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JEE Main 2023 April Question Paper with Answer

6th, 8th, 10th, 11th 12th, 13th & 15th April (Shift 1 & Shift 2)

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FINAL JEE–MAIN EXAMINATION – APRIL, 2023

(Held On Thursday 06th April, 2023) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Let $5f(x)+4f\left(\frac{1}{x}\right)=\frac{1}{x}+3, x>0$. Then $18\int_1^2 f(x)dx$ is equal to:

(1) $10\log_e 2-6$

(2) $10\log_e 2+6$

(3) $5\log_e 2+3$

(4) $5\log_e 2-3$

Official Ans. by NTA (1)

Sol. $5f(x)+4f\left(\frac{1}{x}\right)=\frac{1}{x}+3$ .....(1)

replace $x\to\frac{1}{x}$

$5f\left(\frac{1}{x}\right)+4f(x)=x+3$ .....(2)

Eq. (1) × 5 – eq. (2) × 4

$f(x)=\frac{1}{9}\left(\frac{5}{x}-4x+3\right)$

$I=18\int_1^2\frac{1}{9}\left(\frac{5}{x}-4x+3\right)dx = 10\log_e 2-6$

2. A pair of dice is thrown 5 times. For each throw, a total of 5 is considered a success. If the probability of at least 4 successes is $\frac{k}{3^{11}}$, then k is equal to

(1) 82

(2) 123

(3) 164

(4) 75

Official Ans. by NTA (2)

Sol. Probability of success = $\frac{1}{9}=p$

Probability of failure $q=\frac{8}{9}$

P(at least 4 success) = P (4 success) + P (5 success)

$={}^5C_4\,p^4q+{}^5C_5\,p^5=\frac{41}{3^{10}}=\frac{123}{3^{11}}$

k = 123

3. If ${}^{2n}C_3:{}^nC_3=10:1$, then the ratio $(n^2+3n):(n^2-3n+4)$ is

(1) 35: 16

(2) 65:37

(3) 27:11

(4) 2:1

Official Ans. by NTA (4)

Sol. $\frac{{}^{2n}C_3}{{}^nC_3}=10\Rightarrow\frac{2n(2n-1)(2n-2)}{n(n-1)(n-2)}=10$

n = 8

So $(n^2+3n):(n^2-3n+4)=2$

4. If the ratio of the fifth term from the begining to the fifth term from the end in the expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6}:1$, then the third term from the beginning is:

(1) $60\sqrt{2}$

(2) $60\sqrt{3}$

(3) $30\sqrt{2}$

(4) $30\sqrt{3}$

Official Ans. by NTA (2)

Sol. $\frac{{}^nC_4\,2^{\frac{n-4}{4}}\cdot\left(3^{\frac{-1}{4}}\right)^4}{{}^nC_4\,3^{-\left(\frac{n-4}{4}\right)}\cdot\left(2^{\frac{1}{4}}\right)^4}=\frac{\sqrt{6}}{1}$

$\Rightarrow$ n = 10

So $T_3={}^{10}C_2\;2^{\frac{1}{4}\cdot 8}.3^{-\frac{1}{4}\cdot 2}=\frac{45.4}{\sqrt{3}}=60\sqrt{3}$

5. Let $\vec{a}=2\hat{i}+3\hat{j}+4\hat{k}$, $\vec{b}=2\hat{i}-2\hat{j}-2\hat{k}$ and $\vec{c}=-\hat{i}+4\hat{j}+3\hat{k}$. If $\vec{d}$ is a vector perpendicular to both $\vec{b}$ and $\vec{c}$ and $\vec{a}.\vec{d}=18$, Then $\left|\vec{a}\times\vec{d}\right|^2$ is equal to

(1) 640

(2) 760

(3) 680

(4) 720

Official Ans. by NTA (4)

Sol. $a = \lambda\left(b \times c\right)$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -2 \\ -1 & 4 & 3 \end{vmatrix} = 2\hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{d} = \lambda\left(2\hat{i} - \hat{j} + 2\hat{k}\right)$$

$$\vec{a}.\vec{d} = 18$$

$$\lambda = 2$$

So $\vec{d} = 2\left(2\hat{i} - \hat{j} + 2\hat{k}\right)$

$$\vec{d} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & 4 \\ 2 & 3 & 4 \end{vmatrix} = -20\hat{i} - 8\hat{j} + 16\hat{k}$$

$$\left|\vec{d} \times \vec{a}\right|^2 = 720$$

6. The straight lines l_1 and l_2 pass through the origin and trisect the line segment of the line L: 9x + 5y = 45 between the axes. If m_1 and m_2 are the slopes of the lines l_1 and l_2,then the point of intersection of the line $y = (m_1 + m_2)x$ with L lies on

(1) 6x + y = 10

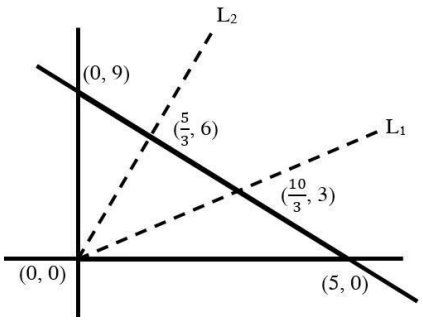
(2) 6x– y = 15

(3) y –x = 5

(4) y –2x = 5

Official Ans. by NTA (3)

Sol.



$$m_{L_1} = \frac{3.3}{10} = \frac{9}{10}$$

$$m_{L_2} = \frac{6.3}{5} = \frac{18}{5}$$

$y = (m_1 + m_2)x$

$$y = \frac{9}{2}x$$

Point of intersection with L is $\left(\frac{10}{7}, \frac{45}{7}\right)$

7. From the top A of a vertical wall AB of height 30 m, the angles of depression of the top P and bottom Q of a vertical tower PQ are 15° and 60° respectively. B and Q are on the same horizontal level. If C is a point on AB such that CB = PQ, then the area (in m^2) of the quadrilateral BCPQ is equal to

(1) $600\left(\sqrt{3} - 1\right)$

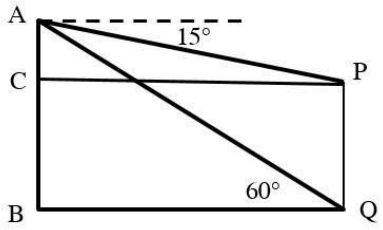
(2) $300\left(\sqrt{3} + 1\right)$

(3) $200\left(3 - \sqrt{3}\right)$

(4) $300\left(\sqrt{3} - 1\right)$

Official Ans. by NTA (1)

Sol.



$$\tan 60° = \sqrt{3} = \frac{30}{BQ}$$

$BQ = 10\sqrt{3}m = CP$

$$\tan 15° = 2 - \sqrt{3} = \frac{AC}{CP}$$

$$AC = 10\sqrt{3}\left(2 - \sqrt{3}\right)$$

Area = $10\sqrt{3}\left(60 - 20\sqrt{3}\right) = 600\left(\sqrt{3} - 1\right)$

8. The sum of the first 20 terms of the series 5 +11 + 19+ 29 + 41 + ... is

(1) 3450

(2) 3250

(3) 3420

(4) 3520

Official Ans. by NTA (4)

Sol. $S_{20} = 5 + 11 + 19 + 29 + \ldots\ldots$

Let $T_r = ar^2 + br + c$

$T_1 = a + b + c = 5$

$T_2 = 4a + 2b + c = 11$

$T_3 = 9a + 3b + c = 19$

$a = 1, b = 3, c = 1$

Hence $S_{20} = \sum_{r=1}^{20} r^2 + 3\sum_{r=1}^{20} r + \sum_{r=1}^{20} 1 = 3520$

9. The mean and variance of a set of 15 numbers are 12 and 14 respectively. The mean and variance of another set of 15 numbers are 14 and σ^2 respectively. If the variance of all the 30 numbers in the two sets is 13, then σ^2 is equal to

(1) 9

(2) 12

(3) 11

(4) 10

Official Ans. by NTA (4)

Sol. **Combine var.** = $\frac{n_1\sigma^2 + n_2\sigma^2}{n_1 + n_2} + \frac{n_1 n_2 (m_1 - m_2)^2}{(n_1 + n_2)^2}$

$13 = \frac{15.14 + 15.\sigma^2}{30} + \frac{15.15(12-14)^2}{30 \times 30}$

$13 = \frac{14 + \sigma^2}{2} + \frac{4}{4}$

$\sigma^2 = 10$

10. Let $A = [a_{ij}]_{2\times 2}$ where $a_{ij} \neq 0$ for all i, j and $A^2 = I$. Let a be the sum of all diagonal elements of A and $b = |A|$, then $3a^2 + 4b^2$ is equal to

(1) 7

(2) 14

(3) 3

(4) 4

Official Ans. by NTA (4)

Sol. Let $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$

$A^2 = \begin{bmatrix} p^2 + qr & pq + qs \\ pr + rs & qs + s^2 \end{bmatrix}$

$\Rightarrow p^2 + qr = 1$ (1) $pq + qs = 0 \Rightarrow q(p+s) = 0$ (3)

$\Rightarrow s^2 + qr = 1$ (2) $pr + rs = 0 \Rightarrow r(p+s) = 0$ (4)

Equation (1) – equation (2)

$p^2 = s^2 \Rightarrow p + s = 0$

Now $3a^2 + 4b^2$

$= 3(p+s)^2 + 4(ps - qr)^2$

$= 3.0 + 4\left(-p^2 - qr\right)^2 = 4\left(p^2 + qr\right)^2 = 4$

11. Let $I(x) = \int \frac{x^2 \; x\sec^2 x + \tan x}{(x\tan x + 1)^2} dx$. If I(0) = 0 the I $\left(\frac{\pi}{4}\right)$ is equal to

(1) $\log_e \frac{(x+4)^2}{16} - \frac{\pi^2}{4(\pi+4)}$

(2) $\log_e \frac{(x+4)^2}{16} + \frac{\pi^2}{4(\pi+4)}$

(3) $\log_e \frac{(x+4)^2}{32} - \frac{\pi^2}{4(\pi+4)}$

(4) $\log_e \frac{(x+4)^2}{32} + \frac{\pi^2}{4(\pi+4)}$

Official Ans. by NTA (3)

Sol. $I(x) = \int \frac{x^2 (x\sec^2 x + \tan x)}{(x\tan x + 1)^2} dx$

Let xtan x + 1= t

$I = x^2\left(\frac{-1}{x\tan x + 1}\right) + \int \frac{2x}{x\tan x + 1} dx$

$I = x^2\left(\frac{-1}{x\tan x + 1}\right) + 2\int \frac{x\cos x}{x\sin x + \cos x} dx$

$I = x^2\left(\frac{-1}{x\tan x + 1}\right) + 2\ln|x\sin x + \cos x| + C$

As I(0) = 0 $\Rightarrow$ C = 0

$I\left(\frac{\pi}{4}\right) = \ln\left(\frac{(\pi+4)^2}{32}\right) - \frac{\pi^2}{4(\pi+4)}$

12. If the equation of the plane passing through the line of intersection of the planes 2x – y + z = 3, 4x– 3y +5z + 9 = 0 and parallel to the line $\frac{x+1}{-2} = \frac{y+3}{4} = \frac{z-2}{5}$ is ax + by + cz + 6 = 0. then a + b + c is equal to

(1) 14

(2) 12

(3) 13

(4) 15

Official Ans. by NTA (1)

Sol. **Equation of family of plane**

$(2x-y+z-3)+\lambda(4x-3y+5z+9)=0$

$x(2+4\lambda)-y(1+3\lambda)+z(1+5\lambda)-3+9\lambda=0$

Parallel to the line

$-2(2+4\lambda)-(1+3\lambda)4+(1+5\lambda)5=0$

$5\lambda=3$

$\lambda=\frac{3}{5}$

equation of plane

$11x-7y+10z+6=0$

a + b + c = 14

13. Statement $(P\Rightarrow Q)\wedge(R\Rightarrow Q)$ is logically equivalent to

(1) $(P\vee R)\Rightarrow Q$

(2) $(P\Rightarrow R)\wedge(Q\Rightarrow R)$

(3) $(P\Rightarrow R)\vee(Q\Rightarrow R)$

(4) $(P\wedge R)\Rightarrow Q$

Official Ans. by NTA (1)

Sol. $(P\Rightarrow Q)\wedge(R\Rightarrow Q)$

We known that $P\Rightarrow Q\equiv\sim P\vee Q$

$\Rightarrow(\sim P\vee Q)\wedge(\sim R\vee Q)$

$\Rightarrow(\sim P\wedge\sim R)\vee Q$

$\Rightarrow\sim(P\vee R)\vee Q$

$\Rightarrow(P\vee R)\Rightarrow Q$

14. The sum of all the roots of the equation $|x^2-8x+15|-2x+7=0$ is:

(1) $9+\sqrt{3}$

(2) $11+\sqrt{3}$

(3) $9-\sqrt{3}$

(4) $11-\sqrt{3}$

Official Ans. by NTA (1)

Sol. **For** $x\le3$ or $x\ge5$

$x^2-8x+15-2x+7=0$

$x=5+\sqrt{3}$

For $3<x<5$, $x^2-8x+15+2x-7=0$

x = 4

Hence sum $=9+\sqrt{3}$

15. Let $a_1, a_2, a_3, \ldots a_n$ be n positive consecutive terms of an arithmetic progression. If d > 0 is its common difference, then

$$\lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\ldots\ldots+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right)$$

(1) 1

(2) $\sqrt{d}$

(3) $\frac{1}{\sqrt{d}}$

(4) 0

Official Ans. by NTA (1)

Sol. $\lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\ldots\ldots+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right)$

On rationalising each term

$$\lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{\sqrt{a_n}-\sqrt{a_1}}{d}\right)$$

$$\lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{(n-1)d}{\left(\sqrt{a_n}+\sqrt{a_1}\right)d}\right)=1$$

16. If the system of equations

x + y + az = b

2x + 5y + 2z = 6

x + 2y + 3z = 3

has infinitely many solutions, then 2a + 3b is equal to

(1) 23

(2) 28

(3) 25

(4) 20

Official Ans. by NTA (1)

Sol. $\Delta=\begin{vmatrix}1&1&a\\2&5&2\\1&2&3\end{vmatrix}=0\Rightarrow11-4-a=0$

a = 7

$\Delta_1=\begin{vmatrix}b&1&a\\6&5&2\\3&2&3\end{vmatrix}=0\Rightarrow11b-12-21=0$

b = 3

2a + 3b = 23

17. If $2x^y + 3y^x = 20$, then $\frac{dy}{dx}$ at (2, 2) is equal to

(1) $-\left(\frac{3+\log_e 8}{2+\log_e 4}\right)$ (2) $-\left(\frac{2+\log_e 8}{3+\log_e 4}\right)$

(3) $-\left(\frac{3+\log_e 16}{4+\log_e 8}\right)$ (4) $-\left(\frac{3+\log_e 4}{2+\log_e 8}\right)$

Official Ans. by NTA (2)

Sol. $2x^y + 3y^x = 20$

$$2x^y\left[\frac{y}{x}+(\ln x)\,y'\right]+3y^x\left[\frac{xy'}{y}+\ln y\right]=0$$

$$y'=\frac{-(12\ln 2+8)}{12+8\ln 2}=-\left(\frac{2+\log_e 8}{3+\log_e 4}\right)$$

18. One vertex of a rectangular parallelopiped is at the origin O and the lengths of its edges along x, y and z axes are 3, 4 and 5 units respectively. Let P be the vertex (3, 4, 5). Then the shortest distance between the diagonal OP and an edge parallel to z axis, not passing through O or P is:

(1) $\frac{12}{\sqrt{5}}$ (2) $\frac{12}{5\sqrt{5}}$

(3) $12\sqrt{5}$ (4) $\frac{12}{5}$

Official Ans. by NTA (4)

Sol Equation of OP is $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$

$a_1 = (0, 0, 0)$ $a_2 = (3, 0, 5)$

$b_1 = (3, 4, 5)$ $b_2 = (0, 0, 1)$

Equation of edge parallel to z axis

$$\frac{x-3}{0}=\frac{y-0}{0}=\frac{z-5}{1}$$

$$\text{S.D} = \frac{\left(\vec{a}_2-\vec{a}_1\right).\left(\vec{b}_1\times\vec{b}_2\right)}{\left|\vec{b}_1\times\vec{b}_2\right|}$$

$$\frac{\begin{vmatrix}3&0&5\\3&4&5\\0&0&1\end{vmatrix}}{\left\|\begin{matrix}\hat{i}&\hat{j}&\hat{k}\\3&4&5\\0&0&1\end{matrix}\right\|}=\frac{3(4)}{\left|4\hat{i}-3\hat{j}\right|}=\frac{12}{5}$$

19. Let the position vectors of the points A, B, C and D be $5\hat{i}+5\hat{j}+2\lambda\hat{k}$, $\hat{i}+2\hat{j}+3\hat{k}$, $-2\hat{i}+\lambda\hat{j}+4\hat{k}$ and $-\hat{i}+5\hat{j}+6\hat{k}$. Let the set $S=\{\lambda\in\mathbb{R}:$ The points A, B, C and D are coplanar$\}$. Then $\sum_{\lambda\in S}(\lambda+2)^2$ is equal to

(1) 41 (2) 25

(3) 13 (4) $\frac{37}{2}$

Official Ans. by NTA (1)

Sol. Since A, B, C, D are coplanner

Hence $\left[\overrightarrow{BA}\quad\overrightarrow{CA}\quad\overrightarrow{DA}\right]=0$

$$\begin{vmatrix}4&3&2\lambda-3\\7&5-\lambda&2\lambda-4\\6&0&2\lambda-6\end{vmatrix}=0$$

$\lambda = 2, 3$ Hence $\sum_{\lambda\in S}(\lambda+2)^2 = 41$

20. Let $A=\{x\in\mathbb{R}:[x+3]+[x+4]\le 3\}$,

$B=\left\{x\in\mathbb{R}:3^x\left(\sum_{r=1}^{\infty}\frac{3}{10^r}\right)^{x-3}<3^{-3x}\right\}$, where [t] denotes greatest integer function. Then,

(1) $A\cap B=\phi$

(2) A = B

(3) $B\subset C, A\neq B$

(4) $A\subset B, A\neq B$

Official Ans. by NTA (2)

Sol. $[x]+3+[x]+4\le 3$

$2[x]\le -4$

$[x]\le -2 \Rightarrow x\in(-\infty,-1)$ ……..(A)

$$3^x\left(\frac{3.\frac{1}{10}}{1-\frac{1}{10}}\right)^{x-3}<3^{-3x}$$

$27 < 3^{-3x}$

$-3x > +3$

$x < -1$ ……..(B)

A = B

SECTION-B

21. Let $a \in \mathbb{Z}$ and [t] be the greatest integer $\leq$ t. Then the number of points, where the function f(x) = [a + 13 sin x], $x \in (0, \pi)$ is not differentiable, is ____

Official Ans. by NTA (25)

Sol. f(x) = [a + 13 sin x], $x \in (0, \pi)$

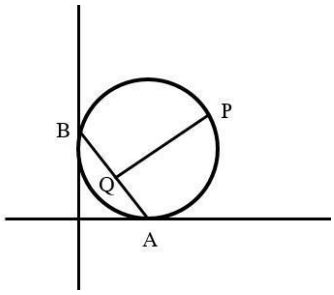
For [n sin x]; Total number of non differentiable points are = 2n –1 for $x \in (0, \pi)$

So number of non differentiable points for [13 sin x] $\Rightarrow$ 25 Points

22. A circle passing through the point $P(\alpha, \beta)$ in the first quadrant touches the two coordinate axes at the points A and B. The point P is above the line AB. The point Q on the line segment AB is the foot of perpendicular from P on AB. If PQ is equal to 11 units, then the value of $\alpha\beta$ is_____

Official Ans. by NTA (121)

Sol.



Let equation of circle is $(x-a)^2 + (y-a)^2 = a^2$

which is passing through P (α,β)

then $(\alpha-a)^2 + (\beta-a)^2 = a^2$

$\alpha^2 + \beta^2 - 2\alpha a - 2\beta a + a^2 = 0$

Here equation of AB is x + y = a

Let Q (α',β') be foot of perpendicular of P on AB

$$\frac{\alpha'-\alpha}{1} = \frac{\beta'-\beta}{1} = \frac{-(\alpha+\beta-a)}{2}$$

$PQ^2 = (\alpha'-\alpha)^2 + (\beta'-\beta) = \frac{1}{4}(\alpha+\beta-a)^2 + \frac{1}{4}(\alpha+\beta-\ )$

$$121 = \frac{1}{2}(\alpha+\beta-a)^2$$

$$242 = \alpha^2 + \beta^2 - 2\alpha a - 2\beta a + a^2 + 2\alpha\beta$$

$242 = 2\alpha\beta$

$\Rightarrow \alpha\beta = 121$

23. The number of ways of giving 20 distinct oranges to 3 children such that each child gets atleast one orange is_____

Official Ans. by NTA (171)

Sol. 20 distinct oranges distributed among 3 children so that each child gets at least one orange

$= 3^{20} - {}^3C_1\, 2^{20} + {}^3C_2\, 1^{20}$

Bonus

24. If the area of the region

$S = \{(x,y) : 2y - y^2 \leq x^2 \leq 2y, x \geq y\}$ is equal to

$\frac{n+2}{n+1} - \frac{\pi}{n-1}$, then the natural number n is equal to _____

Official Ans. by NTA (5)

Sol. $x^2 + y^2 - 2y \geq 0$ & $x^2 - 2y \leq 0$, $x \geq y$

Hence required area = $\frac{1}{2} \times 2 \times 2 - \int_0^2 \frac{x^2}{2} dx - \left(\frac{\pi}{4} - \frac{1}{2}\right)$

$= \frac{7}{6} - \frac{\pi}{4} \Rightarrow n = 5$

25. Let the point (p, p + 1) lie inside the region

$E = \left\{(x,y) : 3 - x \leq y \leq \sqrt{9-x^2}, 0 \leq x \leq 3\right\}$ If the set of all values of p is the interval (a, b). then $b^2 + b - a^2$ is equal to _____

Official Ans. by NTA (3)

Sol. $3-x \le y \le \sqrt{9-x^2}$

Points (p, p + 1) lies on y = x + 1

So point of intersection between

y = x + 1 & y = 3 – x is x = 1, y = 2

and point of intersection between

$x+1=\sqrt{9-x^2}$ is $x=\frac{-1+\sqrt{17}}{2}$

Hence $p \in \left(1, \frac{-1+\sqrt{17}}{2}\right)$

Hence $b^2 + b - a^2 = 3$

26. Let y = y(x) be a solution of the differential equation $(x\cos x)dy + (xy\sin x + y\cos x - 1)dx = 0$, $0 < x < \frac{\pi}{2}$. If $\frac{\pi}{3}y\left(\frac{\pi}{3}\right)=\sqrt{3}$, then

$\left|\frac{\pi}{6}y''\left(\frac{\pi}{6}\right)+2y'\left(\frac{\pi}{6}\right)\right|$ is equal to _____

Official Ans. by NTA (2)

Sol. $(x\cos x)dy + (xy\sin x + y\cos x - 1)dx = 0,\ 0 < x < \frac{\pi}{2}$

$\frac{dy}{dx}+\left(\frac{x\sin x+\cos x}{x\cos x}\right)y=\frac{1}{x\cos x}$

IF = x secx

$y.x\sec x=\int\frac{x\sec x}{x\cos x}dx=\tan x+c$

Since $y\left(\frac{\pi}{3}\right)=\frac{3\sqrt{3}}{\pi}$

Hence $c=\sqrt{3}$

Hence $\left|\frac{\pi}{6}y''\left(\frac{\pi}{6}\right)+y'\left(\frac{\pi}{6}\right)\right|=|-2|=2$

27. The coefficient of x^{18} in the expansion of $\left(x^4-\frac{1}{x^3}\right)^{15}$ is _____

Official Ans. by NTA (5005)

Sol. $\left(x^4-\frac{1}{x^3}\right)^{15}$

$T_{r+1}={}^{15}C_r\left(x^4\right)^{15-r}\left(\frac{-1}{x^3}\right)^r$

60 –7r = 18

r = 6

Hence coeff. of $x^{18} = {}^{15}C_6 = 5005$

28. Let A={1, 2, 3, 4,.....10} and B = {0, 1, 2, 3, 4}. The number of elements in the relation R = {(a, b) ∈ A × A: $2(a-b)^2 + 3(a-b) \in B$} is_____

Official Ans. by NTA (18)

Sol. $A=\{1,2,3,.....10\}$

$B=\{0,1,2,3,4\}$

R = {(a, b) ∈ A × A: $2(a-b)^2 + 3(a-b) \in B$}

Now $2(a-b)^2+3(a-b)=(a-b)(2(a-b)+3)$

⇒ a = b or a– b = –2

When a = b ⇒ 10 order pairs

When a– b = –2 ⇒8 order pairs

Total = 18

29. Let the image of the point P(l, 2, 3) in the plane 2x – y + z = 9 be Q. If the coordinates of the point R are (6, 10, 7), then the square of the area of the triangle PQR is_

Official Ans. by NTA (594)

Sol. Let Q (α,β,γ) be the image of P, about the plane 2x – y + z = 9

$\frac{\alpha-1}{2}=\frac{\beta-2}{-1}=\frac{\gamma-3}{1}=2$

$\Rightarrow \alpha=5,\ \beta=0,\ \gamma=5$

Then area of triangle PQR is $=\frac{1}{2}\left|\overrightarrow{PQ}\times\overrightarrow{PR}\right|$

$=\left|-12\hat{i}-3\hat{j}+21\hat{k}\right| = \sqrt{144+9+441}=\sqrt{594}$

Square of area = 594

30. Let the tangent to the curve $x^2 + 2x - 4y + 9 = 0$ at the point P(1, 3) on it meet the y-axis at A. Let the line passing through P and parallel to the line $x - 3y = 6$ meet the parabola $y^2 = 4x$ at B. If B lies on the line $2x - 3y = 8$. then $(AB)^2$ is equal to

Official Ans. by NTA (292)

Sol. Equation of tangent at P (1, 3) to the curve $x^2 + 2x - 4y + 9 = 0$ is $y - x = 2$

Then the point A is (0, 2)

Equation of line passing through P and parallel to the line $x - 3y = 6$.

The possible coordinate of B are (4, 4) or (16, 8)

But (4, 4) does not satisfy $2x - 3y = 8$

Thus the point B is (16, 8)

Then $(AB)^2 = 292$

SECTION-A

31. For the plane electromagnetic wave given by $E = E_0 \sin(\omega t - kx)$ and $B = B_0 \sin(\omega t - kx)$, the ratio of average electric energy density to average magnetic energy density is

(1) 1 (2) 1/2

(3) 2 (4) 4

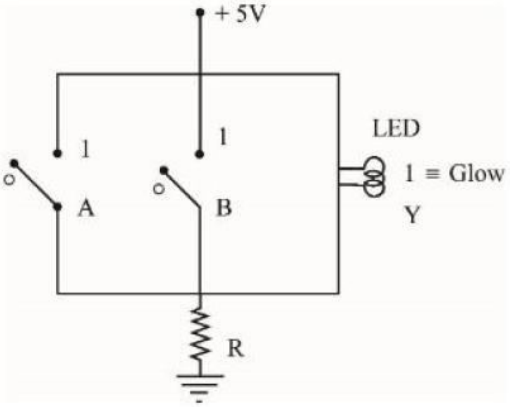
Official Ans. by NTA (1)

Sol. $$\frac{\text{Electric energy density}}{\text{Magnetic energy density}} = \frac{\frac{1}{2}\epsilon_0 E_{rms}^2}{\left(\frac{B_{rms}^2}{2\mu_0}\right)}$$

$$= \left(\frac{E_{rms}}{B_{rms}}\right)^2 .\mu_0 \epsilon_0 \qquad \left[C = \frac{1}{\mu_0 \epsilon_0}\right]$$

$$= \frac{C^2}{C^2} = 1$$

32. Name the logic gate equivalent to the diagram attached



(1) OR (2) NOR

(3) NAND (4) AND

Official Ans. by NTA (2)

Sol. Circuit is closed when neither A nor B is closed ⇒ current flows for A = 0 B = 0 when either or both of A & B is closed we get current bypass from switch

Hence it is "NOR" gate

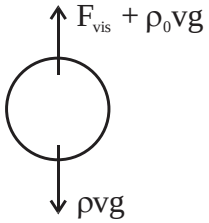
33. A small ball of mass M and density ρ is dropped in a viscous liquid of density ρ_0. After some time, the ball falls with a constant velocity. What is the viscous force on the ball ?

(1) $F = Mg\left(1 - \frac{\rho_0}{\rho}\right)$ (2) $F = Mg\left(1 + \frac{\rho}{\rho_0}\right)$

(3) $F = Mg\left(1 + \frac{\rho_0}{\rho}\right)$ (4) $F = Mg(1 \pm \rho\rho_0)$

Official Ans. by NTA (1)

Sol.



For constant velocity $F_{net} = 0$

$F_{vis} + \rho_0 vg = \rho vg$

$F_{vis} = (\rho - \rho_0)\, vg$

$$= \rho vg\left(1 - \frac{\rho_0}{\rho}\right)$$

$$= Mg\left(1 - \frac{\rho_0}{\rho}\right)$$

34. The number of air molecules per cm^3 increased from 3×10^{19} to 12×10^{19}. The ratio of collision frequency of air molecules before and after the increase in number respectively is

(1) 1.25 (2) 0.25

(3) 0.75 (4) 0.50

Official Ans. by NTA (2)

Sol. Collision frequency,

$$f = \frac{V}{\lambda} = \frac{V}{\left(\frac{1}{\sqrt{2}\pi d^2 n_v}\right)} = \sqrt{2}\pi d^2 v n_v$$

$\therefore$ $f \propto n_v$, n_v is number density

$$\frac{f_1}{f_2} = \frac{n_{v_1}}{n_{v_2}} = \frac{3\times10^{19}}{12\times10^{-19}} = 0.25$$

35. A source supplies heat to a system at the rate of 1000 W. If the system performs work at a rate of 200 W. The rate at which internal energy of the system increases

(1) 1200 W (2) 600 W

(3) 500 W (4) 800 W

Official Ans. by NTA (4)

Sol. $dQ = dU + dw$

$$\frac{dU}{dt} = \frac{dQ}{dt} - \frac{dw}{dt}$$

$$\frac{dU}{dt} = 1000 - 200 = 800 \text{ W}$$

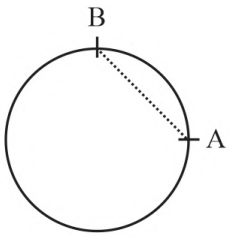
36. A particle is moving with constant speed in a circular path. When the particle turns by an angle 90°, the ratio of instantaneous velocity to its average velocity is $\pi : x\sqrt{2}$. The value of x will be

(1) 2 (2) 5

(3) 1 (4) 7

Official Ans. by NTA (1)

Sol.



AB = R$\sqrt{2}$

Let instantaneous velocity be v. time,

$$t = \frac{\text{Arc length}}{v} = \frac{2\pi\frac{R}{4}}{v} = \frac{\pi R}{2v}$$

average velocity,

$$< v >= \frac{AB}{t} = \frac{R\sqrt{2}(2v)}{\pi R} = \frac{2\sqrt{2}v}{\pi}$$

$$\Rightarrow \frac{V}{< V >} = \frac{\pi}{2\sqrt{2}}.$$

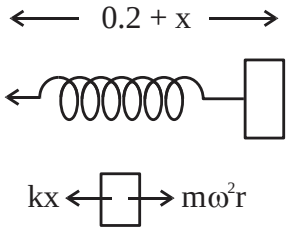
37. A small block of mass 100 g is tied to a spring of spring constant 7.5 N/m and length 20 cm. The other end of spring is fixed at a particular point A. If the block moves in a circular path on a smooth horizontal surface with constant angular velocity 5 rad/s about point A, then tension in the spring is

(1) 1.5 N (2) 0.75 N

(3) 0.25 N (4) 0.50 N

Official Ans. by NTA (2)

Sol.



Let extension in length of spring be x.

Radius of circle r = 0.2 + x

$Kx = m\omega^2 r$

$$7.5x = \left(\frac{1}{10}\right)\left(5^2\right)(0.2 + x)$$

$$\Rightarrow \frac{15}{2}x = \frac{5}{2}\left(x + \frac{1}{5}\right)$$

$$\Rightarrow x = \frac{1}{10}$$

$$\therefore \text{ Tension in spring } = kx = 7.5 \times \frac{1}{10} = 0.75\text{N}$$

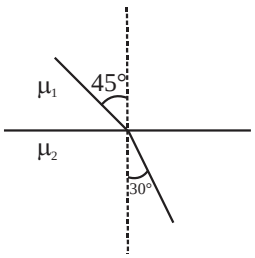
38. A monochromatic light wave with wavelength λ_1 and frequency ν_1 in air enters another medium. If the angle of incidence and angle of refraction at the interface are 45° and 30° respectively, then the wavelength λ_2 and frequency ν_2 of the refracted wave are :

(1) $\lambda_2 = \lambda_1, \nu_2 = \sqrt{2}\nu_1$ (2) $\lambda_2 = \frac{1}{\sqrt{2}}\lambda_1, \nu_2 = \nu_1$

(3) $\lambda_2 = \sqrt{2}\lambda_1, \nu_2 = \nu_1$ (4) $\lambda_2 = \lambda_1, \nu_2 = \frac{1}{\sqrt{2}}\nu_1$

Official Ans. by NTA (2)

Sol.



Snell's law $\mu_1 \sin 45° = \mu_2 \sin 30°$

$$\frac{\mu_1}{\mu_2} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{\mu_1}{\mu_2} = \frac{\lambda_2}{\lambda_1} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \lambda_2 = \frac{\lambda_1}{\sqrt{2}}$$

Frequency doesn't change on change in medium.

39. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A : When a body is projected at an angle 45°, it's range is maximum.

Reason R : For maximum range, the value of sin2θ should be equal to one.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**

(2) Both **A** and **R** are correct **R** is the correct explanation of **A**

(3) **A** is true but **R** is false

(4) **A** is false but **R** is true

Official Ans. by NTA (2)

Sol. $R = \frac{u^2}{g}\sin 2\theta$

R is maximum for $2\theta = 90°$.

40. Two resistances are given as $R_1 = (10 \pm 0.5)\Omega$ and $R_2 = (15 \pm 0.5)\Omega$. The percentage error in the measurement of equivalent resistance when they are connected in parallel is

(1) 6.33 (2) 2.33

(3) 4.33 (4) 5.33

Official Ans. by NTA (3)

Sol. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

Differentiating both sides, we get

$$\frac{\Delta R}{R^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2} \left[R = \frac{R_1 R_2}{R_1 + R_2} = \frac{10 \times 15}{10 + 15} = 6 \right]$$

$$\Rightarrow \frac{\Delta R}{R} = \left(\frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2} \right) R$$

$$= \left(\frac{0.5}{100} + \frac{0.5}{225} \right) 6$$

$$= \left(\frac{6 \times 0.5}{25} \right) \left(\frac{1}{4} + \frac{1}{9} \right) = \frac{13}{300}$$

$$\frac{\Delta R}{R} \times 100 = \frac{13}{3} = 4.33\%$$

41. A planet has double the mass of the earth. Its average density is equal to the that of the earth. An object weighing W on earth will weigh on that planet :

(1) $2^{2/3}$ W

(2) W

(3) $2^{1/3}$ W

(4) 2 W

Official Ans. by NTA (3)

Sol. $m = \rho \times \frac{4}{3}\pi R^3$

$R \propto m^{\frac{1}{3}} (\rho = \text{constant})$

$\text{weight} = W \propto g \propto \frac{Gm}{R^2}$

$W \propto \frac{m}{m^{2/3}} \propto m^{1/3}$

So, $W^1 = (2)^{1/3} W$

42. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A : Earth has atmosphere whereas moon doesn't have any atmosphere.

Reason R : The escape velocity on moon is very small as compared to that on earth.

In the light of the above statement, choose the correct answer from the options given below :

(1) A is true but R is false

(2) A is false but R is true

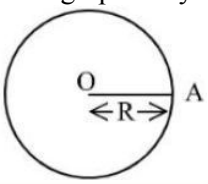
(3) Both A and R are correct but R is NOT the correct explanation of A

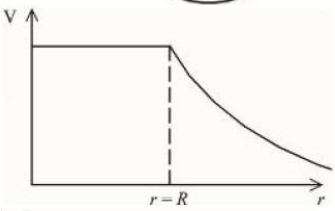
(4) Both A and R are correct and R is correct explanation of A

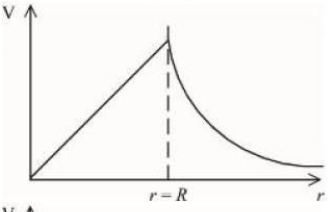
Official Ans. by NTA (4)

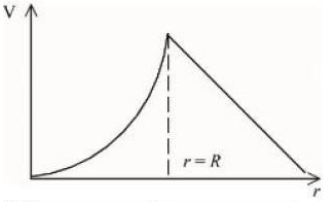
Sol. At Moon, due to low escape velocity, the rms velocity of molecules is greater than escape velocity. Hence molecules escape and there is no atmosphere at Moon.

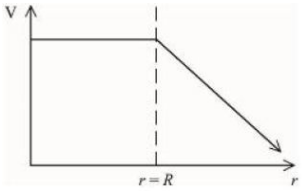
43. For a uniformly charged thin spherical shell, the electric potential (V) radially away from the center (O) of shell can be graphically represented as



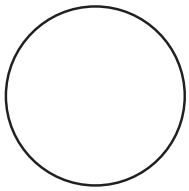
(1)


(2)


(3)


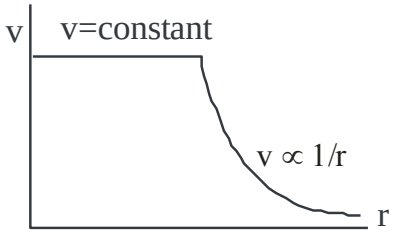
(4)


Official Ans. by NTA (1)

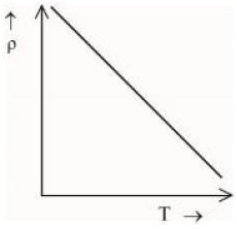
Sol.


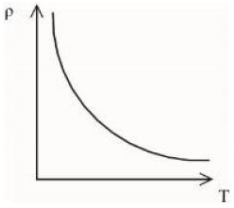
$$V_{inside} = \frac{kQ}{R}$$

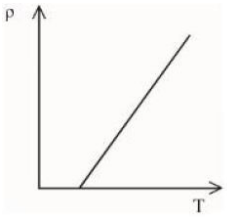
$$V_{outside} = \frac{kQ}{r}$$

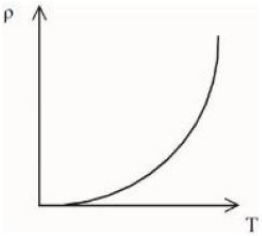


44. The resistivity (ρ) of semiconductor varies with temperature. Which of the following curve represents the correct behaviour

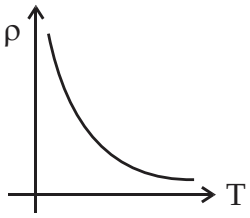
(1)


(2)


(3)


(4)


Official Ans. by NTA (2)

Sol.


$$\rho = \frac{m}{ne^2\tau}$$

With rise in temperature, number density (n) of electrons and holes increases for semiconductors.

As m, e, τ are constant

$$\rho \propto \frac{1}{n} \Rightarrow \rho \propto \frac{1}{T}$$ [Rectangular hyperbola]

45. The kinetic energy of an electron, α-particle and a proton are given as 4K, 2K and K respectively. The de-Broglie wavelength associated with electron (λe) α-particle (λα) and the proton (λp) are as follows :

(1) $\lambda\alpha = \lambda p < \lambda e$

(2) $\lambda\alpha > \lambda p > \lambda e$

(3) $\lambda\alpha < \lambda p < \lambda e$

(4) $\lambda\alpha = \lambda p > \lambda e$

Official Ans. by NTA (3)

Sol.

	Electron	Alpha	Proton
Mass :	$\frac{m}{1840}$	4m	m
Charge :	e	2e	e
Kinetic : energy	4K	2K	K
$\lambda = \frac{h}{\sqrt{2mK}}$	$\frac{h}{\sqrt{2.\frac{m}{1840}.4K}}$	$\frac{h}{\sqrt{2.4m.2K}}$	$\frac{h}{\sqrt{2mK}}$

$\lambda_\alpha < \lambda_p < \lambda_e$

46. By what percentage will the transmission range of a TV tower be affected when the height of the tower is increased by 21% ?

(1) 14% (2) 12%

(3) 10% (4) 15%

Official Ans. by NTA (3)

Sol. $\text{Range}, R = \sqrt{2Rh}$

$R_1 = \sqrt{2Rh_1}$

$h_2 = h_1 + \left(h_1 \times \frac{21}{100}\right) = 1.21h_1$

$\therefore \quad R_2 = \sqrt{2Rh_2} = \sqrt{2R(1.21)h_1} = 1.1\sqrt{2Rh_1}$

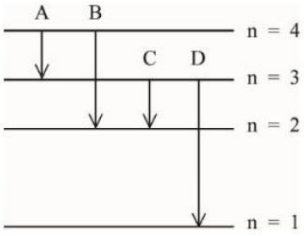
$\therefore \quad R_2 = 1.1\ R_1$

% increase in range

$= \frac{R_2 - R_1}{R_1} \times 100 = \left(\frac{R_2}{R_1} - 1\right) \times 100$

$= (1.1 - 1) \times 100 = 10\%$

47. The energy levels of an hydrogen atom are shown below. The transition corresponding to emission of shortest wavelength is



(1) C (2) D

(3) B (4) A

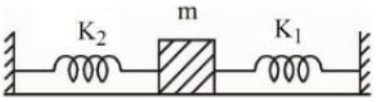
Official Ans. by NTA (2)

Sol. $\Delta E = \frac{hc}{\lambda} \Rightarrow \lambda \alpha \frac{1}{\Delta E}$

For shortest wavelength, energy gap should be maximum.

So, correct choice is transition from n = 3 to n = 1.

48. A mass m is attached to two springs as shown in figure. The spring constants of two springs are K_1 and K_2. For the frictionless surface, the time period of oscillation of mass m is



(1) $\frac{1}{2\pi}\sqrt{\frac{K_1 + K_2}{m}}$

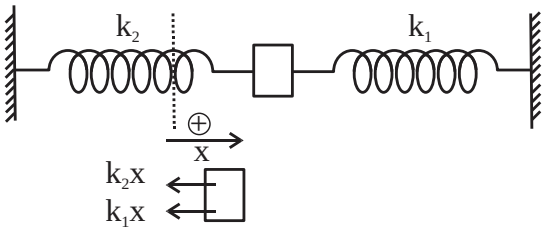
(2) $\frac{1}{2\pi}\sqrt{\frac{K_1 - K_2}{m}}$

(3) $2\pi\sqrt{\frac{m}{K_1 + K_2}}$

(4) $2\pi\sqrt{\frac{m}{K_1 - K_2}}$

Official Ans. by NTA (3)

Sol.



On displacing m to right by x

$F = -(k_1x + k_2x) = -(k_1 + k_2)x$

$a = \frac{F}{m} = -\left(\frac{k_1 + k_2}{m}\right)x = -\omega^2 x$

$\therefore \quad \omega = \sqrt{\frac{k_1 + k_2}{m}} \Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$

49. The induced emf can be produced in a coil by

A. moving the coil with uniform speed inside magnetic field

B. moving the coil with non-uniform speed inside uniform magnetic field

C. rotating the coil inside the uniform magnetic field

D. changing the area of the coil inside the uniform magnetic field

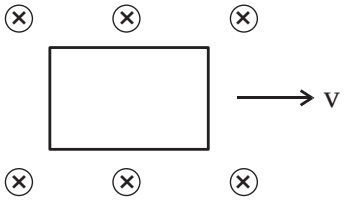
Choose the correct answer from the options given below:

(1) B and D only (2) B and C only

(3) A and C only (4) C and D only

Official Ans. by NTA (4)

Sol.



Moving a coil inside a uniform magnetic field either with uniform or non-uniform speed doesn't changes flux, so, no emf is induced.

50. A long straight wire of circular cross-section (radius a) is carrying steady current I. The current I is uniformly distributed across this cross-section. The magnetic field is

(1) Zero in the region r < a and inversely proportional to r in the region r > a

(2) Inversely proportional to r in the region r < a and uniform throughout in the region r > a

(3) Directly proportional to r in the region r < a and inversely proportional to r in the region r > a

(4) Uniform in the region r < a and inversely proportional to distance r from the axis, in the region r > a

Official Ans. by NTA (3)

Sol.

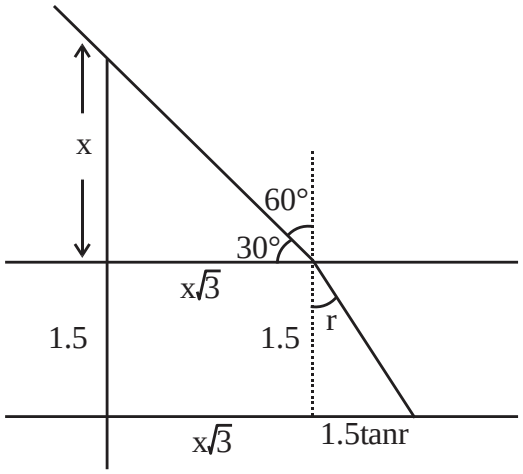
$$B=\begin{cases}\dfrac{\mu_0 Ir}{\pi a^2} & r\le a\\ \dfrac{\mu_0 I}{\pi r^2} & r\ge a\end{cases}$$

SECTION-B

51. A pole is vertically submerged in swimming pool, such that it gives a length of shadow 2.15 m within water when sunlight is incident at an angle of 30° with the surface of water. If swimming pool is filled to a height of 1.5 m, then the height of the pole above the water surface in centimetres is ($n_w = 4/3$) _____ .

Official Ans. by NTA (50)

Sol.



By Snell's law

$$1\sin 60^\circ=\frac{4}{3}\sin r\rightarrow \sin r=\frac{3\sqrt{3}}{8}\rightarrow \tan r=\frac{3\sqrt{3}}{\sqrt{37}}$$

By the diagram

$$x\sqrt{3}+1.5\tan r=2.15$$

$$x\sqrt{3}=2.15-1.5\times\frac{3\sqrt{3}}{\sqrt{37}}$$

$$x=\frac{2.15}{\sqrt{3}}-\frac{1.5\times 3}{\sqrt{37}}$$

$= 1.241 - 0.739$

$= 0.502$

≈ 0.50 meter

x = 50 cm

52. The length of a metallic wire is increased by 20% and its area of cross section is reduced by 4%. The percentage change in resistance of the metallic wire is _____ .

Official Ans. by NTA (25)

Sol. $R=\rho\frac{\ell}{A}$ be the initial resistance new resistance

$$R'=\rho\frac{1.2\ell}{0.96A}=1.25\rho\frac{\ell}{A}=1.25R$$

$$\text{percentage change}=\frac{1.25R-R}{R}\times 100=25\%$$

53. A particle of mass 10 g moves in a straight line with retardation 2x, where x is the displacement in SI units. Its loss of kinetic energy for above displacement is $\left(\frac{10}{x}\right)^{-n}$ J. The value of n will be _____ .

Official Ans. by NTA (2)

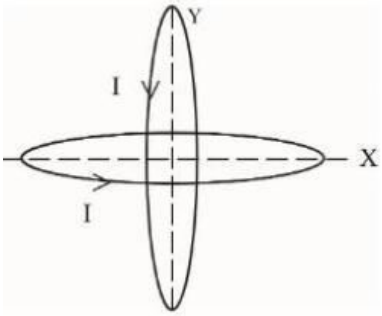
Sol. Loss of K.E = work done against retarding force.

$$= \int_0^x madx = \int_0^x m2xdx = mx^2$$

$$= (10^{-2}\,kg)x^2 J = \left(\frac{10}{x}\right)^{-2} J$$

So n = 2

54. Two identical circular wires of radius 20 cm and carrying current $\sqrt{2}$ A are placed in perpendicular planes as shown in figure. The net magnetic field at the centre of the circular wire is _____ $\times 10^{-8}$ T. (Take $\pi = 3.14$)



Official Ans. by NTA (628)

Sol. Magnetic field B_C at center $= \frac{\mu_0 i}{2r}$

$$= \frac{4\pi\times10^{-7}}{2\times0.2}\times\sqrt{2}T$$

Net magnetic field is

$$B_C\sqrt{2} = \frac{4\pi\times10^{-7}\times\sqrt{2}}{2\times0.2}\times\sqrt{2}T = 2\pi\times10^{-6}T$$

$$= 200\pi\times10^{-8}T$$

$$= 2\times314\times10^{-8}T$$

$$= 628\times10^{-8}T$$

55. A person driving car at a constant speed of 15 m/s is approaching a vertical wall. The person notices a change of 40 Hz in the frequency of his car's horn upon reflection from the wall. The frequency of horn is _____ Hz.

(Given : Speed of sound : 330 m/s)

Official Ans. by NTA (420)

Sol. Frequency of reflected sound $= \left(\frac{v+v_c}{v-v_c}\right)f_0$

$$f = \left(\frac{330+15}{330-15}\right)\times f_0$$

$$= \frac{345}{315}f_0$$

$$\frac{345}{315}f_0 - f_0 = 40$$

$$\frac{30}{315}f_0 = 40$$

$$f_0 = \frac{4\times315}{3} = 420Hz$$

56. The radius of fifth orbit of the Li^{++} is _____ $\times 10^{-12}$ m. Take : radius of hydrogen atom = 0.51Å

Official Ans. by NTA (425)

Sol. $r_n = r_0\frac{n^2}{z} \rightarrow r_n = 0.51\times\frac{25}{3}Å = 4.25\times10^{-10}m$

$= 425 \times 10^{-12}$ m

57. A steel rod has a radius of 20 mm and a length of 2.0 m. A force of 62.8 kN stretches it along its length. Young's modulus of steel is 2.0×10^{11} N/m^2. The longitudinal strain produced in the wire is _____ $\times 10^{-5}$

Official Ans. by NTA (25)

Sol. $\text{Strain} = \dfrac{\text{stress}}{Y} = \dfrac{\frac{62.8\times10^3}{\pi\times(0.02)^2}}{2\times10^{11}}$

$= \dfrac{62.8\times10^3}{3.14\times4\times10^{-4}\times2\times10^{11}}$

$= 2.5 \times 10^{-4}$

$= 25 \times 10^{-5}$

58. An ideal transformer with purely resistive load operates at 12 kV on the primary side. It supplies electrical energy to a number of nearby houses at 120 V. The average rate of energy consumption in the houses served by the transformer is 60 kW. The value of resistive load (Rs) required in the secondary circuit will be _____ mΩ.

Official Ans. by NTA (240)

Sol. $v_p = 12 \times 10^3$ volts

$v_s = 120$ volts

$p_s = 60\ \text{KW} = v_s \times i_s$

$i_s = \dfrac{60\times10^3}{120} = 5\times10^2\,\text{A}$

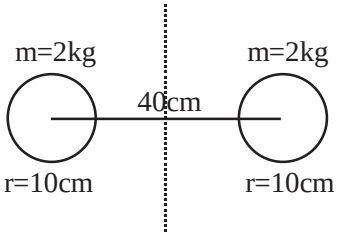
$R_L = \dfrac{v_s}{i_s} = \dfrac{120}{5\times10^2} = 24\times10^{-2} = 240\times10^{-3}\,\Omega$

$= 240\ \text{m}\Omega$

59. Two identical solid spheres each of mass 2 kg and radii 10 cm are fixed at the ends of a light rod. The separation between the centres of the spheres is 40 cm. The moment of inertia of the system about an axis perpendicular to the rod passing through its middle point is _____ $\times 10^{-3}$ kg-m^2

Official Ans. by NTA (176)

Sol.



$I = 2\left(I_{cm} + md^2\right)$

$= 2\left(\dfrac{2}{5}mr^2 + md^2\right)$

$= \dfrac{4}{5}\times2\times(0.1)^2 + 2(2)(0.20)^2$

$= \dfrac{8}{5}\times10^{-2} + 16\times10^{-2}$

$= (1.6+16)\times10^{-2}$

$= 17.6\times10^{-2}$

$I = 176\times10^{-3}\,\text{kg m}^2$

60. A parallel plate capacitor with plate area A and plate separation d is filled with a dielectric material of dielectric constant K = 4. The thickness of the dielectric material is x, where x < d.

Let C_1 and C_2 be the capacitance of the system for $x = \dfrac{1}{3}d$ and $x = \dfrac{2d}{3}$, respectively. If $C_1 = 2\mu F$ the value of C_2 is _____ μF

Official Ans. by NTA (3)

Sol. For $x = \dfrac{d}{3}$

$C_1 = \dfrac{\epsilon_0 A}{\left(\frac{d/3}{k} + \frac{2d}{3}\right)} = \dfrac{\epsilon_0 A}{\frac{d}{12} + \frac{2d}{3}}$

$= \dfrac{\epsilon_0 A}{d} \times \left(\dfrac{12}{9}\right)$

$C_1 = \dfrac{4}{3}\dfrac{\epsilon_0 A}{d} = 2\mu F$

for $x = \dfrac{2d}{3}$

$C_2 = \dfrac{\epsilon_0 A}{\left(\frac{2d/3}{k} + \frac{d}{3}\right)} = \dfrac{\epsilon_0 A}{d} \times 2$

$\Rightarrow \dfrac{6}{4} \times 2 = 3\mu F$

SECTION-A

61. A compound is formed by two elements X and Y. The element Y forms cubic close packed arrangement and those of element X occupy one third of the tetrahedral voids. What is the formula of the compound?

(1) X_2Y_3

(2) X_3Y

(3) X_3Y_2

(4) XY_3

Official Ans. by NTA (1)

Sol. Y : CCP $\Rightarrow$ 4Y

X = 1/3 THV = 1/3 × 8 $\Rightarrow$ 8/3x

$\therefore$ Formula : $X_{8/3}Y_4$ or X_2Y_3

62. **Match List I with List II**

List I Element detected		List II Reagent used/ Product formed	
A	Nitrogen	I.	$Na_2[Fe(CN)_5 NO]$
B	Sulphur	II.	$AgNO_3$
C	Phosphorous	III.	$Fe_4[Fe(CN)_6]_3$
D	Halogen	IV.	$(NH_4)_2 MoO_4$

Choose the correct answer from the options given below:

(1) A-II, B-IV, C-I, D-III

(2) A-IV, B-II, C-I, D-III

(3) A-II, B-I, C-IV, D-III

(4) A-III, B-I, C-IV, D-II

Official Ans. by NTA (4)

Nitrogen detection by lassaigne's method

$Na + C + N \rightarrow NaCN$

$6NaCN + FeSO_4 \rightarrow Na_4[Fe(CN)_6] + Na_2SO_4$

$Na_4[Fe(CN)_6] + Fe^{3+} \rightarrow Fe_4[Fe(CN)_6]_3$

(Prussian blue)

Sulphur detection by Sodium nitroprusside

$Na_2[Fe(CN)_5 NO] + Na_2S \rightarrow Na_4[Fe(CN)_5 NOS]$

[Purple]

Phosphorus detection by ammonium molybdate

$Na_3PO_4 + 3HNO_3 \rightarrow H_3PO_4 + 3NaNO_3$

$H_3PO_4 + 12(NH_4)_2 MoO_4 + 21HNO_3 \rightarrow$

$(NH_4)_3 PO_4.\ 12MoO_3 + 21NH_4NO_3 + 12H_2O$

(canary yellow)

Halogen give specific coloured ppt with $AgNO_3$(aq)

$NaCl + AgNO_3(aq) \rightarrow AgCl + NaNO_3$

(White)

$NaBr + AgNO_3(aq) \rightarrow AgBr + NaNO_3$

(Pale yellow)

$NaI + AgNO_3(aq) \rightarrow AgI + NaNO_3$

(Yellow)

63. The standard electrode potential of M^+/M in aqueous solution does not depend on

(1) Ionisation of a solid metal atom

(2) Sublimation of a solid metal

(3) Ionisation of a gaseous metal atom

(4) Hydration of a gaseous metal ion

Official Ans. by NTA (1)

Sol. Factual

64. Polymer used in orlon is:

(1) Polyacrylonitrile

(2) Polyethene

(3) Polycarbonate

(4) Polyamide

Official Ans. by NTA (1)

Sol. $n\ CH_2 = \underset{\displaystyle CN}{\underset{|}{CH}} \xrightarrow{\text{Polymerisation}} \left[CH_2 - \underset{\displaystyle CN}{\underset{|}{CH}} \right]_n$

Acrylonitrile

Polyacrylonitrile

(Orlon)

65. The difference between electron gain enthalpies will be maximum between:

(1) Ne and F

(2) Ne and Cl

(3) Ar and Cl

(4) Ar and F

Official Ans. by NTA (2)

Sol. Cl has the most negative ΔH_{eg} among all the elements and Ne has the most positive ΔH_{eg}.

66. Match List I with List II

List I Enzymatic reaction		List II Enzyme	
A	Sucrose → Glucose and Fructose	I.	Zymase
B	Glucose→ethyl alcohol and CO_2	II.	Pepsin
C	Starch → Maltose	III.	Invertase
D	Proteins → Amino acids	IV.	Diastase

Choose the correct answer from the options given below:

(1) A-III, B-I, C-II, D-IV

(2) A-I, B-IV, C-III, D-II

(3) A-III, B-I, C-IV, D-II

(4) A-I, B-II, C-IV, D-III

Official Ans. by NTA (3)

Sol. Factual

67. The possibility of photochemical smog formation is more at

(1) The places with healthy vegetation

(2) Himalayan villages in winter

(3) Marshy lands

(4) Industrial areas

Official Ans. by NTA (4)

Sol. Photochemical smog occurs in warm, dry and sunny climate. The main components come from the action of sunlight on unsaturated hydrocarbon and nitrogen oxides produced by automobiles and factories.

68. The setting time of Cement is increased by adding

(1) Clay

(2) Silica

(3) Limestone

(4) Gypsum

Official Ans. by NTA (4)

Sol. Factual

69. Given below are two statements: one is labelled as assertion and the other is labelled as reason .

Assertion: Loss of electron from hydrogen atom results in nucles of ~1.5×10^{-3} pm size.

Reason: Proton (H^+) always exists in combined form

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both A and R are correct and R is the correct explanation of A

(2) A is correct but R is not correct

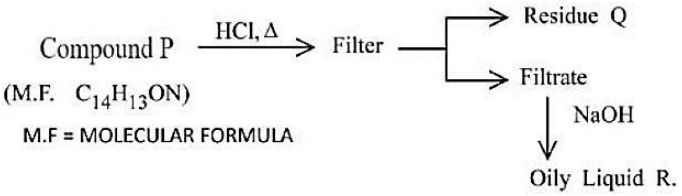
(3) A is not correct but R is correct

(4) Both A and R are correct but R is NOT the correct explanation of A.

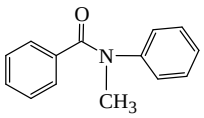
Official Ans. by NTA (4)

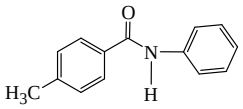
Sol. Factual

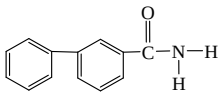
70.

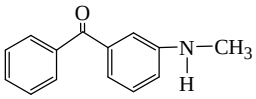


Compound P is neutral. Q gives effervescence with $NaHCO_3$ while R reacts with Hinsbergs reagent to give solid soluble in NaOH. Compound P is

(1) 

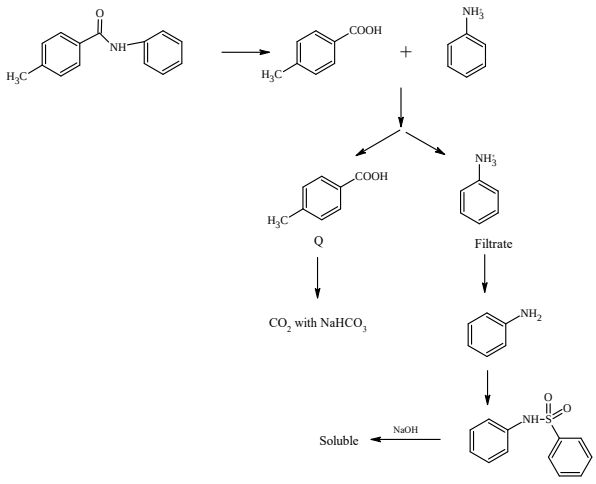
(2) 

(3) 

(4) 

Official Ans. by NTA (2)

Sol.



71. Match List I with List II

List I Name of reaction		List II Reagent used	
A	Hell-Volhard-Zelinsky reaction	I.	$NaOH + I_2$
B	Iodoform reaction	II.	(i) CrO_2Cl_2,CS_2(ii) H_2O
C	Etard reaction	III.	(i) Br_2/red phosphorus (ii) H_2O
D	Gatterman-Koch reaction	IV.	CO, HCl, anhyd. $A1C1_3$

Choose the correct answer from the options given below:

(1) A-III, B-II, C-I, D-IV

(2) A-III, B-I, C-IV, D-II

(3) A-I, B-II, C-III, D-IV

(4) A-III, B-I, C-II, D-IV

Official Ans. by NTA (4)

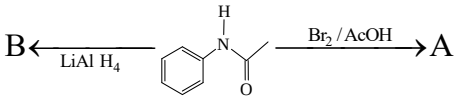
Sol. HVZ reactions = Br_2 / red P

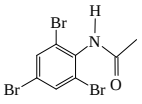
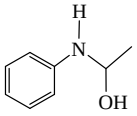
Iodoform reaction = $NaOH + I_2$

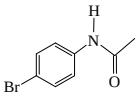
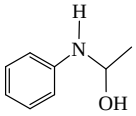
Etard reaction = (i) CrO_2 Cl_2, CS_2(ii) H_2O

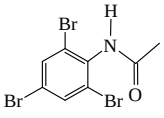
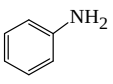
Gatterman-Koch Reaction = CO, HCl, Anhydrous, $AlCl_3$

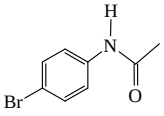
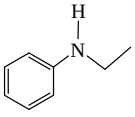
72. The major products A and B from the following reactions are:



(1) A =  B = 

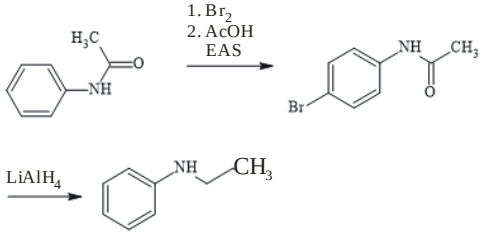
(2) A =  B = 

(3) A =  B = 

(4) A =  B = 

Official Ans. by NTA (4)

Sol.



73. Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A: The spin only magnetic moment value for $[Fe(CN)_6]^{3-}$ is 1.74 BM, whereas for $[Fe(H_2O)_6]^{3+}$ is 5.92 BM.

Reason R : In both complexes, Fe is present in +3 oxidation state.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both A and R are true but R is NOT the correct explanation of A

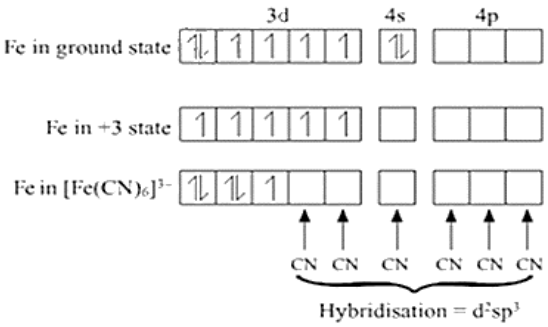
(2) A is false but R is true

(3) A is true but R is false

(4) Both A and R are true and R is the correct explanation of A

Official Ans. by NTA (1)

Sol. $[Fe(CN)_6]^{3-}$



Unpaired electron = 1

$\mu = \sqrt{n(n+2)} = \sqrt{1\times 3} = 1.74$ B.M.

$[Fe(H_2O)_6]^{3+}$ No pairing because H_2O is WFL

Number of unpaired electrons = 5, μ = 5.92 BM

Assertion is true, Reason is true but not correct explanation.

74. Match List I with List II

List I Vitamin		List II Deficiency disease	
A	Vitamin A	I.	Beri-Beri
B	Thiamine	II.	Cheilosis
C	Ascorbic acid	III.	Xeropthalmia
D	Riboflavin	IV.	Scurvy

Choose the correct answer from the options given below:

(1) A-IV, B-II,C-III, D-I (2) A-III, B-II, C-IV, D-I

(3) A-IV, B-I,C-III, D-II (4) A-III,B-I,C-IV, D-II

Official Ans. by NTA (4)

Sol. Factual

75. Which of the following options are correct for the reaction

$2[Au(CN)_2]^-_{(aq)} + Zn(s) \rightarrow 2Au(s) + [Zn(CN)_4]^{2-}_{(aq)}$

A. Redox reaction

B. Displacement reaction

C. Decomposition reaction

D. Combination reaction

Choose the correct answer from the options given below:

(1) A and B only (2) A only

(3) C and D only (4) A and D only

Official Ans. by NTA (1)

Sol. $2\left[\overset{+1}{Au}(CN)_2\right]^- + \overset{0}{Z}n(s) \longrightarrow 2\overset{0}{Au} + \left[\overset{+2}{Zn}(CN)_4\right]^{-2}$

Zn displaced Au^+

Reduction and Oxidation both are taking place.

76. **Match List I with List II**

List I Oxide		List II Type of Bond	
A	N_2O_4	I.	1N = O bond
B	NO_2	II.	1N – O – N bond
C	N_2O_5	III.	1N – N bond
D	N_2O	IV.	1N = N / N ≡ N bond

Choose the correct answer from the options given below:

(1) A-II, B-IV, C-III, D-I

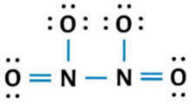
(2) A-II, B-I, C-III, D-IV

(3) A-III, B-I, C-IV, D-II

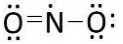
(4) A-III, B-I, C-II, D-IV

Official Ans. by NTA (4)

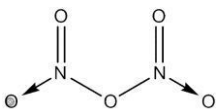
Sol. N_2O_4



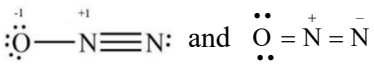
NO_2



N_2O_5



N_2O



77. Strong reducing and oxidizing agents among the following, respectively, are

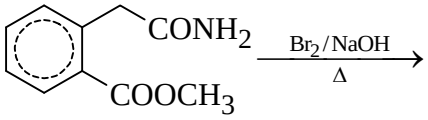
(1) Ce^{4+} and Eu^{2+} (2) Ce^{4+} and Tb^{4+}

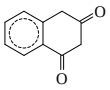
(3) Ce^{3+} and Ce^{4+} (4) Eu^{2+} and Ce^{4+}

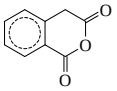
Official Ans. by NTA (4)

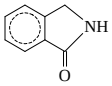
Sol. **Factual**

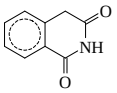
78. The major product formed in the following reaction is



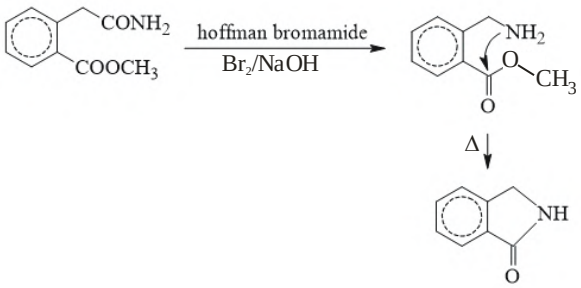
(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (3)

Sol. 

79. For a concentrated solution of a weak electrolyte (K_{eq}= equilibrium constant) A_2B_3 of concentration 'c', the degree of dissociation "α' is

(1) $\left(\frac{K_{eq}}{108c^4}\right)^{\frac{1}{5}}$ (2) $\left(\frac{K_{eq}}{6c^5}\right)^{\frac{1}{5}}$

(3) $\left(\frac{K_{eq}}{5c^4}\right)^{\frac{1}{5}}$ (4) $\left(\frac{K_{eq}}{25c^2}\right)^{\frac{1}{5}}$

Official Ans. by NTA (1)

Sol. $A_2B_3(aq.) \rightleftharpoons 2A^{3+}_{(aq.)} + 3B^{2-}_{(aq)}$

$c(1-\alpha) \quad 2c\alpha \quad 3c\alpha$

$$K_{eq} = \frac{\left[A^{3+}\right]^2\left[B^{2-}\right]^3}{\left[A_2B_3\right]} = \frac{4c^2\alpha^2 \times 27c^3\alpha^3}{c(1-\alpha)}$$

$$K_{eq} == \frac{108c^5\alpha^5}{c} \quad \alpha = \left(\frac{K_{eq}}{108c^4}\right)^{\frac{1}{5}}$$

80. For the reaction:

$$RCH_2Br + I^- \xrightarrow{\text{Acetone}} \underset{\text{major}}{RCH_2I} + Br^-$$

The correct statement is :

(1) The transition state formed in the above reaction is less polar than the localised anion.

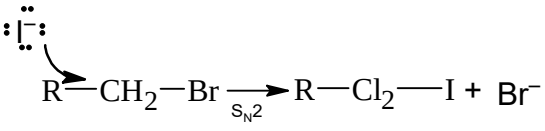
(2) The reaction can occur in acetic acid also.

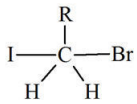
(3) The solvent used in the reaction solvates the ions formed in rate determining step.

(4) Br^- can act as competing nucleophile.

Official Ans. by NTA (1)

Sol. This is finkelstein reaction



Transition state 

Clearly, the transition state is less polar than free anions. $Br^-$and I^-

Acetic acid is protic which does not support S_N2

Acetone does not solvate anion

Br^- gets precipitated and hence can not compete with I^-

So only (1)is correct

SECTION-B

81. The wavelength of an electron of kinetic energy 4.50×10^{-29}J is…… $\times 10^{-5}$ m. (Nearest integer)

Given : mass of electron is 9×10^{-31} kg, h =6.6×10^{-34} J s

Official Ans. by NTA (7)

Sol. $\lambda_d = \frac{h}{mv} = \frac{h}{\sqrt{2mKE}} = \frac{6.6\times10^{-34}}{\sqrt{2\times9\times10^{-31}\times4.5\times10^{-29}}}$

$= \frac{6.6\times10^{-34}}{\sqrt{9^2\times10^{-60}}}$

$= \frac{6.6\times10^{-34}}{9\times10^{-30}} = \frac{6.6}{9}\times10^{-4}$

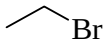
$= 7.3 \times 10^{-5}$ m

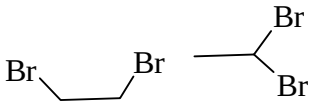
Therefore Ans = 7

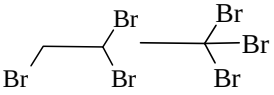
82. Number of bromo derivatives obtained on treating ethane with excess of Br_2, in diffused sunlight is…

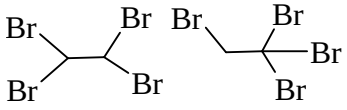
Official Ans. by NTA (9)

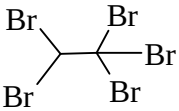
Sol. $CH_3 - CH_3 + Br_2$ (Excess) $\xrightarrow{h\nu}$

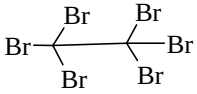
Monobromo 

Dibromo 

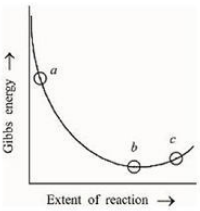
Tribromo 

Tretrabromo 

Pentabromo 

Hexabromo 

83. Consider the graph of Gibbs free energy G vs Extent of reaction. The number of statement/s from the following which are true with respect to points (a), (b) and (c) is……..



A. Reaction is spontaneous at (a) and (b)

B. Reaction is at equilibrium at point (b) and non-spontaneous at point (c)

C. Reaction is spontaneous at (a) and non-spontaneous at (c)

D. Reaction is non-spontaneous at (a) and (b)

Official Ans. by NTA (2)

Sol. For, Spontaneous process dG< 0

For, Equilibrium dG = 0

For, Nonspontaneous process dG > 0

∴ A Wrong

B Correct

C Correct

D Wrong

84. Mass of Urea (NH_2CONH_2) required to be dissolved in 1000 g of water to reduce the vapour pressure of water by 25% is……g. (Nearest integer)

Given: Molar mass of N. C. O and H are 14. 12. 16 and 1 2 mol^{-1} respectively.

Official Ans. by NTA (1111)

Sol. $$\frac{P^0 - P_s}{P_s} = \frac{n_{solute}}{n_{solvent}} = \frac{\frac{x}{60}}{\frac{1000}{18}} = \frac{P^0 - 0.75P^0}{0.75P^0}$$

$$\Rightarrow x = \frac{10000}{9} = 1111\,gm$$

Ans: 1111

85. The value of log K for the reaction A⇋ B at 298 K is ……. (Nearest integer)

Given: $\Delta H^0 = -54.07$ kJ mol^{-1}

$\Delta S° = 10$ JK^{-1} mol^{-1}

(Take 2.303 × 8.314 × 298 = 5705)

Official Ans. by NTA (10)

Sol. $\Delta G^0 = \Delta H^0 - T\Delta S$

$\Rightarrow \Delta G^0 = (-54070 - 10 \times 298)$

Also, $\Delta G^0 = (-2.303\ RT \log K)$

$\Rightarrow (-54070 - 10 \times 298)$

$= (-2.303 \times 8.134 \times 298 \log K)$

$\Rightarrow \log K = 10$ Ans: 10

86. The number of species from the following which have square pyramidal structure is

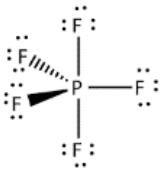
PF_5, BrF_4^-, IF_5; BrF_5, $XeOF_4$, ICl_4^-

Official Ans. by NTA (3)

Sol. PF_5

sp^3d (0 lone pair)

Trigonal bipyramidal



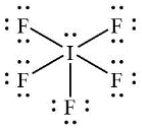
BrF_4^-,

sp^3d^2 (2 lone pair)

 square planar

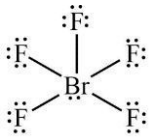
IF_5

sp^3d^2 (1 lone pair)

 square pyramidal

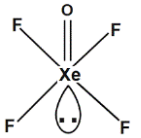
BrF_5

sp^3d^2 (1 lone pair)

 square pyramidal

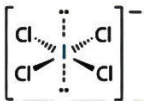
$XeOF_4$

sp^3d^2 (1 lone pair)

 square pyramidal

ICl_4^-

sp^3d^2 (2 lone pair)

 square planar

87. Number of ambidentate ligands in a representative metal complex $[M(en)(SCN)_4]$ is

[en = ethylenediamine]

Official Ans. by NTA (4)

Sol. $[M(en)(SCN)_4]$

$S = C = N^-$

Ambidentate ligand means two ligand site, so ambidentate ligand is SCN^-.

Ans: 4

88. For the adsorption of hydrogen on platinum, the activation energy is 30 kJ mol^{-1} and for the adsorption of hydrogen on nickel, the activation energy is 41.4 kJ mol^{-1}. The logarithm of the ratio of the rates of chemisorption on equal areas of the metals at 300 K is …… (Nearest integer)

Given: ln 10 = 2. 3 R = 8.3 JK^{-1} mol^{-1}

Official Ans. by NTA (2)

Sol. $K = Ae^{-\frac{E_a}{RT}}$

$K_1 = Ae^{-\frac{(E_a)_1}{RT}}$

$K_2 = Ae^{-\frac{(E_a)_2}{RT}}$

$\frac{K_2}{K_1} = e^{\frac{(E_a)_1-(E_a)_2}{RT}}$

$\log\frac{K_2}{K_1} = \frac{(E_a)_1-(E_a)_2}{2.3RT}$

$= \frac{(41.4-30)\times1000}{2.3\times8.3\times300} = 1.99$

Ans: 2

89. If 5 moles of $BaCl_2$ is mixed with 2 moles of Na_3PO_4, the maximum number of moles of $Ba_3(PO_4)_2$ formed is…….

(Nearest integer)

Official Ans. by NTA (1)

Sol. $3BaCl_2 + 2Na_3PO_4 \rightarrow Ba_3(PO_4)_2 + 6NaCl$

5 2

Na_3PO_4 is limiting reagent.

2 mole Na_3PO_4 gives 1 mole of $Ba_3(PO_4)_2$

Ans: 1

90. In ammonium-phosphomolybdate, the oxidation state of Mo is $^+$ ……

Official Ans. by NTA (6)

Sol. $(NH_4)_3PO_4.12MoO_3$

Let X = oxidation state of Mo in MoO_3

$X + (-2) \times 3 = 0$

$X = +6$

Ans: 6

FINAL JEE–MAIN EXAMINATION – APRIL, 2023

(Held On Thursday 06th April, 2023) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Three dice are rolled. If the probability of getting different numbers on the three dice is $\frac{p}{q}$, where p and q are co-prime, then q – p is equal to

(1) 4

(2) 3

(3) 1

(4) 2

Official Ans. by NTA (1)

Sol. Total number of ways = $6^3 = 216$

Favourable outcomes ${}^6p_3 = 120$

$\Rightarrow$ Probability = $\frac{120}{216} = \frac{5}{9}$

$\Rightarrow p = 5, q = 9$

$\Rightarrow q - p = 4$

2. Among the statements:

(S1) : $2023^{2022} - 1999^{2022}$ is divisible by 8.

(S2) : $13(13)^n - 11n - 13$ is divisible by 144 for infinitely many $n \in N$.

(1) both (S1) and (S2) are incorrect

(2) only (S2) is correct

(3) both (S1) and (S2) are correct

(4) only (S1) is correct

Official Ans. by NTA (3)

Sol. $S_1 = (1999 + 24)^{2022} - (1999)^{2022}$

$\Rightarrow {}^{2022}C_1(1999)^{2021}(24) + {}^{2022}C_2(1999)^{2020}(24)^2 + ...$so on

S_1 is divisible by 8

$S_2 : 13(13^n) - 11n - 13$

$13^n = (1+12)^n = 1 + 12n + {}^nC_2 12^2 + {}^nC_3 12^3 .....$

$13(13^n) - 11n - 13 = 145n + {}^nC_2\ 12^2 + {}^nC_3 12^3 .....$

If $(n = 144m,\ m \in N)$, then it is divisible by 144

For infinite value of n.

3. $\lim\limits_{n\to\infty} \left\{ \left(2^{\frac{1}{2}} - 2^{\frac{1}{3}}\right)\left(2^{\frac{1}{2}} - 2^{\frac{1}{5}}\right) ..... \left(2^{\frac{1}{2}} - 2^{\frac{1}{2n+1}}\right) \right\}$

is equal to

(1) $\frac{1}{\sqrt{2}}$

(2) 1

(3) $\sqrt{2}$

(4) 0

Official Ans. by NTA (4)

Sol. $\left(2^{\frac{1}{2}} - 2^{\frac{1}{3}}\right)^n < \left(2^{\frac{1}{2}} - 2^{\frac{1}{3}}\right)\left(2^{\frac{1}{2}} - 2^{\frac{1}{5}}\right)\left(2^{\frac{1}{2}} - 2^{\frac{1}{7}}\right)$

$---- \left(2^{\frac{1}{2}} - 2^{\frac{1}{2n+1}}\right) < \left(2^{\frac{1}{2}} - 2^{\frac{1}{2n+1}}\right)^n$

$\left(2^{\frac{1}{2}} - 2^{\frac{1}{3}}\right)^n < L < \left(2^{\frac{1}{2}} - 2^{\frac{1}{2n+1}}\right)^n$

$\lim\limits_{n\to\infty}\left(2^{\frac{1}{2}} - 2^{\frac{1}{3}}\right)^n = 0$ and $\lim\limits_{n\to\infty}\left(2^{\frac{1}{2}} - 2^{\frac{1}{2n+1}}\right)^n = 0$

$\Rightarrow \lim\limits_{n\to\infty} L = 0$

4. Let $a \neq b$ be two non-zero real numbers.

Then the number of elements in the set $X = \left\{ z \in C : Re(az^2 + bz) = a \text{ and } Re(bz^2 + az) = b \right\}$

is equal to

(1) 1

(2) 3

(3) 0

(4) 2

Official Ans. by NTA (3)

Sol. $Re(az^2 + bz) = a$

$az^2 + bz + a\bar{z}^2 + b\bar{z} = 2a$

$a(z^2 + \bar{z}^2) + b(z + \bar{z}) = 2a$ ....(1)

$Re(bz^2 + az) = b$

$bz^2 + az + b\bar{z}^2 + a\bar{z} = 2b$

$b(z^2 + \bar{z}^2) + a(z + \bar{z}) = 2b$ ....(2)

(1) × b – (2) × (a)

$\Rightarrow (b^2 - a^2)(z + \bar{z}) = 0$

$\Rightarrow \quad (z+\bar{z}) = 0 \quad (a^2 \neq b^2)$

$(1) \times a - (2) \times (b)$

$\Rightarrow \quad (a^2 - b^2)(z+\bar{z}) = 2(a^2 - b^2) \quad (a^2 \neq b^2)$

$z^2 + \bar{z}^2 = 2$

$\Rightarrow \quad (z+\bar{z})^2 - 2z\bar{z} = 2$

$z\bar{z} = -1$

$\Rightarrow \quad 1 + 1^2 = -1$

$\Rightarrow$ No solution

But when $a = -b$,

$\text{Re}(az^2 - az) = a$

$\Rightarrow \quad \text{Re}\left(a(x^2 - y^2 + i2xy) - a(x+iy)\right) = a$

$\Rightarrow \quad a(x^2 - y^2) - ax = a$

$\Rightarrow \quad x^2 - y^2 - x = 1$

$\Rightarrow \quad x^2 - x - 1 = y^2$

For any real values of y there two values of x, hence infinite complex numbers are possible.

5. Let the sets A and B denote the domain and range respectively of the function $f(x) = \frac{1}{\sqrt{\lceil x \rceil - x}}$, where $\lceil x \rceil$ denotes the smallest integer greater than or equal to x . Then among the statements

(S1) : $A \cap B = (1, \infty) - N$ and

(S2) : $A \cup B = (1, \infty)$

(1) only (S1) is true

(2) both (S1) and (S2) are true

(3) neither (S1) nor (S2) is true

(4) only (S2) is true

Official Ans. by NTA (1)

Sol. $f(x) = \frac{1}{\sqrt{\lceil x \rceil - x}}$

If $x \in I \ \lceil x \rceil = [x]$ (greatest integer function)

If $x \notin I \ \lceil x \rceil = [x] + 1$

$$\Rightarrow f(x) = \begin{cases} \frac{1}{\sqrt{[x]-x}}, & x \in I \\ \frac{1}{\sqrt{[x]+1-x}}, & x \notin I \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} \frac{1}{\sqrt{-\{x\}}}, & x \in I, \text{ (does not exist)} \\ \frac{1}{\sqrt{1-\{x\}}}, & x \notin I \end{cases}$$

$\Rightarrow$ domain of $f(x) = R - I$

Now, $f(x) = \frac{1}{\sqrt{1-\{x\}}}, \ x \notin I$

$\Rightarrow 0 < \{x\} < 1$

$\Rightarrow 0 < \sqrt{1-\{x\}} < 1$

$\Rightarrow \frac{1}{\sqrt{1-\{x\}}} > 1$

$\Rightarrow$ Range $(1, \infty)$

$\Rightarrow A = R - I$

$B = (1, \infty)$

So, $A \cap B = (1, \infty) - N$

$A \cup B \neq (1, \infty)$

$\Rightarrow$ S1 is only correct

6. If the solution curve f(x, y) = 0 of the differential equation $(1 + \log_e x)\frac{dx}{dy} - x \log_e x = e^y$, $x > 0$, passes through the points (1,0) and $(\alpha, 2)$ then α^α is equal to

(1) $e^{2e^{\sqrt{2}}}$

(2) $e^{\sqrt{2}e^2}$

(3) e^{e^2}

(4) e^{2e^2}

Official Ans. by NTA (4)

Sol. $(1 + \ln x)\frac{dx}{dy} - x \ln x = e^y$

Let $x \ln x = t$

$(1 + \ln x)\frac{dx}{dy} = \frac{dt}{dy}$

$\frac{dt}{dy} - t = e^y$

If $= e^{\int -dy} = e^{-y}$

$t.e^{-y} = \int e^y e^{-y} dy + c$

$te^{-y} = y + c$

$x \ln x \ e^{-y} = y + c$

$x \ln x = ye^y + ce^y$

$(1, 0) \quad \boxed{0 = C}$

$\Rightarrow x \ln x = ye^y$

$\Rightarrow \alpha \ln \alpha = 2e^2$

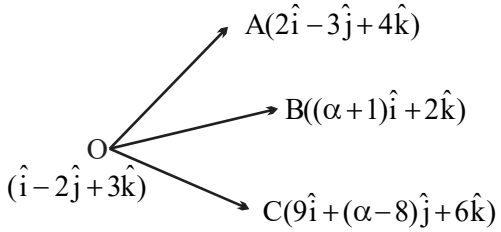
$\alpha^\alpha = e^{2e^2}$

7. The sum of all values of α, for which the points whose position vectors $\hat{i}-2\hat{j}+3\hat{k}$, $2\hat{i}-3\hat{j}+4\hat{k}$, $(\alpha+1)\hat{i}+2\hat{k}$ and $9\hat{i}+(\alpha-8)\hat{j}+6\hat{k}$ are coplanar, is equal to

(1) 6
(2) 4
(3) –2
(4) 2

Official Ans. by NTA (4)

Sol.



$[OA\ \ OB\ \ OC] = 0$

$$\begin{vmatrix} 1 & -1 & 1 \\ \alpha & 2 & -1 \\ 8 & \alpha-6 & 3 \end{vmatrix} = 0$$

$\Rightarrow \alpha^2 - 2\alpha - 8 = 0$

$\Rightarrow (\alpha-4)(\alpha+2) = 0$

$\therefore \alpha = 4, -2$

8. For the system of equations

$x + y + z = 6$

$x + 2y + \alpha z = 10$

$x + 3y + 5z = \beta$, which one of the following is NOT true?

(1) System has a unique solution for $\alpha = 3, \beta \neq 14$.
(2) System has no solution for $\alpha = 3, \beta = 24$.
(3) System has a unique solution for $\alpha = -3, \beta = 14$.
(4) System has infinitely many solutions for $\alpha = 3, \beta = 14$.

Official Ans. by NTA (1)

Sol. $x + y + z$

$x + 2y + \alpha z = 10$

$x + 3y + 5z = \beta$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \alpha \\ 1 & 3 & 5 \end{vmatrix} = 1(10-3\alpha) - 1(5-\alpha) + 1(3-z)$$

$= 10 - 3\alpha - 5 + \alpha + 1$

$= 6 - 2\alpha$

For unique solution $6 - 2\alpha \neq 0 \Rightarrow \alpha \neq 3$

9. The area bounded by the curves $y = |x-1| + |x-2|$ and $y = 3$ is equal to

(1) 3
(2) 4
(3) 5
(4) 6

Official Ans. by NTA (2)

Sol. $y = |x-1| + |x-2|$ and $y = 3$

$\therefore$ Required area $= \frac{1}{2}(1+3)\times 2 = 4$

10. Let P be a square matrix such that $P^2 = I - P$. For $\alpha, \beta, \gamma, \delta \in N$, if $P^\alpha + P^\beta = \gamma I - 29P$ and $P^\alpha - P^\beta = \delta I - 13P$, then $\alpha + \beta + \gamma - \delta$ is equal to

(1) 18
(2) 40
(3) 24
(4) 22

Official Ans. by NTA (3)

Sol. $P^2 = I - P$

$P^\alpha + P^\beta = \gamma I - 29P$, $P^\alpha - P^\beta = \delta I - 13P$

$P^4 = (I-P)^2 = I - 2P + P^2 = 2I - 3P$

$P^6 = (2I - 3P)(I - P) = 5I - 8P$

$P^8 = (2I - 3P)^2 = 4I - 12P + 9(I - P) = 13I - 21P$

$P^8 + P^6 = 18I - 29P$

$P^8 - P^6 = 8I - 13P$

$\alpha = 8; \beta = 6; \gamma = 18, \delta = 8$

$\alpha + \beta + \gamma - \delta = 8 + 6 + 18 - 8 = 24$

11. All the letters of the word PUBLIC are written in all possible orders and these words are written as in a dictionary with serial numbers. Then the serial number of the word PUBLIC is

(1) 580
(2) 582
(3) 578
(4) 576

Official Ans. by NTA (2)

Sol. B ……$\rightarrow$ 5! = 120

C ……$\rightarrow$ 5! = 120

I ……$\rightarrow$ 5! = 120

L ……$\rightarrow$ 5! = 120

PB ……$\rightarrow$ 4! = 24

PC ……$\rightarrow$ 4! = 24

PL ……$\rightarrow$ 4! = 24

PI ……$\rightarrow$ 4! = 24

P $\cup$ BC …..…$\rightarrow$ 2! = 2

$P \cup BI$ …..…. $\rightarrow 2! = 2$

$P \cup BLC$ ……. $\rightarrow 1! = 1$

$P \cup BLIC$ ……. $\rightarrow = 1$

Serial number = 4(120) + 4(24) + 6 = 582

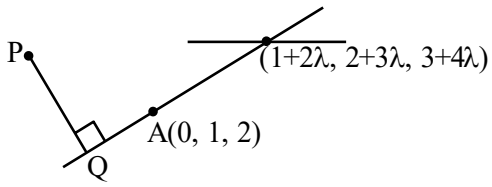
12. Let the line L pass through the point (0, 1, 2), intersect the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and be parallel to the plane 2x + y – 3z = 4. Then the distance of the point P(l, –9, 2) from the line L is

(1) 9 (2) $\sqrt{54}$

(3) $\sqrt{69}$ (4) $\sqrt{74}$

Official Ans. by NTA (4)

Sol.



$\overrightarrow{AB}\cdot\vec{n}$

$\Rightarrow \left[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1+4\lambda)\hat{k}\right]\cdot\left(2\hat{i}+\hat{j}-3\hat{k}\right)$

2 + 4λ + 1 + 3λ – 3 – 12λ = 0

5λ = 0 $\Rightarrow$ λ = 0

Line $\overrightarrow{AB}$, $\vec{r}=\hat{j}+2\hat{k}+\mu\left(\hat{i}+\hat{j}+\hat{k}\right)$

General form: Q(μ, 1 + μ, 2 + μ)

$\therefore \overrightarrow{PQ}\cdot\overrightarrow{AB}=0$

(μ – 1) + (10 + μ) + μ = 0

3μ = – 9 $\Rightarrow$ μ = – 3

$\therefore$ distance = $\sqrt{16+49+9}=\sqrt{74}$

13. A plane P contains the line of intersection of the plane $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})$ = 6 and $\vec{r}\cdot(2\hat{i}+3\hat{j}+4\hat{k})$ = –5. If P passes through the point (0, 2, –2), then the square of distance of the point (12, 12, 18) from the plane P is

(1) 1240 (2) 620

(3) 310 (4) 155

Official Ans. by NTA (2)

Sol. Equation of plane P is

$(x+y+z-6)+\lambda(2x+3y+4z+5)=0$

Plane passes through the point (0, 2, –2)

$\therefore (2-2-6)+\lambda(6-8+5)=0$

$-6+\lambda(3)=0$

$\lambda=2$

Equation of plane p is

$(x+y+z-6)+2(2x+3y+4z+5)=0$

$5x+7y+9z+4=0$

$d=\left|\frac{5\times12+7\times12+9\times18+4}{\sqrt{5^2+7^2+9^2}}\right|$

$d=\left|\frac{60+84+162+4}{\sqrt{25+49+81}}\right|$

$d=\frac{310}{\sqrt{155}}$

$d^2=\frac{310\times310}{155}=620$

14. Let f(x) be a function satisfying f(x) + f(π – x) = π^2, $\forall x \in R$. Then $\int_0^{\pi} f(x)\sin x\,dx$ is equal to

(1) $\frac{\pi^2}{4}$ (2) $\frac{\pi^2}{2}$

(3) $2\pi^2$ (4) π^2

Official Ans. by NTA (4)

Sol. $f(x)+f(\pi-x)=\pi^2$

$I=\int_0^{\pi} f(x)\sin x\,dx$

Applying King's Rule

$I=\int_0^{\pi} f(\pi-x).\sin(\pi-x)\,dx$

$2I=\int_0^{\pi}[f(x)+f(\pi-x)]\sin x\,dx$

$2I=\int_0^{\pi}\pi^2\sin x\,dx$

$2I=\pi^2.\int_0^{\pi}\sin x\,dx$

$2I=\pi^2\times2$

$I=\pi^2$

15. If the coefficients of x^7 in $\left(ax^2+\frac{1}{2bx}\right)^{11}$ and x^{-7} in $\left(ax-\frac{1}{3bx^2}\right)^{11}$ are equal, then

(1) 64ab = 243 (2) 729ab = 32

(3) 243ab = 64 (4) 32ab = 729

Official Ans. by NTA (2)

Sol. $\left(ax^2+\frac{1}{2bx}\right)^{11}$

$T_{r+1}={}^{11}C_r(ax^2)^{11-r}.\left(\frac{1}{2bx}\right)^r$

$={}^{11}C_r\,a^{11-r}.\left(\frac{1}{2b}\right)^r.x^{22-2r-r}={}^{11}C_r\,a^{11-r}.\left(\frac{1}{2b}\right)^r.x^{22-3r}$

$\therefore 22-3r=7$

$3r=15$

$r=5$

Again $\left(ax-\frac{1}{3bx^2}\right)^{11}$

$T_{r+1}={}^{11}C_r(ax)^{11-r}\left(-\frac{1}{3bx^2}\right)^r$

$={}^{11}C_r a^{11-r}.\left(\frac{-1}{3b}\right)^r.x^{11-r-2r}$

$\therefore 11-3r=-7$

$3r=18$

$r=6$

Now, $\frac{{}^{11}C_5a^6}{32b^5}=\frac{{}^{11}C_6.a^5}{3^6.b^6}$

$729ab=32$

16. Among the statements

(S1): $(p\Rightarrow q)\vee(\sim p)\wedge q)$ is a tautology

(S2): $(q\Rightarrow p)\Rightarrow((\sim p)\wedge q)$ is a contradiction

(1) neither (S1) and (S2) is True

(2) only (S1) is True

(3) only (S2) is True

(4) both (S1) and (S2) are True

Official Ans. by NTA (1)

Sol. $(p\to q)\vee((\sim p)\wedge q)$

p	q	$p\to q$	$\sim p\wedge q$	$(p\to q)\vee(\sim p)\wedge q)$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	F	T

Not a tautology

p	q	$q\to p$	$(\sim p)\wedge q$	$(q\to p)\vee(\sim p)\wedge q)$
T	T	T	F	F
T	F	T	F	F
F	T	F	T	T
F	F	T	F	F

Not a contradiction

17. If the tangents at the points P and Q on the circle $x^2+y^2-2x+y=5$ meet at the point R $\left(\frac{9}{4},2\right)$, then the area of the triangle PQR is

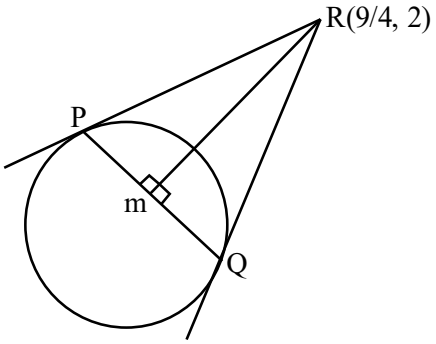
(1) $\frac{13}{4}$ (2) $\frac{13}{8}$

(3) $\frac{5}{4}$ (4) $\frac{5}{8}$

Official Ans. by NTA (4)

Sol. Equation of circle is $x^2+y^2-2x+y-5=0$

$R=\frac{5}{2}$



Length of $PR=QR=\sqrt{S_1}$

$=\sqrt{\frac{81}{16}+4-\frac{2\times9}{4}+2-5}=\frac{5}{4}$

Area of triangle PQR $=\frac{RL^3}{R^2+L^2}=\frac{\frac{5}{2}.\frac{125}{64}}{\frac{25}{4}+\frac{25}{16}}=\frac{5}{8}$

18. Let the vectors $\vec{a},\vec{b},\vec{c}$ represent three coterminous edges of a parallelopiped of volume V. Then the volume of the parallelopiped, whose coterminous edges are represented by $\vec{a},\vec{b}+\vec{c}$ and $\vec{a}+2\vec{b}+3\vec{c}$ is equal to

(1) 3V (2) 6V

(3) V (4) 2V

Official Ans. by NTA (3)

Sol. $V=[\vec{a}\ \vec{b}\ \vec{c}]$

$[\vec{a},\vec{b}+\vec{c},\vec{a}+2\vec{b}+3\vec{c}]$

$=\begin{vmatrix}1&0&0\\0&1&1\\1&2&3\end{vmatrix}[\vec{a}\,\vec{b}\,\vec{c}]=1(3-2)\,V=V.$

19. If gcd (m, n) = 1 and $1^2 - 2^2 + 3^2 - 4^2 + \ldots\ldots + (2021)^2 - (2022)^2 + (2023)^2 = 1012\ m^2n$, then $m^2 - n^2$ is equal to

(1) 200 (2) 240

(3) 220 (4) 180

Official Ans. by NTA (2)

Sol. $1^2-2^2+3^2-4^2+\ldots.(2021)^2-(2022)^2+(2023)^2=1012\ m^2n$

$=(1-2)(1+2)+(3-4)(3+4)+\ldots.+(2021-2022)\ (2021+2022)+(2023)^2$

$=(-1)(1+2+3+4+\ldots..+2022)+(2023)^2$

$=(-1)\cdot\frac{(2022)(2023)}{2}+(2023)^2$

$=2023(2023-1011)=2023\times 1012$

$m^2n=2023=17^2.7$

$m=17,\ n=7$

$m^2-n^2=17^2-7^2=240$

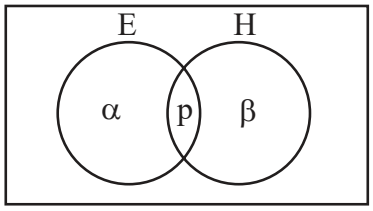
20. In a group of 100 persons 75 speak English and 40 speak Hindi. Each person speaks at least one of the two languages. If the number of persons, who speak only English is α and the number of persons who speak only Hindi is β, then the eccentricity of the ellipse $25(\beta^2x^2+\alpha^2y^2)=\alpha^2\beta^2$ is

(1) $\frac{3\sqrt{15}}{12}$ (2) $\frac{\sqrt{117}}{12}$

(3) $\frac{\sqrt{119}}{12}$ (4) $\frac{\sqrt{129}}{12}$

Official Ans. by NTA (3)

Sol.



$\alpha+p=75$ ....(1)

$\beta+p=40$ ....(2)

$\alpha+\beta+p=100$ ....(3)

From (1), (2) and (3)

P = 15, $\alpha=60$ and $\beta=25$

Now equation of ellipse: $25\left(\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}\right)=1$

$\frac{x^2}{144}+\frac{y^2}{25}=1$

$\Rightarrow e=\frac{\sqrt{119}}{12}$

SECTION-B

21. Let $f(x)=\frac{x}{\left(1+x^n\right)^{\frac{1}{n}}}$, $x\in R-\{-1\}$, $n\in N$, $n>2$.

If $f^n(x)$ = (fofof …… upto n times) (x), then $\lim\limits_{n\to\infty}\int\limits_0^1 x^{n-2}\left(f^n(x)\right)dx$ is equal to_____

Official Ans. by NTA (0)

Sol. Let $f(x)=\frac{x}{\left(1+x^n\right)^{1/x}}$, $x\in R-\{-1\}$, $n\in N$, $n>2$

$F^n(x)$ = (fofof… upto n times)(x),

then $\lim\limits_{n\to\infty}\int_0^1 x^{n-2}\left(f^n(x)\right)dx$

$f(f(x))=\frac{x}{\left(1+2x^n\right)^{1/n}}$

$f(f(f(x)))=\frac{x}{\left(1+3x^n\right)^{1/n}}$

Similarly $f^n(x)=\frac{x}{\left(1+n\cdot x^n\right)^{1/n}}$

Now $\lim\limits_{n\to\infty}\int\frac{x^{n-2}\cdot x\,dx}{\left(1+n\cdot x^n\right)^{1/n}}=\lim\limits_{n\to\infty}\int\frac{x^{n-1}\cdot dx}{\left(1+n\cdot x^n\right)^{1/n}}$

Now $1+nx^n=t$

$n^2\cdot x^{n-1}dx=dt$

$x^{n-1}dx=\frac{dt}{n^2}$

$\Rightarrow\lim\limits_{n\to\infty}\frac{1}{n^2}\int_1^{1+n}\frac{dt}{t^{1/n}}$

$\Rightarrow\lim\limits_{n\to\infty}\frac{1}{n^2}\left[\frac{t^{1-\frac{1}{n}}}{1-\frac{1}{n}}\right]_1^{1+n}$

$\Rightarrow\lim\limits_{n\to\infty}\frac{1}{n(n-1)}\left((1+n)^{\frac{n-1}{n}}-1\right)$ Now let $n=\frac{1}{h}$

$\Rightarrow\lim\limits_{h\to0}\frac{\left(1+\frac{1}{h}\right)^{1-h}-1}{\frac{1}{h}\frac{(1-h)}{h}}$

Using series expansion.

$\Rightarrow 0$

22. The value of tan 9° – tan 27° – tan 63° + tan 81° is _____.

Official Ans. by NTA (4)

Sol. The value of tan9° –tan27°–tan63° + tan81°

$\Rightarrow$ tan9°+cot9°–tan27°–cot27°

$\Rightarrow \dfrac{2}{\sin 18°} - \dfrac{2}{\sin 54°}$

$\Rightarrow \dfrac{2\times 4}{\sqrt{5}-1} - \dfrac{2\times 4}{\sqrt{5}+1}$

$\Rightarrow 4$

23. If the lines $\dfrac{x-1}{2} = \dfrac{2-y}{-3} = \dfrac{z-3}{\alpha}$

and $\dfrac{x-4}{5} = \dfrac{y-1}{2} = \dfrac{z}{\beta}$ intersect,

then the magnitude of the minimum value of $8\alpha\beta$ is _____.

Official Ans. by NTA (18)

Sol. If the lines $\dfrac{x-1}{2} = \dfrac{2-y}{-3} = \dfrac{z-3}{\alpha}$

And $\dfrac{x-4}{5} = \dfrac{y-1}{2} = \dfrac{z}{\beta}$ intersect

Point on first line (1, 2, 3) and point on second line (4, 1, 0).

Vector joining both points is $-3\hat{i} + \hat{j} + 3\hat{k}$

Now vector along first line is $2\hat{i} + 3\hat{j} + \alpha\hat{k}$

Also vector along second line is $5\hat{i} + 2\hat{j} + \beta\hat{k}$

Now these three vectors must be coplanar

$$\Rightarrow \begin{vmatrix} 2 & 3 & \alpha \\ 5 & 2 & \beta \\ -3 & 1 & 3 \end{vmatrix} = 0$$

$\Rightarrow 2(6-\beta) - 3(15+3\beta) + \alpha(11) = 0$

$\Rightarrow \alpha - \beta = 3$

Now $\alpha = 3 + \beta$

Given expression $8(3+\beta)\cdot\beta = 8(\beta^2 + 3\beta)$

$= 8\left(\beta^2 + 3\beta + \dfrac{9}{4} - \dfrac{9}{4}\right) = 8\left(\beta + \dfrac{3}{2}\right)^2 - 18$

So magnitude of minimum value = 18

24. If $(20)^{19} + 2(21)(20)^{18} + 3(21)^2(20)^{17} + \ldots\ldots + 20(21)^{19} = k(20)^{19}$, then k is equal to_____.

Official Ans. by NTA (400)

Sol. If $(20)^{19} + 2(21)(20)^{18} + 3(21)^2(20)^{17} + \ldots + 20(21)^{19} = k(20)^{19}$ then k is

$$20^{19}\left(1 + 2\cdot\left(\frac{21}{20}\right) + 3\left(\frac{21}{20}\right)^2 + \ldots + 20\left(\frac{21}{20}\right)^{19}\right) = k(20)^{19}$$

$$\Rightarrow k = 1 + 2\left(\frac{21}{20}\right) + 3\left(\frac{21}{20}\right)^2 + \ldots + 20\left(\frac{21}{20}\right)^{19} \quad \ldots(1)$$

$$\Rightarrow k\left(\frac{21}{20}\right) = \frac{21}{20} + 2\cdot\left(\frac{21}{20}\right)^2 + \ldots.$$

$$\ldots. + 19\left(\frac{21}{20}\right)^{19} + 20\cdot\left(\frac{21}{20}\right)^{20} \quad \ldots.(2)$$

Subtracting equation (2) from (1)

$$\Rightarrow k\left(\frac{-1}{20}\right) = 1 + \frac{21}{20} + \left(\frac{21}{20}\right)^2 + \ldots. + \left(\frac{21}{20}\right)^{19} - 20\cdot\left(\frac{21}{20}\right)^{20}$$

$$\Rightarrow k\left(\frac{-1}{20}\right) = \frac{1\left(\left(\frac{21}{20}\right)^{20} - 1\right)}{\left(\frac{21}{20} - 1\right)} - 20\cdot\left(\frac{21}{20}\right)^{20}$$

$$\Rightarrow k\left(\frac{-1}{20}\right) = 20\left(\frac{21}{20}\right)^{20} - 20 - 20\cdot\left(\frac{21}{20}\right)^{20}$$

$$\Rightarrow k\left(\frac{-1}{20}\right) = -20$$

$\Rightarrow k = 400$

25. The number of 4-letter words, with or without meaning, each consisting of 2 vowels and 2 consonants, which can be formed from the letters of the word UNIVERSE without repetition is_____

Official Ans. by NTA (432)

Sol. UNIVERSE

Vowels: E, I, U

Consonants: N, V, R, S

$\rightarrow {}^3C_2 \times {}^4C_2 \times 4! = 3 \times 6 \times 24 = 432$

26. The number of points, where the curve $y = x^5 - 20x^3 + 50x + 2$ crosses the x-axis, is _____.

Official Ans. by NTA (5)

Sol. $y = x^5 - 20x^3 + 50x + 2$

$$\frac{dy}{dx} = 5x^4 - 60x^2 + 50 = 5(x^4 - 12x^2 + 10)$$

$$\frac{dy}{dx} = 0 \Rightarrow x^4 - 12x^2 + 10 = 0$$

$$\Rightarrow x^2 = \frac{12 \pm \sqrt{144 - 40}}{2}$$

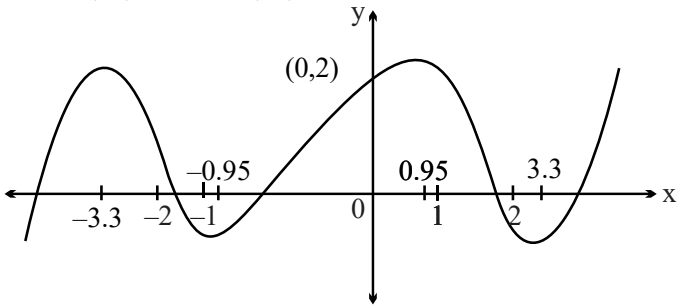
$$\Rightarrow x^2 = 6 \pm \sqrt{26} \Rightarrow x^2 \approx 6 \pm 5.1$$

$$\Rightarrow x^2 \approx 11.1, 0.9$$

$$\Rightarrow x \approx \pm 3.3, \pm 0.95$$

f(0) = 2, f (1) = + ve, f (2) = – ve

f(–1) = – ve, f (–2) = +ve



27. For $\alpha, \beta, z \in C$ and $\lambda > 1$, if $\sqrt{\lambda - 1}$ is the radius of the circle $|z - \alpha|^2 + |z - \beta|^2 = 2\lambda$, then $|\alpha - \beta|$ is equal to _____.

Official Ans. by NTA (2)

Sol. For circle :

$$|z - z_1|^2 + |z - z_2|^2 = |z_1 - z_2|^2$$

$$r = \frac{|z_1 - z_2|}{2} = \frac{|\alpha - \beta|}{2} = \sqrt{\lambda - 1}$$

$$2\lambda = |\alpha - \beta|^2$$

$$|\alpha - \beta| = 2\sqrt{\lambda - 1}$$

$$|\alpha - \beta|^2 = 4\lambda - 4 = 2\lambda$$

$\lambda = 2$

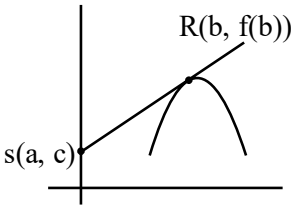
$$\Rightarrow |\alpha - \beta|^2 = 4$$

$$|\alpha - \beta| = 2$$

28. Let a curve y = f(x), x ∈ (0, ∞) pass through the points $P\left(1, \frac{3}{2}\right)$ and $Q\left(a, \frac{1}{2}\right)$. If the tangent at any point $R(b, f(b))$ to the given curve cuts the y-axis at the point S (0, c) such that bc = 3, then $(PQ)^2$ is equal to _____.

Official Ans. by NTA (5)

Sol.



Equation of tangent at R(b, f(2)) is

$$y - f(b) = f'(b).(x - b)$$

which passes through (0, c)

$$\Rightarrow c - f(b) = f'(b).(-b)$$

$$\Rightarrow \frac{3}{b} - f(b) = f'(b).(-b)$$

$$\Rightarrow bf'(b) - f(b) = -\frac{3}{b}$$

$$\Rightarrow \frac{bf'(b) - f(b)}{b^2} = -\frac{3}{b^3}$$

$$\Rightarrow d\left(\frac{f(b)}{b}\right) = -\frac{3}{b^3} \Rightarrow \frac{f(b)}{b} = \frac{3}{2b^2} + \lambda$$

Which passes through (1, 3/2)

$$\Rightarrow \frac{3}{2} = \frac{3}{2} + \lambda \Rightarrow \lambda = 0$$

$$\Rightarrow f(b) = \frac{3}{2b}$$

$$f(a) = \frac{1}{2} \Rightarrow \frac{1}{2} = \frac{3}{2b} \Rightarrow b = 3$$

$$\Rightarrow c = 1 \Rightarrow Q(3, 1/2)$$

$$\Rightarrow PQ^2 = 2^2 + (1)^2 = 5$$

29. Let the eccentricity of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is reciprocal to that of the hyperbola $2x^2 - 2y^2 = 1$. If the ellipse intersects the hyperbola at right angles, then square of length of the latus-rectum of the ellipse is _____.

Official Ans. by NTA (2)

Sol. $e_H = \sqrt{2}$

$e_E = \frac{1}{\sqrt{2}}$

Since the curves intersect each other orthogonally

The ellipse and the hyperbola are confocal

$H: \frac{x^2}{1/2} - \frac{y^2}{1/2} = 1$

$\Rightarrow$ foci $= (1,0)$

For ellipse $a.e_E = 1$

$\Rightarrow a = \sqrt{2}$

$(e_E)^2 = \frac{1}{2} \Rightarrow 1 - \frac{b^2}{a^2} = \frac{1}{2} \Rightarrow \frac{b^2}{a^2} = \frac{1}{2}$

$\Rightarrow b^2 = 1$

Length of L.R. $= \frac{2b^2}{a} = \frac{2}{\sqrt{2}} = \sqrt{2}$

30. If the mean and variance of the frequency distribution

x_i	2	4	6	8	10	12	14	16
f_i	4	4	α	15	8	β	4	5

are 9 and 15.08 respectively, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is _____.

Official Ans. by NTA (25)

Sol.

x_i	f_i	$f_i x_i$	$f_i x_i^2$
2	4	8	16
4	4	16	64
6	α	6α	36α
8	15	120	960
10	8	80	800
12	β	12β	144β
14	4	56	784
16	5	80	1280

$N = \sum f_i = 40 + \alpha + \beta$

$\sum f_i x_i = 360 + 6\alpha + 12\beta$

$\sum f_i x_i^2 = 3904 + 36\alpha + 144\beta$

$\text{Mean}(\bar{x}) = \frac{\sum f_i x_i}{\sum f_i} = 9$

$\Rightarrow 360 + 6\alpha + 12\beta = 9(40 + \alpha + \beta)$

$3\alpha = 3\beta \Rightarrow \alpha = \beta$

$\sigma^2 = \frac{\sum f_i x_1^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2$

$\Rightarrow \frac{3904 + 36\alpha + 144\beta}{40 + \alpha + \beta} - (\bar{x})^2 = 15.08$

$\Rightarrow \frac{3904 + 180\alpha}{40 + 2\alpha} - (9)^2 = 15.08$

$\Rightarrow \alpha = 5$

Now, $\alpha^2 + \beta^2 - \alpha\beta = \alpha^2 = 25$

SECTION-A

31. A 2 meter long scale with least count of 0.2 cm is used to measure the locations of objects on an optical bench. While measuring the focal length of a convex lens, the object pin and the convex lens are placed at 80 cm mark and lm mark, respectively. The image of the object pin on the other side of lens coincides with image pin that is kept at 180 cm mark. The % error in the estimation of focal length is:

(1) 1.02 (2) 0.85

(3) 1.70 (4) 0.51

Official Ans. by NTA (3)

Sol. Least count = 0.2 cm

$u = (100 \pm 0.2) - (80 \pm 0.2) = (20 \pm 0.4)\text{cm}$

$v = (180 \pm 0.2) - (100 \pm 0.2) = (80 \pm 0.4)\text{cm}$

From lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{f} = \frac{1}{80} - \frac{1}{-20}$$

f = 16 cm

Also $\frac{\Delta v}{v^2} + \frac{\Delta u}{u^2} = \frac{\Delta f}{f^2}$

$$\Rightarrow \frac{\Delta f}{f} \times 100 = \left(\frac{\Delta v}{v^2} + \frac{\Delta u}{u^2}\right) \times f \times 100$$

$$\Rightarrow \%f = \left(\frac{0.4}{400} + \frac{0.4}{6400}\right) \times 16 \times 100$$

=1.70

32. A capacitor of capacitance 150.0 μF is connected to an alternating source of emf given by E = 36 sin(120πt) V. The maximum value of current in the circuit is approximately equal to :

(1) 2A (2) $\frac{1}{\sqrt{2}}$A

(3) $\sqrt{2}$A (4) $2\sqrt{2}$A

Official Ans. by NTA (1)

Sol. $I_0 = \frac{E_0}{x_c} = \frac{E_0}{\frac{1}{\omega_c}} = E_0\omega_c$

$\Rightarrow I_0 = 36 \times 120\pi \times 150 \times 10^{-6}$

$\Rightarrow I_0 = 2.03$

$\simeq 2A$

33. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A: When you squeeze one end of a tube to get toothpaste out from the other end, Pascal's principle is observed.

Reason R: A change in the pressure applied to an enclosed incompressible fluid is transmitted undiminished to every portion of the fluid and to the walls of its container.

In the light of the above statements, choose the most appropriate answer from the options given below

(1) **A** is not correct but **R** is correct

(2) **A** is correct but **R** is not correct

(3) Both **A** and **R** are correct and R is the correct explanation of **A**

(4) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**

Official Ans. by NTA (3)

Sol. (R) is the statement of Pascal's principle & which explains the assertion (S)

34. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A: The phase difference of two light waves change if they travel through different media having same thickness, but different indices of refraction.

Reason R: The wavelengths of waves are different in different media.

In the light of the above statements, choose the most appropriate answer from the options given below

(1) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**

(2) **A** is correct but **R** is not correct

(3) Both **A** and **R** are correct and **R** is the correct explanation of **A**

(4) **A** is not correct but **R** is correct

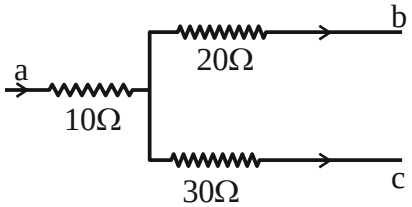
Official Ans. by NTA (3)

Sol. As medium changes, optical path changes.

Also, $\frac{\Delta x}{\lambda}=\frac{\Delta\phi}{2\pi}$

Hence phase difference changes.

35. Figure shows a part of an electric circuit. The potentials at points a, b and c are 30 V, 12 V and 2V respectively. The current through the 20Ω resistor will be.

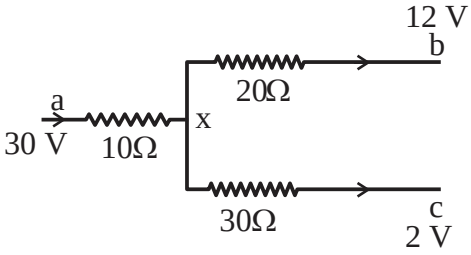


(1) 0.4 A (2) 0.2 A

(3) 0.6 A (4) 1.0 A

Official Ans. by NTA (1)

Sol. Sum of current at junction point will be zero :



$$\frac{x-30}{10}+\frac{x-12}{20}+\frac{x-2}{30}=0$$

$$\Rightarrow x\left(\frac{1}{10}+\frac{1}{20}+\frac{1}{30}\right)=\frac{30}{10}+\frac{12}{20}+\frac{2}{30}$$

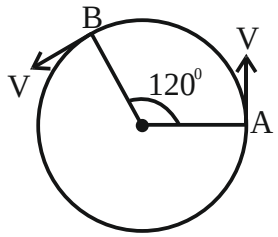
$$\Rightarrow x\left(\frac{6+3+2}{60}\right)=\frac{180+36+4}{60}$$

$$\Rightarrow x=\frac{220}{11}=20V$$

$$\therefore \text{ Current through } 20\Omega=\frac{x-12}{20}$$

$$=\frac{20-12}{20}=\frac{2}{5}=0.4A$$

36. As shown in the figure, a particle is moving with constant speed π m/s. Considering its motion from A to B, the magnitude of the average velocity is:



(1) π m/s (2) $\sqrt{3}$ m/s

(3) $2\sqrt{3}$ m/s (4) $1.5\sqrt{3}$ m/s

Official Ans. by NTA (4)

Sol. $\left|\langle\vec{v}\rangle\right|=\frac{\left|\vec{r}_f-\vec{r}_i\right|}{\Delta t}$

$$=\frac{2R\cos\left[\frac{\pi-\theta}{2}\right]}{\frac{2\pi R}{3v}}=3\cos 30^0$$

$1.5\sqrt{3}\ m/s$

Correct option is (4)

37. The work functions of Aluminium and Gold are 4.1 eV and 5.1 eV respectively. The ratio of the slope of the stopping potential versus frequency plot for Gold to that of Aluminium is

(1) 1.24 (2) 2

(3) 1 (4) 1.5

Official Ans. by NTA (3)

Sol. $eV_s = k_{max}$

$$V_s = \left\{\frac{h}{e}\right\} f + \left\{\frac{-\phi}{e}\right\}$$

Slope is independent of nature of metal

$$slope(V_s)^{Gold} = slope(V_s)^{Aluimiun}$$

38. The ratio of speed of sound in hydrogen gas to the speed of sound in oxygen gas at the same temperature is:

(1) 4 : 1 (2) 1 : 2

(3) 1 : 4 (4) 1 : 1

Official Ans. by NTA (1)

Sol. $C = \sqrt{\frac{\gamma RT}{M}}$

$$C \propto \frac{1}{\sqrt{M}}$$

$$\frac{C_{H_2}}{C_{O_2}} = \sqrt{\frac{32}{2}} = 4:1$$

Correct option (1)

39. A child of mass 5 kg is going round a merry-go-round that makes 1 rotation in 3.14 s. The radius of the merry-go-round is 2 m. The centrifugal force on the child will be

(1) 80 N (2) 50 N

(3) 100 N (4) 40 N

Official Ans. by NTA (4)

Sol. $\omega = \frac{2\pi}{3.14} = 2\ rad/s$

$$\left|\bar{f}_{centrifugal}\right| = \left|-m\bar{a}_{Ref.}\right|$$

$$= M\omega^2 R$$

= 40 N

Correct option (4)

40. A particle starts with an initial velocity of 10.0ms^{-1} along x-direction and accelerates uniformly at the rate of 2.0 ms^{-2}. The time taken by the particle to reach the velocity of 60.0 ms^{-1} is_____.

(1) 6s (2) 3s

(3) 30s (4) 25s

Official Ans. by NTA (4)

Sol. v = u + at

60 = 10 + 2t

t = 25 sec.

Correct option (4)

41. Choose the incorrect statement from the following:

(1) The speed of satellite in a given circular orbit remains constant.

(2) For a planet revolving around the sun in an elliptical orbit, the total energy of the planet remains constant.

(3) When a body fall towards earth, the displacement of earth towards the body is negligible.

(4) The linear speed of a planet revolving around the sun remains constant.

Official Ans. by NTA (4)

Sol. Planets revolve in elliptical paths around sun. Thus their linear speed is not constant

42. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A: Diffusion current in a p-n junction is greater than the drift current in magnitude if the junction is forward biased.

Reason R: Diffusion current in a p-n junction is from the n-side to the p-side if the junction is forward biased.

In the light of the above statements, choose the most appropriate answer from the options given below

(1) Both **A** and **R** are correct and **R** is the correct explanation of **A**

(2) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**

(3) **A** is correct but **R** is not correct

(4) **A** is not correct but **R** is correct

Official Ans. by NTA (3)

Sol. In forward biased condition, diffusion of majority charge carriers takes place from p-side to n-side which constitute the diffusion current.

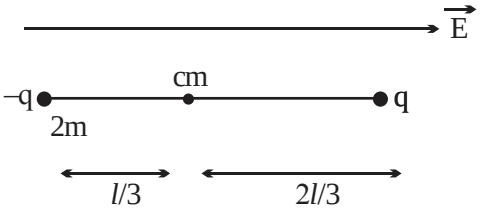
43. A dipole comprises of two charged particles of identical magnitude q and opposite in nature. The mass 'm' of the positive charged particle is half of the mass of the negative charged particle. The two charges are separated by a distance '*l*'. If the dipole is placed in a uniform electric field '$\vec{E}$'; such a way that dipole axis makes a very small angle with the electric field, '$\vec{E}$'. The angular frequency of the oscillations of the dipole when released is given by:

(1) $\sqrt{\frac{8qE}{3ml}}$ (2) $\sqrt{\frac{4qE}{ml}}$

(3) $\sqrt{\frac{4qE}{3ml}}$ (4) $\sqrt{\frac{8qE}{ml}}$

Official Ans. by NTA (3)

Sol.



If released, it will oscillate about centre of mass.

For small 'θ'

$\tau = -PE\,.\,\theta$

$\Rightarrow \left[2m\,\frac{l^2}{9} + m\,\frac{4l^2}{9}\right]\alpha = -qlE.\theta$

$\Rightarrow \frac{2ml^2}{3}\alpha = -qlE.\theta \quad \Rightarrow \alpha = -\frac{3qE}{2ml}\theta$

$\omega = \sqrt{\frac{3qE}{2ml}}$

44. The energy density associated with electric field $\vec{E}$ and magnetic field $\vec{B}$ of an electromagnetic wave in free space is given by (ϵ_0 – permittivity of free space, μ_0 - permeability of free space)

(1) $U_E = \frac{E^2}{2\epsilon_0}, U_B = \frac{B^2}{2\mu_0}$

(2) $U_E = \frac{E^2}{2\epsilon_0}, U_B = \frac{\mu_0 B^2}{2}$

(3) $U_E = \frac{\epsilon_0 E^2}{2}, U_B = \frac{\mu_0 B^2}{2}$

(4) $U_E = \frac{\epsilon_0 E^2}{2}, U_B = \frac{B^2}{2\mu_0}$

Official Ans. by NTA (4)

Sol. $U_E = \frac{1}{2}\epsilon_0 E^2, U_B = \frac{B^2}{2\mu_0}$

45. The temperature of an ideal gas is increased from 200 K to 800 K. If r.m.s. speed of gas at 200K is v_0. Then, r.m.s. speed of the gas at 800 K will be:

(1) v_0 (2) $4v_0$

(3) $\frac{v_0}{4}$ (4) $2v_0$

Official Ans. by NTA (4)

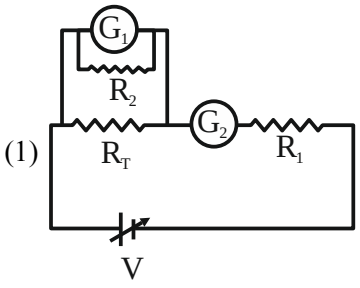
Sol. $V_{rms} = \sqrt{\frac{3RT}{M}}$

$\Rightarrow V_{rms} \propto \sqrt{T}$

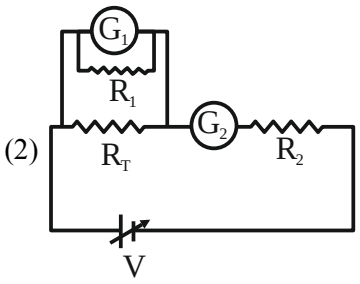
Increasing temperature 4 times, rms speed gets doubled.

46. A student is provided with a variable voltage source V, a test resistor R_r =10Ω, two identical galvanometers G_1 and G_2 and two additional resistors, R_1 = 10MΩ and R_2 = 0.001Ω. For conducting an experiment to verify ohms law, the most suitable circuit is:

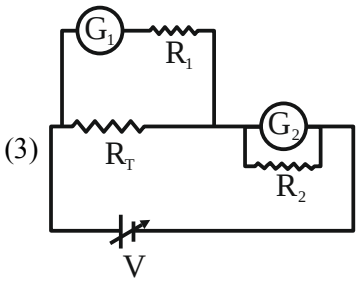
(1)



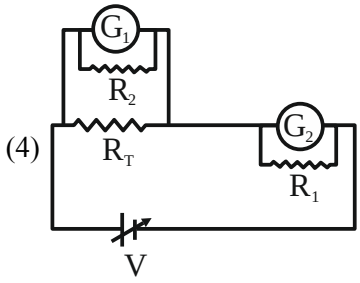
(2)



(3)



(4)



Official Ans. by NTA (3)

Sol. To convert galvanometer into ammeter low resistances should be added into parallel & for voltmeter conversion, a very high resistance should be added in series.

47. A body cools in 7 minutes from 60°C to 40°C. The temperature of the surrounding is 10°C. The temperature of the body after the next 7 minutes will be:

(1) 32°C (2) 30°C

(3) 28°C (4) 34°C

Official Ans. by NTA (3)

Sol. using average rate of Newton's law of cooling

$$\frac{T_1-T_2}{t}=K\left(\frac{T_1+T_2}{2}-T_s\right)$$

Given $\frac{60-40}{7}=K(50-10)$ … (i)

& $\frac{40-T}{7}=K\left(\frac{40+T}{2}-10\right)$ … (ii)

From (i) & (ii)

T = 28°C

48. A small particle of mass m moves in such a way that its potential energy U = $\frac{1}{2}m\omega^2r^2$ where ω is constant and r is the distance of the particle from origin. Assuming Bohr's quantization of momentum and circular orbit, the radius of n^{th} orbit will be proportional to.

(1) $\sqrt{n}$ (2) n

(3) n^2 (4) $\frac{1}{n}$

Official Ans. by NTA (1)

Sol. $U=\frac{1}{2}m\omega^2r^2$

$$F=-\frac{dv}{dr}=-m\omega^2r$$

Now $m\omega^2r=\frac{mv^2}{r}\Rightarrow v=\omega r$ …(i)

& $mvr=\frac{nh}{2\pi}$ …(ii)

From (i) & (ii)

$$m\omega r^2=\frac{nh}{2\pi}$$

$$\Rightarrow r\propto\sqrt{n}$$

49. For an amplitude modulated wave the minimum amplitude is 3V, while the modulation index is 60%. The maximum amplitude of the modulated wave is:

(1) 15V

(2) 12V

(3) 10V

(4) 5V

Official Ans. by NTA (2)

Sol. Modulation index $= \frac{A_m}{A_c} = 0.6$

Minimum amplitude of modulated wav $= A_c - A_m = 3$

$\therefore A_C - 0.6A_c = 3 \Rightarrow 0.4A_c = 3$

$A_c = \frac{3}{0.4} = \frac{15}{2} = 7.5V$

$A_m = 0.6A_c = 4.5V$

$\therefore$ Maximum amplitude = $A_c + A_m$

$= 7.5 + 4.5 = 12V$

$\therefore$ Correct option is (2)

50. The weight of a body on the surface of the earth is 100 N. The gravitational force on it when taken at a height, from the surface of earth, equal to one-fourth the radius of the earth is:

(1) 100 N

(2) 64 N

(3) 50 N

(4) 25 N

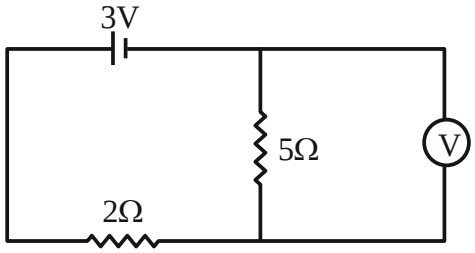
Official Ans. by NTA (2)

Sol. $\Rightarrow g' = \frac{gR^2}{r^2} = \frac{gR^2}{\left(R + \frac{R}{4}\right)^2} = \frac{16g}{25}$

$\therefore Weight = \frac{16}{25} \times 100 = 64N$

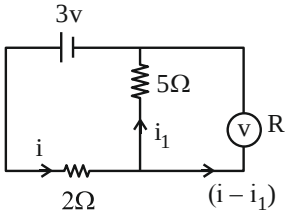
SECTION-B

51. As shown in the figure, the voltmeter reads 2V across 5 Ω resistor. The resistance of the voltmeter is_____Ω.



Official Ans. by NTA (20)

Sol. $i_1 = \frac{2V}{5\Omega} = \frac{2}{5}A$



$i = \frac{1V}{2\Omega} = \frac{1}{2}A$

$\therefore$ Current through voltmeter = $i - i_1$

$= \frac{1}{2} - \frac{2}{5} = \frac{5-4}{10} = \frac{1}{10}A$

$\therefore$ For voltmeter

$2 = \left(\frac{1}{10}\right)R \Rightarrow R = 20\Omega$

52. A metal block of mass m is suspended from a rigid support through a metal wire of diameter 14 mm. The tensile stress developed in the wire under equilibrium state is 7×10^5 Nm^{-2}. The value of mass m is _____ kg.

(Take, g = 9.8 ms^{-2} and $\pi = \frac{22}{7}$)

Official Ans. by NTA (11)

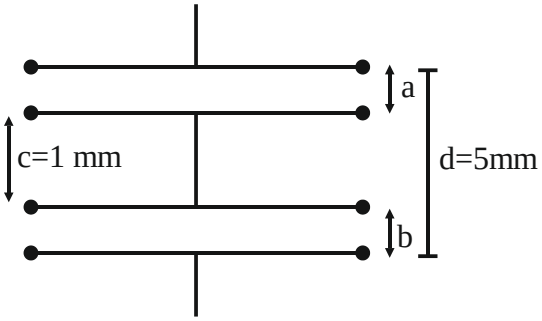
Sol. Tensile stress, $\sigma = \frac{F}{A} = \frac{4mg}{\pi D^2}$

$\therefore m = \frac{\pi D^2 \sigma}{4g}$

$= \frac{22}{7} \times \frac{\left(14\times10^{-3}\right)^2 \times 7\times10^5}{4\times9.8}$

= 11 kg

53. As shown in the figure, two parallel plate capacitors having equal plate area of 200 cm^2 are joined in such a way that a ≠ b. The equivalent capacitance of the combination is x $\in_0$ F. The value of x is_____.



Official Ans. by NTA (5)

Sol. $c = \frac{\varepsilon_0 A}{(d-c)}$

$= \frac{\varepsilon_0 \times 200\times10^{-4}}{4\times10^{-3}}$

$\therefore x = 5$

The situation is equivalent to a conducting slab placed between the plates

54. A ring and a solid sphere rotating about an axis passing through their centers have same radii of gyration. The axis of rotation is perpendicular to plane of ring. The ratio of radius of ring to that of sphere is $\sqrt{\frac{2}{x}}$. The value of x is _____

Official Ans. by NTA (5)

Sol. For ring $I = mR_1^2 = mK_1^2$

$\therefore$ Radius of gyration $K_1 = R_1$

For solid sphere

$I' = \frac{2}{5} m'R_2^2 = m'K_2^2$

$\therefore$ Its radius of gyration $= K_2 = \sqrt{\frac{2}{5}} R_2$

$\because K_1 = K_2$

$\therefore R_1 = \sqrt{\frac{2}{5}} R_2$

$\therefore \frac{R_1}{R_2} = \sqrt{\frac{2}{5}}$

$\therefore x = 5$

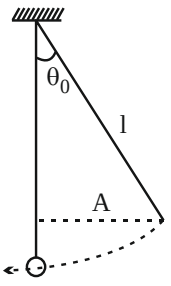
55. A simple pendulum with length 100 cm and bob of mass 250 g is executing S.H.M. of amplitude 10cm. The maximum tension in the string is found to be $\frac{x}{40}$ N. The value of x is_____.

Official Ans. by NTA (99)

Sol. $\sin\theta_0 = \frac{A}{l} = \frac{10}{100} = \frac{1}{10}$

From conservation of energy

$\frac{1}{2}mv^2 = mgl(1-\cos\theta)$



Maximum tension occurs at mean position.

$\therefore T - mg = \frac{mv^2}{l}$

$\Rightarrow T = mg + \frac{mv^2}{l}$

$$\therefore T = mg + 2mg(1-\cos\theta)$$

$$= mg\left[1+2\left(1-\sqrt{1-\sin^2\theta}\right)\right]$$

$$= mg\left[3-2\sqrt{1-\frac{1}{100}}\right]$$

$$= \frac{250}{1000}\times 9.8\left[3-2\left(1-\frac{1}{200}\right)\right] = \frac{99}{40}$$

$$\therefore\ x = 99$$

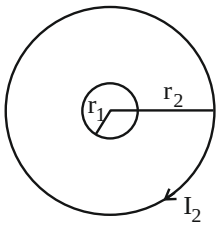
56. Two concentric circular coils with radii 1 cm and 1000 cm, and number of turns 10 and 200 respectively are placed coaxially with centers coinciding. The mutual inductance of this arrangement will be _____ $\times 10^{-8}$ H.

(Take, π^2=10)

Official Ans. by NTA (4)

Sol. $r_1 = 1cm,\ N_1 = 10$

$r_2 = 1000cm,\ N_2 = 200$



$$\phi_{1,2} = MI_2$$

$$N_2\vec{B}_2 \cdot N_1\vec{A}_1 = MI_2$$

$$\Rightarrow N_1N_2\frac{\mu_0 I_2}{2r_2}\cdot\pi r_1^2 = MI_2$$

$$\Rightarrow M = \frac{10\times 200\times 4\pi\times 10^{-7}\times\pi\times(0.01)^2}{2\times 10}$$

$$\Rightarrow M = 4\times 10^{-8}$$

57. A beam of light consisting of two wavelengths 7000 Å and 5500 Å is used to obtain interference pattern in Young's double slit experiment. The distance between the slits is 2.5 mm and the distance between the plane of slits and the screen is 150 cm. The least distance from the central fringe, where the bright fringes due to both the wavelengths coincide, is n × 10^{-5} m. The value of n is_____.

Official Ans. by NTA (462)

Sol. d = 2.5 mm, D = 150 cm

Fringe width $\beta = \frac{\lambda D}{d}$

Let n^{th} bright tringle of λ_1 match with m^{th} bright triangle of λ_2

$$\Rightarrow n\beta_1 = m\beta_2$$

$$\Rightarrow n\lambda_1 = m\lambda_2 \Rightarrow \frac{n}{m} = \frac{\lambda_2}{\lambda_1} = \frac{5500}{7000}$$

$$\Rightarrow \frac{n}{m} = \frac{11}{14}$$

Distance where bright fringe will match

$$= n\beta_1 = \frac{11\times 7000A^0\times 150\ cm}{0.25\ cm}$$

$$= 462\times 10^{-5}$$

58. A body is dropped on ground from a height 'h_1' and after hitting the ground, it rebounds to a height 'h_2' If the ratio of velocities of the body just before and after hitting ground is 4, then percentage loss in kinetic energy of the body is $\frac{x}{4}$. The value of x is_____.

Official Ans. by NTA (375)

Sol. Let V_1 and V_2 are velocity just before and just after hitting the floor.

$$\frac{V_1}{V_2}=4\Rightarrow V_1=4V_2$$

$$KE_{before}=\frac{1}{2}mV_1^2$$

$$KE_{after}=\frac{1}{2}mV_2^2=\frac{1}{2}\frac{m.V_1^2}{16}$$

$$\Delta KE=\frac{1}{2}mV_1^2\left(\frac{1}{16}-1\right)=\frac{-15}{32}mV_1^2$$

$$\%\text{ change }=\frac{\Delta KE}{KE_{before}}\times 100\%$$

$$=\frac{-15}{16}\times 100=\frac{-375}{4}\%$$

59. Experimentally it is found that 12.8 eV energy is required to separate a hydrogen atom into a proton and an electron. So the orbital radius of the electron in a hydrogen atom is $\frac{9}{x}\times 10^{-10}$ m. The value of the x is _____.

(1eV = 1.6 × l0^{-19}J, $\frac{1}{4\pi\in_0}=9\times 10^9\,\text{Nm}^2/\text{C}^2$ and electronic charge = 1.6 × l0^{-19}J C)

Official Ans. by NTA (16)

Sol. Binding energy of system $=\frac{ke^2}{2r}$ joule and

$$\frac{ke^2}{2r}=12.8\ ev$$

$$\frac{9\times 10^9\times\left(1.6\times 10^{-19}\right)^2}{2r}=12.8\times 1.6\times 10^{-19}$$

$$\Rightarrow r=\frac{9\times 10^9\times 1.6\times 10^{-19}}{12.8\times 2}$$

$$\Rightarrow r=\frac{9\times 10^{-10}}{16}$$

60. A proton with a kinetic energy of 2.0 eV moves into a region of uniform magnetic field of magnitude $\frac{\pi}{2}\times 10^{-3}$ T. The angle between the direction of magnetic field and velocity of proton is 60°. The pitch of the helical path taken by the proton is_____cm.

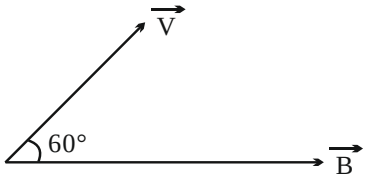
(Take, mass of proton = 1.6 × 10^{-27} kg and Charge on proton = 1.6 × 10^{-19} C).

Official Ans. by NTA (40)

Sol. $B=\frac{\pi}{2}\times 10^{-3}$

$$K.E.=\frac{1}{2}mV^2$$

$$\Rightarrow V=\sqrt{\frac{2KE}{m}}$$



Pitch = v cos 60° × time period of one rotation

$$=v\cos 60^0\times\frac{2\pi m}{eB}$$

$$=\sqrt{\frac{2\times 2\times 1.6\times 10^{-19}}{1.6\times 10^{-27}}}\times\cos 60^0\times\frac{2\pi\times 1.6\times 10^{-27}}{1.6\times 10^{-19}\times\frac{\pi}{2}\times 10^{-3}}$$

$$=2\times 10^4\times\frac{1}{2}\times 4\times 10^{-5}$$

$$=4\times 10^{-1}m=40cm$$

SECTION-A

61. Ion having highest hydration enthalpy among the given alkaline earth metal ions is:-

(1) Be^{2+}

(2) Ba^{2+}

(3) Sr^{2+}

(4) Ca^{2+}

Official Ans. by NTA (1)

Sol. Hydration enthalpy $\propto \frac{1}{\text{size}}$

Down the group as size increases hydration enthalpy decreases

Order : $Be^{2+} > Mg^{+2} > Ca^{+2} > Sr^{+2} > Ba^{+2}$

62. The IUPAC name of $K_3[Co(C_2O_4)_3]$ is :-

(1) Potassium trioxalatocobaltate(III)

(2) Potassium tris(oxalato)cobalt(III)

(3) Potassium tris(oxalato)cobaltate(III)

(4) Potassium trioxalatocobalt(III)

Official Ans. by NTA (1)

Sol. IUPAC name of $K_3[Co(C_2O_4)_3]$ is

Potassium trioxalatocobaltate(III)

63. Match **List I** with **List II**

List I	**List II**
Natural Amino acid	**One Letter Code**
(A) Arginine	(I) D
(B) Aspartic acid	(II) N
(C) Asparagine	(III) A
(D) Alanine	(IV) R

Choose the correct answer from the options given below :-

(1) (A)–IV, (B)–I, (C)–III, (D)–II

(2) (A)–I, (B)–III, (C)–IV, (D)–II

(3) (A)–III, (B)–I, (C)–II, (D)–IV

(4) (A)–IV, (B)–I, (C)–II, (D)–III

Official Ans. by NTA (4)

Sol. Factual.

64. Element not present in Nessler's reagent is:-

(1) Hg

(2) I

(3) K

(4) N

Official Ans. by NTA (4)

Sol. Nessler reagent is – $K_2[HgI_4]$

65. Structure of $BeCl_2$ in solid state, vapour phase and at very high temperature respectively are :-

(1) Dimeric, Polymeric, Monomeric

(2) Polymeric, Dimeric, Monomeric

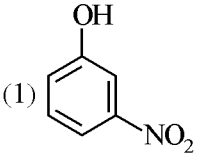
(3) Monomeric, Dimeric, Polymeric

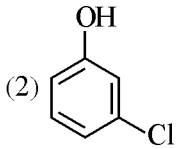
(4) Polymeric, Monomeric, Dimeric

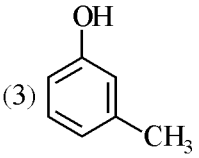
Official Ans. by NTA (2)

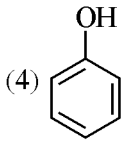
Sol. In solid state $BeCl_2$ as polymer, in vapour state it form chloro-bridged dimer while above 1200K it is monomer.

66. The strongest acid from the following is

(1) 

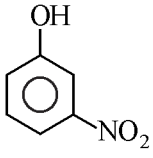
(2) 

(3) 

(4) 

Official Ans. by NTA (1)

Sol. Strongest acid from the following is



$-NO_2$ group has more EWG nature so more acidic,

67. Group-13 elements react with O_2 in amorphous form to form oxides of type M_2O_3 (M = element). Which among the following is the most basic oxide?

(1) Al_2O_3

(2) Ga_2O_3

(3) Tl_2O_3

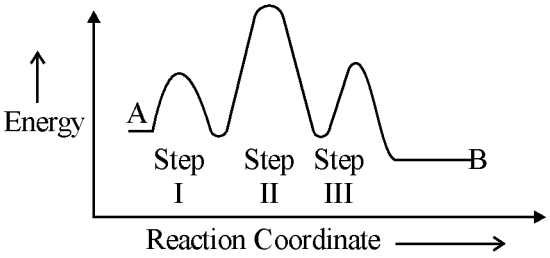
(4) B_2O_3

Official Ans. by NTA (3)

Sol. As electropositive character increases basic character of oxide increases.

$$\underbrace{B_2O_3}_{\text{acidic}} < \underbrace{Al_2O_3 < Ga_2O_3}_{\text{amphoteric}} < \underbrace{In_2O_3 < Tl_2O_3}_{\text{basic}}$$

68. Consider the following reaction that goes from A to B in three steps as shown below:-



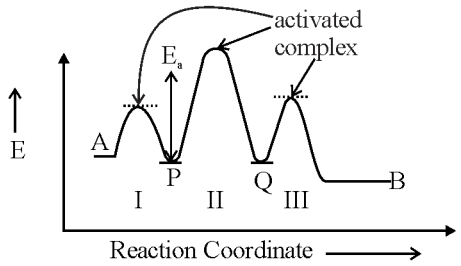
Choose the correct

	Number of intermediates	Number of Activated complexes	Rate determining step
(1)	3	2	II
(2)	2	3	II
(3)	2	3	I
(4)	2	3	III

Official Ans. by NTA (2)

Sol. Step with highest activation energy is RDS, so step II is RDS

No. of activated complex = 3



P and Q are intermediates

(Number of intermediates = 2)

69. Given below are two statements : one is labelled as **"Assertion A"** and the other is labelled as **"Reason R".**

Assertion A : In the complex $Ni(CO)_4$ and $Fe(CO)_5$, the metals have zero oxidation state.

Reason R : Low oxidation states are found when a complex has ligands capable of π-donor character in addition to the σ-bonding.

In the light of the above statements, choose the **most appropriate** answer from the option given below.

(1) **A** is correct but **R** is not correct

(2) **A** is not correct but **R** is correct

(3) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**

(4) Both **A** and **R** are correct and **R** is the correct explanation of **A**.

Official Ans. by NTA (1)

Sol. Low oxidation state of metals can stabilized by synergic bonding so ligand has to be π-acceptor.

70. During the reaction of permanganate with thiosulphate, the change in oxidation of manganese occurs by value of 3. Identify which of the below medium will favour the reaction.

(1) aqueous acidic

(2) aqueous neutral

(3) both aqueous acidic and neutral

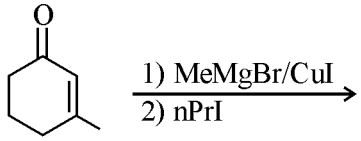
(4) both aqueous acidic and faintly alkaline.

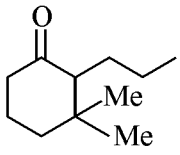
Official Ans. by NTA (2)

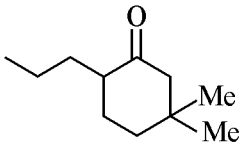
Sol. In neutral or weakly alkaline solution oxidation state of Mn changes by 3 unit

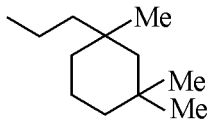
$$\overset{+7}{Mn}O_4^{-1} \rightarrow \overset{+4}{Mn}O_2$$

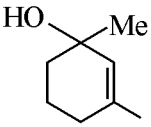
71. Find out the major product from the following reaction



(1) 

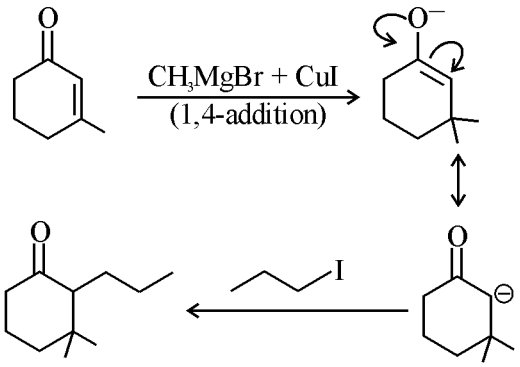
(2) 

(3) 

(4) 

Official Ans. by NTA (1)

Sol.



72. Formation of which complex, among the following, is not a confirmatory test of Pb^{2+} ions

(1) lead chromate

(2) lead iodide

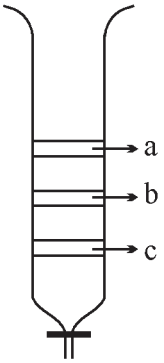
(3) lead nitrate

(4) lead sulphate

Official Ans. by NTA (3)

Sol. $\because$ $Pb(NO_3)_2$ is a soluble colourless compound so it cannot be used in confirmatory test of Pb^{+2} ion.

73. From the figure of column chromatography given below, identify incorrect statements.



A. Compound 'c' is more polar than 'a' and 'b'

B. Compound 'a' is least polar

C. Compound 'b' comes out of the column before 'c' and after 'a'

D. Compound 'a' spends more time in the column

Choose the correct answer from the options given below :-

(1) A, B and C only

(2) B, C and D only

(3) A, B and D only

(4) B and D only

Official Ans. by NTA (1)

74. Given below are two statements :-

Statement-I : Morphine is a narcotic analgesis. It helps in relieving pain without producing sleep.

Statement-II : Morphine and its derivatives are obtained from opium poppy.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are false

(3) Both Statement I and Statement II are true

(4) Statement I is false but Statement II is true

Official Ans. by NTA (4)

Sol. **Statement-I-** Morphine relieves in pain and produce sleep (incorrect)

Statement-II - Correct

75. The volume of 0.02 M aqueous HBr required to neutralize 10.0 mL of 0.01 M aqueous $Ba(OH)_2$ is (Assume complete neutralization)

(1) 2.5 mL

(2) 5.0 mL

(3) 10.0 mL

(4) 7.5 mL

Official Ans. by NTA (3)

Sol. $N_1v_1 = N_2v_2$

$\Rightarrow 0.02v_1 = 0.02 \times 10$

$\Rightarrow v_1 = 10ml$

76. The product, which is not obtained during the electrolysis of brine solution is

(1) NaOH

(2) Cl_2

(3) H_2

(4) HCl

Official Ans. by NTA (4)

Sol. Brine is aq. Solution of NaCl

$NaCl_{(aq)} \rightarrow Na^+ + Cl^-$

Cathode reaction

$2H_2O + 2e^- \xrightarrow{\text{reduction}} H_{2(g)} + 2OH^-$

Anode reaction

$2Cl^- \xrightarrow{\text{oxidation}} Cl_{2(g)} + 2e^-$

So HCl will not form during electrolysis.

77. The group of chemicals used as pesticide is

(1) Sodium chlorate, DDT, PAN

(2) Aldrin, Sodium chlorate, Sodium arsinite

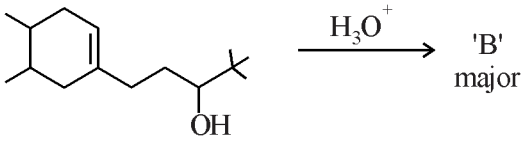
(3) DDT, Aldrin

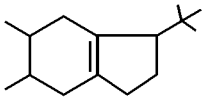
(4) Dieldrin, Sodium arsinite, Tetrachloroethene

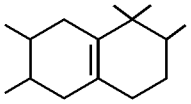
Official Ans. by NTA (3)

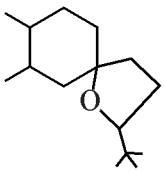
Sol. Pesticides $\rightarrow$ D.D.T and Aldrin

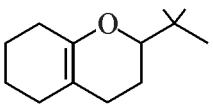
78. In the following reaction, 'B' is



(1) 

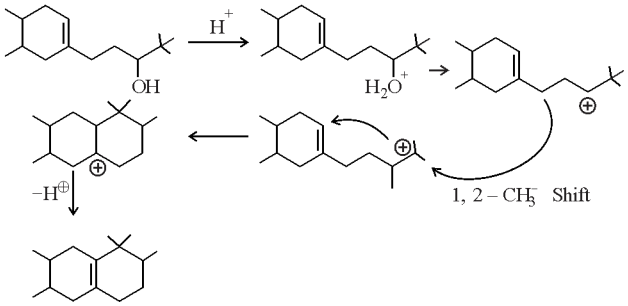
(2) 

(3) 

(4) 

Official Ans. by NTA (2)

Sol.



79. Which one of the following elements will remain as liquid inside pure boiling water ?

(1) Cs

(2) Ga

(3) Li

(4) Br

Official Ans. by NTA (2)

Sol. Li, Cs reacts vigorously with water.

Br_2 changes in vapour state in boiling water (BP = 58°C)

Ga reacts with water above 100°C (MP = 29°C, BP = 2400°C)

80. If the radius of the first orbit of hydrogen atom a_0, then de Broglie's wavelength of electron in 3rd orbit is

(1) $\frac{\pi a_0}{6}$

(2) $\frac{\pi a_0}{3}$

(3) $6\pi a_0$

(4) $3\pi a_0$

Official Ans. by NTA (3)

Sol. $(r_3)_H = \frac{a_0 n^2}{Z} = a_0 \times 3^2 = 9a_0$

$2\pi r = n\lambda$

$\Rightarrow 2\pi \times 9a_0 = 3\lambda$

$\Rightarrow \lambda = 6\pi a_0$

SECTION-B

81. In an ice crystal, each water molecule is hydrogen bonded to ……..neighbouring molecules.

Official Ans. by NTA (4)

Sol. In ice each water molecule is hydrogen bonded with four other water molecules.

82. The equilibrium composition for the reaction $PCl_3 + Cl_2 \rightleftharpoons PCl_5$ at 298 K is given below.

$[PCl_3]_{eq} = 0.2$ mol L^{-1} $[Cl_2]_{eq} = 0.1$ mol L^{-1},

$[PCl_5]_{eq} = 0.40$ mol L^{-1}

If 0.2 mol of Cl_2 is added at the same temperature, the equilibrium concentrations of PCl_5 is_____ $\times 10^{-2}$ mol L^{-1}.

Given : K_c for the reaction at 298 K is 20

Official Ans. by NTA (48)

Sol.

	PCl_3	+ Cl_2	$\rightleftharpoons$	PCl_5
	0.2M	(0.1+0.2)M		0.4M
Eq^m.	0.2–x	0.3–x		0.4+x

$$\frac{(0.4+x)}{(0.2-x)(0.3-x)} = 20$$

$\Rightarrow x \simeq 0.086$

$[PCl_5]_{eq} = 0.486M = 48.6 \times 10^{-2} M$

83. Consider the following pairs of solution which will be isotonic at the same temperature. The number of pairs of solutions is/are…….

A. 1 M aq. NaCl and 2 M aq. Urea

B. 1 M aq. $CaCl_2$ and 1.5 M aq. KCl

C. 1.5 M aq. $AlCl_3$ and 2 M aq. Na_2SO_4

D. 2.5 M aq. KCl and 1 M aq. $Al_2(SO_4)_3$

Official Ans. by NTA (4)

Sol. $\pi = icRT$

A, B, C and D are isotonic pairs.

84. The standard reduction potential at 298 K for the following half cells are given below :-

$NO_3^- + 4H^+ + 3e^- \rightarrow NO(g) + 2H_2O$ $E^0 = 0.97V$

$V^{2+}(aq) + 2e^- \rightarrow V$ $E^0 = -1.19V$

$Fe^{3+}(aq) + 3e^- \rightarrow Fe$ $E^0 = -0.04V$

$Ag^+(aq) + e^- \rightarrow Ag(s)$ $E^0 = 0.80V$

$Au^{3+}(aq) + 3e^- \rightarrow Au(s)$ $E^0 = 1.40V$

The number of metal(s) which will be oxidized by NO_3^- in aqueous solution is _____.

Official Ans. by NTA (3)

Sol. Metal having lower SRP than 0.97V will be oxidised by NO_3^-.

85. The number of colloidal systems from the following, which will have 'liquid' as the dispersion medium, is_____

Gem stones, paints, smoke, cheese, milk, hair cream, insecticide sprays, froth, soap lather.

Official Ans. by NTA (5)

Sol. Paints, milk, hair cream, froth, soap lather.

86. The number of species having a square planar shape from the following is _____

XeF_4, SF_4, SiF_4, BF_4^-, BrF_4^-, $[Cu(NH_3)_4]^{2+}$, $[FeCl_4]^{2-}$, $[PtCl_4]^{2-}$

Official Ans. by NTA (4)

Sol. XeF_4, BrF_4^{-1}, $[Cu(NH_3)_4]^{+2}$, $[PtCl_4]^{-2}$ has square planar shape.

87. Consider the following date

Heat of combustion of $H_2(g) = -241.8$ kJ mol^{-1}

Heat of combustion of $C(s) = -393.5$ kJ mol^{-1}

Heat of combustion of $C_2H_5OH(l) = -1234.7$ kJ mol^{-1}.

The heat of formation of $C_2H_5OH(l)$ is (–) _____ kJ mol^{-1} (Nearest integer)

Official Ans. by NTA (278)

Sol. $2C_{(s)} + 3H_{2(g)} + \frac{1}{2}O_{2(g)} \rightarrow C_2H_5OH_{(l)}$

$(\Delta H_f)_{C_2H_5OH_{(l)}} = \Sigma(\Delta H_{comb})_{reactant} - \Sigma(\Delta H_{comb})_{product}$

$= 2\times(-393.5) + 3(-241.8) - (-1234.7)$

$= -277.7 kJ/mol$

88. Among the following, the number of compounds which will give positive iodoform reaction is_____

(a) 1-Phenylbutan-2-one

(b) 2-Methylbutan-2-ol

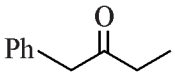
(c) 3-Methylbutan-2-ol

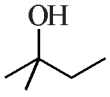
(d) 1-Phenylethanol

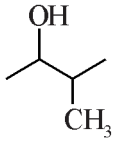
(e) 3,3-dimethylbutan-2-one

(f) 1-Phenylpropan-2-ol

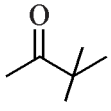
Official Ans. by NTA (4)

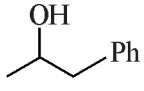
Sol. (a) 

(b) 

(c)  (Positive for iodoform)

(d) Ph–CH(OH)–CH$_3$ (Positive for iodoform)

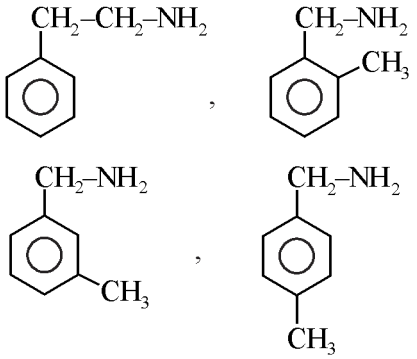
(e)  (Positive for iodoform)

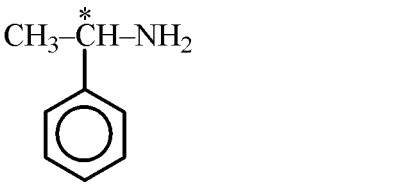
(f)  (Positive for iodoform)

89. Number of isomeric aromatic amines with molecular formula $C_8H_{11}N$, which can be synthesized by Gabriel Phthalimide synthesis is_____

Official Ans. by NTA (5)

Sol.





(d + l)

90. Number of crystal systems from the following where body centred unit cell can be found, is…..

Cubic, tetragonal, orthorhombic, hexagonal, rhombohedral, monoclinic, triclinic.

Official Ans. by NTA (3)

Sol. Cubic, tetragonal and orthorhombic have body centered unit cell.

FINAL JEE–MAIN EXAMINATION – APRIL, 2023

(Held On Saturday 08th April, 2023) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS — TEST PAPER WITH ANSWER

SECTION-A

1. Let $I(x)=\int \frac{(x+1)}{x\left(1+xe^{x}\right)^{2}}dx,\ x>0,$

If $\lim\limits_{x\to\infty} I(x)=0$, then I(1) is equal to

(1) $\frac{e+1}{e+2}-\log_e(e+1)$

(2) $\frac{e+1}{e+2}+\log_e(e+1)$

(3) $\frac{e+2}{e+1}+\log_e(e+1)$

(4) $\frac{e+2}{e+1}-\log_e(e+1)$

Official Ans. by NTA (4)

2. If the equation of the plane containing the line $x+2y+3z-4=0 = 2x+y-z+5$ and perpendicular to the plane $\vec{r}=(\hat{i}-\hat{j})+\lambda(\hat{i}+\hat{j}+\hat{k})+\mu(\hat{i}-2\hat{j}+3k)$ is $ax+by+cz=4$, then (a–b + c) is equal to

(1) 20 (2) 24

(3) 22 (4) 18

Official Ans. by NTA (3)

3. Let R be the focus of the parabola $y^2=20x$ and the line $y=mx+c$ intersect the parabola at two points P and Q. Let the point G(10, 10) be the centroid of the triangle PQR. If c–m = 6, then $(PQ)^2$ is

(1) 325 (2) 317

(3) 296 (4) 346

Official Ans. by NTA (1)

4. Let $C(\alpha,\beta)$ be the circumcenter of the triangle formed by the lines

$4x+3y=69$

$4y-3x=17$ and

$x+7y=61$

Then $(\alpha-\beta)^2+\alpha+\beta$ is equal to

(1) 18 (2) 17

(3) 16 (4) 15

Official Ans. by NTA (2)

5. Let $P=\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$, $A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ and $Q=PQP^T$. If $P^TQ^{2007}P=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$, then $2a+b-3c-4d$ equal to

(1) 2007 (2) 2005

(3) 2006 (4) 2004

Official Ans. by NTA (2)

6. Let α,β,γ be the three roots of the equation $x^3+bx+c=0$. If $\beta\gamma=1=-\alpha$, then $b^3+2c^3-3\alpha^3-6\beta^3-8\gamma^3$ is equal to

(1) 21 (2) $\frac{169}{8}$

(3) 19 (4) $\frac{155}{8}$

Official Ans. by NTA (3)

7. The number of ways , in which 5 girls and 7 boys can be seated at a round table so that no two girls sit together, is

(1) $126(5!)^2$

(2) $7(360)^2$

(3) 720

(4) $7(720)^2$

Official Ans. by NTA (1)

8. In a bolt factory, machines A, B and C manufacture respectively 20%, 30% and 50% of the total bolts. Of their output 3, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from the product. If the bolt drawn is found the defective, then the probability that it is manufactured by the machine C is

(1) $\frac{2}{7}$ (2) $\frac{9}{28}$

(3) $\frac{5}{14}$ (4) $\frac{3}{7}$

Official Ans. by NTA (3)

9. The number of arrangements of the letter of the word "INDEPENDENCE" in which all the vowels always occur together is

(1) 16800 (2) 14800

(3) 18000 (4) 33600

Official Ans. by NTA (1)

10. Let $f(x)=\frac{\sin x+\cos x-\sqrt{2}}{\sin x-\cos x}, x\in[0,\pi]-\left\{\frac{\pi}{}\right\}$.

Then $f\left(\frac{7\pi}{12}\right)f''\left(\frac{7\pi}{12}\right)$ is equal to

(1) $\frac{-2}{3}$ (2) $\frac{2}{9}$

(3) $-\frac{1}{3\sqrt{3}}$ (4) $\frac{-2}{3\sqrt{3}}$

Official Ans. by NTA (2)

11. If the points with vectors $\alpha\hat{i}+10\hat{j}+13\hat{k}$, $6\hat{i}+11\hat{j}+11\hat{k}$, $\frac{9}{2}\hat{i}+\beta\hat{j}-8\hat{k}$ are collinear, then $(19\alpha-6\beta)^2$ is equal to

(1) 36

(2) 16

(3) 25

(4) 49

Official Ans. by NTA (1)

12. If the coefficients of the three consecutive terms in the expansion of $(1+x)^n$ are in the ratio 1 : 5 : 20, then the coefficient of the fourth term is

(1) 3654 (2) 1827

(3) 5481 (4) 2436

Official Ans. by NTA (1)

13. Let $S_k=\frac{1+2+.....+K}{K}$ and

$\sum_{j=1}^{n}S_j^2=\frac{n}{A}\left(Bn^2+Cn+D\right)$, where A, B, C, $D\in N$ and A has least value. Then

(1) A + B is divisible by D

(2) A + B = 5 (D–C)

(3) A + C + D is not divisible by B

(4) A + B + C + D is divisible by 5

Official Ans. by NTA (1)

14. Let $A=\begin{bmatrix}2&1&0\\1&2&-1\\0&-1&2\end{bmatrix}$. If $|\text{adj}(\text{adj}(\text{adj}2A))|=(16)^n$, then n is equal to

(1) 10 (2) 9

(3) 12 (4) 8

Official Ans. by NTA (1)

15. Negation of $p \Rightarrow q \Rightarrow q \Rightarrow p$ is

(1) $(\sim p) \vee q$ (2) $(\sim q) \wedge p$

(3) $q \wedge (\sim p)$ (4) $p \vee (\sim q)$

Official Ans. by NTA (3)

16. The shortest distance between the lines $\frac{x-4}{4} = \frac{y+2}{5} = \frac{z+3}{3}$ and $\frac{x-1}{3} = \frac{y-3}{4} = \frac{z-4}{2}$ is

(1) $3\sqrt{6}$ (2) $6\sqrt{3}$

(3) $6\sqrt{2}$ (4) $2\sqrt{6}$

Official Ans. by NTA (1)

17. The area of the region

$\{(x,y) : x^2 \le y \le 8 - x^2, y \le 7\}$ is

(1) 21 (2) 18

(3) 24 (4) 20

Official Ans. by NTA (4)

18. Let the number of elements in sets A and B be five and two respectively. Then the number of subsets of A × B each having at least 3 and at most 6 element is :

(1) 792 (2) 752

(3) 782 (4) 772

Official Ans. by NTA (1)

19. $\lim\limits_{x \to 0} \left(\left(\frac{1 - \cos^2(3x)}{\cos^3(4x)} \right) \left(\frac{\sin^3(4x)}{(\log_e(2x+1))^5} \right) \right)$ is equal to _____

(1) 9 (2) 18

(3) 15 (4) 24

Official Ans. by NTA (2)

20. If for $z = \alpha + i\beta, |z+2| = z+4\ 1+i$, then $\alpha + \beta$ and $\alpha\beta$ are the roots of the equation

(1) $x^2 + 7x + 12 = 0$

(2) $x^2 + 3x - 4 = 0$

(3) $x^2 + 2x - 3 = 0$

(4) $x^2 + x - 12 = 0$

Official Ans. by NTA (1)

SECTION-B

21. Let [t] denotes the greatest integer $\le t$. Then $\frac{2}{\pi} \int_{\pi/6}^{5\pi/6} (8[\operatorname{cosec} x] - 5[\cot x])\,dx$ is equal to

Official Ans. by NTA (14)

22. Let [t] denotes the greatest integer $\le t$. If the constant term in the expansion of $\left(3x^2 - \frac{1}{2x^5}\right)^7$ is α, then $[\alpha]$ is equal to ____

Official Ans. by NTA (1275)

23. Let $\vec{a} = 6\hat{i} + 9\hat{j} + 12\hat{k}$, $\vec{b} = \alpha\hat{i} + 11\hat{j} - 2\hat{k}$ and $\vec{c}$ be vectors such that $\vec{a} \times \vec{c} = \vec{a} \times \vec{b}$. If $\vec{a}.\vec{c} = -12$, $\vec{c}.(\hat{i} - 2\hat{j} + \hat{k}) = 5$, then $\vec{c}.(\hat{i} + \hat{j} + \hat{k})$ is equal to ___

Official Ans. by NTA (11)

24. The largest natural number n such that 3^n divides 66! is _____

Official Ans. by NTA (31)

25. If a_n is the greatest term in the sequence $a_n = \frac{n^3}{n^4+147}, n = 1,2,3.....$, then α is equal to ____

Official Ans. by NTA (5)

26. Let $A = \{0, 3,4,6,7,8,9,10\}$ and R be the relation defined on A such that $R = \{(x,y) \in A \times A : x - y$ is odd positive integer or x – y = 2}. The minimum number of elements that must be added to the relation R, so that it is a symmetric relation, is equal to _____

Official Ans. by NTA (19)

27. Consider a circle $C_1 : x^2 + y^2 - 4x - 2y = \alpha - 5$. Let its mirror image in the line y = 2x + 1 be another circle $C_2 : 5x^2 + 5y^2 - 10fx - 10gy + 36 = 0$. Let r be the radius of C_2. Then $\alpha + r$ is equal to ___

Official Ans. by NTA (2)

28. If the solution curve of the differential equation $(y - 2\log_e x)dx + (x\log_e x^2)dy = 0,\ x > 1$ passes through the points $\left(e, \frac{4}{3}\right)$ and (e^4, α), then α is equal to ____

Official Ans. by NTA (3)

29. Let λ_1, λ_2 be the values of λ for which the points $\left(\frac{5}{2}, 1, \lambda\right)$ and $(-2,0,1)$ are at equal distance from the plane $2x + 3y - 6z + 7 = 0$. if $\lambda_1 > \lambda_2$, then the distance of the point $(\lambda_1 - \lambda_2, \lambda_2, \lambda_1)$ from the line $\frac{x-5}{1} = \frac{y-1}{2} = \frac{z+7}{2}$ is ____

Official Ans. by NTA (9)

30. Let the mean and variance of 8 numbers x, y, 10, 12, 6, 12, 4, 8, be 9 and 9.25 respectively. If x > y, then 3x – 2y is equal to ___

Official Ans. by NTA (25)

SECTION-A

31. A charge particle moving in magnetic field B, has the components of velocity along B as well as perpendicular to B. The path of the charge particle will be

(1) helical path with the axis perpendicular to the direction of magnetic field B

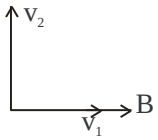
(2) straight along the direction of magnetic field B

(3) helical path with the axis along magnetic field B

(4) circular path

Official Ans. by NTA (3)

Sol.



Due to component v_1,

magnetic force $F = qv_1B\sin\theta = 0$

So v_1 remains unchanged

but due to component v_2 magnetic force act towards centre i.e. moving it circular. So path is helical with the axis parallel to magnetic field B.

32. Two projectiles A and B are thrown with initial velocities of 40 m/s and 60 m/s at angles 30° and 60° with the horizontal respectively. The ratio of their ranges respectively is (g= 10 m/s^2)

(1) $\sqrt{3}:2$ (2) $2:\sqrt{3}$

(3) 1 : 1 (4) 4 : 9

Official Ans. by NTA (4)

Sol. $R_1 = \dfrac{u_1^2 \sin 2\theta_1}{g}; R_2 = \dfrac{u_2^2 \sin 2\theta_2}{g}$

$$\frac{R_1}{R_2} = \frac{u_1^2}{u_2^2}\frac{\sin 2\theta_1}{\sin 2\theta_2} = \frac{40^2 \sin(2\times 30°)}{60^2 \sin(2\times 60°)} = \frac{4}{9}$$

33. Certain galvanometers have a fixed core made of non magnetic metallic material. The function of this metallic material is

(1) to oscillate the coil in magnetic field for longer period of time

(2) to bring the coil to rest quickly

(3) to produce large deflecting torque on the coil

(4) to make the magnetic field radial

Official Ans. by NTA (2)

Sol. Due to motion of the coil eddy current develops thus bringing the coil to rest.

34. A TV transmitting antenna is 98 m high and the receiving antenna is at the ground level. If the radius of the earth is 6400 km, the surface area covered by the transmitting antenna is approximately:

(1) 1240 km^2 (2) 3942 km^2

(3) 4868 km^2 (4) 1549 km^2

Official Ans. by NTA (2)

Sol. $h_T = 98\text{m}, h_R = 0, R = 6400\,\text{km}$

$$d = \sqrt{2h_T.R} + \sqrt{2h_R.R}$$

$$= \sqrt{2\times 98\times 6400\times 10^3} + 0 = \frac{112}{\sqrt{10}}\,\text{km}$$

So area = πd^2

$$= 3.14\times\frac{112^2}{10} = 3942\,\text{km}^2$$

35. In a reflecting telescope, a secondary mirror is used to:

(1) reduce the problem of mechanical support

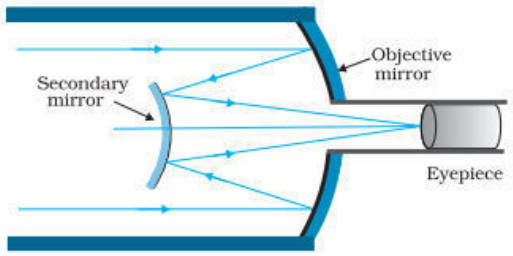
(2) remove spherical aberration

(3) make chromatic aberration zero

(4) move the eyepiece outside the telescopic tube

Official Ans. by NTA (4)

Sol.



It has advantage of a large focal length in a short telescope

36. Given below are two statements:

Statement I: If heat is added to a system, its temperature must increase.

Statement II: If positive work is done by a system in a thermodynamic process, its volume must increase.

In the light of the above statements, choose the correct answer from the options given below

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are true

(3) Both Statement I and Statement II are false

(4) Statement I is false but Statement II is true

Official Ans. by NTA (4)

Sol. **Statement I:** $\Delta Q > 0$

According to 1st law of thermodynamics

$\Delta Q = \Delta U + W$

If $\Delta Q > 0$, $\Delta U < 0$ and $W > 0$ is also possible.

Hence $\Delta T < 0$, so T decreases.

Statement I is false

Statement II: $W > 0$

$\therefore \int Pdv > 0$

Therefore volume of system must increase during positive work done by the system.

Statement II is true

37. The weight of a body on the earth is 400 N. Then weight of the body when taken to a depth half of the radius of the earth will be:

(1) Zero (2) 300 N

(3) 100 N (4) 200 N

Official Ans. by NTA (4)

Sol. $W = mg = 400$ N

At depth d, gravity $g' = g\left(1 - \frac{d}{R}\right)$

For $d = \frac{R}{2}$ $g' = g\left(1 - \frac{R}{2R}\right) = \frac{g}{2}$

$W' = mg' = \frac{mg}{2} = 200N$

38. An aluminium rod with Young's modulus Y = 7.0 $\times$ 10^{10} N/m^2 undergoes elastic strain of 0.04%. The energy per unit volume stored in the rod in SI unit is:

(1) 5600 (2) 8400

(3) 2800 (4) 11200

Official Ans. by NTA (1)

Sol. $Y = 7 \times 10^{10}\ N/m^2$

$\text{Strain} = \frac{0.04}{100}$

$\text{Energy} = \frac{1}{2}\left(\frac{YA}{l}\right)\Delta x^2$

$\text{Energy} = \frac{1}{2} YA\left(\frac{\Delta x}{l}\right)^2 \times l$

$\frac{E}{V} = \frac{1}{2} \times Y \times \text{strain}^2$

$= \frac{1}{2} \times 7 \times 10^{10} \times \frac{0.04 \times 0.04}{10^4} = 56 \times 10^2$

39. At any instant the velocity of a particle of mass 500 g is $\left(2t\,\hat{i}+3t^2\,\hat{j}\right)ms^{-1}$. If the force acting on the particle at t=1s is $\left(\hat{i}+x\hat{j}\right)N$. Then the value of x will be:

(1) 3 (2) 4 (3) 6 (4) 2

Official Ans. by NTA (1)

Sol. $\vec{v}=2t\,\hat{i}+3t^2\hat{j}$

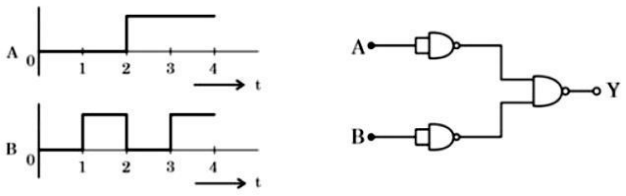
$\vec{a}=2\,\hat{i}+6t\,\hat{j}$

at $t=1, \vec{a}=2\hat{i}+6\hat{j}$

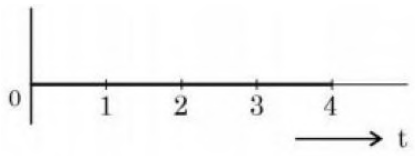
$\vec{F}=m\vec{a}=0.5\left(2\hat{i}+6\hat{j}\right)=\hat{i}+3\hat{j}$

$\vec{F}=\hat{i}+x\,\hat{j}$ Hence x = 3

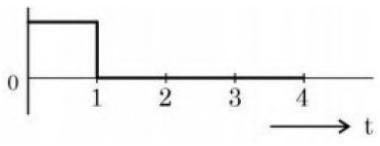
40. For the logic circuit shown, the output waveform at Y is:



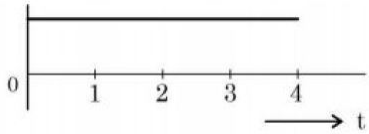
(1)



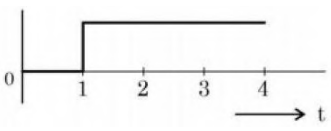
(2)



(3)



(4)

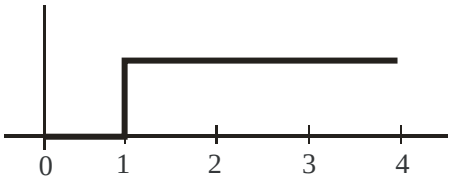


Official Ans. by NTA (4)

Sol.

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

According to truth table , resultant graph is



41. For a nucleus ${}^{A}_{Z}X$ having mass number A and atomic number Z

A. The surface energy per nucleon $\left(b_s\right)=a_1A^{2/3}$

B. The Coulomb contribution to the binding energy $b_c=-a_2\dfrac{Z(Z-1)}{A^{4/3}}$

C. The volume energy $b_v=a_3A$

D. Decrease in the binding energy is proportional to surface area.

E. While estimating the surface energy, it is assumed that each nucleon interacts with 12 nucleons, (a_1, a_2 and a_3 are constants)

Choose the **most appropriate** answer from the options given below:

(1) C, D only (2) B, C, E only

(3) A, B, C, D only (4) B, C only

Official Ans. by NTA (1)

Sol. Surface energy per nucleon $\propto\dfrac{r^2}{A}\propto\dfrac{A^{2/3}}{A}\propto\dfrac{1}{A^{1/3}}$

$\left(\text{Mass number } A\propto r^3\Rightarrow r\propto A^{1/3}\right)$.

A is incorrect

Contribution to binding energy by columbic forces is

$=\dfrac{-a_2Z(Z-1)}{A^{1/3}}$

B is incorrect

Volume energy $\propto$ A
C is correct

For (D) , if we consider only surface energy contribution then option is correct.
For (E) only 3 interactions contribute to surface energy.

42. Given below are two statements:

Statement I :

If E be the total energy of a satellite moving around the earth, then its potential energy will be $\frac{E}{2}$.

Statement II:

The kinetic energy of a satellite revolving in an orbit is equal to the half the magnitude of total energy E.

In the light of the above statements, choose the most appropriate answer from the options given below

(1) Both Statement I and Statement II are correct
(2) Both Statement I and Statement II are incorrect
(3) Statement I is incorrect but Statement II is correct
(4) Statement I is correct but Statement II is incorrect

Official Ans. by NTA (2)

Sol. Energy of satellite in orbit $E = \frac{-GMm}{2R}$.

PE of satellite in orbit $U = \frac{-GMm}{R}$

$\Rightarrow U = 2E$

KE of satellite in orbit $K = E - U$

$$K = \frac{GMm}{2R} = (-E)$$

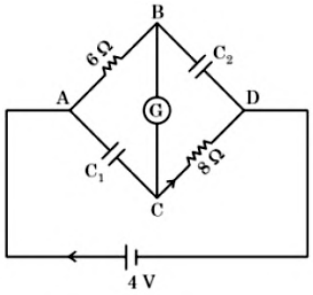
43. Dimension of $\frac{1}{\mu_0 \in_0}$ should be equal to

(1) T^2/L^2 (2) L/T
(3) L^2/T^2 (4) T/L

Official Ans. by NTA (3)

Sol. $$\frac{1}{\mu_0 \in_0} = c^2 \Rightarrow \left[\frac{1}{\mu_0 \in_0}\right] = \left[c^2\right] = \left[L^2T^{-2}\right]$$

44. In this figure the resistance of the coil of galvanometer G is 2 Ω. The emf of the cell is 4 V. The ratio of potential difference across C_1 and C_2 is:



(1) 1 (2) $\frac{4}{5}$ (3) $\frac{3}{4}$ (4) $\frac{5}{4}$

Official Ans. by NTA (2)

Sol. At steady state, current in the circuit is

$$i = \frac{4V}{6+2+8} = \frac{1}{4}A$$

Voltage across C_1 is

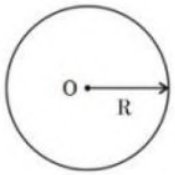
$$V_1 = V_{AC} = i(6\Omega + 2\Omega) = \frac{1}{4} \times 8 = 2V$$

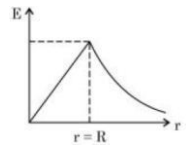
Voltage across C_2 is

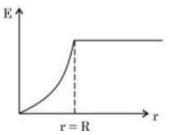
$$V_2 = V_{BD} = i(2\Omega + 8\Omega) = \frac{1}{4} \times 10 = 2.5V$$

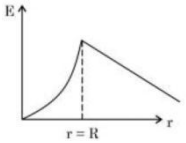
$$\Rightarrow \frac{V_1}{V_2} = \frac{2}{2.5} = \frac{4}{5}$$

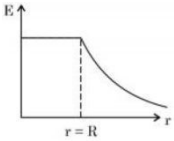
45. Graphical variation of electric field due to a uniformly charged insulating solid sphere of radius R, with distance r from the centre O is represented by:



(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (1)

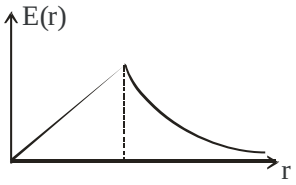
Sol. Electric field of solid sphere (uniformly charged)

$$E(r)\begin{cases}\dfrac{Q}{4\pi\in_0 r^2} & r\geq R\\ \dfrac{Qr}{4\pi\in_0 R^3} & r\leq R\end{cases}$$

Graphically

$E(r)\propto r$ for $r\leq R$

$\propto\dfrac{1}{r^2}$ for $r\geq R$



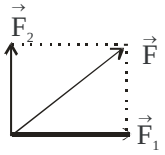
46. Two forces having magnitude A and $\dfrac{A}{2}$ are perpendicular to each other. The magnitude of their resultant is

(1) $\dfrac{\sqrt{5}A}{4}$ (2) $\dfrac{5A}{2}$

(3) $\dfrac{\sqrt{5}A^2}{2}$ (4) $\dfrac{\sqrt{5}A}{2}$

Official Ans. by NTA (4)

Sol.



$\vec{F}=(\vec{F}_1+\vec{F}_2)$

$|\vec{F}|=\sqrt{F_1^2+F_2^2+2F_1F_2\cos 90^\circ}$

$=\sqrt{A^2+\dfrac{A^2}{4}}=\dfrac{A\sqrt{5}}{2}$

47. The engine of a train moving with speed $10ms^{-1}$ towards a platform sounds a whistle at frequency 400Hz. The frequency heard by a passenger inside the train is (neglect air speed. Speed of sound in air $330ms^{-1}$)

(1) 200Hz (2) 400Hz

(3) 412 Hz (4) 388 Hz

Official Ans. by NTA (2)

Sol. The relative velocity of a passenger with source of sound (engine) is 0. So there will be no doppler's effect. So frequency heard is 400 Hz.

48. An air bubble of volume $1cm^3$ rises from the bottom of a lake 40m deep to the surface at a temperature of 12ºC. The atmospheric pressure is 1×10^5 Pa, the density of water is 1000 kg/m^3 and g =10m/s^2. There is no difference of the temperature of water at the depth of 40m and on the surface. The volume of air bubble when it reaches the surface will be

(1) $5cm^3$ (2) $2cm^3$

(3) $4cm^3$ (4) $3cm^3$

Official Ans. by NTA (1)

Sol. $P=P_0+\rho gh=10^5\,Pa+10^3\times10\times40=5\times10^5\,Pa$

At T is constant

$PV=P_0V_0$

$\Rightarrow\ 5\times10^5\,Pa\times1cm^3=10^5\,Pa\times V_0\Rightarrow V_0=5cm^3$

49. A cylindrical wire of mass $(0.4\pm0.01)g$ has length $(8\pm0.04)cm$ and radius $(6\pm0.03)mm$. The maximum error in its density will be

(1) 1% (2) 3.5 %

(3) 4% (4) 5%

Official Ans. by NTA (3)

Sol. $\rho = \frac{m}{\pi r^2 l} \Rightarrow \left|\frac{d\rho}{\rho}\right|_{max} = \left|\frac{dm}{m}\right| + 2\left|\frac{dr}{r}\right| + \left|\frac{dl}{l}\right|$

$= \frac{0.01}{0.4} + \frac{2(0.03)}{6} + \frac{0.04}{8}$

$\Rightarrow$ % error in density $= \left(\frac{d\rho}{\rho}\right) \times 100\%$

$= (2.5 + 1 + 0.5)\% = 4\%$

50. Proton (P) and electron (e) will have same de-Broglie wavelength when the ratio of their momentum is (assume, $m_p = 1849 m_e$)

(1) 1: 43 (2) 43 : 1

(3) 1 : 1849 (4) 1 : 1

Official Ans. by NTA (4)

Sol. De Broglie wavelength is $\lambda = \frac{h}{mv}$

$\lambda_p = \lambda_e \Rightarrow m_p v_p = m_e v_e \Rightarrow p_p = p_e$

SECTION-B

51. An electric dipole of dipole moment is 6.0×10^{-6} Cm placed in a uniform electric field of 1.5×10^3 NC^{-1} in such a way that dipole moment is along electric field. The work done in rotating dipole by 180º in this field will be _____mJ

Official Ans. by NTA (18)

Sol. The work done in rotating the electric dipole = ΔU

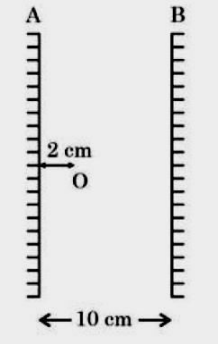
$= U_f - U_i$

$= (-pE\cos(180°)) - (-pE\cos(0°))$

$= pE + pE = 2pE$

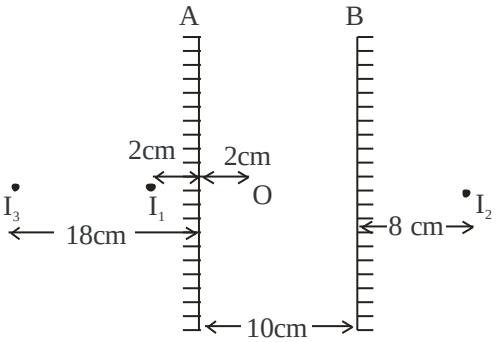
$= 2 \times 6 \times 10^{-6} \times 1.5 \times 10^3 = 18$mJ

52. Two vertical parallel mirrors A and B are separated by 10 cm. A point object O is placed at a distance of 2 cm from mirror A. The distance of the second nearest image behind mirror A from the mirror A is ____cm



Official Ans. by NTA (18)

Sol.



The desired image is 18 cm from A.

53. The momentum of a body is increased by 50%. The percentage increase in the kinetic energy of the body is _____%

Official Ans. by NTA (125)

Sol. Kinetic energy of body $= \frac{p^2}{2m}$

Initial kinetic energy $= \frac{p_i^2}{2m}$

Final kinetic energy $= \frac{p_f^2}{2m} = \frac{(1.5 p_i)^2}{2m}$

$= \frac{2.25 p_i^2}{2m}$

% increase in KE $= \frac{2.25\frac{p_i^2}{2m} - \frac{p_i^2}{2m}}{\frac{p_i^2}{2m}} \times 100 = 125\%$

54. The moment of inertia of semicircular ring about an axis, passing through the center and perpendicular to the plane of ring, is $\frac{1}{x}MR^2$, where R is the radius and M is the mass of semicircular ring. The value of x will be

Official Ans. by NTA (1)

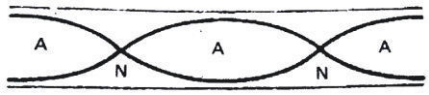
Sol. The moment of inertia of semicircular ring about axis passing through centre of ring and perpendicular to plane of ring is = MR^2

so x = 1

55. An organ pipe 40cm long is open at both ends. The speed of sound in air is $360 ms^{-1}$. The frequency of the second harmonic is ____Hz.

Official Ans. by NTA (900)

Sol.



For second harmonic of open organ pipe

$L = \lambda$

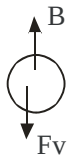
So frequency of vibration is $f = \frac{V}{\lambda}$

$$f = \frac{V}{\lambda} = \frac{V}{L} = \frac{360}{\frac{40}{100}} = 900\,Hz$$

56. An air bubble of diameter 6 mm rises steadily through a solution of density $1750 kg/m^3$ at the rate of 0.35 cm/s. The co-efficient of viscosity of the solution (neglect density of air) is ____Pas (given, $g = 10\,ms^{-2}$)

Official Ans. by NTA (10)

Sol.



Since the bubble is moving at constant speed the force acting on it is zero.

$B = F_V$

$$\frac{4}{3}\pi R^3 \rho g = 6\pi\eta Rv$$

$$\eta = \frac{2R^2\rho g}{9v} = \frac{2\times\left(3\times10^{-3}\right)^2\times1750\times10}{9\times0.35\times10^{-2}} = 10\,Pas$$

57. An oscillating LC circuit consists of a 75 mH inductor and a $1.2\mu F$ capacitor. If the maximum charge to the capacitor is $2.7\mu C$. The maximum current in the circuit will be ____mA.

Official Ans. by NTA (9)

Sol. Maximum energy stored in capacitor is same as maximum energy stored in inductor.

$$\frac{1}{2}Li_{max}^2 = \frac{1}{2}\frac{Q_{max}^2}{C}$$

$$i_{max} = \sqrt{\frac{1}{LC}}Q_{max}$$

$$= \frac{2.7\times10^{-6}}{\sqrt{75\times10^{-3}\times1.2\times10^{-6}}} = 9\,mA$$

58. The magnetic intensity at the centre of a long current carrying solenoid is found to be $1.6\times10^3\,Am^{-1}$. If the number of turns is 8 per cm, then the current flowing through the solenoid is_____A.

Official Ans. by NTA (2)

Sol. $$H = \frac{B}{\mu_0} = \frac{\mu_0 ni}{\mu_0} = ni$$

$$i = \frac{H}{n} = \frac{1.6\times10^3}{\left(\frac{8}{10^{-2}}\right)} = 2A$$

59. A current of 2 A flows through a wire of cross-sectional area 25.0 mm^2. The number of free electrons in a cubic meter are 2.0×10^{28}. The drift velocity of the electrons is $\_\_\_\_\_\times10^{-6}ms^{-1}$ (given, charge on electron $=1.6\times10^{-19}C$)

Official Ans. by NTA (25)

Sol. Drift velocity $v_d=\frac{I}{neA}$

$$=\frac{2}{2\times10^{28}\times1.6\times10^{-19}\times25\times10^{-6}}$$

$$=25\times10^{-6}ms^{-1}$$

60. A nucleus with mass number 242 and binding energy per nucleon as 7.6 MeV breaks into fragment each with mass number 121. If each fragment nucleus has binding energy per nucleon as 8.1 MeV, the total gain in binding energy is ____MeV

Official Ans. by NTA (121)

Sol. Initial binding energy = 242 × 7.6 MeV

Final binding energy

= 121 × 8.1 MeV + 121 × 8.1 MeV

= 242×8.1 MeV

Total gain in binding energy

= 242 (8.1 – 7.6) = 121 MeV

SECTION-A

61. $2IO_3^- + xI^- + 12H^+ \rightarrow 6I_2 + 6H_2O$

What is the value of x?

(1) 12 (2) 2

(3) 6 (4) 10

Official Ans. by NTA (4)

Sol. Number of atoms of iodine on reactant side = number of atoms of Iodine on product side

$2 + x = 6 \times 2$

$X = 10$

$2IO_3^- + 10I^- + 12H^+ \rightarrow 6I_2 + 6H_2O$

62. Which of the following metals can be extracted through alkali leaching technique?

(1) Cu (2) Sn

(3) Pb (4) Au

Official Ans. by NTA (2)

Sol. Reference : NCERT

63. Match List I with List II

A.	Saccharin	I.	High potency sweetener
B.	Aspartame	II.	First artificial sweetening agent
C.	Alitame	III.	Stable at cooking temperature
D.	Sucralose	IV	Unstable at cooking temperature

Choose the correct answer from the options given below:

(1) A-II, B-III, C-IV, D-I

(2) A-II, B-IV, C-III, D-I

(3) A-IV, B-III, C-I, D-II

(4) A-II, B-IV, C-I, D-III

Official Ans. by NTA (4)

Sol. (A) Saccharin II. First artificial sweetener

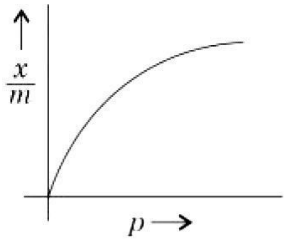
(B) Aspartame IV. Unstable at cooking temperature

(C) Alitame I. High potency sweetener

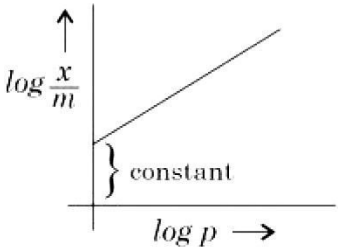
(D) Sucralose III. Stable at cooking temperature

64. Which of the following represent the Freundlich adsorption isotherms?

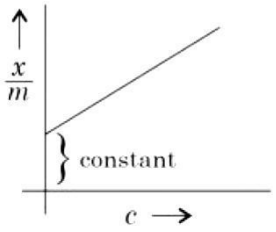
(A)



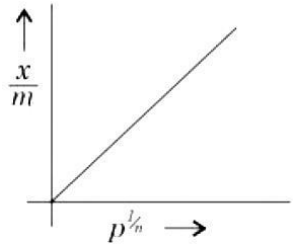
(B)



(C)



(D)



Choose the correct answer from the options given below:

(1) B, C, D only (2) A, B, D only

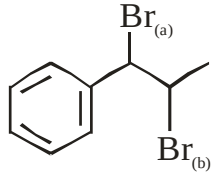
(3) A, B only (4) A, C, D only

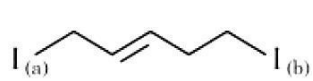
Official Ans. by NTA (2)

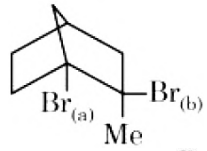
Sol. $\frac{x}{m} = k\ p^{1/n}$

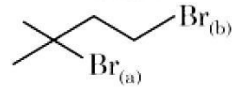
and $\log\frac{x}{m} = \log k + \frac{1}{n}\log P$

65. Choose the halogen which is most reactive towards SN1 reaction in the given compounds (A, B, C & D)

A. 

B. 

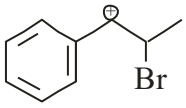
C. 

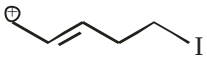
D. 

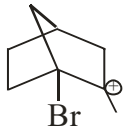
(1) A-$Br_{(b)}$; B-$I_{(b)}$; C-$Br_{(b)}$; D-$Br_{(b)}$
(2) A-$Br_{(a)}$; B-$I_{(a)}$; C-$Br_{(b)}$; D-$Br_{(a)}$
(3) A-$Br_{(b)}$; B-$I_{(a)}$; C-$Br_{(a)}$; D-$Br_{(a)}$
(4) A-$Br_{(b)}$; B-$I_{(a)}$; C-$Br_{(a)}$; D-$Br_{(a)}$

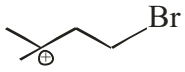
Official Ans. by NTA (2)

Sol. Stable is the carbocation, faster will be rate of SN1 reaction

(A)  (Benzylic carbocation)

(B)  (Allylic carbocation)

(C)  (Non bridgehead tertiary carbocation)

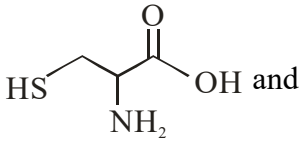
(D)  (Tertiary carbocation)

66. Sulphur (S) containing amino acids from the following are:

(a) isoleucine (b) cysteine (c) lysine
(d) methionine (e) glutamic acid
(1) a, d (2) b, d
(3) b, c, e (4) a, b, c

Official Ans. by NTA (2)

Sol. Sulphur containing amino acids

(b) cysteine  and

(d) methionine $CH_3–S–CH_2–CH_2–\underset{\underset{NH_2}{|}}{CH}–\overset{\overset{O}{||}}{C}–OH$.

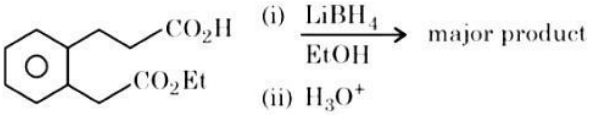
67. The water gas on reacting with cobalt as a catalyst forms

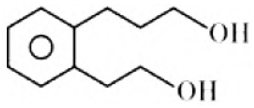
(1) Ethanol (2) Methanoic acid
(3) Methanal (4) Methanol

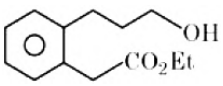
Official Ans. by NTA (4)

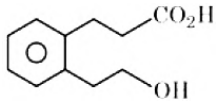
Sol. $CO + 2H_2 \xrightarrow{Co} CH_3OH$

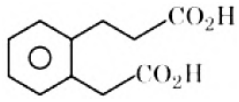
68. The major product formed in the following reaction is:



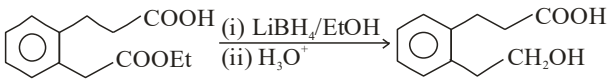
(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (3)

Sol. 

$LiBH_4$ can reduce ester selectively but not carboxylic acids.

Hence correct answer is option (3).

69. Which of the following complex is octahedral, diamagnetic and the most stable?

(1) $Na_3[CoCl_6]$ (2) $[Ni(NH_3)_6]Cl_2$
(3) $K_3[Co(CN)_6]$ (4) $[Co(H_2O)_6]Cl_2$

Official Ans. by NTA (3)

Sol. $Co^{+3} = t_2g^6\ eg^0$

CN^- -strong field ligand

All d-electrons should be paired ($\mu_s = 0$)

Hence diamagnetic.

70. The reaction

$$\frac{1}{2}H_2(g) + AgCl(s) \rightleftharpoons H^+(aq) + Cl^-(aq) + Ag(s)$$

occurs in which of the given galvanic cell.

(1) $Pt|H_2(g)|KCl(sol^n)|AgCl(s)|Ag$

(2) $Pt|H_2(g)|HCl(sol^n)|AgCl(s)|Ag$

(3) $Ag|AgCl(s)|KCl(sol^n)|AgCl(s)|Ag$

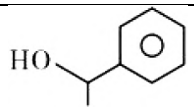
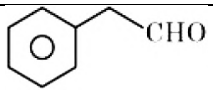
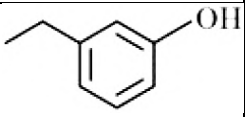
(4) $Pt|H_2(g)|HCl(sol^n)|AgNO_3(sol^n)|Ag$

Official Ans. by NTA (2)

Sol. $\text{Anode}: \frac{1}{2}H_2(g) \rightleftharpoons H^+(aq) + e^-$

$\text{Cathode}: AgCl(s) + e^- \rightarrow Ag(s) + Cl^-(aq)$

71. Match List-I with List-II :

	List-I (Reagents used)		**List-II (Compound with functional group detected)**
A.	Alkaline solution of copper sulphate and sodium citrate	I.	
B.	Neutral $FeCl_3$ solution	II.	
C.	Alkaline chloroform solution	III.	
D.	Potassium iodide and sodium hypochlorite	IV.	

Choose the correct answer from the options given below:

(1) A-II, B-IV, C-III, D-I

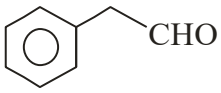
(2) A-IV, B-I, C-II, D-III

(3) A-III, B-IV, C-I, D-II

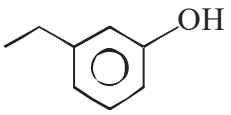
(4) A-III, B-IV, C-II, D-I

Official Ans. by NTA (4)

Sol. **A.** Alkaline solution of copper sulphate and sodium citrate is known as Benedict's solution and it is used to test aliphatic aldehydes. Hence it can be used to test compound (III) i.e.

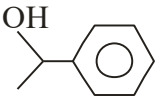


B. Neutral $FeCl_3$ solution is used to test phenolic compound (IV) i.e.



C. Alkaline chloroform solution is used to test primary amines (II) i.e.

D. $2KI + NaOCl + H_2O \rightarrow NaCl + I_2 + 2KOH$

Potassium iodide and sodium hypochlorite gives (I_2 + KOH) which is used to test those compounds which have $CH_3-\overset{\overset{O}{\|}}{C}-$ or $CH_3-\overset{\overset{OH}{|}}{CH}-$ group (iodoform test). Hence the compound is (I)  .

72. Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A: Butan -1- ol has higher boiling point than ethoxyethane.

Reason R: Extensive hydrogen bonding leads to stronger association of molecules.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both A and R are true and R is the correct explanation of A

(2) A is true but R is false

(3) Both A and R are true but R is not the correct explanation of A

(4) A is false but R is true

Official Ans. by NTA (1)

Sol. Butan-1-ol ($CH_3CH_2CH_2CH_2OH$) can undergo hydrogen bonding. Ethoxyethane ($CH_3CH_2-O-CH_2CH_3$) has no hydrogen (attached with F, O, N) which can undergo hydrogen bonding.

More is the extent of intermolecular H-bonding, more will be association of molecules. Thus leading to higher boiling point.

Hence both Assertion (A) and Reason(R) are true and (R) is the correct explanation of (A).

73. In chromyl chloride, the number of d-electrons present on chromium is same as in (Given at no. of Ti : 22, V : 23, Cr : 24, Mn : 25, Fe : 26)

(1) Ti (III) (2) Fe (III)

(3) V (IV) (4) Mn (VII)

Official Ans. by NTA (4)

Sol. In CrO_2Cl_2 oxidation state of Cr is +6

Cr (VI) = $[Ar]^{18}\ 3d^0$

Mn(VII) = $[Ar]^{18}\ 3d^0$

Fe(III) = $[Ar]^{18}\ 3d^5$

Ti(III) = $[Ar]^{18}\ 3d^1$

V(IV) = $[Ar]^{18}\ 3d^1$

Hence Cr (VI) and Mn (VII) have same d^0 configuration.

74. What is the purpose of adding gypsum to cement?

(1) To facilitate the hydration of cement

(2) To speed up the process of setting

(3) To slow down the process of setting

(4) To give a hard mass

Official Ans. by NTA (3)

Sol. Factual

75. The correct order of spin only magnetic moments for the following complex ions is

(1) $[Fe(CN)_6]^{3-} < [CoF_6]^{3-} < [MnBr_4]^{2-} < [Mn(CN)_6]^{3-}$

(2) $[Fe(CN)_6]^{3-} < [Mn(CN)_6]^{3-} < [CoF_6]^{3-} < [MnBr_4]^{2-}$

(3) $[MnBr_4]^{2-} < [CoF_6]^{3-} < [Fe(CN)_6]^{3-} < [Mn(CN)_6]^{3-}$

(4) $[CoF_6]^{3-} < [MnBr_4]^{2-} < [Fe(CN)_6]^{3-} < [Mn(CN)_6]^{3-}$

Official Ans. by NTA (2)

			Unpaired e^-
Sol.	$[Fe(CN)_6]^{3-}$	$Fe^{+3} \Rightarrow t_2g^5\ eg^0$,	1
	$[Mn(CN)_6]^{3-}$	$Mn^{+3} \Rightarrow t_2g^4\ eg^0$,	2
	$[CoF_6]^{3-}$	$Co^{+3} \Rightarrow t_2g^4\ eg^2$,	4
	$[MnBr_4]^{2-}$	$Mn^{+2} \Rightarrow e^2\ t_2^3$,	5

Spin magnetic moment $\mu = \sqrt{n(n+2)}$ B.M

76. Which halogen is known to cause the reaction given below:

$$2Cu^{2+} + 4X^- \rightarrow Cu_2X_2(s) + X_2$$

(1) Only Iodine (2) Only Bromine

(3) All halogens (4) Only Chlorine

Official Ans. by NTA (1)

Sol. $2Cu^{2+} + 4I^- \rightarrow Cu_2I_2(s) + I_2$

77. Match List-I with List-II :

	List-I (Species)		**List-II (Maximum allowed concentration in ppm in drinking water)**
A.	F^-	I.	< 50 ppm
B.	SO_4^{2-}	II.	< 5 ppm
C.	NO_3^-	III.	< 2 ppm
D.	Zn	IV.	< 500 ppm

(1) A-II, B-I, C-III, D-IV

(2) A-IV, B-III, C-II, D-I

(3) A-I, B-II, C-III, D-IV

(4) A-III, B-II, C-I, D-IV

Official Ans. by NTA (4)

Sol. Correct answer

A-III, B-IV, C-I, D-II

78. The correct order of electronegativity for given elements is:

(1) C > P > At > Br

(2) Br > P > At > C

(3) P > Br > C > At

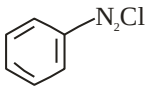
(4) Br > C > At > P

Official Ans. by NTA (4)

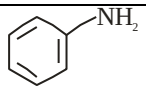
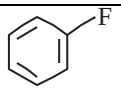
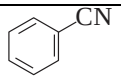
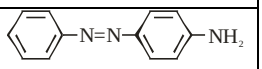
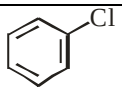
Sol.

Atom	**E.N.**
Br	3.0
C	2.5
At	2.2
P	2.1

79. Match List I with List II:



is reacted with reagents in List I to form products in List II.

	List-I (Reagent)		List-II (Product)
A.		I.	
B.	HBF_4, Δ	II.	
C.	Cu, HCl	III.	
D.	CuCN/KCN	IV.	

Choose the correct answer from the options given below:

(1) A-IV, B-III, C-II, D-I

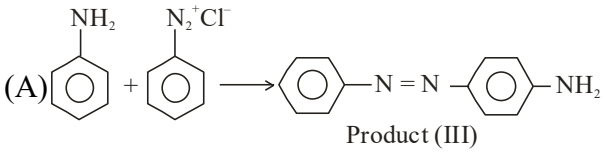
(2) A-I, B-III, C-IV, D-II

(3) A-III, B-I, C-II, D-IV

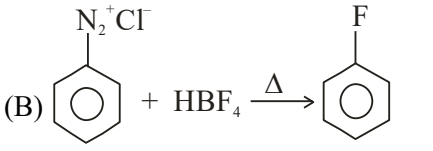
(4) A-III, B-I, C-IV, D-II

Official Ans. by NTA (4)

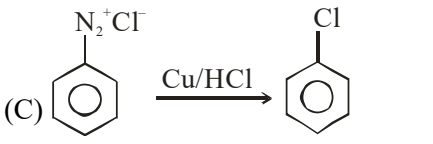
Sol.



Product (III)



Product (I)



Product (IV)

Product (II)

80. Given below are two statements:

Statement I: Lithium and Magnesium do not form superoxide

Statement II: The ionic radius of Li^+ is larger than ionic radius of Mg^{2+}

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Statement I is incorrect but Statement II is correct

(2) Statement I is correct but Statement II is incorrect

(3) Both Statement I and Statement II are correct

(4) Both Statement I and Statement II are incorrect

Official Ans. by NTA (3)

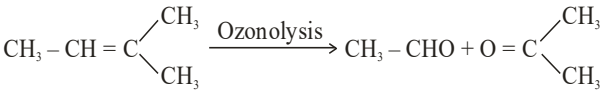
Sol. Li & Mg form oxide and order of size $Li^+ > Mg^{2+}$

SECTION-B

81. Molar mass of the hydrocarbon (X) which on ozonolysis consumes one mole of O_3 per mole of (X) and gives one mole each of ethanal and propanone is _____ g mol^{-1} (Molar mass of C : 12 g mol^{-1}, H : 1 g mol^{-1})

Official Ans. by NTA (70)

Sol.



Hydrocarbon (X)

Hence molar mass of hydrocarbon (X) is 70.

82. The number of following factors which affect the percent covalent character of the ionic bond is____

(a) Polarising power of cation

(b) Extent of distortion of anion

(c) Polarisability of the anion

(d) Polarising power of anion

Official Ans. by NTA (3)

Sol. (a), (b) and (c) are factors which affect the percent covalent character of the ionic bond according to Fajan's rule

83. When a 60 W electric heater is immersed in a gas for 100s in a constant volume container with adiabatic walls, the temperature of the gas rises by 5°C. The heat capacity of the given gas is_____ $J\,K^{-1}$ (Nearest integer)

Official Ans. by NTA (1200)

Sol. Power of heater = 60 W

= 60 J/sec

Total energy emitted

= 60 × 100 = 6000 J

Heat capacity × temp difference = 6000

$$\text{Heat capacity} = \frac{6000}{5} = 1200 JK^{-1}$$

84. The number of given statement/s which is/are correct is____

(A) The stronger the temperature dependence of the rate constant, the higher is the activation energy.

(B) If a reaction has zero activation energy, its rate is independent of temperature.

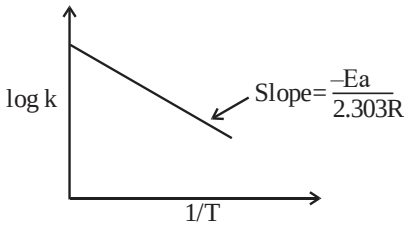
(C) The stronger the temperature dependence of the rate constant, the smaller is the activation energy.

(D) If there is no correlation between the temperature and the rate constant then it means that the reaction has negative activation energy.

Official Ans. by NTA (2)

Sol. $k = A.e^{-Ea/RT}$

$$\log_{10} k = \log_{10} A - \underbrace{\frac{Ea}{2.303R}}_{m}.\underbrace{\frac{1}{T}}_{x}$$

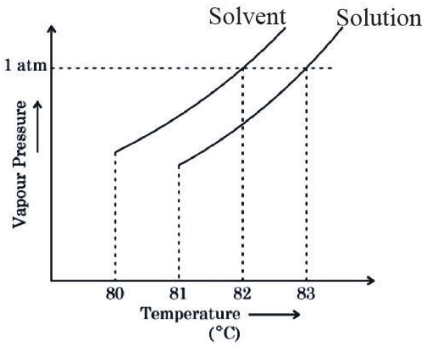


Higher is Ea, stronger is the temperature dependence of k (i.e. steeper the slope)

$$(B) \Rightarrow \frac{1}{k}\frac{dk}{dT} = \frac{Ea}{R}\frac{1}{T^2}$$

$$\Rightarrow \frac{dk}{dT} = A \times e^{-\frac{Ea}{R}}.\frac{Ea}{RT^2}$$

85. The vapour pressure vs. temperature curve for a solution solvent system is shown below.



The boiling point of the solvent is _____°C

Official Ans. by NTA (82)

Sol. Boiling point of solvent is 82ºC

Boiling point of solution is 83ºC

86. XeF_4 reacts with SbF_5 to form

$[XeF_m]^{n+}\,[SbF_y]^{z-}$

m + n + y + z = _____

Official Ans. by NTA (11)

Sol. $XeF_4 + SbF_5 \rightarrow [XeF_3]^+[SbF_6]^-$

m = 3

n = 1

y = 6

z = 1

m + n + y + z = 11

87. 0.5 g of an organic compound (X) with 60% carbon will produce _____ × 10^{-1} g of CO_2 on complete combustion.

Official Ans. by NTA (11)

Sol. Percentage of Carbon

$$= \frac{12}{44} \times \frac{\text{mass of } CO_2 \text{ formed}}{\text{mass of compound taken}} \times 100$$

$$60 = \frac{12}{44} \times \frac{\text{mass of } CO_2 \text{ formed}}{0.5} \times 100$$

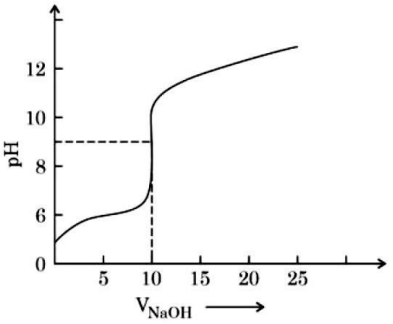
$$\text{Mass of } CO_2 \text{ formed} = \frac{60 \times 44 \times 0.5}{12 \times 100} g$$

= 1.1 gram

= 11 × 10^{-1} gram

88. The titration curve of weak acid vs. strong base with phenolphthalein as indictor) is shown below. The $K_{phenolphthalein} = 4 \times 10^{-10}$.

Given: log2 = 0.3



The number of following statements which is/are correct about phenolphthalein is_____

A. It can be used as an indicator for the titration of weak acid with weak base.

B. It begins to change colour at pH = 8.4

C. It is a weak organic base

D. It is colourless in acidic medium

Official Ans. by NTA (2)

Sol. (B) $pk_{In} = -\log(4 \times 10^{-10}) = 9.4$

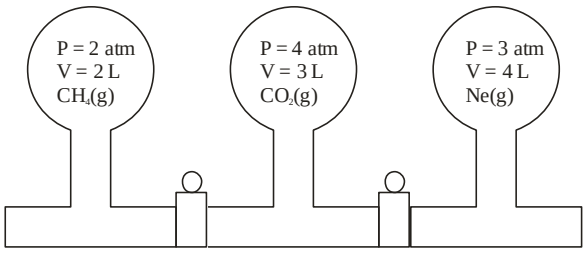
Indicator range

$\Rightarrow pk_{In} \pm 1$

i.e. 8.4 to 10.4

(D) In acidic medium, phenolphthalein is in unionized form and is colourless.

89.



Three bulbs are filled with CH_4, CO_2 and Ne as shown in the picture. The bulbs are connected through pipes of zero volume. When the stopcocks are opened and the temperature is kept constant throughout, the pressure of the system is found to be_____atm. (Nearest integer)

Official Ans. by NTA (3)

Sol. $P_T V_T = n_T RT$

For CH_4

$2 \times 2 = n_1 RT$

$\Rightarrow n_1 = \frac{4}{RT}$

For CO_2

$\Rightarrow n_2 = \frac{12}{RT}$

For Ne

$\Rightarrow n_3 = \frac{12}{RT}$

$\Rightarrow n_T = \frac{1}{RT}[4+12+12] = \frac{28}{RT}$

$P_T = \frac{28}{RT}\frac{RT}{V_T}$

$P_T = \frac{28}{V_T} = 3.11$

90. The number of following statement/s which is/are incorrect is_____

(A) Line emission spectra are used to study the electronic structure

(B) The emission spectra of atoms in the gas phase show a continuous spread of wavelength from red to violet

(C) An absorption spectrum is like the photographic negative of an emission spectrum

(D) The element helium was discovered in the sun by spectroscopic method

Official Ans. by NTA (1)

Sol. Statement (B) is incorrect.

FINAL JEE–MAIN EXAMINATION – APRIL, 2023

(Held On Saturday 08th April, 2023) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS

TEST PAPER WITH ANSWER

SECTION-A

1. Let the mean and variance of 12 observations be $\frac{9}{2}$ and 4 respectively. Later on, it was observed that two observations were considered as 9 and 10 instead of 7 and 14 respectively. If the correct variance is $\frac{m}{n}$, where m and n are co-prime, then m+n is equal to

(1) 316

(2) 314

(3) 317

(4) 315

Official Ans. by NTA (3)

2. Let a_n be the n^{th} term of the series 5 + 8 + 14 + 23 + 35 + 50 + … and $S_n = \sum_{k=1}^{n} a_k$. Then $S_{30} - a_{40}$ is equal to

(1) 11310

(2) 11280

(3) 11290

(4) 11260

Official Ans. by NTA (3)

3. Let P be the plane passing through the line $\frac{x-1}{1} = \frac{y-2}{-3} = \frac{z+5}{7}$ and the point (2, 4, –3). If the image of the point (–1, 3, 4) in the plane P is (α, β, γ), then α + β + γ is equal to

(1) 12

(2) 11

(3) 9

(4) 10

Official Ans. by NTA (4)

4. Let $A = \theta \in (0,2\pi) : \frac{1+2i\sin\theta}{1-i\sin\theta}$ is purely imaginary . Then the sum of the elements in A is

(1) π

(2) 2 π

(3) 4 π

(4) 3 π

Official Ans. by NTA (3)

5. The absolute difference of the coefficients of x^{10} and x^7 in the expansion of $\left(2x^2 + \frac{1}{2x}\right)^{11}$ is equal to

(1) $12^3 - 12$

(2) $11^3 - 11$

(3) $10^3 - 10$

(4) $13^3 - 13$

Official Ans. by NTA (1)

6. If the number of words, with or without meaning, which can be made using all the letters of the word MATHEMATICS in which C and S do not come together, is (6!)k, then k is equal to

(1) 1890

(2) 945

(3) 2835

(4) 5670

Official Ans. by NTA (4)

7. Let S be the set of all values of θ ∈ [–π, π] for which the system of linear equations

$x + y + \sqrt{3}z = 0$

$-x + (\tan\theta)y + \sqrt{7}z = 0$

$x + y + (\tan\theta)z = 0$

has non-trivial solution. Then $\frac{120}{\pi}\sum_{\theta\in s}\theta$ is equal to

(1) 40

(2) 10

(3) 20

(4) 30

Official Ans. by NTA (3)

8. If the probability that the random variable X takes values x is given by $P(X = x) = k(x + 1)3^{-x}$, x = 0, 1, 2, 3… , where k is a constant, then P (X ≥ 2) is equal to

(1) $\frac{7}{27}$

(2) $\frac{11}{18}$

(3) $\frac{7}{18}$

(4) $\frac{20}{27}$

Official Ans. by NTA (1)

9. The value of $36(4\cos^2 9° - 1)(4\cos^2 27° - 1)(4\cos^2 81° - 1)(4\cos^2 243° - 1)$ is

(1) 54

(2) 18

(3) 27

(4) 36

Official Ans. by NTA (4)

10. The integral $\int\left(\left(\frac{x}{2}\right)^x + \left(\frac{2}{x}\right)^x\right)\log_2 x\,dx$ is equal to

(1) $\left(\frac{x}{2}\right)^x + \left(\frac{2}{x}\right)^x + C$

(2) $\left(\frac{x}{2}\right)^x - \left(\frac{2}{x}\right)^x + C$

(3) $\left(\frac{x}{2}\right)^x \log_2\left(\frac{x}{2}\right) + C$

(4) $\left(\frac{x}{2}\right)^x \log_2\left(\frac{2}{x}\right) + C$

Official Ans. by NTA (2)

11. The area of the quadrilateral ABCD with vertices A(2, 1, 1), B(1, 2, 5), C (–2, –3, 5) and D (1, –6, –7) is equal to

(1) 48

(2) $8\sqrt{38}$

(3) 54

(4) $9\sqrt{38}$

Official Ans. by NTA (2)

12. For a, b ∈ Z and $|a - b| \le 10$, let the angle between the plane P: ax + y – z = b and the line l : x – 1 = a –y = z + 1 be $\cos^{-1}\left(\frac{1}{3}\right)$. If the distance of the point (6, –6, 4) from the plane P is $3\sqrt{6}$, then $a^4 + b^2$ is equal to

(1) 25

(2) 85

(3) 48

(4) 32

Official Ans. by NTA (4)

13. $25^{190} - 19^{190} - 8^{190} + 2^{190}$ is divisible by

(1) 34 but not by 14

(2) both 14 and 34

(3) neither 14 nor 34

(4) 14 but not by 34

Official Ans. by NTA (1)

14. Let the vectors $\vec{u}_1 = \hat{i} + \hat{j} + a\hat{k}, \vec{u}_2 = \hat{i} + b\hat{j} + \hat{k}$ and $\vec{u}_3 = c\hat{i} + \hat{j} + \hat{k}$ be coplanar. If the vectors $\vec{v}_1 = (a+b)\hat{i} + c\hat{j} + c\hat{k}$, $\vec{v}_2 = a\hat{i} + (b+c)\hat{j} + a\hat{k}$ and $\vec{v}_3 = b\hat{i} + b\hat{j} + (c+a)\hat{k}$ are also coplanar, then 6 (a + b + c) is equal to

(1) 0

(2) 6

(3) 12

(4) 4

Official Ans. by NTA (3)

15. Let O be the origin and OP and OQ be the tangents to the circle $x^2 + y^2 - 6x + 4y + 8 = 0$ at the point P and Q on it. If the circumcircle of the triangle OPQ passes through the point $\left(\alpha, \frac{1}{2}\right)$, then a value of α is

(1) $\frac{3}{2}$

(2) $\frac{5}{2}$

(3) 1

(4) $-\frac{1}{2}$

Official Ans. by NTA (2)

16. The negation of $p \wedge (\sim q) \vee (\sim p)$ is equivalent to

(1) $p \wedge q$

(2) $p \wedge (\sim q)$

(3) $p \wedge (q \wedge (\sim p))$

(4) $p \vee (q \vee (\sim p))$

Official Ans. by NTA (1)

17. If $\alpha > \beta > 0$ are the roots of the equation $ax^2 + bx + 1 = 0$, and

$$\lim_{x \to \frac{1}{\alpha}} \left(\frac{1 - \cos\left(x^2 + bx + a\right)}{2\left(1 - \alpha x\right)^2} \right)^{\frac{1}{2}} = \frac{1}{k}\left(\frac{1}{\beta} - \frac{1}{\alpha}\right)$$

,then k is equal to

(1) 2β

(2) 2α

(3) α

(4) β

Official Ans. by NTA (2)

18. If $A = \begin{bmatrix} 1 & 5 \\ \lambda & 10 \end{bmatrix}$, $A^{-1} = \alpha A + \beta I$ and $\alpha + \beta = -2$, then $4\alpha^2 + \beta^2 + \lambda^2$ is equal to:

(1) 12

(2) 10

(3) 19

(4) 14

Official Ans. by NTA (4)

19. Let A(0,1), B(1, 1) and C(1, 0) be the mid – points of the sides of a triangle with incentre at the point D. If the focus of the parabola $y^2 = 4ax$ passing through D is $\left(\alpha + \beta\sqrt{2}, 0\right)$, where α and β are rational numbers, then $\frac{\alpha}{\beta^2}$ is equal to

(1) 6

(2) 8

(3) 12

(4) $\frac{9}{2}$

Official Ans. by NTA (2)

20. Let A = {1, 2, 3, 4, 5, 6, 7}.Then the relation R = $\{(x,y) \in A \times A : x + y = 7\}$is

(1) transitive but neither symmetric nor reflexive

(2) reflexive but neither symmetric nor transitive

(3) an equivalence relation

(4) symmetric but neither reflexive nor transitive

Official Ans. by NTA (4)

SECTION-B

21. Let [t] denote the greatest integer function. If $\int_0^{2.4} \left[x^2\right] dx = \alpha + \beta\sqrt{2} + \gamma\sqrt{3} + \delta\sqrt{5}$, then $\alpha + \beta + \gamma + \delta$ is equal to ____

Official Ans. by NTA (6)

22. Let k and m be positive real numbers such that the function $f(x) = \begin{cases} 3x^2 + k\sqrt{x+1}, & 0 < x < 1 \\ mx^2 + k^2, & x \geq 1 \end{cases}$ is differentiable for all x > 0. Then $\frac{8f'(8)}{f'\left(\frac{1}{8}\right)}$ is equal to ____

Official Ans. by NTA (309)

23. Let $0 < z < y < x$ be three real numbers such that $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in an arithmetic progression and x, $\sqrt{2}y$, z are in a geometric progression. If $xy + yz + zx = \frac{3}{\sqrt{2}}xyz$, then $3(x + y + z)^2$ is equal to____

Official Ans. by NTA (150)

24. If domain of the function $\log_e\left(\frac{6x^2+5x+1}{2x-1}\right)+\cos^{-1}\left(\frac{2x^2-3x+4}{3x-5}\right)$ is $(\alpha, \beta) \cup (\gamma, \delta]$, then $18(\alpha^2 + \beta^2 + \gamma^2 + \delta^2)$ is equal to ____

Official Ans. by NTA (20)

25. Let m and n be the numbers of real roots of the quadratic equations $x^2 - 12x + [x] + 31 = 0$ and $x^2 - 5|x + 2| - 4 = 0$ respectively, where [x] denotes the greatest integer $\le x$. Then $m^2 + mn + n^2$ is equal to ____.

Official Ans. by NTA (9)

26. The ordinates of the points P and Q on the parabola with focus (3, 0) and directrix x = –3 are in the ratio 3 : 1. If R(α, β) is the point of intersection of the tangents to the parabola at P and Q, then $\frac{\beta^2}{\alpha}$ is equal to____:

Official Ans. by NTA (16)

27. Let the solution curve x = x(y), $0 < y < \frac{\pi}{2}$, of the differential equation $(\log_e(\cos y))^2 \cos y\, dx - (1 + 3x \log_e(\cos y))\sin y\, dy = 0$ satisfy $x\left(\frac{\pi}{3}\right) = \frac{1}{2\log_e 2}$. If $x\left(\frac{\pi}{6}\right) = \frac{1}{\log_e m - \log_e n}$, where m and n are co-prime, then mn is equal to

Official Ans. by NTA (12)

28. Let P_1 be the plane $3x - y - 7z = 11$ and P_2 be the plane passing through the points (2, –1, 0), (2, 0, –1), and (5, 1, 1). If the foot of the perpendicular drawn from the point (7, 4, –1)on the line of intersection of the planes P_1 and P_2 is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to ____.

Official Ans. by NTA (11)

29. Let R = {a, b, c, d, e} and S = {1, 2, 3, 4}. Total number of onto function f : R $\rightarrow$ S such that f(a) $\neq$ 1, is equal to _____.

Official Ans. by NTA (384)

30. Let the area enclosed by the lines x + y = 2, y = 0, x = 0 and the curve $f(x) = \min\left\{x^2 + \frac{3}{4}, 1 + [x]\right\}$ where [x] denotes the greatest integer $\le$ x, be A. Then the value of 12A is _____

Official Ans. by NTA (17)

SECTION-A

31. Electric potential at a point 'P' due to a point charge of 5×10^{-9} C is 50 V. The distance of 'P' from the point charge is:

(Assume, $\frac{1}{4\pi\varepsilon_0}=9\times10^{+9}\,Nm^2C^{-2}$)

(1) 3 cm (2) 9 cm

(3) 90 cm (4) 0.9 cm

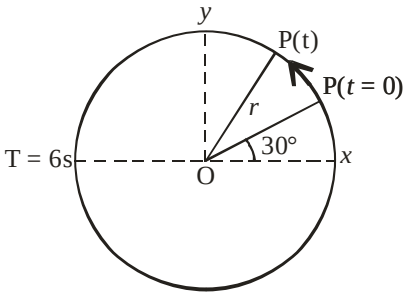
Official Ans. by NTA (3)

Sol. $V_P=\frac{KQ}{r}$

$50=\frac{9\times10^9\times5\times10^{-9}}{r}$

$r=\frac{45}{50}=\frac{9}{10}=0.9m=90cm$

32. For particle P revolving round the centre O with radius of circular path r and angular velocity ω, as shown in below figure, the projection of OP on the x-axis at time t is



(1) $x(t)=r\cos\left(\omega t+\frac{\pi}{6}\right)$

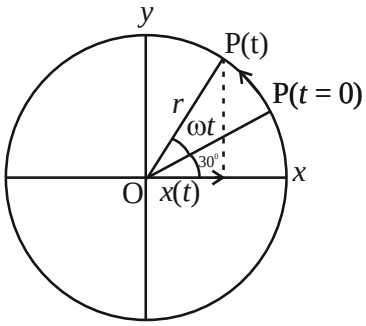
(2) $x(t)=r\cos(\omega t)$

(3) $x(t)=r\sin\left(\omega t+\frac{\pi}{6}\right)$

(4) $x(t)=r\cos\left(\omega t-\frac{\pi}{6}\omega\right)$

Official Ans. by NTA (1)

Sol.



$x(t) = r\cos(\omega t + 30°)$

$x(t) = r\cos(\omega t + \pi/6)$

33. Match List I with List II

LIST-I		LIST-II	
A.	Torque	I.	$ML^{-2}T^{-2}$
B.	Stress	II.	$ML^{2}T^{-2}$
C.	Pressure gradient	III.	$ML^{-1}T^{-1}$
D.	Coefficient of viscosity	IV.	$ML^{-1}T^{-2}$

Choose the correct answer from the options given below:

(1) A-III, B-IV, C-I, D-II

(2) A-IV, B-II, C-III, D-I

(3) A-II, B-IV, C-I, D-III

(4) A-II, B-I, C-IV, D-III

Official Ans. by NTA (3)

Sol. **A.** Torque $\Rightarrow \vec{\tau}=\vec{r}\times\vec{F}$

$[\tau]=[L][MLT^{-2}]$

$\Rightarrow ML^2T^{-2}$

B. $\text{Stress}=\frac{F}{A}\Rightarrow\frac{MLT^{-2}}{L^2}$

$[\text{stress}] = ML^{-1}T^{-2}$

C. Pressure gradient $=\frac{\Delta P}{\Delta X}$

$\Rightarrow\frac{[F/A]}{[L]}\Rightarrow\frac{MLT^{-2}}{L^3}$

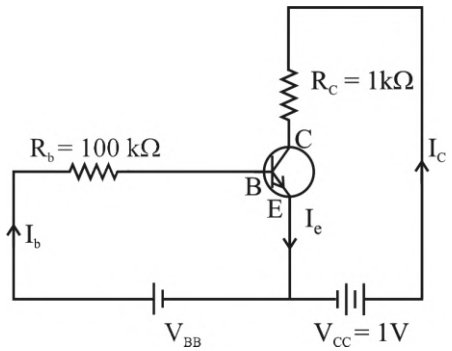
$\Rightarrow ML^{-2}T^{-2}$

D. Coefficient of viscosity $\Rightarrow F = 6\pi\eta rv$

$MLT^{-2} = [\eta]\,L^2T^{-1}$

$[\eta] = ML^{-1}T^{-1}$

34. For a given transistor amplifier circuit in CE configuration V_{CC} = 1 V, R_c = 1 kΩ, R_b = 100 kΩ and β = 100. Value of base current I_b is

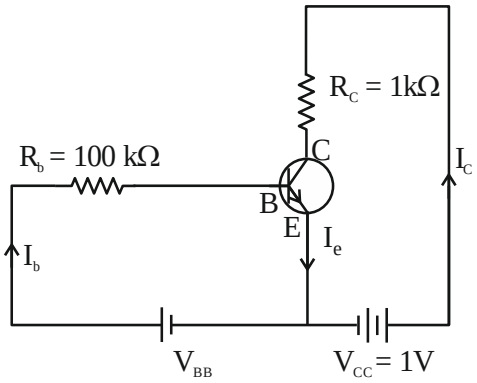


(1) I_b = 1.0 μA (2) I_b = 0.10 μA

(3) I_b = 100 μA (4) I_b = 10 μA

Official Ans. by NTA (4)

Sol.



Considering the transistor in saturation mode

$V_{CE} = 0$

Using KVL

$-I_cR_c + V_{CC} = 0$

$$I_c = \frac{V_{CC}}{R_C} = \frac{1}{1\times10^3}$$

$I_c = 10^{-3}$A

$$\beta = \frac{I_c}{I_b}$$

$$I_b = \frac{10^{-3}}{100} \Rightarrow 10^{-5}\text{A} \Rightarrow I_b = 10\ \mu\text{A}$$

35. The trajectory of projectile, projected from the ground is given by $y = x - \frac{x^2}{20}$. Where x and y are measured in meter. The maximum height attained by the projectile will be.

(1) 5 m (2) $10\sqrt{2}$m

(3) 200 m (4) 10 m

Official Ans. by NTA (1)

Sol. $y = x - \frac{x^2}{20}$

For maximum height,

$$\frac{dy}{dx} = 0 \Rightarrow 1 - \frac{2x}{20} = 0$$

x = 10

$$\text{So, } y_{max} = 10 - \frac{100}{20} = 5\text{m}$$

36. A radio-active material is reduced to 1/8 of its original amount in 3 days. If 8×10^{-3} kg of the material is left after 5 days. The initial amount of the material is

(1) 64 g (2) 40 g

(3) 32 g (4) 256 g

Official Ans. by NTA (4)

Sol. $N = N_0\left(\frac{1}{2}\right)^n$

$$\frac{N_0}{8} = N_0\left(\frac{1}{2}\right)^n$$

n = 3

3 half lives = 3 days

1 half life = 1 day

5 days = 5 half life

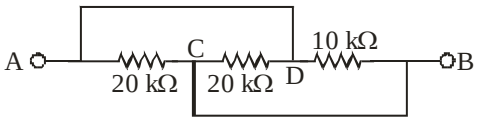
$$N = N_0\left(\frac{1}{2}\right)^n$$

$$8 \times 10^{-3} = N_0\left(\frac{1}{2}\right)^5$$

$N_0 = 256 \times 10^{-3}$ kg

N_0 = 256 g

37. The equivalent resistance between A and B as shown in figure is:

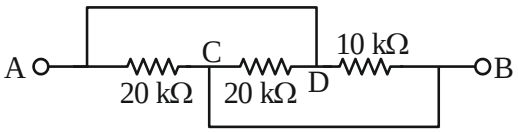


(1) 5 kΩ (2) 30 kΩ

(3) 10 kΩ (4) 20 kΩ

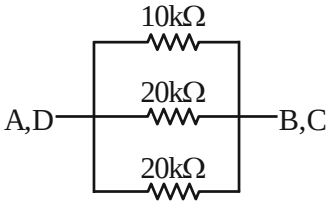
Official Ans. by NTA (1)

Sol.



$V_A = V_D$

$V_C = V_B$



All resistors are in parallel. So,

$$\frac{1}{R_{eq}} = \frac{1}{10} + \frac{1}{20} + \frac{1}{20}$$

$R_{eq} = 5\ k\Omega$.

38. A hydraulic automobile lift is designed to lift vehicles of mass 5000 kg. The area of cross section of the cylinder carrying the load is 250 cm^2. The maximum pressure the smaller piston would have to bear is [Assume g = 10 m/s^2] :

(1) $200 \times 10^{+6}$ Pa (2) $20 \times 10^{+6}$ Pa

(3) $2 \times 10^{+6}$ Pa (4) $2 \times 10^{+5}$ Pa

Official Ans. by NTA (3)

Sol. Force = mg = 5000 g

Area of cross section = 250 cm^2 = 250×10^{-4} m^2

$$\text{maximum pressure} = \frac{\text{Force}}{\text{area of cross section}}$$

$$= \frac{5000g}{250\times10^{-4}} = \frac{20\times g}{10^{-4}} = 2\times10^{6}\,\text{Pa}$$

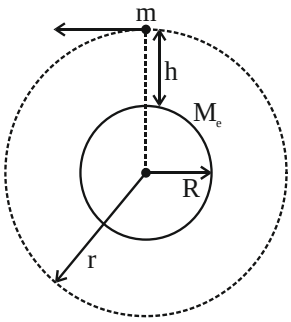
39. The orbital angular momentum of a satellite is L, when it is revolving in a circular orbit at height h from earth surface. If the distance of satellite from the earth centre is increased by eight times to its initial value, then the new angular momentum will be-

(1) 8 L (2) 4 L

(3) 9 L (4) 3 L

Official Ans. by NTA (4)

Sol.



$L = mvr$

$$v = \sqrt{\frac{GM_e}{r}}$$

$$L = m\sqrt{\frac{GM_e}{r}}\cdot r$$

$$L \propto r^{\frac{1}{2}}$$

Now distance from centre is increased by 8 times.

So new distance from centre = r + 8r = 9r

Now angular momentum $L' \propto (9r)^{1/2}$

$$\frac{L}{L'} = \frac{r^{1/2}}{(9r)^{1/2}} = \frac{1}{3}$$

L'= 3 L

40. The temperature at which the kinetic energy of oxygen molecules becomes double than its value at 27°C is

(1) 1227° C (2) 927°C

(3) 327°C (4) 627°C

Official Ans. by NTA (3)

Sol. Kinetic energy = $\frac{f}{2}kT$, T is absolute temperature.

If K_1 is kinetic energy at 27°C.

K_2 is kinetic energy at new temperature T.

$$\frac{K_1}{K_2} = \frac{T_1}{T_2} \Rightarrow \frac{1}{2} = \frac{300}{T}$$

T = 600 K

T = 327°C

41. The acceleration due to gravity at height h above the earth if h <<R (radius of earth) is given by

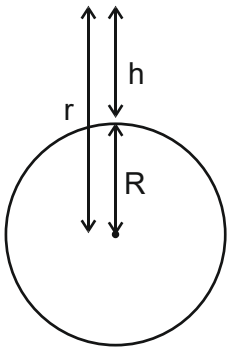
(1) $g' = g\left(1 - \frac{2h}{R}\right)$ (2) $g' = g\left(1 - \frac{2h^2}{R^2}\right)$

(3) $g' = g\left(1 - \frac{h}{2R}\right)$ (4) $g' = g\left(1 - \frac{h^2}{2R^2}\right)$

Official Ans. by NTA (1)

Sol. For point outside the surface of earth

$$g = \frac{GM}{r^2}$$



r = distance from center of earth

$$\Rightarrow g(h) = \frac{GM}{(R+h)^2} \Rightarrow g(h) = \frac{GM}{R^2\left(1+\frac{h}{R}\right)^2}$$

$$\Rightarrow g(h) = \frac{GM}{R^2}\left(1+\frac{h}{R}\right)^{-2}$$

If $h <<< R$, $\left(1+\frac{h}{R}\right)^{-2} \approx 1-\frac{2h}{R}$

$$\Rightarrow g(h) = \frac{GM}{R^2}\left(1-\frac{2h}{R}\right)$$

$$\Rightarrow g(h) = g_{surface}\left(1-\frac{2h}{R}\right), \frac{GM}{R^2} = g_{surface}$$

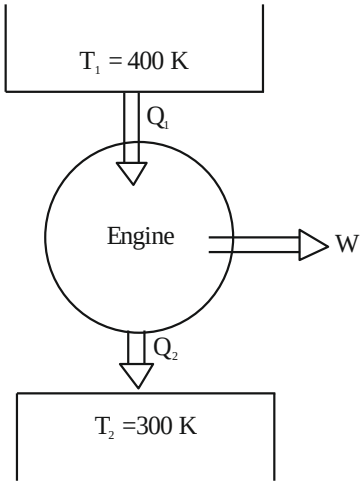
42. Work done by a Carnot engine operating between temperatures 127°C and 27°C is 2 kJ. The amount of heat transferred to the engine by the reservoir is:

(1) 4kJ (2) 2 kJ

(3) 8kJ (4) 2.67 kJ

Official Ans. by NTA (3)

Sol.



Efficiency of carnot engine

$$\eta = 1-\frac{T_2}{T_1} = \frac{W}{Q_1}$$

$$\Rightarrow \frac{W}{Q_1} = 1-\frac{300}{400} = \frac{1}{4}$$

$$\Rightarrow \frac{2kJ}{Q_1} = \frac{1}{4}$$

$$\Rightarrow Q_1 = 8 \text{ kJ}$$

43. Given below are two statements:

Statement I: Area under velocity- time graph gives the distance travelled by the body in a given time.

Statement II: Area under acceleration- time graph is equal to the change in velocity- in the given time.

In the light of given statements, choose the correct answer from the options given below.

(1) Both Statement I and Statement II are true.

(2) Statement I is correct but Statement II is false.

(3) Statement I is incorrect but Statement II is true.

(4) Both Statement I and Statement II are False.

Official Ans. by NTA (1)

Sol. Area under velocity time graph gives displacement of body in given time.

Area under acceleration time graph gives change in velocity in the given time.

So Statement I false

Statement II True

44. The waves emitted when a metal target is bombarded with high energy electrons are

(1) Microwaves (2) X-rays

(3) Infrared rays (4) Radio Waves

Official Ans. by NTA (2)

Sol. X rays are emitted when target metal is bombarded with high energy electron.

45. The width of fringe is 2 mm on the screen in a double slits experiment for the light of wavelength of 400 nm. The width of the fringe for the light of wavelength 600 nm will be:

(1) 4 mm (2) 1.33 mm

(3) 3 mm (4) 2 mm

Official Ans. by NTA (3)

Sol. Fringe width $(\beta) = \frac{D\lambda}{d}$

$\Rightarrow \frac{\beta_2}{\beta_1} = \frac{\lambda_2}{\lambda_1}$

$\Rightarrow \frac{\beta_2}{2mm} = \frac{600nm}{400nm} = \frac{3}{2}$

$\Rightarrow \boxed{\beta_2 = 3mm}$

46. Given below are two statements; one is labelled as Assertion A and the other is labelled as Reason R

Assertion A: Electromagnets are made of soft iron.

Reason R: Soft iron has high permeability and low retentivity.

In the light of above, statements, choose the most appropriate answer from the options given below.

(1) A is not correct but R is correct

(2) Both A and R are correct and R is the correct explanation of A

(3) Both A and R are correct but R is NOT the correct explanation of A

(4) A is correct but R is not correct

Official Ans. by NTA (2)

Sol. Electromagnets are made of soft iron because it has high permeability and low retentivity.

So, Both A and R are correct and R is the correct explanation of A

47. In photo electric effect

A. The photocurrent is proportional to the intensity of the incident radiation.

B. Maximum Kinetic energy with which photoelectrons are emitted depends on the intensity of incident light.

C. Max. K.E with which photoelectrons are emitted depends on the frequency of incident light.

D. The emission of photoelectrons require a minimum threshold intensity of incident radiation.

E. Max. K.E of the photoelectrons is independent of the frequency of the incident light.

Choose the correct answer from the options given below:

(1) A and C only (2) A and E only

(3) B and C only (4) A and B only

Official Ans. by NTA (1)

Sol. Intensity of light $\propto$ number of photons $\propto$ no of photo electrons $\propto$ photo current

So, A is correct

$KE_{max} = h\nu - \phi$

KE_{max} depends on frequency

So, C is correct

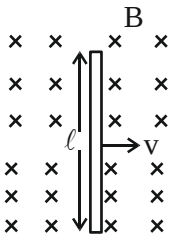
So, A and C are correct

48. An emf of 0.08 V is induced in a metal rod of length 10 cm held normal to a uniform magnetic field of 0.4 T, when moves with a velocity of:

(1) 2 ms^{-1} (2) 3.2 ms^{-1}

(3) 0.5 ms^{-1} (4) 20 ms^{-1}

Sol. Official Ans. by NTA (1)



Induced emf = Bℓv

$\Rightarrow 0.08 = 0.4\left(\frac{10}{100}\right)v$

$\Rightarrow v = \left(\frac{0.08 \times 10}{0.4}\right)$ $\Rightarrow v = 2m/s$

49. A bullet of mass 0.1 kg moving horizontally with speed 400 ms^{-1} hits a wooden block of mass 3.9 kg kept on a horizontal rough surface. The bullet gets embedded into the block and moves 20 m before coming to rest. The coefficient of friction between the block and the surface is_____.

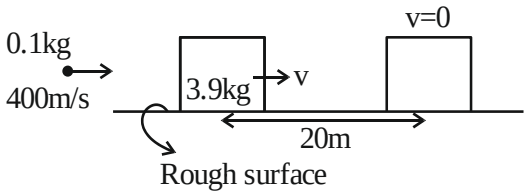
(Given g= 10 ms^2)

(1) 0.50 (2) 0.90

(3) 0.65 (4) 0.25

Official Ans. by NTA (4)

Sol.



$P_i = P_f$ (Collision)

$\Rightarrow (0.1)(400) = (0.1+3.9)v$

$\Rightarrow v = \frac{0.1\times 400}{4} = 10 m/s$

$a = \frac{\mu mg}{m} = \mu g$

Apply equation of motion,

$v^2 = u^2 + 2as$

$\Rightarrow 0 = (10)^2 - 2\mu g \times 20$

$\Rightarrow 40\mu g = 100$

$\Rightarrow \mu = \frac{100}{2\times 10\times 20} = \frac{1}{4}$

50. The power radiated from a linear antenna of length l is proportional to

(Given, λ = Wavelength of wave):

(1) $\frac{l}{\lambda}$ (2) $\frac{l}{\lambda^2}$ (3) $\frac{l^2}{\lambda}$ (4) $\left(\frac{l}{\lambda}\right)^2$

Official Ans. by NTA (4)

Sol. Power radiated form a linear antenna of length

$l \propto \left(\frac{l}{\lambda}\right)^2$

SECTION-B

51. A series combination of resistor of resistance 100 Ω, inductor of inductance 1 H and capacitor of capacitance 6.25 μF is connected to an ac source. The quality factor of the circuit will be _____.

Official Ans. by NTA 4

Sol. Quality factor $= \frac{X_L}{R} = \frac{\omega L}{R}$

$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1\times 6.25\times 10^{-6}}} = \frac{10^3}{2.5} = 400/sec$

Q-factor $= \frac{400\times 1}{100} = 4$

52. A guitar string of length 90 cm vibrates with a fundamental frequency of 120 Hz. The length of the string producing a fundamental frequency of 180 Hz will be _____ cm.

Official Ans. by NTA 60

Sol. $f = \frac{nv}{2\ell}$, for fundamental mode n = 1

$f = \frac{v}{2\ell}$

$f \propto \frac{1}{\ell}$

$\frac{f_1}{f_2} = \frac{\ell_2}{\ell_1}$

$\frac{120}{180} = \frac{\ell_2}{90}$

$\ell_2 = 60cm$

53. The ratio of wavelength of spectral lines H_α and H_β in the Balmer series is $\frac{x}{20}$. The value of x is _____

Official Ans. by NTA 27

Sol. $\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$ for H-atom

For balmer series, n_1=2

$\frac{1}{\lambda} = R\left[\frac{1}{4} - \frac{1}{n_2^2}\right]$

For H_α, $n_2 = 3$

& H_β, $n_2 = 4$

$\frac{1}{\lambda_{H_\alpha}} = R\left[\frac{1}{4} - \frac{1}{9}\right] = \frac{5R}{36}$

$\frac{1}{\lambda_{H_\beta}} = R\left[\frac{1}{4} - \frac{1}{16}\right] = \frac{3R}{16}$

$\frac{\frac{1}{\lambda_{H_\alpha}}}{\frac{1}{\lambda_{H_\beta}}} = \frac{\frac{5R}{36}}{\frac{3R}{16}}$

$\frac{\lambda_{H_\alpha}}{\lambda_{H_\beta}} = \frac{27}{20} = \frac{x}{20}$

$x = 27$

54. The number density of free electrons in copper is nearly $8 \times 10^{28}\ m^{-3}$. A copper wire has its area of cross section $= 2 \times 10^{-6}\ m^2$ and is carrying a current of 3.2 A. The drift speed of the electrons is _____ $\times 10^{-6}\ ms^{-1}$.

Official Ans. by NTA 125

Sol. $n = 8\times10^{28}\ m^{-3}$

Area $= 2\times10^{-6}\ m^2$

$I = 3.2\ A$

$I = neAv_d$

$$V_d = \frac{I}{neA} = 125\times10^{-6}\,m/s$$

55. A steel rod of length 1 m and cross sectional area $10^{-4}\ m^2$ is heated from 0°C to 200°C without being allowed to extend or bend. The compressive tension produced in the rod is _____ $\times 10^4$ N. (Given Young's modulus of steel $= 2 \times 10^{11}\ Nm^{-2}$, coefficient of linear expansion $= 10^{-5}K^{-1}$.

Official Ans. by NTA 4

Sol. Stress = Y × strain

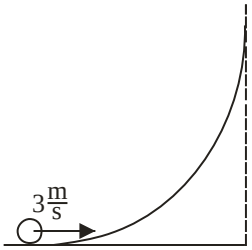
$$\text{Stress} = Y \times \frac{\Delta\ell}{\ell}$$

$$= Y \times \frac{\ell\alpha\Delta T}{\ell} = Y\alpha\Delta T$$

Compressive Tension = Stress × Area of cross section

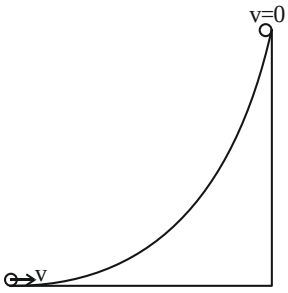
$= YA\alpha\Delta T = 4 \times 10^4$ N

56. A hollow spherical ball of uniform density rolls up a curved surface with an initial velocity 3 m/s (as shown in figure). Maximum height with respect to the initial position covered by it will be _____ cm.



Official Ans. by NTA 75

Sol.



At highest point $KE_f = 0$

Initial KE = Translational KE + Rotational KE

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

In case of rolling $v = R\omega$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\times\frac{2}{3}mR^2\times\frac{v^2}{R^2}$$

$$= \frac{5}{6}mv^2$$

Apply energy conservation

$$KE_i + PE_i = KE_f + PE_f$$

$$\frac{5}{6}mv^2 = mgh$$

$$h = \frac{5}{6\times10}\times9m = \frac{15}{20}m = 75cm$$

57. A body of mass 5 kg is moving with a momentum of 10 kg ms^{-1}. Now a force of 2 N acts on the body in the direction of its motion for 5 s. The increase in the Kinetic energy of the body is _____ J.

Official Ans. by NTA (30)

Sol. Given

$M = 5$ kg

$P_i = 10$ kg m/s (initial momentum)

Impulse $= F\Delta t = \Delta P = P_f - P_i$

$2 \times 5 = P_f - 10$

$P_f = 20$ kg m/s (final momentum)

Increase in $KE = KE_f - KE_i$

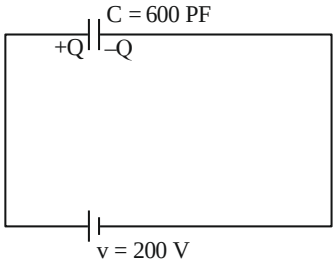
$$= \frac{P_f^2}{2m} - \frac{P_i^2}{2m}$$

$$= \frac{400}{2\times5} - \frac{100}{2\times5} = 40 - 10 = 30J$$

58. A 600 pF capacitor is charged by 200V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. Electrostatic energy lost in the process is _____ μJ.

Official Ans. by NTA (6)

Sol.

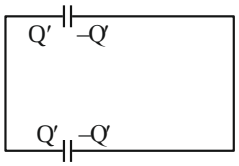


$Q = CV = 600 \times 10^{-12} \times 200 = 12 \times 10^{-8} C$

Initial energy $= \frac{1}{2}CV^2$

$= \frac{1}{2} \times 600 \times 10^{-12} \times (200)^2 = 12\mu J$

When connected to another uncharged capacitor



Charge will be equally distributed on identical capacitor

$Q' = \frac{Q}{2} = 6 \times 10^{-8}$

Final energy $= 2 \times \frac{Q'^2}{2C} = \frac{Q'^2}{C}$

$\frac{(6 \times 10^{-8})^2}{600 \times 10^{-12}} = 6\mu J$

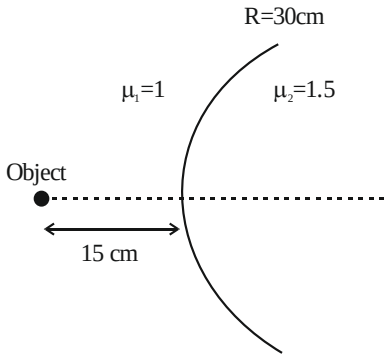
Energy lost = Initial energy – Final energy

= (12–6) μJ = 6μJ

59. Two transparent media having refractive indices 1.0 and 1.5 are separated by a spherical refracting surface of radius of curvature 30 cm. The centre of curvature of surface is towards denser medium and a point object is placed on the principle axis in rarer medium at a distance of 15 cm from the pole of the surface. The distance of image from the pole of the surface is _____ cm.

Official Ans. by NTA 30

Sol.



$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\frac{1.5}{v} - \frac{1}{-15} = \frac{1.5-1}{30} = \frac{1}{60}$

$\frac{1.5}{v} + \frac{1}{15} = \frac{1}{60}$

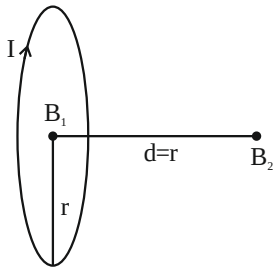
$\frac{1.5}{v} = \frac{1}{60} - \frac{1}{15} = \frac{-1}{20}$

$\frac{1.5}{v} = -\frac{1}{20} \Rightarrow v = -30cm$

60. The ratio of magnetic field at the centre of a current carrying coil of radius *r* to the magnetic field at distance *r* from the centre of coil on its axis is $\sqrt{x} : 1$. The value of x is _____

Official Ans. by NTA 8

Sol.



Magnetic field at centre $(B_1) = \frac{\mu_0 I}{2r}$

Magnetic field on axis $= \frac{\mu_0 I r^2}{2(r^2 + d^2)^{3/2}}$

Value of d = r (given)

$B_2 = \frac{\mu_0 I}{4\sqrt{2}r}$

$\frac{B_1}{B_2} = \frac{\mu_0 I}{2r} \times \frac{4\sqrt{2}r}{\mu_0 I} = \frac{2\sqrt{2}}{1} = \frac{\sqrt{8}}{1}$

x = 8

SECTION-A

61. Which of the following have same number of significant figures ?

(A) 0.00253

(B) 1.0003

(C) 15.0

(D) 163

Choose the correct answer from the options given below

(1) A, B and C only

(2) C and D only

(3) A, C and D only

(4) B and C only

Official Ans. by NTA (3)

Sol. All non zero digits are significant.

0.00253

Significant figures = 3(2, 5, 3)

1.0003

Zeros between non-zero digit are significant.

Thus, 1.0003 has 5 significant figures.

15.0

Significant number = 3

163

Significant number = 3

Options (3) – A, C and D

62. Which of these reactions is not a part of breakdown of ozone in stratosphere ?

(1) $\dot{Cl}O(g) + O(g) \longrightarrow \dot{Cl}(g) + O_2(g)$

(2) $\dot{Cl}(g) + O_3(g) \longrightarrow \dot{Cl}O(g) + O_2(g)$

(3) $2\ \dot{Cl}O \longrightarrow ClO_2(g) + \dot{Cl}(g)$

(4) $CF_2Cl_2(g) \xrightarrow{uv} \dot{Cl}(g) + \dot{C}F_2Cl(g)$

Official Ans. by NTA (3)

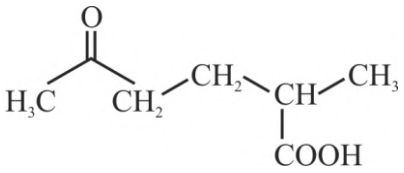
Sol. Ozone destruction

$CF_2Cl_2 \xrightarrow{hv} Cl^{\bullet} + C^{\bullet}F_2Cl(g)$

$Cl^{\bullet} + O_3 \rightarrow ClO^{\bullet} + O_2$

$ClO^{\bullet} + O^{\bullet} \rightarrow Cl^{\bullet} + O_2$

63. The correct IUPAC nomenclature for the following compound is



(1) 5–Formyl–2–methylhexanoic acid

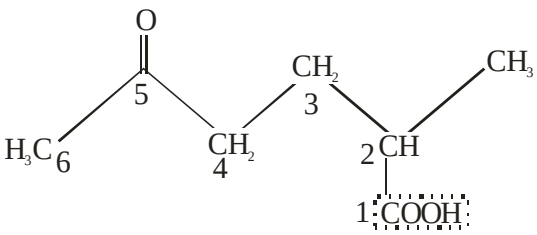
(2) 2–Methyl–5–oxohexanoic acid

(3) 2–Formyl–5–methylhexan–6–oic acid

(4) 5–Methyl–2–oxohexan–6–oic acid

Official Ans. by NTA (2)

Sol.



IUPAC NAME

2–Methyl–5–oxohexanoic acid

64. Arrange the following gases in increasing order of van der Waals constant 'a'

A. Ar

B. CH_4

C. H_2O

D. C_6H_6

Choose the correct option from the following :-

(1) B, C, D and A

(2) C, D, B and A

(3) A, B, C and D

(4) D, C, B and A

Official Ans. by NTA (3)

Sol. Vanderwaal constant – 'a'

(i) Ar = 1.34

(ii) CH_4 = 2.25

(iii) H_2O = 5.46

(iv) C_6H_6 = 18.57

'a' symbolises force of attraction and directly proportional to surface area

65. Given below are two statements :-

Statement I :- Methyl orange is a weak acid.

Statement II :- The benzenoid form of methyl orange is more intense/deeply coloured than the quinonoid form.

In the light of the above statement, choose the most appropriate answer from the options given below :-

(1) Statement I is correct but Statement II is incorrect.

(2) Statement I is incorrect but statement II is correct.

(3) Both Statement I and Statement II are incorrect.

(4) Both statement I and Statement II are correct.

Official Ans. by NTA (3)

Sol. Methyl orange is weak base .

Benzenoid structure $\rightleftharpoons$ Quinonoid structure

(yellow coloured) (Red coloured)

(more intense)

Statement I – FALSE

Statement II – FALSE

66. Given below are two statements :-

Statement I :- In redox titration, the indicators used are sensitive to change in pH of the solution.

Statement II :- In acid-base titration, the indicators used are sensitive to change in oxidation potential.

In the light of the above statements, choose the most appropriate answer from the options given below

(1) Both statement I and statement II are correct.

(2) Statement I is incorrect but Statement II is correct.

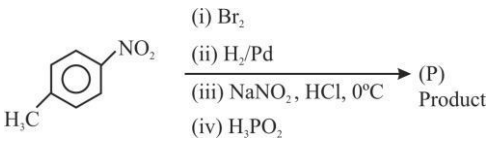
(3) Statement I is correct but Statement II is incorrect.

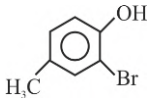
(4) Both statement I and statement II are incorrect.

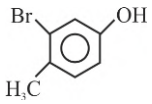
Official Ans. by NTA (4)

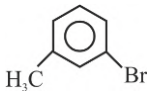
Sol. In redox titration, indicators are sensitive to oxidation potential and in acid base titration, indicators are sensitive to change in pH of solution Both statement are false.

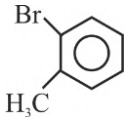
67. The product (P) formed from the following multistep reaction is :-



(1) 

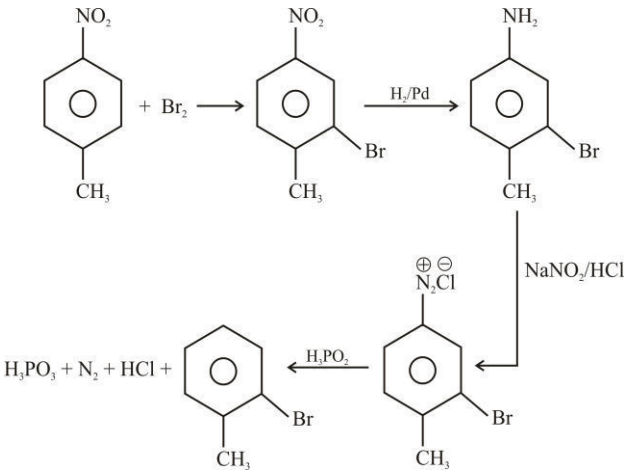
(2) 

(3) 

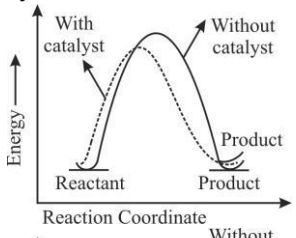
(4) 

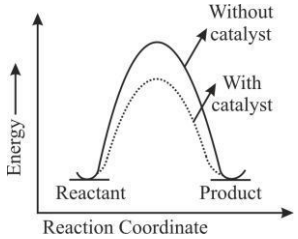
Official Ans. by NTA (4)

Sol.

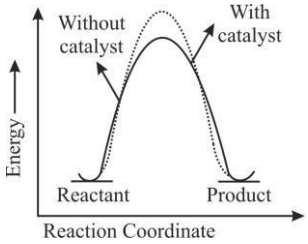


68. The correct reaction profile diagram for a positive catalyst reaction.

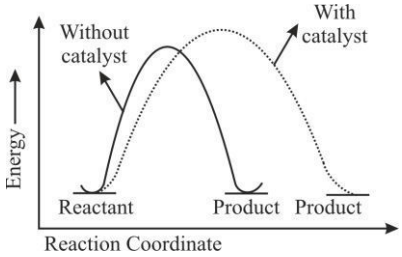
(1) 

(2) 

(3)



(4)



Official Ans. by NTA (2)

Sol. By using positive catalyst :

(i) ΔH does not change

(ii) Activation energy decreases

69. Which of the following can reduce decomposition of H_2O_2 on exposure to light

(1) Alkali

(2) Urea

(3) Dust

(4) Glass containers

Official Ans. by NTA (2)

Sol. Urea acts as a stabilizer in the decomposition of H_2O_2

70. The statement/s which are true about antagonists from the following is/are :-

A. They bind to the receptor site.

B. Get transferred inside the cell for their action.

C. Inhibit the natural communication of the body.

D. Mimic the natural messenger.

Choose the correct answer from the options given below :-

(1) B only

(2) A, C and D

(3) A and B

(4) A and C

Official Ans. by NTA (4)

Sol. Drugs that bind to the receptor site and inhibit its natural function are called antagonists

71. Match List I with List II :-

	List I Coordination Complex		List II Number of unpaired electrons
A.	$[Cr(CN)_6]^{3-}$	I.	0
B.	$[Fe(H_2O)_6]^{2+}$	II.	3
C.	$[Co(NH_3)_6]^{3+}$	III.	2
D.	$[Ni(NH_3)_6]^{2+}$	IV.	4

Choose the correct answer from the options given below :-

(1) A – II, B – IV, C – I, D – III

(2) A – IV, B – III, C – II, D – I

(3) A – III, B – IV, C – I, D – II

(4) A – II, B – I, C – IV, D – III

Official Ans. by NTA (1)

Sol. For option (A)

Cr^{+3} : $3d^3$

$CN^- \rightarrow$ SFL

$\Rightarrow$ No. of unpaired electrons = 3

For option (B)

Fe^{+2} : $3d^6$

H_2O : WFL

No. of unpaired electrons = 4

For option (C)

Co^{+3} : $3d^6$

NH_3 : SFL

No. of unpaired electrons = 0

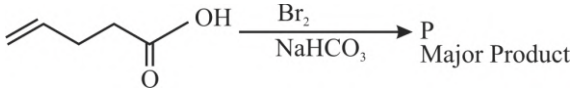
For option (D)

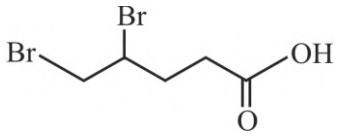
Ni^{+2} : $3d^8$

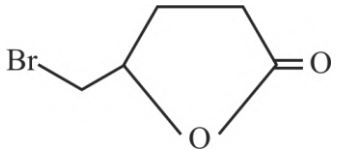
NH_3 : SFL

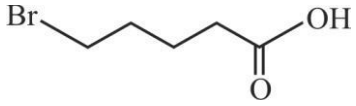
No. of unpaired electrons = 2

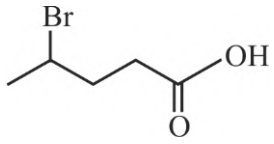
72. Major product 'P' formed in the following reaction is :-



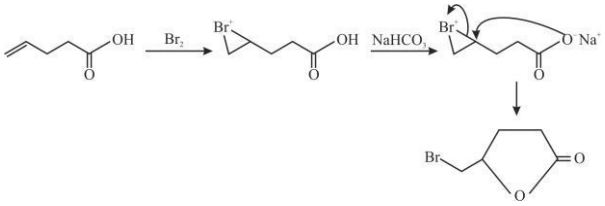
(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (2)

Sol. 

73. In Hall – Heroult process, the following is used for reducing Al_2O_3 :-
(1) Graphite
(2) Magnesium
(3) Na_3AlF_6
(4) CaF_2
Official Ans. by NTA (1)

Sol. In case of Hall's process, reduction of Al_2O_3 to Al can be done using graphite.

74. Given below are two statements : One is labelled as **Assertion A** and the other is labelled as **Reason R**
Assertion A :- Sodium is about 30 times as abundant as potassium in the oceans.
Reason R :- Potassium is bigger in size than sodium.
In the light of above statements, choose the correct answer from the options given below

(1) Both A and R are true and R is the correct explanation of A.
(2) A is true but R is false.
(3) A is false but R is true
(4) Both A and R are true but R is NOT the correct explanation of A.
Official Ans. by NTA (1)

Sol. Due to bigger size of potassium, it forms more efficient lattices as compared to sodium with silicates.
The abundance of sodium in ocean is more due to the more soluble nature of salt of sodium as compared to potassium salts.

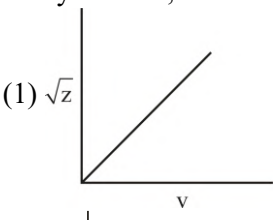
75. Math List I with List II
Choose the correct answer from the options given below :

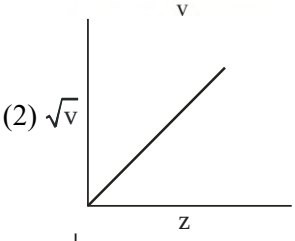
	List I Natural amino acid		**List II One letter code**
A.	Glutamic acid	I.	Q
B.	Glutamine	II.	W
C.	Tyrosine	III.	E
D.	Tryptophan	IV.	Y

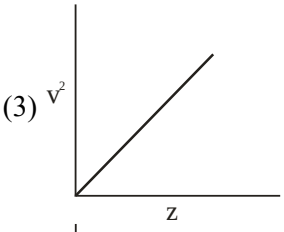
(1) A-II, B-I, C-IV, D-III
(2) A-IV, B-III, C-I, D-II
(3) A-III, B-I, C-IV, D-II
(4) A-III, B-IV, C-I, D-II
Official Ans. by NTA (3)

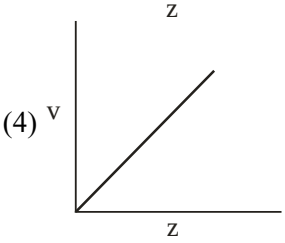
Sol. According to List I and List II option (3) is correct.

76. Henry Moseley studied characteristic X-ray spectra of elements. The graph which represents his observation correctly is : (Given v = frequency of X-ray emitted; Z = atomic number)

(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (2)

Sol. $\sqrt{v} \alpha Z$

77. The descending order of acidity for the following carboxylic acid is :

A. CH_3COOH

B. $F_3C–COOH$

C. $ClCH_2–COOH$

D. $FCH_2–COOH$

E. $BrCH_2–COOH$

Choose the correct answer from the options given below :

(1) D > B > A > E > C

(2) E > D > B > A > C

(3) B > C > D > E > A

(4) B > D > C > E > A

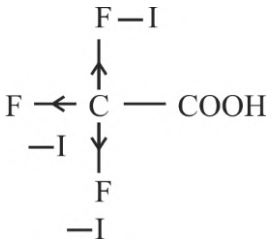
Official Ans. by NTA (4)

Sol. $\text{Acidic Strength } \alpha \frac{1}{+I \text{ effect}}$

$\boxed{\text{Acidic Strength } \alpha -I \text{ effect}}$

$\boxed{F > Cl > Br}$ – I effect order

(A) $CH_3 \rightarrow COOH$ ($+I$)

(B) 

$\Rightarrow$ 3, –I group

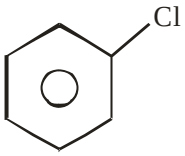
(C) $Cl \leftarrow CH_2 \rightarrow COOH$ ($-I$)

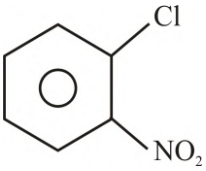
(D) $F \leftarrow CH_2 \rightarrow COOH$ ($-I$)

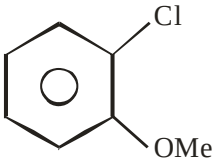
(E) $Br \leftarrow CH_2 \rightarrow COOH$ ($-I$)

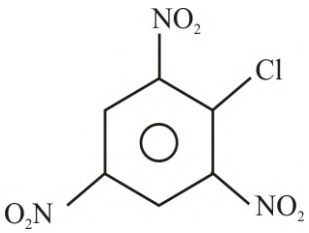
So Option (4) B > D > C > E > A

78. The correct order of reactivity of following haloarenes towards nucleophilic substitution with aqueous NaoH is :

A. 

B. 

C. 

D. 

Choose the correct answer from the options given below :

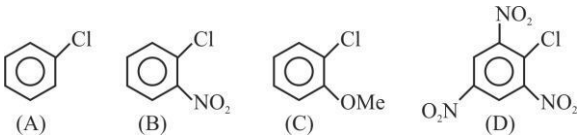
(1) A > B > D > C

(2) C > A > D > B

(3) D > C > B > A

(4) D > B > A > C

Official Ans. by NTA (4)

Sol. 

$\boxed{D > B > A > C}$

Option (4) is correct.

(– M) group increases reactivity where as (+M) group decreases reactivity of Halobenzene towards Nucleophilic substitution reaction.

79. For a good quality cement, the ratio of lime to the total of the oxides of Si, Al and Fe should be as close as to :

(1) 4

(2) 2

(3) 3

(4) 1

Official Ans. by NTA (2)

Sol. $\frac{\%\ CaO}{\%\ SiO_2 + \%Al_2O_3 + \%Fe_2O_3} = 1.9 - 2.1$

Option (2) is correct.

80. A compound 'X' when treated with phthalic anhydride in presence of concentrated H_2SO_4 yields 'Y'. 'Y' is used as an acid/base indicator. 'X' and 'Y' are respectively:

(1) Carbolic acid, Phenolphthalein

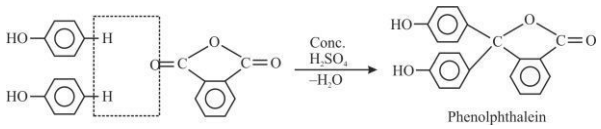
(2) Anisole, methyl orange

(3) Salicylaldehyde, Phenolphthalein

(4) Toludine, Phenolphthalein

Official Ans. by NTA (1)

Sol



Phenolphthalein

SECTION-B

81. The solubility product of $BaSO_4$ is 1×10^{-10} at 298K. The solubility of $BaSO_4$ in 0.1 M K_2SO_4(aq) solution is _____ $\times 10^{-9}$ g L^{-1} (nearest integer).

Given : Molar mass of $BaSO_4$ is 233 g mol^{-1}

Official Ans. by NTA (233)

Sol. $K_2SO_4 \longrightarrow 2K^+ + SO_4^{2-}$

0.1 M 0.2M 0.1M

$BaSO_4 \rightleftharpoons Ba^{+2} + SO_4^{2-}$

a–S S S + 0.1 $\approx$ 0.1

$K_{SP} = S \times 10^{-1}$

$\Rightarrow 1 \times 10^{-10} = S \times 10^{-1}$

$\Rightarrow S = 10^{-9}$ mol L^{-1}

So, $S = 10^{-9} \times 233$ g L^{-1}

So, Answer : 233

82. Coagulating value of electrolytes $AlCl_3$ and NaCl for As_2S_3 are 0.09 and 50.04 respectively. The coagulating power of $AlCl_3$ is x times the coagulating power of NaCl. The value of x is _____:

Official Ans. by NTA (556)

Sol. $\underset{(C.V)}{\text{Coagulating Value}} \propto \dfrac{1}{\underset{(C.P)}{\text{Coagulating Power}}}$

$\Rightarrow \dfrac{(C.V)_{AlCl_3}}{(C.V)_{NaCl}} = \dfrac{(C.P)_{NaCl}}{(C.P)_{AlCl_3}}$

$\Rightarrow \dfrac{0.09}{50.04} = \dfrac{(C.P)_{NaCl}}{(C.P)_{AlCl_3}}$

$\Rightarrow (C.P)_{AlCl_3} = 556(C.P)_{NaCl}$

So, Answer = 556

83. The number of atomic orbitals from the following having 5 radial nodes is _____.

7s, 7p, 6s, 8p, 8d

Official Ans. by NTA (3)

Sol. Radial node = $n - \ell - 1$

7s $\Rightarrow$ R.N = 7 – 0 – 1 = 6

7p $\Rightarrow$ R.N = 7 – 1 – 1 = 5

6s $\Rightarrow$ R.N = 6 – 0 – 1 = 5

8p $\Rightarrow$ R.N = 8 – 1 – 1 = 6

8d $\Rightarrow$ R.N = 8 – 2 – 1 = 5

So, Answer is 3

84. For complete combustion of ethene.

$C_2H_4(g) + 3O_2(g) \rightarrow 2CO_2(g) + 2H_2O(l)$ the amount of heat produced as measured in bomb calorimeter is 1406 kJ mol^{-1} at 300K. The minimum value of $T\Delta S$ needed to reach equilibrium is (–) _____kJ. (Nearest integer)

Given : R = 8.3 JK^{-1} mol^{-1}

Official Ans. by NTA (1411)

Sol. $C_2H_4(g) + 3O_2(g) \longrightarrow 2CO_2(g) + 2H_2O(\ell)$

$\Delta U = -1406$ KJ mol^{-1}, T = 300 K

$\Delta H = \Delta U + \Delta n_g RT$

$\Delta H = -1406 + (-2) \times 8.3 \times 300 = -1406 - 4.98$

$= -1410.98$ KJ $mol^{-1} \approx -1411$

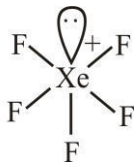
$\Delta H = T\Delta S = -1411$ KJ mol^{-1}

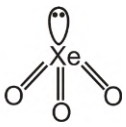
85. The number of species from the following carrying a single lone pair on central atom Xenon is ____:

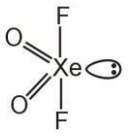
XeF_5^+, XeO_3, XeO_2F_2, XeF_5^-, XeO_3F_2, $XeOF_4$, XeF_4

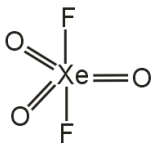
Official Ans. by NTA (4)

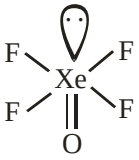
Sol.

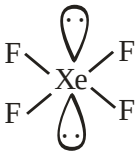
XeF_5^+ 

XeO_3 

XeO_2F_2 

XeO_3F_2 

$XeOF_4$ 

XeF_4 

So, Answer is 4

86. If the boiling points of two solvents X and Y (having same molecular weights) are in the ratio 2 : 1 and their enthalpy of vaporizations are in the ratio 1 : 2, then the boiling point elevation constant of X is m times the boiling point elevation constant of Y. The value of m is _____ (Nearest integer)

Official Ans. by NTA (8)

Sol. $\frac{(T_B)_x}{(T_B)_y}=\frac{2}{1}\quad \frac{(\Delta H)_x}{(\Delta H)_y}=\frac{1}{2}$

$$\frac{(\Delta T_B)_x}{(\Delta T_B)_y}=m=\frac{(K_B)_x\times molality}{(K_B)_y\times molality}$$

$$=\frac{(T.B)_x^2}{(T.B)_y^2}\times\frac{\Delta H_y}{(\Delta H)_x}=(2)^2\times 2=8$$

87. The sum of oxidation state of the metals in $Fe(CO)_5$, VO^{2+} and WO_3 is _____ :

Official Ans. by NTA (10)

Sol. $\overset{(0)}{Fe}(CO)_5 \quad \overset{(+4)}{V}O^{2+} \quad \overset{(+6)}{W}O_3$

So, Sum of oxidation state = 0 + 4 + 6 = 10

88. The observed magnetic moment of the complex $[Mn(\underline{N}CS)_6]^{x-}$ is 6.06 BM. The numerical value of x is _____ :

Official Ans. by NTA (4)

Sol. $[Mn(NCS)_6]^{x-}$

Number of unpaired electron = 5

So, Mn must be in +2 oxidation state (Mn^{+2})

$\Rightarrow 2+(-6)=-x$

$\Rightarrow -4=-x$

$\Rightarrow x=4$

89. The number of incorrect statements from the following is _____

A. The electrical work that a reaction can perform at constant pressure and temperature is equal to the reaction Gibbs energy.

B. E^0_{cell} is dependent on the pressure

C. $\frac{dE^0cell}{dT}=\frac{\Delta_rS^0}{nF}$

D. A cell is operating reversibly if the cell potential is exactly balanced by an opposing source of potential difference.

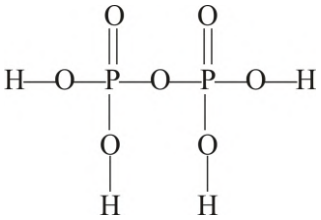
Official Ans. by NTA (1)

Sol. Option B is incorrect

So, Answer is 1

90. The ratio of sigma and π bonds present in pyrophosphoric acid is _____ :

Official Ans. by NTA (6)

Sol. 

$\frac{\sigma}{\pi}=\frac{12}{2}=6$

So, Answer is 6

FINAL JEE–MAIN EXAMINATION – APRIL, 2023

(Held On Monday 10th April, 2023) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Let O be the origin and the position vector of the point P be $-\hat{i}-2\hat{j}+3\hat{k}$. If the position vectors of the points A, B and C are $-2\hat{i}+\hat{j}-3\hat{k}, 2\hat{i}+4\hat{j}-2\hat{k}$ and $-4\hat{i}+2\hat{j}-\hat{k}$ respectively then the projection of the vector $\overrightarrow{OP}$ on a vector perpendicular to the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is

(1) 3 (2) $\frac{8}{3}$

(3) $\frac{10}{3}$ (4) $\frac{7}{3}$

Official Ans. by NTA (1)

Sol. $AB = OB - OA$

$= \left(2\hat{i}+4\hat{j}-2\hat{k}\right)-\left(-2\hat{i}+\hat{j}-3\hat{k}\right)$

$= 4\hat{i}+3\hat{j}+\hat{k}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = -2\hat{i}+\hat{j}+2\hat{k}$

$\overrightarrow{AB}\times\overrightarrow{AC} = 5\hat{i}-10\hat{j}+10\hat{k}$

$\overrightarrow{OP} = -\hat{i}-2\hat{j}+3\hat{k}$

Projection

$$= \frac{\left(\overrightarrow{OP}\right).\left(\overrightarrow{AB}\times\overrightarrow{AC}\right)}{\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|} = 3$$

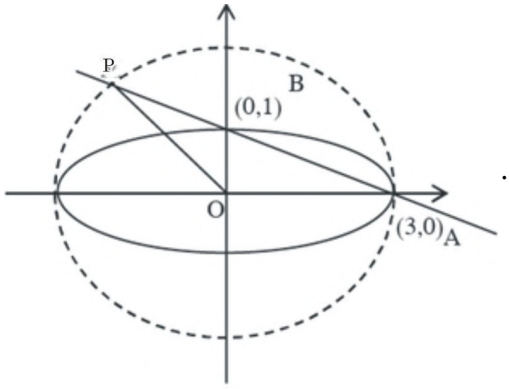
2. Let the ellipse E : $x^2 + 9y^2 = 9$ intersect the positive x- and y-axes at the points A and B respectively Let the major axis of E be a diameter of the circle C. Let the line passing through A and B meet the circle C at the point P. If the area of the triangle which vertices A, P and the origin O is $\frac{m}{n}$, where m and n are coprime, then m – n is equal to

(1) 18 (2) 16

(3) 17 (4) 15

Official Ans. by NTA (3)

Sol.



For line AB $x + 3y = 3$ and circle is $x^2 + y^2 = 9$

$(3 - 3y)^2 + y^2 = 9$

$\Rightarrow 10y^2 - 18y = 0$

$\Rightarrow y = 0, \frac{9}{5}$

$\therefore$ Area $= \frac{1}{2}\times 3\times\frac{9}{5} = \frac{27}{10}$

m – n = 17

3. If $f(x) = \frac{\left(\tan 1^o\right)x + \log_e(123)}{x\log_e(1234) - \left(\tan 1^o\right)}, x > 0,$ then the least value of $f\left(f(x)\right) + f\left(f\left(\frac{4}{x}\right)\right)$ is

(1) 8

(2) 4

(3) 2

(4) 0

Official Ans. by NTA (2)

Sol. Let f(x) = $\frac{Ax+B}{Cx-A}$

$$f\left(f\left(x\right)\right)=\frac{A\left(\frac{Ax+B}{Cx-A}\right)+B}{C\left(\frac{Ax+B}{Cx-A}\right)-A}=x$$

$$f\left(f\left(\frac{4}{x}\right)\right)=\frac{4}{x}$$

$$f\left(f\left(x\right)\right)+f\left(f\left(\frac{4}{x}\right)\right)=x+\frac{4}{x}\geq 4\left(by\ A.M.\geq G.M.\right)$$

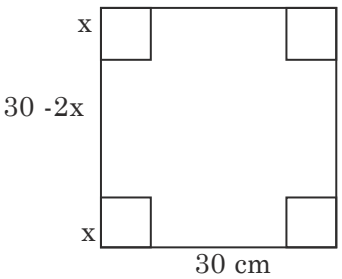
4. A square piece of tin of side 30 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form a box. If the volume of the box is maximum, then its surface area (in cm^2) is equal to

(1) 675 (2) 1025

(3) 800 (4) 900

Official Ans. by NTA (3)

Sol.



Volume (V) = x $(30 - 2x)^2$

$$\frac{dV}{dx}=\left(30-2x\right)\left(30-6x\right)=0$$

x = 5 cm

Surface area = $4 \times 5 \times 20 + (20)^2 = 800\ cm^2$

5. Let f be a differentiable function such that $x^2 f(x) - x = 4\int_0^x t f\left(t\right)dt$, $f\left(1\right)=\frac{2}{3}$.

Then 18 $f(3)$ is equal to

(1) 160 (2) 210

(3) 180 (4) 150

Official Ans. by NTA (1)

Sol. Differentiate the given equation

$$\Rightarrow 2xf\left(x\right)+x^2 f'\left(x\right)-1=4x\,f\left(x\right)$$

$$\Rightarrow x^2\frac{dy}{dx}-2xy=1$$

$$\Rightarrow \frac{dy}{dx}+\left(-\frac{2}{x}\right)y=\frac{1}{x^2}$$

$$I.F.=e^{\int -\frac{2}{x}\ell nx}=\frac{1}{x^2}$$

$$\therefore y\left(\frac{1}{x^2}\right)=\int\frac{1}{x^4}dx$$

$$\Rightarrow \frac{y}{x^2}=\frac{-1}{3x^3}+c$$

$$\Rightarrow y=-\frac{1}{3x^3}+c$$

$$\Rightarrow y=-\frac{1}{3x}+cx^2$$

$$\because f\left(1\right)=\frac{2}{3}=-\frac{1}{3}+c\Rightarrow c=1$$

$$f\left(x\right)=-\frac{1}{3x}+x^2$$

$$18f\left(3\right)=160$$

6. A line segment AB of length λ moves such that the points A and B remain on the periphery of a circle of radius λ. Then the locus of the point, that divides the line segment AB in the ratio 2 : 3, is a circle of radius

(1) $\frac{3}{5}\lambda$ (2) $\frac{\sqrt{19}}{7}\lambda$

(3) $\frac{2}{3}\lambda$ (4) $\frac{\sqrt{19}}{5}\lambda$

Official Ans. by NTA (4)

Sol. $\left(\frac{\lambda}{\sqrt{2}}\sin\theta, \frac{-\lambda}{\sqrt{2}}\cos\theta\right)$ A 3 2 P(h,k) B $\left(\frac{\lambda}{\sqrt{2}}\cos\theta, \frac{\lambda}{\sqrt{2}}\sin\theta\right)$

$$h = \frac{\frac{2\lambda}{\sqrt{2}\sin\theta} + 3\times\frac{\lambda}{\sqrt{2}}\cos\theta}{5}$$

$$k = \frac{\frac{-2\lambda}{\sqrt{2}}2\cos\theta + \frac{3\lambda}{\sqrt{2}}\sin\theta}{5}$$

$$h^2 + k^2 = \frac{19\lambda^2}{5}$$

$$r = \frac{\sqrt{19}\lambda}{5}$$

7. Let the complex number z = x + iy be such that $\frac{2z-3i}{2z+i}$ is purely imaginary. If $x + y^2 = 0$, then $y^4 + y^2 - y$ is equal to :

(1) $\frac{3}{2}$ (2) $\frac{4}{3}$

(3) $\frac{2}{3}$ (4) $\frac{3}{4}$

Official Ans. by NTA (4)

Sol. $\frac{2z-3i}{2z+i}$ is purely imaginary

$$\therefore \frac{2z-3i}{2z+i} + \frac{2\bar{z}+3i}{2\bar{z}-i} = 0$$

$$z = x + iy$$

$$\Rightarrow 4x^2 + 4y^2 - 4y - 3 = 0$$

Given that $x + y^2 = 0$

$y^4 + y^2 - y = 3/4$

8. $96\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{4\pi}{33}\cos\frac{8\pi}{33}\cos\frac{16\pi}{33}$ is equal to

(1) 3 (2) 2 (3) 4 (4) 1

Official Ans. by NTA (1)

Sol. $P = 96\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{4\pi}{33}\cos\frac{8\pi}{33}\cos\frac{16\pi}{33}$

$$2P\times\sin\frac{\pi}{33} = 96\times 2\sin\frac{\pi}{33}\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{4\pi}{33}\cos\frac{8\pi}{33}\cos\frac{16\pi}{33}$$

$$2P\times\sin\frac{\pi}{33} = 6\times\sin\frac{32\pi}{33} = 6\sin\frac{\pi}{33}$$

P = 3

9. If A is a 3 × 3 matrix and |A| = 2, then $\left|3\,adj\left(|3A|A^2\right)\right|$ is equal to

(1) $3^{11}\cdot 6^{10}$ (2) $3^{12}\cdot 6^{10}$

(3) $3^{10}\cdot 6^{11}$ (4) $3^{12}\cdot 6^{11}$

Official Ans. by NTA (1)

Sol. $\left|3\,adj\ |3A|A^2\ \right| = 3^3\left|adj\left(54A^2\right)\right| = 3^3.\left|54A^2\right|^2$

$$= 3^3\times 54^6\times|A|^4 = 3^{11}\times 6^{10}$$

10. The slope of tangent at any point (x, y) on a curve y = y(x) is $\frac{x^2+y^2}{2xy}$, x > 0. If y(2) = 0, then a value of y(8) is

(1) $-2\sqrt{3}$ (2) $4\sqrt{3}$

(3) $2\sqrt{3}$ (4) $-4\sqrt{2}$

Official Ans. by NTA (2)

Sol. $$\frac{dy}{dx} = \frac{1+\left(\frac{y}{x}\right)^2}{2\left(\frac{y}{x}\right)}$$

Let y = tx

$$\Rightarrow t + x\frac{dt}{dx} = \frac{1+t^2}{2t}$$

$$\Rightarrow x\frac{dt}{dx} = \frac{1-t^2}{2t}$$

$$\Rightarrow \int\frac{2t}{1-t^2}dt = \int\frac{dx}{x}$$

$$\Rightarrow \ell n\left|1-t^2\right| = \ell nx + \ell nc$$

$$\Rightarrow \left(1-t^2\right)(cx) = 1$$

$$\Rightarrow \left(1-\frac{y^2}{x^2}\right)cx = 1$$

$$y(2) = 0 \Rightarrow c = \frac{1}{2}$$

$$\left(1-\frac{y^2}{x^2}\right).\frac{1}{2}x = 1$$

at x = 8

$$\left(1-\frac{y^2}{64}\right)\times\frac{8}{2} = 1$$

$$y = \pm 4\sqrt{3}$$

11. For the system of linear equations

$2x - y + 3z = 5$

$3x + 2y - z = 7$

$4x + 5y + \alpha z = \beta$

Which of the following is NOT correct ?

(1) The system has infinitely many solutions for $\alpha = -5$ and $\beta = 9$

(2) The system has a unique solution for $\alpha \neq -5$ and $\beta = 8$

(3) The system has infinitely many solutions for $\alpha = -6$ and $\beta = 9$

(4) The system is inconsistent for $\alpha = -5$ and $\beta = 8$

Official Ans. by NTA (3)

Sol. $$\Delta = \begin{vmatrix} 2 & -1 & 3 \\ 3 & 2 & -1 \\ 4 & 5 & \alpha \end{vmatrix} = 7(\alpha + 5)$$

$$\Delta_1 = \begin{vmatrix} 5 & -1 & 3 \\ 7 & 2 & -1 \\ \beta & 5 & \alpha \end{vmatrix} = 17\alpha - 5\beta + 130$$

$$\Delta_2 = \begin{vmatrix} 2 & 5 & 3 \\ 3 & 7 & -1 \\ 4 & \beta & \alpha \end{vmatrix} = -11\beta + \alpha + 104$$

$$\Delta_3 = \begin{vmatrix} 2 & -1 & 5 \\ 3 & 2 & 7 \\ 4 & 5 & \beta \end{vmatrix} = 7(\beta - 9)$$

For infinitely many solutions

$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

For $\alpha = -5$ and $\beta = 9$

Hence option (3) is incorrect

12. Let N denotes the sum of the numbers obtained when two dice are rolled. If the probability that $2^N < N!$ is $\frac{m}{n}$, where m and n are coprime, then 4m – 3n is equal to

(1) 8 (2) 16

(3) 10 (4) 12

Official Ans. by NTA (1)

Sol. N = Sum of the numbers when two dice are rolled such that $2^N < N!$

$\Rightarrow 4 \le N \le 12$

Probability that $2^N \ge N!$

$$Now\, P(N=2) + P(N=3) = \frac{1}{36} + \frac{2}{36} = \frac{3}{36} = \frac{1}{12}$$

Required probability = $1 - \frac{1}{12} = \frac{11}{12} = \frac{m}{n}$

4m – 3n = 8

13. Let P be the point of intersection of the line $\frac{x+3}{3} = \frac{y+2}{1} = \frac{1-z}{2}$ and the plane x + y + z = 2. If the distance of the point P from the plane 3x – 4y + 12z = 32 is q, then q and 2q are the roots of the equation

(1) $x^2 - 18x - 72 = 0$

(2) $x^2 + 18x + 72 = 0$

(3) $x^2 - 18x + 72 = 0$

(4) $x^2 + 18x - 72 = 0$

Official Ans. by NTA (3)

Sol. P = (3λ – 3, λ – 2, 1 – 2 λ)

P lies on the plane, x + y + z = 2

$\Rightarrow \lambda = 3$

P = (6, 1, –5)

$$q = \left|\frac{18-4-60-32}{\sqrt{9+16+144}}\right| = \frac{78}{13} = 6$$

q = 6, 2q = 12

Equation, $x^2 - 18x + 72 = 0$

14. The negation of the statement $(p \vee q) \wedge (q \vee (\sim r))$ is

(1) $((\sim p) \vee r) \wedge (\sim q)$

(2) $((\sim p) \vee (\sim q)) \wedge (\sim r)$

(3) $((\sim p) \vee (\sim q)) \vee (\sim r)$

(4) $(p \vee r) \wedge (\sim q)$

Official Ans. by NTA (1)

Sol. $\sim[(p \vee q) \wedge (q \vee (\sim p)]$

$\Rightarrow \sim (p \wedge q) \vee \sim (q \vee (\sim p))$

$\Rightarrow (\sim p \wedge \sim q) \vee (\sim q \wedge p)$

Apply distribution law

$\Rightarrow \sim q \wedge (\sim p \vee p)$

$\Rightarrow (\sim p \vee p) \wedge (\sim q)$

15. If the coefficient of x^7 in $\left(ax - \frac{1}{bx^2}\right)^{13}$ and the coefficient of x^{-5} in $\left(ax + \frac{1}{bx^2}\right)^{13}$ are equal, then a^4b^4 is equal to :

(1) 44

(2) 22

(3) 11

(4) 33

Official Ans. by NTA (2)

Sol. $T_{r+1} = {}^{13}C_r (ax)^{13-r} \left(-\frac{1}{bx^2}\right)^r$

$= {}^{13}C_r (a)^{13-r} \left(-\frac{1}{b}\right)^r x^{13-3r}$

13 – 3r = 7 $\Rightarrow r = 2$

Coefficient of $x^7 = {}^{13}C_2 (a)^{11} \cdot \frac{1}{b^2}$

In the other expansion $T_{r+1} = {}^{13}C_r (ax)^{13-r} \left(\frac{1}{bx^2}\right)^r$

13 – 3r = -5 $\Rightarrow$ **r = 6**

Coefficient of $x^{-5} = {}^{13}C_6 (a)^7 \cdot \frac{1}{b^6}$

$${}^{13}C_2 \frac{a^{11}}{b^2} = {}^{13}C_6 \frac{a^7}{b^6}$$

$$a^4b^4 = \frac{{}^{13}C_6}{{}^{13}C_2} = 22$$

16. Let two vertices of triangle ABC be (2, 4, 6) and (0, –2, –5), and its centroid be (2, 1, –1). If the image of third vertex in the plane x + 2y + 4z = 11 is (α, β, γ), then αβ + βγ + γα is equal to

(1) 72 (2) 74

(3) 76 (4) 70

Official Ans. by NTA (2)

Sol. Given, A(2, 4, 6), B(0, -2, -5)

G(2, 1, –1)

Let vertex C(x, y, z)

$$\frac{2+0+x}{3} = 2 \Rightarrow x = 4$$

$$\frac{4-2+y}{3} = 1 \Rightarrow y = 1$$

$$\frac{6-5+z}{3} = -1 \Rightarrow z = -4$$

Third vertex, C(4, 1, -4)

Then image of vertex in the plane let image (α, β, γ)

i.e,. $\frac{\alpha-4}{1} = \frac{\beta-1}{2} = \frac{\gamma+4}{4} = \frac{-2(4+2-16-11)}{21}$

α = 6, β = 5, γ = 4

αβ + βγ + γα = 30 + 20 + 24 = 74

17. The shortest distance between the lines $\frac{x+2}{1}=\frac{y}{-2}=\frac{z-5}{2}$ and $\frac{x-4}{1}=\frac{y-1}{2}=\frac{z+3}{0}$ is

(1) 6 (2) 9

(3) 7 (4) 8

Official Ans. by NTA (2)

Sol. Given lines

$$\frac{x+2}{1}=\frac{y}{-2}=\frac{z-5}{2} \;\&\; \frac{x-4}{1}=\frac{y-1}{2}=\frac{z+3}{0}$$

Formula for shortest distance

$$\text{S.D.}=\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}$$

$$=\frac{\begin{vmatrix} 6 & 1 & -8 \\ 1 & -2 & 2 \\ 1 & 2 & 0 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 1 & 2 & 0 \end{vmatrix}}=\frac{54}{6}=9$$

18. If $I(x)=\int e^{\sin^2 x}(\cos x \sin 2x-\sin x)\,dx$ and $I(0)=1$, then $I\left(\frac{\pi}{3}\right)$ is equal to

(1) $-\frac{1}{2}e^{\frac{3}{4}}$

(2) $e^{\frac{3}{4}}$

(3) $\frac{1}{2}e^{\frac{3}{4}}$

(4) $-e^{\frac{3}{4}}$

Official Ans. by NTA (3)

Sol. $I(x)=\int \underset{II}{e^{\sin x}.\sin 2x}.\underset{I}{\cos x}\,dx-\int e^{\sin^2 x}.\sin x\,dx$

$\Rightarrow I(x)=e^{\sin^2 x}-\int(-\sin x).e^{\sin^2 x}dx-\int e^{\sin^2 x}.\sin x dx$

$\Rightarrow I(x)=e^{\sin^2 x}.\cos x+c$

Put x = 0, c = 0

$\therefore I\left(\frac{\pi}{3}\right)=e^{\frac{3}{4}}.\cos\frac{\pi}{3}=\frac{1}{2}e^{\frac{3}{4}}$

19. Let the first term a and the common ratio r of a geometric progression be positive integers. If the sum of its squares of first three terms is 33033, then the sum of these three terms is equal to

(1) 231

(2) 210

(3) 220

(4) 241

Official Ans. by NTA (1)

Sol. $\Rightarrow a^2 + a^2r^2 + a^2r^4 = 33033$

$\Rightarrow a^2(r^4 + r^2 + 1) = 3 \times 7 \times 11^2 \times 13 \Rightarrow a = 11$

$\Rightarrow r^4 + r^2 + 1 = 273 \quad \Rightarrow r^4 + r^2 - 272 = 0$

$\Rightarrow (r^2 + 17)(r^2 - 16) = 0 \Rightarrow r^2 = 16 \Rightarrow r = \pm 4$

$t_1 + t_2 + t_3 = a + ar + ar^2 = 11 + 44 + 176 = 231$

20. An are PQ of a circle subtends a right angle at its centre O. The mid point of the arc PQ is R. If $\overrightarrow{OP}=\vec{u}$, $\overrightarrow{OR}=\vec{v}$ and $\overrightarrow{OQ}=\alpha\vec{u}+\beta\vec{v}$, then α, β^2 are the roots of the equation

(1) $x^2 - x - 2 = 0$

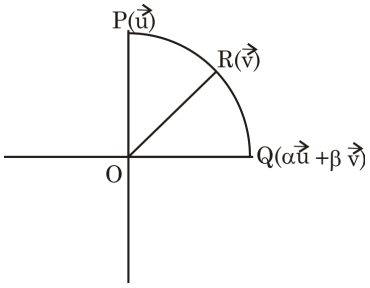
(2) $3x^2 + 2x - 1 = 0$

(3) $x^2 + x - 2 = 0$

(4) $3x^2 - 2x - 1 = 0$

Official Ans. by NTA (1)

Sol.



$$|\vec{u}| = |\vec{v}| = |\alpha\vec{u} + \beta\vec{v}|$$

$$(\vec{u}).(\alpha\vec{u} + \beta\vec{v}) = 0$$

$$\vec{u}.\vec{v} = |u||v|\cos 45^o$$

$$\alpha = -\frac{\beta}{\sqrt{2}}$$

$$= |\alpha\vec{u} + \beta\vec{v}| = r$$

$$\alpha^2 + \beta^2 + \sqrt{2}\alpha\beta = 1$$

$$\alpha = -1,\ \beta^2 = 2$$

SECTION-B

21. The coefficient of x^7 in $\left(1 - x + 2x^3\right)^{10}$ is _____.

Official Ans. by NTA (960)

Sol. General term $= \frac{10!}{r_1!.r_2!.r_3!}(-1)^{r_2}.(2)^{r_3}\ x^{r_2+3r_3}$

where $r_1 + r_2 + r_3 = 10$ and $r_2 + 3r_3 = 7$

r_1	r_2	r_3
3	7	0
5	4	1
7	1	2

Required coefficient

$$= \frac{10!}{3!.7!}(-1)^7 + \frac{10!}{5!.4!}(-1)^4(2) + \frac{10!}{7!.2!}(-1)^1(2)^2$$

$= -120 + 2520 - 1440 = 960$

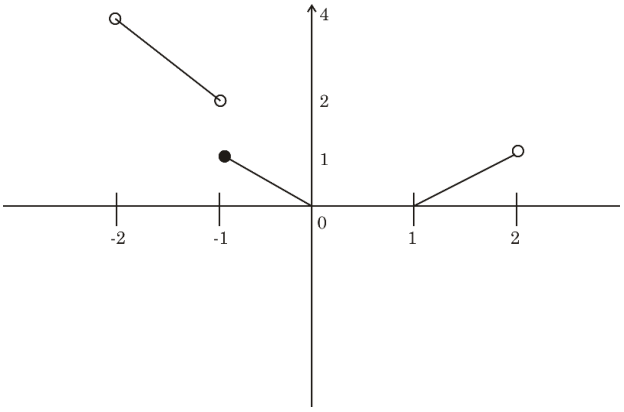
22. Let f : (-2, 2) $\rightarrow$ IR be defined by

$$f(x) = \begin{cases} x[x] & ,-2 < x < 0 \\ (x-1)[x] & ,0 \le x < 2 \end{cases}$$

Where [x] denotes the greatest integer function. If m and n respectively are the number of points in (-2, 2) at which y = $|f(x)|$ is not continuous and not differentiable, then m + n is equal to _____.

Official Ans. by NTA (4)

Sol. $f(x) = \begin{cases} x[x] & ,-2 < x < 0 \\ (x-1)[x] & ,0 \le x < 2 \end{cases}$



$|f(x)|$ = Remain same

m = 1, n = 3

m + n = 4

23. The sum of all those terms, of the arithmetic progression 3, 8, 13,…… 373, which are not divisible by 3, is equal to _____.

Official Ans. by NTA (9525)

Required sum = (3 + 8 + 13 + 18 + …… + 373) – (3 + 18 + 33 + …… + 363)

$$= \frac{75}{2}(3 + 373) - \frac{25}{2}(3 - 363)$$

= 75 × 188 – 25 × 183

= 9525

24. Let a common tangent to the curves $y^2 = 4x$ and $(x - 4)^2 + y^2 = 16$ touch the curves at the points P and Q. Then $(PQ)^2$ is equal to _____.

Official Ans. by NTA (32)

Sol. General tangent of slope m to the circle $(x - 4)^2 + y^2 = 16$ is given by $y = m(x-4) \pm 4\sqrt{1+m^2}$

General tangent of slope m to the parabola $y^2 = 4x$ is given by $y = mx + \frac{1}{m}$

For common tangent $\frac{1}{m} = -4m \pm 4\sqrt{1+m^2}$

$m = \pm\frac{1}{2\sqrt{2}}$

Point of contact on parabola is $\left(8, 4\sqrt{2}\right)$

Length of tangent PQ from $\left(8, 4\sqrt{2}\right)$ on the circle $(x - 4)^2 + y^2 = 16$ is equal to $\sqrt{(8-4)^2 + \left(4\sqrt{2}\right)^2 - 16}$ is equal to $\sqrt{32}$

PQ^2 is equal to 32

25. The number of permutations, of the digits 1, 2, 3, …..7 without repetition, which neither contain the string 153 nor the string 2467, is _____.

Official Ans. by NTA (4898)

Sol. Digits → 1, 2, 3, 4, 5, 6 , 7

Total permutations = 7!

Let A = number of numbers containing string 153

Let B = number of numbers containing string 2467

n(A) = 5! × 1 — 153 2467

n(B) = 4! × 1 — 2467 135

$n(A \cap B) = 2!$ — 153 2467

$n(A \cup B) = 5! + 4! - 2! = 142$

n(neither string 153 nor string 2467)

= Total – $n(A \cup B)$

= 7! – 142 = 4898

26. Let a, b, c be three distinct positive real numbers such that $(2a)^{\log_e a} = (bc)^{\log_e b}$ and $b^{\log_e 2} = a^{\log_e c}$.

Then 6a + 5bc is equal to _____.

Official Ans. by NTA (8)

Sol. $(2a)^{\ln a} = (bc)^{\ln b}$ 2a > 0, bc > 0 $b^{\ln 2} = a^{\ln c}$

ln a (ln 2 + ln a) = ln b (ln b + ln c) | ln2.lnb = lnc.lna

ln 2 = α, ln a = x_1 ln b = y, ln c = z | αy = yz

x (a + x) = y (y + 2)

$\alpha = \frac{xz}{y}$ $(2a)^{\ln a} = (2a)^0$

$x\left(\frac{xz}{y} + x\right) = y(y+z)$

$x^2 (z + y) = y^2 (y + z)$

y + z = 0 or $x^2 = y^2 \Rightarrow x = -y$

bc = 1 or ab = 1

(1) if bc = 1 ⇒ $(2a)^{\ln a} = 1$ → a=1, a=1/2

(a, b, c) = $\left(\frac{1}{2}, \lambda, \frac{1}{\lambda}\right)$, $\lambda \neq 1, 2, \frac{1}{2}$

then 6a + 5bc = 3 + 5 = 8

(II) (a, b, c) = $\left(\lambda, \frac{1}{\lambda}, \frac{1}{2}\right)$, $\lambda \neq 1, 2, \frac{1}{2}$

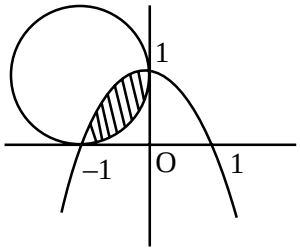
In this situation infinite answer are possible

So, Bonus.

27. Let y = p(x) be the parabola passing through the points (-1, 0), (0, 1) and (1, 0). If the area of the region $\left\{(x, y) : (x+1)^2 + (y-1)^2 \leq 1, y \leq p(x)\right\}$ is A, then 12(π-4A) is equal to _____.

Official Ans. by NTA (16)

Sol. There can be infinitely many parabolas through given points.



$$A = \int_{-1}^{0} \left(1-x^2\right) - \left(x - \sqrt{1-(x+1)^2}\right) dx$$

$$= \int_{-1}^{0} -x^2 + \sqrt{1-(x+1)^2}\, dx$$

$$= \left(-\frac{x^3}{3} + \frac{x+1}{2} = \sqrt{1-(x+1)^2} + \frac{1}{2}.\sin^{-1}\left(\frac{x+1}{1}\right)\right)_{-1}^{0}$$

$$A = \frac{\pi}{4} - \left(\frac{1}{3}\right)$$

$$\therefore \quad 12\,(\pi - 4A) = 12\left(\pi - 4\left(\frac{\pi}{4} - \frac{1}{3}\right)\right) = 16$$

This is possible only when axis of parabola is parallel to Y axis but is not given in question, so it is bonus.

28. If the mean of the frequency distribution

Class :	0-10	10-20	20-30	30-40	40-50
Frequency	2	3	x	5	4

is 28, then its variance is _____.

Official Ans. by NTA (151)

Sol. Given mean is = 28

$$\frac{2\times5+3\times15+x\times25+5\times35+4\times45}{14+x} = 28$$

x = 6

$$\text{Variance} = \left(\frac{\sum x_i^2 f_i}{\sum f_i}\right) - (mean)^2$$

$$\text{Variance} = = \frac{2\times5^2+3\times15^2+6\times25^2+5\times35^2+4\times45^2}{20} - (28)^2$$

= 151

29. Some couples participated in a mixed doubles badminton tournament. If the number of matches played, so that no couple played in a match, is 840, then the total numbers of persons, who participated in the tournament, is _____.

Official Ans. by NTA (16)

Sol. $^{n}C_2 \times {}^{n-2}C_2 \times 2 = 840$

$\Rightarrow n = 8$

Therefore total persons = 16

30. The number of elements in the set $\left\{n \in \mathbb{Z} : \left|n^2 - 10n + 19\right| < 6\right\}$ is _____.

Official Ans. by NTA (6)

Sol. $-6 < n^2 - 10n + 19 < 6$

$\Rightarrow n^2 - 10n + 25 > 0$ and $n^2 - 10n + 13 <$

$(n-5)^2 > 0$ $n \in \left[5 - 2\sqrt{3}, 5 + 2\sqrt{3}\right]$

$n \in R - [5]$

$\therefore n \in [1.3, 8.3]$

$\Rightarrow n = 2,3,4,6,7,8$

SECTION-A

31. A physical quantity P is given as

$$P = \frac{a^2 b^3}{c\sqrt{d}}$$

The percentage error in the measurement of a, b, c and d are 1%, 2%, 3% and 4% respectively. The percentage error in the measurement of quantity P will be

(1) 13% (2) 14%

(3) 12% (4) 16%

Official Ans. by NTA (1)

Sol. $\frac{\Delta P}{P}\times 100\% = \left(2\frac{\Delta a}{a} + 3\frac{\Delta b}{b} + \frac{\Delta c}{c} + \frac{1}{2}\frac{\Delta d}{d}\right)\times 100\%$

$= 2(1\%) + 3(2\%) + 3\% + \frac{1}{2} \times 4\% = 13\%$

32. Assuming the earth to be a sphere of uniform mass density, the weight of a body at a depth $d = \frac{R}{2}$ from the surface of earth, if its weight on the surface of earth is 200 N, will be:

(Given R = Radius of earth)

(1) 400 N (2) 500 N

(3) 300 N (4) 100 N

Official Ans. by NTA (4)

Sol. $M = \frac{W}{g} = \frac{200}{10} = 20\,kg$

Acc. due to gravity at a depth $g' = g\left(1 - \frac{d}{R}\right)$

$d \rightarrow$ depth from surface

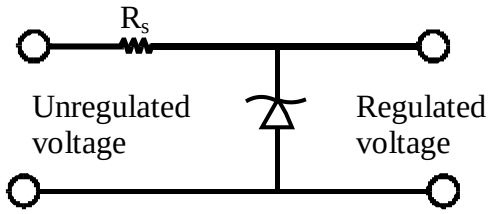
$d = \frac{R}{2}$

$g' = g\left(1 - \frac{R/2}{R}\right) = \frac{g}{2} = 5\ m/s^2$

weight = m × g

at depth R/2 = 20 × 5 = 100 N

33. A zener diode of power rating 1.6 W is to be used as voltage regulator. If the zener diode has a breakdown of 8V and it has to regulate voltage fluctuating between 3V and 10 V. The value of resistance R_s for safe operation of diode will be :

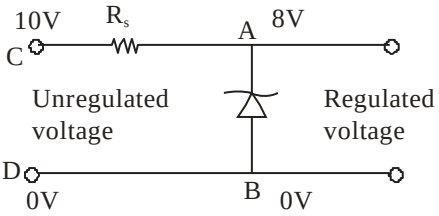


(1) 13.3Ω (2) 12Ω

(3) 10Ω (4) 13Ω

Official Ans. by NTA (3)

Sol.



$V_b = 8$ volt

$V_A - V_B = 8$ volt

Current through zener diode,

$$i = \frac{P}{V} = \frac{1.6\,W}{8V} = 0.2A$$

$V_C - V_A = 10 - 8$ volt

$$\therefore R = \frac{V_C - V_A}{i} = \frac{2V}{0.2A} = 10\Omega$$

[Note : A zener diode can regulate only if input voltage is ≥ zener breakdown voltage the range of input voltage should be 8 to 10 V so that output voltage remains constant = 8 V]

34. The range of the projectile projected at an angle of 15° with horizontal is 50 m. If the projectile is projected with same velocity at an angle of 45° with horizontal, then its range will be :

(1) 50 m (2) $50\sqrt{2}$ m

(3) 100 m (4) $100\sqrt{2}$ m

Official Ans. by NTA (3)

Sol. $R = \frac{\upsilon^2 \sin 2\theta}{g}$

$R \propto \sin(2\theta)$

$\frac{R_1}{R_2} = \frac{\sin(2\theta_1)}{\sin(2\theta_2)} = \frac{\sin(2\times15)}{\sin(2\times45)} = \frac{\sin 30°}{\sin 90°}$

$\frac{50}{R_2} = \frac{1}{2}$

$R_2 = 100m$

35. A carrier wave of amplitude 15V is modulated by a sinusoidal base band signal of amplitude 3V. The ratio of maximum amplitude to minimum amplitude in an amplitude modulated wave is :

(1) 2 (2) $\frac{3}{2}$

(3) 5 (4) 1

Official Ans. by NTA (2)

Sol. Given, $A_c = 15$ V

$A_m = 3V$

Maximum amplitude of modulated wave

$A_{max} = A_c + A_m = 15 + 3 = 18V$

Minimum amplitude of modulated wave

$A_{min} = A_c - A_m = 15 - 3 = 12V$

$\therefore \frac{A_c + A_m}{A_c - A_m} = \frac{18}{12} = \frac{3}{2}$

36. The angular momentum for the electron in Bohr's orbit is L. If the electron is assumed to revolve in second orbit of hydrogen atom, then the change in angular momentum will be :

(1) $\frac{L}{2}$ (2) zero

(3) L (4) 2L

Official Ans. by NTA (3)

Sol. $L = mvr, \quad r \propto n^2, \quad v \propto \frac{1}{n}$

$\therefore L \propto n$

Also, $L = \frac{nh}{2\pi}$, Bohr orbit is, $L_1 = L = \frac{1.h}{2\pi}$

$L_2 = 2[L] = 2L$

$L_2 = \frac{2h}{2\pi}$

So, change $= L_2 - L_1 = 2L - L = L$

37. A particle of mass m moving with velocity v collides with a stationary particle of mass 2m. After collision, they stick together and continue to move together with velocity

(1) v

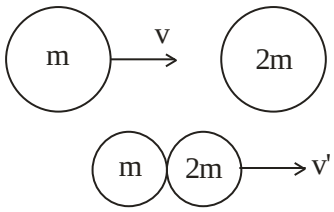
(2) $\frac{v}{2}$

(3) $\frac{v}{3}$

(4) $\frac{v}{4}$

Official Ans. by NTA (3)

Sol.



Applying conservation of linear momentum

$\Rightarrow \vec{P}_i = \vec{P}_f$

$mv + 2m \times 0 = (3m)v'$

$\therefore mv = 3mv'$

$v' = \frac{v}{3}$

38. Given below are two statement :

Statement I : If the number of turns in the coil of a moving coil galvanometer is doubled then the current sensitivity becomes double.

Statement II : Increasing current sensitivity of a moving coil galvanometer by only increasing the number of turns in the coil will also increase its voltage sensitivity in the same ratio:

In the light of the above statement, choose the correct answer from the options given below :

(1) Statement I is false but Statement II is true

(2) Both Statement I and Statement II are true

(3) Both Statement I and Statement II are false

(4) Statement I is true but Statement II is false

Official Ans. by NTA (4)

Sol. For a moving coil galvanometer

$BiNA = k\theta$

$\theta = \left(\frac{BNA}{k}\right)i$; Current sensitive $= \frac{BNA}{k}$

So, if N is doubled then current sensitivity is doubled.

Voltage sensitivity

$B\frac{V}{R}NA = k\theta$

$V = \frac{BNA}{Rk}\theta$, as N is doubled R is also doubled.

So, no change in voltage sensitivity.

Hence, option (4) is right.

39. Match List I with List II :

List-I		List II	
(A)	3 Translational degrees of freedom	(I)	Monoatomic gases
(B)	3 Translational, 2 rotational degrees of freedoms	(II)	Polyatomic gases
(C)	3 Translational, 2 rotational and 1 vibrational degrees of freedom	(III)	Rigid diatomic gases
(D)	3 Translational, 3 rotational and more than one vibrational degrees of freedom	(IV)	Nonrigid diatomic gases

Choose the correct answer from the options given below :

(1) (A) – (IV), (B) – (III), (C) – (II), (D) – (I)

(2) (A) – (IV), (B) – (II), (C) – (I), (D) – (III)

(3) (A) – (I), (B) – (III), (C) – (IV), (D) – (II)

(4) (A) – (I), (B) – (IV), (C) – (III), (D) – (II)

Official Ans. by NTA (3)

Sol. Factual

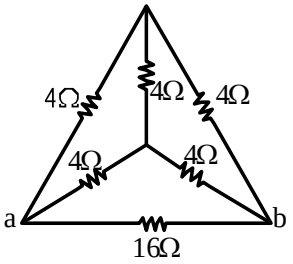
Type of gases	No. of degrees of freedom
Monoatomic gas	3 T
Diatomic + rigid	3T + 2R
Diatomic + non-rigid	3T+ 2R + 1V
Polyatomic	3T + 3R + More than 1V

T = Translational degree of freedom

R = Rotational degree of freedom

V = Vibrational degree of freedom

40. The equivalent resistance of the circuit shown below between points a and b is :

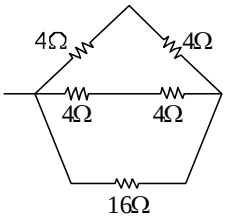


(1) 24Ω (2) 3.2Ω

(3) 20Ω (4) 16Ω

Official Ans. by NTA (2)

Sol. The circuit can be reduced to



$\Rightarrow R_{eq} = \frac{16\times4}{16+4} = \frac{16}{5}\Omega$

$= R_{eq} = 3.2\Omega$

41. Consider two containers A and B containing monoatomic gases at the same Pressure (P), Volume (V) and Temperature (T). The gas in A is compressed isothermally to $\frac{1}{8}$ of its original volume while the gas B is compressed adiabatically to $\frac{1}{8}$ of its original volume. The ratio of final pressure of gas in B to that of gas in A is :

(1) 8 (2) $8^{\frac{3}{2}}$

(3) $\frac{1}{8}$ (4) 4

Official Ans. by NTA (4)

Sol. Isothermal process, T = constant

$PV = nRT$ = constant

$P_1V_1 = P_2V_2$

$PV = P_A(V/8)$

$P_A = 8P$

Adiabatic process, PV^{γ} = constant

γ for monoatomic gas is $\frac{5}{3}$.

$P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$$\frac{P_B}{P} = \left(\frac{V_1}{V_2}\right)^{\gamma} = \left(\frac{V}{V/8}\right)^{\frac{5}{3}}$$

$P_B = 32P$

$$\frac{P_B}{P_A} = \frac{32P}{8P} = 4$$

42. Given below are two statements:

Statement I : Maximum power is dissipated in a circuit containing an inductor, a capacitor and a resistor connected in series with an AC source, when resonance occurs

Statement II : Maximum power is dissipated in a circuit containing pure resistor due to zero phase difference between current and voltage.

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is false but Statement II is true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are true

(4) Both Statement I and Statement II are false

Official Ans. by NTA (3)

Sol. Power will be maximum when impedance is minimum

$$Z = [R^2 + (X_L - X_C)^2]^{\frac{1}{2}}$$

At resonance, $X_L = X_C$

$Z_{min} = R$

43. Two satellites of masses m and 3m revolve around the earth in circular orbits of radii r & 3r respectively. The ratio of orbital speeds of the satellites respectively is :

(1) 1 : 1 (2) 3 : 1

(3) $\sqrt{3}:1$ (4) 9 : 1

Official Ans. by NTA (3)

Sol. $$v = \sqrt{\frac{GM}{r}}$$

$$v \propto \frac{1}{\sqrt{r}} \qquad \Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{r_2}{r_1}} = \sqrt{\frac{3r}{r}}$$

$$= \sqrt{3}:1$$

44. Given below are two statements:

Statement I : Pressure in a reservoir of water is same at all points at the same level of water.

Statement II : The pressure applied to enclosed water is transmitted in all directions equally.

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is false but Statement II is true

(2) Both Statement I and Statement II are true

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are false

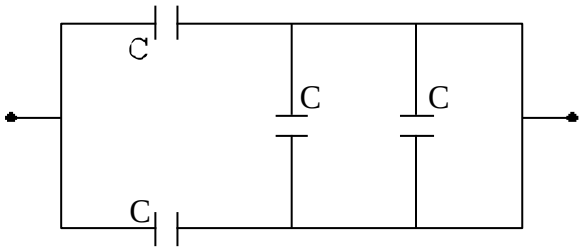
Official Ans. by NTA (2)

Sol. Pressure in a static liquid will be same at each point on same horizontal level.

$\therefore P = P_{atm} + \rho gh$

As per Pascal law, same pressure applied to enclosed water is transmitted in all directions equally.

45. The equivalent capacitance of the combination shown is

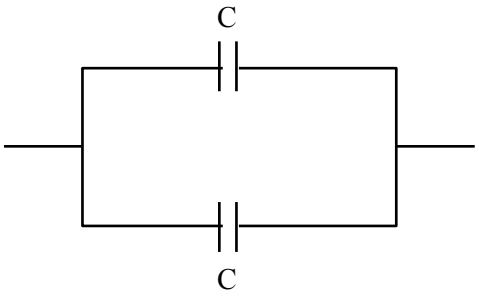


(1) $\frac{C}{2}$ (2) 4C

(3) 2C (4) $\frac{5}{3}C$

Official Ans. by NTA (3)

Sol. The circuit can be reduced to



Parallel combination

$C_{eq} = C + C = 2C$

46. The energy of an electromagnetic wave contained in a small volume oscillates with

(1) zero frequency

(2) half the frequency of the wave

(3) double the frequency of the wave

(4) the frequency of the wave

Official Ans. by NTA (3)

Sol. $E = E_0 \sin(\omega t - kx)$

Energy density $\left(\frac{du}{dv}\right) = \varepsilon_0 E_0^2 \sin^2(\omega t - kx)$

$\frac{\varepsilon_0 E_0^2}{2}\left[1 - \cos(2\omega t - 2kx)\right]$

47. An object is placed at a distance of 12 cm in front of a plane mirror. The virtual and erect image is formed by the mirror. Now the mirror is moved by 4 cm towards the stationary object. The distance by which the position of image would be shifted, will be:

(1) 4 cm towards mirror

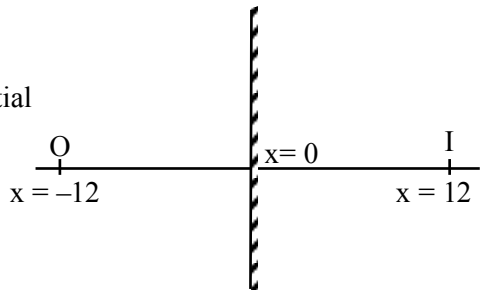
(2) 8 cm towards mirror

(3) 8 cm away from mirror

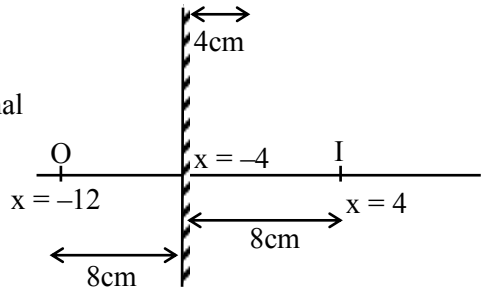
(4) 2 cm towards mirror

Official Ans. by NTA (2)

Sol. Initial



Final



$\therefore$ Shifting of image will be 8 cm towards mirror.

48. The de Broglie wavelength of a molecule in a gas at room temperature (300 K) is λ_1. If the temperature of the gas is increased to 600 K, then the de Broglie wavelength of the same gas molecule becomes

(1) $\frac{1}{\sqrt{2}}\lambda_1$ (2) $2\lambda_1$

(3) $\frac{1}{2}\lambda_1$ (4) $\sqrt{2}\lambda_1$

Official Ans. by NTA (1)

Sol. From K.T.G.

$$v_{RMS} = \sqrt{\frac{3k_B T}{m}}$$

$$v_{RMS} \propto \sqrt{T}$$

and $\frac{h}{mv_{RMS}} = \lambda$ i.e., $\lambda \propto \frac{1}{\sqrt{T}}$

$$\frac{\lambda_2}{\lambda_1} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{300}{600}} = \frac{1}{\sqrt{2}}$$

$$\lambda_2 = \frac{\lambda_1}{\sqrt{2}}$$

49. A particle executes S.H.M. of amplitude A along x-axis. At t = 0, the position of the particle is $x = \frac{A}{2}$ and it moves along positive x-axis the displacement of particle in time t I $x = A\sin(\omega t + \delta)$, then the value δ will be :

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{3}$

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

Official Ans. by NTA (1)

Sol. $X = A\sin(\omega t + \delta)$ $V = A\omega\cos(\omega t + \delta)$

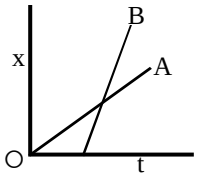
$\frac{A}{2} = A\sin(\omega t + \delta)$ $\therefore$ V is +ve, δ must be

At t = 0 in 1st quadrant or 4th

$\sin\delta = \frac{1}{2} \Rightarrow \delta = \frac{\pi}{6}, \frac{5\pi}{6}$ quadrant

$\therefore$ Common solution is $\delta = \frac{\pi}{6}$

50. The position-time graphs for two students A and B returning from the school to their homes are shown in figure :



(A) A lives closer to the school

(B) B lives closer to the school

(C) A takes lesser time to reach home

(D) A travels faster than B

(E) B travels faster than A

Choose the correct answer from the options given below :

(1) (A) and (E) only

(2) (B) and (E) only

(3) (A), (C) and (E) only

(4) (A), (C) and (D) only

Official Ans. by NTA (1)

Sol. As slope of B > Slope of A

$\therefore V_B > V_A$

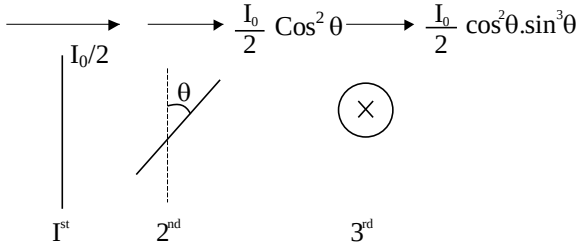
Also, $t_B < t_A$

SECTION-B

51. Unpolarised light of intensity $32\,Wm^{-2}$ passes through the combination of three polaroids such that the pass axis of the last polaroid is perpendicular to that of the pass axis of first polaroid. If intensity of emerging light is $3\,Wm^{-2}$, then the angle between pass axis of first two polaroids is _____ °.

Official Ans. by NTA (30)

Sol. $I_0 = 32w/m^2$



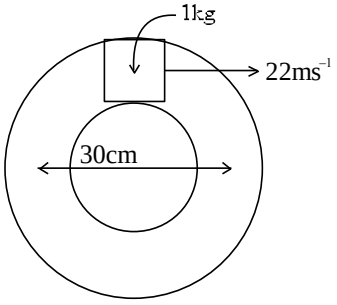
$I_{net} = 3 = \frac{32}{2}\cos^2\theta.\sin^2\theta$

$\frac{3}{4} = 4\sin^2\theta.\cos^2\theta = (\sin 2\theta)^2$

$\frac{\sqrt{3}}{2} = \sin(2\theta)$

Hence, $\theta = 30°$ and $60°$

52. A closed circular tube of average radius 15 cm, whose inner walls are rough, is kept in vertical plane. A block of mass 1 kg just fit inside the tube. The speed of block is 22 m/s, when it is introduced at the top of tube. After completing five oscillations, the block stops at the bottom region of tube. The work done by the tube on the block is _____ J. [Given g = 10 m/s^2]



Official Ans. by NTA (+245)

Sol. $r_{avg} = 15$ cm

$w_f + w_g = \Delta KE$

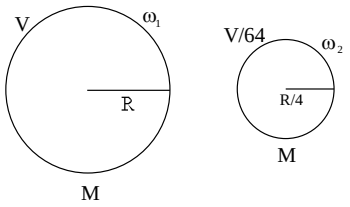
$w_f + 10 \times 0.3 = -\frac{1}{2} \times 484$

$w_f = -245$ J

53. If the earth suddenly shrinks to $\frac{1}{64}$th of its original volume with its mass remaining the same, the period of rotation of earth becomes $\frac{24}{x}$h. The value of x is _____ .

Official Ans. by NTA (16)

Sol. From conservation of angular momentum



$$\frac{2}{5}MR^2\omega_1 = \frac{2}{5}M\left(\frac{R}{4}\right)^2\omega_2$$

$$\Rightarrow MR^2\omega_1 = \frac{MR^2}{16}\omega_2$$

$$\Rightarrow \frac{\omega_1}{\omega_2} = \frac{1}{16} \Rightarrow \frac{T_2}{T_2} = \frac{1}{16} \Rightarrow \frac{T_1}{T_2} = \frac{16}{1} = \frac{t_1}{x}$$

$$\therefore t_1 = 24 \Rightarrow \frac{16}{1} = \frac{24}{t_2} \Rightarrow x = 16$$

54. The current required to be passed through a solenoid of 15 cm length and 60 turns in order to demagnetise a bar magnet of magnetic intensity 2.4×10^3 Am^{-1} is _____ A.

Official Ans. by NTA (6)

Sol. $I = H$

Given, $I = 2.4 \times 10^3$ A/m

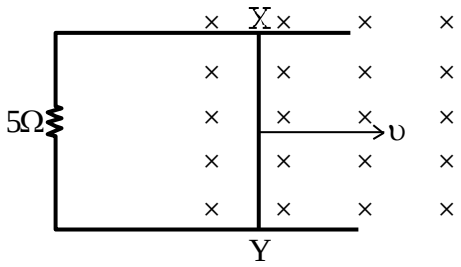
$2.4 \times 10^3 = H = ni$

$n = \frac{N}{\ell}$

$$2.4 \times 10^3 = \frac{60}{15\times10^{-2}}i$$

$$i = \frac{2.4\times15\times10}{60} = \frac{36}{6} = 6A$$

55. A 1m long metal rod XY completes the circuit as shown in figure. The plane of the circuit is perpendicular to the magnetic field of flux density 0.15 T. If the resistance of the circuit is 5Ω, the force needed to move the rod in direction, as indicated, with a constant speed of 4 m/s will be _____ 10^{-3} N.



Official Ans. by NTA (18)

Sol. $F = i\ell B$

$$= \left(\frac{\varepsilon}{R}\right)\ell B = \left(\frac{vB\ell}{R}\right)\ell B = \frac{vB^2\ell^2}{R} = \frac{4}{5}\times\left(\frac{15}{100}\right)^2 \times 1^2$$

$$= \frac{4}{5}\times\frac{225}{10^4}$$

$$= \frac{180}{10^4} = 0.018\,N$$

$= 18 \times 10^{-3}$ N

56. A transverse harmonic wave on a string is given by

$y(x,t) = 5\sin(6t + 0.003x)$

where x and y are in cm and t in sec. The wave velocity is _____ ms^{-1}.

Official Ans. by NTA (20)

Sol. $y(x, t) = 5\sin(6t + 0.003x)$

$k = 0.003$ cm^{-1}, $\omega = 6$ rad/s, $v = \frac{\omega}{k}$

$$\Rightarrow \frac{6}{0.003\times10^2} = 20\ ms^{-1}$$

57. The decay constant for a radioactive nuclide is $1.5\times10^{-5}s^{-1}$. Atomic of the substance is 60 g $mole^{-1}$, $\left(N_A=6\times10^{23}\right)$. The activity of 1.0 μg of the substance is _____ $\times10^{10}$ Bq.

Official Ans. by NTA (15)

Sol. $\lambda = 1.5 \times 10^{-5}\ s^{-1}$

$$\text{No. of mole} = \frac{1\times10^{-6}}{60} = \frac{10^{-7}}{6}$$

No. of atoms = no. of moles × N_A

$$= \frac{10^{-7}}{6}\times6\times10^{23} = 10^{16}$$

$$A = N_0\lambda e^{-\lambda t}$$

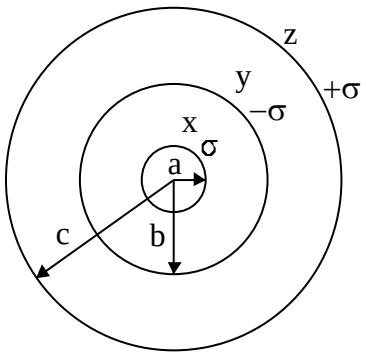
For, t = 0, $A = A_0 = N_0\lambda$

$= 1.5 \times 10^{-5} \times 10^{16} = 15 \times 10^{10}$ Bq.

58. Three concentric spherical metallic shells X, Y and Z of radius a, b and c respectively $[a<b<c]$ have surface charge densities $\sigma, -\sigma$ and σ, respectively. The shells X and Z are at same potential. If the radii of X & Y are 2 cm and 3 cm, respectively. The radius of shell Z is _____ cm.

Official Ans. by NTA (5)

Sol.



$$q_x = \sigma 4\pi a^2$$

$$q_y = -\sigma 4\pi b^2$$

$$q_z = \sigma 4\pi c^2$$

Potential x = potential z

$V_x = V_z$

$$\frac{q_x}{4\pi\varepsilon_0 a}+\frac{q_y}{4\pi\varepsilon_0 b}+\frac{q_z}{4\pi\varepsilon_0 c}=\frac{q_x}{4\pi\varepsilon_0 c}+\frac{q_y}{4\pi\varepsilon_0 c}+\frac{q_z}{4\pi\varepsilon_0 c}$$

$$\frac{\sigma 4\pi a^2}{a}-\frac{\sigma 4\pi b^2}{b}+\frac{\sigma 4\pi c^2}{c}=\frac{4\pi\sigma[a^2-b^2+c^2]}{c}$$

$c(a - b + c) = a^2 - b^2 + c^2$

$c(a - b) = a^2 - b^2$

$c = a + b$

$c = 5$ cm

59. 10 resistors each of resistance 10Ω can be connected in such as to get maximum and minimum equivalent resistance. The ratio of maximum and minimum equivalent resistance will be _____ .

Official Ans. by NTA (100)

Sol. Maximum resistance occurs

When all the resisters are connected in series combination

$\therefore R_{max} = 10\ R$

Here R = 10 ohm

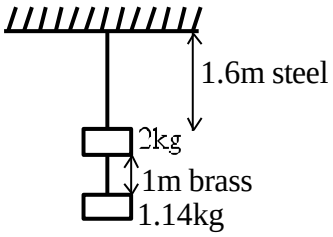
Minimum resistance occurs

When all the resistance are connected in parallel combination

$$R_{min} = \frac{R}{10}$$

$$\therefore \frac{R_{max}}{R_{min}} = 100$$

60. Two wires each of radius 0.2 cm and negligible mass, one made of steel and other made of brass are loaded as shown in the figure. The elongation of the steel wire is _____ $\times 10^{-6}$ m. [Young's modulus for steel = $2\times10^{11}\,Nm^{-2}$ and $g=10\,ms^{-2}$]

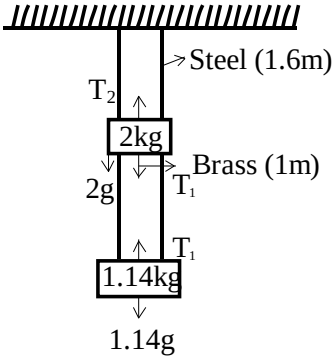


Official Ans. by NTA (20)

Sol. Tension in steel wire $T_2 = 2g + T_1$

$T_2 = 20 + 11.4$

$= 31.4\,N$



Elongation in steel wire $\Delta L = \dfrac{T_2 L}{Ay}$

$$\Delta L = \frac{31.4\times1.6}{\pi\left(0.2\times10^{-2}\right)^2\times2\times10^{11}}$$

$$\Delta L = \frac{16}{2\times4\times10^{-6}\times10^{11}}$$

$= 2\times10^{-5}\,m$

$= 20\times10^{-6}\,m$

SECTION-A

61. Using column chromatography, mixture of two compounds 'A' and 'B' was separated. 'A' eluted first, this indicates 'B' has

(1) low R_f, weaker adsorption

(2) high R_f, stronger adsorption

(3) high R_f, weaker adsorption

(4) low R_f, stronger adsorption

Official Ans. by NTA (4)

Sol. If any component eluted second then it means that its R_f value is low and its adsorption is stronger

$$R_f = \frac{\text{distance covered by substance from base line}}{\text{total distance covered by solvent from base line}}$$

62. Prolonged heating is avoided during the preparation of ferrous ammonium sulphate to

(1) prevent oxidation

(2) prevent reduction

(3) prevent hydrolysis

(4) prevent breaking

Official Ans. by NTA (1)

Sol. Prolonged heating will cause oxidation of Fe^{+2} to Fe^{+3}.

63. Lime reacts exothermally with water to give 'A' which has low solubility in water. Aqueous solution of 'A' is often used for the test of CO_2, a test in which insoluble B is formed. If B is further reacted with CO_2 then soluble compound is formed 'A' is

(1) Quick lime (2) Slaked lime

(3) Lime water (4) White lime

Official Ans. by NTA (2)

Sol. $CaO + H_2O \rightarrow \underset{A\ (\text{less soluble})}{Ca(OH)_2}$

$Ca(OH)_2 + CO_2 \rightarrow \underset{B\ (\text{insoluble})}{CaCO_3} + H_2O$

$\underset{B}{CaCO_3} + H_2O + CO_2 \rightarrow \underset{\text{Soluble}}{Ca(HCO_3)_2}$

64. The pair from the following pairs having both compounds with net non-zero dipole moment is

(1) Benzene, anisidine

(2) 1,4-Dichlorobenzene, 1,3-Dichlorobenzene

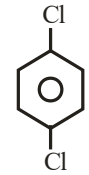
(3) CH_2Cl_2, $CHCl_3$

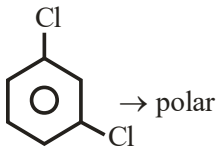
(4) cis-butene, trans-butene

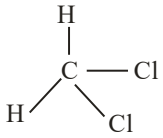
Official Ans. by NTA (3)

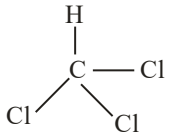
Sol. (1) Benzene → non polar

Anisidine → polar

(2)  → Non polar

 → polar

(3) CH_2Cl_2, $\mu_{net} \neq 0$ polar 

$CHCl_3$, $\mu_{net} \neq 0$ polar 

(4)  ⇒ polar

 ⇒ non polar

65. Match List-I with List-II

List-I Industry	**List-II Waste Generated**
(A) Steel plants	(I) Gypsum
(B) Thermal power plants	(II) Fly ash
(C) Fertilizer industries	(III) Slag
(D) Paper mils	(IV) Bio-degradable Wastes

Choose the correct answer from the options given below:

(1) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

(2) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

(3) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

(4) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

Official Ans. by NTA (1)

Sol. Steel plant produces slag from blast furnace. Thermal power plant produces fly ash, Fertilizer industries produces gypsum. Paper mills produces bio degradable waste

66. Isomeric amines with molecular formula $C_8H_{11}N$ give the following tests

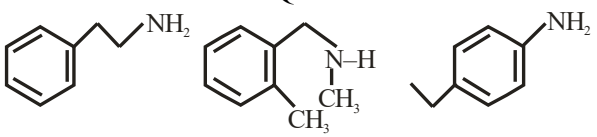
Isomer (P) $\Rightarrow$ Can be prepared by Gabriel phthalimide synthesis

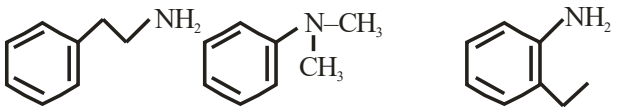
Isomer (Q) $\Rightarrow$ Reacts with Hinsberg's reagent to give solid insoluble in NaOH

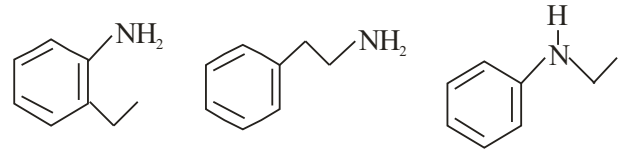
Isomer (R) $\Rightarrow$ Reacts with HONO followed by β-naphthol in NaOH to give red dye.

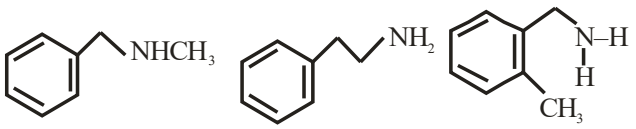
Isomers (P), (Q) and (R) respectively are

P Q R

(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (1)

Sol. (P) Gabriel phthalimide synthesis is used for the preparation of aliphatic primary amines. Aromatic primary amines cannot be prepared by this method.

(Q) 2°-amines reacts with Hinsberg's reagent to give solid insoluble in NaOH

(R) Aromatic primary amine react with nitrous acid at low temperature (273 – 298 K) to form diazonium salts, which form Red dye with β-Naphthol

67. Given below are two statements

Statement I : Aqueous solution of $K_2Cr_2O_7$ is preferred as a primary standard in volumetric analysis over $Na_2Cr_2O_7$ aqueous solution

Statement II : $K_2Cr_2O_7$ has a higher solubility in water than $Na_2Cr_2O_7$

In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

(3) Both Statement I is true but Statement II is false

(4) Both Statement I is false but Statement II is true

Official Ans. by NTA (3)

Sol. (1) $K_2Cr_2O_7$ is used as primary standard. The concentration $Na_2Cr_2O_7$ changes in aq. solution.

(2) It is less soluble than $Na_2Cr_2O_7$.

68. The one that does not stabilize 2° and 3° structures of proteins is

(1) H-bonding (2) –S-S-linkage

(3) –O-O-linkage (4) van der Waals forces

Official Ans. by NTA (3)

Sol. 2° and 3° structure of proteins are stabilized by hydrogen bonding, disulphide linkages, Van der Waals force of attraction and electrostatic force of attraction.

69. Given below are two reactions, involved in the commercial production of dihydrogen (H_2).

The two reactions are carried out at temperature "T_1" and "T_2" respectively

$$C(s) + H_2O(g) \xrightarrow{T_1} CO(g) + H_2(g)$$

$$CO(g) + H_2O(g) \xrightarrow[\text{Catalyst}]{T_2} CO_2(g) + H_2(g)$$

The temperature T_1 and T_2 are correctly related as

(1) $T_1 > T_2$

(2) $T_1 = T_2$

(3) $T_1 = 100$ K, $T_2 = 1270$ K

(4) $T_1 < T_2$

Official Ans. by NTA (1)

Sol. $T_1 = 1270$ K $T_2 = 673$ K

$T_1 > T_2$ on the basis of data

70. Which of the following statements are correct?

(A) The M^{3+}/M^{2+} reduction potential for iron is greater than manganese

(B) The higher oxidation states of first row d-block elements get stabilized by oxide ion.

(C) Aqueous solution of Cr^{2+} can liberate hydrogen from dilute acid.

(D) Magnetic moment of V^{2+} is observed between 4.4-5.2 BM

Choose the correct answer from the options given below:

(1) (B), (C) only (C), (D) only

(2) (A), (B), (D) only (A), (B) only

Official Ans. by NTA (1)

Sol. (A) The M^{3+}/M^{2+} reduction potential for manganese is greater than iron

(B) $E^0_{Fe^{+3}/Fe^{+2}} = +0.77$

$E^0_{Mn^{+3}/Mn^{+2}} = +1.57$

(C) $E^0_{Cr^{+3}/Cr^{+2}} = -0.26$

$\therefore \quad Cr^{2\oplus} + H^{\oplus} \longrightarrow Cr^{3\oplus} + \frac{1}{2}H_2$

(D) $V^{2\oplus}$ = 3 unpaired electron

Magnetic Moment = 3.87 B.M

71. Which of the following is used as a stabilizer during the concentration of sulphide ores?

(1) Pine oils

(2) Xanthates

(3) Fatty acids

(4) Cresols

Official Ans. by NTA (4)

Sol. Cresol is used as stabilizer.

72. The octahedral diamagnetic low spin complex among the following is

(1) $[NiCl_4]^{2-}$

(2) $[CoCl_6]^{3-}$

(3) $[CoF_6]^{3-}$

(4) $[Co(NH_3)_6]^{3+}$

Official Ans. by NTA (4)

Sol. (1) Paramagnetic, High Spin & Tetrahedral

(2) Paramagnetic, High Spin & Octahedral

(3) Paramagnetic, High Spin & Octahedral

(4) Diamagnetic, Low Spin & Octahedral

$[Co(NH_3)_6]^{3+}$, CN = 6 (Octahedral)

NH_3 = SFL

$Co^{+3} = [Ar]3d^6$

Diamagnetic & Low spin complex

73. Given

(A) $2CO(g)+O_2(g) \rightarrow 2CO_2(g) \qquad \Delta H^{\theta}_1 = -x\ kJ\ mol^{-1}$

(B) $C(graphite) + O_2(g) \rightarrow CO_2(g) \quad \Delta H^{\theta}_2 = -y\ kJ\ mol^{-1}$

The ΔH^{θ} for the reaction

$C(graphite) + \frac{1}{2}O_2(g) \rightarrow CO(g)$ is

(1) $\frac{x-2y}{2}$ (3) $\frac{x+2y}{2}$

(3) $\frac{2x-y}{2}$ (4) $2y - x$

Official Ans. by NTA (1)

Sol. Target equation

$C(graphite) + \frac{1}{2}O_{2(g)} \rightarrow CO_{(g)}$ .....(i) ΔH

$C(graphite) + O_{2(g)} \rightarrow CO_{2(g)}$.(ii) ΔH_1 = - y kJ/mole

$CO_{2(g)} \rightarrow CO_{(g)} + \frac{1}{2}O_{2(g)}$ ...(iii) $\Delta H_2 = \frac{x}{2}$ kJ/mole

eq. (i) = eq.(ii) + eq (iii)

$\therefore \Delta H = \frac{x}{2} - y = \frac{x-2y}{2}$

74. The compound which does not exist is

(1) NaO_2

(2) $(NH_4)_2BeF_4$

(3) BeH_2

(4) $PbEt_4$

Official Ans. by NTA (1)

Sol. Sodium superoxide is not stable

75. Match List I with List II

List-I **Polymer**	**List-II** **Type/Class**
(A) Nylon-2-Nylon-6	(I) Thermosetting Polymer
(B) Buna-N	(II) Biodegradable polymer
(C) Urea-formaldehyde	(III) Synthetic rubber resin
(D) Dacron	(IV) Polyester

Choose the correct answer from the options given below:

(1) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)

(2) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(3) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(4) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

Official Ans. by NTA (4)

Sol.

(A) Nylon-2-nylon-6
Biodegradable polymer and polyamides (II)

(B) Buna-N → Butadiene acrylonitrile rubber → synthetic rubber (III)

(C) Urea-formaldehyde resin → Thermosetting polymer (I)

(D) Dacron → Polyester polymer of ethylene glycol and terephthalic acid (IV)

76. The number of molecules and moles in 2.8375 litres of O_2 at STP are respectively

(1) 7.527×10^{22} and 0.250 mol

(2) 1.505×10^{23} and 0.250 mol

(3) 7.527×10^{23} and 0.125 mol

(4) 7.527×10^{22} and 0.125 mol

Official Ans. by NTA (4)

Sol. Number of moles of $O_2 = \frac{2.8375}{22.7} = 0.125$

$\Rightarrow$ Number of molecules $= 0.125\ N_A$

$= 7.525 \times 10^{22}$

77. The enthalpy change for the adsorption process and micelle formation respectively are

(1) $\Delta H_{ads} < 0$ and $\Delta H_{mic} > 0$

(2) $\Delta H_{ads} < 0$ and $\Delta H_{mic} < 0$

(3) $\Delta H_{ads} > 0$ and $\Delta H_{mic} < 0$

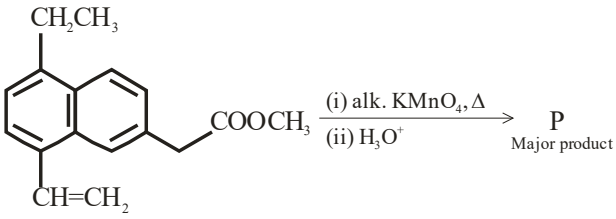
(4) $\Delta H_{ads} > 0$ and $\Delta H_{mic} > 0$

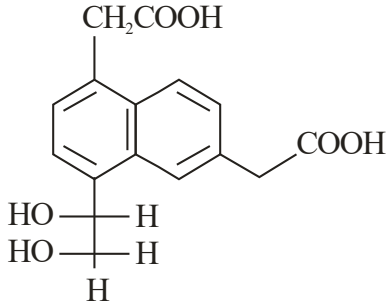
Official Ans. by NTA (1)

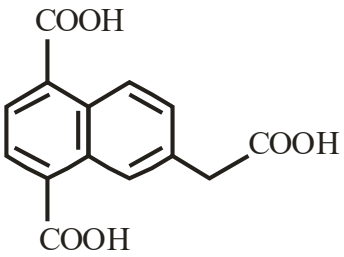
Sol. Adsorption is exothermic process due to decrease in surface energy

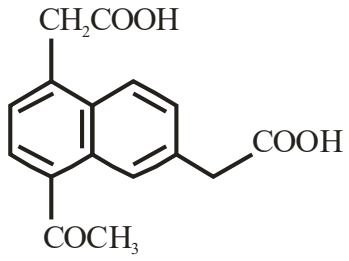
Micelle formation is endothermic

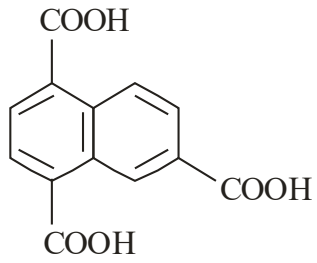
78. The major product 'P' formed in the given reaction is



(1)


(2)


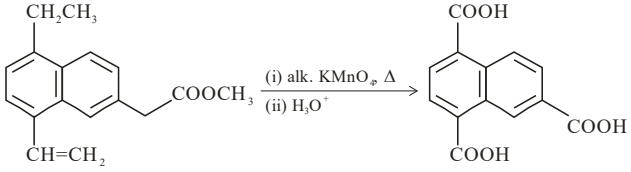
(3)


(4)


Official Ans. by NTA (4)

Sol.

$KMnO_4$ oxidises benzylic carbon containing atleast one α-hydrogen atom to –COOH.



79. Suitable reaction condition for preparation of Methyl phenyl ether is

(1) $Ph-Br$, MeO^-Na^+

(2) PhO^-Na^+, $MeOH$

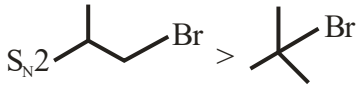
(3) PhO^-Na^+, $MeBr$

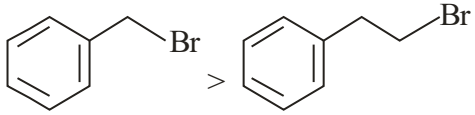
(4) Benzene, MeBr

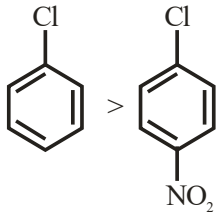
Official Ans. by NTA (3)

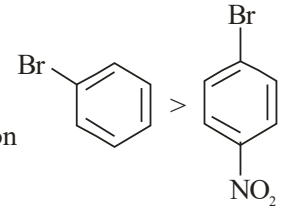
Sol. $PhO^-Na^+ + Me-Br \xrightarrow{S_N2} Ph-O-Me$

80. Identify the correct order of reactivity for the following pairs towards the respective mechanism

(A) S_N2 

(B) S_N1 

(C) Electrophilic substitution 

(D) Nucleophilic substitution 

Choose the correct answer from the options given below :

(1) (A), (B) and (D) only

(2) (A), (B), (C) and (D)

(3) (A), (C) and (D) only

(4) (B), (C) and (D) only

Official Ans. by NTA (2)

Sol.

All are correct

(A) S_N2 reaction decreases with increase in steric crowding.

(B) S_N1 reaction increases with stability of carbocation.

(C) EAS reaction decreases with decrease in electron density.

(D) Presence of electron withdrawing group at ortho and para-position to a halogen in haloarene increase nucleophilic aryl substitution.

SECTION-B

81. The number of correct statement/s involving equilibria in physical process from the following is

(A) Equilibrium is possible only in a closed system at a given temperature

(B) Both the opposing processes occur at the same rate.

(C) When equilibrium is attained at a given temperature, the value of all its parameters became equal

(D) For dissolution of solids in liquids, the solubility is constant at a given temperature

Official Ans. by NTA (3)

Sol. (A) is correct

(B) for equilibrium $r_f = r_b$

$\Rightarrow$ (B) is correct

(C) at equilibrium the value of parameters become constant of a given temperature and not equal

$\Rightarrow$ (C) is incorrect

(D) for a given solid solute and a liquid solvent solubility depends upon temperature only

$\Rightarrow$ (D) is correct

82. The number of bent-shaped molecule/s from the following is _____

N_3^-, NO_2^-, I_3^-, O_3, SO_2

Official Ans. by NTA (3)

Sol. N_3^- linear

NO_2^- bent

I_3^- linear

O_3 bent

SO_2 bent

83. A molecule undergoes two independent first order reactions whose respective half lives are 12 min and 3 min. If both the reactions are occurring then the time taken for the 50% consumption of the reactant is _____ min. (Nearest integer)

Official Ans. by NTA (2)

Sol. $\frac{1}{t_{1/2}} = \frac{1}{3} + \frac{1}{12} = \frac{4+1}{12} = \frac{5}{12}$

$t_{1/2} = \frac{12}{5}\text{min} = 2.4$

Ans. is 2

84. The number of incorrect statement/s about the black body from the following is _____

(A) Emit or absorb energy in the form of electromagnetic radiation

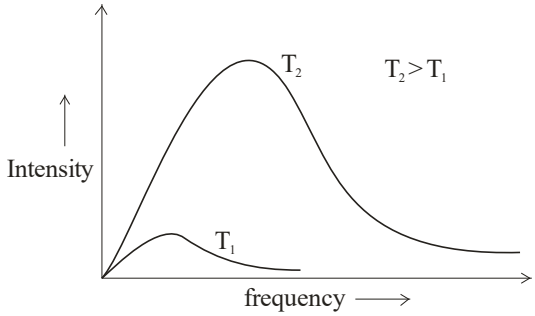
(B) Frequency distribution of the emitted radiation depends on temperature

(C) At a given temperature, intensity vs frequency curve passes through a maximum value

(D) The maximum of the intensity vs frequency curve is at a higher frequency at higher temperature compared to that at lower temperature

Official Ans. by NTA (0)

Sol. A blackbody can emit and absorb all the wavelengths in electromagnetic spectrum $\Rightarrow$ (A) is correct



$\Rightarrow$ (B), (C), (D) correct

Ans (0)

85. In the following reactions, the total number of oxygen atoms in X and Y is _____

$Na_2O + H_2O \rightarrow 2X$

$Cl_2O_7 + H_2O \rightarrow 2Y$

Official Ans. by NTA (5)

Sol. $Na_2O + H_2O \rightarrow 2NaOH$

$Cl_2O_7 + H_2O \rightarrow 2HClO_4$

$1 + 4 = 5$

86. $FeO_4^{2-} \xrightarrow{+2.2\,V} Fe^{3+} \xrightarrow{+0.70\,V} Fe^{2+} \xrightarrow{-0.45V} Fe^0$

$E^{\theta}_{FeO_4^{2-}/Fe^{2+}}$ is x × 10^{-3} V. The value of x is _____

Official Ans. by NTA (1825)

Sol.

$$FeO_4^{2-} \xrightarrow[n=3]{2.2V} Fe^{3+} \xrightarrow[n=1]{0.7V} Fe^{2+} \xrightarrow{-0.45V} Fe$$

$$\underbrace{\qquad\qquad E = ? \qquad\qquad}_{n = 4}$$

$4 \times E = 3 \times 2.2 + 1 \times 0.7$

$E = \frac{7.3}{4} = 1.825\ V = 1825 \times 10^{-3}\ V$

87. If the degree of dissociation of aqueous solution of weak monobasic acid is determined to be 0.3, then the observed freezing point will be _____ % higher than the expected/theoretical freezing point. (Nearest integer)

Official Ans. by NTA (30)

Sol. $i = 1 + \alpha$ (for HA)

$= 1.3$

$$\%\ \text{increase} = \frac{(\Delta T_f)_{obs} - (\Delta T_f)_{cal}}{(\Delta T_f)_{cal}} \times 100$$

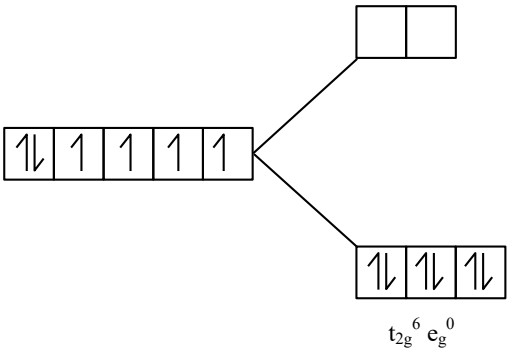
$$= \frac{K_f \times i \times m - K_f \times m}{K_f \times m} \times 100$$

$$= \frac{i-1}{1} \times 100 = 30\%$$

88. In potassium ferrocyanide, there are ____ pairs of electrons in the t_{2g} set of orbitals

Official Ans. by NTA (3)

Sol. $K_4[Fe(CN)_6]$



$t_{2g}^6 e_g^0$

$Fe^{+2} = [Ar]3d^6$

$CN^- = SFL$

t_{2g} contain 6 electron so it become 3 pairs:-

89. At constant temperature a gas is at a pressure of 940.3 mm Hg. The pressure at which its volume decreases by 40% is _____ mm Hg.

(Nearest Integer)

Official Ans. by NTA (1567)

Sol. $P_1V_1 = P_2V_2$

$940.3 \times 100 = P_2 \times 60$

$P_2 = 1567$ mm of Hg

90. The sum of lone pairs present on the central atom of the interhalogen IF_5 and IF_7 is _____

Official Ans. by NTA (1)

Sol. $IF_5 = 1$ lone pair

$IF_7 = 0$ lone pair

$1 + 0 = 1$

FINAL JEE–MAIN EXAMINATION – APRIL, 2023

(Held On Monday 10th April, 2023) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS | **TEST PAPER WITH SOLUTION**

SECTION-A

1. Let f be a continuous function satisfying $\int_0^{t^2}\left(f(x)+x^2\right)dx=\frac{4}{3}t^3,\ \forall\ t>0$. Then $f\left(\frac{\pi^2}{4}\right)$ is equal to :

(1) $\pi\left(1-\frac{\pi^3}{16}\right)$

(2) $-\pi^2\left(1+\frac{\pi^2}{16}\right)$

(3) $-\pi\left(1+\frac{\pi^3}{16}\right)$

(4) $\pi^2\left(1-\frac{\pi^2}{16}\right)$

Official Ans. by NTA (1)

Sol. $\int_0^{t^2}\left(f(x)+x^2\right)dx=\frac{4}{3}t^3,\ \forall\ t>0$

$\left(f(t^2)+t^4\right)=2t$

$f(t^2)=2t-t^4$

$t=\frac{\pi}{2}\Rightarrow f\left(\frac{\pi^2}{4}\right)=\frac{2\pi}{2}-\frac{\pi^4}{16}$

$=\pi-\frac{\pi^4}{16}=\pi\left(1-\frac{\pi^3}{16}\right)$

2. Eight persons are to be transported from city A to city B in three cars of different makes. If each car can accommodate at most three persons, then the number of ways, in which they can be transported, is:

(1) 3360

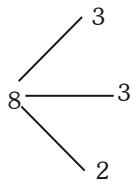
(2) 1680

(3) 560

(4) 1120

Official Ans. by NTA (1)

Sol.



Ways = $\frac{8!}{3!3!2!2!}\times 3!$

$=\frac{8\times7\times6\times5\times4}{4}$

$= 56 \times 30$

$= 1680$

3. For, $\alpha,\beta,\gamma,\delta\in\mathbb{N}$, if

$$\int\left(\left(\frac{x}{e}\right)^{2x}+\left(\frac{e}{x}\right)^{2x}\right)\log_e x\,dx=\frac{1}{\alpha}\left(\frac{x}{e}\right)^{\beta x}-\frac{1}{\gamma}\left(\frac{e}{x}\right)^{\delta x}+C,$$

Where $e=\sum_{n=0}^{\infty}\frac{1}{n!}$ and C is constant of integration, then $\alpha + 2\beta + 3\gamma - 4\delta$ is equal to:

(1) 1

(2) –4

(3) –8

(4) 4

Official Ans. by NTA (4)

Sol. $\left(x=e^{\ln x}\right)$

$\int\left(\left(\frac{x}{e}\right)^{2x}+\left(\frac{e}{x}\right)^{2x}\right)\log_e x\,dx=\int\left[e^{2(x\ln x-x)}+e^{-2(x\ln x-x)}\right]\ln x\,dx$

$x\ln x - x = t$

$\ln x.dx = dt$

$\int\left(e^{2t}+e^{-2t}\right)dt$

$\frac{e^{2t}}{2}-\frac{e^{-2t}}{2}+C$

$=\frac{1}{2}\left(\frac{x}{e}\right)^{2x}-\frac{1}{2}\left(\frac{e}{x}\right)^{2x}+C$

$\alpha=\beta=\gamma=\delta=2$

$\alpha + 2\beta + 3\gamma - 4\delta = 4$

4. Let the image of the point P(1, 2, 6) in the plane passing through the points A(1, 2, 0), B(1, 4, 1) and C(0, 5, 1) be Q (α, β, γ). Then ($\alpha^2+\beta^2+\gamma^2$) is equal to :

(1) 65

(2) 70

(3) 76

(4) 62

Official Ans. by NTA (1)

Sol. Equation of plane $A(x-1)+B(y-2)+C(z-0)=0$

Put (1, 4, 1) $\Rightarrow 2B + C = 0$

Put (0, 5, 1) $\Rightarrow -A + 3B + C = 0$

$$\text{Sub} : B - A = 0 \Rightarrow A = B,\ C = -2B$$

$1(x-1)+1(y-2)-2(z-0)=0$

$x+y-2z-3=0$

Image is (α, β, γ) $pt \equiv (1, 2, 6)$

$$\frac{\alpha-1}{1}=\frac{\beta-2}{1}=\frac{\gamma-6}{-2}=\frac{-2(1+2-12-3)}{6}$$

$$\frac{\alpha-1}{1}=\frac{\beta-2}{1}=\frac{\gamma-6}{-2}=4$$

$\alpha=5, \beta=6, \gamma=-2 \Rightarrow \alpha^2+\beta^2+\gamma^2$

$=25+36+4=65$

5. Let A = {2, 3, 4} and B = {8, 9, 12}. Then the number of elements in the relation

R = $\{((a_1, b_1), (a_2, b_2)) \in (A \times B, A \times B) : a_1$ divides b_2 and a_2 divides $b_1\}$ is :

(1) 36

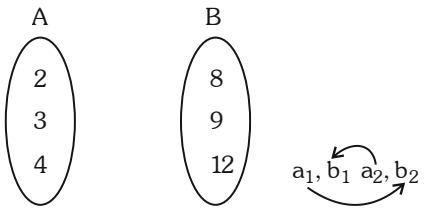
(2) 12

(3) 18

(4) 24

Official Ans. by NTA (1)

Sol.



a_1 divides b_2

Each element has 2 choices

$\Rightarrow 3 \times 2 = 6$

a_2 divides b_1

Each element has 2 choices

$\Rightarrow 3 \times 2 = 6$

Total = $6 \times 6 = 36$

6. If $A=\frac{1}{5!6!7!}\begin{bmatrix}5! & 6! & 7!\\ 6! & 7! & 8!\\ 7! & 8! & 9!\end{bmatrix}$, then $|\text{adj}(\text{adj}(2A))|$ is equal to :

(1) 2^8

(2) 2^{12}

(3) 2^{20}

(4) 2^{16}

Official Ans. by NTA (4)

Sol. $|\text{adj}\,\text{adj}(2A)|=|2A|^{(n-1)^2}$

$=|2A|^4$

$=(2^3|A|)^4$

$= 2^{12}|A|^4 \Rightarrow 2^{16}$

$$|A| = \frac{1}{5!6!7!}5!6!\begin{vmatrix}1 & 6 & 42\\ 1 & 7 & 56\\ 1 & 8 & 72\end{vmatrix}$$

$R_3 \to R_3 \to R_2$

$R_2 \to R_2 \to R_1$

$$|A|=\begin{vmatrix}1 & 8 & 42\\ 0 & 1 & 14\\ 0 & 1 & 16\end{vmatrix}=2$$

7. Let A be the point (1, 2) and B be any point on the curve $x^2+y^2=16$. If the centre of the locus of the point P, which divides the line segment AB in the ratio 3 : 2 is the point C (α, β), then the length of the line segment AC is

(1) $\frac{6\sqrt{5}}{5}$ (2) $\frac{4\sqrt{5}}{5}$

(3) $\frac{2\sqrt{5}}{5}$ (4) $\frac{3\sqrt{5}}{5}$

Official Ans. by NTA (4)

Sol. A(1, 2) — P(h,k) — B(4cosθ, 4sinθ)

$\frac{12\cos\theta+2}{5}=h \quad \Rightarrow \quad 12\cos\theta = 5h-2$

$\frac{12\sin\theta+4}{5}=k \quad \Rightarrow \quad 12\sin\theta = 5k\ \ 4$

Sq & add :

$144 = (5h-2)2 + (5k-4)2$

$\left(x-\frac{2}{5}\right)^2+\left(y-\frac{4}{5}\right)^2=\frac{144}{25}$

Centre $\equiv\left(\frac{2}{5},\frac{4}{5}\right)\equiv(\alpha,\beta)$

$AC = \sqrt{\left(1-\frac{2}{5}\right)^2+\left(2-\frac{4}{5}\right)^2}$

$= \sqrt{\frac{9}{25}+\frac{36}{25}} = \frac{\sqrt{45}}{5}=\frac{3\sqrt{5}}{5}$

8. Let a die be rolled n times. Let the probability of getting odd numbers seven times be equal to the probability of getting odd numbers nine times. If the probability of getting even numbers twice is $\frac{k}{2^{15}}$, then k is equal to :

(1) 30

(2) 90

(3) 15

(4) 60

Official Ans. by NTA (4)

Sol. P(odd number 7 times) = P(odd number 9 times)

$^nC_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^{n-7} = {}^nC_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^{n-9}$

$^nC_7 = {}^nC_9$

$\Rightarrow$ n = 16

Required

$P = {}^{16}C_2\times\left(\frac{1}{2}\right)^{16}$

$= \frac{16\cdot15}{2}\times\frac{1}{2^{16}} = \frac{15}{2^{13}}$

$\Rightarrow \frac{60}{2^{15}} \Rightarrow k = 60$

9. Let $g(x)=f(x)+f(1-x)$ and $f''(x)>0,\ x\in(0,1)$. If g is decreasing in the interval (0, α) and increasing in the interval (α, 1), then $\tan^1(2\alpha)+\tan^{-1}\left(\frac{1}{\alpha}\right)+\tan^{-1}\left(\frac{\alpha+1}{\alpha}\right)$ is equal to :

(1) $\frac{3\pi}{2}$

(2) π

(3) $\frac{5\pi}{4}$

(4) $\frac{3\pi}{4}$

Official Ans. by NTA (2)

Sol. g(x) = f(x) + f(1 – x) & f′′(x) > 0, x ∈ (0, 1)

g′(x) = f′(x) – f′(1 – x) = 0

$\Rightarrow$ f′(x) = f′(1 – x)

x = 1 – x

$x = \frac{1}{2}$

g′(x) = 0

at $x = \frac{1}{2}$

g′′(x) = f′′(x) + f′′(1 – x) > 0

g is concave up

hence $\alpha = \frac{1}{2}$

$\tan^{-1}2\alpha+\tan^{-1}\frac{1}{\alpha}+\tan^{-1}\frac{\alpha+1}{\alpha}$

$\Rightarrow \tan^{-1}1+\tan^{-1}2+\tan^{-1}3=\pi$

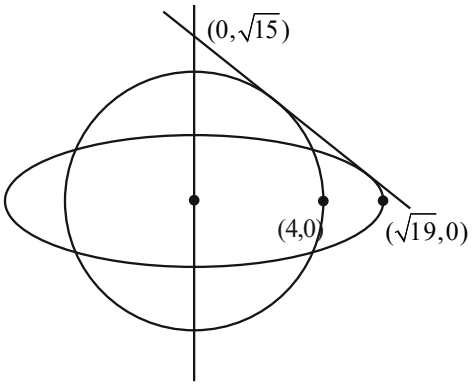
10. Let a circle of radius 4 be concentric to the ellipse $15x^2+19y^2=285$. Then the common tangents are inclined to the minor axis of the ellipse at the angle.

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{3}$

(3) $\frac{\pi}{12}$ (4) $\frac{\pi}{6}$

Official Ans. by NTA (2)

Sol. $\frac{x^2}{19}+\frac{y^2}{15}=1$



Let tang be

$y = mx \pm \sqrt{19m^2+15}$

$mx - y \pm \sqrt{19m^2+15} = 0$

Parallel from (0, 0) = 4

$\left|\frac{\pm\sqrt{19m^2+15}}{\sqrt{m^2+1}}\right| = 4$

$19m^2 + 15 = 16m^2 + 16$

$3m^2 = 1$

$m = \pm\frac{1}{\sqrt{3}}$

$\theta = \frac{\pi}{6}$ with x-axis

Required angle $\frac{\pi}{3}$.

11. Let $\vec{a}=2\hat{i}+7\hat{j}-\hat{k}$, $\vec{b}=3\hat{i}+5\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}+2\hat{k}$. Let $\vec{d}$ be a vector which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}\cdot\vec{d}=12$. Then $\left(-\hat{i}+\hat{j}-\hat{k}\right)\cdot(\vec{c}\times\vec{d})$ is equal to

(1) 48

(2) 42

(3) 44

(4) 24

Official Ans. by NTA (3)

Sol. $\vec{a}=2\hat{i}+7\hat{j}-\hat{k}$

$\vec{b}=3\hat{i}+5\hat{k}$

$\vec{c}=\hat{i}-\hat{j}+2\hat{k}$

$$\vec{d}=\lambda(\vec{a}\times\vec{b})=\lambda\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&7&-1\\3&0&5\end{vmatrix}$$

$\vec{d}=\lambda(35\hat{i}-13\hat{j}-21\hat{k})$

$\lambda(35+13-42)=12$

$\lambda = 2$

$\vec{d}=2(35\hat{i}-13\hat{j}-21\hat{k})$

$(\hat{i}+\hat{j}-\hat{k})(\vec{c}\times\vec{d})$

$$=\begin{vmatrix}-1&1&-1\\1&-1&2\\70&-26&-42\end{vmatrix}=44$$

12. If $S_n = 4 + 11 + 21 + 34 + 50 + \ldots..$ to n terms, then $\frac{1}{60}(S_{29}-S_9)$ is equal to

(1) 226 (2) 220

(3) 223 (4) 227

Official Ans. by NTA (3)

Sol. $S_n = 4+11+21+34+50+....+n$ terms

Difference are in A.P.

Let $T_n = an^2+bn+c$

$T_1 = a+b+c=4$

$T_2 = 4a+2b+c=11$

$T_3 = 9a+3b+c=21$

By solving these 3 equations

$a=\frac{3}{2}, b=\frac{5}{2}, c=0$

So $T_n=\frac{3}{2}n^2+\frac{5}{2}n$

$S_n = \Sigma T_n$

$=\frac{3}{2}\Sigma n^2+\frac{5}{2}\Sigma n$

$=\frac{3}{2}\frac{n(n+1)(2n+1)}{6}=\frac{5}{2}\frac{(n)(n+1)}{2}$

$=\frac{n(n+1)}{4}[2n+1+5]$

$S_n=\frac{n(n+1)}{4}(2n+6)=\frac{n(n+1)(n+3)}{2}$

$\frac{1}{60}\left(\frac{29\times30\times32}{2}-\frac{9\times10\times12}{2}\right)=223$

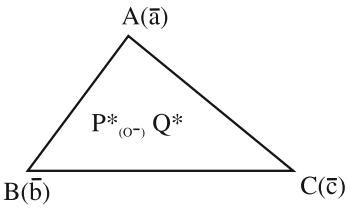
13. If the points P and Q are respectively the circumcentre and the orthocentre of a ΔABC, then $\vec{PA}+\vec{PB}+\vec{PC}$ is equal to

(1) $2\vec{QP}$ (2) $\vec{QP}$

(3) $2\vec{PQ}$ (4) $\vec{PQ}$

Official Ans. by NTA (4)

Sol.



$\overline{PA}+\overline{PB}+\overline{PC}=\bar{a}+\bar{b}+\bar{c}$

$\overline{PG}=\dfrac{\bar{a}+\bar{b}+\bar{c}}{3}$

$\Rightarrow \bar{a}+\bar{b}+\bar{c}=3\overline{PG}=\overline{PQ}$

Ans. (4)

14. The statement $\sim[p \vee (\sim (p \wedge q))]$ is equivalent to

(1) $(\sim(p \wedge q)) \wedge q$

(2) $\sim (p \wedge q)$

(3) $\sim(p \vee q)$

(4) $(p \wedge q) \wedge (\sim p)$

Official Ans. by NTA (4)

Sol. $\sim[pv(\sim (p \wedge q))]$

$\sim p \wedge (p \wedge q)$

15. Let $S=\left\{x \in\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right): 9^{1-\tan^2 x}+9^{\tan^2 x}=10\right\}$ and $\beta=\sum\limits_{x \in s}\tan^2\left(\dfrac{x}{3}\right)$, then $\dfrac{1}{6}(\beta-14)^2$ is equal to

(1) 32

(2) 8

(3) 64

(4) 16

Official Ans. by NTA (1)

Sol. Let $9^{\tan^2 x}=P$

$\dfrac{9}{P}+P=10$

$P^2-10P+9=0$

$(P-9)(P-1)=0$

$P=1, 9$

$9^{\tan^2 x}=1,\ 9^{\tan^2 x}=9$

$\tan^2 x=0, \tan^2 x=1$

$x=0, \pm\dfrac{\pi}{4} \quad \therefore x \in\left(-\dfrac{\pi}{2},\dfrac{p}{2}\right)$

$\beta=\tan^2(0)+\tan^2\left(+\dfrac{\pi}{12}\right)+\tan^2\left(-\dfrac{\pi}{12}\right)$

$=0+2(\tan 15°)^2$

$2(2-\sqrt{3})^2$

$2(7-4\sqrt{3})$

Than $\dfrac{1}{6}(14-8\sqrt{3}-14)^2=32$

16. If the coefficients of x and x^2 in $(1+x)^p(1-x)^q$ are 4 and –5 respectively, then 2p + 3q is equal to

(1) 63

(2) 69

(3) 66

(4) 60

Official Ans. by NTA (1)

Sol. $(1+x)^P(1-x)^q$

$\left(1+px+\dfrac{p(p-1)}{2!}x^2+...\right)$

$\left(1-qx+\dfrac{q(q-1)}{2!}x^2-...\right)$

$p-q=4$

$\dfrac{p(p-1)}{2}+\dfrac{q(q-1)}{2}-pq=-5$

$p^2+q^2-p-q-2pq=-10$

$(q+4)^2+q^2-(q+4)-q-2(4+q)q=-10$

$q^2+8q+16-q^2-q-4-q-8q-2q^2=-10$

$-2q=-22$

$q=11$

$p=15$

$2(15)+3(11)$

$30+33=63$

17. Let the line $\frac{x}{1}=\frac{6-y}{2}=\frac{z+8}{5}$ intersect the lines $\frac{x-5}{4}=\frac{y-7}{3}=\frac{z+2}{1}$ and $\frac{x+3}{6}=\frac{3-y}{3}=\frac{z-6}{1}$ at the points A and B respectively. Then the distance of the mid-point of the line segment AB from the plane $2x - 2y + z = 14$ is

(1) 4 (2) $\frac{10}{3}$

(3) 3 (4) $\frac{11}{3}$

Official Ans. by NTA (1)

Sol. $\frac{x}{1}=\frac{y-6}{-2}=\frac{z+8}{5}=\lambda$ .... (1)

$\frac{x-5}{4}=\frac{y-7}{3}=\frac{z+2}{1}=\mu$ ....(2)

$\frac{x+3}{4}=\frac{y-3}{-3}=\frac{z-6}{1}=\gamma$ .... (3)

Intersection of (1) & (2) "A"

$(\lambda,-2\lambda+6,5\lambda-8)$ & $(4\mu+5,3\mu+7,\mu-2)$

$\lambda = 1, \mu = -1$

A(1, 4, –3)

Intersection of (1) & (3) "B"

$(\lambda,-2\lambda+6,5\lambda-8)$ & $(6\gamma-3,-3\gamma+3,\gamma+6)$

$\lambda = 3$

$\gamma = 1$

B(3, 0, 7)

Mid point of A & B $\Rightarrow$ (2, 2, 2)

Perpendicular distance from the plane

$2x - 2y + z = 14$

$\Rightarrow \left|\frac{2(2)-2(2)+2-14}{\sqrt{4+4+1}}\right|=4$

18. Let $S=\left\{z=x+iy:\frac{2z-3i}{4z+2i} \text{ is a real number}\right\}$.

Then which of the following is NOT correct?

(1) $y+x^2+y^2\neq-\frac{1}{4}$

(2) $x = 0$

(3) $(x, y)=\left(0,-\frac{1}{2}\right)$

(4) $y\in\left(-\infty,-\frac{1}{2}\right)\cup\left(-\frac{1}{2},\infty\right)$

Official Ans. by NTA (3)

Sol. $\frac{2z-3i}{qz+2i}\in R$

$\frac{2(x+iy)-3i}{4(x+it)+2i}=\frac{2x+(2y-3)i}{4x+(4y+2)i}\times\frac{4x-(4y+2)i}{4x-(4y+2)i}$

$4x(2y - 3) - 2x(4y + 2) = 0$

$x = 0$ $\qquad y\neq-\frac{1}{2}$

Ans. = 3

19. Let the number $(22)^{2022} + (2022)^{22}$ leave the remainder α when divided by 3 and β when divided by 7. Then $(\alpha^2 + \beta^2)$ is equal to

(1) 10

(2) 5

(3) 20

(4) 13

Official Ans. by NTA (2)

Sol. $(22)^{2022} + (2022)^{22}$

divided by 3

$(21 + 1)^{2022} + (2022)^{22}$

$= 3k + 1$ $\qquad (\alpha = 1)$

Divided by 7

$(21 + 1)^{2022} + (2023 - 1)^{22}$

$7k + 1 + 1$ $\qquad (\beta = 2)$

$7k + 2$

So $\alpha^2 + \beta^2 \Rightarrow 5$

20. Let μ be the mean and σ be the standard deviation of the distribution

x_i	0	1	2	3	4	5
f_i	$k+2$	$2k$	k^2-1	k^2-1	k^2+1	$k-3$

where $\sum f_i = 62$. if [x] denotes the greatest integer $\le x$, then $[\mu^2 + \sigma^2]$ is equal

(1) 8

(2) 7

(3) 6

(4) 9

Official Ans. by NTA (1)

Sol. $\sum f_i = 62$

$\Rightarrow$ $3k^2 + 16k - 12k - 64 = 0$

$\Rightarrow$ $k = \text{or } -\frac{16}{3}$ (rejected)

$$\mu = \frac{\sum f_i x_i}{\sum f_i}$$

$$\mu = \frac{8 + 2(15) + 3(15) + 4(17) + 5}{62} = \frac{156}{62}$$

$$\sigma^2 = \sum f_i x_i^2 - \left(\sum f_i x_i\right)^2$$

$$= \frac{8 \times 1^2 + 15 \times 13 + 17 \times 16 + 25}{62} - \left(\frac{156}{62}\right)^2$$

$$\sigma^2 = \frac{500}{62} - \left(\frac{156}{62}\right)^2$$

$$\sigma^2 + \mu^2 = \frac{500}{62}$$

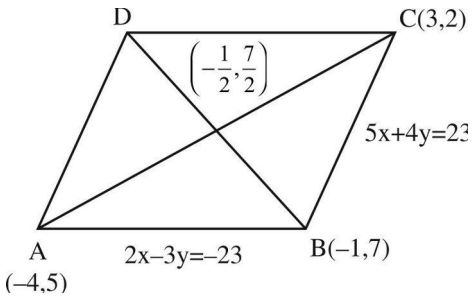
$$\left[\sigma^2 + \mu^2\right] = 8$$

SECTION-B

21. Let the equations of two adjacent sides of a parallelogram ABCD be $2x - 3y = -23$ and $5x + 4y = 23$. If the equation of its one diagonal AC is $3x + 7y = 23$ and the distance of A from the other diagonal is d, then $50\ d^2$ is equal to _____ .

Official Ans. by NTA (529)

Sol.



A & C point will be (–4, 5) & (3, 2)

mid point of AC will be $\left(-\frac{1}{2},\frac{7}{2}\right)$

equation of diagonal BD is

$$y - \frac{7}{2} = \frac{\frac{7}{2}}{-\frac{1}{2}} \qquad \left(x + \frac{1}{2}\right)$$

$\Rightarrow$ $7x + y = 0$

Distance of A from diagonal BD

$$= d = \frac{23}{\sqrt{50}}$$

$\Rightarrow$ $50d^2 = (23)^2$

$50d^2 = 529$

22. Let S be the set of values of λ, for which the system of equations

$6\lambda x - 3y + 3z = 4\lambda^2$,

$2x + 6\lambda y + 4z = 1$,

$3x + 2y + 3\lambda z = \lambda$ has no solution. Then $12\sum_{\lambda \in S} |\lambda|$ is equal to _____ .

Official Ans. by NTA (24)

Sol. $\Delta = \begin{vmatrix} 6\lambda & -3 & 3 \\ 2 & 6\lambda & 4 \\ 3 & 2 & 3\lambda \end{vmatrix} = 0$ (For No Solution)

$2\lambda\,(9\lambda^2 - 4) + (3\lambda - 6) + (2 - 9\lambda) = 0$

$18\lambda^3 - 14\lambda - 4 = 0$

$(\lambda - 1)(3\lambda + 1)(3\lambda + 2) = 0$

$\Rightarrow \lambda = 1, -1/3, -2/3$

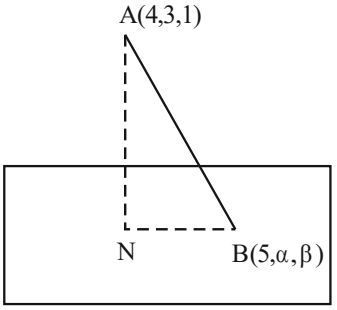
For each λ, $\Delta_1 = \begin{vmatrix} 6\lambda & -3 & 4\lambda^2 \\ 2 & 6\lambda & 1 \\ 3 & 2 & \lambda \end{vmatrix} \neq 0$

Ans. $12\left(1 + \frac{1}{3} + \frac{2}{3}\right) = 24$

23. Let the foot of perpendicular from the point A(4, 3, 1) on the plane P : $x - y + 2z + 3 = 0$ be N. If B(5, α, β), $\alpha, \beta \in \mathbb{Z}$ is a point on plane P such that the area of the triangle ABN in $3\sqrt{2}$, then $\alpha^2 + \beta^2 + \alpha\beta$ is equal to _____ .

Official Ans. by NTA (7)

Sol.



$AN = \sqrt{6}$

$5 - \alpha + 2\beta + 3 = 0$

$\Rightarrow$ $\alpha = 8 + 2\beta$ .... (1)

N is given by

$$\frac{x-4}{1}=\frac{y-3}{-1}=\frac{z-1}{2}=\frac{-(4-3+2+3)}{1+1+4}$$

$\Rightarrow$ x = 3, y = 4, z = –1

$\Rightarrow$ N is (3, 4, –1)

$BN=\sqrt{4+(\alpha-4)^2+(\beta+1)^2}$

$=\sqrt{4+(2\beta+4)^2+(\beta+1)^2}$

Area of $\Delta ABN=\frac{1}{2}AN\times BN=3\sqrt{2}$

$\Rightarrow \frac{1}{2}\times\sqrt{6}\times BN=3\sqrt{2}$

$BN=2\sqrt{3}$

$\Rightarrow 4+(2\beta+4)^2+(\beta+1)^2=12$

$(2\beta+4)^2+(\beta+1)^2-8=0$

$5\beta^2+18\beta+9=0$

$(5\beta+3)(\beta+3)=0$

$\beta=-3$

$\Rightarrow \alpha=2$

$\Rightarrow \alpha^2+\beta^2+\alpha\beta=9+4-6=7$

24. Let quadratic curve passing through the point (–1, 0) and touching the line y = x at (1, 1) be y = f(x). Then the x-intercept of the normal to the curve at the point $(\alpha, \alpha+1)$ in the first quadrant is _____ .

Official Ans. by NTA (11)

Sol. f(x) = (x + 1) (ax + b)

1 = 2a + 2b ____(1)

f'(x) = (ax + b) + a (x + 1)

1 = (3a + b) _____(2)

$\Rightarrow$ b = 1/4, a = 1/4

$f(x)=\frac{(x+1)^2}{4}$

$f'(x)=\frac{x}{2}+\frac{1}{2}$ $\qquad \alpha+1=\frac{(\alpha+1)^2}{4},\ \alpha>-1$

$\alpha+1=4$

$\alpha=3$

normal at (3, 4)

$y-4=-\frac{1}{2}(x-3)$

y = 0 $\qquad$ x = 8 + 3

Ans. 11

25. Let the tangent at any point P on a curve passing through the points (1, 1) and $\left(\frac{1}{10},100\right)$, intersect positive x-axis and y-axis at the points A and B respectively. If PA : PB = 1 : k and y = y(x) is the solution of the differential equation $e^{\frac{dy}{dx}}=kx+\frac{k}{2}$, y(0) = k, then $4y(1)-5\log_e 3$ is equal to _____ .

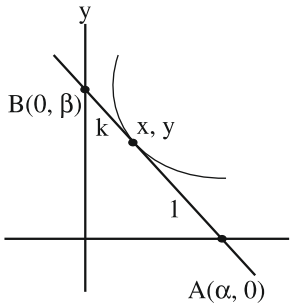
Official Ans. by NTA (6)

Sol. equation of tangent at P (x, y)

$Y-y=\frac{dy}{dx}(X-x)$

Y = 0

$X=\frac{-ydx}{dy}+x$



$\frac{k\alpha+0}{k+1}=x,\ \alpha=\frac{k+1}{k}x$

$\frac{k+1}{k}x=-y\frac{dx}{dy}+x$

$x+\frac{x}{k}=-y\frac{dx}{dy}+x$

$x\frac{dy}{dx}+ky=0$ $\qquad \frac{dy}{dx}+\frac{k}{x}y=0$

$y.x^k=C$

C = 1

$100.\left(\frac{1}{10}\right)^k=1$

K = 2

$\frac{dy}{dx}=\ell n(2x+1)$

$y=\frac{(2x+1)}{2}(\ell n(2x+1)-1)+c$

$2=\frac{1}{2}(0-1)+C$

$C=2+\frac{1}{2}=\frac{5}{2}$

$y(1)=\frac{3}{2}(\ell n3-1)+\frac{5}{2}$

$=\frac{3}{2}\ell n3+1$

$4y(1)=6\ell n3+4$

$4y(1)-5\ell n3=4+\ell n3$

26. Suppose a_1, a_2, 2, a_3, a_4 be in an arithmetico-geometric progression. If the common ratio of the corresponding geometric progression is 2 and the sum of all 5 terms of the arithmetico-geometric progression is $\frac{49}{2}$, then a_4 is equal to _____ .

Official Ans. by NTA (16)

Sol. $\frac{(a-2d)}{4}, \frac{(a-d)}{2}$, a, 2(a + d), 4(a + 2d)

a = 2

$$\left(\frac{1}{4}+\frac{1}{2}+1+6\right)\times 2+(-1+2+8)d=\frac{49}{2}$$

$$2\left(\frac{3}{4}+7\right)+9d=\frac{49}{2}$$

$$9d=\frac{49}{2}-\frac{62}{4}=\frac{98-62}{4}=9$$

d = 1

$\Rightarrow a_4 = 4\,(a + 2d)$

$= 16$

27. If the domain of the function $f(x)=\sec^{-1}\left(\frac{2x}{5x+3}\right)$ is $[\alpha,\beta)\cup(\gamma,\delta]$, then $|3\alpha+10(\beta+\gamma)+21\delta|$ is equal to _____ .

Official Ans. by NTA (24)

Sol. $f(x) = \sec^{-1}\frac{2x}{5x+3}$

$$\left|\frac{2x}{5x+3}\right|$$

$$\left|\frac{2x}{5x+3}\right|\geq 1\Rightarrow |2x|\geq|5x+3|$$

$(2x)^2 - (5x + 3)^2 \geq |5x + 3|$

$(7x + 3)(-3x - 3) \geq 0$

$$\begin{array}{ccccc} - & & + & & - \\ & -1 & & -\frac{3}{7} & \end{array}$$

$\therefore$ domain $\left[-1,\frac{-3}{5}\right)\cup\left(\frac{-3}{5},\frac{-3}{7}\right]$

$\alpha = -1,\ \beta=\frac{-3}{5},\ \gamma=\frac{-3}{5},\ \delta=\frac{-3}{7}$

$3\alpha + 10(\beta + \gamma) + 21\delta = -3$

$$-3+10\left(\frac{-6}{5}\right)+\left(\frac{-3}{7}\right)21=-24$$

28. The sum of all the four-digit numbers that can be formed using all the digits 2, 1, 2, 3 is equal to _____ .

Official Ans. by NTA (26664)

Sol. **2, 1, 2, 3**

$$\_\ \_\ \_\ \ \underline{1}\ \ \frac{3!}{2!}=3$$

$$\_\ \_\ \_\ \ \underline{2}\ \ 3!=6$$

$$\_\ \_\ \_\ \ \underline{3}\ \ \frac{3!}{2!}=3$$

Sum of digits of unit place = 3 × 1 + 6 × 2 + 3 × 3 = 24

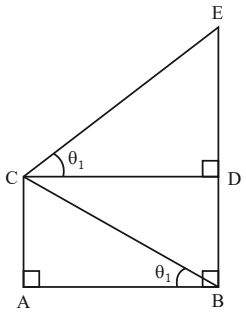
$\therefore$ required sum

= 24 ×1000 + 24 × 100 + 24 × 10 + 24 × 1

= 24 ×1111

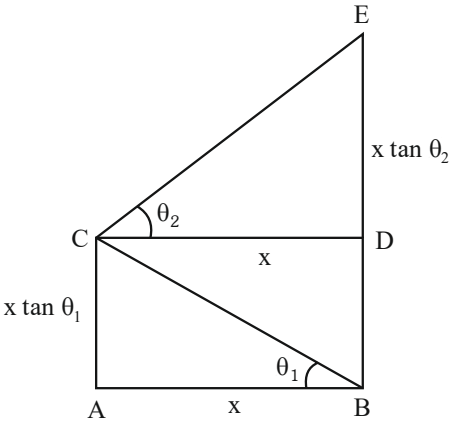
Ans ; 26664

29. In the figure, $\theta_1+\theta_2=\frac{\pi}{2}$ and $\sqrt{3}(BE)=4(AB)$. If the area of ΔCAB is $2\sqrt{3}-3$ unit^2, when $\frac{\theta_2}{\theta_1}$ is the largest, then the perimeter (in unit) of ΔCED is equal to _____.



Official Ans. by NTA (6)

Sol.



$\sqrt{3}$ BE = 4 AB

Ar (ΔCAB) = $2\sqrt{3} - 3$

$\frac{1}{2}x^2 \tan\theta_1 = 2\sqrt{3} - 3$

BE = BD + DE

= x ($\tan\theta_1 + \tan\theta_2$)

BE = AB ($\tan\theta_1 + \cot\theta_1$)

$\frac{4}{\sqrt{3}} \tan\theta_1 + \cot\theta_1 \Rightarrow \tan\theta_1 = \sqrt{3}, \frac{1}{\sqrt{3}}$

$\theta_1 = \frac{\pi}{6}$ $\theta_2 = \frac{\pi}{3}$

$\theta_1 = \frac{\pi}{3}$ $\theta_2 = \frac{\pi}{6}$

as $\frac{\theta_2}{\theta_1}$ is largest $\therefore$ $\theta_1 = \frac{\pi}{6}$ $\theta_2 = \frac{\pi}{3}$

$$\therefore x^2 = \frac{\left(2\sqrt{3}-3\right)\times 2}{\tan\theta_1} = \frac{\sqrt{3}\left(2-\sqrt{3}\right)\times 2}{\tan\frac{\pi}{6}}$$

$x^2 = 12 - 6\sqrt{3} = \left(3-\sqrt{3}\right)^2$

$x = 3 - \sqrt{3}$

Perimeter of ΔCED

= CD + DE + CE

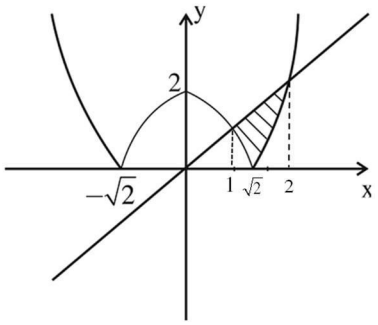
$= 3\sqrt{3} + \left(3-\sqrt{3}\right)\sqrt{3} + \left(3-\sqrt{3}\right)\times 2 = 6$

Ans : 6

30. If the area of the region $\{(x,y) : |x^2 - 2| \le y \le x\}$ is A, then $6A + 16\sqrt{2}$ is equal to _____ .

Official Ans. by NTA (27)

Sol. $|x^2 - 2| \le y \le x$



$$A = \int_1^{\sqrt{2}} (x - (2 - x^2))dx + \int_{\sqrt{2}}^{2} (x - (x^2 - 2))dx$$

$$= \left(1 - 2\sqrt{2} + \frac{2\sqrt{2}}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right) + \left(2 - \frac{8}{3} + 4\right) - \left(1 - \frac{2\sqrt{2}}{3} + 2\sqrt{2}\right)$$

$$= -4\sqrt{2} + \frac{4\sqrt{2}}{3} + \frac{7}{6} + \frac{10}{3} = \frac{-8\sqrt{2}}{3} + \frac{9}{2}$$

$6A = -16\sqrt{2} + 27$ $\therefore$ $6A + 16\sqrt{2} = 27$

Ans : 27

SECTION-A

31. A person travels x distance with velocity v_1 and then x distance with velocity v_2 in the same direction. The average velocity of the person is v, then the relation between v, v_1 and v_2 will be :

(1) $v = v_1 + v_2$

(2) $v = \frac{v_1 + v_2}{2}$

(3) $\frac{2}{v} = \frac{1}{v_1} + \frac{1}{v_2}$

(4) $\frac{1}{v} = \frac{1}{v_1} + \frac{1}{v_2}$

Official Ans. by NTA (3)

Sol. Average velocity $= \frac{x + x}{\frac{x}{v_1} + \frac{x}{v_2}} = v$

$$\frac{1}{v_1} + \frac{1}{v_2} = \frac{2}{v}$$

32. The half-life of a radioactive substance is T. The time taken, for disintegrating $\frac{7}{8}$th part of its original mass will be :

(1) 3T

(2) 8T

(3) T

(4) 2T

Official Ans. by NTA (1)

Sol. $t_{1/2} = T$

$$1 \xrightarrow[T]{} \frac{1}{2} \xrightarrow[T]{} \frac{1}{4} \xrightarrow[T]{} \frac{1}{8}$$

$t_{7/8} = 3T$

33. A gas mixture consists of 2 moles of oxygen and 4 moles of neon at temperature T. Neglecting all vibrational modes, the total internal energy of the system will be :

(1) 8 RT

(2) 16 RT

(3) 4 RT

(4) 11 RT

Official Ans. by NTA (4)

Sol. $(C_v)_{mix} = \frac{n_1 C v_1 + n_2 C v_2}{n_1 + n_2}$

$$(C_v)_{mix} = \frac{2 \times \frac{5}{2} R + 4 \times \frac{3}{2} R}{2 + 4} = \frac{11R}{6}$$

$$\Delta U = n(C_v)_{mix} RT = 6 \frac{11R}{6} \times RT = 11R$$

34. In an experiment with Vernier callipers of least count 0.1 mm, when two jaws are joined together the zero of Vernier scale lies right to the zero of the main scale and 6th division of Vernier scale coincides with the main scale division. While measuring the diameter of a spherical bob, the zero of vernier scale lies in between 3.2 cm and 3.3 cm marks, and 4th division of vernier scale coincides with the main scale division. The diameter of bob is measured as :

(1) 3.18 cm

(2) 3.25 cm

(3) 3.26 cm

(4) 3.22 cm

Official Ans. by NTA (1)

Sol. LC = 0.1mm

Zero Error = 6 × LC = 0.6 mm

Reading = MSR +VSR × LC – Zero Error

= [32mm + (0.1)4mm] – 0.6mm

= 31.8 mm

= 3.18 cm

35. Given below are two statements:

Statement I: For diamagnetic substance $-1 \le \chi < 0$, where χ is the magneticsusceptibility.

Statement II: Diamagnetic substances when placed in an external magnetic field, tend to move from stronger to weaker part of the field.

In the light of the above statements, choose the correct answer from the options given below.

(1) Both Statement I and Statement II are false.

(2) Both Statement I and Statement II are true.

(3) Statement I is incorrect but Statement II is true.

(4) Statement I is correct but Statement II is false.

Official Ans. by NTA (2)

Sol. Both Statements are correct.

36. The distance between two plates of a capacitor is d and its capacitance is C_1, when air is the medium between the plates. If a metal sheet of thickness $\frac{2d}{3}$ and of same area as plate is introduced between the plates, the capacitance of the capacitor becomes C_2. The ratio $\frac{C_2}{C_1}$ is:

(1) 2 : 1

(2) 4 : 1

(3) 3 : 1

(4) 1 : 1

Official Ans. by NTA (3)

Sol. $K_{\text{metal sheet}} = \infty$, $t = \frac{2d}{3}$

$$C_1 = \frac{\epsilon_0 A}{d}$$

$$C_2 = \frac{\epsilon_0 A}{d - t + \frac{t}{k}} = \frac{\epsilon_0 A}{d - \frac{2d}{3} + 0} = 3C_1$$

$$\frac{C_2}{C_1} = 3$$

37. Given below are two statements:

Statement I: Rotation of the earth shows effect on the value of acceleration due to gravity (g).

Statement II: The effect of rotation of the earth on the value of 'g' at the equator is minimum and that at the pole is maximum.

In the light of the above statements, choose the correct answer from the options given below.

(1) Statement I is false but Statement II is true.

(2) Statement I is true but Statement II are false.

(3) Both Statement I and Statement II are true.

(4) Both Statement I and Statement II are false.

Official Ans. by NTA (2)

Sol. Statement I is true due to centrifugal force.

Statement II is incorrect,

At pole $g = g_s$ (no effect)

At equator $g = g_s - r\omega^2\cos^2\lambda = g_s - r\omega^2$

$\therefore$ ($\cos^2\lambda_{\text{maximum}}$ at $\lambda = 0°$ i.e. at equator)

Effect is maximum at equator.

38. The time period of a satellite, revolving above earth's surface at a height equal to R will be

(Given $g = \pi^2$ m/s^2, R = radius of earth)

(1) $\sqrt{4R}$

(2) $\sqrt{8R}$

(3) $\sqrt{32R}$

(4) $\sqrt{2R}$

Official Ans. by NTA (3)

Sol. $\frac{mv^2}{2R} = \frac{GMm}{(2R)^2} \Rightarrow v = \sqrt{\frac{GM}{2R}} = \sqrt{\frac{Rg}{2}}$

$$T = \frac{2\pi(2R)}{v} = \frac{4\pi R\sqrt{2}}{\sqrt{Rg}} = \sqrt{32R}$$

39. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R.**
Assertion A: An electric fan continues to rotate for some time after the current is switched off.
Reason R: Fan continuous to rotate due to inertia of motion.
In the light of above statements, choose the most appropriate answer from the options given below.
(1) **A** is correct but **R** is not correct.
(2) Both **A** and **R** are correct and **R** is the correct explanation of **A**.
(3) **A** is not correct but **R** is correct.
(4) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**.
Official Ans. by NTA (2)

Sol. Fact

40. The amplitude of magnetic field in an electromagnetic wave propagating along y-axis is 6.0×10^{-7} T. The maximum value of electric field in the electromagnetic wave is:
(1) 5×10^{14} Vm^{-1}
(2) 180 Vm^{-1}
(3) 2×10^{15} Vm^{-1}
(4) 6.0×10^{-7} Vm^{-1}
Official Ans. by NTA (2)

Sol. $\frac{E}{B} = C$

$E = BC$

$= 6 \times 10^{-7} \times 3 \times 10^{8}$

$= 18 \times 10$

$E = 180$ Vm^{-1}

41. A gas is compressed adiabatically, which one of the following statement is NOT true.
(1) There is no heat supplied to the system
(2) The temperature of the gas increases
(3) The change in the internal energy is equal to the work done on the gas.
(4) There is no change in the internal energy
Official Ans. by NTA (4)

Sol. (1) $\Delta Q = 0$

(2) $\Delta Q = \Delta U + \Delta W$

$\Rightarrow \Delta U = -\Delta W$

adiabatic compression ($V\downarrow$)

$\Delta W = -ve \Rightarrow \Delta U = +ve$

$\Delta U \uparrow \Rightarrow T \uparrow$

$\Delta U \neq 0$

42. The ratio of intensities at two points P and Q on the screen in a Young's double slit experiment where phase difference between two wave of same amplitude are $\pi/3$ and $\pi/2$, respectively are
(1) 1 : 3 (2) 3 : 1
(3) 3 : 2 (4) 2 :3
Official Ans. by NTA (3)

Sol. $I_{net} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

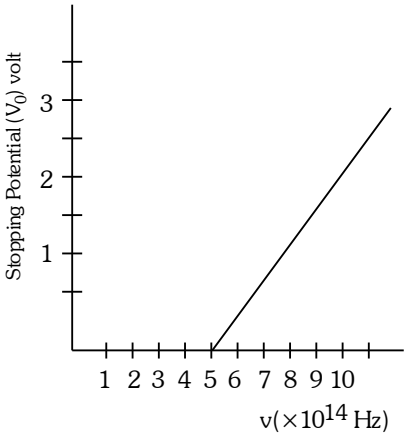
$= I_0 + I_0 + 2I_0 \cos\frac{\pi}{3}$

$= 2I_0 + 2I_0 \times \frac{1}{2} = 3I_0$

$I_{net} = I_0 + I_0 + 2I_0 \cos 90° = 2I_0$

Ratio $= \frac{3}{2}$

43. The variation of stopping potential (V_0) as a function of the frequency (ν) of the incident light for a metal is shown in figure. The work function of the surface is



(1) 18.6 eV (2) 2.98 eV
(3) 2.07 eV (4) 1.36 eV
Official Ans. by NTA (3)

Sol. $eV_0 = h\nu - \phi$

$0 = h\nu - \phi$

$\phi = h\nu$

$= 6.6 \times 10^{-34} \times 5 \times 10^{14}$

$= 33 \times 10^{-20}\ J$

$\phi = \dfrac{33 \times 10^{-20}}{1.6 \times 10^{-19}} = 2.07eV$

44. For a periodic motion represented by the equation

$Y = \sin \omega t + \cos \omega t$

The amplitude of the motion is

(1) 0.5 (2) $\sqrt{2}$

(3) 1 (4) 2

Official Ans. by NTA (2)

Sol. $y = \sin \omega t + \cos \omega t$

$y = \sin \omega t + \sin\left(\omega t + \dfrac{\pi}{2}\right)$

$\Delta\phi = \dfrac{\pi}{2}$

$A_{net} = \sqrt{1^2 + 1^2 + 2 \times 1 \times 1 \times \cos(\Delta\phi)}$

$A_{net} = \sqrt{2}$

45. In a metallic conductor, under the effect of applied electric field, the free electrons of the conductor

(1) drift from higher potential to lower potential.

(2) move in the curved paths from lower potential to higher potential

(3) move with the uniform velocity throughout from lower potential to higher potential

(4) move in the straight line paths in the same direction

Official Ans. by NTA (2)

Sol. Move in curve path

$i = neAV_d$

46. Young's moduli of the material of wires A and B are in the ratio of 1 : 4, while its area of cross sections are in the ratio of 1 : 3. If the same amount of load is applied to both the wires, the amount of elongation produced in the wires A and B will be in the ratio of

[Assume length of wires A and B are same]

(1) 36 : 1 (2) 12 : 1

(3) 1 : 36 (4) 1 : 12

Official Ans. by NTA (2)

Sol. $\Delta L = \dfrac{FL}{AY}$

$\dfrac{\Delta L_A}{\Delta L_B} = \dfrac{A_B}{A_A}\dfrac{Y_B}{Y_A} = 12$

47. Two projectiles are projected at 30° and 60° with the horizontal with the same speed. The ratio of the maximum height attained by the two projectiles respectively is:

(1) $2:\sqrt{3}$ (2) $\sqrt{3}:1$

(3) 1 : 3 (4) $1:\sqrt{3}$

Official Ans. by NTA (3)

Sol. $H_{max} = \dfrac{u^2 \sin^2\theta}{2g}$

$\dfrac{H_1}{H_2} = \dfrac{\sin^2\theta_1}{\sin^2\theta_2} = \dfrac{1}{3}$

48. A message signal of frequency 3kHz is used to modulate a carrier signal of frequency 1.5 MHz. The bandwidth of the amplitude modulated wave is

(1) 3 kHz (2) 6 MHz

(3) 3 MHz (4) 6 kHz

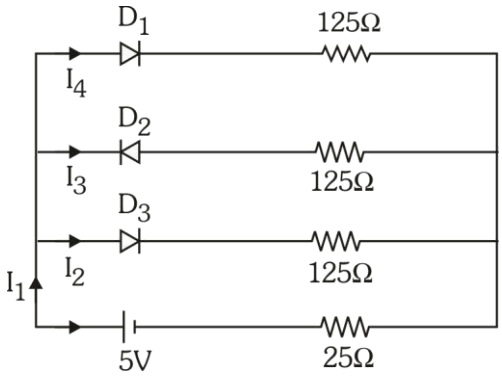
Official Ans. by NTA (4)

Sol. Bandwidth = $2f_m$

$= 2 \times 3$ kHz

$= 6$ kHz

49. If each diode has a forward bias resistance of 25 Ω in the below circuit,



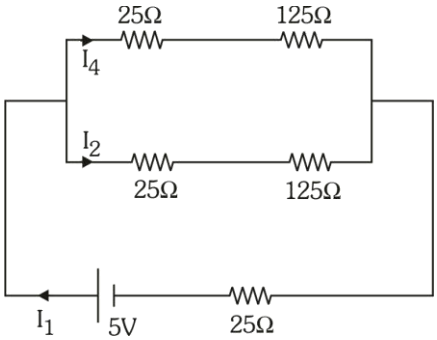
Which of the following options is correct:

(1) $\frac{I_3}{I_4}=1$ (2) $\frac{I_2}{I_3}=1$

(3) $\frac{I_1}{I_2}=1$ (4) $\frac{I_1}{I_2}=2$

Official Ans. by NTA (4)

Sol.



$R_{eq}=\frac{150\times150}{300}+25=100\Omega$

$I_1=\frac{5}{10}=0.05A$

$I_2=I_4=\frac{0.05}{2}=0.025A$

$\frac{I_1}{I_2}=2$

50. A bar magnet is released from rest along the axis of a very long vertical copper tube. After some time the magnet will

(1) Move down with almost constant speed

(2) Oscillate inside the tube

(3) Move down with an acceleration greater than g

(4) Move down with an acceleration equal to g

Official Ans. by NTA (1)

Sol. After some time both force becomes equal.

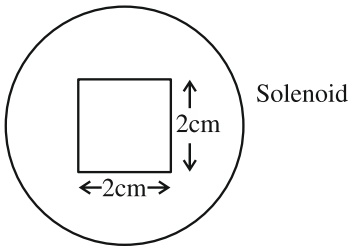
SECTION-B

51. A square loop of side 2.0 cm is placed inside a long solenoid that has 50 turns per centimetre and carries a sinusoidally varying current of amplitude 2.5 A and angular frequency 700 rad s^{-1}. The central axes of the loop and solenoid coincide. The amplitude of the emf induced in the loop is $x\times10^{-4}$ V. The value of x is_____

(Take, $\pi=\frac{22}{7}$)

Official Ans. by NTA (44)

Sol.



$B_{due\ to\ solenoid}=\mu_0 nI$

$\Phi_{through\ square}=\mu_0 nI\times A$ (A = Area)

$Emf=\mu_0 n A\times\frac{dI}{dt}$

$=\mu_0 n A\times I_0\,\omega\cos\omega t$

Emf amplitude $=\mu_0 n A\times I_0\omega$

$=4\pi\times10^{-7}\times\frac{50}{10^{-2}}\times4\times10^{-4}\times2.5\times700$

$=44\times10^{-4}$ V

52. A rectangular block of mass 5 kg attached to a horizontal spiral spring executes simple harmonic motion of amplitude 1 m and time period 3.14 s. The maximum force exerted by spring on block is_____N.

Official Ans. by NTA (20)

Sol. $\therefore T=3.14=\pi$

$T=\pi=\frac{2\pi}{\omega}\Rightarrow\omega=2$

$F_{max}=m\,a_{max}$

$=m(A\omega^2)$

$=mA(2)^2$

$=5\times1\times4$

$=20$ N

53. If 917 Å be the lowest wavelength of Lyman series then the lowest wavelength of Balmer series will be_____Å.

Official Ans. by NTA (3668)

Sol. For lowest wavelength of Lyman series

$$\frac{1}{\lambda} = RZ^2\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] = RZ^2$$

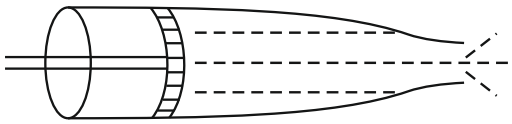
For lowest wavelength of Balmer series

$$\frac{1}{\lambda'} = RZ^2\left[\frac{1}{2^2} - \frac{1}{\infty^2}\right] = \frac{RZ^2}{4}$$

$$\lambda' = \frac{4}{RZ^2} = 4 \times 917$$

$$= 3668\text{Å}$$

54. Figure below shows a liquid being pushed out of the tube by a piston having area of cross section 2.0 cm^2. The area of cross section at the outlet is 10 mm^2. If the piston is pushed at a speed of 4 cm s^{-1}, the speed of outgoing fluidis_____cm s^{-1}.



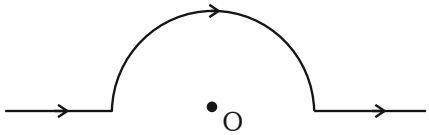
Official Ans. by NTA (80)

Sol. By equation of continuity

$A_1V_1 = A_2V_2$

$$V_2 = \frac{2 \times 4}{10 \times 10^{-2}} = 80\,\text{cm/s}$$

55. A straight wire carrying a current of 14 A is bent into a semicircular are of radius 2.2 cm as shown in the figure. The magnetic field produced by the current at the centre (O) of the arc. is_____$\times 10^{-4}$T

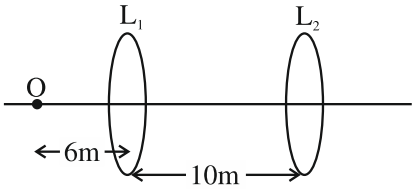


Official Ans. by NTA (2)

Sol. $$B_{\text{at O}} = \frac{\mu_0 I}{4R} = \frac{4\pi \times 10^{-7} \times 14}{4 \times 2.2 \times 10^{-2}}$$

$= 2 \times 10^{-4}$ T

56. A point object, 'O' is placed in front of two thin symmetrical coaxial convex lenses L_1 and L_2 with focal length 24 cm and 9 cm respectively. The distance between two lenses is 10 cm and the object is placed 6 cm away from lens L_1 as shown in the figure. The distance between the object and the image formed by the system of two lenses is_____cm.



Official Ans. by NTA (34)

From Ist lens $\frac{1}{v} + \frac{1}{6} = \frac{1}{24}$

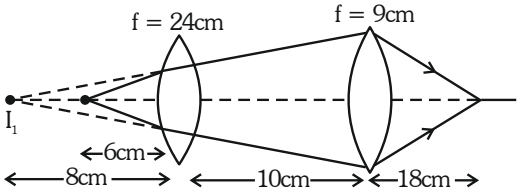
$$\frac{1}{v} = \frac{1}{24} - \frac{1}{6} = -\frac{1}{8}$$

$$v = -8\text{ cm}$$

From IInd lens $\frac{1}{v} + \frac{1}{18} = \frac{1}{9}$

$$\frac{1}{v} = \frac{1}{18}$$

$$v = 18$$



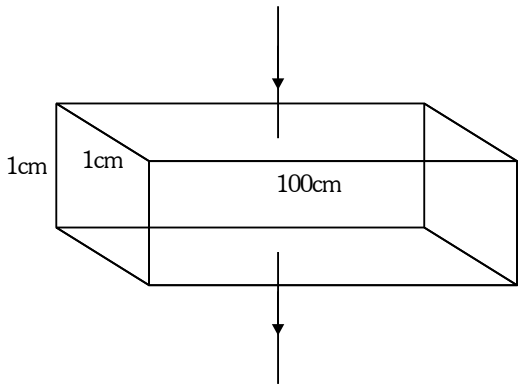
So distance between object and its image

= 6 + 10 + 18 = 34 cm

57. A rectangular parallelopiped is measured as 1 cm × 1 cm × 100 cm. If its specific resistance is 3×10^{-7} Ωm, then the resistance between its two opposite rectangular faces will be_____$\times^{-7}$ Ω.

Official Ans. by NTA (3)

Sol.



$$R = \rho\frac{\ell}{A} = \frac{3\times10^{-7}\times(1\times10^{-2})}{100\times1\times10^{-4}}$$

$= 3 \times 10^{-7}\ \Omega$

58. A force of $-P\hat{k}$ acts on the origin of the coordinate system. The torque about the point (2, –3) is $P\left(a\hat{i}+b\hat{j}\right)$, the ratio of $\frac{a}{b}$ is $\frac{x}{2}$. The value of x is

Official Ans. by NTA (3)

Sol. $\tau = \vec{r}\times\vec{F}$

Where $\vec{r} = -2\hat{i}+3\hat{j}$

$\tau = (-2\hat{i}+3\hat{j})\times(-P\hat{k})$

$= P(-2\hat{j}-3\hat{i}) = P\left(-3\hat{i}-2\hat{j}\right)$

$\Rightarrow$ So $a = -3, b = -2$

$\frac{a}{b} = \frac{3}{2}$

59. If the maximum load carried by an elevator is 1400 kg (600 kg – Passenger + 800 kg – elevator), which is moving up with a uniform speed of 3 ms^{-1} and the frictional force acting on it is 2000 N, then the maximum power used by the motor is_____kW ($g = 10\ m/s^2$).

Official Ans. by NTA (48)

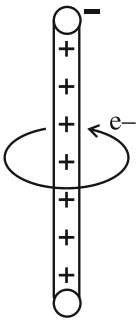
Sol. $P_{max} = F_{max}\times v$

$F_{max} = 1400\ g$ + friction

$= 14000 + 2000 = 16000$

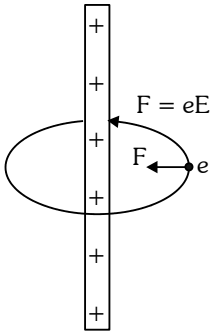
$P_{max} = 16000 \times 3 = 48000\ W = 48KW$

60. An electron revolves around an infinite cylindrical wire having uniform linear change density $2 \times 10^{-8}\ Cm^{-1}$ in circular path under the influence of attractive electrostatic field as shown in the figure. The velocity of electron with which it is revolving is_____$\times 10^6\ ms^{-1}$. Given mass of electron $= 9 \times 10^{-31}$ kg.



Official Ans. by NTA (8)

Sol.



$$eE = \frac{mV^2}{r}$$

$$e\cdot\frac{2K\lambda}{r} = \frac{mV^2}{r}$$

$$V = \sqrt{\frac{e\cdot2k\lambda}{m}}$$

$$= \sqrt{\frac{1.6\times10^{-19}\times2\times9\times10^{9}\times2\times10^{-8}}{9\times10^{-31}}}$$

$= 8 \times 10^6$ m/s

SECTION-A

61. Incorrect method of preparation for alcohols from the following is:
(1) Ozonolysis of alkene.
(2) Reaction of Ketone with RMgBr followed by hydrolysis.
(3) Hydroboration–oxidation of alkene.
(4) Reaction of alkyl halide with aqueous NaOH.
Official Ans. by NTA (1)

Sol. Ozonolysis of alkene, gives aldehyde, ketone & carboxylic acid.

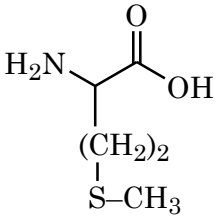
62. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R.**
Assertion A: The energy required to form Mg^{2+} from Mg is much higher than that required to produce Mg^{+}.
Reason R: Mg^{2+} is small ion and carry more charge than Mg^{+}.
In the light of the above statements, choose the **correct** answer from the options given below:
(1) Both A and R are true but R is NOT the correct explanation of A.
(2) A is true but R is false.
(3) A is false but R is true.
(4) Both A and R are true and R is the correct explanation of A.
Official Ans. by NTA (4)

Sol. Assertion & Reason are correct and Reason is correct explanation.
$\because$ Successive I.E. always increases.

63. In Carius tube, an organic compound 'X' is treated with sodium peroxide to form a mineral acid 'Y'. The solution of $BaCl_2$ is added to 'Y' to form a precipitate 'Z'. 'Z' is used for the quantitative estimation of an extra element. 'X' could be:
(1) Cytosine (2) Chloroxylenol
(3) A nucleotide (4) Methionine
Official Ans. by NTA (4)

Sol. $X \xrightarrow{Na_2O_2} Y \xrightarrow{BaCl_2} \underset{[BaSO_4]}{Z}$

Methionine: $C_5H_{11}NO_2S$



64. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R.**
Assertion A: 3.1500g of hydrated oxalic acid dissolved in water to make 250.0 mL solution will result in 0.1 M oxalic acid solution.
Reason R: Molar mass of hydrated oxalic acid is 126 g mol^{-1}.
In the light of the above statements, chose the correct answer from the options given below:
(1) Both A and R are true but R is NOT the correct explanation of A.
(2) A is false but R is true.
(3) A is true but R is false.
(4) Both A and R are true and R is the correct explanation of A.
Official Ans. by NTA (4)

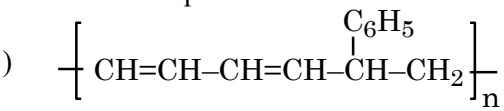
Sol. Assertion is correct.
$H_2C_2O_4.2H_2O$

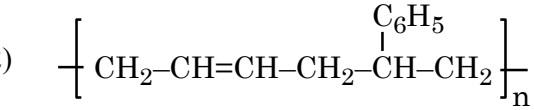
$$M = \frac{3.15 \times 1000}{126 \times 250}$$

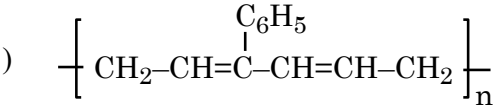
$$= \frac{12.6}{126} = 0.1$$

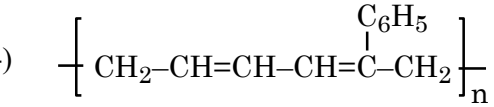
Reason is correct. It is used as a fact in explanation of assertion.

65. Buna–S can be represented as:

(1) 

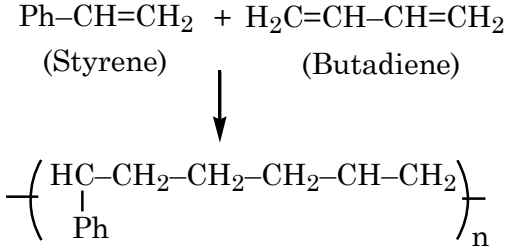
(2) 

(3) 

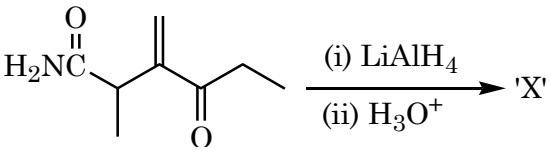
(4) 

Official Ans. by NTA (2)

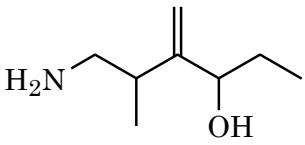
Sol.

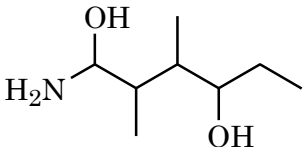


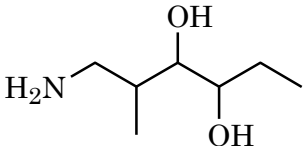
66. In the reaction give below:

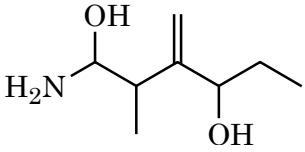


The product 'X' is:

(1) 

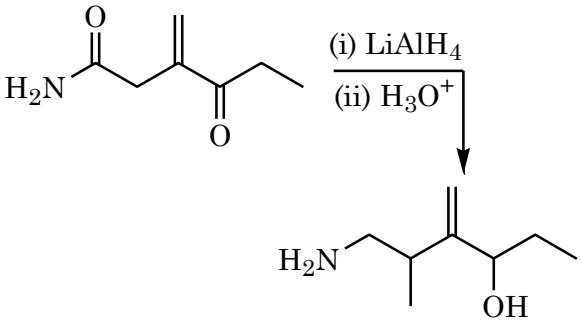
(2) 

(3) 

(4) 

Official Ans. by NTA (1)

Sol.



67. Ferric chloride is applied to stop bleeding because:

(1) Cl^- ions cause coagulation of blood.

(2) Blood absorbs $FeCl_3$ and forms a complex.

(3) Fe^{3+} ions coagulate blood which is a negatively charged sol.

(4) $FeCl_3$ reacts with the constituents of blood which is a positively charged sol.

Official Ans. by NTA (3)

Sol. Fe^{3+} coagulation negatively charged sol blood.

68. The reaction used for preparation of soap from fat is:

(1) reduction reaction

(2) alkaline hydrolysis reaction

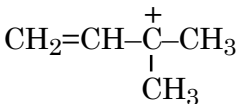
(3) an addition reaction

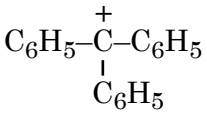
(4) an oxidation reaction

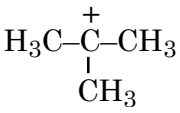
Official Ans. by NTA (2)

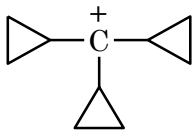
Sol. Saponification: Alkaline hydrolysis.

69. The decreasing order of hydride affinity for following carbocations is:

A. 

B. 

C. 

D. 

Choose the correct answer from the options given below:

(1) A, C, B, D

(2) C, A, B, D

(3) C, A, D, B

(4) A, C, D, B

Official Ans. by NTA (2)

Sol. Stability order of cations is : C < A < B < D

70. The correct relationship between unit cell edge length 'a' and radius of sphere 'r' for face–centred and body centred cubic structures respectively are:

(1) $r = 2\sqrt{2}a$ and $\sqrt{3}r = 4a$

(2) $r = 2\sqrt{2}a$ and $4r = \sqrt{3}a$

(3) $2\sqrt{2}r = a$ and $4r = \sqrt{3}a$

(4) $2\sqrt{2}r = a$ and $\sqrt{3}r = 4a$

Official Ans. by NTA (3)

Sol. FCC.

$a\sqrt{2} = 4r$

$r = \frac{a\sqrt{2}}{4}$

$\Rightarrow a = 2\sqrt{2}r$

BCC

$4r = a\sqrt{3}$

71. Number of water molecules in washing soda and soda ash respectively are:

(1) 10 and 1

(2) 1 and 10

(3) 1 and 0

(4) 10 and 0

Official Ans. by NTA (4)

Sol. Washing soda: $Na_2CO_3.10H_2O$

Soda ash : Na_2CO_3

72. The delicate balance of CO_2 and O_2 is NOT disturbed by:

(1) Burning of Coal (2) Deforestation

(3) Burning of petroleum (4) Respiration

Official Ans. by NTA (4)

Sol. Respiration, is a natural process, So balance of CO_2 and O_2 not disturbed by respiration.

73. The correct order of the number of unpaired electrons in the given complexes is

A. $[Fe(CN)_6]^{3-}$

B. $[FeF_6]^{3-}$

C. $[CoF_6]^{3-}$

D. $[Cr(oxalate)_3]^{3-}$

E. $[Ni(CO)_4]$

Choose the correct answer from the options given below:

(1) A < E < D < C < B (2) E < A < D < C < B

(3) E < A < B < D < C (4) A < E < C < B < D

Official Ans. by NTA (2)

Sol. A. $[Fe(CN)_6]^{3-}$ n = 1

B. $[FeF_6]^{3-}$ n = 5

C. $[CoF_6]^{3-}$ n = 4

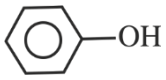
D. $[Cr(oxalate)_3]^{3-}$ n = 3

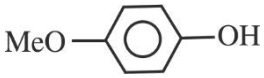
E. $[Ni(CO)_4]$ n = 0

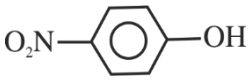
74. The correct order for acidity of the following hydroxyl compound is:

A. CH_3OH

B. $(CH_3)_3COH$

C. 

D. 

E. 

Choose the correct answer from the options given below:

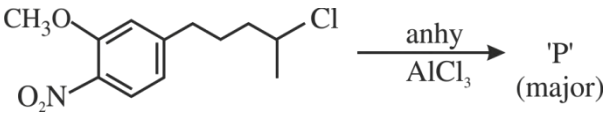
(1) E > C > D > A > B (2) D > E > C > A > B

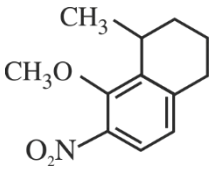
(3) C > E > D > B > A (4) E > D > C > B > A

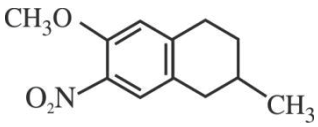
Official Ans. by NTA (1)

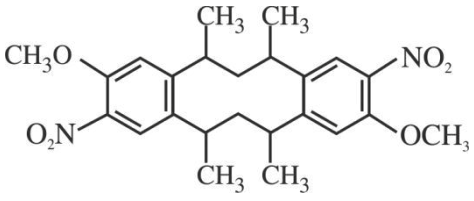
Sol. E > C > D > A > B

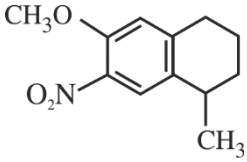
75. The major product 'P' formed in the given reaction is:



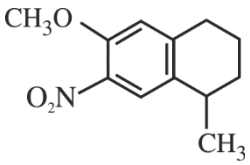
(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (4)

Sol. 

76. Match List I with List II

List I Complex		List II Crystal Field splitting energy (Δ_0)	
A.	$[Ti(H_2O)_6]^{2+}$	I.	–1.2
B.	$[V(H_2O)_6]^{2+}$	II.	–0.6
C.	$[Mn(H_2O)_6]^{3+}$	III.	0
D.	$[Fe(H_2O)_6]^{3+}$	IV	–0.8

Choose the correct answer from the options given below:

(1) A-II, B-IV, C-I, D-III

(2) A-IV, B-I, C-II, D-III

(3) A-IV, B-I, C-III, D-II

(4) A-II, B-IV, C-III, D-I

Official Ans. by NTA (2)

Sol. A-IV, B-I, C-II, D-III

(A) $[Ti(H_2O)_6]^{2+}$

$Ti^{2+} \Rightarrow 3d^2\, 4s^0$

$t_{2g}\, e^- = 2$

$e_g\, e^- = 0$

$CFSE = [-0.4 \times 2 + 0.6 \times 0]\Delta_0$

$= -0.8\, \Delta$

(B) $[V(H_2O)_6]^{2+}$

$V^{2+} \Rightarrow 3d^3\, 4s^0$

$t_{2g}\, e^- = 3$

$e_g\, e^- = 0$

$CFSE = [-0.4 \times 3 + 0.6 \times 0]\, \Delta_0$

$= -1.2\, \Delta_0$

(C) $[Mn(H_2O)_6]^{3+}$

$Mn^{3+} \Rightarrow 3d^4\, 4s^0$

$t_{2g}\, e^- = 3$

$e_g\, e^- = 1$

$CFSE = [-0.4 \times 3 + 0.6 \times 1]\, \Delta_0$

$= -0.6\, \Delta_0$

(D) $[Fe(H_2O)_6]^{3+}$

$Fe^{3+} \Rightarrow 3d^5\, 4s^0$

$t_{2g}\, e^- = 3 \quad e_g = 2$

$CFSE = [-0.4 \times 3 + 0.6 \times 2]\, \Delta_0$

$= 0\, \Delta_0$

77. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A: Physical properties of isotopes of hydrogen are different.

Reason: Mass difference between isotopes of hydrogen is very large.

In the light of the above statements, chose the correct answer from the options given below:

(1) A is false but R is true.

(2) Both A and R are true and R is the NOT the correct explanation of A.

(3) A is true but R is false.

(4) Both A and R are true and R is the correct explanation of A.

Official Ans. by NTA (4)

Sol. Both A and R are true and R is the correct explanation of A.

Due to mass difference in isotopes of hydrogen, these have different physical property.

78. Match List–I with List–II.

	List – I		List –II
A.	16g of $CH_4(g)$	I.	Weighs 28 g
B.	1g of $H_2(g)$	II.	60.2×10^{23} electrons
C.	1 mole of $N_2(g)$	III.	Weighs 32g
D.	0.5 mol of $SO_2(g)$	IV.	Occupies 11.4 L volume at STP

Choose the correct answer from the options given below:

(1) A–I, B–III, C–II, D–IV

(2) A–II, B–III, C–IV, D–I

(3) A–II, B–IV, C–III, D–I

(4) A–II, B–IV, C–I, D–III

Official Ans. by NTA (4)

Sol. 16g CH_4 = 1 mole CH_4 contains $10 \times 6.02 \times 10^{23}$ electrons

$= 60.2 \times 10^{23}$

1g H_2 = 0.5 mole H_2 gas occupy 11.35 litre volume at STP

1 mole of N_2 = 28g

0.5 mole of SO_2 = 32g

79. The correct order of metallic character is:

(1) Be > Ca > K (2) Ca > K > Be

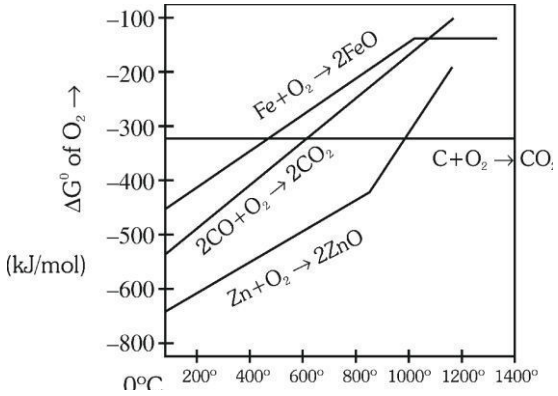
(3) K > Ca > Be (4) K > Be > Ca

Official Ans. by NTA (3)

Sol. On moving from top to bottom metallic character increases while on moving from left to right metallic decreases.

K > Ca > Be.

80. Gibbs energy vs T plot for the formation of oxides is given below:



For the given diagram, the correct statement is-

(1) At 600 ºC, C can reduce ZnO

(2) At 600 ºC, C can reduce FeO

(3) At 600 ºC, CO cannot reduce FeO

(4) At 600 ºC, CO can reduce ZnO

Official Ans. by NTA (2)

Sol. at 600ºC,

$$FeO + C \longrightarrow Fe + CO_2$$

SECTION-B

81. $A(g) \rightleftharpoons 2B(g) + C(g)$

For the given reaction, if the initial pressure is 450 mm Hg and the pressure at time t is 720 mm Hg at a constant temperature T and constant volume V. The fraction of A(g) decomposed under these conditions is $x \times 10^{-1}$. The value of x is _____ (nearest integer)

Official Ans. by NTA (3)

Sol.

$$A_{(g)} \rightleftharpoons 2B_{(g)} + C_{(g)}$$

t = 0	450			
time t	450 - x	2x	x	

$P_T = P_A + P_B + P_C$

$720 = 450 - x + 2x + x$

$2x = 270$

$x = 135$

Fraction of A decomposed = $\frac{135}{450} = 0.3 = 3 \times 10^{-1}$

So, x = 3

82. In alkaline medium, the reduction of permanganate anion involves a gain of _____ electrons.

Official Ans. by NTA (3)

Sol. In faintly alkaline medium,

$$MnO_4^- + 3e^- + 2H_2O \longrightarrow MnO_2 + 4OH^-$$

No. of electrons gained = 3

83. The number of endothermic process/es from the following is _____

A. $I_2(g) \rightarrow 2I(g)$

B. $HCl(g) \rightarrow H(g) + Cl(g)$

C. $H_2O(l) \rightarrow H_2O(g)$

D. $C(s) + O_2(g) \rightarrow CO_2(g)$

E. Dissolution of ammonium chloride in water

Official Ans. by NTA (4)

Sol. A → Endothermic (Atomisation)

B → Endothermic (Atomisation)

C → Endothermic (Vapourisation)

D → Exothermic (Combustion)

E → Endothermic (Dissolution)

84. The number of molecules from the following which contain only two lone pair of electrons is _____

H_2O, N_2, CO, XeF_4, NH_3, NO, CO_2, F_2

Official Ans. by NTA (4)

Sol. H_2O, CO, N_2, NO, has two lone pair of electrons.

85. The difference in the oxidation state of Xe between the oxidised product of Xe formed on complete hydrolysis of XeF_4 and XeF_4 is _____

Official Ans. by NTA (2)

Sol. $$6XeF_4 + 12H_2O \longrightarrow 2XeO_3 + 4Xe + 24HF + 3O_2$$

in XeO_3 , Oxidation state of Xe = +6

in XeF_4 , Oxidation state of Xe = +4

So difference in oxidation state = 2

86. An aqueous solution of volume 300 cm^3 contains 0.63 g of protein. The osmotic pressure of the solution at 300 K is 1.29 mbar. The molar mass of the protein is _____ g mol^{-1}

Given : R = 0.083 L bar $K^{-1}mol^{-1}$

Official Ans. by NTA (40535)

Sol. $\because \pi = CRT$

$$\pi = \frac{n}{V}RT$$

$$\pi = \frac{\omega}{V}\frac{RT}{M}$$

$$M = \frac{\omega RT}{\pi \times V}$$

$$M = \frac{0.63 \times 0.083 \times 300}{1.29 \times 10^{-3} \times 300 \times 10^{-3}}$$

M = 40535 gm/moL

87. For a metal ion, the calculated magnetic moment is 4.90 BM. This metal ion has _____ number of unpaired electons.

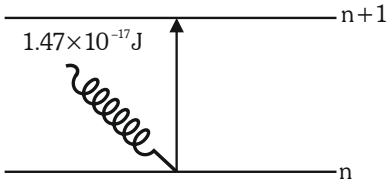
Official Ans. by NTA (4)

Sol. $\mu = \sqrt{n(n+2)}BM$

$$4.90 = \sqrt{n(n+2)}$$

n = 4

88.



The electron in the n^{th} orbit of Li^{2+} is excited to (n + 1) orbit using the radiation of energy 1.47×10^{-17}J (as shown in the diagram). The value of n is _____ .

Given $R_H = 2.18 \times 10^{-18}$J

Official Ans. by NTA (1)

Sol. $$\Delta E = R_H Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$1.47 \times 10^{-17} = 2.18 \times 10^{-18} \times 9\left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right)$$

$$\frac{1.47}{1.96} = \frac{3}{4} = \frac{1}{n^2} - \frac{1}{(n+1)^2}$$

So, n = 1

89. The specific conductance of 0.0025 M acetic acid is 5×10^{-5} S cm^{-1} at a certain temperature. The dissociation constant of acetic acid is _____ $\times 10^{-7}$. (Nearest integer)

Consider limiting molar conductivity of CH_3COOH as 400 S cm^2 mol^{-1}

Official Ans. by NTA (66)

Sol. $$\wedge_m = \frac{k}{C} \times 1000$$

Given $k = 5 \times 10^{-5}$ S cm^{-1}

C = 0.0025 M

$$\wedge_m = \frac{5 \times 10^{-5} \times 10^3}{0.0025} = \frac{5 \times 10^{-2}}{2.5 \times 10^{-3}}$$

$= 20$ S cm^2 mol^{-1}

$$\alpha = \frac{20}{400} = \frac{1}{20}$$

$$K_a = \frac{C\alpha^2}{1-\alpha} = \frac{0.0025 \times \frac{1}{20} \times \frac{1}{20}}{\frac{19}{20}}$$

$$= \frac{0.0025}{19 \times 20} = 6.6 \times 10^{-6}$$

$= 66 \times 10^{-7}$

90. The number of incorrect statement/s from the following is _____

A. The successive half lives of zero order reactions decreases with time.

B. A substance appearing as reactant in the chemical equation may not affect the rate of reaction

C. Order and molecularity of a chemical reaction can be a fractional number

D. The rate constant units of zero and second order reaction are mol L^{-1} s^{-1} and mol^{-1} Ls^{-1} respectively

Official Ans. by NTA (1)

Sol. (A) For zero order $t_{1/2} = \frac{[A]_0}{2K}$ as concentration decreases half life decreases (Correct statement)

(B) If order w.r.t. that reactant is zero then it will not affect rate of reaction. (Correct statement)

(C) Order can be fractional but molecularity can not be (Incorrect statement)

(D) For zero order reaction unit is mol $L^{-1}s^{-1}$ and for second order reaction unit is $mol^{-1}Ls^{-1}$ (Correct statement)

FINAL JEE–MAIN EXAMINATION – APRIL, 2023

(Held On Tuesday 11th April, 2023) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. The value of the integral $\int_{-\log_e 2}^{\log_e 2} e^x\left(\log_e\left(e^x+\sqrt{1+e^{2x}}\right)\right)dx$ is equal to

(1) $\log_e\left(\frac{2(2+\sqrt{5})}{\sqrt{1+\sqrt{5}}}\right)-\frac{\sqrt{5}}{2}$

(2) $\log_e\left(\frac{\sqrt{2}(3-\sqrt{5})^2}{\sqrt{1+\sqrt{5}}}\right)+\frac{\sqrt{5}}{2}$

(3) $\log_e\left(\frac{(2+\sqrt{5})^2}{\sqrt{1+\sqrt{5}}}\right)+\frac{\sqrt{5}}{2}$

(4) $\log_e\left(\frac{\sqrt{2}(2+\sqrt{5})^2}{\sqrt{1+\sqrt{5}}}\right)-\frac{\sqrt{5}}{2}$

Official Ans. by NTA (4)

Sol. $I=\int_{-\ln 2}^{\ln 2} e^x\left(\ln\left(e^x+\sqrt{1+e^{2x}}\right)\right)dx$

Put $e^x=t\Rightarrow e^x dx=dt$

$I=\int_{1/2}^{2}\ln\left(t+\sqrt{1+t^2}\right)dt$

Applying integration by parts.

$=\left[t\ln\left(t+\sqrt{1+t^2}\right)\right]_{\frac{1}{2}}^{2}-\int_{1/2}^{2}\frac{t}{t+\sqrt{1+t^2}}\left(1+\frac{2t}{2\sqrt{1+t^2}}\right)dt$

$=2\ln\left(2+\sqrt{5}\right)-\frac{1}{2}\ln\left(\frac{1+\sqrt{5}}{2}\right)-\int_{1/2}^{2}\frac{t}{\sqrt{1+t^2}}dt$

$=2\ln\left(2+\sqrt{5}\right)-\frac{1}{2}\ln\left(\frac{1+\sqrt{5}}{2}\right)-\frac{\sqrt{5}}{2}$

$=\ln\left(\frac{(2+\sqrt{5})^2}{\left(\frac{\sqrt{5}+1}{2}\right)^{\frac{1}{2}}}\right)-\frac{\sqrt{5}}{2}$

2. If equation of the plane that contains the point (–2,3,5) and is perpendicular to each of the planes $2x+4y+5z=8$ and $3x-2y+3z=5$ is $\alpha x+\beta y+\gamma z+97=0$ then $\alpha+\beta+\gamma=$

(1) 18

(2) 17

(3) 16

(4) 15

Official Ans. by NTA (4)

Sol. The equation of plane through (–2,3,5) is

a(x+2) + b(y-3) + c (z-5) = 0

it is perpendicular to 2x+4y+5z=8 & 3x-2y+3z=5

$\therefore\quad 2a+4b+5c=0$

$3a-2b+3c=0$

$\therefore\quad \frac{a}{\begin{vmatrix}4&5\\-2&3\end{vmatrix}}=\frac{-b}{\begin{vmatrix}2&5\\3&3\end{vmatrix}}=\frac{c}{\begin{vmatrix}2&4\\3&-2\end{vmatrix}}$

$\Rightarrow\quad \frac{a}{22}=\frac{b}{9}=\frac{c}{-16}$

$\therefore$ Equation of Plane is

$22(x+2)+9(y-3)-16(z-5)=0$

$\Rightarrow\quad 22x+9y-16z+97=0$

Comparing with $\alpha x+\beta y+\gamma x+97=0$

We get $\alpha+\beta+\gamma=22+9-16=15$

3. Let R be a rectangle given by the lines x = 0, x = 2, y = 0 and y = 5. Let $A(\alpha,0)$ and $B(0,\beta), \alpha \in [0,2]$ and $\beta \in [0,5]$, be such that the line segment AB divides the area of the rectangle R in the ratio 4:1. Then, the mid-point of AB lies on a

(1) parabola

(2) hyberbola

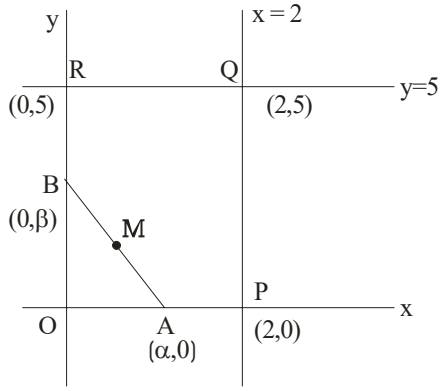
(3) straight line

(4) circle

Official Ans. by NTA (2)

Sol. $\dfrac{ar(OPQR)}{or(OAB)} = \dfrac{4}{1}$

Let M be the mid-point of AB.



$$M(h,k) \equiv \left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$$

$$\Rightarrow \quad \frac{10 - \frac{1}{2}\alpha\beta}{\frac{1}{2}\alpha\beta} = 4$$

$$\Rightarrow \quad \frac{5}{2}\alpha\beta = 10 \Rightarrow \alpha\beta = 4$$

$$\Rightarrow \quad (2h)(2K) = 4$$

$\therefore$ Locus of M is xy = 1

Which is a hyperbola.

4. Let sets A and B have 5 elements each. Let the mean of the elements in sets A and B be 5 and 8 respectively and the variance of the elements in sets A and B be 12 and 20 respectively A new set C of 10 elements is formed by subtracting 3 from each element of A and adding 2 to each element of B. Then the sum of the mean and variance of the elements of C is _____.

(1) 32

(2) 38

(3) 40

(4) 36

Official Ans. by NTA (2)

Sol. $\omega \quad A = \{a_1, a_2, a_3, a_4, a_5\}$

$B = \{b_1, b_2, b_3, b_4, b_5\}$

Given, $\sum_{i=1}^{5} a_i = 25$, $\sum_{i=1}^{5} b_i = 40$

$$\frac{\sum_{i=1}^{5} a_i^2}{5} - \left(\frac{\sum_{i=1}^{5} a_i}{5}\right)^2 = 12, \quad \frac{\sum_{i=1}^{5} b_i^2}{5} - \left(\frac{\sum_{i=1}^{5} b_i}{5}\right)^2 = 20$$

$$\Rightarrow \quad \sum_{i=1}^{5} a_i^2 = 185 \quad , \qquad \sum_{i=1}^{5} b_i^2 = 420$$

Now, $C = \{C_1, C_2, ...C_{10}\}$

s.f. $C_i = \underset{\text{First five elements}}{a_i} = 3 \quad \text{or} \quad \underset{\text{Last five elements}}{b_i} + 2$

$\therefore$ Mean of C, $\bar{C} = \dfrac{\left(\sum a_i - 15\right) + \left(\sum b_i + 10\right)}{10}$

$$\bar{C} = \frac{10 + 50}{10} = 6$$

$\therefore$ $\sigma^2 = \dfrac{\sum_{i=1}^{10} C_i^2}{10} = \left(\bar{C}\right)^2$

$$= \frac{\sum (a_i - 3)^2 + \sum (b_i + 2)^2}{10} - (6)^2$$

$$= \frac{\sum a_i^2 + \sum b_i^2 - 6\sum a_i + 4\sum b_i + 65}{10} - 36$$

$$= \frac{185 + 420 - 150 + 160 + 65}{10} - 36$$

= 32

$\therefore$ Mean + Variance $= \bar{C} + \sigma^2 = 6 + 32 = 38$

5. Let $f(x) = x^2 - x + |-x + [x]|$, where $x \in \mathbb{R}$ and [t] denotes the greatest integer less than or equal to t. Then, f is

(1) continuous at x = 0, but not continuous at x = 1

(2) continuous at x = 0 and x = 1

(3) not continuous at x = 0 and x = 1

(4) continuous at x = 1, but not continuous at x = 0

Official Ans. by NTA (4)

Sol. Here $f(x) = [x(x-1)] + \{x\}$

$f(o^+) = -1 + 0 = -1$	$f(1^+) = 0 + 0 = 0$
$f(o) = 0$	$f(1) = 0$
	$f(1^-) = -1 + 1 = 0$

$\therefore$ f(x) is continuous at x = 1, discontinuous at x = 0

6. The number of triplets (x, y, z). where x, y, z are distinct non negative integers satisfying x + y + z = 15, is

(1) 80

(2) 114

(3) 92

(4) 136

Official Ans. by NTA (2)

Sol. x + y + z = 15

Total no. of solution = $^{15+3-1}C_{3-1} = 136$ ...(1)

Let $x = y \neq z$

$2x + z = 15 \Rightarrow z = 15 - 2t$

$\Rightarrow r \in \{0,1,2,...7\} - \{5\}$

$\therefore$ 7 solutions

$\therefore$ there are 21 solutions in which exactly

Two of x_1 y_1 z are equal ...(2)

There is one solution in which x=y=z ...(3)

Required answer = 136–21–1 = 114

7. For any vector $a = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, with $10|a_i| < 1, i = 1,2,3$, consider the following statements:

(A) : $\max\{|a_1|,|a_2|,|a_3|\} \le |\vec{a}|$

(B) : $|\vec{a}| \le 3\max\{|a_1|,|a_2|,|a_3|\}$

(1) Only (B) is true

(2) Only (A) is true

(3) Neither (A) nor (B) is true

(4) Both (A) and (B) are true

Official Ans. by NTA (4)

Sol. Without loss of generality

Let $|a_1| \le |a_2| \le |a_3|$

$|\vec{a}|^2 = |a_1|^2 + |a_2|^2 + |a_3|^2 \ge (a_3)^2$

$\Rightarrow |\vec{a}| \ge |a_3| = \max\{|a_1|,|a_2|,|a_3|\}$

A is true

$|\vec{a}|^2 = |a_1|^2 + |a_2|^2 + |a_3|^2 \le |a_3|^2 + |a_3|^2 + |a_3|^2$

$\Rightarrow |\vec{a}|^2 \le 3|a_3|^2$

$\Rightarrow |\vec{a}| \le \sqrt{3}|a_3| = \sqrt{3}\max\{|a_1|,|a_2|,|a_3|\}$

$\le 3\max\{|a_1|,|a_2|,|a_3|\}$

(2) is true

8. Let w_1 be the point obtained by the rotation of $z_1 = 5 + 4i$ about the origin through a right angle in the anticlockwise direction, and w_2 be the point obtained by the rotation of $z_2 = 3 + 5i$ about the origin through a right angle in the clockwise direction. Then the principal argument of $w_1 - w_2$ is equal to

(1) $-\pi + \tan^{-1}\frac{33}{5}$

(2) $-\pi - \tan^{-1}\frac{33}{5}$

(3) $-\pi + \tan^{-1}\frac{8}{9}$

(4) $\pi - \tan^{-1}\frac{8}{9}$

Official Ans. by NTA (4)

Sol. $W_1 = z_i i = (5+4i)i = -4+5i$ ...(i)

$W_2 = z_2(-i) = (3+5i)(-i) = 5-3i$ ...(2)

$W_1 - W_2 = -9+8i$

Principal argument = $\pi - \tan^{-1}\left(\frac{8}{9}\right)$

9. An organization awarded 48 medals in event 'A', 25 in event 'B' and 18 in event 'C'. If these medals went to total 60 men and only five men got medals in all the three events, then, how many received medals in exactly two of three events?

(1) 10

(2) 9

(3) 21

(4) 15

Official Ans. by NTA (3)

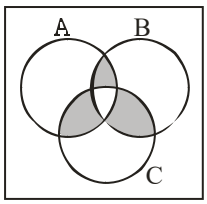
Sol. $|A| = 48$

$|B| = 25$

$|C| = 18$

$|A \cup B \cup C| = 60$ [Total]

$|A \cap B \cap C| = 5$



$|A \cup B \cup C| = \sum|A| - \sum|A \cap B| + |A \cap B \cap C|$

$\Rightarrow \sum|A \cap B| = 48+25+18+5-60$

$= 36$

No. of men who received exactly 2 medals

$= \sum|A \cap B| - 3|A \cap B \cap C|$

$= 36 - 15$

$= 21$

10. Let $S = \{M = [a_{ij}], a_{ij} \in \{0,1,2\}, 1 \le i, j \le 2\}$ be a sample space and $A = \{M \in S : M \text{ is invertible}\}$ be an event. Then P(A) is equal to

(1) $\frac{50}{81}$

(2) $\frac{47}{81}$

(3) $\frac{49}{81}$

(4) $\frac{16}{27}$

Official Ans. by NTA (1)

Sol. $M\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where a, b, c, d, $\in \{0,1,2\}$

n(s) = $3^4 = 81$

we first bound $p(\bar{A})$

$|m| = 0 \Rightarrow ad = bc$

$ad = bc = 0 \Rightarrow$ no. of (a,b,c,d) = $(3^2-2^2)^2 = 25$

$ad = bc = 1 \Rightarrow$ no. of (a,b,c,d) = $1^2 = 1$

$ad = bc = 2 \Rightarrow$ no. of (a,b,c,d) = $2^2 = 4$

$ad = bc = 4 \Rightarrow$ no. of (a,b,c,d) = $1^2 = 1$

$\therefore P(\bar{A}) = \frac{31}{81} \Rightarrow p(A) = \frac{50}{81}$

11. Consider ellipses $E_k : kx^2 + k^2y^2 = 1, k = 1, 2, \ldots, 20$. Let C_k be the circle which touches the four chords joining the end points (one on minor axis and another on major axis) of the ellipse E_k, If r_k is the radius of the circle C_k, then the value of $\sum_{k=1}^{20} \frac{1}{r_k^2}$ is

(1) 3080

(2) 3210

(3) 3320

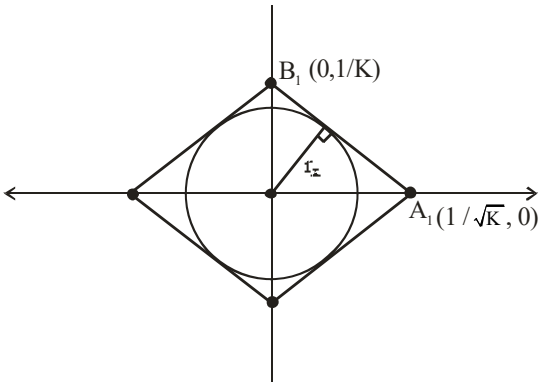
(4) 2870

Official Ans. by NTA (1)

Sol. $Kx^2+K^2y^2=1$

$$\frac{x^2}{1/K}+\frac{y^2}{1/K^2}=1$$

Now



Equation of

$$A_1B_2;\frac{x}{1/\sqrt{K}}+\frac{y}{1/K}=1\Rightarrow\sqrt{K}x+Ky=1$$

$r_K=\perp r$ distance of (0,0) from line A_1B_1

$$r_k=\left|\frac{(0+0-1)}{\sqrt{K+K^2}}\right|=\frac{1}{\sqrt{K+K^2}}$$

$$\frac{1}{r_K^2}=K+K^2\Rightarrow\sum_{k=1}^{20}\frac{1}{r_K^2}=\sum_{K=1}^{20}\left(K+K^2\right)$$

$$=\sum_{K=1}^{20}K+\sum_{K=1}^{20}K^2$$

$$=\frac{20\times21}{2}+\frac{20.21.41}{6}$$

$$=210+10\times7\times41$$

$$=210+2870$$

$$=3080$$

12. The number of integral solutions x of $\log_{\left(x+\frac{7}{2}\right)}\left(\frac{x-7}{2x-3}\right)^2\geq0$ is

(1) 6 (2) 8

(3) 5 (4) 7

Official Ans. by NTA (1)

Sol. $\log_{x+\frac{7}{2}}\left(\frac{x-7}{2x-3}\right)^2\geq0$

Feasible region : $x+\frac{7}{2}>0\Rightarrow x>-\frac{7}{2}$

And $x+\frac{7}{2}\neq1\Rightarrow x\neq-\frac{5}{2}$

And $\frac{x-7}{2x-3}\neq0$ and $2x-3\neq0$

$\Downarrow$ $\Downarrow$

$x\neq7$ $x\neq\frac{3}{2}$

Taking intersection : $x\in\left(\frac{-7}{2},\infty\right)-\left\{-\frac{5}{2},\frac{3}{2},7\right\}$

Now $\log_a b\geq0$ if $a>1$ and $b\geq1$

Or

$a\in(0,1)$ and $b\in(0,1)$

C – I; $x+\frac{7}{2}>1$ and $\left(\frac{x-7}{2x-3}\right)^2\geq1$

$x>-\frac{5}{2}$ $(2x-3)^2-(x-7)^2\leq0$

$(2x-3+n-7)(2x-3-x+7)\leq0$

$(3x-10)(x+4)\leq0$

$x\in\left[-4,\frac{10}{3}\right]$

Intersection : $x\in\left(\frac{-5}{2},\frac{10}{3}\right]$

C-II $x+\frac{7}{2}\in(0,1)$ and $\left(\frac{x-7}{2x-3}\right)^2\in(0,1)$

$0<x+\frac{7}{2}<1$ $\left(\frac{x-7}{2x-3}\right)^2<1$

$-\frac{7}{2}<x<\frac{-5}{2}$ $(x-7)^2<(2x-3)^2$

$x\in(-\infty,-4)\cup\left(\frac{10}{3},\infty\right)$

No common values of x.

Hence intersection with feasible region

We get $x\in\left(\frac{-5}{2},\frac{10}{3}\right]-\left\{\frac{3}{2}\right\}$

Integral value of x are {–2,–1,0,1,2,3}

No. of integral values = 6

13. Area of the region $\{(x,y): x^2+(y-2)^2 \le 4, x^2 \ge 2y\}$ is

(1) $2\pi - \frac{16}{3}$ (2) $\pi - \frac{8}{3}$

(3) $\pi + \frac{8}{3}$ (4) $2\pi + \frac{16}{3}$

Official Ans. by NTA (1)

Sol. $x^2+(y-2)^2 \le 2^2$ and $x^2 \ge 2y$

Solving circle and parabola simultaneously :

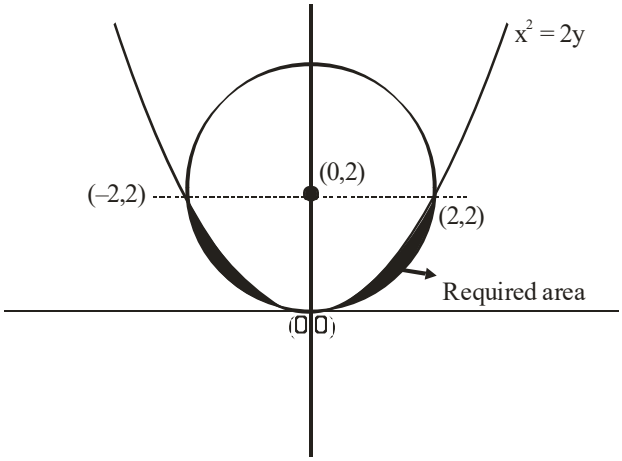
$2y + y^2 - 4y + 4 = 4$

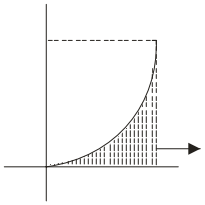
$y^2 - 2y = 0$

$y = 0, 2$

Put y = 2 in $x^2 = 2y \rightarrow x = \pm 2$

$\Rightarrow$ (2,2) and (–2,2)





$= 2\times 2 - \frac{1}{4}\cdot\pi\cdot 2^2 = 4-\pi$

Required area $= 2\left[\int_0^2 \frac{x^2}{2}dx - (4-\pi)\right]$

$= 2\left[\frac{x^3}{6}\Big|_0^2 - 4 + \pi\right]$

$= 2\left[\frac{4}{3} + \pi - 4\right]$

$= 2\left[\pi - \frac{8}{3}\right]$

$= 2\pi - \frac{16}{6}$

14. Let $f:[2,4] \to \mathbb{R}$ be a differentiable function such that $(x\log_e x)f'(x) + (\log_e x)f(x) + f(x) \ge 1$, $x \in [2,4]$ with $f(2) = \frac{1}{2}$ and $f(4) = \frac{1}{4}$.

Consider the following two statements:

(A) : $f(x) \le 1$, for all $x \in [2,4]$

(B) : $f(x) \ge \frac{1}{8}$, for all $x \in [2,4]$

Then,

(1) Only statement (B) is true

(2) Neither statement (A) nor statement (B) is true

(3) Both the statement (A) and (B) are true

(4) Only statement (A) is true

Official Ans. by NTA (3)

Sol. $x\ell nxf'(x) + \ln x\, f(x) + f(x) \ge I, x \in [2,4]$

And $f(2) = \frac{1}{2}, f(4) = \frac{1}{4}$

Now $x\ln x\frac{dy}{dx} + (\ln+1)y \ge 1$

$\frac{d}{dx}(y\cdot x\ln x) \ge 1$

$\frac{d}{dx}(f(x).x\ln x) \ge 1$

$\Rightarrow \frac{d}{dx}(x\ln x\, f(x) - x) \ge 0,\ x \in [2,4]$

$\Rightarrow$ The function g(x) = $x\ln x\, f(x) - x$ is increasing in [2.4]

And $g(2) = 2\ln 2\, f(2) - 2 = \ln 2 - 2$

$g(4) = 4\ln 4\, f(4) - 4 = \ln 4 - 4$

$= 2(\ln 2 - 2)$

Now $g(2) \le g(x) \le g(4)$

$\ln 2 - 2 \le x\ln x\, f(x) - x \le 2(\ln 2 - 2)$

$\frac{\ln 2 - 2}{x\ln x} + \frac{1}{\ln x} \le f(x) \le \frac{2(\ln 2-2)}{x\ln x} + \frac{1}{\ln x}$

Now for $x \in 2,4$

$$\frac{2(\ell n 2-2)}{x \ln x}+\frac{1}{\ell n x}<\frac{2(\ln 2-2)}{2 \ln 2}+\frac{1}{\ln 2}=1-\frac{1}{\ln 2}<1$$

$$\Rightarrow f(x) \leq 1 \text{ for } x \in [2,4]$$

Also for $x \in [2,4]$:

$$\frac{\ln 2-2}{x \ln x}+\frac{1}{\ln x} \geq \frac{\ln 2-2}{4 \ln 4}+\frac{1}{\ln 4}=\frac{1}{8}+\frac{1}{2 \ln 2}>\frac{1}{8}$$

$$\Rightarrow f(x) \geq \frac{1}{8} \text{ for } x \in [2,4]$$

Hence both A and B are true.

LMVT on (yx (lnx)) not satisfied.

Hence no such function exists.

Therefore it should be bonus.

15. Let y = y(x) be a solution curve of the differential equation, $(1-x^2y^2)dx = ydx + xdy$.

If the line x = 1 intersects the curve y = y(x) at y = 2 and the line x = 2 intersects the curve y = y(x) at $y = \alpha$, then a value of α is

(1) $\frac{3e^2}{2(3e^2-1)}$

(2) $\frac{3e^2}{2(3e^2+1)}$

(3) $\frac{1-3e^2}{2(3e^2+1)}$

(4) $\frac{1+3e^2}{2(3e^2-1)}$

Official Ans. by NTA (4)

Sol. $(1-x^2y^2)dx = ydx + x\,dy, y(1) = 2$

$$y(2) = \propto = ?$$

$$dx = \frac{d(xy)}{1-(xy)^2}$$

$$\int dx = \int \frac{d\ xy}{1-(xy)^2}$$

$$x = \frac{1}{2}\ln\left|\frac{1+xy}{1-xy}\right| + C$$

Put x = 1 and y = 2 :

$$1 = \frac{1}{2}\ln\left|\frac{1+2}{1-2}\right| + C$$

$$C = 1 - \frac{1}{2}\ln 3$$

Now put x = 2 :

$$2 = \frac{1}{2}\ln\left|\frac{1+2\alpha}{1-2\alpha}\right| + 1 - \frac{1}{2}\ln 3$$

$$1 + \frac{1}{2}\ln 3 = \frac{1}{2}\ln\left|\frac{1+2\alpha}{1-2\alpha}\right|$$

$$2 + \ln 3 = \ln\left(\frac{1+2\alpha}{1-2\alpha}\right)$$

$$\left|\frac{1+2\alpha}{1-2\alpha}\right| = 3e^2$$

$$\frac{1+2\alpha}{1-2\alpha} = 3e^2 \ , \ -3e^2$$

$$\frac{1+2\alpha}{1-2\alpha} = 3e^2 \Rightarrow \alpha = \frac{3e^2-1}{2(3e^2+1)}$$

And $\frac{1+2\alpha}{1-2\alpha} = -3e^2 \Rightarrow \alpha = \frac{3e^2+1}{2(3e^2-1)}$

16. Let A be a 2 × 2 matrix with real entries such that $A' = \alpha A + I$, where $\alpha \in \mathbb{R} - \{-1,1\}$. If det $(A^2 - A) = 4$, then the sum of all possible values of α is equal to

(1) 0 (2) $\frac{3}{2}$

(3) $\frac{5}{2}$ (4) 2

Official Ans. by NTA (3)

Sol. $A^T = \alpha A + I$

$A = \alpha A^T + I$

$A = \alpha(\alpha A + I) + I$

$A = \alpha^2 A + (\alpha + 1)I$

$A(1-\alpha^2) = (\alpha+1)I$

$A = \dfrac{I}{1-\alpha}$ ...(1)

$|A| = \dfrac{1}{(1-\alpha)^2}$ ...(2)

$|A^2 - A| = |A||A - I|$ ...(3)

$A - I = \dfrac{I}{1-\alpha} - I = \dfrac{\alpha}{1-\alpha} I$

$|A - I| = \left(\dfrac{\alpha}{1-\alpha}\right)^2$ ...(4)

Now $|A^2 - A| = 4$

$|A||A - I| = 4$

$\Rightarrow \dfrac{1}{(1-\alpha)^2} \dfrac{\alpha^2}{(1-\alpha)^2} = 4$

$\Rightarrow \dfrac{\alpha}{(1-\alpha)^2} = \pm 2$

$\Rightarrow 2(1-\alpha)^2 = \pm\alpha$

$(C_1)\ 2(1-\alpha)^2 = \alpha$	$(C_2)\ 2(1-\alpha)^3 = -\alpha$
$2\alpha^2 - 5\alpha + 2 = 0 < \alpha_1 \alpha_2$	$2\alpha^2 - 3\alpha + 2 = 0$
$\alpha_1 + \alpha_2 = \dfrac{5}{2}$	$\alpha \notin R$

Sum of value of $\alpha = \dfrac{5}{2}$

17. Let (α,β,γ) be the image of the point P(2, 3, 5) in the plane 2x + y – 3z = 6. Then $\alpha+\beta+\gamma$ is equal to

(1) 10

(2) 5

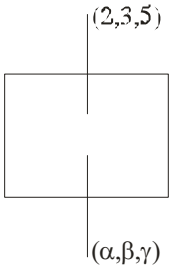
(3) 12

(4) 9

Official Ans. by NTA (1)

Sol. $\dfrac{\alpha-2}{2} = \dfrac{\beta-3}{1} = \dfrac{\gamma-5}{-3} = -2\left(\dfrac{2\times2+3-3\times5-6}{2^2+1^2+1-3^2}\right) = 2$

$\dfrac{\alpha-2}{2} = 2$ $\alpha = 6$	$\beta - 3 = 2$ $\beta = 5$	$\gamma - 5 = -6$ $\gamma = -1$



$\alpha+\beta+\gamma = 10$

18. Let $\vec{a}$ be a non-zero vector parallel to the line of intersection of the two planes described by $\hat{i}+\hat{j}, \hat{i}+\hat{k}$ *and* $\hat{i}-\hat{j}, \hat{j}-\hat{k}$. If θ is the angle between the vector $\vec{a}$ and the vector $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{a}\,.\,\vec{b} = 6$ then the ordered pair $\left(\theta, \left|\vec{a}\times\vec{b}\right|\right)$ is equal to

(1) $\left(\dfrac{\pi}{4}, 3\sqrt{6}\right)$

(2) $\left(\dfrac{\pi}{3}, 3\sqrt{6}\right)$

(3) $\left(\dfrac{\pi}{3}, 6\right)$

(4) $\left(\dfrac{\pi}{4}, 6\right)$

Official Ans. by NTA (4)

Sol. n_1 and n_2 are normal vector to the plane $\hat{i}+\hat{j}, \hat{i}+\hat{k}$ and $\hat{i}-\hat{j}; \hat{j}-\hat{k}$ respectively

$$\vec{n}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{j} \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = \hat{i} - \hat{j} - \hat{k}$$

$$\vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{j} \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix} = \hat{i} + \hat{j} + \hat{k}$$

$\vec{a} = \lambda\left|\vec{n}_2 \times \vec{n}_2\right|$

$$= \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -1 \\ 1 & 1 & 1 \end{vmatrix} = \lambda\left(-2\hat{j}+2\hat{k}\right)$$

$\vec{a}\cdot\vec{b} = \lambda|0+4+2| = 6$

$\Rightarrow \lambda = 1$

$\vec{\alpha} = -2\hat{j}+2\hat{k}$

$\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|a||b|}$

$\cos\theta = \dfrac{6}{2\sqrt{2}\times 3} = \dfrac{1}{\sqrt{2}}$

$\theta = \dfrac{\pi}{4}$

Now $\quad |\vec{a}.\vec{b}|^2 + |\vec{a}\times\vec{b}|^2 = |a|^2|b|^2$

$36 + |\vec{a}\times b^2| = 8\times 9 = 72$

$|\vec{a}\times b|^2 = 36$

$|\vec{a}\times\vec{b}| = 6$

19. The number of elements in the set $S = \{\theta\in[0,2\pi]: 3\cos^4\theta - 5\cos^2\theta - 2\sin^2\theta + 2 = 0\}$ is

(1) 10 (2) 8

(3) 9 (4) 12

Official Ans. by NTA (3)

Sol. $3\cos^4\theta - 5\cos^2\theta - 2\sin^6\theta + 2 = 0$

$\Rightarrow \quad 3\cos^4\theta - 3\cos^2\theta - 2\cos^2\theta - 2\sin^6\theta + 2 = 0$

$\Rightarrow \quad 3\cos^4\theta - 3\cos^2\theta + 2\sin^2\theta - 2\sin^6\theta = 0$

$\Rightarrow \quad 3\cos^2\theta\left(\cos^2\theta - 1\right) + 2\sin^2\theta\left(\sin^4\theta - 1\right) = 0$

$\Rightarrow \quad -3\cos^2\theta\sin^2\theta + 2\sin^2\theta\left(1+\sin^2\theta\right)\cos^2\theta - 1$

$\Rightarrow \quad \sin^2\theta\cos^2\theta\left(2+2\sin^2\theta-3\right) = 0$

$\Rightarrow \quad \sin^2\theta\cos^2\theta\left(2\sin^2\theta - 1\right) = 0$

(C1) $\sin^2\theta = 0 \rightarrow$ 3 solution ; $\theta = \{0,\pi,2\pi\}$

(C2) $\cos^2\theta = 0 \rightarrow$ 2 solution ; $\theta = \left\{\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right\}$

(C3) $\sin^2\theta = \dfrac{1}{2} \rightarrow$ 4 solution ; $\theta = \left\{\dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}\right\}$

No. of solution = 9

20. Let $x_1, x_2 \ldots, x_{100}$ be in an arithmetic progression, with $x_1 = 2$ and their mean equal to 200. If $y_i = i\left(x_i - i\right), 1 \le i \le 100$, then the mean of $y_1, y_2, \ldots,, y_{100}$ is .

(1) 10101.50

(2) 10051.50

(3) 10049.50

(4) 10100

Official Ans. by NTA (3)

Sol. Mean = 200

$\Rightarrow \quad \dfrac{\frac{100}{2}\left(2\times 2 + 99d\right)}{100} = 200$

$\Rightarrow \quad 4 + 99d = 400$

$\Rightarrow d = 4$

$y_i = i\left(xi - i\right)$

$= i\left(2 + (i-1)4 - i\right) = 3i^2 - 2i$

$\text{Mean} = \dfrac{\sum y_i}{100}$

$= \dfrac{1}{100}\sum_{i=1}^{100} 3i^2 - 2i$

$= \dfrac{1}{100}\left\{\dfrac{3\times 100\times 101\times 201}{6} - \dfrac{2\times 100\times 101}{2}\right\}$

$= 101\left\{\dfrac{201}{2} - 1\right\} = 101\times 99.5$

$= 10049\cdot 50$

SECTION-B

21. The mean of the coefficients of $x, x^2, ...x^7$ in the binomial expansion of $(2 + x)^9$ is _____ .

Official Ans. by NTA (2736)

Sol. Coefficient of $x = {}^9C_1 2^8$

Of $x^2 = {}^9C_2 2^7$

Of $x^7 = {}^9C_7 \cdot 2^2$

$$\text{Mean} = \frac{{}^9C_1 \cdot 2^8 + {}^9C_2 \cdot 2^7 ..... + {}^9C_7 \cdot 2^2}{7}$$

$$= \frac{(1+2)^9 - {}^9C_0 \cdot 2^9 - {}^9C_8 \cdot 2^1 - {}^9C_9}{7}$$

$$= \frac{3^9 - 2^9 - 18 - 1}{7}$$

$$= \frac{19152}{7} = 2736$$

22. Let $S = 109 + \frac{108}{5} + \frac{107}{5^2} + ... + \frac{2}{5^{107}} + \frac{1}{5^{108}}$. Then the value of $(16S - (25)^{-54})$ is equal to _____ .

Official Ans. by NTA (2175)

Sol. $$S = 109 + \frac{108}{5} + \frac{107}{5^2} ..... + \frac{1}{5^{108}}$$

$$\frac{S}{5} = \frac{109}{5} + \frac{108}{5^2} ..... + \frac{2}{5^{108}} + \frac{1}{5^{109}}$$

$$\frac{4S}{5} = 109 - \frac{1}{5} - \frac{1}{5^2} ..... - \frac{1}{5^{108}} - \frac{1}{5^{109}}$$

$$= 109 - \left(\frac{\frac{1}{5}\left(1 - \frac{1}{5^{109}}\right)}{\left(1 - \frac{1}{5}\right)} \right)$$

$$= 109 - \frac{1}{4}\left(1 - \frac{1}{5^{109}}\right)$$

$$= 109 - \frac{1}{4} + \frac{1}{4} \times \frac{1}{5^{109}}$$

$$s = \frac{5}{4}\left(109 - \frac{1}{4} + \frac{1}{4.5^{109}}\right)$$

$$16S = 20 \times 109 - 5 + \frac{1}{5^{108}}$$

$$16S - (25)^{-54} = 2180 - 5 = 2175$$

23. For m, n > 0, let $\alpha(m,n) = \int_0^2 t^m (1 + 3t)^n \, dt$. If $11\alpha(10,6) + 18\alpha(11,5) = p(14)^6$, then p is equal to _____ .

Official Ans. by NTA (32)

Sol. $$\alpha(m,n) = \int_0^2 t^m (1+3t)^n \, dt$$

If $11\alpha(10,6) + 18\alpha(11,5) = p(14)^6$ then P

$$= 11 \int_0^2 \frac{t^{10}}{II} \frac{(1+3t)^6}{I} + 10 \int_0^2 t^{11} (1+3t)^5 \, dt$$

$$= 11 \left[(1+3t)^6 \cdot \frac{t^{11}}{11} - \int 6(1+3t)^5 \cdot 3 \frac{t^{11}}{11} \right]_0^2 + 18 \int_0^2 t^{11} (1+3t)^5 \, dt$$

$$= \left(t^{11} (1+3t)^6 \right)_0^2$$

$$= 2^{11} (7)^6$$

$$= 2^5 (14)^6$$

$$= 32 (14)^6$$

24. In an examination, 5 students have been allotted their seats as per their roll numbers. The number of ways, in which none of the students sits on the allotted seat, is _____ .

Official Ans. by NTA (44)

Sol. Derangement of 5 students

$$D_5 = 5! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} \right)$$

$$= 120 \left(\frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120} \right)$$

$$= 60 - 20 + 5 - 1$$

$$= 40 + 4$$

$$= 44$$

25. Let a line l pass through the origin and be perpendicular to the lines

$l_1 : \vec{r} = \left(\hat{i} - 11\hat{j} - 7\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right), \lambda \in \mathbb{R}$

and $l_2 : \vec{r} = \left(-\hat{i} + \hat{k}\right) + \mu\left(2\hat{i} + 2\hat{j} + \hat{k}\right), \mu \in \mathbb{R}$.

If P is the point of intersection of l and l_1, and Q(α, β, γ) is the foot of perpendicular from P on l_2, then $9(\alpha+\beta+\gamma)$ is equal to_____ .

Official Ans. by NTA (5)

Sol. Let $\ell = \left(0\hat{i}+0\hat{j}+0\hat{k}\right)+\gamma\left(a\hat{i}+b\hat{j}+c\hat{k}\right)$

$= \gamma\left(a\hat{i}+b\hat{j}+c\hat{k}\right)$

$$a\hat{i}+b\hat{j}+c\hat{k} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 2 & 1 \end{vmatrix}$$

$= \hat{i}(2-6) - \hat{j}(1-6) + \hat{k}(2-4)$

$= -4\hat{i} - 5\hat{j} - 2\hat{k}$

$\ell = \gamma\left(-4\hat{i}+5\hat{j}-2\hat{k}\right)$

P is intersection of ℓ and ℓ_1

$-4\gamma = 1+\lambda, 5\gamma = -11+2\lambda, -2\gamma = -7+3\lambda$

By solving there equation $\gamma = -1$, P (4,–5,2)

Let $Q(-1+2\mu, 2\mu, 1+\mu)$

$\overrightarrow{PQ}\cdot\left(2\hat{i}+2\hat{j}+\hat{k}\right) = 0$

$-2+4\mu+4\mu+1+\mu = 0$

$9\mu = 1$

$\mu = \frac{1}{9}$

$Q\left(\frac{-7}{9}, \frac{2}{9}, \frac{10}{9}\right)$

$9(\alpha+\beta+\gamma) = 9\left(\frac{-7}{9}+\frac{2}{9}+\frac{10}{9}\right)$

$= 5$

26. The number of integral terms in the expansion of $\left(3^{\frac{1}{2}} + 5^{\frac{1}{4}}\right)^{680}$ is equal to

Official Ans. by NTA (171)

Sol. The number of integral term in the expression of $\left(3^{\frac{1}{2}} + 5^{\frac{1}{4}}\right)^{680}$ is equal to

General term $= {}^{680}C_r \left(3^{\frac{1}{2}}\right)^{680-r} \left(5^{\frac{1}{4}}\right)^{r}$

$= {}^{680}C_r 3^{\frac{680-r}{2}} 5^{\frac{r}{4}}$

Value's of r, where $\frac{r}{4}$ goes to integer

r = 0, 4, 8, 12, …….680

All value of r are accepted for $\frac{680-r}{2}$ as well so

No of integral terms = 171.

27. The number of ordered triplets of the truth values of p, q and r such that the truth value of the statement $(p \vee q) \wedge (p \vee r) \Rightarrow (q \vee r)$ is True, is equal to _____ .

Official Ans. by NTA (7)

Sol.

p	**q**	**r**	**Pvq**	**Pvr**	**(pvq) ∧ (pvr)**	**qvr**	**(pvq) ∧ (pvr) → qvr**
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	T	T	T	F	F
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	F	T	F	T	T
F	F	F	F	F	F	F	T

Hence total no of ordered triplets are 7

28. Let $H_n = \frac{x^2}{1+n} - \frac{y^2}{3+n} = 1,\ n \in N$. Let k be the smallest even value of n such that the eccentricity of H_k is a rational number. If l is length of the latus return of H_k, then $21l$ is equal to _____

Official Ans. by NTA (306)

Sol. $Hn \Rightarrow \frac{x^2}{1+n} - \frac{y^2}{3+n} = 1$

$e = \sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{3+n}{1+n}} = \sqrt{\frac{2n+4}{n+1}}$

$e = \sqrt{\frac{2n+4}{n+1}}$

$n = 48$ (smallest even value for which $e \in Q$)

$\boxed{e = \frac{10}{7}}$

$a^2 = n+1 \quad b^2 = n+3$

$= 49 \quad , \quad = 51$

1 = length of LR $= \frac{2b^2}{a}$

$L = 2 \cdot \frac{51}{7}$

$1 = \frac{102}{7}$

$\boxed{21\ell = 306}$

29. If a and b are the roots of equation $x^2 - 7x - 1 = 0$, then the value of $\frac{a^{21} + b^{21} + a^{17} + b^{17}}{a^{19} + b^{19}}$ is equal to _____ .

Official Ans. by NTA (51)

Sol. $x^2 - 7x - 1 = 0 <_b^a$

By newton's theorem

$S_{n+2} - 7S_{n+1} - S_n = 0$

$S_{21} - 7S_{20} - S_{19} = 0$

$S_{20} - 7S_{19} - S_{18} = 0$

$S_{19} - 7S_{18} - S_{17} = 0$

$\frac{S_{21} + S_{17}}{S_{19}} = \frac{S_{21} + (S_{19} - 7S_{18})}{S_{19}}$

$= \frac{S_{21} + S_{19} - 7(S_{20} - 7S_{19})}{S_{19}}$

$= \frac{50S_{19} + (S_{21} - 7S_{20})}{S_{19}}$

$= 51 \cdot \frac{S_{19}}{S_{19}} = \boxed{51}$

30. Let $A = \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$, where $a, c \in R$. If $A^3 = A$ and the positive value of a belongs to the interval $(n-1, n]$, where $n \in N$, then n is equal to _____ .

Official Ans. by NTA (2)

Sol. $A = \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$

$A^3 = A$

$A^2 = \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} a+2 & 2c & 3 \\ 3 & a+3c & 2a \\ ac & 1 & 2+3c \end{bmatrix}$

$A^3 = \begin{bmatrix} a+2 & 2c & 3 \\ 3 & a+3c & 2a \\ ac & a & 2+3c \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$

$A^3 = \begin{bmatrix} 2ac+3 & a+2+3c & 2a+4+6c \\ a(a+3c)+2a & 3+2ac & 6+3a+9c \\ a+2+3c & ac+c(2+3c) & 2ac+3 \end{bmatrix}$

Given $A^3 = A$

$2ac + 3 = 0$ …(1) and $a+2+3c = 1$

$a + 1 + 3c = 0$

$a + 1 - \frac{9}{2a} = 0$

$2a^2 + 2a - 9 = 0$

$f(1) < 0,\ f(2) > 0$

$a \in (1, 2]$ $\boxed{n = 2}$

SECTION-A

31. The electric field in an electromagnetic wave is given as $\vec{E} = 20\sin\omega\left(t - \frac{x}{c}\right)\hat{j}NC^{-1}$

Where ω and c are angular frequency and velocity of electromagnetic wave respectively. The energy contained in a volume of $5\times10^{-4}m^3$ will be

(Given $\varepsilon_0 = 8.85\times10^{-12}C^2/Nm^2$)

(1) $28.5\times10^{-13}J$ (2) $17.7\times10^{-13}J$

(3) $8.85\times10^{-13}J$ (4) $88.5\times10^{-13}J$

Official Ans. by NTA (3)

Sol. $\vec{E} = 20\sin\omega\left(t - \frac{x}{C}\right)\hat{j}N/C$

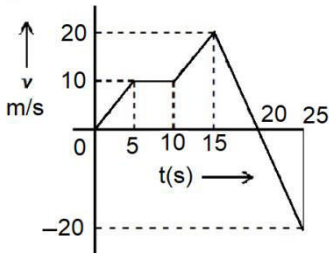
Average energy density of an em wave $= \frac{1}{2}\epsilon_0 E_0^2$

Energy stored $= \left(\frac{1}{2}\epsilon_0 E_0^2\right)(\text{volume})$

$= \frac{1}{2}\times8.85\times10^{-12}\times(20)^2\times\left(5\times10^{-4}\right)J$

$= 8.85\times10^{-13}J$

32. From the v - t graph shown. the ratio of distance to displacement in 25 s of motion



(1) $\frac{3}{5}$ (2) $\frac{1}{2}$

(3) $\frac{5}{3}$ (4) 1

Official Ans. by NTA (3)

Sol. Area under the graph from t = 0 to t = 20 sec = 200 m

Area under the graph from t = 20 to t = 25 sec = 50 m

So distance covered = (200 + 50)m = 250 m

Displacement = (200 – 50)m = 150 m

$\frac{250}{150} = \frac{5}{3}$

33. The radii of two planets 'A' and 'B' are 'R' and '4R' and their densities are ρ and $\rho/3$ respectively. The ratio of acceleration due to gravity at their surfaces (g_A : g_B) will be :

(1) 1 : 16 (2) 3 : 16

(3) 3 : 4 (4) 4 : 3

Official Ans. by NTA (3)

Sol. $g = \frac{GM}{R^2} = \frac{G}{R^2}\times\rho\times\frac{4\pi}{3}R^3 = \left(\frac{4\pi}{3}G\right)\rho R$

$\frac{g_A}{g_B} = \frac{R\times\rho}{4R\times\frac{\rho}{3}} = \frac{3}{4}$

34. A coin placed on a rotating table just slips when it is placed at a distance of 1 cm from the center. If the angular velocity of the table in halved, it will just slip when placed at a distance of_____ from the centre:

(1) 2 cm

(2) 1 cm

(3) 8 cm

(4) 4 cm

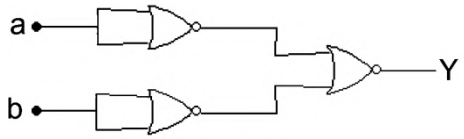
Official Ans. by NTA (4)

Sol. $f_{s\,max} = \mu mg = m\omega^2 R \Rightarrow R = \frac{\mu g}{\omega^2}$

So if ω becomes $\frac{\omega}{2}$, R will become 4R.

So distance from the center will be 4 cm.

35. The logic performed by the circuit shown in figure is equivalent to :

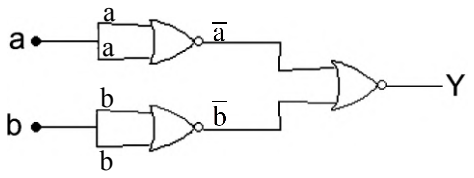


(1) AND (2) NAND

(3) OR (4) NOR

Official Ans. by NTA (1)

Sol.



$Y = \overline{\bar{a} + \bar{b}} = a \cdot b$

The truth table for the given circuit will be

a	b	output
0	0	0
0	1	0
1	0	0
1	1	1

Hence it will be equivalent to AND gate.

36. A parallel plate capacitor of capacitance 2 F is charged to a potential V. The energy stored in the capacitor is E_1. The capacitor is now connected to another uncharged identical capacitor in parallel combination. The energy stored in the combination is E_2. The ratio E_2/E_1 is :

(1) 2 : 1 (2) 1 : 2

(3) 1 : 4 (4) 2 : 3

Official Ans. by NTA (2)

Sol. Initially

$Q_1 = CV = (2)\ V$

$E_1 = 1/2\ CV^2 = 1/2\ (2)V^2 = V^2$

Finally

Charge on each capacitor, $Q_2 = \frac{Q_1}{2} = \frac{2V}{2} = V$

$$E_2 = 2\left(\frac{1}{2}\frac{Q_2^2}{C}\right) = \frac{V^2}{2} \qquad \therefore \frac{E_2}{E_1} = \frac{1}{2}$$

37. Two identical heater filaments are connected first in parallel and then in series. At the same applied voltage, the ratio of heat produced in same time for parallel to series will be:

(1) 4 : 1 (2) 2 : 1 (3) 1 : 2 (4) 1 : 4

Official Ans. by NTA (1)

Sol. Parallel combination

$$H_p = \left[\frac{V^2}{\left(\frac{R}{2}\right)}\right]t = \frac{2V^2t}{R}$$

Series combination

$$H_s = \left(\frac{V^2}{2R}\right)t \qquad \therefore \frac{H_p}{H_s} = 4$$

38. A transmitting antenna is kept on the surface of the earth. The minimum height of receiving antenna required to receive the signal in line of sight at 4 km distance from it is $x \times 10^{-2}$ m. The value of x is (Let. radius of earth R = 6400 km)

(1) 125 (2) 12.5 (3) 1.25 (4) 1250

Official Ans. by NTA (1)

Sol. $d_r = \sqrt{2h_rR} \quad \therefore h_r = \frac{d_r^2}{2R}$

$$= \frac{(4\text{km})^2}{2(6400\text{ km})} = \left(\frac{1}{800}\right)\text{km} = 1.25\text{ m}$$

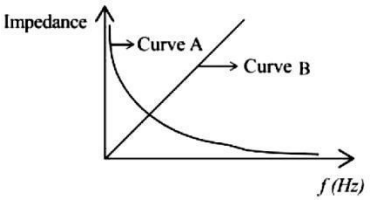
39. As per the given graph choose the correct representation for curve A and curve B.

{Where X_C = reactance of pure capacitive circuit connected with A.C. source

X_L = reactance of pure inductive circuit connected with A.C. source

R = impedance of pure resistive circuit connected with A.C. source

Z = Impedance of the LCR series circuit}



(1) $A = X_C$, $B = R$ (2) $A = X_L$, $B = Z$

(3) $A = X_C$, $B = X_L$ (4) $A = X_L$, $B = R$

Official Ans. by NTA (3)

Sol. $X_C = \frac{1}{\omega C} = \frac{1}{(2\pi f)C}$

$\therefore X_C \propto \frac{1}{f}$

$\therefore$ Curve A

$X_L = \omega L = (2\pi f)L$

$\therefore X_L \propto f$

$\therefore$ Curve B

40. 1 kg of water at 100°C is converted into steam at 100°C by boiling at atmospheric pressure. The volume of water changes from 1.00×10^{-3} m^3 as a liquid to 1.671 m^3 as steam. The change in internal energy of the system during the process will be
(Given latent heat of vaporisaiton = 2257 kJ/kg. Atmospheric pressure = 1×10^5 Pa)

(1) + 2090 kJ (2) – 2090 kJ

(3) – 2426 kJ (4) + 2476 kJ

Official Ans. by NTA (1)

Sol. $\Delta Q = \Delta U + \Delta W$

$\therefore \Delta U = \Delta Q - \Delta W$

$= mL_V - P\Delta V$

$= (1Kg)(2257 \times 10^3 J/kg)$

$-(1 \times 10^5 Pa)(1.671m^3 - 1 \times 10^{-3} m^3)$

$= 2257 \times 10^3 J - 167 \times 10^3 J$

$= 2090$ KJ

41. The critical angle for a denser-rarer interface is 45°. The speed of light in rarer medium is 3×10^8 ms. The speed of light in the denser medium is:

(1) 5×10^7 m/s (2) 2.12×10^8 m/s

(3) 3.12×10^7 m/s (4) $\sqrt{2} \times 10^8$ m/s

Official Ans. by NTA (2)

Sol. $i_C =$ Critical angle

$\frac{v}{C} = \frac{1}{\mu} = \sin i_C = \sin 45° = \frac{1}{\sqrt{2}}$

$\Rightarrow v = \frac{C}{\sqrt{2}} = \frac{3 \times 10^8}{\sqrt{2}} m/s = 2.12 \times 10^8 m/s$

42. A metallic surface is illuminated with radiation of wavelength λ, the stopping potential is V_o. If the same surface is illuminated with radiation of wavelength 2λ, the stopping potential becomes $\frac{V_o}{4}$. The threshold wavelength for this metallic surface will be -

(1) $\frac{\lambda}{4}$ (2) 4λ

(3) $\frac{3}{2}\lambda$ (4) 3λ

Official Ans. by NTA (4)

Sol. From the equation of photoelectric effect

$eV_0 = \frac{hc}{\lambda} - \phi_0 = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$

$\& \frac{eV_0}{4} = \frac{hc}{2\lambda} = \frac{hc}{\lambda_0}$

$\Rightarrow \frac{1}{4}\left(\frac{hc}{\lambda} - \frac{hc}{\lambda_0}\right) = \frac{hc}{2\lambda} - \frac{hc}{\lambda_0}$

$\frac{1}{\lambda_0} - \frac{1}{4\lambda_0} = \frac{1}{2\lambda} - \frac{1}{4\lambda}$

$\frac{3}{4\lambda_0} = \frac{1}{4\lambda}$

$\Rightarrow \lambda_0 = 3\lambda$

43. The free space inside a current carrying toroid is filled with a material of susceptibility 2×10^{-2}. The percentage increase in the value of magnetic field inside the toroid will be

(1) 2% (2) 0.2%

(3) 0.1% (4) 1%

Official Ans. by NTA (1)

Sol. As $X_m = 2 \times 10^{-2}$

$\mu_r = 1 + X_m = 1.02$

$\Rightarrow B = \mu_r B_0 = 1.02 B_0$

So percentage increase in magnetic field

$= \frac{B - B_0}{B_0} \times no\% = 2\%$

44. The current sensitivity of moving coil galvanometer is increased by 25%. This increase is achieved only by changing in the number of turns of coils and area of cross section of the wire while keeping the resistance of galvanometer coil constant. The percentage change in the voltage sensitivity will be:

(1) +25% (2) – 50%

(3) Zero (4) – 25%

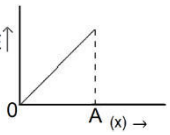
Official Ans. by NTA (1)

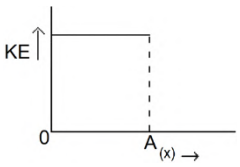
Sol. $I_S = \frac{NBA}{C}$ & $V_S = \frac{NBA}{CG}$

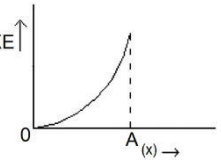
$\Rightarrow V_S = \frac{I_S}{G}$, If G (galvanometer resistance) is constant, then $V_S \propto I_S$

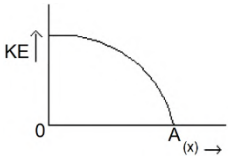
so percentage change in V_S is also 25%.

45. The variation of kinetic energy (KE) of a particle executing simple harmonic motion with the displacement (x) starting from mean position to extreme position (A) is given by

(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (4)

Sol. For a particle executing SHM

$KE = \frac{1}{2}m\omega^2\left(A^2 - x^2\right)$

When x = 0, KE is maximum & when x = A, KE is zero and KE V/S x graph is parabola.

46. On a temperature scale 'X'. The boiling point of water is 65° X and the freezing point is –15°X. Assume that the X scale is linear. The equivalent temperature corresponding to –95° X on the Farenheit scale would be:

(1) –63°F (2) –112°F

(3) –48°F (4) –148°F

Official Ans. by NTA (4)

Sol. $\frac{X - X_{freez}}{X_{boil} - X_{freez}} = \frac{t - 32}{212 - 32}$

$\frac{-95 - (-15)}{65 - (-15)} = \frac{t - 32}{180}$

$\frac{-80}{80} = \frac{t - 32}{180}$

$t = -180 + 32$

$t = -148°f$

47. Given below are two statements :

Statements I : Astronomical unit (Au). Parsec (Pc) and Light year (ly) are units for measuring astronomical distances.

Statements II: Au < Parsec (Pc) < ly

In the light of the above statements. choose the most appropriate answer from the options given below:

(1) Both Statements I and Statements II are correct.

(2) Statements I is correct but Statements II is incorrect.

(3) Both Statements I and Statements II are incorrect.

(4) Statements I is incorrect but statements II is correct.

Official Ans. by NTA (2)

Sol. $1AU = 1.496 \times 10^{11}\,m$

$1\,par\,sec = 3.08 \times 10^{16}\,m$

$1\,light\,year = 9.46 \times 10^{15}\,m$

So, $Au < ly < Per\,sec$

48. Three vessels of equal volume contain gases at the same temperature and pressure. The first vessel contains neon (monoatomic), the second contains chlorine (diatomic) and third contains uranium hexafloride (polyatomic). Arrange these on the basis of their root mean square speed (v_{rms}) and choose the correct answer from the options given below:

(1) $v_{rms}(\text{mono}) = v_{rms}(\text{dia}) = v_{rms}(\text{poly})$

(2) $v_{rms}(\text{mono}) > v_{rms}(\text{dia}) > v_{rms}(\text{poly})$

(3) $v_{rms}(\text{dia}) < v_{rms}(\text{poly}) < v_{rms}(\text{mono})$

(4) $v_{rms}(\text{mono}) < v_{rms}(\text{dia}) < v_{rms}(\text{poly})$

Official Ans. by NTA (2)

Sol. $v_{rms}(\text{mono}) = \sqrt{\dfrac{3RT}{4\times10^{-3}}}$

$v_{rms}(\text{dia}) = \sqrt{\dfrac{3RT}{71\times10^{-3}}}$

$v_{rms}(\text{ply}) = \sqrt{\dfrac{3RT}{146\times10^{-3}}}$

So correct relation is

$v_{rms}(\text{mono}) > v_{rms}(\text{dia}) > v_{rms}(\text{poly})$

49. An average force of 125 N is applied on a machine gun firing bullets each of mass 10 g at the speed of 250 m/s to keep it in position. The number of bullets fired per second by the machine gun is :

(1) 5 (2) 50

(3) 100 (4) 25

Official Ans. by NTA (2)

Sol. $F = n\,m\,v$

where n = number of bullets fired per second

$n = \dfrac{f}{mv} = \dfrac{125}{10\times10^{-3}\times250} = 50$

50. Two radioactive elements A and B initially have same number of atoms. The half life of A is same as the average life of B. If λ_A and λ_B are decay constants of A and B respectively, then choose the correct relation from the given options.

(1) $\lambda_A = \lambda_B$ (2) $\lambda_A = 2\lambda_B$

(3) $\lambda_A = \lambda_B \ln 2$ (4) $\lambda_A \ln 2 = \lambda_B$

Official Ans. by NTA (3)

Sol. $T_{1/2}(A) = T_{av}(B)$

$\dfrac{\ell n 2}{\lambda_A} = \dfrac{1}{\lambda_B}$

$\lambda_A = \lambda_B \ell 2$

SECTION-B

51. A monochromatic light is incident on a hydrogen sample in ground state. Hydrogen atoms absorb a fraction of light and subsequently emit radiation of six different wavelengths. The frequency of incident light is x × 10^{15} Hz. The value of x is _____. (Given h = 4.25 × 10^{-15} eVs)

Official Ans. by NTA (3)

Sol. $6 = {}^4C_2 \qquad \Rightarrow n_2 = 4$

$h\nu = E_4 - E_1$

$\therefore \nu = 13.6\left(\dfrac{1}{1^2} - \dfrac{1}{4^2}\right) \times \dfrac{1}{4.25\times10^{-15}}$

$= 3\times10^{15}\,\text{Hz}$

52. The radius of curvature of each surface of a convex lens having refractive index 1.8 is 20 cm. The lens is now immersed in a liquid of refractive index 1.5. The ratio of power of lens in air to its power in the liquid will be x : 1. The value of x is _____.

Official Ans. by NTA (4)

Sol. $P = (1.8-1)\left(\dfrac{1}{20} + \dfrac{1}{20}\right)$ by lens maker's formula

$P' = \left(\dfrac{1.8}{1.5} - 1\right)\left(\dfrac{1}{20} + \dfrac{1}{20}\right)$

Dividing $\dfrac{P}{P'} = \dfrac{0.8}{1.2-1} = 4$

53. The equation of wave is given by

$$Y = 10^{-2}\sin 2\pi\left(160t - 0.5x + \frac{\pi}{4}\right)$$

Where x and Y are in m and t in s. The speed of the wave is ____ km h^{-1}

Official Ans. by NTA (1152)

Sol. $$V = \frac{\omega}{k} = \frac{2\pi \times 60}{2\pi \times 0.5} = \frac{160}{0.5}\text{ m/s}$$

$$= \frac{160}{0.5} \times \frac{18}{5}\text{ km/h}$$

$$= 1152\text{ km}$$

54. A force $\vec{F} = (2+3x)\hat{i}$ acts on a particle in the x direction where F is in newton and x is in meter. The work done by this force during a displacement from x = 0 to x = 4 m, is ____J.

Official Ans. by NTA (32)

Sol. $$W = \int_0^4 (2+3x)dx$$

$$= \left[2x + \frac{3x^2}{2}\right]_0^4$$

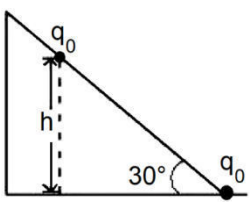
$$= 8 + 3 \times 8$$

$$= 32\text{ J}$$

55. As shown in the figure. a configuration of two equal point charges (q_0 = +2μ C) is placed on an inclined plane. Mass of each point charge is 20 g. Assume that there is no friction between charge and plane. For the system of two point charges to be in equilibrium (at rest) the height h = x × 10^{-3} m

The value of x is ____.

(Take $\frac{1}{4\pi\varepsilon_0} = 9\times10^9\text{ N m}^2\text{C}^{-2}, g = 10\text{ ms}^{-1}$)



Official Ans. by NTA (300)

Sol. For equilibrium along the plane

$$mg\sin\theta = \frac{1}{4\pi\epsilon_0} \times \frac{q_0^2}{(h\text{ cosec }30°)^2}$$

$$\therefore h^2 = \frac{1}{4\pi\epsilon_o} \times \frac{q_0^2}{mg\text{ cosec }30°}$$

$$= 9\times10^9 \times \frac{(2\times10^{-6})^2}{0.02\times10\times2}$$

$$\therefore h = 3\times10^4 \times \frac{2\times10^{-6}}{0.2}$$

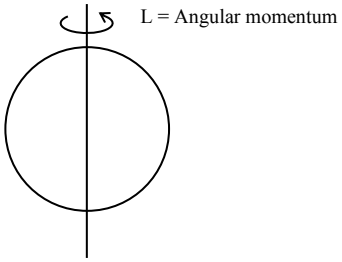
$$= 0.3\text{ m}$$

$$= 300\text{ mm}$$

56. A solid sphere of mass 500 g and radius 5 cm is rotated about one of its diameter with angular speed of 10 rad s^{-1}. If the moment of inertia of the sphere about its tangent is x × 10^{-2} times its angular momentum about the diameter. Then the value of x will be _____.

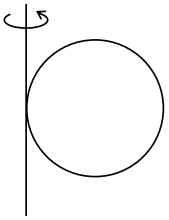
Official Ans. by NTA (35)

Sol.



m = 500 g = 0.5 kg
R = 5cm
ω = 10 rad/sec

moment of inertia about tangent = I_T



$$I_t = x\times10^{-2}L$$

$$\frac{7}{5}mR^2 = x\times10^{-2}\frac{2}{5}mR^2\omega$$

$$\frac{7}{2\omega} = x\times10^{-2} = \frac{7}{2\times10}$$

57. The length of wire becomes l_1 and l_2 when 100N and 120 N tensions are applied respectively. If $10\, l_2 = 11\, l_1$, the natural length of wire will be $\frac{1}{x} l_1$.

Here the value of x is ____.

Official Ans. by NTA (2)

Sol. Let the original length be 'ℓ_0'

When $T_1 = 100$ N, Extension = $\ell_1 - \ell_0$

When $T_2 = 120$ N, Extension = $\ell_2 - \ell_0$

Then $100 = K(\ell_1 - \ell_0)$ ...(1)

And $120 = K(\ell_2 - \ell_0)$ ...(2)

$$\frac{1}{2} \Rightarrow \frac{5}{6} = \frac{\ell_1 - \ell_0}{\ell_2 - \ell_0}$$

$$5\ell_2 - 5\ell_0 = 6\ell_1 - 6\ell_0$$

$$\ell_0 = 6\ell_1 - 5\ell_2$$

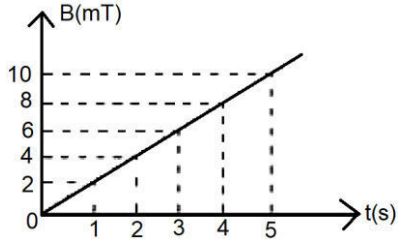
$$\ell_0 = 6\ell_1 - 5\left(\frac{11\ell_1}{10}\right)$$

$$\ell_0 = 6\ell_1 - \frac{11\ell_1}{2}$$

$$\ell_0 = \frac{\ell_1}{2}$$

$$\therefore x = 2$$

58. The magnetic field B crossing normally a square metallic plate of area 4 m^2 is changing with time as shown in figure. The magnitude of induced emf in the plate during t = 2s to t = 4s, is _____ mV

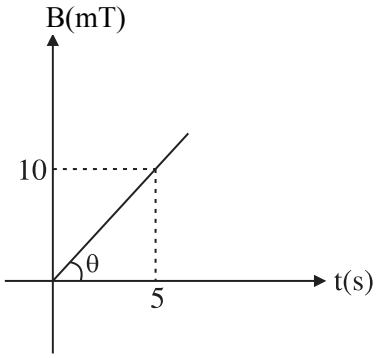


Official Ans. by NTA (8)

Sol. $m = \tan\theta = \frac{10}{5} = 2$

$B = mt$

$B = 2t$



$$\varepsilon = \left|\frac{d\phi}{dt}\right| = \frac{d(BA)}{dt} = \frac{AdB}{dt}$$

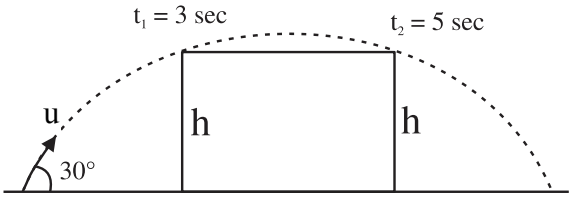
$$\varepsilon = \frac{4d(2t)}{dt} = 4 \times 2 = 8 \text{ mVolt}$$

59. A projectile fired at 30° to the ground is observed to be at same height at time 3s and 5s after projection, during its flight. The speed of projection of the projectile is _____ ms^{-1}

(Given g = 10 m s^{-2})

Official Ans. by NTA (80)

Sol. Time of flight $t_1 + t_2 = 3 + 5 = 8$ sec

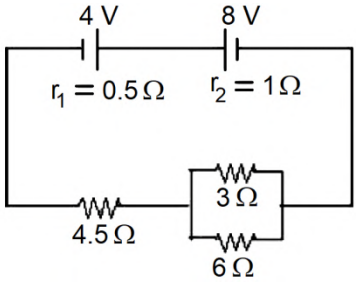


$$T = \frac{2u\sin 30°}{g}$$

$$8 = \frac{2u\sin(30°)}{10}$$

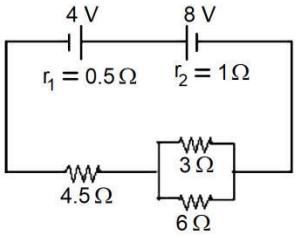
$$u = 80 \text{ m/s}$$

60. In the circuit diagram shown in figure given below, the current flowing through resistance $3\,\Omega$ is $\frac{x}{3}$A. The value of x is ____.

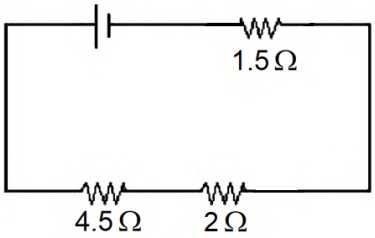


Official Ans. by NTA (1)

Sol.

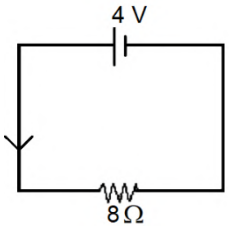


$E_2 - E_1 = 8 - 4 = 4V$

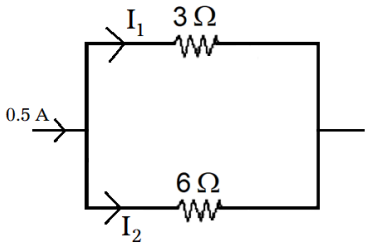


$\frac{1}{3}+\frac{1}{6}=\frac{1}{2}=\frac{1}{R}$

$R = 2\,\Omega$



$I = \frac{4}{8} = 0.5A$



$I_1 = \left(\frac{6}{3+6}\right)\times 0.5$

$I_1 = \frac{2}{3}\times 0.5 = \frac{1}{3}A$

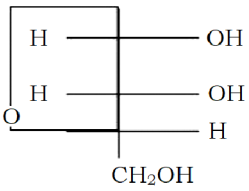
$I_1 = \frac{x}{3} = \frac{1}{3} \therefore x = 1$

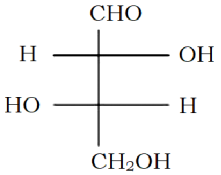
SECTION-A

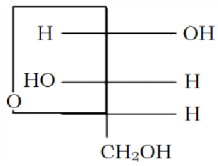
61. L –isomer of tetrose X ($C_4H_8O_4$) gives positive Schiff's test and has two chiral carbons. On acetylation. 'X' yields triacetate. 'X' also undergoes following reactions

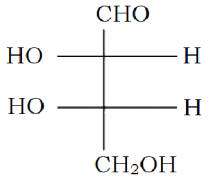
$$'A' \xleftarrow{HNO_3} 'X' \xrightarrow{NaBH_4} \underset{\text{Chiral compound}}{'B'}$$

'X' is

(1) 

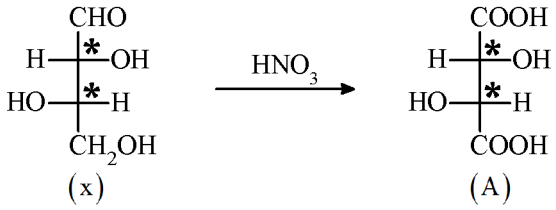
(2) 

(3) 

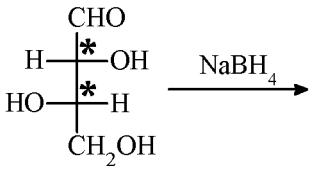
(4) 

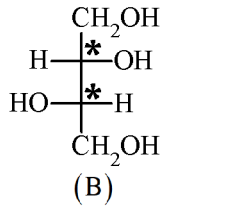
Official Ans. by NTA (2)

Sol.

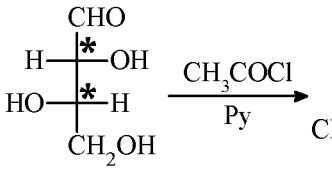


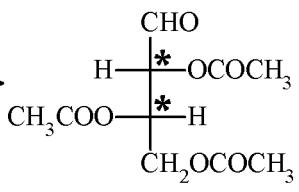
L-tetrose with two chiral centre





optically active





(x) gives positive schiff's test due –CHO group

(x) is L–tetrose.

62. The polymer X – consists of linear molecules and is closely packed. It is prepared in the presence of triethylaluminium and titanium tetrachloride under low pressure. The polymer X is –

(1) Polyacrylonitrile

(2) Low density polythene

(3) Polytetrafluoroethane

(4) High density polythene

Official Ans. by NTA (4)

Sol. Ethene undergoes addition polymerisation to high density polythene in the presence of catalyst such as $AlEt_3$ and $TiCl_4$ (Ziegler – Natta catalyst) at a temperature of 333 K to 343 K and under a pressure of 6–7 atmosphere.

63. When a solution of mixture having two inorganic salts was treated with freshly prepared ferrous sulphate in acidic medium, a dark brown ring was formed whereas on treatment with neutral $FeCl_3$, it gave deep red colour which disappeared on boiling and a brown red ppt was formed. The mixture contains

(1) CH_3COO^- & NO_3^-

(2) $C_2O_4^{2-}$ & NO_3^-

(3) SO_3^{2-} & CH_3COO^-

(4) SO_3^{2-} & $C_2O_4^{2-}$

Official Ans. by NTA (1)

Sol. $CH_3COO^- + FeCl_3 \rightarrow Fe(CH_3COO)_3$ or

$[Fe_3(OH)_2(CH_3COO)_6]^+$

Blood red colour

$\downarrow \Delta$

$Fe(OH)_2(CH_3COO)\downarrow$

Red–brown precipitate

$$2NO_3^- + 4H_2SO_4 + 6Fe^{2+} \rightarrow 6Fe^{3+} + 2NO\uparrow + 4SO_4^{2-} + 4H_2O$$

$$[Fe(H_2O)_6]^{2+} + NO \rightarrow \underset{\text{Brown}}{[Fe(H_2O)_5(NO)]^{2+}} + H_2O$$

64. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R :

Assertion A : In the photoelectric effect, the electrons are ejected from the metal surface as soon as the beam of light of frequency greater than threshold frequency strikes the surface.

Reason R : When the photon of any energy strikes an electron in the atom, transfer of energy from the photon to the electron takes place.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both A and R are correct but R is NOT the correct explanation of A

(2) A is correct but R is not correct

(3) Both A and R are correct and R is the correct explanation of A

(4) A is not correct but R is correct

Official Ans. by NTA (2)

Sol. There is a characteristic minimum frequency, or "threshold frequency," for each metal below which the photoelectric effect is not seen. The ejected electrons leave with a specific amount of kinetic energy at a frequency $v > v_0$ with an increase in light frequency of these electron kinetic energies also rise.

65. 25 mL of silver nitrate solution (1 M) is added dropwise to 25 mL of potassium iodide (1.05 M) solution. The ion(s) present in very small quantity in the solution is/are

(1) NO_3^- only

(2) K^+ only

(3) Ag^+ and I^- both

(4) I^- only

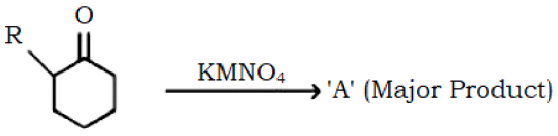
Official Ans. by NTA (3)

Sol. $AgNO_3 + KI \rightarrow AgI\downarrow + KNO_3$

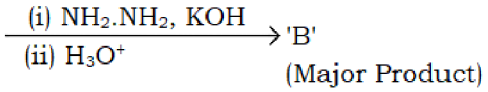
$$AgI \rightarrow \underset{S}{Ag^+} + \underset{S+0.625}{I^-}$$

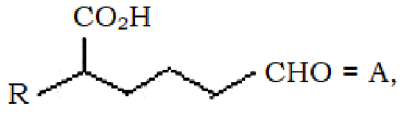
AgI is a insoluble salt so concentration Ag^+ and I^- will be negligible.

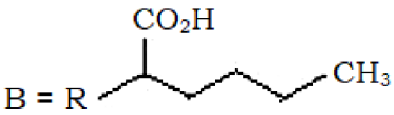
66. 'A' and 'B' in the below reactions are :

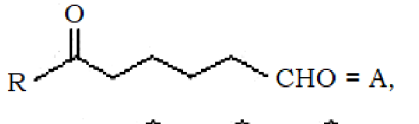


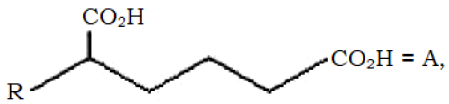
(R = alkyl)

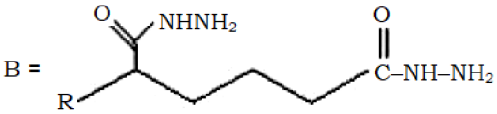


(1) 



(2) 

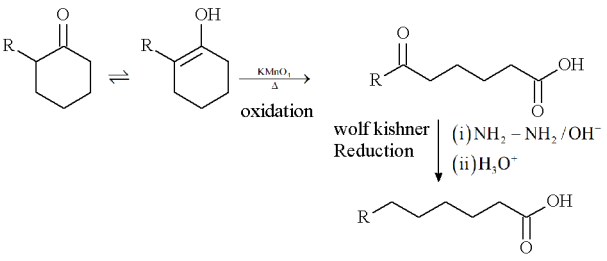
(3) 



(4) 

Official Ans. by NTA (4)

Sol.



67. The set which does not have ambidentate ligand(s) is

(1) $C_2O_4^{2-}$, ethylene diammine, H_2O

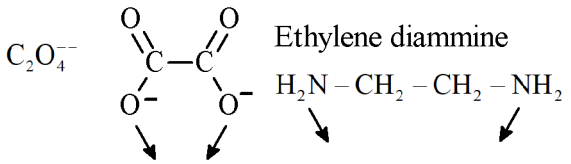
(2) $EDTA^{4-}$, NCS^-, $C_2O_4^{2-}$

(3) NO_2^-, $C_2O_4^{2-}$, $EDTA^{4-}$

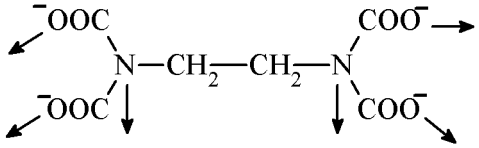
(4) $C_2O_4^{2-}$, NO_2^-, NCS^-

Official Ans. by NTA (1)

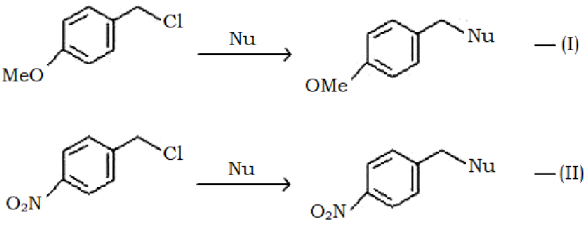
Sol. NO_2^-, NCS^- are ambidentate ligand



EDTA Ethylene diamine tetra acetate



68.



Where Nu = Nucleophile

Find out the correct statement from the options given below for the above 2 reactions.

(1) Reaction (I) is of 2^{nd} order and reaction (II) is of 1^{st} order

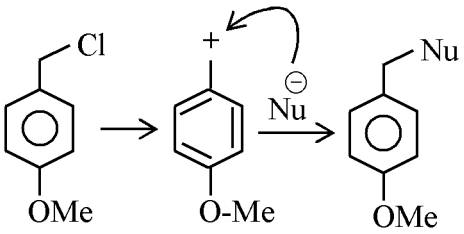
(2) Reaction (I) and (II) both are of 2^{nd} order

(3) Reaction (I) is of 1^{st} order and reaction (II) is of 2^{nd} order

(4) Reactions (I) and (II) both are of 1^{st} order

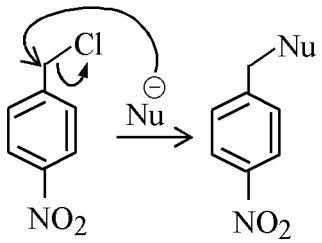
Official Ans. by NTA (3)

Sol.



Electron Donating group

S_N^1 Mech. : I^{st} order



Electron withdrawing group

S_N^2 Mech : 2^{nd} order

69. For elements B, C, N, Li, Be, O and F the correct order of first ionization enthalpy is

(1) Li < Be < B < C < N < O < F

(2) B > Li > Be > C > N > O > F

(3) Li < B < Be < C < O < N < F

(4) Li < Be < B < C < O < N < F

Official Ans. by NTA (3)

Sol. First I.E.

F > N > O > C > Be > B > Li

Li – 520 kJ / mol

Be – 899 kJ / mol

B – 801 kJ / mol

C – 1086 kJ / mol

N – 1402 kJ / mol

O – 1314 kJ / mol

F – 1681 kJ / mol

70. Match List-I with List-II :

List-I Species	List-II Geometry/Shape
A. H_3O^+	I. Tetrahedral
B. Acetylide anion	II. Linear
C. NH_4^+	III. Pyramidal
D. ClO_2^-	IV. Bent

Choose the correct answer from the options given below :

(1) A-III, B-II, C-I, D-IV

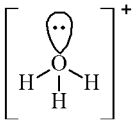
(2) A-III, B-I, C-II, D-IV

(3) A-III, B-IV, C-I, D-II

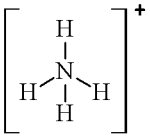
(4) A-III, B-IV, C-II, D-I

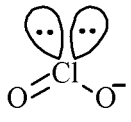
Official Ans. by NTA (1)

Sol. Molecule/Ion Hybridisation Shape

H_3O^+ sp^3 Pyramidal 

Acelylide sp linear $\overline{C} \equiv \overline{C}$

NH_4^+ sp^3 tetrahedral 

ClO_2^- sp^3 Bent 

71. For compound having the formula $GaAlCl_4$, the correct option from the following is

(1) Ga is more electronegative than Al and is present as a cationic part of the salt $GaAlCl_4$

(2) Oxidation state of Ga in the salt $GaAlCl_4$ is +3.

(3) Cl forms bond with both Al and Ga in $GaAlCl_4$

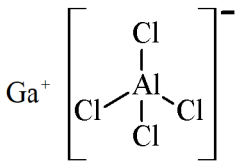
(4) Ga is coordinated with Cl in $GaAlCl_4$

Official Ans. by NTA (1)

Sol. Gallous tetrachloro aluminate $Ga^+AlCl_4^-$

$$2Ga + Ga^+GaCl_4^- + 2Al_2Cl_6 \xrightarrow{190^\circ} 4Ga^+AlCl_4^-$$

Structure of $\overset{+}{G}a\overset{-}{A}lCl_4$



Ga is cationic part of salt $GaAlCl_4$.

72. In the extraction process of copper, the product obtained after carrying out the reactions

(i) $2Cu_2S + 3O_2 \rightarrow 2Cu_2O + 2SO_2$

(ii) $2Cu_2O + Cu_2S \rightarrow 6Cu + SO_2$ is called

(1) Blister copper

(2) Copper scrap

(3) Reduced copper

(4) Copper matte

Official Ans. by NTA (1)

Sol. $2Cu_2S + 3O_2 \rightarrow 2Cu_2O + 3SO_2$

$2Cu_2O + Cu_2S \rightarrow 6Cu + SO_2$

Blister copper

Due to evolution of SO_2, the solidified copper formed has a blistered look and is referred to as blister copper.

73. Match List-I with List-II :

List-I	List-II
A. K	I. Thermonuclear reactions
B. KCl	II. Fertilizer
C. KOH	III. Sodium potassium pump
D. Li	IV. Absorbent of CO_2

Choose the correct answer from the options given below :

(1) A-III, B-II, C-IV, D-I

(2) A-IV, B-I, C-III, D-II

(3) A-IV, B-III, C-I, D-II

(4) A-III, B-IV, C-II, D-I

Official Ans. by NTA (1)

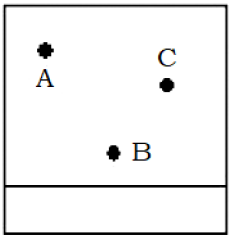
Sol. K^+ – Sodium – Potassium Pump

KCl – Fertiliser

KOH – absorber of CO_2

Li – used in thermonuclear reactions

74. Thin layer chromatography of a mixture shows the following observation :



The correct order of elution in the silica gel column chromatography is

(1) A, C, B

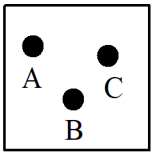
(2) B, C, A

(3) C, A, B

(4) B, A, C

Official Ans. by NTA (1)

Sol.



According to the observation, A is more mobile and interacts with the mobile phase more than C, and C is more drawn to the mobile phase than B.

Hence, the correct order of elution in the silico gel column chromatography is – $B < C < A$

75. Which of the following complex has a possibility to exist as meridional isomer?

(1) $[Co(NH_3)_3(NO_2)_3]$

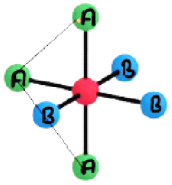
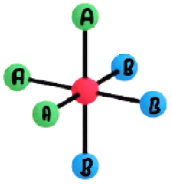
(2) [Co (en)$_3$]

(3) [Co (en)$_2$ Cl$_2$]

(4) [Pt (NH$_3$)$_2$ Cl$_2$]

Official Ans. by NTA (1)

Sol. $[MA_3B_3]$ type of compound exists as facial and meridonial isomer.

76. Given below are two statements :

Statement-I : Methane and steam passed over a heated Ni catalyst produces hydrogen gas.

Statement-II : Sodium nitrite reacts with NH_4Cl to give H_2O, N_2 and NaCl.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both the statements I and II are correct

(2) Both the statements I and II are incorrect

(3) Statement I is incorrect but Statement II is correct

(4) Statement I is correct but Statement II is incorrect

Official Ans. by NTA (1)

Sol. $CH_4(g) + \underset{\text{Steam}}{H_2O(g)} \xrightarrow[1270\,K]{Ni} CO(g) + 3H_2(g)$

$NaNO_2(aq) + NH_4Cl(aq) \rightarrow N_2(g) + NaCl(aq) + 2H_2O(\ell)$

77. Given below are two statements :

Statement I : If BOD is 4 ppm and dissolved oxygen is 8 ppm, then it is a good quality water.

Statement II : If the concentration of zinc and nitrate salts are 5 ppm each, then it can be a good quality water.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both the statements I and II are incorrect

(2) Statement I is incorrect but Statement II is correct

(3) Both the statements I and II are correct

(4) Statement I is correct but Statement II is incorrect

Official Ans. by NTA (3)

Sol. Clean water would have BOD value of less than 5 ppm.

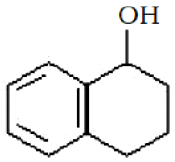
Maximum limit of Zn in clean water

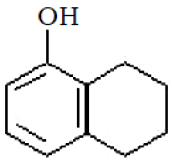
= 5.0 ppm or mg dm^{-3}

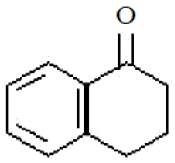
Maximum limit of NO_3^- in clean water

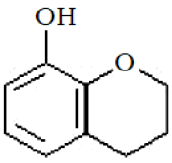
= 50 ppm or mg dm^{-3}

78. Arrange the following compounds in increasing order of rate of aromatic electrophilic substitution reaction


(a)


(b)


(c)


(d)

(1) d, b, c, a

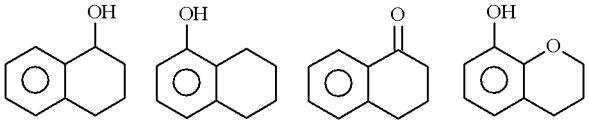
(2) b, c, a, d

(3) c, a, b, d

(4) d, b, a, c

Official Ans. by NTA (3)

Sol. Benzene becomes more reactive towards EAS when any substituent raises the electron density.



a
-CH_2 has +H effect

b
-OH has +R while -CH_2- group has +H effect.

c
$-\overset{O}{\overset{\|}{C}}-$ has -R effect

d
-OH and -O- both show +R effect

Correct order

$c < a < b < d$

79. The complex that dissolves in water is

(1) $Fe_4[Fe(CN)_6]_3$

(2) $[Fe_3(OH)_2(OAc)_6]Cl$

(3) $K_3[Co(NO_2)_6]$

(4) $(NH_4)_3[As(Mo_3O_{10})_4]$

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. $Fe_4\left[Fe(CN)_6\right]_3$ Prussian Blue–water insoluble

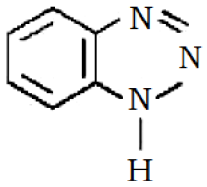
$K_3\left[Co(NO_2)_6\right]$ very poorly water soluble

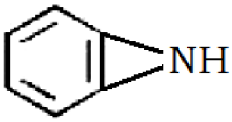
$(NH_4)_3$ [As $(MO_3O_{10})_4$] water insoluble ammonium arseno molybdate

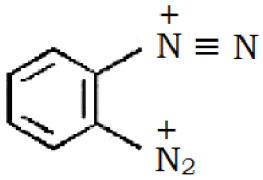
$\left[Fe_3(OH)_2(OAc)_6\right]Cl$ is water soluble.

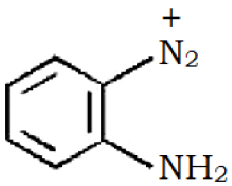
80. o-Phenylenediamine $\xrightarrow{HNO_2}$ 'X'
Major Product

'X' is

(1) 

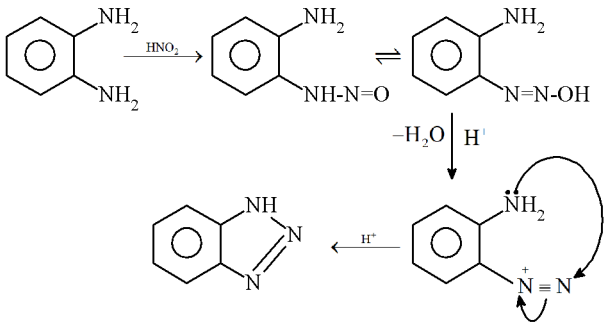
(2) 

(3) 

(4) 

Official Ans. by NTA (1)

Sol. Orthophenyl amine.



SECTION-B

81. A mixture of 1 mole of H_2O and 1 mole of CO is taken in a 10 litre container and heated to 725 K. At equilibrium 40% of water by mass reacts with carbon monoxide according to the equation :

$CO(g) + H_2O(g) \rightleftharpoons CO_2(g) + H_2(g)$.

The equilibrium constant $K_C \times 10^2$ for the reaction is _____. (Nearest integer)

Official Ans. by NTA (44)

Sol. $CO_{(g)} + H_2O_{(g)} \rightleftharpoons CO_{2(g)} + H_{2(g)}$

t = 0	1 mol	1 mol	0	0
at equ.	1–x	1–x	x	x

at equilibrium 40% by mass water reacts with CO

x = 0.4 $\quad$ 1 – x = 0.6

$$K_C = \frac{[CO_2][H_2]}{[CO][H_2O]} = \frac{0.4 \times 0.4}{0.6 \times 0.6} = 0.44$$

$K_C \times 10^2 = 44$

82. The ratio of spin-only magnetic moment values $\mu_{eff}[Cr(CN)_6]^{3-} / \mu_{eff}[Cr(H_2O)_6]^{3+}$ is _____.

Official Ans. by NTA (1)

Sol. Spin magnetic moment of $[Cr(CN)_6]^{3-}(t_{2g}^3 e_g^0)$

$\mu_1 = \sqrt{3(3+2)} = \sqrt{15}$ BM

Spin magnetic moment of $[Cr(H_2O)_6]^{3+}(t_{2g}^3 e_g^0)$

$\mu_2 = \sqrt{3(3+2)} = \sqrt{15}$ BM

$$\frac{\mu_1}{\mu_2} = \frac{\sqrt{15}}{\sqrt{15}} = 1$$

83. An atomic substance A of molar mass 12 g mol^{-1} has a cubic crystal structure with edge length of 300 pm. The no. of atoms present in one unit cell of A is _____. (Nearest integer)

Given the density of A is 3.0 g mL^{-1} and N_A = 6.02 $\times 10^{23}$ mol^{-1}

Official Ans. by NTA (4)

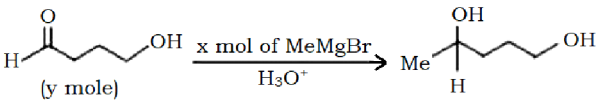
Sol. d = 3 g/cc M = 12 g/mol

a = 300 pm = 3×10^{-8} cm

$$Z = \frac{d \times N_A \times a^3}{M} = \frac{3 \times 6.02 \times 10^{23} \times (3 \times 10^{-8})^3}{12}$$

$$= 4.06 \approx 4$$

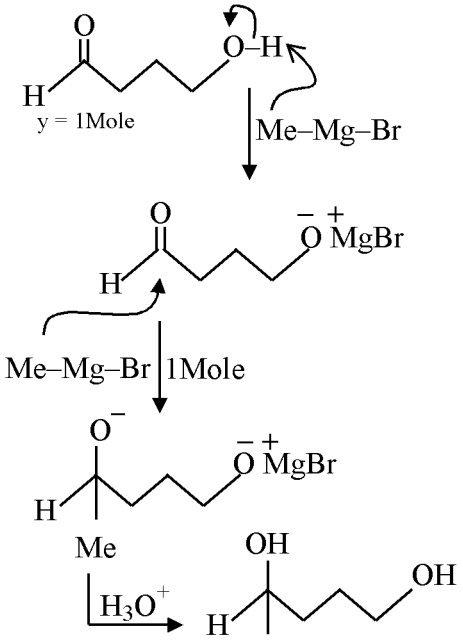
84.



The ratio x/y on completion of the above reaction is _____.

Official Ans. by NTA (2)

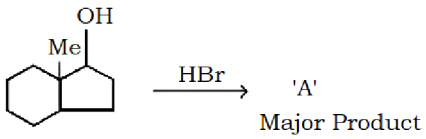
Sol.



$\therefore$ x = 2 mole

$$\frac{x}{y} = \frac{2}{1} = 2$$

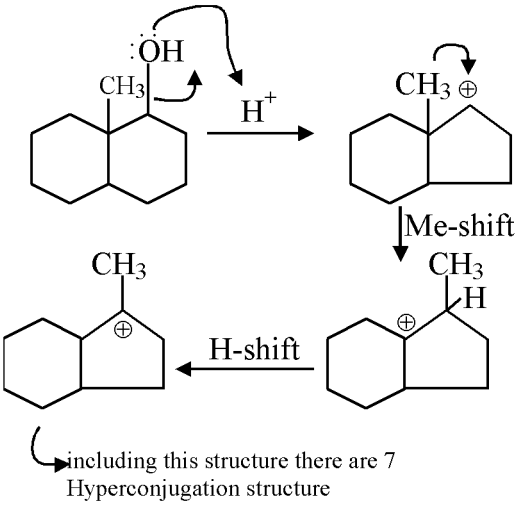
85.



The number of hyperconjugation structures involved to stabilize carbocation formed in the above reaction is _____.

Official Ans. by NTA (7)

Sol.



86. Solid fuel used in rocket is a mixture of Fe_2O_3 and Al (in ratio 1 : 2). The heat evolved (kJ) per gram of the mixture is _____ (Neatest integer)

Given : $\Delta H_f^\theta (Al_2O_3) = -1700\ kJ\ mol^{-1}$

$\Delta H_f^\theta (Fe_2O_3) = -840\ kJ\ mol^{-1}$

Molar mass of Fe, Al and O are 56, 27 and 16 g mol^{-1} respectively.

Official Ans. by NTA (4)

Sol.

$$Fe_2O_3 + 2Al \rightarrow Al_2O_3 + 2Fe$$

Molar mass 160g 27g

$$\left(\Delta H_f^0\right)_{reaction} = \left[\left(\Delta H_f^0\right)_{Al_2O_3} + 2\left(\Delta H_f^0\right)_{Fe}\right] - \left[\left(\Delta H_f^0\right)_{Fe_2O_3} + 2\left(\Delta H_f^0\right)_{Al}\right]$$

= [– 1700 + 0] – [– 840 + 0]
= – 860 kJ/mol
Total mass of mixture = Fe_2O_3 + Al (1 : 2 molar ratio)
= 160 + 2 × 27
= 214 g/mol

Heat evolved per gram $= \frac{860}{214} = 4\ kJ/g$

87. A solution of sugar is obtained by mixing 200 g of its 25% solution and 500 g of its 40% solution (both by mass). The mass percentage of the resulting sugar solution is _____. (Nearest integer)

Official Ans. by NTA (36)

Sol. Total mass of sugar in mixture of 25% of 200 and 40% of 500 g

Sugar solution = $0.25 \times 200 + 0.40 \times 500$

$= 50 + 200 = 250$ g

Total mass of solution = 200 + 500 = 700 g

Mass of sugar in solution = $\frac{250}{700} \times 100 = 35.7\%$

$\approx 36\%$

88. $KClO_3 + 6FeSO_4 + 3H_2SO_4 \rightarrow$

$KCl + 3Fe_2(SO_4)_3 + 3H_2O$

The above reaction was studied at 300 K by monitoring the concentration of $FeSO_4$ in which initial concentration was 10 M and after half an hour became 8.8 M. The rate of production of $Fe_2(SO_4)_3$ is _____ $\times 10^{-6}$ mol L^{-1} s^{-1}.

(Nearest integer)

Official Ans. by NTA (333)

Sol. $KClO_3 + 6FeSO_4 + 3H_2SO_4 \rightarrow KCl + 3Fe_2(SO_4)_3 + 3H_2O$

$$ROR = -\frac{\Delta[KClO_3]}{\Delta t} = \frac{-1}{6}\frac{\Delta[FeSO_4]}{\Delta t} = \frac{+1}{3}\frac{\Delta[Fe_2(SO_4)_3]}{\Delta t}$$

$$\frac{\Delta[Fe_2(SO_4)_3]}{\Delta t} = \frac{1}{2}\frac{-\Delta[FeSO_4]}{\Delta t}$$

$$= \frac{1}{2}\frac{(10-8.8)}{30\times 60}$$

$$= 0.333 \times 10^{-3}$$

$$= 333 \times 10^{-6} \text{ mol litre}^{-1} \text{ sec}^{-1}$$

89. 0.004 M K_2SO_4 solution is isotonic with 0.01 M glucose solution. Percentage dissociation of K_2SO_4 is _____ (Nearest integer)

Official Ans. by NTA (75)

Sol. Isotonic solutions,

$\pi_{K_2SO_4} = \pi_{Glucose}$

$i \times 0.004 \times RT = 0.01 \times RT$

$i = 2.5$

For K_2SO_4 {for dissociation $i = 1 + (n-1)\alpha$}

$$DOD(\alpha) = \frac{i-1}{n-1} = \frac{2.5-1}{3-1} = 0.75$$

% dissociation = 75

90. In an electrochemical reaction of lead, at standard temperature, if $E^0_{(Pb^{2+}/Pb)} = m$ Volt and $E^0_{(Pb^{4+}/Pb)} = n$ Volt, then the value of $E^0_{(Pb^{2+}/Pb^{4+})}$ is given by m – xn. The value of x is _____.

(Nearest integer)

Official Ans. by NTA (2)

Sol. $Pb^{2+} + 2e^- \rightarrow Pb$ $\quad \Delta G^0_1 = -2FE^0_1$

$Pb^{4+} + 4e^- \rightarrow Pb$ $\quad \Delta G^0_2 = -4FE^0_2$

$Pb^{2+} \rightarrow Pb^{4+} + 2e^-$ $\quad \Delta G^0_3 = -2FE^0_3$

$\Delta G^0_3 = \Delta G^0_1 - \Delta G^0_2$

$-2FE^0_3 = 2F(2n-m)$

$E^0_3 = m - 2n = m - xn$

Hence x = 2

FINAL JEE–MAIN EXAMINATION – APRIL, 2023

(Held On Tuesday 11th April, 2023) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. If $\begin{vmatrix} x+1 & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda^2 \end{vmatrix} = \frac{9}{8}(103x+81)$, then λ, $\frac{\lambda}{3}$ are the roots of the equation

(1) $4x^2 + 24x - 27 = 0$ (2) $4x^2 - 24x + 27 = 0$

(3) $4x^2 + 24x + 27 = 0$ (4) $4x^2 - 24x - 27 = 0$

Official Ans. by NTA (2)

Sol. Put $x = 0$

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda^2 \end{vmatrix} = \frac{9}{8} \times 81$$

$$\lambda^3 = \frac{9^3}{8} \therefore \lambda = \frac{9}{2}$$

$$\therefore \frac{\lambda}{3} = \frac{3}{2}$$

$$\therefore \text{Required equation is} : x^2 - x\left(\frac{9}{2} + \frac{3}{2}\right)x + \frac{27}{4} = 0$$

$$4x^2 - 24x + 27 = 0$$

2. Let the line passing through the points, P (2, – 1, 2) and Q (5, 3, 4) meet the plane $x - y + z = 4$ at the point R. Then the distance of the point R from the plane $x + 2y + 3z + 2 = 0$ measured parallel to the line $\frac{x-7}{2} = \frac{y+3}{2} = \frac{z-2}{1}$ is equal to

(1) $\sqrt{31}$ (2) $\sqrt{189}$

(3) $\sqrt{61}$ (4) 3

Official Ans. by NTA (4)

Sol. Line : $\frac{x-5}{3} = \frac{y-3}{4} = \frac{z-4}{2} = \lambda$

$R(3\lambda+5, 4\lambda+3, 2\lambda+4)$

$\therefore 3\lambda + 5 - 4\lambda - 3 + 2\lambda + 4 = 4$

$\lambda + 6 = 4 \therefore \lambda = -2$

$\therefore R \equiv (-1, -5, 0)$

Line : $\frac{x+1}{2} = \frac{y+5}{2} = \frac{z-0}{1} = \mu$

Point $T = (2\mu - 1, 2\mu - 5, \mu)$

It lies on plane

$2\mu - 1 + 2(2\mu - 5) + 3\mu + 2 = 0$

$\mu = 1$

$\therefore T = (1, -3, 1)$

$\therefore RT = 3$

3. If the 1011^{th} term from the end in the binomial expansion of $\left(\frac{4x}{5} - \frac{5}{2x}\right)^{2022}$ is 1024 times 1011^{th} term from the beginning, then $|x|$ is equal to

(1) 12 (2) 8

(3) 10 (4) 4 15

Official Ans. by NTA (3)

Sol. T_{1011} from beginning $= T_{1010+1}$

$= {}^{2022}C_{1010}\left(\frac{4x}{5}\right)^{1012}\left(\frac{-5}{2x}\right)^{1010}$

T_{1011} from end

$= {}^{2022}C_{1010}\left(\frac{-5}{2x}\right)^{1012}\left(\frac{4x}{5}\right)^{1010}$

Given : ${}^{2022}C_{1010}\left(\frac{-5}{2x}\right)^{1012}\left(\frac{4x}{5}\right)^{1010}$

$= 2^{10} \cdot {}^{2022}C_{1010}\left(\frac{-5}{2x}\right)^{1010}\left(\frac{4x}{5}\right)^{1012}$

$\left(\frac{-5}{2x}\right)^2 = 2^{10}\left(\frac{4x}{5}\right)^2$

$x^4 = \frac{5^4}{2^{16}}$

$|x| = \frac{5}{16}$

4. Let the function $f : [0, 2] \to R$ be defined as

$$f(x)=\begin{cases} e^{\min\{x^2, x-[x]\}}, & x\in[0,1) \\ e^{[x-\log_e x]}, & x\in[1,2] \end{cases}$$

where [t] denotes the greatest integer less than or equal to t. Then the value of the integral $\int_0^2 xf(x)dx$ is

(1) $2e-1$ (2) $1+\frac{3e}{2}$

(3) $2e-\frac{1}{2}$ (4) $(e-1)\left(e^2+\frac{1}{2}\right)$

Official Ans. by NTA (3)

Sol. Minimum $\{x^2, \{x\}\} = x^2;\ x\in[0,1)$

$$[x-\log_e x]=1;\ x\in[1,2)$$

$$\therefore f(x)=\begin{cases} e^{x^2}; x\in[0,1) \\ e\ ; x\in[1,2) \end{cases}$$

$$\int_0^2 xf(x)dx=\int_0^1 xe^{x^2}dx+\int_1^2 ex\,dx$$

$$=\frac{1}{2}(e-1)+\frac{1}{2}(4-1)e$$

$$=2e-\frac{1}{2}$$

5. Let y = y (x) be the solution of the differential equations $\frac{dy}{dx}+\frac{5}{x(x^5+1)}y=\frac{(x^5+1)^2}{x^7}, x>0.$ If y(1) = 2, then y(2) is equal to

(1) $\frac{637}{128}$ (2) $\frac{679}{128}$

(3) $\frac{693}{128}$ (4) $\frac{697}{128}$

Official Ans. by NTA (3)

Sol. $I.F = e^{\int\frac{5dx}{x(x^5+1)}} = e^{\int\frac{5x^{-6}dx}{(x^{-5}+1)}}$

Put, $1+x^{-5}=t \Rightarrow -5x^{-6}dx=dt$

$$\Rightarrow e^{\int\frac{-dt}{t}}=\frac{1}{t}=\frac{x^5}{1+x^5}$$

$$y\cdot\frac{x^5}{1+x^5}=\int\frac{x^5}{(1+x^5)}\times\frac{(1+x^5)^2}{x^7}dx$$

$$=\int x^3dx+\int x^{-2}dx$$

$$y\cdot\frac{x^5}{1+x^5}=\frac{x^4}{4}-\frac{1}{x}+c$$

Given that : $x = 1 \Rightarrow y = 2$

$$2\cdot\frac{1}{2}=\frac{1}{4}-1+c$$

$$c=\frac{7}{4}$$

$$y\cdot\frac{x^5}{1+x^5}=\frac{x^4}{4}-\frac{1}{x}+\frac{7}{4}$$

Now put, x = 2

$$y\cdot\left(\frac{32}{33}\right)=\frac{21}{4}$$

$$y=\frac{693}{128}$$

6. If four distinct points with position vectors $\vec{a},\vec{b},\vec{c}$ and $\vec{d}$ are coplanar; then $[\vec{a}\,\vec{b}\,\vec{c}]$ is equal to

(1) $[\vec{d}\,\vec{c}\,\vec{a}]+[\vec{b}\,\vec{d}\,\vec{a}]+[\vec{c}\,\vec{d}\,\vec{b}]$

(2) $[\vec{d}\,\vec{b}\,\vec{a}]+[\vec{a}\,\vec{c}\,\vec{d}]+[\vec{d}\,\vec{b}\,\vec{c}]$

(3) $[\vec{a}\,\vec{d}\,\vec{b}]+[\vec{d}\,\vec{c}\,\vec{a}]+[\vec{d}\,\vec{b}\,\vec{c}]$

(4) $[\vec{b}\,\vec{c}\,\vec{d}]+[\vec{d}\,\vec{a}\,\vec{c}]+[\vec{d}\,\vec{b}\,\vec{a}]$

Official Ans. by NTA (1)

Sol. a, b, c, d are coplanar points.

$\vec{b}-\vec{a},\vec{c}-\vec{a},\vec{d}-\vec{a}$ are coplanar vectors.

So, $[\vec{b}-\vec{a}\ \vec{c}-\vec{a}\ \vec{d}-\vec{a}]=0$

$$(\vec{b}-\vec{a})\cdot((\vec{c}-\vec{a})\times(\vec{d}-\vec{a}))=0$$

$$[\vec{b}\ \vec{c}\ \vec{d}]-[\vec{b}\ \vec{c}\ \vec{a}]-[\vec{b}\ \vec{a}\ \vec{d}]-[\vec{a}\ \vec{c}\ \vec{d}]=0$$

$$\Rightarrow[\vec{a}\,\vec{b}\,\vec{c}]=[\vec{c}\,\vec{d}\,\vec{b}]+[\vec{b}\,\vec{d}\,\vec{a}]+[\vec{d}\ \vec{c}\ \vec{a}]$$

7. If $f:\mathbb{R}\to\mathbb{R}$ be a continuous function satisfying

$$\int_0^{\pi/2} f(\sin 2x)\cdot\sin x dx+\alpha\int_0^{\pi/4} f(\cos 2x)\cdot\cos x\,dx=0,$$

then α is equal to

(1) $-\sqrt{3}$ (2) $\sqrt{2}$

(3) $\sqrt{3}$ (4) $-\sqrt{2}$

Official Ans. by NTA (4)

Sol. $I=\int_0^{\frac{\pi}{4}} f(\sin 2x)\sin x\,dx+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} f(\sin 2x)\sin x\,dx$

$$+\alpha\int_0^{\frac{\pi}{4}} f(\cos 2x)\cos x\,dx=0$$

Apply king in first part and put $x-\frac{\pi}{4}=t$ in second part.

$$I=\int_0^{\frac{\pi}{4}} f(\cos 2x)\sin\left(\frac{\pi}{4}-x\right)dx+\int_0^{\frac{\pi}{4}} f(\cos 2t)\sin\left(\frac{\pi}{4}+t\right)dt$$

$$+\alpha\int_0^{\frac{\pi}{4}} f(\cos 2x)\cos x\,dx = 0$$

$$I=\int_0^{\frac{\pi}{4}} f(\cos 2x)\left[2\sin\frac{\pi}{4}\cdot\cos x+\alpha\cos x\right]dx=0$$

$$I=\left(\alpha+\sqrt{2}\right)\int_0^{\frac{\pi}{4}} f(\cos 2x)\cos x\,dx=0$$

$$\therefore \alpha=-\sqrt{2}$$

8. If the system of linear equations

$7x+11y+\alpha z=13$

$5x+4y+7z=\beta$

$175x+194y+57z=361$

has infinitely many solutions, then $\alpha+\beta+2$ is equal to

(1) 4 (2) 3

(3) 5 (4) 6

Official Ans. by NTA (1)

Sol. $7x+11y+\alpha z=13$ ..... (i)

$5x+4y+7z=\beta$ ..... (ii)

$175x+194y+57z=361$ ..... (iii)

(i) × 10 + (ii) × 21 – (iii)

$z(10\alpha+147-57)=130+21\beta-361$

$\therefore 10\alpha+90=0$

$\alpha=-9$

$130-361+21\beta=0$

$\beta=11$

$\alpha+\beta+2=4$

9. The domain of the function

$f(x)=\dfrac{1}{\sqrt{[x]^2-3[x]-10}}$ is (where [x] denotes the greatest integer less than or equal to x)

(1) $(-\infty,-2)\cup(5,\infty)$ (2) $(-\infty,-3]\cup[6,\infty)$

(3) $(-\infty,-2)\cup[6,\infty)$ (4) $(-\infty,-3]\cup(5,\infty)$

Official Ans. by NTA (3)

Sol. $[x]^2-3[x]-10>0$

$[x]<-2 \text{ or } [x]>5$

10. Let P be the plane passing through the points (5, 3, 0), (13, 3, –2) and (1, 6, 2). For $\alpha\in N$, if the distances of the points A (3, 4, α) and B (2, α, a) from the plane P are 2 and 3 respectively, then the positive value of a is

(1) 6 (2) 4

(3) 3 (4) 5

Official Ans. by NTA (2)

Sol. $\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 8 & 0 & -2\\ 4 & -3 & -2\end{vmatrix}=\hat{i}(-6)+8\hat{j}-24\hat{k}$

Normal of the plane = $3\hat{i}-4\hat{j}+12\hat{k}$

Plane : $3x-4y+12z=3$

Distance from A (3, 4, α)

$\left|\dfrac{9-16+12\alpha-3}{13}\right|=2$

$\alpha=3$

$\alpha=-8(\text{rejected})$

Distance from B (2, 3, a)

$\left|\dfrac{6-12+12a-3}{13}\right|=3$

a = 4

11. The converse of the statement $((\sim p)\wedge q)\Rightarrow r$ is

(1) $(\sim r)\Rightarrow p\wedge q$ (2) $(\sim r)\Rightarrow((\sim p)\wedge q)$

(3) $((\sim p)\vee q)\Rightarrow r$ (4) $(p\vee(\sim q))\Rightarrow(\sim r)$

Official Ans. by NTA (4)

Sol. Converse of $(\sim p)\wedge q)\Rightarrow r$

$\equiv r\Rightarrow(\sim p\wedge q)$

$\equiv\sim r\vee(\sim p\wedge q)$

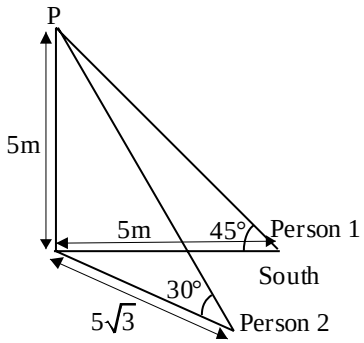
$\equiv\sim r\vee(p\vee\sim q)\equiv(p\vee\sim q)\Rightarrow\sim r$

12. The angle of elevation of the top P of a tower from the feet of one person standing due South of the tower is 45° and from the feet of another person standing due west of the tower is 30°. If the height of the tower is 5 meters, then the distance (in meters) between the two persons is equal to

(1) 10 (2) 5

(3) $5\sqrt{5}$ (4) $\dfrac{5}{2}\sqrt{5}$

Official Ans. by NTA (1)



Sol.
Distance = 10 (By Pythagoras theorem)

13. Let a, b, c and d be positive real numbers such that a + b + c + d = 11. If the maximum value of $a^5b^3c^2d$ is 3750β, then the value of β is

(1) 90 (2) 110

(3) 55 (4) 108

Official Ans. by NTA (1)

Sol. $$\frac{5\left(\frac{a}{5}\right)+3\left(\frac{b}{3}\right)+2\left(\frac{c}{2}\right)+d}{11} \geq \left(\frac{a^5b^3c^2d}{5^5 3^3 2^2}\right)^{1/11}$$

$$1 \geq \left(\frac{a^5b^3c^2d}{5^5 3^3 2^2}\right)^{1/11}$$

$$\beta = 90$$

14. If the radius of the largest circle with centre (2, 0) inscribed in the ellipse $x^2 + 4y^2 = 36$ is r, then $12r^2$ is equal to

(1) 72 (2) 115

(3) 92 (4) 69

Official Ans. by NTA (3)

Sol. $(x-2)^2 + y^2 = r^2$

Solving with ellipse, we get

$$(x-2)^2 + \frac{36-x^2}{4} = r^2$$

$$3x^2 - 16x + 52 - 4r^2 = 0$$

$$D = 0 \Rightarrow 4r^2 = \frac{92}{3}$$

15. Let the mean of 6 observation 1, 2, 4, 5, x and y be 5 and their variance be 10. Then their mean deviation about the mean is equal to

(1) $\frac{10}{3}$ (2) $\frac{7}{3}$

(3) 3 (4) $\frac{8}{3}$

Official Ans. by NTA (4)

Sol. x + y = 18 $\because$ mean = 5 ..... (i)

$$10 = \frac{1+4+16+25+x^2+y^2}{6} - 25$$

$$x^2 + y^2 = 164 \ldots\ldots \text{(ii)}$$

By solving (i) and (ii)

x = 8, y = 10

$$\text{M.D.}(\bar{x}) = \frac{\sum|x_i - \bar{x}|}{6} = \frac{8}{3}$$

16. The sum of the coefficients of three consecutive terms in the binomial expansion of $(1+x)^{n+2}$, which are in the ratio 1 : 3 : 5, is equal to

(1) 25 (2) 63

(3) 41 (4) 92

Official Ans. by NTA (2)

Sol. $${}^{n+2}C_{r-1} : {}^{n+2}C_r : {}^{n+2}C_{r+1} = 1:3:5$$

$$\frac{{}^{n+2}C_{r-1}}{{}^{n+2}C_r} = \frac{1}{3}$$

$$n = 4r - 3 \ldots\ldots \text{(i)}$$

$$\frac{{}^{n+2}C_r}{{}^{n+2}C_{r+1}} = \frac{3}{5}$$

$$8r - 1 = 3n \ldots\ldots \text{(ii)}$$

From, (i) and (ii)

r = 2 and n = 5

Required sum = 63

17. If the letters of the word MATHS are permuted and all possible words so formed are arranged as in a dictionary with serial numbers, then the serial number of the word THAMS is

(1) 103 (2) 104

(3) 101 (4) 102

Official Ans. by NTA (1)

Sol. $4 \times 4! + 1 \times 3! + 1 = 103$

18. For $a \in C$, let $A = \{z \in C : \text{Re}(a + \bar{z}) > \text{Im}(\bar{a} + z)\}$ and $B = \{z \in C : \text{Re}(a + \bar{z}) < \text{Im}(\bar{a} + z)\}$. Then among the two statements :

(S1) : If Re (A), Im (A) > 0, then the set A contains all the real numbers

(S2) : If Re (A), Im (A) < 0, then the set B contains all the real numbers,

(1) Only (S1) is true (2) both are false

(3) Only (S2) is true (4) Both are true

Official Ans. by NTA (2)

Sol. Let $a = x_1 + iy_1$ $z = x + iy$

Now $\mathrm{Re}(a + \bar{z}) > \mathrm{Im}(\bar{a} + z)$

$\therefore x_1 + x > -y_1 + y$

$x_1 = 2, y_1 = 10, x = -12, y = 0$

Given inequality is not valid for these values.

S1 is false.

Now $\mathrm{Re}(a + \bar{z}) < \mathrm{Im}(\bar{a} + z)$

$x_1 + x < -y_1 + y$

$x_1 = -2, y_1 = -10, x = 12, y = 0$

Given inequality is not valid for these values.

S2 is false.

19. Let A = {1, 3, 4, 6, 9} and B = {2, 4, 5, 8, 10}. Let R be a relation defined on A × B such that R = {((a₁, b₁),(a₂, b₂)) : $a_1 \le b_2$ and $b_1 \le a_2$ }. Then the number of elements in the set R is

(1) 26 (2) 160

(3) 180 (4) 52

Official Ans. by NTA (2)

Sol. Let $a_1 = 1 \Rightarrow 5$ choices of b_2

$a_1 = 3 \Rightarrow 4$ choices of b_2

$a_1 = 4 \Rightarrow 4$ choices of b_2

$a_1 = 6 \Rightarrow 2$ choices of b_2

$a_1 = 9 \Rightarrow 1$ choices of b_2

For (a_1, b_2) 16 ways .

Similarly, $b_1 = 2 \Rightarrow 4$ choices of a_2

$b_1 = 4 \Rightarrow 3$ choices of a_2

$b_1 = 5 \Rightarrow 2$ choices of a_2

$b_1 = 8 \Rightarrow 1$ choices of a_2

Required elements in R = 160

20. Let f and g be two functions defined by

$$f(x) = \begin{cases} x+1, & x < 0 \\ |x-1|, & x \ge 0 \end{cases} \text{ and } g(x) = \begin{cases} x+1, & x < 0 \\ 1, & x \ge 0 \end{cases}.$$

Then (gof) (x) is

(1) Differentiable everywhere

(2) Continuous everywhere but not differentiable exactly at one point

(3) Not continuous at x = – 1

(4) Continuous everywhere but not differentiable at x = 1

Official Ans. by NTA (2)

Sol. $f(x) = \begin{cases} x+1, & x < 0 \\ 1-x, & 0 \le x < 1 \\ x-1, & 1 \le x \end{cases}$

$g(x) = \begin{cases} x+1, & x < 0 \\ 1, & x \ge 0 \end{cases}$

$g(f(x)) = \begin{cases} x+2, & x < -1 \\ 1, & x \ge -1 \end{cases}$

$\therefore g(f(x))$ is continuous everywhere

$g(f(x))$ is not differentiable at x = – 1

Differentiable everywhere else

SECTION-B

21. The number of points, where the curve

$f(x) = e^{8x} - e^{6x} - 3e^{4x} - e^{2x} + 1$, $x \in \mathbb{R}$ cuts x-axis, is equal to

Official Ans. by NTA (2)

Sol. Let $e^{2x} = t$

$\Rightarrow t^4 - t^3 - 3t^2 - t + 1 = 0$

$\Rightarrow t^2 + \frac{1}{t^2} - \left(t + \frac{1}{t}\right) - 3 = 0$

$\Rightarrow \left(t + \frac{1}{t}\right)^2 - \left(t + \frac{1}{t}\right) - 5 = 0$

$\Rightarrow t + \frac{1}{t} = \frac{1 + \sqrt{21}}{2}$

Two real values of t.

22. Let the probability of getting head for a biased coin be $\frac{1}{4}$. It is tossed repeatedly until a head appears. Let N be the number of tosses required. If the probability that the equation $64x^2 + 5Nx + 1 = 0$ has no real root is $\frac{p}{q}$, where p and q are co-prime, then q – p is equal to

Official Ans. by NTA (27)

Sol. $64x^2+5Nx+1=0$

$D=25N^2-256<0$

$\Rightarrow N^2<\frac{256}{25}\Rightarrow N<\frac{16}{5}$

$\therefore N=1,2,3$

$\therefore \text{Probability}=\frac{1}{4}+\frac{3}{4}\times\frac{1}{4}+\frac{3}{4}\times\frac{3}{4}\times\frac{1}{4}=\frac{37}{64}$

$\therefore q-p=27$

23. Let $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}-\hat{k}$. If $\vec{c}$ is a vector such that $\vec{a}\cdot\vec{c}=11$, $\vec{b}\cdot(\vec{a}\times\vec{c})=27$ and $\vec{b}\cdot\vec{c}=-\sqrt{3}|\vec{b}|$, then $|\vec{a}\times\vec{c}|^2$ is equal to

Official Ans. by NTA (285)

Sol. $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=\hat{i}+\hat{j}-\hat{k}$

$\vec{b}\cdot(\vec{a}\times\vec{c})=27$, $\vec{a}\cdot\vec{b}=0$

$\vec{b}\times(\vec{a}\times\vec{c})=-3\vec{a}$

Let θ be angle between $\vec{b}$, $\vec{a}\times\vec{c}$

Then $|\vec{b}|\cdot|\vec{a}\times\vec{c}|\sin\theta=3\sqrt{14}$

$|\vec{b}|\cdot|\vec{a}\times\vec{c}|\cos\theta=27$

$\Rightarrow \sin\theta=\frac{\sqrt{14}}{\sqrt{95}}$

$\therefore |\vec{b}|\times|\vec{a}\times\vec{c}|=3\sqrt{95}$

$\Rightarrow |\vec{a}\times\vec{c}|=\sqrt{3}\times\sqrt{95}$

24. Let $S=\left\{z\in C-\{i,2i\}:\frac{z^2+8iz-15}{z^2-3iz-2}\in R\right\}$. If $\alpha-\frac{13}{11}i\in S,\alpha\in\mathbb{R}-\{0\}$, then $242\alpha^2$ is equal to

Official Ans. by NTA (1680)

Sol. $\left(\frac{z^2+8iz-15}{z^2-3iz-2}\right)\in R$

$\Rightarrow 1+\frac{(11iz-13)}{(z^2-3iz-2)}\in R$

Put $z=\alpha-\frac{13}{11}i$

$\Rightarrow (z^2-3iz-2)$ is imaginary

Put $z=x+iy$

$\Rightarrow (x^2-y^2+2xyi-3ix+3y-2)\in$ Imaginary

$\Rightarrow \text{Re}(x^2-y^2+3y-2+(2xy-3x)i)=0$

$\Rightarrow x^2-y^2+3y-2=0$

$x^2=y^2-3y+2$

$x^2=(y-1)(y-2)\ \therefore z=\alpha-\frac{13}{11}i$

Put $x=\alpha$, $y=\frac{-13}{11}$

$\alpha^2=\left(\frac{-13}{11}-1\right)\left(\frac{-13}{11}-2\right)$

$\alpha^2=\frac{(24\times35)}{121}$

$242\alpha^2=48\times35=1680$

25. For $k\in\mathbb{N}$, if the sum of the series

$1+\frac{4}{k}+\frac{8}{k^2}+\frac{13}{k^3}+\frac{19}{k^4}+\ldots$ is 10, then the value of k is

Official Ans. by NTA (2)

Sol. $10 = 1 + \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + ...\text{upto}\infty$

$9 = \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + ...\text{upto}\infty$

$\frac{9}{k} = \frac{4}{k^2} + \frac{8}{k^3} + \frac{13}{k^4} + ...\text{upto}\infty$

$S = 9\left(1 - \frac{1}{k}\right) = \frac{4}{k} + \frac{4}{k^2} + \frac{5}{k^3} + \frac{6}{k^4} + ...\text{upto}\infty$

$\frac{S}{k} = \quad \frac{4}{k^2} + \frac{4}{k^3} + \frac{5}{k^4} + ...\text{upto}\infty$

$\left(1 - \frac{1}{k}\right)S = \frac{4}{k} + \frac{1}{k^3} + \frac{1}{k^4} + \frac{1}{k^5} + ...\infty$

$9\left(1 - \frac{1}{k}\right)^2 = \frac{4}{k} + \frac{\frac{1}{k^3}}{\left(1 - \frac{1}{k}\right)}$

$9(k-1)^3 = 4k(k-1) + 1$

$k = 2$

26. Let A = {1, 2, 3, 4, 5} and B = {1, 2, 3, 4, 5, 6}. Then the number of functions $f : A \to B$ satisfying f(1) + f(2) = f(4) – 1 is equal to

Official Ans. by NTA (360)

Sol. $f(1) + f(2) + 1 = f(4) \le 6$

$f(1) + f(2) \le 5$

Case (i) $f(1) = 1 \Rightarrow f(2) = 1, 2, 3, 4 \Rightarrow 4 \text{ mappings}$

Case (ii) $f(1) = 2 \Rightarrow f(2) = 1, 2, 3 \Rightarrow 3 \text{ mappings}$

Case (iii) $f(1) = 3 \Rightarrow f(2) = 1, 2 \Rightarrow 2 \text{ mappings}$

Case (iv) $f(1) = 4 \Rightarrow f(2) = 1 \Rightarrow 1 \text{ mapping}$

$f(5)$ & $f(6)$ both have 6 mappings each

Number of functions $= (4 + 3 + 2 + 1) \times 6 \times 6 = 360$

27. Let the tangent to the parabola $y^2 = 12x$ at the point $(3, \alpha)$ be perpendicular to the line 2x + 2y = 3. Then the square of distance of the point (6, – 4) from the normal to the hyperbola $\alpha^2x^2 - 9y^2 = 9\alpha^2$ at its point $(\alpha - 1, \alpha + 2)$ is equal to

Official Ans. by NTA (116)

Sol. $\because P(3, \alpha)$ lies on $y^2 = 12x$

$\Rightarrow \alpha = \pm 6$

But, $\left.\frac{dy}{dx}\right|_{(3,\alpha)} = \frac{6}{\alpha} = 1 \Rightarrow \alpha = 6 (\alpha = -6 \text{ reject})$

Now, hyperbola $\frac{x^2}{9} - \frac{y^2}{36} = 1$, normal at $Q(\alpha - 1, \alpha + 2)$ is $\frac{9x}{5} + \frac{36y}{8} = 45$

$\Rightarrow 2x + 5y - 50 = 0$

Now, distance of (6, –4) from 2x + 5y – 50 = 0 is equal to

$\left|\frac{2(6) - 5(4) - 50}{\sqrt{2^2 + 5^2}}\right| = \frac{58}{\sqrt{29}}$

$\Rightarrow \text{Square of distance} = 116$

28. Let the line $\ell : x = \frac{1 - y}{-2} = \frac{z - 3}{\lambda}, \lambda \in \mathbb{R}$ meet the plane P : x + 2y + 3z = 4 at the point (α, β, γ). If the angle between the line ℓ and the plane P is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then $\alpha + 2\beta + 6\gamma$ is equal to

Official Ans. by NTA (11)

Sol. $\ell : x = \frac{y-1}{2} = \frac{z-3}{\lambda}, \lambda \in \mathbb{R}$

DR's of line ℓ (1, 2, λ)

DR's of normal vector of plane P : x + 2y + 3z = 4 are (1, 2, 3)

Now, angle between line ℓ and plane P is given by

$$\sin\theta = \left|\frac{1+4+3\lambda}{\sqrt{5+\lambda^2}\cdot\sqrt{14}}\right| = \frac{3}{\sqrt{14}}\left(\text{given}\cos\theta = \sqrt{\frac{5}{14}}\right)$$

$$\Rightarrow \lambda = \frac{2}{3}$$

Let variable point on line ℓ is $\left(t, 2t+1, \frac{2}{3}t+3\right)$ lies on plane P.

$\Rightarrow t = -1$

$\Rightarrow \left(-1, -1, \frac{7}{3}\right) \equiv (\alpha, \beta, \gamma)$

$\Rightarrow \alpha + 2\beta + 6\gamma = 11$

29. If the line $\ell_1 : 3y - 2x = 3$ is the angular bisector of the lines $\ell_2 : x - y + 1 = 0$ and $\ell_3 : \alpha x + \beta y + 17 = 0$, then $\alpha^2 + \beta^2 - \alpha - \beta$ is equal to

Official Ans. by NTA (348)

Sol. Point of intersection of $\ell_1 : 3y - 2x = 3$

$\ell_2 : x - y + 1 = 0$ is $P \equiv (0,1)$

Which lies on $\ell_3 : \alpha x + \beta y + 17 = 0$,

$\Rightarrow \boxed{\beta = -17}$

Consider a random point $Q \equiv (-1, 0)$

on $\ell_2 : x - y + 1 = 0$, image of Q about

$\ell_2 : x - y + 1 = 0$ is $Q' \equiv \left(\frac{-17}{13}, \frac{6}{13}\right)$ which is calculated by formulae

$$\frac{x-(-1)}{2} = \frac{y-0}{-3} = -2\left(\frac{-2+3}{13}\right)$$

Now, Q′ lies on $\ell_3 : \alpha x + \beta y + 17 = 0$

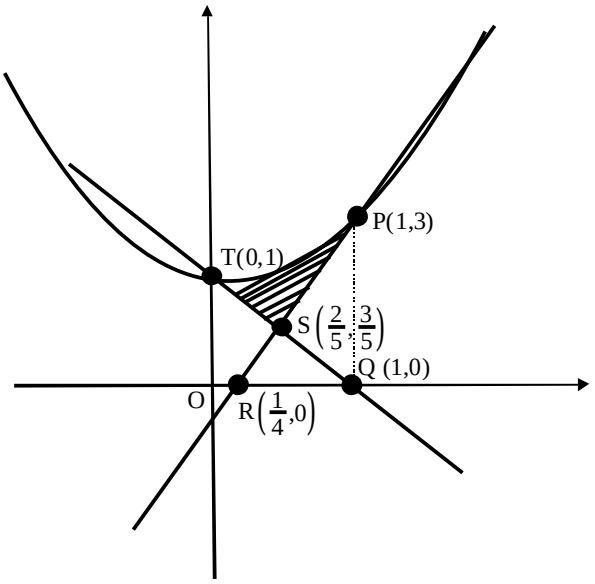
$\Rightarrow \boxed{\alpha = 7}$

Now, $\alpha^2 + \beta^2 - \alpha - \beta = 348$

30. If A is the area in the first quadrant enclosed by the curve $C : 2x^2 - y + 1 = 0$, the tangent to C at the point (1, 3) and the line x + y = 1, then the value of 60A is

Official Ans. by NTA (16)

Sol.



$y = 2x^2 + 1$

Tangent at (1, 3)

y = 4x – 1

$A = \int_0^1 \left(2x^2 + 1\right)dx - \text{area of } (\Delta QOT) - \text{area of } (\Delta PQR) + \text{area of } (\Delta QRS)$

$$A = \left(\frac{2}{3} + 1\right) - \frac{1}{2} - \frac{9}{8} + \frac{9}{40} = \frac{16}{60}$$

SECTION-A

31. Eight equal drops of water are falling through air with a steady speed of 10 cm/s. If the drops coalesce, the new velocity is:-

(1) 10 cm/s (2) 40 cm/s

(3) 16 cm/s (4) 5 cm/s

Official Ans. by NTA (2)

Sol. $v \propto r^2$

$$\frac{v_1}{v_2} = \left(\frac{r}{R}\right)^2$$

$$8.\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$R = 2r$

$$\frac{10}{v_2} = \left(\frac{1}{2}\right)^2$$

$v_2 = 40$ cm/s

32. A car P travelling at 20 ms^{-1} sounds its horn at a frequency of 400 Hz. Another car Q is travelling behind the first car in the same direction with a velocity 40 ms^{-1}. The frequency heard by the passenger of the car Q is approximately [Take, velocity of sound = 360 ms^{-1}]

(1) 514 Hz (2) 421 Hz

(3) 485 Hz (4) 471 Hz

Official Ans. by NTA (2)

Sol. $$f = f_0\left(\frac{c + v_0}{c + v_s}\right)$$

$$f = 400\left(\frac{360 + 40}{360 + 20}\right)$$

$f = 421$ Hz

33. A plane electromagnetic wave of frequency 20 MHz propagates in free space along x-direction. At a particular space and time, $\vec{E} = 6.6\hat{j}\ V/m$. What is $\vec{B}$ at this point ?

(1) $-2.2\times10^{-8}\hat{i}\,T$ (2) $2.2\times10^{-8}\hat{k}\,T$

(3) $-2.2\times10^{-8}\hat{k}\,T$ (4) $2.2\times10^{-8}\hat{i}\,T$

Official Ans. by NTA (2)

Sol. $\vec{E} = 6.6\hat{j}$

$\nu = 20\,MHz$

$\vec{c} = 3\times10^8\hat{i}$

$$|\vec{B}| = \frac{|\vec{E}|}{c} = 2.2\times10^{-8}\,T$$

$\hat{E}\times\hat{B} = \hat{c}$

$\vec{B} = 2.2\times10^{-8}\hat{k}\,T$

34. A capacitor of capacitance C is charged to a potential V. The flux of the electric field through a closed surface enclosing the positive plate of the capacitor is:

(1) $\frac{CV}{2\varepsilon_0}$ (2) $\frac{2CV}{\varepsilon_0}$

(3) $\frac{CV}{\varepsilon_0}$ (4) Zero

Official Ans. by NTA (3)

Sol. $$\phi = \frac{q_{in}}{\epsilon_0}$$

$$= \frac{Q}{\epsilon_0}$$

$$= \frac{CV}{\epsilon_0}$$

35. If force (F), velocity (V) and time (T) are considered as fundamental physical quantity, then dimensional formula of density will be:

(1) $FV^{-2}T^2$ (2) $FV^{-4}T^2$

(3) FV^4T^{-6} (3) $F^2V^{-2}T^6$

Official Ans. by NTA (2)

Sol. $[ML^{-3}] = [MLT^{-2}]^a[LT^{-1}]^b[T]^c$

$= [M^aL^{a+b}T^{-2a-b+c}]$

$a=1,$

$a + b = -3,$

$\Rightarrow b= -4,$

also $-2a - b + c=0$

$c = -2$

36. In satellite communication, the uplink frequency band used is:

(1) 3.7 – 4.2 GHz

(2) 5.925 – 6.425 GHz

(2) 76 – 88 MHz

(4) 420 – 890 MHz

Official Ans. by NTA (2)

Sol. Conceptual

37. If V is the gravitational potential due to sphere of uniform density on it's surface, then it's value at the center of sphere will be:-

(1) $\frac{3V}{2}$ (2) V

(3) $\frac{4}{3}V$ (4) $\frac{V}{2}$

Official Ans. by NTA (1)

Sol. $V = \frac{GM}{2R^3}\left(3R^2 - r^2\right)$ at $r = R \Rightarrow V = \left(\frac{GM}{R}\right)$

at $r = 0,\ V_0 = \frac{3GM}{2R} = \left(\frac{3V}{2}\right)$

38. A body of mass 500 g moves along x-axis such that it's velocity varies with displacement x according to the relation $v = 10\sqrt{x}$ m/s the force acting on the body is:-

(1) 166 N (2) 25 N

(3) 125 N (4) 5 N

Official Ans. by NTA (2)

Sol. $v = 10\sqrt{x} \Rightarrow v^2 = 100x$

$2v\frac{dv}{dx} = 100 \Rightarrow a = 50\,m/s^2$

F= 25 N

39. A projectile is projected at 30° from horizontal with initial velocity 40 ms^{-1}. The velocity of the projectile at t = 2 s from the start will be:

(Given $g = 10\ m/s^2$)

(1) $20\sqrt{3}\,ms^{-1}$ (2) $40\sqrt{3}\,ms^{-1}$

(3) 20 ms^{-1} (4) Zero

Official Ans. by NTA (1)

Sol. At t = 2 particle is at maximum height

moving with velocity $V = 40\cos 30° = 20\sqrt{3}\,ms^{-1}$.

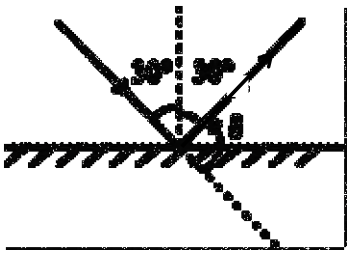
40. When one light ray is reflected from a plane mirror with 30° angle of reflection, the angle of deviation of the ray after reflection is:

(1) 140° (2) 120°

(3) 110° (4) 130°

Official Ans. by NTA (2)

Sol.



$\delta = 180° - 60° = 120°$

41. A spaceship of mass 2 × 10 kg is launched into a circular orbit close to the earth surface. The additional velocity to be imparted to the spaceship in the orbit to overcome the gravitational pull will be (if g = 10 m/s^2 and radius of earth = 6400 km)

(1) $11.2\left(\sqrt{2}-1\right)\text{km/s}$

(2) $7.9\left(\sqrt{2}-1\right)\text{km/s}$

(3) $8\left(\sqrt{2}-1\right)\text{km/s}$

(4) $7.4\left(\sqrt{2}-1\right)\text{km/s}$

Official Ans. by NTA (3)

Sol. $v_{orbit}=\sqrt{\frac{GM}{R}}=\sqrt{gR};$

$v_{escape}=\sqrt{\frac{2GM}{R}}=\sqrt{2gR}$

$\Delta v=\left(\sqrt{2}-1\right)\sqrt{gR}=8\left(\sqrt{2}-1\right)\text{km/s}$

42. The ratio of the de-Broglie wavelengths of proton and electron having same kinetic energy:

(Assume $m_p = m_e \times 1849$)

(1) 1 : 43 (2) 1 : 30

(3) 1 : 62 (4) 2 : 43

Official Ans. by NTA (1)

Sol. $\lambda \propto \frac{1}{\sqrt{m}} \Rightarrow \frac{\lambda_p}{\lambda_e}=\sqrt{\frac{m_e}{m_p}}=1:43$

43. The thermodynamic process, in which internal energy of the system remains constant is

(1) Isochoric (2) Isothermal

(3) Adiabatic (4) Isobaric

Official Ans. by NTA (2)

Sol. T = constant $\Rightarrow$ U = constant

44. The energy of He ion in its first excited state is. (The ground state energy for the Hydrogen atom is –13.6 eV):

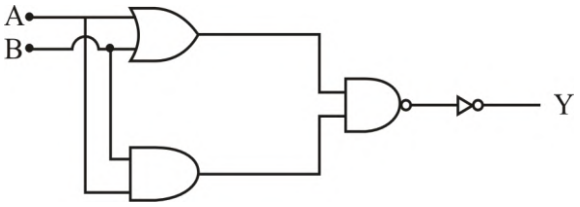
(1) –3.4 eV (2) –54.4 eV

(3) –13.6 eV (4) –27.2 eV

Official Ans. by NTA (3)

Sol. $E_n=\frac{-13.6Z^2}{n^2}=\frac{-13.6\times 4}{4}=-13.6\text{ eV}$

45. The logic operations performed by the given digital circuit is equivalent to:

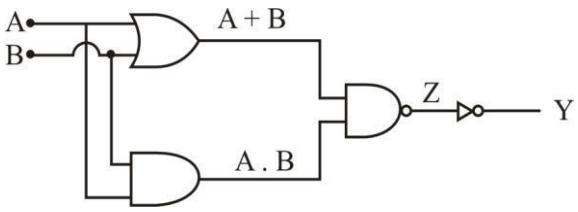


(1) AND (2) NOR

(3) OR (4) NAND

Official Ans. by NTA (1)

Sol.



$Z=\overline{(A+B).(A.B)}$

$Y=\overline{Z}=(A+B).(A.B)$

Y = A. B

∴ It is an AND Gate

46. The root mean square speed of molecules of nitrogen gas at 27°C is approximately:

(Given mass of a nitrogen molecule $=4.6\times10^{-26}$kg and take Boltzmann constant $k_B = 1.4 \times10^{-23}$ JK^{-1})

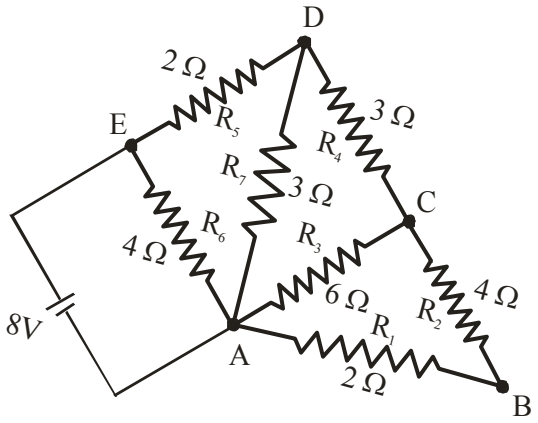
(1) 523 m/s (2) 1260 m/s

(3) 91 m/s (4) 27.4 m/s

Official Ans. by NTA (1)

Sol. $V_{rms}=\sqrt{\frac{3k_BT}{m}}=\sqrt{\frac{3\times1.4\times10^{-23}\times300}{4.6\times10^{-26}}}=523\text{ m/s}$

47.



The current flowing through R_2 is:

(1) $\frac{2}{3}$A

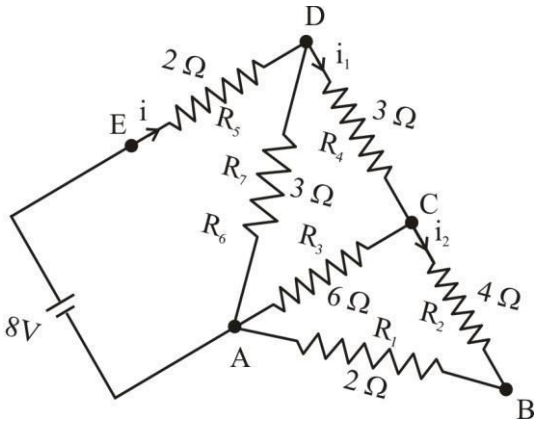
(2) $\frac{1}{4}$A

(3) $\frac{1}{2}$A

(4) $\frac{1}{3}$A

Official Ans. by NTA (4)

Sol.



$R_{eq} = 4\Omega$

$i = \frac{8}{4} = 2A$

$i_1 = \frac{2\times 3}{3+6} = \frac{2}{3}A$

$i_2 = \frac{2/3}{2} = \frac{1}{3}A$

48. When vector $\vec{A} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ is subtracted from vector $\vec{B}$, it gives a vector equal to $2\hat{j}$. Then the magnitude of vector $\vec{B}$ will be:

(1) $\sqrt{13}$

(2) 3

(3) $\sqrt{6}$

(4) $\sqrt{5}$

Official Ans. by NTA (BONUS)

Sol. $\vec{B} - \vec{A} = 2j$

$\vec{B} = 2\hat{i} + 5\hat{j} + 2\hat{k}$

$|\vec{B}| = \sqrt{33}$

49. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A: A bar magnet dropped through a metallic cylindrical pipe takes more time to come down compared to a non–magnetic bar with same geometry and mass.

Reason R: For the magnetic bar, Eddy currents are produced in the metallic pipe which oppose the motion of the magnetic bar.

In the light of the above statements, choose the correct answer from the options given below

(1) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

(2) **A** is true but **R** is false

(3) Both **A** and **R** are true and **R** is the correct explanation of **A**

(4) **A** is false but **R** is true

Official Ans. by NTA (3)

Sol. Conceptual

50. An electron is allowed to move with constant velocity along the axis of current carrying straight solenoid.

A. The electron will experience magnetic force along the axis of the solenoid.

B. The electron will not experience magnetic force.

C. The electron will continue to move along the axis of the solenoid.

D. The electron will be accelerated along the axis of the solenoid.

E. The electron will follow parabolic path–inside the solenoid.

Choose the correct answer from the options given below:

(1) B, C and D only (2) B and C only

(3) A and D only (4) B and E only

Official Ans. by NTA (2)

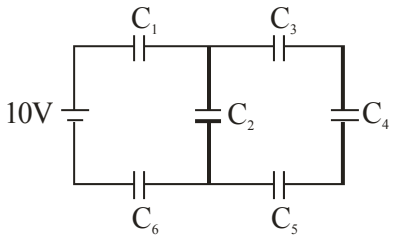
Sol. $\vec{F} = q\left(\vec{v} \times \vec{B}\right)$ as angle between $\vec{v}$ and $\vec{B}$ is 0°

$\vec{F} = 0$

SECTION-B

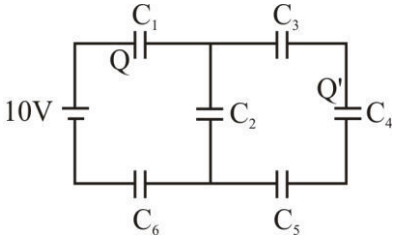
51. In the given circuit,

$C_1 = 2\ \mu F$, $C_2 = 0.2\ \mu F$, $C_3 = 2\ \mu F$, $C_4 = 4\ \mu F$, $C_5 = 2\ \mu F$, $C_6 = 2\ \mu F$, the charge stored on capacitor C_4 is _____ μC.



Official Ans. by NTA (4)

Sol.



$C_{eq} = 0.5\ \mu F$

$Q = 0.5 \times 10 = 5\ \mu C$

$$Q' = \frac{5\mu C \times 0.8}{0.8 + 0.2} = 4\mu C$$

52. A circular plate is rotating in horizontal plane, about an axis passing through its center and perpendicular to the plate, with an angular velocity ω. A person sits at the center having two dumbbells in his hands. When he stretches out his hands, the moment of inertia of the system becomes triple. If E be the initial Kinetic energy of the system, then final Kinetic energy will be $\frac{E}{x}$.

The value of x is

Official Ans. by NTA (3)

Sol. $$KE = \frac{L^2}{2I} \Rightarrow \frac{KE_{final}}{KE_{initial}} = \frac{I_{initial}}{I_{final}} \Rightarrow \frac{KE_{final}}{E} = \frac{1}{3}$$

$$\Rightarrow KE_{final} = \frac{E}{3}$$

53. A nucleus disintegrates into two nuclear parts, in such a way that ratio of their nuclear sizes is $1 : 2^{1/3}$. Their respective speed have a ratio of n : 1. The value of n is _____

Official Ans. by NTA (2)

Sol. $$\frac{v_1}{v_2} = \frac{m_2}{m_1} = \frac{A_2}{A_1} = \frac{2}{1}$$

54. Two identical cells each of emf 1.5 V are connected in series across a 10 Ω resistance. An ideal voltmeter connected across 10 Ω resistance reads 1.5 V. The internal resistance of each cell is _____Ω.

Official Ans. by NTA (5)

Sol. $V = I \times 10$

$$1.5 = \left(\frac{3}{10 + 2r}\right) \times 10$$

$r = 5\ \Omega$

55. A block of mass 5 kg starting from rest pulled up on a smooth incline plane making an angle of 30° with horizontal with an affective acceleration of 1 ms^{-2}. The power delivered by the pulling force at t = 10 s from the start is _____ W.

[Use g = 10 ms^{-2}]

(calculate the nearest integer value)

Official Ans. by NTA (300)

Sol. $F - 5g\sin30° = 5a \Rightarrow F = 5 + 25 = 30N$

$V_{10} = u + at \Rightarrow v_{10} = 0 + 1(10) = 10$ m/s

$P_{10} = Fv = 300$ W

56. A coil has an inductance of 2H and resistance of 4Ω. A 10 V is applied across the coil. The energy stored in the magnetic field after the current has built up to its equilibrium value will be ____$\times 10^{-2}$J

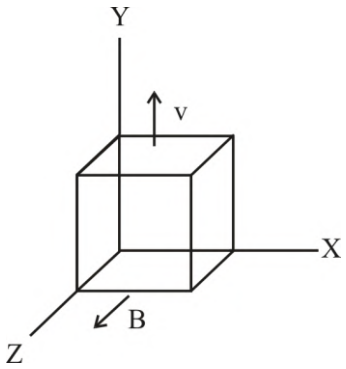
Official Ans. by NTA (625)

Sol. $I = \frac{V}{R} = \frac{5}{2}A$

$$E = \frac{1}{2}LI^2 = \frac{1}{2} \times 2 \times \left(\frac{5}{2}\right)^2$$

$E = 625 \times 10^{-2}$ J

57. A metallic cube of side 15 cm moving along y–axis at a uniform velocity of 2 ms^{-1}. In a region of uniform magnetic field of magnitude 0.5 T directed along z-axis. In equilibrium the potential difference between the faces of higher and lower potential developed because of the motion through the field will be _____ mV.



Official Ans. by NTA (150)

Sol. $\Delta V = (v \times B)d$

$\Delta V = (2 \times 1/2)0.15$

$\Delta V = 150$ mV

58. A wire of density 8×10^3 kg/m^3 is stretched between two clamps 0.5 m apart. The extension developed in the wire is 3.2×10^{-4} m. If $Y = 8 \times 10^{10}$ N/m^2, the fundamental frequency of vibration in the wire will be _____ Hz.

Official Ans. by NTA (80)

Sol. $$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2L}\sqrt{\frac{YA\Delta L}{\rho AL}}$$

$f = 80$ Hz

59. The surface tension of soap solution is 3.5×10^{-2} Nm^{-1}. The amount of work done required to increase the radius of soap bubble from 10 cm to 20 cm is _____$\times 10^{-4}$ J.

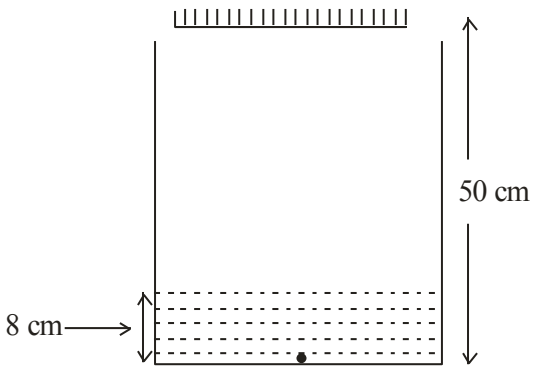
Official Ans. by NTA (264)

Sol. $W = T.(\Delta A)$

$$W = T\left(8\pi\left(r_2^2 - r_1^2\right)\right)$$

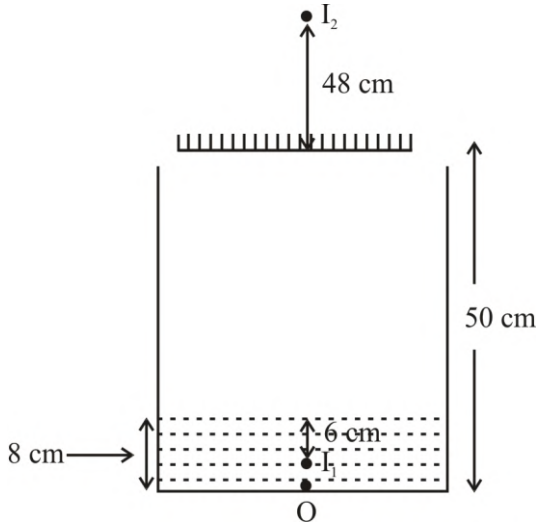
$W = 264 \times 10^{-4}$ J

60. As shown in the figure, a plane mirror is fixed at a height of 50 cm from the bottom of tank containing water $\left(\mu = \frac{4}{3}\right)$. The height of water in the tank is 8 cm. A small bulb is placed at the bottom of the water tank. The distance of image of the bulb formed by mirror from the bottom of the tank is _____ cm.



Official Ans. by NTA (98)

Sol.



Apparent depth of $O = \frac{d}{\mu} = 6$

Distance between O and $I_2 = 48 + 50 = 98$ cm

SECTION-A

61. Which hydride among the following is less stable?

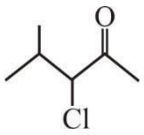
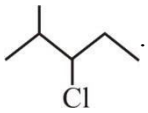
(1) BeH_2 (2) NH_3

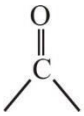
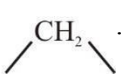
(3) HF (4) LiH

Official Ans. by NTA (1)

Solution : BeH_2 is hypovalent

62. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A :  can be subjected to Wolff-Kishner reduction to give  .

Reason R : Wolff-Kishner reduction is used to convert  into  .

In the light of the above statements, choose the correct answer from the options given below :

(1) Both A and R are true but R is NOT the correct explanation of A.

(2) A is true but R is false.

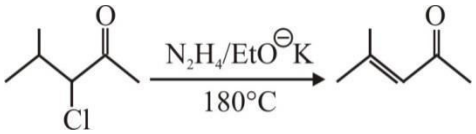
(3) A is false but R is true.

(4) Both A and R are true and R is the correct explanation of A.

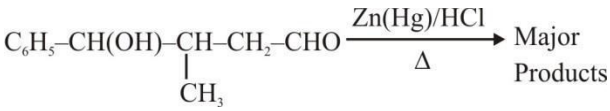
Official Ans. by NTA (3)

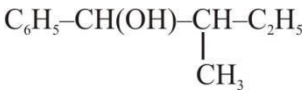
Solution :

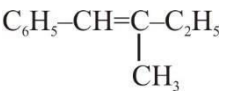
Wolff-Kishner reduction is not suitable for base sensitive group.

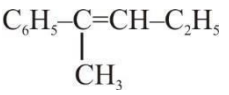


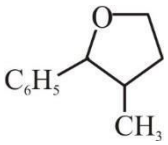
63. The major product formed in the following reaction is:



(A) 

(B) 

(C) 

(D) 

Choose the correct answer from the options given below :

(1) A only

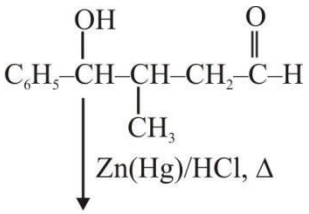
(2) B only

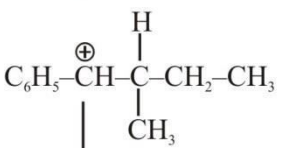
(3) C only

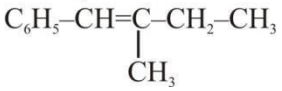
(4) D only

Official Ans. by NTA (2)

Solution :







64. Which of the following compounds is an example of Freon?

(1) $C_2Cl_2F_2$

(2) C_2HF_3

(3) $C_2H_2F_2$

(4) C_2F_4

Official Ans. by NTA (1)

Solution : Freons are chlorofluoro carbon.

65. For a chemical reaction A + B $\rightarrow$ Product, the order is 1 with respect to A and B.

Rate mol L^{-1} s^{-1}	[A] mol L^{-1}	[B] mol L^{-1}
0.10	20	0.5
0.40	*x*	0.5
0.80	40	*y*

What is the value of *x* and *y*?

(1) 80 and 2

(2) 40 and 4

(3) 160 and 4

(4) 80 and 4

Official Ans. by NTA (1)

Solution :

$r = K[A]^1[B]^1$

$0.1 = K(20)^1 (0.5)^1$ …(i)

$0.40 = K(x)^1 (0.5)^1$ …(ii)

$0.80 = K(40)^1 (y)^1$ …(iii)

From (i) and (ii)

$x = 80$

From (i) and (iii)

$y = 2$

66. Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : $[CoCl(NH_3)_5]^{2+}$ absorbs at lower wavelength of light with respect to $[Co(NH_3)_5(H_2O)]^{3+}$

Reason R : It is because the wavelength of the light absorbed depends on the oxidation state of the metal ion.

In the light of the above statements, choose the correct answer from the options given below :

(1) A is false but R is true.

(2) A is true but R is false.

(3) Both A and R are true and R is the correct explanation of A.

(4) Both A and R are true and R is NOT the correct explanation of A.

Official Ans. by NTA (1)

Solution : Since H_2O is strong field ligand compared to chloride and Co^{3+} ion is present.

$\therefore$ CFSE is higher for $[Co(NH_3)_5H_2O]^{+3}$, hence it will absorb at lower wavelength.

67. Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : A solution of the product obtained by heating a mole of glycine with a mole of chlorine in presence of red phosphorous generates chiral carbon atom.

Reason R : A molecule with 2 chiral carbons is always optically active.

In the light of the above statements, choose the correct answer from the options given below :

(1) A is false but R is true.

(2) A is true but R is false.

(3) Both A and R are true and R is the correct explanation of A.

(4) Both A and R are true and R is NOT the correct explanation of A.

Official Ans. by NTA (2)

Solution :

(1) $\underset{\text{Glycine}}{H_2N-CH_2-COOH} \xrightarrow[Cl_2]{\text{Red P}}_{\text{(HVZ)}} \underset{\text{Optically active}}{NH_2-\overset{*}{\underset{\displaystyle Cl}{\underset{|}{C}}}H-COOH}$

(2) Meso compound are optically inactive.

68. $H_3C-CH_2-\underset{\underset{OH}{|}}{CH}-CH_3 \xrightarrow[(iii)\ D_2O]{(i)\ NaI,\ H_3PO_4 \\ (ii)\ Mg,\ Dry\ ether} [X]$ Product

Product [X] formed in the above reaction is :

(1) $H_3C-CH_2-\underset{\underset{D}{|}}{CH}-CH_3$

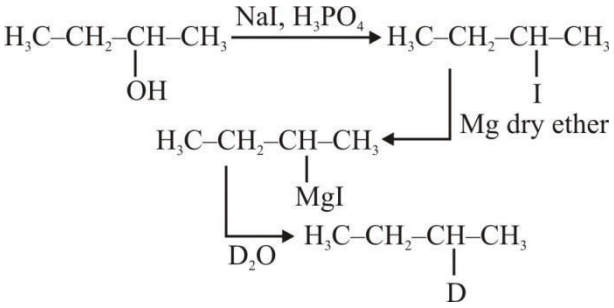
(2) $H_3C-CH_2-\overset{\overset{OH}{|}}{\overset{H}{\underset{|}{C}}}-CH_3$

(3) $H_3C—CH_2—CH=CH_2$

(4) $H_3C—CH=CH—CH_3$

Official Ans. by NTA (1)

Solution :



69. Given below are two statements :

Statements I : Ethene at 333 to 343K and 6-7 atm pressure in the presence of $AlEt_3$ and $TiCl_4$ undergoes addition polymerization to give LDP.

Statement II : Caprolactam at 533-543K in H_2O through step growth polymerizes to give Nylon 6.

In the light of the above Statements, chose the correct answer from the options given below :

(1) Both Statement I and Statement II are true.

(2) Statement I is false but Statement II is true.

(3) Statement I is true but Statement II is false.

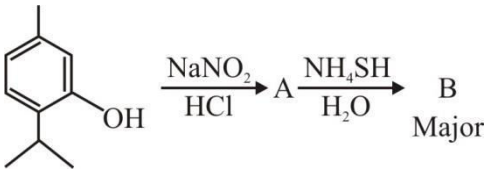
(4) Both Statement I and Statement II are false.

Official Ans. by NTA (2)

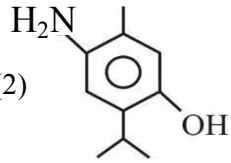
Solution : $S_1 \Rightarrow$ HDPE is formed by $TiCl_4$ & $Al(Et)_3$.

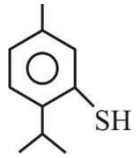
$S_2 \Rightarrow$ Nylon-6 is formed by caprolactam.

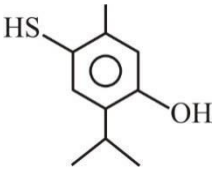
70. Compound 'B' is



(1) 

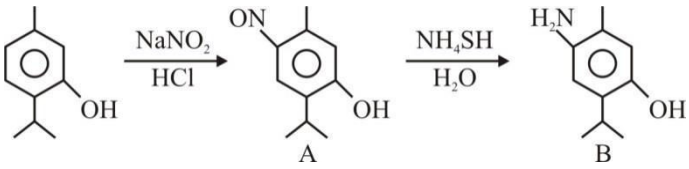
(2) 

(3) 

(4) 

Official Ans. by NTA (2)

Solution :



71. Which one of the following pairs is an example of polar molecular solids?

(1) $SO_2(s)$, $NH_3(s)$

(2) $SO_2(s)$, $CO_2(s)$

(3) HCl(s), AlN(s)

(4) MgO(s), $SO_2(s)$

Official Ans. by NTA (1)

Solution : SO_2 and NH_3 are polar molecules. They are constituent particles of polar molecular solids.

72. One mole of P_4 reacts with 8 moles of $SOCl_2$ to give 4 moles of A, x mole of SO_2 and 2 moles of B. A, B and x respectively are

(1) PCl_3, S_2Cl_2 and 4

(2) $POCl_3$, S_2Cl_2 and 4

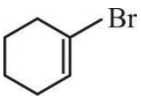
(3) PCl_3, S_2Cl_2 and 2

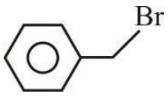
(4) $POCl_3$, S_2Cl_2 and 2

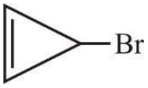
Official Ans. by NTA (1)

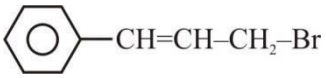
Solution : $P_4 + 8\ SOCl_2 \rightarrow 4PCl_3 + 2S_2Cl_2 + 4SO_2$

73. Compound from the following that will not produce precipitate on reaction with $AgNO_3$ is :

(1) 

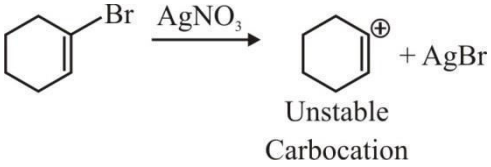
(2) 

(3) 

(4) 

Official Ans. by NTA (1)

Solution :



74. A solution is prepared by adding 2g of "X" of 1 mole of water. Mass percent of "X" in the solution is :

(1) 20%

(2) 5%

(3) 2%

(4) 10%

Official Ans. by NTA (4)

Solution : Solute (X) = 2 g

Solvent (H_2O) = 1 mole = 18 g

Total mass = 2 + 18 = 20 g

$$\% \text{ mass of X} = \frac{2}{20} \times 100 = 10\%$$

75. Given below are two statements :

Statement-I : In the metallurgy process, sulphide ore is converted to oxide before reduction.

Statement-II : Oxide ores in general are easier to reduce.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both Statement I and Statement II are correct.

(2) Statement I is correct but Statement II is incorrect.

(3) Both Statement I and Statement II are incorrect.

(4) Statement I is incorrect but Statement II is correct.

Official Ans. by NTA (1)

Solution : $2ZnS + 3O_2 \rightarrow 2ZnO + 2SO_2$

Oxides on carbon reduction forms CO_2 while sulphide on carbon reduction gives CS_2.

CO_2 is more volatile compared to CS_2 therefore oxides are easy to reduce.

76. Alkali metal from the following with least melting point is :

(1) Rb

(2) K

(3) Na

(4) Cs

Official Ans. by NTA (4)

Solution : On moving down the group in alkali metals melting point decreases.

77. What weight of glucose must be dissolved in 100 g of water to lower the vapour pressure by 0.20 mm Hg?
(Assume dilute solution is being formed)
Given : Vapour pressure of pure water is 54.2 mm Hg at room temperature. Molar mass of glucose is 180 g mol^{-1}.
(1) 4.69 g (2) 3.59 g
(3) 2.59 g (4) 3.69 g
Official Ans. by NTA (4)

Solution : $\frac{P^0 - P_s}{P^0} = \frac{n}{N}$ (for dilute solution)

$\frac{0.2}{54.2} = \frac{n \times 18}{100}$

$n = \frac{100}{271 \times 18}$

$w = \frac{100 \times 180}{271 \times 18}$; $w = 3.69g$

78. The magnetic moment is measured in Bohr Magneton (BM).
Spin only magnetic moment of Fe in $[Fe(H_2O)_6]^{3+}$ and $[Fe(CN)_6]^{3-}$ complexes respectively is :
(1) 6.92 B.M. in both
(2) 4.89 B.M. and 6.92 B.M.
(3) 3.87 B.M. and 1.732 B.M.
(4) 5.92 B.M. and 1.732 B.M
Official Ans. by NTA (4)

Solution : $[Fe(H_2O)_6]^{3+}$
$Fe^{3+} \Rightarrow [Ar]\ 3d^5\ 4s^0$
No pairing | 1 | 1 | 1 | 1 | 1 |
$\therefore$ Unpaired $e^- = 5$
$\mu = \sqrt{n(n+2)}$
$= \sqrt{5(5+2)}$
$\mu = \sqrt{35} = 5.92\,B.M.$
$[Fe(CN)_6]^{-3}$
$Fe^{3+} \Rightarrow [Ar]\ 3d^5\ 4s^\circ$
| 1 | 1 | 1 | 1 | 1 |
Pairing occur due to strong field ligand CN^-
| ⇅ | ⇅ | 1 | | |
$\therefore$ Unpaired $e^- \Rightarrow 1$
$\mu = \sqrt{n(n+2)}$
$= \sqrt{1(1+2)} = \sqrt{3} = 1.732\,B.M.$

79. Match List I with List II.

List I Complex		List II Colour	
A.	$Mg(NH_4)PO_4$	I.	Brown
B.	$K_3[Co(NO_2)_6]$	II.	White
C.	$MnO(OH)_2$	III.	Yellow
D.	$Fe_4[Fe(CN)_6]_3$	IV.	blue

Choose the correct answer from the options given below :
(1) A-II, B-III, C-I, D-IV
(2) A-III, B-IV, C-II, D-I
(3) A-II, B-IV, C-I, D-III
(4) A-II, B-III, C-IV, D-I
Official Ans. by NTA (1)

Solution : $Mg(NH_4)PO_4 \Rightarrow$ White
$K_3[Co(NO_2)_6] \Rightarrow$ Yellow
$MnO(OH)_2 \Rightarrow$ Brown
$Fe_4[Fe(CN)_6]_3 \Rightarrow$ Blue

80. If Ni^{2+} is replaced by Pt^{2+} in the complex $[NiCl_2Br_2]^{2-}$, which of the following properties are expected to get changed?
A. Geometry
B. Geometrical isomerism
C. Optical isomerism
D. Magnetic properties
(1) A, B and C
(2) A, B and D
(3) A and D
(4) B and C
Official Ans. by NTA (2)

Solution : $[NiBr_2Cl_2]^{2-} \rightarrow$ This complex species is tetrahedral as $Br^\ominus$ & $Cl^\ominus$ are weak field ligands.
$[PtBr_2Cl_2]^{2-} \rightarrow$ As Pt belongs to 5d series. This complex species is square planar.
Both the complex species are optically inactive.
$[NiBr_2Cl_2]^{2-}$, being tetrahedral does not show Geometrical Isomerism.
$[PtBr_2Cl_2]^{2-}$ shows two Geometrical Isomers.

SECTION-B

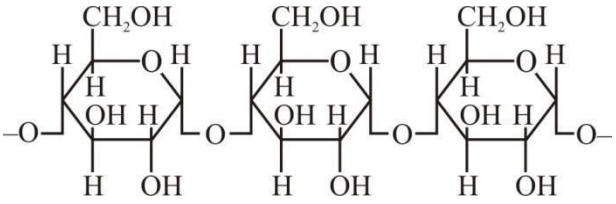
81. Number of compounds from the following which will not produce orange red precipitate with Benedict solution is …….

Glucose, maltose, sucrose, ribose, 2-deoxyribose, amylose, lactose.

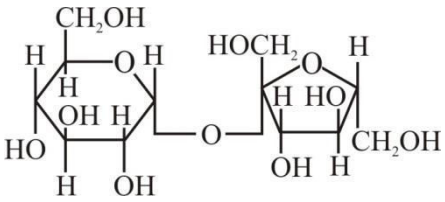
Official Ans. by NTA (3)

Solution :

Amylose



Sucrose :



Both Amylose and Sucrose does not give Benedict's test.

82. 4.5 moles each of hydrogen and iodine is heated in a sealed ten litre vessel. At equilibrium, 3 moles of HI were found. The equilibrium constant for $H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$ is …….

Official Ans. by NTA (1)

Solution :

	$H_{2(g)}$	+ $I_{2(g)}$	$\rightleftharpoons$ $2HI_{(g)}$
t = 0	4.5	4.5	–
t_{eq}	3	3	3

$$K_c = \frac{[HI]^2}{[H_2][I_2]} = \frac{(3)^2}{3\times 3} = \frac{9}{9} = 1$$

83. The number of correct statements about modern adsorption theory of heterogeneous catalysis from the following is …….

A. The catalyst is diffused over the surface of reactants.

B. Reactants are adsorbed on the surface of the catalyst.

C. Occurrence of chemical reaction on the catalyst's surface through formation of an intermediate.

D. It is a combination of intermediate compound formation theory and the old adsorption theory.

E. It explains the action of the catalyst as well as those of catalytic promoters and poisons.

Official Ans. by NTA (3)

Solution : B, C and D are correct.
(NCERT – Surface Chemistry)

84. The number of correct statements from the following _____

A. For 1s orbital, the probability density is maximum at the nucleus.

B. For 2s orbital, the probability density first increases to maximum and then decreases sharply to zero.

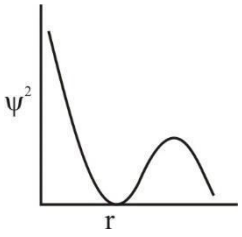
C. Boundary surface diagrams of the orbitals encloses a region of 100% probability of finding the electron.

D. p and d-orbitals have 1 and 2 angular nodes respectively.

E. Probability density of p-orbital is zero at the nucleus.

Official Ans. by NTA (3)

Solution : A, D and E statements are correct.



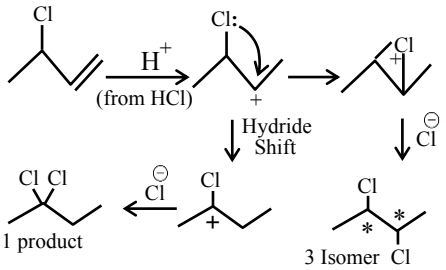
For 2s orbital, the probability density first decreases and then increases.

At any distance from nucleus the probability density of finding electron is never zero and it always have some finite value.

85. The number of possible isomeric products formed when 3-chloro-1-butene reacts with HCl through carbocation formation is _____

Official Ans. by NTA (4)

Solution :



Total Possible Isomeric product = 1+3 = 4

86. $Mg(NO_3)_2 \cdot XH_2O$ and $Ba(NO_3)_2 \cdot YH_2O$, represent formula of the crystalline forms of nitrate salts. Sum of X and Y is _____

Official Ans. by NTA (6)

Solution : $Mg(NO_3)_2 \cdot 6H_2O$ is a hydrated salt whereas $Ba(NO_3)_2$ is a anhydrous salt.

$\therefore \quad x + y = 6$

87. The total number of intensive properties from the following is _____

Volume, Molar heat capacity, Molarity, E^{θ} cell, Gibbs free energy change, Molar mass, Mole

Official Ans. by NTA (4)

Solution : Extensive $\Rightarrow$ Mole, Volume, Gibbs free energy.

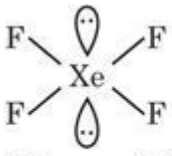
Intensive $\Rightarrow$ Molar mass, Molar heat capacity, Molarity, E^{θ} cell.

88. The maximum number of lone pairs of electrons on the central atom from the following species is ___

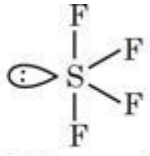
ClO_3^-, XeF_4, SF_4 and I_3^-

Official Ans. by NTA (3)

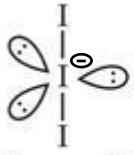
Solution :



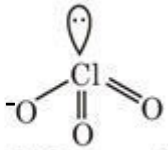
[2 lone pair]



[1 lone pair]



[3 lone pair]



[1 lone pair]

89. The volume of hydrogen liberated at STP by treating 2.4 g of magnesium with excess of hydrochloric acid is _____ $\times 10^{-2}$ L.

Given: Molar volume of gas is 22.4 L at STP.

Molar mass of magnesium is 24 g mol^{-1}.

Official Ans. by NTA (224)

Solution :

$Mg + 2HCl \rightarrow MgCl_2 + H_2 \uparrow$

$w = 2.4$ g

$N = \frac{2.4}{24} = 0.1$ mole

1 mole of gas at STP $\Rightarrow$ 22.4 lit.

$\therefore$ 0.1 mole of gas = 0.1×22.4

$= 2.24$ lit. $= 224 \times 10^{-2}$ litre

90. The number of correct statements from the following is :

A. E_{cell} is an intensive parameter.

B. A negative E^{Θ} means that the redox couple is a stronger reducing agent than the H^+/H_2 couple.

C. The amount of electricity required for oxidation or reduction depends on the stoichiometry of the electrode reaction.

D. The amount of chemical reaction which occurs at any electrode during electrolysis by a current is proportional to the quantity of electricity passed through the electrolyte.

Official Ans. by NTA (4)

Solution : Given statements A, B, C and D are correct.

FINAL JEE–MAIN EXAMINATION – APRIL, 2023

(Held On Wednesday 12th April, 2023) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. The number of five digit numbers, greater than 40000 and divisible by 5, which can be formed using the digits 0, 1, 3, 5, 7 and 9 without repetition, is equal to

(1) 120

(2) 132

(3) 72

(4) 96

Official Ans. by NTA (1)

Sol.

5 x x x 0

7 x x x 0

7 x x x 5

9 x x x 0

9 x x x 5

So Required numbers = $5 \times {}^4P_3 = 120$

2. Let α, β be the roots of the quadratic equation $x^2 + \sqrt{6}x + 3 = 0$. Then $\dfrac{\alpha^{23} + \beta^{23} + \alpha^{14} + \beta^{14}}{\alpha^{15} + \beta^{15} + \alpha^{10} + \beta^{10}}$ is equal to

(1) 729

(2) 72

(3) 81

(4) 9

Official Ans. by NTA (3)

Sol. $\alpha, \beta = \dfrac{-\sqrt{6} \pm \sqrt{6-12}}{2} = \dfrac{-\sqrt{6} \pm \sqrt{6}\ i}{2}$

$= \sqrt{3}e^{\pm\frac{3\pi i}{4}}$

Required expression

$$= \frac{\left(\sqrt{3}\right)^{23}\left(2\cos\frac{69\pi}{4}\right) + \left(\sqrt{3}\right)^{14}\left(2\cos\frac{42\pi}{4}\right)}{\left(\sqrt{3}\right)^{15}\left(2\cos\frac{45\pi}{4}\right) + \left(\sqrt{3}\right)^{10}\left(2\cos\frac{30\pi}{4}\right)}$$

$\left(\sqrt{3}\right)^8 = 81$

3. Let $< a_n >$ be a sequence such that $a_1 + a_2 + ... + a_n = \dfrac{n^2 + 3n}{(n+1)(n+2)}$. If $28\sum_{k=1}^{10} \dfrac{1}{a_k} = p_1 p_2 p_3 .... p_m$, where p_1, p_2, ...pm are the first m prime numbers, then m is equal to

(1) 7

(2) 6

(3) 5

(4) 8

Official Ans. by NTA (2)

Sol. $a_n = S_n - S_{n-1} = \dfrac{n^2 + 3n}{(n+1)(n+2)} - \dfrac{(n-1)(n+2)}{n(n+1)}$

$$\Rightarrow a_n = \frac{4}{n(n+1)(n+2)}$$

$$\Rightarrow 28\sum_{k=1}^{10} \frac{1}{a_k} = 28\sum_{k=1}^{10} \frac{k(k+1)(k+2)}{4}$$

$$= \frac{7}{4}\sum_{k=1}^{10}(k(k+1)(k+2)(k+3) - (k-1)k(k+1)(k+2))$$

$$= \frac{7}{4}.10.11.12.13 = 2.3.5.7.11.13$$

So m = 6

4. Let the lines $l_1 : \dfrac{x+5}{3} = \dfrac{y+4}{1} = \dfrac{z-\alpha}{-2}$ and $l_2 : 3x + 2y + z - 2 = 0 = x - 3y + 2z - 13$ be coplanar. If the point P(a, b, c) on l_1 is nearest to the point Q(-4, -3, 2), then |a| + |b|+|c| is equal to

(1) 12

(2) 14

(3) 10

(4) 8

Official Ans. by NTA (3)

Sol. $(3x + 2y + z - 2) + \mu (x - 3y + 2z - 13) = 0$

$3(3 + \mu) + 1.(2-3\mu) - 2(1+2\mu) = 0$

$9 - 4\mu = 0$

$\mu = \frac{9}{4}$

$4(-15 - 8 + \alpha - 2) + 9(-5 + 12 + 2\alpha - 13) = 0$

$-100 + 4\alpha - 54 + 18\alpha = 0$

$\Rightarrow \alpha = 7$

Let $P \equiv (3\lambda - 5, \lambda - 4, -2\lambda + 7)$

Direction ratio of PQ $(3\lambda - 1, \lambda - 1, -2\lambda + 5)$

But $PQ \perp \ell_1$

$\Rightarrow 3(3\lambda - 1) + 1.(\lambda - 1) - 2(-2\lambda + 5) = 0$

$\Rightarrow \lambda = 1$

$P(-2, -3, 5) \Rightarrow |a|+|b| + |c| = 10$

5. Let $P\left(\frac{2\sqrt{3}}{\sqrt{7}}, \frac{6}{\sqrt{7}}\right)$, Q, R and S be four points on the ellipse $9x^2 + 4y^2 = 36$. Let PQ and RS be mutually perpendicular and pass through the origin. If $\frac{1}{(PQ)^2} + \frac{1}{(RS)^2} = \frac{p}{q}$, where p and q are coprime, then p + q is equal to

(1) 143 (2) 137

(3) 157 (4) 147

Official Ans. by NTA (3)

Sol. Let $R(2\cos\theta, 3\sin\theta)$

as $OP \perp OR$

$$\text{so } \frac{3\sin\theta}{2\cos\theta} \times \frac{\frac{6}{\sqrt{7}}}{\frac{2\sqrt{3}}{\sqrt{7}}} = -1$$

$$\Rightarrow \tan\theta = \frac{-2}{3\sqrt{3}}$$

$$\Rightarrow R\left(\frac{-6\sqrt{3}}{\sqrt{31}}, \frac{6}{\sqrt{31}}\right) \text{ or } R\left(\frac{6\sqrt{3}}{\sqrt{31}}, \frac{-6}{\sqrt{31}}\right)$$

$$\text{Now} = \frac{1}{(PQ)^2} + \frac{1}{(RS)^2} = \frac{1}{4}\left(\frac{1}{(OP)^2} + \frac{1}{(OR)^2}\right)$$

$$= \frac{1}{4}\left(\frac{1}{\frac{48}{7}} + \frac{1}{\frac{144}{31}}\right) = \frac{1}{4}\left(\frac{7}{48} + \frac{31}{144}\right)$$

$$= \frac{13}{144}$$

$$\Rightarrow p + q = 157$$

6. Let a, b, c be three distinct real numbers, none equal to one. If the vectors $a\hat{i} + \hat{j} + \hat{k}, \hat{i} + b\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + c\hat{k}$ are coplanar, then $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$ is equal to

(1) 1

(2) – 1

(3) – 2

(4) 2

Official Ans. by NTA (1)

Sol. $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$

$C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$$\begin{vmatrix} a & 1-a & 1-a \\ 1 & b-1 & 0 \\ 1 & 0 & c-1 \end{vmatrix} = 0$$

$a(b-1)(c-1) - (1-a)(c-1) + (1-a)(1-b) = 0$

$a(1-b)(1-c) + (1-a)(1-c) + (1-a)(1-b) = 0$

$$\frac{a}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0$$

$$\Rightarrow -1 + \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0$$

$$\Rightarrow \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$$

7. If the local maximum value of the function $f(x) = \left(\frac{\sqrt{3e}}{2\sin x}\right)^{\sin^2 x}$, $x \in \left(0, \frac{\pi}{2}\right)$, is $\frac{k}{e}$, then $\left(\frac{k}{e}\right)^8 + \frac{k^8}{e^5} + k^8$ is equal to

(1) $e^5 + e^6 + e^{11}$

(2) $e^3 + e^5 + e^{11}$

(3) $e^3 + e^6 + e^{11}$

(4) $e^3 + e^6 + e^{10}$

Official Ans. by NTA (3)

Sol. **Let** $y = \left(\frac{\sqrt{3e}}{2\sin x}\right)^{\sin^2 x}$

$$\ln y = \sin^2 x . \ln\left(\frac{\sqrt{3e}}{2\sin x}\right)$$

$$\frac{1}{y}y' = \ln\left(\frac{\sqrt{3e}}{2\sin x}\right)2\sin x\cos x + \sin^2 x \frac{2\sin x}{\sqrt{3e}}\frac{\sqrt{3e}}{2}(-\text{cosec}\, x \cot x)$$

$$\frac{dy}{dx} = 0 \Rightarrow \ln\left(\frac{\sqrt{3e}}{2\sin x}\right)2\sin x\cos x - \sin x\cos x = 0$$

$$\Rightarrow \sin x\cos x\left[2\ln\left(\frac{\sqrt{3e}}{2\sin x}\right) - 1\right] = 0$$

$$\Rightarrow \ln\left(\frac{3e}{4\sin^2 x}\right) = 1 \Rightarrow \frac{3e}{4\sin^2 x} = e \Rightarrow \sin^2 x = \frac{3}{4}$$

$$\Rightarrow \sin x = \frac{\sqrt{3}}{2} \quad \left(\text{as } x \in \left(0, \frac{\pi}{2}\right)\right)$$

$$\Rightarrow \text{ local max value} = \left(\frac{\sqrt{3e}}{\sqrt{3}}\right)^{3/4} = e^{3/8} = \frac{k}{e}$$

$$\Rightarrow k^8 = e^{11}$$

$$\Rightarrow \left(\frac{k}{e}\right)^8 + \frac{k^8}{e^5} + k^8 = e^3 + e^6 + e^{11}$$

8. Let D be the domain of the function $f(x) = \sin^{-1}\left(\log_{3x}\left(\frac{6+2\log_3 x}{-5x}\right)\right)$. If the range of the function $g : D \to R$ defined by $g(x) = x - [x]$, ([x] is the greatest integer function), is (α, β), then $\alpha^2 + \frac{5}{\beta}$ is equal to

(1) 46

(2) 135

(3) 136

(4) 45

Official Ans. by NTA (2)

Sol. $\frac{6+2\log_3 x}{-5x} > 0 \;\&\; x > 0 \;\&\; x \neq \frac{1}{3}$

this gives $x \in \left(0, \frac{1}{27}\right)$ ... (1)

$$-1 \le \log_{3x}\left(\frac{6+2\log_3 x}{-5x}\right) \le 1$$

$$3x \le \frac{6+2\log_3 x}{-5x} \le \frac{1}{3x}$$

$15x^2 + 6 + 2\log_3 x \ge 0$ $\quad$ $6 + 2\log_3 x + \frac{5}{3} \ge 0$

$x \in \left(0, \frac{1}{27}\right)$ ... (2) $\quad$ $x \ge 3^{-\frac{23}{6}}$ ... (3)

from (1), (2) & (3)

$$x \in \left[3^{-\frac{23}{6}}, \frac{1}{27}\right)$$

$\therefore$ α is small positive quantity

$$\& \ \beta = \frac{1}{27}$$

$\therefore$ $\alpha^2 + \frac{5}{\beta}$ is just greater than 135

Ans. (Bonus)

9. Let y = y(x), y > 0, be a solution curve of the differential equation $(1 + x^2)\, dy = y\,(x - y)\, dx$.

If $y(0) = 1$ and $y(2\sqrt{2}) = \beta$, then

(1) $e^{3\beta^{-1}} = e(3 + 2\sqrt{2})$

(2) $e^{\beta^{-1}} = e^{-2}\left(5 + \sqrt{2}\right)$

(3) $e^{\beta^{-1}} = e^{-2}\left(3 + 2\sqrt{2}\right)$

(4) $e^{3\beta^{-1}} = e\left(5 + \sqrt{2}\right)$

Official Ans. by NTA (1)

Sol. $(1+x^2)\, dy = y\,(x - y)dx$

$y(0) = 1$. $y(2\sqrt{2}) = \beta$

$$\frac{dy}{dx} = \frac{yx - y^2}{1 + x^2}$$

$$\frac{dy}{dx} + y\left(\frac{-x}{1+x^2}\right) = \left(\frac{-1}{1+x^2}\right)y^2$$

$$\frac{1}{y^2}\frac{dy}{dx} + \frac{1}{y}\left(\frac{-x}{1+x^2}\right) = \frac{-1}{1+x^2}$$

put $\frac{1}{y} = t$ then $\frac{-1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$

$$\frac{dt}{dx}+t\frac{x}{1+x^2}=\frac{1}{1+x^2}$$

$$I.F = e^{\int\frac{x}{1+x^2}dx} = e^{\frac{1}{2}\ln(1+x^2)} = \sqrt{1+x^2}$$

$$t\sqrt{1+x^2} = \int\frac{1}{\sqrt{1+x^2}}dx$$

$$\frac{\sqrt{1+x^2}}{y} = \ln\left(x+\sqrt{x^2+1}\right)+c$$

$$y(0) = 1 \quad \Rightarrow c = 1$$

$$\Rightarrow \sqrt{1+x^2} = y\ln(e(x+\sqrt{x^2+1}))$$

$$\beta = \frac{3}{\ln(e(3+2\sqrt{2}))} \Rightarrow \frac{3}{\beta} = \ln(e(3+2\sqrt{2}))$$

$$e^{\frac{3}{\beta}} = e(3+2\sqrt{2})$$

10. Among the two statements

(S1) : $(p \Rightarrow q)\wedge(q\wedge(\sim q))$ is a contradiction and

(S2) : $(p\wedge q)\vee((\sim p)\wedge q)\vee$

$(p\wedge(\sim q))\vee((\sim p)\wedge(\sim q))$ is a tautology

(1) only (S2) is true

(2) only (S1) is true

(3) both are false.

(4) both are true

Official Ans. by NTA (4)

Sol. $S_1 : (p\to q)\wedge(p\wedge(\sim q))$

p	q	$p\to q$	$p\wedge(\sim q)$	S1
T	T	T	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	F	F

$\Rightarrow$ S_1 is Contradiction

S_2

p	q	$p\wedge q$	$(\sim p\wedge q)$	$(p\wedge\sim q)$	$(\sim p)\wedge(\sim q)$	S_2
T	T	T	F	F	F	T
T	F	F	F	T	F	T
F	T	F	T	F	F	T
F	F	F	F	F	T	T

S_2 is tautology

11. Let $\lambda\in Z, a = \lambda\hat{i}+\hat{j}-\hat{k}$ and $b = 3\hat{i}-\hat{j}+2\hat{k}$. Let $\vec{c}$ be a vector such that $(\vec{a}+\vec{b}+\vec{c})\times\vec{c} = \vec{0}, \vec{a}.\vec{c} = -17$ and $\vec{b}.\vec{c} = -20$.

Then $|\vec{c}\times\left(\lambda\hat{i}+\hat{j}+\hat{k}\right)|^2$ is equal to

(1) 62

(2) 46

(3) 53

(4) 49

Official Ans. by NTA (2)

Sol. $a+b+c\ \times c = 0$

$$\left(\vec{a}+\vec{b}\right)\times\vec{c} = 0$$

$$\vec{c} = \alpha(\vec{a}+\vec{b}) = \alpha(\lambda+3)\hat{i}+\alpha\hat{k}$$

$$\vec{b}.\vec{c} = -20 \Rightarrow 3\alpha(\lambda+3)+2\alpha = -20$$

$$\vec{a}.\vec{c} = -17 \Rightarrow \alpha\lambda(\lambda+3)-\alpha = -17$$

$$\Rightarrow \alpha(3\lambda+9+2) = -20$$

$$\alpha(\lambda^2+3\lambda-1) = -17$$

$$17(3\lambda+11) = 20(\lambda^2+3\lambda-1)$$

$$20\lambda^2+9\lambda-207 = 0$$

$$\lambda = 3 \qquad (\lambda\in Z)$$

$$\Rightarrow \alpha = -1 \qquad \Rightarrow \vec{c} = -(6\hat{i}+\hat{k})$$

$$\vec{v} = \vec{c}\times(3\hat{i}+\hat{j}+\hat{k})$$

$$= \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -6 & 0 & -1\\ 3 & 1 & 1\end{vmatrix} = \hat{i}+3\hat{j}-6\hat{k}$$

$$|\vec{v}|^2 = (-1)^2+3^2+6^2 = 46$$

12. The sum, of the coefficients of the first 50 terms in the binomial expansion of $(1-x)^{100}$, is equal to

(1) $-{}^{101}C_{50}$

(2) ${}^{99}C_{49}$

(3) $-{}^{99}C_{49}$

(4) ${}^{101}C_{50}$

Official Ans. by NTA (3)

Sol. $(1-x)^{100} = Co - C_1x + C_2x^2 - C_3x^3 + \ldots C_{99}x^{99} + C_{100}x^{100}$

$$\Rightarrow Co - C_1 + C_2 - C_3 + \ldots\ldots - C_{99} + C_{100} = 0$$

$$2(Co - C_1 + C_2 + \ldots\ldots - C_9) + C_{50} = 0$$

$$C_0 - C_1 + C_2 + \ldots C_{99} = -\frac{1}{2}\,{}^{100}C_{50}$$

$$-\frac{1}{2}\frac{100!}{50!50!} = -\frac{1}{2}\times\frac{100\times 99!}{50!50!} = -{}^{99}C_{49}$$

13. The area of the region enclosed by the curve $y = x^3$ and its tangent at the point (-1, -1) is

(1) $\frac{27}{4}$

(2) $\frac{19}{4}$

(3) $\frac{23}{4}$

(4) $\frac{31}{4}$

Official Ans. by NTA (1)

Sol. equation of tangent : $y + 1 = 3(x + 1)$

i.e. $y = 3x + 2$

Point of intersection with curve (2, 8)

So Area $= \int_{-1}^{2}((3x+2) - x^3)dx = \frac{27}{4}$

14. Let $A = \begin{bmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{bmatrix}$. If $B = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} A \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$, then the sum of all the elements of the matrix $\sum_{n=1}^{50} B^n$ is equal to

(1) 100

(2) 50

(3) 75

(4) 125

Official Ans. by NTA (1)

Sol. Let $C = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$, $D = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$

$$DC = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$B = CAD$

$B^n = \underbrace{(CAD)(CAD)(CAD)....(CAD)}_{n-\text{times}}$

$\Rightarrow B^n = CA^nD$ ....(1)

$$A^2 = \begin{bmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{2}{51} \\ 0 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & \frac{3}{51} \\ 0 & 1 \end{bmatrix}$$

similarly $A^n = \begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix}$

$$B^n = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \frac{n}{51}+2 \\ -1 & -\frac{n}{51}-1 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{n}{51}+1 & \frac{n}{51} \\ -\frac{n}{51} & 1-\frac{n}{51} \end{bmatrix}$$

$$\sum_{n=1}^{50} B^n = \begin{bmatrix} 25+50 & 25 \\ -25 & -25+50 \end{bmatrix} = \begin{bmatrix} 75 & 25 \\ -25 & 25 \end{bmatrix}$$

Sum of the elements = 100

15. Let the plane P : $4x - y + z = 10$ be rotated by an angle $\frac{\pi}{2}$ about its line of intersection with the plane $x + y - z = 4$. If α is the distance of the point $(2, 3, -4)$ from the new position of the plane P, then 35α is

(1) 90 (2) 85

(3) 105 (4) 126

Official Ans. by NTA (4)

Sol. Let equation in new position is

$(4x - y + z - 10) + \lambda(x + y - z - 4) = 0$

$4(4+\lambda) - 1.(-1+\lambda) + 1.(1-\lambda) = 0$

$\Rightarrow \lambda = -9$

So equation in new position is

$-5x - 10y + 10z + 26 = 0$

$\Rightarrow \alpha = \frac{54}{15}$

16. If $\frac{1}{n+1}\,{}^{n}C_{n} + \frac{1}{n}\,{}^{n}C_{n-1}$

$+ \ldots + \frac{1}{2}\,{}^{n}C_{1} + {}^{n}C_{0} = \frac{1023}{10}$ then n is equal to

(1) 6

(2) 9

(3) 8

(4) 7

Official Ans. by NTA (2)

Sol. $\sum_{r=0}^{n} \frac{{}^{n}C_{r}}{r+1} = \frac{1}{n+1}\sum_{r=0}^{n} {}^{n+1}C_{r+1}$

$= \frac{1}{n+1}(2^{n+1} - 1) = \frac{1023}{10}$

$n + 1 = 10 \Rightarrow n = 9$

17. Let C be the circle in the complex plane with centre $z_0 = \frac{1}{2}(1+3i)$ and radius r = 1. Let z_1 = 1+ i and the complex number z_2 be outside the circle C such that $|z_1 - z_0|\ |z_2 - z_0| = 1$. If z_0, z_1 and z_2 are collinear, then the smaller value of $|z_2|^2$ is equal to

(1) $\frac{13}{2}$

(2) $\frac{5}{2}$

(3) $\frac{3}{2}$

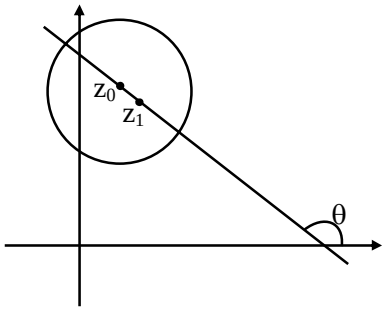
(4) $\frac{7}{2}$

Official Ans. by NTA (2)

Sol. $|z_1 - z_0| = \left|\frac{1-i}{2}\right| = \frac{1}{\sqrt{2}}$

$\Rightarrow |z_2 - z_o| = \sqrt{2}$; centre $\left(\frac{1}{2}, \frac{3}{2}\right)$

$z_o\left(\frac{1}{2}, \frac{3}{2}\right)$ and $z_1(1, 1)$



$\tan\theta = -1 \Rightarrow \theta = 135°$

$z_2\left(\frac{1}{2} + \sqrt{2}\cos 135°, \frac{3}{2} + \sqrt{2}\sin 135°\right)$

or

$\left(\frac{1}{2} - \sqrt{2}\cos 135°, \frac{3}{2} - \sqrt{2}\sin 135°\right)$

$\Rightarrow z_2\left(-\frac{1}{2}, \frac{5}{2}\right)$ or $z_2\left(\frac{3}{2}, \frac{1}{2}\right)$

$\Rightarrow |z_2|^2 = \frac{26}{4}, \frac{5}{2}$

$\Rightarrow |z_2|^2_{min} = \frac{5}{2}$

18. If the point $\left(\alpha, \frac{7\sqrt{3}}{3}\right)$ lies on the curve traced by the mid-points of the line segments of the lines x cos θ + y sin θ = 7, $\theta \in \left(0, \frac{\pi}{2}\right)$ between the co-ordinates axes, then α is equal to

(1) 7

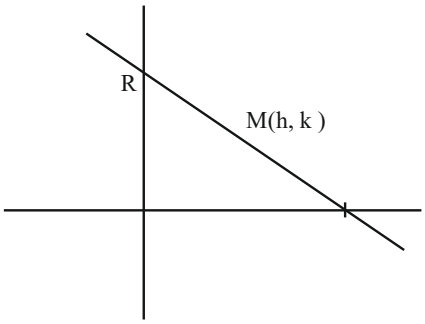
(2) –7

(3) $-7\sqrt{3}$

(4) $7\sqrt{3}$

Official Ans. by NTA (1)

Sol. $pt(\alpha, \frac{7\sqrt{3}}{3})$



$$x\cos\theta + y\sin\theta = 7$$

$$\text{x - intercept} = \frac{7}{\cos\theta}$$

$$\text{y - intercept} = \frac{7}{\sin\theta}$$

$$A:\left(\frac{7}{\cos\theta}, 0\right) \quad B:\left(0, \frac{7}{\sin\theta}\right)$$

Locus of mid pt M : (h, k)

$$h = \frac{7}{2\cos\theta}, k = \frac{7}{2\sin\theta}$$

$$\frac{7}{2\sin\theta} = \frac{7\sqrt{3}}{3} \Rightarrow \sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}$$

$$\alpha = \frac{7}{2\cos\theta} = 7$$

19. Two dice A and B are rolled, Let the numbers obtained on A and B be α and β respectively. If the variance of $\alpha - \beta$ is $\frac{p}{q}$, where p and q are co-prime, then the sum of the positive divisors of p is equal to

(1) 36

(2) 48

(3) 31

(4) 72

Official Ans. by NTA (2)

Sol.

α-β	Case	P
5	(6, 1)	1/36
4	(6, 2) (5, 1)	2/36
3	(6, 3) (5, 2) (4, 1)	3/36
2	(6, 4) (5, 3) (4, 3) (3, 1)	4/36
1	(6, 5) (5, 4) (4, 3) (3, 2) (2, 1)	5/36
0	(6, 6) (5, 5) …… (1, 1)	6/36
-1	-----	5/36
-2	-----	4/36
-3	-----	3/36
-4	(2, 6) (1, 5)	2/36
-5	(1, 6)	1/36

$$\sum(x^2) = \sum x^2 P(x) = 2\left[\frac{25}{36} + \frac{32}{36} + \frac{27}{36} + \frac{16}{36} + \frac{5}{36}\right]$$

$$= \frac{105}{18} = \frac{35}{6}$$

$\mu = \sum(x) = 0$ as data is symmetric

$\sigma^2 = \sum(x^2) = \sum x^2 P(x) = \frac{35}{6}$ $\quad P = 35 = 5 \times 7$

Sum of divisors = $(5^0 + 5^1)(7^0 + 7^1) = 6 \times 8 = 48$

20. In a triangle ABC, if cos A + 2 cos B + cos C = 2 and the lengths of the sides opposite to the angles A and C are 3 and 7 respectively, then cos A – cos C is equal to

(1) $\frac{3}{7}$

(2) $\frac{9}{7}$

(3) $\frac{10}{7}$

(4) $\frac{5}{7}$

Official Ans. by NTA (3)

Sol. $\cos A + \cos C = 2(1 - \cos B)$

$$2\cos\frac{A+C}{2}\cos\frac{A-C}{2} = 4\sin^2 B/2$$

as $\cos\left(\frac{A+C}{2}\right) = \sin\frac{B}{2}$

so $\cos\frac{A-C}{2} = 2\sin\frac{B}{2}$

$$2\cos B/2\cos\frac{A-C}{2} = 4\sin B/2\cos B/2$$

$$2\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right) = 4\sin B/2\cos B/2$$

Sin A + sin C = 2 sin B

a + c = 2b $\Rightarrow$ a = 3, c = 7, b = 5

$$\cos A - \cos C = \frac{b^2+c^2-a^2}{2bc} - \frac{a^2+b^2-c^2}{2ab}$$

$$= \frac{25+49-9}{70} - \frac{9+25-49}{30}$$

$$= \frac{65}{70} + \frac{1}{2} = \frac{20}{14} = \frac{10}{7}$$

SECTION-B

21. A fair n (n > 1) faces die is rolled repeatedly until a number less than n appears. If the mean of the number of tosses required is $\frac{n}{9}$, then n is equal to _____.

Official Ans. by NTA (10.00)

Sol. Mean = $1.\frac{n-1}{n} + 2\frac{1}{n}\left(\frac{n-1}{n}\right) + 3\left(\frac{1}{n}\right)^2\left(\frac{n-1}{n}\right)$

...

$$\frac{n}{9} = \left(\frac{n-1}{n}\right)\left(1 + 2\left(\frac{1}{n}\right) + 3\left(\frac{1}{n}\right)^2 .....\right)$$

$$\frac{n}{9} = \left(\frac{n-1}{n}\right)\left(1-\frac{1}{n}\right)^{-2} = \left(\frac{n-1}{n}\right)\cdot\frac{n^2}{(n-1)^2}$$

$$\frac{n}{9} = \frac{n}{n-1} \Rightarrow n = 10$$

22. Let the digits a, b, c be in A.P. Nine-digit numbers are to be formed using each of these three digits thrice such that three consecutive digits are in A.P. at least once. How many such numbers can be formed?

Official Ans. by NTA (1260)

Sol. **a**bc or cba

$$\_\_\_\frac{a}{c}\frac{b}{b}\frac{c}{a}\_\_\_$$

$$\frac{{}^7C_1 \times 2 \times 6!}{2!2!2!} = 1260$$

23. Let [x] be the greatest integer ≤ x. Then the number of points in the interval (–2,1), where the function f(x) = | [x] | + $\sqrt{x-[x]}$ is discontinuous is_____.

Official Ans. by NTA (2.00)

Sol. Need to check at doubtful points

discont at x ∈ I only

at x = – 1 $\Rightarrow$ f (– 1^+) = 1 + 0 = 1

$\Rightarrow$ f (– 1^-) = 2 + 1 = 3

at x = 0 $\Rightarrow$ f (0^+) = 0 + 0 = 0

$\Rightarrow$ f (0^-) = 1 + 1 = 2

at x = 1 $\Rightarrow$ f (1^+) = 1 + 0 = 1

$\Rightarrow$ f (1^-) = 0 + 1 = 1

discont. at two points

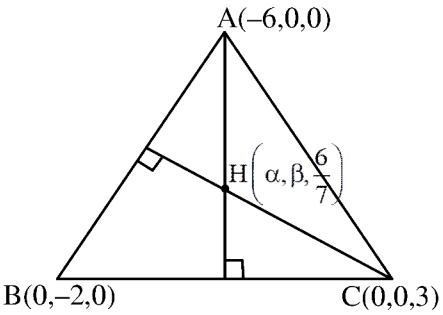
24. Let the plane x + 3y – 2z + 6 = 0 meet the co-ordinate axes at the points A,B,C. If the orthocentre of the triangle ABC is $\left(\alpha, \beta, \frac{6}{7}\right)$, then 98 $(\alpha + \beta)^2$ is equal to_____.

Official Ans. by NTA (288.00)

Sol. A (– 6, 0, 0) B (0, – 2, 0) C = (0, 0, 3)

$\overrightarrow{AB} = 6\hat{i} - 2\hat{j}, \quad \overrightarrow{BC} = 2\hat{j} + 3\hat{k},$

$\overrightarrow{AC} = 6\hat{i} + 3\hat{k}$



$\overrightarrow{AH} \cdot \overrightarrow{BC} = 0$

$\left(\alpha + 6, \beta, \frac{6}{7}\right) \cdot (0, 2, 3) = 0$

$\boxed{\beta = \frac{-9}{7}}$

$\overrightarrow{CH} \cdot \overrightarrow{AB} = 0$

$\left(\alpha, \beta, \frac{-15}{7}\right) \cdot (6, -2, 0) = 0$

$6\alpha - 2\beta = 0$

$\alpha = \frac{-3}{7}$

$98(\alpha + \beta)^2 = (98)\frac{(144)}{49} = 288$

25. Let I (x) = $\int \sqrt{\frac{x+7}{x}}\, dx$ and I (9) = 12 + 7 $\log_e$ 7.

If I (1) = α + 7 $\log_e$ $(1 + 2\sqrt{2})$, then α^4 is equal to_____.

Official Ans. by NTA (64.00)

Sol. $\int \sqrt{\frac{x+7}{x}}\, dx$

Put $x = t^2$

dx = 2tdt

$\int 2\sqrt{t^2+7}\, dt = 2\int \sqrt{t^2 + \sqrt{7}^2}\, dt$

$I(t) = 2\left[\frac{t}{2}\sqrt{t^2+7} + \frac{7}{2}\ln\left|t + \sqrt{t^2+7}\right|\right] + C$

$I(x) = \sqrt{x}\sqrt{x+7} + 7\, ln\left|\sqrt{x} + \sqrt{x+7}\right| + C$

$I(9) = 12 + 7\, ln7 = 12 + 7\,(ln\,(3+4)) + C$

$\Rightarrow C = 0$

$I(x) = \sqrt{x}\sqrt{x+7} + 7\, ln\left(\sqrt{x} + \sqrt{x+7}\right)$

$I(1) = 1\sqrt{8} + 7\, ln\left(1 + \sqrt{8}\right)$

$I(1) = \sqrt{8} + 7\, ln\,(1 + 2\sqrt{2})$

$\alpha = \sqrt{8}$

$\alpha^4 = (8^{1/2})^4$

$\alpha^4 = 8^2 = 64$

26. Let $D_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$. If $\sum_{k=1}^{n}$ D_k = 96, then n is equal to

Official Ans. by NTA (6.00)

Sol. $D_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$

$\sum_{k=1}^{n} D_k = 96 \Rightarrow$

$\begin{vmatrix} \sum_{k=1}^{n} 1 & \Sigma 2k & \Sigma(2k-1) \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix} = 96$

$\Rightarrow \begin{vmatrix} n & n^2+n & n^2 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix} = 96$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\begin{vmatrix} n & n^2+n & n^2 \\ 0 & 2 & 0 \\ 0 & 0 & n+2 \end{vmatrix} = 96$

$\Rightarrow n(2n+4) = 96 \Rightarrow n(n+2) = 48 \Rightarrow n = 6$

27. Let the positive numbers a_1, a_2, a_3, a_4 and a_5 be in a G.P. Let their mean and variance be $\frac{31}{10}$ and $\frac{m}{n}$ respectively, where m and n are co-prime. If the mean of their reciprocals is $\frac{31}{40}$ and $a_3 + a_4 + a_5 = 14$, then m + n is equal to_____.

Official Ans. by NTA (211)

Sol. Let $\frac{a}{r^2}, \frac{a}{r}$, a, ar, ar^2

Given $\frac{a}{r^2}+\frac{a}{r}+a+ar+ar^2 = 5\times\frac{31}{10}$ ...(1)

And $\frac{r^2}{a}+\frac{r}{a}+\frac{1}{a}+\frac{1}{ar}+\frac{1}{ar^2} = 5\times\frac{31}{40}$ ...(2)

(1) ÷ (2) $a^2 = 4 \Rightarrow a = 2 \quad \therefore r+\frac{1}{r} = 5/2 \quad (a \neq -2)$

$\Rightarrow r = 2$

$\therefore$ Now $\frac{1}{2}$, 1, 2. 4, 8

$\therefore \sigma^2 = \frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2$

$= \frac{186}{25} = \frac{M}{N} \Rightarrow 211 = m + n$

28. The number of relations, on the set {1,2,3} containing (1,2) and (2,3), which are reflexive and transitive but not symmetric, is_____

Official Ans. by NTA (4.00)

Sol. A = {1,2, 3}

For Reflexive (1, 1) (2, 2), (3, 3) $\in$ R

For transitive : (1, 2) and (2, 3) $\in$R $\Rightarrow$ (1, 3) $\in$ R

Not symmetric : (2, 1) and (3, 2) $\notin$ R

R_1 = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)}

R_2 = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3) (2, 1)}

R_3 = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3) (3, 2)}

29. If $\int_{-0.15}^{0.15} |\,100x^2 - 1\,|\,dx = \frac{k}{3000}$, then k is equal to_____.

Official Ans. by NTA (575)

Sol. $\int_{-0.15}^{0.15} |100x^2 - 1|\,dx = 2\int_{0}^{0.15} |100x^2 - 1|dx$

Now $100x^2 - 1 = 0 \Rightarrow x^2 = \frac{1}{100} \Rightarrow x = 0.1$

$$I = 2\left[\int_0^{0.1} (1 - 100x^2)\,dx + \int_{0.1}^{0.15} (100x^2 - 1)\,dx\right]$$

$$I = 2\left[x - \frac{100}{3}x^3\right]_0^{0.1} + 2\left[\frac{100x^3}{3} - x\right]_{0.1}^{0.15}$$

$$= 2\left[0.1 - \frac{0.1}{3}\right] + 2\left[\frac{0.3375}{3} - 0.15 - \frac{0.1}{3} + 0.1\right]$$

$$= 2\left[0.2 - \frac{0.2}{3} + 0.1125 - 0.15\right]$$

$$= 2\left[\frac{5}{100} - \frac{2}{30} + \frac{1125}{10000}\right] = 2\left(\frac{1500 - 2000 + 3375}{30000}\right)$$

$$= \frac{575}{3000} \Rightarrow k = 575$$

30. Two circles in the first quadrant of radii r_1 and r_2 touch the coordinate axes. Each of them cuts off an intercept of 2 units with the line x + y = 2. Then $r_1^2 + r_2^2 - r_1r_2$ is equal to_____.

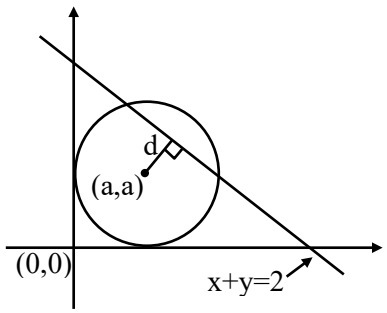
Official Ans. by NTA (7.00)

Sol. Circle $(x - a)^2 + (y - a)^2 = a^2$

$x^2 + y^2 - 2ax - 2ay + a^2 = 0$

intercept = 2

$\Rightarrow 2\sqrt{a^2 - d^2} = 2$



Where d = perpendicular distance of centre from line x + y = 2

$\Rightarrow 2\sqrt{a^2 - \left(\frac{a + a - 2}{\sqrt{2}}\right)^2} = 2$

$\Rightarrow a^2 - \frac{(2a-2)^2}{2} = 1 \Rightarrow 2a^2 - 4a^2 + 8a - 4 = 2$

$\Rightarrow 2a^2 - 8a + 6 = 0 \Rightarrow a^2 - 4a + 3 = 0$

$\therefore r_1 + r_2 = 4$ and $r_1r_2 = 3$

$\therefore r_1^2 + r_2^2 - r_1r_2 = (r_1 + r_2)^2 - 3r_1r_2$

$= 16 - 9 = 7$

SECTION-A

31. An ice cube has a bubble inside. When viewed from one side the apparent distance of the bubble is 12 cm. when viewed from the opposite side, the apparent distance of the bubble is observed as 4 cm. If the side of the ice cube is 24 cm, the refractive index of the ice cube is

(1) $\frac{4}{3}$ (2) $\frac{3}{2}$

(3) $\frac{2}{3}$ (4) $\frac{6}{5}$

Official Ans. by NTA (2)

Sol. $d_{apparent} = \frac{d_{actual}}{\mu_{rel}}$

$12 = \frac{x}{\mu}$ …(1)

$4 = \frac{24 - x}{\mu}$ …(2)

On solving we get $\mu = 1.5$

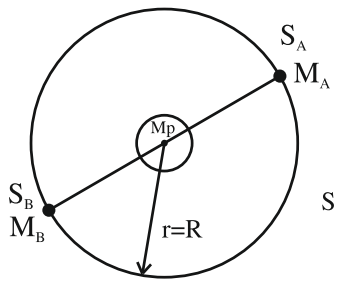
32. Two satellites A and B move round the earth in the same orbit. The mass of A is twice the mass of B. The quantity which is same for the two satellites will be :

(1) Potential energy (2) Total energy

(3) Kinetic energy (4) Speed

Official Ans. by NTA (4)

Sol.



$P.E = -\frac{GM_pM_A}{R}$

$K.E = +\frac{GM_pM_A}{2R}$

$T.E = -\frac{GM_pM_A}{2R}$

$Speed = v = \sqrt{\frac{GM_p}{R}}$

Speed of satellite in Independent of mass of satellite.

33. The amplitude of 15 sin (1000 πt) is modulated by 10 sin (4πt) signal. The amplitude modulated signal contains frequencies of

1. 500 Hz. 2. 2 Hz

3. 250 Hz 4. 498 Hz

5. 502 Hz

Choose the correct answer from the options given below:

(1) (1) and (3) only

(2) (1) and (4) only

(3) (1) and (2) only

(4) (1), (4) and (5) only

Official Ans. by NTA (4)

Sol. Equation of Carrier wave

$c(t) = 15 \sin (1000\ \pi t)$

$f_i = \frac{\omega_c}{2\pi} = \frac{1000\pi}{2\pi} = 500\ Hz$

Equation of modulated wave

$m(t) = 10 \sin (4\ \pi t)$

$f_m = \frac{\omega_m}{2\pi} = \frac{4\pi}{2\pi} = 2\ Hz$

Frequencies contained in resultant Amplitude modulated wave are (500–2)Hz, 500 Hz and (500+2) Hz.

Correct ans is (4)

34. In an n-p-n common emitter (CE) transistor the collector current changes from 5 mA to 16 mA for the change in base current from 100 μA and 200 μA, respectively. The current gain of transistor is_____.

(1) 110 (2) 0.9

(3) 210 (4) 9

Official Ans. by NTA (1)

Sol. Current gain in common emitter transistor

$$\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{16\text{mA} - 5\text{mA}}{200\,\mu\text{A} - 100\mu\text{A}} = \frac{11\,\text{mA}}{100\mu\text{A}} = 110$$

35. If the r.m.s. speed of chlorine molecule is 490 m/s at 27° C, the r.m.s. speed of argon molecules at the same temperature will be (Atomic mass of argon = 39.9u, molecular mass of chlorine = 70.9u)

(1) 751.7 m/s (2) 451.7 m/s

(3) 651.7 m/s (4) 551.7 m/s

Official Ans. by NTA (3)

Sol. $V_{rms} = \sqrt{\frac{3RT}{M}}$ $\qquad \frac{\upsilon_{Ar}}{\upsilon_{Cl}} = \sqrt{\frac{M_{Cl}}{M_{Ar}}}$

$$\Rightarrow \upsilon_{Ar} = 1.33 \times 490 = 651.7\,\text{m/s}$$

36. A proton and an α-particle are accelerated from rest by 2V and 4V potentials, respectively. The ratio of their de-Broglie wavelength is:

(1) 4:1 (2) 2:1

(3) 8:1 (4) 16:1

Official Ans. by NTA (1)

Sol. $\lambda = \frac{h}{m\upsilon} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mq\Delta V}}$

$$\frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{m_p V_p q_p}{m_\alpha V_\alpha q_\alpha}}$$

$$\Rightarrow \frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{1\times 2\times 1}{4\times 4\times 2}} = \frac{1}{4}$$

$$\Rightarrow \lambda_p : \lambda_\alpha = 4:1$$

37. Given below are two statements:

Statement I : The diamagnetic property depends on temperature.

Statement II : The included magnetic dipole moment in a diamagnetic sample is always opposite to the magnetizing field.

In the light of given statement, choose the correct answer from the options given below:

(1) Statement I is incorrect but Statement II is true

(2) Both Statement I and Statement II are true.

(3) Both Statement I and Statement II are false.

(4) Statement I is correct but Statement II is false.

Official Ans. by NTA (1)

Sol. **Conceptual**

38. A wire of resistance 160 Ω is melted and drawn in wire of one-fourth of its length. The new resistance of the wire will be

(1) 10 Ω (2) 640 Ω

(3) 40 Ω (4) 16 Ω

Official Ans. by NTA (1)

Sol. Volume = Constant

$A_1L_1 = A_2L_2$

$$A_1 L = A_2 \frac{L}{4}$$

$$\boxed{4A_1 = A_2}$$

$$R_1 = \frac{\rho L_1}{A_1} \qquad R_2 = \frac{\rho L_2}{A_2}$$

$$\frac{R_2}{R_1} = \frac{L_2 A_1}{A_2 L_1} = \frac{L}{4}\frac{A_1}{4A_1 L}$$

$$R_2 = \frac{1}{16} R_1 = 10\,\Omega$$

39. Match List I with List II

List I		List II	
A.	Spring constant	I.	(T^{-1})
B.	Angular speed	II.	(MT^{-2})
C.	Angular momentum	III.	(ML^2)
D.	Moment of Inertia	IV.	(ML^2T^{-1})

Choose the correct answer from the options given below:

(1) A-II, B-I, C-IV, D-III

(2) A-IV, B-I, C-III, D-II

(3) A-II, B-III, C-I, D-IV

(4) A-I, B-III, C-II, D-IV

Official Ans. by NTA (1)

Sol. Spring Constant

$$[K]=\frac{[F]}{[x]}=\frac{MLT^{-2}}{L}=MT^{-2}$$

$$[\omega]=\frac{[\theta]}{[t]}=\frac{1}{T}=T^{-1}$$

40. Three force $F_1 = 10N$, $F_2 = 8$ N, $F_3 = 6$ N are acting on a particle of mass 5 kg. The forces F_2 and F_3 are applied perpendicular so that particle remains at rest. If the force F_1 is removed, then the acceleration of the particle is:

(1) 2 ms^{-2} (2) 0.5 ms^{-2}

(3) 4.8 ms^{-2} (4) 7 ms^{-2}

Official Ans. by NTA (1)

Sol. Resultant of $\vec{F_2}$ and $\vec{F_3}$ should be opposite to $\vec{F_1}$

$$a=\frac{10}{5}=2m/s^2$$

41. A body cools from 80°C to 60°C in 5 minutes. The temperature of the surrounding is 20°C. The time it takes to cool from 60°C to 40°C is:

(1) 500 s (2) $\frac{25}{3}$ s

(3) 450 s (4) 420 s

Official Ans. by NTA (1)

Sol. Rate of cooling α Temperature difference

$$\frac{80-60}{5}=k\{70-20\}\ \text{--- (1)}$$

$$\frac{60-40}{t}=k[50-20]\ \text{---- (2)}$$

$$\frac{4t}{20}=\frac{50}{30}$$

$$t=\frac{25}{3}\min=500\sec$$

$$\Rightarrow t=500\,\text{seconds}$$

42. An engine operating between the boiling and freezing points of water will have

1. efficiency more than 27%

2. efficiency less than the efficiency a Carnot engine operating between the same two temperatures.

3. efficiency equal to 27%

4. efficiency less than 27%

(1) 2, 3 and 4 only (2) 2 and 3 only

(3) 2 and 4 only (4) 1 and 2 only

Official Ans. by NTA (3)

Sol. $\eta=\left(1-\frac{273}{373}\right)\times 100=26.8\%$

43. Given below are two statements:

Statement I : A truck and a car moving with same kinetic energy are brought to rest by applying brakes which provide equal retarding forces. Both come to rest in equal distance.

Statement II : A car moving towards east takes a turn and moves towards north, the speed remains unchanged. The acceleration of the car is zero.

In the light of given statements, choose the most appropriate answer from the options given below.

(1) Statement I is correct but Statement II is incorrect

(2) Statement I is incorrect but Statement II is correct

(3) Both Statement I is correct but Statement II are incorrect

(4) Both Statement I is correct but Statement II are correct

Official Ans. by NTA (1)

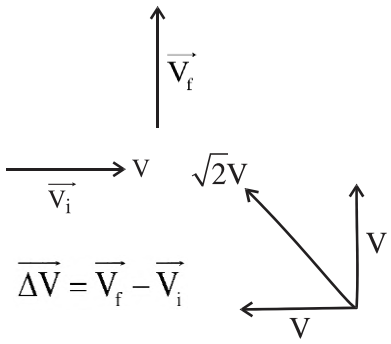
Sol. Work done = ΔKE

Work done = –FS = 0 – K

$$S = \frac{K}{F}$$

Statement 1 → correct

Statement 2 → incorrect



Velocity is changing $\Rightarrow \vec{a} \neq 0$

Ans. 1

44. A particle is executing Simple Harmonic Motion (SHM). The ratio of potential energy and kinetic energy of the particle when its displacement is half of its amplitude will be:

(1) 1 : 1 (2) 2 : 1

(3) 1 : 4 (4) 1 : 3

Official Ans. by NTA (4)

Sol. $x = \frac{A}{2}, \quad P.E. = \frac{1}{2}kx^2$

$$K.E. = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$$

$$\frac{P.E}{K.E} = \frac{x^2}{A^2 - x^2} = \frac{A^2}{4\left(\frac{3A^2}{4}\right)} = \frac{1}{3}$$

45. A ball is thrown vertically upward with an initial velocity of 150 m/s. The ratio of velocity after 3 s and 5s is $\frac{x+1}{x}$. The value of x is _____.

Take (g = 10 m/s^2).

(1) 6 (2) 5

(3) –5 (4) 10

Official Ans. by NTA (2)

Sol. $\vec{v} = \vec{u} + \vec{a}t$

$V = 150 - 10t$

V(3) = 150 – 30 = 120

V(5) = 150 – 50 = 100

$$\frac{120}{100} = \frac{x+1}{x} = \frac{6}{5} \Rightarrow x = 5$$

Ans. (2)

46. Given below are two statement: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : If an electric dipole of dipole moment 30×10^{-5} Cm is enclosed by a closed surface, the net flux coming out of the surface will be zero.

Reason R : Electric dipole consists of two equal and opposite charges.

In the light of above, statements, choose the correct answer from the options given below:

(1) Both A and R are true and R is the correct explanation of A

(2) A is true but R is false

(3) Both A and R true but R is NOT the correct explanation of A

(4) A is false but R is true

Official Ans. by NTA (1)

Sol. $\vec{P} = 30\times10^{-5}$ Cm

Using Gauss law

$$\phi = \frac{Q_{in}}{\varepsilon_0} \text{ and } Q_{in} = 0$$

$\Rightarrow \phi = 0$

Statement 1 and Statement 2 are correct.

Ans. (1)

47. Given below are two statement : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : EM waves used for optical communication have longer wavelengths than that of microwave, employed in Radar technology.

Reason R : Infrared EM waves are more energetic than microwaves, (used in Radar)

In the light of given statements, choose the correct answer from the options given below:

(1) A is false but R is true

(2) A is true but R is false

(3) Both A and R true but R is NOT the correct explanation of A

(4) Both A and R true and r is the correct explanation of A

Official Ans. by NTA (1)

Sol. Optical communication is performed in the frequency range of 1THz to 1000 THz.
(Microwave to UV)
So, EM waves used for optical communication have shorter wavelength than that of microwaves used in RADAR.

Also, $\upsilon_{INFRARED} > \upsilon_{MICROWAVE}$

$\therefore$ Infrared EM waves are more energetic than microwave

48. A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. The number of spectral lines emitted will be:

(1) 2 (2) 1

(3) 3 (4) 4

Official Ans. by NTA (3)

Sol. According to Bohr's postulates, an electron makes jump to higher energy orbital if it absorbs a photon of energy equal to difference between the energies of an excited state and the ground state. Assuming that collided electron takes energy equal to 10.2 eV or 12.09 eV from incoming electron beam (some part lost due to collision). The maximum excited state is n = 3. So, number of spectral lines is

$$\frac{3(3-1)}{2} = 3$$

49. The ratio of escape velocity of a planet to the escape velocity of earth will be:

Given : Mass of the planet is 16 times mass of earth and radius of the planet is 4 times the radius of earth.

(1) 4 : 1 (2) 2 : 1

(3) 1 : $\sqrt{2}$ (4) 1 : 4

Official Ans. by NTA (1)

Sol. $V_{escape} = \sqrt{\frac{2GM}{R}}$

$\therefore$ V_{escape} for planet $= \sqrt{\frac{2G(16M_E)}{(4R_E)}} = 2\sqrt{\frac{2GM_E}{R_E}}$

$= 2(V_{escape}$ for Earth)

50. Given below are two statements :

Statement I : When the frequency of an a.c. source in a series LCR circuit increases, the current in the circuit first increases, attains a maximum value and then decreases.

Statement II : In a series LCR circuit, the value of power factor at resonance is one.

In the light of given statements, choose the most appropriate answer from the options given below:

(1) Statement I is incorrect but Statement II is true.

(2) Both Statement I and Statement II are false.

(3) Statement I is correct but Statement II is false.

(4) Both Statement I and Statement II are true.

Official Ans. by NTA (4)

Sol. Both statements are correct. Theory based.

SECTION-B

51. For a certain organ pipe, the first three resonance frequencies are in the ratio of 1:3:5 respectively. If the frequency of fifth harmonic is 405 Hz and the speed of sound in air is 324 ms^{-1} the length of the organ pipe is _____m.

Official Ans. by NTA (1)

Sol. For 5th harmonic in closed organ pipe,

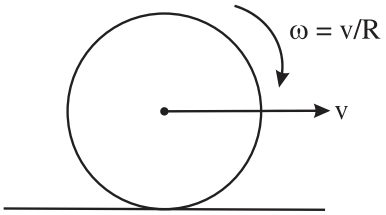
$$f_5 = \frac{5V}{4\ell} \Rightarrow 405 = \frac{5\times 324}{4\ell}$$

$$\Rightarrow \ell = 1m$$

52. For a rolling spherical shell, the ratio of rotational kinetic energy and total kinetic energy is $\frac{x}{5}$. The value of x is_____.

Official Ans. by NTA (2)

Sol.



$$\frac{K_{rot}}{K_{Total}} = \frac{\frac{1}{2}\left(\frac{2}{3}mR^2\right)\left(\frac{V}{R}\right)^2}{\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{3}mR^2\right)\left(\frac{V}{R}\right)^2}$$

$$\Rightarrow \frac{x}{5} = \frac{2}{5} \quad \Rightarrow x = 2$$

53. A compass needle oscillates 20 times per minute at a place where the dip is 30° and 30 times per minute where the dip is 60°. The ratio of total magnetic field due to the earth at two place respectively is $\frac{4}{\sqrt{x}}$. The value of x is

Official Ans. by NTA (243)

Sol. Period of oscillation $\alpha \frac{1}{\sqrt{B_H}}$

$$T\alpha \frac{1}{\sqrt{B\cos\theta}} \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{B_2\cos\theta_2}{B_1\cos\theta_1}}$$

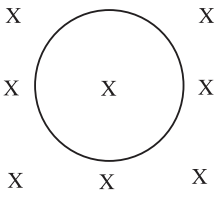
$$\Rightarrow \frac{60/20}{60/30} = \sqrt{\frac{B_2}{B_1}\frac{\cos 60^\circ}{\cos 30^\circ}} \Rightarrow \frac{3}{2} = \sqrt{\frac{B_2}{\sqrt{3}B_1}}$$

$$\Rightarrow \frac{9}{4} = \frac{B_2}{\sqrt{3}B_1} \Rightarrow \frac{B_1}{B_2} = \frac{4}{9\sqrt{3}} = \frac{4}{\sqrt{243}}$$

54. A conducting circular loop is placed in a uniform magnetic field of 0.4 T with its plane perpendicular to the field. Somehow, the radius of the loop starts expanding at a constant rate of 1 mm/s. The magnitude of induced emf in the loop at an instant when the radius of the loop is 2 cm will be _____ μV.

Official Ans. by NTA (50)

Sol.



$$\frac{dr}{dt} = 10^{-3}\ m/s$$

$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$$

$$\varepsilon = \left|\frac{-d\phi}{dt}\right| = \left|\frac{BdA}{dt}\right|$$

$$= 0.4 \times 2 \times \pi \times 2 \times 10^{-2} \times 10^{-3} V$$

$$= 16\pi\mu V = 50.24\ \mu V$$

55. To maintain a speed of 80 km/h by a bus of mass 500 kg on a plane rough road for 4 km distance, the work done by the engine of the bus will be _____KJ. [The coefficient of friction between tyre of bus and road is 0.04].

Official Ans. by NTA (784)

Sol. For constant speed, WD by engine + WD by friction = 0 [by WET]

$WD_{engine} = -WD_{friction} = -[-\mu mgx]$

$= 0.04 \times 500 \times 9.8 \times 4 \times 10^3$

$= 784$ KJ

56. A common example of alpha decay is

$$^{238}_{92}U \longrightarrow {}^{234}_{90}Th + {}_2He^4 + Q$$

Given :

$^{238}_{92}U = 238.05060u,$

$^{234}_{90}Th = 234.04360u,$

$^{4}_{2}He = 4.00260u,$ and

$1u = 931.5\frac{MeV}{c^2}$

The energy released (Q) during the alpha decay of $^{238}_{92}U$ is _____ MeV

Official Ans. by NTA (4)

Sol. Energy released = $(\Delta m)_{amu} \times 931.5$ MeV

$= (m_u - m_{Th} - m_{He})_{amu} \times 931.5$ MeV

$= 0.0044 \times 931.5$ MeV $= 4.0986$ MeV

57. The current flowing through a conductor connected across a source is 2A and 1.2 A at 0°C and 100°C respectively. The current flowing through the conductor at 50°C will be _____$\times 10^2$ mA.

Official Ans. by NTA (15)

Sol. $i_o R_o = i_{100} R_{100}$ [For same source]

$\Rightarrow 2 R_o = 1.2 R_o [1 + 100\alpha]$ --- (1)

$\Rightarrow 1 + 100\alpha = \frac{5}{3} \Rightarrow 100\alpha = \frac{2}{3}$

$\Rightarrow 50\,\alpha = \frac{1}{3}$

$\therefore \; i_{50} R_{50} = i_o R_o$

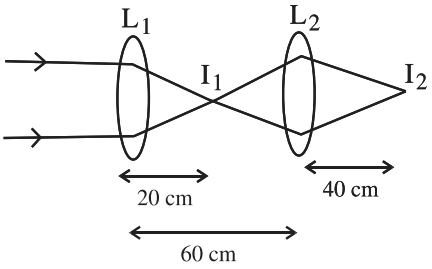
$\Rightarrow i_{50} = \frac{i_o R_o}{R_{50}} = \frac{2 \times R_o}{R_o(1+50\alpha)} = \frac{2}{1+\frac{1}{3}} = 1.5A$

$= 15 \times 10^2$ mA

58. Two convex lenses of focal length 20 cm each are placed coaxially with a separation of 60 cm between them. The image of the distant object formed by the combination is at _____ cm from the first lens.

Official Ans. by NTA (100)

Sol. **$f_1 = 20$ cm** **$f_2 = 20$ cm**



1st refraction in $L_1(I_1)$

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} - \frac{1}{\infty} = \frac{1}{f}$

$\therefore \; v = f$

2nd refraction in L_2

$I_1 \rightarrow$ object

$I_2 \rightarrow$ image

$u = -40$ cm $\qquad$ $f = 20$ cm

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} - \frac{1}{(-40)} = \frac{1}{20}$

$\frac{1}{v} = \frac{1}{20} - \frac{1}{40} = \frac{6-3}{120}$

$\frac{1}{v} = \frac{3}{120} = \frac{1}{40}$

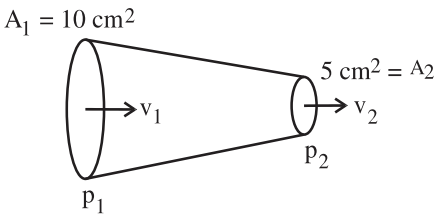
$\therefore \; v = 40$ cm

Correct Answer is 100.

59. Glycerine of density 1.25×10^3 kg m^{-3} is flowing through the conical section of pipe. The area of cross-section of the pipe at its ends is 10 cm^2 and 5 cm^2 and pressure drop across its length is 3 Nm^{-2}. The rate of flow of glycerine through the pipe is $x \times 10^{-5}$ m^3 s^{-1}. The value of x is ____.

Official Ans. by NTA (4)

Sol.



$\Delta P = P_1 - P_2 = 3$ N/m^2 (given)

By continuity eqn

$A_1 v_1 = A_2 v_2$

$\therefore \; v_1 = \frac{A_2}{A_1} v_2$ ----- (1)

By Bernoulli's eq^{n}

$$P_1 + \frac{1}{2}\rho v_1^{\,2} = P_2 + \frac{1}{2}\rho v_2^{\,2}$$

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$$\Delta P = \frac{1}{2}\rho\left(v_2^2 - \frac{A_2^2}{A_1^2}v_2^2\right)$$

$$\Delta P = \frac{1}{2}\rho\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]v_2^2$$

$$3 = \frac{1}{2}\times 1.25\times 10^3\left[1 - \left(\frac{5}{10}\right)^2\right]v_2^2$$

$$3 = \frac{1}{2}\times 1.25\times 10^3\left[1 - \frac{1}{4}\right]v_2^2$$

$$3 = \frac{1}{2}\times 1.25\times 10^3 \times \frac{3}{4}v_2^2$$

$$\therefore \quad v_2 = 8 \times 10^{-2}\ \text{m/s}$$

So discharge rate = $A_2 v_2$

$$= 5 \times 10^{-4} \times 8 \times 10^{-2}$$

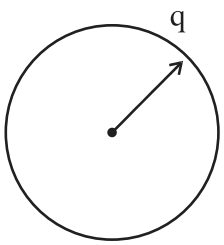
$$= 4 \times 10^{-5}\ \text{m}^3/\text{s}$$

Correct ans is x = 4

60. 64 identical drops each charged upto potential of 10 mV are combined to form a bigger dorp. The potential of the bigger drop will be _____ mV.

Official Ans. by NTA (160)

Sol.



Let q = charge on each drop

$$V = \frac{Kq}{r} \text{---- (1)}$$

Now for combination of 64 drop

$$64 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

R = 4r

And Q = 64 q

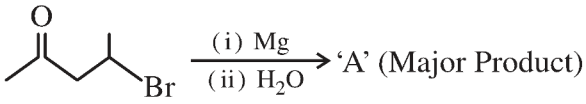
Potential of bigger drop

$$= \frac{KQ}{R} = \frac{K64q}{4r} = 16\frac{Kq}{r}$$

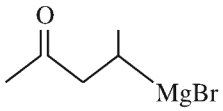
= 16 × 10 mV = 160 mV.

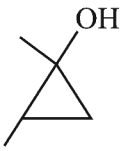
Correct answer is 160.

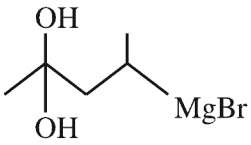
SECTION-A

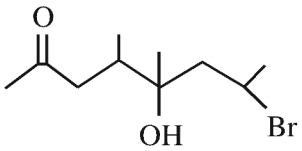
61. 

A is

(1) 

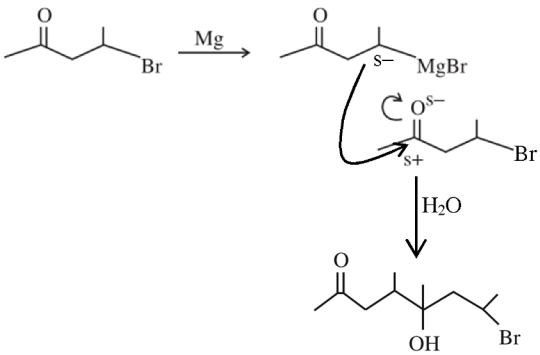
(2) 

(3) 

(4) 

Official Ans. by NTA (4)

Sol.



62. Four gases A, B, C and D have critical temperatures 5.3, 33.2, 126.0 and 154.3K respectively.

For their adsorption on fixed amount of charcoal, the correct order is :

(1) C > B > D > A

(2) C > D > B > A

(3) D > C > A > B

(4) D > C > B > A

Official Ans. by NTA (4)

Sol. Extent of adsorption α critical temp.

63. Given below are two statement: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A: 5f electrons can participate in bonding to a far greater extent than 4f electrons

Reason R: 5f orbitals are not as buried as 4f orbitals

In the light of the above statements, choose the *correct* answer from the options given below

(1) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**

(2) Both **A** and **R** are true and **R** is the correct explanation of **A**

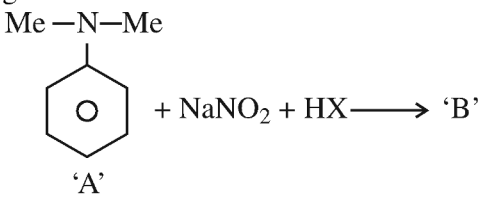
(3) **A** is false but **R** is true

(4) **A** is true but **R** is false

Official Ans. by NTA (2)

Sol. 5f orbital not buried as 4f orbitals so e^- present in 5f orbital experience less nuclear attraction than e^- present in 4f orbital. Hence electrons of 5f orbital can take part in bonding to a far greater extent.

64. The <u>incorrect</u> statement regarding the reaction given below is



(1) The electrophile involved in the reaction is NO^+

(2) 'B' is N-nitroso ammonium compound

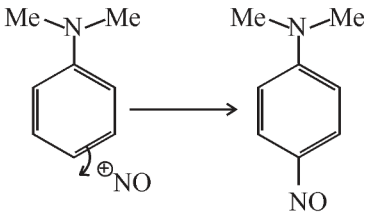
(3) The reaction occurs at low temperature

(4) The product 'B' formed in the above reaction is p-nitroso compound at low temperature

Official Ans. by NTA (2)

Sol. $NaNO_2 + HX \rightarrow HNO_2 + NaX$

$H-O-N=O \xrightarrow[-H_2O]{H^{\oplus}} NO^{\oplus}$ (Nitrosonium ion)



P – Nitroso product

65. Match List I with List II

LIST I Complex		**LIST II CFSE(Δ_0)**	
A.	$[Cu(NH_3)_6]^{2+}$	I.	-0.6
B.	$[Ti(N_2O)_6]^{3+}$	II.	-2.0
C.	$[Fe(CN)_6]^{3-}$	III.	-1.2
D.	$[NiF_6]^{4-}$	IV.	-0.4

Choose the correct answer from the options given below :

(1) A-I, B-IV, C-II, D-III

(2) A-II, B-III, C-I, D-IV

(3) A-I, B-II, C-IV, D-III

(4) A-III, B-IV, C-I, D-II

Official Ans. by NTA (1)

Sol. CFSE = $(-0.4\ nt_{2g} + 0.6\ n_{eg})\ \Delta_0$

nt_{2g} = number of electrons in t_{2g} orbital

n_{eg} = number of electrons in eg orbital

Complex	No.of at electrons	CFSE(Δ_0)
$[Cu(NH_3)_6]^{+2}$	d^9 (S.L.) $t_{2g}^{2,2,2}eg^{2,1}$	-0.6
$[Ti(H_2O)_6]^{+3}$	d^1 (W.L.) $t_{2g}^{1,0,0}eg^{0,0}$	-0.4
$[Fe(CN)_6]^{3-}$	d^5 (S.L.) $t_{2g}^{2,2,1}eg^{0,0}$	-2.0
$[NiF_6]^{4-}$	d^8 (W.L.) $t_{2g}^{2,2,2}eg^{1,1}$	-1.2

66. Match List I with List II

LIST I (Examples)		**LIST I (Examples)**	
A.	2-Chloro-1, 3 - butadiene	I.	Biodegradable polymer
B.	Nylon 2-nylon 6	II.	Synthetic Rubber
C.	Polyacrylonitrile	III.	Polyester
D.	Dacron	IV.	Addition Polymer

Choose the correct answer from the options given below :

(1) A-IV, B-I, C-III, D-II

(2) A-IV, B-III, C-I, D-II

(3) A-II, B-IV, C-I, D-III

(4) A-II, B-I, C-IV, D-III

Official Ans. by NTA (4)

Sol. FACT

67. The density of alkali metals is in the order

(1) Na < K < Cs < Rb

(2) K < Na < Rb < Cs

(3) K < Cs < Na < Rb

(4) Na < Rb < K < Cs

Official Ans. by NTA (2)

Sol. In general moving down the group, mass increases more prominently as compared to volume (size) hence density increases for Group I metal. Due to empty 3d subshell in K increase in size is more prominent as compare to mass.

Li < K < Na < Rb < Cs

68. Given below are two statements :

Statements : $SbCl_5$ is more covalent than $SbCl_3$

Statements : The higher oxides of halogens also tend to be more stable than the lower ones.

In the light of the above statements, choose the most appropriate answer from the options given below

(1) Both statement I and Statement II are correct

(2) Both statement I and Statement II are incorrect

(3) Statement I is correct but Statement II is incorrect

(4) Statement I is incorrect but Statement II is correct

Official Ans. by NTA (1)

Sol. Statement I : Is correct according to Fajan's rule Sb^{+5} more polarising power than Sb^{+3}.

Statement II : Stability of higher oxides of halogen is primarily due to

a) Higher oxidation state

b) More EN halogen

c) Resonance stabilization

69. A metal chloride contains 55.0% of chlorine by weight. 100 mL vapours of the metal chloride at STP weigh 0.57 g. The molecular formula of the metal chloride is

(Given : Atomic mass of chlorine is 35.5u)

(1) MCl_2

(2) MCl_4

(3) MCl_3

(4) MCl

Official Ans. by NTA (1)

Sol. Molecular. weight of metal chloride

$$= \frac{0.57}{100} \times 22700$$

$$= 129.39$$

$$\text{weight of Cl} = 129.39 \times 0.55$$

$$= 71.1645$$

$$\therefore \text{Mole of Cl} = \frac{71.1645}{35.5} \cong 2$$

Hence MCl_2

70. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : In the Ellingham diagram, a sharp change in slope of the line is observed for $Mg \rightarrow MgO$ at $\sim 1120\,^{\circ}C$

Reason R : There is a large change of entropy associated with the change of state

In the light of the above statements, choose the *correct* answer from the options given below

(1) Both A and R are true but R is NOT the correct explanation of A

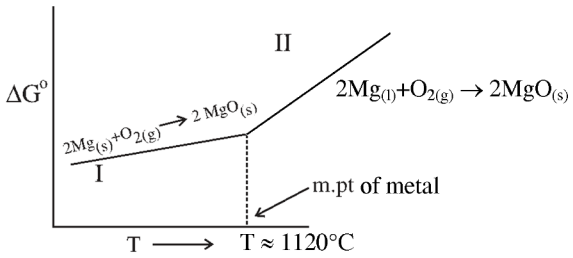
(2) Both A and R are true and R is the correct explanation of A

(3) A is false but R is true

(4) A is true but R is false

Official Ans. by NTA (2)

Sol.



For line II, ΔS is more –ve than line I. hence higher slope.

For I $\Delta S_I = (S_{solid}) - (S_{solid} + S_{gas})$

For II $\Delta S_{II} = (S_{solid}) - (S_{liq} + S_{gas})$

Hence ΔS_{II} more –ve than ΔS_I

71. Match List I with List II

	LIST I		LIST II
A.	Nitrogen oxides in air	I.	Eutrophication
B.	Methane in air	II.	pH of rain water becomes 5.6.
C.	Carbon dioxide	III.	Global warming
D.	Phosphate fertilisers in water	IV.	Acid rain

Choose the correct answer from the options given below :

(1) A-IV, B-III, C-II, D-I

(2) A-II, B-III, C-I, D-IV

(3) A-I, B-II, C-III, D-IV

(4) A-IV, B-II, C-III, D-I

Official Ans. by NTA (1)

Sol.:

i. $4NO_2(g) + O_2(g) + 2H_2O(\ell) \rightarrow 4HNO_3(aq)$

SO_2 & NO_2 have major contribution in acid rain

ii. CO_2, CH_4, O_3, CFC are responsible for global warming

iii. $H_2O(\ell) + CO_2(g) \rightleftharpoons H_2CO_3(aq.)$

$H_2CO_3(aq.) \rightleftharpoons H^+(aq.) + HCo_3^-(aq.)$

Rain water has pH of 5.6 due to the Presence of H^+ ions formed by the reaction of rain water with CO_2

iv. Phosphates present in fertilizers contribution for Eutrophication (Process in which nutrient enriched water bodies support a dense plant population, which kills animal life by depriving it of oxygen and results in subsequent loss of biodiversity is known as Eutrophication.)

72. For lead storage battery pick the correct statements

A. During charging of battery, $PbSO_4$ on anode is converted into PbO_2

B. During charging of battery, $PbSO_4$ on cathode is converted into PbO_2

C. Lead storage battery, consists of grid of lead packed with PbO_2 as anode

D. Lead storage battery has $\sim$ 38% solution of sulphuric acid as an electrolyte

Choose the correct answer from the options given below :

(1) B, D only

(2) B, C, D only

(3) A, B, D only

(4) B, C only

Official Ans. by NTA (1)

Sol. Lead storage battery consists of lead anode and a grid of lead packed with lead oxide (PbO_2) as cathode, a 38% solution of H_2SO_4 is used as an electrolyte.

On charging the battery the reaction is reversed and $PbSO_4(s)$ on anode and cathode is converted into Pb and PbO_2 respectively.

73. $2-\text{hexene} \xrightarrow[(ii)H_2O]{(i)O_3} \text{Products}$

The two products formed in above reaction are -

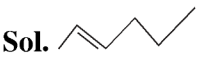
(1) Butanoic acid and acetic acid

(2) Butanal and acetic acid

(3) Butanal and acetaldehyde

(4) Butanoic acid and acetaldehyde

Official Ans. by NTA (1)

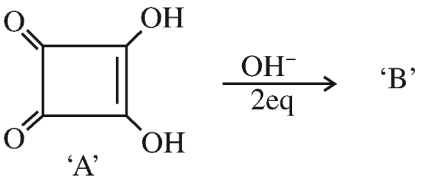
Sol. 

$\xrightarrow[ii)H_2O]{i)O_3} CH_3COOH + CH_3CH_2CH_2COOH$

Acetic acid Butanoic acid

it is oxidative ozonolysis.

74. Correct statements for the given reaction are :



A. Compound 'B' is aromatic

B. The completion of above reaction is very slow

C. 'A' shows tautomerism

D. The bond lengths C-C in compound B are found to be same

Choose the correct answer from the options given below :

(1) A, B and D only

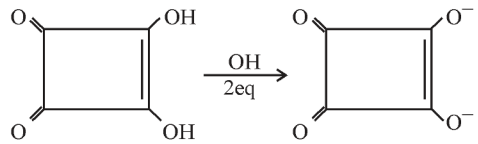
(2) A, B and C only

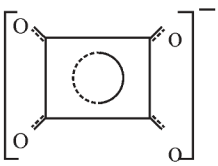
(3) B, C and D only

(4) A, C and D only

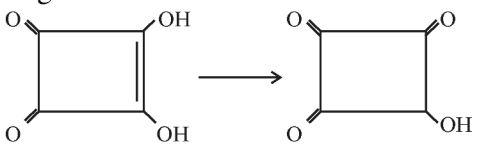
Official Ans. by NTA (4)

Sol.





Resonance hybrid of B showing all C-C bond length same



75. The bond order and magnetic property of acetylide ion are same as that of

(1) NO^+

(2) O_2^+

(3) O_2^-

(4) N_2^+

Official Ans. by NTA (1)

Sol. Acetylide ion $\rightarrow C_2^{2-}(\overline{C} \equiv \overline{C})$

Bond order = 3 & Diamagnetic

NO^+ $14e^-$ $\rightarrow$ Bond order = 3 & Diamagnetic

76. In the given reaction cycle

$CaCl_2 + Na_2CO_3 \longrightarrow \underline{X} + \underline{Y}$

Z

X, Y and Z respectively are

(1)

X	Y	Z
CaO	$NaCl + CO_2$	KCl

(2)

X	Y	Z
$CaCO_3$	NaCl	KCl

(3)

X	Y	Z
$CaCO_3$	NaCl	HCl

(4)

X	Y	Z
CaO	$NaCl + CO_2$	NaCl

Official Ans. by NTA (3)

Sol.

$CaCl_2 + Na_2CO_3 \longrightarrow \underset{X}{CaCO_3} + \underset{Y}{NaCl}$

HCl
Z

77. Given below are two statements :

Statement I : Boron is extremely hard indicating its high lattice energy

Statement II : Boron has highest melting and boiling point compared to its other group members.

In the light of the above statements, choose the *most appropriate* answer from the options given below

(1) Statement I is incorrect but Statement II is correct

(2) Both Statement I and Statement II is correct

(3) Statement I is correct but Statement II is incorrect

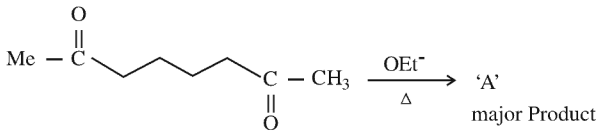
(4) Both Statement I and Statement II is incorrect

Official Ans. by NTA (2)

Sol. Boron is non- metallic in nature. It is extremely hard and black coloured solid. It exists in many allotropic forms. Due to very strong crystalline lattice, boron has unusually high melting point and boiling point.

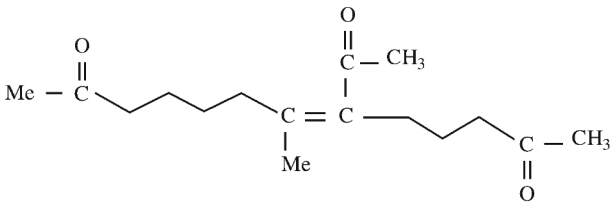
Element					
	B	Al	Ga	In	Tl
Melting point/K	2453	933	303	430	576
Boiling point/K	3923	2740	2676	2353	1730

78.

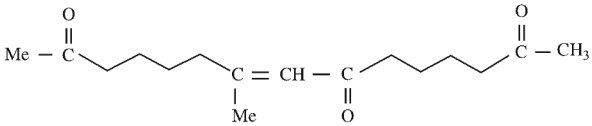


A in the above reaction is :

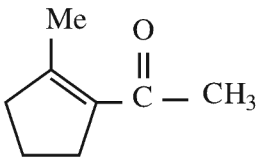
(1)



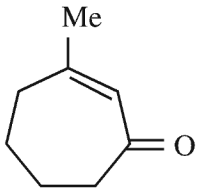
(2)



(3)

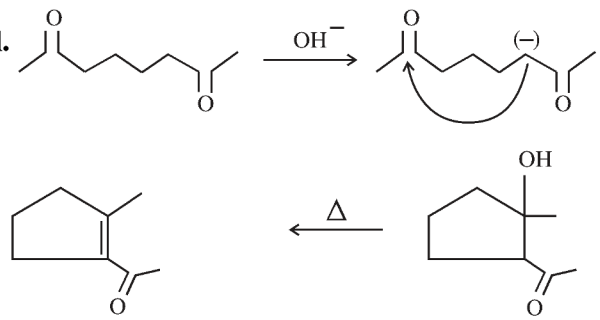


(4)



Official Ans. by NTA (3)

Sol.



79. Match List I with List II

LIST I Type of Hydride		LIST II Example	
A.	Electron deficient hydride	I.	MgH_2
B.	Electron rich hydride	II.	HF
C.	Electron precise hydride	III.	B_2H_6
D.	Saline hydride	IV.	CH_4

Choose the correct answer from the options given below :

(1) A-III, B-II, C-IV, D-I

(2) A-II, B-III, C-IV, D-I

(3) A-II, B-III, C-I, D-IV

(4) A-III, B-II, C-I, D-IV

Official Ans. by NTA (1)

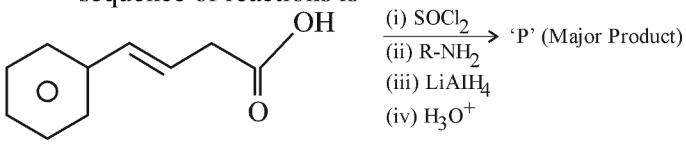
Sol. $B_2H_6 \Rightarrow e^-$ deficient hydride

$HF \Rightarrow e^-$ rich hydride

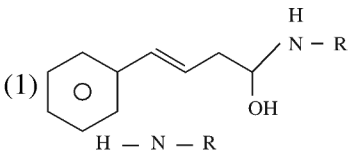
$CH_4 \Rightarrow e^-$ Precise hydride

$MgH_2 \Rightarrow$ Saline hydride

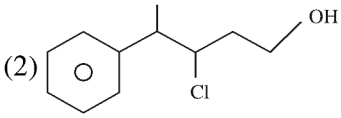
80. The major product 'P' formed in the following sequence of reactions is



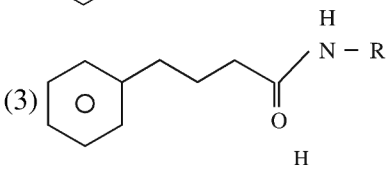
(1)



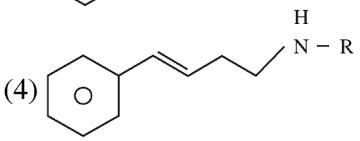
(2)



(3)

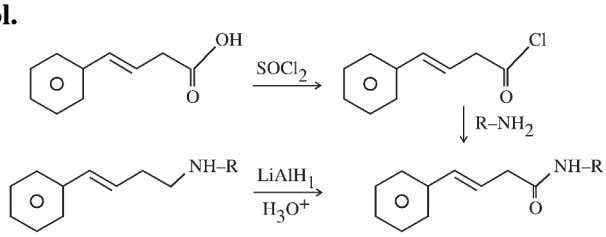


(4)



Official Ans. by NTA (4)

Sol.



SECTION-B

81. One mole of an ideal gas at 350K is in a 2.0 L vessel of thermally conducting walls, which are in contact with the surroundings. It undergoes isothermal reversible expansion from 2.0L to 3.0L against a constant pressure of 4 atm. The change in entropy of the surroundings (ΔS) is _____ $J\ K^{-1}$ (Nearest integer)

Given : R = 8.314 $J\ K^{-1}\ Mol^{-1}$.

Official Ans. by NTA (3)

Sol. $\Delta S_{System} = nR\ell n\left(\frac{V_2}{V_1}\right)$

$= 1 \times 8.314\, \ell n\left(\frac{3}{2}\right)$

$\Delta S_{System} = 3.37$

$\Delta S_{Surr.} = 3.37$

Correct Ans : 3

82. The mass of NH_3 produced when 131.8 kg of cyclohexanecarbaldehyde undergoes Tollen's test is _____ kg. (Nearest Integer)

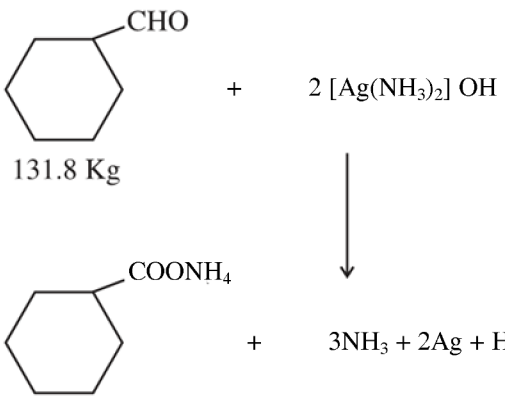
Molar Mass of C = 12g/mol

N = 14g/mol

O = 16g/mol

Official Ans. by NTA (60)

Sol.



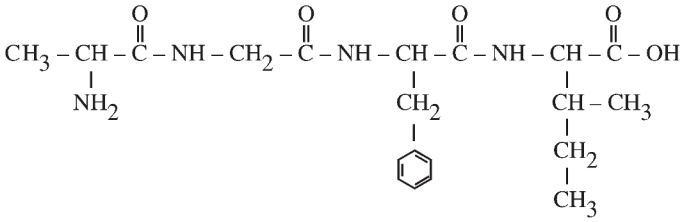
$$W_{NH_3} = \frac{131.8 \times 1000}{112} \times 3 \times 17$$

$= 60$ Kg

83. In an oligopeptide named Alanylglycylphenyl alanyl isoleucine, the number of sp^2 hybridised carbons is _____.

Official Ans. by NTA (10)

Sol.



84. An analyst wants to convert. 1L HCl of pH = 1 to a solution of HCl of pH 2. The volume of water needed to do this dilution is _____ mL. (Nearest Integer)

Official Ans. by NTA (9000)

Sol.

$$\underset{10^{-1} \times 1}{(M_1 \times V_1)} = \underset{10^{-2} \times V_2}{(M_2 \times V_2)}$$

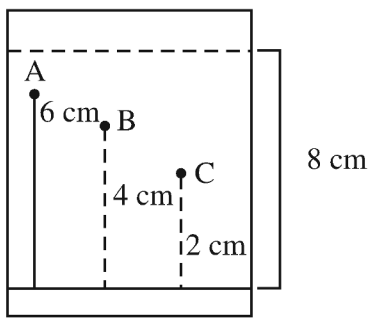
$V_2 = 10$L

Water added = 10 – 1

= 9 Litre

= 9000 mL

85. Three organic compounds A, B and C were allowed to run in thin layer chromatography using hexane and gave the following result (see figure). The R_f value of the most polar compound is _____ $\times 10^{-2}$



Official Ans. by NTA (25)

Sol. More R_f, less its polarity

$$R_f = \frac{\text{Distance travelled by compound 'X'}}{\text{Distance travelled by solvent 'Y'}}$$

$$= \frac{2}{8} = 0.25 = 25 \times 10^{-2}$$

86. 80 mole percent of $MgCl_2$ is dissociated in aqueous solution. The vapour pressure of 1.0 molal aqueous solution of $MgCl_2$ at 38°C is _____ mm Hg. (Nearest integer)

Given : Vapour pressure of water at 38°C is 50 mm Hg

Official Ans. by NTA (48)

Sol.

$$MgCl_2 \rightarrow Mg^{+2} + 2Cl^-$$
$$1-\alpha \qquad \alpha \qquad 2\alpha$$

$$i = 1 + 2\alpha \quad (\alpha = 0.8)$$

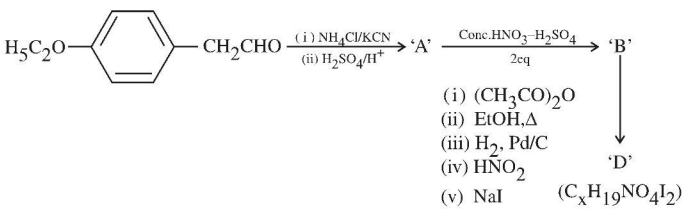
$$i = 2.6$$

$$\frac{\Delta p}{p^\circ} = \frac{i \times n_2}{n_1}$$

$$\Delta p = 2.34$$

$$p_s = 47.66$$
$$p_s \cong 48$$

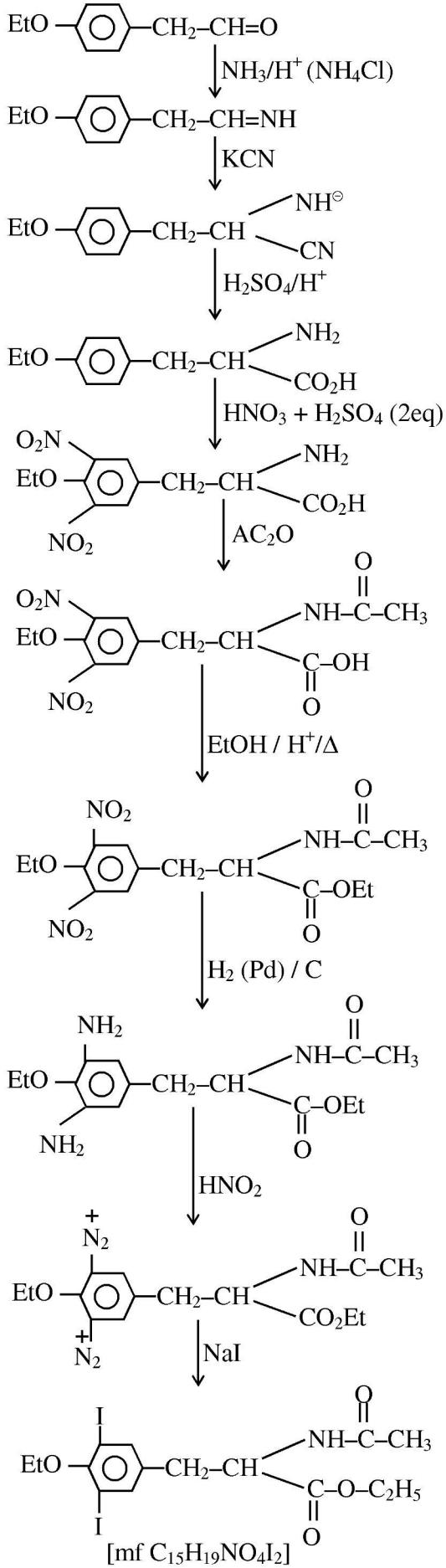
87.



The value of x in compound 'D' is ____

Official Ans. by NTA (15)

Sol.



88. At 600K, the root mean square (rms) speed of gas X (molar mass = 40) is equal to the most probable speed of gas Y at 90K. The molar mass of the gas Y is _____ g mol^{-1}. (Nearest integer)

Official Ans. by NTA (4)

Sol. $(U_{rms})_{X,600} = (U_{mp})_{Y,90}$

$$\sqrt{\frac{3\times R\times 600}{40}} = \sqrt{\frac{2\times R\times 90}{M}}$$

$M = 4$

89. The reaction $2NO + Br_2 \rightarrow 2NOBr$

takes places through the mechanism given below :

$NO + Br_2 \Leftrightarrow NOBr_2$ (fast)
$NOBr_2 + NO \rightarrow 2NOBr$ (slow)

The overall order of the reaction is _____.

Official Ans. by NTA (3)

Sol. RDS : $NOBr_2 + NO \rightarrow 2NOBr$

$r = K[NOBr_2][NO]$ ----(i)

$$Keq = \frac{[NOBr_2]}{[NO][Br_2]} \quad -----(ii)$$

From (i) & (ii)

r = K. Keq. [NO] [Br_2] [NO]

$r = K'[NO]^2[Br_2]$

Overall order = 3

Ans. 3

90. Values of work function (W_0) for a few metals are given below

Metal	Li	Na	K	Mg	Cu	Ag
W_o/eV	2.42	2.3	2.25	3.7	4.8	4.3

The number of metals which will show photoelectric effect when light of wavelength 400nm falls on it is _____

Given : h = 6.6×10^{-34} J s

c = 3×10^8 m s^{-1}

e = 1.6×10^{-19} C

Official Ans. by NTA (3)

Sol. $E(ev) = \frac{1240}{400} = 3.1$ ev

Mg, Cu, Ag

Ans.3

FINAL JEE–MAIN EXAMINATION – APRIL, 2023

(Held On Thursday 13th April, 2023) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. $\int_0^{\infty} \frac{6}{e^{3x}+6e^{2x}+11e^{x}+6}dx$

(1) $\log_e\left(\frac{512}{81}\right)$

(2) $\log_e\left(\frac{32}{27}\right)$

(3) $\log_e\left(\frac{256}{81}\right)$

(4) $\log_e\left(\frac{64}{27}\right)$

Official Ans. by NTA (2)

Sol.

$$I=\int_0^{\infty}\frac{6}{(e^x+1)(e^x+2)(e^x+3)}dx$$

$$=6\int_0^{\infty}\left(\frac{\frac{1}{2}}{e^x+1}+\frac{-1}{e^x+2}+\frac{\frac{1}{2}}{e^x+3}\right)dx$$

$$=3\int_0^{\infty}\frac{e^{-x}}{1+e^{-x}}dx-6\int_0^{\infty}\frac{e^{-x}dx}{1+2e^{-x}}+3\int_0^{\infty}\frac{e^{-x}}{1+3e^{-x}}dx$$

$$=3\left[-\ln\left(1+e^{-x}\right)\right]_0^{\infty}+6\frac{1}{2}\left[\ln\left(1+2e^{-x}\right)\right]_0^{\infty}$$

$$-\frac{3}{3}\left[\ln\left(1+3e^{-x}\right)\right]_0^{\infty}$$

$$=3\ln 2-3\ln 3+\ln 4$$

$$=3\ln\frac{2}{3}+\ln 4$$

$$=\ln\frac{32}{27}$$

2. $\max\limits_{0\le x\le\pi}\left\{x-2\sin x\cos x+\frac{1}{3}\sin 3x\right\}=$

(1) $\frac{5\pi+2+3\sqrt{3}}{6}$

(2) $\frac{\pi+2-3\sqrt{3}}{6}$

(3) π

(4) 0

Official Ans. by NTA (1)

Sol.

$$f(x)=x-\sin 2x+\frac{1}{3}\sin 3x$$

$$f'(x)=1-2\cos 2x+\cos 3x=0$$

$$x=\frac{5\pi}{6},\frac{\pi}{6}$$

$$\therefore f''(x)=4\sin 2x-3\sin 3x$$

$$f''\left(\frac{5\pi}{6}\right)<0$$

$$\Rightarrow\left(\frac{5\pi}{6}\right)\text{ is point of maxima}$$

$$f\left(\frac{5\pi}{6}\right)=\frac{5\pi}{6}+\frac{\sqrt{3}}{2}+\frac{1}{3}$$

3. The set of all $a\in\mathbb{R}$ for which the equation $x|x-1|+|x+2|+a=0$ has exactly one real root is :

(1) $(-6,-3)$

(2) $(-\infty,\infty)$

(3) $(-6,\infty)$

(4) $(-\infty,-3)$

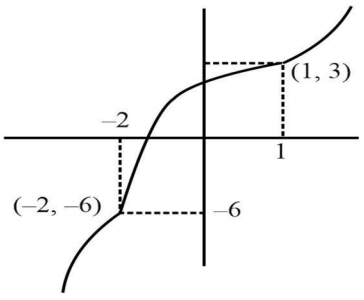
Official Ans. by NTA (2)

Sol.

$$f(x)=x|x-1|+|x+2|$$

$$x|x-1|+|x+2|+a=0$$

$$x|x-1|+|x+2|=-a$$



All values are increasing.

4. The negation of the statement

$\big((A \wedge (B \vee C)) \Rightarrow (A \vee B)\big) \Rightarrow A$ is

(1) equivalent to $\sim A$

(2) equivalent to $\sim C$

(3) equivalent to $B \vee \sim C$

(4) a fallacy

Official Ans. by NTA (1)

Sol.

$p : \big((A \wedge (B \vee C)) \Rightarrow (A \vee B)\big) \Rightarrow A$

$[\sim(A \wedge (B \vee C)) \vee (A \vee B)] \Rightarrow A$

$[(A \wedge (B \vee C)) \wedge \sim (A \vee B)] \vee A$

$(f \vee A) = A$

$\sim p \equiv \sim A$

5. The distance of the point $(-1,2,3)$ from the plane $\vec{r}.(\hat{i}-2\hat{j}+3\hat{k}) = 10$ parallel to the line of the shortest distance between the lines $\vec{r} = (\hat{i}-\hat{j}) + \lambda(2\hat{i}+\hat{k})$ and $\vec{r} = (2\hat{i}-\hat{j}) + \mu(\hat{i}-\hat{j}+\hat{k})$ is :

(1) $3\sqrt{6}$

(2) $3\sqrt{5}$

(3) $2\sqrt{6}$

(4) $2\sqrt{5}$

Official Ans. by NTA (3)

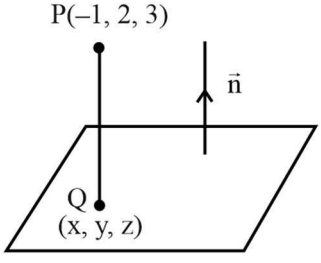
Sol.

Let $L_1 : \vec{r} = (\hat{i}-\hat{j}) + \lambda(2\hat{i}+\hat{k})$

$L_2 : \vec{r} = (2\hat{i}-\hat{j}) + \mu(\hat{i}-\hat{j}+\hat{k})$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & -1 & 1 \end{vmatrix}$$

$\vec{n} = \hat{i}-\hat{j}-2\hat{k}$



Equation of line along shortest distance of L_1 and L_2

$$\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-3}{-2} = r$$

$\Rightarrow (x,y,z) \equiv (r-1, 2-r, 3-2r)$

$\Rightarrow (r-1) - 2(2-r) + 3(3-2r) = 10$

$\Rightarrow r = -2$

$\Rightarrow Q(x,y,z) \equiv (-3,4,7)$

$\Rightarrow PQ = \sqrt{4+4+16} = 2\sqrt{6}$

6. A coin is biased so that the head is 3 times as likely to occur as tail. This coin is tossed until a head or three tails occur. If X denotes the number of tosses of the coin, then the mean of X is

(1) $\frac{21}{16}$

(2) $\frac{81}{64}$

(3) $\frac{15}{16}$

(4) $\frac{37}{16}$

Official Ans. by NTA (1)

Sol.

$P(H) = \frac{3}{4}$

$P(T) = \frac{1}{4}$

X	1	2	3
P(X)	$\frac{3}{4}$	$\frac{1}{4}\times\frac{3}{4}$	$\left(\frac{1}{4}\right)^3 + \left(\frac{1}{4}\right)^2 \times \frac{3}{4}$

Mean $\bar{X} = \frac{3}{4} + \frac{3}{8} + 3\left(\frac{1}{64} + \frac{3}{64}\right)$

$= \frac{3}{4} + \frac{3}{8} + \frac{3}{16}$

$= 3\left(\frac{7}{16}\right)$

$= \frac{21}{16}$

7. For the system of linear equations

$2x+4y+2az=b$

$x+2y+3z=4$

$2x-5y+2z=8$

which of the following is **NOT** correct ?

(1) It has infinitely many solutions if a =3, b = 6

(2) It has unique solution if a = b = 6

(3) It has unique solution if a = b = 8

(4) It has infinitely many solution if a = 3, b = 8

Official Ans. by NTA (1)

Sol.

$$\Delta=\begin{vmatrix}2 & 4 & 2a\\ 1 & 2 & 3\\ 2 & -5 & 2\end{vmatrix}=18(3-a)$$

$$\Delta_x=\begin{vmatrix}b & 4 & 2a\\ 4 & 2 & 3\\ 8 & -5 & 2\end{vmatrix}=(64+19b-72a)$$

For unique solution $\Delta=0$

$\Rightarrow$ $\boxed{a\neq 3}$ and $\boxed{b\in R}$

For Infinitely many solution ;

$\Delta=\Delta_x=\Delta_y=\Delta_z=0$

$\Rightarrow a=3 \quad \because \Delta=0$

and b = 8 $\because \Delta_x=0$

8. For the differentiable function

$f:\mathbb{R}-\{0\}\to\mathbb{R}$, let $3f(x)+2f\left(\frac{1}{x}\right)=\frac{1}{x}-10$, then

$\left|f(3)+f'\left(\frac{1}{4}\right)\right|$ is equal to

(1) 7

(2) $\frac{33}{5}$

(3) $\frac{29}{5}$

(4) 13

Official Ans. by NTA (4)

Sol.

$$\left[3f(x)+2f\left(\frac{1}{x}\right)=\frac{1}{x}-10\right]\times 3$$

$$\left[2f(x)+3f\left(\frac{1}{x}\right)=x-10\right]\times 2$$

$$5f(x)=\frac{3}{x}-2x-10$$

$$f(x)=\frac{1}{5}\left(\frac{3}{x}-2x-10\right)$$

$$f'(x)=\frac{1}{5}\left(-\frac{3}{x^2}-2\right)$$

$$\left|f(3)+f'\left(\frac{1}{4}\right)\right|=\left|\frac{1}{5}(1-6-10)+\frac{1}{5}(-48-2)\right|$$

$= |-3-10| \quad = 13$

9. Let the tangent and normal at the point $(3\sqrt{3},1)$ on the ellipse $\frac{x^2}{36}+\frac{y^2}{4}=1$ meet the y-axis at the points A and B respectively. Let the circle C be drawn taking AB as a diameter and the line $x=2\sqrt{5}$ intersect C at the points P and Q. If the tangents at the points P and Q on the circle intersect at the point (α,β), then $\alpha^2-\beta^2$ is equal to

(1) $\frac{314}{5}$

(2) $\frac{304}{5}$

(3) 60

(4) 61

Official Ans. by NTA (2)

Sol.

Given ellipse $\frac{x^2}{36}+\frac{y^2}{4}=1$

$\frac{x}{4\sqrt{3}}+\frac{y}{4}=1$

$y=4$

$\frac{x}{4}-\frac{4}{4\sqrt{3}}=\frac{2}{\sqrt{3}}$

$y=-8$

$x^2 + y^2 + 4y - 32 = 0$

$hx + ky + 2(y + k) - 32 = 0$

$k = -2$

$hx + 2k - 32 = 0$

$hx = 36$

$\alpha = h = \frac{36}{2\sqrt{5}}$

$\beta = k = -2$

$\alpha^2 - \beta^2 = \frac{304}{5}$

10. The area of the region enclosed by the curve $f(x) = \max\{\sin x, \cos x\}, -\pi \le x \le \pi$ and the x–axis is

(1) $2(\sqrt{2}+1)$

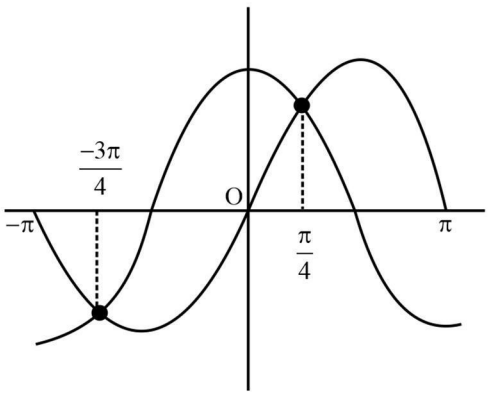
(2) $2\sqrt{2}(\sqrt{2}+1)$

(3) $4(\sqrt{2})$

(4) 4

Official Ans. by NTA (4)

Sol.



Area =

$$\left|\int_{-\pi}^{\frac{-3\pi}{4}} \sin x\,dx\right| + \left|\int_{\frac{-3\pi}{4}}^{\frac{-\pi}{2}} \cos x\,dx\right| + \int_{\frac{-\pi}{2}}^{\frac{\pi}{4}} \cos x\,dx + \int_{\frac{\pi}{4}}^{\pi} \sin x\,dx$$

= 4

11. The number of symmetric matrices of order 3, with all the entries from the set {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, is :

(1) 6^{10}

(2) 9^{10}

(3) 10^9

(4) 10^6

Official Ans. by NTA (4)

Sol.

$$A = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}, a, b, c, d, e, f \in \{0, 1, 2, ...9\}$$

Number of matrices = 10^6

12. Among :

(S1) : $\lim_{n\to\infty} \frac{1}{n^2}(2 + 4 + 6 + ... + 2n) = 1$

(S2) : $\lim_{n\to\infty} \frac{1}{n^{16}}(1^{15} + 2^{15} + 3^{15} + ... + n^{15}) = \frac{1}{16}$

(1) Both (S1) and (S2) are true

(2) Both (S1) and (S2) are false

(3) Only (S2) is true

(4) Only (S1) is true

Official Ans. by NTA (1)

Sol.

$S_1 : \lim_{n\to\infty} \frac{n(n+1)}{n^2} = 1 \Rightarrow$ True

$S_2 : \lim_{n\to\infty} \frac{1}{n^{16}}\left(\sum r^{15}\right) = \lim_{n\to\infty} \frac{1}{n}\sum\left(\frac{r}{n}\right)^{15}$

$= \int_0^1 x^{15}dx = \frac{1}{16} \Rightarrow$ True

13. Let PQ be a focal chord of the parabola $y^2 = 36x$ of length 100, making an acute angle with the positive x-axis. Let the ordinate of P be positive and M be the point on the line segment PQ such that PM:MQ=3:1. Then which of the following points does NOT lie on the line passing through M and perpendicular to the line PQ?

(1) $(-3, 43)$

(2) $(-6, 45)$

(3) $(3, 33)$

(4) (6, 29)

Official Ans. by NTA (1)

Sol.

$9\left(t + \frac{1}{t}\right)^2 = 100$

$t = 3$

$\Rightarrow P(81, 54)$ & $Q(1, -6)$

M (21, 9)

$\Rightarrow L$ is $(y - 9) = \frac{-4}{3}(x - 21)$

$3y - 27 = -4x + 84$

$4x + 3y = 111$

For $x \in \mathbb{R}$, two real valued functions f(x) and g(x) are such that, $g(x)=\sqrt{x}+1$ and $fog(x)=x+3-\sqrt{x}$. Then f(0) is equal to

(1) 1

(2) –3

(3) 5

(4) 0

Official Ans. by NTA (3)

Sol.

$g(x)=\sqrt{x}+1$

$fog(x)=x+3-\sqrt{x}$

$=\left(\sqrt{x}+1\right)^2-3\left(\sqrt{x}+1\right)+5$
$=g^2(x)-3g(x)+5$
$\Rightarrow f(x)=x^2-3x+5$
$\therefore\ f(0)=5$

But, if we consider the domain of the composite function fog(x) then in that case f(0) will be not defined as g(x) cannot be equal to zero.

15. Fractional part of the number $\frac{4^{2022}}{15}$ is equal to

(1) $\frac{4}{15}$

(2) $\frac{1}{15}$

(3) $\frac{14}{15}$

(4) $\frac{8}{15}$

Official Ans. by NTA (2)

Sol.

$\left\{\frac{4^{2022}}{15}\right\}=\left\{\frac{2^{4044}}{15}\right\}$

$=\left\{\frac{(1+15)^{1011}}{15}\right\}$

$=\frac{1}{15}$

16. Let $a=\hat{i}+4\hat{j}+2\hat{k}, b=3\hat{i}-2\hat{j}+7\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$. If a vector $\vec{d}$ satisfies $\vec{d}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{d}.\vec{a}=24$, then $|\vec{d}|^2$ is equal to

(1) 413

(2) 423

(3) 323

(4) 313

Official Ans. by NTA (1)

Sol.:

$\vec{d}\times\vec{b}=\vec{c}\times\vec{b}$

$\Rightarrow(\vec{d}-\vec{c})\times\vec{b}=0$

$\Rightarrow\vec{d}=\vec{c}+\lambda\vec{b}$

Also $\vec{d}.\vec{a}=24$

$\Rightarrow\left(\vec{c}+\lambda\vec{b}\right).\vec{a}=24$

$\lambda=\frac{24-\vec{a}.\vec{c}}{\vec{b}.\vec{a}}=\frac{24-6}{9}=2$

$\Rightarrow\vec{d}=\vec{c}+2\left(\vec{b}\right)$

$=8\hat{i}-5\hat{j}+18\hat{k}$

$\Rightarrow\left|\vec{d}\right|^2=64+25+324=413$

17. Let $B=\begin{bmatrix}1 & 3 & \alpha\\ 1 & 2 & 3\\ \alpha & \alpha & 4\end{bmatrix}, \alpha>2$ be the adjoint of a matrix A and |A| = 2, then $[\alpha\ -2\alpha\ \alpha]B\begin{bmatrix}\alpha\\ -2\alpha\\ \alpha\end{bmatrix}$ is equal to :-

(1) 16

(2) 32

(3) -16

(4) 0

Official Ans. by NTA (3)

Sol.

Given, $B=\begin{bmatrix}1 & 3 & \alpha\\ 1 & 2 & 3\\ \alpha & \alpha & 4\end{bmatrix}$

$|B|=4$

$1(8-3\alpha)-3(4-3\alpha)+\alpha(\alpha-2\alpha)=4$

$-\alpha^2+6\alpha-8=0$

$\alpha=2,4$

Given , $\alpha>2$

So, $\alpha=2$ is rejected

$[4\quad -8\quad 4]\begin{bmatrix}1 & 3 & 4\\ 1 & 2 & 3\\ 4 & 4 & 4\end{bmatrix}\begin{bmatrix}4\\ -8\\ 4\end{bmatrix}=[-16]_{1\times1}$

18. Let $s_1, s_2, s_3, \ldots, s_{10}$ respectively be the sum to 12 terms of 10 A.P.s whose first terms are 1, 2, 3, …. , 10 and the common differences are 1, 3, 5, …. , 19 respectively. Then $\sum_{i=1}^{10} s_i$ is equal to

(1) 7380

(2) 7220

(3) 7360

(4) 7260

Official Ans. by NTA (4)

Sol.

$S_k = 6(2k + (11)(2k-1))$

$S_k = 6(2k + 22k - 11)$

$S_k = 144k - 66$

$$\sum_{1}^{10} S_k = 144\sum_{k=1}^{10} k - 66\times 10$$

$$= 144 \times \frac{10\times 11}{2} - 660$$

$$= 7920 - 660$$

$$= 7260$$

19. Let $y = y_1(x)$ and $y = y_2(x)$ be the solution curves of the differential equation $\frac{dy}{dx} = y + 7$ with initial conditions $y_1(0) = 0, y_2(0) = 1$ respectively. Then the curves $y = y_1(x)$ and $y = y_2(x)$ intersect at

(1) Two points

(2) no point

(3) infinite number of points

(4) one point

Official Ans. by NTA (2)

Sol.

$$\frac{dy}{dx} = y + 7 \Rightarrow \frac{dy}{dx} - y = 7$$

$I.F. = e^{-x}$

$$ye^{-x} = \int 7e^{-x}dx$$

$$\Rightarrow ye^{-x} = -7e^{-x} + c$$

$$\Rightarrow y = -7 + ce^{x}$$

$$-7 + 7e^{x} = -7 + 8e^{x}$$

$\Rightarrow e^{x} = 0$

No solution

20. Let the equation of plane passing through the line of intersection of the planes $x + 2y + az = 2$ and $x - y + z = 3$ be $5x - 11y + bz = 6a - 1$. For $c \in \mathbb{Z}$, if the distance of this plane from the point $(a, -c, c)$ is $\frac{2}{\sqrt{a}}$, then $\frac{a+b}{c}$ is equal to

(1) -2

(2) 2

(3) -4

(4) 4

Official Ans. by NTA (3)

Sol.

$$(x + 2y + az - 2) + \lambda(x - y + z - 3) = 0$$

$$\frac{1+\lambda}{5} = \frac{2-\lambda}{-11} = \frac{a+\lambda}{b} = \frac{2+3\lambda}{6a-1}$$

$$\lambda = -\frac{7}{2}, a = 3, b = 1$$

$$\frac{2}{\sqrt{a}} = \left|\frac{5a + 11c + bc - 6a + 1}{\sqrt{25 + 121 + 1}}\right|$$

$c = -1$

$$\therefore \frac{a+b}{c} = \frac{3+1}{-1} = -4$$

SECTION-B

21. Let α be the constant term in the binomial expansion of $\left(\sqrt{x} - \frac{6}{x^{\frac{3}{2}}}\right)^n$, $n \le 15$. If the sum of the coefficients of the remaining terms in the expansion is 649 and the coefficient of x^{-n} is $\lambda\alpha$, then λ is equal to _____.

Official Ans. by NTA (36)

Sol.

$$T_{k+1} = {}^{n}C_{k}(x)^{\frac{n-k}{2}}(-6)^{k}(x)^{\frac{-3}{2}k}$$

$$\frac{n-k}{2}-\frac{3}{2}k=0$$

$$n-4k=0$$

$$(-5)^{n}-\left({}^{n}C_{\frac{n}{4}}(-6)^{\frac{n}{4}}\right)=649$$

By observation $(625+24=649)$, we get $n=4$

$\because\ n=4\ \&\ k=1$

Required is coefficient of x^{-4} is $\left(\sqrt{4}-\frac{6}{x^{\frac{3}{2}}}\right)^{4}$

${}^{4}C_{1}(-6)^{3}$

By calculating we will get $\lambda=36$

22. If

$$S=\left\{x\in\mathbb{R}:\sin^{-1}\left(\frac{x+1}{\sqrt{x^2+2x+2}}\right)-\sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right)=\frac{\pi}{4}\right\},$$

then

$$\sum_{x\in R}\left(\sin\left(\left(x^2+x+5\right)\frac{\pi}{2}\right)-\cos\left(\left(x^2+x+5\right)\pi\right)\right)$$ is

equal to _____.

Official Ans. by NTA (4)

Sol.

$$\sin^{-1}\left(\frac{(x+1)}{\sqrt{(x+1)^2+1}}\right)-\sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right)=\frac{\pi}{4}$$

$\because\ \frac{t}{\sqrt{t^2+1}}\in(-1,1)$

$$\sin^{-1}\left(\frac{(x+1)}{\sqrt{(x+1)^2+1}}\right)=\sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right)+\frac{\pi}{4}$$

$$\frac{(x+1)}{\sqrt{(x+1)^2+1}}=\left(\frac{1}{\sqrt{2}}\right)\cos\left(\sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right)\right)+\frac{1}{\sqrt{2}}\left(\frac{x}{\sqrt{x^2+1}}\right)$$

$$=\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{x^2+1}}+\frac{x}{\sqrt{x^2+1}}\right)$$

$$\frac{(x+1)}{\sqrt{(x+1)^2+1}}=\frac{1}{\sqrt{2}}\left(\frac{1+x}{\sqrt{x^2+1}}\right)$$

After solving this equation, we get

$$x=-1 \text{ or } x=0$$

$S=\{-1,0\}$

$$\sum_{x\in R}\left(\sin\left(\left(x^2+x+5\right)\frac{\pi}{2}\right)-\cos\left(\left(x^2+x+5\right)\pi\right)\right)$$

$$=\left[\sin\left(\frac{5\pi}{2}\right)-\cos(5\pi)\right]+\left[\sin\left(\frac{5\pi}{2}\right)-\cos(5\pi)\right]$$

$=4$

23. Let $\omega=z\bar{z}+k_1z+k_2iz+\lambda(1+i), k_1\ ,\ k_2\in\mathbb{R}$. Let $\mathrm{Re}(\omega)=0$ be the circle C of radius 1 in the first quadrant touching the line y =1 and the y-axis. If the curve Im (ω) = 0 intersects C at A and B, then 30 $(AB)^2$ is equal to _____.

Official Ans. by NTA (24)

Sol.

$$\omega=z\bar{z}+k_1z+k_2iz+\lambda(1+i)$$

$$\mathrm{Re}(w)=x^2+y^2+k_1x-k_2y+\lambda=0$$

$$\text{Centre}\equiv\left(\frac{-k_1}{2},\frac{k_2}{2}\right)\equiv(1,2)$$

$\Rightarrow k_1=-2,\ k_2=4$

radius $=1\Rightarrow\lambda=4$

$$\mathrm{Im}=k_1y+k_2x+\lambda=0$$

$\therefore\ 2x-y+2=0$

$$d=\frac{2}{\sqrt{5}}$$

$$\frac{l^2}{4}=1-\frac{4}{5}=\frac{1}{5}$$

$\therefore\ 30l^2=24$

24. Let for $x \in \mathbb{R}, S_0(x) = x$,

$S_k(x) = C_k x + k\int_0^x S_{k-1}(t)dt$, where

$C_0 = 1, C_k = 1 - \int_0^1 S_{k-1}(x)dx$, k = 1, 2, 3….. Then

$S_2(3)+6C_3$ is equal to _____.

Official Ans. by NTA (18)

Sol.

Given,

$S_k(x) = C_k x + k\int_0^x S_{k-1}(t)dt,$

Put k = 2 and x = 3

$S_2(3) = C_2(3) + 2\int_0^3 S_1(t)dt$ …..(1)

Also,

$S_1(x) = C_1(x) + \int_0^x S_0(t)dt$

$= C_1 x + \frac{x^2}{2}$

$S_2(3) = 3C_2 + 2\int_0^3 \left(C_1 t + \frac{t^2}{2}\right) dt$

$= 3C_2 + 9C_1 + 9$

Also,

$C_1 = 1 - \int_0^1 S_0(x)dx = \frac{1}{2}$

$C_2 = 1 - \int_0^1 S_1(x)dx = 0$

$C_3 = 1 - \int_0^1 S_2(x)dx$

$= 1 - \int_0^1 \left(C_2 x + C_1 x^2 + \frac{x^3}{3}\right)dx$

$= \frac{3}{4}$

$S_2(x) = C_2 x + 2\int_0^x S_1(t)dt$

$= C_2 x + C_1 x^2 + \frac{x^3}{3}$

$\Rightarrow S_2(3) + 6C_3 = 6C_3 + 3C_2 + 9C_1 + 9$

$= 18$

25. The sum to 20 terms of the series

$2.2^2 - 3^2 + 2.4^2 - 5^2 + 2.6^2 -$ …… is equal to _____.

Official Ans. by NTA (1310)

Sol.

$(2^2 - 3^2 + 4^2 - 5^2 + 20 \text{ terms}) +$

$(2^2 + 4^2 + ... + 10 \text{ terms})$

$-(2 + 3 + 4 + 5 + ... + 11) + 4[1 + 2^2 + ...10^2]$

$-\left[\frac{21 \times 22}{2} - 1\right] + 4 \times \frac{10 \times 11 \times 21}{6}$

$= 1 - 231 + 14 \times 11 \times 10$

$= 1540 + 1 - 231$

$= 1310$

26. The number of seven digit positive integers formed using the digits 1,2,3 and 4 only and sum of the digits equal to 12 is _____.

Official Ans. by NTA (413)

Sol.

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 12, x_i \in \{1,2,3,4\}$

No. of solutions $= {}^{5+7-1}C_{7-1} - \frac{7!}{6!} - \frac{7!}{5!} = 413$

27. Let m_1 and m_2 be the slopes of the tangents drawn from the point P(4,1) to the hyperbola $H: \frac{y^2}{25} - \frac{x^2}{16} = 1$. If Q is the point from which the tangents drawn to H have slopes $|m_1|$ and $|m_2|$ and they make positive intercepts α and β on the x-axis, then $\frac{(PQ)^2}{\alpha\beta}$ is equal to _____.

Official Ans. by NTA (8)

Sol.

Equation of tangent to the hyperbola $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$

$y=mx\pm\sqrt{a^2-b^2m^2}$

passing through (4, 1)

$1=4m\pm\sqrt{25-16m^2} \Rightarrow 4m^2-m-3=0$

$\Rightarrow m=1,\frac{-3}{4}$

Equation of tangent with positive slopes 1 & $\frac{3}{4}$.

$\left.\begin{matrix}4y=3x-16\\ y=x-3\end{matrix}\right\}$ with positive intercept on x-axis.

$\alpha=\frac{16}{3},\beta=3$

Intersection points:

$Q:(-4,-7)$

$P:(4,1)$

$PQ^2 = 128$

$\frac{PQ^2}{\alpha\beta}=\frac{128}{16}=8$

28. Let the image of the point $\left(\frac{5}{3},\frac{5}{3},\frac{8}{3}\right)$ in the plane $x-2y+z-2=0$ be P. If the distance of the point $Q(6,-2,\alpha),\alpha>0$, from P is 13, then α is equal to _____.

Official Ans. by NTA (15)

Sol.

Image of point $\left(\frac{5}{3},\frac{5}{3},\frac{8}{3}\right)$

$$\frac{x-\frac{5}{3}}{1}=\frac{y-\frac{5}{3}}{-2}=\frac{z-\frac{8}{3}}{1}=\frac{-2\left(1\times\frac{5}{3}+(-2)\times\frac{8}{3}+1\times\frac{8}{3}-2\right)}{1^2+2^2+1^2}$$

$$=\frac{1}{3}$$

$\therefore\ x=2,\ y=1,\ z=3$

$13^2=(6-2)^2+(-2-1)^2+(\alpha-3)^2$

$\Rightarrow(\alpha-3)^2=144 \Rightarrow \alpha=15\ (\because \alpha>0)$

29. Let $a=3\hat{i}+\hat{j}-\hat{k}$ and $c=2\hat{i}-3\hat{j}+3\hat{k}$. If b is a vector such that $\vec{a}=\vec{b}\times\vec{c}$ and $|\vec{b}|^2=50$, then $|72-|\vec{b}+\vec{c}|^2|$ is equal to _____.

Official Ans. by NTA (66)

Sol.

$|\vec{a}|=\sqrt{11},\ |\vec{c}|=\sqrt{22}$

$|\vec{a}|=|\vec{b}\times\vec{c}|=|\vec{b}||\vec{c}|\sin\theta$

$\sqrt{11}=\sqrt{50}\sqrt{22}\sin\theta$

$\Rightarrow\sin\theta=\frac{1}{10}$

$$|\vec{b}+\vec{c}|^2=|\vec{b}|^2+|\vec{c}|^2+2\vec{b}.\vec{c}$$

$$=|\vec{b}|^2+|\vec{c}|^2+2|\vec{b}||\vec{c}|\cos\theta$$

$$=50+22+2\times\sqrt{50}\times\sqrt{22}\times\frac{\sqrt{99}}{10}$$

$$=72+66$$

$\left|72-|\vec{b}+\vec{c}|^2\right|=66$

30. Let the mean of the data

x	1	3	5	7	9
Frequency(f)	4	24	28	α	8

be 5. If m and σ^2 are respectively the mean deviation about the mean and the variance of the data, then $\frac{3\alpha}{m+\sigma^2}$ is equal to _____.

Official Ans. by NTA (8)

Sol.

$$5=\bar{x}=\frac{\sum x_if_i}{\sum f_i}=\frac{4+72+140+7\alpha+72}{64+\alpha}$$

$\Rightarrow 320+5\alpha=288+7\alpha \Rightarrow 2\alpha=32 \Rightarrow \alpha=16$

$M.D.(\bar{x})=\frac{\sum f_i|x_i-\bar{x}|}{\sum f_i}$ where $\sum f_i=64+16=80$

$$M.D.(\bar{x})=\frac{4\times4+24\times2+28\times0+16\times2+8\times4}{80}$$

$$=\frac{8}{5}$$

$$\text{Variance}=\frac{\sum f_i(x_i-\bar{x})^2}{\sum f_i}$$

$$=\frac{4\times16+24\times4+0+16\times4+8\times16}{80}=\frac{352}{80}$$

$$\therefore\ \frac{3\alpha}{m+\sigma^2}=\frac{3\times16}{\frac{128}{80}+\frac{352}{80}}=8$$

SECTION-A

31. Which of the following Maxwell's equations is valid for time varying conditions but not valid for static conditions :

(1) $\oint \vec{B}.\vec{dl} = \mu_0 I$ (2) $\oint \vec{E}.\vec{dl} = 0$

(3) $\oint \vec{E}.\vec{dl} = -\frac{\partial \phi_B}{\partial t}$ (4) $\oint \vec{D}.\vec{dA} = Q$

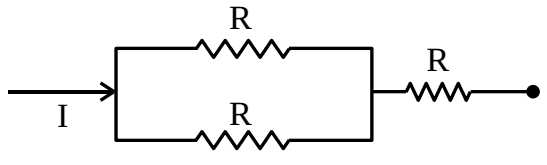
Official Ans. by NTA (3)

Sol. Based on equations of Maxwell

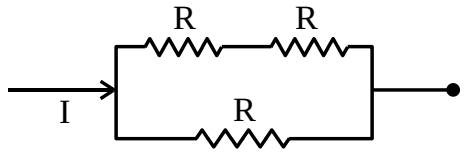
32. Different combination of 3 resistors of equal resistance R are shown in the figures.

The increasing order for power dissipation is:

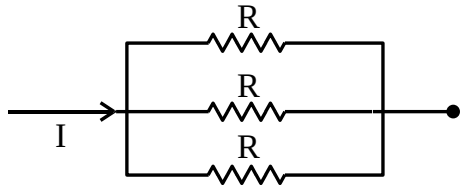
(A)



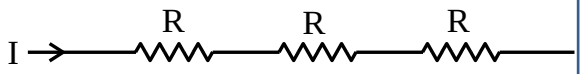
(B)



(C)



(D)



(1) $P_A < P_B < P_C < P_D$

(2) $P_C < P_D < P_A < P_B$

(3) $P_B < P_C < P_D < P_A$

(4) $P_C < P_B < P_A < P_D$

Official Ans. by NTA (4)

Sol. $P = I^2R$

$$R_1 = \frac{3R}{2}, R_2 = \frac{2R}{3}, R_3 = \frac{R}{3}, R_4 = 3R$$

Since i is same, hence $P \alpha R$ so options (4) is correct

33. A vessel of depth 'd' is half filled with oil of refractive index n_1 and the other half is filled with water of refractive index n_2. The apparent depth of this vessel when viewed from above will be-

(1) $\frac{d\, n_1 n_2}{(n_1 + n_2)}$

(2) $\frac{d(n_1 + n_2)}{2n_1 n_2}$

(3) $\frac{d\, n_1 n_2}{2(n_1 + n_2)}$

(4) $\frac{2d(n_1 + n_2)}{n_1 n_2}$

Official Ans. by NTA (2)

Sol. Formula used $d_{app} = \frac{d_1}{n_1} + \frac{d_2}{n_2}$

$$d_{app} = \frac{d}{2}\left[\frac{n_1 + n_2}{n_1 n_2}\right]$$

34. The source of time varying magnetic field may be

(A) a permanent magnet

(B) an electric field changing linearly with time

(C) direct current

(D) a decelerating charge particle

(E) an antenna fed with a digital signal

Choose the correct answer from the options given below:

(1) (D) only

(2) (C) and (E) only

(3) (A) only

(4) (B) and (D) only

Official Ans. by NTA (1)

Sol. Source of time varying magnetic field may be

$\rightarrow$accelerated or retarded charge which produces varying electric and magnetic fields.

$\rightarrow$ An electric field varying linearly with time will not produce variable magnetic field as current will be constant

35. Two trains 'A' and 'B' of length 'l' and '$4l$' are travelling into a tunnel of length 'L' in parallel tracks from opposite directions with velocities 108 km/h and 72 km/h, respectively. If train 'A' takes 35s less time than train 'B' to cross the tunnel then, length 'L' of tunnel is :

(Given L = 60 l)

(1) 1200 m

(2) 2700 m

(3) 1800 m

(4) 900 m

Official Ans. by NTA (3)

Sol. $\frac{60\ell+4\ell}{20}-\frac{61\ell}{30}=35$

$\Rightarrow \ell=\frac{1050}{35}$

$\Rightarrow L=60\ell=\frac{1050}{35}\times 60=1800\text{ m}$

36. The ratio of powers of two motors is $\frac{3\sqrt{x}}{\sqrt{x}+1}$, that are capable of raising 300 kg water in 5 minutes and 50 kg water in 2 minutes respectively from a well of 100 m deep. The value of x will be

(1) 2

(2) 4

(3) 2.4

(4) 16

Official Ans. by NTA (4)

Sol. Average Power = $\frac{\textit{total work done}}{\textit{total time}}$

So $P=\frac{mgh}{t}$

$$\frac{P_1}{P_2}=\frac{\frac{m_1gh}{t_1}}{\frac{m_2gh}{t_2}}=\frac{m_1}{t_1}\frac{t_2}{m_2}$$

$$\frac{P_1}{P_2}=\frac{300\times 2}{5\times 50}=\frac{12}{5}=\frac{3\sqrt{x}}{\sqrt{x}+1}$$

$$12\sqrt{x}+12=15\sqrt{x}$$

$$3\sqrt{x}=12$$

$$x=16$$

37. A planet having mass 9 Me and radius $4R_e$, where Me and Re are mass and radius of earth respectively, has escape velocity in km/s given by:

(Given escape velocity on earth

$V_e=11.2\times 10^3\text{ m/s}$)

(1) 67.2

(2) 16.8

(3) 33.6

(4) 11.2

Official Ans. by NTA (2)

Sol. $V_p=\sqrt{\frac{2GM_p}{R_p}}\quad V_E=\sqrt{\frac{2GM_E}{R_E}}$

$$\frac{V_P}{V_E}=\frac{\sqrt{\frac{2GM_P}{R_P}}}{\sqrt{\frac{2GM_E}{R_E}}}=\sqrt{\frac{R_E}{R_P}\times\frac{M_P}{M_E}}$$

$$V_P=\sqrt{\frac{1}{4}\times 9}\times V_E=\frac{3}{2}V_E$$

$$V_P=\frac{3}{2}\times 11.2\text{ km/sec}$$

$$=16.8\text{ km/sec}$$

38. The difference between threshold wavelengths for two metal surfaces A and B having work function $\phi_A=9\text{eV}$ and $\phi_B=4.5\text{eV}$ in nm is:

(Given, hc = 1242 eV nm)

(1) 264

(2) 138

(3) 276

(4) 540

Official Ans. by NTA (2)

Sol. $\lambda_A=\left(\frac{1242}{9}\right)=138\text{ nm}$

$$\lambda_B=\left(\frac{1242}{4.5}\right)=276\text{ nm}$$

$$\lambda_B-\lambda_A=138\text{ nm}$$

39. A bullet 10 g leaves the barrel of gun with a velocity of 600 m/s. If the barrel of gun is 50 cm long and mass of gun is 3 kg, then value of impulse supplied to the gun will be :

(1) 12 Ns (2) 6 Ns

(3) 36 Ns (4) 4 Ns

Official Ans. by NTA (2)

Sol. By momentum conservation

$0 = 3(-v) + 0.01(600 - v)$

$v \simeq 2 m/s$

Impulse on gun $= 3 \times 2 = 6$ Ns

40. Two charges each of magnitude 0.01 C and separated by a distance of 0.4 mm constitute an electric dipole. If the dipole is placed in an uniform electric field $'\vec{E}'$ of 10 dyne/C making 30^0 angle with $\vec{E}$, the magnitude of torque acting on dipole is :

(1) 4.0×10^{-10} Nm (2) 2.0×10^{-10} Nm

(3) 1.0×10^{-8} Nm (4) 1.5×10^{-9} Nm

Official Ans. by NTA (2)

Sol. $|\vec{P}| = qd$

$= 0.01 \times 0.4 \times 10^{-3}$

$= 4 \times 10^{-6}$

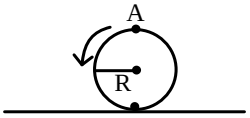
$|\vec{\tau}| = PE \sin\theta$

$= 4 \times 10^{-6} \times 10 \times 10^{-5} \times \sin 30$

$= 4 \times 10^{-6-5+1} \times \frac{1}{2}$

$= 2 \times 10^{-10}$

41. A disc is rolling without slipping on a surface. The radius of the disc is R. At t = 0, the top most point on the disc is A as shown in figure. When the disc completes half of its rotation, the displacement of point A from its initial position is

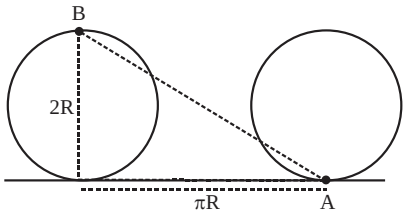


(1) $R\sqrt{(\pi^2 + 4)}$ (2) $R\sqrt{(\pi^2 + 1)}$

(3) 2 R (4) $2R\sqrt{(1 + 4\pi^2)}$

Official Ans. by NTA (1)

Sol.



Displacement $= \sqrt{(2R)^2 + (\pi R)^2} = R\sqrt{4 + \pi^2}$

42. Match List – I with List – II

List - I (Layer of atmosphere)	List – II (Approximate height over earth's surface)
(A) F_1 - Layer	(I) 10 km
(B) D – Layer	(II) 170 – 190 km
(C) Troposphere	(III) 100 km
(D) E-layer	(IV) 65 – 75 km

Choose the correct answer from the options given below:

(1) A – III, B – IV, C – I, D – II

(2) A – II, B – IV, C – III, D – I

(3) A – II, B – I, C – IV, D – III

(4) A – II, B – IV, C – I, D – III

Official Ans. by NTA (4)

Sol. Based on Theory

43. The rms speed of oxygen molecule in a vessel at particular temperature is $\left(1 + \frac{5}{x}\right)^{\frac{1}{2}} \upsilon$, where υ is the average speed of the molecule. The value of x will be:(Take $\pi = \frac{22}{7}$)

(1) 28

(2) 27

(3) 8

(4) 4

Official Ans. by NTA (1)

Sol. $\sqrt{\frac{3RT}{M}} = \left(1 + \frac{5}{x}\right)^{\frac{1}{2}} \sqrt{\frac{8RT}{\pi M}}$

$\Rightarrow \frac{3 \times 22}{7 \times 8} = 1 + \frac{5}{x}$

$\Rightarrow x = 28$

44. A body of mass (5 ± 0.5) kg is moving with a velocity of (20 ± 0.4) m/s. Its kinetic energy will be

(1) (1000 ± 140)J

(2) (1000 ± 0.14)J

(3) (500 ± 0.14)J

(4) (500 ± 140)J

Official Ans. by NTA (1)

Sol. $k=\frac{1}{2}mv^2$

$k=\frac{1}{2}\times5\times400=5\times200=1000\text{ J}$

$\frac{\Delta k}{2k}=\frac{\Delta m}{m}+\frac{2\,\Delta v}{v}=\frac{0.5}{5}+\frac{2\times0.4}{20}$

$\Delta k=1000\left(\frac{1}{10}+\frac{4}{100}\right)=1000\left(\frac{10+4}{100}\right)=140\text{ J}$

45. Two bodies are having kinetic energies in the ratio 16 : 9. If they have same linear momentum, the ratio of their masses respectively is :

(1) 4 : 3 (2) 3 : 4

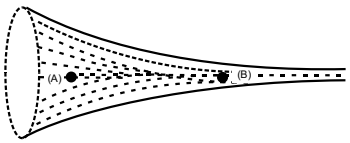
(3) 16 : 9 (4) 9 : 16

Official Ans. by NTA (4)

Sol. $\frac{K_1}{K_2}=\frac{p_1^2}{2m_1}\times\frac{2m_2}{p_2^2}=\frac{m_2}{m_1}=\frac{16}{9}$

$\frac{m_1}{m_2}=\frac{9}{16}$

46.



The figure shows a liquid of given density flowing steadily in horizontal tube of varying cross-section. Cross sectional areas at A is 1.5 cm^2, and B is 25 mm^2, if the speed of liquid at B is 60 cm/s then $(P_A - P_B)$ is :

(Given P_A and P_B are liquid pressures at A and B points.

Density $\rho=1000\text{ kg m}^{-3}$

A and B are on the axis of tube

(1) 175 Pa

(2) 27 Pa

(3) 135 Pa

(4) 36 Pa

Official Ans. by NTA (1)

Sol. From continuity theorem $A_1V_1=A_2V_2$

$1.5\times V_1=25\times10^{-2}\times60$

$V_1=\frac{25\times60\times10^{-2}\times10}{1.5}$

$V_1=10\text{ cm/s}$

By Bernoulli's theorem

$P_1+\frac{1}{2}\times1000\times(0.1)^2=P_2+\frac{1}{2}\times1000\times(0.6)^2$

$P_1+5=P_2+\frac{1}{2}\times1000\times36\times10^{-2}$

$P_1+5=P_2+180$

$P_1-P_2=175\text{ Pa}$

47. Under isothermal condition, the pressure of a gas is given by $P=aV^{-3}$, where a is a constant and V is the volume of the gas. The bulk modulus at constant temperature is equal to

(1) $\frac{P}{2}$ (2) 3 P

(3) 2 P (4) P

Official Ans. by NTA (2)

Sol. $B=-\frac{dP}{dv/v}$

$Pv^3=a$

Differentiating w.r.t to pressure

$v^3+P3v^2\frac{dv}{dP}=0$

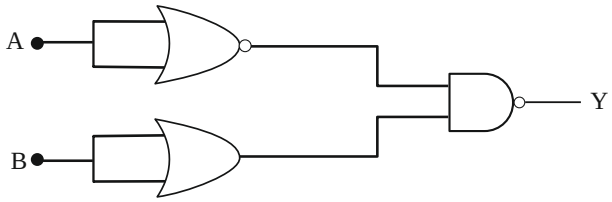
$v=-3\frac{Pdv}{dP}=0$

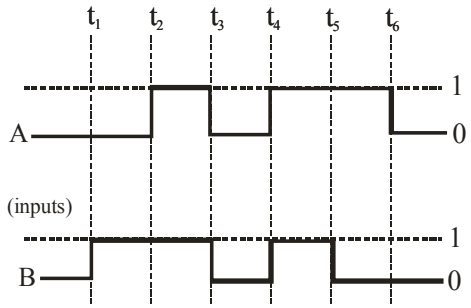
$v=-3\frac{Pdv}{dP}$

$\frac{dP.v}{dv}=-3P$

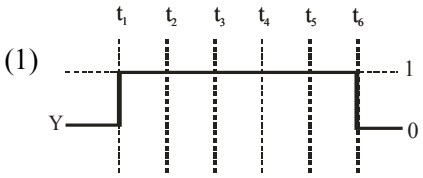
$B=-\left(\frac{dPv}{dv}\right)=-(-3P)=3P$

48. For the following circuit and given inputs A and B, choose the correct option for output 'Y'

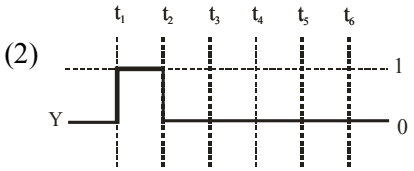




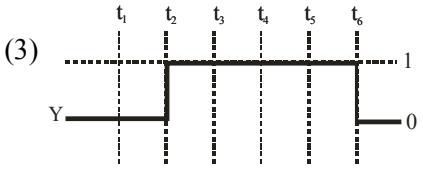
(1)



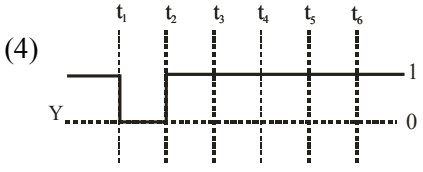
(2)



(3)



(4)

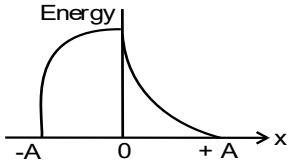


Official Ans. by NTA (4)

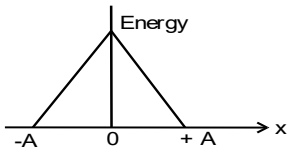
Sol. $Y = \overline{\overline{A} \cdot B} = A + \overline{B}$

49. Which graph represents the difference between total energy and potential energy of a particle executing SHM Vs it's distance from mean position?

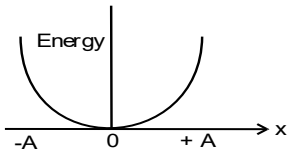
(1)



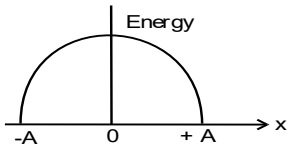
(2)



(3)



(4)



Official Ans. by NTA (4)

Sol. T.E. – P.E. = K.E.

$$\text{K.E.} = \frac{1}{2}m\omega^2\left(A^2 - x^2\right)$$

Which is the equation of downward parabola.

50. $${}^{238}_{92}A \rightarrow {}^{234}_{90}B + {}^{4}_{2}D + Q$$

In the given nuclear reaction, the approximate amount of energy released will be :

[Given, mass of ${}^{238}_{92}A = 238.05079 \times 931.5\ \text{MeV}/c^2$,

mass of ${}^{234}_{90}B = 234.04363 \times 931.5\ \text{MeV}/c^2$,

mass of ${}^{4}_{2}D = 4.00260 \times 931.5\ \text{MeV}/c^2$]

(1) 3.82 MeV (2) 5.9 MeV

(3) 2.12 MeV (4) 4.25 MeV

Official Ans. by NTA (4)

Sol. $Q = \left(m_A - m_B - m_D\right) \times 931.5\ MeV$

$$= \left(238.05079 - 234.04363 - 4.00260\right) \times 931.5$$

$$\Rightarrow 4.25\ Mev$$

Section - B

51. The elastic potential energy stored in a steel wire of length 20 m stretched through 2 cm is 80 J. The cross sectional area of the wire is_____ mm^2.

(Given, $y = 2.0\times10^{11}\,Nm^{-2}$)

Official Ans. by NTA (40)

Sol. Energy per unit volume $= \frac{1}{2}\,stress\times strain$

$$\text{Energy} = \frac{1}{2}stress\times strain\times volume$$

$$80 = \frac{1}{2}\times Y\times strain^2\ A\times\ell$$

$$80 = \frac{1}{2}\times 2\times10^{11}\times\frac{\left(2\times10^{-2}\right)^2}{400}\times A\times 20$$

$$20 = \frac{10^{+7}}{20}\times A$$

$$40\times10^{-6}\ m^2 = A$$

$$A = 40\ mm^2$$

52. A potential V_0 is applied across a uniform wire of resistance R. The power dissipation is P_1. The wire is then cut into two equal halves and a potential of V_0 is applied across the length of each half. The total power dissipation across two wires is P_2. The ratio P_2 : P_1 is $\sqrt{x}:1$. The value of x is _____.

Official Ans. by NTA (16)

Sol. $P = VI = I^2R = \frac{V^2}{R}$

Now $R = \frac{\rho l}{A}$

If wire is cut in two equal half

$$R' = \frac{R}{2}$$

Initial $P_1 = \frac{V_0^2}{R}$

After $P_2 = \frac{V_0^2}{R'}\times2 \Rightarrow \frac{V_0^2}{R}\times4$

$$\frac{P_2}{P_1} = 4 = \frac{\sqrt{x}}{1}$$

$$x = 16$$

53. At a given point of time the value of displacement of a simple harmonic oscillator is given as $y = A\cos(30^0)$. If amplitude is 40 cm and kinetic energy at that time is 200 J, the value of force constant is $1.0\times10^{x}\,Nm^{-1}$. The value of x is _____.

Official Ans. by NTA (4)

Sol. General equation for displacement is given by

$$x = A\sin(\omega t+\phi)$$

at given time

$$\Rightarrow \omega t+\phi = 30^0$$

$$\Rightarrow x = 40\times\frac{\sqrt{3}}{2} \Rightarrow 20\sqrt{3}\ cm$$

$$\Rightarrow A = 40\ cm$$

$$\Rightarrow K.E = \frac{1}{2}k\left(A^2-x^2\right) = 200$$

$$200 = \frac{1}{2}k\left(\frac{1600-1200}{100\times100}\right)$$

$$400\times100\times100 = k\times400$$

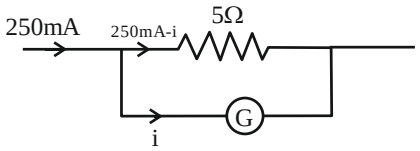
$$k = 10^4$$

$$x = 4$$

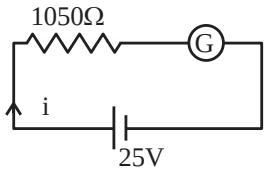
54. When a resistance of 5Ω is shunted with a moving coil galvanometer, it shows a full scale deflection for a current of 250 mA, however when 1050 Ω resistance is connected with it in series, it gives full scale deflection for 25 volt. The resistance of galvanometer is _____ Ω.

Official Ans. by NTA (50)

Sol.



$$\frac{250\ mA\times 5}{5+R_G}=i \qquad \ldots\ldots\ldots \text{(i)}$$



$$i=\frac{25}{1050+R_G} \qquad \ldots\ldots\ldots \text{(ii)}$$

From (i) and (ii)

$$\frac{25}{1050+R_G}=\frac{5}{4\left(5+R_G\right)}$$

$$100\left(5+R_G\right)=1050\times 5+R_G\times 5$$

$$95\ R_G=4750$$

$$R_G=50\Omega$$

55. The radius of 2nd orbit of He^+ of Bohr's model is r_1 and that of fourth orbit of Be^{3+} is represented as r_2. Now the ratio $\frac{r_2}{r_1}$ is x : 1. The value of x is _____

Official Ans. by NTA (2)

Sol. $r\propto\frac{n^2}{z}$

$$\frac{r_{He^+}}{r_{Be^{3+}}}=\frac{2^2\times 4}{2\times 4\times 4}=\frac{1}{2}$$

56. A thin infinite sheet charge and an infinite line charge of respective charge densities $+\sigma$ and $+\lambda$ are placed parallel at 5 m distance from each other. Points 'P' and 'Q' are at $\frac{3}{\pi}$ m and $\frac{4}{\pi}$ m perpendicular distance from line charge towards sheet charge, respectively. 'E_P' and 'E_Q' are the magnitudes of resultant electric field intensities at point 'P' and 'Q', respectively. If $\frac{E_P}{E_Q}=\frac{4}{a}$ for $2|\sigma|=|\lambda|$. Then the value of a is _____.

Official Ans. by NTA (6)

Sol. $E_A=\frac{\lambda}{2\pi\varepsilon_0 r_A}-\frac{\sigma}{2\varepsilon_0}\left\{r_A=\frac{3}{\pi}\right\}$

$$=\frac{1}{2\varepsilon_0}\left[\frac{\lambda}{3}-\sigma\right]$$

$$E_B=\frac{\lambda}{2\pi\varepsilon_0 r_A}-\frac{\sigma}{2\varepsilon_0}\left\{r_B=\frac{4}{\pi}\right\}$$

$$=\frac{1}{2\varepsilon_0}\left[\frac{\lambda}{4}-\sigma\right]$$

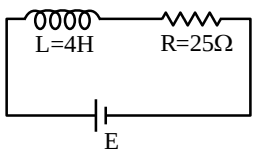
$$\frac{E_A}{E_B}=\frac{4}{3}\left(\frac{\lambda-3\sigma}{\lambda-4\sigma}\right)$$

$$=\frac{4}{3}\left[\frac{2\sigma-3\sigma}{2\sigma-4\sigma}\right]$$

$$=\frac{4}{3}\left[\frac{-\sigma}{-2\sigma}\right]$$

$$=\frac{4}{6}$$

57. In the given figure, an inductor and a resistor are connected in series with a battery of emf E volt. $\frac{E^a}{2b}$ J/s represents the maximum rate at which the energy is stored in the magnetic field (inductor). The numerical value of $\frac{b}{a}$ will be _____



Official Ans. by NTA (25)

Sol. $E=\frac{1}{2}LI^2$

Rate of energy storing $=\frac{dE}{dt}=LI\frac{dI}{dt}$

Now we Know for $R-L$ circuit

$I=\frac{E}{R}\left(1-e^{-t\frac{R}{L}}\right)$

So $\frac{dI}{dt}=\frac{E}{L}e^{-\frac{tR}{L}}$

$\frac{dE}{dt}=\frac{E^2}{R}\left(1-e^{-\frac{tR}{L}}\right)\left(e^{-t\frac{R}{L}}\right)$

Time at which rate of power storing will be max,

$t=\frac{L}{R\ln 2}$

So $\frac{dE}{dt}=\frac{E^2}{R}\left(1-\frac{1}{2}\right)\times\frac{1}{2}$

$\Rightarrow\frac{E^2}{4R}=\frac{E^2}{100}=\frac{E^2}{2\times 50}$

$a=2, b=50$

So $\frac{b}{a}=25$

58. A fish rising vertically upward with a uniform velocity of 8 ms^{-1}, observes that a bird is diving vertically downward towards the fish with the velocity of 12 ms^{-1}. If the refractive index of water is $\frac{4}{3}$, then the actual velocity of the diving bird to pick the fish, will be _____ ms^{-1}.

Official Ans. by NTA (3)

Sol. $\frac{V_{b/f}}{\frac{4}{3}}=\frac{-8}{\frac{4}{3}}+\frac{(-v)}{1}$

$\Rightarrow\frac{-12}{\frac{4}{3}}=\frac{-8}{\frac{4}{3}}+\frac{(-v)}{1}$

$\Rightarrow v=3\,m/s$

59. A solid sphere is rolling on a horizontal plane without slipping. If the ratio of angular momentum about axis of rotation of the sphere to the total energy of moving sphere is $\pi:22$ then, the value of its angular speed will be _____ rad/s.

Official Ans. by NTA (4)

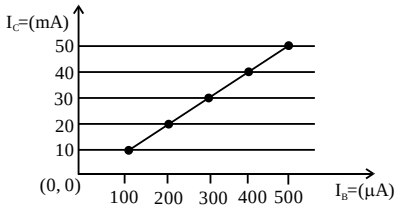
Sol. $L=(I_{com})(\omega)$ and $K=\frac{1}{2}(I_{com})(\omega^2)+\frac{1}{2}MV_{com}^2$

$L=\frac{2}{5}MR^2\frac{V_{com}}{R}\qquad K=\frac{1}{2}\left(\frac{2}{5}MR^2\right)\frac{V_{com}^2}{R^2}+\frac{1}{2}MV_{com}^2$

$L=\frac{2MRV_{com}}{5}\qquad K=\frac{7}{10}MV_{com}^2$

Ratio $\frac{L}{K}=\frac{4}{7}\frac{R}{V_{com}}=\frac{\pi}{22}\Rightarrow\omega=\frac{4}{7}\times\frac{22}{22}\times 7=4$

60. From the given transfer characteristic of a transistor in CE configuration, the value of power gain of this configuration is 10^x, for R_B = 10 kΩ, and R_C = 1 kΩ. The value of x is _____.



Official Ans. by NTA (3)

Sol. Power gain

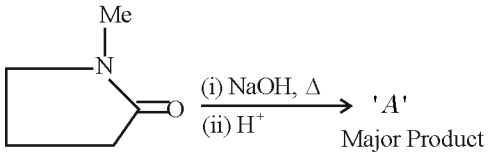
$\Rightarrow A_v.A_1=B\frac{R_C}{R_B}.B=B^2\frac{R_C}{R_B}$

$=\left(\frac{(20-10)\times10^{-3}}{(200-100)\times10^{-6}}\right)\times\frac{1\times10^3}{10\times10^3}=10^3$

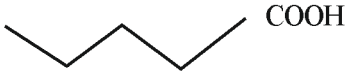
Hence $x=3$

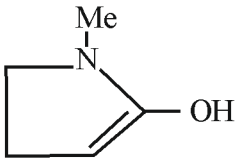
SECTION-A

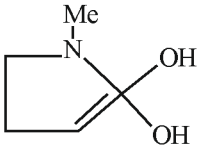
61. In the reaction given below

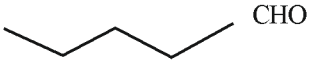


'A' is

(1) 

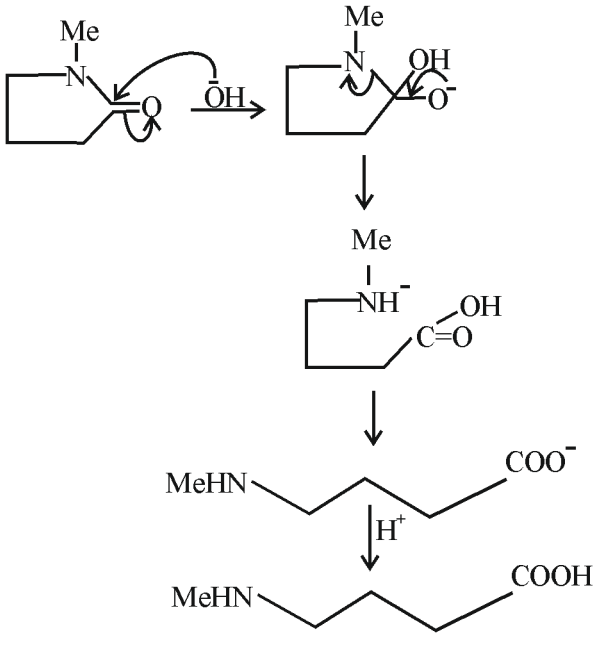
(2) 

(3) 

(4) 

Official Ans. by NTA (1)

Sol.



62. Given below are two statements:

Statement-I Permutit process is more efficient compared to the synthetic resin method for the softening of water.

Statement-II: Synthetic resin method results in the formation of soluble sodium salts.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both the Statements I and II are correct

(2) Statement I is correct but Statement II is incorrect

(3) Statement I is incorrect but Statement II is correct

(4) Both the Statements I and II are incorrect

Official Ans. by NTA (4)

Sol. Nowadays hard water is softened by using synthetic ion exchangers. This method is more efficient than zeolite process/Permutit process

63. The mismatched combinations are

A. Chlorophyll - Co

B. Water hardness - EDTA

C. Photography - $\left[Ag(CN)_2\right]^-$

D. Wilkinson catalyst - $\left[(Ph_3P)_3\,RhCl\right]$

E. Chelating ligand – D - Penicillamine

Choose the correct answer from the options given below:

(1) A and C Only

(2) A and E Only

(3) D and E Only

(4) A, C and E Only

Official Ans. by NTA (1)

Sol. Mg is present in chlorophyll and in black and white photography the developed film is fixed by washing with hypo solution which dissolves the undecomposed AgBr to form a complex ion $[Ag(S_2O_3)_2]^{3-}$

64. In which of the following processes, the bond order increases and paramagnetic character changes to diamagnetic one?

(1) $O_2 \rightarrow O_2^{2-}$

(2) $NO \rightarrow NO^+$

(3) $N_2 \rightarrow N_2^+$

(4) $O_2 \rightarrow O_2^+$

Official Ans. by NTA (2)

Sol. NO is paramagnetic with BO = 2.5, NO^+ is diamagnetic with BO = 3

65. The incorrect statement from the following for borazine is:

(1) It has electronic delocalization

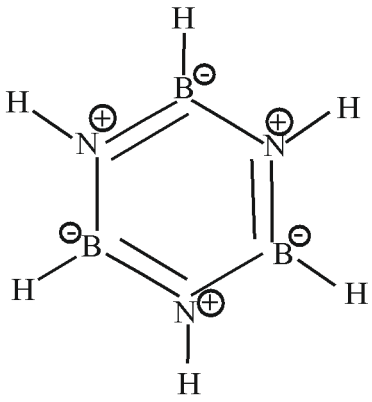
(2) It contains banana bonds.

(3) It can react with water.

(4) It is a cyclic compound.

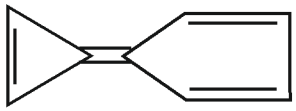
Official Ans. by NTA (3)

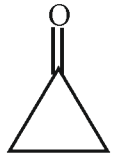
Sol. Borazine is $B_3N_3H_6$

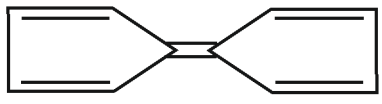


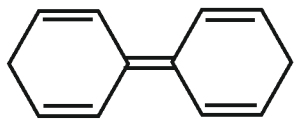
$$B_3N_3H_6 + 9H_2O \rightarrow 3NH_3 + 3H_3BO_3 + 3H_2$$

66. Among the following compounds, the one which shows highest dipole moment is

(1) 

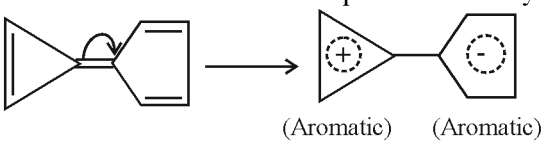
(2) 

(3) 

(4) 

Official Ans. by NTA (1)

Sol. Among the given compounds, the following compound has the highest dipole moment because both the +ve and –ve ends acquire aromaticity.



(Aromatic) (Aromatic)

67. Match the following

Column –A		Column-B	
a	Nylon 6	**I**	Natural Rubber
b	Vulcanized Rubber	**II**	Cross Linked
c	cis-1,4-polyisoprene	**III**	Caprolactam
d	Polychloroprene	**IV**	Neoprene

Choose the correct answer from option given below:

(1) a → IV,b-III, c → II,d → I

(2) a → III, b → IV, c → I,d → II

(3) a → II, b → III, c → IV, d → I

(4) a → III, b → II, c → I, d → IV

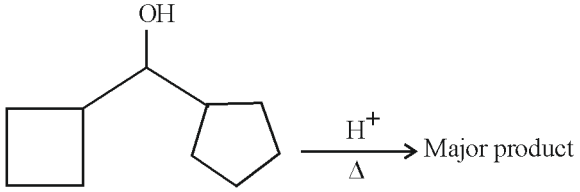
Official Ans. by NTA (4)

Sol. Nylon-6 – Caprolactum (Monomer)

Natural rubber – Isoprene (Monomer)

Vulcanized rubber – Sulphur containing rubber

Neoprene – Chloroprene (Monomer)

68. 

In the above reaction. Left hand side and right hand side rings are named as 'A' and 'B' respectively. They undergo ring expansion. The correct statement for this process is:

(1) Finally both rings will become six membered each.

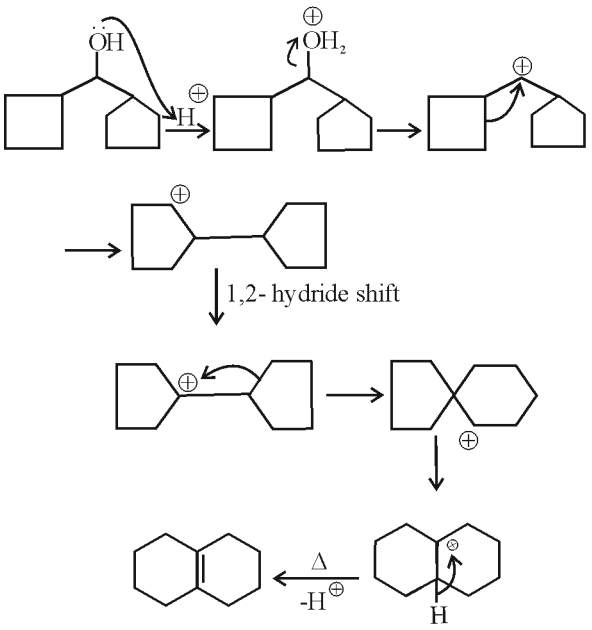
(2) Finally both rings will become five membered each.

(3) Only 'A' will become 6 membered.

(4) Ring expansion can go upto seven membered rings

Official Ans. by NTA (1)

Sol.



69. The radical which mainly causes ozone depletion in the presence of UV radiations is:

(1) $CH_3^{\bullet}$ (2) $NO^{\bullet}$

(3) $Cl^{\bullet}$ (4) $\dot{O}H$

Official Ans. by NTA (3)

Sol.

$$O_2(g) \xrightarrow{UV} O(g) + O(g)$$

$$O_2(g) + O(g) \longrightarrow O_3(g)$$

$$CF_2Cl_2(g) \xrightarrow{UV} \dot{C}l(g) + \dot{C}F_2Cl(g)$$

$$\dot{C}l(g) + O_3(g) \longrightarrow Cl\dot{O}(g) + O_2(g)$$

$$Cl\dot{O}(g) + O(g) \longrightarrow \dot{C}l(g) + O_2(g)$$

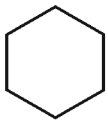
70. In the following reaction 'X' is

$$CH_3(CH_2)_4CH_3 \xrightarrow[HCl,\,\Delta]{Anhy.AlCl_3} \underset{\text{major product}}{'X}$$

(1) $CH_3(CH_2)_4CH_2Cl$

(2) $Cl-CH_2-(CH_2)_4-CH_2-Cl$

(3) $CH_3\underset{\displaystyle CH_3}{\underset{|}{C}}H-(CH_2)_2CH_3$

(4) 

Official Ans. by NTA (3)

Sol. n-alkanes on heating in this presence of anhydrous $AlCl_3$ and hydrogen chloride gas isomerise to branched chain alkanes. The major product has one methyl side chain.

$$CH_3-(CH_2)_4-CH_3 \xrightarrow[HCl,\Delta]{Anhy.AlCl_3} CH_3-\underset{\displaystyle CH_3}{\underset{|}{C}}H-(CH_2)_2-CH_3$$

2 – methylpentane
(major)

71. 2-Methyl propyl bromide reacts with $C_2H_5O^-$ and gives 'A' whereas on reaction with C_2H_5OH it gives 'B'. The mechanism followed in these reactions and the products 'A' and 'B' respectively are:

(1) S_N2. A = iso-butyl ethyl ether; S_N1, B= tert-butyl ethyl ether

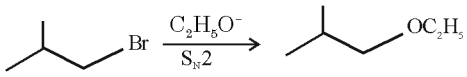
(2) S_N1, A = tert-butyl ethyl ether; S_N1, B = 2-butyl ethyl ether

(3) S_N1, A = tert-butyl ethyl ether; S_N2, B = iso-butyl ethyl ether

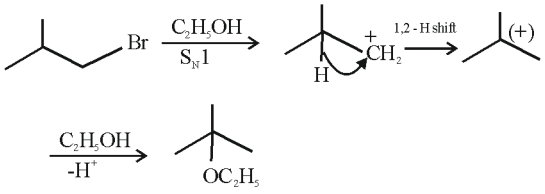
(4) S_N2, A =2-butyl ethyl ether; S_N2, B= iso-butyl ethyl ether

Official Ans. by NTA (1)

Sol.

(i) 

$C_2H_5O^-$ is strong nucleophile.

(ii) 

C_2H_5OH is weak nucleophile.

72. D- (+)- Glyceraldehyde $\xrightarrow[(iii)\ HNO_3]{(i)\ HCN \quad (ii)\ H_2O/H^+}$

The products formed in the above reaction are

(1) Two optically active products

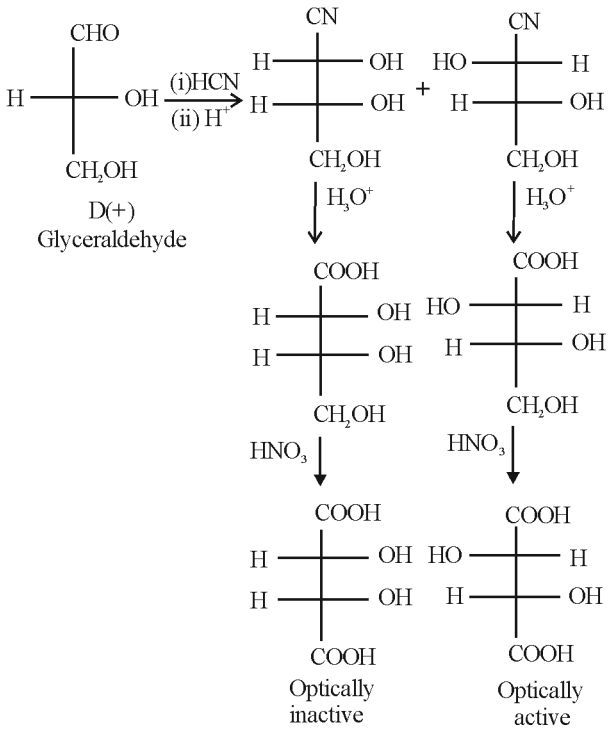
(2) One optically active and one meso product

(3) One optically inactive and one meso product.

(4) Two optically inactive products

Official Ans. by NTA (2)

Sol.



73. Which one of the following is most likely a mismatch?

(1) Zinc- Liquation

(2) Titanium – van Arkel method

(3) Nickel – Mond process

(4) Copper - Electrolysis

Official Ans. by NTA (1)

Sol. Zinc is refined by distillation method, which is used for metals having low boiling point.

74. ClF_5 at room temperature is a:

(1) Colourless gas with trigonal bipyramidal geometry.

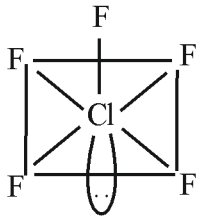
(2) Colourless gas with square pyramidal geometry

(3) Colourless liquid with square pyramidal geometry

(4) Colourless liquid with trigonal bipyramidal geometry.

Official Ans. by NTA (3)

Sol.



ClF_5 is colourless liquid.

75. $Be(OH)_2$ react with $Sr(OH)_2$ to yield an ionic salt. Choose the incorrect option related to this reaction from the following:

(1) Be is tetrahedrally coordinated in the ionic salt.

(2) The reaction is an example of acid - base neutralization reaction.

(3) Both Sr and Be elements are present in the ionic salt.

(4) The elements Be is present in the cationic part of the ionic salt.

Official Ans. by NTA (4)

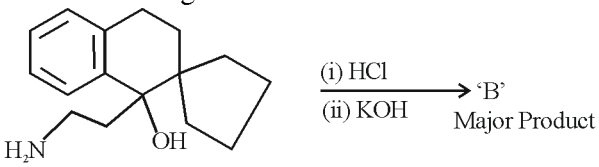
Sol. $Be(OH)_2$ is amphoteric in nature.

$Sr(OH)_2$ is basic in nature.

These two undergo acid – base reaction to form a salt.

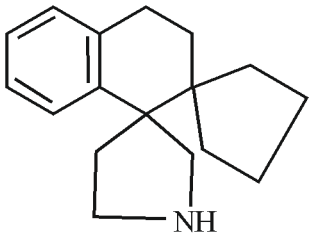
$$Be(OH)_2 + Sr(OH)_2 \rightarrow \underset{\text{(salt)}}{Sr[Be(OH)_4]}$$

76. In the reaction given below

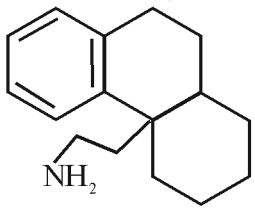


'B' is:

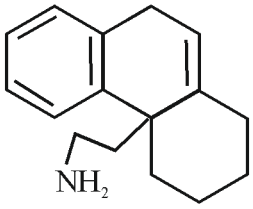
(1)



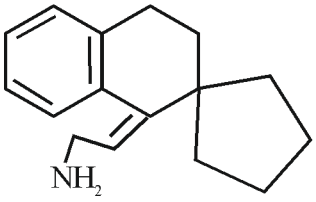
(2)



(3)

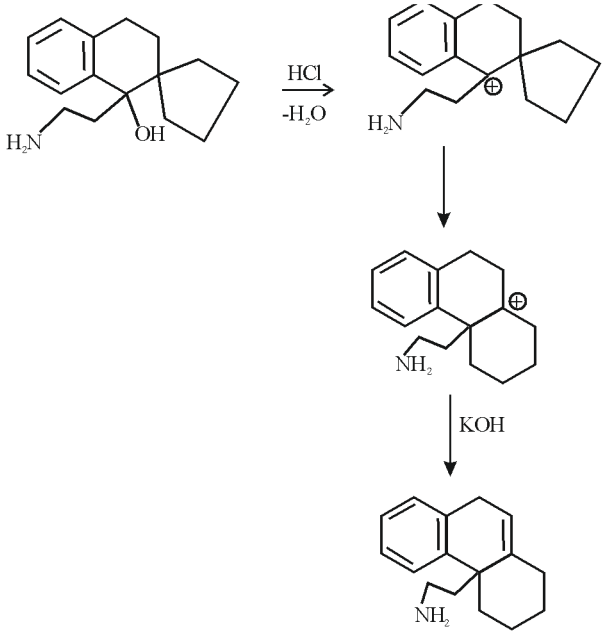


(4)



Official Ans. by NTA (3)

Sol.



77. Which of the following statements are **not** correct?

A. The electron gain enthalpy of F is more negative than that of Cl

B. Ionization enthalpy decreases in a group of periodic table

C. The electronegativity of an atom depends upon the atoms bonded to it.

D. Al_2O_3 and NO are examples of amphoteric oxides.

Choose the most appropriate answer from the options given below:

(1) A, B, C, and D

(2) A, C and D Only

(3) B and D Only

(4) A, B and D Only

Official Ans. by NTA (2)

Sol. Electronegativity of an element depends on the atom with which it is attached.

NO = neutral oxide

Al_2O_3 = amphoteric oxide

78. The energy of an electron in the first Bohr orbit of hydrogen atom is $-2.18\times10^{-18}J$. Its energy in the third Bohr orbit is_____.

(1) $\frac{1}{27}$ of this value

(2) One third of this value

(3) Three times of this value

(4) $\frac{1}{9}$ th of this value

Official Ans. by NTA (4)

Sol.

$$E_n = \frac{-2.18\times10^{-18}Z^2}{n^2}$$

$$\text{i.e. } E_n \propto \frac{1}{n^2}$$

79. What happens when a lyophilic sol is added to a lyophobic sol?

(1) Lyophilic sol is dispersed in lyophobic sol.

(2) Film of lyophobic sol is formed over lyophilic sol.

(3) Lyophobic sol is coagulated

(4) Film of lyophilic sol is formed over lyophobic sol.

Official Ans. by NTA (4)

Sol. Lyophilic sol is used as protective action for lyophobic sol. It forms a layer / film around the lyophobic sol.

80. The pair of lanthanides in which both elements have high third –ionization energy is:

(1) Eu, Gd

(2) Eu, Yb

(3) Lu, Yb

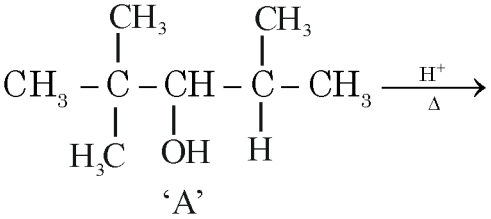
(4) Dy, Gd

Official Ans. by NTA (2)

Sol. $Eu^{+2}:[Xe]4f^7$, $Yb^{+2}:[Xe]4f^{14}$ } High IE due to half filled & fully filled configurations

SECTION-B

81. For the given reaction

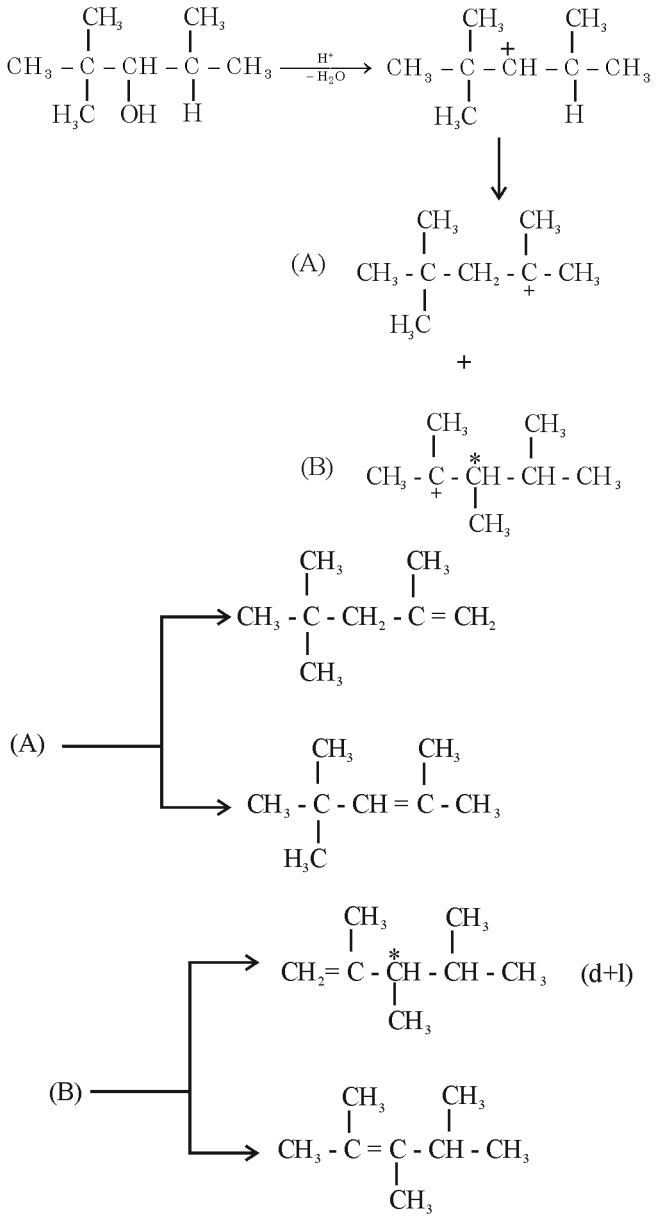


'A'

The total number of possible products formed by tertiary carbocation of A is _____.

Official Ans. by NTA (4)

Sol.



82. Solution of 12 g of non – electrolyte (A) prepared by dissolving it in 1000 mL of water exerts the same osmotic pressure as that of 0.05 M glucose solution at the same temperature. The empirical formula of A is CH_2O. The molecular mass of A is _____ g. (Nearest integer)

Official Ans. by NTA (240)

Sol. For Isotonic solutions

$\pi_1 = \pi_2$

$\Rightarrow C_1 = C_2$

$\frac{12}{x} = 0.05 \left[x \rightarrow \text{Molar Mass of A}\right]$

X = 240

83. 25.0 mL of 0.050 M $Ba(NO_3)_2$ is mixed with 25.0 mL of 0.020 M NaF. K_{sp} of BaF_2 is 0.5×10^{-6} at 298 K. The ratio of $\left[Ba^{2+}\right]\left[F^-\right]^2$ and K_{sp} is _____ (Nearest integer)

Official Ans. by NTA (5)

Sol. $\left[Ba^{+2}\right] = \frac{25\times0.05}{50} = 0.025M$

$\left[F^-\right] = \frac{25\times0.02}{50} = 0.01M$

$\left[Ba^{+2}\right]\left[F^-\right]^2 = 25\times10^{-7}$

$K_{sp} = 5\times10^{-7}$ (given)

$\text{Ratio} = \frac{\left[Ba^{+2}\right]\left[F^-\right]^2}{K_{sp}} = 5$

84. $A_2 + B_2 \rightarrow 2AB.\ \Delta H_f^0 = -200\ kJmol^{-1}$

AB, A_2 and B_2 are diatomic molecule. If the bond enthalpies of A_2, B_2 and AB are in the ratio 1:0.5:1, then the bond enthalpy of A_2 is _____ $kJmol^{-1}$ (Nearest integer)

Official Ans. by NTA (400)

Sol. $A_2 + B_2 \rightarrow 2AB\ ;\ \Delta H_f^0 = -200\ kJ\ mol^{-1}$

$\Rightarrow \Delta H_f^0\,(AB) = -200\ kJ\ mol^{-1}$

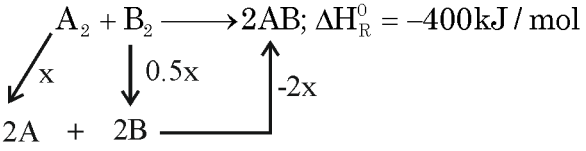
$\therefore \Delta H_R^0$ for reaction $A_2 + B_2 \rightarrow 2AB$ is $-400\ kJ\ mol^{-1}$

Given: Bond Enthalpy of A_2, B_2 and AB is 1 : 0.5 : 1

Assuming bond enthalpy of A_2 be x kJ mol^{-1}

$\therefore$ Bond enthalpy $B_2 = 0.5$ x kJ mol^{-1}

$\therefore$ Bond enthalpy $AB = (x)\,kJ\ mol^{-1}$



-400 = x + 0.5 x -2 x

-400 = – 0.5 x

$\therefore$ x = 800 kJ / mol

85. An organic compound gives $0.220\,g$ of CO_2 and $0.126\,g$ of H_2O on complete combustion. If the % of carbon is 24 then the % hydrogen is _____ $\times 10^{-1}$. (Nearest integer)

Official Ans. by NTA (56)

Sol. Moles of $CO_2 = \frac{0.22}{44} = \frac{1}{200}$

$\therefore$ Moles of carbon

$= (\text{Moles of } CO_2) \times 1$

$= \frac{1}{200}$

$\therefore$ wt. of C $= \frac{1}{200} \times 12 = 0.06$

% of C $= \frac{0.06}{W} \times 100 = 24$

(W = Wt. of Organic Compound)

$W = 0.25$

Moles of $H_2O = \frac{0.126}{18} = 0.007$

$\therefore$ Moles of H atom $= 2 \times 0.007 = 0.014$

% of Hydrogen $= \frac{0.014 \times 1}{W} \times 100$

$= \frac{0.014 \times 1}{0.25} \times 100$

$= 5.6$

$= 56 \times 10^{-1}$

86. 20 mL of calcium hydroxide was consumed when it was reacted with 10 mL of unknown solution of H_2SO_4. Also 20 mL standard solution of 0.5 M HCl containing 2 drops of phenolphthalein was titrated with calcium hydroxide the mixture showed pink colour when burette displayed the value of 35.5 mL whereas the burette showed 25.5 mL initially. The concentration of H_2SO_4 is _____ M (Nearest integer)

Official Ans. by NTA (1)

Sol. Reaction with HCl

$Ca(OH)_2 + 2HCl \rightarrow CaCl_2 + 2H_2O$

Volume of $Ca(OH)_2 = 10$ ml

Volume of HCl = 20 ml

Concentration of HCl = 0.5 M.

No. of milli moles of HCl= 10

No. of milli moles of $Ca(OH)_2 = 5$.

i.e. $M_{Ca(OH)_2} = \frac{\text{no. of milli moles}}{V(ml)} = \frac{5}{10}$

= 0.5 M.

Reaction with H_2SO_4

$Ca(OH)_2 + H_2SO_4 \rightarrow CaSO_4 + 2H_2O$.

No. of milli moles of $Ca(OH)_2 = 20 \times 0.5$

$= 10$

i.e. no. of milli moles of $H_2SO_4 = 10$

$\Rightarrow M_{H_2SO_4} = \frac{\text{no. of mil lim oles}}{V(ml)}$

$= \frac{10}{10}$

= 1 M

87. A certain quantity of real gas occupies a volume of 0.15 dm^3 at 100 atm and 500 K when its compressibility factor is 1.07. Its volume at 300 atm and 300K (When its compressibility factor is 1.4) is _____ $\times 10^{-4} dm^3$ (Nearest integer)

Official Ans. by NTA (392)

Sol. $z = \frac{PV}{nRT}$; $n = \frac{PV}{ZRT}$

$Z_1 = 1.07, P_1 = 100 atm, V_1 = 0.15 L, T_1 = 500 K$

$Z_2 = 1.4, P_2 = 300 atm, T_2 = 300 K, V_2 = ?$

$\frac{P_1V_1}{Z_1RT_1} = \frac{P_2V_2}{Z_2RT_2} = n$

$V_2 = \frac{1.4}{1.07} \times .03$

$= 392 \times 10^{-4}\ dm^3$

88. $t_{87.5}$ is the time required for the reaction to undergo 87.5% completion and t_{50} is the time required for the reaction to undergo 50% completion. The relation between $t_{87.5}$ and t_{50} for a first order reaction is $t_{87.5} = x \times t_{50}$

The value of x is_____ (Nearest integer)

Official Ans. by NTA (3)

Sol. $A_t = A_0 \times \frac{12.5}{100} = \frac{A_0}{8}$ [87.5% complete]

$$A_0 \xrightarrow{t_{1/2}} \frac{A_0}{2} \xrightarrow{t_{1/2}} \frac{A_0}{4} \xrightarrow{t_{1/2}} \frac{A_0}{8}$$

$t_{87.5} = 3t_{1/2}$

$\therefore\ x = 3$

89. $KMnO_4$ is titrated with ferrous ammonium sulphate hexahydrate in presence of dilute H_2SO_4. Number of water molecules produced for 2 molecules of $KMnO_4$ is _____.

Official Ans. by NTA (68)

Sol. By balancing the redox reaction we get

$$10\left[FeSO_4.(NH_4)_2SO_4.6H_2O\right] + 2KMnO_4 + 8H_2SO_4$$

$$\downarrow$$

$$5Fe_2(SO_4)_3 + 2MnSO_4 + 10(NH_4)_2SO_4 + K_2SO_4 + 68H_2O$$

90. A metal surface of $100\ cm^2$ area has to be coated with nickel layer of thickness 0.001mm. A current of 2A was passed through a solution of $Ni(NO_3)_2$ for 'x' seconds to coat the desired layer. The value of x is _____ (Nearest integer)

(ρ_{Ni} (density of Nickel) is $10\ gmL^{-1}$, Molar mass of Nickel is $60\ gmol^{-1}$ $F = 96500\ C\ mol^{-1}$)

Official Ans. by NTA (161)

Sol. $W = z \times i \times t.$

$$\text{Density} \times \text{volume} = \frac{E \times i \times t}{96500}$$

$$10 \times 100 \times 0.0001 = \frac{\left(\frac{\text{atomic wt.}}{\text{v.f}}\right) \times 2 \times x}{96500} \quad (\text{v.f} = 2)$$

$\therefore$ x = 161 sec.

FINAL JEE–MAIN EXAMINATION – APRIL, 2023

(Held On Thursday 13th April, 2023) **TIME :3 : 00 PM to 6 : 00 PM**

MATHEMATICS | TEST PAPER WITH SOLUTION

SECTION-A

1. If the system of equations

$2x + y - z = 5$

$2x - 5y + \lambda z = \mu$

$x + 2y - 5z = 7$

has infinitely many solutions, then $(\lambda + \mu)^2 + (\lambda - \mu)^2$ is equal to

(1) 916

(2) 912

(3) 920

(4) 904

Official Ans. By NTA (1)

Sol.

$$\begin{vmatrix} 2 & 1 & -1 \\ 2 & -5 & \lambda \\ 1 & 2 & -5 \end{vmatrix} = 0$$

$2(25 - 2\lambda) - (-10 - \lambda) - (4 + 5) = 0$

$50 - 4\lambda + 10 + \lambda - 9 = 0$

$51 = 3\lambda \Rightarrow \lambda = 17$

$$\begin{vmatrix} 2 & 1 & 5 \\ 2 & -5 & \mu \\ 1 & 2 & 7 \end{vmatrix} = 0$$

$\Rightarrow 2(-35 - 2\mu) - (14 - \mu) + 5(4 + 5) = 0$

$-70 - 4\mu - 14 + \mu + 45 = 0$

$-3\mu = 39$

$-\mu = 13$

$(\lambda + \mu)^2 + (\lambda - \mu)^2 = 2(\lambda^2 + \mu^2)$

$= 2(17^2 + 13^2) = 916$

2. The coefficient of x^5 in the expansion of $\left(2x^3 - \frac{1}{3x^2}\right)^5$ is

(1) 8

(2) 9

(3) $\frac{80}{9}$

(4) $\frac{26}{3}$

Official Ans. By NTA (3)

Sol. $\left(2x^3 - \frac{1}{3x^2}\right)^5$

$$T_{r+1} = {}^5C_r\ (2x^3)^{5-r}\left(\frac{-1}{3x^2}\right)^r = {}^5C_r\ \frac{(2)^{5-r}}{(-3)^r}(x)^{15-5r}$$

$\therefore 15 - 5r = 5$

$\therefore r = 2$

$$T_3 = 10\left(\frac{8}{9}\right)x^5$$

So, coefficient is $\frac{80}{9}$

3. The plane, passing through the points (0, –1, 2) and (–1, 2, 1) and parallel to the line passing through (5, 1, –7) and (1, –1, –1), also passes through the point.

(1) (1, –2, 1)

(2) (0, 5, –2)

(3) (–2, 5, 0)

(4) (2, 0, 1)

Official Ans. By NTA (3)

Sol. Points (0, –1, 2) and (–1, 2, 1) parallel to the line of (5, 1, –7) and (1, –1, –1)

$$\text{Normal Vector} = \begin{vmatrix} i & j & k \\ 4 & 2 & -6 \\ -1 & 3 & -1 \end{vmatrix}$$

$\vec{n} = 16\hat{i} + 10\hat{j} + 14\hat{k}$

$16x + 10y + 14z = d$

Point (0, –1, 2)

$0 - 10 + 28 = d \Rightarrow d = 18$

$8x + 5y + 7z = 9$ is equation of plane.

4. Let α, β be the roots of the equation $x^2 - \sqrt{2}x + 2 = 0$. Then $\alpha^{14} + \beta^{14}$ is equal to

(1) $-64\sqrt{2}$

(2) $-128\sqrt{2}$

(3) -64

(4) -128

Official Ans. By NTA (4)

Sol. $x^2 - \sqrt{2}x + 2 = 0$

$$x = \frac{\sqrt{2} \pm \sqrt{2-8}}{2} = \frac{\sqrt{2} \pm \sqrt{6}i}{2}$$

$$\alpha = \frac{\sqrt{2} + \sqrt{6}i}{2} = \sqrt{2}\, e^{\frac{i\pi}{3}} \;\&\; \beta = \sqrt{2}\, e^{\frac{-i\pi}{3}}$$

$$\alpha^{14} = 2^7 e^{\frac{i14\pi}{3}} = 128\left[e^{\frac{i2\pi}{3}}\right]$$

$$\beta^{14} = 128\left[e^{\frac{-i2\pi}{3}}\right]$$

$$\alpha^{14} + \beta^{14} = 128(2)\cos\left(\frac{2\pi}{3}\right) = -128$$

5. Let $a_1, a_2, a_3, \ldots$ be a G.P. of increasing positive numbers. Let the sum of its 6th and 8th terms be 2 and the product of its 3rd and 5th terms be $\frac{1}{9}$. Then $6(a_2 + a_4)(a_4 + a_6)$ is equal to

(1) $2\sqrt{2}$

(2) 2

(3) $3\sqrt{3}$

(4) 3

Official Ans. By NTA (4)

Sol.

$ar^5 + ar^7 = 2$

$(ar^2)(ar^4) = \frac{1}{9}$

$a^2 r^6 = \frac{1}{9}$

Now, $r > 0$

$ar^5 (1 + r^2) = 2$

Now, $ar^3 = \frac{1}{3}$ or $-\frac{1}{3}$ (rejected)

$r^2 = 2$

$r = \sqrt{2}$

$a = \frac{1}{6\sqrt{2}}$

Now, $6(a_2 + a_4)(a_4 + a_6)$

$6(ar + ar^3)(ar^3 + ar^5)$

$6\, a^2 r^4 (1 + r^2)$

$6\left(\frac{1}{36.2}\right)(4)(9) = 3$

6. Let (α, β) be the centroid of the triangle formed by the lines $15x - y = 82$, $6x - 5y = -4$ and $9x + 4y = 17$. Then $\alpha + 2\beta$ and $2\alpha - \beta$ are the roots of the equation

(1) $x^2 - 7x + 12 = 0$

(2) $x^2 - 13x + 42 = 0$

(3) $x^2 - 14x + 48 = 0$

(4) $x^2 - 10x + 25 = 0$

Official Ans. By NTA (2)

Sol. upon solving we get coordinates as (6, 8), (1, 2) and (5, –7)

So centroid : (α, β) is

$$\alpha = \frac{6+1+5}{3} = 4$$

$$\beta = \frac{8+2-7}{3} = 1$$

$\alpha + 2\beta = 6$

$2\alpha - \beta = 7$

Ans. $x^2 - 13x + 42 = 0$

7. **Let** $|\vec{a}|=2$, $|\vec{b}|=3$ and the angle between the vectors $\vec{a}$ and $\vec{b}$ be $\frac{\pi}{4}$. Then $\left|(\vec{a}+2\vec{b})\times(2\vec{a}-3\vec{b})\right|^2$ is equal to

(1) 482

(2) 441

(3) 841

(4) 882

Official Ans. By NTA (4)

Sol. $|\vec{a}|=2, |\vec{b}|=3$

$$\left|(\vec{a}+2\vec{b})\times(2\vec{a}-3\vec{b})\right|^2$$

$$\left|-3\vec{a}\times\vec{b}+4\vec{b}\times\vec{a}\right|^2$$

$$\left|-3\vec{a}\times\vec{b}-4\vec{a}\times\vec{b}\right|^2$$

$$\left|-7\vec{a}\times\vec{b}\right|^2$$

$$\left(-7|\vec{a}|\times|\vec{b}|\sin\left(\frac{\pi}{4}\right)\right)^2$$

$$49\times4\times9\times\frac{1}{2}=882$$

8. Let N be the foot of perpendicular from the point P (1, –2, 3) on the line passing through the points (4, 5, 8) and (1, –7, 5). Then the distance of N from the plane $2x-2y+z+5=0$ is

(1) 6

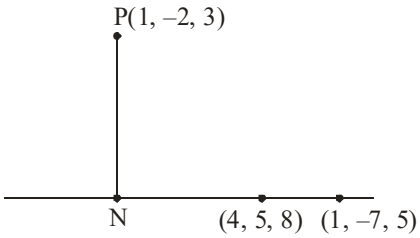
(2) 9

(3) 7

(4) 8

Official Ans. By NTA (3)

Sol.



Equation of line

$$\frac{x-4}{4-1}=\frac{y-5}{5-(-7)}=\frac{z-8}{8-5}$$

$$\frac{x-4}{3}=\frac{y-5}{12}=\frac{z-8}{3}$$

Let point $N(3\lambda+4, 12\lambda+5, 3\lambda+8)$

$$\overrightarrow{PN}=(3\lambda+4-1)\hat{i}+(12\lambda+5-(-2))\hat{j}+(3\lambda+8-3)\hat{k}$$

$$\overrightarrow{PN}=(3\lambda+3)\hat{i}+(12\lambda+7)\hat{j}+(3\lambda+5)\hat{k}$$

And parallel vector to line (say $\vec{a}=3\hat{i}+12\hat{j}+3\hat{k}$)

Now, $\overrightarrow{PN}\cdot\vec{a}=0$

$(3\lambda+3)3+(12\lambda+7)12+(3\lambda+5)3=0$

$$162\lambda+108=0\Rightarrow\lambda=\frac{-108}{162}=\frac{-2}{3}$$

So point N is (2, –3, 6)

Now distance is $=\left|\frac{2(2)-2(-3)+6+5}{\sqrt{4+4+1}}\right|=7$

9. If $\lim\limits_{x\to0}\dfrac{e^{ax}-\cos(bx)-\dfrac{cxe^{-cx}}{2}}{1-\cos(2x)}=17$, then $5a^2+b^2$ is equal to

(1) 72

(2) 76

(3) 68

(4) 64

Official Ans. By NTA (3)

Sol. $\lim\limits_{x\to 0}\dfrac{e^{ax}-\cos(bx)-\dfrac{cxe^{-cx}}{2}}{\dfrac{(1-\cos 2x)}{4x^2}\times 4x^2}=17$

On expansion,

$$\lim_{x\to 0}\frac{\left(1+ax+\frac{a^2x^2}{2}\right)-\left(1-\frac{b^2x^2}{2}\right)-\frac{cx}{2}(1-cx)}{2x^2}=17$$

$$\lim_{x\to 0}\frac{\left(a-\frac{c}{2}\right)x+x^2\left(\frac{a^2}{2}+\frac{b^2}{2}+\frac{c^2}{2}\right)}{2x^2}=17$$

For limit to be exist $a-\dfrac{c}{2}=0$

$$a=\frac{c}{2}$$

and $\dfrac{a^2+b^2+c^2}{4}=17$

$a^2 + b^2 + 4a^2 = 17 \times 4$

$5a^2 + b^2 = 68$

10. Let the centre of a circle C be (α, β) and its radius $r < 8$. Let $3x + 4y = 24$ and $3x - 4y = 32$ be two tangents and $4x + 3y = 1$ be a normal to C. Then $(\alpha - \beta + r)$ is equal to

(1) 7

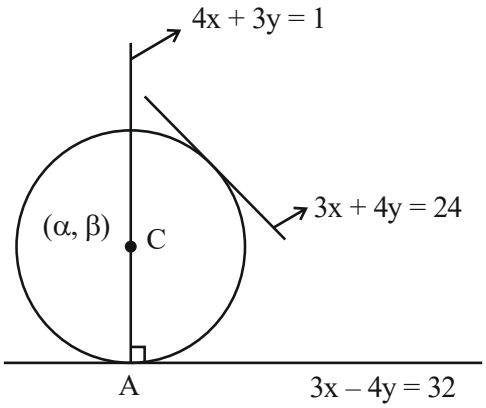
(2) 9

(3) 5

(4) 6

Official Ans. By NTA (1)

Sol.



First find point A by solving $4x + 3y = 1$ and $3x - 4y = 32$

After solving, point A is $(4, -5)$

centre (α, β) lie on $4x + 3y = 1$

$$4\alpha + 3\beta = 1 \Rightarrow \beta = \frac{1-4\alpha}{3}$$

Now distance from centre to line $3x - 4y - 32 = 0$ and $3x + 4y - 24 = 0$ are equal.

$$\left|\frac{3\alpha-4\left(\frac{1-4\alpha}{3}\right)-32}{5}\right|=\left|\frac{3\alpha+4\left(\frac{1-4\alpha}{3}\right)-24}{5}\right|$$

after solving $\alpha = 1$ and $\alpha = \dfrac{28}{3}$

For $\alpha = 1$, centre $(1, -1) \Rightarrow$ radius $= 5$

For $\alpha = \dfrac{28}{3}$, centre $\left(\dfrac{28}{3}, \dfrac{-109}{2}\right)$

$\Rightarrow$ radius ≈ 49.78 (rejected)

Hence, $\alpha = 1, \beta = -1, r = 5$

$\alpha - \beta + r = 7$

11. All words, with or without meaning, are made using all the letters of the word MONDAY. These words are written as in a dictionary with serial numbers. The serial number of the word MONDAY is

(1) 327

(2) 326

(3) 328

(4) 324

Official Ans. By NTA (1)

Sol. First arrange in alphabetical order

i.e. ADMNOY

A _ _ _ _ _ = 5!

D _ _ _ _ _ = 5!

M A _ _ _ _ = 4!

M D _ _ _ _ = 4!

M N _ _ _ _ = 4!

M O A _ _ _ = 3!

M O D _ _ _ = 3!

M O N A _ _ = 2!

M O N D A Y = 1

= 327

12. The range of $f(x)=4\sin^{-1}\left(\frac{x^2}{x^2+1}\right)$ is

(1) [0, π]

(2) [0, 2π)

(3) [0, π)

(4) [0, 2π]

Official Ans. By NTA (2)

Sol. $f(x) = 4\sin^{-1}\left(\frac{x^2}{x^2+1}\right)$

$$\frac{x^2+1-1}{x^2+1}=1-\frac{1}{x^2+1}\Rightarrow[0,1)$$

Range of f(x) = [0, 2π)

13. The statement $(p\wedge(\sim q))\vee((\sim p)\wedge q)\vee((\sim p)\wedge(\sim q))$ is equivalent to

(1) $(\sim p)\vee(\sim q)$

(2) $p\vee(\sim q)$

(3) $(\sim p)\vee q$

(4) $p\vee q$

Official Ans. By NTA (1)

Sol. $(p\wedge(\sim q))\vee((\sim p)\wedge q)\vee((\sim p)\wedge(\sim q))$

(p ^ (~q)) ∨ ((~p) ^ (q ∨ (~q)))

(p ^ (~q)) ∨ ((~p) ^ t)

(p ^ (~q)) ∨ (~p)

(~p) ∨ (p ^ ~q)

(~p ∨ p) ^ (~p ∨ ~q)

t ^ (~p ∨ ~q)

= ~p ∨ ~q

14. The random valuable X follows binomial distribution B (n, p) for which the difference of the mean and the variance is 1. If 2P(X = 2) = 3P(X = 1), then n^2 P(X > 1) is equal to

(1) 12

(2) 15

(3) 11

(4) 16

Official Ans. By NTA (3)

Sol. np – npq = 1

$\Rightarrow$ $np^2 = 1$

$^nC_2p^2q^{n-2} = {}^nC_1pq^{n-1}$

$\Rightarrow$ np – p = 3q $\quad (\because q = 1 - p)$

$\Rightarrow p = \frac{1}{2}$

Hence n = 4

P (x > 1) = 1 – (p(x = 0) + p(x = 1)

$$= 1-\left({}^4C_0\left(\frac{1}{2}\right)^4+{}^4C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^3\right)=\frac{11}{16}$$

15. Let for $A=\begin{bmatrix}1 & 2 & 3\\ a & 3 & 1\\ 1 & 1 & 2\end{bmatrix}$, |A| = 2. If |2adj (2adj (2A))| = 32^n, then 3n + α is equal to

(1) 10

(2) 9

(3) 12

(4) 11

Official Ans. By NTA (4)

Sol. $A=\begin{bmatrix}1 & 2 & 3\\ a & 3 & 1\\ 1 & 1 & 2\end{bmatrix}$ $|A| = 2,$

$1(6 - 1) - 2(2\alpha - 1) + 3(\alpha - 3) = 2$

$5 - 4\alpha + 2 + 3\alpha - 9 = 2$

$-\alpha - 4 = 0$

$\alpha = -4$

$8|\text{Adj}(2\text{Adj}(2A))|$

$8|\text{Adj}(2\times 2^2 \text{ Adj }(A))|$

$8|\text{Adj}(2^3 \text{AdjA})|$

$8|2^6 \text{Adj}(\text{AdjA})|$

$2^3(2^6)^3|\text{Adj}(\text{Adj})|$

$2^3 \cdot 2^{18} |A|^4$

$2^{21} \cdot 2^4 = 2^{25} = (2^5)^5 = (32)^5$

n = 5

α = –4

16. Let $S=\left\{Z\in C:\bar{z}=i(z^2+\text{Re}(\bar{z}))\right\}$. Then $\sum_{z\in S}|z|^2$ is equal to

(1) $\frac{7}{2}$

(2) 4

(3) $\frac{5}{2}$

(4) 3

Official Ans. By NTA (2)

Sol. Let Z = x + iy, x ∈ R, y ∈ R

$x - iy = i(x^2 - y^2 + (2xy)i + x)$

$x = -2xy$ **…..(1)**

$-y = -y^2 + x^2 + x$ **…..(2)**

$\Rightarrow x = 0, y = -\frac{1}{2}$ (from (1))

If $x \neq 0$, then y = 0, 1

If $y = -\frac{1}{2}$, then $x = \frac{1}{2}, -\frac{3}{2}$

$Z = 0 + i0, 0 + i, \frac{1}{2} - \frac{i}{2}, -\frac{3}{2} - \frac{i}{2}$

17. The area of the region $\left\{(x, y): x^2 \le y \le |x^2 - 4|, y \ge 1\right\}$ is

(1) $\frac{3}{4}\left(4\sqrt{2}-1\right)$

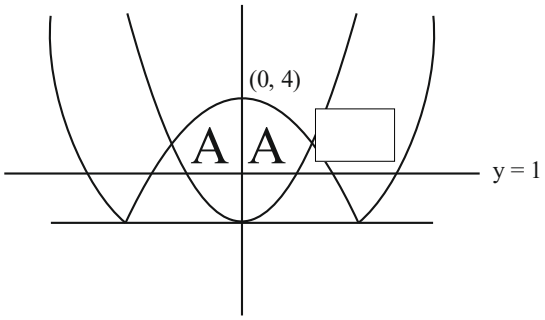
(2) $\frac{4}{3}\left(4\sqrt{2}-1\right)$

(3) $\frac{4}{3}\left(4\sqrt{2}+1\right)$

(4) $\frac{3}{4}\left(4\sqrt{2}+1\right)$

Official Ans. By NTA (2)

Sol.



Required area = $2\left[\int_1^2 \sqrt{y}\,dy + \int_2^4 \sqrt{4-y}\,dy\right] = \frac{4}{3}\left[4\sqrt{2}-1\right]$

18. Let for a triangle ABC,

$\overrightarrow{AB} = -2\hat{i}+\hat{j}+3\hat{k}$

$\overrightarrow{CB} = \alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$

$\overrightarrow{CA} = 4\hat{i}+3\hat{j}+\delta\hat{k}$

If $\delta > 0$ and the area of the triangle ABC is $5\sqrt{6}$, then $\overrightarrow{CB}.\overrightarrow{CA}$ is equal to

(1) 60

(2) 120

(3) 108

(4) 54

Official Ans. By NTA (1)

Sol. $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA} = \vec{0}$

$\alpha = 2, \beta = 4, \gamma - \delta = 3$

$\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right| = 5\sqrt{6}$

$(\delta - 9)^2 + (2\delta + 12)^2 + 100 = 600$

$\Rightarrow \delta = 5, \gamma = 8$

Hence $\overrightarrow{CB}\cdot\overrightarrow{CA} = 60$

19. The line, that is coplanar to the line $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$, is

(1) $\frac{x+1}{1} = \frac{y-2}{2} = \frac{z-5}{5}$

(2) $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$

(3) $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{4}$

(4) $\frac{x-1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$

Official Ans. By NTA (2)

Sol. Condition of co-planarity

$$\begin{vmatrix} x_2-x_1 & a_1 & a_2 \\ y_2-y_1 & b_1 & b_2 \\ z_2-z_1 & c_1 & c_2 \end{vmatrix} = 0$$

Where a_1, b_1, c_1 are direction cosine of 1st line and a_2, b_2, c_2 are direction cosine of 2nd line.

Now, solving options

Point (–3, 1, 5) & point (–1, 2, 5)

(1) $\begin{vmatrix} -3 & 1 & 5 \\ 1 & 2 & 5 \\ -2 & -1 & 0 \end{vmatrix}$

$= -3(5) - (10) + 5(-1 + 4)$

$= -15 - 10 + 15 = -10$

(2) Point (–1, 2, 5)

$\begin{vmatrix} -3 & 1 & 5 \\ -1 & 2 & 5 \\ -2 & -1 & 0 \end{vmatrix}$

$= 3(5) - (10) + 5(1 + 4)$

$-25 + 25 = 0$

(3) Point (–1, 2, 5)

$\begin{vmatrix} -3 & 1 & 5 \\ -1 & 2 & 4 \\ -2 & -1 & 0 \end{vmatrix}$

$-3(4) - (8) + 5\,(1 + 4)$

$-12 - 8 + 25 = 5$

(4) Point (–1, 2, 5)

$\begin{vmatrix} -3 & 1 & 5 \\ -1 & 2 & 5 \\ 4 & 1 & 0 \end{vmatrix}$

$-3\,(-5) - (-20) + 5(-1 - 8)$

$15 + 20 - 45 = -10$

20. The value of $\dfrac{e^{-\frac{\pi}{4}}+\int_0^{\frac{\pi}{4}} e^{-x}\tan^{50}x\,dx}{\int_0^{\frac{\pi}{4}} e^{-x}\left(\tan^{49}x+\tan^{51}x\right)dx}$ is

(1) 50

(2) 49

(3) 51

(4) 25

Official Ans. By NTA (1)

Sol. $\int_0^{\pi/4} e^{-x}\tan^{50}x\,dx$

$$\left[-e^{-y}(\tan x)^{50}\right]_0^{\pi/4}+\int_0^{\pi/4} e^{-x}\,(50)\,(\tan x)^{49}\sec^2 x$$

$$=-e^{-\pi/4}+0+50\int_0^{\pi/4} e^{-x}(\tan x)^{49}(\tan^2 x+1)$$

$$=-e^{-\pi/4}+50\left(\int_0^{\pi/4} e^{-x}(\tan x)^{51}+(\tan x)^{49}\right)dx$$

Now, $\dfrac{-e^{-\pi/4}+\int_0^{\pi/4} e^{-x}(\tan x)^{50}dx}{\int_0^{\pi/4} e^{-x}(\tan^{49}x+\tan^{51}x)dx}$

$$\frac{50\int_0^{\pi/4} e^{-x}\left((\tan x)^{51}+(\tan x)^{49}\right)dx}{\int_0^{\pi/4} e^{-x}(\tan^{49}x+\tan^{51}x)dx}=50$$

SECTION-B

21. The mean and standard deviation of the marks of 10 students were found to be 50 and 12 respectively. Later, it was observed that two marks 20 and 25 were wrongly read as 45 and 50 respectively. Then the correct variance is _____.

Official Ans. by NTA (269)

Sol. $\overline{x}=50$

$\sum x_i = 500$

$\sum x_{i\,correct} = 500+20+25-45-50=450$

$\sigma^2 = 144$

$\dfrac{\sum x_i^2}{10}-(50)^2=144$

$\sum x_{icorrect}^2=\left(144+(50)^2\right)\times 10-(45)^2-(50)^2+(20)^2+(25)^2$
$= 22940$

Correct variance $= \dfrac{\sum(x_{icorrect})^2}{10}-\left(\dfrac{\sum x_{icorrect}}{10}\right)^2$

$= 2294 - (45)^2$

$= 2294 - 2025 = 269$

22. Let A= {–4, –3, –2, 0, 1, 3, 4} and R = {(a, b) ∈ A × A : b = |a| or $b^2 = a + 1$} be a relation on A. Then the minimum number of elements, that must be added to the relation R so that it becomes reflexive and symmetric, is_____.

Official Ans. by NTA (7)

Sol. R = [(–4, 4), (–3, 3), (3, –2), (0, 1), (0, 0), (1, 1), (4, 4), (3, 3)}

For reflexive, add ⇒ (–2, –2), (–4, –4), (–3, –3)

For symmetric, add ⇒ (4, –4), (3, –3), (–2, 3), (1, 0)

23. Let $f(x)=\sum_{k=1}^{10} kx^k$, $x \in R$. If $2f(2)+f'(2)=119(2)^n+1$ then n is equal to _____.

Official Ans. by NTA (10)

Sol. $f(x)=\sum_{k=1}^{10} k\ x^k$

$f(x) = x + 2x^2 + \ldots\ldots\ldots + 10\,x^{10}$

$f(x).\ x = x^2 + 2x^3 + \ldots\ldots\ldots + 9\,x^{10} + 10x^{11}$

$f(x)\,(1-x) = x + x^2 + x^3 + \ldots\ldots\ldots + x^{10} - 10\,x^{11}$

$$f(x)=\frac{x(1-x^{10})}{(1-x)^2}-\frac{10x^{11}}{(1-x)}$$

$$f(x)=\frac{x-x^{11}-10x^{11}+10x^{12}}{(1-x)^2} \Rightarrow \frac{10x^{12}-11x^{11}+x}{(1-x)^2}$$

Hence $2f(2)+f'(2) = 119.2^{10}+1$

$\Rightarrow$ So, n = 10

24. Total numbers of 3-digit numbers that are divisible by 6 and can be formed by using the digits 1, 2, 3, 4, 5 with repetition, is____.

Official Ans. by NTA (16)

Sol. For number to be divisible by '6' unit digit should be even and sum of digit is divisible by 3.

(2, 1, 3), (2, 3, 4), (2, 5, 5), (2, 2, 5), (2, 2, 2), (4, 1, 1), (4, 4, 1), (4, 4, 4), (4, 3, 5)

2, 1, 3 $\Rightarrow$ 312, 132

2, 3, 4 $\Rightarrow$ 342, 432, 234, 324

2, 5, 5 $\Rightarrow$ 552

2, 2, 5 $\Rightarrow$ 252, 522

2, 2, 2 $\Rightarrow$ 222

4, 1, 1 $\Rightarrow$ 114

4, 4, 1 $\Rightarrow$ 414, 144

4, 4, 4 $\Rightarrow$ 444

4, 3, 5 $\Rightarrow$ 354, 534

Total 16 numbers.

25. Let $[\alpha]$ denote the greatest integer $\le \alpha$. Then $\left[\sqrt{1}\right]+\left[\sqrt{2}\right]+\left[\sqrt{3}\right]+.....+\left[\sqrt{120}\right]$ is equal to.

Official Ans. by NTA (825)

Sol. $\left[\sqrt{1}\right]+\left[\sqrt{2}\right]+\left[\sqrt{3}\right]+.....\left[\sqrt{120}\right]$

$\Rightarrow$ 1 + 1 + 1 + 2 + 2 + 2 +2 + 2 + 3 + 3 + ……. + 3 = 7 times

+ 4 + 4 + …….. + 4 = 9 times + ……. 10 + 10 + ……. + 10 = 21 times

$$\Rightarrow \sum_{r=1}^{10} (2r+1)\,.r$$

$$\Rightarrow 2\sum_{r=1}^{10} r^2 + \sum_{r=1}^{10} r$$

$$\Rightarrow 2\times\frac{10\times11\times21}{6}+\frac{10\times11}{2}$$

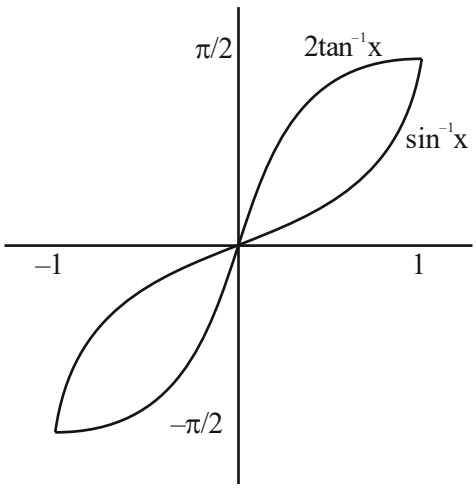
$\Rightarrow$ 770 + 55

$\Rightarrow$ 825

26. For $x \in (-1, 1]$, the number of solutions of the equation $\sin^{-1}x = 2\tan^{-1}x$ is equal to

Official Ans. by NTA (2)

Sol.



27. If y = y (x) is the solution of the differential equation $\frac{dy}{dx}+\frac{4x}{(x^2-1)}y=\frac{x+2}{(x^2-1)^{\frac{5}{2}}}$, x > 1 such that $y(2)=\frac{2}{9}\log_e\left(2+\sqrt{3}\right)$ and $y\left(\sqrt{2}\right)$ = $\alpha\log_e\left(\sqrt{\alpha}+\beta\right)+\beta-\sqrt{\gamma},\alpha,\beta,\gamma\in N$, then αβγ is equal to ____.

Official Ans. by NTA (6)

Sol. $\frac{dy}{dx}+\frac{4x}{(x^2-1)}y=\frac{x+2}{(x^2-1)^{\frac{5}{2}}}$, $x>1$

I.F. = $e^{\int\frac{4x}{x^2-1}dx}$

I.F. = $(x^2-1)^2$

$\Rightarrow d\left(y.\left(x^2-1\right)^2\right)=\frac{x+2}{(x^2-1)^{\frac{5}{2}}}.\left(x^2-1\right)^2$

$\Rightarrow \int d\left(y.\left(x^2-1\right)^2\right)=\int\frac{x+2}{(x^2-1)^{\frac{1}{2}}}dx$ _____ (1)

$y(x^2-1)^2=\sqrt{x^2-1}+2\ln\left(x+\sqrt{x^2-1}\right)+C$

$\Rightarrow C=-\sqrt{3}$

So $\left(x^2-1\right)^2=\sqrt{x^2-1}+2\ln\left(x+\sqrt{x^2-1}\right)-\sqrt{3}$

$\Rightarrow \alpha\beta\gamma = 6$

28. The foci of a hyperbola are (±2,0) and its eccentricity is $\frac{3}{2}$. A tangent, perpendicular to the line 2x + 3y = 6, is drawn at a point in the first quadrant on the hyperbola. If the intercepts made by the tangent on the x- and y-axes are a and b respectively, then |6a| + |5b| is equal to_____.

Official Ans. by NTA (12)

Sol. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$ae = 2$ & $e=\frac{3}{2}\Rightarrow a=\frac{4}{3}$

also $b^2 = a^2e^2 - a^2 \Rightarrow 4-\frac{16}{9}$

$\Rightarrow b^2=\frac{20}{9}$

Slope of tangent = $\frac{3}{2}$

So tangent equation will be

$y = mx \pm \sqrt{a^2m^2-b^2}$

$\Rightarrow y=\frac{3x}{2}\pm\sqrt{\frac{16}{9}.\frac{9}{4}-\frac{20}{9}}$

$\Rightarrow y=\frac{3x}{2}\pm\frac{4}{3}\Rightarrow |x_{intercept}|=\frac{8}{9}$

$|y_{intercept}|=\frac{4}{3}$

$\Rightarrow |6a|+|5b|=\frac{48}{9}+\frac{60}{9}=\frac{109}{9}=12$

29. Let $f_n=\int_0^{\frac{\pi}{2}}\left(\sum_{k=1}^{n}\sin^{k-1}x\right)\left(\sum_{k=1}^{n}(2k-1)\sin^{k-1}x\right)\cos x\,dx$, n ∈ N. Then $f_{21}-f_{20}$ is equal to____.

Official Ans. by NTA (41)

Sol.

$f_n(x)=\int_0^{\frac{\pi}{2}}(1+\sin x+\sin^2 x+\sin^3 x+...+\sin^{n-1}(x))$

$(1 + 3\sin x + 5\sin^2x + ... + (2n-1))\sin^{n-1} x.\cos x dx$

Multiply & divide by $\sqrt{\sin x}$

$\int_0^{\frac{\pi}{2}}\left((\sin x)^{\frac{1}{2}}+(\sin x)^{\frac{3}{2}}+(\sin x)^{\frac{5}{2}}+(\sin x)^{\frac{7}{2}}+...(\sin x)^{\frac{2n-1}{2}}\right)$

$(1 + 3\sin x + 5\sin^2x + ... + (2n-1)\sin^{n-1}(x))\frac{\cos x}{\sqrt{\sin x}}dx$

Put $(\sin x)^{1/2} + (\sin x)^{3/2} + (\sin x)^{5/2} + ... + (\sin x)^{n-1/2} = t$

$\frac{1}{2}\frac{(1+3\sin x+5\sin^2 x+...(2n-1)\sin^{n-1}x)}{\sqrt{\sin x}}\cos x dx = dt$

$f_n = 2\int_0^n t\,dt$

$f_n = n^2$

$f_{21} - f_{20} = (21)^2 - (20)^2$

$= 441 - 400$

$= 41$

30. The remainder, when 7^{103} is divided by 17 is ___.

Official Ans. by NTA (12)

Sol. $7^{103} = 7 \times 7^{102}$

$= 7 \times (49)^{51}$

$= 7 \times (51 - 2)^{51}$

Remainder :- $7 \times (-2)^{51}$

$\Rightarrow -7\,(2^3 \cdot (16)^{12})$

$\Rightarrow -56\,(17 - 1)^{12}$

Remainder $= -56 \times (-1)^{12} = -56 + 68 = 12$

SECTION-A

31. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : The binding energy per nucleon is practically independent of the atomic number for nuclei of mass number in the range 30 to 170.

Reason R : Nuclear force is short ranged.

In the light of the above statements, choose the ***correct*** answer from the options given below

Option :

(1) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**

(2) **A** is true but **R** is false

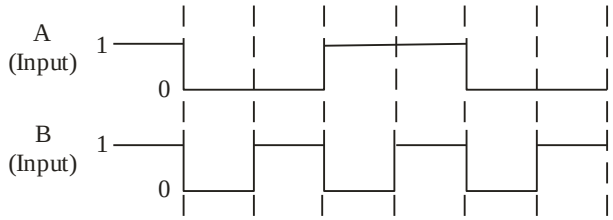
(3) **A** is false but **R** is true

(4) Both **A** and **R** are true and **R** is the correct explanation of **A**

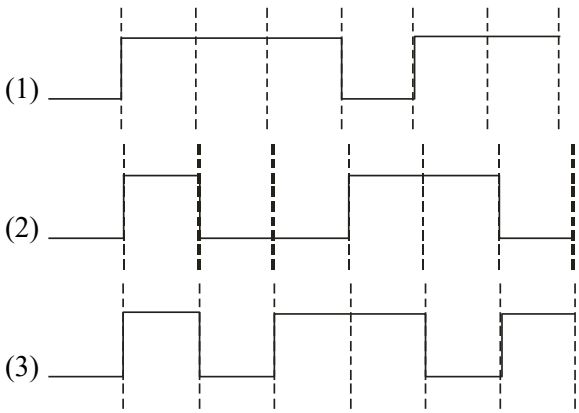
Official Ans. by NTA (4)

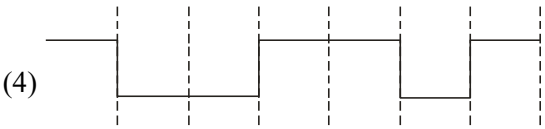
Sol. Binding energy per nucleon is almost same for nuclei of mass number ranging 30 to 170.

32. The output from a NAND gate having inputs A and B given below will be.



Option :





Official Ans. by NTA (1)

Sol. Truth table for NAND gate is

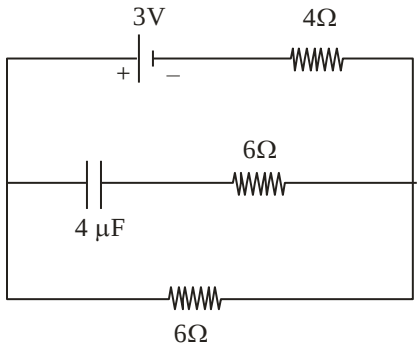
A	B	$Y = \overline{A \cdot B}$
0	0	1
0	1	1
1	0	1
1	1	0

On the basis of given input A and B the truth table is

A	B	Y
1	1	0
0	0	1
0	1	1
1	0	1
1	1	0
0	0	1
0	1	1

So the correct answer is Option 1.

33. In the network shown below, the charge accumulated in the capacitor in steady state will be :



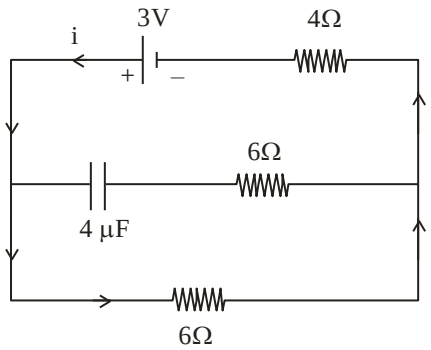
Option :

(1) 7.2 μC (2) 4.8 μC

(3) 10.3 μC (4) 12 μC

Official Ans. by NTA (1)

Sol.



No current will flow in capacitor in steady state, current flowing in the circuit in steady state

$$I = \frac{3}{6+4} = \frac{3}{10}$$

Potential difference on 6Ω resistance

$$V = 6 \times \frac{3}{10} = 1.8 \text{ volt}$$

Capacitor will have same potential so charge,

$q = CV = (4\ \mu F) \cdot (1.8 \text{ volt}) = 7.2\ \mu C.$

34. Given below are two statements :

Statement I: For a planet, if the ratio of mass of the planet to its radius increases, the escape velocity from the planet also increases.

Statement II: Escape velocity is independent of the radius of the planet.

In the light of above statements, choose the ***most appropriate*** answer from the options given below

Option :

(1) Both Statement I and Statement II are incorrect

(2) Statement I is correct but statement II is incorrect

(3) Statement I is incorrect but statement II is correct

(4) Both Statement I and Statement II are correct

Official Ans. by NTA (2)

Sol. $V_e = \sqrt{\frac{2GM}{R}} \Rightarrow V_e \propto \sqrt{\frac{M}{R}}$

As $\frac{M}{R}$ increases $\Rightarrow V_e$ increases

Statement (1) is correct

Also $V_e \propto \frac{1}{\sqrt{R}}$

As V_e depends upon R

$\Rightarrow$ Statement (2) is incorrect

Option (2) is correct

35. A particle executes SHM of amplitude A. The distance from the mean position when its's kinetic energy becomes equal to its potential energy is :

Option :

(1) $\sqrt{2A}$ (2) 2A

(3) $\frac{1}{\sqrt{2}}A$ (4) $\frac{1}{2}A$

Official Ans. by NTA (3)

Sol. KE = PE

$$\frac{1}{2}M\omega^2(A^2 - x^2) = \frac{1}{2}M\omega^2x^2$$

$$A^2 - x^2 = x^2 \Rightarrow A^2 = 2 \times 2$$

$$\Rightarrow x = \pm \frac{A}{\sqrt{2}}$$

36. A passenger sitting in a train A moving at 90 km/h observes another train B moving in the opposite direction for 8 s. If the velocity of the train B is 54 km/h, then length of train B is :

Option :

(1) 80 m (2) 200 m

(3) 120 m (4) 320 m

Official Ans. by NTA (4)

Sol. Velocity of train A

$$V_A = 90\frac{km}{hr} = 90 \times \frac{5}{18} = 25 \text{ m/s}$$

Velocity of train B

$$V_B = 54\frac{km}{hr} = 54 \times \frac{5}{18} = 15 \text{ m/s}$$

Velocity of train B w.r.t. train A $= \vec{V}_B - \vec{V}_A$

$= 15 - (-25)$ m/s

$= 40$ m/s

$$\text{Time of crossing} = \frac{\text{length of train}}{\text{relative velocity}}$$

$$(8) = \frac{\ell}{40}$$

$\ell = 8 \times 40 = 320$ meter.

37. The initial pressure and volume of an ideal gas are P_0 and V_0. The final pressure of the gas when the gas is suddenly compressed to volume $\frac{V_0}{4}$ will be :

(Given γ = ratio of specific heats at constant pressure and at constant volume)

Option :

(1) $P_0(4)^{\frac{1}{\gamma}}$ (2) $P_0(4)^{\gamma}$

(3) P_0 (4) $4P_0$

Official Ans. by NTA (2)

Sol. As gas is suddenly compressed, the processes is adiabatic.

Equation of gas for adiabatic process is PV^{γ} = constant.

$\Rightarrow P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$\Rightarrow P_0V_0^{\gamma} = P_2\left(\frac{V_0}{4}\right)^{\gamma}$

$\Rightarrow P_2 = P_0(4)^{\gamma}$

Option (2) is correct

38. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : A spherical body of radius (5 ± 0.1) mm having a particular density is falling through a liquid of constant density. The percentage error in the calculation of its terminal velocity is 4%.

Reason R : The terminal velocity of the spherical body falling through the liquid is inversely proportional to its radius.

In the light of the above statements, choose the ***correct*** answer from the options given below

Option :

(1) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**

(2) Both **A** and **R** are true and **R** is the correct explanation of **A**

(3) **A** is false but **R** is true

(4) **A** is true but **R** is false

Official Ans. by NTA (4)

Sol. Terminal velocity of a spherical body in liquid

$\Rightarrow V_t \propto r^2$

$\Rightarrow \frac{\Delta V_t}{V_t} = 2 \cdot \frac{\Delta r}{r}$

$\Rightarrow \frac{\Delta V_t}{V_t} \times 100\,\% = 2\frac{(0.1)}{5} \times 100 = 4\,\%$

Also $V_t \propto r^2$

Reason R is false

Option (4) is correct

39. In an electromagnetic wave, at an instant and at a particular position, the electric field is along the negative z-axis and magnetic field is along the positive x-axis. Then the direction of propagation of electromagnetic wave is :

Option :

(1) at 45° angle from positive y-axis

(2) negative y-axis

(3) positive z-axis

(4) positive y-axis

Official Ans. by NTA (2)

Sol. Direction of propagation of EM wave will be in the direction of $\vec{E} \times \vec{B}$.

40. The distance travelled by an object in time t is given by $s = (2{\cdot}5)t^2$. The instantaneous speed of the object at t = 5 s will be :

Option :

(1) 12.5 ms^{-1} (2) 62.5 ms^{-1}

(3) 5 ms^{-1} (4) 25 ms^{-1}

Official Ans. by NTA (4)

Sol. Distance (s) = $(2.5)t^2$

Speed (v) = $\frac{ds}{dt} = \frac{d}{dt}\{(2.5)t^2\}$

$v = 5t$

At t = 5, $v = 5 \times 5 = 25$ m/s.

Option (4) is correct

41. An electron is moving along the positive x-axis. If the uniform magnetic field is applied parallel to the negative z-axis. then

A. The electron will experience magnetic force along positive y-axis

B. The electron will experience magnetic force along negative y-axis

C. The electron will not experience any force in magnetic field

D. The electron will continue to move along the positive x-axis

E. The electron will move along circular path in magnetic field

Choose the correct answer from the options given below :

Option :

(1) B and E only (2) A and E only

(3) C and D only (4) B and D only

Official Ans. by NTA (1)

Sol. $\vec{F} = -e(\vec{v} \times \vec{B})$

Force will be along –ve y-axis.

As magnetic force is $\perp$ to velocity, path of electron must be a circle.

42. Two planets A and B of radii R and 1.5 R have densities ρ and $\rho/2$ respectively. The ratio of acceleration due to gravity at the surface of B to A is :

Option :

(1) 2 : 3 (2) 2 : 1

(3) 3 : 4 (4) 4 : 3

Official Ans. by NTA (3)

Sol. $g = \frac{GM}{R^2} = \frac{4}{3}\pi G\rho R$

$\therefore \frac{g_2}{g_1} = \frac{\rho_2}{\rho_1} \times \frac{R_2}{R_1} = \frac{1}{2} \times 1.5 = \frac{3}{4}$

43. Given below are two statements:

Statement I : An AC circuit undergoes electrical resonance if it contains either a capacitor or an inductor.

Statement II : An AC circuit containing a pure capacitor or a pure inductor consumes high power due to its non-zero power factor.

In the light of above statements, choose the ***correct*** answer from the options given below :

Option :

(1) Both Statement I and Statement II are false

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are true

(4) Statement I is false but Statement II is true

Official Ans. by NTA (1)

Sol. For resonance, $\phi = 0$, hence both inductor & capacitor must be present. Also power factor is zero for pure inductor or pure capacitor hence both the component consume zero average power.

44. A vehicle of mass 200 kg is moving along a levelled curved road of radius 70 m with angular velocity of 0.2 rad/s. The centripetal force acting on the vehicle is :

Option :

(1) 560 N (2) 2800 N

(3) 14 N (4) 2240 N

Official Ans. by NTA (1)

Sol. $F_c = m\omega^2 r = 200 \times (0.2)^2 \times 70 = 560$ N

45. To radiate EM signal of wavelength λ with high efficiency, the antennas should have a minimum size equal to :

Option :

(1) $\frac{\lambda}{2}$ (2) $\frac{\lambda}{4}$

(3) 2λ (4) λ

Official Ans. by NTA (2)

Sol. Minimum length of antenna should be $\frac{\lambda}{4}$.

46. Given below are two statements:

Statement I : Out of microwaves, infrared rays and ultraviolet rays, ultraviolet rays are the most effective for the emission of electrons from a metallic surface.

Statement II : Above the threshold frequency, the maximum kinetic energy of photoelectrons is inversely proportional to the frequency of the incident light.

In the light of above statements, choose the ***correct*** answer from the options given below

Option :

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are true

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are false

Official Ans. by NTA (1)

Sol. UV rays have maximum frequency hence are most effective for emission of electrons from a metallic surface.

$$KE_{max.} = hf - hf_0$$

47. A 10 μC charge is divided into two parts and placed at 1 cm distance so that the repulsive force between them is maximum. The charges of the two parts are :

Option :

(1) 9 μC, 1 μC (2) 5 μC, 5 μC

(3) 7 μC, 3 μC (4) 8 μC, 2 μC

Official Ans. by NTA (2)

Sol. Divide q = 10 μC into two parts x & q – x.

$$F = \frac{Kx(q-x)}{r^2}$$

For F to be maximum

$$\frac{dF}{dx} = \frac{K}{r^2}(q - 2x) = 0$$

$$x = \frac{q}{2}$$

48. In the equation $\left[X + \frac{a}{Y^2}\right][Y - b] = RT$, X is pressure, Y is volume, R is universal gas constant and T is temperature. The physical quantity equivalent to the ratio $\frac{a}{b}$ is :

Option :

(1) Energy

(2) Impulse

(3) Pressure gradient

(4) Coefficient of viscosity

Official Ans. by NTA (1)

Sol. X and $\frac{a}{Y^2}$ have same dimensions

Y and b have same dimensions

$\therefore [a] = [ML^5T^{-2}]$

$[b] = [L^3]$

$\frac{[a]}{[b]} = [ML^2T^{-2}]$ has dimensions of energy

49. In a Young's double slits experiment, the ratio of amplitude of light coming from slits is 2:1. The ratio of the maximum to minimum intensity in the interference pattern is :

Option :

(1) 9 : 4 (2) 9 : 1

(3) 2 : 1 (4) 25 : 9

Official Ans. by NTA (2)

Sol. Given that $\frac{A_1}{A_2} = \frac{2}{1}$

$$\frac{I_{max}}{I_{min}} = \left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2 = \frac{9}{1} = 9:1$$

50. The mean free path of molecules of a certain gas at STP is 1500d, where d is the diameter of the gas molecules. While maintaining the standard pressure, the mean free path of the molecules at 373K is approximately :

Option :

(1) 1098d (2) 2049d

(3) 750d (4) 1500d

Official Ans. by NTA (2)

Sol. Mean free path

$$\lambda = \frac{RT}{\sqrt{2}\pi d^2 N_A P}$$

$\lambda \propto T$

$$\frac{1500d}{\lambda} = \frac{273}{373}$$

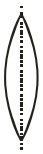
$\lambda = 2049d$

SECTION-B

51. A bi convex lens of focal length 10 cm is cut in two identical parts along a plane perpendicular to the principal axis. The power of each lens after cut is _____ D.

Official Ans. by NTA (5)

Sol.



Let power of each part is P_1, then

$$P_1 + P_1 = P = \frac{1}{f}$$

$$2P_1 = \frac{1}{0.1} = 10$$

$$P_1 = 5D$$

52. An atom absorbs a photon of wavelength 500 nm and emits another photon of wavelength 600 nm. The net energy absorbed by the atom in this process is $n \times 10^{-4}$eV. The value of n is _____.

[Assume the atom to be stationary during the absorption and emission process]

(Take $h = 6.6 \times 10^{-34}$ Js and $c = 3 \times 10^8$ m/s).

Official Ans. by NTA (4125)

Sol. $E = E_1 - E_2 = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2} = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$

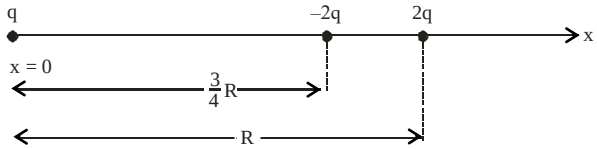
$$= 6.6 \times 10^{-34} \times 3 \times 10^8 \left(\frac{1}{500 \times 10^{-9}} - \frac{1}{600 \times 10^{-9}}\right)$$

$$= 6.6 \times 10^{-20} \text{ J}$$

$$= \frac{6.6 \times 10^{-20}}{1.6 \times 10^{-19}} \text{eV} = 4.125 \times 10^{-1} \text{ eV}$$

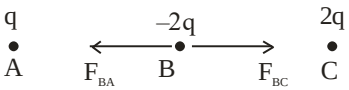
$$= 4125 \times 10^{-4} \text{ eV}$$

53. Three point charges q, –2q and 2q are placed on x-axis at a distance $x = 0$, $x = \frac{3}{4}R$ and $x = R$ respectively from origin as shown. If $q = 2 \times 10^{-6}$ C and R = 2 cm, the magnitude of net force experienced by the charge –2q is _____ N.



Official Ans. by NTA (5440)

Sol.



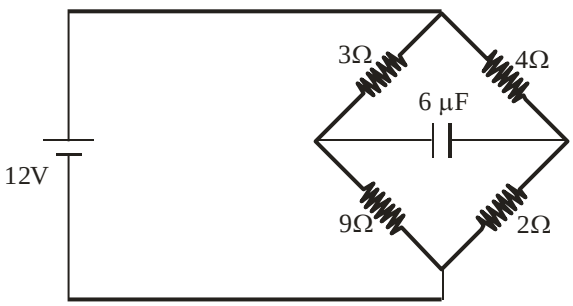
$$F_{BA} = \frac{Kq(2q)}{\left(\frac{3}{4}R\right)^2} = \frac{32Kq^2}{9R^2}$$

$$F_{BC} = \frac{K(2q)(2q)}{\left(\frac{R}{4}\right)^2} = \frac{64Kq^2}{R^2}$$

$$F_B = F_{BC} - F_{BA} = \frac{544Kq^2}{9R^2}$$

$$= \frac{544 \times 9 \times 10^9 \times (2 \times 10^{-6})^2}{9 \times (2 \times 10^{-2})^2} = 5440 \text{ N}$$

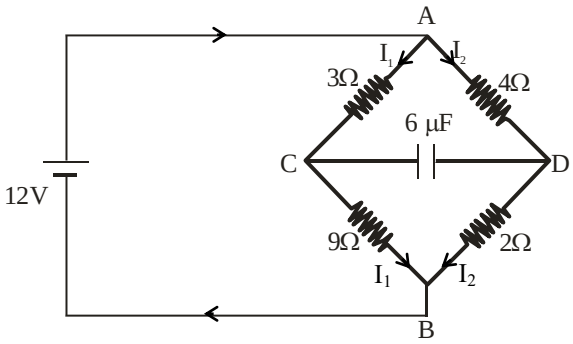
54. In the circuit shown, the energy stored in the capacitor is n μJ. The value of n is _____.



Official Ans. by NTA (75)

Sol. $I_1 = \dfrac{12}{3+9} = 1A$

$I_2 = \dfrac{12}{4+2} = 2A$



$V_A - V_C = 3I_1 = 3V$ ....(1)

$V_A - V_D = 2 \times 4 = 8V$ ....(2)

Subtracting eq. (1) from eq. (2)

$V_C - V_D = 5V \Rightarrow V = 5V$

$U = \dfrac{1}{2}CV^2 = \dfrac{1}{2} \times 6 \times 5^2 = 75\ \mu J$

55. An insulated copper wire of 100 turns is wrapped around a wooden cylindrical core of the cross-sectional area 24 cm^2. The two ends of the wire are connected to a resistor. The total resistance in the circuit is 12Ω. If an externally applied uniform magnetic field in the core along its axis changes from 1.5 T in one direction to 1.5 T in the opposite direction, the charge flowing through a point in the circuit during the change of magnetic field will be _____ mC.

Official Ans. by NTA (60)

Sol. $\Delta Q = -\dfrac{\Delta\phi}{R} = -\left(\dfrac{\phi_2 - \phi_1}{R}\right)$

$\phi_1 = NBA$

$\phi_2 = -NBA$

$\therefore \Delta Q = \dfrac{2NBA}{R} = \dfrac{2 \times 100 \times 1.5 \times 24 \times 10^{-4}}{12}$

$= 6 \times 10^{-2}\ C = 60\ mC$

56. In an experiment with sonometer when a mass of 180 g is attached to the string, it vibrates with fundamental frequency of 30 Hz. When a mass m is attached, the string vibrates with fundamental frequency of 50 Hz. The value of m is _____g.

Official Ans. by NTA (500)

Sol. $f = \dfrac{1}{2\ell}\sqrt{\dfrac{T}{\mu}}$ (T : Tension)

$\dfrac{f_2}{f_1} = \sqrt{\dfrac{T_2}{T_1}}$

$\left(\dfrac{50}{30}\right)^2 = \dfrac{mg}{180g} \Rightarrow m = \dfrac{25}{9} \times 180 = 500\ gram$

57. A light rope is wound around a hollow cylinder of mass 5 kg and radius 70 cm. The rope is pulled with a force of 52.5 N. The angular acceleration of the cylinder will be _____ rad s^{-2}.

Official Ans. by NTA (15)

Sol. $\tau = I\alpha$

$\Rightarrow FR = mR^2\alpha$

$\alpha = \dfrac{F}{mR} = \dfrac{52.5}{5 \times 0.7} = 15\ rad\ s^{-2}$

58. A car accelerates from rest to u m/s. The energy spent in this process is E J. The energy required to accelerate the car from u m/s to 2u m/s is nE J. The value of n is _____.

Official Ans. by NTA (3)

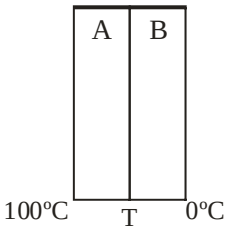
Sol. $E_1 = \dfrac{1}{2}mu^2 - 0 = \dfrac{1}{2}mu^2 = E$

$E_2 = \dfrac{1}{2}m(2u)^2 - \dfrac{1}{2}mu^2 = \dfrac{3}{2}mu^2 = 3E$

59. Two plates A and B have thermal conductivities 84 $Wm^{-1}K^{-1}$ and 126 $Wm^{-1}K^{-1}$ respectively. They have same surface area and same thickness. They are placed in contact along their surfaces. If the temperatures of the outer surfaces of A and B are kept at 100 °C and 0 ºC respectively, then the temperature of the surface of contact in steady state is _____°C.

Official Ans. by NTA (40)

Sol.



Let the temperature of contact surface is T, then

$$H_A = H_B$$

$$\frac{K_A A(T_A - T)}{L} = \frac{K_B A(T - T_B)}{L}$$

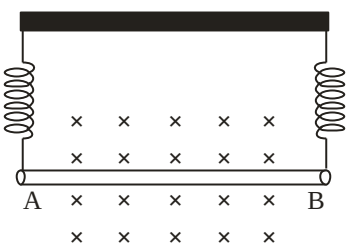
$84(100 - T) = 126\,(T - 0)$

$2(100 - T) = 3T$

$200 - 2T = 3T$

$T = 40ºC$

60. A straight wire AB of mass 40 g and length 50 cm is suspended by a pair of flexible leads in uniform magnetic field of magnitude 0.40 T as shown in the figure. The magnitude of the current required in the wire to remove the tension in the supporting leads is _____A. (Take g = 10 ms^{-2}).



Official Ans. by NTA (2)

Sol. For equilibrium

$Mg = I\ell B$

$$I = \frac{mg}{\ell B} = \frac{40 \times 10^{-3} \times 10}{50 \times 10^{-2} \times 0.4} = 2A$$

SECTION-A

61. In the wet tests for detection of various cations by precipitation, Ba^{2+} cations are detected by obtaining precipitate of

(1) Ba(ox) : Barium oxalate

(2) $BaCO_3$

(3) $Ba(OAc)_2$

(4) $BaSO_4$

Official Ans. by NTA (2)

Sol. In wet testing, $(NH_4)_2CO_3$ is used as group reagent for 5th group cations (Ba^{2+}, Ca^{2+}, Sr^{2+})

$$Ba^{+2} + (NH_4)_2CO_3 \rightarrow \underset{\text{(white precipitate)}}{BaCO_3 \downarrow} + NH_4^{\oplus}$$

62. The naturally occurring amino acid that contains only one basic functional group in its chemical structure is

(1) arginine

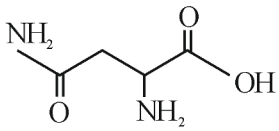
(2) lysine

(3) asparagine

(4) histidine

Official Ans. by NTA (3)

Sol. Asparagine has only one basic functional group in its chemical structure.



Others are basic amino acid with more than one basic functional group.

63. Given below are two statements related to Ellingham diagram:

Statement-I : Ellingham diagrams can be constructed for formation of oxides, sulfides and halides of metals.

Statement-II : It consists of plots of $\Delta_f H^0$ vs T for formation of oxides of elements.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both Statement I and Statement II are incorrect

(2) Statement I is incorrect but Statement II is. correct

(3) Both Statement I and Statement II are correct

(4) Statement I is correct but Statement II is incorrect

Official Ans. by NTA (1)

Sol. Statement I is correct, Ellingham diagram can be constructed for formation of oxides, sulphides and halides of metals. (Ref: NCERT)

Statement II is incorrect because Ellingham diagram consists of $\Delta_f G^0$ vs T for formation of oxides of elements.

64. Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : The diameter of colloidal particles in solution should not be much smaller than wavelength of light to show Tyndall effect.

Reason R : The light scatters in all directions when the size of particles is large enough.

In the light of the above statements, choose the correct answer from the options given below:

(1) A is true but R is false

(2) A is false but R is true

(3) Both A and R are correct and R is the correct explanation of A

(4) Both A and R are correct but R is NOT the correct explanation of A

Official Ans. by NTA (3)

Sol. Tyndall effect is observed only when the following two conditions are satisfied

(a) The diameter of the dispersed particle is not much smaller than the wave length of light used.

(b) Refractive indices of dispersed phase and dispersion medium differ greatly in magnitude.

65. The total number of stereoisomers for the complex $[Cr(ox)_2\ ClBr]^{3-}$ (where ox = oxalate) is:

(1) 2

(2) 3

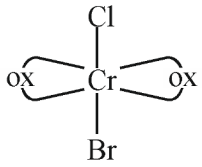
(3) 1

(4) 4

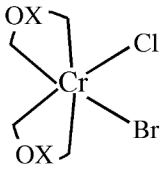
Official Ans. by NTA (2)

Sol. $[Cr(Ox)_2\ ClBr]^{-3}$

- No. of isomers –



- This structure has plane of symmetry, So no optical isomerism will be shown.



- This structure does not contain plane of symmetry, So two forms d as well as l will be shown.

66. Better method for preparation of BeF_2, among the following is

(1) $(NH_4)_2BeF_4 \xrightarrow{\Delta} BeF_2$

(2) $BeH_2 + F_2 \xrightarrow{\Delta} BeF_2$

(3) $Be + F_2 \xrightarrow{\Delta} BeF_2$

(4) $BeO + C + F_2 \xrightarrow{\Delta} BeF_2$

Official Ans. by NTA (1)

Sol. As per NCERT (s block), the better method of preparation of BeF_2 is heating $(NH_4)_2BeF_4$

$(NH_4)_2BeF_4 \xrightarrow{\Delta} BeF_2 + NH_4F$

67. Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : Isotopes of hydrogen have almost same chemical properties, but difference in their rates of reaction.

Reason R : Isotopes of hydrogen have different enthalpy of bond dissociation.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both A and R are correct but R is NOT the correct explanation of A

(2) Both A and R are correct and R is the correct explanation of A

(3) A is not correct but R is correct

(4) A is correct but R is not correct

Official Ans. by NTA (2)

Sol. Source NCERT

Since the isotopes have the same electronic configuration, they have almost same chemical properties. The only difference is in their rates of reactions, mainly due to their different enthalpy of bond dissociation.

68. Given below are two statements:

Statement I: Tropolone is an aromatic compound and has 8π electrons.

Statement II: π electrons of >C = O group in tropolone is involved in aromaticity.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are true

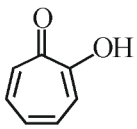
(2) Statement I is true but Statement II is false

(3) Statement I is false but Statement II is true

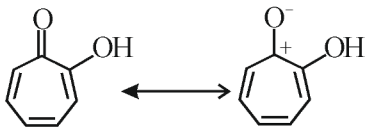
(4) Both Statement I and Statement II are false

Official Ans. by NTA (2)

Sol.

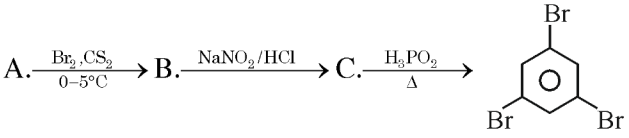


Tropolone is an aromatic compound and has 8π electrons ($6\pi e^-$ are endocyclic and $2\pi e^-$ are exocyclic) and π electrons of >C = O group in tropolone is not involved in aromaticity.



aromatic compound ($6\pi e^-$)

69. Compound A from the following reaction sequence is:



(1) Benzoic Acid

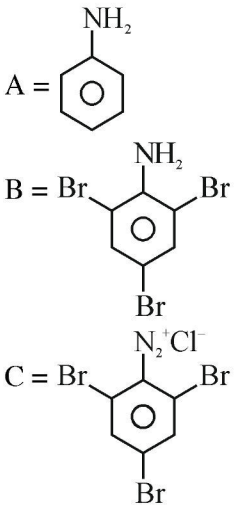
(2) Phenol

(3) Salicylic Acid

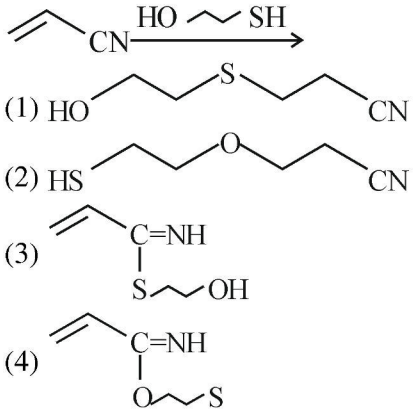
(4) Aniline

Official Ans. by NTA (4)

Sol.

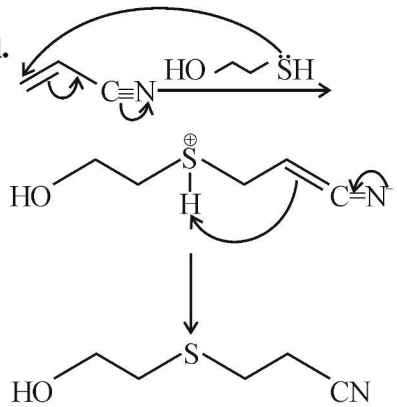


70. The major product for the following reaction is:



Official Ans. by NTA (1)

Sol.



71. Which of the following are the Green house gases?

A. Water vapour

B. Ozone

C. I_2

D. Molecular hydrogen

Choose the most appropriate answer from the options given

(1) B and C only

(2) C and D only

(3) A and D only

(4) A and B only

Official Ans. by NTA (4)

Sol. Green house gases are CO_2, CH_4, water vapour, nitrous oxide, CFC_s and ozone.

72. Match List I with List II

	LIST I		**LIST II**
A.	Weak intermolecular forces of attraction	I.	Hexamethylenediamine + adipic acid
B.	Hydrogen bonding	II.	$AlEt_3 + TiCl_4$
C.	Heavily branched polymer	III.	2–chloro–1, 3-butadiene
D.	High density polymer	IV.	Phenol + formaldehyde

Choose the correct answer from the options given below:

(1) A-II, B-IV, C-I, D-III

(2) A-III, B-I, C-IV, D-II

(3) A-IV, B-I, C-III, D-II

(4) A-IV, B-II, C-III, D-I

Official Ans. by NTA (2)

Sol.

- Hexamethylenediamine on reaction with adipic acid forms Nylon 6, 6 which shows H-bonding due to presence of amide group.
- $AlEt_3 + TiCl_4$ is Ziegler-Natta catalyst used to prepare high density polyethylene.
- 2-chloro-1, 3-butadiene (chloroprene) is monomer of neoprene which is a rubber (an elastomer)
- Phenol – formaldehyde forms Bakelite which is heavily branched (cross-linked) polymer

73. Given below are two statements :

Statement I: SO_2 and H_2O both possess V-shaped structure.

Statement II: The bond angle of SO_2 is less than that of H_2O.

In the light of the above statements, choose the most appropriate answer from the options given below:

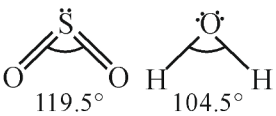
(1) Both Statement I and Statement II are correct

(2) Statement I is correct but Statement II is incorrect

(3) Both Statement I and Statement II are incorrect

(4) Statement I is incorrect but Statement II is correct

Official Ans. by NTA (2)

Sol. 

Both are bent in shape.

Bond angle of SO_2 (sp^2) is greater than that of H_2O (sp^3) due to higher repulsion of multiple bonds.

74. The correct group of halide ions which can be oxidised by oxygen in acidic medium is

(1) Br^- only

(2) Cl^-, Br^- and I^- only

(3) Br^- and I^- only

(4) I^- only

Official Ans. by NTA (4)

Sol. Only I^- among halides can be oxidised to Iodine by oxygen in acidic medium

$$4I^-(aq) + 4H^+(aq) + O_2(g) \rightarrow 2I_2(s) + 2H_2O(l)$$

75. What happens when methane undergoes combustion in systems A and B respectively?

Adiabatic system	Diathermic container
System A	System B

(1)

System A	System B
Temperature rises	Temperature remains same

(2)

System A	System B
Temperature falls	Temperature rises

(3)

System A	System B
Temperature falls	Temperature remains same

(4)

System A	System B
Temperature remains same	Temperature rises

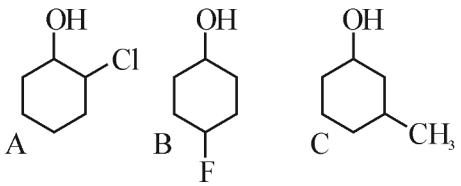
Official Ans. by NTA (1)

Sol. Adiabatic boundary does not allow heat exchange thus heat generated in container can't escape out thereby increasing the temperature.

In case of Diathermic container, heat flow can occur to maintain the constant temperature.

76. Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : Order of acidic nature of the following compounds is A> B > C.



Reason R : Fluoro is a stronger electron withdrawing group than Chloro group.

In the light of the above statements, choose the correct answer from the options given below:

(1) A is false but R is true

(2) Both A and R are correct and R is the correct explanation of A

(3) Both A and R are correct but R is NOT the correct explanation of A

(4) A is true but R is false

Official Ans. by NTA (3)

Sol. Acidic strength α – I effect

$$\alpha \frac{1}{+I} \text{ effect}$$

F, Cl exerts –I effect, Methyl exerts +I effect, C is least acidic.

Among A and B; since inductive effect is distance dependent, Extent of –I effect is higher in A followed by B even though F is stronger electron withdrawing group than Cl. Thus, A is more acidic than B.

77. Identify the correct order of standard enthalpy of formation of sodium halides.

(1) NaI < NaBr < NaCl < NaF

(2) NaF < NaCl < NaBr < NaI

(3) NaCl < NaF < NaBr < NaI

(4) NaI < NaBr < NaF < NaCl

Official Ans. by NTA (1)

Sol. For a given metal $\Delta_f H^0$ always becomes less negative from fluoride to iodide.

78. Match List I with List II

1 - Bromopropane is reacted with reagents in List I to give product in List II

	LIST I - Reagent		**LIST II - Product**
A.	KOH (alc)	I.	Nitrile
B.	KCN (alc)	II.	Ester
C.	$AgNO_2$	III.	Alkene
D.	$H_3CCOOAg$	IV.	Nitroalkane

(1) A-IV, B-III, C-II, D-I

(2) A-III, B-I, C-IV, D-II

(3) A-I, B-II, C-III, D-IV

(4) A-I, B-III, C-IV, D-II

Official Ans. by NTA (2)

Sol.

$$CH_3-CH_2-CH_2-Br + KOH(Alc) \rightarrow \underset{(Alkene)}{CH_3-CH=CH_2}$$

$$CH_3-CH_2-CH_2-Br + KCN(Alc) \rightarrow CH_3-\underset{(Nitrile)}{CH_2}-CH_2-CN$$

$$CH_3-CH_2-CH_2-Br + AgNO_2 \rightarrow CH_3-\underset{(Nitroalkane)}{CH_2}-CH_2-NO_2 + AgBr\downarrow$$

$$CH_3-CH_2-CH_2-Br + CH_3-COOAg \rightarrow CH_3-COO-\underset{(Ester)}{CH_2}-CH_2-CH_3 + AgBr\downarrow$$

79. The covalency and oxidation state respectively of boron in $[BF_4]^-$, are

(1) 4 and 3

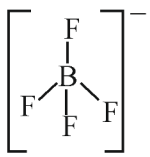
(2) 4 and 4

(3) 3 and 4

(4) 3 and 5

Official Ans. by NTA (1)

Sol.



Number of covalent bond formed by Boron is 4

Oxidation number of fluorine is –1,

Oxidation number of B + 4 × (-1) = –1,

Thus, Oxidation number of B = +3

80. Which of the following complexes will exhibit maximum attraction to an applied magnetic field?

(1) $[Zn(H_2O)_6]^{2+}$

(2) $[Co(H_2O)_6]^{2+}$

(3) $[Co(en)_3]^{3+}$

(4) $[Ni(H_2O_6]^{2+}$

Official Ans. by NTA (2)

Sol. Complex with maximum number of unpaired electron will exhibit maximum attraction to an applied magnetic field

$[Zn(H_2O)_6]^{2+} \rightarrow d^{10}$ system $\rightarrow t_{2g}^6\ eg^4$,0 unpaired e^-

$[Co(H_2O)_6]^{2+} \rightarrow d^7$ system $\rightarrow t_{2g}^5\ eg^2$, 3 unpaired e^-

$[Co(en)_3]^{3+} \rightarrow d^6$ system $\rightarrow t_{2g}^6\ eg^0$, 0 unpaired e^-

$[Ni(H_2O_6]^{2+} \rightarrow d^8$ system $\rightarrow t_{2g}^6\ eg^2$, 2 unpaired e^-

SECTION-B

81. 0.400 g of an organic compound (X) gave 0.376 g of AgBr in Carius method for estimation of bromine. % of bromine in the compound (X) is_____.

(Given: Molar mass AgBr = 188 g mol^{-1} Br = 80 g mol^{-1})

Official Ans. by NTA (40)

Sol. mole of $AgBr = \frac{0.376}{188}$

mole of Br^- = mole of $AgBr = \frac{0.376}{188}$

mass of $Br^- = \frac{0.376}{188} \times 80$

$\%$ of $Br^- = \frac{0.376 \times 80}{188 \times 0.4} \times 100 = 40\%$

82. 1g of a carbonate (M_2CO_3) on treatment with excess HCl produces 0.01 mol of CO_2 The molar mass of M_2CO_3 is _____ g mol^{-1}. (Nearest integer)

Official Ans. by NTA (100)

Sol. $\underset{1gm}{M_2CO_3} + \underset{Excess}{2HCl} \rightarrow \underset{0.02\ mole}{2MCl} + H_2O + \underset{0.01\ mole}{CO_2}$

From principle of atomic conservation of carbon atom,

Mole of $M_2CO_3 \times 1$ = Mole of $CO_2 \times 1$

$$\frac{1gm}{\text{molar mass of } M_2CO_3} = 0.01 \times 1$$

$\therefore$ Molar mass of M_2CO_3 = 100 gm/mole

83. See the following chemical reaction:

$$Cr_2O_7^{2-} + XH^+ + 6Fe^{2+} \rightarrow YCr^{3+} + 6Fe^{3+} + ZH_2O$$

The sum of X. Y and Z is _____.

Official Ans. by NTA (23)

Sol.

$$Cr_2O_7^{2-} + 14H^+ + 6Fe^{2+} \rightarrow 6Fe^{3+} + 2Cr^{3+} + 7H_2O$$

x = 14

y = 2

z = 7

Hence (x + y + z) = 14 + 2 + 7 = 23

84. If the formula of Borax is $Na_2B_4O_x(OH)_y \cdot zH_2O$, then x + y + z = _____.

Official Ans. by NTA (17)

Sol. Formula of borax is $Na_2B_4O_5(OH)_4 \cdot 8H_2O$

85. At 298 K, the standard reduction potential for Cu^{2+} / Cu electrode is 0.34 V.

Given : K_{sp} $Cu(OH)_2 = 1 \times 10^{-20}$

$$\text{Take } \frac{2.303RT}{F} = 0.059V$$

The reduction potential at pH = 14 for the above couple is $(-)x \times 10^{-2}$ V. The value of x is _____.

Official Ans. by NTA (25)

Sol. $Cu(OH)_2(s) \rightleftharpoons Cu^{2+}(aq) + 2OH^-(aq)$

$Ksp = [Cu^{2+}][OH^-]^2$

pH = 14 ; pOH = 0 ; $[OH^-] = 1M$

$$\therefore \quad [Cu^{2+}] = \frac{Ksp}{[1]^2} = 10^{-20}M$$

$Cu^{2+}(aq) + 2e^- \rightarrow Cu(s)$

$$E = E^\circ - \frac{0.059}{2}\log_{10}\frac{1}{[Cu^{2+}]}$$

$$= 0.34 - \frac{0.059}{2}\log_{10}\frac{1}{10^{-20}}$$

$$= -0.25 = -25 \times 10^{-2}$$

86. 20 mL of 0.1 M NaOH is added to 50 mL of 0.1 M acetic acid solution. The pH of the resulting solution is _____ $\times 10^{-2}$ (Nearest integer)

Given : pKa (CH_3COOH) = 4.76

log 2 = 0.30

log 3 = 0.48

Official Ans. by NTA (448)

Sol. $CH_3COOH + NaOH \rightarrow CH_3COONa + H_2O$

Initially	5mmol	2mmol	0	0
after Rxn	3mmol	0	2 mmole	2 mmole

$$pH = pKa + \log_{10}\frac{[salt]}{[acid]}$$

$$pH = 4.76 + \log_{10}\frac{2}{3}$$

$$pH = 4.58 = 458 \times 10^{-2}$$

87. $A(g) \rightarrow 2B(g) + C(g)$ is a first order reaction. The initial pressure of the system was found to be 800 mm Hg which increased to 1600 mm Hg after 10 min. The total pressure of the system after 30 min will be _____ mm Hg. (Nearest integer)

Official Ans. by NTA (2200)

Sol. $t_{\frac{1}{2}}$ = 10 minutes

$$(P_A)_{30min} = (P_A)_0\left(\frac{1}{2}\right)^{30/10}$$

$$(P_A)_{30min} = 100 \text{ mm Hg}$$

	A(g) →	2B(g)	+	C(g)
at t = 0	800mm	0		0
at t = 30	100mm	1400mm		700mm

Total pressure after 30 minutes = 2200 mm Hg

88. The orbital angular momentum of an electron in 3s orbital is $\frac{xh}{2\pi}$. The value of .x is

Official Ans. by NTA (0)

Sol. Orbital angular momentum = $\sqrt{l(l+1)}\frac{h}{2\pi}$

Value of l for s = 0

89. Sodium metal crystallizes in a body centred cubic lattice with unit cell edge length of 4 Å. The radius of sodium atom is _____ $\times 10^{-1}$ Å (Nearest integer)

Official Ans. by NTA (17)

Sol. $\sqrt{3}a = 4r$

$\sqrt{3} \times 4 = 4r$

$r = 1.732Å$

$= 17.32 \times 10^{-1}$

90. Sea water contains 29.25% NaCl and 19% $MgCl_2$ by weight of solution. The normal boiling point of the sea water is _____°C (Nearest integer)

Assume 100% ionization for both NaCl and $MgCl_2$

Given : $K_b(H_2O) = 0.52$ K kg mol^{-1}

Molar mass of NaCl and $MgCl_2$ is 58.5 and 95 g mol^{-1} respectively.

Official Ans. by NTA (116)

Sol.

Amount of solvent = 100 – (29.25 + 19) = 51.75g

$$\Delta T_b = \left[\frac{2 \times 29.25 \times 1000}{58.5 \times 51.75} + \frac{3 \times 19 \times 1000}{95 \times 51.75}\right] \times 0.52$$

$\Delta Tb = 16.075$

$\Delta Tb = (T_b)_{solution} - (T_b)_{solvent}$

$(T_b)_{solution} = 100 + 16.07$

$= 116.07°C$

FINAL JEE–MAIN EXAMINATION – APRIL, 2023

(Held On Saturday 15th April, 2023) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. The total number of three-digit numbers, divisible by 3, which can be formed using the digits 1, 3, 5, 8, if repetition of digits is allowed, is:

(1) 22 (2) 18

(3) 21 (4) 20

Official Ans. by NTA (1)

Sol. (1,1,1) (3,3,3) (5,5,5) (8,8,8)

(5,5,8) (8,8,5) (1,3,5) (1,3,8)

Total number = $1+1+1+1+\frac{3!}{2!}+\frac{3!}{2!}+3!+3! = 22$

2. Let S be the set of all values of λ, for which the shortest distance between the lines $\frac{x-\lambda}{0}=\frac{y-3}{4}=\frac{z+6}{1}$ and $\frac{x+\lambda}{3}=\frac{y}{-4}=\frac{z-6}{0}$ is 13.

Then $8\left|\sum_{\lambda\in S}\lambda\right|$ is equal to

(1) 304 (2) 308

(3) 306 (4) 302

Official Ans. by NTA (3)

Sol. Shor test distance = $\frac{\left|\begin{vmatrix}0&4&1\\3&-4&0\\2\lambda&3&-12\end{vmatrix}\right|}{\left|\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\0&4&1\\3&-4&0\end{vmatrix}\right|}$

$13=\frac{|153+8\lambda|}{|4\hat{i}+3\hat{j}-12\hat{k}|}$

$=\frac{|153+8\lambda|}{13}$

$|153 + 8\lambda| = 169$

$153 + 8\lambda = 169, -169$

$\lambda=\frac{16}{8},\frac{-322}{8}$

$8\left|\sum_{\lambda\in S}\lambda\right|=306$

3. The mean and standard deviation of 10 observations are 20 and 8 respectively. Later on, it was observed that one observation was recorded as 50 instead of 40. Then the correct variance is:

(1) 14 (2) 13

(3) 12 (4) 11

Official Ans. by NTA (2)

Sol. $\mu = 20, \sigma = 8$

$\mu_{Corrected}=\frac{200-50+40}{10}=19$

$\sigma^2=\frac{1}{10}\sum x_i^2-20^2$

$(64 + 400)\,10 = \sum x_i^2$

$\sigma^2_{Corrected}=\frac{1}{10}\left[(64+400)10-2500+1600\right]-19^2$

$= 374 - 361$

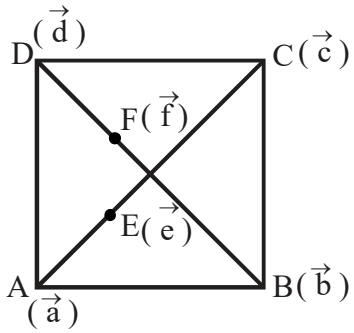
$= 13$

4. Let ABCD be a quadrilateral. If E and F are the mid points of the diagonals AC and BD respectively and $\left(\overrightarrow{AB}-\overrightarrow{BC}\right)+\left(\overrightarrow{AD}-\overrightarrow{DC}\right)=k\overrightarrow{FE}$, then k is equal to

(1) 2 (2) –2

(3) –4 (4) 4

Official Ans. by NTA (3)



Sol.

$\overrightarrow{AB}-\overrightarrow{BC}+\overrightarrow{AB}-\overrightarrow{DC}=k\overrightarrow{FE}$

$\left(\vec{b}-\vec{a}\right)-\left(\vec{c}-\vec{b}\right)+\left(\vec{d}-\vec{a}\right)-\left(\vec{c}-\vec{d}\right)=k\overrightarrow{FE}$

$2\left(\vec{b}+\vec{d}\right)-2\left(\vec{a}-\vec{c}\right)=k\overrightarrow{FE}$

$2\left(2f - 2(2e)\right) = kFE$

$4\left(\vec{f} - \vec{e}\right) = k\overrightarrow{FE}$

$-4\overrightarrow{FE} = k\overrightarrow{FE}$

$k = -4$

5. Let $x = x(y)$ be the solution of the differential equation $2(y+2)\log_e(y+2)dx + (x + 4 - 2\log_e(y+2))dy = 0$, $y > -1$ with $x(e^4 - 2) = 1$. Then $x(e^9 - 2)$ is equal to

(1) $\frac{4}{9}$ (2) $\frac{10}{3}$

(3) $\frac{32}{9}$ (4) 3

Official Ans. by NTA (3)

Sol. $2(y+2)\ln(y+2)dx + (x + 4 - 2\ln(y+2))dy = 0$

$2\ln(y+2) + (x + 4 - 2\ln(y+2))\frac{1}{y+2}.\frac{dy}{dx} = 0$

let, $\ln(y+2) = t$

$\frac{1}{y+2}.\frac{dy}{dx} = \frac{dt}{dx}$

$2t + (x + 4 - 2t).\frac{dt}{dx} = 0$

$(x + 4 - 2t)\frac{dt}{dx} = -2t$

$\frac{dx}{dt} = \frac{2t - 4 - x}{2t}$

$\frac{dx}{dt} + \frac{x}{2t} = \frac{2t-4}{2t}$

$x.t^{1/2} = \int \frac{2t-4}{2t}.t^{1/2}.dt$

$x.t^{1/2} = \int\left(t^{1/2} - \frac{2}{t^{1/2}}\right).dt$

$= \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - 2.\frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C$

$x.t^{\frac{1}{2}} = \frac{2t^{\frac{3}{2}}}{3} - 4t^{\frac{1}{2}} + C$

$x = \frac{2}{3}.t - 4 + C.t^{\frac{-1}{2}}$

$x = \frac{2}{3}\ln(y+2) - 4 + C.\left(\ln(y+2)\right)^{\frac{-1}{2}}$

Put $y = e^4 - 2$, $x = 1$

$1 = \frac{2}{3} \times 4 - 4 + C \times \frac{1}{2}$

$\frac{C}{2} = 5 - \frac{8}{3} = \frac{7}{3}$

$C = \frac{14}{3}$

$x = \frac{2}{3} \times 9 - 4 + \frac{14}{3} \times \frac{1}{3}$

$= 2 + \frac{14}{9}$

$= \frac{32}{9}$

6. Let $[x]$ denote the greatest integer function and $f(x) = \max\{1 + x + [x], 2 + x, x + 2[x]\}$, $0 \le x \le 2$. Let m be the number of points in $[0,2]$, where f is not continuous and n be the number of points in $(0,2)$, where f is not differentiable. Then $(m+n)^2 + 2$ is equal to:

(1) 11 (2) 2

(3) 6 (4) 3

Official Ans. by NTA (4)

Sol. $\text{Let } g(x) = 1 + x + [x] = \begin{cases} 1+x; & x \in [0,1) \\ 2+x; & x \in [1,2) \\ 5; & x = 2 \end{cases}$

$\lambda(x) = x + 2[x] = \begin{cases} x; & x \in [0,1) \\ x+2; & x \in [1,2) \\ 6; & x = 2 \end{cases}$

$r(x) = 2 + x$

$f(x) = \begin{cases} 2+x; & x \in [0,2) \\ 6; & x = 2 \end{cases}$

$f(x)$ is discontinuous only at $x = 2 \Rightarrow m = 1$

$f(x)$ is differentiable in $(0,2) \Rightarrow n = 0$

$(m+n)^2 + 2 = 3$

7. The number of real roots of the equation $x|x| - 5|x+2| + 6 = 0$, is

(1) 5 (2) 3

(3) 6 (4) 4

Official Ans. by NTA (2)

Sol. $x|x| - 5|x+2| + 6 = 0$

C–1 :- $x \in [0,\infty]$

$x^2 - 5x - 4 = 0$

$x = \frac{5 \pm \sqrt{25+16}}{2} = \frac{5+\sqrt{41}}{2}$

$x = \frac{5 \pm \sqrt{41}}{2}$

C–2 :- :- $x \in [-2, 0)$

$-x^2 - 5x - 4 = 0$

$x^2 + 5x + 4 = 0$

$x = -1, -4$

$x = -1$

C–3 : $x \in [-\infty, -2)$

$-x^2 + 5x + 16 = 0$

$x^2 - 5x - 16 = 0$

$x = \frac{5 \pm \sqrt{25+64}}{2}$

$\frac{5 \pm \sqrt{89}}{2}$

$x = \frac{5-\sqrt{89}}{2}$

8. Let $(a + bx + cx^2)^{10} = \sum_{i=0}^{20} p_i x^i$, $a, b, c \in \mathbb{N}$. If $p_1 = 20$ and $p_2 = 210$, then $2(a + b + c)$ is equal to

(1) 8 (2) 12

(3) 15 (4) 6

Official Ans. by NTA (2)

Sol. $(a + bx + cx^2)^{10} = \sum_{i=0}^{20} p_i x^i$

Coefficient of $x^1 = 20$

$20 = \frac{10!}{9!1!} \times a^9 \times b^1$

$a^9 . b = 2$

$a = 1, b = 2$

Coefficient of $x^2 = 210$

$210 = \frac{10!}{9!1!} \times a^9 \times c^1 + \frac{10!}{8!2!} \times a^8 b^2$

$210 = 10.c + 45 \times 4$

$10c = 30$

$c = 3$

$2(a + b = c) = 12$

9. Let the determinant of a square matrix A of order m be m – n, where m and n satisfy 4m + n = 22 and 17m + 4n = 93. If det (n adj(adj(mA))) = $3^a 5^b 6^c$. then a + b + c is equal to:

(1) 96 (2) 101

(3) 109 (4) 84

Official Ans. by NTA (1)

Sol. $|A| = m - n$

$4m + n = 22$

$17m + 4n = 93$

$m = 5, n = 2$

$|A| = 3$

$|2 \text{ adj } (\text{adj } 5A))| = 2^5 |5A|^{16}$

$= 2^5 . 5^{80} |A|^{16}$

$= 2^5 . 5^{80} . 3^{16}$

$= 3^{11} . 5^{80} . 6^5$

$a + b + c = 96$

10. Let A_1 and A_2 be two arithmetic means and G_1, G_2, G_3 be three geometric means of two distinct positive numbers. The $G_1^4 + G_2^4 + G_3^4 + G_1^2 G_3^2$ is equal to

(1) $2(A_1 + A_2)G_1 G_3$

(2) $(A_1 + A_2)^2 G_1 G_3$

(3) $(A_1 + A_2)G_1^2 G_3^2$

(4) $2(A_1 + A_2)G_1^2 G_3^2$

Official Ans. by NTA (2)

Sol. a, A_1, A_2, b are in A.P.

$d = \frac{b-a}{3}$; $A_1 = a + \frac{b-a}{3} = \frac{2a+b}{3}$

$A_2 = \frac{a+2b}{3}$

$A_1 + A_2 = a + b$

a, G_1, G_2, G_3, b are in G.P.

$r = \left(\frac{b}{a}\right)^{\frac{1}{4}}$

$G_1 = \left(a^3 b\right)^{\frac{1}{4}}$

$G_2 = \left(a^2 b^2\right)^{\frac{1}{4}}$

$G_3 = \left(a b^3\right)^{\frac{1}{4}}$

$G_1^4 + G_2^4 + G_3^4 + G_1^2 G_3^2 =$

$a^3 b + a^2 b^2 + ab^3 + \left(a^3 b\right)^{\frac{1}{2}} . \left(ab^3\right)^{\frac{1}{2}}$

$= a^3 b + a^2 b^2 + ab^3 + a^2 . b^2$

$= ab(a^2 + 2ab + b^2)$

$= ab(a + b)^2$

$= G_1 . G_3 . (A_1 + A_2)^2$

11. If the set $\left\{\mathrm{Re}\left(\frac{z-\bar{z}+z\bar{z}}{2-3z+5\bar{z}}\right): z \in \mathbb{C}, \mathrm{Re}(z)=3\right\}$ is equal to the interval $(\alpha,\beta]$, then $24(\beta-\alpha)$ is equal to

(1) 36 (2) 42

(3) 27 (4) 30

Official Ans. by NTA (4)

Sol. Let $z_1 = \left(\frac{z-\bar{z}+z\bar{z}}{2-3z+5\bar{z}}\right)$

Let $z = 3+iy$

$\bar{z} = 3-iy$

$$z_1 = \frac{2iy+\left(9+y^2\right)}{2-3(3+iy)+5(3-iy)}$$

$$= \frac{9+y^2+i(2y)}{8-8iy}$$

$$= \frac{\left(9+y^2\right)+i(2y)}{8(1-iy)}$$

$$\mathrm{Re}(z_1) = \frac{\left(9+y^2\right)-2y^2}{8\left(1+y^2\right)}$$

$$= \frac{9-y^2}{8\left(1+y^2\right)}$$

$$= \frac{1}{8}\left[\frac{10-\left(1+y^2\right)}{\left(1+y^2\right)}\right]$$

$$= \frac{1}{8}\left[\frac{10}{1+y^2}-1\right]$$

$1 + y^2 \in [1, \infty]$

$$\frac{1}{1+y^2} \in (0,1]$$

$$\frac{10}{1+y^2} \in (0,10]$$

$$\frac{10}{1+y^2}-1 \in (-1,9]$$

$$\mathrm{Re}(z_1) \in \left(\frac{-1}{8},\frac{9}{8}\right]$$

$$\alpha = \frac{-1}{8}, \beta = \frac{9}{8}$$

$$24(\beta-\alpha) = 24\left(\frac{9}{8}+\frac{1}{8}\right) = 30$$

12. The number of common tangents, to the circles $x^2 + y^2 - 18x - 15y + 131 = 0$ and $x^2 + y^2 - 6x - 6y - 7 = 0$, is :

(1) 3 (2) 2

(3) 1 (4) 4

Official Ans. by NTA (1)

Sol. $C_1\left(9,\frac{15}{2}\right)$ $r_1 = \sqrt{81+\frac{225}{4}-131} = \frac{5}{2}$

$C_2\ (3,3)$ $r_2 = 5$

$$C_1C_2 = \sqrt{6^2+\frac{81}{4}} = \frac{15}{2}$$

$$r_1 + r_2 = \frac{15}{2}$$

$C_1C_2 = r_1 + r_2$

Number of common tangents = 3

13. Negation of $p \wedge (q \wedge \sim(p \wedge q))$ is

(1) $\sim(p \vee q)$ (2) $p \vee q$

(3) $(\sim(p \wedge q)) \wedge q$ (4) $(\sim(p \wedge q)) \vee p$

Official Ans. by NTA (4)

Sol. $\sim\left[p \wedge \left(q \wedge \sim(p \wedge q)\right)\right]$

$\sim p \vee \left(\sim q \vee (p \wedge q)\right)$

$\sim p \vee \left((\sim q \vee p) \wedge (\sim q \vee q)\right)$

$\sim p \vee (\sim q \vee p)$

$\sim(p \wedge q) \vee p$

14. Let the system of linear equations

$-x + 2y - 9z = 7$

$-x + 3y + 7z = 9$

$-2x + y + 5z = 8$

$-3x + y + 13z = \lambda$

has a unique solution $x = \alpha$, $y = \beta$, $z = \gamma$. Then the distance of the point (α, β, γ) from the plane $2x - 2y + z = \lambda$ is

(1) 9 (2) 11

(3) 13 (4) 7

Official Ans. by NTA (4)

Sol. $-x + 2y - 9z = 7$ –(1)

$-x + 3y - 7z = 9$ –(2)

$-2x + y + 5z = 8$ –(3)

(2) – (1)

$y + 16z = 2$ (4)

(3) – 2 × (1)

$-3y + 23z = -6 - (5)$

$3 \times (4) + (5)$

$71z = 0 \Rightarrow z = 0$

$y = 2$

$x = -3$

$(-3, 2, 0) \rightarrow (\alpha, \beta, \gamma)$

Put in $-3x + y + 13z = \lambda$

$\lambda = 9 + 2 = 11$

$$d = \left|\frac{-6-4-11}{3}\right| = 7$$

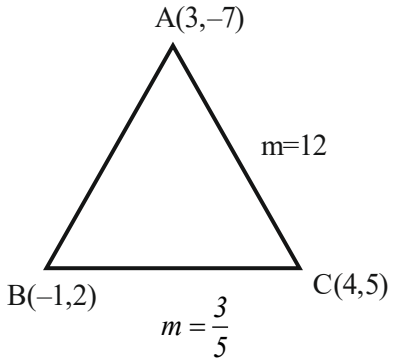
15. If (α, β) is the orthocentre of the triangle ABC with vertices A(3, –7), B(–1, 2) and C(4, 5), then $9\alpha - 6\beta + 60$ is equal to:

(1) 30 (2) 25

(3) 40 (4) 35

Official Ans. by NTA (2)

Sol.



Altitude of BC: $y + 7 = \frac{-5}{3}(x-3)$

$3y + 21 = -5x + 15$

$5x + 3y + 6 = 0$

Altitude of AC: $y - 2 = \frac{-1}{12}(x + 1)$

$12y - 24 = -x - 1$

$x + 12y = 23$

$$\alpha = \frac{-47}{19}, \quad \beta = \frac{121}{57}$$

$9\alpha - 6\beta + 60 = 25$

16. Let the foot of perpendicular of the point P(3, –2, –9) on the plane passing through the points (–1, –2, –3), (9, 3, 4), (9, –2, 1) be Q (α, β, γ). Then the distance of Q from the origin is:

(1) $\sqrt{29}$ (2) $\sqrt{35}$

(3) $\sqrt{42}$ (4) $\sqrt{38}$

Official Ans. by NTA (3)

Sol. P(3, –2, –9)

Equation of plane through A,B,C.

$$\begin{vmatrix} x+1 & y+2 & z+3 \\ 10 & 5 & 7 \\ 10 & 0 & 4 \end{vmatrix} = 0$$

$2x + 3y - 5z - 7 = 0$

Foot of I^r of P (3, –2, –9) is

$$\frac{x-3}{2} = \frac{y+2}{3} = \frac{z+9}{-5} = -\frac{(6-6+45-7)}{4+9+25}$$

$$\frac{x-3}{2} = \frac{y+2}{3} = \frac{z+9}{-5} = -1$$

$Q(1, -5, -4) \equiv (\alpha, \beta, \gamma)$

$OQ = \sqrt{\alpha^2 + \beta^2 + \gamma^2} = \sqrt{42}$

17. A bag contains 6 white and 4 black balls. A die is rolled once and the number of balls equal to the number obtained on the die are drawn from the bag at random. The probability that all the balls drawn are white is:

(1) $\frac{1}{4}$ (2) $\frac{9}{50}$

(3) $\frac{1}{5}$ (4) $\frac{11}{50}$

Official Ans. by NTA (3)

Sol.

6	W
4	R

$$\frac{1}{6} \times \left[\frac{{}^6C_1}{{}^{10}C_1} + \frac{{}^6C_2}{{}^{10}C_2} + \frac{{}^6C_3}{{}^{10}C_3} + \frac{{}^6C_4}{{}^{10}C_4} + \frac{{}^6C_5}{{}^{10}C_5} + \frac{{}^6C_6}{{}^{10}C_6} \right]$$

$$= \frac{1}{6}\left(\frac{126+70+35+15+5+1}{210}\right) = \frac{42}{210} = \frac{1}{5}$$

18. If

$$\int_0^1 \frac{1}{\left(5+2x-2x^2\right)\left(1+e^{(2-4x)}\right)}dx = \frac{1}{\alpha}\log_e\left(\frac{\alpha+1}{\beta}\right),$$

$\alpha, \beta > 0$, then $\alpha^4 - \beta^4$ is equal to:

(1) 21 (2) 0

(3) 19 (4) –21

Official Ans. by NTA (1)

Sol. $$I = \int_0^1 \frac{dx}{(5+2x-2x^2)(1+e^{2-4x})} \quad \ldots(i)$$

$x \to 1 - x$

$$I = \int_0^1 \frac{e^{2-4x}dx}{\left(5+2x-2x^2\right)\left(1+e^{2-4x}\right)} \quad \ldots(ii)$$

Add (i) and (ii)

$$2I = \int_0^1 \frac{dx}{5+2x-2x^2} = \int_0^1 \frac{dx}{2\left(\frac{11}{4}-\left(x-\frac{1}{2}\right)^2\right)}$$

$$I = \frac{1}{\sqrt{11}}\ln\left(\frac{\sqrt{11}+1}{\sqrt{10}}\right) \qquad \alpha = \sqrt{11}, \quad \beta = \sqrt{10}$$

$$\alpha^4 - \beta^4 = 121 - 100 = 21$$

19. Let S be the set of all (λ, μ) for which the vectors $\lambda\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} + \mu\hat{k}$ and $3\hat{i} - 4\hat{j} + 5\hat{k}$, where $\lambda - \mu = 5$, are coplanar, then $\sum_{(\lambda,\mu)\in S} 80(\lambda^2+\mu^2)$ is equal to :

(1) 2370 (2) 2130

(3) 2290 (4) 2210

Official Ans. by NTA (3)

Sol. $$\begin{vmatrix} \lambda & -1 & 1 \\ 1 & 2 & \mu \\ 3 & -4 & 5 \end{vmatrix} = 0 \qquad \& \ \lambda - \mu = 5$$

$\lambda(10 + 4\mu) + (5 - 3\mu) + (-10) = 0$

$(\mu + 5)(4\mu + 10) + 5 - 3\mu - 10 = 0$

$\mu = -15; \ \lambda = 5/4$

$\mu = -3; \ \lambda = 2$

Hence $\sum_{(\lambda,\mu)\in S} 80(\lambda^2+\mu^2)$

$$= 80\left(\frac{250}{16}+13\right)$$

$= 1250 + 1040$

$= 2290$

20. If the domain of the function

$f(x) = \log_e\left(4x^2+11x+6\right) + \sin^{-1}$

$(4x+3) + \cos^{-1}\left(\frac{10x+6}{3}\right)$ is $(\alpha, \beta]$,

Then $36 | \alpha + \beta|$ is equal to :

(1) 63 (2) 45

(3) 72 (4) 54

Official Ans. by NTA (2)

Sol. $f(x) = \ln(4x^2 + 11x + 6) + \sin^{-1}(4x + 3)$

$$+\cos^{-1}\left(\frac{10x+6}{3}\right)$$

(i) $4x^2 + 11x + 6 > 0$

$4x^2 + 8x + 3x + 6 > 0$

$(4x + 3)(x + 2) > 0$

$x \in (-\infty, -2) \cup \left(-\frac{3}{4}, \infty\right)$

(ii) $4x + 3 \in [-1, 1]$

$x \in [-1, -1/2]$

$$(iii)\ \frac{10x+6}{3} \in [-1,1]$$

$$x \in \left[-\frac{9}{10}, -\frac{3}{10}\right]$$

$$x \in \left(-\frac{3}{4}, -\frac{1}{2}\right] \qquad \alpha = -\frac{3}{4}, \beta = -\frac{1}{2}$$

$$\alpha + \beta = -\frac{5}{4}$$

$36 |\alpha + \beta| = 45$

SECTION-B

21. If the sum of the series

$$\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{2^2}-\frac{1}{2.3}+\frac{1}{3^2}\right)+$$

$$\left(\frac{1}{2^3}-\frac{1}{2^2.3}+\frac{1}{2.3^2}-\frac{1}{3^3}\right)+$$

$$\left(\frac{1}{2^4}-\frac{1}{2^3.3}+\frac{1}{2^2.3^2}-\frac{1}{2.3^3}+\frac{1}{3^4}\right)+\ldots. \text{ is } \frac{\alpha}{\beta}, \text{ where}$$

α and β are co-prime, then $\alpha + 3\beta$ is equal to….

Official Ans. by NTA (7)

Sol. $P=\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{2^2}-\frac{1}{2.3}+\frac{1}{3^2}\right)+$

$\left(\frac{1}{2^3}-\frac{1}{2^2.3}+\frac{1}{2.3^2}-\frac{1}{3^3}\right)+.....$

$P\left(\frac{1}{2}+\frac{1}{3}\right)=\left(\frac{1}{2^2}-\frac{1}{3^2}\right)+\left(\frac{1}{2^3}+\frac{1}{3^3}\right)+\left(\frac{1}{2^4}-\frac{1}{3^4}\right)+...$

$\frac{5P}{6}=\frac{\frac{1}{4}}{1-\frac{1}{2}}-\frac{\frac{1}{9}}{1+\frac{1}{3}}$

$\frac{5P}{6}=\frac{1}{2}-\frac{1}{12}=\frac{5}{12}$

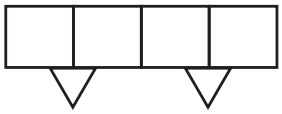
$\therefore P=\frac{1}{2}=\frac{\alpha}{\beta}$ $\quad\quad\quad$ $\therefore\ \alpha = 1, \beta = 2$

$\alpha + 3\beta = 7$

22. A person forgets his 4-digit ATM pin code. But he remembers that in the code all the digits are different, the greatest digit is 7 and the sum of the first two digits is equal to the sum of the last two digits. Then the maximum number of trials necessary to obtain the correct code is____

Official Ans. by NTA (72)

Sol.



Sum of first two digits

Sum of last two digits = α

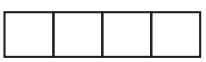
Case-I : $\alpha = 7$

$2 \times 12 = 24$ ways.

7 0	
0 7	1 6
	2 5
	3 4
	4 3
	5 2
	6 1

Case-II : $\alpha = 8$

17	26
71	62
	35
	53

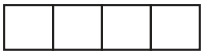


2×8 ways

$= 16$ ways

Case-III : $\alpha = 9$

27	36
72	63
	45
	54



2×8 ways

$= 16$ ways

Case-IV : $\alpha = 10$

37	46
73	64

2×4 ways

8 ways

Case-V : $\alpha = 11$

47	56
74	65

2×4 ways

$= 8$ ways

Ans. $24 + 16 + 16 + 8 + 8 = 72$

23. Let the plane P contain the line $2x + y - z - 3 = 0 = 5x - 3y + 4z + 9$ and be parallel to the line $\frac{x+2}{2}=\frac{3-y}{-4}=\frac{z-7}{5}$. Then the distance of the point A(8, –1, –19) from the plane P measured parallel to the line $\frac{x}{-3}=\frac{y-5}{4}=\frac{2-z}{-12}$ is equal to____

Official Ans. by NTA (26)

Sol. Plane $P \equiv P_1 + \lambda P_2 = 0$

$(2x + y - z - 3) + \lambda(5x - 3y) + 4z + 9) = 0$

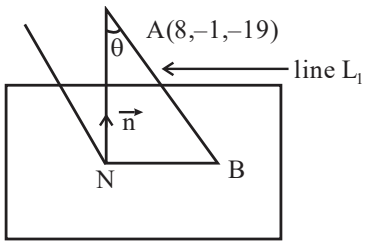
$(5\lambda + 2)x + (1 - 3\lambda)y + (4\lambda - 1)z + 9\lambda - 3 = 0$

$\vec{n}.\vec{b} = 0$ where $\vec{b}$ (2, 4, 5)

$2(5\lambda + 2) + 4(1 - 3\lambda) + 5(4\lambda - 1) = 0$

$\lambda = -\frac{1}{6}$

Plane $7x + 9y - 10z - 27 = 0$



Equation of line AB is

$$\frac{x-8}{-3}=\frac{y+1}{4}=\frac{z+19}{12}=\lambda$$

Let B = (8–3λ,–1+4λ, –19 + 12λ) lies on plane P

$\therefore$ 7 (8 – 3λ) + 9 (4λ – 1) – 10 (12λ – 19) = 27

λ = 2

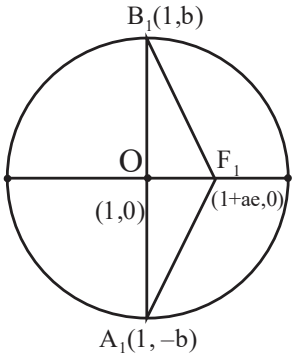
$\therefore$ Point B = (2, 7, 5)

$$AB=\sqrt{6^2+8^2+24^2}=26$$

24. Let an ellipse with centre (1, 0) and latus rectum of length $\frac{1}{2}$ have its major axis along x-axis. If its minor axis subtends an angle 60° at the foci, then the square of the sum of the lengths of its minor and major axes is equal to___

Official Ans. by NTA (9)

Sol.



$$\text{L.R.}=\frac{2b^2}{a}=\frac{1}{2}$$

$4b^2 = a$ …(i)

Ellipse $\frac{(x-1)^2}{a^2}+\frac{y^2}{b^2}=1$

$$m_{B_2F_1}=\frac{1}{\sqrt{3}}$$

$$\frac{b}{ae}=\frac{1}{\sqrt{3}}$$

$3b^2 = a^2e^2 = a^2 – b^2$

$4b^2 = a^2$ …(ii)

From (i) and (ii)

$a = a^2$

$\therefore$ a = 1

$$b^2=\frac{1}{4}$$

$((2a) + (2b))^2 = 9$

25. Let A = {1, 2, 3, 4} and R be a relation on the set A × A defined by $R=\{((a,b),(c,d)):2a+3b=4c+5d\}$. Then the number of elements in R is:

Official Ans. by NTA (6)

Sol. A = {1, 2, 3, 4}

R = {(a, b), (c, d)}

2a + 3b = 4c + 5d = α let

2a = {2, 4, 6, 8} 4c = {4, 8, 12, 16}

3b = {3, 6, 9, 12} 5d = {5, 10, 15, 20}

$$2a+3b=\begin{Bmatrix}5,8,11,14\\7,10,13,16\\9,12,15,18\\11,14,17,20\end{Bmatrix}\quad 4c+5d\begin{Bmatrix}9,14,19,24\\13,18....\\17,22....\\21,26....\end{Bmatrix}$$

Possible value of α = 9, 13, 14, 14, 17, 18

Pairs of {(a, b), (c, d)} = 6

26. The number of elements in the set

$\{n\in\mathbb{N}:10\le n\le 100$ and $3^n - 3$ is a multiple of 7$\}$ is____

Official Ans. by NTA (15)

Sol. n∈[10, 100]

$3^n – 3$ is multiple of 7

$3^n = 7\lambda + 3$

n = 1, 7, 13, 20,……97

Number of possible values of n = 15

27. If the line x = y = z intersects the line x sin A + y sin B + z sin C – 18 = 0 = x sin 2A + y sin 2B + z sin 2C – 9, where A, B, C are the angles of a triangle ABC, then $80\left(\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right)$ is equal to_____

Official Ans. by NTA (5)

Sol. $\sin A + \sin B + \sin C = \frac{18}{x}$

$\text{Sin}2A + \sin 2B + \sin 2C = \frac{9}{x}$

$\therefore \sin A + \sin B + \sin C = 2(\sin 2A + \sin 2B + \sin 2C)$

$4\cos A/2 \cos B/2 \cos C/2 = 2(4\sin A \sin B \sin C)$

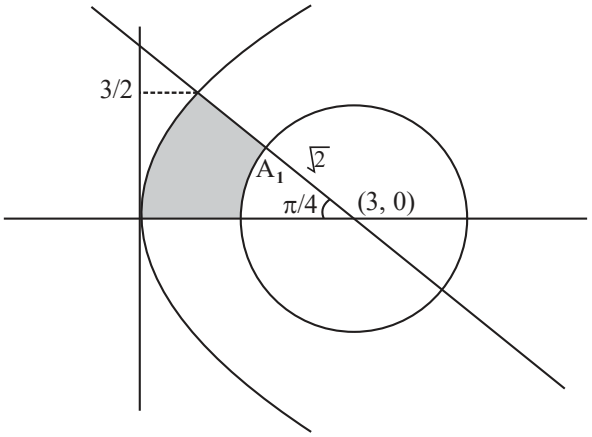
$16\sin A/2 \sin B/2 \sin C/2 = 1$

Hence Ans. = 5.

28. If the area bounded by the curve $2y^2 = 3x$, lines $x + y = 3$, $y = 0$ and outside the circle $(x-3)^2 + y^2 = 2$ is A, then $4(\pi + 4A)$ is equal to___

Official Ans. by NTA (42)

Sol.



$y^2 = \frac{3x}{2}$, $x + y = 3$, $y = 0$

$2y^2 = 3(3 - y)$

$2y^2 + 3y - 9 = 0$

$2y^2 - 3y + 6y - 9 = 0$

$(2y - 3)(y + 2) = 0$; $y = 3/2$

$$\text{Area}\left(\int_0^{\frac{3}{2}}(x_R - x_2)dy\right) - A_1$$

$$= \int_0^{\frac{3}{2}}\left((3-y) - \frac{2y^2}{3}\right)dy - \frac{\pi}{8}(2)$$

$$A = \left(3y - \frac{y^2}{2} - \frac{2y^3}{9}\right)_0^{\frac{3}{2}} - \frac{\pi}{4}$$

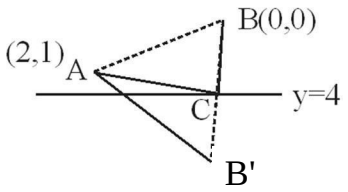
$$4A + \pi = 4\left[\frac{9}{2} - \frac{9}{8} - \frac{3}{4}\right] = \frac{21}{2} = 10.50$$

$\therefore 4(4A + \pi) = 42$

29. Consider the triangles with vertices A(2, 1) B (0, 0) and C (t, 4), $t \in [0, 4]$. It the maximum and the minimum perimeters of such triangles are obtained at $t = \alpha$ and $t = \beta$ respectively, then $6\alpha + 21\beta$ is equal to____

Official Ans. by NTA (48)

Sol. A (2,1), B (0,0), C (t, 4) : $t \in [0,4]$



$B_1(0,8) \equiv$ image of B w.r.t. $y = 4$

for AC+BC+AB to be minimum.

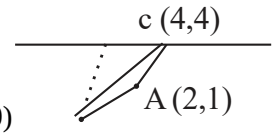
$$m_{AB'} = \frac{-7}{2}$$

line $AB_1 \equiv 7x + 2y = 16$

$$C\left(\frac{8}{7}, 4\right)$$

$$\beta = \frac{8}{7}$$

For max. perimeter



B(0, 0) $\alpha = 4$

AB = $\sqrt{5}$: BC = $4\sqrt{2}$, AC = $\sqrt{13}$

$6\alpha + 21\beta = 24 + 24 = 48$

30. Let $f(x)=\int\frac{dx}{\left(3+4x^2\right)\sqrt{4-3x^2}},|x|<\frac{2}{\sqrt{3}}$. If f(0) = 0 and $f(1)=\frac{1}{\alpha\beta}\tan^{-1}\left(\frac{\alpha}{\beta}\right)$, α, β > 0, then $\alpha^2+\beta^2$ is equal to____

Official Ans. by NTA (28)

Sol. $f(x)=\int\frac{dx}{(3+4x^2)\sqrt{4-3x^2}}$

$x=\frac{1}{t}$

$=\int\frac{\frac{-1}{t^2}dt}{\frac{(3t^2+4)}{t^2}\frac{\sqrt{4t^2-3}}{t}}$

$=\int\frac{-dt.t}{(3t^2+4)\sqrt{4t^2-3}}$: Put $4t^2-3=z^2$

$=-\frac{1}{4}\int\frac{z\ dz}{\left(3\left(\frac{z^2+3}{4}\right)+4\right)z}$

$=\int\frac{-\ dz}{3z^2+25}=-\frac{1}{3}\int\frac{dz}{z^2+\left(\frac{5}{\sqrt{3}}\right)^2}$

$=-\frac{1}{3}\frac{\sqrt{3}}{5}\tan^{-1}\left(\frac{\sqrt{3}z}{5}\right)+C$

$=-\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}}{5}\sqrt{4t^2-3}\right)+C$

$f(x)=-\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}}{5}\sqrt{\frac{4-3x^2}{x^2}}\right)+C$

$\because f(0)=0\ \because c=\frac{\pi}{10\sqrt{3}}$

$f(1)=-\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}}{5}\right)+\frac{\pi}{10\sqrt{3}}$

$f(1)=\frac{1}{5\sqrt{3}}\cot^{-1}\left(\frac{\sqrt{3}}{5}\right)=\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{5}{\sqrt{3}}\right)$

$\alpha=5:\beta=\sqrt{3}\ \therefore\ \alpha^2+\beta^2=28$

SECTION-A

31. The electric field due to a short electric dipole at a large distance (r) from center of dipole on the equatorial plane varies with distance as :

(1) r (2) $\frac{1}{r}$

(3) $\frac{1}{r^3}$ (4) $\frac{1}{r^2}$

Official Ans. by NTA (3)

Sol. Electric field due to a dipole at point on its axis

$$E = \frac{2kp}{r^3}$$

32. In a linear simple harmonic motion (SHM)

(A) Restoring force is directly proportional to the displacement.

(B) The acceleration and displacement are opposite in direction.

(C) The velocity is maximum at mean position.

(D) The acceleration is minimum at extreme points.

Choose the correct answer from the options given below :

(1) (A), (B) and (C) only

(2) (C) and (D) only

(3) (A), (B) and (D) only

(4) (A), (C) and (D) only

Official Ans. by NTA (1)

Sol.

$F = -kx$	A true
$a = -\omega^2 x$	B true
Velocity is maximum at mean position	C true
Acceleration is maximum at extreme points	D false

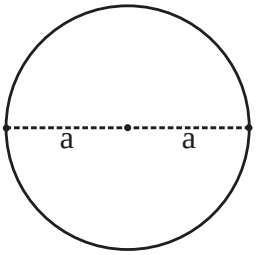
33. Two identical particles each of mass 'm' go round a circle of radius *a* under the action of their mutual gravitational attraction. The angular speed of each particle will be :

(1) $\sqrt{\frac{Gm}{2a^3}}$ (2) $\sqrt{\frac{Gm}{8a^3}}$

(3) $\sqrt{\frac{Gm}{4a^3}}$ (4) $\sqrt{\frac{Gm}{a^3}}$

Official Ans. by NTA (3)

Sol.



$$F = m\omega^2 r$$

$$\Rightarrow \frac{Gmm}{(2a)^2} = m\omega^2 a$$

$$\Rightarrow \omega = \sqrt{\frac{Gm}{4a^3}}$$

34. The height of transmitting antenna is 180 m and the height of the receiving antenna is 245 m. The maximum distance between them for satisfactory communication in line of sight will be :

(given R = 6400 km)

(1) 48 km (2) 56 km

(3) 96 km (4) 104 km

Official Ans. by NTA (4)

Sol. $d_{max} = \sqrt{2Rh_t} + \sqrt{2Rh_r}$

$$= \sqrt{2\times64\times10^5\times180} + \sqrt{2\times64\times10^5\times245}$$

$$= \{(8\times6\times10^3) + (8\times7\times10^3)\}\ m$$

$$= (48 + 56)\ km$$

$$= 104\ km$$

35. The half-life of a radioactive nucleus is 5 years, The fraction of the original sample that would decay in 15 years is :

(1) $\frac{1}{8}$ (2) $\frac{1}{4}$

(3) $\frac{7}{8}$ (4) $\frac{3}{4}$

Official Ans. by NTA (3)

Sol. 15 year = 3 half lives

Number of active nuclei = $\frac{N_0}{8}$

Number of decay = $\frac{7N_0}{8}$

36. The de Broglie wavelength of an electron having kinetic energy E is λ. If the kinetic energy of electron becomes $\frac{E}{4}$, then its de-Broglie wavelength will be :

(1) $\frac{\lambda}{\sqrt{2}}$ (2) $\frac{\lambda}{2}$

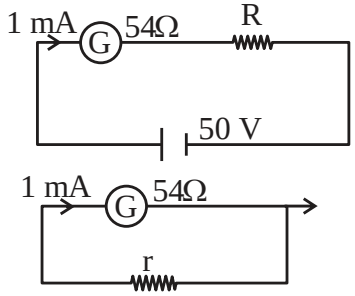
(3) 2λ (4) $\sqrt{2}\lambda$

Official Ans. by NTA (3)

Sol. $\lambda = \frac{h}{\sqrt{2mE}}$

$\lambda' = \frac{h}{\sqrt{2m\left(\frac{E}{4}\right)}} = \frac{2h}{\sqrt{2mE}} = 2\lambda$

37. For designing a voltmeter of range 50 V and an ammeter of range 10 mA using a galvanometer which has a coil of resistance 54 Ω showing a full scale deflection for 1 mA as in figure.



(A) for voltmeter R ≈ 50 kΩ

(B) for ammeter r ≈ 0.2 Ω

(C) for ammeter r ≈ 6 Ω

(D) for voltmeter R ≈ 5 kΩ

(E) for voltmeter R ≈ 500 Ω

Choose the correct answer from the options given below :

(1) (C) and (E) (2) (C) and (D)

(3) (A) and (C) (4) (A) and (B)

Official Ans. by NTA (3)

Sol. For voltmeter

$R = \frac{V}{I_g} - G$

$= \frac{50}{10^{-3}} - 54 \approx 50k\Omega$ (A)

For ammeter

$S = \frac{I_g G}{I - I_g} = \frac{10^{-3} \times 54}{(10-1)\times 10^{-3}} = 6\Omega$ (C)

38. (A flask contains Hydrogen and Argon in the ratio 2:1 by mass. The temperature of the mixture is 30°C. The ratio of average kinetic energy per molecule of the two gases (K argon/K hydrogen) is: (Given: Atomic Weight of Ar = 39.9)

(1) 1 (2) 2

(3) $\frac{39.9}{2}$ (4) 39.9

Official Ans. by NTA (1)

Sol. Average KE per molecule = $\frac{3}{2}kT$

$\frac{K_{Ar}}{K_H} = \frac{1}{1}$

39. Given below are two statements:

Statement I : The equivalent resistance of resistors in a series combination is smaller than least resistance used in the combination.

Statement II : The resistivity of the material is independent of temperature.

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is false but Statement II is true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are true

Official Ans. by NTA (2)

Sol. $R_{eq} = R_1 + R_2 + R_3$ So St–1 False

Resistivity depends on temperature. St–2 False

40. A body is released from a height equal to the radius (R) of the earth. The velocity of the body when it strikes the surface of the earth will be :

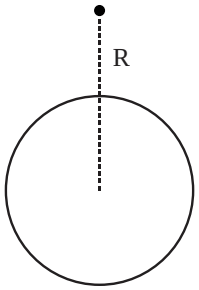
(Given g = acceleration due to gravity on the earth.)

(1) $\sqrt{gR}$ (2) $\sqrt{4gR}$

(3) $\sqrt{2gR}$ (4) $\sqrt{\frac{gR}{2}}$

Official Ans. by NTA (1)

Sol.



By conservation of mechanical energy

$U_i + K_i = U_f + K_i$

$$-\frac{GMm}{2R} + 0 = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$\frac{GMm}{2R} = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

41. A 12 V battery connected to a coil of resistance 6 Ω through a switch, drives a constant current in the circuit. The switch is opened in 1 ms. The emf induced across the coil is 20 V. The inductance of the coil is :

(1) 5 mH (2) 12 mH

(3) 8 mH (4) 10 mH

Official Ans. by NTA (D)

Sol. Induced emf $= -L\frac{dI}{dt}$

$$\Rightarrow 20 = -L\frac{(0-2)}{10^{-3}}$$

$\Rightarrow L = 10$ mH

42. A wire of length 'L' and radius 'r' is clamped rigidly at one end. When the other end of the wire is pulled by a force f, its length increases by 'ℓ'.

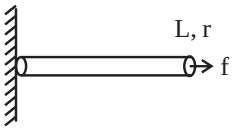
Another wire of same material of length '2L' and radius '2r' is pulled by a force '2f'. Then the increase in its length will be :

(1) 2ℓ (2) ℓ

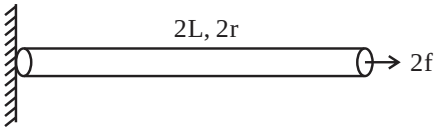
(3) 4ℓ (4) $\ell/2$

Official Ans. by NTA (2)

Sol.



$$\frac{f}{\pi r^2} = Y\frac{\ell}{L}$$



$$\frac{2f}{\pi(2r)^2} = Y\frac{\ell'}{2L}$$

$$\Rightarrow \frac{2}{1} = \frac{2\ell'}{\ell} \Rightarrow \ell' = \ell$$

43. The position of a particle related to time is given by $x = (5t^2 - 4t + 5)$m. The magnitude of velocity of the particle at t = 2s will be :

(1) 10 ms^{-1} (2) 14 ms^{-1}

(3) 16 ms^{-1} (4) 06 ms^{-1}

Official Ans. by NTA (3)

Sol. $x = 5t^2 - 4t + 5$

$v = 10t - 4$

At t = 2s v = 16m/s

44. The position vector of a particle related to time t is given by

$$\vec{r} = \left(10t\hat{i} + 15t^2\hat{j} + 7\hat{k}\right)m$$

The direction of net force experienced by the particle is :

(1) Positive y-axis (2) Positive x-axis

(3) Positive z-axis (4) In x-y plane

Official Ans. by NTA (1)

Sol. $\vec{r} = 10t\hat{i} + 15t^2\hat{j} + 7\hat{k}$

$\vec{v} = 10\hat{i} + 30t\,\hat{j}$

$\vec{a} = 30\hat{j}$

So Net force is along +y direction

45. Match List I with List II of Electromagnetic waves with corresponding wavelength range :

List I	**List II**
(A) Microwave	(I) 400 nm to 1 nm
(B) Ultraviolet	(II) 1 nm to 10^{-3} nm
(C) X-Ray	(III) 1 mn to 700 nm
(D) Infra-red	(IV) 0.1 m to 1mm

Choose the correct answer from the options given below :

(1) (A)-(I), (B)-(IV), (C)-(II), (D)-(III)

(2) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

(3) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

(4) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)

Official Ans. by NTA (2)

Sol. Increasing order of wave length

X-ray	1 nm to 10^{-3} nm
Ultra Violet	400 nm to 1 nm
Intra red	1 mm to 700 nm
Micro wave	0.1 m to 1mm

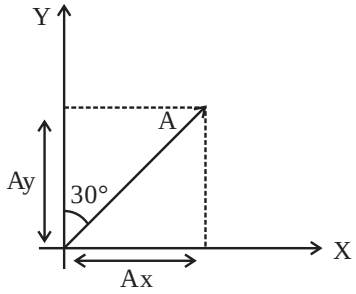
46. A vector in x-y plane makes an angle of 30° with y-axis The magnitude of y-component of vector is $2\sqrt{3}$. The magnitude of x-component of the vector will be :

(1) $\frac{1}{\sqrt{3}}$ (2) 6

(3) $\sqrt{3}$ (4) 2

Official Ans. by NTA (4)

Sol.



$A_y = A\cos 30° = 2\sqrt{3}$

$\Rightarrow A\frac{\sqrt{3}}{2} = 2\sqrt{3}$

$\Rightarrow A = 4$

Now $A_x = A\sin 30° = 4 \times \frac{1}{2} = 2$

47. The speed of a wave produced in water is given by $\upsilon = \lambda^a g^b \rho^c$. Where λ, g and ρ are wavelength of wave, acceleration due to gravity and density of water respectively. The values of a, b and c respectively, are :

(1) $\frac{1}{2}, \frac{1}{2}, 0$ (2) 1, 1, 0

(3) 1, –1, 0 (4) $\frac{1}{2}, 0, \frac{1}{2}$

Official Ans. by NTA (1)

Sol. $\nu = \lambda^a g^b \rho^c$

using dimension formula

$\Rightarrow [M^0L^1T^{-1}] = [L^1]^a [L^1T^{-2}]^b [M^1L^{-3}]^c$

$\Rightarrow [M^0L^1T^{-1}] = [M^c L^{a+b-3c} T^{-2b}]$

$\therefore c = 0, a + b - 3c = 1, -2b = -1 \Rightarrow b = \frac{1}{2}$

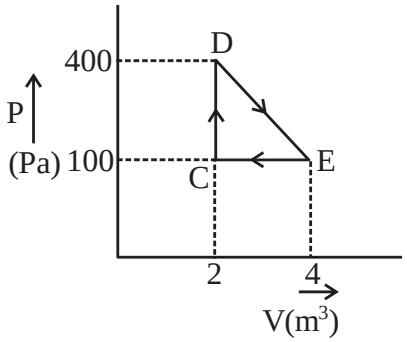
Now $a + b - 3c = 1$

$\Rightarrow a + \frac{1}{2} - 0 = 1$

$\Rightarrow a = \frac{1}{2}$

$\therefore a = \frac{1}{2}, b = \frac{1}{2}, c = 0$

48. A thermodynamic system is taken through cyclic process. The total work done in the process is :



(1) 100 J (2) 300 J

(3) Zero (4) 200 J

Official Ans. by NTA (2)

Sol. On P–V scale area of loop = work done

$\Rightarrow W = +\frac{1}{2}(2) \times 300$

$W = 300J$

49. A single slit of width *a* is illuminated by a monochromatic light of wavelength 600 nm. The value of 'a' for which first minimum appears at $\theta = 30°$ on the screen will be :

(1) 0.6 μ m (2) 1.2 μ m

(3) 1.8 μ m (4) 3 μ m

Official Ans. by NTA (2)

Sol. As for first minima

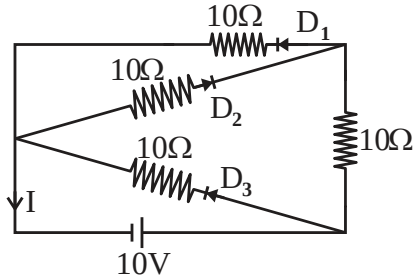
$a \sin\theta = \lambda$

$\Rightarrow a \sin 30° = 600 \times 10^{-9}$

$\Rightarrow a = 1200 \times 10^{-9}$ m

$\Rightarrow a = 1.2$ μm

50. In the given circuit, the current (I) through the battery will be :

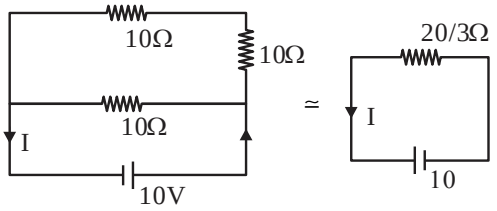


(1) 1.5 A (2) 1 A

(3) 2.5 A (4) 2 A

Official Ans. by NTA (1)

Sol. In the circuit D_1 and D_3 are forward biased and D_2 is reverse biased.



$\therefore I = \frac{10}{20/3} = \frac{3}{2} A = 1.5A$

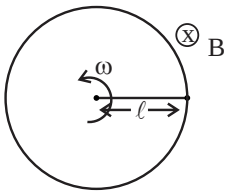
SECTION-B

51. A 20 cm long metallic rod is rotated with 210 rpm about an axis normal to the rod passing through its one end. The order end of the rod is in contact with a circular metallic ring. A constant and uniform magnetic field 0.2T parallel to the axis exists everywhere. The emf developed between the centre and the ring is _____ mV.

Take $\pi = \frac{22}{7}$

Official Ans. by NTA (88)

Sol.



Here $\omega = 210$ rpm

$= 210 \times \frac{2\pi}{60}$ rad/s

$\Rightarrow \omega = 7\pi$ rad/s

& $\ell = 0.2$m

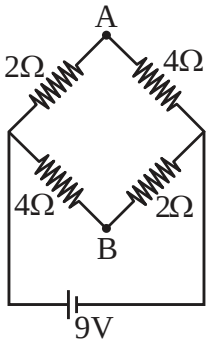
& $B = 0.2T$

emf developed across rod is $= \frac{1}{2} B\omega\ell^2$

$\frac{1}{2} \times 0.2 \times 7\pi \times (0.2)^2$

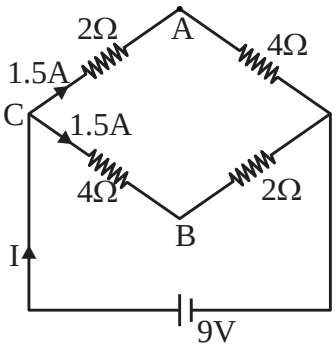
$= 88$ mV

52. A network of four resistances is connected to 9V battery, as shown in figure. The magnitude of voltage difference between the points A and B is _____V.



Official Ans. by NTA (3)

Sol.



In the circuit $I = \frac{9}{3} = 3A$

$V_C - V_A = 2 \times 1.5 = 3$ .....(I)

$V_C - V_B = 4 \times 1.5 = 6$ .....(I)

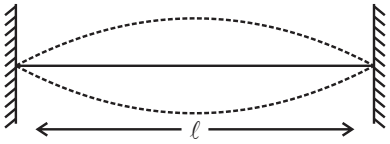
Eq^n (II) – Eq^n (I)

$V_A - V_B = 6 - 3 = 3$ Volt

53. The fundamental frequency of vibration of a string stretched between two rigid support is 50 Hz. The mass of the string is 18 g and its linear mass density is 20 g/m. The speed of the transverse waves so produced in the string is_____ ms^{-1}.

Official Ans. by NTA (90)

Sol.



Fundamental frequency = 50 Hz

mass/length = 20g/m

mass = 18g

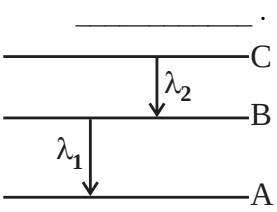
length of string = $\frac{18}{20}$m = $\frac{9}{10}$m

from diagram $\frac{\lambda}{2} = \ell$

$\Rightarrow \lambda = 2\ell = \frac{9}{5}$m

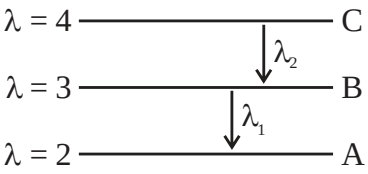
again speed $v = f\lambda = 50 \times \frac{9}{5} = 90$ m/s

54. As per given figure A, B and C are the first, second and third excited energy level of hydrogen atom respectively. If the ratio of the two wavelengths $\left(\text{i.e.}\frac{\lambda_1}{\lambda_2}\right)$ is $\frac{7}{4n}$, then the value of n will be _____.



Official Ans. by NTA (5)

Sol.



As $\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$\frac{1}{\lambda_1} = R(1)^2\left[\frac{1}{(2)^2} - \frac{1}{(3)^2}\right] = R\left(\frac{5}{36}\right)$ (i)

& $\frac{1}{\lambda_2} = R(1)^2\left[\frac{1}{(3)^2} - \frac{1}{(4)^2}\right] = R\left(\frac{7}{144}\right)$ (ii)

(ii) ÷ (i) gives

$\frac{\lambda_1}{\lambda_2} = \frac{7/144}{5/36} = \frac{7}{20} = \frac{7}{4\times5}$

$\therefore$ n = 5

55. A solid sphere and a solid cylinder of same mass and radius are rolling on a horizontal surface without slipping. The ratio of their radius of gyrations respectively (k_{sph} : k_{cyl}) is $2:\sqrt{x}$, then value of x is _____ .

Official Ans. by NTA (5)

Sol. For solid sphere $\frac{2}{5}mR^2 = mk_{sph}^2$

$$k_{sph} = \sqrt{\frac{2}{5}}R$$

For solid cylinder $\frac{mR^2}{2} = mk_{cyl}^2$

$$\Rightarrow k_{cyl} = \frac{R}{\sqrt{2}}$$

$$\frac{k_{sph}}{k_{cyl}} = \frac{\sqrt{\frac{2}{5}}}{\frac{1}{\sqrt{2}}} = \frac{2}{\sqrt{5}} = \frac{2}{\sqrt{x}}$$

$\therefore x = 5$

56. The refractive index of a transparent liquid filled in an equilateral hollow prism is $\sqrt{2}$. The angle of minimum deviation for the liquid will be _____°.

Official Ans. by NTA (30)

Sol. As $\mu = \frac{\sin\left(\frac{D_{min}+A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

$$\sqrt{2} = \frac{\sin\left(\frac{D_{min}+60}{2}\right)}{\sin\left(\frac{60}{2}\right)}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \sin\left(\frac{D_{min}+60}{2}\right)$$

$$\Rightarrow \frac{D_{min}+60}{2} = 45$$

$$\Rightarrow D_{min} = 30$$

57. An electron in a hydrogen atom revolves around its nucleus with a speed of 6.76×10^6 ms^{-1} in an orbit of radius 0.52 A°. The magnetic field produced at the nucleus of the hydrogen atom is _____T.

Official Ans. by NTA (40)

Sol. Magnetic field due to moving charge

$$B = \frac{\mu_0}{4\pi}\frac{q\,v\sin\theta}{r^2}$$

$$B = \frac{\mu_0}{4\pi}\frac{ev\sin(\pi/2)}{r^2}$$

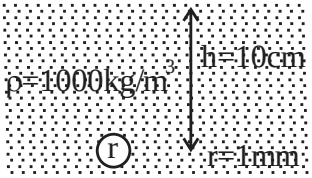
$$B = \frac{10^{-7}\times 1.6\times 10^{-19}\times 6.76\times 10^6}{0.52\times 0.52\times 10^{-20}}$$

B = 40 T

58. There is an air bubble of radius 1.0 mm in a liquid of surface tension 0.075 Nm^{-1} and density 1000 kg m^{-3} at a depth of 10 cm below the free surface. The amount by which the pressure inside the bubble is greater than the atmospheric pressure is_____ Pa ($g = 10\ ms^{-2}$)

Official Ans. by NTA (1150)

Sol.



Pressure inside the bubble

$$P = P_0 + h\rho g + \frac{2T}{r}$$

$$P - P_0 = h\rho g + \frac{2T}{r}$$

$$= 0.1\times 1000\times 10 + \frac{2\times .075}{10^{-3}}$$

= 1000 + (0.15) (1000)

= 1150 Pa

59. A block of mass 10 kg is moving along x-axis under the action of force F = 5x N. The work done by the force in moving the block from x = 2m to 4m will be _____J.

Official Ans. by NTA (30)

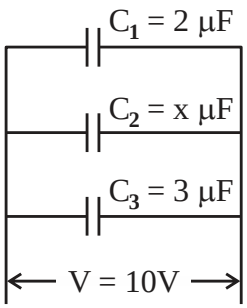
Sol. Work done = $\int F dx$

$$\int_2^4 5x dx = 5\left[\frac{x^2}{2}\right]_2^4$$

$$= \frac{5}{2}[16-4]$$

= 30 J

60. In the given figure the total charge stored in the combination of capacitors is 100 μC. The value of 'x' is_____ .



Official Ans. by NTA (5)

Sol. Charge on C_1 is $Q_1 = 2 \times 10 = 20\mu C$ (i)

Charge on C_2 is $Q_2 = x \times 10 = 10x\mu C$ (ii)

Charge on C_3 is $Q_3 = 3 \times 10 = 30\mu C$ (iii)

Total charge $20 + 10x + 30 = 100$

$\Rightarrow x = 5$

SECTION-A

61. Match List I with List II:

List I-(Monomer)	**List II-(Polymer)**
(A) Tetrafluoroethene	(i) Orlon
(B) Acrylonitrile	(ii) Natural rubber
(C) Caprolactam	(iii) Teflon
(D) Isoprene	(IV) Nylon-6

Choose the correct answer from the options given below :

(1) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

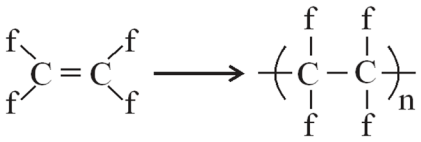
(2) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

(3) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

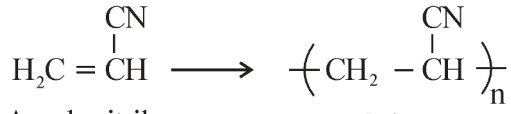
(4) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

Official Ans. by NTA (1)

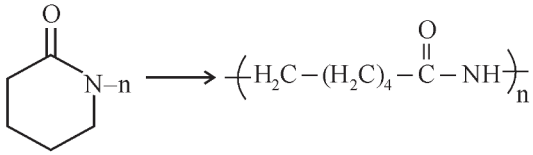
Sol.



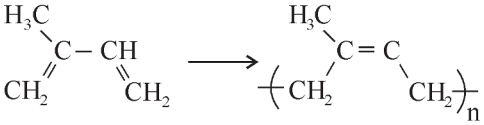
Tetra Fluoroethene Teflon



Acrylonitrile Orlon



Caprolactum Nylon - 6



Isoprene Natural rubber

62. The product formed in the following multistep reaction is:

$$CH_3-CH=CH_2 \xrightarrow[\text{iii) PCC}]{\text{i) } B_2H_6 \text{ ii) } H_2O_2,NaOH} \xrightarrow[\text{iv) } CH_3MgBr]{}$$

(1) $CH_3-CH_2-\underset{\displaystyle OH}{\underset{|}{CH}}-CH_3$

(2) $CH_3-CH_2-CH_2-CH_2-OH$

(3) $CH_3-CH_2-\overset{\displaystyle O}{\overset{\|}{C}}-OCH_3$

(4) $CH_3-\underset{\displaystyle CH_3}{\underset{|}{\overset{\displaystyle OH}{\overset{|}{C}}}}-CH_3$

Official Ans. by NTA (1)

Sol.

$$CH_3-CH=CH_2 \xrightarrow[\text{ii) } H_2O_2,NaOH]{\text{i) } B_2H_6} CH_3-CH_2-\underset{\displaystyle OH}{\underset{|}{CH_2}}$$

$$\downarrow \text{P.C.C}$$

$$CH_3-CH_2-\overset{\displaystyle O}{\overset{\|}{C}}-H$$

$$\downarrow CH_3MgBr$$

$$CH_3-CH_2-\underset{\displaystyle OH}{\underset{|}{CH}}-CH_3$$

63. The possibility of photochemical smog formation will be minimum at

(1) Kolkata in October

(2) Mumbai in May

(3) New-Delhi in August (Summer)

(4) Srinagar, Jammu and Kashmir in January

Official Ans. by NTA (4)

Sol. Photochemical smog occurs in warm, dry and sunny climate.

64. Which one of the following is not an example of calcination ?

(1) $Fe_2O_3 \cdot xH_2O \xrightarrow{\Delta} Fe_2O_3 + xH_2O$

(2) $CaCO_3 \xrightarrow{\Delta} CaO + CO_2$

(3) $CaCO_3 \cdot MgCO_3 \xrightarrow{\Delta} CaO + MgO + 2CO_2$

(4) $2PbS + 3O_2 \xrightarrow{\Delta} 2PbO + 2SO_2$

Official Ans. by NTA (4)

Sol. $2\ PbS + 3\ O_2(g) \xrightarrow{\Delta} 2\ PbO + 2\ SO_2(g)$

It is a roasting reaction.

65. Consider the following statements:

(A) NF_3 molecule has a trigonal planar structure.

(B) Bond length of N_2 is shorter than O_2.

(C) Isoelectronic molecules or ions have identical bond order.

(D) Dipole moment of H_2S is higher than that of water molecule.

Choose the correct answer from the option below:

(1) (A) and (D) are correct

(2) (C) and (D) are correct

(3) (A) and (B) are correct

(4) (B) and (C) are correct

Official Ans. by NTA (4)

Sol. (A) NF_3 has trigonal pyramidal shape.

(B) Bond order $\Rightarrow N_2 > O_2$

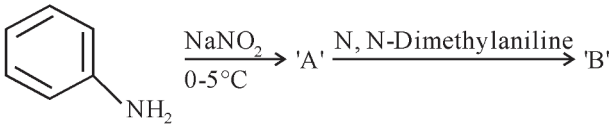
Bond length $\Rightarrow N_2 < O_2$

$\Rightarrow$ (C)

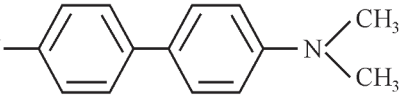
(D) Dipole moment $H_2O > H_2S$

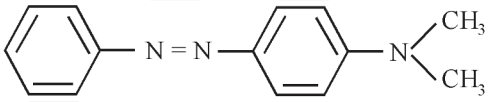
Due to Electronegativity difference.

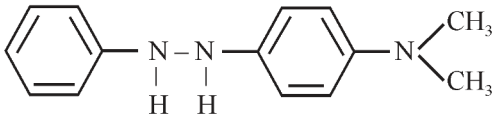
66. Consider the following sequence of reactions:

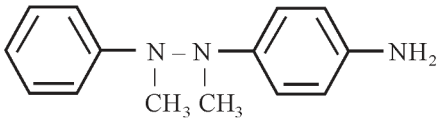


The product 'B' is

(1) H_2NHN– 

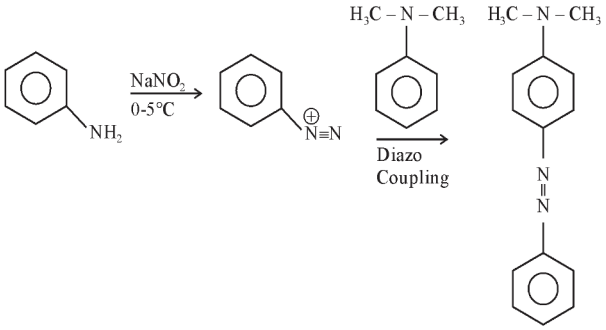
(2) 

(3) 

(4) 

Official Ans. by NTA (2)

Sol.



67. The number of P – O – P bonds in $H_4P_2O_7$, $(HPO_3)_3$ and P_4O_{10} are respectively.

(1) 1, 3, 6

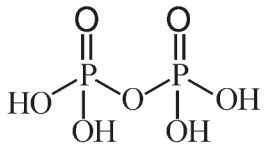
(2) 0, 3, 6

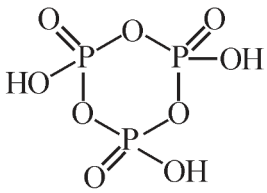
(3) 0, 3, 4

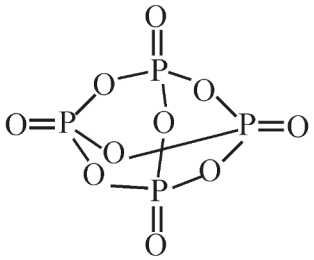
(4) 1, 2, 4

Official Ans. by NTA (1)

Sol.







Molecule	Number of P-O-P Bond
$H_4P_2O_7$	1
$(HPO_3)_3$	3
P_4O_{10}	6

68. Given below are two statements:

Statement I: According to Bohr's model of hydrogen atom, the angular momentum of an electron in a given stationary state is quantised.

Statement II : The concept of electron in Bohr's orbit, violates the Heisenberg uncertainty principle.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both Statement I and Statement II are correct.

(2) Statement I is correct but Statement II is incorrect.

(3) Statement I is incorrect but Statement II is correct

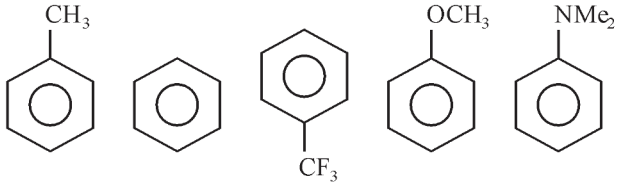
(4) Both Statement I and Statement II are incorrect.

Official Ans. by NTA (1)

Sol. According to Bohr's model the angular momentum is quantised and equal to $\frac{nh}{2\pi}$.

Heisenberg uncertainty principle explains orbital concept, which is based on probability of finding electron.

69. Decreasing order of reactivity towards electrophilie substitution for the following compounds is :



(a) (b) (c) (d) (e)

(1) c > b > a > d > e

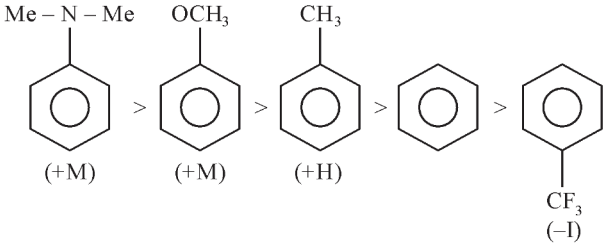
(2) e > d > a > b > c

(3) a > d > e > b > c

(4) d > a > e > c > b

Official Ans. by NTA (2)

Sol. Higher the electron density on Benzene Ring, Higher its Reactivity towards electrophilic substitution Reaction



70. Which of the following statement(s) is/are correct?

(A) The pH of 1×10^{-8} M HCl solution is 8.

(B) The conjugate base $H_2PO_4^-$ is HPO_4^{2-}.

(C) K_w increases with increase in temperature.

(D) When a solution of weak monoprotic acid is titrated against a strong base at half neutralisation point, $pH = \frac{1}{2}pK_a$

Choose the correct answer from the option given below.

(1) (B), (C), (D)

(2) (A), (D)

(3) (A), (B), (C)

(4) (B), (C)

Official Ans. by NTA (4)

Sol. (A) pH of 10^{-8} M HCl is in acidic range (6.98).

(B) Conjugate Base of $H_2PO_4^-$ is HPO_4^{2-}

(C) K_w increases with increasing Temperature, as the temperature increases, the dissociation of water increases.

(D) At half neutralisation point, half of the acid is present in the form of salt.

$$pH = Pk_a + \log\frac{1}{1} = Pk_a$$

71. Given below are two statements : One is labelled as Assertion A and the other is labelled as Reason R :

Assertion (A) : $BeCl_2$ and $MgCl_2$ produce characteristic flame.

Reason (R) : The excitation energy is high in $BeCl_2$ and $MgCl_2$

In the light of the above statements, choose the correct answer from the option given below :

(1) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

(2) (A) is false but (R) is true

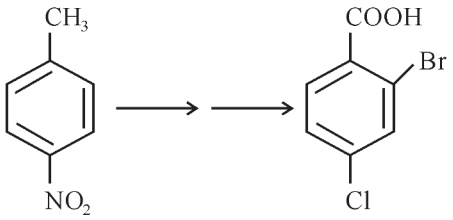
(3) Both (A) and (R) are true and (R) is the correct explanation of (A)

(4) (A) is true but (R) is false.

Official Ans. by NTA (2)

Sol. Be, Mg do not give colour to flame due to high excitation energy.

72.



In the above conversion the correct sequence of reagents to be added is

(1) (i) Fe/H^+, (ii) HONO, (iii) CuCl, (iv) $KMnO_4$, (v) Br_2

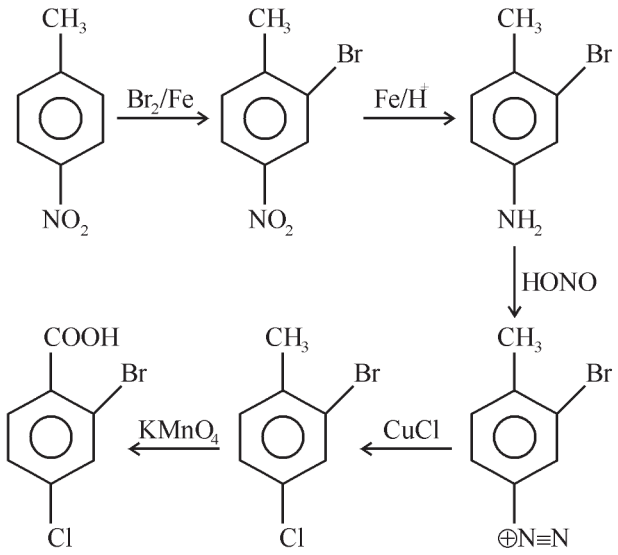
(2) (i) $KMnO_4$, (ii) Br_2/Fe, (iii) Fe/H^+, (iv) Cl_2

(3) (i) Br_2/Fe, (ii) Fe/H^+, (iii) HONO, (iv) CuCl, (v) $KMnO_4$

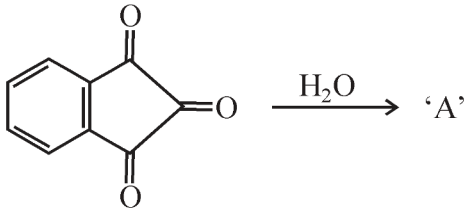
(4) Br_2/Fe, (ii) Fe/H^+, (iii) $KMnO_4$, (iv) Cl_2

Official Ans. by NTA (3)

Sol.

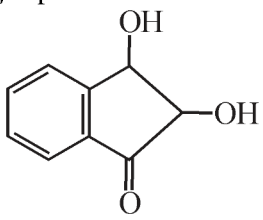


73.

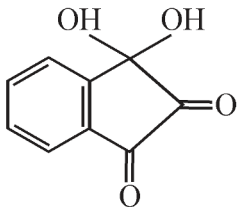


major product 'A' formed in the above reaction is

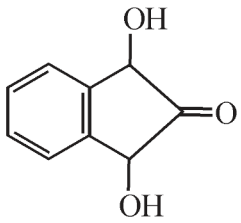
(1)



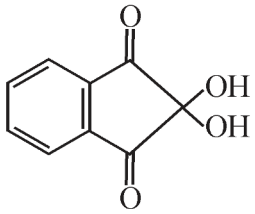
(2)



(3)

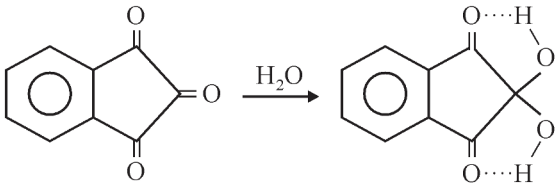


(4)



Official Ans. by NTA (4)

Sol.



74. Which is not true for arginine?

(1) It is a crystalline solid.

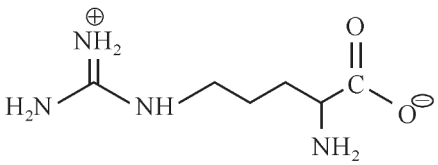
(2) It is associated with more than one pK_a values.

(3) It has a fairly high melting point.

(4) It has high solubility in benzene.

Official Ans. by NTA (4)

Sol.



Arginine exist is zwitterion, so solid in nature and soluble in polar solvent.

75. During water-gas shift reaction

(1) carbon monoxide is oxidized to carbon dioxide.

(2) carbon is oxidized to carbon monoxide.

(3) carbon dioxide is reduced to carbon monoxide.

(4) water is evaporated in presence of catalyst.

Official Ans. by NTA (1)

Sol. Water gas shift reaction

$$\underbrace{CO(g) + H_2}_{\text{water gas}} + \underbrace{H_2O}_{\text{Steam}} \xrightarrow[\text{Catalyst}]{\text{Iron Chromate}} CO_2(g) + 2H_2(g)$$

76. For a good quality cement, the ratio of silica to alumina is found to be

(1) 3

(2) 4.5

(3) 2

(4) 1.5

Official Ans. by NTA (1)

Sol. For good quality cement, the ratio of silica (SlO_2) to Alumina (Al_2O_3) should be between 2.5 to 4.

77. Which of the following statement is correct for paper chromatography ?

(1) Water present in the mobile phase gets absorbed by the paper which then forms the stationary phase.

(2) Water present in the pores of the paper forms the stationary phase.

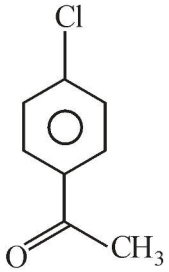
(3) Paper sheet forms the stationary phase.

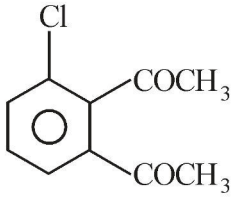
(4) Paper and water present in its pores together form the stationary phase.

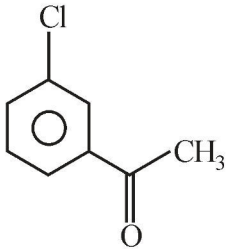
Official Ans. by NTA (2)

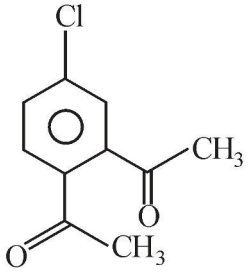
Sol. In paper chromatography, a special quality paper known as chromatography paper is used. Paper contains water trapped in it, which acts as the stationary phase.

78. The major product formed in the Friedel-Craft acylation of chlorobenzene is .

(1) 

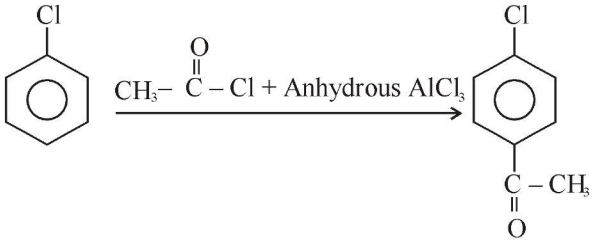
(2) 

(3) 

(4) 

Official Ans. by NTA (1)

Sol.



Chlorine is ortho/para directing, para is major.

79. The complex with highest magnitude of crystal field splitting energy (Δ_o) is

(1) $[Cr(OH_2)_6]^{3+}$

(2) $[Ti(OH_2)_6]^{3+}$

(3) $[Fe(OH_2)_6]^{3+}$

(4) $[Mn(OH_2)_6]^{3+}$

Official Ans. by NTA (1)

Sol. Ti^{+3} = 67 pm radius

Cr^{3+} = 62 pm radius

Mn^{+3} = 65 pm radius

Fe^{+3} = 65 pm radius

So, Cr^{3+} has highest tendency to attract ligand.

80. Which of the following expressions is correct in case of a CsCl unit cell (edge length'a')?

(1) $r_{Cs^+} + r_{Cl^-} = \frac{a}{\sqrt{2}}$

(2) $r_{Cs^+} + r_{Cl^-} = a$

(3) $r_{Cs^+} + r_{Cl^-} = \frac{\sqrt{3}}{2}a$

(4) $r_{Cs^+} + r_{Cl^-} = \frac{a}{2}$

Official Ans. by NTA (3)

Sol. For CsCl, $Cs^{\oplus}$ is present at Body centre and $Cl^{\ominus}$ at all corner. $\frac{\sqrt{3}a}{2} = r_{cs^{\oplus}} + r_{Cl^{\ominus}}$

SECTION-B

81. The homoleptic and octahedral complex of Co^{2+} and H_2O has_____unpaired electron(s) in the t_{2g} set of orbitals.

Official Ans. by NTA (1)

Sol. Co^{2+} : $3d^7$ configuration

t_{2g}^{221} e_g^{11}

82. The volume (in mL) of 0.1 M $AgNO_3$ required for complete precipitation of chloride ions present in 20 mL of 0.01 M solution of $[Cr(H_2O)_5Cl]Cl_2$ as silver chloride is_____

Official Ans. by NTA (4)

Sol. $[Cr(H_2O)_5Cl]Cl_2 + 2\ AgNO_3 \rightarrow$

0.01M, 20mL 0.1M

For 0.2 milimole $AgNO_3$ required

= 0.4 milimole

$0.4 = 0.1 \times V(ml)$

V = 4mL

83. The total change in the oxidation state of manganese involved in the reaction of $KMnO_4$ and potassium iodide in the acidic medium is_____

Official Ans. by NTA (5)

Sol. $KMnO_4 \rightarrow Mn^{2+}$

↓ ↓

+7 **+2**

Change in oxidation state of Mn= 5

84. In Chromyl chloride, the oxidation state of chromium is (+)_____

Official Ans. by NTA (6)

Sol. $CrO_2Cl_2 x - 4 - 2 = 0$

Oxidation State = +6

85. The total number of isoelectronic species from the given set is _____

O^{2-} ,F^- ,Al ,Mg^{2+} ,Na^+ ,O^+ ,Mg ,Al^{3+} ,F

Official Ans. by NTA (5)

Sol. Isoelectronic species $O^{2\ominus}, F^{\ominus}, Mg^{2+}, Na^{\oplus}, Al^{3+}$

86. The vapour pressure of 30% (w/v) aqueous solution of glucose is_____mm Hg at 25°C.

[Given : The density of 30% (w/v), aqueous solution of glucose is 1.2 g cm^{-3} and vapour pressure of pure water is 24 mm Hg.]

(Molar mass of glucose is 180 g mol^{-1}.)

Official Ans. by NTA (23)

Sol. $\frac{24-P_s}{P_s}=\frac{m\times 18}{1000}$

wt of solute = 30 gm

Volume of solution = 100 mL

wt. of solution = 1.2 × 100 = 120 gm

wt. of solvent = 120 – 30 = 90 gm

$m=\frac{30\times 1000}{180\times 90}=1.85$

$\frac{24-P_s}{P_s}=\frac{1.85\times 18}{1000}$

$24-P_s = 0.0333\ P_s$

$P_s(1.033) = 24$

$P_s = 23.22$

87. 20 mL of 0.5 M NaCl is required to coagulate 200 mL of As_2S_3 solution in 2 hours. The coagulating value of NaCl is _____

Official Ans. by NTA (50)

Sol. Coagulating value is required milimole of electrolyte needed to coagulate 1 L sol in 2 hours.

Coagulating value $=\frac{20\times 0.5}{200}\times 1000 = 50$

88. For a reversible reaction $A \rightleftharpoons B$, the $\Delta H_{forward\ reaction}$ =20 kJ mol^{-1} . The activation energy of the uncatalysed forward reaction is 300 kJ mol^{-1}. When the reaction is catalysed keeping the reactant concentration same, the rate of the catalysed forward reaction at 27°C is found to be same as that of the uncatalysed reaction at 327°C. The activation energy of the cataysed backward reactoion is _____kJ mol^{-1}.

Official Ans. by NTA (130)

Sol. E_a = 300 kJ mol^{-1}

$\frac{E_a}{T}=\frac{E_a'}{T'}$

(Since rate of catalysed and uncatalysed reaction is same)

$\frac{300}{600}=\frac{E'_{a,f}}{300}$

$E'_{a,f} = 150$

$20 = 150 - E'_{a,b}$

$E'_{a,b} = 130$

89. The number of correct statements from the following is _____

(A) Conductivity always decreases with decrease in concentration for both strong and weak electrolytes.

(B) The number of ions per unit volume that carry current in a solution increases on dilution.

(C) Molar conductivity increases with decrease in concentration.

(D) The variation in molar conductivity is different for strong and weak electrolytes.

(E) For weak electrolytes, the change in molar conductivity with dilution is due to decrease in degree of dissociation.

Official Ans. by NTA (3)

Sol.

(A) Conductivity decreases with dilution for strong electrolyte as well as weak electrolyte.

(B) On dilution, The number of ions per unit volume that carry current in a solution decreases.

(C) Molar conductivity increases with dilution.

(D) Molar conductivity of strong electrolyte follows DHO equation but it is not applicable for weak electrolyte.

(E) On dilution degree of dissociation of weak electrolyte increases.

So answer is (A), (C) & (D).

90. 30.4 kJ of heat is required to melt one mole of sodium chloride and the entropy change at the melting point is 28.4 J K^{-1} mol^{-1} at 1 atm. The melting point of sodium chloride is _____K (Nearest Integer)

Official Ans. by NTA (1070)

Sol. $\Delta S=\frac{\Delta H}{T_{mp}}$

$28.4=\frac{30.4\times 1000}{T_{mp}}$

$T_{mp} = 1070.422\ K.$