4) 4, 6 & 5

**Sol. 4**



$(\text{CH}_3\text{CO})_2\text{O}$  reacts with  $-\text{OH}$  group to form acetyl derivative, so as the no. of  $-\text{OH}$  group no. of eq. of  $(\text{CH}_3\text{CO})_2\text{O}$  will be used

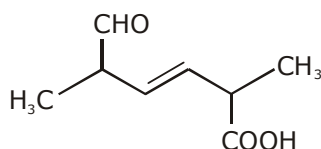
So,  $x = 4$

$y = 6$

$z = 5$

So, option (4) will be correct answer.

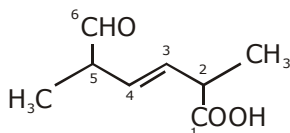
**7.** The IUPAC name for the following compound is:



(1) 2,5-dimethyl-5-carboxy-hex-3-enal  
(3) 6-formyl-2-methyl-hex-3-enoic acid

(2) 2,5-dimethyl-6-oxo-hex-3-enoic acid  
(4) 2,5-dimethyl-6-carboxy-hex-3-enal

**Sol. 2**



2,5-Dimethyl-6-oxohex-3-enoic acid

**8.** For the following Assertion and Reason, the correct option is

**Assertion (A):** When Cu (II) and sulphide ions are mixed, they react together extremely quickly to give a solid.

**Reason (R):** The equilibrium constant of  $\text{Cu}^{2+}(\text{aq}) + \text{S}^{2-}(\text{aq}) \rightleftharpoons \text{CuS}(\text{s})$  is high because the solubility product is low.

- (1) **(A)** is false and **(R)** is true.
- (2) Both **(A)** and **(R)** are false.
- (3) Both **(A)** and **(R)** are true but **(R)** is not the explanation for **(A)**.
- (4) Both **(A)** and **(R)** are true but **(R)** is the explanation for **(A)**.

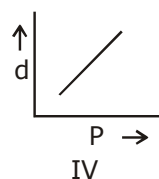
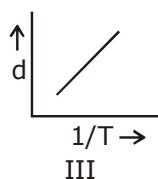
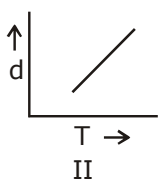
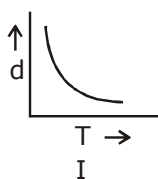
**Sol. 4**

(A) is (B) true &

(R) is correct explanation of (A)

Ans. 4

**9.** Which one of the following graphs is not correct for ideal gas?



d = Density, P = Pressure, T = Temperature

(1) I

(2) IV

(3) III

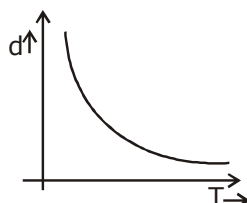
(4) II

**Sol. 4**

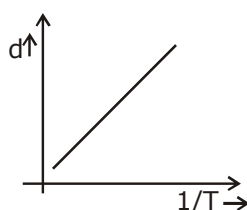
For ideal Gas

$$d = \frac{P \times M}{RT}$$

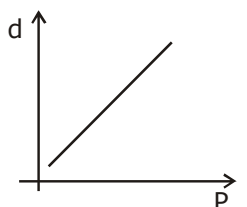
$d \propto T \rightarrow$  Hyperbolic



$d \propto \frac{1}{T} \rightarrow$  St. line



$d \propto p \rightarrow$  St line



$\therefore$  'II' Graph is incorrect

Ans (4)

**10.** While titrating dilute HCl solution with aqueous NaOH, which of the following will not be required?

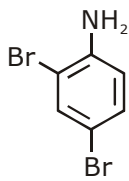
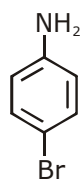
- |  |                                 |
|--|---------------------------------|
| (1) Bunsen burner and measuring cylinder | (2) Burette and porcelain tile  |
| (3) Clamp and phenolphthalein            | (4) Pipette and distilled water |

**Sol. 1**

Bunsen Burner & measuring cylinder are not Required. As titration is already an exothermic process

Ans.(1)

**11.** In Carius method of estimation of halogen, 0.172 g of an organic compound showed presence of 0.08 g of bromine. Which of these is the correct structure of the compound?

- |                                    |   |   |  |
|------------------------------------|---|---|--|
| (1) $\text{H}_3\text{C}-\text{Br}$ | (2)  | (3)  | (4) $\text{H}_3\text{C}-\text{CH}_2-\text{Br}$ |
|------------------------------------|---|---|--|

**Sol. 3**

carius method

$$\text{mass \% of 'Br'} = \frac{0.08}{0.172} \times 100 = \frac{8000}{172} = 46.51\%$$

$$\text{option (1) mass \%} = \frac{80}{95} \times 100$$

$$(2) \text{ mass \%} = \frac{2 \times 80 \times 100}{252}$$

$$(3) \text{ mass \%} = \frac{1 \times 80 \times 100}{80 + 72 + 6 + 14} = \frac{8000}{172} \%$$

$$(4) \text{ mass \%} = \frac{1 \times 80 \times 100}{109} \%$$

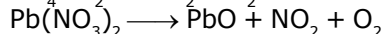
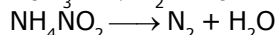
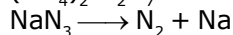
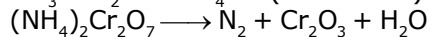
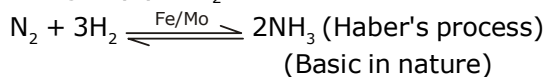
Option (3) matches with the given mass percentage value

Ans (3)

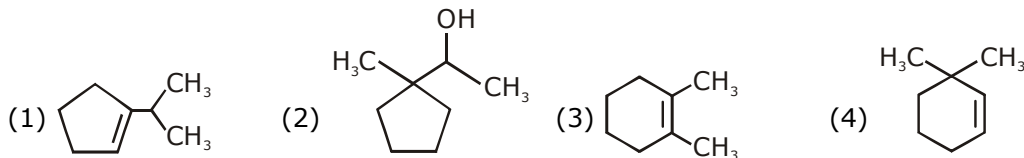
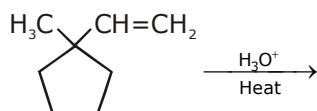
- 12.** On heating compound (A) gives a gas (B) which is a constituent of air. This gas when treated with  $\text{H}_2$  in the presence of a catalyst gives another gas (C) which is basic in nature. (A) should not be:  
 (1)  $(\text{NH}_4)_2\text{Cr}_2\text{O}_7$       (2)  $\text{NaN}_3$       (3)  $\text{NH}_4\text{NO}_2$       (4)  $\text{Pb}(\text{NO}_3)_2$

**Sol. 4**

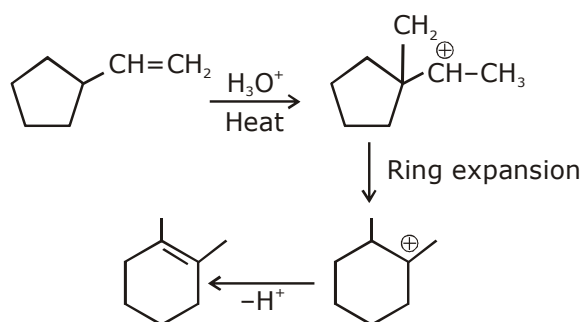
The gas (B) is  $\text{N}_2$  which is found in air



- 13.** The major product in the following reaction is:



**Sol. 3**



Option (3) is correct answer.

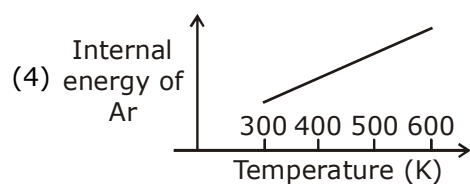
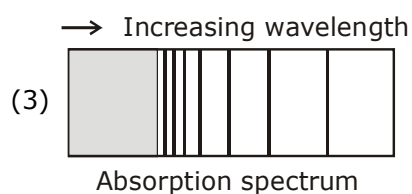
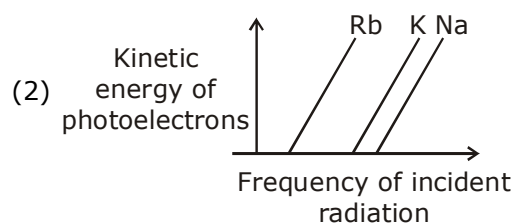
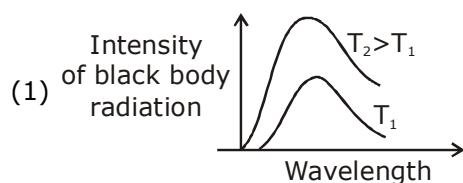
- 14.** In general, the property (magnitudes only) that shows an opposite trend in comparison to other properties across a period is:

- |                         |                            |
|-------------------------|----------------------------|
| (1) Ionization enthalpy | (2) Electronegativity      |
| (3) Atomic radius       | (4) Electron gain enthalpy |

**Sol. 3**

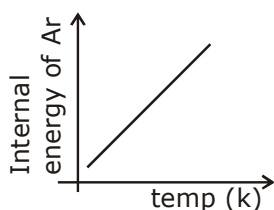
Ionisation energy, electronegativity & electron gain enthalpy increase across a period but atomic radius decreases

- 15.** The figure that is not a direct manifestation of the quantum nature of atoms is:



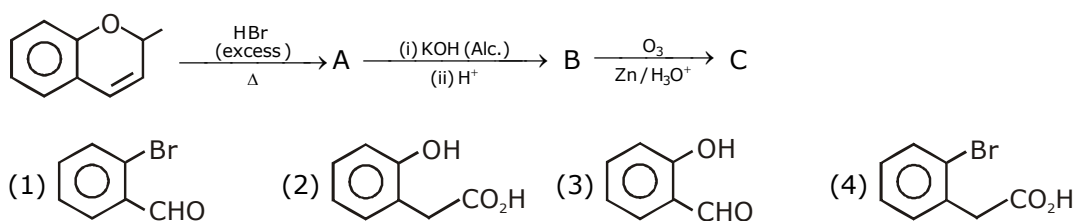
**Sol. 4**

Internal energy of 'Ar' or any gas, has nothing to do with Quantum nature of atom hence

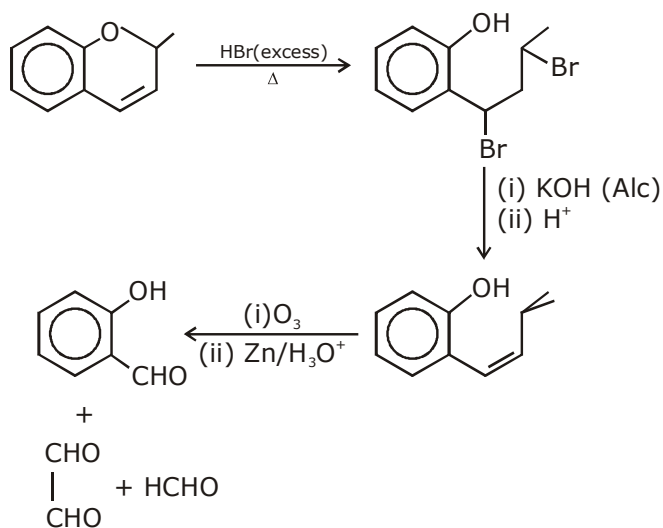


Ans. option (4)

**16.** The major aromatic product C in the following reaction sequence will be :



**Sol. 3**



Option (3) is correct answer.



**Sol. 1**

In  $\text{CH}_3-\underset{\text{C}_2\text{H}_5}{\text{CH}}-\text{CH}_2\text{Br}$  attack of  $\text{OH}^-$  is not on chiral carbon, it is adjacent to chiral carbon, so configuration of chiral carbon remains constant.

**20.** The metal mainly used in devising photoelectric cells is:

- (1) Li (2) Cs (3) Rb (4) Na

**Sol. 2**

'Cs' is used in photoelectric cell as its ionisation energy is lowest  
Hence Ans (2)

**21.** The mass of gas adsorbed,  $x$ , per unit mass of adsorbate,  $m$ , was measured at various pressures,  $p$ .

A graph between  $\log \frac{x}{m}$  and  $\log p$  gives a straight line with slope equal to 2 and the intercept equal

to 0.4771. The value of  $\frac{x}{m}$  at a pressure of 4 atm is: (Given  $\log 3 = 0.4771$ )

**Sol. 48**

$$\frac{x}{m} = KP^{1/n}$$

$$\log (x / m) = \log_{(k)} + \frac{1}{n} \log(p)$$

$$y = c + mx$$

$$\text{Intercept } C = \log_k = 0.4771$$

$$\text{slop} = \frac{1}{n} = 2, k = 3$$

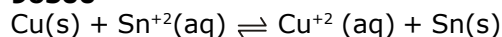
$$\begin{aligned} \frac{x}{m} &= k(P)^{1/n} \quad \text{at } P = 4 \text{ atm} \\ &= 3(4)^2 \end{aligned}$$

$$\frac{x}{m} = 3 \times 16 = 48 \text{ Ans}$$

**22.** The Gibbs energy change (in J) for the given reaction at  $[\text{Cu}^{2+}] = [\text{Sn}^{2+}] = 1 \text{ M}$  and 298 K is:  
 $\text{Cu(s)} + \text{Sn}^{2+}(\text{aq.}) \rightarrow \text{Cu}^{2+}(\text{aq.}) + \text{Sn(s)}$

$$(E_{\text{Sn}^{2+}|\text{Sn}}^0 = -0.16\text{V}, E_{\text{Cu}^{2+}|\text{Cu}}^0 = 0.34\text{V}, \text{Take } F = 96500 \text{ C mol}^{-1})$$

**Sol. 96500**



$$E_{\text{cell}}^0 = -0.16 - 0.34$$

$$= -0.50$$

$$\Delta G^0 = -nF E_{\text{cell}}^0$$

$$= -2 \times 96500 \times (-0.5)$$

$$= +96500$$

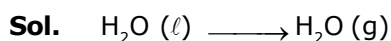
$$\Delta G = \Delta G^0 + RT \ln Q$$

$$= 96500 + \frac{25}{3} \times 298 \times 2.303 \log (1)$$

$$\Delta G = 96500 \text{ Joules}$$

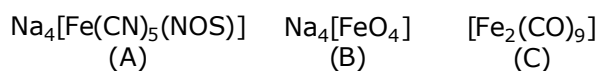
- 23.** The internal energy change (in J) when 90 g of water undergoes complete evaporation at 100° C is \_\_\_\_\_.

(Given:  $\Delta H_{\text{vap}}$  for water at 373 K = 41 kJ/mol,  $R = 8.314 \text{ JK}^{-1} \text{ mol}^{-1}$ )

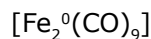
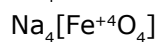
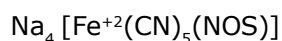


$$\begin{aligned}\Delta E_{\text{vap}} &= \Delta H_{\text{vap}} - \Delta n g R T \\ &= 41000 \times 5 - 5 \times 8.314 \times 373 \\ &= 189494.39\end{aligned}$$

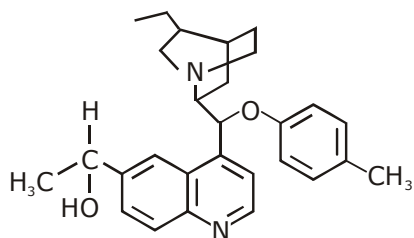
- 24.** The oxidation states of iron atoms in compounds (A), (B) and (C), respectively, are x, y and z. The sum of x, y and z is \_\_\_\_\_.



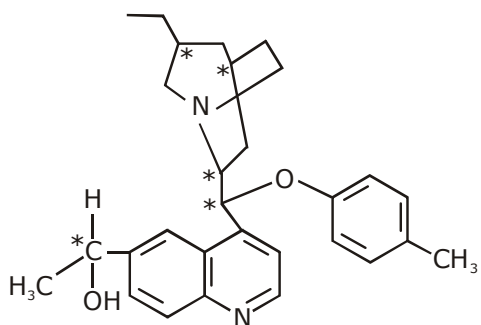
**Sol.** **6**



- 25.** The number of chiral carbons present in the molecule given below is \_\_\_\_\_.



**Sol.** **5**



Total chiral carbon = 5

# QUESTION PAPER WITH SOLUTION

## MATHEMATICS - 2 Sep 2020 - SHIFT - 1

**Q.1** A line parallel to the straight line  $2x-y=0$  is tangent to the hyperbola  $\frac{x^2}{4} - \frac{y^2}{2} = 1$  at the point

$(x_1, y_1)$ . Then  $x_1^2 + 5y_1^2$  is equal to :

- (1) 6 (2) 10 (3) 8 (4) 5

**Sol. 1**

$$T : \frac{xx_1}{4} - \frac{yy_1}{2} = 1 \quad \dots(1)$$

$t : 2x - y = 0$  is parallel to  $T$

$$\Rightarrow T : 2x - y = \lambda \quad \dots\dots(2)$$

Now compare (1) & (2)

$$\frac{x_1}{4} = \frac{y_1}{1} = \frac{1}{\lambda}$$

$$x_1 = 4/\lambda \text{ \& } y_1 = 1/\lambda$$

$$(x_1, y_1) \text{ lies on hyperbola} \Rightarrow \frac{64}{4\lambda^2} - \frac{1}{2\lambda^2} = 1$$

$$\Rightarrow 14 = \lambda^2$$

$$\text{Now } = x_1^2 + 5y_1^2$$

$$= \frac{64}{\lambda^2} + 5 \frac{1}{\lambda^2}$$

$$= \frac{84}{14}$$

$$= 6 \text{ Ans.}$$

**Q.2** The domain of the function  $f(x) = \sin^{-1}\left(\frac{|x|+5}{x^2+1}\right)$  is  $(-\infty, -a] \cup [a, \infty)$ . Then  $a$  is equal to :

- (1)  $\frac{\sqrt{17}-1}{2}$  (2)  $\frac{\sqrt{17}}{2}$  (3)  $\frac{1+\sqrt{17}}{2}$  (4)  $\frac{\sqrt{17}}{2} + 1$

**Sol. 3**

$$-1 \leq \frac{|x|+5}{x^2+1} \leq 1$$

$$-x^2-1 \leq |x|+5 \leq x^2+1$$

**case - I**

$$-x^2-1 \leq |x|+5$$

this inequality is always right  $\forall x \in \mathbb{R}$

**case - II**

$$|x|+5 \leq x^2+1$$

$$x^2 - |x| \geq 4$$

$$|x|^2 - |x| - 4 \geq 0$$

$$\left( |x| - \left( \frac{1 + \sqrt{17}}{2} \right) \right) \left( |x| - \left( \frac{1 - \sqrt{17}}{2} \right) \right) \geq 0$$

$$|x| \leq \frac{1 - \sqrt{17}}{2} \cup |x| \geq \frac{1 + \sqrt{17}}{2}$$

not possible

$$x \in \left( -\infty, \frac{-1 - \sqrt{17}}{2} \right] \cup \left[ \frac{1 + \sqrt{17}}{2}, \infty \right)$$

$$a = \frac{1 + \sqrt{17}}{2}$$

**Q.3** If a function  $f(x)$  defined by  $f(x) = \begin{cases} ae^x + be^{-x}, & -1 \leq x < 1 \\ cx^2, & 1 \leq x \leq 3 \\ ax^2 + 2cx, & 3 < x \leq 4 \end{cases}$  be continuous for some  $a, b, c \in \mathbb{R}$  and

$f'(0) + f'(2) = e$ , then the value of  $a$  is :

(1)  $\frac{1}{e^2 - 3e + 13}$

(2)  $\frac{e}{e^2 - 3e - 13}$

(3)  $\frac{e}{e^2 + 3e + 13}$

(4)  $\frac{e}{e^2 - 3e + 13}$

**Sol.**

**4**  
 $f(x)$  is continuous

$$\text{at } x=1 \Rightarrow \boxed{ae + \frac{b}{e} = c}$$

$$\text{at } x=3 \Rightarrow 9c = 9a + 6c \Rightarrow c = 3a$$

$$\text{Now } f'(0) + f'(2) = e$$

$$\Rightarrow a - b + 4c = e$$

$$\Rightarrow a - e(3a - ae) + 4.3a = e$$

$$\Rightarrow a - 3ae + ae^2 + 12a = e$$

$$\Rightarrow 13a - 3ae + ae^2 = e$$

$$\Rightarrow \boxed{a = \frac{e}{13 - 3e + e^2}}$$

**Q.4** The sum of the first three terms of a G.P. is  $S$  and their product is 27. Then all such  $S$  lie in :

(1)  $(-\infty, -9] \cup [3, \infty)$

(2)  $[-3, \infty)$

(3)  $(-\infty, 9]$

(4)  $(-\infty, -3] \cup [9, \infty)$

**Sol.**

**4**

$$\frac{a}{r} \cdot aar = 27 \Rightarrow a = 3$$

$$\frac{a}{r} + a + ar = S$$

$$\frac{1}{r} + 1 + r = \frac{S}{3}$$

$$r + \frac{1}{r} = \frac{S}{3} - 1$$

$$r + \frac{1}{r} \geq 2 \text{ or } r + \frac{1}{r} \leq -2$$

$$\frac{S}{3} \geq 3 \text{ or } \frac{S}{3} \leq -1$$

$$S \geq 9 \text{ or } S \leq -3$$

$$S \in (-\infty, -3] \cup [9, \infty)$$

**Q.5** If  $R = \{(x, y) : x, y \in \mathbb{Z}, x^2 + 3y^2 \leq 8\}$  is a relation on the set of integers  $\mathbb{Z}$ , then the domain of  $R^{-1}$  is :

- (1)  $\{-1, 0, 1\}$  (2)  $\{-2, -1, 1, 2\}$  (3)  $\{0, 1\}$  (4)  $\{-2, -1, 0, 1, 2\}$

**Sol. 1**

$$3y^2 \leq 8 - x^2$$

$$R : \{(0, 1), (0, -1), (1, 0), (-1, 0), (1, 1), (1, -1), (-1, 1), (-1, -1), (2, 0), (-2, 0), (-2, 1), (2, -1), (-2, -1), (-2, -1)\}$$

$$\Rightarrow R : \{-2, -1, 0, 1, 2\} \rightarrow \{-1, 0, 1\}$$

$$\text{Hence } R^{-1} : \{-1, 0, 1\} \rightarrow \{-2, -1, 0, 1, 2\}$$

**Q.6** The value of  $\left( \frac{1 + \sin \frac{2\pi}{9} + i \cos \frac{2\pi}{9}}{1 + \sin \frac{2\pi}{9} - i \cos \frac{2\pi}{9}} \right)^3$  is :

- (1)  $-\frac{1}{2}(1 - i\sqrt{3})$  (2)  $\frac{1}{2}(1 - i\sqrt{3})$  (3)  $-\frac{1}{2}(\sqrt{3} - i)$  (4)  $\frac{1}{2}(\sqrt{3} - i)$

**Sol. 3**

$$\left( \frac{1 + \sin \frac{2\pi}{9} + i \cos \frac{2\pi}{9}}{1 + \sin \frac{2\pi}{9} - i \cos \frac{2\pi}{9}} \right)^3$$

$$= \left( \frac{1 + \cos \left( \frac{\pi}{2} - \frac{2\pi}{9} \right) + i \sin \left( \frac{\pi}{2} - \frac{2\pi}{9} \right)}{1 + \cos \left( \frac{\pi}{2} - \frac{2\pi}{9} \right) - i \sin \left( \frac{\pi}{2} - \frac{2\pi}{9} \right)} \right)^3$$

$$\begin{aligned}
 &= \left( \frac{1 + \cos \frac{5\pi}{18} + i \sin \frac{5\pi}{18}}{1 + \cos \frac{5\pi}{18} - i \sin \frac{5\pi}{18}} \right)^3 \\
 &= \left( \frac{2 \cos \frac{5\pi}{36} \left\{ \cos \frac{5\pi}{36} + i \sin \frac{5\pi}{36} \right\}}{2 \cos \frac{5\pi}{36} \left\{ \cos \frac{5\pi}{36} - i \sin \frac{5\pi}{36} \right\}} \right)^3 \\
 &= \left( \frac{\text{cis} \left( \frac{5\pi}{36} \right)}{\text{cis} \left( -\frac{5\pi}{36} \right)} \right)^3 \\
 &= \text{cis} \left( \frac{5\pi}{36} \times 3 + \frac{5\pi}{36} \times 3 \right) \\
 &= \text{cis} \left( \frac{10\pi}{12} \right) \\
 &= \text{cis} \left( \frac{5\pi}{6} \right) = \boxed{-\frac{\sqrt{3}}{2} + \frac{i}{2}}
 \end{aligned}$$

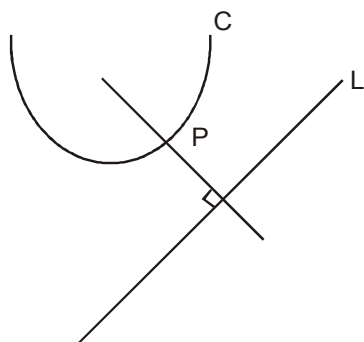
**Q.7** Let P(h,k) be a point on the curve  $y=x^2+7x+2$ , nearest to the line,  $y=3x-3$ . Then the equation of the normal to the curve at P is:

- (1)  $x+3y-62=0$       (2)  $x-3y-11=0$       (3)  $x-3y+22=0$       (4)  $x+3y+26=0$

**Sol.**

**4**  
C :  $y = x^2 + 7x + 2$

Let P : (h, k) lies on



Curve =  $k = h^2 + 7h + 2$

Now for shortest distance

$$M_T \big|_p^c = m_L = 2h+7 = 3$$

$$h = -2$$

$$k = -8$$

$$P : (-2, -8)$$

equation of normal to the curve is perpendicular to  $L : 3x - y = 3$

$$N : x + 3y = \lambda$$

$$\downarrow \text{Pass } (-2, -8)$$

$$\lambda = -26$$

$$N : x + 3y + 26 = 0$$

**Q.8** Let  $A$  be a  $2 \times 2$  real matrix with entries from  $\{0, 1\}$  and  $|A| \neq 0$ . Consider the following two statements:

(P) If  $A \neq I_2$ , then  $|A| = -1$

(Q) If  $|A| = 1$ , then  $\text{tr}(A) = 2$ ,

where  $I_2$  denotes  $2 \times 2$  identity matrix and  $\text{tr}(A)$  denotes the sum of the diagonal entries of  $A$ . Then:

(1) Both (P) and (Q) are false

(2) (P) is true and (Q) is false

(3) Both (P) and (Q) are true

(4) (P) is false and (Q) is true

**Sol. 4**

$$P : A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \neq I_2 \text{ \& } |A| \neq 0 \text{ \& } |A| = 1 (\text{false})$$

$$Q : A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = I_2 \text{ then } \text{Tr}(A) = 2 \text{ (true)}$$

**Q.9** Box I contains 30 cards numbered 1 to 30 and Box II contains 20 cards numbered 31 to 50. A box is selected at random and a card is drawn from it. The number on the card is found to be a non-prime number. The probability that the card was drawn from Box I is:

(1)  $\frac{4}{17}$

(2)  $\frac{8}{17}$

(3)  $\frac{2}{5}$

(4)  $\frac{2}{3}$

**Sol. 2**

1 to 30

box I

Prime on I

$\{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$

31 to 50

box II

Prime on II

$\{31, 37, 41, 43, 47\}$

$A$  : selected number on card is non - prime

$$P(A) = P(I) \cdot P(A/I) + P(II) \cdot P(A/II)$$

$$= \frac{1}{2} \times \frac{20}{30} + \frac{1}{2} \cdot \frac{15}{20}$$

$$\text{Now, } P(I/A) = \frac{P(II) \cdot P(A/I)}{P(A)}$$

$$= \frac{\frac{1}{2} \cdot \frac{20}{30}}{\frac{1}{2} \cdot \frac{20}{30} + \frac{1}{2} \cdot \frac{15}{20}} = \frac{\frac{2}{3}}{\frac{2}{3} + \frac{3}{4}} = \frac{8}{17}$$

**Q.10** If  $p(x)$  be a polynomial of degree three that has a local maximum value 8 at  $x=1$  and a local minimum value 4 at  $x=2$ ; then  $p(0)$  is equal to :

- (1) 12                      (2) -12                      (3) -24                      (4) 6

**Sol. 2**

$$p'(1) = 0 \text{ \& } p'(2) = 0$$

$$p'(x) = a(x-1)(x-2)$$

$$p(x) = a \left( \frac{x^3}{3} - \frac{3x^2}{2} + 2x \right) + b$$

$$p(1) = 8 \Rightarrow a \left( \frac{1}{3} - \frac{3}{2} + 2 \right) + b = 8 \quad \dots(i)$$

$$p(2) = 4 \Rightarrow a \left( \frac{8}{3} - \frac{3 \cdot 4}{2} + 2 \cdot 2 \right) + b = 4 \quad \dots(ii)$$

from equation (i) and (ii)

$$a = 24 \text{ \& } b = -12$$

$$p(0) = b = \boxed{-12}$$

**Q.11** The contrapositive of the statement "If I reach the station in time, then I will catch the train" is:

- (1) If I will catch the train, then I reach the station in time.  
 (2) If I do not reach the station in time, then I will catch the train.  
 (3) If I do not reach the station in time, then I will not catch the train.  
 (4) If I will not catch the train, then I do not reach the station in time.

**Sol. 4**

Statement  $p$  and  $q$  are true

Statement, then the contra positive of the implication

$$p \rightarrow q = (\sim q) \rightarrow (\sim p)$$

hence correct Ans. is 4

**Q.12** Let  $\alpha$  and  $\beta$  be the roots of the equation,  $5x^2+6x-2=0$ . If  $S_n = \alpha^n + \beta^n$ ,  $n=1,2,3,\dots$ , then:

- (1)  $5S_6+6S_5+2S_4=0$                       (2)  $6S_6+5S_5=2S_4$   
 (3)  $6S_6+5S_5+2S_4=0$                       (4)  $5S_6+6S_5=2S_4$

**Sol. 4**

$$5x^2 + 6x - 2 = 0 \quad \alpha = 5\alpha^2 + 6\alpha = 2$$

$$6\alpha - 2 = -5\alpha^2$$

Similarly

$$6\beta - 2 = -5\beta^2$$

$$S_6 = \alpha^6 + \beta^6$$

$$S_5 = \alpha^5 + \beta^5$$

$$S_4 = \alpha^4 + \beta^4$$

$$\text{Now } 6S_5 - 2S_4$$

$$= 6\alpha^5 - 2\alpha^4 + 6\beta^5 - 2\beta^4$$

$$= \alpha^4(6\alpha - 2) + \beta^4(6\beta - 2)$$

$$= \alpha^4(-5\alpha^2) + \beta^4(-5\beta^2)$$

$$= -5(\alpha^6 + \beta^6)$$

$$= -5S_6$$

$$= 6S_5 + 5S_6 = 2S_4$$

**Q.13** If the tangent to the curve  $y = x + \sin y$  at a point  $(a, b)$  is parallel to the line joining  $\left(0, \frac{3}{2}\right)$  and

$\left(\frac{1}{2}, 2\right)$ , then:

(1)  $b = \frac{\pi}{2} + a$

(2)  $|a+b|=1$

(3)  $|b-a|=1$

(4)  $b=a$

**Sol. 3**

$$\left. \frac{dy}{dx} \right|_{p(a,b)} = \frac{2 - \frac{3}{2}}{\frac{1}{2} - 0}$$

$$\begin{aligned} 1 + \cos b &= 1 \\ \cos b &= 0 \end{aligned} \quad \left| \begin{array}{l} p : (a, b) \text{ lies on curve} \\ b = a + \sin b \end{array} \right.$$

$$\boxed{b = a \pm 1}$$

$$b - a = \pm 1$$

$$\boxed{|b - a| = 1}$$

**Q.14** Area (in sq. units) of the region outside  $\frac{|x|}{2} + \frac{|y|}{3} = 1$  and inside the ellipse  $\frac{x^2}{4} + \frac{y^2}{9} = 1$  is:

(1)  $3(\pi - 2)$

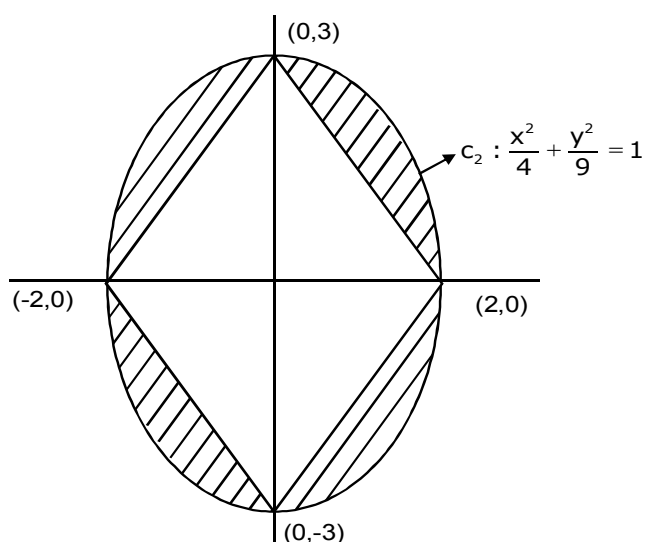
(2)  $6(\pi - 2)$

(3)  $6(4 - \pi)$

(4)  $3(4 - \pi)$

**Sol. 2**

$$C_1 : \frac{|x|}{2} + \frac{|y|}{3} = 1$$



$$A = 4 \left( \frac{\pi ab}{4} - \frac{1}{2} \cdot 2 \cdot 3 \right)$$

$$A = \pi \cdot 2 \cdot 3 - 12$$

$$A = 6(\pi - 2)$$

**Q.15** If  $|x| < 1, |y| < 1$  and  $x \neq y$ , then the sum to infinity of the following series  $(x+y) + (x^2+xy+y^2) + (x^3+x^2y+xy^2+y^3) + \dots$  is:

(1)  $\frac{x+y+xy}{(1-x)(1-y)}$       (2)  $\frac{x+y-xy}{(1-x)(1-y)}$       (3)  $\frac{x+y+xy}{(1+x)(1+y)}$       (4)  $\frac{x+y-xy}{(1+x)(1+y)}$

**Sol. 2**

$$(x+y) + (x^2+xy+y^2) + (x^3+x^2y+xy^2+y^3) + \dots \infty$$

$$= \frac{1}{(x-y)} \{ (x^2 - y^2) + (x^3 - y^3) + (x^4 - y^4) + \dots \infty \}$$

$$= \frac{\frac{x^2}{1-x} - \frac{y^2}{1-y}}{x-y}$$

$$= \frac{x^2(1-y) - y^2(1-x)}{(1-x)(1-y)(x-y)}$$

$$= \frac{(x^2 - y^2) - xy(x-y)}{(1-x)(1-y)(x-y)} = \frac{((x+y) - xy)(x-y)}{(1-x)(1-y)(x-y)}$$

$$= \frac{x+y-xy}{(1-x)(1-y)}$$

**Q.16** Let  $\alpha > 0, \beta > 0$  be such that  $\alpha^3 + \beta^2 = 4$ . If the maximum value of the term independent of  $x$  in

the binomial expansion of  $\left(\alpha x^{\frac{1}{9}} + \beta x^{-\frac{1}{6}}\right)^{10}$  is  $10k$ , then  $k$  is equal to:

- (1) 176                      (2) 336                      (3) 352                      (4) 84

**Sol.**

**2**  
For term independent of  $x$

$$T_{r+1} = {}^{10}C_r \left(\alpha x^{\frac{1}{9}}\right)^{10-r} \cdot \left(\beta x^{-\frac{1}{6}}\right)^r$$

$$T_{r+1} = {}^{10}C_r \alpha^{10-r} \beta^r x^{\frac{10-r}{9}} x^{-\frac{r}{6}}$$

$$\therefore \frac{10-r}{9} - \frac{r}{6} = 0 \Rightarrow r=4$$

$$T_5 = {}^{10}C_4 \alpha^6 \beta^4$$

$$\therefore AM \geq GM$$

$$\text{Now } \frac{\left(\frac{\alpha^3}{2} + \frac{\alpha^3}{2} + \frac{\beta^2}{2} + \frac{\beta^2}{2}\right)}{4} \geq \sqrt[4]{\frac{\alpha^6 \beta^4}{2^4}}$$

$$\left(\frac{4}{4}\right)^4 \geq \frac{\alpha^6 \beta^4}{2^4}$$

$$\alpha^6 \beta^4 \leq 2^4$$

$${}^{10}C_4 \cdot \alpha^6 \beta^4 \leq {}^{10}C_4 2^4$$

$$T_5 \leq {}^{10}C_4 2^4$$

$$T_5 \leq \frac{10!}{6!4!} \cdot 2^4$$

$$T_5 \leq \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 2^4}{4 \cdot 3 \cdot 2 \cdot 1}$$

$$\text{maximum value of } T_5 = 10 \cdot 3 \cdot 7 \cdot 16 = 10k$$

$$k = 16 \cdot 7 \cdot 3$$

$$k = 336$$

**Q.17** Let  $S$  be the set of all  $\lambda \in \mathbb{R}$  for which the system of linear equations

$$2x - y + 2z = 2$$

$$x - 2y + \lambda z = -4$$

$$x + \lambda y + z = 4$$

has no solution. Then the set  $S$

(1) is an empty set.

(2) is a singleton.

(3) contains more than two elements.

(4) contains exactly two elements.

**Sol. 4**

For no solution

$$\Delta = 0 \text{ \& } \Delta_1 | \Delta_2 | \Delta_3 \neq 0$$

$$\Delta = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & \lambda \\ 1 & \lambda & 1 \end{vmatrix} = 0$$

$$2(-2 - \lambda^2) + 1(1 - \lambda) + 2(\lambda + 2) = 0$$

$$-4 - 2\lambda^2 + 1 - \lambda + 2\lambda + 4 = 0$$

$$-2\lambda^2 + \lambda + 1 = 0$$

$$2\lambda^2 - \lambda - 1 = 0 \Rightarrow \lambda = 1, -1/2$$

Equation has exactly 2 solution

**Q.18** Let  $X = \{x \in \mathbb{N} : 1 \leq x \leq 17\}$  and  $Y = \{ax + b : x \in X \text{ and } a, b \in \mathbb{R}, a > 0\}$ . If mean and variance of elements of  $Y$  are 17 and 216 respectively then  $a + b$  is equal to:

(1) -27

(2) 7

(3) -7

(4) 9

**Sol. 3**

$$X : \{1, 2, \dots, 17\}$$

$$Y : \{ax + b : x \in X \text{ \& } a, b \in \mathbb{R}, a > 0\}$$

$$\text{Given Var}(Y) = 216$$

$$\frac{\sum y_i^2}{n} - (\text{mean})^2 = 216$$

$$\frac{\sum y_i^2}{17} - 289 = 216$$

$$\sum y_i = 8585$$

$$(a+b)^2 + (2a+b)^2 + \dots + (17a+b)^2 = 8585$$

$$105a^2 + b^2 + 18ab = 505 \dots (1)$$

$$\text{Now } \sum y_i = 17 \times 17$$

$$a(17 \times 9) + 17b = 17 \times 17$$

$$9a + b = 17 \dots (2)$$

from equation (1) \& (2)

$$a = 3 \text{ \& } b = -10$$

$$a + b = -7$$

**Q.19** Let  $y=y(x)$  be the solution of the differential equation,  $\frac{2+\sin x}{y+1} \cdot \frac{dy}{dx} = -\cos x$ ,  $y > 0$ ,  $y(0) = 1$ . If

$y(\pi) = a$ , and  $\frac{dy}{dx}$  at  $x = \pi$  is  $b$ , then the ordered pair  $(a,b)$  is equal to:

- (1)  $\left(2, \frac{3}{2}\right)$                       (2)  $(1,1)$                       (3)  $(2,1)$                       (4)  $(1,-1)$

**Sol. 2**

$$\int \frac{dy}{y+1} = \int \frac{-\cos x \, dx}{2+\sin x}$$

$$\ln |y+1| = -\ln |2+\sin x| + k$$

$$\downarrow (0,1)$$

$$k = \ln 4$$

$$\text{Now } C : (y+1)(2+\sin x) = 4$$

$$y(\pi)=a \Rightarrow (a+1)(2+0)=4 \Rightarrow (a=1)$$

$$\left. \frac{dy}{dx} \right|_{x=\pi} = b \Rightarrow b = -(-1) \left( \frac{2+0}{1+1} \right)$$

$$\Rightarrow b = 1$$

$$(a,b) = (1,1)$$

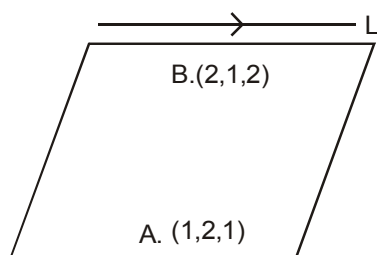
**Q.20** The plane passing through the points  $(1,2,1)$ ,  $(2,1,2)$  and parallel to the line,  $2x=3y$ ,  $z=1$  also passes through the point:

- (1)  $(0,-6,2)$                       (2)  $(0,6,-2)$                       (3)  $(-2,0,1)$                       (4)  $(2,0,-1)$

**Sol. 3**

$$L : \begin{cases} 2x = 3y \\ z = 1 \end{cases} \begin{matrix} P : (0,0,1) \\ Q : (3,2,1) \end{matrix}$$

$$\vec{V}_L \text{ Dr of line } (3,2,0)$$



$$\vec{n}_p = \vec{AB} \times \vec{V}_L$$

$$\vec{n}_p = \langle 1, -1, 1 \rangle \times \langle 3, 2, 0 \rangle$$

$$\vec{n}_p = \langle -2, +3, 5 \rangle$$

$$\text{Plane : } -2(x-1)+3(y-2)+5(z-1)=0$$

Plane :  $-2x+3y+5z+2-6-5=0$

Plane :  $2x - 3y - 5z = -9$

**Q.21** The number of integral values of  $k$  for which the line,  $3x+4y=k$  intersects the circle,  $x^2+y^2-2x-4y+4=0$  at two distinct points is.....

**Sol.** **9**

$c : (1,2) \text{ \& } r = 1$

$|cp| < r$

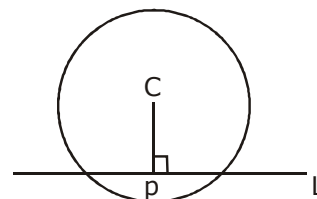
$$\left| \frac{3 \cdot 1 + 4 \cdot 2 - k}{5} \right| < 1$$

$|11-k| < 5$

$-5 < k-11 < 5$

$6 < k < 16$

$k = 7, 8, 9, \dots, 15 \Rightarrow$  total 9 value of  $k$



**Q.22** Let  $\vec{a}, \vec{b}$  and  $\vec{c}$  be three unit vectors such that  $|\vec{a} - \vec{b}|^2 + |\vec{a} - \vec{c}|^2 = 8$ . Then  $|\vec{a} + 2\vec{b}|^2 + |\vec{a} + 2\vec{c}|^2$  is equal to :

**Sol.** **2**

$$|\vec{a} - \vec{b}|^2 + |\vec{a} - \vec{c}|^2 = 8$$

$$(\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b}) + (\vec{a} - \vec{c}) \cdot (\vec{a} - \vec{c}) = 8$$

$$a^2 + b^2 - 2\vec{a} \cdot \vec{b} + a^2 + c^2 - 2\vec{a} \cdot \vec{c} = 8$$

$$2a^2 + b^2 + c^2 - 2\vec{a} \cdot \vec{b} - 2\vec{a} \cdot \vec{c} = 8$$

$$\vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} = -2$$

Now  $|\vec{a} + 2\vec{b}|^2 + |\vec{a} + 2\vec{c}|^2$

$$= 2a^2 + 4b^2 + 4c^2 + 4\vec{a} \cdot \vec{b} + 4\vec{a} \cdot \vec{c}$$

$$= 2 + 4 + 4 + 4(-2)$$

$$= 2$$

**Q.23** If the letters of the word 'MOTHER' be permuted and all the words so formed (with or without meaning) be listed as in a dictionary, then the position of the word 'MOTHER' is.....

**Sol.** **309**

E H M O R T

E - - - - = 5!

H - - - - = 5!

M E - - - = 4!

M H - - - = 4!

M O E - - = 3!

M O H - - = 3!

M O R - - = 3!

M O T E - - = 2!

M O T H E R = 1

309

**Q.24.** If  $\lim_{x \rightarrow 1} \frac{x + x^2 + x^3 + \dots + x^n - n}{x - 1} = 820, (n \in \mathbb{N})$  then the value of  $n$  is equal to :

**Sol.** 40

$$\lim_{x \rightarrow 1} \frac{(x-1)}{x-1} + \frac{(x^2-1)}{x-1} + \dots + \frac{(x^n-1)}{x-1} = 820$$

$$\Rightarrow 1 + 2 + 3 + \dots + n = 820$$

$$\Rightarrow \sum n = 820$$

$$\Rightarrow \frac{n(n+1)}{2} = 820$$

$$\Rightarrow n = 40$$

**Q.25** The integral  $\int_0^2 ||x-1|-x| dx$  is equal to :

**Sol.** 1.5

$$\int_0^2 ||x-1|-x| dx$$

$$= \int_0^1 |1-x-x| dx + \int_1^2 |x-1-x| dx$$

$$= \int_0^1 |2x-1| dx + \int_1^2 1 dx$$

$$= \int_0^{\frac{1}{2}} (1-2x) dx + \int_{\frac{1}{2}}^1 (2x-1) dx + \int_1^2 1 dx$$

$$= \left[ \left( \frac{1}{2} - 0 \right) - \left( \frac{1}{4} - 0 \right) \right] + \left( 1 - \frac{1}{4} \right) - \left( 1 - \frac{1}{2} \right) + 1$$

$$= \frac{1}{2} - \frac{1}{4} + \frac{3}{4} - \frac{1}{2} + 1$$

$$= \frac{3}{2}$$

# QUESTION PAPER WITH SOLUTION

## PHYSICS \_ 2 Sep. \_ SHIFT - 2

1. If momentum (P), area (A) and time (T) are taken to be the fundamental quantities then the dimensional formula for energy is:

(1)  $[P^{1/2} AT^{-1}]$  (2)  $[PA^{1/2}T^{-1}]$  (3)  $[PA^{1/2}T^{-1}]$  (4)  $[P^2AT^{-2}]$

Sol.

(2)

$[P] = MLT^{-1} \leftarrow$  momentum

$[A] = M^0L^2T^0 \leftarrow$  Area

$[T] = M^0L^0T^1 \leftarrow$  Time

Let  $[E] = P^x A^y T^z$

$ML^2T^{-2} = [MLT^{-1}]^x [L^2]^y [T]^z$

$= M^x L^{x+2y} T^{z-x}$

Comparing both sides :-

$x = 1$  .....(i)

$x + 2y = 2 \Rightarrow 1 + 2y = 2$  or,  $y = \frac{1}{2}$  .....(ii)

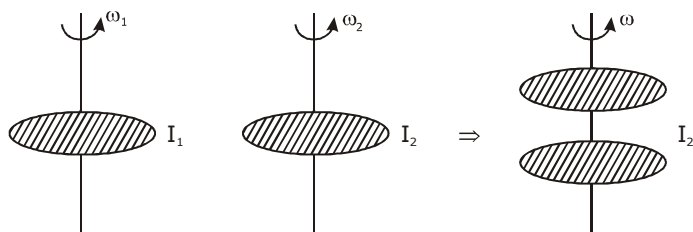
$z - x = -2 \Rightarrow z - 1 = -2$  or  $z = -1$  .....(iii)

$\therefore [E] = [P^1 A^{1/2} T^{-1}]$

2. Two uniform circular discs are rotating independently in the same direction around their common axis passing through their centres. The moment of inertia and angular velocity of the first disc are  $0.1 \text{ kg-m}^2$  and  $10 \text{ rad s}^{-1}$  respectively while those for the second one are  $0.2 \text{ kg-m}^2$  and  $5 \text{ rad s}^{-1}$  respectively. At some instant they get stuck together and start rotating as a single system about their common axis with some angular speed. The Kinetic energy of the combined system is:

(1)  $\frac{2}{3} \text{ J}$  (2)  $\frac{10}{3} \text{ J}$  (3)  $\frac{5}{3} \text{ J}$  (4)  $\frac{20}{3} \text{ J}$

Sol. (4)



$$I_1 \omega_1 + I_2 \omega_2 = (I_1 + I_2) \omega$$

$$\omega = \frac{I_1 \omega_1 + I_2 \omega_2}{I_1 + I_2} = \frac{0.1 \times 10 + 0.2 \times 5}{0.1 + 0.2} = \frac{1 + 1}{0.3} = \frac{2}{0.3}$$

$$\omega = \frac{20}{3}$$

Now find  $KE = \frac{1}{2} I_1 \omega^2 + \frac{1}{2} I_2 \omega^2$

$$= \frac{1}{2} (I_1 + I_2) \omega^2 = \frac{1}{2} \times 0.3 \times \left( \frac{20}{3} \right)^2$$

$$= \frac{1}{2} \times \frac{3}{10} \times \frac{20}{3} \times \frac{20}{3}$$

$$\frac{0.3}{2} \times \frac{400}{9}$$

$$\boxed{(K.E.)_f = \frac{20}{3}}$$

- 3.** A particle is moving 5 times as fast as an electron. The ratio of the de-Broglie wavelength of the particle to that of the electron is  $1.878 \times 10^{-4}$ . The mass of the particle is close to:

(1)  $4.8 \times 10^{-27}$  kg    (2)  $9.1 \times 10^{-31}$  kg    (3)  $9.7 \times 10^{-28}$  kg    (4)  $1.2 \times 10^{-28}$  kg

**Sol. (3)**

$$p = \frac{h}{\lambda}$$

$$\lambda = \frac{h}{p}$$

$$\frac{\lambda_{\text{Particle}}}{\lambda_e} = 1.878 \times 10^{-4}$$

$$\Rightarrow \frac{h}{p_{\text{particle}}} \times \frac{p_e}{h} = 1.878 \times 10^{-4} \Rightarrow \frac{p_e}{p_{\text{particle}}} = 1.878 \times 10^{-4}$$

$$\Rightarrow \frac{M_e \cdot V_e}{M_p \cdot V_p} = 1.878 \times 10^{-4} \Rightarrow M_p = \frac{M_e}{1.878 \times 10^{-4}} \times \left( \frac{V_e}{V_p} \right)$$

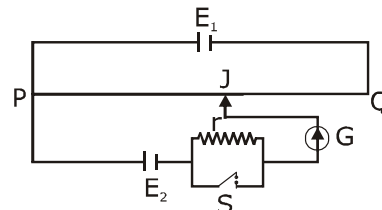
$$= \frac{9.11 \times 10^{-31}}{1.878 \times 10^{-4}} \times \frac{1}{5}$$

$$= \frac{7.11 \times 10^{-31}}{1.878 \times 10^{-4}} \times \frac{1}{5}$$

$$= 0.97 \times 10^{-27} \text{ kg}$$

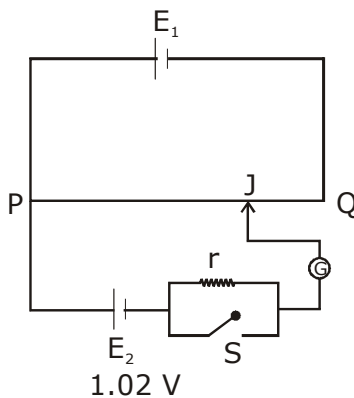
$$= 9.7 \times 10^{-28} \text{ kg}$$

4. A potentiometer wire PQ of 1 m length is connected to a standard cell  $E_1$ . Another cell  $E_2$  of emf 1.02 V is connected with a resistance 'r' and switch S (as shown in figure). With switch S open, the null position is obtained at a distance of 49 cm from Q. The potential gradient in the potentiometer wire is:



- (1) 0.03V/cm      (2) 0.02 V/cm      (3) 0.04 V/cm      (4) 0.01 V/cm

**Sol. (2)**



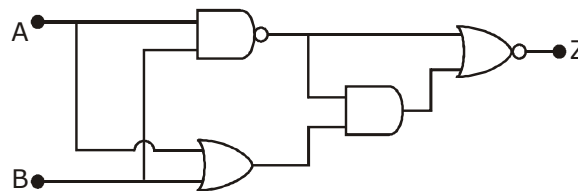
$$PQ = 1\text{m}$$

$$QJ = 49\text{ cm}$$

$$\therefore PJ = 51\text{ cm}$$

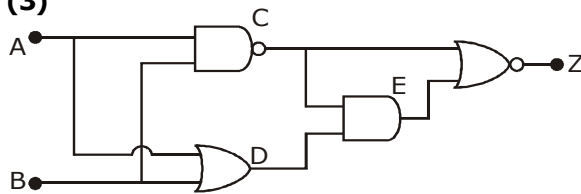
$$\frac{v}{\ell} = \frac{1.02}{51} = 0.02\text{ v/cm}$$

5. In the following digital circuit, what will be the output at 'Z', when the input (A,B) are (1,0), (0,0), (1,1), (0,1):



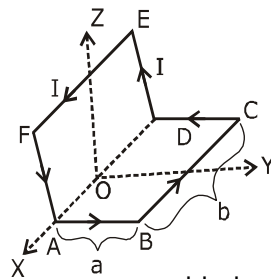
- (1) 0,1,0,0      (2) 1,1,0,1      (3) 0,0,1,0      (4) 1,0,1,1

**Sol. (3)**



A	B	$C = \overline{A \cdot B}$	$D = A + B$	$E = C \cdot D$	$Z = \overline{C + E}$
1	0	1	1	1	0
0	0	1	0	0	0
0	0	1	0	0	0
1	1	0	1	0	1
0	1	1	1	1	0

6. A wire carrying current  $I$  is bent in the shape ABCDEFA as shown, where rectangle ABCDA and ADEFA are perpendicular to each other. If the sides of the rectangles are of lengths  $a$  and  $b$ , then the magnitude and direction of magnetic moment of the loop ABCDEFA is:



(1)  $\sqrt{2} abI$ , along  $\left( \frac{\hat{j}}{\sqrt{5}} + \frac{2\hat{k}}{\sqrt{5}} \right)$

(2)  $abl$ , along  $\left( \frac{\hat{j}}{\sqrt{5}} + \frac{2\hat{k}}{\sqrt{5}} \right)$

(3)  $\sqrt{2} abI$ , along  $\left( \frac{\hat{j}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}} \right)$

(4)  $abl$ , along  $\left( \frac{\hat{j}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}} \right)$

**Sol. (3)**

LOOP = ABCD

$$\vec{M}_1 = (abI) \hat{k}$$

Loop DEFA

$$\vec{M}_2 = (abI) \hat{j}$$

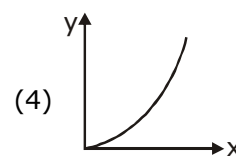
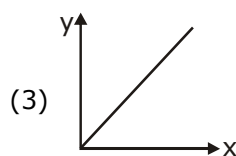
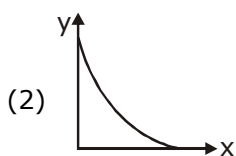
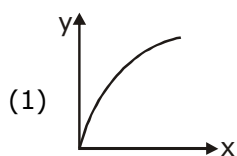
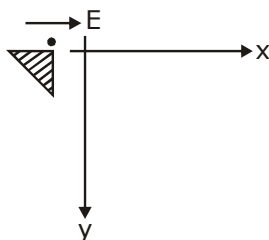
$$\vec{M} = \vec{M}_1 + \vec{M}_2 = abI (\hat{j} + \hat{k})$$

$$|\vec{M}| = \sqrt{2} abI$$

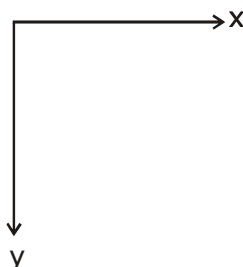
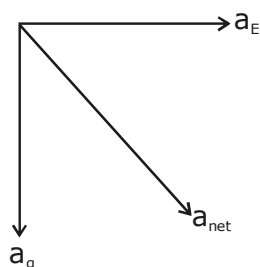
$$\text{direction} = \text{along} \left( \frac{\hat{j}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}} \right)$$

$$\sqrt{2} abI, \text{ along } \left( \frac{\hat{j}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}} \right)$$

7. A small point mass carrying some positive charge on it, is released from the edge of a table. There is a uniform electric field in this region in the horizontal direction. Which of the following options then correctly describe the trajectory of the mass? (Curves are drawn schematically and are not to scale).



Sol. (3)



Since it is released from rest.

And  $a_{\text{net}}$  is constant.

It will have straight line path along net 'a'.

8. In a plane electromagnetic wave, the directions of electric field and magnetic field are represented by  $\hat{k}$  and  $2\hat{i} - 2\hat{j}$ , respectively. What is the unit vector along direction of propagation of the wave.

(1)  $\frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$       (2)  $\frac{1}{\sqrt{5}}(2\hat{i} + \hat{j})$       (3)  $\frac{1}{\sqrt{5}}(\hat{i} + 2\hat{j})$       (4)  $\frac{1}{\sqrt{2}}(\hat{j} + \hat{k})$

Sol. (1)

$$\vec{E} \times \vec{B} = \hat{k} \times (2\hat{i} - 2\hat{j}) = 2\hat{k} \times \hat{i} - 2\hat{k} \times \hat{j} = 2\hat{j} + 2\hat{i}$$

$$\text{unit vector along } \vec{E} \times \vec{B} = \frac{1}{2\sqrt{2}}(2\hat{i} + 2\hat{j}) = \frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$$

$$C = \frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$$

9. An inductance coil has a reactance of  $100 \Omega$ . When an AC signal of frequency  $1000 \text{ Hz}$  is applied to the coil, the applied voltage leads the current by  $45^\circ$ . The self-inductance of the coil is:

(1)  $6.7 \times 10^{-7} \text{ H}$       (2)  $5.5 \times 10^{-5} \text{ H}$       (3)  $1.1 \times 10^{-1} \text{ H}$       (4)  $1.1 \times 10^{-2} \text{ H}$

**Sol. (4)**

L-R circuit

$$\tan 45^\circ = \frac{X_L}{R}$$

$$1 = \frac{X_L}{R} \Rightarrow X_L = R$$

$$\text{Now } Z = \sqrt{R^2 + X_L^2}$$

$$\text{or } Z = \sqrt{X_L^2 + X_L^2} = \sqrt{2X_L^2} = \sqrt{2}X_L$$

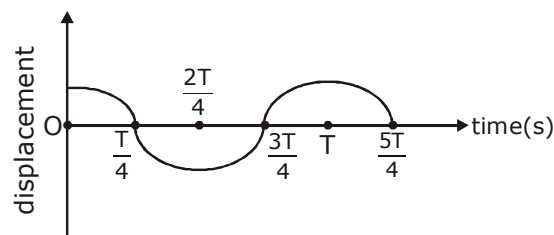
$$100 = \sqrt{2}X_L$$

$$X_L = \frac{100}{\sqrt{2}}$$

$$\omega L = \frac{100}{\sqrt{2}} \Rightarrow L = \frac{100}{\sqrt{2}\omega} = \frac{100}{\sqrt{2} \times 2\pi f} = \frac{100}{\sqrt{2} \times 2 \times 3.14 \times 1000}$$

$$= 1.1 \times 10^{-2} \text{ H}$$

10. This displacement time graph of a particle executing S.H.M. is given in figure : (sketch is schematic and not to scale)



Which of the following statements is/are true for this motion?

- (A) The force is zero at  $t = \frac{3T}{4}$       (B) The acceleration is maximum at  $t=T$
- (C) The speed is maximum at  $t = \frac{T}{4}$       (D) The P.E. is equal to K.E. of the oscillation at  $t = \frac{T}{2}$
- (1) (B), (C) and (D)    (2) (A), (B) and (D)    (3) (A) and (D)      (4) (A), (B) and (C)

**Sol. (A,B,C)**

(A) at  $t = \frac{3T}{4}$

Particle is at mean position

$$a = 0$$

$$F = 0$$

(B) at  $t = T$ ,

Particle is at extreme.

F is maximum

$$a = \max$$

(C) at  $t = \frac{T}{4}$ ; mean position

so, maximum velocity

(d)  $KE = PE$

$$\frac{1}{2}k(A^2 - x^2) = \frac{1}{2}kx^2$$

$$A^2 - x^2 = x^2$$

$$A^2 = 2x^2$$

$$A = \sqrt{2}x$$

$$x = \frac{A}{\sqrt{2}} = A \cos \omega t$$

$$\cos \omega t = \frac{1}{\sqrt{2}}$$

$$\omega t = \frac{\pi}{4}$$

$$\frac{2\pi}{T} \cdot t = \frac{\pi}{4} \Rightarrow t = \frac{T}{8}$$

- 11.** In a Young's double slit experiment, 16 fringes are observed in a certain segment of the screen when light of wavelength 700 nm is used. If the wavelength of light is changed to 400 nm, the number of fringes observed in the same segment of the screen would be:

- (1) 28                      (2) 24                      (3) 30                      (4) 18

**Sol. (1)**

$$y = \frac{D\lambda}{d}$$

$$\text{or } n_1 \frac{D\lambda_1}{d} = n_2 \frac{D\lambda_2}{d}$$

$$n_1\lambda_1 = n_2\lambda_2$$

$$\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1}$$

$$n_2 = n_1 \cdot \frac{\lambda_1}{\lambda_2} \Rightarrow 16 \times \frac{700}{400} = 28$$

- 12.** A heat engine is involved with exchange of heat of 1915 J, -40J, + 125J and -QJ, during one cycle achieving an efficiency of 50.0%. The value of Q is:

(1) 980 J                      (2) 640 J                      (3) 40 J                      (4) 400 J

**Sol. (1)**

$$\eta = \frac{W}{\sum Q_+} = \frac{Q_1 + Q_2 + Q_3 + Q_4}{Q_1 + Q_3}$$

$$0.5 = \frac{1915 - 40 + 125 - Q}{1915 + 125}$$

$$1020 = 1915 - 40 + 125 - Q$$

$$Q = 2000 - 1020 = 980 \text{ J}$$

- 13.** In a hydrogen atom the electron makes a transition from  $(n + 1)^{\text{th}}$  level to the  $n^{\text{th}}$  level. If  $n \gg 1$ , the frequency of radiation emitted is proportional to :

(1)  $\frac{1}{n^2}$                       (2)  $\frac{1}{n}$                       (3)  $\frac{1}{n^3}$                       (4)  $\frac{1}{n^4}$

**Sol. (3)**

$$E_n = \frac{-Rhc}{n^2}$$

$$E_{n+1} = \frac{-Rhc}{(n+1)^2}$$

$$\Delta E = E_{n+1} - E_n$$

$$h\nu = Rhc \left[ \frac{1}{n^2} - \frac{1}{(n+1)^2} \right]$$

$$v = Rc \left[ \frac{(n+1)^2 - n^2}{n^2 \cdot (n+1)^2} \right]$$

$$= Rc \left[ \frac{1+2n}{n^2 (n+1)^2} \right]$$

if  $n \gg 1$

$$v = \frac{2n}{n^2 \times n^2} = \frac{2n}{n^4} = \frac{2}{n^3}$$

$$\boxed{v \propto \frac{1}{n^3}}$$

- 14.** When the temperature of a metal wire is increased from  $0^\circ\text{C}$  to  $10^\circ\text{C}$ , its length increases by 0.02%. The percentage change in its mass density will be closest to :  
 (1) 0.06                      (2) 0.008                      (3) 2.3                      (4) 0.8

**Sol. (A)**

$$\Delta \ell = \ell \propto \Delta t$$

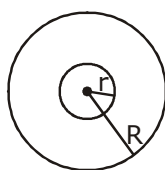
$$\alpha = \frac{\Delta \ell}{\ell \Delta t} = \frac{0.02}{100 \times 10} = 2 \times 10^{-5}$$

$$v = 3\alpha = 6 \times 10^{-5}$$

$$\text{Now, } \frac{\Delta v}{v} \times 100 = \gamma \cdot \Delta t \cdot 100 = 6 \times 10^{-5} \times 10 \times 100$$

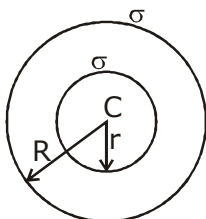
$$= 6 \times 10^{-2} = 0.06$$

- 15.** A charge  $Q$  is distributed over two concentric conducting thin spherical shells radii  $r$  and  $R$  ( $R > r$ ). If the surface charge densities on the two shells are equal, the electric potential at the common centre is:



- (1)  $\frac{1}{4\pi\epsilon_0} \frac{(2R+r)}{(R^2+r^2)} Q$                       (2)  $\frac{1}{4\pi\epsilon_0} \frac{(R+r)}{(R^2+r^2)} Q$
- (3)  $\frac{1}{4\pi\epsilon_0} \frac{(R+r)}{2(R^2+r^2)} Q$                       (4)  $\frac{1}{4\pi\epsilon_0} \frac{(R+2r)}{2(R^2+r^2)} Q$

**Sol. (2)**



$$Q_1 = \sigma \cdot 4\pi r^2$$

$$Q_2 = \sigma \cdot 4\pi R^2$$

$$Q = 4\pi\sigma(R^2 + r^2)$$

$$\Rightarrow \sigma = \frac{Q}{4\pi(R^2 + r^2)}$$

$$V_c = \frac{KQ_1}{r} + \frac{KQ_2}{R}$$

$$= \frac{K\sigma 4\pi r^2}{r} + \frac{K\sigma 4\pi R^2}{R} = K\sigma 4\pi(r + R)$$

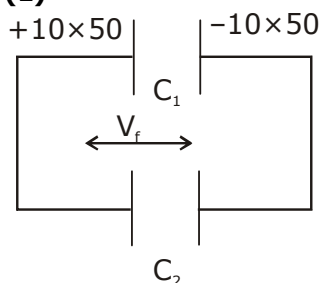
$$= 4\pi K \frac{Q}{4\pi(R^2 + r^2)}(r + R) = \frac{KQ(r + R)}{(R^2 + r^2)}$$

$$\frac{1}{4\pi\epsilon_0} \frac{(R + r)}{(R^2 + r^2)} \times Q$$

- 16.** A  $10 \mu\text{F}$  capacitor is fully charged to a potential difference of 50V. After removing the source voltage it is connected to an uncharged capacitor in parallel. Now the potential difference across them becomes 20 V. The capacitance of the second capacitor is:

(1)  $15 \mu\text{F}$                       (2)  $20 \mu\text{F}$                       (3)  $10 \mu\text{F}$                       (4)  $30 \mu\text{F}$

**Sol. (1)**



$$C_1 = 10 \mu\text{F}$$

$$V_f = 20 \text{ V}$$

$$\frac{500}{C_1 + C_2} = 20$$

$$\frac{500}{10 + C_2} = 20$$

$$10 + C_2 = \frac{500}{20} = 25$$

$$C_2 = 15 \mu\text{F}$$

- 17.** An ideal gas in a closed container is slowly heated. As its temperature increases, which of the following statements are true?  
 (A) the mean free path of the molecules decreases.  
 (B) the mean collision time between the molecules decreases.  
 (C) the mean free path remains unchanged.  
 (D) the mean collision time remains unchanged.  
 (1) (B) and (C)      (2) (A) and (B)      (3) (C) and (D)      (4) (A) and (D)

**Sol. (1) B,C**

$$\lambda = \frac{1}{\sqrt{2} \left( \frac{N}{V} \right) \pi d^2}$$

$\lambda \rightarrow$  Mean free path

$N \rightarrow$  No. of molecules

$V =$  volume of container

$d =$  diameter of molecule

$\therefore N$  and  $V$  are constant

$\therefore$  Mean free path remains unchanged.

Now, If  $T \uparrow$  no. of collisions increases.

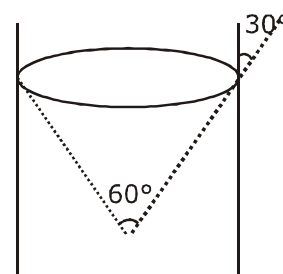
- 18.** A capillary tube made of glass of radius 0.15 mm is dipped vertically in a beaker filled with methylene iodide (surface tension =  $0.05 \text{ Nm}^{-1}$ , density =  $667 \text{ kg m}^{-3}$ ) which rises to height  $h$  in the tube. It is observed that the two tangents drawn from liquid-glass interfaces (from opp. sides of the capillary) make an angle of  $60^\circ$  with one another. Then  $h$  is close to ( $g = 10 \text{ ms}^{-2}$ ).  
 (1) 0.172 m      (2) 0.049 m      (3) 0.087 m      (4) 0.137 m

**Sol.** (3)

$$h = \frac{2T \cos \theta}{\rho g r} \quad \{ \theta = 30^\circ \}$$

$$= \frac{2 \times 0.05 \times \frac{1}{2}}{667 \times 10 \times 0.15 \times 10^{-3}}$$

$$= 0.087 \text{ m option (3)}$$



**19.** The height 'h' at which the weight of a body will be the same as that at the same depth 'h' from the surface of the earth is (Radius of the earth is R and effect of the rotation of the earth is neglected):

(1)  $\frac{\sqrt{3}R - R}{2}$

(2)  $\frac{\sqrt{5}}{2}R - R$

(3)  $\frac{\sqrt{5}R - R}{2}$

(4)  $\frac{R}{2}$

**Sol.** (3)

$$\frac{g_0}{\left(1 + \frac{h}{R}\right)^2} = g_0 \left(1 - \frac{h}{R}\right)$$

$$\frac{R^2}{(R+h)^2} = \frac{(R-h)}{R}$$

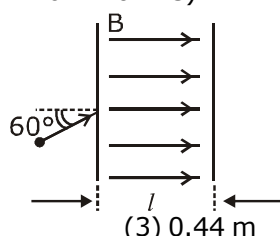
$$R^3 = (R-h)(R+h)^2$$

$$R^3 = (R-h)(R^2 + 2hR + h^2)$$

on solving we get,

$$h = \frac{\sqrt{5}R - R}{2}$$

**20.** The figure shows a region of length 'l' with a uniform magnetic field of 0.3 T in it and a proton entering the region with velocity  $4 \times 10^5 \text{ ms}^{-1}$  making an angle  $60^\circ$  with the field. If the proton completes 10 revolution by the time it cross the region shown, 'l' is close to (mass of proton =  $1.67 \times 10^{-27} \text{ kg}$ , charge of the proton =  $1.6 \times 10^{-19} \text{ C}$ )



(1) 0.11 m

(2) 0.22 m

(3) 0.44 m

(4) 0.88 m

**Sol. (3)**

$$\ell = 10 \times \text{pitch}$$

$$= 10 \times v \cos 60^\circ \times \frac{2\pi m}{qB}$$

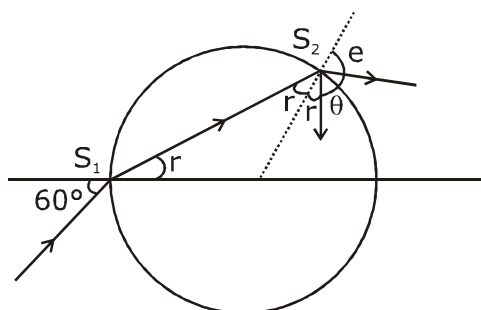
$$= 10 \times v \times \frac{1}{2} \times \frac{2\pi m}{qB}$$

$$\boxed{\ell = \frac{10v\pi m}{qB}} = \frac{10 \times 4 \times 10^5 \times 3.14 \times 1.67 \times 10^{-27}}{1.6 \times 10^{-19} \times 0.3}$$

$$\simeq 0.44 \text{ m}$$

- 21.** A light ray enters a solid glass sphere of refractive index  $\mu = \sqrt{3}$  at an angle of incidence  $60^\circ$ . The ray is both reflected and refracted at the farther surface of the sphere. The angle (in degrees) between the reflected and refracted rays at this surface is \_\_\_\_\_.

**21. 90**



At  $S_1$

$$1 \times \sin 60^\circ = \sqrt{3} \sin r$$

$$r = 30^\circ$$

$$\therefore r_1 = 30^\circ \text{ \{from geometry\}}$$

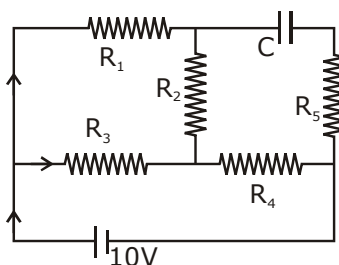
$$\text{As } S_2 \quad \sqrt{3} \sin r_1 = 1 \sin e$$

$$e = 60^\circ$$

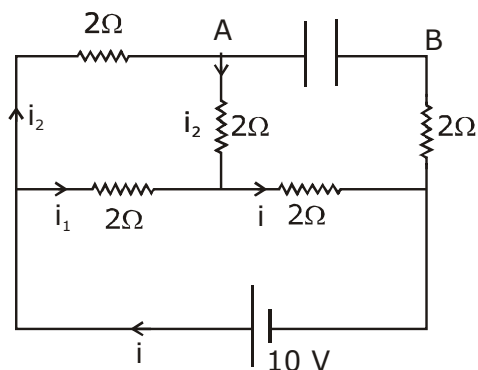
$$\text{Now, } r_1 + \theta + e = 180^\circ$$

$$\boxed{\theta = 90^\circ}$$

- 22.** An ideal cell of emf 10 V is connected in circuit shown in figure. Each resistance is  $2\Omega$ . The potential difference (in V) across the capacitor when it is fully charged is \_\_\_\_\_.



**Sol. 8**



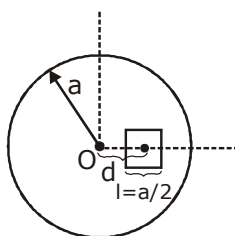
$$i = \frac{10}{\frac{4}{3} + 2} = \frac{10 \times 3}{10} = 3 \text{ Amp}$$

$$i_1 = 2 \text{ Amp}$$

$$i_2 = 1 \text{ Amp}$$

$$V_{AB} = 1 \times 2 + 3 \times 2 = 8 \text{ V}$$

- 23.** A square shaped hole of side  $l = \frac{a}{2}$  is carved out at a distance  $d = \frac{a}{2}$  from the centre 'O' of a uniform circular disk of radius  $a$ . If the distance of the centre of mass of the remaining portion from O is  $-\frac{a}{X}$ , value of X (to the nearest integer) is \_\_\_\_\_.



**Sol. 23**

$$X_{cm} = \frac{\pi a^2 \times 0 - \frac{a^2}{4} \times \frac{a}{2}}{\pi a^2 - \frac{a^2}{4}}$$

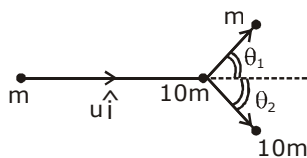
$$= \frac{-a}{2(4\pi-1)} = \frac{-a}{8\pi-2}$$

$$X = (8\pi-2) = 8 \times 3.14 - 2$$

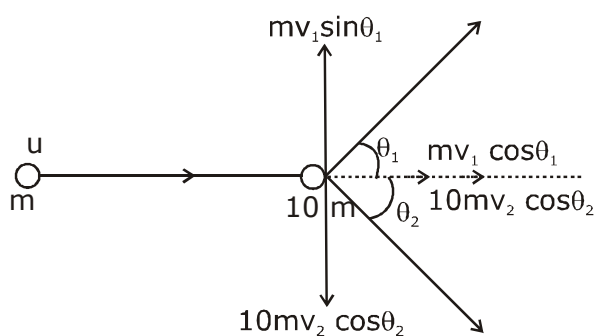
$$= 23.12$$

$$\text{Nearest Integer} = 23$$

- 24.** A particle of mass  $m$  is moving along the  $x$ -axis with initial velocity  $u\hat{i}$ . It collides elastically with a particle of mass  $10m$  at rest and then moves with half its initial kinetic energy (see figure). If  $\sin\theta_1 = \sqrt{n} \sin\theta_2$  then value of  $n$  is



**24. 10**



$$\frac{1}{2}mv_1^2 = \frac{1}{2}\left(\frac{1}{2}mu^2\right)$$

$$v_1^2 = \frac{u^2}{2}$$

$$\boxed{v_1 = \frac{u}{\sqrt{2}}} \quad \dots(i)$$

$$\text{Also, } \frac{1}{2}mu^2 = \frac{1}{2}mv_1^2 + \frac{1}{2} \times (10m) \times v_2^2$$

$$\frac{1}{2} \times 10m \times v_2^2 = \frac{1}{2} \times \frac{1}{2}mu^2$$

$$v_2^2 = \frac{u^2}{20} \quad \text{or, } v_2 = \frac{u}{\sqrt{20}}$$

$$\text{now, } mv_1 \sin \theta_1 = 10mv_2 \sin \theta_2$$

$$\frac{u}{\sqrt{2}} \sin \theta_1 = 10 \times \frac{u}{\sqrt{20}} \sin \theta_2$$

$$\sin \theta_1 = \frac{10}{\sqrt{10}} \sin \theta_2$$

$$\sin \theta_1 = \sqrt{10} \sin \theta_2$$

$$\therefore \boxed{n=10} \text{ ans}$$

- 25.** A wire of density  $9 \times 10^{-3} \text{ kg cm}^{-3}$  is stretched between two clamps 1 m apart. The resulting strain in the wire is  $4.9 \times 10^{-4}$ . The lowest frequency of the transverse vibrations in the wire is (Young's modulus of wire  $Y = 9 \times 10^{10} \text{ Nm}^{-2}$ ), (to the nearest integer), \_\_\_\_\_.

**Sol. 35**

$$f = \frac{1}{2\ell} \sqrt{\frac{T}{\mu}} = \frac{1}{2\ell} \sqrt{\frac{T}{\rho A}} = \frac{1}{2\ell} \sqrt{\frac{YA\ell}{\rho \ell}}$$

$$f = \frac{1}{2 \times 1} \sqrt{\frac{9 \times 10^{10} \times 4.9 \times 10^{-4}}{9000 \times 1}}$$

$$= \frac{1}{2} \sqrt{49 \times 100} = 35 \text{ Hz ans}$$

# QUESTION PAPER WITH SOLUTION

## CHEMISTRY - 2 Sep 2020 - SHIFT - 2

1. Cast iron is used for the manufacture of :

- (1) Wrought iron and steel (2) Wrought iron and pig iron  
(3) Wrought iron, pig iron and steel (4) Pig iron, scrap iron and steel

Sol. 1

Refer topic metallurgy

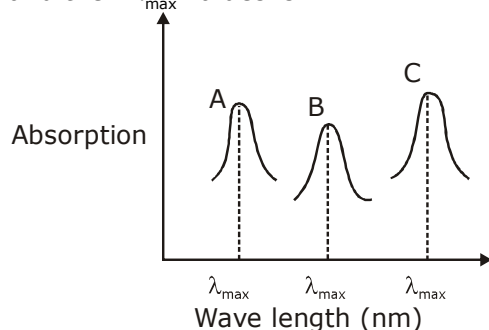
2. The shape/structure of  $[\text{XeF}_5]^-$  and  $\text{XeO}_3\text{F}_2$ , respectively, are :

- (1) Pentagonal planar and trigonal bipyramidal  
(2) Trigonal bipyramidal and trigonal bipyramidal  
(3) Octahedral and square pyramidal  
(4) Trigonal bipyramidal and pentagonal planar

Sol. 1

$[\text{XeF}_5]^-$  5BP + 2LP = 7VSEP  $\Rightarrow$   $sp^3d^3$  hybridisation  
 $\text{XeO}_3\text{F}_2$  5BP + 0LP = 5VSEP  $\Rightarrow$   $sp^3d$  hybridisation

3. Simplified absorption spectra of three complexes ((i), (ii) and (iii)) of  $M^{n+}$  ion are provided below; their  $\lambda_{\text{max}}$  values are marked as A, B and C respectively. The correct match between the complexes and their  $\lambda_{\text{max}}$  values is :



(i)  $[\text{M}(\text{NCS})_6]^{(-6+n)}$

(ii)  $[\text{MF}_6]^{(-6+n)}$

(iii)  $[\text{M}(\text{NH}_3)_6]^{n+}$

(1) A-(i), B-(ii), C-(iii)

(2) A-(iii), B-(i), C-(ii)

(3) A-(ii), B-(iii), C-(i)

(4) A-(ii), B-(i), C-(iii)

Sol. 2

$$\Delta = \frac{hc}{\lambda_{\text{absorbedf(max)}}$$

A  $\rightarrow$   $\text{NH}_3$  comp (iii)

B  $\rightarrow$   $\text{NCS}$  comp (i)

C  $\rightarrow$   $\text{F}^-$  comp (ii)

using spectrochemical series of ligand

$\text{F}^- < \text{NCS}^- < \text{NH}_3$  order of  $\Delta + e$

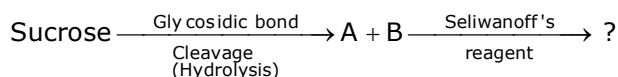
crystal field splitting energy

So.  $\text{NH}_3$  complex  $\rightarrow$  A

$\text{F}^-$  complex  $\rightarrow$  C

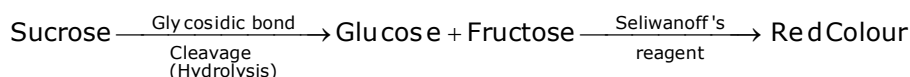
$\text{NCS}^-$  complex  $\rightarrow$  B

4. The correct observation in the following reactions is :



- (1) Formation of red colour (2) Formation of blue colour  
(3) Formation of violet colour (4) Gives no colour

Sol. 1



5. The results given in the below table were obtained during kinetic studies of the following reaction :  
 $2\text{A} + \text{B} \rightarrow \text{C} + \text{D}$

Experiment	[A]/ molL <sup>-1</sup>	[B]/ molL <sup>-1</sup>	Initial rate/ molL <sup>-1</sup> min <sup>-1</sup>
I	0.1	0.1	$6.00 \times 10^{-3}$
II	0.1	0.2	$2.40 \times 10^{-2}$
III	0.2	0.1	$1.20 \times 10^{-2}$
IV	X	0.2	$7.20 \times 10^{-2}$
V	0.3	Y	$2.88 \times 10^{-1}$

X and Y in the given table are respectively :

- (1) 0.4, 0.4 (2) 0.3, 0.4 (3) 0.4, 0.3 (4) 0.3, 0.3

Sol. 2



$$\begin{aligned} \text{Exp. (I)} \quad & 6 \times 10^{-3} = K (0.1)^p (0.1)^q \\ \text{(II)} \quad & 2.4 \times 10^{-2} = K (0.1)^p (0.2)^q \\ \text{(III)} \quad & 1.2 \times 10^{-2} = K (0.2)^p (0.1)^q \end{aligned}$$

$$\frac{\text{exp(I)}}{\text{exp(II)}} \quad \frac{1}{4} = \left(\frac{1}{2}\right)^q \Rightarrow q = 2$$

$$\frac{\text{Exp.(I)}}{\text{Exp.(III)}} \quad \frac{1}{2} = \left(\frac{1}{2}\right)^p \Rightarrow p = 1$$

exp. (I) ÷ exp (IV)

$$\frac{0.6 \times 10^{-2}}{7.2 \times 10^{-2}} = \left(\frac{0.1}{x}\right)^1 \cdot \left[\frac{0.1}{0.2}\right]^2$$

$$\frac{1}{12} = \frac{0.1}{x} - \frac{1}{4}$$

$$[x] = 0.3$$

exp (I) ÷ exp(V)

$$\frac{0.6 \times 10^{-2}}{2.88 \times 10^{-1}} = \left(\frac{0.1}{0.3}\right)^1 \times \left(\frac{0.1}{y}\right)^2$$

$$\frac{1}{48} = \frac{1}{3} \times \frac{10^{-2}}{y^2} \Rightarrow y^2 = 0.16$$

$$y = 0.4$$

Ans(2)

6. Match the type of interaction in column A with the distance dependence of their interaction energy in column B :

A		B	
(I)	ion-ion	(a)	$\frac{1}{r}$
(II)	dipole-dipole	(b)	$\frac{1}{r^2}$
(III)	London dispersion	(c)	$\frac{1}{r^3}$
		(d)	$\frac{1}{r^6}$
(1)	(I)-(a), (II)-(b), (III)-(d)	(2)	(I)-(a), (II)-(b), (III)-(c)
(3)	(I)-(b), (II)-(d), (III)-(c)	(4)	(I)-(a), (II)-(c), (III)-(d)

Sol.

4

$$\text{ion - ion} \propto \frac{1}{r}$$

$$\text{dipole - dipole} \propto \frac{1}{r^3}$$

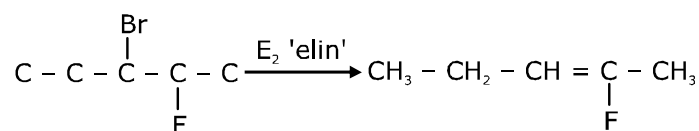
$$\text{Londong dispersion} \propto \frac{1}{r^6}$$

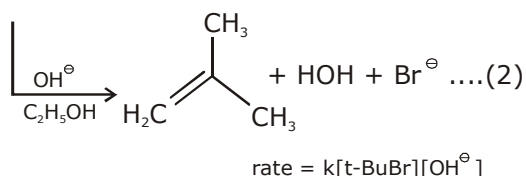
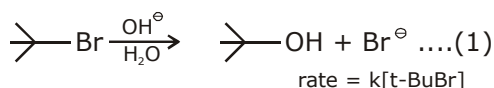
7. The major product obtained from  $E_2$ - elimination of 3-bromo-2-fluoropentane is :



Sol.

1

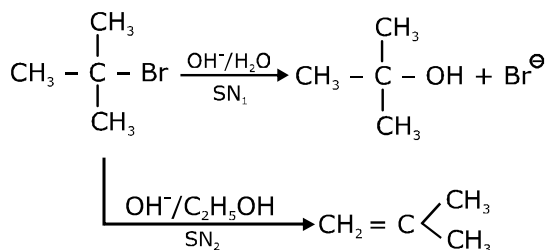




Which of the following statements is true :

- (1) Changing the concentration of base will have no effect on reaction (1).
- (2) Doubling the concentration of base will double the rate of both the reactions.
- (3) Changing the base from  $\text{OH}^\ominus$  to  $^\ominus\text{OR}$  will have no effect on reaction (2).
- (4) Changing the concentration of base will have no effect on reaction (2).

**Sol. 1**



- 9.** The size of a raw mango shrinks to a much smaller size when kept in a concentrated salt solution. Which one of the following process can explain this ?

- (1) Diffusion
- (2) Osmosis
- (3) Reverse osmosis
- (4) Dialysis

**Sol. 2**

Theoretical  
Ans. Osmosis  
Option (2)

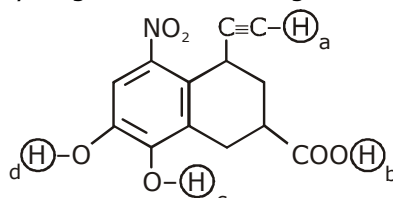
- 10.** If you spill a chemical toilet cleaning liquid on your hand, your first aid would be :

- (1) Aqueous  $\text{NH}_3$
- (2) Aqueous  $\text{NaHCO}_3$
- (3) Aqueous  $\text{NaOH}$
- (4) Vinegar

**Sol. 2**

Fact

11. Arrange the following labelled hydrogens in decreasing order of acidity :



(1)  $b > a > c > d$

(3)  $c > b > d > a$

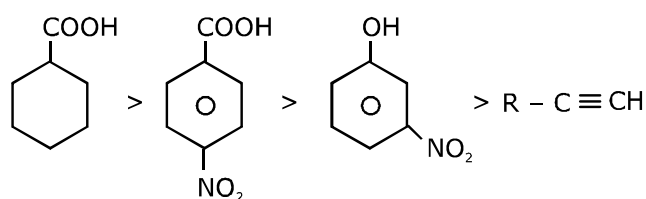
(2)  $b > c > d > a$

(4)  $c > b > a > d$

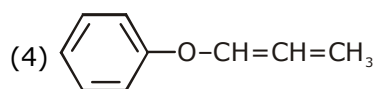
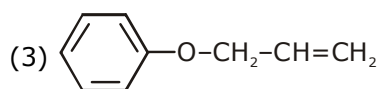
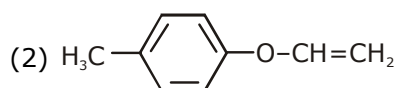
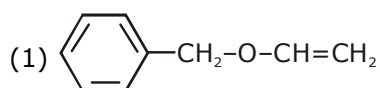
Sol.

2

Order of acidic strength

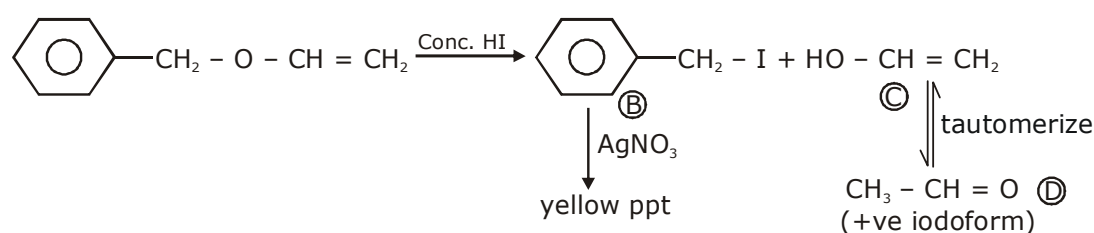


12. An organic compound 'A' ( $C_9H_{10}O$ ) when treated with conc. HI undergoes cleavage to yield compounds 'B' and 'C'. 'B' gives yellow precipitate with  $AgNO_3$  whereas 'C' tautomerizes to 'D'. 'D' gives positive iodoform test. 'A' could be :



Sol.

1



13. Two elements A and B have similar chemical properties. They don't form solid hydrogencarbonates, but react with nitrogen to form nitrides. A and B, respectively, are :

(1) Na and Ca

(2) Cs and Ba

(3) Na and Rb

(4) Li and Mg

Sol.

4

$LiHCO_3$  &  $Mg(HCO_3)_2$  does not exist in solid form but both form nitrides with nitrogen gas

**14.** The number of subshells associated with  $n = 4$  and  $m = -2$  quantum numbers is :

- (1) 4                                      (2) 8                                      (3) 2                                      (4) 16

**Sol. 3**

$$n = 4$$

$$\ell = 0 \qquad m = 0$$

$$\ell = 1 \qquad m = -1, 0, +1$$

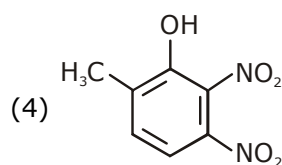
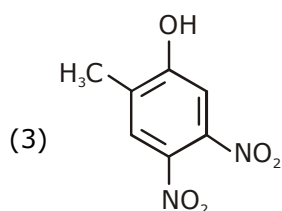
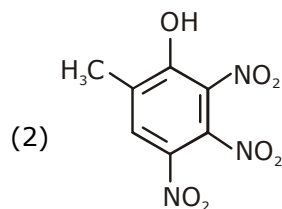
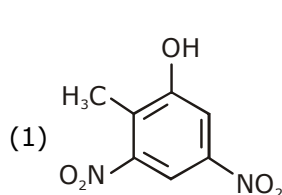
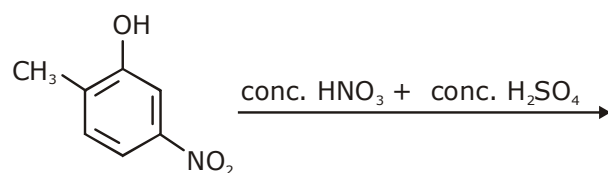
$$\ell = 2 \qquad m = -2, +2, -1, +1, 0$$

$$\ell = 3 \qquad m = \pm 3, \pm 2, \pm 1, 0$$

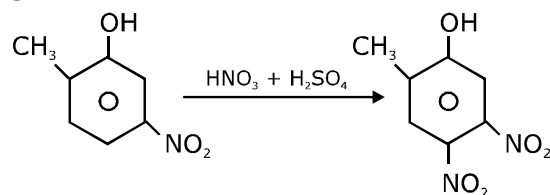
Ans. '2' Subshells

Option (3)

**15.** The major product of the following reaction is :



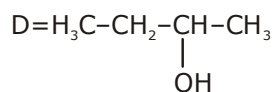
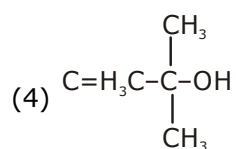
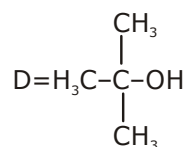
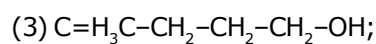
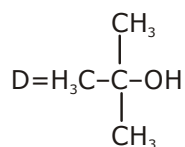
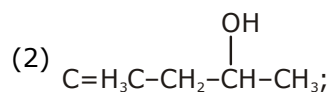
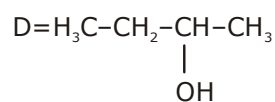
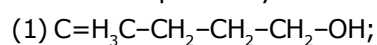
**Sol. 3**



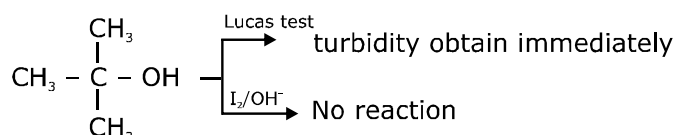
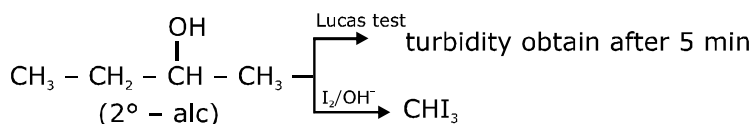
16. Two compounds A and B with same molecular formula ( $C_3H_6O$ ) undergo Grignard's reaction with methylmagnesium bromide to give products C and D. Products C and D show following chemical tests.

Test	C	D
Ceric ammonium nitrate Test	Positive	Positive
Lucas Test	Turbidity obtained after five minutes	Turbidity obtained immediately
Iodoform Test	Positive	Negative

C and D respectively are :



**Sol. 2**



- 17.** Three elements X, Y and Z are in the 3<sup>rd</sup> period of the periodic table. The oxides of X, Y and Z, respectively, are basic, amphoteric and acidic. The correct order of the atomic numbers of X, Y and Z is :

- (1)  $X < Y < Z$  (2)  $Y < X < Z$   
(3)  $Z < Y < X$  (4)  $X < Z < Y$

**Sol. 1**

x	<	y	<	z
Mg		Al		Si
Basic oxide		amphoteric		acidic oxide

- 18.** The one that is not expected to show isomerism is :

- (1)  $[\text{Ni}(\text{NH}_3)_4(\text{H}_2\text{O})_2]^{2+}$  (2)  $[\text{Ni}(\text{en})_3]^{2+}$   
(3)  $[\text{Pt}(\text{NH}_3)_2\text{Cl}_2]$  (4)  $[\text{Ni}(\text{NH}_3)_2\text{Cl}_2]$

**Sol. 4**

$[\text{Ni}(\text{NH}_3)_2\text{Cl}_2]$   $\text{Ni}^{2+}$  is  $\text{sp}^3$  hybridised & such tetrahedral complex does not show either of geometrical or optical isomerism

$[\text{Ni}(\text{en})_3]^{2+}$  shows only optical isomers while other three shows geometrical isomerism

- 19.** Amongst the following statements regarding adsorption, those that are valid are :

- (a)  $\Delta H$  becomes less negative as adsorption proceeds.  
(b) On a given adsorbent, ammonia is adsorbed more than nitrogen gas.  
(c) On adsorption, the residual force acting along the surface of the adsorbent increases.  
(d) With increase in temperature, the equilibrium concentration of adsorbate increases.  
(1) (b) and (c) (2) (c) and (d)  
(3) (a) and (b) (4) (d) and (a)

**Sol.** Statement 'a' & 'b'

- 20.** The molecular geometry of  $\text{SF}_6$  is octahedral. What is the geometry of  $\text{SF}_4$  (including lone pair(s) of electrons, if any) ?

- (1) Pyramidal (2) Trigonal bipyramidal  
(3) Tetrahedral (4) Square planar

**Sol. 2**

$\text{SF}_4$  is  $\text{Sp}^3\text{d}$  hybridised in which hybrid orbitals have TBP arrangement but its shape is sea-saw

- 21.** The ratio of the mass percentages of 'C & H' and 'C & O' of a saturated acyclic organic compound 'X' are 4 : 1 and 3 : 4 respectively. Then, the moles of oxygen gas required for complete combustion of two moles of organic compound 'X' is \_\_\_\_\_.

**Sol.** Mass ratio of C : H is 4 : 1  $\Rightarrow$  12 : 3

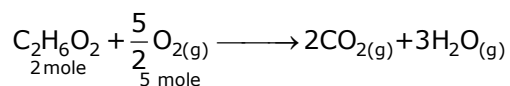
& C : O is 3 : 4  $\Rightarrow$  12 : 16

So,

	mass	mole	molar ratio
C	12	1	1
H	3	3	3
O	16	1	1

Empirical formula  $\Rightarrow$  CH<sub>3</sub>O

as compound is saturated a cyclic so, molecular formula is C<sub>2</sub>H<sub>6</sub>O<sub>2</sub>.



So, required moles of O<sub>2</sub> is  $\Rightarrow$  5

- 22.** For the disproportionation reaction  $2\text{Cu}^+(\text{aq}) \rightleftharpoons \text{Cu}(\text{s}) + \text{Cu}^{2+}(\text{aq})$  at K,  $\ln K$  (where K is the equilibrium constant) is \_\_\_\_\_  $\times 10^{-1}$ .

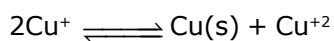
Given :

$$(E^0_{\text{Cu}^{2+}/\text{Cu}^+} = 0.16 \text{ V})$$

$$E^0_{\text{Cu}^+/\text{Cu}} = 0.52 \text{ V}$$

$$\frac{RT}{F} = 0.025$$

**Sol. 144**



$$E^0 = 0.52 - 0.16 = 0.36$$

$$E^0 = \frac{RT}{nF} \ln(k_{\text{eq}})$$

$$\ln(k_{\text{eq}}) = \frac{0.36}{0.025} \times \frac{1}{1}$$

$$= \frac{360}{25} = 14.4$$

$$= 144 \times 10^{-1}$$

Ans. 144

- 23.** The work function of sodium metal is  $4.41 \times 10^{-19} \text{ J}$ . If photons of wavelength 300 nm are incident on the metal, the kinetic energy of the ejected electrons will be ( $h = 6.63 \times 10^{-34} \text{ J s}$ ;  $c = 3 \times 10^8 \text{ m/s}$ )  $\times 10^{-21} \text{ J}$ .

**Sol. 222**

$$\phi = 4.41 \times 10^{-19} \text{ J}$$

$$\lambda = 300 \text{ nm}$$

$$KE_{\text{max}} = \frac{hc}{\lambda} - \phi$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{300 \times 10^{-9}} - 4.41 \times 10^{-19}$$

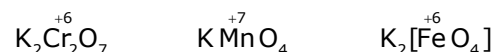
$$= 6.63 \times 10^{-19} - 4.41 \times 10^{-19}$$

$$= 222 \times 10^{-21}$$

Ans. 222

- 24.** The oxidation states of transition metal atoms in  $\text{K}_2\text{Cr}_2\text{O}_7$ ,  $\text{KMnO}_4$  and  $\text{K}_2\text{FeO}_4$ , respectively, are x, y and z. The sum of x, y and z is \_\_\_\_\_.

**Sol. 19**



- 25.** The heat of combustion of ethanol into carbon dioxide and water is  $-327 \text{ kcal}$  at constant pressure. The heat evolved (in cal) at constant volume and  $27^\circ\text{C}$  (if all gases behave ideally) is ( $R = 2 \text{ cal mol}^{-1} \text{ K}^{-1}$ ) \_\_\_\_\_.

**Sol.**  $\Delta H_c^0 [\text{C}_2\text{H}_5\text{OH}] = -327 \text{ kcal}$



$$\Delta E_c^0 = \Delta H_c^0 - \Delta n gRT$$

$$= -327 \times 1000 - (-1) \times 2 \times 300$$

$$= -327000 + 600$$

$$= -326400$$

# QUESTION PAPER WITH SOLUTION

## MATHEMATICS - 2 Sep 2020 - SHIFT - 2

**Q.1** Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a function which satisfies  $f(x+y) = f(x) + f(y) \forall x, y \in \mathbb{R}$ . If  $f(1) = 2$  and

$g(n) = \sum_{k=1}^{(n-1)} f(k), n \in \mathbb{N}$  then the value of  $n$ , for which  $g(n) = 20$ , is:

- (1) 9 (2) 5 (3) 4 (4) 20

**Sol.**

**(2)**

$$f(1) = 2; f(x+y) = f(x) + f(y)$$

$$x = y = 1 \Rightarrow f(2) = 2 + 2 = 4$$

$$x = 2, y = 1 \Rightarrow f(3) = 4 + 2 = 6$$

$$g(n) = f(1) + f(2) + \dots + f(n-1)$$

$$= 2 + 4 + 6 + \dots + 2(n-1)$$

$$= 2 \sum (n-1)$$

$$= 2 \frac{(n-1) \cdot n}{2}$$

$$= n^2 - n$$

$$\text{Given } g(n) = 20 \Rightarrow n^2 - n = 20$$

$$n^2 - n - 20 = 0$$

$$n = 5$$

**Q.2** If the sum of first 11 terms of an A.P.,  $a_1, a_2, a_3, \dots$  is 0 ( $a_1 \neq 0$ ) then the sum of the A.P.,  $a_1, a_3, a_5, \dots, a_{23}$  is  $ka_1$ , where  $k$  is equal to:

- (1)  $-\frac{121}{10}$  (2)  $-\frac{72}{5}$  (3)  $\frac{72}{5}$  (4)  $\frac{121}{10}$

**Sol.**

**(2)**

$$\sum_{k=1}^{11} a_k = 0 \Rightarrow 11a + 55d = 0$$

$$a + 5d = 0$$

$$\text{Now } a_1 + a_3 + \dots + a_{23} = ka_1$$

$$12a + d(2+4+6+\dots+22) = ka$$

$$12a + 2d \cdot 66 = ka$$

$$12(a+11d) = ka$$

$$12 \left( a + 11 \left( -\frac{a}{5} \right) \right) = ka$$

$$12 \left( 1 - \frac{11}{5} \right) = k$$

$$k = -\frac{72}{5}$$

**Q.3** Let  $E^c$  denote the complement of an event  $E$ . Let  $E_1, E_2$  and  $E_3$  be any pairwise independent events with  $P(E_1) > 0$  and  $P(E_1 \cap E_2 \cap E_3) = 0$ . Then  $P(E_2^c \cap E_3^c / E_1)$  is equal to:

- (1)  $P(E_3^c) - P(E_2^c)$  (2)  $P(E_3) - P(E_2^c)$   
 (3)  $P(E_3^c) - P(E_2)$  (4)  $P(E_2^c) + P(E_3)$

**Sol. (3)**

$$\begin{aligned} P(E_2^c \cap E_3^c / E_1) &= \frac{P(E_2^c \cap E_3^c \cap E_1)}{P(E_1)} \\ &= \frac{P(E_1) - P(E_1 \cap E_2) - P(E_1 \cap E_3) + P(E_1 \cap E_2 \cap E_3)}{P(E_1)} \\ &= \frac{P(E_1) - P(E_1) \cdot P(E_2) - P(E_1) \cdot P(E_3) + 0}{P(E_1)} = 1 - P(E_2) - P(E_3) \\ &= P(E_3^c) - P(E_2) \end{aligned}$$

**Q.4** If the equation  $\cos^4 \theta + \sin^4 \theta + \lambda = 0$  has real solutions for  $\theta$ , then  $\lambda$  lies in the interval:

- (1)  $\left[-\frac{1}{2}, -\frac{1}{4}\right]$  (2)  $\left[-1, -\frac{1}{2}\right]$  (3)  $\left[-\frac{3}{2}, -\frac{5}{4}\right]$  (4)  $\left(-\frac{5}{4}, -1\right)$

**Sol. (2)**

$$\cos^4 \theta + \sin^4 \theta + \lambda = 0$$

$$\lambda = -\left\{1 - \frac{1}{2} \sin^2 2\theta\right\}$$

$$2(\lambda + 1) = \sin^2 2\theta$$

$$0 \leq 2(\lambda + 1) \leq 1$$

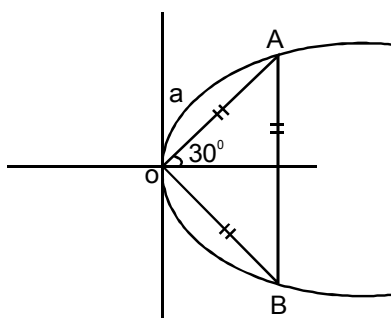
$$0 \leq \lambda + 1 \leq \frac{1}{2}$$

$$\boxed{-1 \leq \lambda \leq -\frac{1}{2}}$$

**Q.5** The area (in sq. units) of an equilateral triangle inscribed in the parabola  $y^2 = 8x$ , with one of its vertices on the vertex of this parabola, is:

- (1)  $128\sqrt{3}$                       (2)  $192\sqrt{3}$                       (3)  $64\sqrt{3}$                       (4)  $256\sqrt{3}$

**Sol. (2)**



A :  $(a \cos 30^\circ, a \sin 30^\circ)$   
lies on parabola

$$\frac{a^2}{4} = 8 \cdot \frac{a \cdot \sqrt{3}}{2}$$

$$\boxed{a = 16\sqrt{3}}$$

$$\text{Area of equilateral } \Delta = \frac{\sqrt{3}}{4} a^2$$

$$\Delta = \frac{\sqrt{3}}{4} \cdot 16 \cdot 16 \cdot 3$$

$$\Delta = 192\sqrt{3}$$

**Q.6** The imaginary part of  $(3 + 2\sqrt{-54})^{1/2} - (3 - 2\sqrt{-54})^{1/2}$  can be :

- (1)  $\sqrt{6}$                       (2)  $-2\sqrt{6}$                       (3) 6                      (4)  $-\sqrt{6}$

**Sol. (2)**

$$(3 + 2i\sqrt{54})^{1/2} - (3 - 2i\sqrt{54})^{1/2}$$

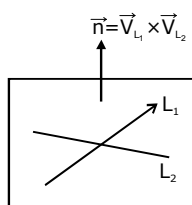
$$= (9 + 6i^2 + 2 \cdot 3i\sqrt{6})^{1/2} - (9 + 6i^2 - 2 \cdot 3i\sqrt{6})^{1/2}$$

$$= \left( (3 + \sqrt{6}i)^2 \right)^{1/2} - \left( (3 - \sqrt{6}i)^2 \right)^{1/2}$$

$$= \pm(3 + \sqrt{6}i) \mp (3 - \sqrt{6}i) = -2\sqrt{6} i$$

- Q.7** A plane passing through the point  $(3,1,1)$  contains two lines whose direction ratios are  $1, -2, 2$  and  $2, 3, -1$  respectively. If this plane also passes through the point  $(\alpha, -3, 5)$ , then  $\alpha$  is equal to:
- (1)  $-5$  (2)  $10$  (3)  $5$  (4)  $-10$

**Sol. (3)**



$$\vec{n}_p = (-4, 5, 7)$$

Equation of plane :

$$P : -4(x-3) + 5(y-1) + 7(z-1) = 0$$

$$P : -4x + 5y + 7z + 12 - 5 - 7 = 0$$

$$P : 4x - 5y - 7z = 0$$

Pass  $(\alpha, -3, 5)$

$$4\alpha + 15 - 35 = 0$$

$$4\alpha = 20$$

$$\alpha = 5$$

- Q.8** Let  $A = \{X = (x, y, z)^T : PX = 0 \text{ and } x^2 + y^2 + z^2 = 1\}$ , where  $P = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix}$ , then the set A:

- (1) contains more than two elements  
(3) contains exactly two elements

- (2) is a singleton.  
(4) is an empty set.

**Sol. (3)**

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ \& } x^2 + y^2 + z^2 = 1$$

$$PX = 0$$

$$\begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x + 2y + z = 0 \dots\dots(1)$$

$$-2x + 3y - 4z = 0 \dots\dots(2)$$

$$x + 9y - z = 0 \dots\dots(3)$$

from (1) & (3)

$$\Rightarrow 2x + 11y = 0$$

from (1) & (2)

$$\Rightarrow 2x + 11y = 0$$

from (2) & (3)  
 $-6x - 33y = 0$   
 $\Rightarrow 2x + 11y = 0$   
 put in (1)  
 $-7y + 2z = 0$

Now  $\left(\frac{11y}{2}\right)^2 + y^2 + \left(\frac{7y}{2}\right)^2 = 1$

$y^2(121 + 4 + 49) = 4$   
 $y^2(171) = 4$

$y = \pm \frac{2}{\sqrt{171}} \Rightarrow x = \pm \frac{7}{\sqrt{171}} \Rightarrow z = \mp \frac{11}{\sqrt{171}} \Rightarrow$  Only two pair possible

- Q.9** The equation of the normal to the curve  $y = (1+x)^{2y} + \cos^2(\sin^{-1}x)$  at  $x=0$  is:  
 (1)  $y+4x=2$  (2)  $2y+x=4$  (3)  $x+4y=8$  (4)  $y=4x+2$

**Sol.** (3)  
 at  $x = 0 \Rightarrow y = 1 + \cos^2(0) = 2$   
 $p : (0, 2)$

Now  $y' = (1+x)^{2y} \left\{ \frac{2y}{1+x} + \ln(1+x) \cdot 2y \right\} - \sin 2(\sin^{-1}x) \cdot \frac{1}{\sqrt{1-x^2}}$

$y \Big|_{(0,2)} = 4 - 0$

$N_o : y - 2 = -\frac{1}{4}(x-0)$

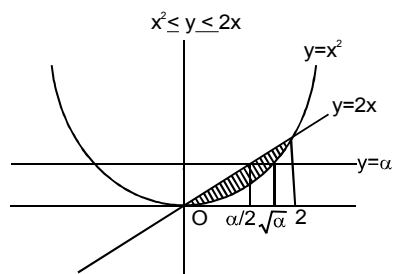
$N_o : 4y - 8 = -x$

$N_o : \boxed{x + 4y = 8}$

- Q.10** Consider a region  $R = \{(x, y) \in \mathbb{R}^2 : x^2 \leq y \leq 2x\}$ . If a line  $y = \alpha$  divides the area of region R into two equal parts, then which of the following is true.?

- (1)  $\alpha^3 - 6\alpha^2 + 16 = 0$  (2)  $3\alpha^2 - 8\alpha^{3/2} + 8 = 0$   
 (3)  $\alpha^3 - 6\alpha^{3/2} - 16$  (4)  $3\alpha^2 - 8\alpha + 8 = 0$

**Sol.** (2)



$$A = \text{Area} = \int_0^2 (2x - x^2) dx = 4 - \frac{8}{3} = \frac{4}{3}$$

$$\text{Now } \int_0^{\alpha/2} (2x - x^2) dx + \int_{\alpha/2}^{\sqrt{\alpha}} (\alpha - x^2) dx = \frac{1}{2} A$$

$$\frac{\alpha^2}{4} - \frac{\alpha^3}{24} + \alpha \left( \sqrt{\alpha} - \alpha/2 \right) - \left( \frac{\alpha\sqrt{\alpha}}{3} - \frac{\alpha^3}{24} \right) = \frac{4}{6}$$

$$\frac{\alpha^2}{4} + \alpha\sqrt{\alpha} - \frac{\alpha^2}{2} - \frac{\alpha\sqrt{\alpha}}{3} = \frac{4}{6}$$

$$-3\alpha^2 + 8\alpha\sqrt{\alpha} = 8$$

$$3\alpha^2 - 8\alpha\sqrt{\alpha} + 8 = 0$$

**Q.11** Let  $f : (-1, \infty) \rightarrow \mathbb{R}$  be defined by  $f(0)=1$  and  $f(x) = \frac{1}{x} \log_e(1+x)$ ,  $x \neq 0$ . Then the function  $f$ :

- (1) increases in  $(-1, \infty)$
- (2) decreases in  $(-1, 0)$  and increases in  $(0, \infty)$
- (3) increases in  $(-1, 0)$  and decreases in  $(0, \infty)$
- (4) decreases in  $(-1, \infty)$ .

**Sol. (4)**

$$f(x) = \frac{1}{x} \ln(1+x)$$

$$f' = \frac{x - \frac{1}{1+x} - \ln(1+x)}{x^2}$$

$$f' = \frac{1 - \frac{1}{1+x} - \ln(1+x)}{x^2}$$

$$f' < 0 \quad \forall x \in (-1, \infty)$$

**Q.12** Which of the following is a tautology?

- (1)  $(p \rightarrow q) \wedge (q \rightarrow p)$
- (2)  $(\sim p) \wedge (p \vee q) \rightarrow q$
- (3)  $(q \rightarrow p) \vee \sim(p \rightarrow q)$
- (4)  $(\sim q) \vee (p \wedge q) \rightarrow q$

**Sol. (2)**

$\sim p$	$p \vee q$	$\sim p \wedge (p \vee q)$	$\sim p \wedge (p \vee q) \rightarrow q$
F	T	F	T
F	T	F	T
T	T	T	T
T	F	F	T

**Q.13** Let  $f(x)$  be a quadratic polynomial such that  $f(-1)+f(2)=0$ . If one of the roots of  $f(x)=0$  is 3, then its other roots lies in:

- (1) (0,1)                      (2) (1,3)                      (3) (-1,0)                      (4) (-3,-1)

**Sol. (3)**

$$\begin{aligned}\text{Let } f(x) &= a(x-3)(x-\alpha) \\ f(-1)+f(2) &= 0 \\ a[(-1-3)(-1-\alpha)+(2-3)(2-\alpha)] &= 0 \\ a[4+4\alpha-2+\alpha] &= 0 \\ 5\alpha+2 &= 0\end{aligned}$$

$$\boxed{\alpha = -\frac{2}{5}}$$

**Q.14** Let  $S$  be the sum of the first 9 terms of the series :

$$\{x+ka\} + \{x^2+(k+2)a\} + \{x^3+(k+4)a\} + \{x^4+(k+6)a\} + \dots \quad \text{where } a \neq 0 \quad \text{and} \quad a \neq 1.$$

$$\text{If } S = \frac{x^{10} - x + 45a(x-1)}{x-1}, \text{ then } k \text{ is equal to:}$$

- (1) 3                      (2) -3                      (3) 1                      (4) -5

**Sol. (2)**

$$\begin{aligned}S &= \{x+ka\} + \{x^2+(k+2)a\} + \{x^3+(k+4)a\} \text{ up to 9 term} \\ S &= (x+x^2+\dots+x^9) + a\{k+(k+2)+(k+4)+\dots\text{up to 9 term}\}\end{aligned}$$

$$S = \frac{x(1-x^9)}{1-x} + a\{9k+2.36\}$$

$$S = \frac{x^{10} - x}{x-1} + 9ak + 72a$$

$$S = \frac{x^{10} - x + 45a(x-1)}{x-1} = \frac{x^{10} - x + (9k+72)a(x-1)}{x-1}$$

$$= 45 = 9k + 72$$

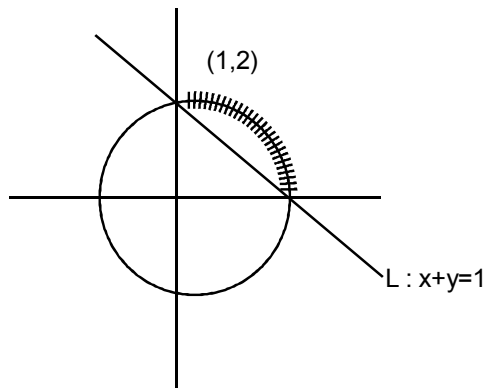
$$9k = -27$$

$$k = -3$$

**Q.15** The set of all possible values of  $\theta$  in the interval  $(0, \pi)$  for which the points  $(1,2)$  and  $(\sin \theta, \cos \theta)$  lie on the same side of the line  $x+y=1$  is:

- (1)  $\left(0, \frac{\pi}{4}\right)$                       (2)  $\left(0, \frac{\pi}{2}\right)$                       (3)  $\left(0, \frac{3\pi}{4}\right)$                       (4)  $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

**Sol. (2)**



$(\sin\theta, \cos\theta)$  lie on  $x^2 + y^2 = 1$

Shaded points satisfy

$$\Rightarrow \theta \in (0, \pi/2)$$

**Q.16** Let  $n > 2$  be an integer. Suppose that there are  $n$  Metro stations in a city located along a circular path. Each pair of stations is connected by a straight track only. Further, each pair of nearest stations is connected by blue line, whereas all remaining pairs of stations are connected by red line. If the number of red lines is 99 times the number of blue lines, then the value of  $n$  is:

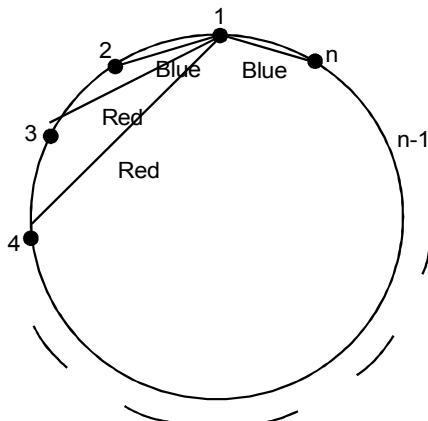
(1) 201

(2) 199

(3) 101

(4) 200

**Sol. (1)**



Red line = 99 blue line

$${}^nC_2 - n = 99n$$

$$\frac{n(n-1)}{2} = 100n$$

$$n-1 = 200$$

$$\boxed{n = 201}$$

**Q.17** If a curve  $y=f(x)$ , passing through the point  $(1,2)$  is the solution of the differential equation,  $2x^2dy=(2xy+y^2)dx$ , then  $f\left(\frac{1}{2}\right)$  is equal to:

- (1)  $\frac{-1}{1+\log_e 2}$       (2)  $1+\log_e 2$       (3)  $\frac{1}{1+\log_e 2}$       (4)  $\frac{1}{1-\log_e 2}$

**Sol. (3)**

$$2 \frac{dy}{dx} = 2 \frac{y}{x} + \left(\frac{y}{x}\right)^2 \rightarrow \text{HDE}$$

$$\therefore y = vx$$

$$2 \left( v + x \frac{dv}{dx} \right) = 2v + v^2$$

$$2 \frac{dv}{v^2} = \frac{dx}{x}$$

$$-\frac{2}{v} = \ln x + c$$

$$-\frac{2x}{y} = \ln x + c$$

$$\downarrow (1,2)$$

$$c = -1$$

$$c : \ln x + \frac{2x}{y} = 1$$

$$\text{For } f(1/2) \Rightarrow \ln \left( \frac{1}{2} \right) + \frac{2}{2y} = 1$$

$$y = \frac{1}{1+\ln 2}$$

**Q.18** For some  $\theta \in \left(0, \frac{\pi}{2}\right)$ , if the eccentricity of the hyperbola,  $x^2 - y^2 \sec^2 \theta = 10$  is  $\sqrt{5}$  times the eccentricity of the ellipse,  $x^2 \sec^2 \theta + y^2 = 5$ , then the length of the latus rectum of the ellipse, is:

- (1)  $\frac{4\sqrt{5}}{3}$       (2)  $\frac{2\sqrt{5}}{3}$       (3)  $2\sqrt{6}$       (4)  $\sqrt{30}$

**Sol. (1)**

$$H : x^2 - y^2 \sec^2 \theta = 10$$

$$E : x^2 \sec^2 \theta + y^2 = 5$$

$$\sqrt{1 + \frac{10 \cos^2 \theta}{10}} = \sqrt{5} \sqrt{1 - \frac{5 \cos^2 \theta}{5}}$$

$$1 + \cos^2 \theta = 5 - 5 \cos^2 \theta$$

$$6 \cos^2 \theta = 4$$

$$\cos \theta = \pm \sqrt{\frac{2}{3}}$$

$$l(\text{LR}) \text{ of ellipse} = \frac{2.5 \cos^2 \theta}{\sqrt{5}}$$

$$= 2\sqrt{5} \cdot \frac{2}{3} = \boxed{\frac{4\sqrt{5}}{3}}$$

**Q.19**  $\lim_{x \rightarrow 0} \left( \tan \left( \frac{\pi}{4} + x \right) \right)^{1/x}$  is equal to:

(1) e

(2)  $e^2$

(3) 2

(4) 1

**Sol.**

**(2)**

$$\lim_{x \rightarrow 0} \left( \tan \left( \frac{\pi}{4} + x \right) \right)^{1/x} (1^\infty) = e^L$$

$$L = \lim_{x \rightarrow 0} \frac{\tan \left( \frac{\pi}{4} + x \right) - 1}{x}$$

$$L = \lim_{x \rightarrow 0} \frac{\frac{1 + \tan x}{1 - \tan x} - 1}{x}$$

$$L = \lim_{x \rightarrow 0} 2 \left( \frac{\tan x}{x} \right) \cdot \left( \frac{1}{1 - \tan x} \right)$$

$$L = +2$$

$$\text{Ans. } e^2$$

**Q.20** Let  $a, b, c \in \mathbb{R}$  be all non-zero and satisfy  $a^3 + b^3 + c^3 = 2$ . If the matrix

$$A = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$$

satisfies  $A^T A = I$ , then a value of  $abc$  can be:

$$(1) \frac{2}{3}$$

$$(2) 3$$

$$(3) -\frac{1}{3}$$

$$(4) \frac{1}{3}$$

**Sol.**

**(4)**

$$a^3 + b^3 + c^3 = 2$$

$$A^T A = I$$

$$\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= a^2 + b^2 + c^2 = 1$$

$$\& ab + bc + ca = 0$$

$$\text{Now } (a+b+c)^2 = \sum a^2 + 2\sum ab$$

$$(\sum a)^2 = 1 + 0 \Rightarrow (\sum a)^2 = 1 \Rightarrow \sum a = \pm 1$$

$$\text{Now } \sum a^3 - 3abc = (\sum a)(\sum a^2 - \sum ab)$$

$$2 - 3abc = \pm 1 (1 - 0)$$

$$2 - 3abc = \pm 1$$

$$\begin{array}{c} (+) \quad (-) \\ \swarrow \quad \searrow \\ 3abc = 1 \quad 3abc = 3 \\ \boxed{abc = \frac{1}{3}} \quad \boxed{abc = 1} \end{array}$$

**Q.21** Let the position vectors of points 'A' and 'B' be  $\hat{i} + \hat{j} + \hat{k}$  and  $2\hat{i} + \hat{j} + 3\hat{k}$ , respectively. A point 'P' divides the line segment AB internally in the ratio  $\lambda : 1 (\lambda > 0)$ . If O is the origin and

$$\vec{OB} \cdot \vec{OP} - 3 \left| \vec{OA} \times \vec{OP} \right|^2 = 6, \text{ then } \lambda \text{ is equal to } \underline{\hspace{2cm}}$$

**Sol.**

**0.8**

$$\vec{OA} = \langle 1, 1, 1 \rangle, \vec{OB} = \langle 2, 1, 3 \rangle$$

$$\begin{array}{c} \lambda \quad \quad 1 \\ \text{A} \quad \quad \text{B} \\ \quad \quad \text{P} \end{array}$$

$$\vec{OP} = \left( \frac{2\lambda + 1}{\lambda + 1}, 1, \frac{3\lambda + 1}{\lambda + 1} \right)$$

$$\vec{OB} \cdot \vec{OP} = \frac{2(2\lambda + 1)}{\lambda + 1} + 1 + \frac{3(3\lambda + 1)}{\lambda + 1}$$

$$= \frac{14\lambda + 6}{\lambda + 1}$$

$$\begin{aligned}
 |\overline{OA} \times \overline{OP}|^2 &= |\overline{OA}|^2 |\overline{OP}|^2 - (\overline{OA} \cdot \overline{OP})^2 \\
 3 \cdot \left( \frac{(2\lambda + 1)^2 + (\lambda + 1)^2 + (3\lambda + 1)^2}{(\lambda + 1)^2} \right) - \left( \frac{2\lambda + 1 + \lambda + 1 + 3\lambda + 1}{\lambda + 1} \right)^2 \\
 &= \frac{1}{(\lambda + 1)^2} \{ 3(14\lambda^2 + 12\lambda + 3) - (6\lambda + 3)^2 \} \\
 &= \frac{1}{(\lambda + 1)^2} \{ 6\lambda^2 \}
 \end{aligned}$$

$$\text{Now } \frac{14\lambda + 6}{\lambda + 1} - 3 \left( \frac{6\lambda^2}{(\lambda + 1)^2} \right) = 6$$

$$(14\lambda + 6)(\lambda + 1) - 18\lambda^2 = 6(\lambda + 1)^2$$

$$-4\lambda^2 + 20\lambda + 6 = 6\lambda^2 + 12\lambda + 6$$

$$10\lambda^2 - 8\lambda = 0$$

$$\lambda(10\lambda - 8) = 0$$

$$\therefore \lambda > 0$$

$$\boxed{\lambda = .8}$$

**Q.22** Let  $[t]$  denote the greatest integer less than or equal to  $t$ . Then the value of

$$\int_1^2 |2x - [3x]| dx \text{ is } \underline{\hspace{2cm}}$$

**Sol. 1**

$$\int_1^2 |2x - [3x]| dx$$

$$3x = t$$

$$= \frac{1}{3} \int_3^6 \left| \frac{2t}{3} - [t] \right| dt$$

$$= \frac{1}{9} \left[ \int_3^6 |2t - 3[t]| dt \right]$$

$$= \frac{1}{9} \left[ \int_3^4 |2t - 9| + \int_4^5 |2t - 12| + \int_5^6 |2t - 15| \right] dt$$

$$= \frac{1}{9} \left[ \int_3^4 (9 - 2t) + \int_4^5 (12 - 2t) + \int_5^6 (15 - 2t) \right] dt$$

$$\begin{aligned}
 &= \frac{1}{9} [9 \cdot 1 + 12 \cdot 1 + 15 \cdot 1 - [4^2 - 3^2] - [5^2 - 4^2] - [6^2 - 5^2]] \\
 &= \frac{1}{9} [36 - [4^2 - 3^2 + 5^2 - 4^2 + 6^2 - 5^2]] \\
 &= \frac{1}{9} [36 - 36 + 9] = 1
 \end{aligned}$$

**Q.23** If  $y = \sum_{k=1}^6 k \cos^{-1} \left\{ \frac{3}{5} \cos kx - \frac{4}{5} \sin kx \right\}$ , then  $\frac{dy}{dx}$  at  $x=0$  is \_\_\_\_\_

**Sol. 91**

$$y = \sum_{k=1}^6 k \cos^{-1} \{ \cos(kx + \theta) \}$$

$$\text{where } \tan \theta = \frac{4}{3}$$

$$y = \cos^{-1}(\cos(x+\theta)) + 2\cos^{-1}(\cos(2x+\theta)) + \dots + 6\cos^{-1}(\cos(6x+\theta))$$

$$\left. \frac{dy}{dx} \right|_{x=0} = \frac{\sin(\theta)}{\sqrt{1 - \cos^2 \theta}} + \dots$$

$$= 1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3 + \dots + 6 \cdot 6$$

$$= \sum 6^2 = \frac{6 \cdot 7 \cdot 13}{6} = 91$$

**Q.24** If the variance of the terms in an increasing A.P.,  $b_1, b_2, b_3, \dots, b_{11}$  is 90, then the common difference of this A.P. is \_\_\_\_\_

**Sol. 3**

$$\text{Var}(x) = \frac{\sum bi^2}{11} - \left( \frac{\sum bi}{11} \right)^2$$

$$90 = \frac{a^2 + (a+d)^2 + (a+2d)^2 + \dots + (a+10d)^2}{11}$$

$$\left( \frac{a + a + d + a + 2d + \dots + (a+10d)}{11} \right)^2$$

$$10890 = 11 \{ 11a^2 + 385d^2 + 110ad \} - \{ 11a + 55d \}^2$$

$$10890 = 1210d^2$$

$$d = 3$$

**Q.25** For a positive integer  $n$ ,  $\left(1 + \frac{1}{x}\right)^n$  is expanded in increasing powers of  $x$ . If three consecutive coefficients in this expansion are in the ratio, 2:5:12, then  $n$  is equal to \_\_\_\_\_

**Sol. 118**

Let 3 consecutive coH are

$${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} :: 2:5:12$$

$$\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{2}{5} \quad \& \quad \frac{{}^nC_r}{{}^nC_{r+1}} = \frac{5}{12}$$

$$\frac{r}{n-r+1} = \frac{2}{5} \quad \& \quad \frac{r+1}{(n-r)} = \frac{5}{12}$$

$$7r = 2n + 2 \quad \& \quad 17r = 5n - 12$$

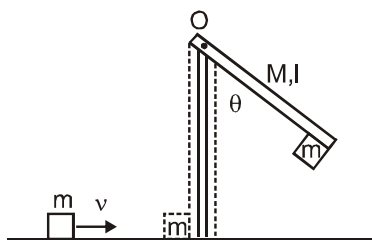
$$\Rightarrow \frac{2n+2}{7} = \frac{5n-12}{17}$$

$$= 34n + 34 = 35n - 84$$

$$\Rightarrow n = 118$$

## PHYSICS \_ 3 Sep. \_ SHIFT - 1

1. A block of mass  $m = 1 \text{ kg}$  slides with velocity  $v = 6 \text{ m/s}$  on a frictionless horizontal surface and collides with a uniform vertical rod and sticks to it as shown. The rod is pivoted about O and swings as a result of the collision making angle  $\theta$  before momentarily coming to rest. If the rod has mass  $M = 2 \text{ kg}$ , and length  $l = 1 \text{ m}$ , the value of  $\theta$  is approximately :  
(take  $g = 10 \text{ m/s}^2$ )



(1)  $49^\circ$

(2)  $63^\circ$

(3)  $69^\circ$

(4)  $55^\circ$

**Sol.**

**2**

$$mv\ell = I\omega$$

$$\omega = \frac{mv\ell}{I}$$

$$\frac{1}{2}I\omega^2 = (m+M)g\ell_{\text{com}}(1 - \cos\theta)$$

$$\frac{1}{2}\frac{(mv\ell)^2}{I} = (m+M)g\ell_{\text{com}}(1 - \cos\theta)$$

$$I = \left(\frac{M\ell^2}{3} + m\ell^2\right)$$

$$I = \left(\frac{2}{3} + 1\right) = \frac{5}{3}$$

$$\frac{36 \times 3}{2 \times 5} = \frac{3 \times 10 \times 2}{3}(1 - \cos\theta)$$

$$\frac{27}{50} = (1 - \cos\theta)$$

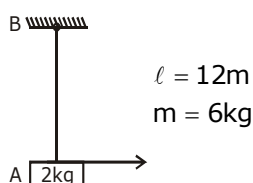
$$\cos\theta = 1 - \frac{27}{50}$$

$$\cos\theta = \frac{23}{50}$$

$$\theta = 63^\circ$$

2. A uniform thin rope of length 12 m and mass 6 kg hangs vertically from a rigid support and a block of mass 2 kg is attached to its free end. A transverse short wavetrain of wavelength 6 cm is produced at the lower end of the rope. What is the wavelength of the wavetrain (in cm) when it reaches the top of the rope ?  
 (1) 12 (2) 3 (3) 9 (4) 6

Sol. 1



$$\mu = \frac{6}{12} = \frac{1}{2} \text{ kg/m}$$

$$A \rightarrow V = \sqrt{\frac{T}{\mu}} = f\lambda$$

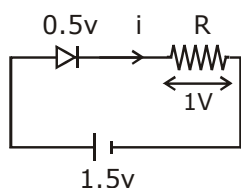
$$B \rightarrow \sqrt{\frac{T'}{\mu}} = f\lambda'$$

$$\sqrt{\frac{T}{T'}} = \frac{\lambda}{\lambda'}$$

$$\sqrt{\frac{20}{80}} = \frac{6}{\lambda'} = \lambda' = 12$$

3. When a diode is forward biased, it has a voltage drop of 0.5 V. The safe limit of current through the diode is 10 mA. If a battery of emf 1.5 V is used in the circuit, the value of minimum resistance to be connected in series with the diode so that the current does not exceed the safe limit is :  
 (1) 300  $\Omega$  (2) 200  $\Omega$  (3) 50  $\Omega$  (4) 100  $\Omega$

Sol. 4



$$V = iR$$

$$R = \frac{V}{i} = \frac{1}{10\text{mA}}$$

$$= \frac{1000}{10} = 100\Omega$$

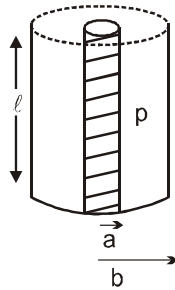
4. Using screw gauge of pitch 0.1 cm and 50 divisions on its circular scale, the thickness of an object is measured. It should correctly be recorded as :  
 (1) 2.125 cm                      (2) 2.124 cm                      (3) 2.123 cm                      (4) 2.121 cm

**Sol. 2**

$$\text{Least count} = \frac{0.1}{50} = \frac{1}{500} = 0.2 \times 10^{-2} = 0.002$$

when we multiply by division no.'s  
 it must be even because L.C. is 0.002.

5. Model a torch battery of length  $\ell$  to be made up of a thin cylindrical bar of radius 'a' and a concentric thin cylindrical shell of radius 'b' filled in between with an electrolyte of resistivity  $\rho$  (see figure). If the battery is connected to a resistance of value R, the maximum joule heating in R will take place for :



(1)  $R = \frac{\rho}{2\pi\ell} \ln\left(\frac{b}{a}\right)$       (2)  $R = \frac{2\rho}{\pi\ell} \ln\left(\frac{b}{a}\right)$       (3)  $R = \frac{\rho}{\pi\ell} \ln\left(\frac{b}{a}\right)$       (4)  $\frac{\rho}{2\pi\ell} \ln\left(\frac{b}{a}\right)$

**Sol. 4**

$$dR = \rho \frac{dr}{2\pi r\ell}$$

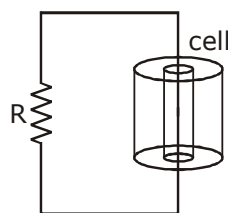
$$\int dR = \frac{\rho}{2\pi\ell} \int_a^b \frac{1}{r} dr$$

$$R = \frac{\rho}{2\pi\ell} [\ln(r)]_a^b$$

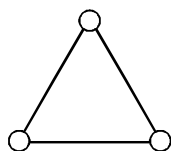
$$R = \frac{\rho}{2\pi\ell} \ln\left(\frac{b}{a}\right)$$

$$r = R$$

For Max heat transfer



6. Consider a gas of triatomic molecules. The molecules are assumed to be triangular and made of massless rigid rods whose vertices are occupied by atoms. The internal energy of a mole of the gas at temperature  $T$  is :



- (1)  $\frac{3}{2} RT$       (2)  $3RT$       (3)  $\frac{5}{2} RT$       (4)  $\frac{9}{2} RT$

**Sol. 2**

$$U = \frac{f}{2} nRT$$

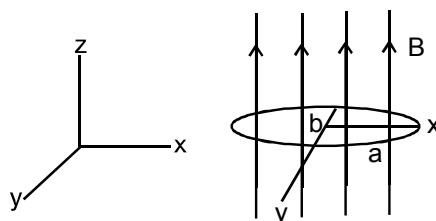
$$U = \frac{6}{2} nRT$$

$$U = 3nRT$$

$$U = 3RT$$

$$n = 1$$

7. An elliptical loop having resistance  $R$ , of semi major axis  $a$ , and semi minor axis  $b$  is placed in magnetic field as shown in the figure. If the loop is rotated about the  $x$ -axis with angular frequency  $\omega$ , the average power loss in the loop due to Joule heating is :



- (1)  $\frac{\pi ab B \omega}{R}$       (2)  $\frac{\pi^2 a^2 b^2 B^2 \omega^2}{R}$       (3)  $\frac{\pi^2 a^2 b^2 B^2 \omega^2}{2R}$       (4) Zero

**Sol. 3**

$$\phi = BA \cos \omega t$$

$$\epsilon = \frac{d\phi}{dt} = BA \omega \sin \omega t$$

$$\epsilon = B \pi a b \omega \sin \omega t$$

$$\epsilon_0 = B \pi a b \omega$$

$$P = \frac{\epsilon^2}{R} = \frac{B^2 \pi^2 a^2 b^2 \omega^2}{R} (\sin^2 \omega t)$$

$$P_{av} = \frac{B^2 \pi^2 a^2 b^2 \omega^2}{2R}$$

8. A balloon filled with helium (32° C and 1.7 atm.) bursts. Immediately afterwards the expansion of helium can be considered as :

(1) reversible isothermal (2) irreversible isothermal  
(3) reversible adiabatic (4) irreversible adiabatic

Sol. 4

irreversible adiabatic → Because Energy can not be restored.

9. When the wavelength of radiation falling on a metal is changed from 500 nm to 200 nm, the maximum kinetic energy of the photoelectrons becomes three times larger. The work function of the metal is close to :

(1) 1.02 eV (2) 0.61 eV (3) 0.52 eV (4) 0.81 eV

Sol. 2

$$KE_{\max} = \frac{hc}{\lambda_1} - \phi$$

$$3KE_{\max} = \frac{hc}{\lambda_2} - \phi$$

$$3\left(\frac{hc}{\lambda_1} - \phi\right) = \frac{hc}{\lambda_2} - \phi$$

$$\frac{3hc}{\lambda_1} - \frac{hc}{\lambda_2} = 2\phi$$

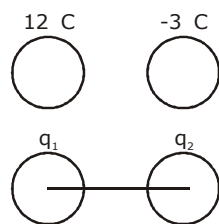
$$\frac{3 \times 1240}{500} - \frac{1240}{200} = 2\phi$$

$$\phi = 0.6$$

10. Two isolated conducting spheres  $S_1$  and  $S_2$  of radius  $\frac{2}{3}R$  and  $\frac{1}{3}R$  have 12  $\mu\text{C}$  and  $-3\mu\text{C}$  charges, respectively, and are at a large distance from each other. They are now connected by a conducting wire. A long time after this is done the charges on  $S_1$  and  $S_2$  are respectively :

(1) 6  $\mu\text{C}$  and 3  $\mu\text{C}$  (2) 4.5  $\mu\text{C}$  on both  
(3) + 4.5  $\mu\text{C}$  and  $-4.5 \mu\text{C}$  (4) 3  $\mu\text{C}$  and 6  $\mu\text{C}$

Sol. 1



$$q_1 + q_2 = 9\mu\text{C}$$

..(1)

$$\frac{Kq_1}{\frac{2}{3}R} = \frac{Kq_2}{\frac{1}{3}R}$$

$$q_1 = 2q_2$$

$$3q_2 = 9\mu C$$

$$q_2 = 3\mu C$$

$$q_1 = 6\mu C$$

- 11.** In a radioactive material, fraction of active material remaining after time  $t$  is  $9/16$ . The fraction that was remaining after  $t/2$  is :

(1)  $\frac{3}{4}$                       (2)  $\frac{7}{8}$                       (3)  $\frac{4}{5}$                       (4)  $\frac{3}{5}$

**Sol. 1**

$$N = N_0 e^{-\lambda t}$$

$$\left(\frac{N}{N_0}\right) = e^{-\lambda t}$$

$$\frac{9}{16} = e^{-\lambda t}$$

$$\frac{N}{N_0} = e^{-\lambda t/2}$$

$$\frac{N}{N_0} = \left(\frac{9}{16}\right)^{\frac{1}{2}} = \frac{3}{4}$$

- 12.** Moment of inertia of a cylinder of mass  $M$ , length  $L$  and radius  $R$  about an axis passing through its centre and perpendicular to the axis of the cylinder is  $I = M \left( \frac{R^2}{4} + \frac{L^2}{12} \right)$ . If such a cylinder is to be made for a given mass of a material, the ratio  $L/R$  for it to have minimum possible  $I$  is :

(1)  $\frac{2}{3}$                       (2)  $\frac{3}{2}$                       (3)  $\sqrt{\frac{2}{3}}$                       (4)  $\sqrt{\frac{3}{2}}$

**Sol. 4**

$$M = d.V$$

$$M = d\pi R^2 L$$

$$I = M \left( \frac{R^2}{4} + \frac{L^2}{12} \right)$$

$$I = M \left( \frac{M}{4d\pi L} + \frac{L^2}{12} \right)$$

$$I = \left( \frac{M^2}{4d\pi L} + \frac{ML^2}{12} \right)$$

$$\frac{dI}{dL} = \frac{M^2}{4d\pi} \left( \frac{-1}{L^2} \right) + \frac{2LM}{12} = 0$$

$$\frac{M^2}{4d\pi L^2} = \frac{2LM}{12}$$

$$\frac{d\pi R^2 L}{4d\pi L^2} = \frac{L}{6}$$

$$\frac{R^2}{L^2} = \frac{2}{3}$$

$$\frac{R}{L} = \sqrt{\frac{2}{3}} \Rightarrow \frac{L}{R} = \sqrt{\frac{3}{2}}$$

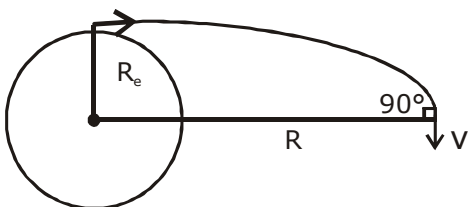
- 13.** A satellite is moving in a low nearly circular orbit around the earth. Its radius is roughly equal to that of the earth's radius  $R_e$ . By firing rockets attached to it, its speed is instantaneously increased

in the direction of its motion so that it become  $\sqrt{\frac{3}{2}}$  times larger. Due to this the farthest distance from the centre of the earth that the satellite reaches is  $R$ . Value of  $R$  is :

- (1)  $2R_e$  (2)  $3R_e$  (3)  $4R_e$  (4)  $2.5 R_e$

**Sol.** **2**

$$V_0 = \sqrt{\frac{GM}{R_e}} \times \sqrt{\frac{3}{2}}$$



$$mV_0 R_e = mVR$$

$$\sqrt{\frac{3}{2}} \sqrt{\frac{GM}{R_e}} R_e = VR$$

$$-\frac{GMm}{R_e} + \frac{1}{2}mv_0^2 = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$-\frac{GMm}{R_e} + \frac{1}{2}m \frac{3}{2} \frac{GM}{R_e} = -\frac{GMm}{R} + \frac{1}{2}m \frac{3}{2} \frac{GM R_e^2}{R^2}$$

$$-\frac{1}{R_e} + \frac{3}{4R_e} = -\frac{1}{R} + \frac{3}{4} \frac{R_e}{R^2}$$

$$-\frac{1}{4R_e} = -\frac{1}{R} + \frac{3}{4} \frac{R_e}{R^2}$$

By further calculating

$$R = 3R_e$$

- 14.** Pressure inside two soap bubbles are 1.01 and 1.02 atmosphere, respectively. The ratio of their volumes is :

(1) 4 : 1                      (2) 2 : 1                      (3) 0.8 : 1                      (4) 8 : 1

**Sol. 4**

$$P_{in} = P_0 + \frac{4T}{R_1}$$

$$1.01 = 1 + \frac{4T}{R_1}$$

$$0.01 = \frac{4T}{R_1}$$

$$0.02 = \frac{4T}{R_2}$$

$$\frac{1}{2} = \frac{R_2}{R_1}$$

$$\frac{V_1}{V_2} = \frac{8}{1}$$

- 15.** In a Young's double slit experiment, light of 500 nm is used to produce an interference pattern. When the distance between the slits is 0.05 mm, the angular width (in degree) of the fringes formed on the distance screen is close to :

(1) 0.17°                      (2) 0.07°                      (3) 0.57°                      (4) 1.7°

**Sol. 3**

$$B = \frac{\lambda D}{d}$$

$$\theta = \frac{\beta}{D} = \left( \frac{\lambda}{d} \right)$$

$$\theta = \frac{500 \times 10^{-9}}{(0.05 \times 10^{-3})}$$

$$\theta = \frac{5 \times 10^{-2}}{5} = \frac{5}{100 \times 5}$$

$$\theta^\circ = \frac{5}{100} \times \frac{180}{5 \times \pi}$$

$$\theta^\circ = 0.57^\circ$$

- 16.** A 750 Hz, 20 V (rms) source is connected to a resistance of  $100\ \Omega$ , an inductance of  $0.1803\ \text{H}$  and a capacitance of  $10\ \mu\text{F}$  all in series. The time in which the resistance (heat capacity  $2\ \text{J}/^\circ\text{C}$ ) will get heated by  $10^\circ\text{C}$ . (assume no loss of heat to the surroundings) is close to :

(1) 245 s                      (2) 365 s                      (3) 418 s                      (4) 348 s

**Sol. 4**

$$f = 750\text{Hz}, V_{\text{rms}} = 20\text{V}, R = 100\Omega, L = 0.1803\text{H}$$

$$C = 10\mu\text{f}$$

$$S = 2\text{J} / ^\circ\text{C}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$$

$$|Z| = \sqrt{(100)^2 + \left(2 \times 3.14 \times 750 \times 0.1803 - \frac{1}{2 \times 3.14 \times 750 \times 10^{-5}}\right)^2}$$

$$|Z| = 834\Omega$$

In AC

$$P = V_{\text{rms}} i_{\text{rms}} \cos \phi$$

$$P = \left( V_{\text{rms}} \cdot \frac{V_{\text{rms}}}{|Z|} \cdot \frac{R}{|Z|} \right)$$

$$P = \left( \frac{V_{\text{rms}}}{|Z|} \right)^2 R$$

$$P = \left( \frac{20}{834} \right)^2 \times 100 = 0.0575\text{J} / \text{S}$$

$$P \times t = S \Delta \theta$$

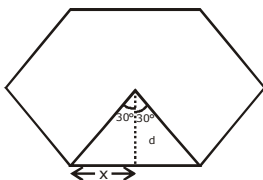
$$t = \frac{2(10)}{0.0575}$$

$$t = 348\text{ sec}$$

- 17.** Magnitude of magnetic field (in SI units) at the centre of a hexagonal shape coil of side 10 cm, 50 turns and carrying current  $I$  (Ampere) in units of  $\frac{\mu_0 I}{\pi}$  is :

(1)  $250\sqrt{3}$                       (2)  $50\sqrt{3}$                       (3)  $500\sqrt{3}$                       (4)  $5\sqrt{3}$

**Sol. 3**



$$\tan 30 = \frac{x}{d}$$

$$d = \frac{x}{\tan 30}$$

$$d = \frac{5 \times 10^{-2}}{\frac{1}{\sqrt{3}}}$$

$$d = 5\sqrt{3} \times 10^{-2}$$

$$B = \frac{6 \times \mu_0 I N}{4\pi d} (\sin \theta_1 + \sin \theta_2)$$

$$B = \frac{\mu_0 I}{4\pi} \times \frac{50 \times 6}{5\sqrt{3} \times 10^{-2}} (\sin 30 + \sin 30)$$

$$B = \frac{10 \times 6 \times 100}{\sqrt{3} \times 4} \left( \frac{\mu_0 I}{\pi} \right)$$

$$B = 500\sqrt{3}$$

- 18.** The magnetic field of a plane electromagnetic wave is

$$\vec{B} = 3 \times 10^{-8} \sin[200\pi(y + ct)] \hat{i} \text{ T}$$

where  $c = 3 \times 10^8 \text{ ms}^{-1}$  is the speed of light.

The corresponding electric field is :

(1)  $\vec{E} = -9 \sin[200\pi(y + ct)] \hat{k} \text{ V/m}$

(2)  $\vec{E} = 9 \sin[200\pi(y + ct)] \hat{k} \text{ V/m}$

(3)  $\vec{E} = -10^{-6} \sin[200\pi(y + ct)] \hat{k} \text{ V/m}$

(4)  $\vec{E} = 3 \times 10^{-8} \sin[200\pi(y + ct)] \hat{k} \text{ V/m}$

**Sol.**

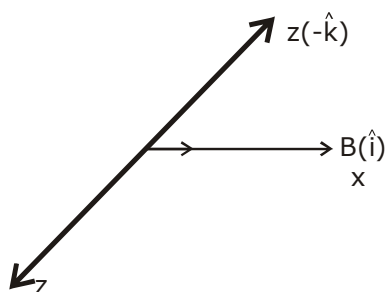
**1**

$$E = BC$$

$$E = 3 \times 10^{-8} \times 3 \times 10^8$$

$$E = 9$$

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{r}$$



$$E = 9 \sin[200\pi(y + ct)(-\hat{k})]$$

- 19.** A charged particle carrying charge  $1 \mu\text{C}$  is moving with velocity  $(2\hat{i} + 3\hat{j} + 4\hat{k}) \text{ ms}^{-1}$ . If an external magnetic field of  $(5\hat{i} + 3\hat{j} - 6\hat{k}) \times 10^{-3} \text{ T}$  exists in the region where the particle is moving then the force on the particle is  $\vec{F} \times 10^{-9} \text{ N}$ . The vector  $\vec{F}$  is :

(1)  $-0.30\hat{i} + 0.32\hat{j} - 0.09\hat{k}$

(2)  $-3.0\hat{i} + 3.2\hat{j} - 0.9\hat{k}$

(3)  $-30\hat{i} + 32\hat{j} - 9\hat{k}$

(4)  $-300\hat{i} + 320\hat{j} - 90\hat{k}$

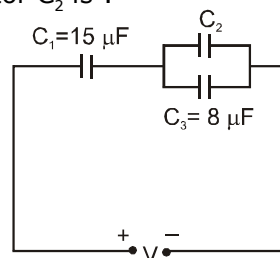
**Sol. 3**

$$\vec{F} = q(\vec{V} \times \vec{B})$$

$$\vec{F} = 10^{-6}(2\hat{i} + 3\hat{j} + 4\hat{k}) \times (5\hat{i} + 3\hat{j} - 6\hat{k}) \times 10^{-3}$$

$$\vec{F} = (-30\hat{i} + 32\hat{j} - 9\hat{k}) \times 10^{-9}$$

- 20.** In the circuit shown in the figure, the total charge is  $750 \mu\text{C}$  and the voltage across capacitor  $C_2$  is  $20 \text{ V}$ . Then the charge on capacitor  $C_2$  is :



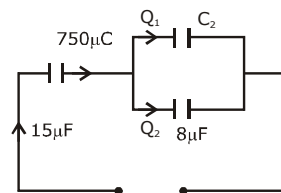
(1)  $650 \mu\text{C}$

(2)  $450 \mu\text{C}$

(3)  $590 \mu\text{C}$

(4)  $160 \mu\text{C}$

**Sol. 3**



$$Q_2 = CV = 8\mu\text{f} \times 20 = 160\mu\text{C}$$

$$Q_1 = 750 - 160 = 590\mu\text{C}$$

- 21.** A person of  $80 \text{ kg}$  mass is standing on the rim of a circular platform of mass  $200 \text{ kg}$  rotating about its axis at  $5 \text{ revolutions per minute (rpm)}$ . The person now starts moving towards the centre of the platform. What will be the rotational speed (in rpm) of the platform when the person reaches its centre \_\_\_\_\_.

**Sol. 9**

$$I_1\omega_1 = I_2\omega_2$$

$$\left(\frac{MR^2}{2} + mR^2\right)\omega_1 = \omega_2 \frac{MR^2}{2}$$

$$\left(1 + \frac{2mR^2}{MR^2}\right)\omega_1 = \omega_2$$

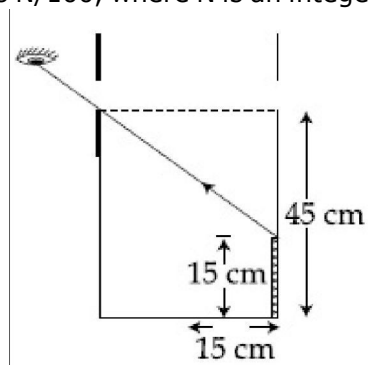
$$\left(1 + \frac{2 \times 80}{200}\right)\omega_1 = \omega_2$$

$$\omega_2 = \omega_1 \times 1.8$$

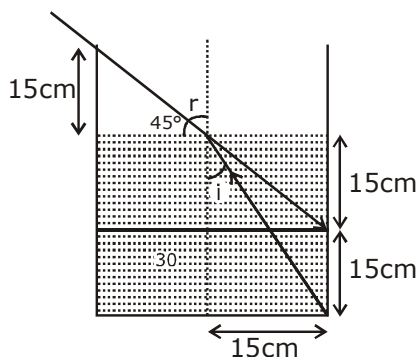
$$2\pi f_2 = 2\pi f_1 \times 1.8$$

$$f_2 = 5 \times 1.8 = 9$$

22. An observer can see through a small hole on the side of a jar (radius 15 cm) at a point at height of 15 cm from the bottom (see figure). The hole is at a height of 45 cm. When the jar is filled with a liquid up to a height of 30 cm the same observer can see the edge at the bottom of the jar. If the refractive index of the liquid is  $N/100$ , where  $N$  is an integer, the value of  $N$  is \_\_\_\_\_.



**Sol. 1.58**



$$\mu \sin i = 1 \sin 45$$

$$\mu \frac{15}{\sqrt{1125}} = \frac{1}{\sqrt{2}}$$

$$\mu = \frac{\sqrt{1125}}{15\sqrt{2}}$$

$$\mu = \sqrt{\frac{1125}{450}}$$

$$\mu = 1.58$$

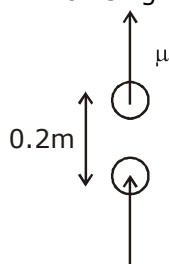
$$\mu = \frac{N}{100} = 1.58$$

$$\Rightarrow N = 158$$

- 23.** A cricket ball of mass 0.15 kg is thrown vertically up by a bowling machine so that it rises to a maximum height of 20 m after leaving the machine. If the part pushing the ball applies a constant force  $F$  on the ball and moves horizontally a distance of 0.2 m while launching the ball, the value of  $F$  (in N) is ( $g = 10 \text{ ms}^{-2}$ )\_\_\_\_\_.

**Sol.** **150**

$$m = 0.15 \text{ kg}$$



$$\mu = \sqrt{2gh}$$

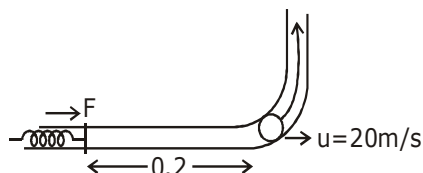
$$\mu = \sqrt{2 \times 10 \times 20}$$

$$\mu = 20 \text{ m / sec}$$

$$Fd = \frac{1}{2}mv^2$$

$$F \times 0.2 = \frac{1}{2} \times \frac{0.15}{100} \times 400$$

$$F = \frac{15 \times 200 \times 10}{100 \times 0.2} = 150$$

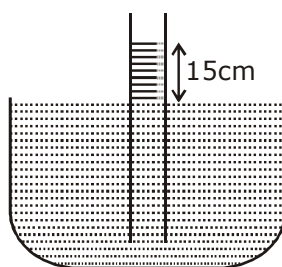


- 24.** When a long glass capillary tube of radius 0.015 cm is dipped in a liquid, the liquid rises to a height of 15 cm within it. If the contact angle between the liquid and glass is close to  $0^\circ$ , the surface tension of the liquid, in milliNewton  $\text{m}^{-1}$ , is

[ $\rho_{\text{(liquid)}} = 900 \text{ kgm}^{-3}$ ,  $g = 10 \text{ ms}^{-2}$ ] (Give answer in closest integer)\_\_\_\_\_.

**Sol.** **101**

$$r = 0.015$$



$$h = \frac{2T}{\rho g r}$$

$$T = \frac{h \rho g r}{2}$$

$$T = \frac{15 \times 10^{-2} \times 900 \times 10 \times 0.015 \times 10^{-2}}{2 \times 10^3}$$

$$T = 101 \text{ milli N/m}$$

- 25.** A bakelite beaker has volume capacity of 500 cc at 30°C. When it is partially filled with  $V_m$  volume (at 30°C) of mercury, it is found that the unfilled volume of the beaker remains constant as temperature is varied. If  $\gamma_{\text{(beaker)}} = 6 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$  and  $\gamma_{\text{(mercury)}} = 1.5 \times 10^{-4} \text{ } ^\circ\text{C}^{-1}$ , where  $\gamma$  is the coefficient of volume expansion, then  $V_m$  (in cc) is close to \_\_\_\_\_.

**Sol. 20**

$$\Delta V = V \gamma \Delta T$$

$$V_1 \gamma_1 = V_2 \gamma_2$$

$$500\text{cc} \times 6 \times 10^{-6} = V_m \times 1.5 \times 10^{-4}$$

$$V_m = \frac{500 \times 6 \times 10^{-6}}{1.5 \times 10^{-4}} = \frac{30}{1.5}$$

$$V_m = 20\text{cc}$$

## CHEMISTRY \_ 3 Sep. \_ SHIFT - 1

1. It is true that :
- (1) A second order reaction is always a multistep reaction
  - (2) A first order reaction is always a single step reaction
  - (3) A zero order reaction is a multistep reaction
  - (4) A zero order reaction is a single step reaction

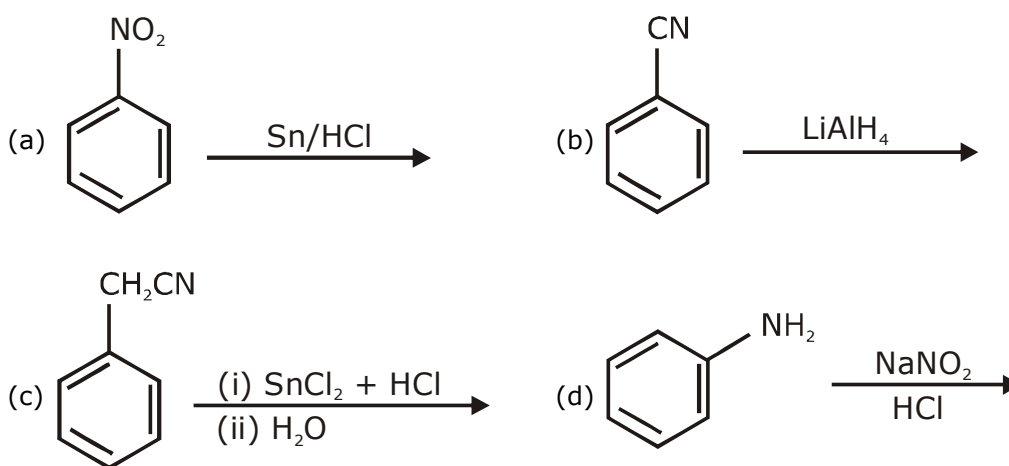
**Sol. 3**  
**Factual**

2. An acidic buffer is obtained on mixing :
- (1) 100 mL of 0.1 M HCl and 200 mL of 0.1 M  $\text{CH}_3\text{COONa}$
  - (2) 100 mL of 0.1 M HCl and 200 mL of 0.1 M NaCl
  - (3) 100 mL of 0.1 M  $\text{CH}_3\text{COOH}$  and 100 mL of 0.1 M NaOH
  - (4) 100 mL of 0.1 M  $\text{CH}_3\text{COOH}$  and 200 mL of 0.1 M NaOH

**Sol. 1**

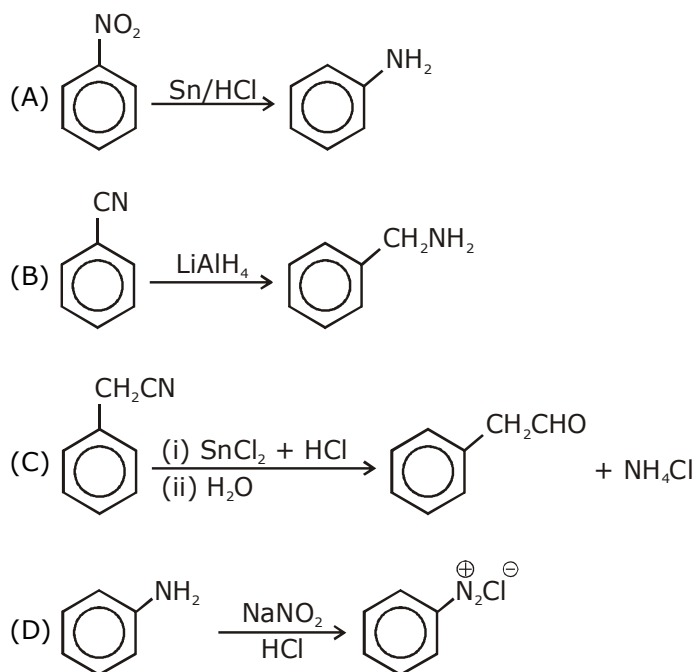
$$\begin{array}{rcl}
 2\text{HCl} + \text{CH}_3\text{COO}^- & \longrightarrow & \text{CH}_3\text{COOH} + \text{OH}^- \\
 10 \quad 20 & & \\
 \text{X} \quad 10 \quad 10 & & \\
 \hline
 & \text{Acidic buffer} &
 \end{array}$$

3. The Kjeldahl method of Nitrogen estimation fails for which of the following reaction products?



- (1) (a), (c) and (d)
- (2) (b) and (c)
- (3) (c) and (d)
- (4) (a) and (d)

**Sol. 3**



Diazo compound and inorganic nitrogen can't be estimated by kjeldal method.

**4.** If the boiling point of  $\text{H}_2\text{O}$  is 373 K, the boiling point of  $\text{H}_2\text{S}$  will be :

- (1) greater than 300 K but less than 373 K
- (2) equal to 373 K
- (3) more than 373 K
- (4) less than 300 K

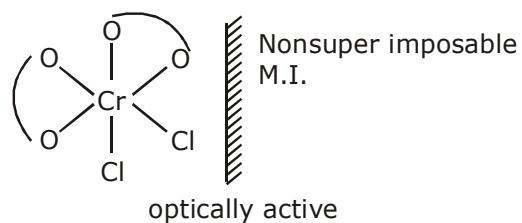
**Sol. 4**

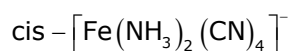
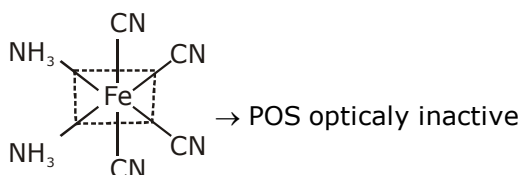
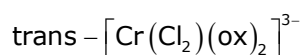
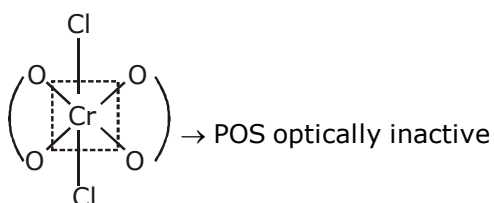
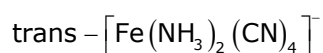
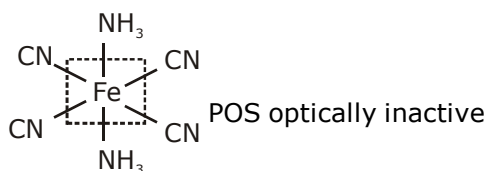
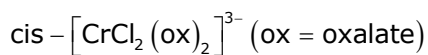
Less than 300 K (factual)

**5.** The complex that can show optical activity is :

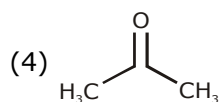
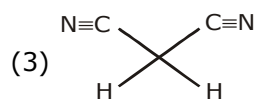
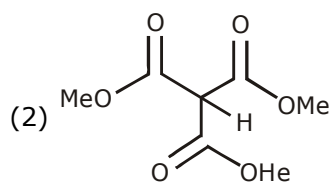
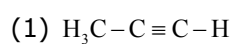
- (1)  $\text{cis} - [\text{CrCl}_2(\text{ox})_2]^{3-}$  (ox = oxalate)
- (2)  $\text{trans} - [\text{Fe}(\text{NH}_3)_2(\text{CN})_4]^-$
- (3)  $\text{trans} - [\text{Cr}(\text{Cl}_2)(\text{ox})_2]^{3-}$
- (4)  $\text{cis} - [\text{Fe}(\text{NH}_3)_2(\text{CN})_4]^-$

**Sol. 1**

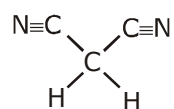




6. Which one of the following compounds possesses the most acidic hydrogen?



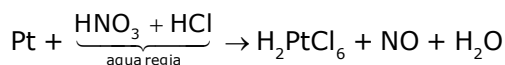
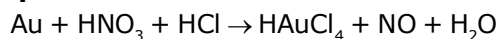
**Sol. 3**



has most acidic hydrogen among given compound, this is due to strong -M effect of -CN group which stabilize -ve charge significantly.

7. Aqua regia is used for dissolving noble metals (Au, Pt, etc.). The gas evolved in this process is :  
 (1)  $N_2O_3$  (2)  $N_2$  (3)  $N_2O_5$  (4) NO

Sol. 4

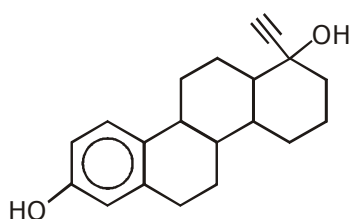


8. The antifertility drug "Novestrol" can react with :

- (1)  $Br_2$ /water;  $ZnCl_2$ /HCl;  $FeCl_3$  (2)  $Br_2$ /water;  $ZnCl_2$ /HCl; NaOCl  
 (3) Alcoholic HCN; NaOCl;  $ZnCl_2$ /HCl (4)  $ZnCl_2$ /HCl;  $FeCl_3$ ; Alcoholic HCN

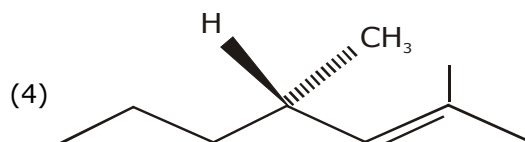
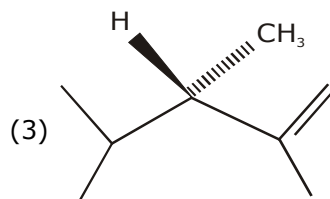
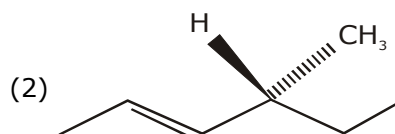
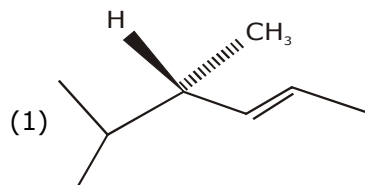
Sol. 1

Novestrol

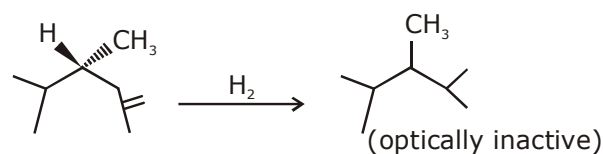


It can react with  $Br_2$ /water due to presence of unsaturation, with  $ZnCl_2$ /HCl due to  $-OH$  group and with  $FeCl_3$  due to phenol.

9. Which of the following compounds produces an optically inactive compound on hydrogenation?



Sol. 3



10. Of the species, NO,  $NO^+$ ,  $NO^{2+}$  and  $NO^-$ , the one with minimum bond strength is :  
 (1)  $NO^-$  (2)  $NO^+$  (3)  $NO^{2+}$  (4) NO

**Sol. 1**

B.O.  $\text{NO}^- = 2$   
 BO  $\text{NO}^+ = 3$   
 BO  $\text{NO}^{2+} = 2.5$   
 BO  $\text{NO} = 2.5$

$$\text{B.O} \propto \frac{1}{\text{B.L}}$$

**11.** Glycerol is separated in soap industries by :

- (1) Fractional distillation (2) Distillation under reduced pressure  
 (3) Differential extraction (4) Steam distillation

**Sol. 2**

conceptual

Glycerol is separated in soap industries by distillation under reduced pressure

**12.** Thermal power plants can lead to :

- (1) Ozone layer depletion (2) Blue baby syndrome  
 (3) Eutrophication (4) Acid rain

**Sol. 4**

Refer environmental chemistry

It emits  $\text{CO}_2$  that combine with moisture of atmosphere and forms  $\text{H}_2\text{CO}_3$  (carbonic acid)

**13.** Henry's constant (in kbar) for four gases  $\alpha$ ,  $\beta$ ,  $\gamma$  and  $\delta$  in water at 298 K is given below :

	$\alpha$	$\beta$	$\gamma$	$\delta$
$K_H$	50	2	$2 \times 10^{-5}$	0.5

(density of water =  $10^3 \text{ kg m}^{-3}$  at 298 K)

This table implies that :

- (1) solubility of  $\gamma$  at 308 K is lower than at 298 K  
 (2) The pressure of a 55.5 molal solution of  $\delta$  is 250 bar  
 (3)  $\alpha$  has the highest solubility in water at a given pressure  
 (4) The pressure of a 55.5 molal solution of  $\gamma$  is 1 bar

**Sol. 1**

$p = K_H X$  mol fraction of gas in liquid.

On increasing temp, ' $K_H$ ' increases

Hence solubility  $\downarrow$

therefore, option 1

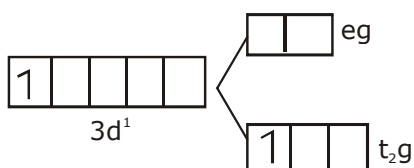
- 14.** The electronic spectrum of  $[\text{Ti}(\text{H}_2\text{O})_6]^{3+}$  shows a single broad peak with a maximum at  $20,300 \text{ cm}^{-1}$ . The crystal field stabilization energy (CFSE) of the complex ion, in  $\text{kJ mol}^{-1}$ , is :

(1  $\text{kJ mol}^{-1} = 83.7 \text{ cm}^{-1}$ )

- (1) 83.7                      (2) 242.5                      (3) 145.5                      (4) 97

**Sol. 4**

$[\text{Ti}(\text{H}_2\text{O})_6]^{3+}$   $\text{Ti}^{3+}$   $3d^1$  in octahedral field of ligand



$$\text{CFSE} = -0.4 \Delta_0$$

$$\text{CFSE} = \frac{-0.4 \times 20300}{83.7}$$

$$= 97 \text{ kJ mol}^{-1}$$

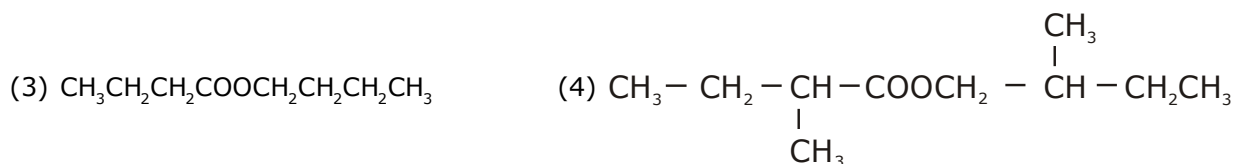
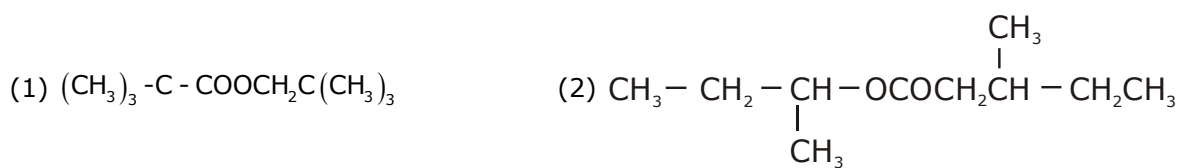
- 15.** The atomic number of the element unnilennium is :

- (1) 109                      (2) 102                      (3) 119                      (4) 108

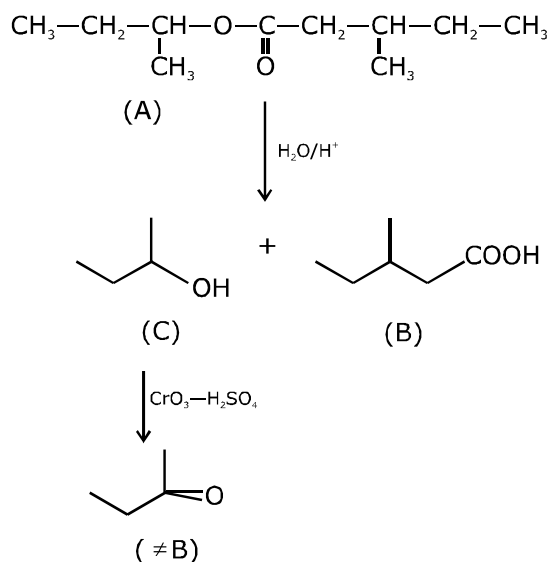
**Sol. 1**

Unnilennium 109

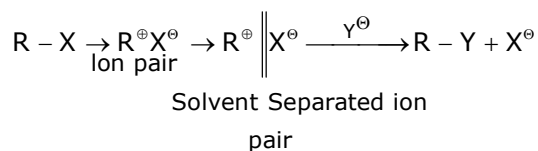
- 16.** An organic compound [A], molecular formula  $\text{C}_{10}\text{H}_{20}\text{O}_2$  was hydrolyzed with dilute sulphuric acid to give a carboxylic acid [B] and an alcohol [C]. Oxidation of [C] with  $\text{CrO}_3 - \text{H}_2\text{SO}_4$  produced [B]. Which of the following structures are not possible for [A]?



**Sol. 2**



**17.** The mechanism of  $\text{S}_\text{N}1$  reaction is given as :



A student writes general characteristics based on the given mechanism as :

- (a) The reaction is favoured by weak nucleophiles.
- (b)  $\text{R}^+$  would be easily formed if the substituents are bulky.
- (c) The reaction is accompanied by racemization.
- (d) The reaction is favoured by non-polar solvents.

Which observations are correct?

- (1) (a) and (b)
- (2) (a), (b) and (c)
- (3) (a) and (c)
- (4) (b) and (d)

**Sol. 2**

Statement (a), (b) & (c) are correct for  $\text{S}_\text{N}1$  reaction mechanism.

**18.** Tyndall effect is observed when:

- (1) The diameter of dispersed particles is much smaller than the wavelength of light used.
- (2) The diameter of dispersed particles is much larger than the wavelength of light used.
- (3) The refractive index of dispersed phase is greater than that of the dispersion medium.
- (4) The diameter of dispersed particles is similar to the wavelength of light used.

**Sol. 4**

Diameter of dispersed particles should not be much smaller than wavelength of light used.  
Refer topic surface chemistry

- 19.** Let  $C_{\text{NaCl}}$  and  $C_{\text{BaSO}_4}$  be the conductances (in S) measured for saturated aqueous solutions of NaCl and  $\text{BaSO}_4$ , respectively, at a temperature T. Which of the following is false?

- (1)  $C_{\text{NaCl}}(T_2) > C_{\text{NaCl}}(T_1)$  for  $T_2 > T_1$
- (2)  $C_{\text{BaSO}_4}(T_2) > C_{\text{BaSO}_4}(T_1)$  for  $T_2 > T_1$
- (3) Ionic mobilities of ions from both salts increase with T.
- (4)  $C_{\text{NaCl}} \gg C_{\text{BaSO}_4}$  at a given T

**Sol. 4**

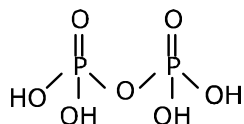
Ionic

$C_{\text{NaCl}} \gg C_{\text{BaSO}_4}$  at temp 'T'

- 20.** In a molecule of pyrophosphoric acid, the number of P-OH, P = O and P - O - P bonds/moiety(ies) respectively are :

- (1) 3, 3 and 3
- (2) 4, 2 and 1
- (3) 2, 4 and 1
- (4) 4, 2 and 0

**Sol. 2**



P - OH bonds = 4

P = O bonds = 2

P - O - P linkage = 1

Ans. 4, 2, 1

- 21.** The mole fraction of glucose ( $\text{C}_6\text{H}_{12}\text{O}_6$ ) in an aqueous binary solution is 0.1. The mass percentage of water in it, to the nearest integer, is \_\_\_\_\_.

**Sol. 47 %**

$$x_{\text{Glucose}} = 0.1$$

$$\text{mass\% of glucose} = \frac{0.1 \times 180}{0.1 \times 180 + 0.9 \times 18} \times 100$$

$$= \frac{1800}{18 + 16.2}$$

$$= \frac{1800}{34.2} \%$$

$$= 52.63\%$$

$$= 53\%$$

$$\therefore \text{mass \% of H}_2\text{O} = 47\%$$

- 22.** The volume strength of 8.9 M  $\text{H}_2\text{O}_2$  solution calculated at 273 K and 1 atm is \_\_\_\_\_. ( $R = 0.0821 \text{ L atm K}^{-1} \text{ mol}^{-1}$ ) (rounded off to the nearest integer)

**Sol. 100**

$$\begin{aligned}\text{Vol. strength} &= \frac{8.9}{2} \times \frac{0.0821 \times 273}{1} \\ &= 99.73 \\ &= 100\end{aligned}$$

- 23.** An element with molar mass  $2.7 \times 10^{-2} \text{ kg mol}^{-1}$  forms a cubic unit cell with edge length 405 pm. If its density is  $2.7 \times 10^3 \text{ kg m}^{-3}$ , the radius of the element is approximately \_\_\_\_\_  $\times 10^{-12} \text{ m}$  (to the nearest integer).

**Sol. 143**

$$\text{Density} = \frac{Z \times \text{GMM}}{N_A \times a^3}$$

$$2.7 \times 10^3 = \frac{Z \times 2.7 \times 10^{-2}}{6.023 \times 10^{23} \times (405 \times 10^{-12})^3}$$

$$Z = 6.023 \times 405 \times 405 \times 405 \times 10^{23-36+3+2}$$

$$Z = 6.023 \times 405 \times 405 \times 405 \times 10^{-8}$$

$$Z = 4$$

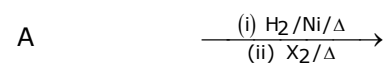
FCC

$$4R = \sqrt{2} \times a$$

$$R = \frac{405}{2\sqrt{2}} \times 10^{-12} = 143.21 \times 10^{-12} \text{ m}$$

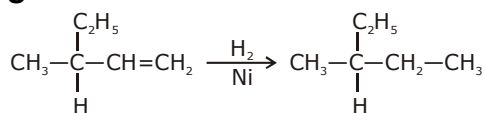
$$= 143 \text{ ans}$$

- 24.** The total number of monohalogenated organic products in the following (including stereoisomers) reaction is \_\_\_\_\_.

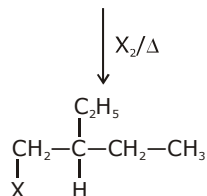


(Simplest  
optically  
active  
alkene)

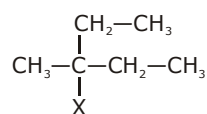
**Sol. 8**



(Simplest optically active alkene)

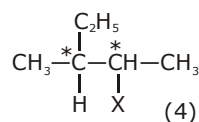


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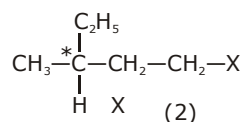


+

Total 8 organic products are possible



+



**25.** The photoelectric current from Na (Work function,  $w_0 = 2.3 \text{ eV}$ ) is stopped by the output voltage of the cell  $\text{Pt(s)} \mid \text{H}_2(\text{g}, 1 \text{ Bar}) \mid \text{HCl (aq. pH} = 1) \parallel \text{AgCl(s)} \mid \text{Ag(s)}$ .

The pH of aq. HCl required to stop the photoelectric current from K ( $w_0 = 2.25 \text{ eV}$ ), all other conditions remaining the same, is \_\_\_\_\_  $\times 10^{-2}$  (to the nearest integer).

Given,

$$2.303 \frac{RT}{F} = 0.06 \text{ V}; E_{\text{AgCl}|\text{Ag}|\text{Cl}^-}^0 = 0.22 \text{ V}$$

**Sol. 58**

Energy of photon =  $2.3 - E_{\text{cell}} \{\text{for Na}\}$

Energy of photon =  $2.25 - E_{\text{cell}} \{\text{for K}\}$

$E_{\text{cell}} \{\text{for 'Na'}\} + 0.05 = E_{\text{cell}} \{\text{for 'K'}\}$

$0.22 + 0.06 \log [\text{H}^+][\text{Cl}^-] + 0.05 = 0.22 + 0.06 \log [\text{H}^+][\text{Cl}^-]$

$6 \log (10^{-2}) + 5 = 6 \log [\text{H}^+][\text{Cl}^-]$

$\log (10^{-12}) + \log (10^5) = \log \{[\text{H}^+][\text{Cl}^-]\}^6$

$\{[\text{H}^+][\text{Cl}^-]\}^6 = 10^{-7}$

$[\text{H}^+]^{12} = 10^{-7}$

$$\text{pH} = \frac{7}{12} = 0.58$$

$$= 58 \times 10^{-2} = 58 \text{ Ans}$$

# QUESTION PAPER WITH SOLUTION

## MATHEMATICS \_ 3 Sep. \_ SHIFT - 1

**Q.1** The value of  $(2 \cdot {}^1P_0 - 3 \cdot {}^2P_1 + 4 \cdot {}^3P_2 - \dots$  up to  $51^{\text{th}}$  term)  $+(1! - 2! + 3! - \dots$  up to  $51^{\text{th}}$  term) is equal to:

- (1)  $1 - 51(51)!$       (2)  $1 + (52)!$       (3)  $1$       (4)  $1 + (51)!$

**Sol. 2**

$$2. {}^1P_0 = \underline{2}$$

$$3. {}^2P_1 = \underline{3}$$

$$4. {}^3P_2 = \underline{4}$$

$$(\underline{2} - \underline{3} + \underline{4} - \underline{5} + \dots \dots \dots \underline{52}) + (\underline{1} - \underline{2} + \underline{3} - \underline{4} \dots \dots \dots + \underline{51})$$

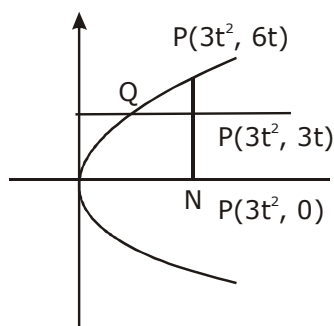
$$= \underline{52} + 1$$

**Q.2** Let P be a point on the parabola,  $y^2 = 12x$  and N be the foot of the perpendicular drawn from P on the axis of the parabola. A line is now drawn through the mid-point M of PN, parallel to its axis

which meets the parabola at Q. If the y-intercept of the line NQ is  $\frac{4}{3}$ , then:

- (1)  $PN=4$       (2)  $MQ=\frac{1}{3}$       (3)  $PN=3$       (4)  $MQ=\frac{1}{4}$

**Sol. 4**



Q (h, 3t) lie on

Parabola

$$9t^2 = 12h$$

$$h = \frac{3t^2}{4}$$

$$Q = \left( \frac{3t^2}{4}, 3t \right)$$

Equation of NQ

$$y = \frac{3t}{\left(\frac{3t^2}{4} - 3t^2\right)} \quad (x - 3t^2)$$

$$y = \frac{-4t}{3t^2} (x - 3t^2)$$

$$\text{put } x = 0$$

$$y = \frac{-4}{3t} (-3t^2) = 4t$$

$$4t = \frac{4}{3} \Rightarrow t = \frac{1}{3}$$

$$PN = 6t = 6 \cdot \frac{1}{3} = 2$$

$$M = \left[\frac{1}{3}, 1\right], Q = \left[\frac{1}{12}, 1\right]$$

$$MQ = \frac{1}{3} - \frac{1}{12} = \frac{1}{4}$$

**Q.3** If  $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3+Bx^2+Cx+D$ , then  $B+C$  is equal to:

(1) 1

(2) -1

(3) -3

(4) 9

**Sol.**

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3+Bx^2+Cx+D$$

$$R_2 \rightarrow R_2 - 2R_1, R_3 \rightarrow R_3 - 3R_1$$

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 1 & -x+2 & -2x+3 \\ 1 & -x+1 & x-5 \end{vmatrix}$$

$$R_3 \rightarrow R_3 - R_2$$

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 1 & -x+2 & -2x+3 \\ 0 & -1 & 3x-8 \end{vmatrix} = Ax^3+Bx^2+Cx+D$$

$$\Rightarrow -1[(3-2x)(x-2) - (3x-4)] + (3x-8)[(x-2)(-x+2) - (2x-3)] = Ax^3+Bx^2+Cx+D$$

$$\Rightarrow 3x - 2x^2 - 6 + 4x - 3x + 4 + (3x-8)[-x^2 + 4x - 4 - 2x + 3] = Ax^3+Bx^2+Cx+D$$

$$A = -3, B = 12, C = -15$$

$$B + C = -3$$

**Q.4** The foot of the perpendicular drawn from the point (4,2,3) to the line joining the points (1,-2,3) and (1,1,0) lies on the plane:

(1)  $x-y-2z=1$

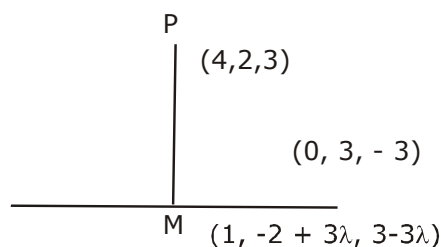
(2)  $x-2y+z=1$

(3)  $2x+y-z=1$

(4)  $x+2y-z=1$

**Sol. 3**

$$\vec{r} = (1, -2, 3) + \lambda(0, 3, -3)$$



$$\vec{PM} \perp \vec{b}$$

$$\vec{PM} \cdot \vec{b} = 0$$

$$(-3, 3\lambda - 4, -3\lambda) \cdot (0, 3, -3) = 0$$

$$\Rightarrow 0 + 9\lambda - 12 + 9\lambda = 0 \Rightarrow \lambda = \frac{12}{18} = \frac{2}{3}$$

$$m = (1, 0, 1) \text{ are on } 2x + y - z = 1$$

**Q.5** If  $y^2 + \log_e(\cos^2 x) = y$ ,  $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ , then

(1)  $|y'(0)| + |y''(0)| = 1$

(2)  $y''(0) = 0$

(3)  $|y'(0)| + |y''(0)| = 3$

(4)  $|y''(0)| = 2$

**Sol. 4**

$$2yy' + 2(-\tan x) = y'$$

diff. w.r.t. x

$$2yy'' + 2(y')^2 - 2\sec^2 x = y''$$

Put  $x = 0$  in given equation we get  $y = 0, 1$

from (1)  $x = 0, y = 0 \Rightarrow y'(0) = 0$

$x = 0, y = 1, \Rightarrow y'(0) = 0$

$$\dots(1)$$

$$\dots(2)$$

from (2)  $x = 0, y = 0, y'(0) = 0 \Rightarrow y''(0) = -2$   
 $x = 0, y = 1, y'(0) = 0 \Rightarrow y''(0) = 2$   
 $|y''(0)| = 2$

**Q.6**  $2\pi - \left( \sin^{-1} \frac{4}{5} + \sin^{-1} \frac{5}{13} + \sin^{-1} \frac{16}{25} \right)$  is equal to:

- (1)  $\frac{5\pi}{4}$  (2)  $\frac{3\pi}{2}$  (3)  $\frac{7\pi}{4}$  (4)  $\frac{\pi}{2}$

**Sol. 2**

$$2\pi - \left[ \tan^{-1} \left( \frac{4}{3} \right) + \tan^{-1} \left( \frac{5}{12} \right) + \tan^{-1} \frac{16}{63} \right]$$

$$2\pi - \tan^{-1} \left( \frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3} \cdot \frac{5}{12}} \right) - \tan^{-1} \left( \frac{16}{63} \right)$$

$$\Rightarrow 2\pi - \tan^{-1} \left( \frac{48 + 15}{36 - 20} \right) - \tan^{-1} \left( \frac{16}{63} \right)$$

$$\Rightarrow 2\pi - \left[ \tan^{-1} \left( \frac{63}{16} \right) + \cot^{-1} \left( \frac{63}{16} \right) \right]$$

$$\Rightarrow 2\pi - \frac{\pi}{2}$$

$$= \frac{3\pi}{2}$$

**Q.7** A hyperbola having the transverse axis of length  $\sqrt{2}$  has the same foci as that of the ellipse  $3x^2 + 4y^2 = 12$ , then this hyperbola does not pass through which of the following points ?

- (1)  $\left( \sqrt{\frac{3}{2}}, \frac{1}{\sqrt{2}} \right)$  (2)  $\left( 1, -\frac{1}{\sqrt{2}} \right)$  (3)  $\left( \frac{1}{\sqrt{2}}, 0 \right)$  (4)  $\left( -\sqrt{\frac{3}{2}}, 1 \right)$

**Sol. 1**

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

$$b_1^2 = a_1^2 (1 - e_1^2)$$

$$3 = 4(1 - e_1^2)$$

$$e_1 = \frac{1}{2}$$

$$\text{focus} = (\pm a_1 e_1, 0) \\ = (\pm 1, 0)$$

$$\text{Length of transverse axis } 2a_2 = \sqrt{2} \rightarrow a_2 = \frac{1}{\sqrt{2}}$$

$$a_2 e_2 = 1$$

$$= e_2 = \sqrt{2}$$

$$b_2^2 = a_2^2 (e_2^2 - 1)$$

$$b_2^2 = \frac{1}{2} (2 - 1) = \frac{1}{2}$$

equation of Hyperbola

$$x^2 - y^2 = \frac{1}{2}$$

**Q.8** For the frequency distribution:

Variate(x):	$x_1$	$x_2$	$x_3 \dots x_{15}$
Frequency(f):	$f_1$	$f_2$	$f_3 \dots f_{15}$

where  $0 < x_1 < x_2 < x_3 < \dots < x_{15} = 10$  and  $\sum_{i=1}^{15} f_i > 0$ , the standard deviation cannot be:

(1) 1

(2) 4

(3) 6

(4) 2

**Sol. 3**

$$\sigma^2 \leq \frac{1}{4} (M - m)^2$$

(M = upper bound of value of any random variable,  
m = Lower bound of value of any random variable)

$$\sigma^2 \leq \frac{1}{4} (10 - 0)^2$$

$$\sigma^2 < 25$$

$$-5 < \sigma < 5$$

$$\sigma \neq 6$$

**Q.9** A die is thrown two times and the sum of the scores appearing on the die is observed to be a multiple of 4. Then the conditional probability that the score 4 has appeared atleast once is:

(1)  $\frac{1}{3}$

(2)  $\frac{1}{4}$

(3)  $\frac{1}{8}$

(4)  $\frac{1}{9}$

**Sol. 4**

Total Possibilities = (1, 3), (3, 1), (2, 2),  
(2, 6), (6, 2) (4, 4)  
(3, 5), (5, 3) (6, 6)  
fav. = 1 = (4, 4)

$$\text{prob.} = \frac{1}{9}$$

**Q.10** If the number of integral terms in the expansion of  $\left(3^{\frac{1}{2}} + 5^{\frac{1}{8}}\right)^n$  is exactly 33, then the least value

of n is:

(1) 128

(2) 248

(3) 256

(4) 264

**Sol. 3**

$$T_{r+1} = {}^nC_r \left(3^{\frac{1}{2}}\right)^{n-r} \left(5^{\frac{1}{8}}\right)^r$$

$$\left. \begin{array}{l} \frac{n-r}{2} \rightarrow n-r = 0, 2, 4, 6, 8, \dots \end{array} \right\}$$

$$\left. \begin{array}{l} \frac{r}{8} \rightarrow r = 0, 8, 16, 24, \dots \end{array} \right\}$$

common r = 0, 8, 16, 24, .....

no. of integral term = 33.

$$L = 0 + (33 - 1) \times 8 \rightarrow L = 32 \times 8 = 256$$

**Q.11**  $\int_{-\pi}^{\pi} |\pi - |x|| dx$  is equal to:

(1)  $\pi^2$

(2)  $\frac{\pi^2}{2}$

(3)  $\sqrt{2}\pi^2$

(4)  $2\pi^2$

**Sol. 1**

$$\int_{-\pi}^{\pi} |\pi - |x|| dx$$

even function

$$2 \int_0^{\pi} |\pi - x| dx$$

$$= 2 \int_0^{\pi} (\pi - x) dx \Rightarrow 2 \left[ \pi x - \frac{x^2}{2} \right]_0^{\pi}$$

$$= 2 \left[ \frac{\pi^2}{2} \right] = \pi^2$$

Q.12 Consider the two sets:

$A = \{m \in \mathbb{R} : \text{both the roots of } x^2 - (m+1)x + m+4 = 0 \text{ are real}\}$  and  $B = [-3, 5]$ .

Which of the following is not true ?

(1)  $A - B = (-\infty, -3) \cup (5, \infty)$

(2)  $A \cap B = \{-3\}$

(3)  $B - A = (-3, 5)$

(4)  $A \cup B = \mathbb{R}$

**Sol. 1**

$$D \geq 0$$

$$(m+1)^2 - 4(m+4) \geq 0$$

$$\Rightarrow m^2 - 2m - 15 \geq 0$$

$$(m-5)(m+3) \geq 0$$

$$m \in (-\infty, -3] \cup [5, \infty)$$

$$A = (-\infty, -3] \cup [5, \infty)$$

$$B = [-3, 5]$$

$$A - B = (-\infty, -3) \cup (5, \infty)$$

$$A \cup B = \mathbb{R}$$

Q.13 The proposition  $p \rightarrow \neg(p \wedge \sim q)$  is equivalent to :

(1)  $(\sim p) \vee (\sim q)$

(2)  $(\sim p) \wedge q$

(3)  $q$

(4)  $(\sim p) \vee q$

**Sol. 4**

$$\sim(p \wedge \sim q) \rightarrow \sim p \vee q$$

$$p \rightarrow (\sim p \vee q)$$

$$\Rightarrow \sim p \vee (\sim p \vee q)$$

$$\Rightarrow \sim p \vee q$$

Q.14 The function,  $f(x) = (3x-7)x^{2/3}$ ,  $x \in \mathbb{R}$  is increasing for all  $x$  lying in:

(1)  $\left(-\infty, -\frac{14}{15}\right) \cup (0, \infty)$

(2)  $\left(-\infty, \frac{14}{15}\right)$

(3)  $(-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$

(4)  $(-\infty, 0) \cup \left(\frac{3}{7}, \infty\right)$

**Sol. 3**

$$f(x) = (3x-7) \cdot \frac{2}{3x^{1/3}} + x^{2/3} \cdot 3$$

$$= \frac{6x - 14 + 9x}{3x^{1/3}}$$

$$= \frac{15x - 14}{3x^{\frac{1}{3}}}$$

+	-	+
0		14/15

$$f(x) > 0 \Rightarrow x \in (-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$$

**Q.15** If the first term of an A.P. is 3 and the sum of its first 25 terms is equal to the sum of its next 15 terms, then the common difference of this A.P. is:

- (1)  $\frac{1}{6}$                       (B)  $\frac{1}{5}$                       (C)  $\frac{1}{4}$                       (D)  $\frac{1}{7}$

**Sol. 1**  
 $a = 3$

$$\frac{25}{2} [2a + 24d] = \frac{15}{2} [2 \times (a + 25d) + 14d]$$

$$\Rightarrow 50a + 600d = 15 [2a + 50d + 14d]$$

$$\Rightarrow 20a + 600d = 960d$$

$$\Rightarrow 60 = 360d$$

$$d = \frac{1}{6}$$

**Q.16** The solution curve of the differential equation,  $(1 + e^{-x})(1 + y^2) \frac{dy}{dx} = y^2$ , which passes through the point (0,1), is:

(1)  $y^2 = 1 + y \log_e \left( \frac{1 + e^{-x}}{2} \right)$

(2)  $y^2 + 1 = y \left( \log_e \left( \frac{1 + e^{-x}}{2} \right) + 2 \right)$

(3)  $y^2 + 1 = y \left( \log_e \left( \frac{1 + e^x}{2} \right) + 2 \right)$

(4)  $y^2 = 1 + y \left( \log_e \left( \frac{1 + e^x}{2} \right) \right)$

**Sol. 4**

$$\int \left( \frac{1 + y^2}{y^2} \right) dy = \int \left( \frac{1}{1 + e^{-x}} \right) dx$$

$$\int \frac{1}{y^2} dy + \int dy = \int \left( \frac{e^x}{e^x + 1} \right) dx$$

$$\Rightarrow \frac{-1}{y} + y = \ln|e^x + 1| + c$$

$$x = 0, y = 1$$

$$\Rightarrow -1 + 1 = \ln 2 + c \Rightarrow c = -\ln 2$$

$$\Rightarrow \frac{-1}{y} + y = \ln|e^x + 1| - \ln 2$$

$$\Rightarrow y^2 = 1 + y \left[ \ln \left( \frac{e^x + 1}{2} \right) \right]$$

**Q.17** The area (in sq. units) of the region  $\left\{ (x, y) : 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, \frac{1}{2} \leq x \leq 2 \right\}$  is

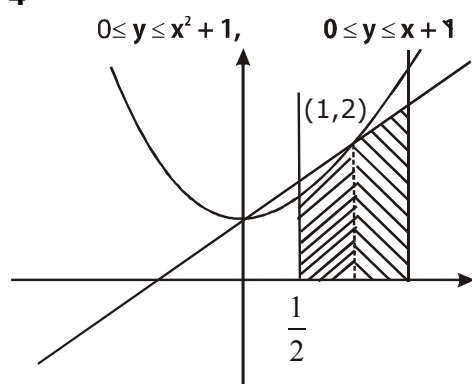
(1)  $\frac{23}{16}$

(2)  $\frac{79}{16}$

(3)  $\frac{23}{6}$

(4)  $\frac{79}{24}$

**Sol. 4**



$$A = \int_{\frac{1}{2}}^1 (x^2 + 1) dx + \int_1^2 (x + 1) dx$$

$$\left( \frac{x^3}{3} + x \right) \Big|_{\frac{1}{2}}^1 + \left( \frac{x^2}{2} + x \right) \Big|_1^2$$

$$= \left( \frac{1}{3} + 1 \right) - \left( \frac{1}{24} + \frac{1}{2} \right) + \left( (2 + 2) - \left( \frac{3}{2} \right) \right)$$

$$= \left( \frac{4}{3} - \frac{13}{24} \right) + \left( \frac{5}{2} \right)$$

$$= \left( \frac{32-13}{24} \right) + \left( \frac{5}{2} \right) = \frac{19+60}{24} = \frac{79}{24}$$

**Q.18** If  $\alpha$  and  $\beta$  are the roots of the equation  $x^2+px+2=0$  and  $\frac{1}{\alpha}$  and  $\frac{1}{\beta}$  are the roots of the

equation  $2x^2+2qx+1=0$ , then  $\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$  is equal to :

(1)  $\frac{9}{4}(9+p^2)$       (2)  $\frac{9}{4}(9+q^2)$       (3)  $\frac{9}{4}(9-p^2)$       (4)  $\frac{9}{4}(9-q^2)$

**Sol. 3**

$$\alpha + \beta = -p, \alpha\beta = 2$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = -q, \frac{1}{\alpha\beta} = \frac{1}{2}$$

$$\frac{\alpha + \beta}{\alpha\beta} = -q \Rightarrow \frac{-p}{2} = -q$$

$$\Rightarrow p = 2q$$

$$\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right) = \alpha\beta + \frac{1}{\alpha\beta} + 2$$

$$= 2 + \frac{1}{2} + 2 = \frac{9}{2}$$

$$\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right) = \alpha\beta + \frac{1}{\alpha\beta} - \frac{\alpha}{\beta} - \frac{\beta}{\alpha}$$

$$= 2 + \frac{1}{2} - \left[ \frac{\alpha^2 + \beta^2}{\alpha\beta} \right]$$

$$= \frac{5}{2} - \left[ \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} \right]$$

$$= \frac{5}{2} - \left[ \frac{p^2 - 4}{2} \right]$$

$$= \frac{9 - p^2}{2}$$

$$\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right) = \left(\frac{9 - p^2}{2}\right)\left(\frac{9}{2}\right)$$

$$= \frac{9}{4}(9 - p^2)$$

**Q.19** The lines  $\vec{r} = (\hat{i} - \hat{j}) + l(2\hat{i} + \hat{k})$  and  $\vec{r} = (2\hat{i} - \hat{j}) + m(\hat{i} + \hat{j} - \hat{k})$

(1) do not intersect for any values of  $l$  and  $m$

(2) intersect when  $l=1$  and  $m=2$

(3) intersect when  $l=2$  and  $m=\frac{1}{2}$

(4) intersect for all values of  $l$  and  $m$

**Sol. 1**

$$\frac{2}{1} \neq \frac{0}{1} \neq \frac{1}{-1} \rightarrow \text{lines are intersecting}$$

$$\vec{r} = (1 + 2l)\hat{i} - \hat{j} + l\hat{k} \quad \dots(1)$$

$$\vec{r} = (2 + m)\hat{i} + (m - 1)\hat{j} - m\hat{k} \quad \dots(2)$$

compare coeff. of  $\hat{i}, \hat{j}, \hat{k}$

$$1 + 2l = 2 + m \quad \left| \begin{array}{l} -1 = m - 1 \\ m = 0 \end{array} \right| \quad l = 0$$

Lines do not intersect

**Q.20** Let  $[t]$  denote the greatest integer  $\leq t$ . if for some  $\lambda \in \mathbb{R} - \{0, 1\}$

$$\lim_{x \rightarrow 0} \left| \frac{1 - x + |x|}{\lambda - x + [x]} \right| = L, \text{ then } L \text{ is equal to:}$$

(1) 0

(2) 2

(3)  $\frac{1}{2}$

(4) 1

**Sol. 2**

$$\lim_{x \rightarrow 0} \left| \frac{1-x+|x|}{\lambda-x+[x]} \right| = L$$

$$\lim_{h \rightarrow 0} \left| \frac{1-h+h}{\lambda-h+[h]} \right|$$

$$\lim_{h \rightarrow 0} \left| \frac{1}{\lambda-h+0} \right| = \left| \frac{1}{\lambda} \right| \quad [x] = 0$$

$$\lim_{h \rightarrow 0} \left| \frac{1+h+h}{\lambda+h+[-h]} \right|$$

$$= \left| \frac{1}{\lambda-1} \right| \quad [-h] = -1$$

$$\therefore |\lambda| = |\lambda-1|$$

$$\lambda^2 = \lambda^2 - 2\lambda + 1 \Rightarrow \lambda = \frac{1}{2}$$

$$L = 2$$

**Q.21** If  $\lim_{x \rightarrow 0} \left\{ \frac{1}{x^8} \left( 1 - \cos \frac{x^2}{2} - \cos \frac{x^2}{4} + \cos \frac{x^2}{2} \cos \frac{x^2}{4} \right) \right\} = 2^{-k}$ , then the value of k is .....

**Sol. 8**

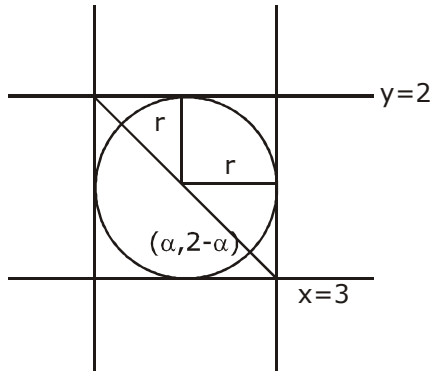
$$\lim_{x \rightarrow 0} \frac{\left( 1 - \cos \frac{x^2}{2} \right) \left( 1 - \cos \frac{x^2}{4} \right) \left( \frac{x^2}{2} \right) \cdot \left( \frac{x^2}{4} \right)^2}{\left( \frac{x^2}{2} \right)^2 \left( \frac{x^2}{4} \right)^2 \cdot x^8}$$

$$\lim_{x \rightarrow 0} \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{16} \Rightarrow \frac{1}{256} = 2^{-k}$$

$$2^{-8} = 2^{-k} \Rightarrow k = 8$$

**Q.22** The diameter of the circle, whose centre lies on the line  $x + y = 2$  in the first quadrant and which touches both the lines  $x=3$  and  $y=2$ , is .....

**Sol.** 2



$$p = r$$

$$\text{for } y = 2$$

$$r = \left| \frac{2 - \alpha - 2}{1} \right| = |\alpha|$$

$$\text{for } x = 3$$

$$r = \left| \frac{\alpha - 3}{1} \right| = |\alpha - 3|$$

$$|\alpha| = |\alpha - 3|$$

$$\Rightarrow \alpha^2 + \alpha^2 - 6\alpha + 9 \Rightarrow \alpha = \frac{3}{2}$$

$$2\alpha = 3 = 2r$$

**Q.23** The value of  $(0.16)^{\log_{2.5} \left( \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots \text{to } \infty \right)}$  is equal to.....

**Sol.** 4

$$\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots \text{to } \infty = \frac{\frac{1}{3}}{1 - \frac{1}{3}} = \frac{1}{2}$$

$$\log_{2.5} \left( \frac{1}{2} \right) \Rightarrow \log_{\frac{5}{2}} \frac{1}{2}$$

$$.16 = \frac{16}{100} = \frac{4}{25} = \left( \frac{2}{5} \right)^2$$

$$\Rightarrow \left(\frac{2}{5}\right)^{2 \log_{\frac{5}{2}} \frac{1}{2}} = \left(\frac{5}{2}\right)^{-2 \log_{\frac{5}{2}} \frac{1}{2}}$$

$$\Rightarrow \left(\frac{5}{2}\right)^{\log_{\frac{5}{2}} \left(\frac{1}{2}\right)^{-2}}$$

$$= 4$$

**Q.24** Let  $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$ ,  $x \in \mathbb{R}$  and  $A^4 = [a_{ij}]$ . If  $a_{11} = 109$ , then  $a_{22}$  is equal to .....

**Sol. 10**

$$A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x^2 + 1 & x \\ x & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} x^2 + 1 & x \\ x & 1 \end{bmatrix} \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} x^3 + x + x & x^2 + 1 \\ x^2 + 1 & x \end{bmatrix}$$

$$A^4 = \begin{bmatrix} x^3 + 2x & x^2 + 1 \\ x^2 + 1 & x \end{bmatrix} \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} x^4 + 2x^2 + x^2 + 1 & x^3 + 2x \\ x^3 + x + x & x^2 + 1 \end{bmatrix}$$

$$a_{11} \Rightarrow x^4 + 3x^2 + 1 = 109$$

$$x^4 + 3x^2 - 108 = 0$$

$$\Rightarrow (x^2 + 12)(x^2 - 9) = 0$$

$$x = \pm 3$$

$$a_{11} = x^2 + 1 = 10$$

**Q.25** If  $\left(\frac{1+i}{1-i}\right)^{\frac{m}{2}} = \left(\frac{1+i}{i-1}\right)^{\frac{n}{3}} = 1$ , ( $m, n \in \mathbb{N}$ ) then the greatest common divisor of the least values of  $m$  and  $n$  is .....

**Sol.** 4

$$\left[\frac{(1+i)(1+i)}{(1+i)(1-i)}\right]^{\frac{m}{2}} = \left[\left(\frac{1+i}{-1+i}\right)\left(\frac{-1-i}{-1-i}\right)\right]^{\frac{n}{3}} = 1$$

$$= \left(\frac{2i}{2}\right)^{\frac{m}{2}} = 1 \quad \left| \quad \left(\frac{-1-i-i+1}{1+1}\right)^{\frac{n}{3}} = 1\right.$$

$$m = 8$$

$$\begin{aligned} (-i)^{n/3} &= 1 \\ n &= 12 \end{aligned}$$

greatest common divisor of  $m$  &  $n$  is 4

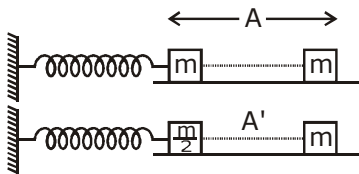
## PHYSICS \_ 3 Sep. \_ SHIFT - 2

1. A block of mass  $m$  attached to a massless spring is performing oscillatory motion of amplitude ' $A$ ' on a frictionless horizontal plane. If half of the mass of the block breaks off when it is passing through its equilibrium point, the amplitude of oscillation for the remaining system become  $fA$ . The value of  $f$  is :

- (1)  $\frac{1}{\sqrt{2}}$                       (2)  $\frac{1}{2}$                       (3) 1                      (4)  $\sqrt{2}$

**Sol. 4**

$$V_1 = V_{\max} = A\omega$$



$$V_2 = V_{\max} = A'\omega'$$

$$A\omega = A'\omega'$$

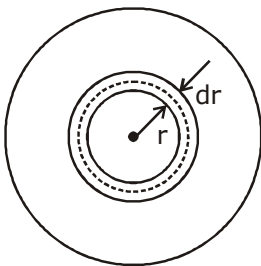
$$A\sqrt{\frac{k}{m}} = A'\sqrt{\frac{2k}{m}}$$

$$A' = \frac{A}{\sqrt{2}}$$

2. The mass density of a planet of radius  $R$  varies with the distance  $r$  from its centre as  $\rho(r) = \rho_0 \left(1 - \frac{r^2}{R^2}\right)$ . Then the gravitational field is maximum at :

- (1)  $r = \frac{1}{\sqrt{3}}R$                       (2)  $r = \sqrt{\frac{3}{4}}R$                       (3)  $r = R$                       (4)  $r = \sqrt{\frac{5}{9}}R$

**Sol. 4**



$$dm = \rho dv$$

$$\int dm = \int \rho_0 \left(1 - \frac{r^2}{R^2}\right) 4\pi r^2 dr$$

$$M = 4\pi\rho_0 \left( \frac{r^3}{3} - \frac{r^5}{5R^2} \right)$$

$$E_g = \frac{GM}{r^2}$$

$$E_g = G4\pi\rho_0 \left( \frac{r}{3} - \frac{r^3}{5R^2} \right)$$

$$\frac{dE_g}{dr} = \frac{1}{3} - \frac{3r^2}{5R^2} = 0$$

$$\frac{1}{3} = \frac{3r^2}{5R^2}$$

$$r = \sqrt{\frac{5}{9}} R$$

- 3.** Two sources of light emit X-rays of wavelength 1 nm and visible light of wavelength 500 nm, respectively. Both the sources emit light of the same power 200 W. The ratio of the number density of photons of X-rays to the number density of photons of the visible light of the given wavelengths is :

- (1)  $\frac{1}{500}$                       (2)  $\frac{1}{250}$                       (3) 500                      (4) 250

**Sol. 1**

$$P = \frac{nhc}{\lambda}$$

$$\frac{n}{\lambda} = \text{const}$$

$$\frac{n_1}{n_2} = \frac{\lambda_1}{\lambda_2} = \frac{1\text{nm}}{500\text{nm}} = \frac{1}{500}$$

- 4.** If a semiconductor photodiode can detect a photon with a maximum wavelength of 400 nm, then its band gap energy is :

Planck's constant  $h = 6.63 \times 10^{-34}$  J.s. Speed of light  $c = 3 \times 10^8$  m/s

- (1) 1.5 eV                      (2) 2.0 eV                      (3) 3.1 eV                      (4) 1.1 eV

**Sol. 3**

$$E = \frac{hc}{\lambda}$$

$$E = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{400 \times 10^{-9}}$$

$$E \approx \frac{1240}{400} \text{ eV}$$

$$E = 3.1 \text{ eV}$$

5. Amount of solar energy received on the earth's surface per unit area per unit time is defined a solar constant. Dimension of solar constant is :

(1)  $ML^0T^{-3}$  (2)  $MLT^{-2}$  (3)  $M^2L^0T^{-1}$  (4)  $ML^2T^{-2}$

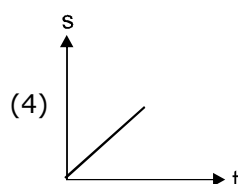
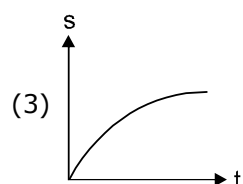
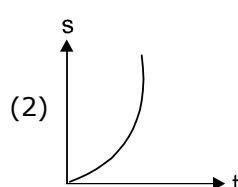
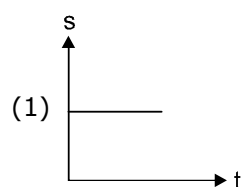
Sol. 1

$$E = \frac{Q}{At}$$

$$E = \frac{ML^2T^{-2}}{L^2T}$$

$$E = MT^{-3}$$

6. A particle is moving unidirectionally on a horizontal plane under the action of a constant power supplying energy source. The displacement (s) - time (t) graph that describes the motion of the particle is (graphs are drawn schematically and are not to scale) :



Sol. 2

$$P = FV$$

$$P = m \frac{dv}{dt} v$$

$$v dv = \frac{P}{m} dt$$

$$V^2 = k't$$

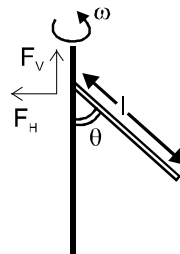
$$V = k'' \sqrt{t}$$

$$s \propto t^{3/2}$$

7. Which of the following will NOT be observed when a multimeter (operating in resistance measuring mode) probes connected across a component, are just reversed ?
- (1) Multimeter shows NO deflection in both cases i.e. before and after reversing the probes if the chosen component is metal wire.
  - (2) Multimeter shows a deflection, accompanied by a splash of light out of connected component in one direction and NO deflection on reversing the probes if the chosen component is LED.
  - (3) Multimeter shows an equal deflection in both cases i.e. before and after reversing the probes if the chosen component is resistor.
  - (4) Multimeter shows NO deflection in both cases i.e. before and after reversing the probes if the chosen component is capacitor.

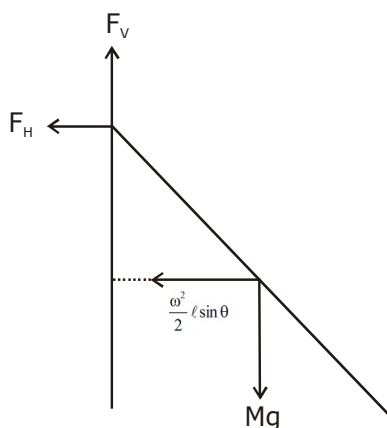
**Sol. 4**  
By Theory

8. A uniform rod of length 'l' is pivoted at one of its ends on a vertical shaft of negligible radius. When the shaft rotates at angular speed  $\omega$  the rod makes an angle  $\theta$  with it (see figure). To find  $\theta$  equate the rate of change of angular momentum (direction going into the paper)  $\frac{ml^2}{12} \omega^2 \sin\theta \cos\theta$  about the centre of mass (CM) to the torque provided by the horizontal and vertical forces  $F_H$  and  $F_V$  about the CM. The value of  $\theta$  is then such that :



- (1)  $\cos\theta = \frac{2g}{3l\omega^2}$       (2)  $\cos\theta = \frac{3g}{2l\omega^2}$       (3)  $\cos\theta = \frac{g}{2l\omega^2}$       (4)  $\cos\theta = \frac{g}{l\omega^2}$

**Sol. 2**



$$F_v = mg$$

$$F_H = m\omega^2 \frac{\ell}{2} \sin \theta$$

$$\therefore \tau_{\text{net}} \text{ about COM} = F_v \cdot \frac{\ell}{2} \sin \theta - F_H \frac{\ell}{2} \cos \theta$$

$$= \frac{m\ell^2}{12} \omega^2 \sin \theta \cos \theta$$

$$mg \frac{\ell}{2} \sin \theta - m\omega^2 \frac{\ell}{2} \sin \theta \frac{\ell}{2} \cos \theta = \frac{m\ell^2}{12} \omega^2 \sin \theta \cos \theta$$

$$\Rightarrow \frac{g\ell}{2} - \frac{\omega^2 \ell^2}{4} \cos \theta = \frac{\ell^2}{12} \omega^2 \cos \theta$$

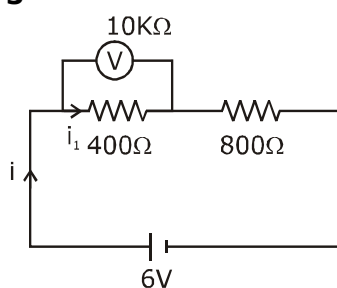
$$\frac{g\ell}{2} = \omega^2 \ell^2 \cos \theta \left( \frac{1}{12} + \frac{1}{4} \right)$$

$$\frac{g\ell}{2} = \frac{\omega^2 \ell^2 \cos \theta}{3}$$

$$\cos \theta = \frac{3g}{2\omega^2 \ell}$$

9. Two resistors  $400\Omega$  and  $800\Omega$  are connected in series across a 6 V battery. The potential difference measured by a voltmeter of  $10\text{ k}\Omega$  across  $400\Omega$  resistor is close to :  
 (1) 2.05 V (2) 2 V (3) 1.95 V (4) 1.8 V

**Sol. 3**



$$i = \frac{6}{800 + \frac{400 \times 10000}{400 + 10000}}$$

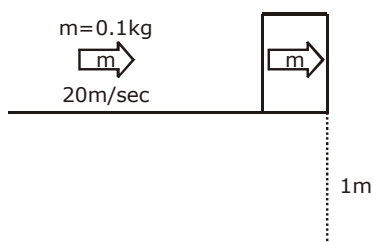
$$i = \frac{6}{800 + \frac{40000}{104}}$$

$$i = \frac{6}{800 + 384.61} = \frac{6}{1184.61} = 0.00506$$

$$V_v = 6 - 800 \times 0.00506 = 6 - 4.05 = 1.95$$

- 10.** A block of mass 1.9 kg is at rest at the edge of a table, of height 1 m. A bullet of mass 0.1 kg collides with the block and sticks to it. If the velocity of the bullet is 20 m/s in the horizontal direction just before the collision then the kinetic energy just before the combined system strikes the floor, is [Take  $g = 10 \text{ m/s}^2$ . Assume there is no rotational motion and loss of energy after the collision is negligible.]  
 (1) 23 J (2) 21 J (3) 20 J (4) 19 J

**Sol. 2**



$$0.1 \times 20 = (1.9 + 0.1)V$$

$$2 = 2V$$

$$V = 1 \text{ m/sec}$$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2} \times 2 \times (1)^2 = 1 \text{ J}$$

$$TE = KE + Mgh = 1 + 2 \times 10 \times 1 = 21 \text{ J}$$

- 11.** A metallic sphere cools from  $50^\circ\text{C}$  to  $40^\circ\text{C}$  in 300 s. If atmospheric temperature around is  $20^\circ\text{C}$ , then the sphere's temperature after the next 5 minutes will be close to :  
 (1)  $35^\circ\text{C}$  (2)  $31^\circ\text{C}$  (3)  $33^\circ\text{C}$  (4)  $28^\circ\text{C}$

**Sol. 3**

$$\frac{\Delta T}{\Delta t} = k \left( \frac{T_f + T_i}{2} - T_0 \right)$$

$$\frac{50 - 40}{300} = k \left( \frac{50 + 40}{2} - 20 \right)$$

$$\frac{40 - T}{300} = k \left( \frac{40 + T}{2} - 20 \right)$$

$$\frac{10}{40 - T} = \frac{25 \times 2}{40 + T - 40}$$

$$\frac{1}{40 - T} = \frac{5}{T}$$

$$T = 200 - 5T$$

$$6T = 200$$

$$T = 33^\circ\text{C}$$

- 12.** To raise the temperature of a certain mass of gas by  $50^{\circ}\text{C}$  at a constant pressure, 160 calories of heat is required. When the same mass of gas is cooled by  $100^{\circ}\text{C}$  at constant volume, 240 calories of heat is released. How many degrees of freedom does each molecule of this gas have (assume gas to be ideal) ?

(1) 6 (2) 7 (3) 5 (4) 3

**Sol. 1**

$$Q = nC_p\Delta T$$

$$160 = nC_p 50$$

$$240 = nC_v 100$$

$$\frac{16}{24} = \frac{C_p}{2C_v}$$

$$r = \frac{4}{3}$$

$$r = 1 + \frac{2}{f}$$

$$\frac{4}{3} - 1 = \frac{2}{f}$$

$$f = 6$$

- 13.** The radius  $R$  of a nucleus of mass number  $A$  can be estimated by the formula  $R = (1.3 \times 10^{-15})A^{1/3}$  m. It follows that the mass density of a nucleus is of the order of :

$$(M_{\text{prot.}} \cong M_{\text{neut.}} \cong 1.67 \times 10^{-27} \text{ kg})$$

(1)  $10^{17} \text{ kg m}^{-3}$  (2)  $10^{10} \text{ kg m}^{-3}$  (3)  $10^{24} \text{ kg m}^{-3}$  (4)  $10^3 \text{ kg m}^{-3}$

**Sol. 1**

$$R = (1.3 \times 10^{-15}) A^{1/3}$$

$$m = \rho V$$

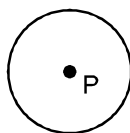
$$\rho = \frac{m}{V}$$

$$\rho = \frac{m_p A}{\frac{4}{3} \pi R^3}$$

$$\rho = \frac{m_p A}{\frac{4}{3} \pi \times (1.3 \times 10^{-15})^3 A}$$

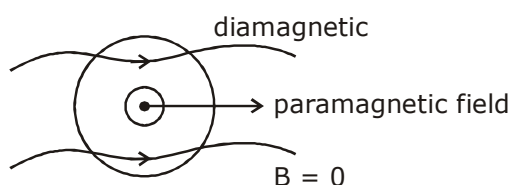
$$\rho \approx 10^{17} \text{ kg / m}^3$$

- 14.** A perfectly diamagnetic sphere has a small spherical cavity at its centre, which is filled with a paramagnetic substance. The whole system is placed in a uniform magnetic field  $\vec{B}$ . Then the field inside the paramagnetic substance is :



- (1) much large than  $|\vec{B}|$  and parallel to  $\vec{B}$       (2) zero  
(3)  $\vec{B}$       (4) much large than  $|\vec{B}|$  but opposite to  $\vec{B}$

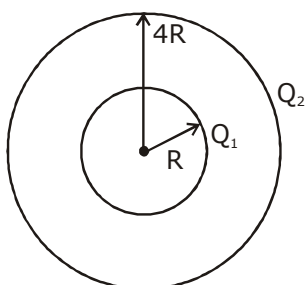
**Sol. 2**



- 15.** Concentric metallic hollow spheres of radii  $R$  and  $4R$  hold charges  $Q_1$  and  $Q_2$  respectively. Given that surface charge densities of the concentric spheres are equal, the potential difference  $V(R) - V(4R)$  is :

- (1)  $\frac{3Q_2}{4\pi\epsilon_0 R}$       (2)  $\frac{3Q_1}{4\pi\epsilon_0 R}$       (3)  $\frac{3Q_1}{16\pi\epsilon_0 R}$       (4)  $\frac{Q_2}{4\pi\epsilon_0 R}$

**Sol. 3**



$$\sigma = \frac{Q_1}{4\pi R^2} = \frac{Q_2}{4\pi 16R^2}$$

$$16Q_1 = Q_2$$

$$V_R - V_{4R} = \frac{KQ_1}{R} + \frac{KQ_2}{4R} - \frac{KQ_1}{4R} - \frac{KQ_2}{4R}$$

$$= \frac{3KQ_1}{4R} = \frac{3Q_1}{16\pi\epsilon_0 R}$$

- 16.** The electric field of a plane electromagnetic wave propagating along the x direction in vacuum is  $\vec{E} = E_0 \hat{j} \cos(\omega t - kx)$ . The magnetic field  $\vec{B}$ , at the moment  $t = 0$  is :

(1)  $\vec{B} = \frac{E_0}{\sqrt{\mu_0 \epsilon_0}} \cos(kx) \hat{j}$

(2)  $\vec{B} = \frac{E_0}{\sqrt{\mu_0 \epsilon_0}} \cos(kx) \hat{k}$

(3)  $\vec{B} = E_0 \sqrt{\mu_0 \epsilon_0} \cos(kx) \hat{j}$

(4)  $\vec{B} = E_0 \sqrt{\mu_0 \epsilon_0} \cos(kx) \hat{k}$

**Sol.**

**4**  
 $E = E_0 \cos(\omega t - kx) \hat{j}$

$E = BC$

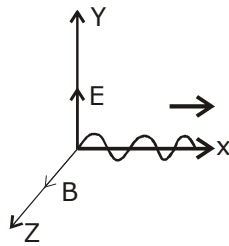
$B = \frac{E}{C} = \frac{E_0}{\frac{1}{\sqrt{\mu_0 \epsilon_0}}}$

$B = E_0 \sqrt{\mu_0 \epsilon_0}$

$B = E_0 \sqrt{\mu_0 \epsilon_0} \cos(\omega t - kx) \hat{k}$

at  $t = 0$

$B = E_0 \sqrt{\mu_0 \epsilon_0} \cos(kx) \hat{k}$



- 17.** A uniform magnetic field  $B$  exists in a direction perpendicular to the plane of a square loop made of a metal wire. The wire has a diameter of 4 mm and a total length of 30 cm. The magnetic field changes with time at a steady rate  $dB/dt = 0.032 \text{ Ts}^{-1}$ . The induced current in the loop is close to (Resistivity of the metal wire is  $1.23 \times 10^{-8} \Omega \text{m}$ )

(1) 0.53 A

(2) 0.61 A

(3) 0.34 A

(4) 0.43 A

**Sol.**

**2**  
 $\phi = BA$

$E = \frac{d\phi}{dt} = \frac{A dB}{dt}$

$E = \ell^2 \frac{dB}{dt}$

$i = \frac{E}{R} = \frac{\ell^2}{\rho \ell} \frac{dB}{dt} \text{ A}$

$i = \frac{30}{4} \times \frac{30}{4} \times \frac{10^{-4} \times 0.032 \times 4 \times 10^{-6} \times \pi}{1.23 \times 10^{-8} \times 30 \times 10^{-2} \times 10^3}$

$i = \frac{240 \times \pi \times 10^{-10}}{1.23 \times 10^{-7}}$

$i = \frac{240 \times 3.14 \times 10^{-3}}{1.23} = \frac{753.6}{1.23} \times 10^{-3}$

$i = 612.68 \times 10^{-3} = 0.61 \text{ A}$

- 18.** Hydrogen ion and singly ionized helium atom are accelerated, from rest, through the same potential difference. The ratio of final speeds of hydrogen and helium ions is close to :

(1) 2 : 1                      (2) 1 : 2                      (3) 5 : 7                      (4) 10 : 7

**Sol. 1**

$$K = \frac{p^2}{2m}$$

$$qV = \frac{p^2}{2m} = \frac{m^2 v^2}{2m}$$

$$v = \sqrt{\frac{2qV}{m}}$$

$$v \propto \sqrt{\frac{q}{m}}$$

$$\frac{v_H}{v_{He}} = \frac{\sqrt{\frac{e}{m}}}{\sqrt{\frac{e}{4m}}} = \frac{2}{1}$$

- 19.** Two light waves having the same wavelength  $\lambda$  in vacuum are in phase initially. Then the first wave travels a path  $L_1$  through a medium of refractive index  $n_1$  while the second wave travels a path of length  $L_2$  through a medium of refractive index  $n_2$ . After this the phase difference between the two waves is :

(1)  $\frac{2\pi}{\lambda} (n_1 L_1 - n_2 L_2)$                       (2)  $\frac{2\pi}{\lambda} \left( \frac{L_1}{n_1} - \frac{L_2}{n_2} \right)$   
 (3)  $\frac{2\pi}{\lambda} \left( \frac{L_2}{n_1} - \frac{L_1}{n_2} \right)$                       (4)  $\frac{2\pi}{\lambda} (n_2 L_1 - n_1 L_2)$

**Sol. 1**

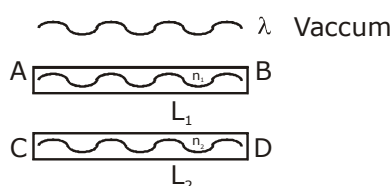
$$\lambda_{n_1} = \frac{\lambda}{n_1}$$

$$\lambda_{n_2} = \frac{\lambda}{n_2}$$

$$(\Delta\phi)_1 = \frac{2\pi}{\lambda_{n_1}} L_1$$

$$(\Delta\phi)_2 = \frac{2\pi}{\lambda_{n_2}} L_2$$

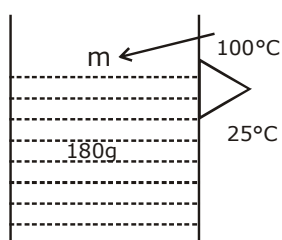
$$(\Delta\phi_1 - \Delta\phi_2) = \frac{2\pi}{\lambda} (n_1 L_1 - n_2 L_2)$$



- 20.** A calorimeter of water equivalent 20 g contains 180 g of water at 25°C. 'm' grams of steam at 100°C is mixed in it till the temperature of the mixture is 31°C. The value of 'm' is close to (Latent heat of water = 540 cal g<sup>-1</sup>, specific heat of water = 1 cal g<sup>-1</sup> °C<sup>-1</sup>)  
 (1) 2 (2) 2.6 (3) 4 (4) 3.2

**Sol. 1**

$$m_c s_c = 20g$$



Temp of mixture → 31°C

$$180 \times 1 \times (31 - 25) + 20 \times (31 - 25) = m \times 540 + m \times 1 \times (100 - 31)$$

$$180 \times 6 + 20 \times 6 = 540m + 100m - 31m$$

$$1080 + 120 = 640m - 31m$$

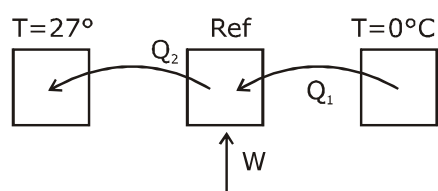
$$1200 = 609m$$

$$m = \frac{1200}{609} = 1.97$$

- 21.** If minimum possible work is done by a refrigerator in converting 100 grams of water at 0°C to ice, how much heat (in calories) is released to the surroundings at temperature 27°C (Latent heat of ice = 80 Cal/gram) to the nearest integer ?

**Sol. 8791**

$$Q_1 = mL = 8000 \text{ cal}$$



$$Q_1 = W + Q_2$$

$$\text{C.O.P.} = \frac{Q_1}{W} = \frac{Q_1}{Q_2 - Q_1} = \frac{T_2}{T_2 - T_1}$$

$$\frac{Q_1}{W} = \frac{273}{300 - 273}$$

$$\frac{Q_1}{W} = \frac{273}{27}$$

$$W = \frac{27}{273} Q_1$$

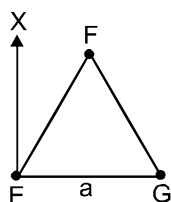
$$W = \frac{27}{273} mL$$

$$W = \frac{27}{273} \times 80 \times 100$$

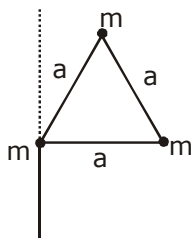
$$Q_2 = \frac{27}{273} \times 80 \times 100 + 80 \times 100$$

$$= 8791.2 \text{ cal}$$

- 22.** An massless equilateral triangle EFG of side 'a' (As shown in figure) has three particles of mass m situated at its vertices. The moment of inertia of the system about the line EX perpendicular to EG in the plane of EFG is  $\frac{N}{20} ma^2$  where N is an integer. The value of N is \_\_\_\_\_.



**Sol. 25**



$$I = ma^2 + \frac{ma^2}{4} = \frac{5}{4} ma^2$$

$$\frac{5}{4} \times ma^2 = \frac{N}{20} ma^2$$

$$N = 25$$

- 23.** A galvanometer coil has 500 turns and each turn has an average area of  $3 \times 10^{-4} \text{ m}^2$ . If a torque of 1.5 Nm is required to keep this coil parallel to a magnetic field when a current of 0.5 A is flowing through it, the strength of the field (in T) is \_\_\_\_\_.

**Sol. 20**

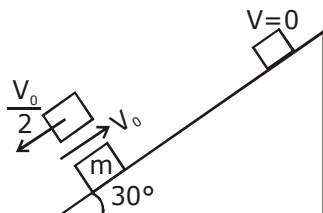
$$\tau = BINA \sin \theta$$

$$1.5 = B \times 0.5 \times 500 \times 3 \times 10^{-4}$$

$$B = \frac{10000}{500} = 20 \text{ Tesla}$$

- 24.** A block starts moving up an inclined plane of inclination  $30^\circ$  with an initial velocity of  $v_0$ . It comes back to its initial position with velocity  $\frac{v_0}{2}$ . The value of the coefficient of kinetic friction between the block and the inclined plane is close to  $\frac{I}{1000}$ . The nearest integer to I is \_\_\_\_\_.

**Sol. 346**



$$a = g \sin 30 + \mu g \cos 30$$

$$V_0^2 = 2ad$$

$$d = \frac{V_0^2}{2a}$$

$$W_f = k_f - k_i$$

$$-2\mu mg \cos 30 \frac{V_0^2}{2a} = \frac{1}{2} m \frac{V_0^2}{4} - \frac{1}{2} m V_0^2$$

$$\frac{+2\mu g \cos 30}{a} = + \frac{3}{4}$$

$$8\mu g \cos 30 = 3g \sin 30 + 3\mu g \cos 30$$

$$5\mu g \cos 30 = 3g \sin 30$$

$$\mu = \frac{3 \tan 30}{5} = \frac{\sqrt{3}}{5}$$

$$\frac{\sqrt{3}}{5} = \frac{I}{1000}$$

$$I = 346$$

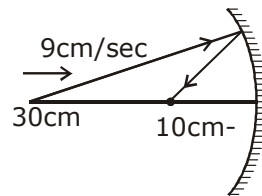
- 25.** When an object is kept at a distance of 30 cm from a concave mirror, the image is formed at a distance of 10 cm from the mirror. If the object is moved with a speed of  $9 \text{ cm s}^{-1}$ , the speed (in  $\text{cm s}^{-1}$ ) with which image moves at that instant is \_\_\_\_\_.

**Sol. 1**

$$V_I = -\frac{V^2}{u^2} V_0$$

$$V_I = -\frac{10 \times 10}{30 \times 30} \times 9$$

$$V_I = 1 \text{ cm / sec}$$



## CHEMISTRY \_ 3 Sep. \_ SHIFT - 2

1. The five successive ionization enthalpies of an element are 800, 2427, 3658, 25024 and 32824 kJ mol<sup>-1</sup>. The number of valence electrons in the element is:

(1) 2                                      (2) 4                                      (3) 3                                      (4) 5

**Sol. 3**

Fourth & Fifth I.E. are very high (periodic properties) indicating presence of three valence shell electrons

2. The incorrect statement is:

(1) Manganate and permanganate ions are tetrahedral  
 (2) In manganate and permanganate ions, the  $\pi$ -bonding takes place by overlap of p-orbitals of oxygen and d-orbitals of manganese  
 (3) Manganate and permanganate ions are paramagnetic  
 (4) Manganate ion is green in colour and permanganate ion is purple in colour

**Sol. 3**

$\text{MnO}_4^{+7-}$                        $d^0 \rightarrow$  diamagnetic

$\text{MnO}_4^{+6-2-}$                        $d^1 \rightarrow$  Paramagnetic

3. Match the following drugs with their therapeutic actions:

(i) Ranitidine	(a) Antidepressant
(ii) Nardil (Phenelzine)	(b) Antibiotic
(iii) Chloramphenicol	(c) Antihistamine
(iv) Dimetane (Brompheniramine)	(d) Antacid
	(e) Analgesic

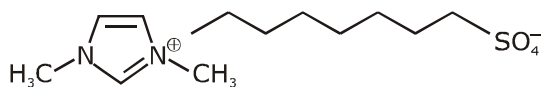
(1) (i)-(d); (ii)-(a); (iii)-(b); (iv)-(c)                      (2) (i)-(d); (ii)-(c); (iii)-(a); (iv)-(e)  
 (3) (i)-(a); (ii)-(c); (iii)-(b); (iv)-(e)                      (4) (i)-(e); (ii)-(a); (iii)-(c); (iv)-(d)

**Sol. 1**

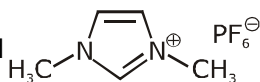
4. An ionic micelle is formed on the addition of:

(1) liquid diethyl ether to aqueous NaCl solution  
 (2) sodium stearate to pure toluene

(3) excess water to liquid



(4) excess water to liquid



**Sol. 3**

ionic micelles formed by addition of water to soap {sodium stearate}

Ans. (3)

5. Among the statements (I-IV), the correct ones are:

(I) Be has smaller atomic radius compared to Mg.  
 (II) Be has higher ionization enthalpy than Al.  
 (III) Charge/radius ratio of Be is greater than that of Al.  
 (IV) Both Be and Al form mainly covalent compounds.

(1) (I), (II) and (IV)                                      (2) (I), (II) and (III)  
 (3) (II), (III) and (IV)                                      (4) (I), (III) and (IV)

**Sol. 1**

Refer S-Block

6. Complex A has a composition of  $H_{12}O_6Cl_3Cr$ . If the complex on treatment with conc.  $H_2SO_4$  loses 13.5% of its original mass, the correct molecular formula of A is:

[Given: atomic mass of Cr = 52 amu and Cl = 35 amu]

- (1)  $[Cr(H_2O)_5Cl]Cl_2 \cdot H_2O$  (2)  $[Cr(H_2O)_4Cl_2]Cl \cdot 2H_2O$   
 (3)  $[Cr(H_2O)_3Cl_3] \cdot 3H_2O$  (4)  $[Cr(H_2O)_6]Cl_3$

**Sol. 2**

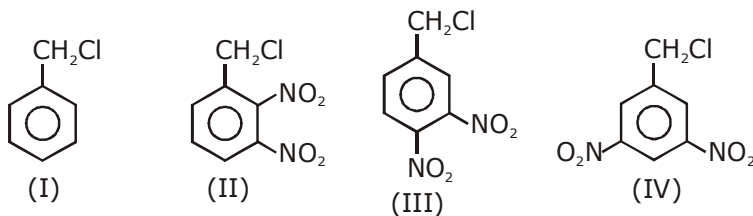
Let x molecule of water are lost then

$$13.5 = \left[ \frac{x \times 18}{6 \times 18 + 3 \times 35 + 52} \right] \times 100$$

$$x = 1.99 \approx 2$$

so, complex is  $[Cr(H_2O)_4Cl_2] \cdot 2H_2O$

7. The decreasing order of reactivity of the following compounds towards nucleophilic substitution ( $S_N2$ ) is:



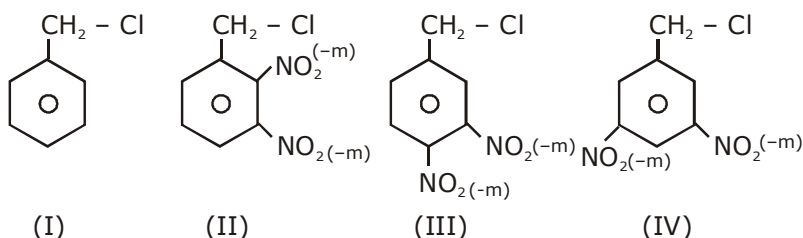
- (1) (III) > (II) > (IV) > (I)

- (2) (IV) > (II) > (III) > (I)

- (3) (II) > (III) > (IV) > (I)

- (4) (II) > (III) > (I) > (IV)

**Sol. 3**



8. The increasing order of the reactivity of the following compounds in nucleophilic addition reaction is:  
 Propanal, Benzaldehyde, Propanone, Butanone

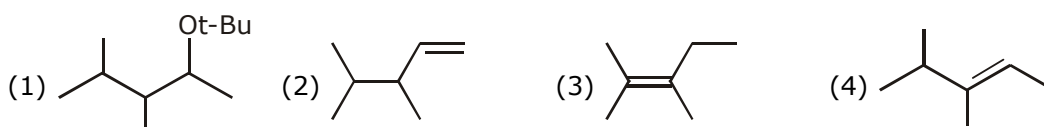
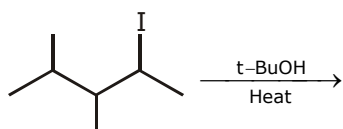
- (1) Benzaldehyde < Propanal < Propanone < Butanone  
 (2) Propanal < Propanone < Butanone < Benzaldehyde  
 (3) Butanone < Propanone < Benzaldehyde < Propanal  
 (4) Benzaldehyde < Butanone < Propanone < Propanal

**Sol. 3**

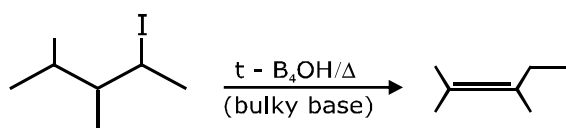
Rate of Nucleophilic addition  $\Rightarrow$  Aldehyde > Ketone

Aliphatic aldehyde > Aromatic aldehyde

9. The major product in the following reaction is:



Sol. 3



10. The incorrect statement(s) among (a) – (d) regarding acid rain is (are):

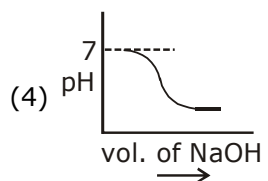
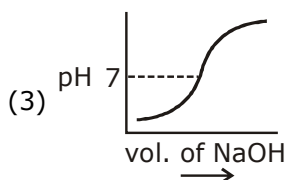
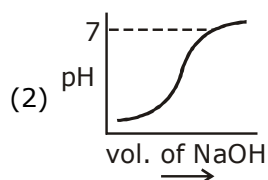
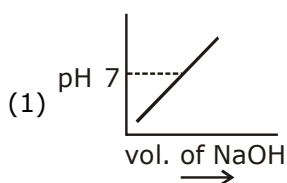
- (a) It can corrode water pipes.  
 (b) It can damage structures made up of stone.  
 (c) It cannot cause respiratory ailments in animals  
 (d) It is not harmful for trees

- (1) (a), (b) and (d) (2) (a), (c) and (d) (3) (c) and (d) (4) (c) only

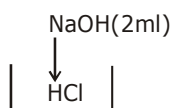
Sol. 3

Acid rain can cause respiratory ailments in animals and also harmful for trees and plant.

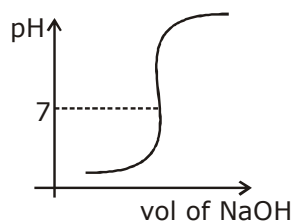
11. 100 mL of 0.1 M HCl is taken in a beaker and to it 100 mL of 0.1 M NaOH is added in steps of 2 mL and the pH is continuously measured. Which of the following graphs correctly depicts the change in pH?



**Sol. 3**



initially pH will be acidic  $< 7$   
at eq pH  $\text{pH} = 7$   
& finally pH will be basic  $> 7$



option (3)

- 12.** Consider the hypothetical situation where the azimuthal quantum number,  $l$ , takes values 0, 1, 2, .....  $n + 1$ , where  $n$  is the principal quantum number. Then, the element with atomic number:
- (1) 13 has a half-filled valence subshell      (2) 9 is the first alkali metal  
(3) 8 is the first noble gas      (4) 6 has a 2p-valence subshell

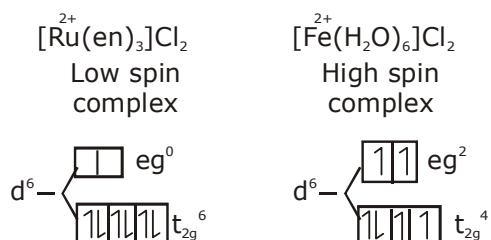
**Sol. 1**

- (1)  ${}_{13}\text{X} = 1s^2 1p^6 1d^5$  - half filled  
(2)  ${}_{9}\text{X} = 1s^2 1p^6 1d^1$  - not alkali metal  
(3)  ${}_{8}\text{X} = 1s^2 1p^6$  - Second noble gas  
Option (1)

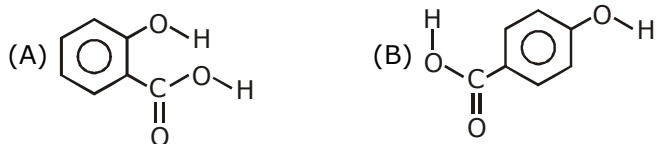
- 13.** The d-electron configuration of  $[\text{Ru}(\text{en})_3]\text{Cl}_2$  and  $[\text{Fe}(\text{H}_2\text{O})_6]\text{Cl}_2$ , respectively are:

- (1)  $t_{2g}^4 e_g^2$  and  $t_{2g}^6 e_g^0$       (2)  $t_{2g}^6 e_g^0$  and  $t_{2g}^6 e_g^0$   
(3)  $t_{2g}^4 e_g^2$  and  $t_{2g}^4 e_g^2$       (4)  $t_{2g}^6 e_g^0$  and  $t_{2g}^4 e_g^2$

**Sol. 4**



14. Consider the following molecules and statements related to them:



- (a) (B) is more likely to be crystalline than (A)  
 (b) (B) has higher boiling point than (A)  
 (c) (B) dissolves more readily than (A) in water  
 Identify the correct option from below:

- (1) (a) and (c) are true (2) only (a) is true  
 (3) (b) and (c) are true (4) (a) and (b) are true

Sol.

Bonus

All answer are correct

15. The strengths of 5.6 volume hydrogen peroxide (of density 1 g/mL) in terms of mass percentage and molarity (M), respectively, are:

(Take molar mass of hydrogen peroxide as 34 g/mol)

- (1) 0.85 and 0.5 (2) 0.85 and 0.25  
 (3) 1.7 and 0.25 (4) 1.7 and 0.5

Sol.

4

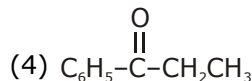
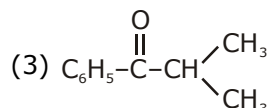
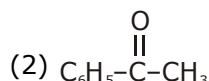
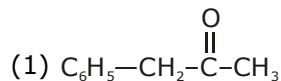
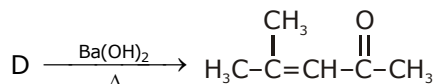
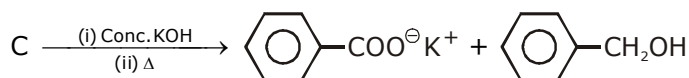
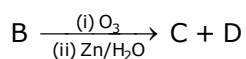
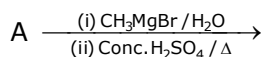
Volume strength = 5.6V

$$\text{molarity} = \frac{5.6}{11.2} = 0.5 \text{ mol/l}$$

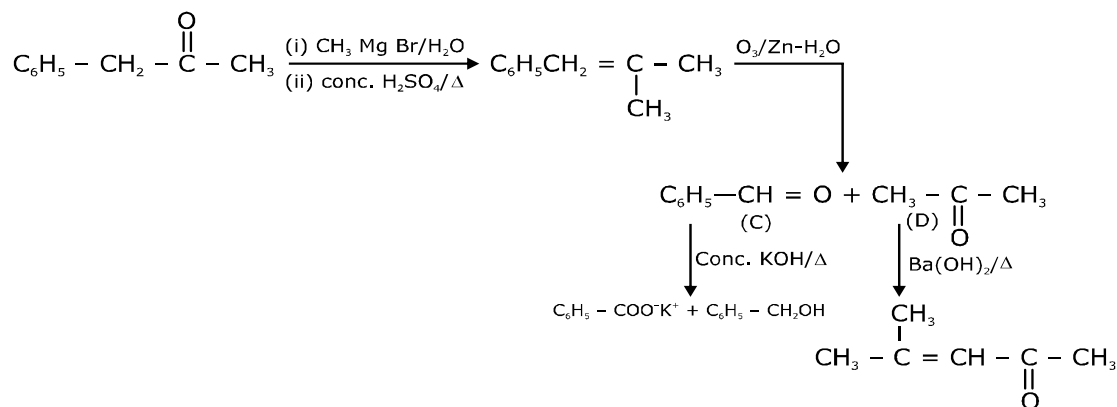
$$\text{mass \%} = \left[ \frac{0.5 \times 34}{10} \right] \times \frac{1}{1 \text{ g/ml}} = 1.7 \%$$

Ans. 1.7 & 0.5 option (4)

16. The compound A in the following reactions is:



**Sol. 1**



- 17.** A mixture of one mole each of  $\text{H}_2$ , He and  $\text{O}_2$  each are enclosed in a cylinder of volume V at temperature T. If the partial pressure of  $\text{H}_2$  is 2 atm, the total pressure of the gases in the cylinder is:  
 (1) 6 atm (2) 14 atm (3) 38 atm (4) 22 atm

**Sol. 1**

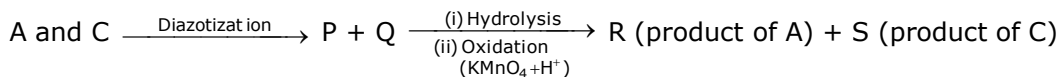
$$p_{\text{H}_2} = 2 \text{ atm} = x_{\text{H}_2} \times p_{\text{total}}$$

$$2 \text{ atm} = \frac{1}{1+1+1} \times P_{\text{total}}$$

$$P_{\text{total}} = 6 \text{ atm}$$

Ans. option (1)

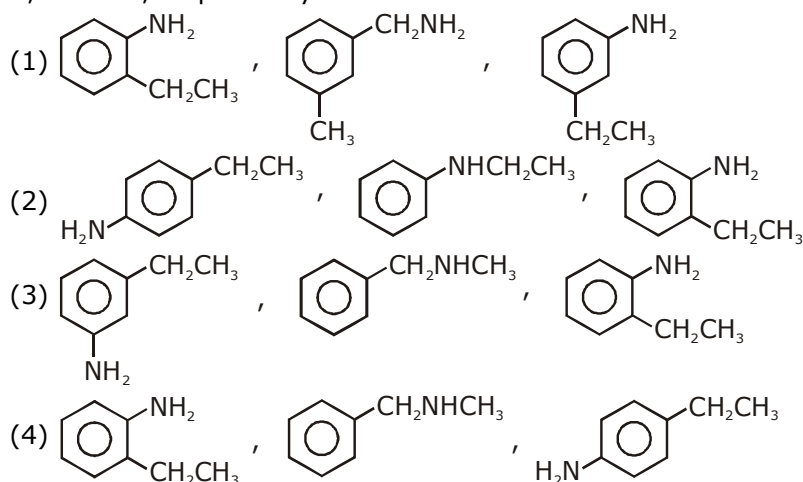
- 18.** Three isomers A, B and C (mol. formula  $\text{C}_8\text{H}_{11}\text{N}$ ) give the following results:



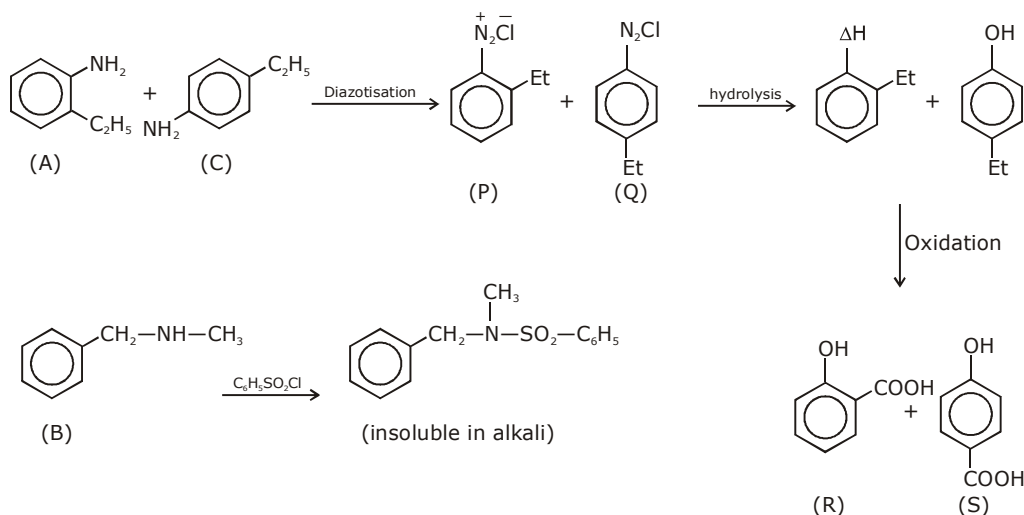
R has lower boiling point than S



A, B and C, respectively are:



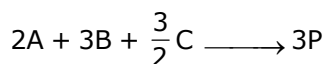
**Sol. 2**



**19.** For the reaction  $2A + 3B + \frac{3}{2}C \rightarrow 3P$ , which statement is correct?

- (1)  $\frac{dn_A}{dt} = \frac{dn_B}{dt} = \frac{dn_C}{dt}$       (2)  $\frac{dn_A}{dt} = \frac{3}{2} \frac{dn_B}{dt} = \frac{3}{4} \frac{dn_C}{dt}$   
 (3)  $\frac{dn_A}{dt} = \frac{2}{3} \frac{dn_B}{dt} = \frac{4}{3} \frac{dn_C}{dt}$       (4)  $\frac{dn_A}{dt} = \frac{2}{3} \frac{dn_B}{dt} = \frac{3}{4} \frac{dn_C}{dt}$

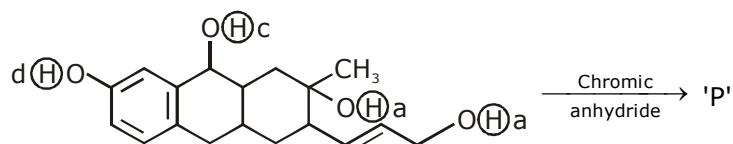
**Sol. 3**



$$\text{ROR} = \frac{1}{2} \left[ \frac{-dn_A}{dt} \right] = \frac{1}{3} \left[ \frac{-dn_B}{dt} \right] = \frac{2}{3} \left[ \frac{-dn_C}{dt} \right] = \frac{1}{3} \left[ \frac{+dn_P}{dt} \right]$$

$$\left[ \frac{-dn_A}{dt} \right] = \frac{2}{3} \left[ \frac{-dn_B}{dt} \right] = \frac{4}{3} \left[ \frac{-dn_C}{dt} \right]$$

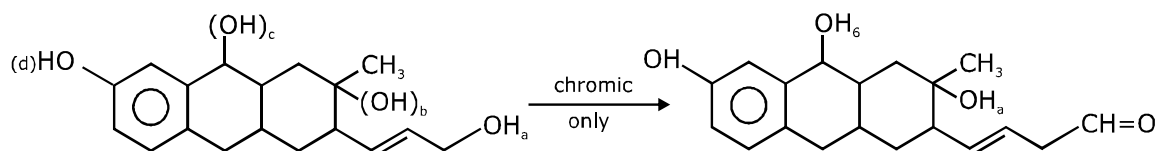
**20.** Consider the following reaction:



The product 'P' gives positive ceric ammonium nitrate test. This is because of the presence of which of these -OH group(s)?

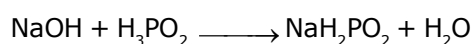
- (1) (b) only      (2) (b) and (d)      (3) (c) and (d)      (4) (d) only

**Sol. 1**



**21.** The volume (in mL) of 0.1 N NaOH required to neutralise 10 mL of 0.1 N phosphinic acid is \_\_\_\_\_.

**Sol. 10 ml**

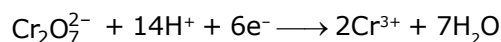


Phosphinic

$$\text{Vol.} \times 0.1 = 0.1 \times 10$$

$$\text{vol} = 10 \text{ ml Ans.}$$

**22.** An acidic solution of dichromate is electrolyzed for 8 minutes using 2A current. As per the following equation



The amount of  $\text{Cr}^{3+}$  obtained was 0.104 g. The efficiency of the process (in %) is (Take:  $F = 96000$  C, At. mass of chromium = 52) \_\_\_\_\_.

**Sol. 60 %**

$$[\text{moles of Cr}^{3+}] \times 3 = \frac{8 \times 60 \times 2}{96000}$$

$$\text{moles of Cr}^{3+} = \frac{8 \times 4}{9600} = \frac{1}{300} \text{ mol}$$

$$\text{mass of Cr}^{3+} = \frac{52}{300} \text{ g}$$

$$\% \text{ efficiency} = \frac{\text{Actual obtained Amt}}{\text{Theo. obtained Amt}} \times 100$$

$$= \frac{0.104}{\frac{52}{300}} \times 100 = 30 \times \frac{104}{52} = 60\%$$

**23.** If 250 cm<sup>3</sup> of an aqueous solution containing 0.73 g of a protein A is isotonic with one litre of another aqueous solution containing 1.65 g of a protein B, at 298 K, the ratio of the molecular masses of A and B is \_\_\_\_\_  $\times 10^{-2}$  (to the nearest integer).

**Sol. 177**

$$\frac{0.73}{M_A} \times \frac{1000}{250} = \frac{1.65}{M_B}$$

$$\frac{M_A}{M_B} = \frac{73 \times 4}{165} = 1.769$$

$$= 176.9 \times 10^{-2}$$

$$= 177 \times 10^{-2}$$

- 24.**  $6.023 \times 10^{22}$  molecules are present in 10 g of a substance 'x'. The molarity of a solution containing 5 g of substance 'x' in 2 L solution is \_\_\_\_\_  $\times 10^{-3}$ .

**Sol.** **25**

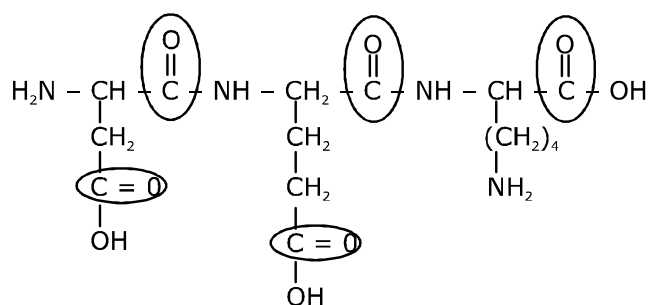
$$\text{Mol. wt of 'x'} = \frac{10}{6.023 \times 10^{22}} \times 6.023 \times 10^{23} \\ = 100 \text{ g/mol}$$

$$M = \frac{5/100}{2} = \left( \frac{5}{200} \times 1000 \right) \times 10^{-3}$$

$$M = 25 \times 10^{-3} \text{ mol/lit}$$

- 25.** The number of  $\text{>C=O}$  groups present in a tripeptide Asp-Glu-Lys is \_\_\_\_\_.

**Sol.** **5**



# QUESTION PAPER WITH SOLUTION

## MATHEMATICS \_ 3 Sep. \_ SHIFT - 2

**Q.1** If  $x^3 dy + xy dx = x^2 dy + 2y dx$ ;  $y(2) = e$  and  $x > 1$ , then  $y(4)$  is equal to:

- (1)  $\frac{\sqrt{e}}{2}$                       (2)  $\frac{3}{2}\sqrt{e}$                       (3)  $\frac{1}{2} + \sqrt{e}$                       (4)  $\frac{3}{2} + \sqrt{e}$

**Sol.**

$$(x^3 - x^2)dy = (2 - x)ydx$$

$$\int \frac{dy}{y} = \int \frac{2 - x}{x^2(x - 1)} dx$$

$$\int \frac{dy}{y} = -\int \frac{x - 1 - 1}{x^2(x - 1)} dx$$

$$\int \frac{dy}{y} = -\int \frac{dx}{x^2} = \int \frac{x^2 - 1 - x^2}{x^2(x - 1)}$$

$$= \frac{1}{x} - \int \frac{x + 1}{x^2} dx + \int \frac{dx}{x - 1}$$

$$\ln|y| = \frac{2}{x} - \ln|x| + \ln|x - 1| + c$$

$$x = 2, y = e$$

$$1 = 1 - \ln 2 + c \Rightarrow c = \ln 2$$

$$\ln|y| = \frac{2}{x} - \ln|x| + \ln|x - 1| + \ln 2$$

$$\text{put } x = 4$$

$$\ln|y| = \frac{1}{2} - 2\ln 2 + \ln 3 + \ln 2$$

$$\ln y = \ln\left(\frac{3}{2}\right) + \frac{1}{2}$$

$$y = \frac{3}{2} \cdot e^{\frac{1}{2}} = \frac{3}{2}\sqrt{e}$$

**Q.2** Let A be a  $3 \times 3$  matrix such that  $\text{adj } A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{bmatrix}$  and  $B = \text{adj}(\text{adj } A)$ .

If  $|A| = \lambda$  and  $|(B^{-1})^T| = \mu$ , then the ordered pair,  $(|\lambda|, \mu)$  is equal to:

- (1)  $\left(9, \frac{1}{81}\right)$       (2)  $\left(9, \frac{1}{9}\right)$       (3)  $\left(3, \frac{1}{81}\right)$       (4)  $(3, 81)$

**Sol. 3**

$$\text{adj } A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{bmatrix} \Rightarrow |\text{adj } A| = 9$$

$$\Rightarrow |A|^2 = 9 \Rightarrow |A| = 3 = |\lambda|$$

$$B = \text{adj}(\text{adj } A) = |A| \cdot A = 3A$$

$$|(B^T)^{-1}| = \frac{1}{|B^T|} = \frac{1}{|B|} = \frac{1}{|3A|} = \frac{1}{27 \times 3} = \frac{1}{81} = \mu$$

$$|\lambda|, \mu = \left(3, \frac{1}{81}\right)$$

**Q.3** Let  $a, b, c \in \mathbb{R}$  be such that  $a^2 + b^2 + c^2 = 1$ , If  $a \cos \theta = b \cos \left(\theta + \frac{2\pi}{3}\right) = c \cos \left(\theta + \frac{4\pi}{3}\right)$ , where  $\theta = \frac{\pi}{9}$ , then the angle between the vectors  $a\hat{i} + b\hat{j} + c\hat{k}$  and  $b\hat{i} + c\hat{j} + a\hat{k}$  is

- (1)  $\frac{\pi}{2}$       (2)  $\frac{2\pi}{3}$       (3)  $\frac{\pi}{9}$       (4) 0

**Sol. 1**

$$\cos \alpha = \frac{ab + bc + ca}{a^2 + b^2 + c^2}$$

$$a \cos \theta = b \cos \left(\theta + \frac{2\pi}{3}\right) = c \cos \left(\theta + \frac{4\pi}{3}\right) = \lambda$$

$$\frac{1}{a} = \frac{\cos \theta}{\lambda}, \frac{1}{b} = \frac{\cos\left(\theta + 2\frac{\pi}{3}\right)}{\lambda}, \frac{1}{c} = \frac{\cos\left(\theta + \frac{4\pi}{3}\right)}{\lambda}$$

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{\lambda} \left[ \cos \theta + \cos\left(\theta + \frac{2\pi}{3}\right) + \cos\left(\theta + \frac{4\pi}{3}\right) \right]$$

$$= \frac{1}{\lambda} \frac{\sin\left[\left(3\right)\left(\frac{\pi}{3}\right)\right]}{\sin\left(\frac{\pi}{3}\right)} \cdot \cos\left[\frac{\theta + \theta + \frac{4\pi}{3}}{2}\right]$$

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$$

$$\sum ab = 0$$

$$\cos \alpha = 0$$

$$\alpha = \frac{\pi}{2}$$

**Q.4** Suppose  $f(x)$  is a polynomial of degree four, having critical points at  $-1, 0, 1$ . If  $T = \{x \in \mathbb{R} \mid f(x) = f(0)\}$ , then the sum of squares of all the elements of  $T$  is:

(1) 6

(2) 2

(3) 8

(4) 4

**Sol.**

$$f'(x) = k(x+1)x(x-1)$$

$$f'(x) = k[x^3 - x]$$

Integrating both sides

$$f(x) = k \left[ \frac{x^4}{4} - \frac{x^2}{2} \right] + c$$

$$f(0) = c$$

$$f(x) = f(0) \Rightarrow k \left( \frac{x^4}{4} - \frac{x^2}{2} \right) + c = c$$

$$\Rightarrow k \frac{x^2}{4} (x^2 - 2) = 0$$

$$\Rightarrow x = 0, \pm \sqrt{2}$$

$$\text{sum of all of squares of elements} = 0^2 + (\sqrt{2})^2 + (-\sqrt{2})^2 = 4$$

**Q.5** If the value of the integral  $\int_0^{1/2} \frac{x^2}{(1-x^2)^{3/2}} dx$  is  $\frac{k}{6}$ , then k is equal to:

- (1)  $2\sqrt{3} + \pi$       (2)  $3\sqrt{2} + \pi$       (3)  $3\sqrt{2} - \pi$       (4)  $2\sqrt{3} - \pi$

**Sol. 4**

$$\int_0^{1/2} \frac{x^2}{(1-x^2)^{3/2}} dx$$

$$x = \sin \theta$$

$$\int_0^{\pi/6} \frac{\sin^2 \theta}{\cos^3 \theta} \cdot \cos \theta d\theta$$

$$\int_0^{\pi/6} \tan^2 \theta d\theta = [\tan \theta - \theta]_0^{\pi/6}$$

$$\Rightarrow \left( \frac{1}{\sqrt{3}} - \frac{\pi}{6} \right) = \frac{k}{6}$$

$$\frac{2\sqrt{3} - \pi}{6} = \frac{k}{6}$$

$$k = 2\sqrt{3} - \pi$$

**Q.6** If the term independent of x in the expansion of  $\left( \frac{3}{2}x^2 - \frac{1}{3x} \right)^9$  is k, then 18 k is equal to:

- (1) 5      (2) 9      (3) 7      (4) 11

**Sol. 3**

$$T_{r+1} = {}^9C_r \left( \frac{3}{2}x^2 \right)^{9-r} \left( \frac{-1}{3x} \right)^r$$

$$= {}^9C_r \frac{3^{9-2r}}{2^{9-r}} (-1)^r \cdot x^{18-3r}$$

$$18 - 3r = 0$$

$$\Rightarrow r=6$$

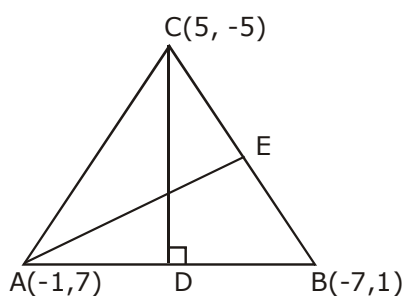
$$= {}^9C_r \left( \frac{3^{-3}}{2^3} \right) = k$$

$$= \frac{7}{18} = k \Rightarrow 18k = 7$$

7. If a  $\triangle ABC$  has vertices  $A(-1,7)$ ,  $B(-7,1)$  and  $C(5,-5)$ , then its orthocentre has coordinates:

- (1)  $(-3,3)$       (2)  $\left(-\frac{3}{5}, \frac{3}{5}\right)$       (3)  $\left(\frac{3}{5}, -\frac{3}{5}\right)$       (4)  $(3,-3)$

**Sol. 1**



equation of CD

$$y + 5 = -1(x - 5)$$

$$x + y = 0$$

.....(1)

equation of AE

$$y - 7 = 2(x + 1)$$

$$2x - y = -9$$

....(2)

from (1) & (2)

$$x = -3, y = 3$$

$$\text{Orthocentre} = (-3, 3)$$

**Q.8.** Let  $e_1$  and  $e_2$  be the eccentricities of the ellipse,  $\frac{x^2}{25} + \frac{y^2}{b^2} = 1$  ( $b < 5$ ) and the hyperbola,  $\frac{x^2}{16} - \frac{y^2}{b^2} = 1$  respectively satisfying  $e_1 e_2 = 1$ . If  $\alpha$  and  $\beta$  are the distances between the foci of the ellipse and the foci of the hyperbola respectively, then the ordered pair  $(\alpha, \beta)$  is equal to:

- (1)  $(8,12)$       (2)  $\left(\frac{24}{5}, 10\right)$       (3)  $\left(\frac{20}{3}, 12\right)$       (4)  $(8,10)$

**Sol. 4**

$$\begin{aligned} \alpha &= 10e_1 \\ \beta &= 8e_2 \end{aligned}$$

$$\begin{aligned} b^2 &= 25(1 - e_1^2) \\ b^2 &= 16(e_2^2 - 1) \end{aligned}$$

$$(e_1 e_2)^2 = 1$$

$$\left(1 - \frac{b^2}{25}\right)\left(1 + \frac{b^2}{16}\right) = 1$$

$$\Rightarrow 1 + \frac{b^2}{25} - \frac{b^2}{25} - \frac{b^4}{400} = 1$$

$$\Rightarrow \frac{9}{16.25} b^2 = \frac{b^4}{400} \Rightarrow b^2 = 9$$

$$\left[ \begin{array}{l} e_1 = \frac{4}{5} \\ e_2 = \frac{5}{4} \end{array} \right] = \left[ \begin{array}{l} \alpha = 2ae_1 = 10 \times \frac{4}{5} = 8 \\ \beta = 2ae_2 = 8 \times \frac{5}{4} = 10 \end{array} \right] = (\alpha, \beta) = (8, 10)$$

**Q.9** If  $z_1, z_2$  are complex numbers such that  $\operatorname{Re}(z_1) = |z_1 - 1|$ ,  $\operatorname{Re}(z_2) = |z_2 - 1|$  and  $\arg(z_1 - z_2) = \frac{\pi}{6}$ , then  $\operatorname{Im}(z_1 + z_2)$  is equal to:

- (1)  $2\sqrt{3}$                       (2)  $\frac{2}{\sqrt{3}}$                       (3)  $\frac{1}{\sqrt{3}}$                       (4)  $\frac{\sqrt{3}}{2}$

**Sol. 1**

$$z_1 = x_1 + iy_1, z_2 = x_2 + iy_2$$

$$x_1^2 = (x_1 - 1)^2 + y_1^2 \quad \dots(1)$$

$$\Rightarrow y_1^2 - 2x_1 + 1 = 0$$

$$x_2^2 = (x_2 - 1)^2 + y_2^2$$

$$y_2^2 - 2x_2 - 1 = 0 \quad \dots(2)$$

$$\text{from equation (1) - (2)}$$

$$(y_1^2 - y_2^2) + 2(x_2 - x_1) = 0$$

$$(y_1 + y_2)(y_1 - y_2) = 2(x_1 - x_2)$$

$$y_1 + y_2 = 2 \left( \frac{x_1 - x_2}{y_1 - y_2} \right)$$

$$\arg(z_1 - z_2) = \frac{\pi}{6}$$

$$\tan^{-1} \left( \frac{y_1 - y_2}{x_1 - x_2} \right) = \frac{\pi}{6}$$

$$\Rightarrow \frac{y_1 - y_2}{x_1 - x_2} = \frac{1}{\sqrt{3}}$$

$$\therefore y_1 + y_2 = 2\sqrt{3}$$



$\Rightarrow 2x - 6y + 4z = 4$   
passes through (4, 0, -1)

**Q.13**  $\lim_{x \rightarrow a} \frac{(a+2x)^{\frac{1}{3}} - (3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}} - (4x)^{\frac{1}{3}}} (a \neq 0)$  is equal to :

- (1)  $\left(\frac{2}{9}\right)^{\frac{4}{3}}$       (2)  $\left(\frac{2}{3}\right)^{\frac{4}{3}}$       (3)  $\left(\frac{2}{3}\right)\left(\frac{2}{9}\right)^{\frac{1}{3}}$       (4)  $\left(\frac{2}{9}\right)\left(\frac{2}{3}\right)^{\frac{1}{3}}$

**Sol. 3**  
Apply L-H Rule

$$\lim_{x \rightarrow a} \frac{\frac{2}{3}(a+2x)^{-\frac{2}{3}} - \frac{1}{3} \cdot \frac{1}{3} x^{-\frac{2}{3}}}{\frac{1}{3}(3a+x)^{-\frac{2}{3}} - \frac{1}{3} \cdot \frac{1}{3} x^{-\frac{2}{3}}}$$

$$\Rightarrow \frac{\frac{2}{3}(3a)^{-\frac{2}{3}} - \frac{1}{3} \cdot \frac{1}{3} \left(a^{-\frac{2}{3}}\right)}{\frac{1}{3}(4a)^{-\frac{2}{3}} - \frac{1}{3} \cdot \frac{1}{3} \left(a^{-\frac{2}{3}}\right)}$$

$$= \frac{2}{3} \cdot \left(\frac{2}{9}\right)^{\frac{1}{3}}$$

**Q.14** Let  $x_i (1 \leq i \leq 10)$  be ten observations of a random variable X. If  $\sum_{i=1}^{10} (x_i - p) = 3$  and  $\sum_{i=1}^{10} (x_i - p)^2 = 9$

where  $0 \neq p \in \mathbb{R}$ , then the standard deviation of these observations is :

- (1)  $\frac{7}{10}$       (2)  $\frac{9}{10}$       (3)  $\sqrt{\frac{3}{5}}$       (4)  $\frac{4}{5}$

**Sol. 2**  
Standard deviation  
is free from shifting  
of origin

$$S.D = \sqrt{\text{variance}}$$

$$= \sqrt{\frac{9}{10} - \left(\frac{3}{10}\right)^2}$$

$$= \sqrt{\frac{9}{10} - \frac{9}{100}}$$

$$= \sqrt{\frac{81}{100}} = \frac{9}{10}$$

**Q.15** The probability that a randomly chosen 5-digit number is made from exactly two digits is :

- (1)  $\frac{134}{10^4}$                       (2)  $\frac{121}{10^4}$                       (3)  $\frac{135}{10^4}$                       (4)  $\frac{150}{10^4}$

**Sol. 3**

$$\begin{aligned} \text{Total case} &= 9(10^4) \\ \text{fav. case} &= {}^9C_2 (2^5 - 2) + {}^9C_1 (2^4 - 1) \\ &= 1080 + 135 = 1215 \end{aligned}$$

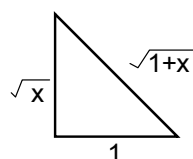
$$\text{Prob} = \frac{1215}{9 \times 10^4} = \frac{135}{10^4}$$

**Q.16** If  $\int \sin^{-1} \left( \sqrt{\frac{x}{1+x}} \right) dx = A(x) \tan^{-1}(\sqrt{x}) + B(x) + C$ , where C is a constant of integration, then the ordered pair (A(x), B(x)) can be:

- (1)  $(x+1, -\sqrt{x})$                       (2)  $(x-1, -\sqrt{x})$                       (3)  $(x+1, \sqrt{x})$                       (4)  $(x-1, \sqrt{x})$

**Sol. 1**

$$\int \sin^{-1} \sqrt{\frac{x}{1+x}} dx$$



$$\int \tan^{-1} \sqrt{x} \cdot \frac{1}{2\sqrt{x}} dx$$

$$\left( \tan^{-1} \sqrt{x} \right) \cdot x - \int \frac{x}{1+x} \cdot \frac{1}{2\sqrt{x}} dx$$

$$\text{put } x = t^2 \Rightarrow dx = 2t dt$$

$$\begin{aligned}
 &= x \tan^{-1} \sqrt{x} - \int \frac{(t^2)(2t dt)}{(1+t^2)(2t)} \\
 &= x \tan^{-1} \sqrt{x} - t + \tan^{-1} t + c \\
 &= x \tan^{-1} \sqrt{x} - \sqrt{x} + \tan^{-1} \sqrt{x} + c \\
 A(x) &= x + 1, B(x) = -\sqrt{x}
 \end{aligned}$$

**Q.17** If the sum of the series  $20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} + \dots$  upto  $n^{\text{th}}$  term is 488 and the  $n^{\text{th}}$  term is negative, then:

- (1)  $n=60$                       (2)  $n=41$                       (3)  $n^{\text{th}}$  term is  $-4$                       (4)  $n^{\text{th}}$  term is  $-4\frac{2}{5}$

**Sol. 3**

$$20 + \frac{98}{5} + \frac{96}{5} + \dots$$

$$S_n = 488$$

$$\Rightarrow \frac{n}{2} \left[ 2 \times 20 + (n-1) \left( \frac{-2}{5} \right) \right] = 488$$

$$\Rightarrow 20n - \frac{n^2}{5} + \frac{n}{5} = 488$$

$$\Rightarrow 100n - n^2 + n = 2440$$

$$= n^2 - 101n + 2440 = 0$$

$$\Rightarrow n = 61 \text{ or } 40$$

$$\text{for } n = 40, T_n = 20 + 39 \left( \frac{-2}{5} \right) = +ve$$

$$n = 61, T_n = 20 + 60 \left( \frac{-2}{5} \right) = 20 - 24 = -4$$

**Q.18** Let  $p, q, r$  be three statements such that the truth value of  $(p \wedge q) \rightarrow (\sim p \vee r)$  is F. Then the truth values of  $p, q, r$  are respectively :

- (1) F, T, F                      (2) T, F, T                      (3) T, T, F                      (4) T, T, T

**Sol.** **3**

$$(p \wedge q) \rightarrow (\sim p \vee r)$$

Possible when

$$p \wedge q \rightarrow T$$

$$\sim p \vee r \rightarrow F$$

$$p \rightarrow T$$

$$q \rightarrow T$$

$$r \rightarrow F$$

$$p \wedge q \Rightarrow T$$

$$\sim p \vee r \rightarrow F \vee F \Rightarrow F$$

$$T \rightarrow F \Rightarrow F$$

**Q.19** If the surface area of a cube is increasing at a rate of  $3.6 \text{ cm}^2/\text{sec}$ , retaining its shape; then the rate of change of its volume (in  $\text{cm}^3/\text{sec}$ ), when the length of a side of the cube is  $10 \text{ cm}$ , is :

- (1) 9                      (2) 10                      (3) 18                      (4) 20

**Sol.** **1**

$$A = 6a^2$$

$a \rightarrow$  side of cube

$$\frac{dA}{dt} = 6 \left( 2a \frac{da}{dt} \right) \Rightarrow 3.6 = 12 \times 10 \frac{da}{dt} \Rightarrow \frac{da}{dt} = \frac{3}{100}$$

$$v = a^3$$

$$\frac{dV}{dt} = 3a^2 \frac{da}{dt}$$

$$= 3 \times 100 \times \frac{3}{100}$$

$$= 9 \text{ cm}^3 / \text{sec}$$

**Q.20** Let  $R_1$  and  $R_2$  be two relations defined as follows:

$$R_1 = \{(a, b) \in \mathbb{R}^2 : a^2 + b^2 \in \mathbb{Q}\} \text{ and}$$

$$R_2 = \{(a, b) \in \mathbb{R}^2 : a^2 + b^2 \notin \mathbb{Q}\}, \text{ where } \mathbb{Q} \text{ is the set of all rational numbers. Then :}$$

(1)  $R_1$  is transitive but  $R_2$  is not transitive

(2)  $R_1$  and  $R_2$  are both transitive

(3)  $R_2$  is transitive but  $R_1$  is not transitive

(4) Neither  $R_1$  nor  $R_2$  is transitive

**Sol.** **4**

for  $R_1$

$$\text{Let } a = 1 + \sqrt{2}, b = 1 - \sqrt{2}, c = \frac{1}{8^4}$$

$$aR_1b \quad a^2 + b^2 = (1 + \sqrt{2})^2 + (1 - \sqrt{2})^2 = 6 \in \mathbb{Q}$$

$$bR_1c \quad b^2 + c^2 = (1 - \sqrt{2})^2 + \left(8^{\frac{1}{4}}\right)^2 = 3 \in Q$$

$$aR_1c \Rightarrow a^2 + c^2 = (1 + \sqrt{2})^2 + (8^{\frac{1}{4}})^2 = 3 + 4\sqrt{2} \notin Q$$

$R_1$  is not transitive

$R_2$

$$\text{let } a = 1 + \sqrt{2}, b = \sqrt{2}, c = 1 - \sqrt{2}$$

$$aR_2b \quad a^2 + b^2 = 5 + 2\sqrt{2} \notin Q$$

$$bR_2c \quad b^2 + c^2 = 5 - 2\sqrt{2} \notin Q$$

$$aR_2c \quad a^2 + c^2 = 6 \in Q$$

$R_2$  is not transitive

**Q.21** If  $m$  arithmetic means (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4<sup>th</sup> A.M. is equal to 2<sup>nd</sup> G.M., then  $m$  is equal to \_\_\_\_

**Sol.** **39**

3, ....., 243  
m A.M.

$$d = \frac{b - a}{n + 1} = \frac{243 - 3}{m + 1} = \frac{240}{m + 1}$$

$$4^{\text{th}} \text{ A.M} = 3 + 4d = 3 + 4\left(\frac{240}{m + 1}\right)$$

$$3 + \frac{960}{m + 1} = 27$$

$$= \frac{960}{m + 1} = 24$$

$$\Rightarrow m = 39$$

3, ....., 243  
3 G.M

$$243 = 3(r)^4$$

$$r = 3$$

$$2^{\text{nd}} \text{ G.M.} = ar^2 = 27$$

**Q.22** Let a plane  $P$  contain two lines  $\vec{r} = \hat{i} + \lambda(\hat{i} + \hat{j}), \lambda \in \mathbb{R}$  and  $\vec{r} = -\hat{j} + \mu(\hat{j} - \hat{k}), \mu \in \mathbb{R}$ . If  $Q(\alpha, \beta, \gamma)$  is the foot of the perpendicular drawn from the point  $M(1, 0, 1)$  to  $P$ , then  $3(\alpha + \beta + \gamma)$  equals \_\_\_\_

**Sol.** **5**

$$\left[ \begin{array}{l} \vec{r} = \hat{i} + \lambda(\hat{i} + \hat{j}) \\ \vec{r} = -\hat{j} + \mu(\hat{j} - \hat{k}) \end{array} \right]$$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix}$$

$$= (-1, 1, 1)$$

equation of plane

$$-1(x - 1) + 1(y - 0) + 1(z - 0) = 0$$

$$\Rightarrow x - y - z - 1 = 0$$

foot of  $\perp$  from  $m(1, 0, 1)$

$$\frac{x-1}{1} = \frac{y-0}{-1} = \frac{z-1}{-1} = -\frac{(1-0-1-1)}{3}$$

$$x-1 = \frac{1}{3} \quad \left| \frac{y}{-1} = \frac{1}{3} \right| = \frac{z-1}{-1} = \frac{1}{3}$$

$$x = \frac{4}{3}, y = \frac{-1}{3}, z = \frac{2}{3}$$

$$\Rightarrow \left[ \begin{array}{l} \alpha = \frac{4}{3} \\ \beta = \frac{-1}{3} \\ \gamma = \frac{2}{3} \end{array} \right]$$

$$\alpha + \beta + \gamma = \frac{4}{3} - \frac{1}{3} + \frac{2}{3} = \frac{5}{3}$$

$$3(\alpha + \beta + \gamma) = 5$$

**Q.23** Let S be the set of all integer solutions,  $(x, y, z)$ , of the system of equations

$$x - 2y + 5z = 0$$

$$-2x + 4y + z = 0$$

$$-7x + 14y + 9z = 0$$

such that  $15 \leq x^2 + y^2 + z^2 \leq 150$ . Then, the number of elements in the set S is equal to \_\_\_\_

**Sol. 8**

$$x - 2y + 5z = 0 \quad \dots(1)$$

$$-2x + 4y + z = 0 \quad \dots(2)$$

$$-7x + 14y + 9z = 0 \quad \dots(3)$$

$$2.(1) + (2) \text{ we get } z = 0, x = 2y$$

$$15 \leq 4y^2 + y^2 \leq 150$$

$$\Rightarrow 3 \leq y^2 \leq 30$$

$$y \in [-\sqrt{30}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{30}]$$

$$y = \pm 2, \pm 3, \pm 4, \pm 5$$

no. of integer's in S is 8

**Q.24** The total number of 3-digit numbers, whose sum of digits is 10, is \_\_\_\_\_

**Sol.** **54**

Let xyz be 3 digit number

$$x + y + z = 10 \text{ where } x \geq 1, y \geq 0, z \geq 0$$

$$\Rightarrow t + y + z = 9$$

$$\left. \begin{array}{l} x - 1 \geq 0 \\ t \geq 0 \end{array} \right\} x - 1 = t$$

$${}^{9+3-1}C_{3-1} = {}^{11}C_2 = 55$$

but for  $t = 9, x = 10$  not possible

$$\text{total numbers} = 55 - 1 = 54$$

**Q.25** If the tangent to the curve,  $y=e^x$  at a point  $(c, e^c)$  and the normal to the parabola,  $y^2=4x$  at the point  $(1,2)$  intersect at the same point on the x-axis, then the value of c is \_\_\_\_\_

**Sol.** **4**

$$\text{Tangent at } (c, e^c) \quad y - e^c = e^c (x - c) \quad \dots(1)$$

$$\text{normal to parabola } y - 2 = -1 (x - 1)$$

$$x + y = 3$$

$$\text{at x-axis } y = 0$$

$$\text{in (1), } x = c - 1$$

$$c - 1 = 3 \Rightarrow c = 4$$

$$\dots(2)$$

$$\text{at x-axis } y = 0$$

$$\text{in (2), } x = 3$$

## PHYSICS \_ 4 Sep. \_ SHIFT - 1

- Q.1** Starting from the origin at time  $t = 0$ , with initial velocity  $5\hat{j}\text{ms}^{-1}$ , a particle moves in the x-y plane with a constant acceleration of  $(10\hat{i} + 4\hat{j})\text{ms}^{-2}$ . At time  $t$ , its coordinates are  $(20\text{ m}, y_0\text{ m})$ . The values of  $t$  and  $y_0$  are, respectively:  
 (1) 5s and 25 m      (2) 2s and 18 m      (3) 2s and 24 m      (4) 4s and 52 m

**Sol. 2**

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$y = 5t + \frac{1}{2} (4) t^2$$

$$y = 5t + 2t^2$$

$$\text{and } x = 0 + \frac{1}{2} (10) (t^2) = 20$$

$$t = 2\text{ s}$$

$$\Rightarrow y = 10 + 8 = 18\text{m}$$

- Q.2** A small bar magnet placed with its axis at  $30^\circ$  with an external field of 0.06 T experiences a torque of 0.018 Nm. The minimum work required to rotate it from its stable to unstable equilibrium position is:

- (1)  $7.2 \times 10^{-2}\text{ J}$       (2)  $6.4 \times 10^{-2}\text{ J}$       (3)  $9.2 \times 10^{-3}\text{ J}$       (4)  $11.7 \times 10^{-3}\text{ J}$

**Sol. 1**

$$\tau = MB \sin 30^\circ$$

$$0.018 = MB \left( \frac{1}{2} \right)$$

$$MB = 0.036$$

$$w = \Delta U = 2MB = 0.072\text{ J}$$

- Q.3** Choose the correct option relating wave lengths of different parts of electromagnetic wave spectrum:

- (1)  $\lambda_{\text{radio waves}} > \lambda_{\text{micro waves}} > \lambda_{\text{visible}} > \lambda_{\text{x-rays}}$       (2)  $\lambda_{\text{visible}} > \lambda_{\text{x-rays}} > \lambda_{\text{radio waves}} > \lambda_{\text{micro waves}}$   
 (3)  $\lambda_{\text{visible}} < \lambda_{\text{micro waves}} < \lambda_{\text{radio waves}} < \lambda_{\text{x-rays}}$       (4)  $\lambda_{\text{x-rays}} < \lambda_{\text{micro waves}} < \lambda_{\text{radio waves}} < \lambda_{\text{visible}}$

**Sol. 1**

By property of electromagnetic wave spectrum.

- Q.4** On the x-axis and at a distance  $x$  from the origin, the gravitational field due a mass distribution is given by  $\frac{Ax}{(x^2 + a^2)^{3/2}}$  in the x-direction. The magnitude of gravitational potential on the x-axis at a distance  $x$ , taking its value to be zero at infinity, is:

- (1)  $A(x^2 + a^2)^{3/2}$       (2)  $\frac{A}{(x^2 + a^2)^{1/2}}$       (3)  $A(x^2 + a^2)^{1/2}$       (4)  $\frac{A}{x(x^2 + a^2)^{3/2}}$

**Sol. 2**

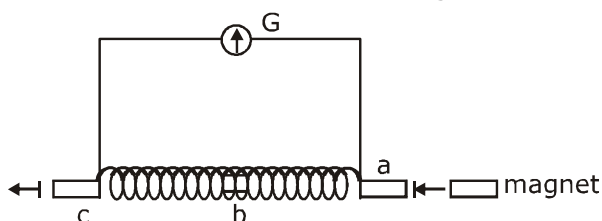
$$E_x = \frac{Ax}{(x^2 + a^2)^{3/2}}$$

$$\frac{-dv}{dx} = \frac{Ax}{(x^2 + a^2)^{3/2}}$$

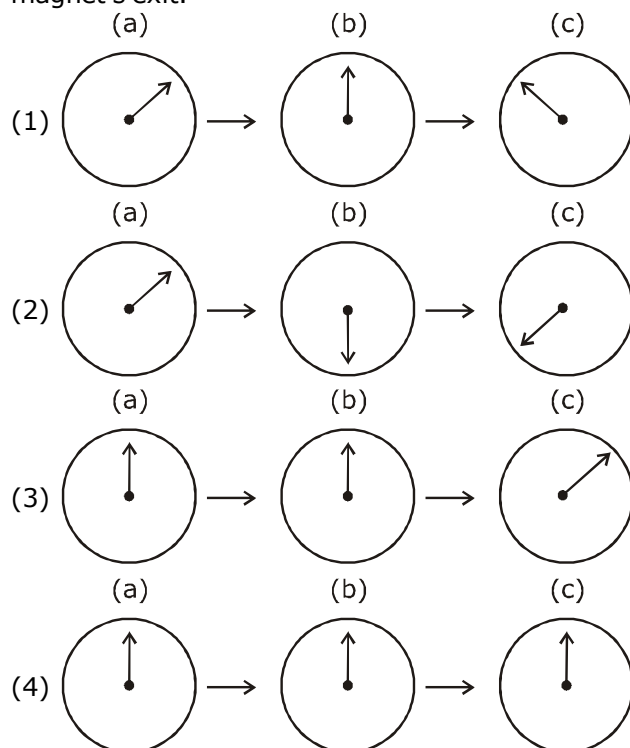
$$\int_0^V dv = - \int_{\infty}^x \frac{Ax}{(x^2 + a^2)^{3/2}} dx$$

$$V = \frac{A}{(x^2 + a^2)^{1/2}}$$

**Q.5** A small bar magnet is moved through a coil at constant speed from one end to the other. Which of the following series of observations will be seen on the galvanometer G attached across the coil?



Three positions shown describe: (a) the magnet's entry (b) magnet is completely inside and (c) magnet's exit.



**Sol. 1**

Let  $\boxed{N \quad S}$

→ When bar magnet enter

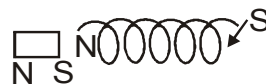


→ When completely inside



$i = 0$

→ when exit



**Q.6** A battery of 3.0V is connected to a resistor dissipating 0.5 W of power. If the terminal voltage of the battery is 2.5V, the power dissipated within the internal resistance is:

- (1) 0.072 W      (2) 0.10 W      (3) 0.125 W      (4) 0.50 W

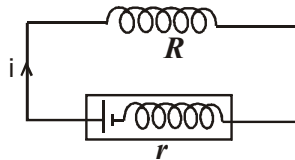
**Sol. 2**

$$P_0 = 0.5 \text{ w}$$

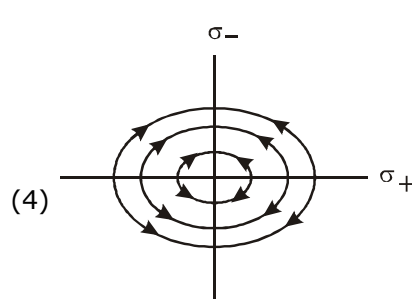
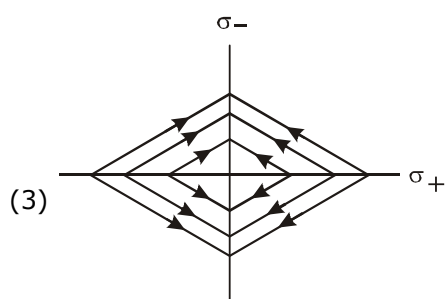
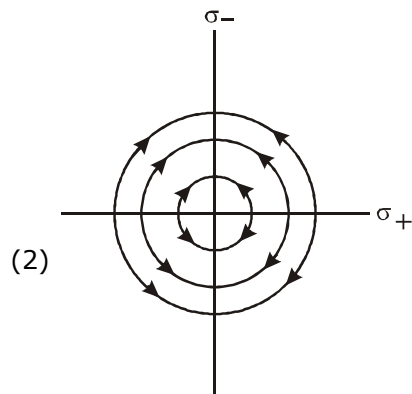
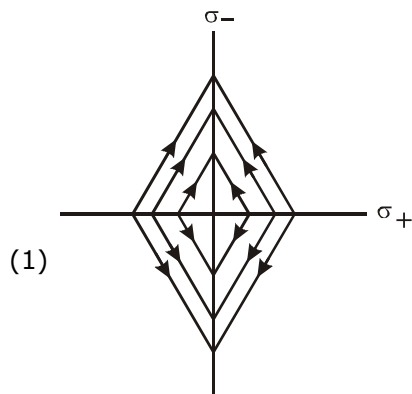
$$i \cdot (2.5) = 0.5$$

$$i = 1/5 \text{ A}$$

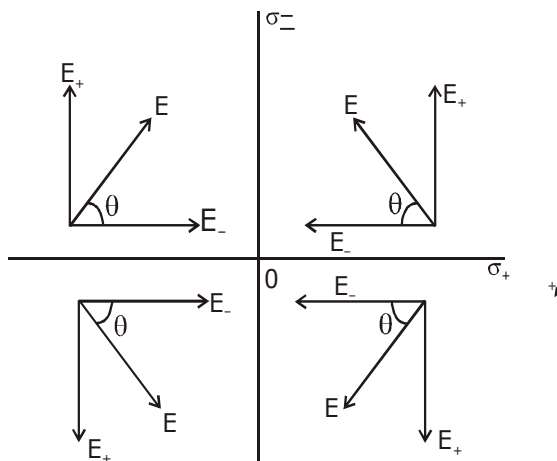
$$P_r = \left(\frac{1}{5}\right)(0.5) = 0.1 \text{ W}$$



**Q.7** Two charged thin infinite plane sheets of uniform surface charge density  $\sigma_+$  and  $\sigma_-$ , where  $|\sigma_+| > |\sigma_-|$ , intersect at right angle. Which of the following best represents the electric field lines for this system:



**Sol. 1**



$$|\vec{E}_+| > |\vec{E}_-|$$

$$\theta > 45^\circ$$

**Q.8** A air bubble of radius 1 cm in water has an upward acceleration  $9.8 \text{ cm s}^{-2}$ . The density of water is  $1 \text{ gm cm}^{-3}$  and water offers negligible drag force on the bubble. The mass of the bubble is ( $g = 980 \text{ cm/s}^2$ ).

- (1) 1.52 gm                      (2) 4.51 gm                      (3) 3.15 gm                      (4) 4.15 gm

**Sol. 4**

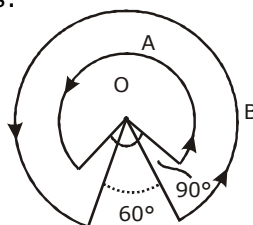


$$F_b - mg = ma \quad \Rightarrow \quad m = \frac{F_b}{g + a}$$

$$m = \frac{V \cdot \rho_w g}{g + a}$$

$$m = \frac{(4/3)\pi r^3 \cdot \rho_w \cdot g}{g + a} = 4.15 \text{ gm}$$

- Q.9** A wire A, bent in the shape of an arc of a circle, carrying a current of 2A and having radius 2 cm and another wire B, also bent in the shape of arc of a circle, carrying a current of 3 A and having radius of 4 cm, are placed as shown in the figure. The ratio of the magnetic field due to the wires A and B at the common centre O is:



- Sol.** (1) 2 : 5 (2) 6 : 5 (3) 6 : 4 (4) 4 : 6

$$B_A = \frac{\mu(2)\left(\frac{3\pi}{2}\right)}{2(a)(2\pi)} = \frac{3\mu}{4a}$$

$$B_B = \frac{\mu(3)\left(\frac{5\pi}{3}\right)}{2(2a)(2\pi)} = \frac{5\mu}{8a}$$

$$\frac{B_A}{B_B} = \frac{3\mu}{4a} \times \frac{8a}{5\mu} = 6 : 5$$

- Q.10** Particle A of mass  $m_A = \frac{m}{2}$  moving along the x-axis with velocity  $v_0$  collides elastically with another particle B at rest having mass  $m_B = \frac{m}{3}$ . If both particles move along the x-axis after the collision, the change  $\Delta\lambda$  in de-Broglie wavelength of particle A, in terms of its de-Broglie wavelength ( $\lambda_0$ ) before collision is:

- (1)  $\Delta\lambda = \frac{5}{2}\lambda_0$  (2)  $\Delta\lambda = 2\lambda_0$  (3)  $\Delta\lambda = 4\lambda_0$  (4)  $\Delta\lambda = \frac{3}{2}\lambda_0$

**Sol. 3**

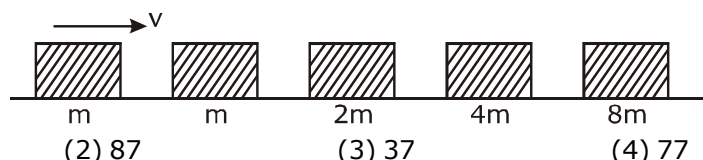
$$V_1 = \frac{m_1 - m_2}{m_1 + m_2} \cdot u_1 + \frac{2m_2}{m_1 + m_2} \cdot u_2$$

$$V_1 = \frac{\frac{m}{2} - m/3}{\frac{m}{2} + m/3} V_0 = V_0/5$$

$$\lambda' = \frac{h}{\frac{m}{2} \cdot \frac{V_0}{5}} = 5 \cdot \frac{h}{\frac{m}{2} \cdot V_0} = 5\lambda_0$$

$$\Delta\lambda = 4\lambda_0$$

- Q.11** Blocks of masses  $m$ ,  $2m$ ,  $4m$  and  $8m$  are arranged in a line on a frictionless floor. Another block of mass  $m$ , moving with speed  $v$  along the same line (see figure) collides with mass  $m$  in perfectly inelastic manner. All the subsequent collisions are also perfectly inelastic. By the time the last block of mass  $8m$  starts moving the total energy loss is  $p\%$  of the original energy. Value of ' $p$ ' is close to:



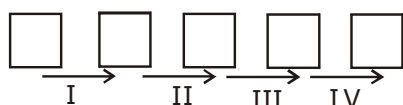
(1) 94

(2) 87

(3) 37

(4) 77

**Sol. 1**

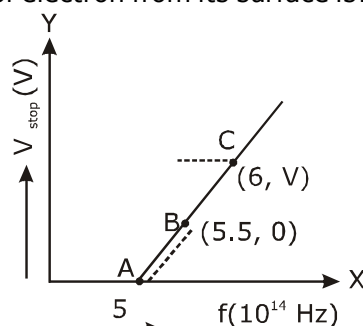


There will be total 4 collisions in each collision K.E. decreasing by 50%

$$E_f = \frac{1}{2^4} E_i = \frac{E_i}{16} = 6.25\%$$

i.e. 93.75 % loss

- Q.12** Given figure shows few data points in a phot electric effect experiment for a certain metal. The minimum energy for ejection of electron from its surface is: (Plancks constant  $h = 6.62 \times 10^{-34} \text{J.s}$ )



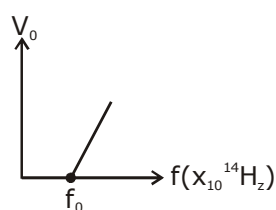
(1) 2.10 eV

(2) 2.27 eV

(3) 2.59 eV

(4) 1.93 eV

**Sol. 2**



$$\phi = hf_0 = 6.62 \times 10^{-34} \times 5.5 \times 10^{14} \\ = 36.41 \times 10^{-20} \text{J} = 2.27 \text{ eV}$$

**Q.13** The specific heat of water =  $4200 \text{ J kg}^{-1}\text{K}^{-1}$  and the latent heat of ice =  $3.4 \times 10^5 \text{ J kg}^{-1}$ . 100 grams of ice at  $0^\circ\text{C}$  is placed in 200 g of water at  $25^\circ\text{C}$ . The amount of ice that will melt as the temperature of water reaches  $0^\circ\text{C}$  is close to (in grams):

- (1) 63.8                      (2) 64.6                      (3) 61.7                      (4) 69.3

**Sol. 3**

Heat loss by water

$$Q = m_w s \Delta\theta$$

$$= \left( \frac{200}{1000} \right) \cdot (4200) (25) = 21000 \text{ J}$$

$$\text{and } \Delta m_i L = 21000$$

$$\Delta m_i = \frac{21000}{3.4 \times 10^5} \times 10^3 \text{ gm} = 61.7 \text{ grams}$$

**Q.14** A beam of plane polarised light of large cross-sectional area and uniform intensity of  $3.3 \text{ Wm}^{-2}$  falls normally on a polariser (cross sectional area  $3 \times 10^{-4} \text{ m}^2$ ) which rotates about its axis with an angular speed of  $31.4 \text{ rad/s}$ . The energy of light passing through the polariser per revolution, is close to:

- (1)  $1.0 \times 10^{-4} \text{ J}$               (2)  $1.0 \times 10^{-5} \text{ J}$               (3)  $5.0 \times 10^{-4} \text{ J}$               (4)  $1.5 \times 10^{-4} \text{ J}$

**Sol. 1**

$$p = p_0 \cos^2 \omega t$$

$$E_{\text{avg}} = \langle p \rangle \cdot T = \frac{p_0}{2} T$$

$$E_{\text{avg}} = \langle P \rangle \cdot T = \frac{p_0}{2} \cdot \frac{2\pi}{\omega} = \frac{10^{-3} \times 3.14}{31.4} = 10^{-4} \text{ J}$$

**Q.15** For a transverse wave travelling along a straight line, the distance between two peaks (crests) is 5m, while the distance between one crest and one trough is 1.5m. The possible wavelengths (in m) of the waves are:

- (1) 1, 3, 5, .....              (2) 1, 2, 3, .....              (3)  $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \dots$               (4)  $\frac{1}{1}, \frac{1}{3}, \frac{1}{5}, \dots$

**Sol. 4**

$$1.5 = (2n_1 + 1) \lambda / 2 \quad \dots(1)$$

$$5 = n_2 \lambda \quad \dots(2)$$

$n_1$  &  $n_2$  are integer

$$n_1 = 1, n_2 = 5$$

$$n_1 = 2, n_2 \text{ is not integer}$$

$$n_1 = 3, n_2 \text{ is not integer}$$

$$n_1 = 4, n_2 = 15, \quad \lambda = 1/3$$

**Q.16** Match the  $C_p/C_v$  ratio for ideal gases with different type of molecules:

Molecule Type	$C_p/C_v$
(A) Monoatomic	(I) 7/5
(B) Diatomic rigid molecules	(II) 9/7
(C) Diatomic non-rigid molecules	(III) 4/3
(D) Triatomic rigid molecules	(IV) 5/3
(1) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)	
(2) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)	
(3) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)	
(4) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)	

**Sol. 4**

$$\gamma = C_p/C_v$$

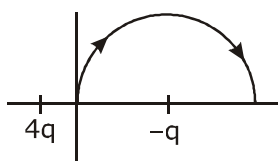
$$\gamma_A = 1 + \frac{2}{3} = 5/3$$

$$\gamma_B = 1 + \frac{2}{5} = 7/5$$

$$\gamma_C = 1 + \frac{2}{7} = 9/7$$

$$\gamma_D = 1 + \frac{2}{6} = 4/3$$

**Q.17** A two point charges  $4q$  and  $-q$  are fixed on the x-axis at  $x = -\frac{d}{2}$  and  $x = \frac{d}{2}$ , respectively. If a third point charge ' $q$ ' is taken from the origin to  $x = d$  along the semicircle as shown in the figure, the energy of the charge will:



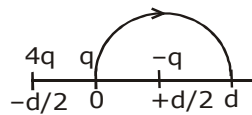
(1) decrease by  $\frac{q^2}{4\pi \epsilon_0 d}$

(2) decrease by  $\frac{4q^2}{3\pi \epsilon_0 d}$

(3) increase by  $\frac{3q^2}{4\pi \epsilon_0 d}$

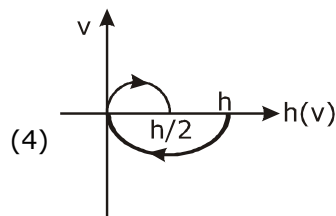
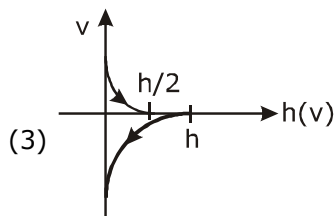
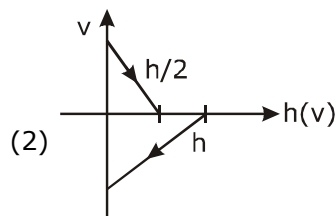
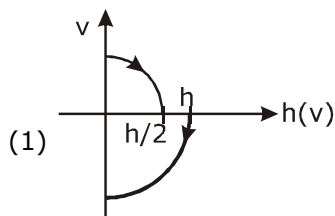
(4) increase by  $\frac{2q^2}{3\pi \epsilon_0 d}$

**Sol. 2**



$$\begin{aligned}\Delta U &= \frac{1}{4\pi\epsilon_0} \cdot \frac{4q \cdot q}{(3d/2)} - \frac{1}{4\pi\epsilon_0} \cdot \frac{4q \cdot q}{(d/2)} \\ &= \frac{4q^2}{4\pi\epsilon_0} \left( \frac{2}{d} \right) \left( -\frac{2}{3} \right) \\ &= (-) \frac{4q^2}{3\pi\epsilon_0 d} \\ &= \text{decrease by } (-)\end{aligned}$$

**Q.18** A Tennis ball is released from a height  $h$  and after freely falling on a wooden floor it rebounds and reaches height  $\frac{h}{2}$ . The velocity versus height of the ball during its motion may be represented graphically by: (graph are drawn schematically and on not to scale)



**Sol. 1**

- $v, h$  curve will be parabolic
- downward velocity is negative and upward is positive
- when ball is coming down graph will be in IV quadrant and when going up graph will be in I quadrant

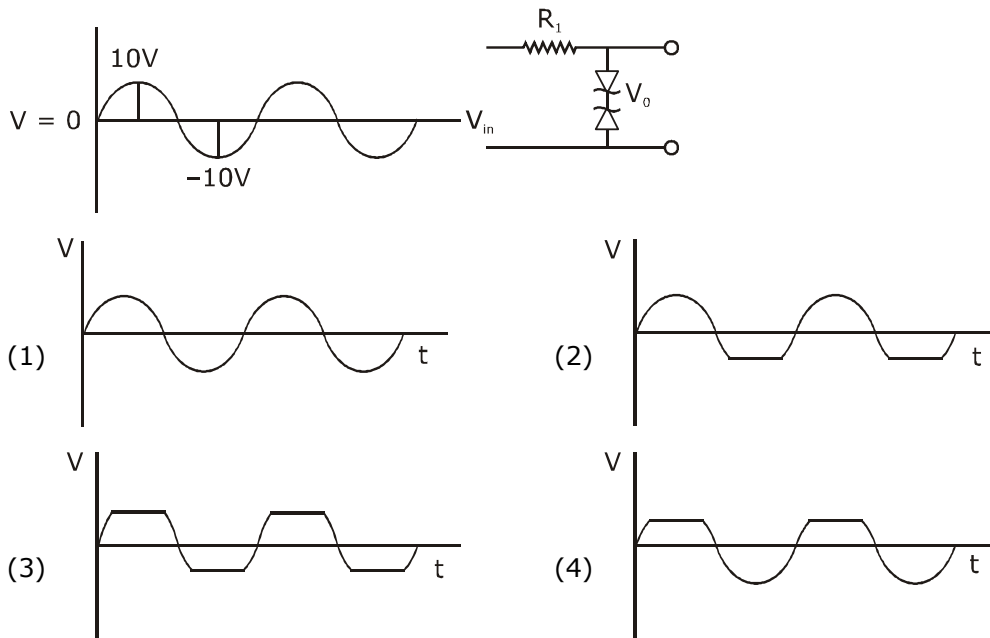
**Q.19** Dimensional formula for thermal conductivity is (here  $K$  denotes the temperature):

- (1)  $MLT^{-3}K^{-1}$       (2)  $MLT^{-2}K^{-2}$       (3)  $MLT^{-2}K$       (4)  $MLT^{-3}K$

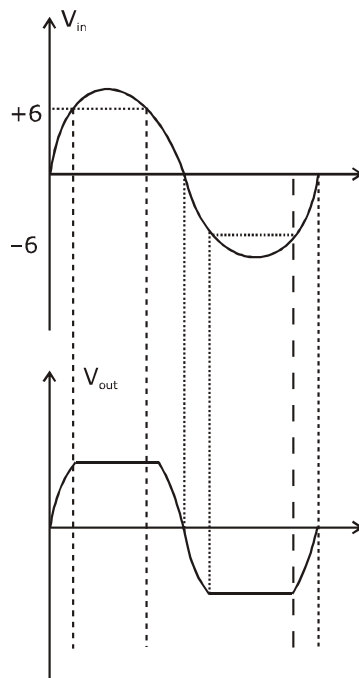
**Sol. 1**

$$\frac{dQ}{dt} = \frac{Kl\Delta T}{A}$$

**Q.20** Take the breakdown voltage of the zener diode used in the given circuit as 6V. For the input voltage shown in figure below, the time variation of the output voltage is : (Graphs drawn are schematic and not to scale)



**Sol. 3**



**Q.21** In the line spectra of hydrogen atoms, difference between the largest and the shortest wavelengths of the Lyman series is  $304\text{\AA}$ . The corresponding difference for the Paschen series in  $\text{\AA}$  is :

**Sol.** 10553

$$\frac{1}{R} = 912 \text{\AA}$$

in Paschen series

$$\frac{1}{\lambda_s} = R \left( \frac{1}{3^2} \right) = \frac{R}{9}$$

$$\frac{1}{\lambda_l} = R \left( \frac{1}{3^2} - \frac{1}{4^2} \right) = \frac{7R}{144}$$

$$(\lambda_l - \lambda_s) = \left( \frac{144}{7} - 9 \right) R = 10553 \text{\AA}$$

**Q.22** A closed vessel contains 0.1 mole of a monoatomic ideal gas at 200 K. If 0.05 mole of the same gas at 400 K is added to it, the final equilibrium temperature (in K) of the gas in the vessel will be close to \_\_\_\_\_.

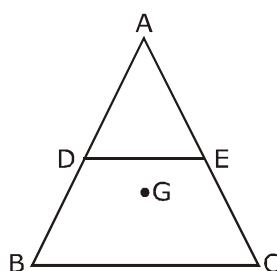
**Sol.** 266

$$(0.1) \left( \frac{3}{2} R \right) (T - 200) = (0.05) \left( \frac{3}{2} R \right) (400 - T)$$

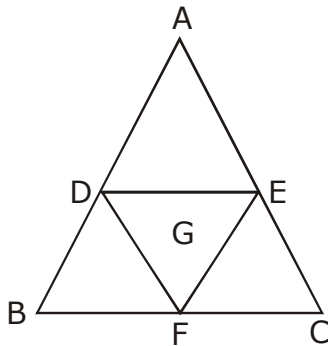
$$T = 266.6 \text{ K}$$

**Q.23** ABC is a plane lamina of the shape of an equilateral triangle. D, E are mid points of AB, AC and G is the centroid of the lamina. Moment of inertia of the lamina about an axis passing through G and perpendicular to the plane ABC is  $I_0$ . If part ADE is removed, the moment of inertia of the remaining

part about the same axis is  $\frac{NI_0}{16}$  where N is an integer. Value of N is \_\_\_\_\_.



**Sol. 11**



Let  $I_0 = kmc^2$

$$I_{DEF} = K \left( \frac{m}{\ell} \right) \left( \frac{\ell}{2} \right)^2 = \left( \frac{I_0}{16} \right)$$

and  $I_{ADE} = I_{BDE} = I_{EFC} = I$

$$3I = I_0 - \frac{I_0}{16} = \frac{15I_0}{16}$$

$$\Rightarrow I = \frac{5I_0}{16}$$

$$I_{\text{remaining}} = 2I + \frac{I_0}{16} = \frac{11I_0}{16}$$

**Q.24** In a compound microscope, the magnified virtual image is formed at a distance of 25 cm from the eye-piece. The focal length of its objective lens is 1 cm. If the magnification is 100 and the tube length of the microscope is 20 cm, then the focal length of the eye-piece lens (in cm) is \_\_\_\_\_.

**Sol. 6.25**

$$L = 20, f_o = 1\text{cm}, M = 100$$

$$M = \frac{v_o}{u_o} \left( 1 + \frac{D}{f_e} \right)$$

$$M = \frac{L}{f_o} \left( 1 + \frac{D}{f_e} \right) \quad [v_o \approx L, u_o \approx f_o]$$

on solving we get  
 $f_e = 6.25 \text{ cm}$

- Q.25** A circular disc of mass  $M$  and radius  $R$  is rotating about its axis with angular speed  $\omega_1$ . If another stationary disc having radius  $\frac{R}{2}$  and same mass  $M$  is dropped co-axially on to the rotating disc. Gradually both discs attain constant angular speed  $\omega_2$  the energy lost in the process is  $p\%$  of the initial energy. Value of  $p$  is \_\_\_\_\_.

**Sol. 20**

$$I_f \omega_f = I_i \omega_i$$

$$I_i = \frac{MR^2}{2}$$

$$I_f = \frac{MR^2}{2} + \frac{M(R/2)^2}{2}$$

$$= \frac{5}{4} \cdot \frac{MR^2}{2}$$

$$\left[ \frac{MR^2}{2} + \frac{M}{2} \left( \frac{R}{2} \right)^2 \right] \omega' = \left( \frac{MR^2}{2} \right) \omega$$

$$\left[ \frac{MR^2}{2} \cdot \left( \frac{5}{4} \right) \right] \omega' = \frac{MR^2}{2} \omega$$

$$\omega' = \frac{4}{5} \omega$$

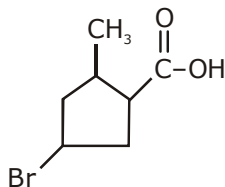
$$\text{loss of K.E.} = \frac{\text{Loss}}{K_i} \times 100 = \frac{\omega^2 - \omega'^2 (5/4)}{\omega^2} \times 100$$

$$\frac{\omega^2 - \frac{16}{25} \omega^2 \left( \frac{5}{4} \right)}{\omega^2} = \left( 1 - \frac{80}{100} \right) \times 100$$

$$= 20\%$$

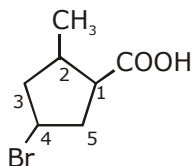
## CHEMISTRY \_ 4 Sep. \_ SHIFT - 1

1. The IUPAC name of the following compound is :



- (1) 3-Bromo-5-methylcyclopentane carboxylic acid
- (2) 4-Bromo-2-methylcyclopentane carboxylic acid
- (3) 5-Bromo-3-methylcyclopentanoic acid
- (4) 3-Bromo-5-methylcyclopentanoic acid

**Sol. 2**

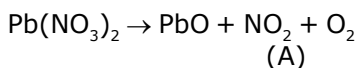


4-Bromo-2-methylcyclopentane carboxylic acid

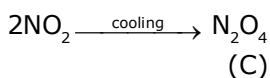
2. On heating, lead(II) nitrate gives a brown gas (A). The gas (A) on cooling changes to a colourless solid/liquid (B). (B) on heating with NO changes to a blue solid (C). The oxidation number of nitrogen in solid (C) is :

- (1) +3
- (2) +4
- (3) +2
- (4) +5

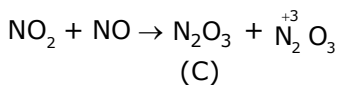
**Sol. 1**



Brown gas



colourless solid

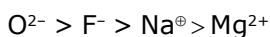


blue solid

3. The ionic radii of  $\text{O}^{2-}$ ,  $\text{F}^-$ ,  $\text{Na}^+$  and  $\text{Mg}^{2+}$  are in the order :

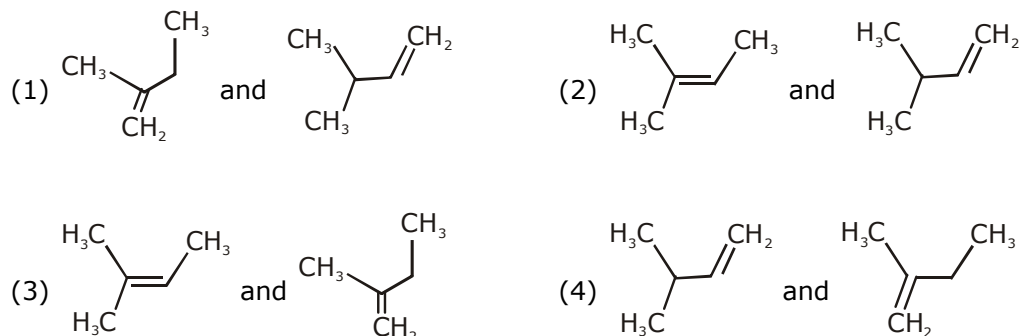
- (1)  $\text{F}^- > \text{O}^{2-} > \text{Na}^+ > \text{Mg}^{2+}$
- (2)  $\text{Mg}^{2+} > \text{Na}^+ > \text{F}^- > \text{O}^{2-}$
- (3)  $\text{O}^{2-} > \text{F}^- > \text{Na}^+ > \text{Mg}^{2+}$
- (4)  $\text{O}^{2-} > \text{F}^- > \text{Mg}^{2+} > \text{Na}^+$

**Sol. 3**

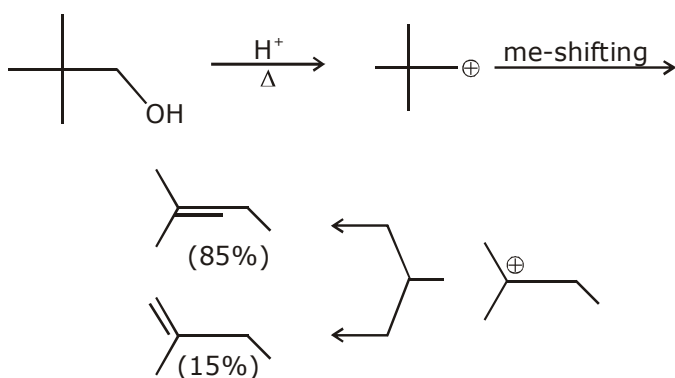


Ans. option (3)

4. When neopentyl alcohol is heated with an acid, it slowly converted into an 85 : 15 mixture of alkenes A and B, respectively. What are these alkenes ?



**Sol. 3**



5. The region in the electromagnetic spectrum where the Balmer series lines appear is :  
 (1) Microwave (2) Infrared (3) Ultraviolet (4) Visible

**Sol. 4**

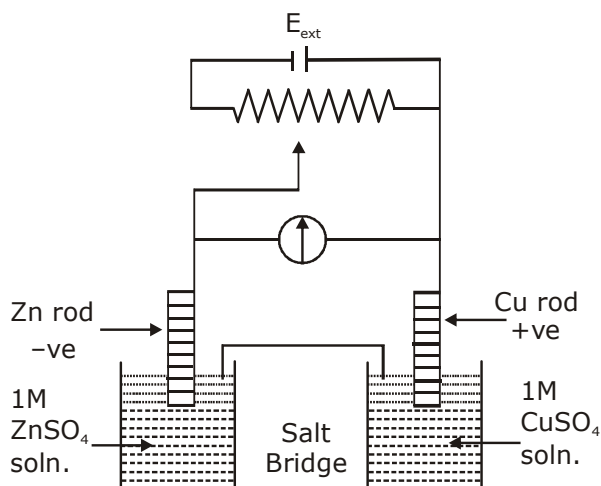
Question should be Bonous

As lines of Balmer series belongs to both UV as well visible region of EM spectrum.

However most appropriate should be visible region

Ans. (4)

6.



$$E_{\text{Cu}^{2+}|\text{Cu}}^{\circ} = + 0.34 \text{ V}$$

$$E_{\text{Zn}^{2+}|\text{Zn}}^{\circ} = - 0.76 \text{ V}$$

Identify the incorrect statement from the options below for the above cell :

- (1) If  $E_{\text{ext}} = 1.1 \text{ V}$ , no flow of  $e^{-}$  or current occurs
- (2) If  $E_{\text{ext}} > 1.1 \text{ V}$ , Zn dissolves at Zn electrode and Cu deposits at Cu electrode
- (3) If  $E_{\text{ext}} > 1.1 \text{ V}$ ,  $e^{-}$  flows from Cu to Zn
- (4) If  $E_{\text{ext}} < 1.1 \text{ V}$ , Zn dissolves at anode and Cu deposits at cathode

Sol. 2

Direction NCERT Text theoretical questions

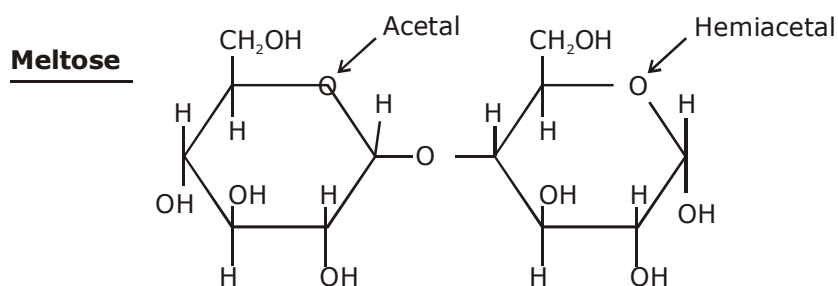
Ans. (2)

7.

What are the functional groups present in the structure of maltose ?

- (1) One acetal and one hemiacetal
- (2) One acetal and one ketal
- (3) One ketal and one hemiketal
- (4) Two acetals

Sol. 1



8. Match the following :

- |               |                |
|---------------|----------------|
| (i) Foam      | (a) smoke      |
| (ii) Gel      | (b) cell fluid |
| (iii) Aerosol | (c) jellies    |
| (iv) Emulsion | (d) rubber     |
|               | (e) froth      |
|               | (f) milk       |

(1) (i)-(e), (ii)-(c), (iii)-(a), (iv)-(f)

(2) (i)-(b), (ii)-(c), (iii)-(e), (iv)-(d)

(3) (i)-(d), (ii)-(b), (iii)-(a), (iv)-(e)

(4) (i)-(d), (ii)-(b), (iii)-(e), (iv)-(f)

Sol. 1

Foam → Froth, whipped cream, soaplather

Gel → Cheese, butter, jellies

Aerosol → smoke dust

Emulsion → milk

Sol → Cell fluid

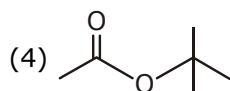
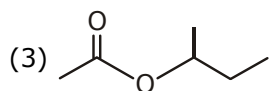
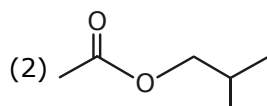
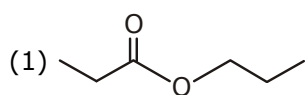
rubber → Solid form

froth → form

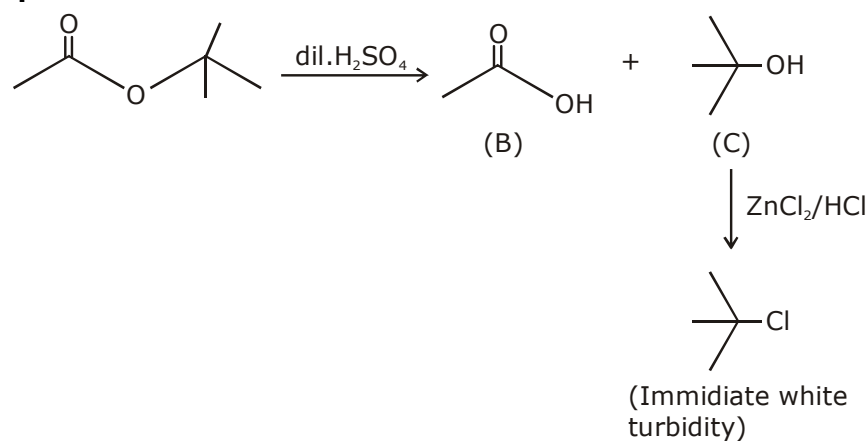
(i) - e, (ii) - c, (iii) - a, (iv) - f

Ans. 1

9. An organic compound (A) (molecular formula  $C_6H_{12}O_2$ ) was hydrolysed with dil.  $H_2SO_4$  to give a carboxylic acid (B) and an alcohol (C). 'C' gives white turbidity immediately when treated with anhydrous  $ZnCl_2$  and conc.  $HCl$ . The organic compound (A) is :

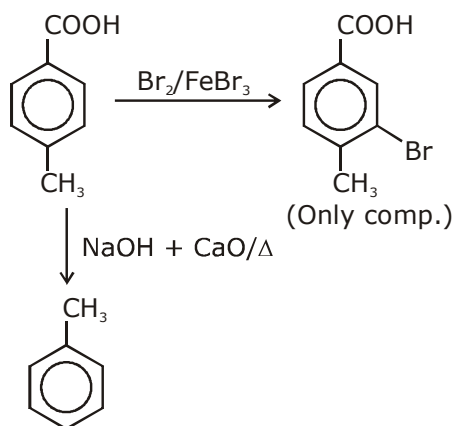


Sol. 4

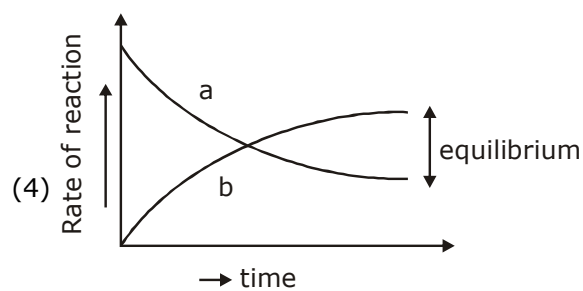
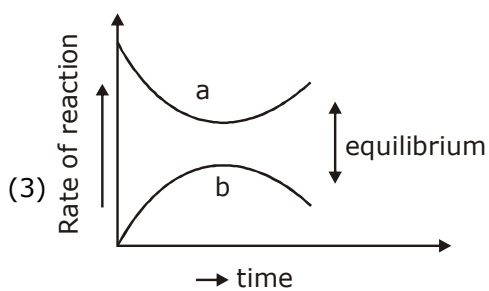
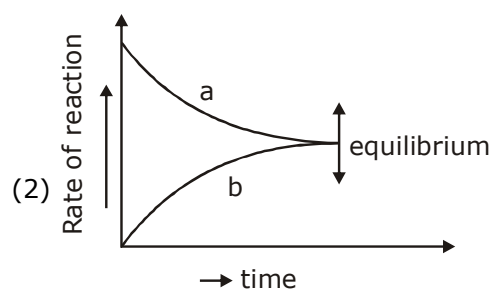
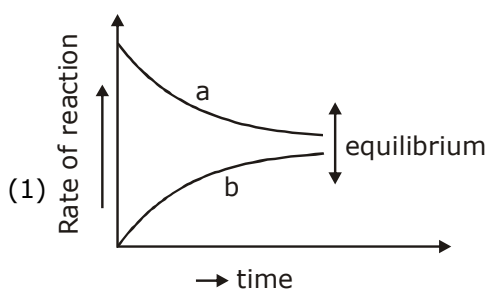




Sol. 2



13. For the equilibrium  $A \rightleftharpoons B$  the variation of the rate of the forward (a) and reverse (b) reaction with time is given by :



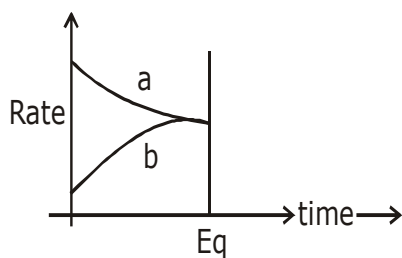
**Sol. 2**

At equilibrium

Rate of forward = Rate of backward

$a = b$

Hence

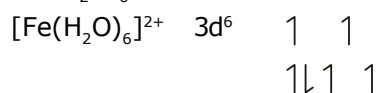
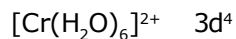


Ans. option (2)

**14.** The pair in which both the species have the same magnetic moment (spin only) is :

- (1)  $[\text{Co}(\text{OH})_4]^{2-}$  and  $[\text{Fe}(\text{NH}_3)_6]^{2+}$       (2)  $[\text{Mn}(\text{H}_2\text{O})_6]^{2+}$  and  $[\text{Cr}(\text{H}_2\text{O})]^{2+}$   
 (3)  $[\text{Cr}(\text{H}_2\text{O})_6]^{2+}$  and  $[\text{CoCl}_4]^{2-}$       (4)  $[\text{Cr}(\text{H}_2\text{O})_6]^{2+}$  and  $[\text{Fe}(\text{H}_2\text{O})_6]^{2+}$

**Sol. 4**



Both has 4 unpaired electron

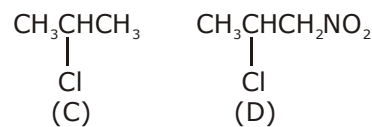
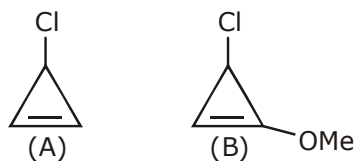
**15.** The number of isomers possible for  $[\text{Pt}(\text{en})(\text{NO}_2)_2]$  is :

- (1) 2      (2) 3      (3) 4      (4) 1

**Sol. 2**

Three linkage isomer  $\text{NO}_2^-$ ;  $\text{ONO}^-$

**16.** The decreasing order of reactivity of the following organic molecules towards  $\text{AgNO}_3$  solution is :



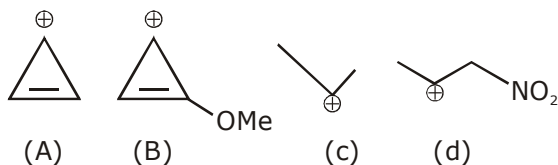
(1)  $(\text{B}) > (\text{A}) > (\text{C}) > (\text{D})$

(3)  $(\text{A}) > (\text{B}) > (\text{D}) > (\text{C})$

(2)  $(\text{A}) > (\text{B}) > (\text{C}) > (\text{D})$

(4)  $(\text{C}) > (\text{D}) > (\text{A}) > (\text{B})$

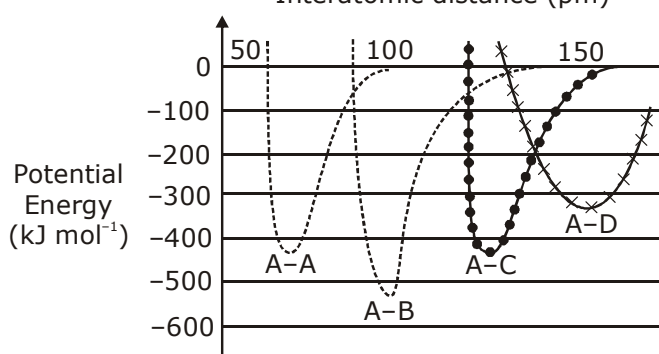
**Sol. 1**



Order of stability

(B) > (A) > (C) > (D)

- 17.** The intermolecular potential energy for the molecules A, B, C and D given below suggests that :  
Interatomic distance (pm)



- (1) A-A has the largest bond enthalpy. (2) D is more electronegative than other atoms.  
(3) A-D has the shortest bond length. (4) A-B has the stiffest bond.

**Sol. 4**

Acc. to Diagram

Ans option (4)

As  $E_{A-B}$  is Highest

- 18.** Which of the following will react with  $\text{CHCl}_3 + \text{alc. KOH}$  ?

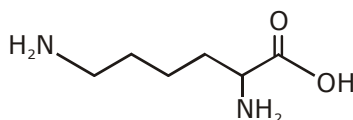
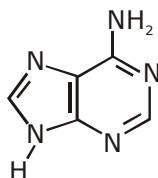
- (1) Thymine and proline (2) Adenine and thymine  
(3) Adenine and lysine (4) Adenine and proline

**Sol. 3**

$\text{CHCl}_3 + \text{Alc. KOH}$  reacts with those compound which have  $-\text{NH}_2$  group

Adenine

Lysin



**19.** The elements with atomic numbers 101 and 104 belong to, respectively, :

- (1) Actinoids and Group 6 (2) Group 11 and Group 4  
(3) Group 6 and Actinoids (4) Actinoids and Group 4

**Sol. 4**

$$Z = 101 \rightarrow [R_n]^{86} 7s^2 5f^{13}$$

↓

Actinoids

$$Z = 104 \rightarrow [R_n]^{86} 7s^2 5f^{14} 6d^2$$

↓

4<sup>th</sup> group element

Ans Actinoids & 4<sup>th</sup> group

Ans. (4)

**20.** On combustion of Li, Na and K in excess of air, the major oxides formed, respectively, are :

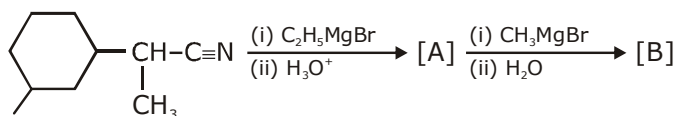
- (1) Li<sub>2</sub>O<sub>2</sub>, Na<sub>2</sub>O<sub>2</sub> and K<sub>2</sub>O<sub>2</sub> (2) Li<sub>2</sub>O, Na<sub>2</sub>O<sub>2</sub> and KO<sub>2</sub>  
(3) Li<sub>2</sub>O, Na<sub>2</sub>O and K<sub>2</sub>O<sub>2</sub> (4) Li<sub>2</sub>O, Na<sub>2</sub>O<sub>2</sub> and K<sub>2</sub>O

**Sol. 2**

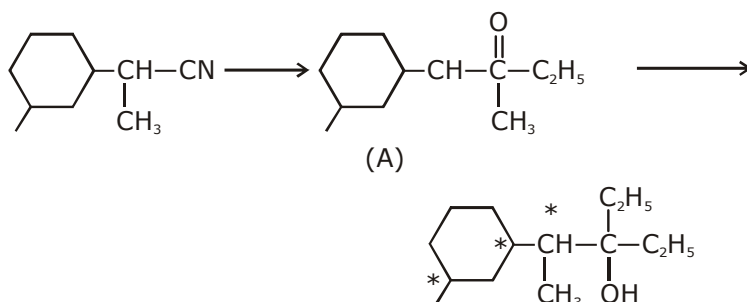
Li<sub>2</sub>O, Na<sub>2</sub>O<sub>2</sub>, K<sub>2</sub>O<sub>2</sub>

option (2)

**21.** The number of chiral centres present in [B] is \_\_\_\_\_.



**Sol. 3**



3 chiral center is present in final products

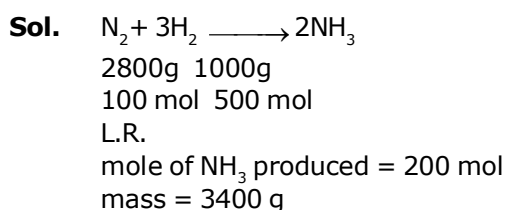
**22.** At 300 K, the vapour pressure of a solution containing 1 mole of n-hexane and 3 moles of n-heptane is 550 mm of Hg. At the same temperature, if one more mole of n-heptane is added to this solution, the vapour pressure of the solution increases by 10 mm of Hg. What is the vapour pressure in mm Hg of n-heptane in its pure state \_\_\_\_\_?

**Sol.**  $550 = \frac{1}{4} \times p_{C_6H_{14}}^0 + \frac{3}{4} \times p_{C_7H_{16}}^0$

$$560 = \frac{1}{5} \times p_{C_6H_{14}}^0 + \frac{4}{5} \times p_{C_7H_{16}}^0$$

$$p_{C_7H_{16}}^0 = [560 \times 5 - 550 \times 4] \\ = 550 + 50 = 600 \text{ mm of Hg}$$

- 23.** The mass of ammonia in grams produced when 2.8 kg of dinitrogen quantitatively reacts with 1 kg of dihydrogen is \_\_\_\_\_.



- 24.** If 75% of a first order reaction was completed in 90 minutes, 60% of the same reaction would be completed in approximately (in minutes) \_\_\_\_\_.  
 (take :  $\log 2 = 0.30$ ;  $\log 2.5 = 0.40$ )

**Sol. 60**

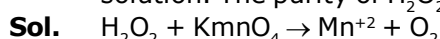
$$t_{75\%} = 90 \text{ min} = 2 \times t_{1/2} \\ t_{1/2} = 45 \text{ min}$$

$$\frac{\ln(2)}{45} \times t_{60\%} = \ln \left\{ \frac{100}{40} \right\}$$

$$t_{60\%} = 45 \times \frac{0.4}{0.3}$$

$$t_{60\%} = 60 \text{ min}$$

- 25.** A 20.0 mL solution containing 0.2 g impure  $H_2O_2$  reacts completely with 0.316 g of  $KMnO_4$  in acid solution. The purity of  $H_2O_2$  (in %) is \_\_\_\_\_ (mol. wt. of  $H_2O_2 = 34$ ; mole wt. of  $KMnO_4 = 158$ )



$$[\text{moles of } H_2O_2] \times 2 = \frac{0.316}{158} \times 5$$

$$\text{moles of } H_2O_2 = 5 \times 10^{-3}$$

$$\text{mass of } H_2O_2 = 170 \times 10^{-3} \text{ g}$$

$$\% \text{ purity} = \frac{170 \times 10^{-3}}{0.2} \times 100 = 85\%$$

# QUESTION PAPER WITH SOLUTION

## MATHEMATICS \_ 4 Sep. \_ SHIFT - 1

1. Let  $y=y(x)$  be the solution of the differential equation,  $xy'-y=x^2(x\cos x+\sin x)$ ,  $x > 0$ . if  $y(\pi) = \pi$ , then

$y''\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right)$  is equal to

(1)  $2 + \frac{\pi}{2} + \frac{\pi^2}{4}$

(2)  $2 + \frac{\pi}{2}$

(3)  $1 + \frac{\pi}{2}$

(4)  $1 + \frac{\pi}{2} + \frac{\pi^2}{4}$

**Sol. (2)**

$$xy' - y = x^2(x \cos x + \sin x) \quad x > 0, y(\pi) = \pi$$

$$y' - \frac{1}{x}y = x\{x\cos x + \sin x\}$$

$$\text{I.F.} = e^{-\int \frac{1}{x} dx} = e^{-\ln x} = \frac{1}{x}$$

$$\therefore y \cdot \frac{1}{x} = \int \frac{1}{x} \cdot x(x \cos x + \sin x) dx$$

$$\frac{y}{x} = \int (x \cos x + \sin x) dx$$

$$\frac{y}{x} = \int \frac{d}{dx} (x \sin x) dx$$

$$\frac{y}{x} = x \sin x + C$$

$$\Rightarrow y = x^2 = \sin x + cx$$

$$x = \pi, y = \pi$$

$$\pi = \pi C \Rightarrow C = 1$$

$$y = x^2 \sin x + x \Rightarrow y\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4} + \frac{\pi}{2}$$

$$y' = 2x \sin x + x^2 \cos x + 1$$

$$y'' = 2 \sin x + 2x \cos x + 2x \cos x - x^2 \sin x$$

$$y''\left(\frac{\pi}{2}\right) = 2 - \frac{\pi^2}{4} \Rightarrow y\left(\frac{\pi}{2}\right) + y''\left(\frac{\pi}{2}\right) = 2 + \frac{\pi}{2}$$

2. The value of  $\sum_{r=0}^{20} {}^{50-r}C_6$  is equal to:

(1)  ${}^{51}C_7 - {}^{30}C_7$

(2)  ${}^{51}C_7 + {}^{30}C_7$

(3)  ${}^{50}C_7 - {}^{30}C_7$

(4)  ${}^{50}C_6 - {}^{30}C_6$

**Sol. (1)**

$$\sum_{r=0}^{20} {}^{50-r}C_6$$

$$\Rightarrow {}^{50}C_6 + {}^{49}C_6 + {}^{48}C_6 + \dots + {}^{31}C_6 + {}^{30}C_6$$

add and subtract  ${}^{30}C_7$

Using

$${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r \Rightarrow {}^{30}C_6 + {}^{30}C_7 = {}^{31}C_7$$

$${}^{31}C_6 + {}^{31}C_7 = {}^{32}C_7$$

Similarly solving

$${}^{51}C_7 - {}^{30}C_7$$

3. Let  $[t]$  denote the greatest integer  $\leq t$ . Then the equation in  $x, [x]^2 + 2[x + 2] - 7 = 0$  has :
- (1) exactly four integral solutions. (2) infinitely many solutions.  
 (3) no integral solution. (4) exactly two solutions.

Sol. (2)

$$[x]^2 + 2[x + 2] - 7 = 0$$

$$[x]^2 + 2[x] - 3 = 0$$

$$\text{let } [x] = y$$

$$y^2 + 3y - y - 3 = 0$$

$$(y - 1)(y + 3) = 0$$

$$[x] = 1 \text{ or } [x] = -3$$

$$x \in [1, 2) \quad \& \quad x \in [-3, -2)$$

4. Let  $P(3,3)$  be a point on the hyperbola,  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ . If the normal to it at  $P$  intersects the  $x$ -axis at  $(9,0)$  and  $e$  is its eccentricity, then the ordered pair  $(a^2, e^2)$  is equal to :

- (1)  $(9,3)$  (2)  $\left(\frac{9}{2}, 2\right)$  (3)  $\left(\frac{9}{2}, 3\right)$  (4)  $\left(\frac{3}{2}, 2\right)$

Sol. (3)

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad P(3,3)$$

$$\frac{9}{a^2} - \frac{9}{b^2} = 1 \quad \dots(1)$$

$$\text{Equation of normal} \Rightarrow \frac{a^2x}{3} + \frac{b^2y}{3} = a^2e^2$$

$$\text{at } x - \text{axis} \Rightarrow y = 0$$

$$\frac{a^2x}{3} = a^2e^2 \Rightarrow x = 3e^2 = 9$$

$$e^2 = 3$$

$$e = \sqrt{3}$$

$$e^2 = 1 + \frac{b^2}{a^2} = 3$$

$$b^2 = 2a^2 \quad \dots(2)$$

put in equation 1

$$\frac{9}{a^2} - \frac{9}{2a^2} = 1 \Rightarrow \frac{9}{2a^2} = 1 \Rightarrow a^2 = \frac{9}{2}$$

$$\therefore (a^2, e^2) = \left(\frac{9}{2}, 3\right)$$

5. Let  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  ( $a > b$ ) be a given ellipse, length of whose latus rectum is 10. If its eccentricity is the maximum value of the function,  $\phi(t) = \frac{5}{12} + t - t^2$ , then  $a^2 + b^2$  is equal to

- (1) 135                      (2) 116                      (3) 126                      (4) 145

**Sol. (3)**

$$L.R = \frac{2b^2}{a} = 10 \quad \dots(1)$$

$$\phi(t) = \frac{5}{12} - \left(t - \frac{1}{2}\right)^2 + \frac{1}{4} = \frac{8}{12} - \left(t - \frac{1}{2}\right)^2$$

$$\therefore \phi(t)_{\max} = \frac{2}{3} = e$$

$$e^2 = 1 - \frac{b^2}{a^2} = \frac{4}{9} \Rightarrow \frac{b^2}{a^2} = \frac{5}{9}$$

$$\frac{b^2}{a^2} = \frac{5}{9} \text{ from (1)}$$

$$\frac{5}{a} = \frac{5}{9} \Rightarrow a = 9$$

$$\therefore b^2 = 45$$

$$a^2 + b^2 = 45 + 81 = 126$$

6. Let  $f(x) = \int \frac{\sqrt{x}}{(1+x)^2} dx$  ( $x \geq 0$ ). Then  $f(3) - f(1)$  is equal to :

- (1)  $-\frac{\pi}{6} + \frac{1}{2} + \frac{\sqrt{3}}{4}$                       (2)  $\frac{\pi}{6} + \frac{1}{2} - \frac{\sqrt{3}}{4}$                       (3)  $-\frac{\pi}{12} + \frac{1}{2} + \frac{\sqrt{3}}{4}$                       (4)  $\frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

**Sol. (4)**

$$f(x) = \int \frac{\sqrt{x}}{(1+x)^2} dx$$

$$x = \tan^2 t$$

$$dx = 2 \tan t \sec^2 t dt$$

$$f(x) = \int \frac{\tan t \cdot 2 \tan t \sec^2 t dt}{\sec^4 t}$$

$$= 2 \int \sin^2 t dt$$

$$x = 3 \Rightarrow t = \frac{\pi}{3}$$

$$x = 1 \Rightarrow t = \frac{\pi}{4}$$

$$\therefore f(3) - f(1) = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (1 - \cos 2t) dt \Rightarrow \left( t - \frac{1}{2} \sin 2t \right)_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$$

7. If  $1 + (1 - 2^2 \cdot 1) + (1 - 4^2 \cdot 3) + (1 - 6^2 \cdot 5) + \dots + (1 - 20^2 \cdot 19) = \alpha - 220\beta$ , then an ordered pair  $(\alpha, \beta)$  is equal to:

(1) (10, 97)                      (2) (11, 103)                      (3) (11, 97)                      (4) (10, 103)

Sol.

$$\begin{aligned} T_n &= 1 - (2n)^2(2n - 1) \\ &= 1 - 4n^2(2n - 1) \\ &= 1 - 8n^3 + 4n^2 \end{aligned}$$

$$S_n = \sum_{n=1}^{10} T_n = n - \sum 8n^3 + \sum 4n^2$$

$$= n - 8 \times \frac{n^2(n+1)^2}{4} + \frac{4n(n+1)(2n+1)}{6}$$

$$= 10 - 2 \times 100 \times 121 + \frac{2}{3} \times 10 \times 11 \times 21$$

$$= 10 - 24200 + 1540$$

$$= 10 - 22660$$

$$\therefore \text{Sum of series} = 11 - 22660 = \alpha - 220\beta$$

$$\alpha = 11, \beta = 103$$

8. The integral  $\int \left( \frac{x}{x \sin x + \cos x} \right)^2 dx$  is equal to

(where C is a constant of integration):

$$(1) \tan x - \frac{x \sec x}{x \sin x + \cos x} + C$$

$$(2) \sec x - \frac{x \tan x}{x \sin x + \cos x} + C$$

$$(3) \sec x + \frac{x \tan x}{x \sin x + \cos x} + C$$

$$(4) \tan x + \frac{x \sec x}{x \sin x + \cos x} + C$$

**Sol. (1)**

$$\int \left( \frac{x}{x \sin x + \cos x} \right)^2 dx$$

$$\int \underbrace{x \sec x}_I \cdot \underbrace{\frac{x \cos x}{(x \sin x + \cos x)^2}}_{II} dx$$

$$x \sec x \left( \frac{-1}{x \sin x + \cos x} \right) + \int \frac{\sec x + x \sec x \tan x}{(x \sin x + \cos x)} dx$$

$$\Rightarrow \frac{-x \sec x}{x \sin x + \cos x} + \int \frac{(\cos x + x \sin x)}{\cos^2 x (x \sin x + \cos x)} dx \Rightarrow \frac{-x \sec x}{x \sin x + \cos x} + \tan x + C$$

**9.** Let  $f(x) = |x-2|$  and  $g(x) = f(f(x))$ ,  $x \in [0, 4]$ . Then  $\int_0^3 (g(x) - f(x)) dx$  is equal to:

(1)  $\frac{1}{2}$

(2) 0

(3) 1

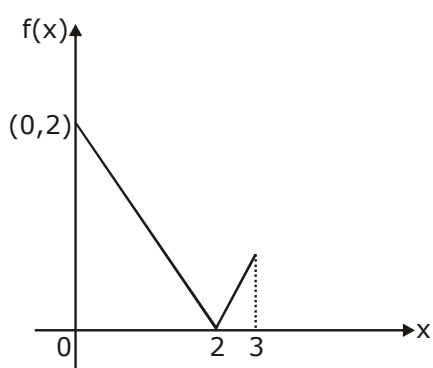
(4)  $\frac{3}{2}$

**Sol. (3)**

$$f(x) = |x - 2|$$

$$g(x) = ||x - 2| - 2| = \begin{cases} \text{if } x \geq 2 \Rightarrow |x - 4| \\ \text{if } x < 2 \Rightarrow |-x| \end{cases}$$

$$\therefore \int_0^3 (g(x) - f(x)) dx$$



$$\begin{aligned}
 &= \int_0^3 g(x) - \int_0^3 f(x) dx \\
 &= \int_0^2 x dx + \int_2^3 (4-x) dx - \int_0^2 (2-x) dx - \int_2^3 (x-2) dx \\
 &\Rightarrow \left( \frac{x^2}{2} \right)_0^2 + \left( 4x - \frac{x^2}{2} \right)_2^3 + \left( \frac{x^2}{2} - 2x \right)_0^2 - \left( \frac{x^2}{2} - 2x \right)_2^3 \\
 &\Rightarrow 2 + \left\{ 12 - \frac{9}{2} - 8 + 2 \right\} + \{ 2 - 4 \} - \left( \frac{9}{2} - 6 - 2 + 4 \right) \\
 &= 2 + \left\{ 6 - \frac{9}{2} \right\} - 2 - \left\{ \frac{9}{2} - 4 \right\} = 2 + \frac{3}{2} - \left( 2 + \frac{1}{2} \right) = \frac{7}{2} - \frac{5}{2} = 1
 \end{aligned}$$

- 10.** Let  $x_0$  be the point of Local maxima of  $f(x) = \vec{a} \cdot (\vec{b} \times \vec{c})$ , where  $\vec{a} = x\hat{i} - 2\hat{j} + 3\hat{k}$ ,  $\vec{b} = -2\hat{i} + x\hat{j} - \hat{k}$  and

$\vec{c} = 7\hat{i} - 2\hat{j} + x\hat{k}$ . Then the value of  $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$

at  $x=x_0$  is :

- (1) -22                      (2) -4                      (3) -30                      (4) 14

**Sol. (1)**

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} x & -2 & 3 \\ -2 & x & -1 \\ 7 & -2 & x \end{vmatrix}$$

$$\Rightarrow x\{x^2 - 2\} + 2\{-2x + 7\} + 3\{4 - 7x\}$$

$$= x^3 - 2x - 4x + 14 + 12 - 21x$$

$$f(x) = x^3 - 27x + 26$$

$$f'(x) = 3x^2 - 27 = 0 \Rightarrow x = \pm 3$$

$$\text{Max at } x_0 = -3$$

$$\therefore \vec{a} = (-3, -2, 3), \vec{b} = (-2, -3, -1), \vec{c} = (7, -2, -3)$$

$$\begin{aligned} \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} &= 6 + 6 - 3 - 14 + 6 + 3 - 21 + 4 - 9 \\ &= 25 - 47 = -22 \end{aligned}$$

- 11.** A triangle ABC lying in the first quadrant has two vertices as A(1,2) and B(3,1) If  $\angle BAC = 90^\circ$ , and  $\text{ar}(\triangle ABC) = 5\sqrt{5}$  s units, then the abscissa of the vertex C is :

- (1)  $1 + \sqrt{5}$                       (2)  $1 + 2\sqrt{5}$                       (3)  $2\sqrt{5} - 1$                       (4)  $2 + \sqrt{5}$

**Sol. (2)**

$$AB = \sqrt{4+1} = \sqrt{5}$$

$$\frac{1}{2} \times \sqrt{5} \times x = 5\sqrt{5}$$

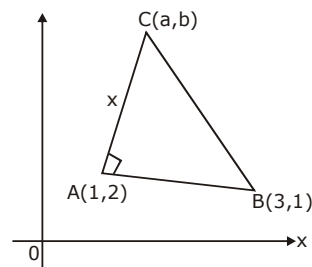
$$x = 10$$

$$m_{AB} = \frac{1}{-2}$$

$$m_{AC} = 2 = \tan \theta$$

$$\therefore \sin \theta = \frac{2}{\sqrt{5}}, \cos \theta = \frac{1}{\sqrt{5}}$$

$$\text{by parametric co-ordinates } a = 1 + 10 \times \frac{1}{\sqrt{5}} = 1 + 2\sqrt{5}$$



**12.** Let  $f$  be a twice differentiable function on  $(1, 6)$ . If  $f(2)=8$ ,  $f'(2)=5$ ,  $f'(x) \geq 1$  and  $f''(x) \geq 4$ , for all  $x \in (1, 6)$ , then:

$$(1) f(5)+f'(5) \geq 28$$

$$(2) f'(5)+f''(5) \leq 20$$

$$(3) f(5) \leq 10$$

$$(4) f(5)+f'(5) \leq 26$$

**Sol. (1)**

$$f(2) = 8, f'(2) = 5, f'(x) \geq 1, f''(x) \geq 4$$

$$x \in (1, 6)$$

$$\int_2^5 f'(x) \geq \int_2^5 1 dx$$

$$f(5) - f(2) \geq 3$$

$$f(5) \geq 11 \quad \dots(1)$$

$$\text{also } \int_2^5 f''(x) dx \geq \int_2^5 4 dx$$

$$f'(5) - f'(2) \geq 12$$

$$f'(5) - 5 \geq 12$$

$$f'(5) \geq 17 \quad \dots(2)$$

**13.** Let  $\alpha$  and  $\beta$  be the roots of  $x^2-3x+p=0$  and  $\gamma$  and  $\delta$  be the roots of  $x^2-6x+q=0$ . If  $\alpha, \beta, \gamma, \delta$  form a geometric progression. Then ratio  $(2q+p):(2q-p)$  is:

$$(1) 33:31$$

$$(2) 9:7$$

$$(3) 3:1$$

$$(4) 5:3$$

**Sol. (2)**

$$x^2 - 3x + p = 0 (\alpha, \beta)$$

$$x^2 - 6x + q = 0 (\gamma, \delta)$$

$$\alpha + \beta = 3$$

$$\gamma + \delta = 6$$

$$\alpha = a, \beta = ar, \gamma = ar^2, \delta = ar^3$$

$$a(1+r) = 3 \quad \dots(1)$$

$$ar^2(1+r) = 6 \quad \dots(2)$$

Divide (2) by (1)

$$r^2 = 2, r = \sqrt{2} \Rightarrow a = \frac{3}{\sqrt{2}+1}$$

$$\alpha = \frac{3}{\sqrt{2}+1}, \beta = \frac{3\sqrt{2}}{\sqrt{2}+1}, \gamma = \frac{3.2}{\sqrt{2}+1}, \delta = \frac{3.2\sqrt{2}}{\sqrt{2}+1}$$

$$\alpha\beta = p = \frac{9\sqrt{2}}{(\sqrt{2}+1)^2}, \gamma\delta = \frac{36\sqrt{2}}{(\sqrt{2}+1)^2} \Rightarrow \frac{72+9}{72-9} = \frac{81}{63} = 9/7$$

- 14.** Let  $u = \frac{2z+i}{z-ki}$ ,  $z = x + iy$  and  $k > 0$ . If the curve represented by  $\text{Re}(u) + \text{Im}(u) = 1$  intersects the y-axis at the points P and Q where  $PQ = 5$ , then the value of k is :  
 (1) 4 (2) 1/2 (3) 2 (4) 3/2

**Sol. (3)**

$$u = \frac{2z+i}{z-ki}, \quad z = x + iy$$

$$= \frac{2x+i(2y+1)}{x+i(y-k)} \times \frac{x-i(y-k)}{x-i(y-k)}$$

$$\Rightarrow \frac{2x^2 + (2y+1)(y-k) + i\{2xy + x - 2xy + 2xk\}}{x^2 + (y-k)^2}$$

$$\text{Re}(u) + \text{Im}(u) = 1$$

$$2x^2 + (2y+1)(y-k) + x + 2xk = x^2 + (y-k)^2$$

at y - axis,  $x = 0$

$$(2y+1)(y-k) = (y-k)^2$$

$$2y^2 + y - 2yk - k = y^2 + k^2 - 2yk$$

$$y^2 + y - (k + k^2) = 0 \quad (y_1, y_2)$$

diff. of roots = 5

$$\sqrt{1 + 4k + 4k^2} = 5$$

$$4k^2 + 4k = 24$$

$$k^2 + k - 6 = 0$$

$$(k+3)(k-2) = 0$$

$$k = 2$$

- 15.** If  $A = \begin{bmatrix} \cos \theta & i \sin \theta \\ i \sin \theta & \cos \theta \end{bmatrix}$ ,  $\left(\theta = \frac{\pi}{24}\right)$  and  $A^5 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ , where  $i = \sqrt{-1}$ , then which one of the following is not true?

(1)  $a^2 - d^2 = 0$                       (B)  $a^2 - c^2 = 1$                       (C)  $0 \leq a^2 + b^2 \leq 1$                       (D)  $a^2 - b^2 = \frac{1}{2}$

**Sol. (4)**

$$\begin{bmatrix} c & is \\ is & c \end{bmatrix} \begin{bmatrix} c & is \\ is & c \end{bmatrix} = z \begin{bmatrix} c^2 - s^2 & 2ics \\ 2ics & c^2 - s^2 \end{bmatrix} = \begin{bmatrix} \cos 2\theta & i \sin 2\theta \\ i \sin 2\theta & \cos 2\theta \end{bmatrix} \quad (\text{where } c = \cos \theta, s = \sin \theta)$$

$$A^5 = \begin{bmatrix} \cos(2^4 \theta) & i \sin(2^4 \theta) \\ i \sin(2^4 \theta) & \cos(2^4 \theta) \end{bmatrix}$$

$$a = d = \cos(16\theta)$$

$$b = c = i \sin(16\theta)$$

$$a^2 - b^2 = \cos^2(16\theta) + \sin^2 16\theta = 1$$

- 16.** The mean and variance of 8 observations are 10 and 13.5, respectively. If 6 of these observations are 5, 7, 10, 12, 14, 15, then the absolute difference of the remaining two observations is:

(1) 3                      (2) 9                      (3) 7                      (4) 5

**Sol. 3**

$$\frac{5 + 7 + 10 + 12 + 14 + 15 + x + y}{8} = 10$$

$$x + y = 17 \quad \dots(1)$$

$$\text{variance} = \frac{739 + x^2 + y^2}{8} - 100 = 13.5$$

$$x^2 + y^2 = 169 \quad \dots(2)$$

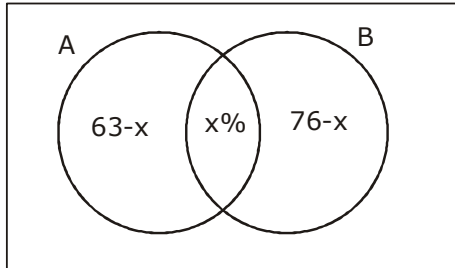
$$\therefore x = 12, y = 5$$

$$|x - y| = 7$$

- 17.** A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. If  $x\%$  of the people read both the newspapers, then a possible value of  $x$  can be:

(1) 37                      (2) 29                      (3) 65                      (4) 55

**Sol. (4)**



$$A \cup B = 13 - x \leq 100$$

$$x \geq 39$$

$$\text{also } x \leq 63$$

**18.** Given the following two statements:

(S<sub>1</sub>):  $(q \vee p) \rightarrow (P \leftrightarrow \sim q)$  is a tautology

(S<sub>2</sub>):  $\sim q \wedge (\sim p \leftrightarrow q)$  is a fallacy. Then:

(1) only (S<sub>1</sub>) is correct.

(2) both (S<sub>1</sub>) and (S<sub>2</sub>) are correct.

(3) only (S<sub>2</sub>) is correct

(4) both (S<sub>1</sub>) and (S<sub>2</sub>) are not correct.

**Sol. (4)**

	p	q	$\sim q$	$q \vee p$	$p \leftrightarrow \sim q$	$(q \vee p) \rightarrow (p \leftrightarrow \sim q)$
$S_1 =$	T	T	F	T	F	F
	T	F	T	T	T	T
	F	T	F	T	T	T
	F	F	T	F	F	T

S<sub>1</sub> is not correct

	p	q	$\sim q$	$\sim p$	$\sim p \leftrightarrow q$	$\sim q \wedge (\sim p \leftrightarrow q)$
$S_2 =$	T	T	F	F	F	F
	T	F	T	F	T	T
	F	T	F	T	T	F
	F	F	T	T	F	F

S<sub>2</sub> is false

**19.** Two vertical poles AB=15 m and CD=10 m are standing apart on a horizontal ground with points A and C on the ground. If P is the point of intersection of BC and AD, then the height of P (in m) above the line AC is:

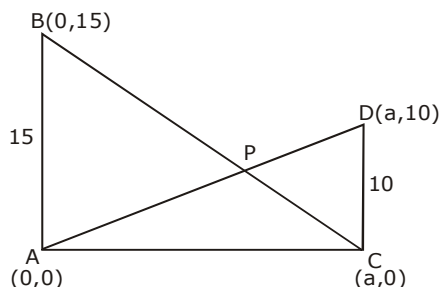
(1) 5

(2) 20/3

(3) 10/3

(4) 6

**Sol. (4)**



equation of AD :  $y = \frac{10x}{a}$

equation of BC :  $\frac{x}{a} + \frac{y}{15} = 1$

$$\Rightarrow \frac{a.y}{10a} + \frac{y}{15} = 1 \Rightarrow \frac{3y + 2y}{30} = 1$$

$$5y = 30 \Rightarrow y = 6$$

**20.** If  $(a + \sqrt{2} b \cos x)(a - \sqrt{2} b \cos y) = a^2 - b^2$ , where  $a > b > 0$ , then  $\frac{dx}{dy}$  at  $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$  is:

(1)  $\frac{a+b}{a-b}$

(2)  $\frac{a-2b}{a+2b}$

(3)  $\frac{a-b}{a+b}$

(4)  $\frac{2a+b}{2a-b}$

**Sol. (1)**

$$(a + \sqrt{2} b \cos x)(a - \sqrt{2} b \cos y) = a^2 - b^2$$

diff both sides w.r.t y

$$-\sqrt{2} b \sin x \cdot \frac{dx}{dy} (a - \sqrt{2} b \cos y) + (a + \sqrt{2} b \cos x) (\sqrt{2} b \sin y) = 0$$

$$x = y = \frac{\pi}{4} \Rightarrow \frac{-b dx}{dy} (a - b) + (a + b)(b) = 0$$

$$\frac{dx}{dy} = \frac{a+b}{a-b}$$

**21.** Suppose a differentiable function  $f(x)$  satisfies the identity  $f(x+y) = f(x) + f(y) + xy^2 + x^2y$ , for all real x

and y. If  $\lim_{x \rightarrow 0} \frac{f(x)}{x} = 1$ , then  $f'(3)$  is equal to.....

**Sol.**  $f(x+y) = f(x) + f(y) + xy^2 + x^2y$

$$x = y = 0$$

$$f(0) = 2f(0) \Rightarrow f(0) = 0$$

Partially diff. w.r.t. x  
 $f'(x+y) = f'(x) + y^2 + 2xy$   
 $x=0, y=x$

$$f'(x) = f'(0) + x^2 \quad \text{given } \lim_{x \rightarrow 0} \frac{f(x)}{x} = 1$$

$$f'(x) = 1 + x^2 \quad \text{by L' hospital}$$

$$\therefore f(x) = x + \frac{x^3}{3} + c \quad \lim_{x \rightarrow 0} \frac{f'(x)}{1} = 1$$

$$\text{put } x=0 \Rightarrow c=0 \quad f'(0)=1$$

$$f'(3)=10$$

- 22.** If the equation of a plane P, passing through the intersection of the planes,  $x+4y-z+7=0$  and  $3x+y+5z=8$  is  $ax+by+6z=15$  for some  $a, b \in \mathbb{R}$ , then the distance of the point  $(3, 2, -1)$  from the plane P is.....

**Sol.**  $p_1 + \lambda p_2 = 0$   
 $(x+4y-z+7) + \lambda(3x+y+5z-8) = ax+by+6z-15$

$$\frac{1-3\lambda}{a} = \frac{4+\lambda}{b} = \frac{-1+5\lambda}{6} = \frac{7-8\lambda}{-15}$$

$$\therefore 15-75\lambda = 42-48\lambda$$

$$-27 = 27\lambda$$

$$\lambda = -1$$

$$\therefore \text{plane is } -2x + 3y - 6z + 15 = 0$$

$$d = \left| \frac{-6+6+6+15}{\sqrt{4+9+36}} \right| = 3$$

- 23.** If the system of equations

$$x-2y+3z=9$$

$$2x+y+z=b$$

$$x-7y+az=24, \text{ has infinitely many solutions, then } a-b \text{ is equal to.....}$$

**Sol.**  $D=0$

$$\begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & 1 \\ 1 & -7 & a \end{vmatrix} = 0$$

$$1(a+7) + 2(2a-1) + 3(-14-1) = 0$$

$$a+7+4a-2-45=0$$

$$5a=40$$

$$a=8$$

$$D_1 = \begin{vmatrix} 9 & -2 & 3 \\ b & 1 & 1 \\ 24 & -7 & 8 \end{vmatrix} = 0$$

$$\Rightarrow 9(8 + 7) + 2(8b - 24) + 3(-7b - 24) = 0$$

$$\Rightarrow 135 + 16b - 48 - 21b - 72 = 0$$

$$15 = 5b \Rightarrow b = 3$$

$$a - b = 5$$

**24.** Let  $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$ . Then  $\frac{a_7}{a_{13}}$  is equal to .....

**Sol. 8**

$$(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$$

$$a_7 = \text{coeff of } x^7$$

$$a_{13} = \text{coeff of } x^{13}$$

$$\frac{10!}{p!q!r!} (2x^2)^p (3x)^q (4)^r$$

for  $x^7$

$$p \quad q \quad r$$

$$3 \quad 1 \quad 6$$

$$2 \quad 3 \quad 5$$

$$1 \quad 5 \quad 4$$

$$0 \quad 7 \quad 3$$

$$a_7 = \frac{2^3 \cdot 3 \cdot 10!}{3!6!} + \frac{10! \cdot 2^2 \cdot 3^3}{2!3!5!} + \frac{10! \cdot 2 \cdot 3^5}{5!4!} + \frac{10! \cdot 3^7}{7!3!}$$

for  $x^{13}$

$$p \quad q \quad r$$

$$6 \quad 1 \quad 3$$

$$5 \quad 3 \quad 2$$

$$4 \quad 5 \quad 1$$

$$3 \quad 7 \quad 0$$

$$a_{13} = \frac{2^6 \cdot 3 \cdot 10!}{6!3!} + \frac{2^5 \cdot 3^3 \cdot 10!}{5!3!2!} + \frac{2^4 \cdot 3^5 \cdot 10!}{4!5!} + \frac{2 \cdot 3^7 \cdot 10!}{3!7!} \therefore \frac{a_7}{a_{13}} = 8$$

- 25.** The probability of a man hitting a target is  $\frac{1}{10}$ . The least number of shots required, so that the probability of his hitting the target at least once is greater than  $\frac{1}{4}$ , is .....

**Sol. 3**

$$P(H) = \frac{1}{10} ; P(M) = \frac{9}{10}$$

$$P(H) + P(M) \cdot P(H) + P(M) \cdot P(M) \cdot P(H) + \dots$$

$$= 1 - P(M)^n \geq \frac{1}{4}$$

$$= 1 - \left(\frac{9}{10}\right)^n \geq \frac{1}{4}$$

$$\left(\frac{9}{10}\right)^n \leq \frac{3}{4} ; n \geq 3$$

# QUESTION PAPER WITH SOLUTION

## PHYSICS \_ 4 Sep. \_ SHIFT - 2

1. A circular coil has moment of inertia  $0.8 \text{ kg m}^2$  around any diameter and is carrying current to produce a magnetic moment of  $20 \text{ Am}^2$ . The coil is kept initially in a vertical position and it can rotate freely around a horizontal diameter. When a uniform magnetic field of  $4 \text{ T}$  is applied along the vertical, it starts rotating around its horizontal diameter. The angular speed the coil acquires after rotating by  $60^\circ$  will be:

(1)  $10 \pi \text{ rad s}^{-1}$       (2)  $20 \text{ rad s}^{-1}$       (3)  $20 \pi \text{ rad s}^{-1}$       (4)  $10 \text{ rad s}^{-1}$

Sol.

4

By energy conservation

$$U_i + K_i = U_f + K_f$$

$$-MB \cos 60^\circ + 0 = -MB \cos 0^\circ + \frac{1}{2} I \omega^2$$

$$-\frac{MB}{2} + MB = \frac{1}{2} I \omega^2$$

$$\frac{MB}{2} = \frac{1}{2} I \omega^2$$

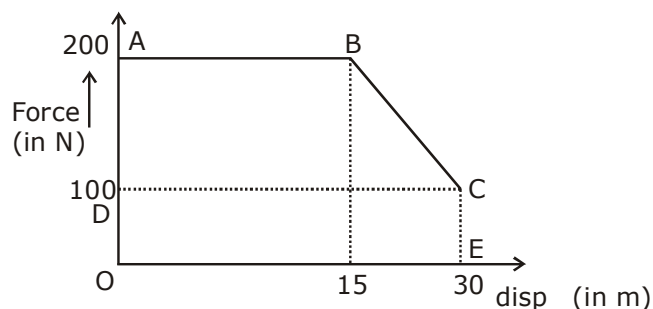
$$\omega = \sqrt{\frac{MB}{I}} = \sqrt{\frac{20 \times 4}{0.8}} = \sqrt{100} = 10 \text{ rad/s}$$

2. A person pushes a box on a rough horizontal platform surface. He applies a force of  $200 \text{ N}$  over a distance of  $15 \text{ m}$ . Thereafter, he gets progressively tired and his applied force reduces linearly with distance to  $100 \text{ N}$ . The total distance through which the box has been moved is  $30 \text{ m}$ . What is the work done by the person during the total movement of the box?

(1)  $5690 \text{ J}$       (2)  $5250 \text{ J}$       (3)  $2780 \text{ J}$       (4)  $3280 \text{ J}$

Sol.

2



Work done = area of ABCE  
= area of trap. ABCD + area of rect. ODCE

$$= \frac{1}{2} \times 45 \times 30 + 100 \times 30 = 5250 \text{ J}$$

3. Match the thermodynamic processes taking place in a system with the correct conditions. In the table :  $\Delta Q$  is the heat supplied,  $\Delta W$  is the work done and  $\Delta U$  is change in internal energy of the system.

Process	Condition
(I) Adiabatic	(1) $\Delta W = 0$
(II) Isothermal	(2) $\Delta Q = 0$
(III) Isochoric	(3) $\Delta U \neq 0, \Delta W \neq 0,$ $\Delta Q \neq 0$
(IV) Isobaric	(4) $\Delta U = 0$
(1) (I) - (1), (II) - (1), (III) - (2), (IV) - (3)	
(2) (I) - (1), (II) - (2), (III) - (4), (IV) - (4)	
(3) (I) - (2), (II) - (4), (III) - (1), (IV) - (3)	
(4) (I) - (2), (II) - (1), (III) - (4), (IV) - (3)	

**Sol.**

**3**

adiabatic,  $\Delta Q = 0$

Isothermal,  $\Delta U = 0$

Isochoric,  $\int p dV = 0$

$\Delta W = 0$

Isobaric,  $\Delta Q \neq 0, \Delta U \neq 0, \Delta W \neq 0$

4. The driver of a bus approaching a big wall notices that the frequency of his bus's horn changes from 420 Hz to 490 Hz when he hears it after it gets reflected from the wall. Find the speed of the bus if speed of the sound is  $330 \text{ ms}^{-1}$ .

(1)  $81 \text{ kmh}^{-1}$       (2)  $91 \text{ kmh}^{-1}$       (3)  $71 \text{ kmh}^{-1}$       (4)  $61 \text{ kmh}^{-1}$

**Sol.**

**2**

Freq received by wall,

$$f_w = \left( \frac{330}{330 - v} \right) f_0$$

$$\text{freq. after reflection, } f' = \left( \frac{330 + v}{330} \right) f_w$$

$$= \left( \frac{330 + v}{330} \right) \times \left( \frac{330}{330 - v} \right) f_0$$

$$490 = \left( \frac{330 + v}{330 - v} \right) 420$$

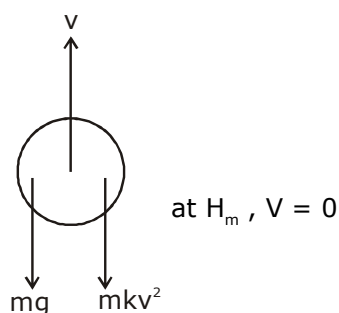
$$\therefore v = 25.2 \text{ m/s}$$

$$= 91 \text{ km/h}$$

5. A small ball of mass  $m$  is thrown upward with velocity  $u$  from the ground. The ball experiences a resistive force  $mkv^2$  where  $v$  is its speed. The maximum height attained by the ball is:

(1)  $\frac{1}{k} \tan^{-1} \frac{ku^2}{2g}$       (2)  $\frac{1}{2k} \ln \left( 1 + \frac{ku^2}{g} \right)$       (3)  $\frac{1}{k} \ln \left( 1 + \frac{ku^2}{2g} \right)$       (4)  $\frac{1}{2k} \tan^{-1} \frac{ku^2}{g}$

Sol. 2



$$F_{\text{net}} = ma$$

$$-mg - mKv^2 = mv \frac{dv}{ds}$$

$$\int_{s=0}^H ds = (-1) \int_{v=u}^{v=0} \frac{v dv}{g + kv^2}$$

$$\int \frac{x dx}{a + bx^2}$$

$$H_{\text{max}} = \frac{1}{2K} \ln \left( \frac{g + ku^2}{g} \right)$$

$$H_m = \frac{1}{2K} \ln \left( 1 + \frac{Ku^2}{g} \right)$$

6. Consider two uniform discs of the same thickness and different radii  $R_1 = R$  and  $R_2 = \alpha R$  made of the same material. If the ratio of their moments of inertia  $I_1$  and  $I_2$ , respectively, about their axes is  $I_1 : I_2 = 1 : 16$  then the value of  $\alpha$  is :

(1)  $\sqrt{2}$       (2) 2      (3)  $2\sqrt{2}$       (4) 4

**Sol. 2**

$$\text{Moment of inertia of disc, } I = \frac{MR^2}{2} = \frac{[\rho(\pi R^2)t]R^2}{2}$$

$$I = KR^4$$

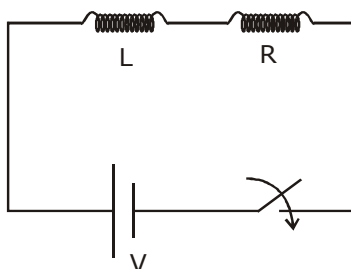
$$\frac{I_1}{I_2} = \left(\frac{R_1}{R_2}\right)^4$$

$$\frac{1}{16} = \left(\frac{R}{\alpha R}\right)^4 \Rightarrow \alpha = (16)^{\frac{1}{4}} = 2$$

- 7.** A series L-R circuit is connected to a battery of emf  $V$ . If the circuit is switched on at  $t = 0$ , then the time at which the energy stored in the inductor reaches  $\left(\frac{1}{n}\right)$  times of its maximum value, is :

(1)  $\frac{L}{R} \ln\left(\frac{\sqrt{n}}{\sqrt{n}+1}\right)$       (2)  $\frac{L}{R} \ln\left(\frac{\sqrt{n}}{\sqrt{n}-1}\right)$       (3)  $\frac{L}{R} \ln\left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)$       (4)  $\frac{L}{R} \ln\left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)$

**Sol. 2**



$$\text{P.E. in inductor, } U = \frac{1}{2}LI^2$$

$$U \propto I^2$$

$$\frac{U}{U_0} = \left(\frac{I}{I_0}\right)^2$$

$$\frac{1}{n} = \left(\frac{I}{I_0}\right)^2$$

$$I = \frac{I_0}{\sqrt{n}}$$

$$I = I_0 \left( 1 - e^{-\frac{R}{L}t} \right)$$

$$\frac{I_0}{\sqrt{n}} = I_0 \left( 1 - e^{-\frac{R}{L}t} \right)$$

taking  $\ell n$  & solving we get,

$$t = \frac{L}{R} \ell n \left( \frac{\sqrt{n}}{\sqrt{n} - 1} \right)$$

8. The electric field of a plane electromagnetic wave is given by

$$\vec{E} = E_0 (\hat{x} + \hat{y}) \sin(kz - \omega t)$$

Its magnetic field will be given by :

$$(1) \frac{E_0}{c} (\hat{x} + \hat{y}) \sin(kz - \omega t)$$

$$(2) \frac{E_0}{c} (\hat{x} - \hat{y}) \sin(kz - \omega t)$$

$$(3) \frac{E_0}{c} (\hat{x} - \hat{y}) \cos(kz - \omega t)$$

$$(4) \frac{E_0}{c} (-\hat{x} + \hat{y}) \sin(kz - \omega t)$$

Sol. (4)

$\vec{E} \times \vec{B}$  should be in direction of  $\vec{v}$

$$\therefore \vec{B} = \frac{E_0}{c} (-\hat{x} + \hat{y}) \sin(Kz - \omega t)$$

9. A cube of metal is subjected to a hydrostatic pressure of 4 GPa. The percentage change in the length of the side of the cube is close to :

(Given bulk modulus of metal,  $B = 8 \times 10^{10}$  Pa)

$$(1) 0.6$$

$$(2) 20$$

$$(3) 1.67$$

$$(4) 5$$

Sol. (3)

$$(-) \frac{\Delta P}{\frac{\Delta V}{V}} = B$$

$$\Delta P = \left( \frac{\Delta V}{V} \right) \cdot B$$

$$= \frac{3\Delta L}{L} \times B$$

$$\therefore \frac{\Delta L}{L} = \frac{\Delta P}{3B} \quad \therefore \% \text{ we get, } \frac{\Delta L}{L} \times 100\%$$

Putting values we get = 1.67

- 10.** A paramagnetic sample shows a net magnetisation of 6 A/m when it is placed in an external magnetic field of 0.4 T at a temperature of 4 K. When the sample is placed in an external magnetic field of 0.3 T at a temperature of 24 K, then the magnetisation will be:

(1) 4 A/m                      (2) 1 A/m                      (3) 0.75 A/m                      (4) 2.25 A/m

**Sol. (3)**

$$M = \frac{CB_{\text{ext}}}{T}$$

$$6 = \frac{C \times 0.4}{4}$$

$$\Rightarrow C = 60$$

$$\therefore \text{case - II :- } M = \frac{60 \times 0.3}{24} = \frac{60 \times 3}{240} = \frac{3}{4} = 0.75 \text{ A/m}$$

- 11.** A body is moving in a low circular orbit about a planet of mass M and radius R. The radius of the orbit can be taken to be R itself. Then the ratio of the speed of this body in the orbit to the escape velocity from the planet is:

(1) 2                      (2)  $\sqrt{2}$                       (3) 1                      (4)  $\frac{1}{\sqrt{2}}$

**Sol. (4)**

$$V_0 = \sqrt{\frac{GM}{r}}, \quad V_e = \sqrt{\frac{2GM}{r}}$$

$$\frac{v_0}{v_e} = \sqrt{\frac{GM}{r} \times \frac{r}{2GM}} = \frac{1}{\sqrt{2}}$$

- 12.** A particle of charge q and mass m is subjected to an electric field  $E = E_0 (1 - ax^2)$  in the x-direction, where a and  $E_0$  are constants. Initially the particle was at rest at  $x = 0$ . Other than the initial position the kinetic energy of the particle becomes zero when the distance of the particle from the origin is:

(1)  $\sqrt{\frac{2}{a}}$                       (2) a                      (3)  $\sqrt{\frac{3}{a}}$                       (4)  $\sqrt{\frac{1}{a}}$

**Sol. (3)**

$$W = \Delta KE$$

$$\int_0^x F dx = 0$$

$$\int_0^x qE dx = 0$$

$$q \int_0^x E_0 (1 - ax^2) dx = 0$$

$$qE_0 \left[ \int_0^x dx - a \int_0^x x^2 dx \right] = 0$$

$$qE_0 \left[ x - \frac{ax^3}{3} \right] = 0$$

$$x \left( 1 - \frac{ax^2}{3} \right) = 0$$

$$x=0, 1 - \frac{ax^2}{3} = 0$$

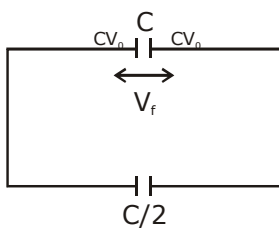
$$\frac{ax^2}{3} = 1$$

$$x = \sqrt{\frac{3}{a}}$$

- 13.** A capacitor  $C$  is fully charged with voltage  $V_0$ . After disconnecting the voltage source, it is connected in parallel with another uncharged capacitor of capacitance  $\frac{C}{2}$ . The energy loss in the process after the charge is distributed between the two capacitors is:

(1)  $\frac{1}{2} CV_0^2$       (2)  $\frac{1}{4} CV_0^2$       (3)  $\frac{1}{3} CV_0^2$       (4)  $\frac{1}{6} CV_0^2$

**Sol. (4) Our Answer**  
**NTA Answer (2)**



$$V_f = \frac{CV_0}{3 \frac{C}{2}} = \frac{2V_0}{3}$$

$$u_i = \frac{1}{2}cv_0^2$$

$$u_f = \frac{1}{2}\left(\frac{3c}{2}\right) \cdot \frac{4v_0^2}{9} = \frac{CV_0^2}{3}$$

$$u_i - u_f = \frac{1}{2}cv_0^2 - \frac{cv_0^2}{3}$$

$$= cv_0^2\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{cv_0^2}{6}$$

- 14.** Find the Binding energy per nucleon for  $^{120}_{50}\text{Sn}$ . Mass of proton  $m_p = 1.00783$  U, mass of neutron  $m_n = 1.00867$  U and mass of tin nucleus  $m_{\text{Sn}} = 119.902199$  U. (take  $1\text{U} = 931$  MeV)  
 (1) 8.0 MeV                      (2) 9.0 MeV                      (3) 7.5 MeV                      (4) 8.5 MeV

**Sol. (4)**

$$\text{B.E.} = \Delta mc^2$$

$$= \Delta m \times 931$$

$$\Delta m = (50 \times 1.00783) + (70 \times 1.00867) - \{119.902199\}$$

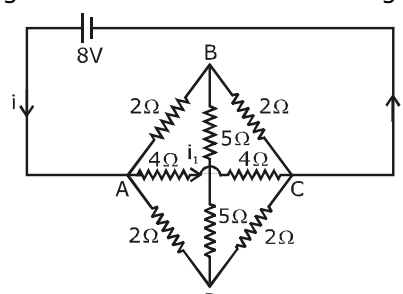
$$= \{120.9984 - 119.902199\} \text{ U}$$

$$= 1.1238 \text{ U}$$

$$\text{BE} = 1.1238 \times 931 = 1046.2578 \text{ MeV}$$

$$\text{BE per nucleon} \simeq 1046/120 \approx 8.5 \text{ MeV}$$

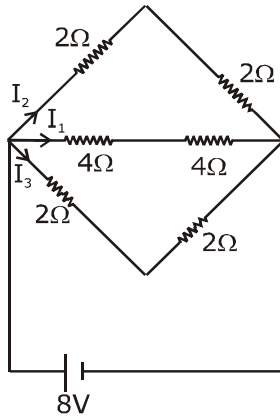
- 15.** The value of current  $i_1$  flowing from A to C in the circuit diagram is:



- (1) 4 A                      (2) 5 A                      (3) 2 A                      (4) 1 A

**Sol. (4)**

eq circuit  $\Rightarrow$

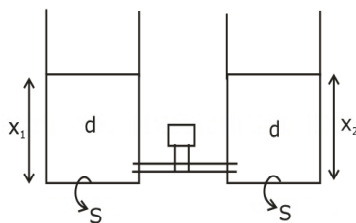


$$I_2 = \frac{8}{4+4} = 1 \text{ amp}$$

- 16.** Two identical cylindrical vessels are kept on the ground and each contain the same liquid of density  $d$ . The area of the base of both vessels is  $S$  but the height of liquid in one vessel is  $x_1$  and in the other,  $x_2$ . When both cylinders are connected through a pipe of negligible volume very close to the bottom, the liquid flows from one vessel to the other until it comes to equilibrium at a new height. The change in energy of the system in the process is:

(1)  $gdS(x_2 + x_1)^2$     (2)  $gdS(x_2^2 + x_1^2)$     (3)  $\frac{1}{4}gdS(x_2 - x_1)^2$     (4)  $\frac{3}{4}gdS(x_2 - x_1)^2$

**Sol. (3)**



$$u_i = \left[ dsx_1 \cdot \frac{x_1}{2} + dsx_2 \cdot \frac{x_2}{2} \right] g \quad \left\{ dsx_1 \rightarrow m, \frac{x_1}{2} \rightarrow h(\text{com}) \right\}$$

$$u_f = \left[ ds \left( \frac{x_1 + x_2}{2} \right) \times \left( \frac{x_1 + x_2}{4} \right) \times 2 \right] g$$

$$u_i - u_f = dsg \left[ \frac{x_1^2}{2} + \frac{x_2^2}{2} - \frac{(x_1 + x_2)^2}{4} \right]$$

$$= dsg \frac{(x_1 - x_2)^2}{4}$$

- 17.** A quantity  $x$  is given by  $(IFv^2/WL^4)$  in terms of moment of inertia  $I$ , force  $F$ , velocity  $v$ , work  $W$  and Length  $L$ . The dimensional formula for  $x$  is same as that of :

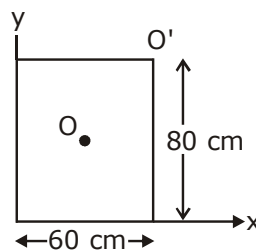
- (1) coefficient of viscosity (2) energy density  
(3) force constant (4) planck's constant

**Sol. (2)**

$$[x] = \frac{IFv^2}{WL^4} = \frac{(M^1L^2)(MLT^{-2})(LT^{-2})^2}{(ML^2T^{-2})L^4}$$

$$= ML^{-1}T^{-2} = \text{Energy density}$$

- 18.** For a uniform rectangular sheet shown in the figure, the ratio of moments of inertia about the axes perpendicular to the sheet and passing through  $O$  (the centre of mass) and  $O'$  (corner point) is:



- (1) 1/2 (2) 2/3 (3) 1/4 (4) 1/8

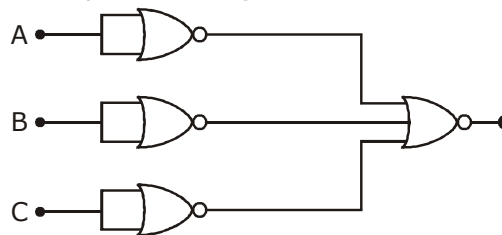
**Sol. (3)**

$$I_0 = \frac{M}{12}(a^2 + b^2)$$

$$I_{O'} = \frac{M}{12}(a^2 + b^2) + M\left(\frac{a^2}{4} + \frac{b^2}{4}\right)$$

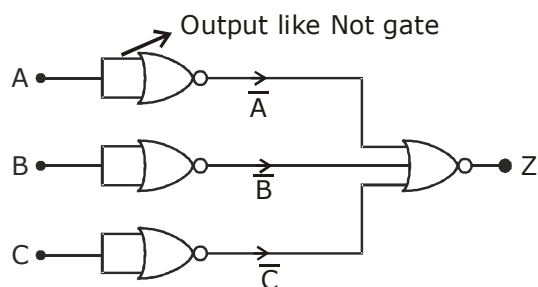
$$\frac{I_0}{I_{O'}} = \frac{\frac{M}{12}(a^2 + b^2)}{\frac{M}{12}(a^2 + b^2) + \frac{M}{4}(a^2 + b^2)} = \frac{\frac{1}{12}}{\frac{1}{12} + \frac{1}{4}} = \frac{1}{12} \times \frac{3}{1} = \frac{1}{4}$$

- 19.** Identify the operation performed by the circuit given below:



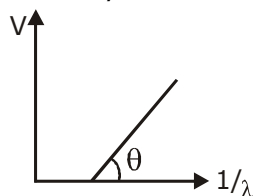
- (1) NOT (2) OR (3) AND (4) NAND

19. (3)



$$Z = \bar{A} + \bar{B} + \bar{C} = A.B.C \text{ (AND gate)}$$

20. In a photoelectric effect experiment, the graph of stopping potential  $V$  versus reciprocal of wavelength obtained is shown in the figure. As the intensity of incident radiation is increased:



- (1) Straight line shifts to right
- (2) Straight line shifts to left
- (3) Slope of the straight line get more steep
- (4) Graph does not change

Sol. (4)

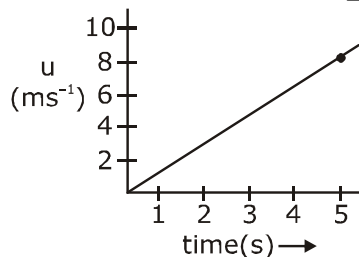
$$eV = h\nu - w \text{ (w = work function)}$$

$$V = \frac{h\nu}{e} - \frac{w}{e}$$

$$\text{as } \frac{h}{e} \& \frac{w}{e} \rightarrow \text{constant}$$

Therefore no change in graph.

21. The speed versus time graph for a particle is shown in the figure. The distance travelled (in m) by the particle during the time interval  $t = 0$  to  $t = 5$  s will be \_\_\_\_\_.

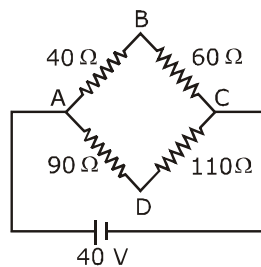


**Sol. 20**

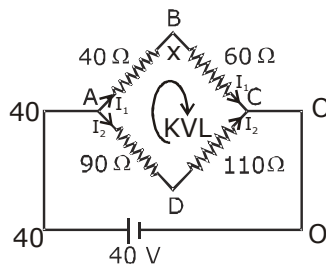
Distance = Area under speed – time graph

$$= \frac{1}{2} \times 8 \times 5 = 20\text{m}$$

- 22.** Four resistances  $40\ \Omega$ ,  $60\ \Omega$ ,  $90\ \Omega$  and  $110\ \Omega$  make the arms of a quadrilateral ABCD. Across AC is a battery of emf  $40\ \text{V}$  and internal resistance negligible. The potential difference across BD in V is \_\_\_\_\_.



**Sol. 2**



$$I_1 = \frac{40}{100}$$

$$I_2 = \frac{40}{200}$$

$$V_B - \frac{40}{100} \times 60 + 110 \times \frac{40}{200}$$

$$= V_D$$

$$V_B - V_D = \frac{40 \times 60}{100} - \frac{100 \times 40}{200}$$

$$= 24 - 22$$

$$= 2\text{V}$$

- 23.** The change in the magnitude of the volume of an ideal gas when a small additional pressure  $\Delta P$  is applied at a constant temperature, is the same as the change when the temperature is reduced by a small quantity  $\Delta T$  at constant pressure. The initial temperature and pressure of the gas were 300 K and 2 atm. respectively. If  $|\Delta T| = C|\Delta P|$  then value of C in (K/atm.) is \_\_\_\_\_.

**Sol. 150**

1st case

$$PV = nRT$$

$$PdV + VdP = 0$$

$$P\Delta V + V\Delta P = 0 \quad \Delta v = \frac{-\Delta P}{P} v$$

2nd case

$$P\Delta V = -nR\Delta T$$

$$\Delta V = -\frac{nR\Delta T}{P}$$

$$-\frac{\Delta P}{P} V = \frac{-nR\Delta T}{P} \Rightarrow \Delta T = \Delta P \frac{V}{nR}$$

$$\Rightarrow \frac{\Delta T}{\Delta P} = \frac{V}{nR}$$

Now, given  $|\Delta T| = C|\Delta P|$

$$C = \frac{\Delta T}{\Delta P} = \frac{V}{nR}$$

$$C = \frac{T}{P} = \frac{300}{2} = 150$$

- 24.** Orange light of wavelength  $6000 \times 10^{-10}$  m illuminates a single slit of width  $0.6 \times 10^{-4}$  m. The maximum possible number of diffraction minima produced on both sides of the central maximum is \_\_\_\_\_.

**Sol. 200**

For minima

$$d \sin \theta = n\lambda$$

$$\text{or } \sin \theta = \frac{n\lambda}{d}$$

$\therefore$  maximum value of  $\sin \theta$  is 1

$$\therefore \frac{n\lambda}{d} \leq 1$$

$$n \leq \frac{d}{\lambda}$$

$$n \leq \frac{0.6 \times 10^{-4}}{6000 \times 10^{-10}}$$

$$n \leq 100$$

for both sides  $100 + 100 = 200$

- 25.** The distance between an object and a screen is 100 cm. A lens can produce real image of the object on the screen for two different positions between the screen and the object. The distance between these two positions is 40 cm. If the power of the lens is close to  $\left(\frac{N}{100}\right)D$  where N is an integer, the value of N is \_\_\_\_\_.

**Sol. 5**

$$\therefore f = \frac{D^2 - d^2}{4D} = \frac{100^2 - 40^2}{400}$$

$$= \frac{10000 - 1600}{400}$$

$$= \frac{100 - 16}{4} = \frac{84}{4} = 21$$

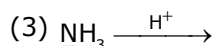
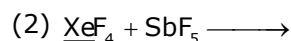
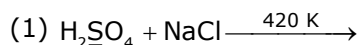
$$p = \frac{1}{f} = \frac{1}{21} = \frac{1}{21} \times \frac{100}{100} = \left(\frac{4.76}{100}\right) = \frac{N}{100}$$

$$\therefore \boxed{N \approx 5}$$

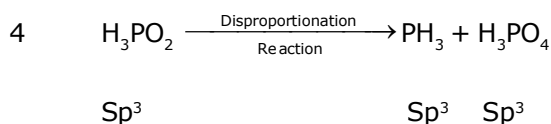
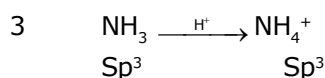
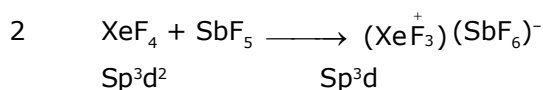
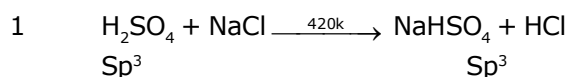
# QUESTION PAPER WITH SOLUTION

## CHEMISTRY \_ 4 Sep. \_ SHIFT - 2

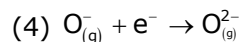
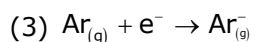
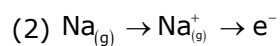
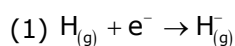
1. The reaction in which the hybridisation of the underlined atom is affected is :



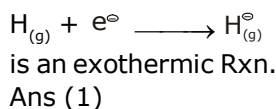
Sol. 2



2. The process that is NOT endothermic in nature is :



Sol. 1



3. If the equilibrium constant for  $\text{A} \rightleftharpoons \text{B} + \text{C}$  is  $K_{\text{eq}}^{(1)}$  and that of  $\text{B} + \text{C} \rightleftharpoons \text{P}$  is  $K_{\text{eq}}^{(2)}$ , the equilibrium constant for  $\text{A} \rightleftharpoons \text{P}$  is :

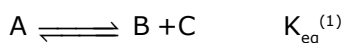
(1)  $K_{\text{eq}}^{(1)} K_{\text{eq}}^{(2)}$

(2)  $K_{\text{eq}}^{(2)} - K_{\text{eq}}^{(1)}$

(3)  $K_{\text{eq}}^{(1)} + K_{\text{eq}}^{(2)}$

(4)  $K_{\text{eq}}^{(1)} / K_{\text{eq}}^{(2)}$

Sol. 1



$K_{\text{eq}} = K_{\text{eq}}^{(1)} \times K_{\text{eq}}^{(2)}$   
 Ans.(1)

4. A sample of red ink (a colloidal suspension) is prepared by mixing eosin dye, egg white, HCHO and water. The component which ensures stability of the ink sample is :

(1) HCHO (2) Water (3) Eosin dye (4) Egg white

**Sol. 4**

Surface theoretical eggwhite

5. The one that can exhibit highest paramagnetic behaviour among the following is :  
gly = glycinato; bpy = 2, 2'-bipyridine

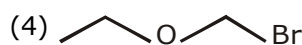
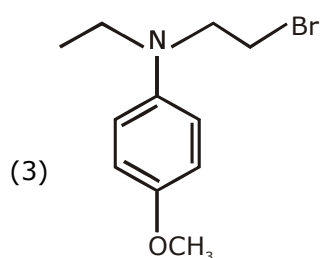
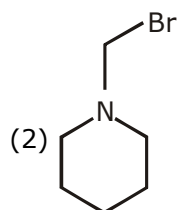
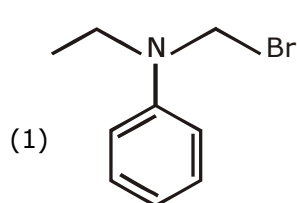
(1)  $[\text{Ti}(\text{NH}_3)_6]^{3+}$  (2)  $[\text{Co}(\text{OX})_2(\text{OH})_2]^-$  ( $\Delta_0 > P$ )

(3)  $[\text{Pd}(\text{gly})_2]$  (4)  $[\text{Fe}(\text{en})(\text{bpy})(\text{NH}_3)_2]^{2+}$

**Sol. 2**

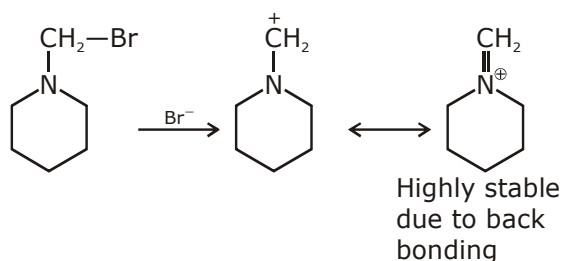
- $(\text{Ti}(\text{NH}_3)_6)^{3+} \Rightarrow \text{Ti}^{3+} (3d^1) \Rightarrow \mu = \sqrt{3}$
- $[\text{Co}(\text{OX}_2)(\text{OH})_2]^- (\Delta_0 > P) \Rightarrow \text{Co}^{+5} (3d^4) \Rightarrow t_2g^4 eg^0$   
 $n = 2, \mu = \sqrt{8}$
- $(\text{Pd}(\text{gly})_2) \Rightarrow \text{Pd}^{2+} (4d^8) \rightarrow \text{Square planar}$   
 $n = 0, \mu = 0$  diamagnetic
- $(\text{Fe}(\text{en})(\text{bpy})(\text{NH}_3)_2)^{2+}$   
 $\text{Fe}^{2+} \Rightarrow 3d^6 (t_2g^6 eg^0) \Rightarrow n = 0, \mu = 0$

6. Which of the following compounds will form the precipitate with aq.  $\text{AgNO}_3$  solution most readily?



**Sol. 2**

Rate of reaction  $\propto$  stability of carbocation.



7. Five moles of an ideal gas at 1 bar and 298 K is expanded into vacuum to double the volume. The work done is :

(1) zero                      (2)  $C_v (T_2 - T_1)$                       (3)  $-RT(V_2 - V_1)$                       (4)  $-RT \ln V_2/V_1$

**Sol. 1**

As it is free expansion against zero ext. pressure

$\therefore$  Work Done = zero

Ans. (1)

8. 250 mL of a waste solution obtained from the workshop of a goldsmith contains 0.1 M  $\text{AgNO}_3$  and 0.1 M  $\text{AuCl}$ . The solution was electrolyzed at 2 V by passing a current of 1 A for 15 minutes. The metal/metals electrodeposited will be:

$$(E_{\text{Ag}^+/\text{Ag}}^0 = 0.80 \text{ V}, E_{\text{Au}^+/\text{Au}}^0 = 1.69 \text{ V})$$

- (1) Silver and gold in proportion to their atomic weights  
(2) Silver and gold in equal mass proportion  
(3) only silver  
(4) only gold

**Sol. 1**

$$\text{Amount of charge transferred} = \frac{1 \times 15 \times 60}{96500} = \frac{9}{965} \approx 10 \times 10^{-3}$$

$$\text{moles of gold deposited} = \frac{0.1 \times 250}{1000} = 25 \times 10^{-3}$$

Both will be deposited

Ans.(1)

9. The mechanism of action of "Terfenadine" (Seldane) is :

- (1) Helps in the secretion of histamine                      (2) Activates the histamine receptor  
(3) Inhibits the secretion of histamine                      (4) Inhibits the action of histamine receptor

**Sol. 4**

The mechanism of action of "Terfenadine" (Seldane) is to inhibit the action of histamine receptor.

10. The shortest wavelength of H atom in the Lyman series is  $\lambda_1$ . The longest wavelength in the Balmer series of  $\text{He}^+$  is :

(1)  $\frac{9\lambda_1}{5}$                       (2)  $\frac{27\lambda_1}{5}$                       (3)  $\frac{36\lambda_1}{5}$                       (4)  $\frac{5\lambda_1}{9}$

Sol. 1

$$\frac{1}{\lambda_1} = R_H \times (1)^2 \times \left\{ 1 - \frac{1}{\infty^2} \right\} = R_H$$

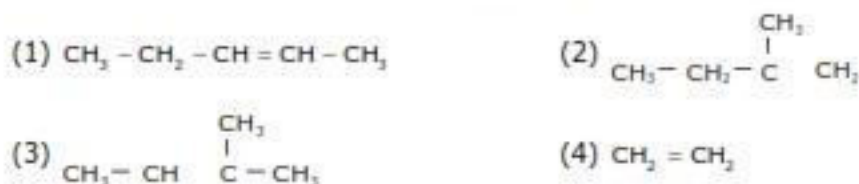
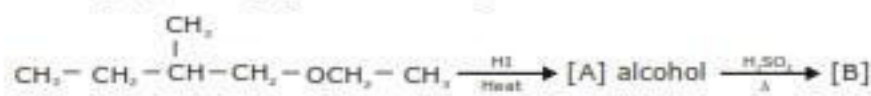
$$\frac{1}{\lambda_2} = R_H \times (2)^2 \times \left\{ \frac{1}{4} - \frac{1}{\infty} \right\} = R_H \left\{ \frac{5}{4} \right\}$$

$$\frac{\lambda_2}{\lambda_1} = \frac{9}{5}$$

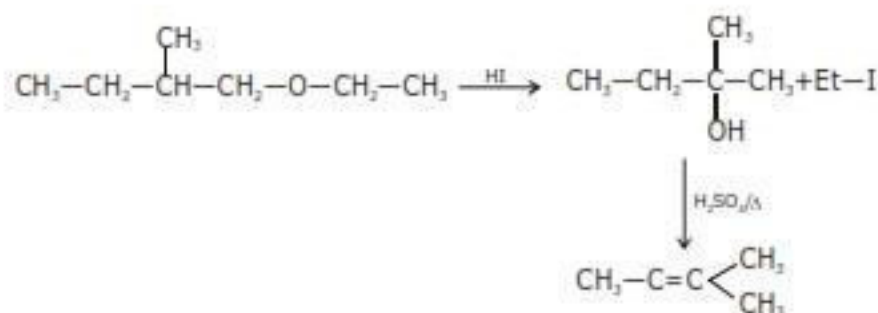
$$\lambda_2 = \frac{9}{5} \lambda_1$$

Ans. (1)

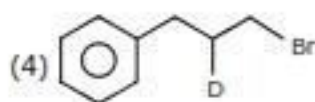
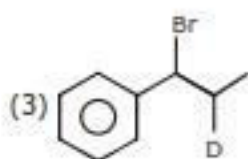
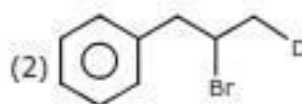
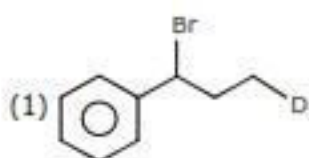
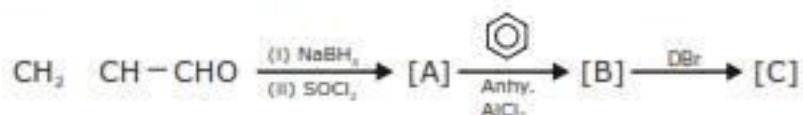
11. The major product [B] in the following reactions is :



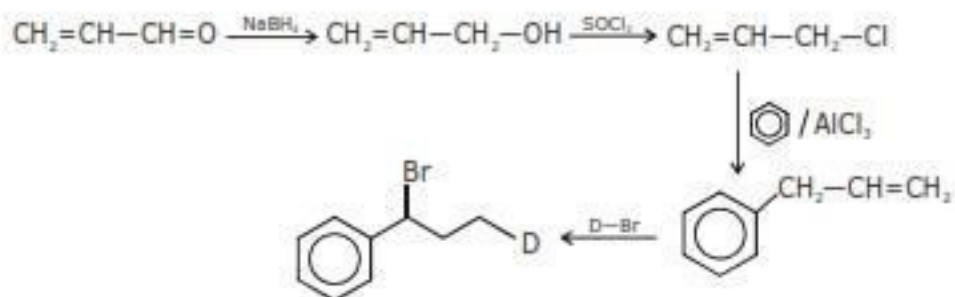
Sol. 3



12. The major product [C] of the following reaction sequence will be :



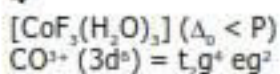
Sol. 1



13. The Crystal Field Stabilization Energy (CFSE) of  $[\text{CoF}_3(\text{H}_2\text{O})_3]$  ( $\Delta_0 < P$ ) is:

- (1)  $-0.8 \Delta_0$       (2)  $-0.8 \Delta_0 + 2P$       (3)  $-0.4 \Delta_0 + P$       (4)  $-0.4 \Delta_0$

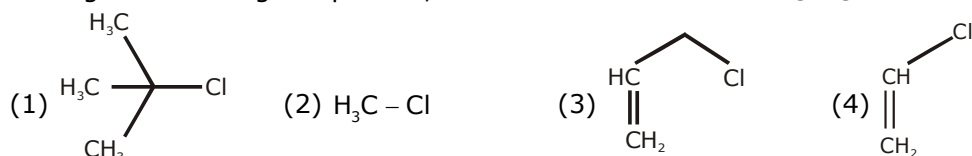
Sol. 4



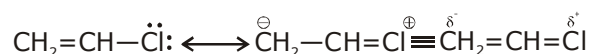
$$\text{CFSE} = \left( -\frac{2}{5} \times 4 + \frac{3}{5} \times 2 \right) \Delta_0$$

$$= -0.4 \Delta_0$$

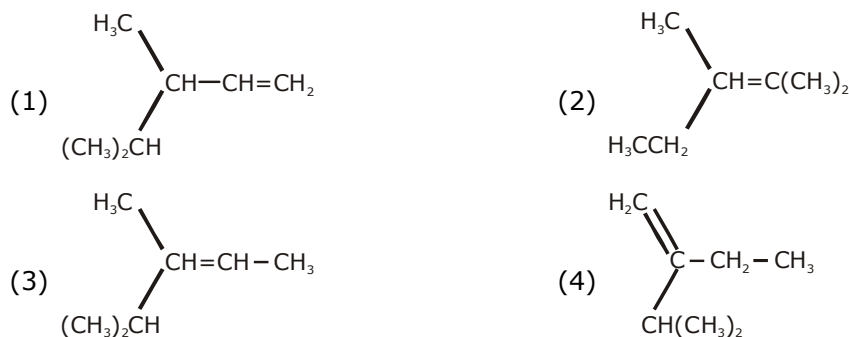
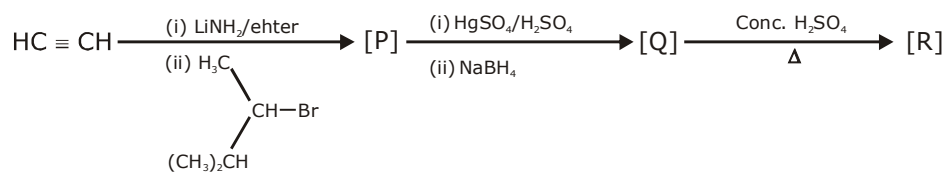
14. Among the following compounds, which one has the shortest C – Cl bond?



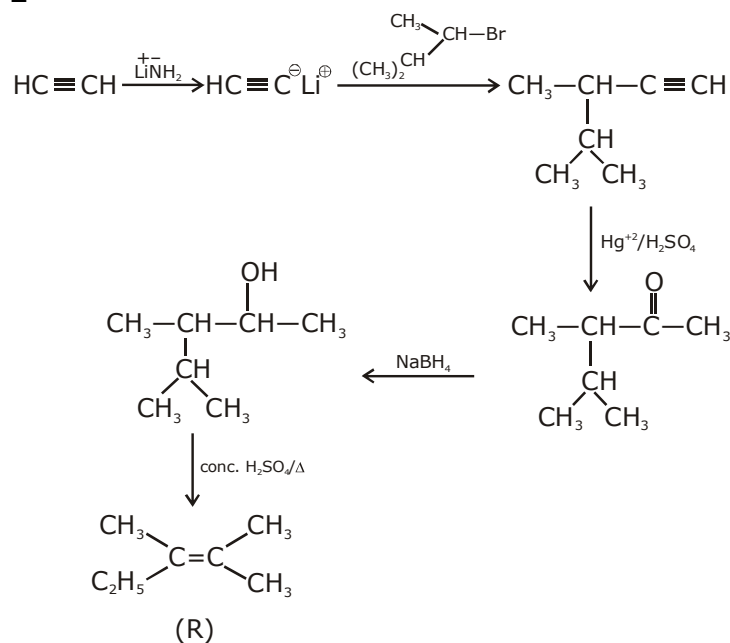
Sol. 4



15. The major product [R] in the following sequence of reactions is :



Sol. 2



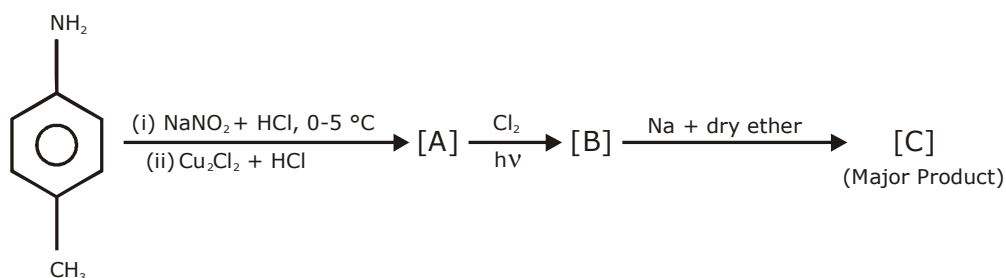
16. The molecule in which hybrid MOs involve only one d-orbital of the central atom is :

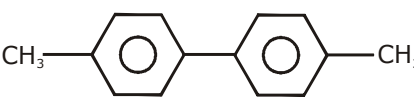
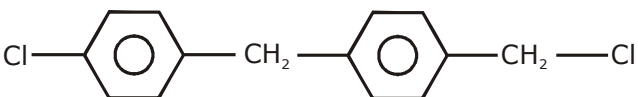
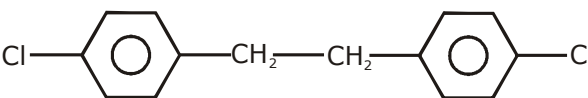
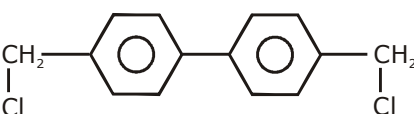
- (1)  $[\text{CrF}_6]^{3-}$                       (2)  $\text{XeF}_4$                       (3)  $\text{BrF}_5$                       (4)  $[\text{Ni}(\text{CN})_4]^{2-}$

Sol. 4

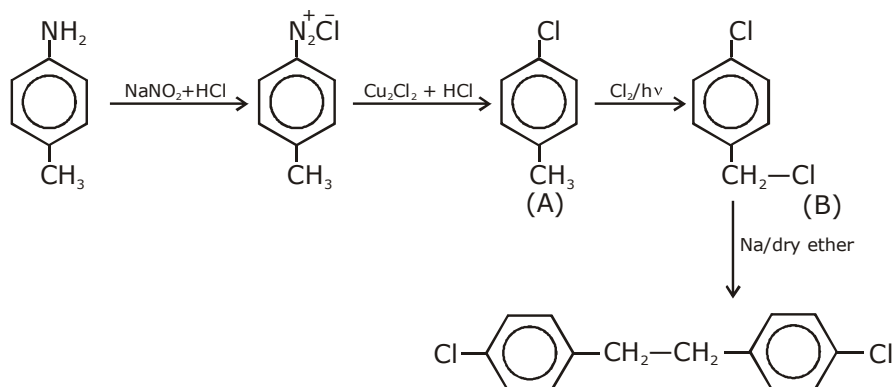
- (1)  $(\text{CrF}_6)^{3-} - d^2\text{sp}^3$   
 (2)  $\text{XeF}_4 - \text{sp}^3d^2$   
 (3)  $\text{BrF}_5 - \text{sp}^3d^2$   
 (4)  $[\text{Ni}(\text{CN})_4]^{2-} \rightarrow dsp^2$

17. In the following reaction sequence, [C] is :



- (1) 
- (2) 
- (3) 
- (4) 

**Sol. 3**



- 18.** The processes of calcination and roasting in metallurgical industries, respectively, can lead to :
- (1) Photochemical smog and ozone layer depletion
  - (2) Photochemical smog and global warming
  - (3) Global warming and photochemical smog
  - (4) Global warming and acid rain

**Sol. 4**

Environmental

Calcination Releases  $\rightarrow \text{CO}_2 \rightarrow$  Global warming

Roasting Releases  $\rightarrow \text{SO}_2 \rightarrow$  Acid Rain

Ans. (4)

- 19.** The incorrect statement(s) among (a) - (c) is (are) :
- (a) W(VI) is more stable than Cr(VI).
  - (b) in the presence of HCl, permanganate titrations provide satisfactory results.
  - (c) some lanthanoid oxides can be used as phosphors.
- (1) (a) only
  - (2) (b) and (c) only
  - (3) (a) and (b) only
  - (4) (b) only

**Sol. 4**

Fact

- 20.** An alkaline earth metal 'M' readily forms water soluble sulphate and water insoluble hydroxide. Its oxide MO is very stable to heat and does not have rock-salt structure. M is :
- (1) Ca
  - (2) Be
  - (3) Mg
  - (4) Sr

**Sol. 2**

Fact

- 21.** The osmotic pressure of a solution of NaCl is 0.10 atm and that of a glucose solution is 0.20 atm.

The osmotic pressure of a solution formed by mixing 1 L of the sodium chloride solution with 2 L of the glucose solution is  $x \times 10^{-3}$  atm.  $x$  is \_\_\_\_\_. (nearest integer)

**Sol. 167**

$$\frac{0.1 \times 1 + 0.2 \times 2}{3}$$

$$= \frac{0.5}{3} = \frac{500}{3} \times 10^{-3} = 167 \text{ Ans.}$$

**22.** The number of molecules with energy greater than the threshold energy for a reaction increases five fold by a rise of temperature from 27 °C to 42 °C. Its energy of activation in J/mol is \_\_\_\_\_. (Take  $\ln 5 = 1.6094$  ;  $R = 8.314 \text{ J mol}^{-1}\text{K}^{-1}$ )

**Sol.**  $\frac{1}{5} = \frac{e^{-E_a/300R}}{e^{-E_a/315R}}$

$$5 = e^{\frac{E_a}{R} \left( \frac{1}{300} - \frac{1}{315} \right)}$$

$$\frac{E_a}{R} \left( \frac{15}{300 \times 315} \right) = \ln(5)$$

$$E_a = 1.6094 \times 315 \times 20 \times 8.314$$

$$E_a = 84297.47 \text{ J/mol Ans.}$$

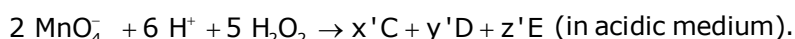
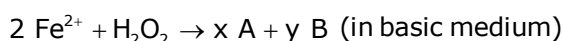
**23.** A 100 mL solution was made by adding 1.43 g of  $\text{Na}_2\text{CO}_3 \cdot x\text{H}_2\text{O}$ . The normality of the solution is 0.1 N. The value of  $x$  is \_\_\_\_\_. (The atomic mass of Na is 23 g/mol).

**Sol.**  $\frac{0.1}{2} \times \frac{100}{1000} = \frac{1.43}{1.6 + 18x}$

$$106 + 18x = 286$$

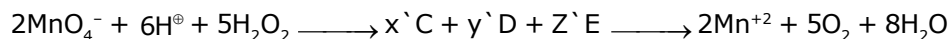
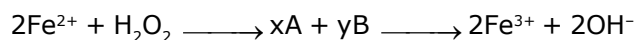
$$18x = 180 \Rightarrow x = 10 \text{ Ans.}$$

**24.** Consider the following equations :



The sum of the stoichiometric coefficients  $x$ ,  $y$ ,  $x'$ ,  $y'$  and  $z'$  for products A, B, C, D and E, respectively, is \_\_\_\_\_.

**Sol. 19**



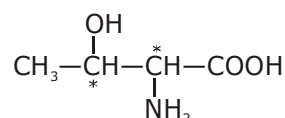
$$x = 2 ; y = 2 ; x' = 2, y' = 5, z' = 8$$

$$2 + 2 + 2 + 5 + 8 = 19$$

Ans. 19

**25.** The number of chiral centres present in threonine is \_\_\_\_\_.

**Sol. 2**



## MATHEMATICS \_ 4 Sep. \_ SHIFT - 2

- Q.1** Suppose the vectors  $x_1, x_2$  and  $x_3$  are the solutions of the system of linear equations,  $Ax=b$  when the vector  $b$  on the right side is equal to  $b_1, b_2$  and  $b_3$  respectively. if

$$x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, x_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, x_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, b_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, b_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} \text{ and } b_3 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \text{ then the determinant of } A \text{ is equal to}$$

- (1) 2                      (2)  $\frac{1}{2}$                       (3)  $\frac{3}{2}$                       (4) 4

**Sol. (1)**

$$A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}_{3 \times 3}$$

$$a_1 + a_2 + a_3 = 1 \quad 2a_2 + a_3 = 0$$

$$a_4 + a_5 + a_6 = 0 \quad 2a_5 + a_6 = 2$$

$$a_7 + a_8 + a_9 = 0 \quad 2a_8 + a_9 = 0$$

$$a_3 = 0, a_6 = 0, a_9 = 2$$

$$\therefore a_8 = -1, a_5 = 1, a_2 = 0 \Rightarrow a_1 = \phi, a_4 = -1, a_7 = -1$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & -1 & 2 \end{bmatrix}$$

$$|A| = 2(1) = 2$$

- Q.2** If  $a$  and  $b$  are real numbers such that  $(2+\alpha)^4 = a + b\alpha$ , where  $\alpha = \frac{-1+i\sqrt{3}}{2}$  then  $a+b$  is equal to:

- (1) 33                      (2) 57                      (3) 9                      (4) 24

**Sol. (3)**

$$(2+\alpha)^4 = a + b\alpha$$

$$\left(2 + \frac{\sqrt{3}i - 1}{2}\right)^4 = a + b\alpha$$

$$\left(\frac{3 + \sqrt{3}i}{2}\right)^4 = 9\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^4$$

$$9\{e^{i\pi/6}\}^4 = 9e^{i2\pi/3} = 9\left(\frac{-1}{2} + \frac{\sqrt{3}i}{2}\right) = \frac{-9}{2} + \frac{9\sqrt{3}}{2}i$$

$$-\frac{9}{2} + \frac{9\sqrt{3}}{2}i = a + b\left(\frac{-1}{2} + \frac{i\sqrt{3}}{2}\right)$$

$$= a - \frac{b}{2} + \frac{bi\sqrt{3}}{2}$$

$$\therefore \frac{b\sqrt{3}}{2} = \frac{9\sqrt{3}}{2} \Rightarrow b = 9$$

$$a = 0 \therefore a + b = 9$$

**Q.3** The distance of the point (1, -2, 3) from the plane  $x - y + z = 5$  measured parallel to the line  $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$  is:

(1)  $\frac{1}{7}$

(2) 7

(3)  $\frac{7}{5}$

(4) 1

**Sol.**

(4)  
Equation of line through (1, -2, 3) whose dr's are (2, 3, -6)

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = \lambda$$

any point on line  $(2\lambda + 1, 3\lambda - 2, -6\lambda + 3)$

put in  $(x - y + z = 5)$

$$2\lambda + 1 - 3\lambda + 2 - 6\lambda + 3 = 5$$

$$-7\lambda = -1$$

$$\lambda = \frac{1}{7}$$

$$\text{distance} = \sqrt{(2\lambda)^2 + (3\lambda)^2 + (6\lambda)^2}$$

$$\sqrt{4\lambda^2 + 9\lambda^2 + 36\lambda^2} = 7\lambda = 1$$

**Q.4** Let  $f : (0, \infty) \rightarrow (0, \infty)$  be a differentiable function such that  $f(1) = e$  and  $\lim_{t \rightarrow x} \frac{t^2 f^2(x) - x^2 f^2(t)}{t - x} = 0$ .

If  $f(x) = 1$ , then  $x$  is equal to :

- (1)  $e$                       (2)  $2e$                       (3)  $\frac{1}{e}$                       (4)  $\frac{1}{2e}$

**Sol. (3)**

$$f(1) = e$$

$$\lim_{t \rightarrow x} \frac{t^2 f^2(x) - x^2 f^2(t)}{t - x}$$

L' Hospital

$$\lim_{t \rightarrow x} (2tf^2(x) - 2x^2 f(t) \cdot f'(t))$$

$$\Rightarrow 2xf^2(x) - 2x^2 f(x) \cdot f'(x) = 0$$

$$2xf(x) \{f(x) - xf'(x)\} = 0$$

$$\Rightarrow \frac{f'(x)}{f(x)} = \frac{1}{x}$$

$$\ln f(x) = \ln x + \ln c$$

$$f(x) = cx$$

$$\text{if } x = 1, e = c$$

$$y = ex$$

$$\therefore \text{ if } f(x) = 1 \Rightarrow x = \frac{1}{e}$$

**Q.5** Contrapositive of the statement :

'If a function  $f$  is differentiable at  $a$ , then it is also continuous at  $a$ ', is:

- (1) If a function  $f$  is not continuous at  $a$ , then it is not differentiable at  $a$ .  
 (2) If a function  $f$  is continuous at  $a$ , then it is differentiable at  $a$ .  
 (3) If a function  $f$  is continuous at  $a$ , then it is not differentiable at  $a$ .  
 (4) If a function  $f$  is not continuous at  $a$ , then it is differentiable at  $a$ .

**Sol. (1)**

Contrapositive of  $P \rightarrow q = \sim q \rightarrow \sim p$

**Q.6** The minimum value of  $2^{\sin x} + 2^{\cos x}$  is:

- (1)  $2^{1-\sqrt{2}}$                       (2)  $2^{1-\frac{1}{\sqrt{2}}}$                       (3)  $2^{-1+\sqrt{2}}$                       (4)  $2^{-1+\frac{1}{\sqrt{2}}}$

**Sol. (2)**

$$y = 2^{\sin x} + 2^{\cos x}$$

by AM  $\geq$  GM

$$\frac{2^{\sin x} + 2^{\cos x}}{2} \geq \sqrt{2^{\sin x + \cos x}}$$

$$2^{\sin x} + 2^{\cos x} \geq 2^{1.2} \frac{\sin x + \cos x}{2}$$

$$2^{\sin x} + 2^{\cos x} \geq 2^{\frac{2 + \sin x + \cos x}{2}} \therefore (2^{\sin x} + 2^{\cos x})_{\min} = 2^{\frac{2 - \sqrt{2}}{2}} = 2^{\frac{-1}{\sqrt{2}} + 1}$$

**Q.7** If the perpendicular bisector of the line segment joining the points P(1, 4) and Q(k, 3) has y-intercept equal to -4, then a value of k is:

- (1) -2                      (2)  $\sqrt{15}$                       (3)  $\sqrt{14}$                       (4) -4

**Sol.** (4)

$$m_{PQ} = \frac{4-3}{1-k} \Rightarrow m_{\perp} = k-1$$

$$\text{mid point of } PQ = \left( \frac{k+1}{2}, \frac{7}{2} \right)$$

equation of perpendicular bisector

$$y - \frac{7}{2} = (k-1) \left( x - \frac{k+1}{2} \right)$$

for y intercept put x = 0

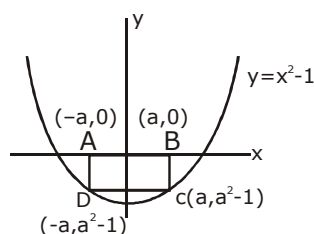
$$y = \frac{7}{2} - \left( \frac{k^2 - 1}{2} \right) = -4$$

$$\frac{k^2 - 1}{2} = \frac{15}{2} \Rightarrow k = \pm 4$$

**Q.8** The area (in sq. units) of the largest rectangle ABCD whose vertices A and B lie on the x-axis and vertices C and D lie on the parabola,  $y = x^2 - 1$  below the x-axis, is:

- (1)  $\frac{2}{3\sqrt{3}}$                       (2)  $\frac{4}{3}$                       (3)  $\frac{1}{3\sqrt{3}}$                       (4)  $\frac{4}{3\sqrt{3}}$

**Sol.** (4)



$$\text{Area} = 2a(a^2 - 1)$$

$$A = 2a^3 - 2a$$

$$\frac{dA}{da} = 6a^2 - 2 = 0$$

$$a = \pm 1\sqrt{3}$$

$$A_{\max} = \frac{-2}{3\sqrt{3}} + \frac{2}{\sqrt{3}} = \frac{-2+6}{3\sqrt{3}} = \frac{4}{3\sqrt{3}}$$

**Q.9** The integral  $\int_{\pi/6}^{\pi/3} \tan^3 x \cdot \sin^2 3x (2 \sec^2 x \cdot \sin^2 3x + 3 \tan x \cdot \sin 6x) dx$  is equal to:

- (1)  $\frac{9}{2}$                       (2)  $-\frac{1}{18}$                       (3)  $-\frac{1}{9}$                       (4)  $\frac{7}{18}$

**Sol. (2)**

$$I = \int_{\pi/6}^{\pi/3} 2 \tan^3 x \sec^2 x \sin^4 3x + 3 \tan^4 x \sin^2 3x \cdot 2 \sin 3x \cos 3x dx$$

$$= \frac{1}{2} \int_{\pi/6}^{\pi/3} 4 \tan^3 x \sec^2 x \sin^4 3x + 3 \cdot 4 \tan^4 x \sin^3 3x \cos 3x dx$$

$$= \frac{1}{2} \int_{\pi/6}^{\pi/3} \frac{d}{dx} (\tan^4 x \sin^4 3x) dx$$

$$= \frac{1}{2} \left[ \tan^4 x \sin^4 3x \right]_{\pi/6}^{\pi/3}$$

$$= \frac{1}{2} \left[ 9 \cdot (0) - \frac{1}{3} \cdot \frac{1}{3} (1) \right] = -\frac{1}{18}$$

**Q.10** If the system of equations

$$x+y+z=2$$

$$2x+4y-z=6$$

$$3x+2y+\lambda z=\mu$$

has infinitely many solutions, then

- (1)  $\lambda - 2\mu = -5$                       (2)  $2\lambda + \mu = 14$                       (3)  $\lambda + 2\mu = 14$                       (4)  $2\lambda - \mu = 5$

**Sol. (2)**

$$D = 0 \quad \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -1 \\ 3 & 2 & \lambda \end{vmatrix} = 0$$

$$(4\lambda + 2) - 1(2\lambda + 3) + 1(4 - 12) = 0$$

$$4\lambda + 2 - 2\lambda - 3 - 8 = 0$$

$$2\lambda = 9 \Rightarrow \lambda = \frac{9}{2}$$

$$D_x = \begin{vmatrix} 2 & 1 & 1 \\ 6 & 4 & -1 \\ \mu & 2 & -9/2 \end{vmatrix} = 0$$

$$\Rightarrow \mu = 5$$

Now check option

$$2\lambda + \mu = 14$$

**Q.11** In a game two players A and B take turns in throwing a pair of fair dice starting with player A and total of scores on the two dice, in each throw is noted. A wins the game if he throws total a of 6 before B throws a total of 7 and B wins the game if he throws a total of 7 before A throws a total of six. The game stops as soon as either of the players wins. The probability of A winning the game is:

(1)  $\frac{5}{31}$

(2)  $\frac{31}{61}$

(3)  $\frac{30}{61}$

(4)  $\frac{5}{6}$

**Sol. 2**

sum total 7 = (1,6)(2,5)(3,4)(4,3)(5,2)(6,1)

$$P(\text{sum}) = \frac{6}{36}$$

sum total 6  $\Rightarrow$  (1,5)(2,4)(3,3)(4,2)(5,1)

$$P(\text{sum } 6) = \frac{5}{36}$$

$$P(A_{\text{win}}) = P(6) + P(\bar{6}) \cdot P(\bar{7}) \cdot P(6) + \dots$$

$$= \frac{5}{36} + \frac{31}{36} \times \frac{30}{36} \times \frac{5}{36} + \dots$$

$$= \frac{\frac{5}{36}}{1 - \frac{31 \times 30}{36 \times 36}} \Rightarrow \frac{5 \times 36}{36 \times 36 - 31 \times 30} \Rightarrow \frac{5 \times 36}{1296 - 930} = \frac{5 \times 36}{366} \Rightarrow \frac{30}{61}$$

**Q.12** If for some positive integer  $n$ , the coefficients of three consecutive terms in the binomial expansion of  $(1+x)^{n+5}$  are in the ratio 5:10:14, then the largest coefficient in this expansion is :

- (1) 792                      (2) 252                      (3) 462                      (4) 330

**Sol. 3**

$$T_r : T_{r+1} : T_{r+2}$$

$${}^{n+5}C_{r-1} : {}^{n+5}C_r : {}^{n+5}C_{r+1} = 5 : 10 : 14$$

$$\frac{(n+5)!}{(r-1)!(n+6-r)!} : \frac{(n+5)!}{r!(n+5-r)!} = \frac{5}{10}$$

$$\frac{r}{n+6-r} = \frac{1}{2} \qquad \frac{(r+1)!(n+4-r)!}{r!(n+5-r)!} = \frac{5}{7}$$

$$2r = n+6-r$$

$$3r = n+6 \qquad \dots(1) \qquad \frac{r+1}{n+5-r} = \frac{5}{7}$$

$$7r+7 = 5n+25-5r$$

$$12r = 5n+18 \qquad \dots(2)$$

$$\therefore 4(n+6) = 5n+18$$

$$n = 6$$

$$\therefore (1+x) \quad \text{largest coeff} = {}^{11}C_5 = 462$$

**Q.13** The function  $f(x) = \begin{cases} \frac{\pi}{4} + \tan^{-1} x, & |x| \leq 1 \\ \frac{1}{2}(|x| - 1), & |x| > 1 \end{cases}$  is :

- (1) both continuous and differentiable on  $\mathbb{R} - \{-1\}$   
 (2) continuous on  $\mathbb{R} - \{-1\}$  and differentiable on  $\mathbb{R} - \{-1, 1\}$   
 (3) continuous on  $\mathbb{R} - \{1\}$  and differentiable on  $\mathbb{R} - \{-1, 1\}$   
 (4) both continuous and differentiable on  $\mathbb{R} - \{1\}$

**Sol. (3)**

$$f(x) = \begin{cases} \frac{\pi}{4} + \tan^{-1} x & x \in [-1, 1] \\ \frac{1}{2}(x-1) & x > 1 \\ \frac{1}{2}(-x-1) & x < -1 \end{cases}$$

at  $x = 1$

$$f(1) = \frac{\pi}{4} \quad f(1^+) = 0$$

$\therefore$  discontinuous  $\Rightarrow$  non diff.

at  $x = -1$

$$f(-1) = 0 \quad f(-1^-) = \frac{1}{2}\{+1-1\} = 0$$

cont. at  $x = -1$

$$f'(x) = \begin{cases} \frac{1}{1+x^2} & x \in [-1, 1] \\ \frac{1}{2} & x > 1 \\ -\frac{1}{2} & x < -1 \end{cases}$$

**Q.14** The solution of the differential equation  $\frac{dy}{dx} - \frac{y+3x}{\log_e(y+3x)} + 3 = 0$  is:  
(where  $c$  is a constant of integration)

(1)  $x - \log_e(y+3x) = C$

(2)  $x - \frac{1}{2}(\log_e(y+3x))^2 = C$

(3)  $x - 2\log_e(y+3x) = C$

(4)  $y + 3x - \frac{1}{2}(\log_e x)^2 = C$

**Sol. (2)**

$$\frac{dy}{dx} - \frac{y+3x}{\ln(y+3x)} + 3 = 0$$

Let  $\ln(y+3x) = t$

$$\frac{1}{y+3x} \cdot \left( \frac{dy}{dx} + 3 \right) = \frac{dt}{dx}$$

$$\Rightarrow \left( \frac{dy}{dx} + 3 \right) = \frac{y+3x}{\ln(y+3x)}$$

$$\therefore (y+3x) \frac{dt}{dx} = \frac{y+3x}{t}$$

$$\Rightarrow t dt = dx$$

$$\frac{t^2}{2} = x + c$$

$$\frac{1}{2} (\ln(y+3x))^2 = x + c$$

**Q.15** Let  $\lambda \neq 0$  be in  $\mathbb{R}$ . If  $\alpha$  and  $\beta$  are the roots of the equation,  $x^2 + x + 2\lambda = 0$  and  $\alpha$  and  $\gamma$  are the

roots of the equation,  $3x^2 - 10x + 27\lambda = 0$ , then  $\frac{\beta\gamma}{\lambda}$  is equal to:

(1) 27

(2) 9

(3) 18

(4) 36

**Sol.**

**(3)**

$$x^2 - x + 2\lambda = 0 \quad (\alpha, \beta)$$

$$3x^2 - 10x + 27\lambda = 0 \quad (\alpha, \gamma)$$

$$3x^2 - 3x + 6\lambda = 0$$

$$\begin{array}{r} - \quad + \quad - \\ \hline \end{array}$$

$$-7x + 21\lambda = 0$$

$$\therefore \alpha = 3\lambda$$

Put in equation

$$9\lambda^2 - 3\lambda + 2\lambda = 0$$

$$9\lambda^2 - \lambda = 0 \Rightarrow \lambda = \frac{1}{9} \Rightarrow \alpha = \frac{1}{3}$$

$$\alpha, \beta = \frac{2}{9} \Rightarrow \beta = \frac{2}{3}$$

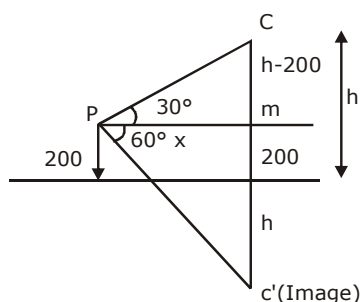
$$\alpha, \gamma = 1 \Rightarrow \gamma = 3$$

$$\therefore \frac{\beta\gamma}{\lambda} \Rightarrow \frac{\frac{2}{3} \cdot 3}{\frac{1}{9}} = 18$$

**Q.16** The angle of elevation of a cloud C from a point P, 200 m above a still lake is  $30^\circ$ . If the angle of depression of the image of C in the lake from the point P is  $60^\circ$ , then PC (in m) is equal to :

- (1)  $200\sqrt{3}$                       (2)  $400\sqrt{3}$                       (3) 400                      (4) 100

**Sol. (3)**



$$\frac{h-200}{x} = \tan 30^\circ \quad \frac{h+200}{x} = \tan 60^\circ$$

$$\frac{h+200}{h-200} = 3$$

$$h+200 = 3h-600$$

$$2h = 800$$

$$h = 400$$

$$\therefore \frac{h-200}{PC} = \sin 30^\circ$$

$$PC = 400 \text{ m}$$

**Q.17** Let  $\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^n Y_i = T$ , where each  $X_i$  contains 10 elements and each  $Y_i$  contains 5 elements. If each element of the set T is an element of exactly 20 of sets  $X_i$ 's and exactly 6 of sets  $Y_i$ 's, then n is equal to :

- (1) 15                      (2) 30                      (3) 50                      (4) 45

**Sol. (2)**

$$\frac{50 \times 10}{20} = \frac{n \times 5}{6}$$

$$\frac{50}{2} \times \frac{6}{5} = n \Rightarrow n = 30$$

**Q.18** Let  $x=4$  be a directrix to an ellipse whose centre is at the origin and its eccentricity is  $\frac{1}{2}$ . If

$P(1, \beta)$ ,  $\beta > 0$  is a point on this ellipse, then the equation of the normal to it at P is :

- (1)  $8x-2y=5$                       (2)  $4x-2y=1$                       (3)  $7x-4y=1$                       (4)  $4x-3y=2$

**Sol.** (2)

$$e = \frac{1}{2} \quad x = \frac{a}{e} = 4$$

$$\Rightarrow a = 2$$

$$e^2 = 1 - \frac{b^2}{a^2} \Rightarrow \frac{1}{4} = 1 - \frac{b^2}{4}$$

$$\frac{b^2}{4} = \frac{3}{4} \Rightarrow b^2 = 3$$

$$\therefore \text{Ellipse } \frac{x^2}{4} + \frac{y^2}{3} = 1$$

$$P(1, \beta)$$

$$x = 1 ; \frac{1}{4} + \frac{\beta^2}{3} = 1$$

$$\frac{\beta^2}{3} = \frac{3}{4} \Rightarrow \beta = \frac{3}{2}$$

$$\Rightarrow P\left(1, \frac{3}{2}\right)$$

$$\text{Equation of normal } \frac{a^2 x}{x_1} - \frac{b^2 y}{y_1} = a^2 - b^2$$

$$\frac{4x}{1} - \frac{3y}{\frac{3}{2}} = 4 - 3$$

$$4x - 2y = 1$$

**Q.19** Let  $a_1, a_2, \dots, a_n$  be a given A.P. whose common difference is an integer and  $S_n = a_1 + a_2 + \dots + a_n$ . If  $a_1 = 1$ ,  $a_n = 300$  and  $15 \leq n \leq 50$ , then the ordered pair  $(S_{n-4}, a_{n-4})$  is equal to:

- (1) (2480, 248)                      (2) (2480, 249)                      (3) (2490, 249)                      (4) (2490, 248)

**Sol.** 4

$$a_1 = 1, a_n = 300, 15 \leq n \leq 50$$

$$300 = 1 + (n-1)d$$

$$(n-1) = \frac{299}{d}$$

d can 23 or 13

if  $n-1 = 13$

$n = 14$

reject

or  $d = 13$

$n-1 = 23$

$n = 24$

$$S_{20} = \frac{20}{2} \{2 + 19.13\}$$

$$a_{20} = 1 + 19.13$$

$$= 10\{249\} = 2490$$

$$a_{20} = 248$$

$$(S_{20}, a_{20}) = (2490, 248)$$

**Q.20** The circle passing through the intersection of the circles,  $x^2 + y^2 - 6x = 0$  and  $x^2 + y^2 - 4y = 0$ , having its centre on the line,  $2x - 3y + 12 = 0$ , also passes through the point:

(1)  $(-1, 3)$

(2)  $(1, -3)$

(3)  $(-3, 6)$

(4)  $(-3, 1)$

**Sol. (3)**

$$S_1 + \lambda(S_2 - S_1) = 0$$

$$x^2 + y^2 - 6x + \lambda(4y - 6x) = 0$$

$$x^2 + y^2 - 6x(1 + \lambda) + 4\lambda y = 0$$

Centre  $(3(1 + \lambda), -2\lambda)$  put in  $2x - 3y + 12 = 0$

$$6 + 6\lambda + 6\lambda + 12 = 0$$

$$12\lambda = -18$$

$$\lambda = -3/2$$

$$\therefore \text{Circle is } x^2 + y^2 + 3x - 6y = 0$$

Check options

**Q.21** Let  $\{x\}$  and  $[x]$  denote the fractional part of  $x$  and the greatest integer  $\leq x$  respectively of a real number  $x$ . If  $\int_0^n \{x\} dx$ ,  $\int_0^n [x] dx$  and  $10(n^2 - n)$ , ( $n \in \mathbb{N}$ ,  $n > 1$ ) are three consecutive terms of a G.P., then  $n$  is equal to \_\_\_\_\_

**Sol. 21**

$$\int_0^n \{x\} dx = n \int_0^1 x dx = n \left( \frac{x^2}{2} \right) = \frac{n}{2}$$

$$\int_0^n [x] dx = \int_0^1 0 dx + \int_1^2 1 dx + \int_2^3 2 dx \dots + \int_{n-1}^n (n-1) dx$$

$$= 1 + 2 + \dots + n - 1 \Rightarrow \frac{n(n-1)}{2}$$

$$= \frac{n}{2}, \frac{n(n-1)}{2}, 10(n^2 - n) \rightarrow G.P$$

$$= \frac{n^2(n-1)^2}{4} = \frac{n}{2} \cdot 10 \cdot n(n-1)$$

$$n - 1 = 20 ; n = 21$$

**Q.22** A test consists of 6 multiple choice questions, each having 4 alternative answers of which only one is correct. The number of ways, in which a candidate answers all six questions such that exactly four of the answers are correct, is \_\_\_\_\_

**Sol. 135**

$${}^6C_4 \times 1 \times 3^2 = 15 \times 9 = 135$$

**Q.23** If  $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ , then the value of  $|\hat{i} \times (\vec{a} \times \hat{i})|^2 + |\hat{j} \times (\vec{a} \times \hat{j})|^2 + |\hat{k} \times (\vec{a} \times \hat{k})|^2$  is equal to \_\_\_\_\_

**Sol. 18**

$$|\hat{i} \times (\vec{a} \times \hat{i})|^2 = |\vec{a} - (a\hat{i})\hat{i}|^2$$

$$= |\hat{j} + 2\hat{k}|^2 = 1 + 4 = 5$$

Similarly

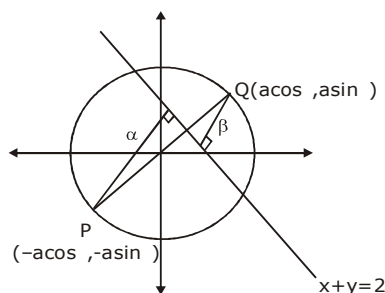
$$|\hat{j} \times (\vec{a} \times \hat{j})|^2 = |2\hat{i} + 2\hat{k}|^2 = 4 + 4 = 8$$

$$|\hat{k} \times (\vec{a} \times \hat{k})|^2 = |2\hat{i} + \hat{j}|^2 = 4 + 1 = 5$$

$$\Rightarrow 5 + 8 + 5 = 18$$

**Q.24** Let PQ be a diameter of the circle  $x^2 + y^2 = 9$ . If  $\alpha$  and  $\beta$  are the lengths of the perpendiculars from P and Q on the straight line,  $x + y = 2$  respectively, then the maximum value of  $\alpha\beta$  is \_\_\_\_\_

**Sol. 7**



$$\alpha = \left| \frac{3 \cos \theta + 3 \sin \theta - 2}{\sqrt{2}} \right|$$

$$\beta = \left| \frac{+3 \cos \theta + 3 \sin \theta + 2}{\sqrt{2}} \right|$$

$$\alpha\beta = \left| \frac{(3 \cos \theta + 3 \sin \theta)^2 - 4}{2} \right| \Rightarrow \alpha\beta = \left| \frac{9 + 9 \sin 2\theta - 4}{2} \right| \Rightarrow \alpha\beta = \left| \frac{5 + 9 \sin 2\theta}{2} \right|$$

$$\alpha\beta_{\max} = \frac{9+5}{2} = 7$$

**Q.25** If the variance of the following frequency distribution :

Class	:	10-20	20-30	30-40
Frequency	:	2	x	2

is 50, then x is equal to \_\_\_\_\_

**Sol.** **4**

$$6^2 = \frac{\sum f_i x_i^2}{\sum f_i} - \left( \frac{\sum f_i x_i}{\sum f_i} \right)^2$$

$x_i$	$f_i$	$x - \bar{x}$	$(x - \bar{x})^2$	$f_i (x - \bar{x})^2$
15	2	-10	100	200
25	x	0	0	0
35	2	10	100	200
<u>4 + x</u>				<u>400</u>

$$\bar{x} = \frac{100 + 25x}{4 + x}$$

$$\bar{x} = 25$$

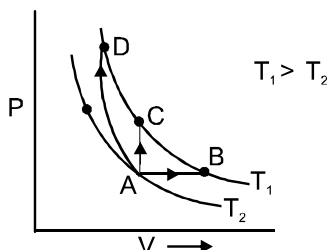
$$\therefore \frac{400}{4 + x} = 50$$

$$x = 4$$

# QUESTION PAPER WITH SOLUTION

## PHYSICS \_ 5 Sep. \_ SHIFT - 1

1. Three different processes that can occur in an ideal monoatomic gas are shown in the P vs V diagram. The paths are labelled as A → B, A → C and A → D. The change in internal energies during these process are taken as  $E_{AB}$ ,  $E_{AC}$  and  $E_{AD}$  and the workdone as  $W_{AB}$ ,  $W_{AC}$  and  $E_{AD}$ . The correct relation between these parameters are :



- (1)  $E_{AB} = E_{AC} < E_{AD}$ ,  $W_{AB} > 0$ ,  $W_{AC} = 0$ ,  $W_{AD} < 0$
- (2)  $E_{AB} > E_{AC} > E_{AD}$ ,  $W_{AB} < W_{AC} < W_{AD}$
- (3)  $E_{AB} < E_{AC} < E_{AD}$ ,  $W_{AB} > 0$ ,  $W_{AC} > W_{AD}$
- (4)  $E_{AB} = E_{AC} = E_{AD}$ ,  $W_{AB} > 0$ ,  $W_{AC} = 0$ ,  $W_{AD} > 0$

**Sol. 1 (Bonus)**

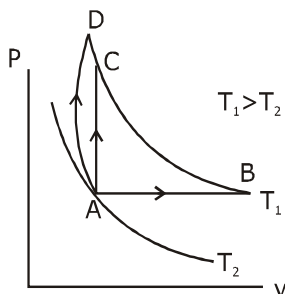
$$E_{AB} = E_{AC} = E_{AD}$$

$$dU = \frac{nfR}{2} (T_f - T_i)$$

$$W_{AB} > 0 (+) V \uparrow$$

$$W_{AC} = 0 V \text{ const.}$$

$$W_{AD} < 0 (-) V \downarrow$$



2. With increasing biasing voltage of a photodiode, the photocurrent magnitude :
- (1) increases initially and saturates finally
  - (2) remains constant
  - (3) increases linearly
  - (4) increases initially and after attaining certain value, it decreases

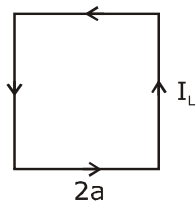
**Sol. 1**

By theory

3. A square loop of side  $2a$ , and carrying current  $I$ , is kept in  $XZ$  plane with its centre at origin. A long wire carrying the same current  $I$  is placed parallel to the  $z$ -axis and passing through the point  $(0, b, 0)$ , ( $b > a$ ). The magnitude of the torque on the loop about  $z$ -axis is given by :

(1)  $\frac{2\mu_0 I^2 a^3}{\pi b^2}$       (2)  $\frac{\mu_0 I^2 a^3}{2\pi b^2}$       (3)  $\frac{2\mu_0 I^2 a^2}{\pi b}$       (4)  $\frac{\mu_0 I^2 a^2}{2\pi b}$

**Sol. 3**

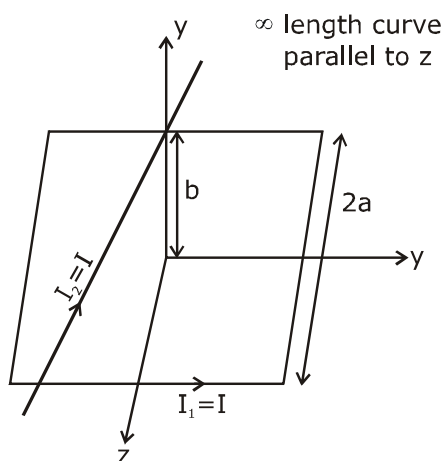


$$M = I_2 (2a)^2 = 4a^2 I_2$$

(magnetic moment)

$$B = \frac{\mu_0 I_1}{2\pi b}$$

$$\tau = MB \sin \theta$$



$\theta$  angle between  $B$  and  $M$  [ $\theta = 90^\circ$ ]

$$\tau = 4 (a^2 I_2) \frac{\mu_0 I_1}{2\pi b}$$

$$\tau = \frac{2\mu_0 I_1 I_2 a^2}{\pi b} = \frac{2\mu_0 I^2 a^2}{\pi b}$$

4. Assume that the displacement ( $s$ ) of air is proportional to the pressure difference ( $\Delta p$ ) created by a sound wave. Displacement( $s$ ) further depends on the speed of sound ( $v$ ), density of air ( $\rho$ ) and the frequency ( $f$ ). If  $\Delta p \sim 10\text{Pa}$ ,  $v \sim 300\text{ m/s}$ ,  $\rho \sim 1\text{ kg/m}^3$  and  $f \sim 1000\text{ Hz}$ , then  $s$  will be of the order of (take the multiplicative constant to be 1)

- (1) 1 mm                      (2) 10 mm                      (3)  $\frac{1}{10}$  mm                      (4)  $\frac{3}{100}$  mm

**Ans. 4**

$$S_0 = \frac{\Delta P}{\beta k} = \frac{\Delta P}{\rho v^2 \frac{\omega}{v}} = \frac{\Delta P}{\rho v \omega} = \frac{\Delta P}{\rho v 2\pi f}$$

$\therefore$  Proportionally constant = 1

$$\begin{aligned} S_0 &= \frac{\Delta P}{\rho v f} \\ &= \frac{10}{1 \times 300 \times 1000} \text{ m} \\ &= \frac{10}{300} \text{ mm} \\ &= \frac{3}{90} \\ &\sim \frac{3}{100} \text{ mm} \end{aligned}$$

5. Two capacitors of capacitances  $C$  and  $2C$  are charged to potential differences  $V$  and  $2V$ , respectively. These are then connected in parallel in such a manner that the positive terminal of one is connected to the negative terminal of the other. The final energy of this configuration is :

- (1) zero                      (2)  $\frac{9}{2} CV^2$                       (3)  $\frac{25}{6} CV^2$                       (4)  $\frac{3}{2} CV^2$

**Sol. 4**

$$\begin{aligned} \text{Circuit diagram:} & \quad U_i = \frac{1}{2} C V^2 + \frac{1}{2} (2C) (2V)^2 \\ & \quad = \frac{9}{2} C V^2 \\ & \quad q_1 + q_2 = q_1' + q_2' \\ & \quad -CV + (2C)(2V) = (C + 2C)V' \\ & \quad V' = \frac{3CV}{3C} = V \end{aligned}$$

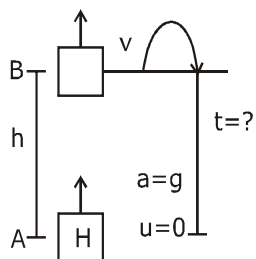
$$U_f = \frac{1}{2} C V^2 + \frac{1}{2} (2C) V^2$$

$$U_f = \frac{3}{2} C V^2$$

6. A helicopter rises from rest on the ground vertically upwards with a constant acceleration  $g$ . A food packet is dropped from the helicopter when it is at a height  $h$ . The time taken by the packet to reach the ground is close to [ $g$  is the acceleration due to gravity] :

(1)  $t = 3.4 \sqrt{\left(\frac{h}{g}\right)}$       (2)  $t = \sqrt{\frac{2h}{3g}}$       (3)  $t = \frac{2}{3} \sqrt{\left(\frac{h}{g}\right)}$       (4)  $t = 1.8 \sqrt{\frac{h}{g}}$

**Sol. 1**



$$V_B^2 = 0^2 + 2gh$$

$$V_B = \sqrt{2gh}$$

$$-h = (V_B)t - \frac{1}{2}gt^2$$

$$-h = \sqrt{2gh}t - \frac{1}{2}gt^2$$

$$gt^2 - 2\sqrt{2gh}t - 2h = 0$$

$$t = \frac{\sqrt{2gh} \pm \sqrt{8gh + 8gh}}{2g} = \frac{2\sqrt{2gh} \pm \sqrt{16gh}}{2g} = \frac{\sqrt{2gh} + 2\sqrt{gh}}{g}$$

$$t = \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{h}{g}} = \sqrt{\frac{h}{g}} (\sqrt{2} + 2) = 3.4 \sqrt{\frac{h}{g}}$$

7. A bullet of mass 5 g, travelling with a speed of 210 m/s, strikes a fixed wooden target. One half of its kinetic energy is converted into heat in the bullet while the other half is converted into heat in the wood. The rise of temperature of the bullet if the specific heat of its material is 0.030 cal/(g - °C) (1 cal = 4.2 × 10<sup>7</sup> ergs) close to :

(1) 38.4°C      (2) 87.5°C      (3) 83.3°C      (4) 119.2°C

**Sol. 2**

$$\left(\frac{1}{2}mv^2\right) \times \frac{1}{2} = ms\Delta T$$

$$s = 0.03 \text{ cal/g}^\circ\text{C}$$

$$\begin{aligned}\frac{v^2}{4} &= 126 \times \Delta T &= \frac{0.03 \times 4.2J}{10^{-3} \text{kgC}} \\ v^2 &= 4 \times 126 \times \Delta T &= 126 \text{ J/kgC} \\ (210)^2 &= 4 \times 126 \times \Delta T \\ 210 \times 210 &= 4 \times 126 \times \Delta T \\ 44100 &= 504 \times \Delta T \\ \Delta T &= \frac{44100}{504} = 87.5^\circ\text{C}\end{aligned}$$

8. A wheel is rotating freely with an angular speed  $\omega$  on a shaft. The moment of inertia of the wheel is  $I$  and the moment of inertia of the shaft is negligible. Another wheel of moment of inertia  $3I$  initially at rest is suddenly coupled to the same shaft. The resultant fractional loss in the kinetic energy of the system is :

(1)  $\frac{3}{4}$                       (2) 0                      (3)  $\frac{5}{6}$                       (4)  $\frac{1}{4}$

**Sol. 1**

$$K_i = \frac{1}{2} I \omega^2$$

$$K_f = \frac{1}{2} (4I) (\omega')^2$$

$$= 2I \left( \frac{\omega}{4} \right)^2 = \frac{1}{8} I \omega^2$$

A.M.C

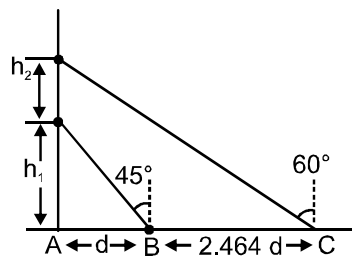
$$I\omega = (I+3I)\omega'$$

$$\omega' = \frac{I\omega}{4I} = \frac{\omega}{4}$$

$$= \frac{K_i - K_f}{K_i} \Rightarrow \frac{\frac{1}{2} I \omega^2 - \frac{1}{8} I \omega^2}{\frac{1}{2} I \omega^2}$$

$$\frac{\frac{3}{8} I \omega^2}{\frac{1}{2} I \omega^2} = \frac{3}{4}$$

9. A balloon is moving up in air vertically above a point A on the ground. When it is at a height  $h_1$ , a girl standing at a distance  $d$  (point B) from A (see figure) sees it at an angle  $45^\circ$  with respect to the vertical. When the balloon climbs up a further height  $h_2$ , it is seen at an angle  $60^\circ$  with respect to the vertical if the girl moves further by a distance  $2.464 d$  (point C). Then the height  $h_2$  is (given  $\tan 30^\circ = 0.5774$ ) :



(1)  $0.464 d$

(2)  $d$

(3)  $0.732 d$

(4)  $1.464 d$

**Sol.**

**2**

$\triangle ABD$

$$\tan 45 = \frac{h_1}{d}$$

$$\Rightarrow 1 = \frac{h_1}{d} \Rightarrow h_1 = d$$

$\triangle ACE$

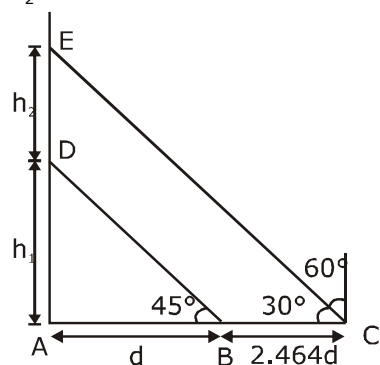
$$\tan 30 = \frac{h_1 + h_2}{d + 2.464d}$$

$$0.5774 = \frac{d + h_2}{3.464d}$$

$$d + h_2 = 0.5774 \times 3.464 \times d$$

$$h_2 = 2.0001136d - d$$

$$h_2 = 2.000d - d = d$$



- 10.** An electrical power line, having a total resistance of  $2\ \Omega$ , delivers 1 kW at 220 V. The efficiency of the transmission line is approximately :

(1) 72%                      (2) 91 %                      (3) 85%                      (4) 96%

**Sol. 4**

$$\eta = \frac{P_{out}}{(P_{out} + P_{loss})} \times 100$$

$$I = \frac{P}{V}$$

$$= \frac{1000}{220} = \frac{50}{11} \text{ A}$$

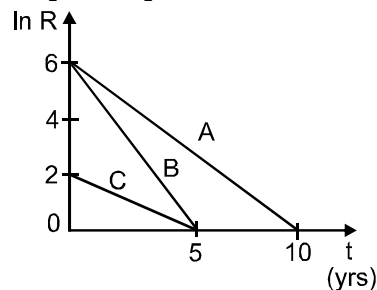
$$P_{loss} = I^2 R$$

$$= \left(\frac{50}{11}\right)^2 \times 2 = 41.322$$

$$\eta = \frac{1000}{1000 + 41.322} \times 100$$

$$\eta = 96\%$$

- 11.** Activities of three radioactive substances A, B and C are represented by the curves A, B and C, in the figure. Then their half-lives  $T_{\frac{1}{2}}(A) : T_{\frac{1}{2}}(B) : T_{\frac{1}{2}}(C)$  are in the ratio :



(1) 3 : 2 : 1                      (2) 2 : 1 : 1                      (3) 4 : 3 : 1                      (4) 2 : 1 : 3

**Sol. 4**

$$R_A = R_0 A e^{-\frac{\ln 2}{T}(t)}$$

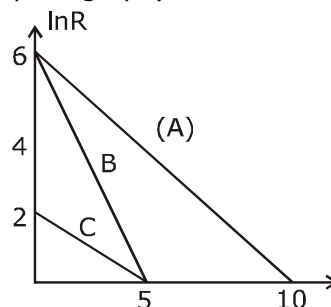
$$\ln(R_A) = \ln(R_0 A) - \lambda t$$

( $\lambda$  = slope of graph)

$$\lambda_A = \frac{6}{10} = \frac{\ln 2}{T_A}$$

$$\lambda_B = \frac{6}{5} = \frac{\ln 2}{T_B}$$

$$\lambda_C = \frac{2}{5} = \frac{\ln 2}{T_C}$$



$$\left. \begin{aligned} T_A &= \frac{5}{3} \ln 2 \\ T_B &= \frac{5}{6} \ln 2 \\ T_C &= \frac{5}{2} \ln 2 \end{aligned} \right\} \Rightarrow T_A : T_B : T_C = \frac{1}{3} : \frac{1}{6} : \frac{1}{2}$$

$$= 2 : 1 : 3$$

- 12.** The value of the acceleration due to gravity is  $g_1$  at a height  $h = \frac{R}{2}$  ( $R$  = radius of the earth) from the surface of the earth. It is again equal to  $g_1$  at a depth  $d$  below the surface of the earth. The ratio  $\left(\frac{d}{R}\right)$  equals :

(1)  $\frac{4}{9}$                       (2)  $\frac{1}{3}$                       (3)  $\frac{5}{9}$                       (4)  $\frac{7}{9}$

**Sol. 3**

$$g_{\text{at high}} = g_{\text{at depth}}$$

$$g_{\text{surface}} = \frac{GM}{R^2}$$

$$g \left(1 - \frac{d}{R}\right) = \frac{GM_e}{(R+h)^2}$$

$$g \left(1 - \frac{d}{R}\right) = \frac{GM}{R^2 \left(1 + \frac{h}{R}\right)^2} = \frac{g}{\left(1 + \frac{R}{2R}\right)^2} = \frac{4g}{9}$$

$$\frac{d}{R} = 1 - \frac{4}{9} = \frac{5}{9}$$

- 13.** A hollow spherical shell at outer radius  $R$  floats just submerged under the water surface. The inner radius of the shell is  $r$ . If the specific gravity of the shell material is  $\frac{27}{8}$  w.r.t water, the value of  $r$  is:

(1)  $\frac{4}{9} R$                       (2)  $\frac{8}{9} R$                       (3)  $\frac{1}{3} R$                       (4)  $\frac{2}{3} R$

**Sol. 2**

$$F_B = mg$$

$$\rho_l V_{\text{body}} g$$

(displaced water)

$$= \rho_b V_b g$$

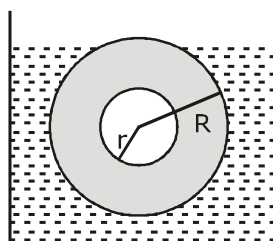
where mater present

$$\frac{4}{3} \pi R^3 = \frac{\rho_b}{\rho_l} \left( \frac{4}{3} \pi R^3 - \frac{4}{3} \pi r^3 \right)$$

$$R^3 = \frac{27}{8} (R^3 - r^3)$$

$$\frac{8}{27} R^3 = R^3 - r^3 \Rightarrow r^3 = R^3 - \frac{8R^3}{27} = \frac{19}{27} R^3$$

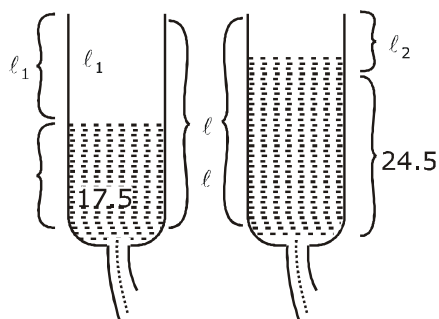
$$r = \frac{(19)^{1/3}}{3} R \approx 0.88 \approx \frac{8}{9} R$$



- 14.** In a resonance tube experiment when the tube is filled with water up to a height of 17.0 cm from bottom, it resonates with a given tuning fork. When the water level is raised the next resonance with the same tuning fork occurs at a height of 24.5 cm. If the velocity of sound in air is 330 m/s, the tuning fork frequency is :

(1) 2200 Hz                      (2) 550 Hz                      (3) 3300 Hz                      (4) 1100 Hz

**Sol. 1**



$$l_1 = l - 17$$

$$l_2 = l - 24.5$$

$$v = 2f (l_1 - l_2)$$

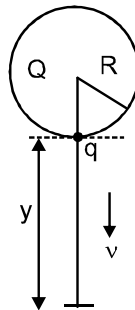
$$330 = 2 \times f \times [(l - 17) - (l - 24.5)] \times 10^{-2}$$

$$165 = f \times 7.5 \times 10^{-2}$$

$$f = \frac{165 \times 1000}{7.5}$$

$$f = 2200 \text{ Hz}$$

- 15.** A solid sphere of radius  $R$  carries a charge  $Q + q$  distributed uniformly over its volume. A very small point like piece of it of mass  $m$  gets detached from the bottom of the sphere and falls down vertically under gravity. This piece carries charge  $q$ . If it acquires a speed  $v$  when it has fallen through a vertical height  $y$  (see figure), then : (assume the remaining portion to be spherical).



$$(1) v^2 = 2y \left[ \frac{qQ}{4\pi\epsilon_0 R(R+y)m} + g \right]$$

$$(2) v^2 = 2y \left[ \frac{QqR}{4\pi\epsilon_0 (R+y)^3 m} + g \right]$$

$$(3) v^2 = y \left[ \frac{qQ}{4\pi\epsilon_0 R(R+y)m} + g \right]$$

$$(4) v^2 = y \left[ \frac{qQ}{4\pi\epsilon_0 R^2 y m} + g \right]$$

**Sol.**

**1**

M.E.C.

$$K_A + U_A = K_B + U_B$$

$$0 + mgy + qV_A = \frac{1}{2} mv^2 + 0 + (+qv_B)$$

$$mgy + qV_A = \frac{1}{2} mv^2 + q(V_B)$$

$$mgy + \frac{qk(Q)}{R} = \frac{1}{2} mv^2 + \frac{qk(Q)}{R+y}$$

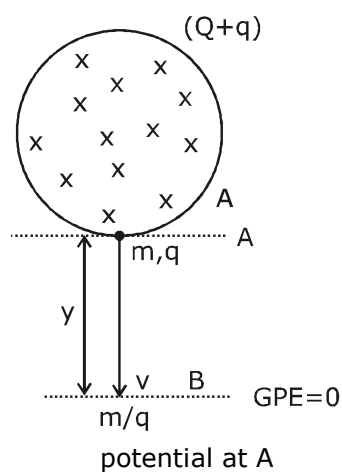
$$\frac{1}{2} mv^2 = -\frac{kq(Q)}{R+y} + \frac{kq(Q)}{R} + mgy$$

$$\frac{mv^2}{2} = \frac{-kq(Q)R + kq(Q)(R+y)}{R(R+y)} + mgy$$

$$v^2 = \frac{2}{m} \left[ \frac{-kqQR + kqQR + kqQ}{R(R+y)} + mgy \right]$$

$$v^2 = \frac{2}{m} \left[ \frac{kq(Q)y}{R(R+y)} + mgy \right]$$

$$v^2 = 2y \left[ \frac{q(Q)}{4\pi\epsilon_0 R(R+y)m} + g \right] = 2y \left[ \frac{qQ}{4\pi\epsilon_0 R(R+y)m} + g \right]$$



$$V_A = \frac{k(Q+q)}{R}$$

$$V_B = \frac{k(Q+q)}{R+y}$$

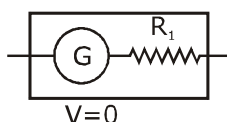
- 16.** A galvanometer of resistance  $G$  is converted into a voltmeter of range  $0 - 1V$  by connecting a resistance  $R_1$  in series with it. The additional resistance that should be connected in series with  $R_1$  to increase the range of the voltmeter to  $0 - 2V$  will be :

(1)  $G$  (2)  $R_1$  (3)  $R_1 - G$  (4)  $R_1 + G$

**Sol.** 4

$$V = I(R_1 + G)$$

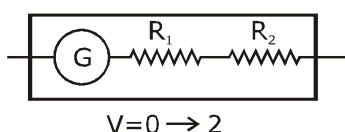
$$\frac{1}{2} = \frac{I(R_1 + G) \dots (i)}{I(R_1 + R_2 + G) \dots (ii)}$$



$$\frac{1}{2} = \frac{R_1 + G}{R_1 + R_2 + G}$$

$$R_1 + R_2 + G = 2R_1 + 2G$$

$$R_2 = R_1 + G$$



- 17.** Number of molecules in a volume of  $4 \text{ cm}^3$  of a perfect monoatomic gas at some temperature  $T$  and at a pressure of  $2 \text{ cm}$  of mercury is close to ? (Given, mean kinetic energy of a molecule (at  $T$ ) is  $4 \times 10^{-14} \text{ erg}$ ,  $g = 980 \text{ cm/s}^2$ , density of mercury =  $13.6 \text{ g/cm}^3$ )

(1)  $4.0 \times 10^{18}$  (2)  $4.0 \times 10^6$  (3)  $5.8 \times 10^{16}$  (4)  $5.8 \times 10^{18}$

**Sol.** 1

$$KE = \frac{3}{2} kT \Rightarrow \left( T = \frac{2E}{3k} \right), PV = NkT$$

$$P = \rho gh, V = 4 \text{ cm}^3$$

$$13.6 \times 10^3 \times 9.8 \times 2 \times 10^{-2} \times 4 \times 10^{-6}$$

$$= Nk \times \frac{2E}{3k} = \frac{N \times 2}{3} \times 4 \times 10^{-14} \times 10^7$$

$$N = \frac{13.6 \times 9.8 \times 2 \times 4 \times 10^{-5} \times 3 \times 10}{8}$$

$$N = 399.84 \times 10^{16}$$

$$= 3.99 \times 10^{18}$$

$$N = 4 \times 10^{18}$$

- 18.** An electron is constrained to move along the  $y$ -axis with a speed of  $0.1 c$  ( $c$  is the speed of light) in the presence of electromagnetic wave, whose electric field is  $\vec{E} = 30 \hat{j} \sin(1.5 \times 10^7 t - 5 \times 10^{-2} x)$  V/m. The maximum magnetic force experienced by the electron will be :

(given  $c = 3 \times 10^8 \text{ ms}^{-1}$  and electron charge =  $1.6 \times 10^{-19} \text{ C}$ )

(1)  $2.4 \times 10^{-18} \text{ N}$  (2)  $4.8 \times 10^{-19} \text{ N}$  (3)  $3.2 \times 10^{-18} \text{ N}$  (4)  $1.6 \times 10^{-19} \text{ N}$

**Sol. 2**

$v_e = 0.1C$  along y-axis direction of emwave - along (x)

$$\vec{E} = \vec{v} \times \vec{B}$$

$$E = CB \Rightarrow B = E/C$$

$\therefore$  force on  $e^-$  will be max.

If  $B$  is  $\perp$  to y-along z-axis

[ $\because E$  also  $\perp B$ ,  $B$  also  $\perp$  to direction of motion of wave]

$\therefore B \rightarrow$  along  $B_z$  ( $-z$ ) as

$$B = \frac{30}{C} \sin(1.5 \times 10^7 t - 5 \times 10^{-2} x)$$

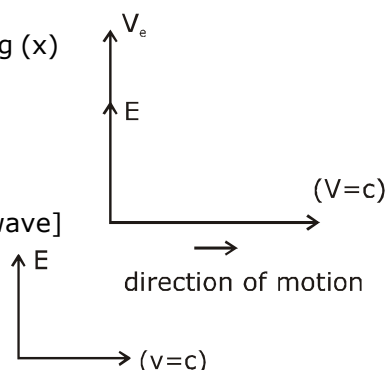
$$B_{\max} = \frac{30}{3 \times 10^8} = 10^{-7} \text{ T}$$

$\theta = 90$  between  $v_e$  &  $B$  so  $F_{\max} = qvB$

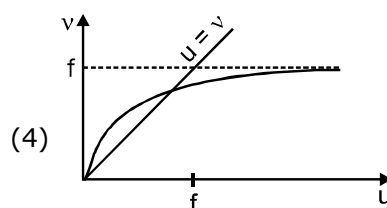
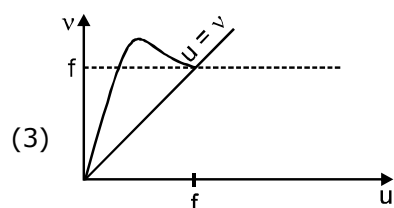
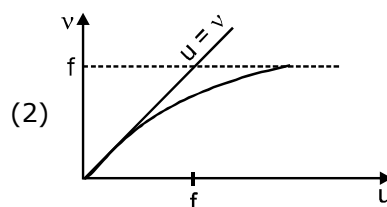
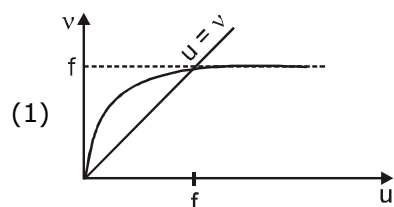
$$F_{\max} = e \times (0.1 \times C) \times \frac{30}{C}$$

$$= 1.6 \times 10^{-19} \times 3$$

$$F_{\max} = 4.8 \times 10^{-19} \text{ N}$$

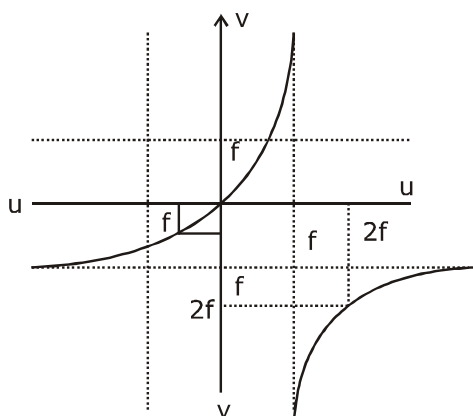


- 19.** For a concave lens of focal length  $f$ , the relation between object and image distances  $u$  and  $v$ , respectively, from its pole can best be represented by ( $u = v$  is the reference line) :



**Sol. 4**

Concave lens graph  $u$  v/s  $v$  by  $u$ - $v$  graph theory



- 20.** A physical quantity  $z$  depends on four observables  $a, b, c$  and  $d$ , as  $z = \frac{a^2 b^{\frac{2}{3}}}{\sqrt{c} d^3}$ . The percentages of error in the measurement of  $a, b, c$  and  $d$  are 2%, 1.5%, 4% and 2.5% respectively. The percentage of error in  $z$  is :
- (1) 16.5 %                      (2) 12.25 %                      (3) 13.5%                      (4) 14.5%

**Sol.**

**4**

$$z = a^2 b^{2/3} c^{-1/2} d^{-3}$$

$$100 \times \frac{dz}{z} = \left( 2 \frac{da}{a} + \frac{2}{3} \frac{db}{b} + \frac{1}{2} \frac{dc}{c} + 3 \frac{d(d)}{d} \right) \times 100$$

% error in  $z$

$$= \left( 2 \times 2 + \frac{2}{3} \times 1.5 + \frac{1}{2} \times 4 + 3 \times 2.5 \right) \%$$

$$= 4 + 1 + 2 + 7.5$$

$$= 14.5 \%$$

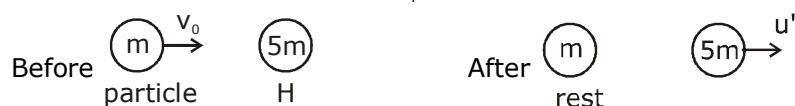
- 21.** A particle of mass  $200 \text{ MeV}/c^2$  collides with a hydrogen atom at rest. Soon after the collision the particle comes to rest, and the atom recoils and goes to its first excited state. The initial kinetic energy of the particle (in eV) is  $\frac{N}{4}$ . The value of  $N$  is :

(Given the mass of the hydrogen atom to be  $1 \text{ GeV}/c^2$ ) \_\_\_\_\_.

**Sol.**

**51**

$$m_H = 1 \text{ GeV}/c^2 = 1000 \text{ MeV}/c^2, m_{\text{particle}} = 200 \text{ MeV}/c^2 = m$$



$$\therefore mv_0 + 0 = 0 + 5mV' \Rightarrow v' = \frac{v_0}{5}$$

loss in KE

$$= \frac{1}{2} m v_0^2 - \frac{1}{2} (5m) \left( \frac{v_0}{5} \right)^2$$

$$= \frac{4}{5} \left( \frac{m v_0^2}{2} \right) = \frac{4}{5} k$$

$$\frac{4}{5} k = 10.2$$

$$k = 12.75 \text{ eV} = \frac{12.75}{100} = \frac{51}{4}$$

$$\text{so } n = 51$$

- 22.** Two concentric circular coils,  $C_1$  and  $C_2$ , are placed in the XY plane.  $C_1$  has 500 turns, and a radius of 1 cm.  $C_2$  has 200 turns and radius of 20 cm.  $C_2$  carries a time dependent current

$I(t) = (5t^2 - 2t + 3) \text{ A}$  where  $t$  is in s. The emf induced in  $C_1$  (in mV), at the instant  $t = 1 \text{ s}$  is  $\frac{4}{x}$ . The value of  $x$  is \_\_\_\_\_.

**Sol.** **5**

$$B_2 = \frac{\mu_0 I_2 N_2}{2R_2}$$

$$\phi = N_1 B_2 \pi R_1^2 = N_1 N_2 \frac{\mu_0 I}{2R_2} \pi R_1^2$$

$$e = \frac{d\phi}{dt}$$

$$\phi = \frac{500 \times 200 \times 4\pi \times 10^{-7} \times (5t^2 - 2t + 3) \pi (10^{-2})^2}{2 \times 20 \times 10^{-2}}$$

$$\frac{10^5 \times 4\pi^2 \times 10^{-7} (5t^2 - 2t + 3) \times 10^{-4}}{40 \times 10^{-2}}$$

$$\phi = (5t^2 - 2t + 3) \times 10^{-4}$$

$$e = \left| \frac{d\phi}{dt} \right| = (10t - 2) \times 10^{-4}$$

$$t = 1 \text{ sec}$$

$$e = 8 \times 10^{-4} = 0.8 \text{ mV} = \frac{0.8}{10} = \frac{4}{5}$$

$$x = 5$$

- 23.** A beam of electrons of energy  $E$  scatters from a target having atomic spacing of  $1\text{\AA}$ . The first maximum intensity occurs at  $\theta = 60^\circ$ . Then  $E$  (in eV) is \_\_\_\_\_.

(Planck constant  $h = 6.64 \times 10^{-34} \text{ Js}$ ,  $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$ , electron mass  $m = 9.1 \times 10^{-31} \text{ kg}$ )

**Sol. 5**

$$2d \sin \theta = n\lambda = 1 \times \sqrt{\frac{150}{V}} \times 10^{-10}, \theta = 90 - \frac{\phi}{2}$$

$$2 \times 10^{-10} \times \sin 60 = \sqrt{\frac{150}{V}} \times 10^{-10}, \theta = 90 - \frac{60}{2} = 60$$

$$2 \times \frac{\sqrt{3}}{2} = \sqrt{\frac{150}{V}}$$

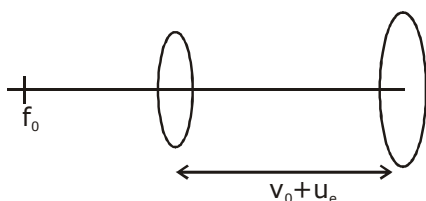
$$V = \frac{150}{3} = 50 \text{ volt}$$

$$E = eV = 50 \text{ eV}$$

- 24.** A compound microscope consists of an objective lens of focal length  $1 \text{ cm}$  and an eye piece of focal length  $5 \text{ cm}$  with a separation of  $10 \text{ cm}$ . The distance between an object and the objective lens, at

which the strain on the eye is minimum is  $\frac{n}{40} \text{ cm}$ . The value of  $n$  is \_\_\_\_\_.

**Sol. 50**



$$f_o = 1 \text{ cm}, f_e = 5 \text{ cm}, u_o = ?$$

final image at ( $\infty$ )

$$(v_e = \infty)$$

$$v_o + u_e = 10 \text{ cm} \quad \dots(i)$$

$$L = v_o + u_e = 10 \text{ cm}$$

$$v_o + 5 = 10$$

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

$$v_o = 5 \text{ cm}$$

$$\frac{1}{\infty} - \frac{1}{u_e} = \frac{1}{5}$$

$$\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o}$$

$$u_e = -5 \text{ cm}$$

$$\frac{1}{5} - \frac{1}{u_o} = \frac{1}{1}$$

$$|u_e| = 5$$

$$\frac{1}{u_0} = \frac{1}{5} - 1 = -\frac{4}{5} \Rightarrow u_0 = -\frac{4}{5}$$

$$|u_0| = \frac{5}{4} = \frac{50}{40} = \frac{n}{40}$$

$$\therefore n = 50$$

- 25.** A force  $\vec{F} = (\hat{i} + 2\hat{j} + 3\hat{k})$  N acts at a point  $(4\hat{i} + 3\hat{j} - \hat{k})$  m. Then the magnitude of torque about the point  $(\hat{i} + 2\hat{j} + \hat{k})$  m will be  $\sqrt{x}$  N-m. The value of x is \_\_\_\_\_.

**Sol. 195**

$$\vec{\tau} = \vec{r} \times \vec{F} = (3\hat{i} + \hat{j} - 2\hat{k}) \times (\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & 2 & 3 \end{vmatrix}$$

$$\hat{i}(3 + 4) - \hat{j}(9 + 2) + \hat{k}(6 - 1)$$

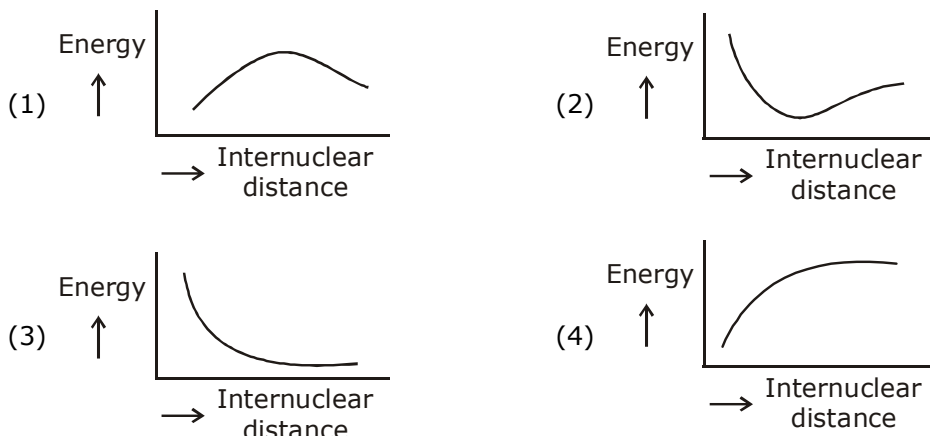
$$\vec{\tau} = 7\hat{i} - 11\hat{j} + 5\hat{k}$$

$$|\vec{\tau}| = \sqrt{49 + 121 + 25} = \sqrt{195}$$

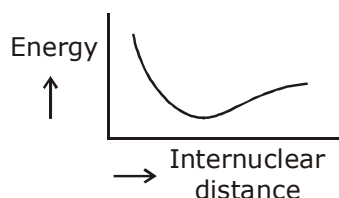
$$x = 195$$

## CHEMISTRY \_ 5 Sep. \_ SHIFT - 1

1. The potential energy curve for the  $H_2$  molecule as a function of internuclear distance is:



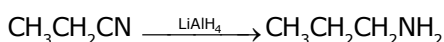
**Sol. 2**



2. The most appropriate reagent for conversion of  $C_2H_5CN$  into  $CH_3CH_2CH_2NH_2$  is:

(1)  $NaBH_4$  (2)  $Na(CN)BH_3$  (3)  $CaH_2$  (4)  $LiAlH_4$

**Sol. 4**



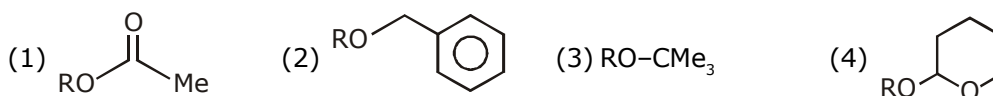
3. Which of the following is not an essential amino acid?

(1) Valine (2) Tyrosine (3) Lysine (4) Leucine

**Sol. 2**

Tyrosine is not an essential amino acid

4. Which of the following derivatives of alcohols is unstable in an aqueous base?



**Sol. 1**

Hydrolysis of ester occurs in basic medium.

5. The structure of  $PCl_5$  in the solid state is:

(1) Square planar  $[PCl_4]^+$  and octahedral  $[PCl_6]^-$   
 (2) Tetrahedral  $[PCl_4]^+$  and octahedral  $[PCl_6]^-$   
 (3) Trigonal bipyramidal  
 (4) Square pyramidal

**Sol. 2**

In solid state  $\text{PCl}_5$  exist in Ionpair i.e.  $(\text{PCl}_4^+)$  and  $(\text{PCl}_6^-)$   
 $\text{PCl}_4^+$  ( $\text{sp}^3$  tetrahedral)  
 $\text{PCl}_6^-$  ( $\text{sp}^3\text{d}^2$ ) – octahedral)

- 6.** A diatomic molecule  $\text{X}_2$  has a body-centred cubic (bcc) structure with a cell edge of 300 pm. The density of the molecule is  $6.17 \text{ g cm}^{-3}$ . The number of molecules present in 200 g of  $\text{X}_2$  is: (Avogadro constant ( $N_A$ ) =  $6 \times 10^{23} \text{ mol}^{-1}$ )

(1)  $8 N_A$  (2)  $2 N_A$  (3)  $40 N_A$  (4)  $4 N_A$

**Sol. 4**

$\text{X}_2 \rightarrow \text{BCC}$   
 $a = 300 \text{ pm}$

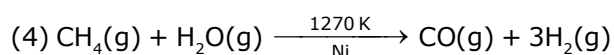
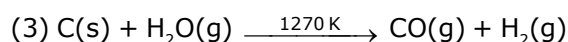
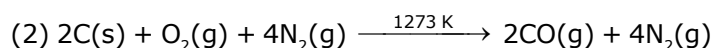
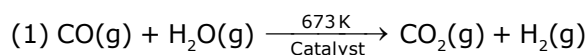
$$d = 6.17 \text{ g/cm}^3 = \frac{2 \times \text{GMM}}{6 \times 10^{23} \times (300 \times 10^{-10})^3}$$

$$\text{GMM} = \frac{6.17 \times 6 \times 9 \times 3 \times 10^{-1}}{2}$$

$$\text{GMM} = 81 \times 6.17 \times 10^{-1} \\ = 49.97 \text{ g/mol}$$

$$\text{No. of molecules} = \frac{200 \text{ g}}{49.97 \text{ g/mol}} = 4 \text{ mol} \\ = 4 N_A$$

- 7.** The equation that represents the water-gas shift reaction is:



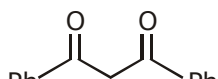
**Sol. 1**

Fact

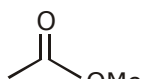
- 8.** The increasing order of the acidity of the  $\alpha$ -hydrogen of the following compounds is:



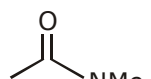
(A)



(B)



(C)



(D)

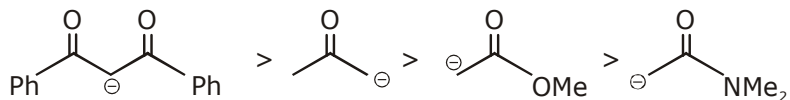
(1) (D) < (C) < (A) < (B)

(3) (C) < (A) < (B) < (D)

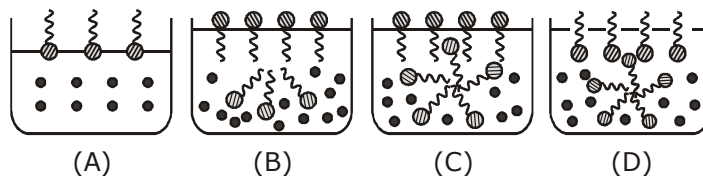
(2) (A) < (C) < (D) < (B)

(4) (B) < (C) < (A) < (D)

**Sol. 1**  
Stability order



**9.** Identify the correct molecular picture showing what happens at the critical micellar concentration (CMC) of an aqueous solution of a surfactant (● polar head; ~ non-polar tail; ● water).



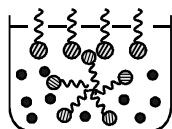
(1) (B)

(2) (A)

(3) (C)

(4) (D)

**Sol. 4**



**10.** If a person is suffering from the deficiency of nor-adrenaline, what kind of drug can be suggested?  
 (1) Antihistamine (2) Antidepressant  
 (3) Anti-inflammatory (4) Analgesic

**Sol. 2**

If nor-adrenaline is low, person may suffer from depression. Hence, anti depressant drug is suggested.

**11.** The values of the crystal field stabilization energies for a high spin  $d^6$  metal ion in octahedral and tetrahedral fields, respectively, are:

- (1)  $-2.4 \Delta_o$  and  $-0.6 \Delta_t$  (2)  $-1.6 \Delta_o$  and  $-0.4 \Delta_t$   
 (3)  $-0.4 \Delta_o$  and  $-0.27 \Delta_t$  (4)  $-0.4 \Delta_o$  and  $-0.6 \Delta_t$

**Sol. 4**

$d^6(\text{octahedral}) \rightarrow \text{high spin complex}$

$$= t_{2g}^4 e_g^2$$

$$\text{CFSE} = \left( -\frac{2}{5} \times 4 + \frac{3}{5} \times 2 \right) \Delta_o$$

$$= \left( \frac{-8+6}{5} \right) \Delta_o$$

$$= -0.4 \Delta_o$$

$d^6(\text{tetrahedral}) \rightarrow \text{high spin complex}$

$$= e_g^3 t_{2g}^3$$

$$\text{CFSE} = \left( -\frac{3}{5} \times 3 + \frac{2}{5} \times 3 \right) \Delta_t$$

$$= -0.6 \Delta_t$$

- 12.** A flask contains a mixture of compounds A and B. Both compounds decompose by first-order kinetics. The half-lives for A and B are 300 s and 180 s, respectively. If the concentrations of A and B are equal initially, the time required for the concentration of A to be four times that of B (in s) is: (Use  $\ln 2 = 0.693$ )

(1) 180                      (2) 300                      (3) 120                      (4) 900

**Sol. 4**

$$A_t = A_0 \cdot e^{-k_1 t}$$

$$B_t = B_0 \cdot e^{-k_2 t}$$

$$k_1 = \frac{\ln 2}{300}$$

$$k_2 = \frac{\ln 2}{180}$$

$A_t$  and  $B_t$  are related as  $[A] = 4[B]$

$$A_0 \cdot e^{-k_1 t} = 4 \times B_0 \cdot e^{-k_2 t}$$

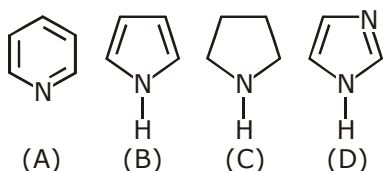
$$\frac{t}{180} - \frac{t}{300} = 2$$

$$\frac{t}{3} - \frac{t}{5} = 120$$

$$\frac{2t}{15} = 120$$

$$t = 900 \text{ sec}$$

- 13.** The increasing order of basicity of the following compounds is:



(1) (D) < (A) < (B) < (C)

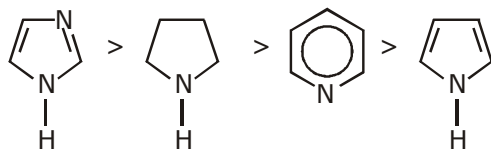
(2) (A) < (B) < (C) < (D)

(3) (B) < (A) < (D) < (C)

(4) (B) < (A) < (C) < (D)

**Sol. 4**

Correct order of basicity



- 14.** The condition that indicates a polluted environment is:  
 (1) pH of rain water to be 5.6 (2) BOD value of 5 ppm  
 (3) 0.03% of CO<sub>2</sub> in the atmosphere (4) eutrophication

**Sol. 4**

Eutrophication is the condition in which excessive richness of nutrients in a lake or water body, which causes dense growth of plant life and BOD increases.

- 15.** In the sixth period, the orbitals that are filled are:  
 (1) 6s, 5d, 5f, 6p (2) 6s, 4f, 5d, 6p (3) 6s, 6p, 6d, 6f (4) 6s, 5f, 6d, 6p

**Sol. 2**

(Fact) → energy order of orbital's according to Aufbau principle  
 6s < 4f < 5d < 6p

- 16.** The difference between the radii of 3<sup>rd</sup> and 4<sup>th</sup> orbits of Li<sup>2+</sup> is ΔR<sub>1</sub>. The difference between the radii of 3<sup>rd</sup> and 4<sup>th</sup> orbits of He<sup>+</sup> is ΔR<sub>2</sub>. Ratio ΔR<sub>1</sub> : ΔR<sub>2</sub> is:  
 (1) 8 : 3 (2) 3 : 8 (3) 3 : 2 (4) 2 : 3

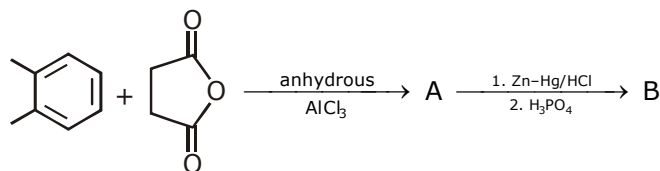
**Sol. 4**

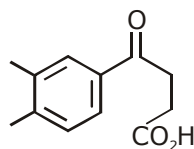
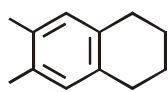
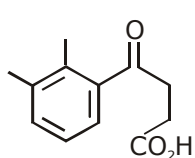
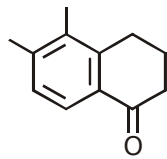
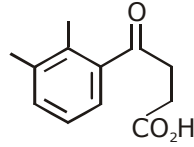
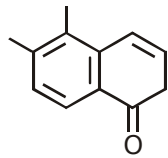
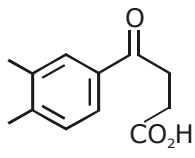
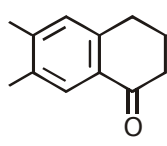
$$(R_4 - R_3)_{\text{Li}^{+2}} = \frac{0.529}{3} \{4^2 - 3^2\} = \Delta R_1$$

$$(R_4 - R_3)_{\text{He}^{+2}} = \frac{0.529}{2} \{4^2 - 3^2\} = \Delta R_2$$

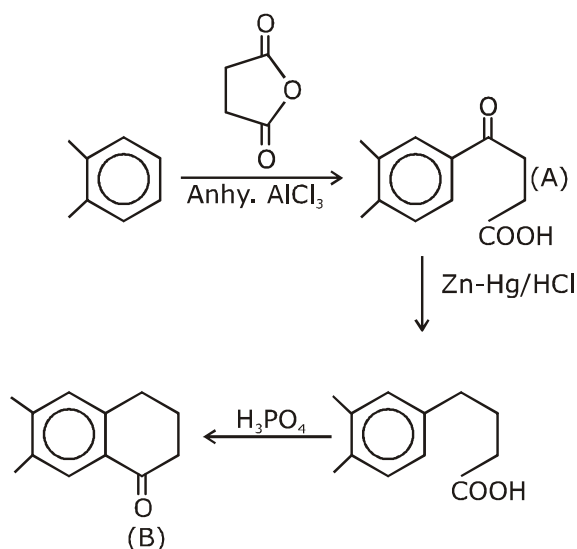
$$\frac{\Delta R_1}{\Delta R_2} = \frac{1/3}{1/2} = \frac{2}{3}$$

- 17.** In the following reaction sequence the major products A and B are:



- (1) A =  ; B =  (2) A =  ; B = 
- (3) A =  ; B =  (4) A =  ; B = 

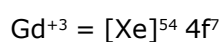
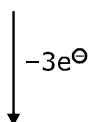
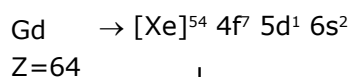
**Sol. 4**



**18.** The correct electronic configuration and spin-only magnetic moment (BM) of  $\text{Gd}^{3+}$  ( $Z = 64$ ), respectively, are:

- (1)  $[\text{Xe}] 5f^7$  and 7.9 (2)  $[\text{Xe}] 4f^7$  and 7.9 (3)  $[\text{Xe}] 5f^7$  and 8.9 (4)  $[\text{Xe}] 4f^7$  and 8.9

**Sol. 2**



$$\mu = \sqrt{7(7+2)} = \sqrt{63}$$

$$= 7.9 \text{ BM}$$

**19.** An Ellingham diagram provides information about:

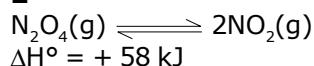
- (1) The pressure dependence of the standard electrode potentials of reduction reactions involved in the extraction of metals.  
 (2) The conditions of pH and potential under which a species is thermodynamically stable.  
 (3) The kinetics of the reduction process.  
 (4) The temperature dependence of the standard Gibbs energies of formation of some metal oxides.

**Sol. 4**

Fact

- 20.** Consider the following reaction:  
 $\text{N}_2\text{O}_4(\text{g}) \rightleftharpoons 2\text{NO}_2(\text{g}); \Delta H^\circ = +58 \text{ kJ}$   
 For each of the following cases (a, b), the direction in which the equilibrium shifts is:  
 (a) Temperature is decreased.  
 (b) Pressure is increased by adding  $\text{N}_2$  at constant T.  
 (1) (a) towards reactant, (b) towards product  
 (2) (a) towards reactant, (b) no change  
 (3) (a) towards product, (b) towards reactant  
 (4) (a) towards product, (b) no change

**Sol. 2**

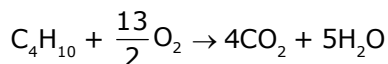
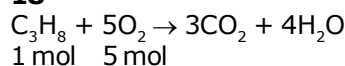


(towards reactant)

- (a) temp  $\downarrow \Rightarrow$  Backward shift as it is endothermic reaction  
 (b) As ' $\text{N}_2$ ' will not react with both  $\text{N}_2\text{O}_4$  &  $\text{NO}_2$ , as moles increases in reactants, as much as in products,  $\Delta =$  hence there is no change in equilibria.  
 $\therefore$  no change

- 21.** The minimum number of moles of  $\text{O}_2$  required for complete combustion of 1 mole of propane and 2 moles of butane is \_\_\_\_\_.

**Sol. 18**

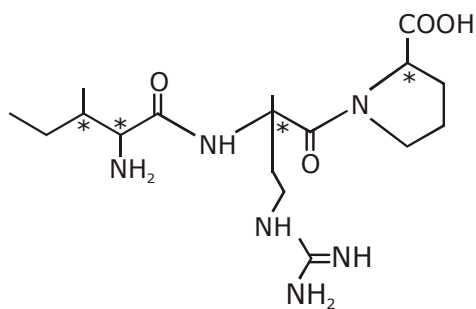


2 mol    13 mol

Total required mol of  $\text{O}_2 = 5 + 13 = 18$

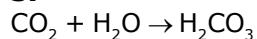
- 22.** The number of chiral carbon(s) present in peptide, Ile-Arg-Pro, is \_\_\_\_\_.

**Sol. 4**



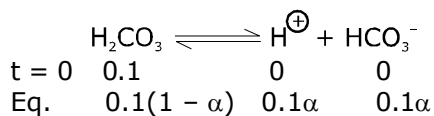
- 23.** A soft drink was bottled with a partial pressure of  $\text{CO}_2$  of 3 bar over the liquid at room temperature. The partial pressure of  $\text{CO}_2$  over the solution approaches a value of 30 bar when 44 g of  $\text{CO}_2$  is dissolved in 1 kg of water at room temperature. The approximate pH of the soft drink is \_\_\_\_\_  $\times 10^{-1}$ .  
 (First dissociation constant of  $\text{H}_2\text{CO}_3 = 4.0 \times 10^{-7}$ ;  $\log 2 = 0.3$ ; density of the soft drink =  $1 \text{ g mL}^{-1}$ )

**Sol. 37**



30 bar .....  $\rightarrow$  1 m/lit.

3 bar .....  $\rightarrow$  0.1 m/lit



$$4 \times 10^{-7} = \frac{0.1\alpha^2}{1 - \alpha}$$

$$(1 - \alpha) \approx 1$$

$$\alpha^2 = 4 \times 10^{-6}$$

$$\alpha = 2 \times 10^{-3}$$

$$[\text{H}^+] = 2 \times 10^{-4} \text{M}$$

$$\text{pH} = -[-4 \times \log(2)] = 3.7 = 37 \times 10^{-1}$$

- 24.** An oxidation-reduction reaction in which 3 electrons are transferred has a  $\Delta G^\circ$  of 17.37 kJ mol<sup>-1</sup> at 25°C. The value of  $E^\circ_{\text{cell}}$  (in V) is \_\_\_\_\_  $\times 10^{-2}$ .  
(1 F = 96,500 C mol<sup>-1</sup>)

**Sol. 6**

$$\Delta G^\circ = -nFE^\circ$$

$$17.37 \times 1000 = -3 \times 96500 \times E^\circ$$

$$E^\circ = \frac{17370}{3 \times 96500}$$

$$E^\circ = \frac{579}{9650} \text{ volt}$$

$$= 0.06 = 6 \times 10^{-2} \text{ volt}$$

Ans. 6

- 25.** The total number of coordination sites in ethylenediaminetetraacetate ( $\text{EDTA}^{4-}$ ) is \_\_\_\_\_.

**Sol. 6**

$\text{EDTA}^{4-}$  is hexadentate ligand

# QUESTION PAPER WITH SOLUTION

## MATHEMATICS \_ 5 Sep. \_ SHIFT - 1

**Q.1** If the volume of a parallelopiped, whose coterminus edges are given by the vectors

$\vec{a} = \hat{i} + \hat{j} + n\hat{k}$ ,  $\vec{b} = 2\hat{i} + 4\hat{j} - n\hat{k}$  and  $\vec{c} = \hat{i} + n\hat{j} + 3\hat{k}$  ( $n \geq 0$ ), is 158 cu. units, then:

- (1)  $\vec{a} \cdot \vec{c} = 17$  (2)  $\vec{b} \cdot \vec{c} = 10$  (3)  $n = 9$  (4)  $n = 7$

**Sol.** 2

$$\begin{vmatrix} 1 & 1 & n \\ 2 & 4 & -n \\ 1 & n & 3 \end{vmatrix} = 158$$

$$(12 + n^2) - (6 + n) + n(2n - 4) = 158$$

$$3n^2 - 5n + 6 - 158 = 0$$

$$3n^2 - 5n - 152 = 0$$

$$3n^2 - 24n + 19n - 152 = 0$$

$$(3n + 19)(n - 8) = 0$$

$$\Rightarrow n = 8$$

$$\Rightarrow \vec{b} \cdot \vec{c} = 10$$

**Q.2** A survey shows that 73% of the persons working in an office like coffee, whereas 65% like tea. If  $x$  denotes the percentage of them, who like both coffee and tea, then  $x$  cannot be:

- (1) 63 (2) 54 (3) 38 (4) 36

**Sol.** 4

$$n(\text{coffee}) = \frac{73}{100}$$

$$n(\text{tea}) = \frac{65}{100}$$

$$n(T \cap C) = \frac{x}{100}$$

$$n(C \cup T) = n(C) + n(T) - x \leq 100$$

$$= 73 + 65 - x \leq 100$$

$$\Rightarrow x \geq 38$$

Ans. 36

**Q.3** The mean and variance of 7 observations are 8 and 16, respectively. If five observations are 2, 4, 10, 12, 14, then the absolute difference of the remaining two observations is:

- (1) 1 (2) 4 (3) 3 (4) 2

**Sol.** 4

$$\text{Var}(x) = \sum \frac{x_i^2}{n} - (\bar{x})^2$$

$$16 = \frac{x_1^2 + x_2^2 + x_4^2 + x_5^2 + x_6^2 + x_7^2}{7} - 64$$

$$80 \times 7 = x_1^2 + x_2^2 + x_3^2 + \dots + x_7^2$$

$$\text{Now, } x_6^2 + x_7^2 = 560 - (x_1^2 + \dots + x_5^2)$$

$$x_6^2 + x_7^2 = 560 - (4 + 16 + 100 + 144 + 196)$$

$$x_6^2 + x_7^2 = 100 \quad \dots\dots(1)$$

$$\text{Now, } \frac{x_1 + x_2 + \dots + x_7}{7} = 8$$

$$x_6 + x_7 = 14 \quad \dots\dots(2)$$

from (1) & (2)

$$(x_6 + x_7)^2 - 2x_6 x_7 = 100$$

$$2x_6 x_7 = 96 \quad \Rightarrow x_6 x_7 = 48 \quad \dots\dots(3)$$

$$\text{Now, } |x_6 - x_7| = \sqrt{(x_6 + x_7)^2 - 4x_6 x_7}$$

$$= \sqrt{196 - 192} = 2$$

**Q.4** If  $2^{10} + 2^9 \cdot 3^1 + 2^8 \cdot 3^2 + \dots + 2 \cdot 3^9 + 3^{10} = S - 2^{11}$ , then S is equal to:

- (1)  $3^{11}$                       (2)  $\frac{3^{11}}{2} + 2^{10}$                       (3)  $2 \cdot 3^{11}$                       (4)  $3^{11} - 2^{12}$

**Sol. 1**

let

$$S' = 2^{10} + 2^9 \cdot 3^1 + 2^8 \cdot 3^2 + \dots + 2 \cdot 3^9 + 3^{10}$$

$$\frac{3 \times S'}{2} = 2^9 \times 3^1 + 2^8 \cdot 3^2 + \dots + 3^{10} + \frac{3^{11}}{2}$$

---


$$\frac{-S'}{2} = 2^{10} - \frac{3^{11}}{2}$$

$$S' = 3^{11} - 2^{11}$$

$$\text{Now } S' = S - 2^{11}$$

$$S = 3^{11}$$

**Q.5** If  $3^{2 \sin 2\alpha - 1}$ , 14 and  $3^{4 - 2 \sin 2\alpha}$  are the first three terms of an A.P. for some  $\alpha$ , then the sixth terms of this A.P. is:

- (1) 65                      (2) 81                      (3) 78                      (4) 66

**Sol. 4**

$$28 = 3^{2\sin 2\alpha - 1} + 3^{4 - 2\sin 2\alpha}$$

$$28 = \frac{9^{\sin 2\alpha}}{3} + \frac{81}{9^{\sin 2\alpha}}$$

$$\text{Let } 9^{\sin 2\alpha} = t$$

$$28 = \frac{t}{3} + \frac{81}{t}$$

$$t^2 - 84t + 243 = 0$$

$$t = 81, 3$$

$$9^{\sin 2\alpha} = 9^2 \text{ or } 3$$

$$\sin 2\alpha = 2 \text{ or } \sin 2\alpha = 1/2$$

(Not possible)

Now three terms in A.P. are

1, 14, 27

Next term are

40, 53, 66

**Q.6** If the common tangent to the parabolas,  $y^2=4x$  and  $x^2=4y$  also touches the circle,  $x^2+y^2 = c^2$ , then  $c$  is equal to:

(1)  $\frac{1}{2}$

(2)  $\frac{1}{4}$

(3)  $\frac{1}{\sqrt{2}}$

(4)  $\frac{1}{2\sqrt{2}}$

**Sol. 3**

$$y = mx + \frac{1}{m}$$

$$x^2 = 4\left(mx + \frac{1}{m}\right)$$

$$x^2 - 4mx - \frac{4}{m} = 0$$

$$D = 0$$

$$16m^2 + \frac{16}{m} = 0$$

$$16\left(\frac{m^3 + 1}{m}\right) = 0$$

$$m = -1$$

$$\Rightarrow y + x = -1$$

$$\text{Now, } \left| \frac{-1}{\sqrt{2}} \right| = c$$

$$c = \frac{1}{\sqrt{2}}$$

**Q.7** If the minimum and the maximum values of the function  $f : \left[ \frac{\pi}{4}, \frac{\pi}{2} \right] \rightarrow \mathbb{R}$ , defined by

$$f(\theta) = \begin{vmatrix} -\sin^2 \theta & -1 - \sin^2 \theta & 1 \\ -\cos^2 \theta & -1 - \cos^2 \theta & 1 \\ 12 & 10 & -2 \end{vmatrix}$$
 are  $m$  and  $M$  respectively, then the ordered pair  $(m, M)$  is

equal to :

- (1)  $(0, 4)$  (2)  $(-4, 0)$   
 (3)  $(-4, 4)$  (4)  $(0, 2\sqrt{2})$

**Sol. 2**

$$f(\theta) = \begin{vmatrix} -\sin^2 \theta & -1 - \sin^2 \theta & 1 \\ -\cos^2 \theta & -1 - \cos^2 \theta & 1 \\ 12 & 10 & -2 \end{vmatrix}$$

$$C_1 \rightarrow C_1 - C_2, C_3 \rightarrow C_3 + C_2$$

$$\begin{vmatrix} 1 & -1 - \sin^2 \theta & -\sin^2 \theta \\ 1 & -1 - \cos^2 \theta & -\cos^2 \theta \\ 2 & 10 & 8 \end{vmatrix}$$

$$C_2 \rightarrow C_2 - C_3$$

$$\begin{vmatrix} 1 & -1 & -\sin^2 \theta \\ 1 & -1 & -\cos^2 \theta \\ 2 & 2 & 8 \end{vmatrix}$$

$$1(2\cos^2\theta - 8) + (8 + 2\cos^2\theta) - 4\sin^2\theta$$

$$f(\theta) = 4\cos 2\theta$$

**Q.8** Let  $\lambda \in \mathbb{R}$ . The system of linear equations

$$2x_1 - 4x_2 + \lambda x_3 = 1$$

$$x_1 - 6x_2 + x_3 = 2$$

$$\lambda x_1 - 10x_2 + 4x_3 = 3$$

is inconsistent for:

- (1) exactly two values of  $\lambda$   
 (2) exactly one negative value of  $\lambda$ .  
 (3) every value of  $\lambda$ .  
 (4) exactly one positive value of  $\lambda$ .

**Sol. 2**

$$D = \begin{vmatrix} 2 & -4 & \lambda \\ 1 & -6 & 1 \\ \lambda & -10 & 4 \end{vmatrix} = 0$$

$$2(-14) + 4(4 - \lambda) + \lambda(6\lambda - 10) = 0$$

$$-28 + 16 - 4\lambda + 6\lambda^2 - 10\lambda = 0$$

$$6\lambda^2 - 14\lambda - 12 = 0$$

$$3\lambda^2 - 7\lambda - 6 = 0$$

$$3\lambda^2 - 9\lambda + 2\lambda - 6 = 0$$

$$(3\lambda + 2)(\lambda - 3) = 0$$

$$\lambda = -2/3, 3$$

$$D_1 = \begin{vmatrix} 1 & -4 & \lambda \\ 2 & -6 & 1 \\ 3 & -10 & 4 \end{vmatrix}$$

$$\Rightarrow -14 + 4(5) + \lambda(-2)$$

$$\Rightarrow -2\lambda + 6$$

$$D_2 = \begin{vmatrix} 2 & 1 & \lambda \\ 1 & 2 & 1 \\ \lambda & 3 & 4 \end{vmatrix}$$

$$\Rightarrow 2(5) - 1(4 - \lambda) + \lambda(3 - 2\lambda)$$

$$\Rightarrow 10 - 4 + \lambda + 3\lambda - 2\lambda^2$$

$$\Rightarrow -2\lambda^2 + 4\lambda + 6$$

$$\Rightarrow -2(\lambda^2 - 2\lambda - 3)$$

$$\Rightarrow -2[\lambda^2 - 3\lambda + \lambda - 3]$$

$$\Rightarrow -2(\lambda - 3)(\lambda + 1)$$

$$D_3 = \begin{vmatrix} 2 & -4 & 1 \\ 1 & -6 & 2 \\ \lambda & -10 & 3 \end{vmatrix} \Rightarrow 2(-18 + 20) + 4(3 - 2\lambda) + 1(-10 + 6\lambda)$$

$$= 4 + 12 - 8\lambda - 10 + 6\lambda$$

$$= -2\lambda + 6$$

$$\Rightarrow \lambda = -2/3 \text{ is answer}$$

**Q.9** If the point P on the curve,  $4x^2 + 5y^2 = 20$  is farthest from the point Q(0, -4), then  $PQ^2$  is equal to:

(1) 48

(2) 29

(3) 21

(4) 36

**Sol. 4**

Let P be  $(\sqrt{5} \cos \theta, 2 \sin \theta)$

$$\text{Now, } PQ = \sqrt{(\sqrt{5} \cos \theta)^2 + (2 \sin \theta + 4)^2}$$

$$PQ = \sqrt{5 \cos^2 \theta + (2 \sin \theta + 4)^2}$$

$$\frac{d(PQ)}{d\theta} = 0 \Rightarrow -10 \sin \theta \cos \theta + (4 \sin \theta + 8) \cos \theta = 0$$

$$\Rightarrow -6 \sin \theta \cos \theta + 8 \cos \theta = 0$$

$$\cos \theta = 0 \quad \text{or} \quad \sin \theta = \frac{4}{3}$$

Not possible

So P is either (0,2) or (0,-2)

$$PQ^2 = 36$$

**Q.10** The product of the roots of the equation  $9x^2 - 18|x| + 5 = 0$  is :

(1)  $\frac{25}{81}$

(2)  $\frac{5}{9}$

(3)  $\frac{5}{27}$

(4)  $\frac{25}{9}$

**Sol. 1**

$$9t^2 - 18t + 5 = 0$$

$$9t^2 - 15t - 3t + 5 = 0$$

$$(3t - 5)(3t - 1) = 0$$

$$|x| = \frac{5}{3}, \frac{1}{3}$$

$$\Rightarrow x = \frac{5}{3}, \frac{-5}{3}, \frac{1}{3}, \frac{-1}{3}$$

$$\Rightarrow P = \frac{25}{81}$$

**Q.11** If  $y=y(x)$  is the solution of the differential equation  $\frac{5+e^x}{2+y} \cdot \frac{dy}{dx} + e^x = 0$  satisfying

$y(0)=1$ , then a value of  $y(\log_e 13)$  is:

(1) 1

(2) 0

(3) 2

(4) -1

**Sol. 4**

$$\frac{dy}{dx} + \left( e^x \times \frac{y+2}{e^x+5} \right) = 0$$

$$\frac{dy}{dx} + \left( \frac{e^x}{e^x + 5} \right) y = \frac{-2e^x}{e^x + 5}$$

$$\text{I.F.} = e^{\int \frac{e^x}{e^x + 5} dx}$$

$$= e^{\int \left( 1 - \frac{5}{e^x + 5} \right) dx}$$

$$= e^{\int \left( 1 - \frac{5e^{-x}}{1 + 5e^{-x}} \right) dx}$$

$$= e^{x + \ln(1 + 5e^{-x})}$$

$$= e^x \cdot (1 + 5e^{-x}) \Rightarrow (e^x + 5)$$

$$y(e^x + 5) = -\int 2e^x dx$$

$$y(e^x + 5) = -2e^x + C$$

$$\Downarrow x=0$$

$$(6) = -2 + C \Rightarrow C = 8$$

$$y(\ln 13) = \frac{8 - 2 \times 13}{13 + 5} = \frac{-18}{18} = -1$$

**Q.12** If  $S$  is the sum of the first 10 terms of the series  $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) + \tan^{-1}\left(\frac{1}{21}\right) + \dots$ , then  $\tan(S)$  is equal to :

(1)  $\frac{5}{11}$

(2)  $\frac{5}{6}$

(3)  $-\frac{6}{5}$

(4)  $\frac{10}{11}$

**Sol. 2**

$$S = \tan^{-1}\left(\frac{1}{1+1 \times 2}\right) + \tan^{-1}\left(\frac{1}{1+2 \times 3}\right) + \dots$$

$$T_r = \tan^{-1}\left(\frac{1}{1+r(r+1)}\right)$$

$$T_r = \tan^{-1}(r+1) - \tan^{-1}r$$

$$T_1 = \tan^{-1}2 - \tan^{-1}1$$

$$T_2 = \tan^{-1}3 - \tan^{-1}2$$

$$T_3 = \tan^{-1}4 - \tan^{-1}3$$

$$T_{10} = \tan^{-1}11 - \tan^{-1}10$$

$$\Rightarrow S = \tan^{-1}11 - \tan^{-1}1$$

$$\Rightarrow \tan S = \frac{10}{12} = \frac{5}{6}$$

**Q.13** The value of  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{1+e^{\sin x}} dx$  is:

- (1)  $\frac{\pi}{2}$                       (2)  $\frac{\pi}{4}$                       (3)  $\pi$                       (4)  $\frac{3\pi}{2}$

**Sol. 1**

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{1+e^{\sin x}} dx$$

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{e^{\sin x}}{1+e^{\sin x}} dx \Rightarrow 2I = \pi$$

$$I = \frac{\pi}{2}$$

**Q.14** If (a, b, c) is the image of the point (1, 2, -3) in the line,  $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$ , then a+b+c is

- (1) 2                      (2) 3                      (3) -1                      (4) 1

**Sol. 1**

$$\overrightarrow{PM} \perp (2\hat{i} - 2\hat{j} - \hat{k})$$

$$\Rightarrow (2\lambda - 2) \cdot 2 + (1 - 2\lambda)(-2) + (3 - \lambda)(-1) = 0$$

$$\Rightarrow 4\lambda - 4 + 4\lambda - 2 + \lambda - 3 = 0$$

$$\Rightarrow 9\lambda = 9 \Rightarrow \lambda = 1$$

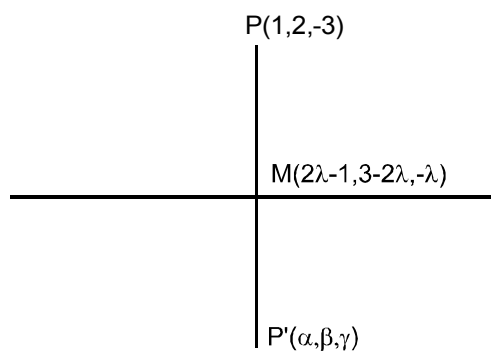
$$\Rightarrow m(1, 1, -1)$$

$$\text{Now, } p' = 2M - P$$

$$= 2(1, 1, -1) - (1, 2, -3)$$

$$= (1, 0, 1)$$

$$a + b + c = 2$$



**Q.15** If the function  $f(x) = \begin{cases} k_1(x - \pi)^2 - 1, & x \leq \pi \\ k_2 \cos x, & x > \pi \end{cases}$  is twice differentiable, then the ordered pair  $(k_1, k_2)$  is equal to:

- (1) (1,1)                      (2) (1,0)                      (3)  $\left(\frac{1}{2}, -1\right)$                       (4)  $\left(\frac{1}{2}, 1\right)$

**Sol. 4**

$$f(x) = \begin{cases} 2k_1(x - \pi); & x \leq \pi \\ -k_2 \sin x & ; x > \pi \end{cases}$$

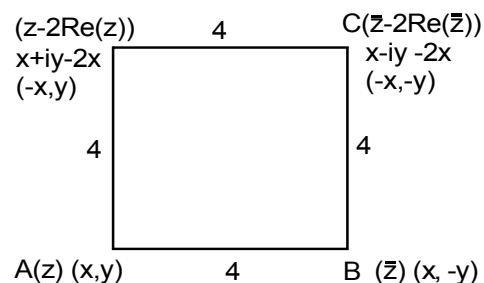
$$f''(x) = \begin{cases} 2k_1 & ; x \leq \pi \\ -k_2 \cos x; & x > \pi \end{cases}$$

$$2k_1 = k_2$$

**Q.16** If the four complex numbers  $z, \bar{z}, \bar{z} - 2\text{Re}(\bar{z})$  and  $z - 2\text{Re}(z)$  represent the vertices of a square of side 4 units in the Argand plane, then  $|z|$  is equal to:

- (1) 2                      (2) 4                      (3)  $4\sqrt{2}$                       (4)  $2\sqrt{2}$

**Sol. 4**



$$\begin{aligned} \text{Let } z &= x + iy \\ CA^2 &= AB^2 + BC^2 \\ 2^2x^2 + 2^2y^2 &= 32 \\ x^2 + y^2 &= 8 \\ \sqrt{x^2 + y^2} &= 2\sqrt{2} \end{aligned}$$

**Q.17** If  $\int (e^{2x} + 2e^x - e^{-x} - 1)e^{(e^x + e^{-x})} dx = g(x)e^{(e^x + e^{-x})} + c$ , where  $c$  is a constant of integration, then  $g(0)$  is equal to :

- (1) 2                      (2)  $e$   
(3) 1                      (4)  $e^2$

**Sol. 1**

$$\begin{aligned}
 & \int (e^{2x} + 2e^x - e^{-x} - 1) e^{(e^x + e^{-x})} dx \\
 & \int (e^{2x} + e^x - 1) e^{(e^x + e^{-x})} dx + \int (e^x - e^{-x}) e^{(e^x + e^{-x})} dx \\
 & \int (e^x + 1 - e^{-x}) e^{(e^x + e^{-x} + x)} dx + \int (e^x - e^{-x}) e^{(e^x + e^{-x})} dx \\
 & e^{(e^x + e^{-x} + x)} + e^{e^x + e^{-x}} + C \\
 & (e^{e^x + e^{-x}}) [e^x + 1] + C \\
 & \Downarrow \\
 & g(x) \\
 & \Rightarrow g(0) = 2
 \end{aligned}$$

**Q.18** The negation of the Boolean expression  $x \leftrightarrow \sim y$  is equivalent to :

- (1)  $(x \wedge y) \wedge (\sim x \vee \sim y)$
- (2)  $(x \wedge y) \vee (\sim x \wedge \sim y)$
- (3)  $(x \wedge \sim y) \vee (\sim x \wedge y)$
- (4)  $(\sim x \wedge y) \vee (\sim x \wedge \sim y)$

**Sol. 2**

As we know

$$\begin{aligned}
 \sim (p \leftrightarrow q) &= (p \wedge \sim q) \vee (\sim p \wedge q) \\
 \Rightarrow \text{so, } \sim (x \leftrightarrow \sim y) &= (x \wedge y) \vee (\sim x \wedge \sim y)
 \end{aligned}$$

**Q.19** If  $\alpha$  is positive root of the equation,  $p(x) = x^2 - x - 2 = 0$ , then  $\lim_{x \rightarrow \alpha^+} \frac{\sqrt{1 - \cos(p(x))}}{x + \alpha - 4}$  is equal to :

- (1)  $\frac{1}{2}$
- (2)  $\frac{3}{\sqrt{2}}$
- (3)  $\frac{3}{2}$
- (4)  $\frac{1}{\sqrt{2}}$

**Sol. 2**

$$f(x) = x^2 - x - 2 \left\langle \begin{matrix} 2 \\ -1 \end{matrix} \right\rangle = \alpha$$

$$\lim_{x \rightarrow 2^+} \frac{\sqrt{1 - \cos(x-2)(x+1)}}{x + \alpha - 4}$$

$$\lim_{x \rightarrow 2^+} \frac{\sqrt{1 - \cos(x-2)(x+1)}}{(x-2)}$$

$$\lim_{h \rightarrow 0} \frac{\sqrt{1 - \cos(h \times (h+3))}}{h}$$

$$\lim_{h \rightarrow 0} \sqrt{\frac{1 - \cos(h \times (h+3))}{h^2 \times (h+3)^2}} \times (h+3)^2 \Rightarrow \sqrt{\frac{1}{2}} \times 9 = \frac{3}{\sqrt{2}}$$

- Q.20** If the co-ordinates of two points A and B are  $(\sqrt{7}, 0)$  and  $(-\sqrt{7}, 0)$  respectively and P is any point on the conic,  $9x^2 + 16y^2 = 144$ , then PA+PB is equal to :
- (1) 6 (2) 16  
(3) 9 (4) 8

**Sol.** 4

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

$$e = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$$

$$F_1(\sqrt{7}, 0), F_2(-\sqrt{7}, 0)$$

$$PF_1 + PF_2 = 2a$$

$$PA + PB = 2 \times 4 = 8$$

- Q.21** The natural number m, for which the coefficient of x in the binomial expansion of

$$\left(x^m + \frac{1}{x^2}\right)^{22} \text{ is 1540, is .....}$$

**Sol.** 13

$$T_{r+1} = {}^{22}C_r (x^m)^{22-r} \left(\frac{1}{x^2}\right)^r$$

$$= {}^{22}C_r (x)^{22m-mr-2r}$$

$$\text{Given } {}^{22}C_r = 1540 = {}^{22}C_{19} \Rightarrow r=19$$

$$\therefore 22m - mr - 2r = 1$$

$$\Rightarrow m = \frac{2r+1}{22-r}$$

$$m = 13 (\text{At } r=19)$$

**Q.22** Four fair dice are thrown independently 27 times. Then the expected number of times, at least two dice show up a three or a five, is .....

**Sol. 11**

$$\begin{aligned}
 (\text{at least 2 or 3}) &= {}^4C_2 \left(\frac{2}{6}\right)^2 \left(\frac{4}{6}\right)^2 + {}^4C_3 \left(\frac{2}{6}\right)^3 \left(\frac{4}{6}\right)^1 + {}^4C_4 \left(\frac{2}{6}\right)^4 \\
 &= 6 \times \frac{1}{9} \times \frac{4}{9} + 4 \times \frac{1}{27} \times \frac{2}{3} + \frac{1}{81} \\
 &= \frac{33}{81} = \frac{11}{27} \Rightarrow nP \Rightarrow 11
 \end{aligned}$$

**Q.23** Let  $f(x) = x \cdot \left[ \frac{x}{2} \right]$ , for  $-10 < x < 10$ , where  $[t]$  denotes the greatest integer function. Then the number of points of discontinuity of  $f$  is equal to.....

**Sol. 8**

$$f(x) = x \left[ \frac{x}{2} \right], -10 < x < 10$$

$$-5 < \frac{x}{2} < 5$$

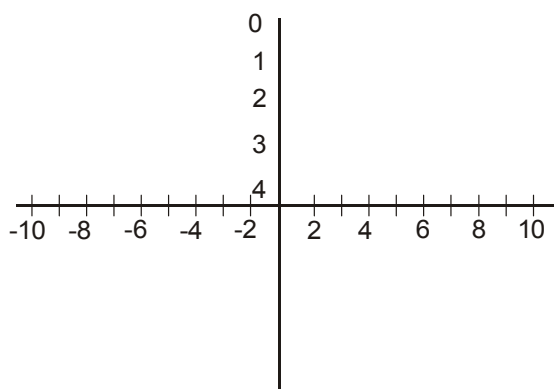
$$-5x \quad -5 < \frac{x}{2} < -4$$

$$-4x \quad -4 < \frac{x}{2} < -3$$

$$-3x \quad -3 < \frac{x}{2} < -2$$

$$-2x \quad -2 < \frac{x}{2} < -1$$

$$-x \quad -1 < \frac{x}{2} < 0$$



Number of point of discontinuity = 8

**Q.24** The number of words, with or without meaning, that can be formed by taking 4 letters at a time from the letters of the word 'SYLLABUS' such that two letters are distinct and two letters are alike, is

**Sol. 240**

SS, Y, LL, A, B, U

$$\begin{array}{|c|c|c|c|} \hline S & S & & \\ \hline \end{array} \Rightarrow {}^5C_2 \times \frac{4!}{2!} \times {}^2C_1$$

$$\Rightarrow 120 \times 2$$

$$= 240$$

**Q.25** If the line,  $2x - y + 3 = 0$  is at a distance  $\frac{1}{\sqrt{5}}$  and  $\frac{2}{\sqrt{5}}$  from the lines  $4x - 2y + \alpha = 0$  and  $6x - 3y + \beta = 0$ , respectively, then the sum of all possible values of  $\alpha$  and  $\beta$  is

**Sol. 30**

$$L_1 : 2x - y + 3 = 0$$

$$L_2 : 4x - 2y + \alpha = 0$$

$$L_3 : 6x - 3y + \beta = 0$$

$$\frac{\left| \frac{\alpha}{2} - 3 \right|}{\sqrt{5}} = \frac{1}{\sqrt{5}} \Rightarrow \frac{\alpha}{2} - 3 = 1, -1$$

$$\Rightarrow \alpha = 8, 4$$

$$\frac{\left| \frac{\beta}{3} - 3 \right|}{\sqrt{5}} = \frac{2}{\sqrt{5}} \Rightarrow \frac{\beta}{3} - 3 = 2, -2$$

$$\Rightarrow \beta = 15, 3$$

# QUESTION PAPER WITH SOLUTION

## PHYSICS \_ 5 Sep. \_ SHIFT - 2

1. A ring is hung on a nail. It can oscillate, without slipping or sliding (i) in its plane with a time period  $T_1$  and, (ii) back and forth in a direction perpendicular to its plane, with a period  $T_2$ . The ratio  $\frac{T_1}{T_2}$  will be:

(1)  $\frac{3}{\sqrt{2}}$

(2)  $\frac{\sqrt{2}}{3}$

(3)  $\frac{2}{\sqrt{3}}$

(4)  $\frac{2}{3}$

**Sol. 3**

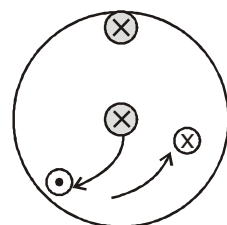
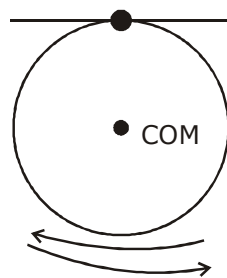
$$T_1 = 2\pi \sqrt{\frac{(mR^2 + mR^2)}{mgR}}$$

$$T_1 = 2\pi \sqrt{\frac{2R}{g}}$$

$$T_2 = 2\pi \sqrt{\frac{I}{mgL_{cm}}}$$

$$T_2 = 2\pi \sqrt{\frac{3mR^2/2}{mgR}} = 2\pi \sqrt{\frac{3R}{2g}}$$

$$\frac{T_1}{T_2} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$$



Back and forth

2. The correct match between the entries in column I and column II are:

**I**

**Radiation**

- (a) Microwave  
(b) Gamma rays  
(c) A.M. radio waves  
(d) X-rays

- (1) (a) - (ii), (b) - (i), (c) - (iv), (d) - (iii)  
(3) (a) - (iv), (b) - (ii), (c) - (i), (d) - (iii)

**II**

**Wavelength**

- (i) 100 m  
(ii)  $10^{-15}$  m  
(iii)  $10^{-10}$  m  
(iv)  $10^{-3}$  m  
(2) (a) - (iii), (b) - (ii), (c) - (i), (d) - (iv)  
(4) (a) - (i), (b) - (iii), (c) - (iv), (d) - (ii)

**Sol. 3**

By theory

3. In an experiment to verify Stokes law, a small spherical ball of radius  $r$  and density  $\rho$  falls under gravity through a distance  $h$  in air before entering a tank of water. If the terminal velocity of the ball inside water is same as its velocity just before entering the water surface, then the value of  $h$  is proportional to : (ignore viscosity of air)

(1)  $r^4$

(2)  $r$

(3)  $r^3$

(4)  $r^2$

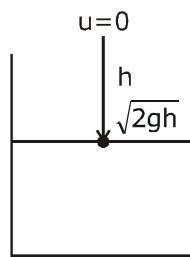
**Sol. 1**

$$V_T = \sqrt{2gh}$$

$$\frac{2}{9} r^2 \frac{(\rho_b - \rho_l)g}{\eta} = \sqrt{2gh}$$

$$r^2 \propto \sqrt{h} \Rightarrow r^4 \propto h$$

$$h \propto r^4$$



- 4.** Ten charges are placed on the circumference of a circle of radius  $R$  with constant angular separation between successive charges. Alternate charges 1, 3, 5, 7, 9 have charge  $(+q)$  each, while 2, 4, 6, 8, 10 have charge  $(-q)$  each. The potential  $V$  and the electric field  $E$  at the centre of the circle are respectively: (Take  $V = 0$  at infinity)

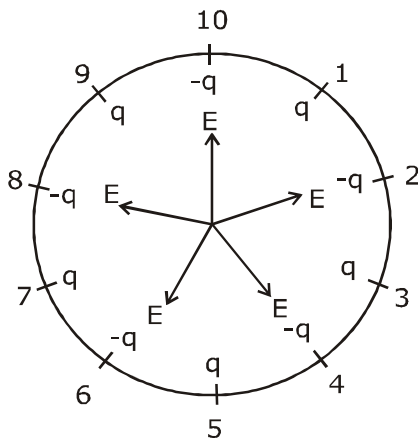
(1)  $V = 0; E = 0$

(2)  $V = \frac{10q}{4\pi\epsilon_0 R}; E = \frac{10q}{4\pi\epsilon_0 R^2}$

(3)  $V = 0; E = \frac{10q}{4\pi\epsilon_0 R^2}$

(4)  $V = \frac{10q}{4\pi\epsilon_0 R}; E = 0$

**Sol. 1**



$$V_{\text{net}} = 5 \left( \frac{kq}{R} \right) + \left( \frac{5k(-q)}{R} \right)$$

$$V_{\text{net}} = 0 [Q_{\text{net}} = 0]$$

$$E_{\text{net}} = 0 \text{ by symmetry}$$

5. A spaceship in space sweeps stationary interplanetary dust. As a result, its mass increases at a rate  $\frac{dM(t)}{dt} = bv^2(t)$ , where  $v(t)$  is its instantaneous velocity. The instantaneous acceleration of the satellite is :

(1)  $-bv^3(t)$                       (2)  $-\frac{bv^3}{M(t)}$                       (3)  $-\frac{2bv^3}{M(t)}$                       (4)  $-\frac{bv^3}{2M(t)}$

**Sol. 2**

$$\frac{dM(t)}{dt} = -bv^2$$

in free space  
no external force  
so there is only thrust force on rocket

$$f_{in} = \frac{dM}{dt} (V_{rel})$$

$$Ma = \left( \frac{-bv^2}{(t)} \right) v$$

$$a = \frac{-bv^3}{M(t)}$$

6. Two different wires having lengths  $L_1$  and  $L_2$ , and respective temperature coefficient of linear expansion  $\alpha_1$  and  $\alpha_2$ , are joined end-to-end. Then the effective temperature coefficient of linear expansion is:

(1)  $\frac{\alpha_1 L_1 + \alpha_2 L_2}{L_1 + L_2}$                       (2)  $2\sqrt{\alpha_1 \alpha_2}$

(3)  $4 \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2} \frac{L_2 L_1}{(L_2 + L_1)^2}$                       (4)  $\frac{\alpha_1 + \alpha_2}{2}$

**Sol. 1**

$$L'_1 = L_1 (1 + \alpha_1 \Delta T)$$

$$L'_2 = L_2 (1 + \alpha_2 \Delta T)$$

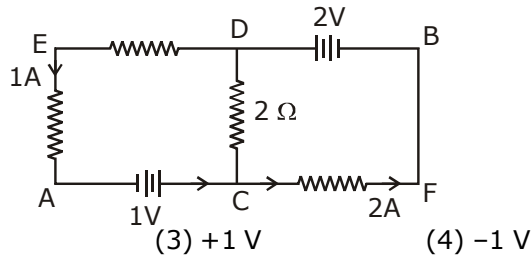
$$L'_1 + L'_2 = L_1 + L_2 + L_1 \alpha_1 \Delta T + L_2 \alpha_2 \Delta T$$

$$= (L_1 + L_2) \left[ 1 + \left[ \frac{L_1 \alpha_1 + L_2 \alpha_2}{L_1 + L_2} \right] \Delta T \right]$$

$$= (L_1 + L_2) [1 + \alpha_{eq} \Delta T]$$

$$\text{So, } \alpha_{eq} = \frac{L_1 \alpha_1 + L_2 \alpha_2}{L_1 + L_2}$$

7. In the circuit, given in the figure currents in different branches and value of one resistor are shown. Then potential at point B with respect to the point A is:



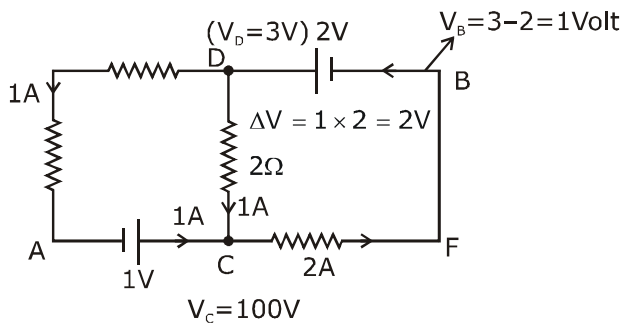
(1) +2 V

(2) -2 V

(3) +1 V

(4) -1 V

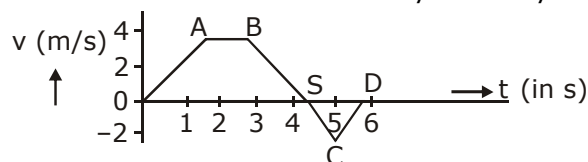
Sol. 3



Let  $V_A = 0$

$$V_B - V_A = 1 - 0 = 1 \text{ volt}$$

8. The velocity ( $v$ ) and time ( $t$ ) graph of a body in a straight line motion is shown in the figure. The point S is at 4.333 seconds. The total distance covered by the body in 6 s is:



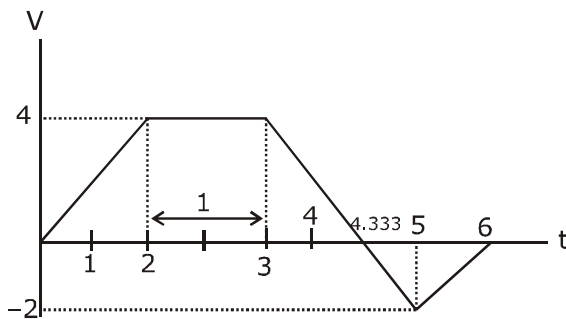
(1)  $\frac{37}{3}$  m

(2)  $\frac{49}{4}$  m

(3) 12 m

(4) 11 m

Sol. 1



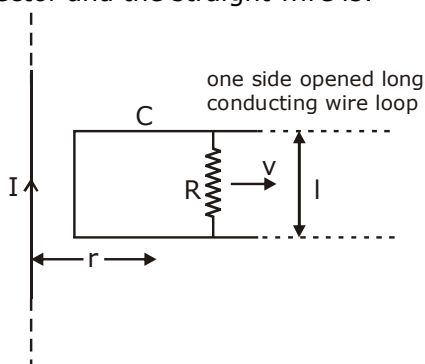
distance = area under graph

$$= \frac{1}{2} (4) \left( \frac{13}{3} + 1 \right) + \left[ \frac{1}{2} \left( 6 - \frac{13}{3} \right) \times 2 \right]$$

$$= 2 \times \frac{16}{3} + \frac{5}{3}$$

$$= \frac{32}{3} + \frac{5}{3} = \frac{37}{3} \text{ m}$$

9. An infinitely long straight wire carrying current  $I$ , one side opened rectangular loop and a conductor  $C$  with a sliding connector are located in the same plane, as shown in the figure. The connector has length  $l$  and resistance  $R$ . It slides to the right with a velocity  $v$ . The resistance of the conductor and the self inductance of the loop are negligible. The induced current in the loop, as a function of separation  $r$ , between the connector and the straight wire is:



(1)  $\frac{\mu_0}{2\pi} \frac{Ivl}{Rr}$

(2)  $\frac{\mu_0}{\pi} \frac{Ivl}{Rr}$

(3)  $\frac{2\mu_0}{\pi} \frac{Ivl}{Rr}$

(4)  $\frac{\mu_0}{4\pi} \frac{Ivl}{Rr}$

**Sol. 1**

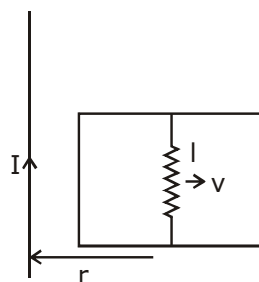
$$B = \left( \frac{\mu_0 I}{2\pi r} \right)$$

induced emf  
 $e = Bvl$

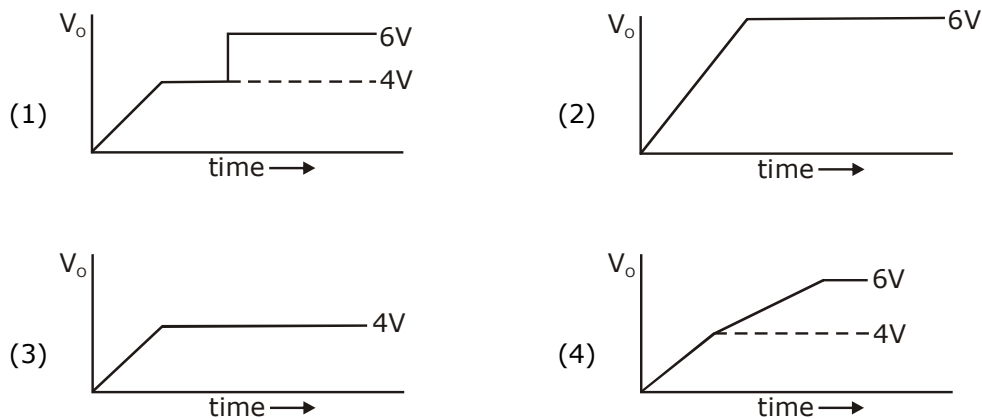
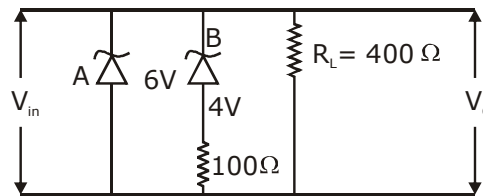
$$= \frac{\mu_0 I}{2\pi r} v.l$$

$$= \frac{\mu_0 Ivl}{2\pi r}$$

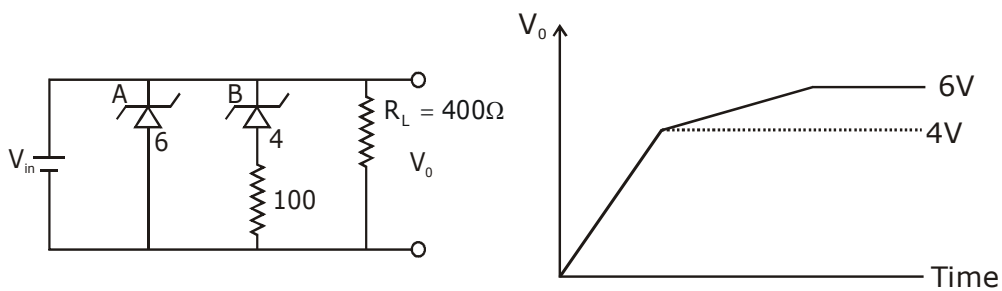
$$\text{induced current } i = \frac{e}{R} = \frac{\mu_0 Ivl}{2\pi rR}$$



- 10.** Two zener diodes (A and B) having breakdown voltages of 6 V and 4 V respectively, are connected as shown in the circuit below. The output voltage  $V_o$  variation with input voltage linearly increasing with time, is given by: ( $V_{\text{input}} = 0$  V at  $t = 0$ ) (figures are qualitative)



**Sol. (4)**  
 $t = 0$   
 $V_i = 0$   
 $V_i \propto t$   
 Given  
 $\therefore$  Zener diode maintain constant breakdown voltage.



- 11.** In an adiabatic process, the density of a diatomic gas becomes 32 times its initial value. The final pressure of the gas is found to be  $n$  times the initial pressure. The value of  $n$  is:

(1) 32                      (2)  $\frac{1}{32}$                       (3) 326                      (4) 128

**Sol. 4**

$$PV^r = \text{const.}$$

$$p (\rho^{-r}) = \text{const.}$$

$$P_1 \rho_1^{-r} = P_2 \rho_2^{-r} \quad r = \frac{7}{5} \text{ for diatomic}$$

$$P_0 \rho_0^{-7/5} = (n P_0) (32 \rho_0)^{-7/5}$$

$$\rho_0^{-7/5} = \frac{n}{(32)^{7/5}} (\rho_0^{-7/5})$$

$$n = (2^5)^{7/5} = 2^7 = 128$$

- 12.** A galvanometer is used in laboratory for detecting the null point in electrical experiments. If, on passing a current of 6 mA it produces a deflection of  $2^\circ$ , its figure of merit is close to:

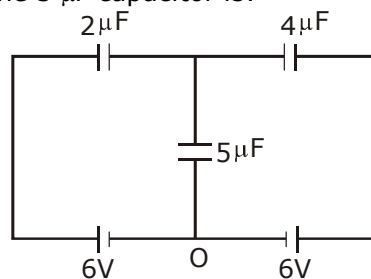
(1)  $6 \times 10^{-3}$  A/div.    (2)  $3 \times 10^{-3}$  A/div.    (3)  $666^\circ$  A/div.    (4)  $333^\circ$  A/div.

**Sol. 2**

$$\text{figure of merit} = \frac{I}{\theta} \Rightarrow \text{A/div.}$$

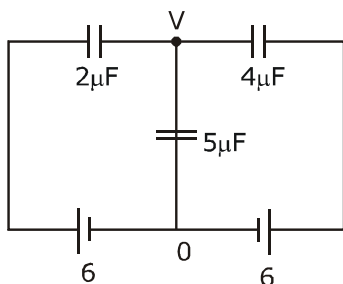
$$= \frac{6 \times 10^{-3}}{2} = 3 \times 10^{-3} \text{ A/div.}$$

- 13.** In the circuit shown, charge on the  $5 \mu\text{F}$  capacitor is:



(1)  $5.45 \mu\text{C}$                       (2)  $18.00 \mu\text{C}$                       (3)  $10.90 \mu\text{C}$                       (4)  $16.36 \mu\text{C}$

**Sol. 2**



$$(V - 6) \times 2 + (V - 0) \times 5 + (V - 6) 4 = 0$$

$$2V - 12 + 5V + 4V - 24 = 0$$

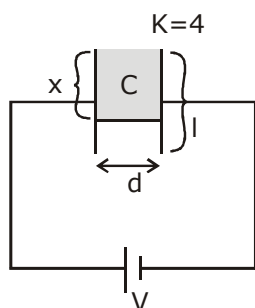
$$11V = 36$$

$$V = \frac{36}{11}$$

$$q = CV = 5 \times \frac{36}{11} \approx 18.00 \mu\text{C}$$

- 14.** A parallel plate capacitor has plate of length 'l', width 'w' and separation of plates is 'd'. It is connected to a battery of emf V. A dielectric slab of the same thickness 'd' and of dielectric constant  $k=4$  is being inserted between the plates of the capacitor. At what length of the slab inside plates, will the energy stored in the capacitor be two times the initial energy stored?
- (1)  $2l/3$                       (2)  $l/2$                       (3)  $l/4$                       (4)  $l/3$

**Sol. 4**

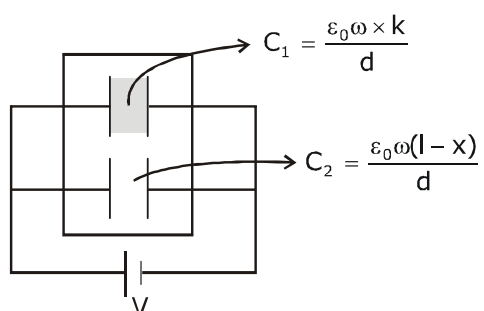


area of plate =  $lw$

$$C = \frac{\epsilon_0 A}{d} = \frac{\epsilon_0 l w}{d}$$

$$U_1 = \frac{1}{2} C V^2 = \frac{1}{2} \frac{\epsilon_0 l w}{d} V^2$$

$$C_{eq} = C_1 + C_2$$



$$C_{eq} = \frac{\epsilon_0 \omega x k}{d} + \frac{\epsilon_0 \omega (l-x)}{d}$$

$$C_{eq} = \frac{\epsilon_0 \omega}{d} [kx + l - x]$$

$$U_f = \frac{1}{2} C_{eq} V^2$$

$$U_f = 2U_i \Rightarrow \frac{1}{2} \frac{\epsilon_0 \omega}{d} [kx + l - x] V^2 = 2 \times \frac{1}{2} \frac{\epsilon_0 l \omega}{d} V^2$$

$$kx + l - x = 2l$$

$$4x - x = l$$

$$3x = l$$

$$x = \frac{l}{3}$$

- 15.** A radioactive nucleus decays by two different processes. The half life for the first process is 10 s and that for the second is 100 s. The effective half life of the nucleus is close to:

(1) 55 sec. (2) 6 sec. (3) 12 sec. (4) 9 sec.

**Sol. 4**

$$T_1 = 10 \text{ sec} \quad \lambda_1 = \frac{\ln 2}{T_1}$$

$$T_2 = 100 \text{ s}, \lambda_2 = \frac{\ln 2}{T_2}, \lambda_{eq} = \frac{\ln 2}{T_{eq}}$$

we know

$$\lambda_{eq} = \lambda_1 + \lambda_2$$

$$\frac{\ln 2}{T_{eq}} = \frac{\ln 2}{T_1} + \frac{\ln 2}{T_2}$$

$$\frac{1}{T_{eq}} = \frac{1}{10} + \frac{1}{100} = \frac{10+1}{100} = \frac{11}{100}$$

$$T_{eq} = \frac{100}{11} = 9 \text{ s}$$

- 16.** A driver in a car, approaching a vertical wall notices that the frequency of his car horn, has changed from 440 Hz to 480 Hz, when it gets reflected from the wall. If the speed of sound in air is 345 m/s, then the speed of the car is:

(1) 24 km/hr (2) 36 km/hr (3) 54 km/hr (4) 18 km/hr

**Sol. 3**

car towards

$$f_1 = \left( \frac{v-0}{v-v_c} \right) f_0 \quad \dots(i)$$

$$480 = \left( \frac{v+v_c}{v-0} \right) f_i \Rightarrow \left( \frac{v+v_c}{v} \right) \left( \frac{v}{v-v_c} \right) f_0$$

$$480 = (350 + v_c) \times \left( \frac{440}{350 - v_c} \right)$$

$$12 = \left( \frac{350 + v_c}{350 - v_c} \right) \times 11$$

$$12 \times 350 - 12 \times v_c = 350 \times 11 + 11 v_c$$

$$23v_c = 4200 - 3850 = 350$$

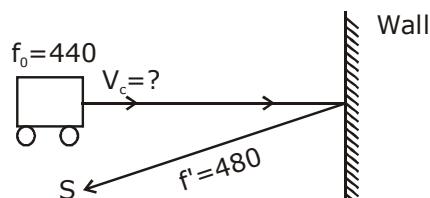
$$v_c = \frac{350}{23} \text{ m}$$

$$v_c = \frac{350}{23} \times \frac{18}{5} \text{ km/h}$$

$$= \frac{70 \times 18}{23}$$

$$= 54.78$$

$$= 54 \text{ km/hr}$$



- 17.** An iron rod of volume  $10^{-3} \text{ m}^3$  and relative permeability 1000 is placed as core in a solenoid with 10 turns/cm. If a current of 0.5 A is passed through the solenoid, then the magnetic moment of the rod will be:

- (1)  $0.5 \times 10^2 \text{ Am}^2$     (2)  $50 \times 10^2 \text{ Am}^2$     (3)  $5 \times 10^2 \text{ Am}^2$     (4)  $500 \times 10^2 \text{ Am}^2$

**Sol. 3**

$$\text{magnetic moment } \vec{M} = NIA(\mu_r - 1)$$

$$n = 10 \text{ turns/cm} \quad = (nl) IA (\mu_r - 1)$$

$$= \frac{10}{10^{-2}} \text{ turn/m} \quad = nI (Al) (\mu_r - 1)$$

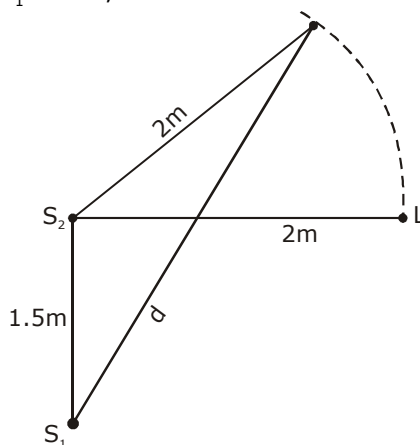
$$= 1000 \text{ turn/m} \quad = 1000 \times 0.5 \times 10^{-3} (1000 - 1)$$

$$V = 10^{-3} \text{ m}^3 = Al \quad = 0.5 \times (999) = 499.5$$

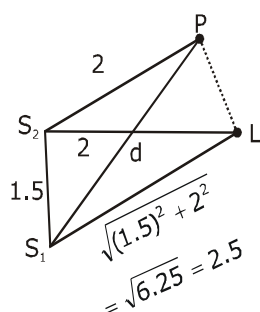
$$I = 0.5 \text{ A}, \mu_r \quad = 500$$

$$N = nl \quad = 5 \times 10^2$$

- 18.** Two coherent sources of sound,  $S_1$  and  $S_2$ , produce sound waves of the same wavelength,  $\lambda = 1$  m, in phase.  $S_1$  and  $S_2$  are placed 1.5 m apart (see fig). A listener, located at L, directly in front of  $S_2$  finds that the intensity is at a minimum when he is 2 m away from  $S_2$ . The listener moves away from  $S_1$ , keeping his distance from  $S_2$  fixed. The adjacent maximum of intensity is observed when the listener is at a distance  $d$  from  $S_1$ . Then,  $d$  is :



- Sol.** (1) 12 m (2) 2 m (3) 3 m (4) 5 m



For min at (L)

$$S_1L - S_2L = \Delta x = \frac{\lambda}{2} (2n + 1); (n = 0, 1, 2, \dots)$$

$$2.5 - 2 = \frac{1}{2} (2n + 1)$$

$$0.5 \times 2 = (2n + 1)$$

$$2n = 0$$

$$n = 0 \text{ (first minima)}$$

so at 'P'  $\rightarrow$  first maxima

$$S_1P - S_2P = \lambda \quad [n = 1] \text{ for first maxima}$$

$$S_1P - 2 = 1$$

$$S_1P = 1 + 2$$

$$d = 3 \text{ m}$$

- 19.** The quantities  $x = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ ,  $y = \frac{E}{B}$  and  $z = \frac{I}{CR}$  are defined where C-capacitance, R-Resistance, l-length, E-Electric field, B-magnetic field and  $\epsilon_0$ ,  $\mu_0$  -free space permittivity and permeability respectively. Then:  
 (1) Only y and z have the same dimension (2) x, y and z have the same dimension  
 (3) Only x and y have the same dimension (4) Only x and z have the same dimension

**Sol. 2**

$$x = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = (\text{speed})$$

$$[x] = LT^{-1} \quad y = \frac{E}{B} = \text{speed}$$

$$Z = \frac{I}{CR} = \frac{m}{\text{sec}} = m/s \quad [y] = LT^{-1}$$

$$[RC = T]$$

$$[Z] = LT^{-1}$$

So, x,y,z has same dimension

- 20.** The acceleration due to gravity on the earth's surface at the poles is g and angular velocity of the earth about the axis passing through the pole is  $\omega$ . An object is weighed at the equator and at a height h above the poles by using a spring balance. If the weights are found to be same, then h is : ( $h \ll R$ , where R is the radius of the earth)

$$(1) \frac{R^2 \omega^2}{g} \quad (2) \frac{R^2 \omega^2}{8g} \quad (3) \frac{R^2 \omega^2}{4g} \quad (4) \frac{R^2 \omega^2}{2g}$$

**Sol. 4**

$\therefore$  weight same at poles and at h (so  $g_1 = g_2$ )

$$g_1 = g - R\omega^2$$

$$g_2 = g \left( 1 - \frac{2h}{R} \right)$$

$$\therefore g_1 = g_2$$

$$g - R\omega^2 = g \left( 1 - \frac{2h}{R} \right) \Rightarrow g - \frac{2gh}{R}$$

$$R\omega^2 = \frac{2gh}{R}$$

$$h = \frac{R^2 \omega^2}{2g}$$

- 21.** Nitrogen gas is at  $300^{\circ}\text{C}$  temperature. The temperature (in K) at which the rms speed of a  $\text{H}_2$  molecule would be equal to the rms speed of a nitrogen molecule, is \_\_\_\_\_.  
(Molar mass of  $\text{N}_2$  gas 28 g).

**Sol. 41**

$$V_{\text{rms}} = \sqrt{\frac{3RT}{m}}$$

$$V_{\text{N}_2} = \sqrt{\frac{3R(573)}{28}}$$

$$V_{\text{H}_2} = \sqrt{\frac{3RT}{2}}$$

$$V_{\text{H}_2} = V_{\text{N}_2}$$

$$\sqrt{\frac{3RT}{2}} = \sqrt{\frac{3R(573)}{28}}$$

$$\frac{T}{2} = \frac{573}{28}$$

$$T = 41\text{ K}$$

- 22.** The surface of a metal is illuminated alternately with photons of energies  $E_1 = 4\text{ eV}$  and  $E_2 = 2.5\text{ eV}$  respectively. The ratio of maximum speeds of the photoelectrons emitted in the two cases is 2. The work function of the metal in (eV) is \_\_\_\_\_.

**Sol. 2**

$$\frac{\frac{1}{2}mV_1^2}{\frac{1}{2}mV_2^2} = \frac{E_1 - \phi_0}{E_2 - \phi_0} = \frac{4 - \phi_0}{2.5 - \phi_0}$$

$$\left(\frac{V_1}{V_2}\right)^2 = \frac{4 - \phi_0}{2.5 - \phi_0}$$

$$(2)^2 = \frac{4 - \phi_0}{2.5 - \phi_0}$$

$$10 - 4\phi_0 = 4 - \phi_0$$

$$3\phi_0 = 10 - 4 = 6$$

$$\phi_0 = 2\text{ eV}$$

- 23.** A prism of angle  $A = 1^{\circ}$  has a refractive index  $\mu = 1.5$ . A good estimate for the minimum angle of deviation (in degrees) is close to  $N/10$ . Value of  $N$  is

**Sol. 5**

$$\begin{aligned}
 A &= 1^\circ \\
 \delta &= (\mu - 1) A \\
 &= (1.5 - 1) A \\
 &= 0.5 \times 1 \\
 &= \frac{5}{10} = \frac{N}{10} \text{ so } N = 5
 \end{aligned}$$

- 24.** A body of mass 2 kg is driven by an engine delivering a constant power of 1 J/s. The body starts from rest and moves in a straight line. After 9 seconds, the body has moved a distance (in m)

**Sol. 18**

$$P = Fv = mav$$

$$a = \frac{p}{mv}$$

$$\frac{dv}{dt} = \frac{p}{mv}$$

$$\int_0^u v dv = \frac{p}{m} \int_0^t dt$$

$$\frac{u^2}{2} = \frac{p}{m} t$$

$$u = \sqrt{\frac{2p}{m}} \sqrt{t}$$

$$\frac{dx}{dt} = \sqrt{\frac{2p}{m}} \sqrt{t}$$

$$\int_0^x dx = \sqrt{\frac{2p}{m}} \int_0^9 \sqrt{t} dt$$

$$x = \frac{2}{3} \left[ (9)^{3/2} \right]$$

$$= \frac{2}{3} \times 27$$

$$x = 18$$

II-method

$$Pt = w = \frac{1}{2} mv^2 - 0$$

$$1 \times t = \frac{1}{2} \times 2 \times u^2$$

$$u = \sqrt{t}$$

$$\frac{dx}{dt} = \sqrt{t} = \int_0^1 dx = \int_0^9 \sqrt{t} dt$$

$$x = \frac{[t^{3/2}]_0^9}{\frac{3}{2}} = 18 \text{ m}$$

- 25.** A thin rod of mass 0.9 kg and length 1 m is suspended, at rest, from one end so that it can freely oscillate in the vertical plane. A particle of mass 0.1 kg moving in a straight line with velocity 80 m/s hits the rod at its bottom most point and sticks to it (see figure). The angular speed (in rad/s) of the rod immediately after the collision will be \_\_\_\_\_.



**Sol.**

**20**

$$L_i = L_f$$

$$0.1 \times 80 \times 1 = \frac{0.9 \times 1^2}{3} \times \omega + (0.1) 1^2 \omega$$

$$8 = (0.3 + 0.1) \omega$$

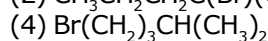
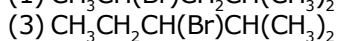
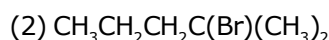
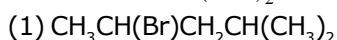
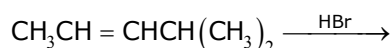
$$8 = (0.4) \omega$$

$$\omega = \frac{80}{4} = 20$$

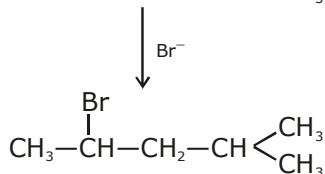
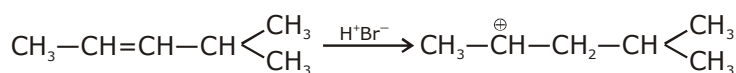
# QUESTION PAPER WITH SOLUTION

## CHEMISTRY \_ 5 Sep. \_ SHIFT - 2

1. The major product formed in the following reaction is :



Sol. 1



2. Hydrogen peroxide, in the pure state, is :

(1) Linear and blue in color

(2) Linear and almost colorless

(3) Non-planar and almost colorless

(4) Planar and blue in color

Sol. 3

$\text{H}_2\text{O}_2$  has openbook structure it is non planar

3. Boron and silicon of very high purity can be obtained through :

(1) Liquefaction

(2) Electrolytic refining

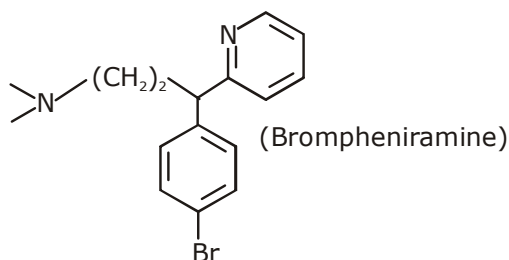
(3) Zone refining

(4) Vapour phase refining

Sol. 3

Fact

4. The following molecule acts as an :



(1) Anti-histamine

(2) Antiseptic

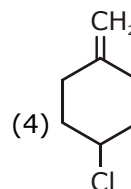
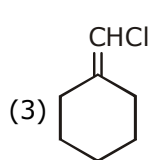
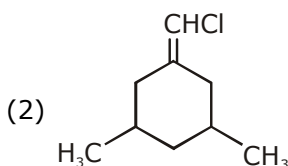
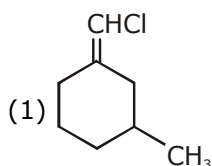
(3) Anti-depressant

(4) Anti-bacterial

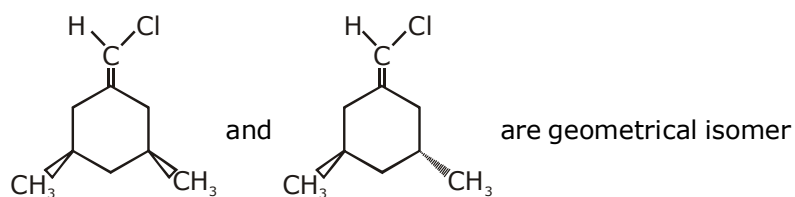
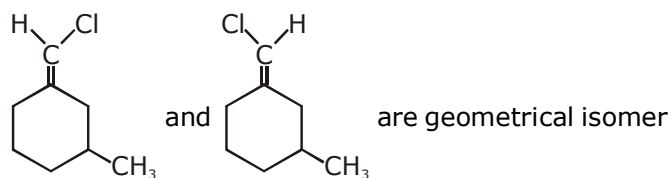
Sol. 1

Anti-histamine

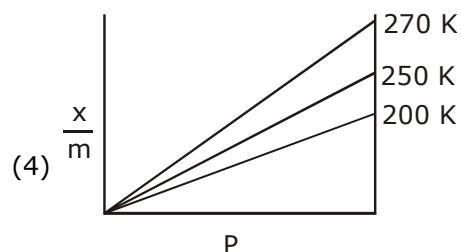
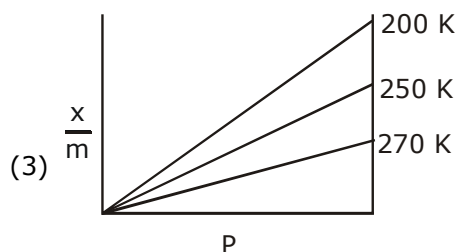
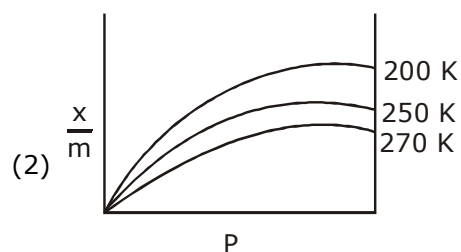
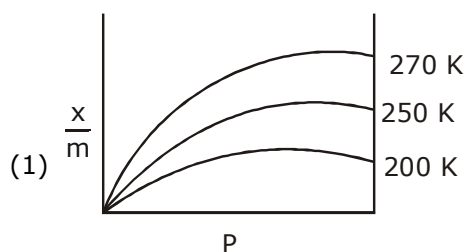
5. Among the following compounds, geometrical isomerism is exhibited by :



**Sol. 1 & 2**



- 6.** Adsorption of a gas follows Freundlich adsorption isotherm. If  $x$  is the mass of the gas adsorbed on mass  $m$  of the adsorbent, the correct plot of  $\frac{x}{m}$  versus  $p$  is :



**Sol. 2**

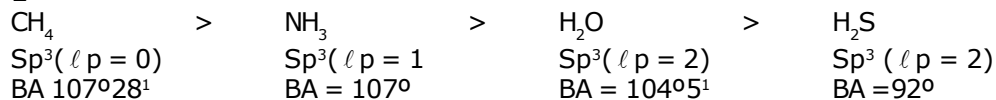
As temp. increases extent of Adsorption decreases  
Therefore correct option (2)

$$\frac{x}{m} = Kp^{1/n}$$

$\frac{x}{m}$  v/s  $p \rightarrow$  non linear curve

7. The compound that has the largest H-M-H bond angle (M=N, O, S, C) is :  
 (1) CH<sub>4</sub> (2) H<sub>2</sub>S (3) NH<sub>3</sub> (4) H<sub>2</sub>O

Sol. 1



8. The correct statement about probability density (except at infinite distance from nucleus) is :  
 (1) It can be zero for 3p orbital (2) It can be zero for 1s orbital  
 (3) It can never be zero for 2s orbital (4) It can negative for 2p orbital

Sol. 1

$$\psi_{R/S}^2 > 0 \text{ always}$$

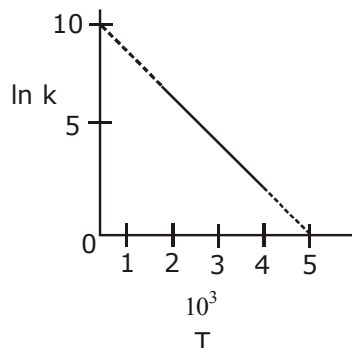
$$\psi_{R/S}^2 \text{ can be } = 0; \text{ As '2s' has 1 Radial Node}$$

$$\psi_R^2 \text{ can never be negative}$$

$$\psi_R^2 (3P) \text{ can be } = 0 \text{ as 3P has Radial Nodes}$$

Ans. Option (1)

9. The rate constant (k) of a reaction is measured at different temperatures (T), and the data are plotted in the given figure. The activation energy of the reaction in kJ mol<sup>-1</sup> is : (R is gas constant)



- Sol. 4 (1) R (2) 2/R (3) 1/R (4) 2R

$$\ln(k) = \ln(A) - \frac{E_a}{R} \left( \frac{1}{T} \right)$$

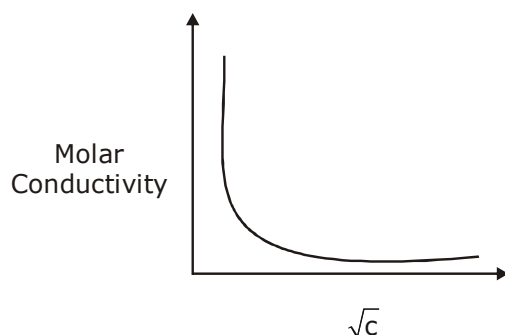
$$\ln(A) = 10$$

$$\text{Slope} = \frac{-E_a}{R} \times 10^{-3} = -10/5$$

$$E_a = 2000R \text{ J/mol}$$

$$E_a = 2R \text{ KJ/mol}$$

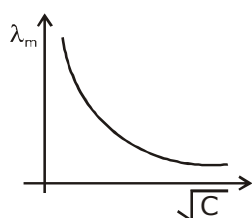
10. The variation of molar conductivity with concentration of an electrolyte (X) in aqueous solution is shown in the given figure.



The electrolyte X is :

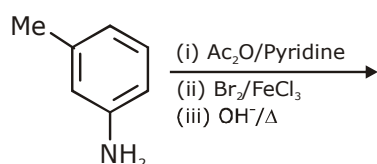
- (1) HCl                      (2) CH<sub>3</sub>COOH                      (3) NaCl                      (4) KNO<sub>3</sub>

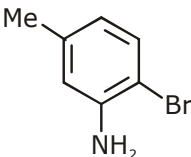
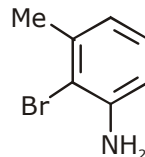
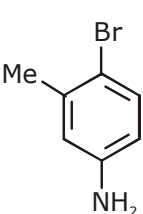
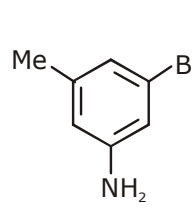
Sol.



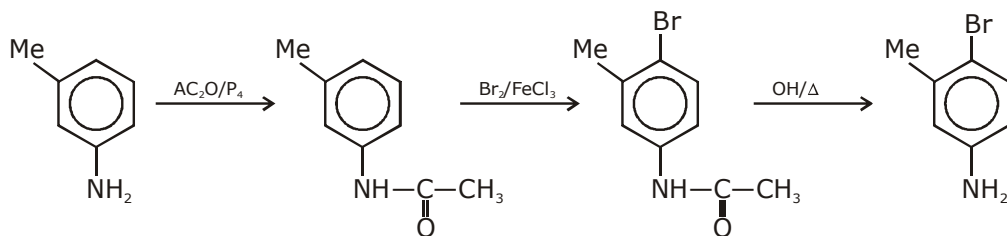
Such type of variation is always for weak electrolyte  
Hence Ans (2) CH<sub>3</sub>COOH

11. The final major product of the following reaction is :

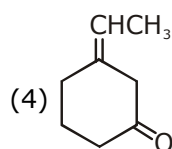
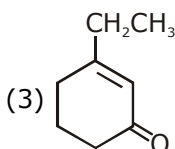
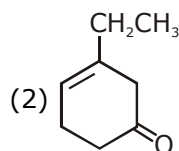
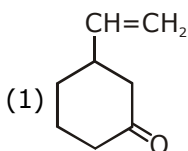
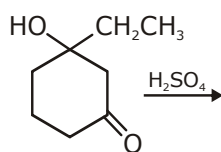


- (1) 
- (2) 
- (3) 
- (4) 

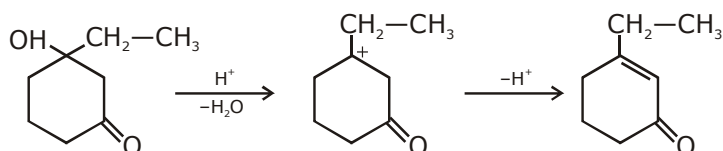
**Sol. 3**



**12.** The major product of the following reaction is :



**Sol. 3**



**13.** Lattice enthalpy and enthalpy of solution of NaCl are  $788 \text{ kJ mol}^{-1}$ , and  $4 \text{ kJ mol}^{-1}$ , respectively. The hydration enthalpy of NaCl is :

(1)  $-780 \text{ kJ mol}^{-1}$

(2)  $784 \text{ kJ mol}^{-1}$

(3)  $-784 \text{ kJ mol}^{-1}$

(4)  $780 \text{ kJ mol}^{-1}$

**Sol. 3**

$$\Delta H_{\text{sol}} = \text{L.E.} + \Delta H_{\text{hyd}}$$

$$4 = 788 + \Delta H_{\text{hyd}}$$

$$\Delta H_{\text{hyd}} = -784 \text{ kJ/mol Ans}$$

**14.** Reaction of ammonia with excess  $\text{Cl}_2$  gives :

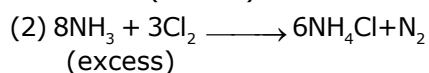
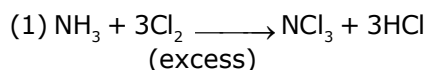
(1)  $\text{NH}_4\text{Cl}$  and  $\text{N}_2$

(2)  $\text{NH}_4\text{Cl}$  and  $\text{HCl}$

(3)  $\text{NCl}_3$  and  $\text{HCl}$

(4)  $\text{NCl}_3$  and  $\text{NH}_4\text{Cl}$

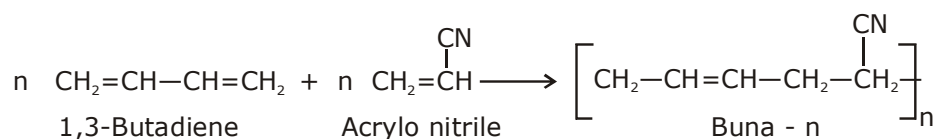
**Sol. 3**



**15.** Which one of the following polymers is not obtained by condensation polymerisation ?

- (1) Bakelite (2) Nylon 6  
(3) Buna-N (4) Nylon 6, 6

**Sol. 2**



**16.** Consider the complex ions,  
trans-[Co(en)<sub>2</sub>Cl<sub>2</sub>]<sup>+</sup> (A) and  
cis-[Co(en)<sub>2</sub>Cl<sub>2</sub>]<sup>+</sup> (B)

The correct statement regarding them is :

- (1) Both (A) and (B) can be optically active.  
(2) (A) can be optically active, but (B) cannot be optically active.  
(3) Both (A) and (B) cannot be optically active.  
(4) (A) cannot be optically active, but (B) can be optically active.

**Sol. 4**

Due to presence of Pos (A) cannot be optically active, but (B) can be optically active

**17.** An element crystallises in a face-centred cubic (fcc) unit cell with cell edge a. The distance between the centres of two nearest octahedral voids in the crystal lattice is :

- (1) a                                  (2)  $\frac{a}{2}$                                   (3)  $\sqrt{2}a$                                   (4)  $\frac{a}{\sqrt{2}}$

**Sol. 4**

Nearest octahedral voids

One along edge center & other at Body centre

$$\text{Distance} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \sqrt{2} \frac{a}{2}$$

$$= \frac{a}{\sqrt{2}} \text{ Ans.}$$

**18.** The correct order of the ionic radii of O<sup>2-</sup>, N<sup>3-</sup>, F<sup>-</sup>, Mg<sup>2+</sup>, Na<sup>+</sup> and Al<sup>3+</sup> is :

- (1) N<sup>3-</sup> < O<sup>2-</sup> < F<sup>-</sup> < Na<sup>+</sup> < Mg<sup>2+</sup> < Al<sup>3+</sup>                  (2) N<sup>3-</sup> < F<sup>-</sup> < O<sup>2-</sup> < Mg<sup>2+</sup> < Na<sup>+</sup> < Al<sup>3+</sup>  
(3) Al<sup>3+</sup> < Na<sup>+</sup> < Mg<sup>2+</sup> < O<sup>2-</sup> < F<sup>-</sup> < N<sup>3-</sup>                  (4) Al<sup>3+</sup> < Mg<sup>2+</sup> < Na<sup>+</sup> < F<sup>-</sup> < O<sup>2-</sup> < N<sup>3-</sup>

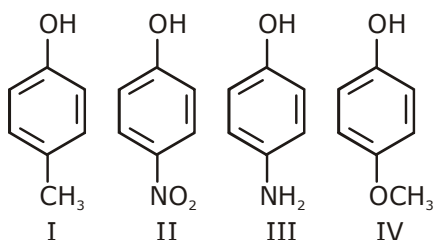
**Sol. 4**

all are Isoelectronic

$$(1) \frac{\text{N}^{3-} \text{O}^{2-} \text{F}^- \text{Na}^+ \text{Mg}^{2+} \text{Al}^{3+}}{\text{Z} \uparrow, \text{Zeff} \uparrow, \text{Ionic Radii} \downarrow}$$

$$(2) \text{Al}^{3+} < \text{Mg}^{2+} < \text{Na}^+ < \text{F}^- < \text{O}^{2-} < \text{N}^{3-}$$

19. The increasing order of boiling points of the following compounds is :



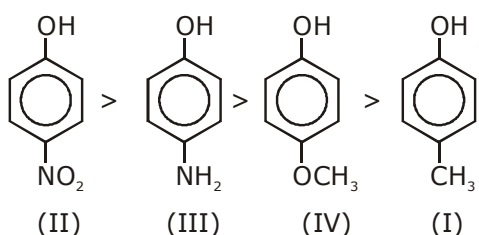
(1) I < III < IV < II

(2) IV < I < II < III

(3) I < IV < III < II

(4) III < I < II < IV

Sol. 3



20. The one that is NOT suitable for the removal of permanent hardness of water is :

(1) Ion-exchange method

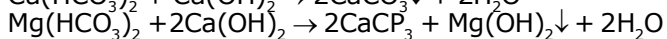
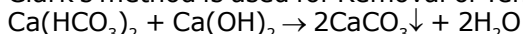
(2) Calgon's method

(3) Treatment with sodium carbonate

(4) Clark's method

Sol. 4

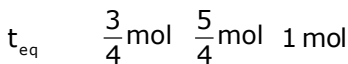
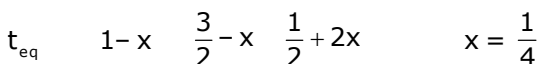
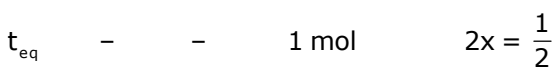
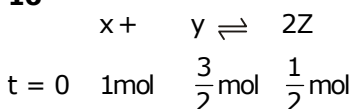
Clark's method is used for Removal of Temporary hardness



21. For a reaction  $\text{X} + \text{Y} \rightleftharpoons 2\text{Z}$ , 1.0 mol of X, 1.5 mol of Y and 0.5 mol of Z were taken in a 1 L vessel and allowed to react. At equilibrium, the concentration of Z was  $1.0 \text{ mol L}^{-1}$ . The equilibrium

constant of reaction is  $\frac{x}{15}$ . The value of x is \_\_\_\_\_.

Sol. 16

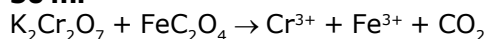


$$K_{\text{eq}} = \frac{(1)^2}{\frac{5}{4} \times \frac{3}{4}} = \frac{16}{15}$$

x = 16 Ans.

- 22.** The volume, in mL, of 0.02 M  $K_2Cr_2O_7$  solution required to react with 0.288 g of ferrous oxalate in acidic medium is \_\_\_\_\_.  
(Molar mass of Fe = 56 g mol<sup>-1</sup>)

**Sol.** **50 ml**

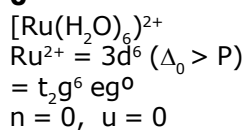


$$\frac{0.02 \times \text{vol} \times 6}{1000} = 3 \times \frac{0.288}{144} \times 100$$

$$\text{Vol.} = \frac{200}{4} = 50 \text{ ml Ans.}$$

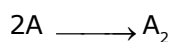
- 23.** Considering that  $\Delta_0 > P$ , the magnetic moment (in BM) of  $[Ru(H_2O)_6]^{2+}$  would be \_\_\_\_\_.

**Sol.** **0**



- 24.** For a dimerization reaction,  $2A(g) \rightarrow A_2(g)$  at 298 K,  $\Delta U^\ominus = -20 \text{ kJ mol}^{-1}$ ,  $\Delta S^\ominus = -30 \text{ kJ mol}^{-1}$ , then the  $\Delta G^\ominus$  will be \_\_\_\_\_ J.

**Sol.** **-13538 J**



$$\Delta U^\ominus = -20 \text{ kJ}$$

$$\Delta H^\ominus = -20000 + (-1) R \times 298$$

$$\Delta G^\ominus = -20000 - 298R + 30 \times 298$$

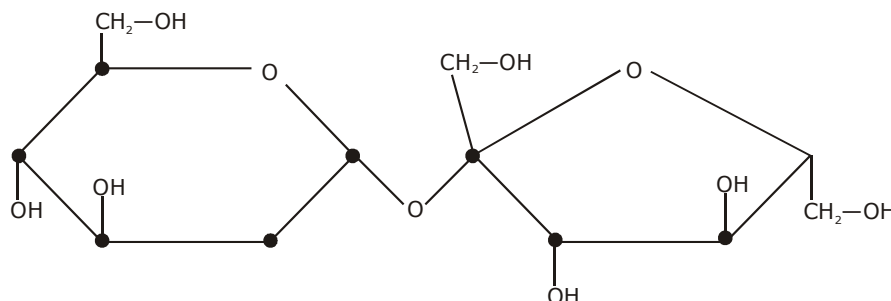
$$\Delta G^\ominus = -20,000 + 298 \left( \frac{90 - 25}{3} \right)$$

$$\Delta G^\ominus = 20,000 + \frac{298 \times 65}{3}$$

$$\Delta G^\ominus = -13538 \text{ J}$$

- 25.** The number of chiral carbons present in sucrose is \_\_\_\_\_.

**Sol.** **9**



# QUESTION PAPER WITH SOLUTION

## MATHEMATICS \_ 5 Sep. \_ SHIFT - 2

**Q.1** If  $x=1$  is a critical point of the function  $f(x)=(3x^2+ax-2-a)e^x$ , then:

(1)  $x=1$  is a local minima and  $x = -\frac{2}{3}$  is a local maxima of  $f$ .

(2)  $x=1$  is a local maxima and  $x = -\frac{2}{3}$  is a local minima of  $f$ .

(3)  $x=1$  and  $x = -\frac{2}{3}$  are local minima of  $f$ .

(4)  $x=1$  and  $x = -\frac{2}{3}$  are local maxima of  $f$ .

**Sol. 1**

$$f(x) = (3x^2+ax-2-a)e^x$$

$$f'(x) = (3x^2+ax-2-a)e^x + (6x+a)e^x = 0$$

$$e^x [3x^2 + (a+6)x - 2] = 0$$

$$\text{at } x = 1, 3 + a + 6 - 2 = 0$$

$$a = -7$$

$$f(x) = (3x^2 - 7x + 5)e^x$$

$$f'(x) = (6x-7)e^x + (3x^2-7x+5)e^x$$

$$= e^x(3x^2-x-2) = 0$$

$$= 3x^2 - 3x + 2x - 2 = 0$$

$$= (3x+2)(x-1) = 0$$

$$x = 1, -2/3$$



**Q.2**

$$\lim_{x \rightarrow 0} \frac{x \left( e^{\left( \frac{\sqrt{1+x^2+x^4}-1}{x} \right)} - 1 \right)}{\sqrt{1+x^2+x^4} - 1}$$

(1) is equal to  $\sqrt{e}$

(2) is equal to 1

(3) is equal to 0

(4) does not exist

**Sol. 2**

$$\lim_{x \rightarrow 0} \frac{x \left[ e^{\left( \frac{\sqrt{1+x^2+x^4}-1}{x} \right)} - 1 \right]}{\left( \sqrt{1+x^2+x^4} - 1 \right)}$$

$$\lim_{x \rightarrow 0} \frac{x \left[ e^{\left[ \frac{(\sqrt{1+x^2+x^4})^2 - 1}{x \times 2} \right]} - 1 \right] \times (\sqrt{1+x^2+x^4} + 1)}{(x^2 + x^4)}$$

$$\lim_{x \rightarrow \infty} \frac{e^{\left( \frac{x^3 + x}{2} \right)} - 1}{\left( \frac{x^3 + x}{2} \right) \times 2} \times 2$$

$$= 1$$

**Q.3** The statement  $(p \rightarrow (q \rightarrow p)) \rightarrow (p \rightarrow (p \vee q))$  is:

(1) equivalent to  $(p \vee q) \wedge (\sim p)$

(2) equivalent to  $(p \wedge q) \vee (\sim p)$

(3) a contradiction

(4) a tautology

**Sol. 4**

p	q	$q \rightarrow p$	$p \rightarrow (q \rightarrow p)$
T	T	T	T
T	T	T	T
F	F	T	T
F	T	T	T

$\Rightarrow$  (Tautology)

p	$p \vee q$	$p \rightarrow (p \vee q)$
T	T	T
T	T	T
F	T	T
F	F	T

**Q.4** If  $L = \sin^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$  and  $M = \cos^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$ , then:

$$(1) M = \frac{1}{2\sqrt{2}} + \frac{1}{2} \cos \frac{\pi}{8}$$

$$(2) M = \frac{1}{4\sqrt{2}} + \frac{1}{4} \cos \frac{\pi}{8}$$

$$(3) L = -\frac{1}{2\sqrt{2}} + \frac{1}{2} \cos \frac{\pi}{8}$$

$$(4) L = \frac{1}{4\sqrt{2}} - \frac{1}{4} \cos \frac{\pi}{8}$$

**Sol. 1**

$$\ell = \sin\left(\frac{3\pi}{16}\right) \sin\left(\frac{-\pi}{16}\right)$$

$$\ell = \frac{-1}{2} \left[ \cos \frac{\pi}{8} - \cos \frac{\pi}{4} \right]$$

$$\ell = \frac{1}{2\sqrt{2}} - \frac{1}{2} \cos \frac{\pi}{8}$$

$$M = \cos\left(\frac{3\pi}{16}\right) \cos\left(\frac{\pi}{16}\right)$$

$$M = \frac{1}{2} \left[ \cos \frac{\pi}{4} + \cos \frac{\pi}{8} \right]$$

$$M = \frac{1}{2\sqrt{2}} + \frac{1}{2} \cos \frac{\pi}{8} \dots (1)$$

**Q.5** If the sum of the first 20 terms of the series  $\log_{(7^{1/2})} x + \log_{(7^{1/3})} x + \log_{(7^{1/4})} x + \dots$  is 460, then x is equal to:

- (1)  $7^{1/2}$  (2)  $7^2$  (3)  $e^2$  (4)  $7^{46/21}$

**Sol.**

$$(2 + 3 + 4 + \dots + 21) \log_7 x = 460$$

$$\Rightarrow \frac{20 \times (21 + 2)}{2} \log_7 x = 460$$

$$\Rightarrow 230 \log_7 x = 460 \Rightarrow \log_7 x = 2 \Rightarrow x = 7^2$$

**Q.6** There are 3 sections in a question paper and each section contains 5 questions. A candidate has to answer a total of 5 questions, choosing at least one question from each section. Then the number of ways, in which the candidate can choose the questions, is:

- (1) 2250 (2) 2255 (3) 1500 (4) 3000

**Sol.**

	S-1	S-2	S-3
	1,2,3,4,5	1,2,3,4,5	1,2,3,4,5
6 case	1	1	3
	1	2	2
	1	3	1
	2	2	1
	2	1	2
	3	1	1

$$3(5_{c_1} \times 5_{c_1} \times 5_{c_3}) + 3(5_{c_1} \times 5_{c_2} \times 5_{c_2}) = 3(25 \times 10) + (100 \times 5)3 = 750 + 1500 = 2250$$

**Q.7** If the mean and the standard deviation of the data 3, 5, 7, a, b are 5 and 2 respectively, then a and b are the roots of the equation:

(1)  $x^2 - 20x + 18 = 0$       (2)  $x^2 - 10x + 19 = 0$       (3)  $2x^2 - 20x + 19 = 0$       (4)  $x^2 - 10x + 18 = 0$

**Sol. 2**

$$S.D. = \sqrt{\frac{\sum x_i^2}{n} - (\bar{x})^2}$$

$$(2)^2 = \frac{83 + a^2 + b^2}{5} - \left(\frac{15 + a + b}{5}\right)^2$$

$$4 = \frac{83 + a^2 + b^2}{5} - 25$$

$$29 \times 5 - 83 = a^2 + b^2 \Rightarrow a^2 + b^2 = 62 \quad (1)$$

$$\frac{a + b + 15}{5} = 5 \Rightarrow \boxed{a + b = 10}$$

$$2ab = 100 - 62 = 38$$

$$\boxed{ab = 19} \quad (2)$$

**Q.8** The derivative of  $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$  with respect to  $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$  at  $x = \frac{1}{2}$  is:

(1)  $\frac{2\sqrt{3}}{3}$       (2)  $\frac{2\sqrt{3}}{5}$       (3)  $\frac{\sqrt{3}}{12}$       (4)  $\frac{\sqrt{3}}{10}$

**Sol. 4**

$$x = \tan \theta$$

$$u = \tan^{-1}\left(\frac{\sec \theta - 1}{\tan \theta}\right) = \tan^{-1}(\tan \theta / 2) = \frac{\theta}{2} = \frac{\tan^{-1} x}{2}$$

$$v = \tan^{-1}\left(\frac{2 \sin \theta \cos \theta}{\cos 2\theta}\right) = 2\theta$$

$$= 2 \sin^{-1} x$$

$$\frac{du}{dv} = \frac{1}{2(1+x^2)} \times \frac{\sqrt{1-x^2}}{2}$$

$$= \frac{\sqrt{3}}{2 \times 2} \times \frac{4}{5 \times 2} = \frac{\sqrt{3}}{10}$$

**Q.9** If  $\int \frac{\cos \theta}{5 + 7 \sin \theta - 2 \cos^2 \theta} d\theta = A \log_e |B(\theta)| + C$  where  $C$  is a constant of integration, then  $\frac{B(\theta)}{A}$  can be:

- (1)  $\frac{5(2 \sin \theta + 1)}{\sin \theta + 3}$  (2)  $\frac{5(\sin \theta + 3)}{2 \sin \theta + 1}$  (3)  $\frac{2 \sin \theta + 1}{\sin \theta + 3}$  (4)  $\frac{2 \sin \theta + 1}{5(\sin \theta + 3)}$

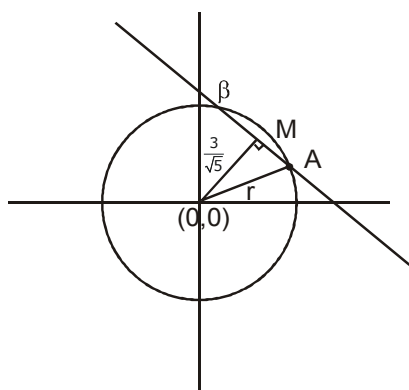
**Sol. 1**

$$\begin{aligned} & \int \frac{\cos \theta}{5 + 7 \sin \theta - 2 + 2 \sin^2 \theta} d\theta \\ & \int \frac{dt}{2t^2 + 7t + 3} \\ & = \frac{1}{2} \int \frac{dt}{t^2 + \frac{7}{2}t + \frac{3}{2}} = \frac{1}{2} \int \frac{dt}{t^2 + \frac{7}{2}t + \left(\frac{7}{4}\right)^2 - \frac{49}{16} + \frac{24}{16}} \\ & = \frac{1}{2} \int \frac{dt}{\left(t + \frac{7}{4}\right)^2 - \left(\frac{5}{4}\right)^2} \\ & \frac{1}{2} \times \frac{1}{2 \cdot \frac{5}{4}} \ln \left| \frac{t + \frac{7}{4} - \frac{5}{4}}{t + \frac{7}{4} + \frac{5}{4}} \right| \\ & \frac{1}{5} \ln \left| \frac{\sin \theta + \frac{1}{2}}{\sin \theta + 3} \right| + C \\ & \frac{B(\theta)}{A} = 5 \left( \frac{2 \sin \theta + 1}{\sin \theta + 3} \right) \end{aligned}$$

**Q.10** If the length of the chord of the circle,  $x^2 + y^2 = r^2$  ( $r > 0$ ) along the line,  $y - 2x = 3$  is  $r$ , then  $r^2$  is equal to:

- (1) 12 (2)  $\frac{24}{5}$  (3)  $\frac{9}{5}$  (4)  $\frac{12}{5}$

**Sol. 4**



$$AB = 2\sqrt{r^2 - 9/5} = r$$

$$r^2 - 9/5 = \frac{r^2}{4}$$

$$3r^2/4 = 9/5$$

$$\boxed{r^2 = \frac{12}{5}}$$

**Q.11** If  $\alpha$  and  $\beta$  are the roots of the equation,  $7x^2 - 3x - 2 = 0$ , then the value of  $\frac{\alpha}{1 - \alpha^2} + \frac{\beta}{1 - \beta^2}$  is equal to:

- (1)  $\frac{27}{32}$                       (2)  $\frac{1}{24}$                       (3)  $\frac{27}{16}$                       (4)  $\frac{3}{8}$

**Sol.** 3  
 $\alpha + \beta = 3/7, \alpha\beta = -2/7$

$$\frac{(\alpha + \beta) - \alpha\beta(\alpha + \beta)}{1 - (\alpha^2 + \beta^2) + (\alpha\beta)^2}$$

$$\frac{\frac{3}{7} + \frac{2}{7} \times \frac{3}{7}}{1 - \left\{ \frac{9}{49} + \frac{4}{7} \right\} + \frac{4}{49}}$$

$$\frac{\left( \frac{21 + 6}{49} \right)}{\frac{16}{49}} \Rightarrow \frac{27}{16}$$

**Q.12** If the sum of the second, third and fourth terms of a positive term G.P. is 3 and the sum of its sixth, seventh and eighth terms is 243, then the sum of the first 50 terms of this G.P. is:

- (1)  $\frac{2}{13}(3^{50} - 1)$                       (2)  $\frac{1}{26}(3^{49} - 1)$                       (3)  $\frac{1}{13}(3^{50} - 1)$                       (4)  $\frac{1}{26}(3^{50} - 1)$

**Sol.** 4

$$\frac{ar + ar^2 + ar^3}{ar^5 + ar^6 + ar^7} = \frac{3}{243}$$

$$\frac{1 + r + r^2}{r^4(1 + r + r^2)} = \frac{1}{81}$$

$$\boxed{r = 3}$$

$$a(3 + 9 + 27) = 3$$

$$a = \frac{3}{39} = \boxed{\frac{1}{13}}$$

$$S_{50} = a \left( \frac{r^{50} - 1}{r - 1} \right)$$

$$= \frac{1}{13} \left\{ \frac{3^{50} - 1}{2} \right\} \dots\dots\dots(4)$$

- Q.13** If the line  $y=mx+c$  is a common tangent to the hyperbola  $\frac{x^2}{100} - \frac{y^2}{64} = 1$  and the circle  $x^2+y^2=36$ , then which one of the following is true?  
 (1)  $4c^2=369$  (2)  $c^2=369$  (3)  $8m+5=0$  (4)  $5m=4$

**Sol. 1**

$$c = \pm \sqrt{a^2 m^2 - b^2}$$

$$c = \pm \sqrt{100m^2 - 64}$$

$$y = mx \pm \sqrt{100m^2 - 64}$$

$$d|_{(0,0)} = 6$$

$$\left| \frac{\sqrt{100m^2 - 64}}{\sqrt{m^2 + 1}} \right| = 6$$

$$100m^2 - 64 = 36m^2 + 36$$

$$64m^2 = 100$$

$$m = 10/8$$

$$c^2 = 100 \times \frac{100}{64} - 64 \Rightarrow \frac{(164)(36)}{64} \quad \boxed{4c^2 = 369}$$

- Q.14** The area (in sq. units) of the region  $A = \{(x, y) : (x-1)[x] \leq y \leq 2\sqrt{x}, 0 \leq x \leq 2\}$  where  $[t]$  denotes the greatest integer function, is:

(1)  $\frac{4}{3}\sqrt{2} - \frac{1}{2}$

(2)  $\frac{8}{3}\sqrt{2} - \frac{1}{2}$

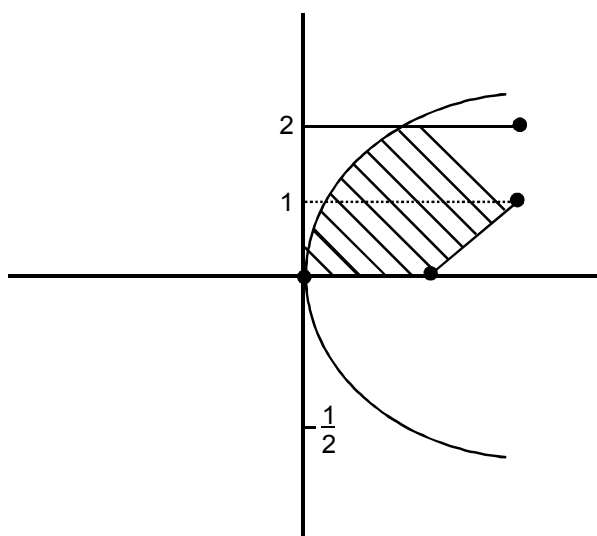
(3)  $\frac{8}{3}\sqrt{2} - 1$

(4)  $\frac{4}{3}\sqrt{2} + 1$

**Sol. 2**

$$y = f(x) = (x-1)[x] = \begin{cases} 0 & 0 \leq x < 1 \\ x-1 & 1 \leq x < 2 \\ 2(x-1) & x = 2 \end{cases}$$

$$y^2 \leq 4x$$



$$\int_0^1 (2\sqrt{x} - 0) + \int_1^2 (2\sqrt{x} - (x-1))$$

$$\frac{2}{3} \times 2x^{3/2} \Big|_0^1 + \left( \frac{4}{3} x^{3/2} - \frac{x^2}{2} + x \right) \Big|_1^2$$

$$\frac{4}{3} + \left\{ \left( \frac{4}{3} \times 2\sqrt{2} - 2 + 2 \right) - \left( \frac{4}{3} + \frac{1}{2} \right) \right\}$$

$$\frac{4}{3} + \frac{8\sqrt{2}}{3} - \frac{4}{3} - \frac{1}{2} = \frac{8\sqrt{2}}{3} - \frac{1}{2}$$

**Q.15** If  $a+x=b+y=c+z+1$ , where  $a, b, c, x, y, z$  are non-zero distinct real numbers. then  $\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix}$  is

equal to:

(1)  $y(a-b)$

(2) 0

(3)  $y(b-a)$

(4)  $y(a-c)$

**Sol. 1**

$$\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} + \begin{vmatrix} x & y & x+a \\ y & y & y+b \\ z & y & z+c \end{vmatrix}$$

$$\begin{vmatrix} x & y & a \\ y & y & b \\ z & y & c \end{vmatrix} \Rightarrow y \begin{vmatrix} x & 1 & a \\ y & 1 & b \\ z & 1 & c \end{vmatrix}$$

$$y \begin{vmatrix} x & 1 & a \\ y-x & 0 & b-a \\ z-x & 0 & c-a \end{vmatrix}$$

$$yx \times 0 - 1 \{ (y-x)(c-a) - (b-a)(z-x) \} + a \times 0 \}$$

$$y [bz - bx - az + ax - (cy - ay - cx + ax)]$$

$$y [bz - bx - az - cy + ay + cx]$$

$$y [b(z-x) + a(y-z) + c(x-y)]$$

$$y [b\{a-c-1\} + a\{c-b+1\} + c\{b-a\}]$$

$$y [ab - bc - b + ac - ab + a + bc - ac]$$

$$\boxed{y(a-b)}$$

**Q.16** If for some  $\alpha \in \mathbb{R}$ , the lines  $L_1 : \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$  and  $L_2 : \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$  are coplanar, then

the line  $L_2$  passes through the point:

(1)  $(2, -10, -2)$  (2)  $(10, -2, -2)$  (3)  $(10, 2, 2)$  (4)  $(-2, 10, 2)$

**Sol. 1**

A  $(-1, 2, 1)$ , B  $(-2, -1, -1)$

$$[\overrightarrow{AB} \ \overrightarrow{b_1} \ \overrightarrow{b_2}] = 0$$

$$\begin{vmatrix} -1 & -3 & -2 \\ 2 & -1 & 1 \\ \alpha & 5-\alpha & 1 \end{vmatrix} = 0$$

$$-1(-1+\alpha-5) + 3(2-\alpha) - 2(10-2\alpha+\alpha) = 0$$

$$6-\alpha + 6-3\alpha + 2\alpha - 20 = 0$$

$$-8 - 2\alpha = 0$$

$$\boxed{\alpha = -4}$$

$$L_2 : \frac{x+2}{-4} = \frac{y+1}{9} = \frac{z+1}{1}$$

any point on  $L_2$  is

$$(-4\lambda-2, 9\lambda-1, \lambda-1) = A$$

**Q.17** The value of  $\left( \frac{-1+i\sqrt{3}}{1-i} \right)^{30}$  is:

(1)  $2^{15}i$

(2)  $-2^{15}$

(3)  $-2^{15}i$

(4)  $6^5$

**Sol. 3**

$$\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30} \Rightarrow \left[\left(\frac{-1+i\sqrt{3}}{2}\right)(1+i)\right]^{30}$$

$$\omega^{30} (1+i)^{30} = 2^{15} (-i)$$

**Q.18** Let  $y=y(x)$  be the solution of the differential equation  $\cos x \frac{dy}{dx} + 2y \sin x = \sin 2x$ ,  $x \in \left(0, \frac{\pi}{2}\right)$ .

If  $y(\pi/3) = 0$ , then  $y(\pi/4)$  is equal to:

- (1)  $2 + \sqrt{2}$                       (2)  $\sqrt{2} - 2$                       (3)  $\frac{1}{\sqrt{2}} - 1$                       (4)  $2 - \sqrt{2}$

**Sol. 2**

$$\frac{dy}{dx} + (2 \tan x) y = 2 \sin x$$

$$\text{I.F.} = e^{2 \int \tan x dx} = \sec^2 x$$

$$y(\sec^2 x) = 2 \int \frac{\sin x}{\cos^2 x} dx$$

$$= 2 \int \sec x \tan x dx = 2 \sec x + c$$

$$y\left(\frac{\pi}{3}\right) = 0$$

$$0 = 2 \times 2 + c = C = -4$$

$$y(\sec^2 x) = 2 \sec x - 4$$

$$x = \pi/4$$

$$2y = 2\sqrt{2} - 4$$

$$y = \boxed{\sqrt{2} - 2}$$

**Q.19** If the system of linear equations

$$x+y+3z=0$$

$$x+3y+k^2z=0$$

$$3x+y+3z=0$$

has a non-zero solution  $(x,y,z)$  for some  $k \in \mathbb{R}$ , then  $x + \left(\frac{y}{z}\right)$  is equal to:

- (1) -9                      (2) 9                      (3) -3                      (4) 3

**Sol. 3**

$$\begin{vmatrix} 1 & 1 & 3 \\ 1 & 3 & k^2 \\ 3 & 1 & 3 \end{vmatrix} = 0$$

$$(9-k^2)-(3-3k^2) + 3(-8)=0$$

$$9-k^2-3+3k^2-24=0$$

$$2k^2-18=0$$

$$K^2 = 9$$

$$\boxed{K = 3, -3}$$

$$x+y+3z=0$$

$$x+3y+9z=0$$

$$2y+6z=0$$

$$\boxed{y = -3z}$$

$$\boxed{y/z = -3}$$

$$2x=0$$

$$\boxed{x = 0}$$

$$x + \left(\frac{y}{z}\right) = -3$$

**Q.20** Which of the following points lies on the tangent to the curve  $x^4 e^y + 2\sqrt{y+1} = 3$  at the point (1,0)?

(1) (2,6)

(2) (2,2)

(3) (-2,6)

(4) (-2,4)

**Sol. 3**

$$4x^3 e^y + x^4 e^y y' + \frac{2y'}{2\sqrt{y+1}} = 0$$

at (1,0)

$$4 + y' + \frac{2y'}{2} = 0$$

$$2y' = -4 \Rightarrow y' = -2$$

E.O.T. :

$$y = -2(x-1)$$

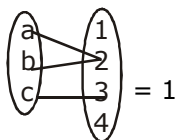
$$\boxed{2x + y = 2}$$

**Q.21** Let  $A = \{a, b, c\}$  and  $B = \{1, 2, 3, 4\}$ . Then the number of elements in the set  $C = \{f : A \rightarrow B \mid 2 \in f(A) \text{ and } f \text{ is not one-one}\}$  is \_\_\_\_\_

**Sol. 19**

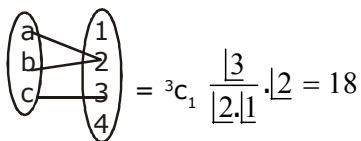
**case - I**

set B only have '2'



**case - II**

set B have more element with 2



total  $18 + 1 = 19$

**Q.22** The coefficient of  $x^4$  in the expansion of  $(1+x+x^2+x^3)^6$  in powers of  $x$ , is \_\_\_\_\_

**Sol.** **120**

$$(1+x)^6(1+x^2)^6$$

$${}^6C_r x^r \quad {}^6C_s x^{2s}$$

$${}^6C_r {}^6C_s x^{r+2s}$$

r	s
0	2
4	0
2	1

$$\Rightarrow {}^6C_0 {}^6C_2 + {}^6C_4 {}^6C_0 + {}^6C_2 {}^6C_1$$

$$\Rightarrow 15+15+15 \times 6$$

$$\Rightarrow 120$$

**Q.23** Let the vectors  $\vec{a}, \vec{b}, \vec{c}$  be such that  $|\vec{a}| = 2, |\vec{b}| = 4$  and  $|\vec{c}| = 4$ . If the projection of  $\vec{b}$  on  $\vec{a}$  is equal to the projection of  $\vec{c}$  on  $\vec{a}$  and  $\vec{b}$  is perpendicular to  $\vec{c}$ , then the value of  $|\vec{a} + \vec{b} - \vec{c}|$  is \_\_\_\_\_

**Sol.** **6**

$$\frac{\vec{b} \cdot \vec{a}}{2} = \frac{\vec{c} \cdot \vec{a}}{2} \quad \boxed{\vec{b} \cdot \vec{a} = \vec{c} \cdot \vec{a}}$$

$$\boxed{\vec{b} \cdot \vec{c} = 0}$$

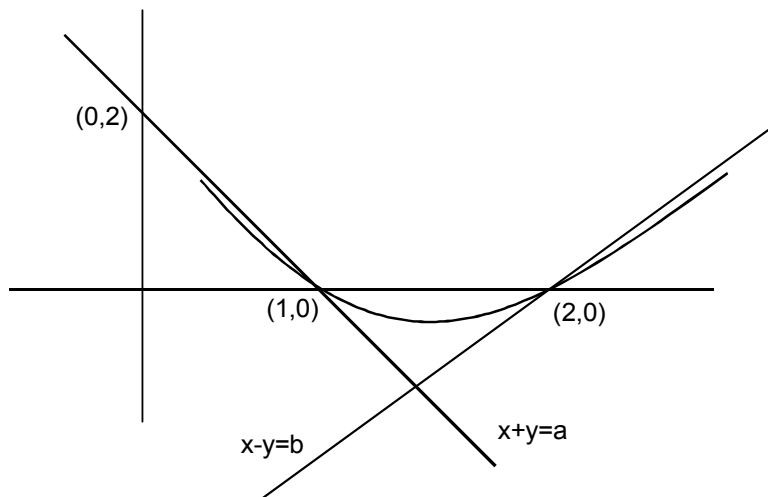
$$|\vec{a} + \vec{b} - \vec{c}| = \sqrt{a^2 + b^2 + c^2 + 2\vec{a} \cdot \vec{b} - 2\vec{b} \cdot \vec{c} - 2\vec{a} \cdot \vec{c}}$$

$$= \sqrt{4 + 16 + 16}$$

$$= 6$$

**Q.24** If the lines  $x+y=a$  and  $x-y=b$  touch the curve  $y=x^2-3x+2$  at the points where the curve intersects the  $x$ -axis, then  $\frac{a}{b}$  is equal to \_\_\_\_\_

**Sol. 0.5**



$$\begin{aligned} y - 0 &= -1(x-1) \\ x + y &= 1 \Rightarrow a = 1 \\ y - 0 &= x - 2 \\ x - y &= 2 = b = 2 \end{aligned}$$

$$\boxed{\frac{a}{b} = \frac{1}{2}}$$

**Q.25** In a bombing attack, there is 50% chance that a bomb will hit the target. At least two independent hits are required to destroy the target completely. Then the minimum number of bombs, that must be dropped to ensure that there is at least 99% chance of completely destroying the target, is \_\_\_\_\_

**Sol. 11**

Let 'n' is total no. of bombs being dropped  
at least 2 bombs should hit

$$\Rightarrow \text{prob} \geq 0.99$$

$$P(x \geq 2) \geq 0.99$$

$$1 - p(x < 2) \geq 0.99$$

$$1 - (p(x=0) + p(x=1)) \geq 0.99$$

$$1 - [{}^nC_0 (p)^0 q^n + {}^nC_1 (p)^1 (q)^{n-1}] \geq 0.99$$

$$1 - [q^n + pnq^{n-1}] \geq 0.99$$

$$1 - \left[ \frac{1}{2^n} + \frac{1}{2} \times \frac{1}{2^{n-1}} \right] \geq 0.99$$

$$1 - \frac{1}{2^n} (n+1) \geq 0.99$$

$$0.01 \geq \frac{1}{2^n} (n+1)$$

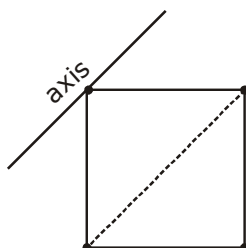
$$2^n \geq 100 + 100n$$

$$n \geq 11$$

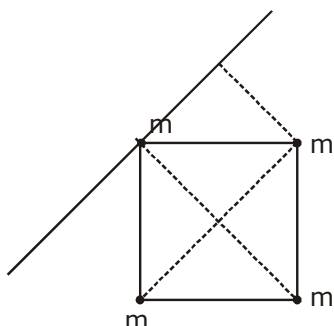
# QUESTION PAPER WITH SOLUTION

## PHYSICS \_ 6 Sep. \_ SHIFT - 1

1. Four point masses, each of mass  $m$ , are fixed at the corners of a square of side  $l$ . The square is rotating with angular frequency  $\omega$ , about an axis passing through one of the corners of the square and parallel to its diagonal, as shown in the figure. The angular momentum of the square about this axis is :



- (1)  $4 ml^2\omega$  (2)  $2 ml^2\omega$  (3)  $3 ml^2\omega$  (4)  $ml^2\omega$
- Sol. (3)



$$L = I\omega$$

$$I = m \left( \frac{a}{\sqrt{2}} \right)^2 \times 2 + m (\sqrt{2}a)^2$$

$$= ma^2 + 2ma^2$$

$$\therefore L = I\omega = 3ml^2\omega \quad (a = l)$$

2. A screw gauge has 50 divisions on its circular scale. The circular scale is 4 units ahead of the pitch scale marking, prior to use. Upon one complete rotation of the circular scale, a displacement of 0.5mm is noticed on the pitch scale. The nature of zero error involved and the least count of the screw gauge, are respectively :

- (1) Positive, 0.1 mm (2) Positive, 0.1  $\mu\text{m}$   
(3) Positive, 10  $\mu\text{m}$  (4) Negative, 2  $\mu\text{m}$

2. (3)

$$= L.C = \frac{0.5}{50} \text{ mm} = 1 \times 10^{-5} \text{ m} = 10 \mu\text{m}$$

3. An electron, a doubly ionized helium ion ( $\text{He}^{++}$ ) and a proton are having the same kinetic energy. The relation between their respective de-Broglie wavelengths  $\lambda_e$ ,  $\lambda_{\text{He}^{++}}$  and  $\lambda_p$  is :

- (1)  $\lambda_e > \lambda_p > \lambda_{\text{He}^{++}}$  (2)  $\lambda_e > \lambda_{\text{He}^{++}} > \lambda_p$   
 (3)  $\lambda_e < \lambda_p < \lambda_{\text{He}^{++}}$  (4)  $\lambda_e < \lambda_{\text{He}^{++}} = \lambda_p$

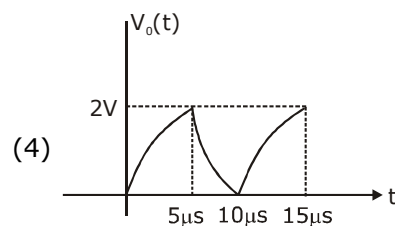
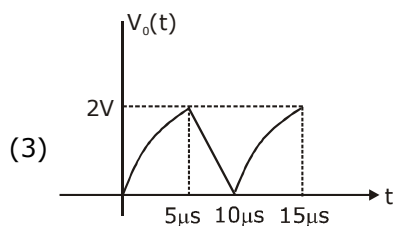
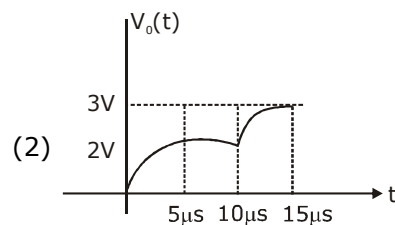
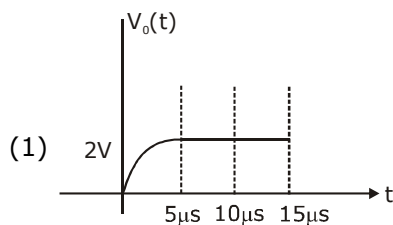
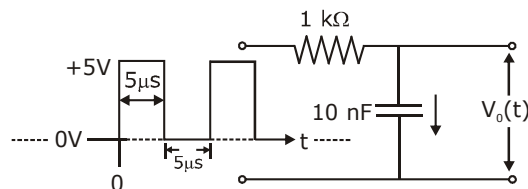
**Sol. (1)**

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK.E}} \quad \frac{C}{q} = \frac{2.27}{1.5}$$

$$m_{\text{He}} > m_p > m_e$$

$$\lambda_{\text{He}} < \lambda_p < \lambda_e$$

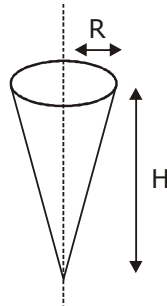
4. For the given input voltage waveform  $V_{\text{in}}(t)$ , the output voltage waveform  $V_o(t)$ , across the capacitor is correctly depicted by :



**Sol. (2)**

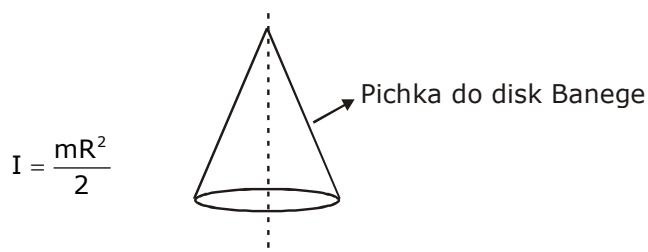
Answer is (2) because capacitor is charging then discharging then again charging. But during discharging not possible to discharge 100%.

5. Shown in the figure is a hollow icecream cone (it is open at the top). If its mass is  $M$ , radius of its top,  $R$  and height,  $H$ , then its moment of inertia about its axis is :



- (1)  $\frac{MR^2}{2}$       (2)  $\frac{MR^2}{3}$       (3)  $\frac{M(R^2 + H^2)}{4}$       (4)  $\frac{MH^2}{3}$

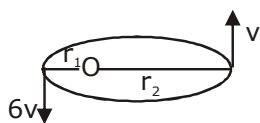
**Sol. (1)**



6. A satellite is in an elliptical orbit around a planet P. It is observed that the velocity of the satellite when it is farthest from the planet is 6 times less than that when it is closest to the planet. The ratio of distances between the satellite and the planet at closest and farthest points is:

- (1) 1 : 2      (2) 1 : 3      (3) 1 : 6      (4) 3 : 4

**Sol. (3)**



$$\begin{aligned} L_i &= L_f \\ m \cdot 6v r_1 &= m \cdot v r_2 \\ 6r_1 &= r_2 \\ \Rightarrow \frac{r_1}{r_2} &= \frac{1}{6} \end{aligned}$$

7. You are given that Mass of  ${}^7_3\text{Li} = 7.0160\text{u}$ ,

Mass of  ${}^4_2\text{He} = 4.0026\text{u}$

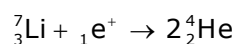
and Mass of  ${}^1_1\text{H} = 1.0079\text{u}$ .

When 20 g of  ${}^7_3\text{Li}$  is converted into  ${}^4_2\text{He}$  by proton capture, the energy liberated, (in kWh), is:

[Mass of nucleon = 1 GeV/c<sup>2</sup>]

(1)  $6.82 \times 10^5$  (2)  $4.5 \times 10^5$  (3)  $8 \times 10^6$  (4)  $1.33 \times 10^6$

**Sol. (4)**



$$\Delta m \Rightarrow [m_{\text{Li}} + m_{\text{H}}] - 2 [M_{\text{He}}]$$

$$\rightarrow \Delta m = (7.0160 + 1.0079) - 2 \times 4.0003$$

$$= 0.0187$$

Energy released in 1 reaction  $\Rightarrow \Delta mc^2$

In use of 7.016 u Li energy is  $\Delta mc^2$

$$\text{In use of 1gm Li energy is } \frac{\Delta mc^2}{m_{\text{Li}}}$$

$$\text{In use of 20gm energy is } \Rightarrow \frac{\Delta mc^2}{m_{\text{Li}}} \times 20\text{gm}$$

$$\frac{0.0187 \times 931.5 \times 10^6 \times 1.6 \times 10^{-19} \times \frac{20}{7} \times 6.023 \times 10^{23}}{36 \times 10^5}$$

$$= 1.33 \times 10^6$$

8. If the potential energy between two molecules is given by  $U = -\frac{A}{r^6} + \frac{B}{r^{12}}$ , then at equilibrium, separation between molecules, and the potential energy are :

(1)  $\left(\frac{2B}{A}\right)^{1/6}$ ,  $-\frac{A^2}{4B}$  (2)  $\left(\frac{2B}{A}\right)^{1/6}$ ,  $-\frac{A^2}{2B}$  (3)  $\left(\frac{B}{A}\right)^{1/6}$ , 0 (4)  $\left(\frac{B}{2A}\right)^{1/6}$ ,  $-\frac{A^2}{2B}$

**Sol. (1)**

$$F = \frac{-dU}{dr} = \frac{-d}{dr}(-Ar^{-6} + Br^{-12}) \quad \text{for equation } F = 0$$

$$= \frac{A(-6)}{r^7} + \frac{B \cdot 12}{r^{13}} = 0$$

$$\frac{12B}{r^{13}} = \frac{6A}{r^7}$$

$$r = \left(\frac{2B}{A}\right)^{1/6}$$

$$U = \frac{-A}{\frac{2B}{A}} + \frac{B}{\left(\frac{2B}{A}\right)^2}$$

$$= \frac{-A^2}{2B} + \frac{A^2}{4B} = \frac{-A^2}{4B}$$

∴ Answer (1)

9. A clock has a continuously moving second's hand of 0.1 m length. The average acceleration of the tip of the hand (in units of  $\text{ms}^{-2}$ ) is of the order of:

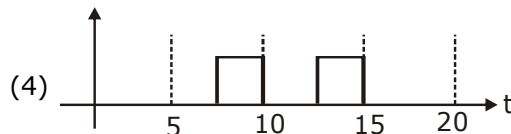
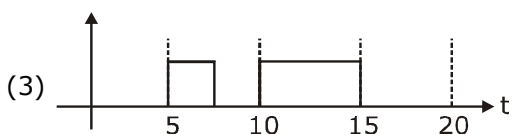
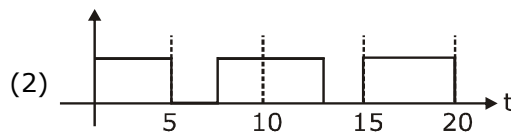
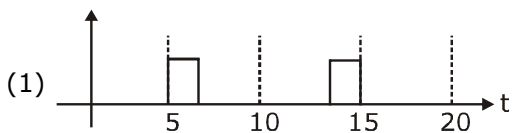
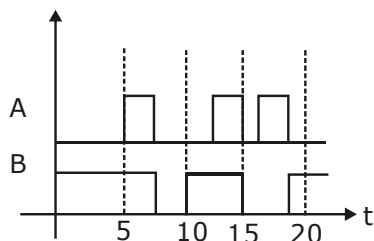
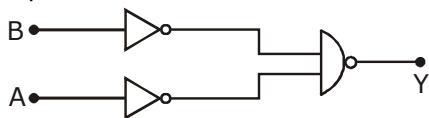
(1)  $10^{-3}$                       (2)  $10^{-1}$                       (3)  $10^{-2}$                       (4)  $10^{-4}$

Sol. (1)

$$a = \frac{v^2}{R} \quad v = \frac{2\pi R}{60}$$

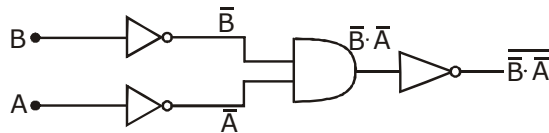
$$= \frac{4\pi^2 \cdot R^2}{(60)^2 R} = \frac{4\pi^2 R}{(60)^2} = \frac{4}{(60)^2} \times 10 \times 0.1 \approx 10^{-3}$$

10. Identify the correct output signal Y in the given combination of gates (as shown) for the given inputs A and B.



**Sol.** None of the option is correct

$$\overline{\overline{A \cdot B}} = \overline{\overline{A} + \overline{B}} = A + B$$

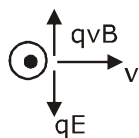


- 11.** An electron is moving along +x direction with a velocity of  $6 \times 10^6 \text{ ms}^{-1}$ . It enters a region of uniform electric field of 300 V/cm pointing along +y direction. The magnitude and direction of the magnetic field set up in this region such that the electron keeps moving along the x direction will be:

- (1)  $3 \times 10^{-4} \text{ T}$ , along -z direction      (2)  $5 \times 10^{-3} \text{ T}$ , along -z direction  
(3)  $5 \times 10^{-3} \text{ T}$ , along +z direction      (4)  $3 \times 10^{-4} \text{ T}$ , along +z direction

**Sol.** (3)

$\vec{B}$  must be in +z axis.

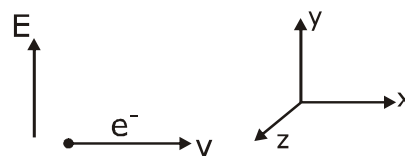


$$qE = qvB$$

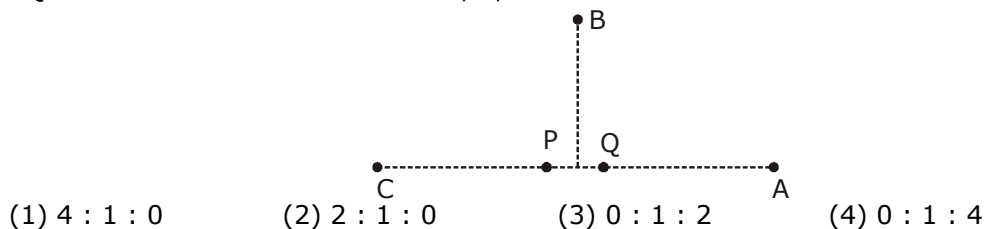
$$E = 300 \frac{\text{V}}{10^{-2} \text{ m}}$$

$$= 30000 \text{ V/m}$$

$$B = \frac{E}{v} = \frac{3 \times 10^4}{6 \times 10^6} = 5 \times 10^{-3} \text{ T}$$



- 12.** In the figure below, P and Q are two equally intense coherent sources emitting radiation of wavelength 20 m. The separation between P and Q is 5 m and the phase of P is ahead of that of Q by  $90^\circ$ . A, B and C are three distinct points of observation, each equidistant from the midpoint of PQ. The intensities of radiation at A, B, C will be in the ratio:



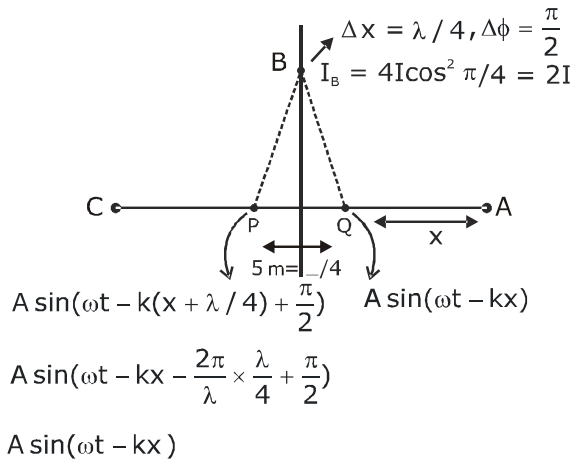
(1) 4 : 1 : 0

(2) 2 : 1 : 0

(3) 0 : 1 : 2

(4) 0 : 1 : 4

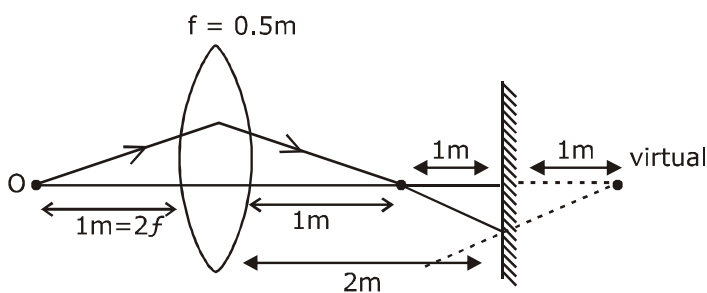
**Sol. (2)**



$\left  \begin{array}{l} \Delta x = \lambda/2 \\ \Delta \phi = \pi \\ I_c = 0 \end{array} \right $	$\therefore$ at A $\Delta x_{\text{effective}} = 0$ or phase difference = 0
	$\therefore I_A = 4I$
	{Same logic as A point but opposites}
	$\therefore$ Answer is 2.

- 13.** A point like object is placed at a distance of 1 m in front of a convex lens of focal length 0.5 m. A plane mirror is placed at a distance of 2 m behind the lens. The position and nature of the final image formed by the system is:
- |                                  |                                    |
|----------------------------------|------------------------------------|
| (1) 1 m from the mirror, virtual | (2) 2.6 m from the mirror, virtual |
| (3) 1 m from the mirror, real    | (4) 2.6 m from the mirror, real    |

**Sol. (1, 2 Both are correct)**



for III<sup>rd</sup> Refraction,  $u = -3$

$$\frac{1}{v} + \frac{1}{3} = \frac{2}{1}$$

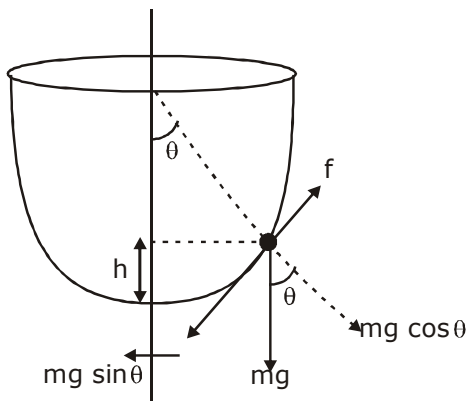
$$v = \frac{3}{5} = 0.6$$

from mirror = 2.6m

- 14.** An insect is at the bottom of a hemispherical ditch of radius 1 m. It crawls up the ditch but starts slipping after it is at height  $h$  from the bottom. If the coefficient of friction between the ground and the insect is 0.75, then  $h$  is: ( $g = 10 \text{ ms}^{-2}$ )

(1) 0.45 m                      (2) 0.60 m                      (3) 0.20 m                      (4) 0.80 m

**Sol. (3)**



$$f = mg \sin \theta$$

$$f = \mu mg \cos \theta$$

$$\mu mg \cos \theta = mg \sin \theta$$

$$\tan \theta = \mu$$

$$\tan \theta = \frac{3}{4}$$

$$\cos \theta = \frac{4}{\sqrt{16+9}} = \frac{4}{5}$$

$$h = 1(1 - \cos \theta) = 1 - \frac{4}{5} = \frac{1}{5}$$

$$h = \frac{1}{5} = 0.2\text{m}$$

- 15.** Molecules of an ideal gas are known to have three translational degrees of freedom and two rotational degrees of freedom. The gas is maintained at a temperature of  $T$ . The total internal energy,  $U$  of a mole of this gas, and the value of  $\gamma \left( = \frac{C_p}{C_v} \right)$  are given, respectively by:

(1)  $U = \frac{5}{2}RT$  and  $\gamma = \frac{7}{5}$

(2)  $U = 5RT$  and  $\gamma = \frac{6}{5}$

(3)  $U = 5RT$  and  $\gamma = \frac{7}{5}$

(4)  $U = \frac{5}{2}RT$  and  $\gamma = \frac{6}{5}$

**Sol. (1)**

$$U = \frac{f}{2}nRT = \frac{5}{2}nRT \left( \begin{array}{l} C_p - C_v = R \\ C_v = \frac{f}{2}R \end{array} \right), \gamma = \frac{C_p}{C_v} \Rightarrow 1 + \frac{2}{f} = 1 + \frac{2}{5} = \frac{7}{5}$$

- 16.** An object of mass  $m$  is suspended at the end of a massless wire of length  $L$  and area of cross-section  $A$ . Young modulus of the material of the wire is  $Y$ . If the mass is pulled down slightly its frequency of oscillation along the vertical direction is:

(1)  $f = \frac{1}{2\pi} \sqrt{\frac{YA}{mL}}$

(2)  $f = \frac{1}{2\pi} \sqrt{\frac{mL}{YA}}$

(3)  $f = \frac{1}{2\pi} \sqrt{\frac{YL}{mA}}$

(4)  $f = \frac{1}{2\pi} \sqrt{\frac{mA}{YL}}$

**Sol. (1)**

$$Y = \frac{F/A}{\Delta L/L}$$

$$Y = \frac{FL}{A\Delta L}$$

$$F = \frac{YA\Delta L}{L}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{YA}{Lm}}$$

$$\left( \frac{YA}{L} = k \right)$$

$$\therefore T = 2\pi \sqrt{\frac{m}{k}}$$

- 17.** An AC circuit has  $R = 100 \Omega$ ,  $C = 2 \mu F$  and  $L = 80 \text{ mH}$ , connected in series. The quality factor of the circuit is:

(1) 20

(2) 2

(3) 0.5

(4) 400

**Sol. (2)**

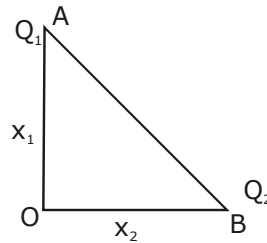
$$Q = \frac{\omega L}{R}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$= \frac{L}{R\sqrt{LC}} = \frac{1}{R} \sqrt{\frac{L}{C}}$$

$$= \frac{1}{100} \sqrt{\frac{80 \times 10^{-3}}{2 \times 10^{-6}}} = 2$$

- 18.** Charges  $Q_1$  and  $Q_2$  are at points A and B of a right angle triangle OAB (see figure). The resultant electric field at point O is perpendicular to the hypotenuse, then  $Q_1/Q_2$  is proportional to:



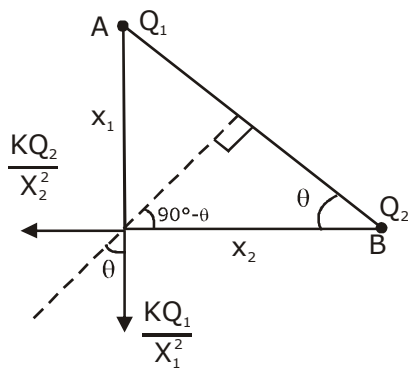
(1)  $\frac{x_2}{x_1}$

(2)  $\frac{x_2^2}{x_1^2}$

(3)  $\frac{x_1^3}{x_2^3}$

(4)  $\frac{x_1}{x_2}$

**Sol. (4)**

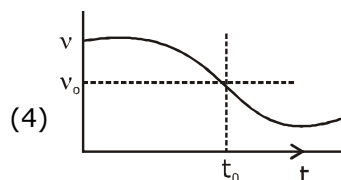
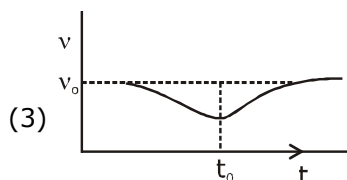
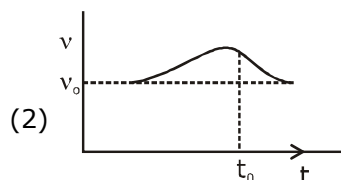
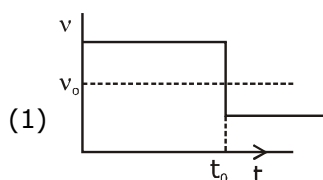


$$\tan \theta = \frac{kQ_2 / x_2^2}{kQ_1 / x_1^2} = \frac{x_1}{x_2}$$

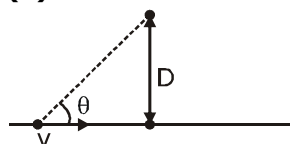
$$\frac{Q_2 \cdot x_1^2}{Q_1 \cdot x_2^2} = \frac{X_1}{X_2}$$

$$\frac{Q_1}{Q_2} = \frac{x_1}{x_2}$$

- 19.** A sound source S is moving along a straight track with speed  $v$ , and is emitting, sound of frequency  $v_0$  (see figure). An observer is standing at a finite distance, at the point O, from the track. The time variation of frequency heard by the observer is best represented by: ( $t_0$  represents the instant when the distance between the source and observer is minimum)



**Sol. (4)**



$$f_{\text{observed}} \Rightarrow \left( \frac{v_{\text{sound}}}{v_{\text{sound}} - v \cos \theta} \right) f_0$$

initially  $\theta$  will be less  $\Rightarrow \cos \theta$  more  
 $\therefore f_{\text{observed}}$  more, then it will decrease.  
 $\therefore$  Ans. 4

- 20.** A particle of charge  $q$  and mass  $m$  is moving with a velocity  $-v\hat{i}$  ( $v \neq 0$ ) towards a large screen placed in the Y-Z plane at a distance  $d$ . If there is a magnetic field  $\vec{B} = B_0\hat{k}$ , the minimum value of  $v$  for which the particle will not hit the screen is:

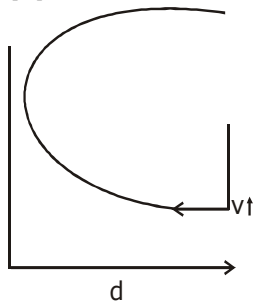
(1)  $\frac{qdB_0}{m}$

(2)  $\frac{qdB_0}{3m}$

(3)  $\frac{2qdB_0}{m}$

(4)  $\frac{qdB_0}{2m}$

**Sol. (1)**



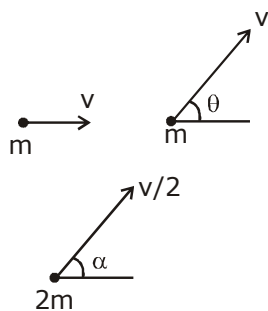
$$d > R$$

$$d > \frac{mv}{qB_0}$$

$$v < \frac{qB_0 d}{m}$$

- 21.** Two bodies of the same mass are moving with the same speed, but in different directions in a plane. They have a completely inelastic collision and move together thereafter with a final speed which is half of their initial speed. The angle between the initial velocities of the two bodies (in degree) is \_\_\_\_\_.

**21. 120**



∴ In Horizontal Direction  
By Momentum conservation.

$$mv + mv \cos \theta = 2m \frac{v}{2} \cos \alpha$$

$$1 + \cos \theta = \cos \alpha \quad \dots(1)$$

In vertical direction  
By Momentum conservation.

$$0 + mv \sin \theta = 2m \frac{v}{2} \sin \alpha$$

$$\sin \theta = \sin \alpha$$

$$1 + \cos \theta = \sqrt{1 - \sin^2 \theta}$$

$$\theta = 120^\circ$$

- 22.** Suppose that intensity of a laser is  $\left(\frac{315}{\pi}\right) \text{ W/m}^2$ . The rms electric field, in units of V/m associated with this source is close to the nearest integer is \_\_\_\_\_.  
( $\epsilon_0 = 8.86 \times 10^{-12} \text{ C}^2\text{Nm}^{-2}$ ;  $c = 3 \times 10^8 \text{ ms}^{-1}$ )

**Sol. 275**

$$I = \frac{1}{2} \epsilon_0 c E_{\text{rms}}^2$$

$$\frac{3.15}{\pi} = \frac{1}{2} \times 8.86 \times 10^{-12} \times 3 \times 10^8 \times E_{\text{rms}}^2$$

$$E_{\text{rms}} = 275$$

- 23.** The density of a solid metal sphere is determined by measuring its mass and its diameter. The maximum error in the density of the sphere is  $\left(\frac{x}{100}\right)\%$ . If the relative errors in measuring the mass and the diameter are 6.0% and 1.5% respectively, the value of x is\_\_\_\_\_.

**Sol. 1050**

$$\rho = \frac{m}{\frac{4}{3} \pi \left(\frac{d}{2}\right)^3}$$

$$\rho = k \cdot \frac{m}{d^3}$$

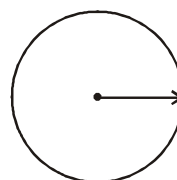
$$\log \rho = \log k + \log m - 3 \log d$$

diff.

$$\frac{d\rho}{\rho} = \frac{dm}{m} - 3 \cdot \frac{dd}{d}$$

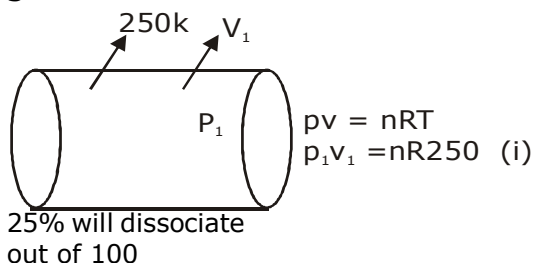
$$= 6.0 + 3 \times 1.5 = 10.5\%$$

$$= x = 1050$$



- 24.** Initially a gas of diatomic molecules is contained in a cylinder of volume  $V_1$  at a pressure  $P_1$  and temperature 250 K. Assuming that 25% of the molecules get dissociated causing a change in number of moles. The pressure of the resulting gas at temperature 2000 K, when contained in a volume  $2V_1$  is given by  $P_2$ . The ratio  $P_2/P_1$  is \_\_\_\_\_.

**Sol. 5**



$$\frac{3n}{4} \text{ molecules will remain same}$$

S

$$\frac{n}{4} \text{ mole become } \rightarrow \frac{n}{2}$$

$\therefore$  Total molecules used

$$\rightarrow \frac{3n}{4} + \frac{n}{2} = \frac{5n}{4}$$

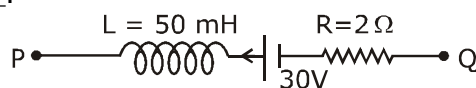
$$P_2 2V_1 = \frac{5n}{4} \cdot R \cdot 2000 \text{ -- (ii)}$$

Eq. (ii/i)

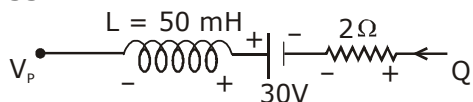
$$\frac{2P_2 V_1}{P_1 V_1} = \frac{5nR \times 2000}{4nR \times 250}$$

$$\frac{P_2}{P_1} = 5$$

- 25.** A part of a complete circuit is shown in the figure. At some instant, the value of current  $I$  is 1A and it is decreasing at a rate of  $10^2 \text{ A s}^{-1}$ . The value of the potential difference  $V_p - V_Q$ , (in volts) at that instant, is \_\_\_\_\_.



**Sol. 33**



$$V_p + L \cdot \frac{di}{dt} - 30 + 2i = V_Q$$

$$V_p + 50 \times 10^{-3} (-10^2) - 30 + 2 \times 1 = V_Q$$

$$V_p - V_Q = 35 - 2 = 33$$

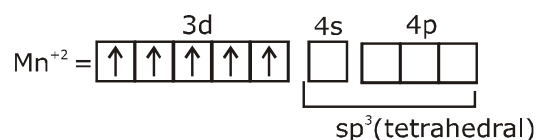
1. The INCORRECT statement is :
- (1) Cast iron is used to manufacture wrought iron.
  - (2) Brass is an alloy of copper and nickel.
  - (3) German silver is an alloy of zinc, copper and nickel.
  - (4) Bronze is an alloy of copper and tin

**Sol. 2**

Brass - (copper Zinc)  
Bronze - (copper tin)

2. The species that has a spin-only magnetic moment of 5.9 BM, is : ( $T_d$  = tetrahedral)
- (1)  $[\text{Ni}(\text{CN})_4]^{2-}$  (square planar)
  - (2)  $\text{Ni}(\text{CO})_4(T_d)$
  - (3)  $[\text{MnBr}_4]^{2-}(T_d)$
  - (4)  $[\text{NiCl}_4]^{2-}(T_d)$

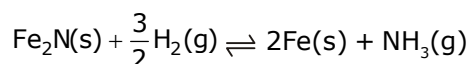
**Sol. 3**



$sp^3(\text{tetrahedral})$

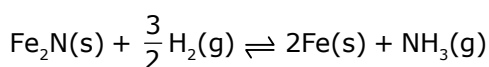
$$\mu = \sqrt{5(5+2)} = 5.9 \text{ BM}$$

3. For the reaction



- (1)  $K_c = K_p(RT)^{1/2}$
- (2)  $K_c = K_p(RT)^{-1/2}$
- (3)  $K_c = K_p(RT)^{\frac{3}{2}}$
- (4)  $K_c = K_p(RT)$

**Sol. 1**

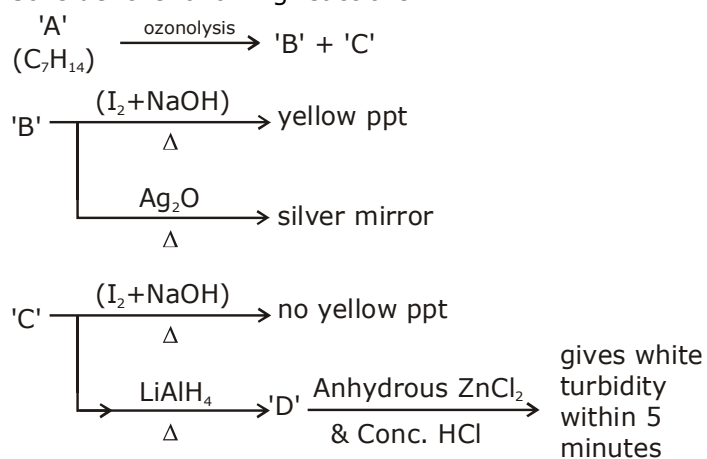


$$\Delta n_g = 1 - \frac{3}{2} = -\frac{1}{2}$$

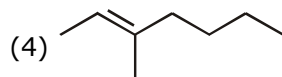
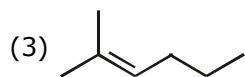
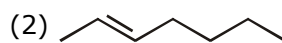
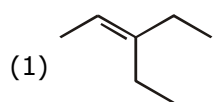
$$\frac{K_p}{K_c} = (RT)^{\Delta n_g} = (RT)^{-1/2}$$

$$K_c = \frac{K_p}{(RT)^{-1/2}} = K_p \cdot (RT)^{1/2}$$

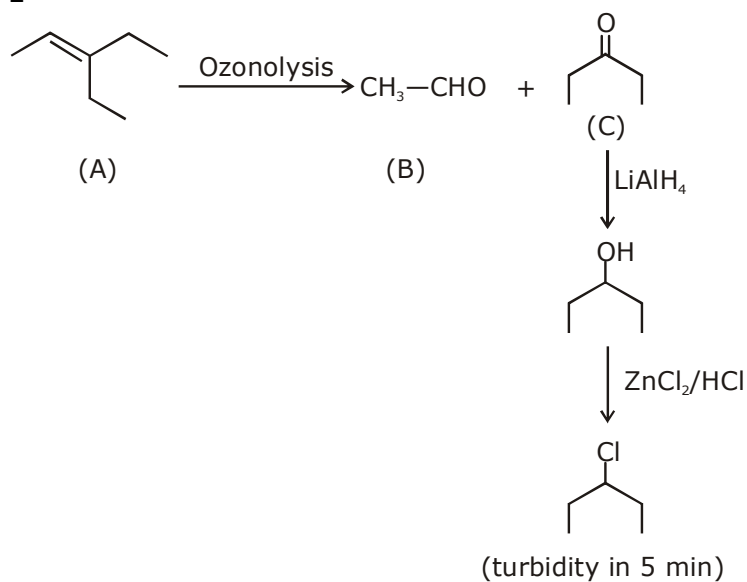
4. Consider the following reactions :



'A' is :



**Sol. 1**



5. Arrange the following solutions in the decreasing order of pOH :

- (A) 0.01 M HCl (B) 0.01 M NaOH  
(C) 0.01 M  $\text{CH}_3\text{COONa}$  (D) 0.01 M NaCl  
(1) (A) > (C) > (D) > (B) (2) (B) > (D) > (C) > (A)  
(3) (B) > (C) > (D) > (A) (4) (A) > (D) > (C) > (B)

Sol. 4

- (i)  $10^{-2} \text{ M HCl} \Rightarrow [\text{H}^+] = 10^{-2} \text{ M} \rightarrow \text{pH} = 2$   
(ii)  $10^{-2} \text{ M NaOH} \Rightarrow [\text{OH}^-] = 10^{-2} \text{ M} \rightarrow \text{pOH} = 2$   
(iii)  $10^{-2} \text{ M CH}_3\text{COO}^-\text{Na}^+ \Rightarrow [\text{OH}^-] > 10^{-7} \Rightarrow \text{pOH} < 7$   
(iv)  $10^{-2} \text{ M NaCl} \Rightarrow \text{Neutral pOH} = 7$   
(i) > (iv) > (iii) > (ii)

6. The variation of equilibrium constant with temperature is given below :

**Temperature** **Equilibrium Constant**

$T_1 = 25^\circ\text{C}$   $K_1 = 10$

$T_2 = 100^\circ\text{C}$   $K_2 = 100$

The value of  $\Delta H^\circ$ ,  $\Delta G^\circ$  at  $T_1$  and  $\Delta G^\circ$  at  $T_2$  (in  $\text{KJ mol}^{-1}$ ) respectively, are close to  
[use  $R = 8.314 \text{ JK}^{-1} \text{ mol}^{-1}$ ]

- (1) 28.4, -7.14 and -5.71 (2) 0.64, -7.14 and -5.71  
(3) 28.4, -5.71 and -14.29 (4) 0.64, -5.71 and -14.29

Sol. 3

$$\ln \left[ \frac{k_2}{k_1} \right] = \frac{\Delta H^\circ}{R} \left\{ \frac{1}{T_1} - \frac{1}{T_2} \right\}$$

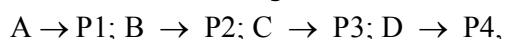
$$\ln(10) = \frac{\Delta H^\circ}{R} \left\{ \frac{1}{298} - \frac{1}{373} \right\}$$

$$\frac{373 \times 298 \times 8.314 \times 2.303}{75} = \Delta H^\circ = 28.37 \text{ kJ mol}^{-1}$$

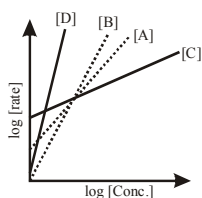
$$\Delta G^\circ_{T_1} = -RT_1 \ln(K_1) = -298R \ln(10) = -5.71 \text{ kJ mol}^{-1}$$

$$\Delta G^\circ_{T_2} = -RT_2 \ln(K_2) = -373R \ln(100) \\ = -14.283 \text{ kJ/mol}$$

7. Consider the following reactions



The order of the above reactions are a,b,c and d, respectively. The following graph is obtained when  $\log[\text{rate}]$  vs.  $\log[\text{conc.}]$  are plotted :



Among the following the correct sequence for the order of the reactions is :

- (1)  $c > a > b > d$  (2)  $d > a > b > c$   
 (3)  $d > b > a > c$  (4)  $a > b > c > d$

**Sol. 3**



$\text{Rate} = K (\text{conc.})^{\text{order}}$

$\log(\text{rate}) = \log(K) + \text{order} \log(\text{case})$

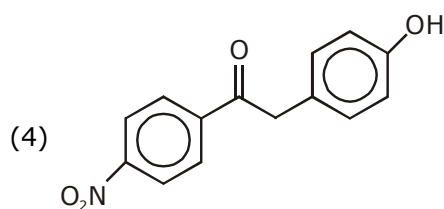
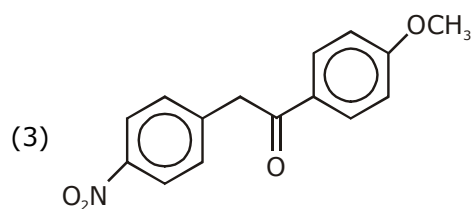
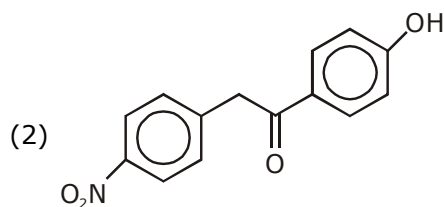
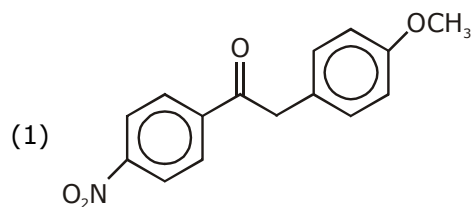
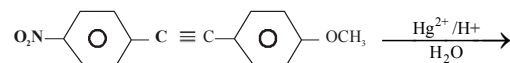
$\underbrace{\quad y \quad \quad c \quad + \quad m.x \quad}_{\text{Straight line}}$

Slope = order

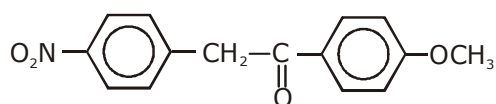
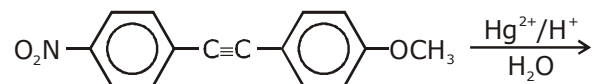
According graph

$d > b > a > c$  order of slope

**8.** The major product obtained from the following reactions is :

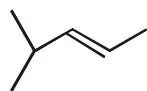


**Sol. 3**



9. Which of the following compounds shows geometrical isomerism ?  
 (1) 2-methylpent-1-ene (2) 4-methylpent-2-ene  
 (3) 2-methylpent-2-ene (4) 4-methylpent-1-ene

Sol. 2



4-Methylpent-2-ene

Can show G.I.

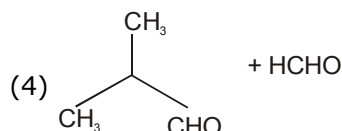
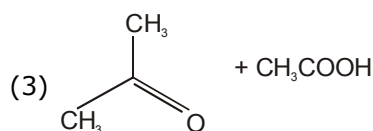
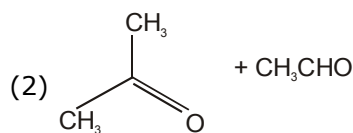
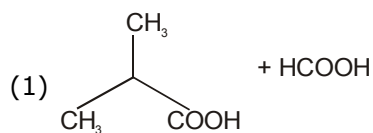
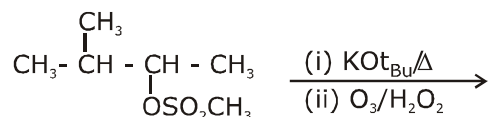
10. The lanthanoid that does NOT show +4 oxidation state is :

- (1) Dy (2) Ce  
 (3) Tb (4) Eu

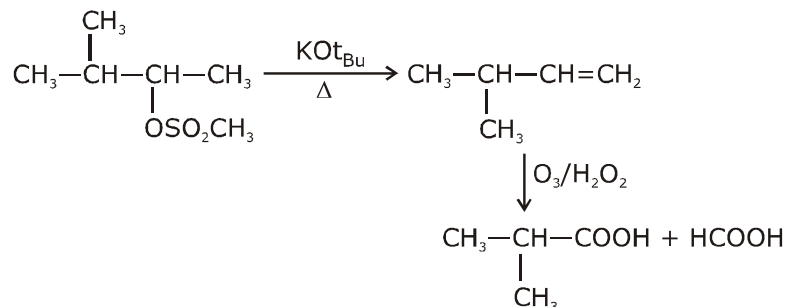
Sol. 4

Fact

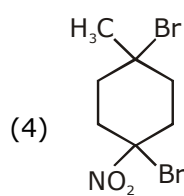
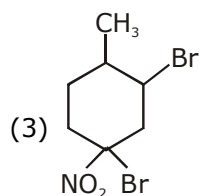
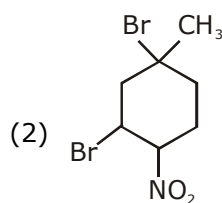
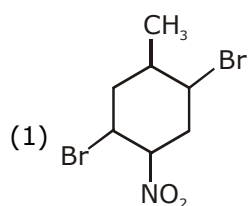
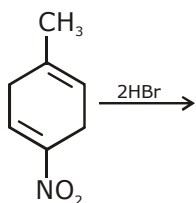
11. The major products of the following reactions are :



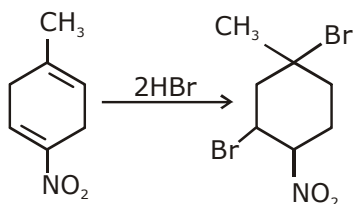
Sol. 1



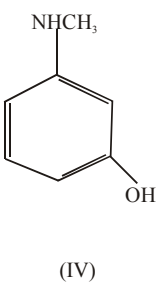
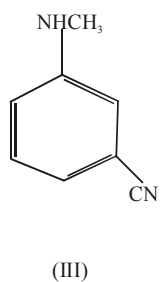
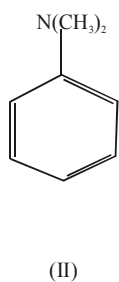
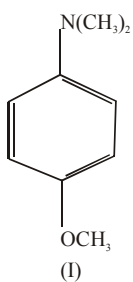
12. The major product of the following reaction is :



Sol. 2



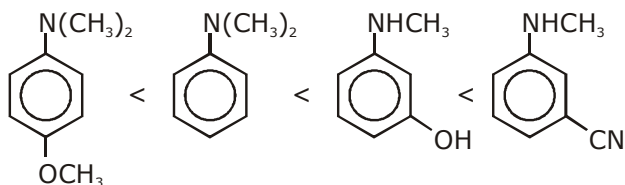
13. The increasing order of  $pK_b$  values of the following compounds is :



- (1) I < II < III < IV  
(3) I < II < IV < III

- (2) II < IV < III < I  
(4) II < I < III < IV

**Sol. 3**  
Order of  $pK_b$



- 14.** Kraft temperature is the temperature :  
 (1) Above which the aqueous solution of detergents starts boiling  
 (2) Below which the formation of micelles takes place.  
 (3) Above which the formation of micelles takes place.  
 (4) Below which the aqueous solution of detergents starts freezing.

**Sol. 3**  
 $T_K$  + temp. above which formation of micelles takes place.

- 15.** The set that contains atomic numbers of only transition elements, is ?  
 (1) 9, 17, 34, 38 (2) 21, 25, 42, 72  
 (3) 37, 42, 50, 64 (4) 21, 32, 53, 64

**Sol. 2**  
 Transition elements = 21 to 30  
                                   37 to 48  
                                   57 & 72 to 80  
 Ans. 21, 25, 42 & 72

- 16.** Consider the Assertion and Reason given below.  
 Assertion (A) : Ethene polymerized in the presence of Ziegler Natta Catalyst at high temperature and pressure is used to make buckets and dustbins.  
 Reason (R) : High density polymers are closely packed and are chemically inert.  
 Choose the correct answer from the following :  
 (1) (A) and (R) both are wrong.  
 (2) Both (A) and (R) are correct and (R) is the correct explanation of (A)  
 (3) (A) is correct but (R) is wrong  
 (4) Both (A) and (R) are correct but (R) is not the correct explanation of (A).

**Sol. 2**  
 From Ziegler - Natta catalyst HDPE is produced, HDPE is closely packed and is chemically inert, so used to make bucket and dustbin.

- 17.** A solution of two components containing  $n_1$  moles of the 1<sup>st</sup> component and  $n_2$  moles of the 2<sup>nd</sup> component is prepared.  $M_1$  and  $M_2$  are the molecular weights of component 1 and 2 respectively. If  $d$  is the density of the solution in  $\text{g mL}^{-1}$ ,  $C_2$  is the molarity and  $x_2$  is the mole fraction of the 2<sup>nd</sup> component, then  $C_2$  can be expressed as :

$$(1) C_2 = \frac{dx_1}{M_2 + x_2(M_2 - M_1)} \quad (2) C_2 = \frac{1000x_2}{M_1 + x_2(M_2 - M_1)}$$

$$(3) C_2 = \frac{dx_2}{M_2 + x_2(M_2 - M_1)} \quad (4) C_2 = \frac{1000dx_2}{M_1 + x_2(M_2 - M_1)}$$

**Sol. 4**

$$C_2 = \frac{x_2}{[x_2M_1 + (1 - x_2)M_2] / d} \times 1000$$

$$C_2 = \frac{1000 dx_2}{M_1 + (M_2 - M_1)x_2}$$

- 18.** The correct statement with respect to dinitrogen is ?

- (1) Liquid dinitrogen is not used in cryosurgery.
- (2)  $N_2$  is paramagnetic in nature
- (3) It can combine with dioxygen at  $25^\circ\text{C}$
- (4) It can be used as an inert diluent for reactive chemicals.

**Sol. 4**

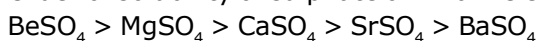
- (1) Liquid nitrogen is used as a refrigerant to preserve biological material food items and in cryosurgery.
- (2)  $N_2$  is diamagnetic, with no unpaired electrons.
- (3)  $N_2$  does not combine with oxygen, hydrogen or most other elements. Nitrogen will combine with oxygen, however ; in the presence of lightning or a spark.
- (4) In iron and chemical Industry inert diluent for reactive chemicals.

- 19.** Among the sulphates of alkaline earth metals, the solubilities of  $\text{BeSO}_4$  and  $\text{MgSO}_4$  in water, respectively, are :

- (1) Poor and high
- (2) High and high
- (3) Poor and poor
- (4) High and poor

**Sol. 2**

Order of solubility of sulphate of Alkaline earth metals



**20.** The presence of soluble fluoride ion upto 1ppm concentration in drinking water, is :

- (1) Harmful to skin (2) Harmful to bones  
(3) Safe for teeth (4) Harmful for teeth

**Sol. 3**

Environmental chemistry - safe for teeth

**21.** A spherical balloon of radius 3cm containing helium gas has a pressure of  $48 \times 10^{-3}$  bar. At the same temperature, the pressure, of a spherical balloon of radius 12cm containing the same amount of gas will be.....  $\times 10^{-6}$  bar.

**Sol. 750**

$$\text{moles} = \frac{48 \times 10^{-3} \times \frac{4}{3\pi} (3\text{cm})^3}{R \times T}$$

$$\text{moles} = \frac{P \times \frac{4}{3\pi} (12\text{cm})^3}{R T}$$

$$P \times 144 \times 12 = 48 \times 9 \times 3 \times 10^{-3}$$

$$P = \frac{27}{36} \times 10^{-3}$$

$$P = \frac{27000}{36} \times 10^{-6}$$

$$P = \frac{3000}{4} \times 10^{-6}$$

$$P = 750 \times 10^{-6} \text{ bar}$$

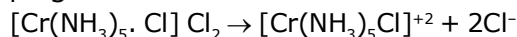
**22.** The elevation of boiling point of 0.10m aqueous  $\text{CrCl}_3 \cdot x\text{NH}_3$  solution is two times that of 0.05 m aqueous  $\text{CaCl}_2$  solution. The value of  $x$  is.....  
[Assume 100% ionisation of the complex and  $\text{CaCl}_2$ , coordination number of Cr as 6, and that all  $\text{NH}_3$  molecules are present inside the coordination sphere]

**Sol. 5**

$$\Delta T_b = i \times K_b \times m$$

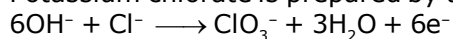
$$i \times 0.1 \times K_b = 3 \times 0.05 \times K_b \times 2$$

$$i = 3$$



$$x = 5$$

**23.** Potassium chlorate is prepared by the electrolysis of KCl in basic solution



If only 60% of the current is utilized in the reaction, the time (rounded to the nearest hour) required to produce 10g of  $\text{KClO}_3$  using a current of 2A is .....

(Given :  $F = 96,500 \text{ C mol}^{-1}$ ; molar mass of  $\text{KClO}_3 = 122 \text{ g mol}^{-1}$ )

**Sol. 11**

$$\frac{10}{122} \times 6 = \frac{2 \times t(\text{hr}) \times 3600 \times 60\%}{96500}$$

$$t(\text{hr}) = \frac{96500}{122 \times 72} = 10.98 \text{ hr}$$

= 11 hours

- 24.** In an estimation of bromine by Carius method, 1.6 g of an organic compound gave 1.88 g of AgBr. The mass percentage of bromine in the compound is ..... (Atomic mass, Ag=108, Br=80 g mol<sup>-1</sup>)

**Sol. 50 %**

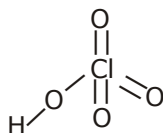
Carius method

$$\% \text{ of Br} = \frac{\text{wt of AgBr}}{\text{wt. of organic compound}} \times 100 \times \frac{\text{molar mass of Br}}{\text{AgBr}}$$

$$= \frac{1.88}{1.6} \times \frac{80}{188} \times 100 = \frac{15040}{300.8} = 50\%$$

- 25.** The number of Cl = O bonds in perchloric acid is, "....."

**Sol. 3**



# QUESTION PAPER WITH SOLUTION

## MATHEMATICS \_ 6 Sep. \_ SHIFT - 1

**Q.1** The region represented by  $\{z = x + iy \in \mathbb{C} : |z| - \operatorname{Re}(z) \leq 1\}$  is also given by the inequality:  
 $\{z = x + iy \in \mathbb{C} : |z| - \operatorname{Re}(z) \leq 1\}$

(1)  $y^2 \leq 2\left(x + \frac{1}{2}\right)$       (2)  $y^2 \leq x + \frac{1}{2}$       (3)  $y^2 \geq 2(x + 1)$       (4)  $y^2 \geq x + 1$

**Sol. 1**

$$\{z = x + iy \in \mathbb{C} : |z| - \operatorname{Re}(z) \leq 1\}$$

$$|z| = \sqrt{x^2 + y^2}$$

$$\operatorname{Re}(z) = x$$

$$|z| - \operatorname{Re}(z) \leq 1$$

$$\Rightarrow \sqrt{x^2 + y^2} - x \leq 1$$

$$\Rightarrow \sqrt{x^2 + y^2} \leq 1 + x$$

$$\Rightarrow x^2 + y^2 \leq 1 + x^2 + 2x$$

$$\Rightarrow y^2 \leq 2\left(x + \frac{1}{2}\right)$$

**Q.2** The negation of the Boolean expression  $p \vee (\sim p \wedge q)$  is equivalent to:

(1)  $p \wedge \sim q$       (2)  $\sim p \vee \sim q$       (3)  $\sim p \vee q$       (4)  $\sim p \wedge \sim q$

**Sol. 4**

$$p \vee (\sim p \wedge q)$$

$$(p \wedge \sim p) \vee (p \wedge q)$$

$$f \wedge (p \vee q)$$

$$p \vee q$$

$$\sim (p \vee (\sim p \wedge q)) = \sim (p \vee q)$$

$$= (\sim p) \wedge (\sim q)$$

**Q.3** The general solution of the differential equation  $\sqrt{1+x^2+y^2+x^2y^2} + xy \frac{dy}{dx} = 0$  is:  
 (where C is a constant of integration)

(1)  $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2} \log_e \left( \frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1} \right) + C$

(2)  $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2} \log_e \left( \frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1} \right) + C$

(3)  $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2} \log_e \left( \frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1} \right) + C$

(4)  $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2} \log_e \left( \frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1} \right) + C$

**Sol. 3**

$$\sqrt{1+x^2+y^2+x^2y^2} + xy \frac{dy}{dx} = 0$$

$$\sqrt{(1+x^2)(1+y^2)} + xy \frac{dy}{dx} = 0$$

$$\frac{\sqrt{(1+x^2)}dx}{x} = -\frac{y}{\sqrt{1+y^2}} dy$$

Integrate the equation

$$\int \frac{\sqrt{1+x^2}}{x} dx = -\int \frac{y}{\sqrt{1+y^2}} dy$$

$$1+x^2 = t^2$$

$$1+y^2 = z^2$$

$$2x dx = 2t dt$$

$$dx = \frac{t}{x} dt$$

$$2y dy = 2z dz$$

$$\int \frac{t \cdot t dt}{t^2 - 1} = -\int \frac{z dz}{z}$$

$$\int \frac{t^2 - 1 + 1}{t^2 - 1} dt = -z + c$$

$$\int 1 dt + \int \frac{1}{t^2 - 1} dt = -z + c$$

$$t + \frac{1}{2} \ln \left( \frac{t-1}{t+1} \right) = -z + c$$

$$\sqrt{1+x^2} + \frac{1}{2} \ln \left( \frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1} \right) = -\sqrt{1+y^2} + c$$

$$\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2} \ln \left( \frac{\sqrt{x^2+1}+1}{\sqrt{x^2+1}-1} \right) + c$$

**Q.4** Let  $L_1$  be a tangent to the parabola  $y^2 = 4(x+1)$  and  $L_2$  be a tangent to the parabola  $y^2 = 8(x+2)$  such that  $L_1$  and  $L_2$  intersect at right angles. Then  $L_1$  and  $L_2$  meet on the straight line:

(1)  $x + 2y = 0$

(2)  $x + 2 = 0$

(3)  $2x + 1 = 0$

(4)  $x + 3 = 0$

**Sol. 4**

Let tangent of  $y^2 = 4(x + 1)$

$$L_1 : t_1 y = (x + 1) + t_1^2 \dots\dots(i)$$

and tangent of  $y^2 = 8(x + 2)$

$$L_2 : t_2 y = (x + 2) + 2 t_2^2$$

$$L_1 \perp L_2$$

$$\frac{1}{t_1} \cdot \frac{1}{t_2} = -1$$

$$t_1 t_2 = -1$$

$$t_2(i) - t_1(ii)$$

$$t_1 t_2 y = t_2 (x + 1) + t_2 \cdot t_1^2$$

$$t_1 t_2 y = t_1 (x + 2) + 2 t_2^2 \cdot t_1$$

$$(t_2 - t_1) x + (t_2 - 2t_1) + t_2 t_1 (t_1 - 2t_2) = 0$$

$$(t_2 - t_1) x + (t_2 - 2t_1) - (t_1 - 2t_2) = 0$$

$$(t_2 - t_1) x + 3t_2 - 3t_1 = 0$$

$$\Rightarrow x + 3 = 0$$

**Q.5** The area (in sq. units) of the region  $A = \{(x, y) : |x| + |y| \leq 1, 2y^2 \geq |x|\}$

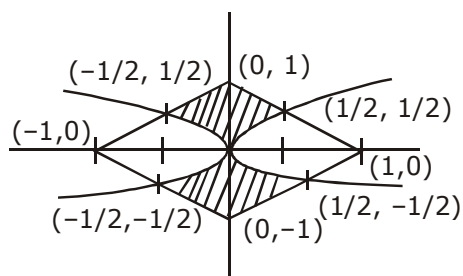
(1)  $\frac{1}{6}$

(2)  $\frac{5}{6}$

(3)  $\frac{1}{3}$

(4)  $\frac{7}{6}$

**Sol. 2**



$$\text{Total area} = 4 \int_0^{1/2} \left[ (1-x) - \left( \sqrt{\frac{x}{2}} \right) \right] dx$$

$$= 4 \left[ x - \frac{x^2}{2} - \frac{1}{\sqrt{2}} \frac{x^{3/2}}{3/2} \right]_{0}^{1/2}$$

$$= 4 \left[ \frac{1}{2} - \frac{1}{8} - \frac{\sqrt{2}}{3} \left( \frac{1}{2} \right)^{3/2} \right]$$

$$= 4 \times \frac{5}{24} = \frac{5}{6}$$

**Q.6** The shortest distance between the lines  $\frac{x-1}{0} = \frac{y+1}{-1} = \frac{z}{1}$  and  $x + y + z + 1 = 0$ ,  $2x - y + z + 3 = 0$  is:

- (1) 1                      (2)  $\frac{1}{\sqrt{2}}$                       (3)  $\frac{1}{\sqrt{3}}$                       (4)  $\frac{1}{2}$

**Sol. 3**

Plane through line of intersection is

$$x + y + z + 1 + \lambda (2x - y + z + 3) = 0$$

It should be parallel to given line

$$0(1 + 2\lambda) - 1(1 - \lambda) + 1(1 + \lambda) = 0 \Rightarrow \lambda = 0$$

$$\text{Plane: } x + y + z + 1 = 0$$

Shortest distance of  $(1, -1, 0)$  from this plane

$$= \frac{|1 - 1 + 0 + 1|}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{3}}$$

**Q.7** Let  $a, b, c, d$  and  $p$  be any non zero distinct real numbers such that  $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) = 0$ . Then:

- (1)  $a, c, p$  are in G.P.                      (2)  $a, b, c, d$  are in G.P.  
(3)  $a, b, c, d$  are in A.P.                      (4)  $a, c, p$  are in A.P.

**Sol. 2**

$$(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) = 0$$

$$(a^2p^2 - 2abp + b^2) + [b^2p^2 - 2bcp + c^2] + [c^2p^2 - 2cdp + d^2]$$

$$(ap - b)^2 + (bp - c)^2 + (cp - d)^2 = 0$$

$$ap = b \quad \frac{b}{a} = \frac{c}{b} = \frac{d}{c} = p$$

$$bp = c$$

$$cp = d$$

$a, b, c, d$  are in G.P.

**Q.8** Two families with three members each and one family with four members are to be seated in a row. In how many ways can they be seated so that the same family members are not separated?

- (1)  $2! 3! 4!$                       (2)  $(3!)^3 \cdot (4!)$                       (3)  $3! (4!)^3$                       (4)  $(3!)^2 \cdot (4!)$

**Sol. 2**

$$F_1 \rightarrow 3 \text{ members}$$

$$F_2 \rightarrow 3 \text{ members}$$

$$F_3 \rightarrow 4 \text{ members}$$

No. of ways can they be seated so that the same family members are not separated

$$= 3! \times 3! \times 3! \times 4! = (3!)^3 \cdot 4!$$

**Q.9** The values of  $\lambda$  and  $\mu$  for which the system of linear equations

$$x + y + z = 2$$

$$x + 2y + 3z = 5$$

$$x + 3y + \lambda z = \mu$$

has infinitely many solutions are, respectively:

(1) 6 and 8

(2) 5 and 8

(3) 5 and 7

(4) 4 and 9

**Sol. 2**

$$x + y + z = 2$$

$$x + 2y + 3z = 5$$

$$x + 3y + \lambda z = \mu$$

has infinitely many solutions

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} = 0$$

$$R_2 \rightarrow R_2 - R_1$$

$$R_3 \rightarrow R_3 - R_1$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & \lambda - 1 \end{vmatrix} = 0$$

$$(\lambda - 1 - 4) = 0$$

$$\Rightarrow \lambda = 5$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 5 \\ 1 & 3 & \mu \end{vmatrix} = 0$$

$$R_2 \rightarrow R_2 - R_1$$

$$R_3 \rightarrow R_3 - R_1$$

$$\begin{vmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 2 & \mu - 2 \end{vmatrix} = 0$$

$$(\mu - 2) - 6 = 0$$

$$\Rightarrow \mu = 8$$

$$\lambda = 5, \mu = 8$$

**Q.10** Let  $m$  and  $M$  be respectively the minimum and maximum values of

$$\begin{vmatrix} \cos^2 x & 1 + \sin^2 x & \sin 2x \\ 1 + \cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1 + \sin 2x \end{vmatrix}$$

Then the ordered pair  $(m, M)$  is equal to:

- (1)  $(-3, -1)$                       (2)  $(-4, -1)$                       (3)  $(1, 3)$                       (4)  $(-3, 3)$

**Sol. 1**

$$\begin{vmatrix} \cos^2 x & 1 + \sin^2 x & \sin 2x \\ 1 + \cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1 + \sin 2x \end{vmatrix}$$

$$R_1 \rightarrow R_1 - R_2, R_3 \rightarrow R_3 - R_2$$

$$\begin{vmatrix} -1 & 1 & 0 \\ 1 + \cos^2 x & \sin^2 x & \sin 2x \\ -1 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow -1(\sin^2 x) - 1(1 + \cos^2 x + \sin 2x)$$

$$\Rightarrow -\sin^2 x - \cos^2 x - 1 - \sin 2x$$

$$= -2 - \sin 2x$$

$$\therefore \text{minimum value when } \sin 2x = 1$$

$$m = -2 - 1 = -3$$

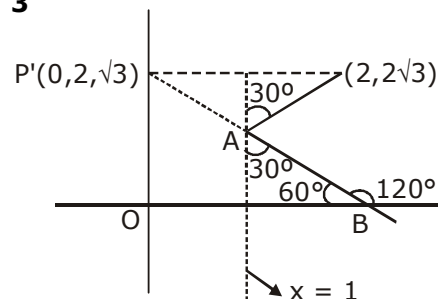
$$\therefore \text{Maximum value when } \sin 2x = -1$$

$$(m, M) = (-3, -1)$$

**Q.11** A ray of light coming from the point  $(2, 2\sqrt{3})$  is incident at an angle  $30^\circ$  on the line  $x = 1$  at the point A. The ray gets reflected on the line  $x = 1$  and meets x-axis at the point B. Then, the line AB passes through the point:

- (1)  $(4, -\sqrt{3})$                       (2)  $\left(3, -\frac{1}{\sqrt{3}}\right)$                       (3)  $(3, -\sqrt{3})$                       (4)  $\left(4, -\frac{\sqrt{3}}{2}\right)$

**Sol. 3**



Equation of P'B  $\rightarrow y - 2\sqrt{3} = \tan 120^\circ (x - 0)$

$$\sqrt{3}x + y = 2\sqrt{3}$$

$(3, -\sqrt{3})$  satisfy the line

**Q.12** Out of 11 consecutive natural numbers if three numbers are selected at random (without repetition), then the probability that they are in A.P. with positive common difference, is:

- (1)  $\frac{10}{99}$                       (2)  $\frac{5}{33}$                       (3)  $\frac{15}{101}$                       (4)  $\frac{5}{101}$

**Sol. 2**

**Case-1**

E, O, E, O, E, O, E, O, E, O, E

$2b = a + c \rightarrow$  Even

$\Rightarrow$  Both a and c should be either even or odd.

$$P = \frac{{}^6C_2 + {}^5C_2}{{}^{11}C_3} = \frac{5}{33}$$

**Case -2**

O, E, O, E, O, E, O, E, O, E, O

$$P = \frac{{}^5C_2 + {}^6C_2}{{}^{11}C_3} = \frac{5}{33}$$

$$\text{Total probability} = \frac{1}{2} \times \frac{5}{33} + \frac{1}{2} \times \frac{5}{33} = \frac{5}{33}$$

**Q.13** If  $f(x + y) = f(x) f(y)$  and  $\sum_{x=1}^{\infty} f(x) = 2$ ,  $x, y \in \mathbb{N}$ , where  $\mathbb{N}$  is the set of all natural number, then the

value of  $\frac{f(4)}{f(2)}$  is :

- (1)  $\frac{2}{3}$                       (2)  $\frac{1}{9}$                       (3)  $\frac{1}{3}$                       (4)  $\frac{4}{9}$

**Sol. 4**

$$f(x + y) = f(x) f(y)$$

\* Put  $x = 1, y = 1$

$$f(2) = (f(1))^2$$

\* Put  $x = 2, y = 1$

$$f(3) = f(2) \cdot f(1) = f(1)^3$$

\* Put  $x = 2, y = 2$

$$f(4) = f(2)^2 = f(1)^4$$

$$f(n) = (f(1))^n$$

$$\sum_{x=1}^{\infty} f(x) = f(1) + f(2) + f(3) + \dots + f(\infty) = 2$$

$$\Rightarrow f(1) + f((1))^2 + f((1))^3 + \dots = 2$$

$$\frac{f(1)}{1 - f(1)} = 2$$

$$f(1) = 2/3$$

$$f(2) = \left(\frac{2}{3}\right)^2, f(4) = \left(\frac{2}{3}\right)^4$$

$$\frac{f(4)}{f(2)} = \frac{(2/3)^4}{(2/3)^2} = \frac{4}{9}$$

**Q.14** If  $\{p\}$  denotes the fractional part of the number  $p$ , then  $\left\{\frac{3^{200}}{8}\right\}$ , is equal to :

(1)  $\frac{5}{8}$

(2)  $\frac{1}{8}$

(3)  $\frac{7}{8}$

(4)  $\frac{3}{8}$

**Sol. 2**

$$\left\{\frac{3^{200}}{8}\right\} = \left\{\frac{9^{100}}{8}\right\} = \left\{\frac{(8+1)^{100}}{8}\right\}$$

$$\left\{\frac{{}^{100}C_0 1^{100} + {}^{100}C_1 (8) 1^{99} + {}^{100}C_2 (8^2) 1^{98} + \dots + {}^{100}C_{100} 8^{100}}{8}\right\}$$

$$= \left\{\frac{{}^{100}C_0 1^{100} + 8k}{8}\right\}$$

$$= \left\{\frac{1+8k}{8}\right\} = \left\{\frac{1}{8} + k\right\} \quad K \in I$$

$$= \frac{1}{8}$$

**Q.15** Which of the following points lies on the locus of the foot of perpendicular drawn upon any tangent to the ellipse,  $\frac{x^2}{4} + \frac{y^2}{2} = 1$  from any of its foci ?

(1)  $(-1, \sqrt{3})$

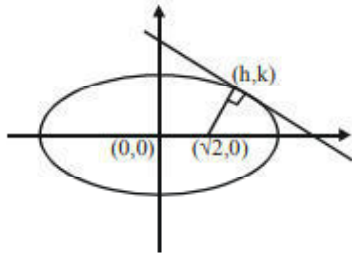
(2)  $(-2, \sqrt{3})$

(3)  $(-1, \sqrt{2})$

(4)  $(1, 2)$

**Sol. 4**

Let foot of perpendicular is  $(h, k)$



$$\frac{x^2}{4} + \frac{y^2}{2} = 1 \quad (\text{Given})$$

$$a = 2, b = \sqrt{2}, e = \sqrt{1 - \frac{b^2}{a^2}} = \frac{1}{\sqrt{2}}$$

$$\therefore \text{Focus } (ae, 0) = (\sqrt{2}, 0)$$

Equation of tangent

$$y = mx + \sqrt{a^2 m^2 + b^2}$$

$$y = mx + \sqrt{4m^2 + 2}$$

Passes through  $(h, k)$   $(k - mh)^2 = 4m^2 + 2$

line perpendicular to tangent will have slope

$$-\frac{1}{m}$$

$$y - 0 = -\frac{1}{m}(x - \sqrt{2})$$

$$my = -x + \sqrt{2}$$

$$(h + mk)^2 = 2$$

Add equation (1) and (2)  $k^2(1 + m^2) + h^2(1 + m^2) = 4(1 + m^2)$

$$h^2 + k^2 = 4$$

$$x^2 + y^2 = 4 \quad (\text{Auxiliary circle})$$

$\therefore (-1, \sqrt{3})$  lies on the locus.

**Q.16**  $\lim_{x \rightarrow 1} \left( \frac{\int_0^{(x-1)^2} t \cos(t^2) dt}{(x-1) \sin(x-1)} \right)$

(1) is equal to 1      (2) is equal to  $\frac{1}{2}$       (3) does not exist      (4) is equal to  $-\frac{1}{2}$

**Sol**      **Bouns**

$$\lim_{x \rightarrow 1} \left( \frac{\int_0^{(x-1)^2} t \cos(t^2) dt}{(x-1) \sin(x-1)} \right)$$

Using L-Hopital rule

$$= \lim_{x \rightarrow 1} \frac{2(x-1) \cdot (x-1)^2 \cos(x-1)^4 - 0}{(x-1) \cdot \cos(x-1) + \sin(x-1)} \left( \frac{0}{0} \right)$$

$$= \lim_{x \rightarrow 1} \frac{2(x-1)^3 \cdot \cos(x-1)^4}{(x-1) \left[ \cos(x-1) + \frac{\sin(x-1)}{(x-1)} \right]}$$

$$= \lim_{x \rightarrow 1} \frac{2(x-1)^2 \cos(x-1)^4}{(x-1) \left[ \cos(x-1) + \frac{\sin(x-1)}{(x-1)} \right]}$$

$$= \lim_{x \rightarrow 1} \frac{2(x-1)^2 \cos(x-1)^4}{\cos(x-1) + \frac{\sin(x-1)}{(x-1)}}$$

on taking limit

$$= \frac{0}{1+1} = 0$$

**Q.17** If  $\sum_{i=1}^n (x_i - a) = n$  and  $\sum_{i=1}^n (x_i - a)^2 = na$ , ( $n, a > 1$ ) then the standard deviation of  $n$  observations  $x_1, x_2, \dots, x_n$  is :

- (1)  $n\sqrt{a-1}$                       (2)  $\sqrt{na-1}$                       (3)  $a-1$                       (4)  $\sqrt{a-1}$

**Sol. 4**

$$\begin{aligned} \text{S.D.} &= \sqrt{\frac{\sum (x_i - a)^2}{n} - \left(\frac{\sum (x_i - a)}{n}\right)^2} \\ &= \sqrt{\left(\frac{na}{n}\right) - \left(\frac{n}{n}\right)^2} = \sqrt{a-1} \end{aligned}$$

**Q.18** If  $\alpha$  and  $\beta$  be two roots of the equation  $x^2 - 64x + 256 = 0$ . Then the value of

$$\left(\frac{\alpha^3}{\beta^5}\right)^{1/8} + \left(\frac{\beta^3}{\alpha^5}\right)^{1/8} \text{ is :}$$

- (1) 1                      (2) 3                      (3) 2                      (4) 4

**Sol. 3**

$$x^2 - 64x + 256 = 0$$

$$\alpha + \beta = 64$$

$$\alpha\beta = 256$$

$$\left(\frac{\alpha^3}{\beta^5}\right)^{1/8} + \left(\frac{\beta^3}{\alpha^5}\right)^{1/8}$$

$$= \frac{\alpha + \beta}{(\alpha\beta)^{5/8}} = \frac{64}{(256)^{5/8}} = \frac{64}{32} = 2$$

**Q.19** The position of a moving car at time  $t$  is given by  $f(t) = at^2 + bt + c$ ,  $t > 0$ , where  $a, b$  and  $c$  are real numbers greater than 1. Then the average speed of the car over the time interval  $[t_1, t_2]$  is attained at the point :

- (1)  $(t_1 + t_2)/2$                       (2)  $2a(t_1 + t_2) + b$                       (3)  $(t_2 - t_1)/2$                       (4)  $a(t_2 - t_1) + b$

**Sol. 1**

$$f'(t) = V_{av} = \frac{f(t_2) - f(t_1)}{t_2 - t_1}$$

$$= \frac{a(t_2^2 - t_1^2) + b(t_2 - t_1)}{t_2 - t_1}$$

$$= a(t_1 + t_2) + b = 2at + b$$

$$t = \frac{t_1 + t_2}{2}$$

**Q.20** If  $I_1 = \int_0^1 (1 - x^{50})^{100} dx$  and  $I_2 = \int_0^1 (1 - x^{50})^{101} dx$  such that  $I_2 = \alpha I_1$  then  $\alpha$  equals to :

- (1)  $\frac{5050}{5049}$                       (2)  $\frac{5050}{5051}$                       (3)  $\frac{5051}{5050}$                       (4)  $\frac{5049}{5050}$

**Sol. 2**

$$I_1 = \int_0^1 (1 - x^{50})^{100} dx$$

$$I_2 = \int_0^1 (1 - x^{50})(1 - x^{50})^{100} dx$$

$$= \int_0^1 (1 - x^{50})^{100} dx - \int_0^1 x^{50}(1 - x^{50})^{100} dx$$

$$I_2 = I_1 - \int_0^1 \underbrace{x^{50}}_I \underbrace{(1 - x^{50})^{100}}_{II} dx$$

By using by parts

$$1 - x^{50} = t$$

$$\Rightarrow x^{49} dx = \frac{-dt}{50}$$

$$I_2 = I_1 - \left[ x \left( \frac{-1}{50} \right) \frac{(1 - x^{50})^{101}}{101} \right]_0^1 + \int_0^1 \left( \frac{-1}{50} \right) \frac{(1 - x^{50})^{101}}{101} dx$$

$$I_2 = I_1 - 0 + \frac{\int_0^1 (1 - x^{50})^{101} dx}{(-5050)}$$

$$I_2 = I_1 - \frac{I_2}{5050}$$

$$\frac{5051}{5050} I_2 = I_1$$

$$I_2 = \frac{5050}{5051} I_1$$

$$\alpha = \frac{5050}{5051}$$

**Q.21** If  $\vec{a}$  and  $\vec{b}$  are unit vectors, then the greatest value of  $\sqrt{3} |\vec{a} + \vec{b}| + |\vec{a} - \vec{b}|$  is\_\_\_\_\_.

**Sol. 4**

$$\sqrt{3} |\vec{a} + \vec{b}| + |\vec{a} - \vec{b}|$$

$$= \sqrt{3}(\sqrt{2 + 2 \cos \theta}) + \sqrt{2 - 2 \cos \theta}$$

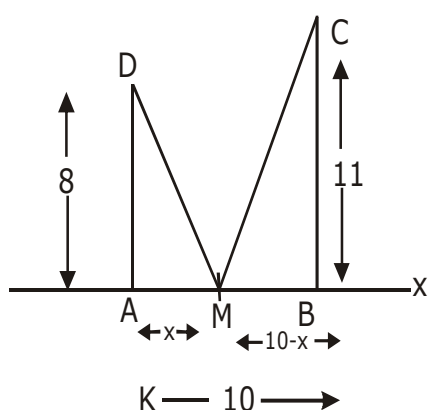
$$= \sqrt{6}(\sqrt{1 + \cos \theta}) + \sqrt{2}(\sqrt{1 - \cos \theta})$$

$$= 2\sqrt{3} \left| \cos \frac{\theta}{2} \right| + 2 \left| \sin \frac{\theta}{2} \right|$$

$$\leq \sqrt{(2\sqrt{3})^2 + (2)^2} = 4$$

**Q.22** Let AD and BC be two vertical poles at A and B respectively on a horizontal ground. If AD = 8 m, BC = 11 m and AB = 10 m; then the distance (in meters) of a point M on AB from the point A such that MD<sup>2</sup> + MC<sup>2</sup> is minimum is \_\_\_\_\_.

**Sol.** 5



$$(MD)^2 = x^2 + 8^2 = x^2 + 64$$

$$(MC)^2 = (10-x)^2 + (11)^2 = (x-10)^2 + 121$$

$$f(x) = (MD)^2 + (MC)^2 = x^2 + 64 + (x-10)^2 + 121$$

Differentiate

$$f'(x) = 0$$

$$2x + 2(x-10) = 0$$

$$4x = 20 \Rightarrow x = 5$$

$$f''(x) = 4 > 0$$

at x = 5 point of minima

**Q.23** Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be defined as

$$f(x) = \begin{cases} x^5 \sin\left(\frac{1}{x}\right) + 5x^2, & x < 0 \\ 0, & x = 0 \\ x^5 \cos\left(\frac{1}{x}\right) + \lambda x^2, & x > 0 \end{cases}$$

The value of  $\lambda$  for which  $f''(0)$  exists, is \_\_\_\_\_.

**Sol. 5**

$$f(x) = \begin{cases} x^5 \sin\left(\frac{1}{x}\right) + 5x^2, & x < 0 \\ 0, & x = 0 \\ x^5 \cos\left(\frac{1}{x}\right) + \lambda x^2, & x > 0 \end{cases}$$

$$f'(x) = \begin{cases} 5x^4 \sin\left(\frac{1}{x}\right) - x^3 \cos\left(\frac{1}{x}\right) + 10x, & x < 0 \\ 0, & x = 0 \\ 5x^4 \cos\left(\frac{1}{x}\right) + x^3 \sin\left(\frac{1}{x}\right) + 2\lambda x, & x > 0 \end{cases}$$

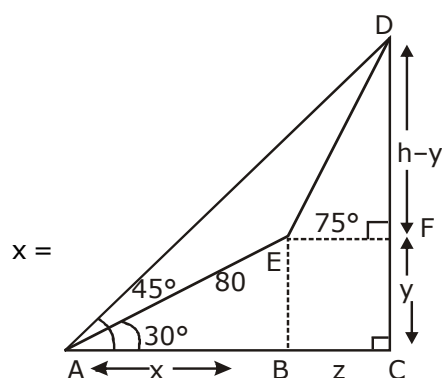
$$f''(x) = \begin{cases} 20x^3 \sin\left(\frac{1}{x}\right) - 5x^2 \cos\left(\frac{1}{x}\right) - 3x^2 \cos\left(\frac{1}{x}\right) - x \sin\left(\frac{1}{x}\right) + 10, & x < 0 \\ 0, & x = 0 \\ 20x^3 \cos\left(\frac{1}{x}\right) + 5x^2 \sin\left(\frac{1}{x}\right) + 3x^2 \sin\left(\frac{1}{x}\right) - x \cos\left(\frac{1}{x}\right) + 2\lambda, & x > 0 \end{cases}$$

$$f''(0^+) = f''(0^-)$$

$$2\lambda = 10 \Rightarrow \lambda = 5$$

**Q.24** The angle of elevation of the top of a hill from a point on the horizontal plane passing through the foot of the hill is found to be  $45^\circ$ . After walking a distance of 80 meters towards the top, up a slope inclined at an angle of  $30^\circ$  to the horizontal plane, the angle of elevation of the top of the hill becomes  $75^\circ$ . Then the height of the hill (in meters) is \_\_\_\_\_.

**Sol. 80**



$$x = 80 \cos 30^\circ = 40\sqrt{3}$$

$$y = 80 \sin 30^\circ = 40$$

In  $\triangle ADC$

$$\tan 45^\circ = \frac{h}{x+z} \Rightarrow h = x + z$$

$$\Rightarrow h = 40\sqrt{3} + z \dots (i)$$

In  $\triangle EDF$

$$\tan 75^\circ = \frac{h-y}{z}$$

$$2 + \sqrt{3} = \frac{h-40}{z} \Rightarrow z = \frac{h-40}{2+\sqrt{3}} \dots (ii)$$

Put the value of  $z$  from (i)

$$h - 40\sqrt{3} = \frac{h-40}{2+\sqrt{3}}$$

$$h(1 + \sqrt{3}) = 40(2\sqrt{3} + 3 - 1)$$

$$h(1 + \sqrt{3}) = 80(1 + \sqrt{3})$$

$$h = 80$$

**Q.25** Set A has  $m$  elements and set B has  $n$  elements. If the total number of subsets of A is 112 more than the total number of subsets of B, then the value of  $m.n$  is \_\_\_\_\_.

**Sol. 28**

A & B are set

No. of subset of A =  $2^m$

No. of subset of B =  $2^n$

$$2^m = 2^n + 112$$

$$2^m - 2^n = 112$$

$$2^n(2^{m-n}-1) = 112$$

$$2^n(2^{m-n}-1) = 2^4(2^3-1)$$

$$n = 4 \quad m - n = 3$$

$$m - 4 = 3 \Rightarrow m = 7$$

$$m.n = 28$$