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JEE Main 2023 January Question Paper with Answer

24th, 25th, 29th, 30th, 31st January & 1st February (Shift 1 & Shift 2)

JEE–MAIN EXAMINATION – JANUARY, 2023

(Held On Thursday 24th January, 2023)	TIME : 9 : 00 AM to 12 : 00 NOON

Physics

SECTION - A

1. A circular loop of radius r is carrying current I A. The ratio of magnetic field at the center of circular loop and at a distance r from the center of the loop on its axis is:

(1) $2\sqrt{2}:1$ (2) $1:3\sqrt{2}$ (3) $1:\sqrt{2}$ (4) $3\sqrt{2}:2$

Sol. **1**

Magnetic field at centre of coil $B_1 = \frac{\mu_0 I}{2r}$

on the axis at $x = r \Rightarrow B_2 = \frac{\mu_0 I r^2}{2\left(r^2 + x^2\right)^{3/2}}$

$$B_2 = \frac{\mu_0 I r^2}{2\left(r^2 + r^2\right)^{3/2}}$$

$$B_2 = \frac{\mu_0 I}{2(2\sqrt{2}\,r)}$$

$$\frac{B_1}{B_2} = 2\sqrt{2}$$

2. The weight of a body at the surface of earth is 18 N. The weight of the body at an altitude of 3200 km above the earth's surface is (given, radius of earth $R_e = 6400$ km):

(1) 8 N (2) 4.9 N (3) 9.8 N (4) 19.6 N

Sol. **1**

Weight on earth surface W = mg = 18 N

Above earth surface $\Rightarrow W_2 = m\frac{GM}{(R+h)^2}$

h = 3200 km = R/2

$$W_2 = m\frac{GM}{\left(\frac{3R}{2}\right)^2} \Rightarrow W_2 = \frac{4}{9}mg$$

$$W_2 = \frac{4}{9}\times 18 \Rightarrow W_2 = 8\,\text{N}$$

3. Two long straight wires P and Q carrying equal current 10 A each were kept parallel to each other at 5 cm distance. Magnitude of magnetic force experienced by 10 cm length of wire P is F_1 - If distance between wires is halved and currents on them are doubled, force F_2 on 10 cm length of wire P will be:

(1) $\frac{F_1}{8}$ (2) $8\ F_1$ (3) $10\ F_1$ (4) $\frac{F_1}{10}$

Sol. **2**

$$F = \frac{\mu_0 I_1 I_2}{2\pi r} \Rightarrow F = \frac{\mu_0 I^2 \ell}{2\pi r}$$

$\ell = 10$ cm (Both) $\Rightarrow F \propto \frac{I^2}{r}$

$$\frac{F_1}{F_2} = \left(\frac{I}{2I}\right)^2\left(\frac{5/2}{5}\right) \Rightarrow \frac{F_1}{F_2} = \frac{1}{8} \Rightarrow F_2 = 8F_1$$

4. Given below are two statements :

Statement I : The temperature of a gas is $-73°\mathrm{C}$. When the gas is heated to $527°\mathrm{C}$, the root mean square speed of the molecules is doubled.

Statement II : The product of pressure and volume of an ideal gas will be equal to translational kinetic energy of the molecules.

In the light of the above statements, choose the correct answer from the options given below:

(1) Statement I is false but Statement II is true
(2) Both Statement I and Statement II are false
(3) Statement I is true but Statement II is false
(4) Both Statement I and Statement II are true

Sol. 3

Statements-1

$$v_{rms} \propto \sqrt{T} \Rightarrow v_{rms_1} \propto \sqrt{273-73}$$

$$v_{rms_2} \propto \sqrt{273+527}$$

$$\frac{v_{rms_1}}{v_{rms_2}} = \sqrt{\frac{200}{800}} \Rightarrow v_{rms_2} = 2v_{rms_1} \qquad \text{(True)}$$

Statements-2

$$\text{Translation K.E.} = \frac{3}{2}nRT = \frac{3}{2}PV \qquad \text{(False)}$$

5. The maximum vertical height to which a man can throw a ball is 136 m. The maximum horizontal distance upto which he can throw the same ball is:

(1) 272 m (2) 68 m (3) 192 m (4) 136 m

Sol. 1

$$\text{Max vertical height } H = \frac{v^2}{2g} = 136\,m$$

$$\text{Max horizontal distance } R = \frac{v^2}{g} \Rightarrow R = 2 \times 136 = 272 \text{ m}$$

6. Given below are two statements :

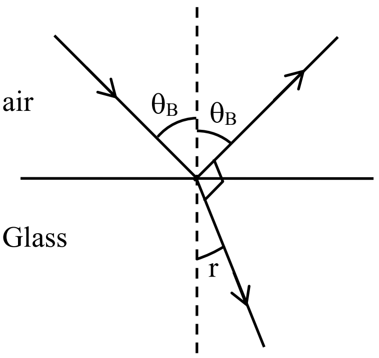
Statement I : If the Brewster's angle for the light propagating from air to glass is θ_B, then the Brewster's angle for the light propagating from glass to air is $\frac{\pi}{2} - \theta_B$

Statement II : The Brewster's angle for the light propagating from glass to air is $\tan^{-1}(\mu_g)$ where μ_g is the refractive index of glass.

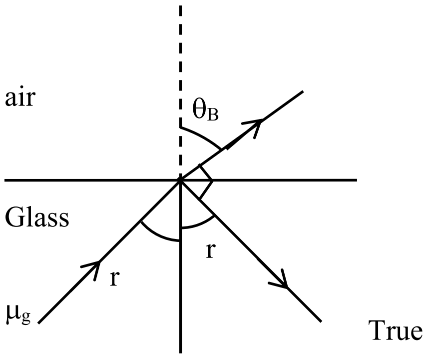
In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are false
(2) Statement I is true but Statement II is false
(3) Statement I is false but Statement II is true
(4) Both Statement I and Statement II are true

Sol. **2**

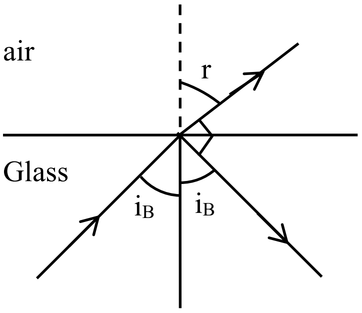


$$\theta_B + r = \frac{\pi}{2}$$



$$r = \frac{\pi}{2} - \theta_B$$

For glass to air



$$\mu_g \sin i_B = 1 \cdot \sin r$$

$$r + i_B = \frac{\pi}{2}$$

$$\mu_g \sin i_B = \cos i_B \Rightarrow \tan i_B = \frac{1}{\mu_g} \Rightarrow i_B = \tan^{-1}\left(\frac{1}{\mu_g}\right)$$

7. A 100 m long wire having cross-sectional area 6.25×10^{-4} m^2 and Young's modulus is $10^{10} Nm^{-2}$ is subjected to a load of 250 N, then the elongation in the wire will be:

(1) 4×10^{-3} m (2) 6.25×10^{-3} m (3) 6.25×10^{-6} m (4) 4×10^{-4} m

Sol. **1**

$$\text{Stress} = \text{y strain} \Rightarrow \frac{W}{A} = y\frac{\Delta\ell}{\ell}$$

$$\Delta\ell = \frac{W\ell}{yA} \Rightarrow \Delta\ell = \frac{250 \times 100}{10^{10} \times 6.25 \times 10^{-4}}$$

$$\Delta\ell = 4 \times 10^{-3} \text{ m}$$

8. If two charges q_1 and q_2 are separated with distance 'd' and placed in a medium of dielectric constant K. What will be the equivalent distance between charges in air for the same electrostatic force?

(1) $2d\sqrt{k}$ (2) $1.5\,d\sqrt{k}$ (3) $d\sqrt{k}$ (4) $k\sqrt{d}$

Sol. **3**

For same force

$$\frac{q_1q_2}{4\pi\varepsilon_0 kd^2} = \frac{q_1q_2}{4\pi\varepsilon_0 r^2} \Rightarrow r = d\sqrt{K}$$

9. Consider the following radioactive decay process

$$^{218}_{84}A \xrightarrow{\alpha} A_1 \xrightarrow{\beta^-} A_2 \xrightarrow{\gamma} A_3 \xrightarrow{\alpha} A_4 \xrightarrow{\beta^+} A_5 \xrightarrow{\gamma} A_6$$

The mass number and the atomic number of A_6 are given by:

(1) 210 and 84 (2) 210 and 82 (3) 211 and 80 (4) 210 and 80

Sol. **4**

$$^{218}_{84}A \xrightarrow{\alpha} {}^{214}_{82}A_1 \xrightarrow{\beta^{\ominus}} {}^{214}_{83}A_2 \xrightarrow{\gamma} {}^{214}_{83}A_3 \xrightarrow{\alpha} {}^{210}_{81}A_4 \xrightarrow{\beta^{\oplus}} {}^{210}_{80}A_5 \xrightarrow{\gamma} {}^{210}_{80}A_6$$

10. From the photoelectric effect experiment, following observations are made. Identify which of these are correct.

A. The stopping potential depends only on the work function of the metal.

B. The saturation current increases as the intensity of incident light increases.

C. The maximum kinetic energy of a photo electron depends on the intensity of the incident light.

D. Photoelectric effect can be explained using wave theory of light.

Choose the correct answer from the options given below:

(1) A, C, D only (2) B, C only (3) B only (4) A, B, D only

Sol. **3**

$$v_{sp} = \frac{h\nu - \phi}{e} \quad (\nu \text{ and } \phi \text{ both})$$

Intensity ↑ current ↑

$kE_{max} = h\nu - \phi$

Photoelectric effect is not explained by wave theory

11. Given below are two statements:

Statement I: An elevator can go up or down with uniform speed when its weight is balanced with the tension of its cable.

Statement II: Force exerted by the floor of an elevator on the foot of a person standing on it is more than his/her weight when the elevator goes down with increasing speed.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are true

(2) Statement I is false but Statement II is true

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are false

Sol. **3**

Statement-1

When force balance it can move with uniform velocity (Uniform speed) True

Statement-2

Elevator going down with increasing speed means its acceleration is downwards

mg – N = ma (on person)

N = mg – ma (False)

12. 1 g of a liquid is converted to vapour at 3×10^5 Pa pressure. If 10% of the heat supplied is used for increasing the volume by 1600 cm^3 during this phase change, then the increase in internal energy in the process will be:

(1) 432000 J (2) 4320 J (3) 4800 J (4) 4.32×10^8 J

Sol. **2**

10% of ΔQ = PΔV (W/D by gas)

$$\frac{\Delta Q}{10} = 3\times10^5(1600\times10^{-6})$$

$\Delta Q = 4800$ J

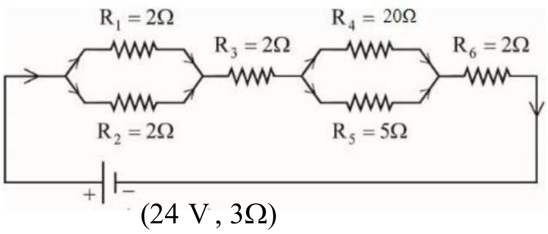
Using first law of the thermodynamics

$\Delta Q = \Delta u + W$

$$\Delta Q = \Delta u + \frac{\Delta Q}{10} \Rightarrow \Delta u = \frac{9}{10}\Delta Q$$

$$\Delta u = \frac{9}{10}\times 4800 \Rightarrow \Delta u = 4320\,J$$

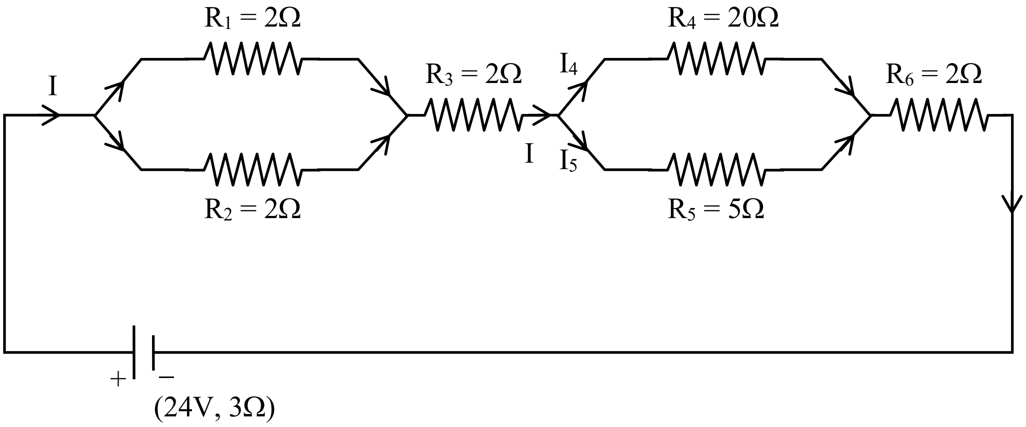
13. As shown in the figure, a network of resistors is connected to a battery of 24 V with an internal resistance of 3Ω. The currents through the resistors R_4 and R_5 are I_4 and I_5 respectively. The values of I_4 and I_5 are:



(1) $I_4 = \frac{2}{5}$ A and $I_5 = \frac{8}{5}$ A (2) $I_4 = \frac{24}{5}$ A and $I_5 = \frac{6}{5}$ A

(3) $I_4 = \frac{8}{5}$ A and $I_5 = \frac{2}{5}$ A (4) $I_4 = \frac{6}{5}$ A and $I_5 = \frac{24}{5}$ A

Sol. **1**



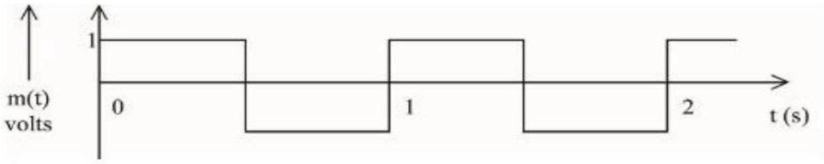
$$R_{eq} = 3 + 1 + 2 + \frac{20 \times 5}{25} + 2 \Rightarrow R_{eq} = 12\Omega$$

Current from battery $I = \frac{24}{12} \Rightarrow I = 2A$

$$I_4 + I_5 = 2A$$

$I_4(20) = I_5(5) \Rightarrow I_5 = 4I_4 \Rightarrow I_4 = \frac{2}{5}A \;\; I_5 = \frac{8}{5}A$

14. A modulating signal is a square wave, as shown in the figure.



If the carrier wave is given as $c(t) = 2\sin(8\pi t)$ volts, the modulation index is:

(1) $\frac{1}{4}$ (2) $\frac{1}{2}$ (3) 1 (4) $\frac{1}{3}$

Sol. 2

Modulation index $\mu = \frac{A_m}{A_c}$

$A_m = 1$ & $A_c = 2$

$\mu = \frac{1}{2}$

15. A conducting circular loop of radius $\frac{10}{\sqrt{\pi}}$ cm is placed perpendicular to a uniform magnetic field of 0.5 T. The magnetic field is decreased to zero in 0.5 s at a steady rate. The induced emf in the circular loop at 0.25 s is:

(1) emf = 1mV (2) emf = 5mV (3) emf = 100mV (4) emf = 10mV

Sol. 4

$$\text{emf} = -\frac{d\phi}{dt} \Rightarrow \varepsilon = \frac{-d(BA)}{dt}$$

$$\varepsilon = -A\frac{dB}{dt} \Rightarrow \varepsilon = -\pi R^2\left(\frac{0-B}{\Delta t}\right)$$

$$\varepsilon = \frac{\pi R^2 B}{\Delta t} \Rightarrow \varepsilon = \frac{\pi\left(\frac{10}{\sqrt{\pi}}\times 10^{-2}\right)^2 \times 0.5}{0.5}$$

$\varepsilon = 10^{-2}$ volt = 10 m volt

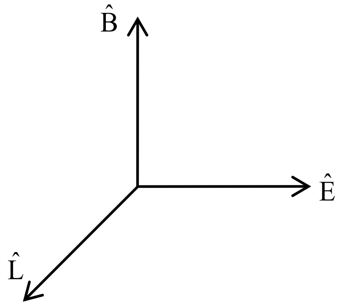
16. In $\vec{E}$ and $\vec{K}$ represent electric field and propagation vectors of the EM waves in vacuum, then magnetic field vector is given by :

(ω - angular frequency):

(1) $\omega(\bar{E}\times\bar{K})$ (2) $\omega(\bar{K}\times\bar{E})$ (3) $\bar{K}\times\bar{E}$ (4) $\frac{1}{\omega}(\bar{K}\times\bar{E})$

Sol. **4**

$$\vec{E} \ \& \ \vec{K} = \frac{W}{C}\hat{L}$$



$$\hat{B} = \hat{L}\times\hat{E}$$

$$\vec{B} = B\hat{B}\left\{\frac{E}{B} = C\right\}$$

$$\vec{B} = \frac{E}{C}(\hat{L}\times\hat{E})$$

$$\vec{B} = \frac{\omega}{C}\left(\frac{\hat{L}\times E\hat{E}}{\omega}\right) \Rightarrow \vec{B} = \frac{\vec{K}\times\vec{E}}{\omega}$$

17. Match List I with List II:

LIST I		LIST II	
A.	Planck's constant (h)	I.	$[M^1 L^2 T^{-2}]$
B.	Stopping potential (Vs)	II.	$[M^1 L^1 T^{-1}]$
C.	Work function (Ø)	III.	$[M^1 L^2 T^{-1}]$
D.	Momentum (p)	IV.	$[M^1 L^2 T^{-3} A^{-1}]$

Choose the correct answer from the options given below:

(1) A-I, B-III, C-IV, D-II
(2) A-III, B-I, C-II, D-IV
(3) A-II, B-IV, C-III, D-I
(4) A-III, B-IV, C-I, D-II

Sol. 4

(A) Planck's constant $h = \frac{E}{\nu}$

$$[h] = \frac{\left[M^1L^2T^{-2}\right]}{\left[T^{-1}\right]} \Rightarrow [h] = \left[M^1L^2T^{-1}\right]$$

(B) Stopping potential $V = \frac{W}{q}$

$$[v] = \frac{ML^2T^{-2}}{AT} \Rightarrow [v] = \left[ML^2T^{-3}A^{-1}\right]$$

(C) Work function = $[ML^2T^{-2}]$

(D) Momentum $[P] = [MLT^{-1}]$

18. A travelling wave is described by the equation

$$y(x, t) = [0.05\sin(8x - 4t)]\text{m}$$

The velocity of the wave is : [all the quantities are in SI unit]

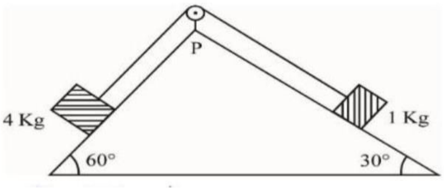
(1) 8 ms^{-1}
(2) 4 ms^{-1}
(3) 0.5 ms^{-1}
(4) 2 ms^{-1}

Sol. 3

$y = 0.05 \sin(8x - 4t)$

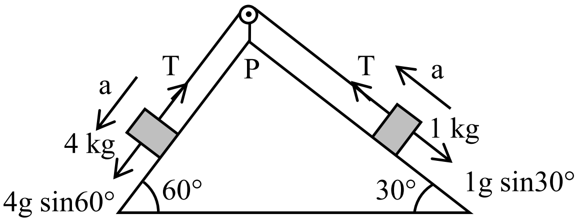
$$v = \frac{\omega}{k} \Rightarrow v = \frac{4}{8} \Rightarrow v = \frac{1}{2} \text{ m/s}$$

19. As per given figure, a weightless pulley P is attached on a double inclined frictionless surfaces. The tension in the string (massless) will be (if $g = 10 \text{ m/s}^2$)



(1) $(4\sqrt{3} + 1)N$
(2) $4(\sqrt{3} + 1)N$
(3) $(4\sqrt{3} - 1)N$
(4) $4(\sqrt{3} - 1)N$

Sol. **2**



$$4g\frac{\sqrt{3}}{2} - T = 4a \qquad \dots (1)$$

$$T - \frac{g}{2} = 1a \qquad \dots (2)$$

$$2\sqrt{3}g - T = 4\left(T - \frac{g}{2}\right) \Rightarrow 5T = (2\sqrt{3} + 2)g$$

$$T = \frac{10}{5}\left(2\sqrt{3} + 2\right) \Rightarrow T = 4\left(\sqrt{3} + 1\right)\text{N}$$

20. Given below are two statements: one is labelled as Assertion **A** and the other is labelled as Reason **R**

Assertion A: Photodiodes are preferably operated in reverse bias condition for light intensity measurement.

Reason : The current in the forward bias is more than the current in the reverse bias for a $p - n$ junction diode.

In the light of the above statements, choose the correct answer from the options given below:

(1) A is true but **R** is false

(2) **A** is false but **R** is true

(3) Both **A** and **R** are true and **R** is the correct explanation of **A**

(4) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

Sol. **4**

Photodiode works in reverse bias and its is used as a intensity detector . (True)

Forward bias current is more as compaired to reverse bias current (True)

SECTION - B

21. Vectors $a\hat{\imath} + b\hat{\jmath} + \hat{k}$ and $2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ are perpendicular to each other when $3a + 2b = 7$, the ratio of a to b is $\frac{x}{2}$ The value of x is

Sol. **1**

$a\hat{i} + b\hat{j} + \hat{k}$ is $\perp$ to $\left(2\hat{i} - 3\hat{j} + 4\hat{k}\right)$

$\vec{A}\cdot\vec{B} = 0 \Rightarrow \quad 2a - 3b - 4 = 0$

$2a - 3b = -4$

Given $\quad 3a + 2b = 7$

$$\frac{2\left(\frac{a}{b}\right) - 3}{3\left(\frac{a}{b}\right) + 2} = \frac{-4}{7} \Rightarrow 14\frac{a}{b} - 21 = -12\frac{a}{b} - 8$$

$$26\frac{a}{b} = 13 \Rightarrow \frac{a}{b} = \frac{1}{2} = \frac{x}{2}$$

$x = 1$

22. Assume that protons and neutrons have equal masses. Mass of a nucleon is 1.6×10^{-27} kg and radius of nucleus is $1.5 \times 10^{-15}\ A^{1/3}$ m. The approximate ratio of the nuclear density and water density is $n \times 10^{13}$. The value of n is

Sol. **11**

$$\rho_{\text{Nucleus}} = \frac{A(m)}{\frac{4}{3}\pi R^3} \Rightarrow$$

$$\rho_N = \frac{3}{4\pi}\frac{Am}{\left(1.5 \times 10^{-15} A^{\frac{1}{3}}\right)^3}$$

$$\frac{\rho_N}{\rho_W} = \frac{3}{4\pi}\frac{(1.6) \times 10^{-27}}{(1.5)^3 \times 10^{-45} \times 10^3}$$

$$\frac{\rho_N}{\rho_W} = 11 \times 10^{13}$$

23. A hollow cylindrical conductor has length of 3.14 m, while its inner and outer diameters are 4 mm and 8 mm respectively. The resistance of the conductor is $n \times 10^{-3}\Omega$. If the resistivity of the material is $2.4 \times 10^{-8}\Omega$m. The value of n is

Sol. **2**

$$R = \frac{\rho\ell}{A} \Rightarrow R = \frac{\rho\ell}{\pi\left(r_2^2 - r_1^2\right)}$$

$$R = \frac{2.4 \times 10^{-8} \times 3.14}{\pi(4^2 - 2^2) \times 10^{-6}}$$

$R = 2 \times 10^{-3}\ \Omega$

24. A stream of a positively charged particles having $\frac{q}{m} = 2 \times 10^{11} \frac{C}{kg}$ and velocity $\vec{v}_0 = 3 \times 10^7 \hat{i} m/s$ is deflected by an electric field $1.8\hat{j}$kV/m. The electric field exists in a region of 10 cm along x direction. Due to the electric field, the deflection of the charge particles in the y direction is ______ mm

Sol. **2**

$$y = \frac{1}{2} a t^2$$

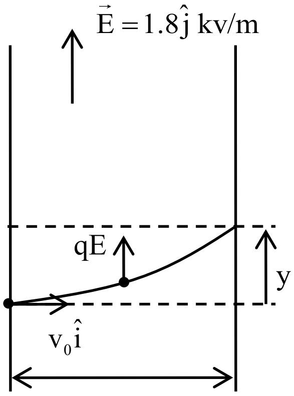
$$y = \frac{1}{2} \frac{qE}{m} t^2$$

$$\ell = v_0 t$$

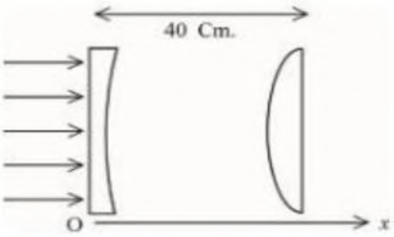
$$y = \frac{1}{2} \frac{qE}{m} \left(\frac{\ell}{v_0} \right)^2$$

$$y = \frac{1}{2} (2 \times 10^{11})(1.8 \times 10^3) \left(\frac{0.1}{3 \times 10^7} \right)^2$$

$$y = 2 \text{ mm}$$



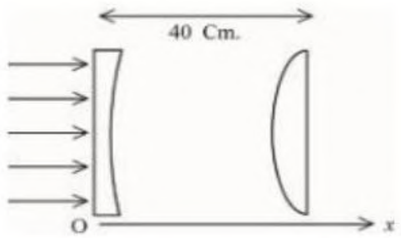
25. As shown in the figure, a combination of a thin plano concave lens and a thin plano convex lens is used to image an object placed at infinity. The radius of curvature of both the lenses is 30 cm and refraction index of the material for both the lenses is 1.75. Both the lenses are placed at distance of 40 cm from each other. Due to the combination, the image of the object is formed at distance = ___cm, from concave lens.



Sol. **120**

Magnitude of focal length of both lens

$$f=\frac{R}{\mu-1}\Rightarrow f=\frac{30}{1.75-1}\Rightarrow f=40\text{ cm}$$



$f=-40$ cm $\qquad$ $f=+40$ cm

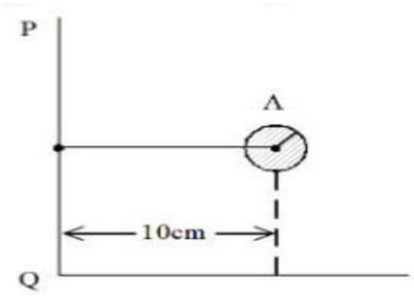
Concave lens will form image at its focus for convex lens $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{v}-\frac{1}{-80}=\frac{1}{+40}$

V = +80 cm

From concave lens distance of image of d = 80 + 40

d = 120 cm

26. Solid sphere A is rotating about an axis PQ. If the radius of the sphere is 5 cm then its radius of gyration about PQ will be $\sqrt{x}$ cm. The value of x is ________



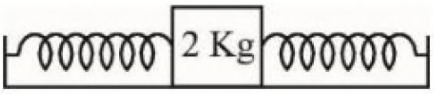
Sol. **110**

$I_{PQ}=I_{cm}+md^2$

$$mk^2=\frac{2}{5}mR^2+md^2\Rightarrow k=\sqrt{\frac{2}{5}(5)^2+(10)^2}$$

$k=\sqrt{110}$ cm

27. A block of a mass 2 kg is attached with two identical springs of spring constant 20 N/m each. The block is placed on a frictionless surface and the ends of the springs are attached to rigid supports (see figure). When the mass is displaced from its equilibrium position, it executes a simple harmonic motion. The time period of oscillation is $\frac{\pi}{\sqrt{x}}$ in SI unit. The value of x is __________



Sol. **5**

$$T = 2\pi\sqrt{\frac{m}{k_{eq}}}$$

$$T = 2\pi\sqrt{\frac{2}{2k}} \Rightarrow T = 2\pi\sqrt{\frac{1}{20}}$$

$$T = \frac{\pi}{\sqrt{5}}$$

28. A hole is drilled in a metal sheet. At 27°C, the diameter of hole is 5 cm. When the sheet is heated to 177°C, the change in the diameter of hole is $d \times 10^{-3}$ cm. The value of d will be __________ if coefficient of linear expansion of the metal is $1.6 \times 10^{-5}/\,°C$.

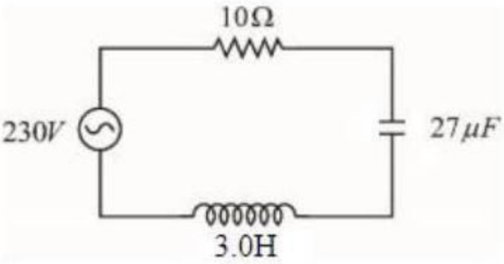
Sol. **12**

$\Delta D = D \propto \Delta T$

$\Delta D = 5 \times 1.6 \times 10^{-5} \times (177 - 27)$

$\Delta D = 12 \times 10^{-3}$ cm

29. In the circuit shown in the figure, the ratio of the quality factor and the band width is _______ S.



Sol. **10**

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \text{ \& bandwidth } = \frac{R}{L}$$

$$\frac{Q}{\text{Bandwidth}} = \frac{L}{R^2}\sqrt{\frac{L}{C}}$$

$$= \frac{3}{100} \times \sqrt{\frac{3}{27 \times 10^{-6}}}$$

$= 10$

30. A spherical body of mass 2 kg starting from rest acquires a kinetic energy of 10000 J at the end of 5th second. The force acted on the body is _______ N.

Sol. **40**

Impulse = ΔP

$F\Delta T = P - 0 \Rightarrow F\Delta T = \sqrt{2mk}$

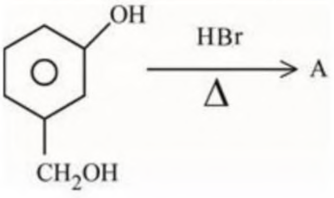
$F(5) = \sqrt{2 \times 2 \times 10000}$

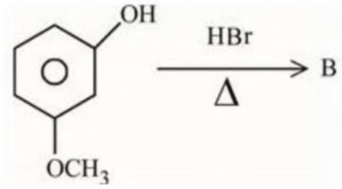
$F = 40$ N

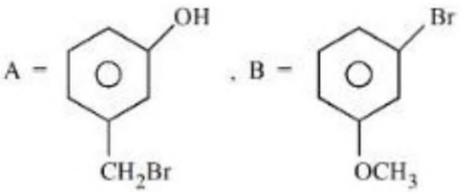
Chemistry

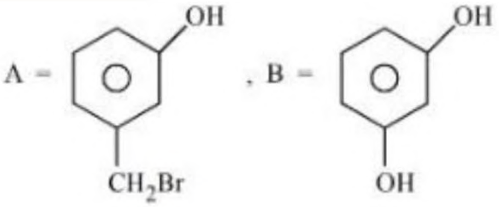
SECTION - A

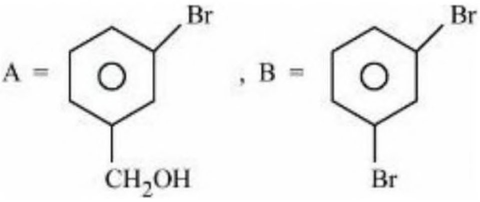
31. ' A ' and ' B ' formed in the following set of reactions are:

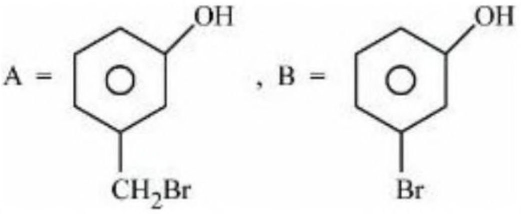




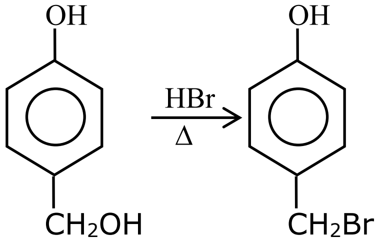
(1) 

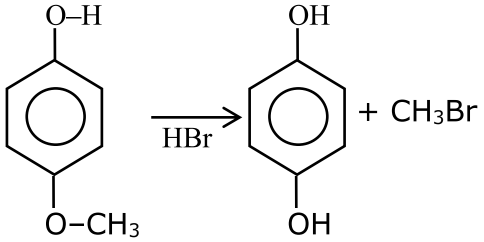
(2) 

(3) 

(4) 

Sol. **2**





32. Decreasing order of the hydrogen bonding in following forms of water is correctly represented by
A.Liquid water
B. Ice
C. Impure water
Choose the correct answer from the options given below:
(1) B > A > C (2) $A > B > C$ (3) A = B > C (4) C > B > A

Sol. **1**

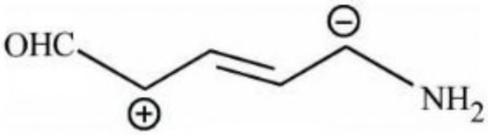
ice > liquid water > impure water

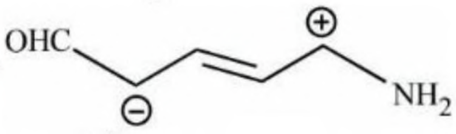
OR

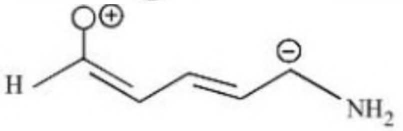
ice > H_2O liq. > impure H_2O

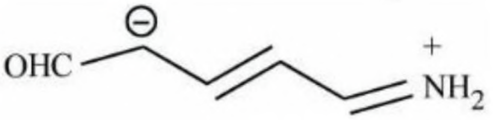
Hydrogen bond $\propto \frac{1}{\text{Temp}}$

33. Increasing order of stability of the resonance structures is:

A. 

B. 

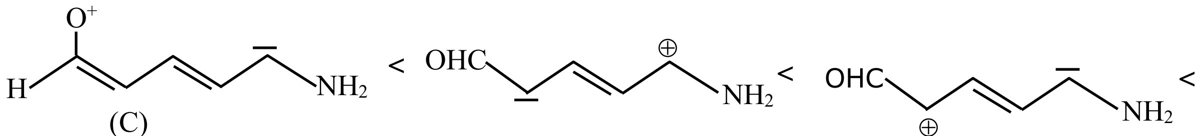
C. 

D. 

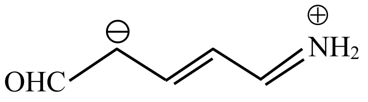
Choose the correct answer from the options given below:

(1) D, C, A, B (2) D, C, B, A (3) C, D, A, B (4) 4. C, D, B, A

Sol. **Bonus**



(C) (+ve on O) (B) (Back bonding) (A)

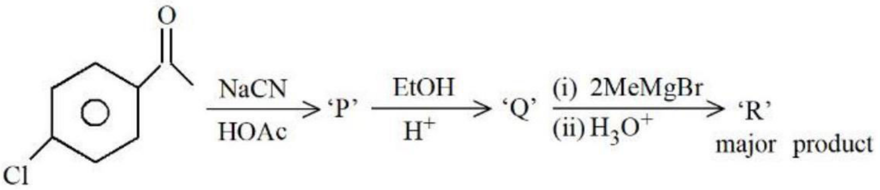


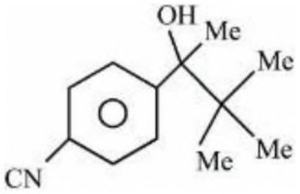
(D)

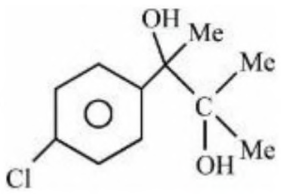
more covalent bond and (+M)

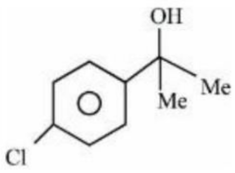
Final correct order C<B<A<D

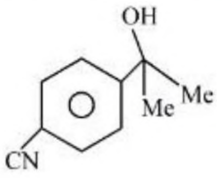
34. ' *R* ' formed in the following sequence of reactions is:



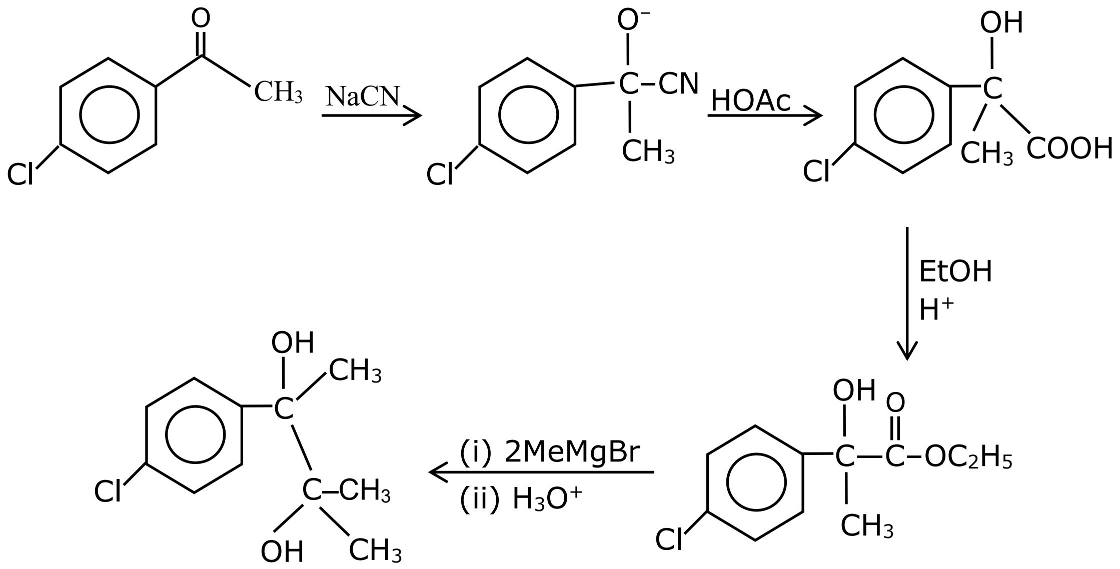
(1) 

(2) 

(3) 

(4) 

Sol. **2**



35. The primary and secondary valencies of cobalt respectively in $[Co(NH_3)_5Cl^{Cl}Cl_2$ are:
(1) 3 and 6 (2) 2 and 6 (3) 3 and 5 (4) 2 and 8

Sol. **1**
$[CO(NH_3)_5Cl]Cl_2$

36. An ammoniacal metal salt solution gives a brilliant red precipitate on addition of dimethylglyoxime. The metal ion is:
(1) Co^{2+} (2) Ni^{2+} (3) Fe^{2+} (4) Cu^{2+}

Sol. **2**
$NiCl_2+NH_4OH+dmg \rightarrow$ Rosy red ppt
$[Ni(dmg)_2]$

37. Reaction of BeO with ammonia and hydrogen fluoride gives A which on thermal decomposition gives BeF_2 and NH_4 F. What is 'A' ?
(1) $(NH_4)_2BeF_4$ (2) H_3NBeF_3 (3) $(NH_4)Be_2\ F_5$ (4) $(NH_4)BeF_3$

Sol. **1**
$(NH_4)_2\ BeF_4 \xrightarrow{\Delta} BeF_2+NH_4F$

38. Match List I with List II

LIST I		LIST II	
A.	Reverberatory furnace	I.	Pig Iron
B.	Electrolytic cell	II.	Aluminum
C.	Blast furnace	III.	Silicon
D.	Zone Refining furnace	IV.	Copper

Choose the correct answer from the options given below:
(1) A-IV, B-II, C-I, D-III (2) A-I, B-III, C-II, D-IV
(3) A-III, B-IV, C-I, D-II (4) A-I, B-IV, C-II, D-III

Sol. **1**

Reverberatory furnance → Cr

Electrolysis cell → Ar

Blast furnance → Pig iron

Zone refining furnance → silicon

39. Match List I with List II

LIST I		LIST II	
A.	Chlorophyll	I.	Na_2CO_3
B.	Soda ash	II.	$CaSO_4$
C.	Dentistry, Ornamental work	III.	Mg^{2+}
D.	Used in white washing	IV.	$Ca(OH)_2$

Choose the correct answer from the options given below:

(1) A-II, B-I, C-III, D-IV (2) A-III, B-I, C-II, D-IV

(3) A-II, B-III, C-IV, D-I (4) A-III, B-IV, C-I, D-II

Sol. **2**

Chlrophyl → Mg^{2+}

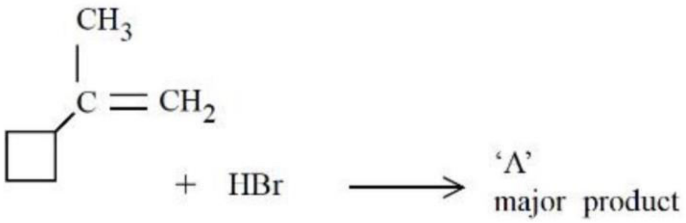
Sodaash → Na_2CO_3

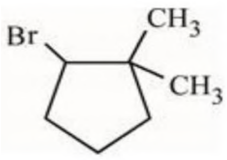
Destistry &

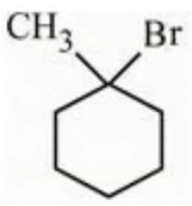
ornamental work → $CaSO_4$

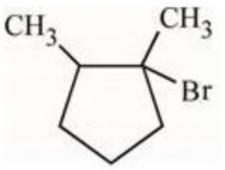
White washing → $Ca(OH)_2$

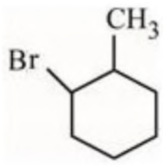
40. In the following given reaction, ' A ' is



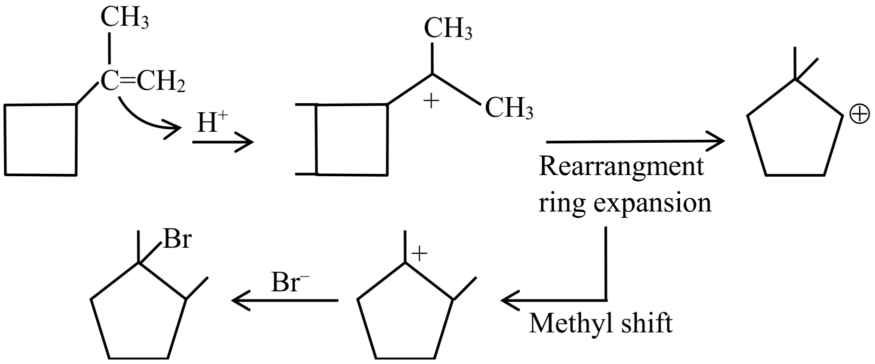
(1)


(2)


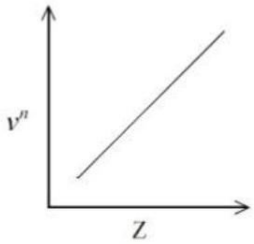
(3)


(4)


Sol. **3**



41. It is observed that characteristic X-ray spectra of elements show regularity. When frequency to the power " n " i.e. υ^n of X-rays emitted is plotted against atomic number " Z ", following graph is obtained.



The value of " n " is

(1) 3 (2) 2 (3) 1 (4) $\frac{1}{2}$

Sol. **4**

$\sqrt{\text{v}} \propto \text{z}$

$\text{v}^{\text{n}} \propto \text{z}$

n = 1/2

42. Given below are two statements:

Statement I : Noradrenaline is a neurotransmitter.

Statement II : Low level of noradrenaline is not the cause of depression in human.

In the light of the above statements, choose the correct answer from the options given below

(1) Statement I is correct but Statement II is incorrect

(2) Both Statement I and Statement II are correct

(3) Both Statement I and Statement II are incorrect

(4) Statement I is incorrect but Statement II is correct

Sol. **1**

Fact

43. Which of the Phosphorus oxoacid can create silver mirror from $AgNO_3$ solution?

(1) $(HPO_3)_n$ (2) $H_4P_2O_6$ (3) $H_4P_2O_5$ (4) $H_4P_2O_7$

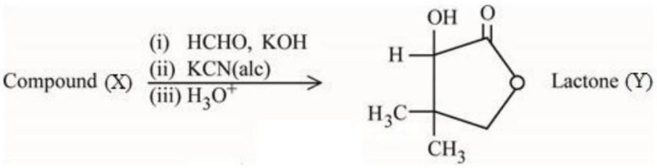
Sol. **3**

Silver mirror test can gives by p^{+3}, p^{+1} ox acid

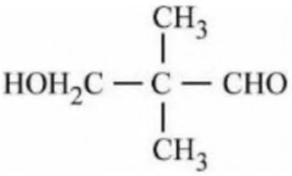
$H_4\overset{+3}{P_2}O_5 + Ag_2O \rightarrow Ag$

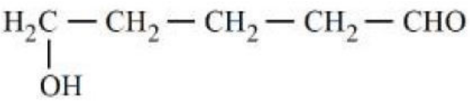
Silver mirror

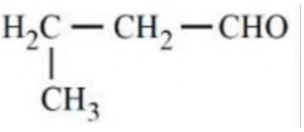
44. Compound (X) undergoes following sequence of reactions to give the Lactone (Y).

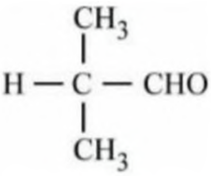


Compound (X) is

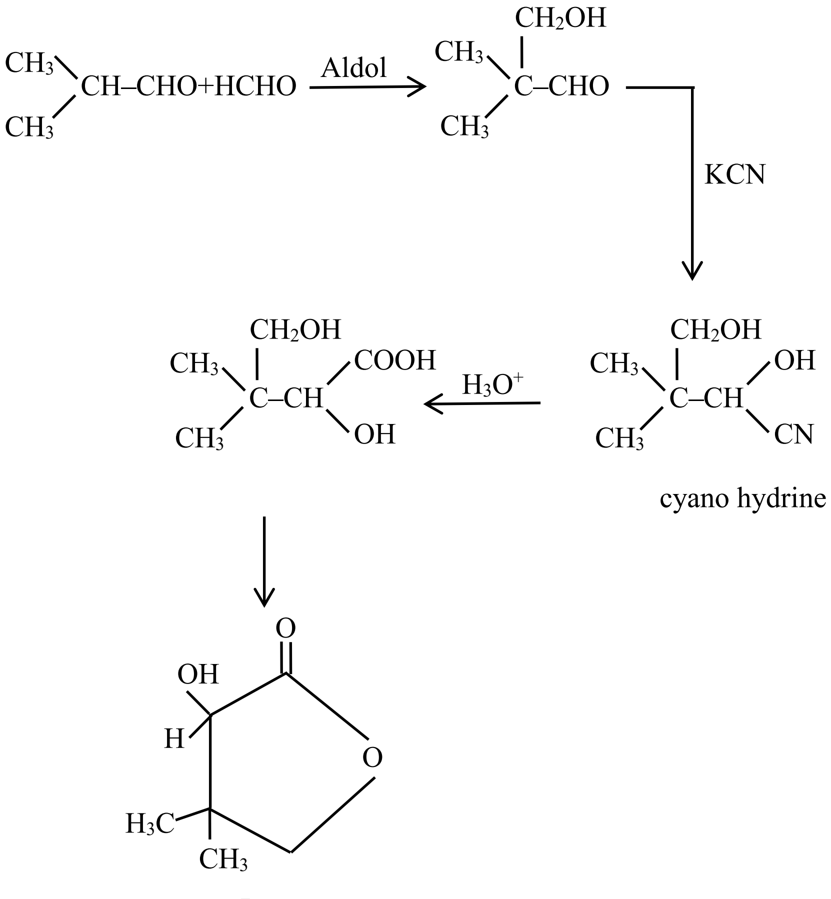
(1)

$\mathrm{HOH_2C-\overset{CH_3}{\underset{CH_3}{C}}-CHO}$

(2)

$\mathrm{H_2\underset{OH}{C}-CH_2-CH_2-CH_2-CHO}$

(3)

$\mathrm{H_2\underset{CH_3}{C}-CH_2-CHO}$

(4)

$\mathrm{H-\overset{CH_3}{\underset{CH_3}{C}}-CHO}$

Sol. **4**



Lactone

45. Order of Covalent bond;

A. KF > KI; LiF > KF

B. KF < KI; LiF > KF

C. $SnCl_4 > SnCl_2$; CuCl > NaCl

D. LiF > KF; CuCl < NaCl

E. KF < KI; CuCl > NaCl

Choose the correct answer from the options given below:

(1) C, E only (2) B, C, E only (3) A, B only (4) B, C only

Sol. **2**

$$\left.\begin{array}{l}\text{Small size of + ve ion}\\ \text{longer size of – ve ion}\end{array}\right\rangle \text{move covalent according to fajan's rule}$$

46. Which of the following is true about freons?
(1) These are radicals of chlorine and chlorine monoxide
(2) These are chemicals causing skin cancer
(3) These are chlorofluorocarbon compounds
(4) All radicals are called freons

Sol. **3**
Freons → chlorofluorocarbon compounds

47. In the depression of freezing point experiment
A. Vapour pressure of the solution is less than that of pure solvent
B. Vapour pressure of the solution is more than that of pure solvent
C. Only solute molecules solidify at the freezing point
D. Only solvent molecules solidify at the freezing point
Choose the most appropriate answer from the options given below:
(1) A and C only (2) A only (3) A and D only (4) B and C only

Sol. **3**
On adding non-volatile solute to pure solvent, depression in freezing point and lowering in vapour pressure occurs.

48. **Statement I :** For colloidal particles, the values of colligative properties are of small order as compared to values shown by true solutions at same concentration.
Statement II : For colloidal particles, the potential difference between the fixed layer and the diffused layer of same charges is called the electrokinetic potential or zeta potential.
In the light of the above statements, choose the correct answer from the options given below
(1) Options 1. Statement I is false but Statement II is true
(2) Statement I is true but Statement II is false
(3) Both Statement I and Statement II are true
(4) Both Statement I and Statement II are false

Sol. **2**
These layers should be of opposite charges

49. **Assertion A :** Hydrolysis of an alkyl chloride is a slow reaction but in the presence of NaI, the rate of the hydrolysis increases.
Reason R : $I^-$is a good nucleophile as well as a good leaving group.
In the light of the above statements, choose the correct answer from the options given below
(1) **A** is false but **R** is true
(2) **A** is true but **R** is false
(3) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**
(4) Both **A** and **R** are true and **R** is the correct explanation of **A**

Sol. **3**
The rate of hydrolysis of alkyl chloride improves because of better Nucleophilicity of I^-.

50. The magnetic moment of a transition metal compound has been calculated to be 3.87 B.M. The metal ion is
(1) Cr^{2+} (2) Ti^{2+} (3) V^{2+} (4) Mn^{2+}

Sol. **3**
$\sqrt{n(n+2)} = 3.87$
n = 3 = no. of unpaired e^-
$Cr^{2+} = [Ar]\ 3d^4$
$Tr^{-2+} = [Ar]\ 3d^2$
$V^{2+} = [Ar]\ 3d^3$

Mn^{2+} = [Ar] $3d^5$

SECTION - B

51. When $Fe_{0.93}O$ is heated in presence of oxygen, it converts to Fe_2O_3. The number of correct statement/s from the following is

A. The equivalent weight of $Fe_{0.93}O$ is $\frac{\text{Molecular weight}}{0.79}$

B. The number of moles of Fe^{2+} and Fe^{3+} in 1 mole of $Fe_{0.93}O$ is 0.79 and 0.14 respectively

C. $Fe_{0.93}O$ is metal deficient with lattice comprising of cubic closed packed arrangement of O^{2-} ions

D. The % composition of Fe^{2+} and Fe^{3+} in $Fe_{0.93}O$ is 85% and 15% respectively

Sol. **4**

$Fe_{0.93}O$

$2x+(0.93-x)3 = 2$

$-x+3\times0.93=2$

$x = 0.79$

0.79 = no. of Fe^{2+} ion

0.14 = no. of Fe^{3+} ion

nf = 0.79

Equivalent wt = $\frac{\text{Molecular weight}}{0.79}$

Due to presence of Fe^{3+} in FeO lattice, Metal deficiency occurs.

% Composition :- Fe^{2+} ions $= \frac{0.79}{0.93}\times100$

85%

Fe^{3+} ion = $\frac{0.14}{0.93}\times100$

15%

52. The number of correct statement/s from the following is

A. Larger the activation energy, smaller is the value of the rate constant.

B. The higher is the activation energy, higher is the value of the temperature coefficient.

C. At lower temperatures, increase in temperature causes more change in the value of k than at higher temperature

D. A plot of ln kvv$\frac{1}{T}$ is a straight line with slope equal to $-\frac{E_a}{R}$

Sol. 4

$K = Ae^{-Ea/RT}$

Here, Ea ↑ K↓

lnK = lnA –Ea/RT

slope of lnK vs 1/T = -Ea/R

The higher is the activation energy, higher is the value of the temperature coefficient.

53. For independent processes at 300 K

Process	$\Delta H/kJ\ mol^{-1}$	$\Delta S/J\ K^{-1}$
A	−25	−80
B	−22	40
C	25	−50
D	22	20

The number of non-spontaneous processes from the following is

Sol. **2**

For process A

$\Delta G = -25\times10^3-300(-80)$

$= -25000+24000$

$= -1000 \Rightarrow \Delta G<0$ spontaneous

For process B

$\Delta G = -22\times10^3-300(40)$

$= -22000-12000 \Rightarrow \Delta G<0$ spontaneous

For process C

$\Delta G = 25\times10^3-300\ (-50)$

$= 25000+15000=40000$ J

$\Delta G>0 \Rightarrow$ Non-spontaneous

For process D

$\Delta G = 22\times10^3-300(20)$

$\Delta G>0 \Rightarrow$ Non-spontaneous.

54. 5 g of NaOH was dissolved in deionized water to prepare a 450 mL stock solution. What volume (in mL) of this solution would be required to prepare 500 mL of 0.1M solution?

Given: Molar Mass of Na, O and H is 23,16 and 1 g mol^{-1} respectively

Sol. **180**

Molarity of stock solution

$$= \frac{5/40}{450}\times1000$$

$$= \frac{50}{4\times45} = \frac{10}{36}\text{M}$$

$M_1V_1 = M_2V_2$

$$\frac{10}{36}\times V = 0.1\times500$$

$$V = \frac{50\times36}{10} = 180 \text{ ml}$$

55. If wavelength of the first line of the Paschen series of hydrogen atom is 720 nm, then the wavelength of the second line of this series is nm. (Nearest integer)

Sol. **492**

Paschen series :-

z = 1

I^{st} line :- 4→3

$$\frac{1}{\lambda} = R\times(1)^2\left(\frac{1}{3^2}-\frac{1}{4^2}\right)$$

$$\frac{1}{720} = R\left(\frac{7}{144}\right) \qquad (1)$$

II^{nd} line → 5 → 3

$$\frac{1}{\lambda} = R\times(1)\left(\frac{1}{3^2}-\frac{1}{5^2}\right)$$

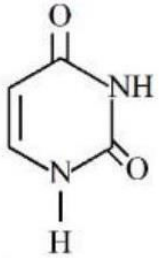
$$\frac{1}{\lambda} = R\left(\frac{16}{225}\right) \qquad (2)$$

Equation (2) ÷ equation (1)

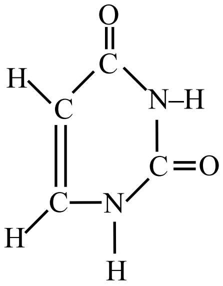
$$\frac{\lambda}{720} = \frac{7}{144}\times\frac{225}{16}$$

$\lambda = 492.18$

56. Uracil is a base present in RNA with the following structure. % of N in uracil is



Sol. **25**



Moleculer formula of uracil

= $C_4N_2H_4O_2$

% of N = $\frac{28}{112}\times 100 = 25\%$

57. The dissociation constant of acetic acid is $x \times 10^{-5}$. When 25 mL of $0.2 M CH_3COONa$ solution is mixed with 25 mL of $0.02 M CH_3COOH$ solution, the pH of the resultant solution is found to be equal to 5 . The value of x is

Sol. **10**

$K_a = x\times10^{-5}$

$CH_3COOH \rightarrow 0.02$ M & 25 ml

$CH_3COONa \rightarrow 0.2$ M and 25 ml

$$pH = p^{K_a} + \log\frac{[salt]}{[acid]}$$

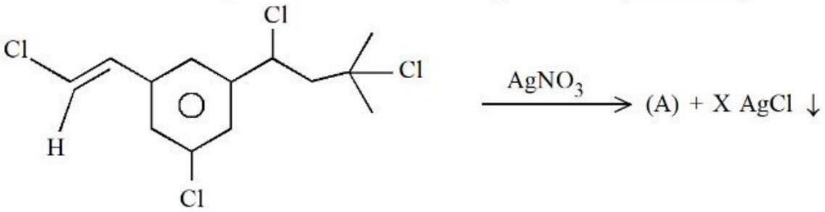
$$5 = p^{K_a} + \log\frac{0.2\times25}{0.02\times25} = p^{K_a} + \log 10$$

$p^{K_a} = 4$

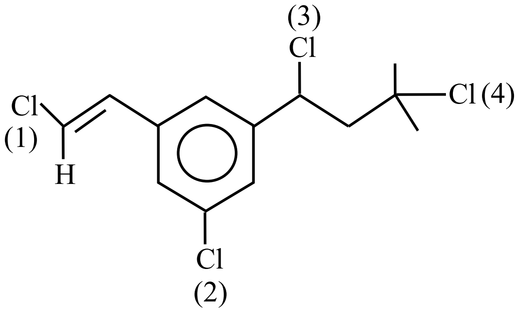
$K_a = 10^{-4} = 10\times10^{-5}$

Hence x = 10

58. Number of moles of AgCl formed in the following reaction is ________



Sol. **2**

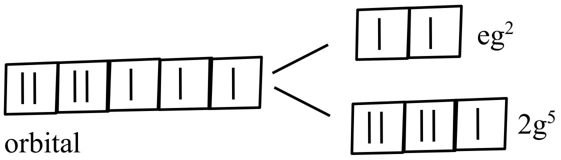


After treat with $AgNO_3$, Cl^- remove and possible carbocation will form
Position 1 : Vinyllic carbocation forms, unstable, So not possible
Position 2 : sp^2 hybridised carbocation is unstable, so not possible
Position 3 : Forms 2° carbocation which will be in conjugation with ring
Position 4 : 3° stable carbocation will form.

59. The d-electronic configuration of $[CoCl_4]^{2-}$ in tetrahedral crystal field is $e^m t_2^n$. Sum of "m" and "number of unpaired electrons" is

Sol.

$CO^{+3} \rightarrow 3d^7$



60. At 298 K, a 1 litre solution containing 10mmol of $Cr_2O_7^{2-}$ and 100mmol of Cr^{3+} shows a pH of 3.0.
Given: $Cr_2O_7^{2-} \rightarrow Cr^{3+}$; $E^\circ = 1.330$ V and $\frac{2.303RT}{F} = 0.059$ V
The potential for the half cell reaction is $x \times 10^{-3}$ V. The value of x is

Sol. **917**

$14H^+ + Cr_2O_7^{2-} + 6e^- \rightarrow 2Cr^{3+} + 7H_2O$

$$E = E^o - \frac{2.303RT}{6F}\log\frac{\left[Cr^{3+}\right]^2}{\left[Cr_2O_7^{2-}\right]\left[H^+\right]^{14}}$$

pH = 3

$[H^+]=10^{-3}$

$$E = 1.330 - \frac{0.059}{6}\log\frac{10^{-2}}{10^{-2}\left(10^{-42}\right)}$$

$E = 0.917$

$= 917\times10^{-3}$

$x = 917$

Mathematics

SECTION - A

61. Let $\vec{u}=\hat{i}-\hat{j}-2\hat{k}, \vec{v}=2\hat{i}+\hat{j}-\hat{k}, \vec{v}\cdot\vec{w}=2$ and $\vec{v}\times\vec{w}=\vec{u}+\lambda\vec{v}$. Then $\vec{u}\cdot\vec{w}$ is equal to

(1) 2 (2) $\frac{3}{2}$ (3) 1 (4) $-\frac{2}{3}$

Sol. **(3)**

$$\bar{v}\cdot\bar{w}=\bar{u}+\lambda\bar{v}$$

$$\bar{v}\cdot\bar{w}\ \ \cdot\bar{v}=\bar{u}\cdot\bar{v}+\lambda\bar{v}\cdot\bar{v}$$

$$0=\ 2-1+2\ \ +\lambda\ \ 4+1+1$$

$$\lambda=\frac{-3}{6}\Rightarrow\lambda=\frac{-1}{2}$$

Now

$$\bar{v}\times\bar{w}\ \ =\bar{u}+\lambda\bar{v}$$

$$\bar{v}\times\bar{w}\ \ \cdot\bar{w}=\bar{u}\cdot\bar{w}+\lambda\bar{v}\cdot\bar{w}$$

$$0=\bar{u}\cdot\bar{w}+\lambda\ \ 2$$

$$\bar{u}\cdot\bar{w}=-2\lambda=1$$

62. $\lim\limits_{t\to0}\left(1^{\frac{1}{\sin^2 t}}+2^{\frac{1}{\sin^2 t}}+\cdots+n^{\frac{1}{\sin^2 t}}\right)^{\sin^2 t}$ is equal to

(1) n^2 (2) $\frac{n(n+1)}{2}$ (3) n (4) n^2+n

Sol. **(3)**

$$\underset{t\to0}{\mathrm{lt}}\left(1^{\frac{1}{\sin^2 t}}+2^{\frac{1}{\sin^2 t}}+.......+n^{\frac{1}{\sin^2 t}}\right)^{\sin^2 t}$$

$$=\underset{t\to0}{\mathrm{lt}}\ n\left(\left(\frac{1}{n}\right)^{\frac{1}{\sin^2 t}}+\left(\frac{2}{n}\right)^{\frac{1}{\sin^2 t}}+.......+\left(\frac{n-1}{n}\right)^{\frac{1}{\sin^2 t}}+1\right)^{\sin^2 t}$$

$$=n\cdot[0+0+...+1]^0$$

$$=\boxed{n}$$

63. Let α be a root of the equation $(a-c)x^2+(b-a)x+(c-b)=0$ where a, b, c are distinct real numbers such that the matrix $\begin{bmatrix}\alpha^2 & \alpha & 1\\ 1 & 1 & 1\\ a & b & c\end{bmatrix}$ is singular. Then, the value of $\frac{(a-c)^2}{(b-a)(c-b)}+\frac{(b-a)^2}{(a-c)(c-b)}+\frac{(c-b)^2}{(a-c)(b-a)}$ is

(1) 12 (2) 9 (3) 3 (4) 6

Sol. **(3)**

$(a-c)\,x^2 + (b-a)\,x + (c-b) = 0$ $\qquad (a \neq c)$

$\boxed{x=1}$ is one root & other root is $\boxed{\dfrac{c-b}{a-c}}$ ….(1)

now $\begin{vmatrix} \alpha^2 & \alpha & 1 \\ 1 & 1 & 1 \\ a & b & c \end{vmatrix}$ is singular

$\Rightarrow \begin{vmatrix} \alpha^2 & \alpha & 1 \\ 1 & 1 & 1 \\ a & b & c \end{vmatrix} = 0 \Rightarrow \alpha^2(c-b) - \alpha\,(c-a) + (b-a) = 0$

$\Rightarrow \alpha^2\,(c-b) + \alpha\,(a-c) + (b-a) = 0$

satisfied by $\boxed{\alpha=1}$ or $\boxed{\alpha = \dfrac{b-a}{c-b}}$ …..(2)

Now, if $\alpha = 1$ then $\forall\ a \neq b \neq c$

$$\sum \frac{(a-c)^2}{(b-a)(c-b)} = \frac{\Sigma(a-c)^3}{(a-b)(b-c)(c-a)}$$

$$= \frac{3(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)}$$

$= 3$

[if $A+B+C=0 \quad \Rightarrow \quad A^3+B^3+C^3 = 3ABC$]

64. The area enclosed by the curves $y^2 + 4x = 4$ and $y - 2x = 2$ is :

(1) 9 (2) $\frac{22}{3}$ (3) $\frac{23}{3}$ (4) $\frac{25}{3}$

Sol. **(1)**

$y^2 + 4x = 4$ & $y = 2 + 2x$

P: $y^2 = 4\,(1-x)$ L : $y = 2\,(1+x)$

Now

$y^2 + 4\left(\dfrac{y}{2} - 1\right) = 4$

$y^2 + 2y - 8 = 0$

$(y+4)\,(y-2) = 0$

required Area $\quad A = \displaystyle\int_{-4}^{2} \left[\left(\frac{4-y^2}{4}\right) - \left(\frac{y-2}{2}\right)\right] dy$

$$A = \int_{-4}^{2} \left(2 - \frac{y^2}{4} - \frac{y}{2}\right) dy$$

$$= \left[2y - \frac{y^3}{12} - \frac{y^2}{4}\right]_{-4}^{2}$$

$$= \left(4 - \frac{8}{12} - 1\right) - \left(-8 + \frac{64}{12} - 4\right)$$

$A = \boxed{9}$

65. Let p, q ∈ ℝ and $(1-\sqrt{3}i)^{200} = 2^{199}(p+iq), i=\sqrt{-1}$ Then $p + q + q^2$ and $p - q + q^2$ are roots of the equation

(1) $x^2 - 4x - 1 = 0$ (2) $x^2 - 4x + 1 = 0$ (3) $x^2 + 4x - 1 = 0$ (4) $x^2 + 4x + 1 = 0$

Sol. **(2)**

$$\left(1-\sqrt{3}i\right)^{200} = 2^{199}\left(p+iq\right)$$

$$\Rightarrow 2^{200}\text{cis}\left(\frac{-\pi}{3}\right)^{200} = 2^{199}\left(p+iq\right)$$

$$\Rightarrow 2^{200}\left(\text{cis}\left(-\frac{200\pi}{3}\right)\right) = 2^{199}\left(p+iq\right)$$

$$\Rightarrow 2\left(\text{cis}\left(-66\pi-\frac{2\pi}{3}\right)\right) = \left(p+iq\right)$$

$$\Rightarrow 2\left[\text{cis}\left(\frac{-2\pi}{3}\right)\right] = \left(p+iq\right)$$

$$\Rightarrow 2\left[\frac{-1}{2}-\frac{\sqrt{3}i}{2}\right] = \left(p+iq\right)$$

$$\Rightarrow p=-1,\ q=-\sqrt{3}$$

Now

$\alpha = p + q + q^2 = 2 - \sqrt{3}$

$\beta = p - q + q^2 = 2 + \sqrt{3}$

req. quad is $x^2 - 4x + 1 = 0$

66. Let N denote the number that turns up when a fair die is rolled. If the probability that the system of equations

$x + y + z = 1$

$2x + Ny + 2z = 2$

$3x + 3y + Nz = 3$

has unique solution is $\frac{k}{6}$, then the sum of value of k and all possible values of N is

(1) 21 (2) 18 (3) 20 (4) 19

Sol. **(3)**

for unique solu.

$\Delta \neq 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & N & 2 \\ 3 & 3 & N \end{vmatrix} \neq 0$$

$\Rightarrow (N^2 - 6) - (2N - 6) + (6 - 3N) \neq 0$

$\Rightarrow N^2 - 5N + 6 \neq 0$

$\Rightarrow N \neq 3$ & $N \neq 2$

Hence N can be {1, 4, 5, 6} Fav case : $\frac{4}{6} = \frac{k}{6} \Rightarrow \boxed{k=4}$ Sum = $\boxed{20}$

67. For three positive integers p, q, r, $x^{pq^2} = y^{qr} = z^{p^2 r}$ and r = pq + 1 such that 3,$3\log_y x$, $3\log_z y$, $7\log_x z$ are in A.P. with common difference $\frac{1}{2}$. Then r – p – q is equal to

(1) –6 (2) 12 (3) 6 (4) 2

Sol. **(4)**

$x^{pq^2} = y^{qr} = z^{p^2 r}$ & r = pq + 1

$3,\ 3\log_y^x,\ 3\log_z^y,\ 7\log_x^z$ are in A.P.

Now

$$3\log_y^x = 3 + \frac{1}{2} = \frac{7}{2} \Rightarrow \log_y^x = \frac{7}{6}$$

$x^6 = y^7$ ......(i)

$$3\log_z^y = 3 + 1 = 4 \Rightarrow \log_z^y = \frac{4}{3}$$

$y^3 = z^4$ ......(2)

$$7\log_x^z = 3 + \frac{3}{2} = \frac{9}{2} \Rightarrow \log_x^z = \frac{19}{14}$$

$z^{14} = x^9$ ......(3)

Now

$$x^{pq^2} = x^{\frac{6}{7}qr} = x^{\frac{9p^2r}{14}}$$

$$pq^2 = \frac{6}{7}qr = \frac{9}{14}p^2 r$$

$pq = \frac{6}{7}r$ $q^2 = \frac{9}{14}pr$

$r = pq + 1$ $\Rightarrow q^3 = \frac{9}{14}\frac{6}{7}r \cdot r$

$\Rightarrow r = \frac{6}{7}r + 1$

$\Rightarrow \boxed{r = 7}$ $\Rightarrow \boxed{q = 3}$

Now

$r - p - q$

$= 7 - 2 - 3$

$= \boxed{2}$

68. The relation $R = \{(a, b) : \gcd(a, b) = 1, 2a \neq b, a, b \in Z\}$ is :

(1) reflexive but not symmetric

(2) transitive but not reflexive

(3) symmetric but not transitive

(4) neither symmetric nor transitive

Sol. **(4)**

gcd (a, b) = 1, 2a ≠ b

reflexive gcd (a, a)= a Not possible

symmetric gcd (b, a) = 1 & 2a ≠ b Not possible

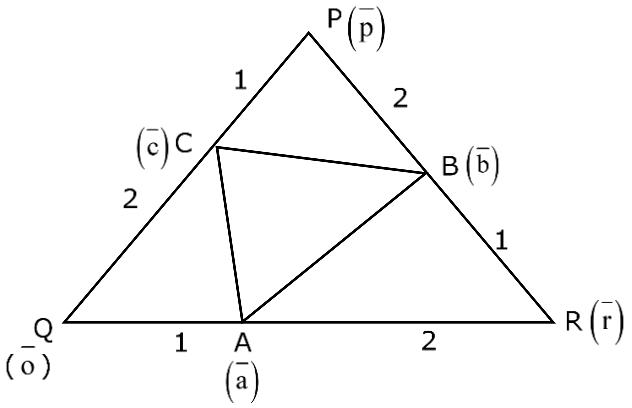
$\boxed{a = 2, b = 1}$

transitive (a, b) = (2, 3) gcd {a, b}= 1, 2a ≠ b
(b, c) = (3, 4) gcd {c, d}= 1, 2a ≠ c
(a, c) = (2, 4) gcd {2, 4}= 2, 2a = c
Not possible

69. Let PQR be a triangle. The points A, B and C are on the sides QR, RP and PQ respectively such that $\frac{QA}{AR} = \frac{RB}{BP} = \frac{PC}{CQ} = \frac{1}{2}$. Then $\frac{\text{Area }(\Delta PQR)}{\text{Area }(\Delta ABC)}$ is equal to

(1) 4 (2) 3 (3) (4) 2

Sol. **(2)**



$$\bar{a} = \frac{\bar{r}}{3}$$

$$\bar{b} = \frac{\bar{b} + 2\bar{r}}{3}$$

$$\bar{c} = \frac{2\bar{p}}{3}$$

$$\Delta\,PQR = \frac{1}{2}\left|\bar{r} \times \bar{p}\right|$$

$$\Delta\,PQR = \frac{1}{2}\left|\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a}\right|$$

$$= \frac{1}{2}\left|\frac{\bar{r} \times \bar{p}}{9} + \frac{4\left(\bar{r} \times \bar{p}\right)}{9} + \frac{2}{9}\bar{p} \times \bar{r}\right|$$

$$= \frac{1}{18}\left|3\left(\bar{r} \times \bar{p}\right)\right|$$

Hence $\frac{|\Delta PQR|}{|\Delta ABC|} = 3$

70. Let $y = y(x)$ be the solution of the differential equation $x^3 dy + (xy - 1)dx = 0, x > 0, y\left(\frac{1}{2}\right) = 3 - \text{e}$. Then y(1) is equal to

(1) 1 (2) e (3) 3 (4) 2 – e

Sol. **(1)**

$$x^3\, dy + (xy - 1)\, dx = 0$$

$$x^3 \frac{dy}{dx} = 1 - xy$$

$$x^3 \frac{dy}{dx} + xy = 1$$

$$\frac{dy}{dx} + \frac{y}{x^2} = \frac{1}{x^3}\Big|_{\text{LDE}}$$

$$\text{I.F} = e^{\int \frac{1}{x^2}dx} = e^{-\frac{1}{x}}$$

$$y \cdot e^{\frac{-1}{x}} = \int e^{\frac{-1}{x}} \cdot \frac{1}{x^3}dx$$

$$\frac{-1}{x} = t$$

$$= -\int e^t t\, dt$$

$$y \cdot e^{\frac{-1}{x}} = -e^t(t-1) + k$$

$$y \cdot e^{\frac{-1}{x}} = -e^{\frac{-1}{x}}\left(\frac{-1}{x} - 1\right) + k$$

$$y = \left(\frac{1}{x} + 1\right) + k\, e^{\frac{1}{x}}$$

put $x = \frac{1}{2}$ $\Rightarrow$ $3 - e = (2+1) + k\, e^2$

$$k = -\frac{1}{e}$$

Now $y(1) = 2 + \frac{-1}{e}e$

$= \boxed{1}$

71. If A and B are two non-zero n × n matrics such that $A^2 + B = A^2\, B$, then

(1) $A^2 = I$ or $B = I$ (2) $A^2B = I$ (3) $AB = I$ (4) $A^2B = BA^2$

Sol. **(4)**

$A^2 + B = A^2B$ …..(1)

$A^2 - A^2B + B = 0$

$A^2 - A^2B - (I - B) = -I$

$(I - A^2)\,(I - B) = I$

So,

$(I - A^2)$ & $(I - B)$ are inverses of each other

So

$(I - B)\,(I - A^2) = I$

$I - B - A^2 + BA^2 = I$

$BA^2 = B + A^2$ …..(2)

So, from (1) & (2)

$A^2B = BA^2$

72. The equation $x^2 - 4x + [x] + 3 = x[x]$, where $[x]$ denotes the greatest integer function, has :

(1) a unique solution in $(-\infty, 1)$ (2) no solution

(3) exactly two solutions in $(-\infty, \infty)$ (4) a unique solution in $(-\infty, \infty)$

Sol. **(4)**

$x^2 - 4x + [x] + 3 = x\,[x]$

$x^2 - 4x + 3 = (x - 1)\,[x]$

$(x - 1)\,(x - 3) = (x - 1)\,[x]$

$\boxed{x = 1}$ or $x - 3 = [x]$

$x - [x] = 3$

$\{x\} = 3$

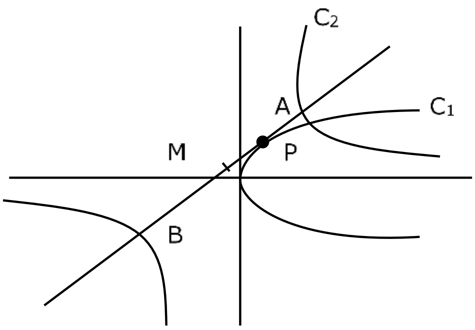
73. Let a tangent to the curve $y^2 = 24x$ meet the curve $xy = 2$ at the points A and B. Then the mid points of such line segments AB lie on a parabola with the

(1) Length of latus rectum $\frac{3}{2}$ (2) directrix $4x = -3$

(3) length of latus rectum 2 (4) directrix $4x = 3$

Sol. **(4)**

$c_1 : y^2 = 24\,x$ & $c_2 : xy = 2$



AB : [Tangent to parabola at p(t)]

$ty = x + 6t^2$ …..(1)

AB : [chord with given mid point of hyperbola]

$T = S_1$

$\frac{x}{h} + \frac{y}{k} = 2$ …..(2)

from (1) & (2)

$$\frac{-1}{\frac{1}{h}} = \frac{t}{\frac{1}{k}} = \frac{6t^2}{2}$$

$-h = kt = 3t^2$

$h = -3t^2$ & $k = 3t$

$h = -3\frac{k^2}{9} \Rightarrow \boxed{y^2 = -3x}$

ℓ (LR) = 3 & directrix is $\boxed{x = \frac{3}{4}}$

74. Let Ω be the sample space and $A \subseteq \Omega$ be an event.

Given below are two statements :

(S1) : If $P(A) = 0$, then $A = \emptyset$

(S2): If $P(A) = 1$, then $A = \Omega$

Then

(1) both (S1) and (S2) are true

(2) only (S1) is true

(3) only (S2) is true

(4) both (S1) and (S2) are false

Sol. **(4)**

Let $\Omega = [0, 1]$

Let A → selecting $\frac{1}{2}$

$A = \left\{\frac{1}{2}\right\}$

then, $P(A) = 0$ but $A \neq \phi$

$B = A^c = [0,1] - \left\{\frac{1}{2}\right\}$

$P(B) = 1$

but $B \neq \Omega$

Ans = 4

75. The value of $\sum_{r=0}^{22} {}^{22}C_r\ {}^{23}C_r$ is

(1) ${}^{44}C_{23}$	(2) ${}^{45}C_{23}$
(3) ${}^{44}C_{22}$	(4) ${}^{45}C_{24}$

Sol. **(2)**

$$\sum_{r=0}^{22} {}^{22}C_r \cdot {}^{23}C_r$$

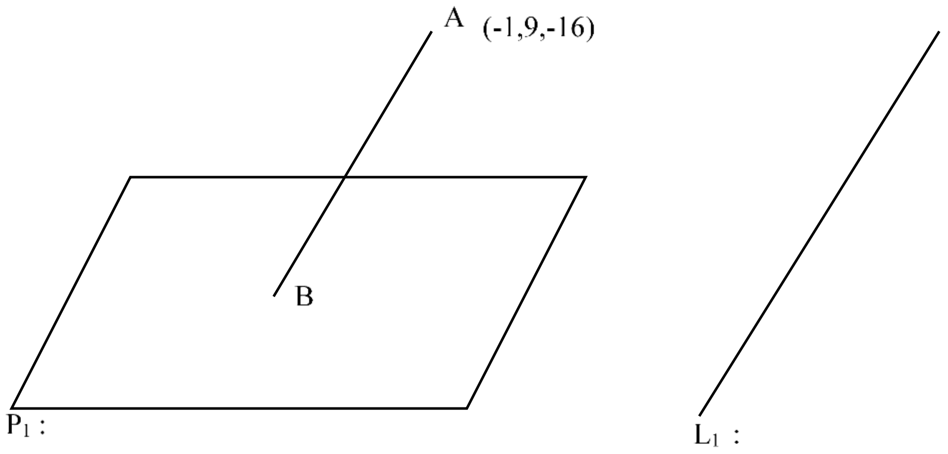
$$= \sum_{r=0}^{22} {}^{22}C_r \cdot {}^{23}C_r$$

$${}^{45}C_{22} = {}^{45}C_{23}$$

76. The distance of the point $(-1,9,-16)$ from the plane $2x + 3y - z = 5$ measured parallel to the line $\frac{x+4}{3} = \frac{2-y}{4} = \frac{z-3}{12}$ is

(1) 31	(2) $13\sqrt{2}$
(3) $20\sqrt{2}$	(4) 26

Sol. **(4)**



L_{AB} : $\frac{x+1}{3}=\frac{y-9}{-4}=\frac{z+16}{12}=t$

B : (3t – 1, 9 – 4t, 12t – 16) lies on plane

$2(3t-1)+3(9-4t)-(12t-16)=5$

$-18t-2+27+16=5$

$-18t+25+16=5$

$-18t=-36 \quad \Rightarrow \boxed{t=2}$

B : (5, 1, 8) & A : (–1, 9,–16)

$l(AB)=\sqrt{36+64+576} = \sqrt{676}=\boxed{26}$

77. $\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right)+\sec^{-1}\left(\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}\right)$ is equal to :

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{2}$

Sol. **(1)**

$$\tan^{-1}\left(\frac{1+\sqrt{3}}{\sqrt{3}\left(1+\sqrt{3}\right)}\right)+\sec^{-1}\left(\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}\right)$$

$$=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)+\sec^{-1}\sqrt{\frac{4\left(2+\sqrt{3}\right)}{3\left(2+\sqrt{3}\right)}}$$

$$=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)+\sec^{-1}\frac{2}{\sqrt{3}}$$

$$=\frac{\pi}{6}+\frac{\pi}{6}=\boxed{\frac{\pi}{3}}$$

78. Let $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & , x \neq 0 \\ 0 & , x = 0 \end{cases}$

Then at $x = 0$

(1) f is continuous but not differentiable

(2) f and f' both are continuous

(3) f' is continuous but not differentiable

(4) f is continuous but f' is not continuous

Sol. **(4)**

$$f(x) = \begin{cases} x^2 \sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$\to$ cont. of f(x) at x = 0

$$\left.\begin{aligned} &\text{LHL} = \underset{h\to 0}{\text{lt}}\ h^2 \sin\frac{1}{(-h)} = 0 \\ &\text{RHL} = \underset{h\to 0}{\text{lt}}\ h^2 \sin\frac{1}{(-h)} = 0 \\ &f(0) = 0 \end{aligned}\right\} \text{continuous at } x = 0$$

$\to$ Diff. of f(x) at x = 0

$$\text{RHD} = \underset{h\to 0}{\text{dt}} \frac{h^2 \sin\left(\frac{1}{h}\right) - 0}{h} = \underset{h\to 0}{\text{dt}}\ h\ \sin\frac{1}{h} = 0$$

$$\text{LHD} = \underset{h\to 0}{\text{dt}} \frac{h^2 \sin\left(\frac{-1}{h}\right) - 0}{-h} = \underset{h\to 0}{\text{dt}}\ h\ \sin\frac{1}{h} = 0$$

Hence f(x) is diff. at x = 0

Now diff. f(s) at x = 0

$$f'(x) = \begin{bmatrix} 2x \sin\left(\frac{1}{x}\right) + x^2 \cos\left(\frac{1}{x}\right)\left(\frac{-1}{x^2}\right) & x \neq 0 \\ 0 & x = 0 \end{bmatrix}$$

$$f'(x) = \begin{bmatrix} 2x \sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{bmatrix}$$

hence f'(x) limit ossicilate at x = 0

hence f'(x) is D.C. at $\boxed{x = 0}$

79. The compound statement $(\sim (P \wedge Q)) \vee ((\sim P) \wedge Q) \Rightarrow ((\sim P) \wedge (\sim Q))$ is equivalent to

(1) $(\sim Q) \vee P$

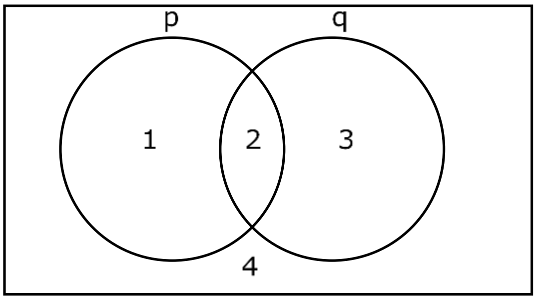
(2) $\left((\sim P) \vee Q\right) \wedge (\sim Q)$

(3) $(\sim P) \vee Q$

(4) $((\sim P) \vee Q) \wedge ((\sim Q) \vee P)$

Sol. **(4)**

$(\sim(p \wedge q) \vee (\sim p \wedge q)) \to \sim p \wedge \sim q$



$(1 + 2 + 4) + 2 \Rightarrow 4$

$(1 + 2 + 4) \Rightarrow 4 \qquad = \sim(1 + 2 + 4) + 4$

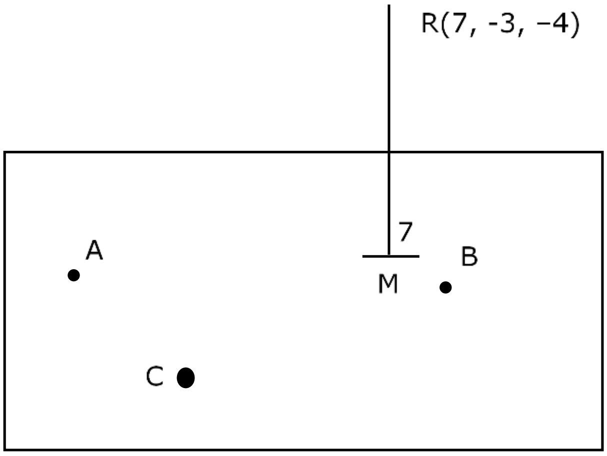
$= 3 + 4$

Ans. $\boxed{4}$

80. The distance of the point $(7,-3,-4)$ from the plane passing through the points $(2,-3,1), (-1,1,-2)$ and $(3,-4,2)$ is :

(1) 5 (2) 4 (3) $5\sqrt{2}$ (4) $4\sqrt{2}$

Sol. **(3)**



$$\bar{n}_p = \overline{AB} \times \overline{DC} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 3 & -4 & 3 \\ -1 & 1 & -1 \end{vmatrix} = <1, 0, -1>$$

Equation of plane P : $1(x-2) + 0\,(y+3) - (z-1) = 0$

P : $x - z - 1 = 0$

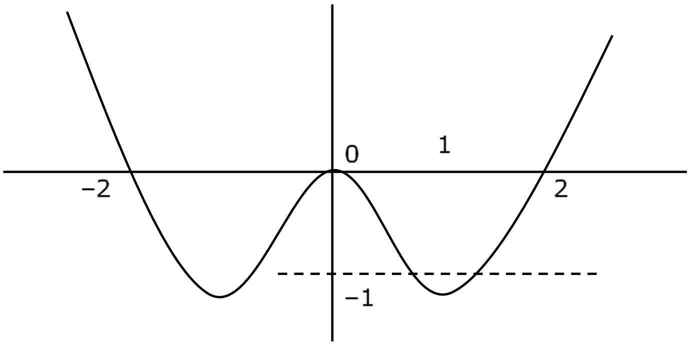
$$d(RM) = \left|\frac{7+4-1}{\sqrt{2}}\right| = 5\sqrt{2}$$

SECTION - B

81. Let $\lambda \in \mathbb{R}$ and let the equation E be $|x|^2 - 2|x| + |\lambda - 3| = 0$. Then the largest element in the set $S = \{x + \lambda : x$ is an integer solution of E$\}$ is

Sol. **5**

$|x|^2 - 2|x| + |\lambda - 3| = 0$



$|x|^2 - 2|x| = -|\lambda - 3|$

$$\left.\begin{array}{ll} \text{LHS} & = -1 \le |x|^2 - 2|x| < \infty \\ \text{RHS} & \quad\quad -|\lambda - 3| \le 0 \end{array}\right]$$

Now LHS = RHS only when

$0 \le |\lambda - 3| \le 1$ & $\boxed{x \in [-2, 2]}$

$-1 \le \lambda - 3 \le 1$

$\boxed{2 \le \lambda \le 4}$

82. Let a tangent to the curve $9x^2 + 16y^2 = 144$ intersect the coordinate axes at the points A and B. Then, the minimum length of the line segment AB is

Sol. **7**

$\frac{x^2}{16} + \frac{y^2}{9} = 1$

Elipse

Let P = $(4\cos\theta, 3\sin\theta)$

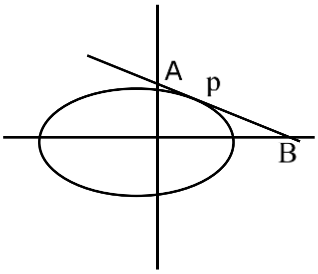
Tp: $\frac{x}{4}\cos\theta + \frac{y}{3}\sin\theta = 1$

A: $(0, 3\text{cosec}\theta)$, B : $(4\sec\theta, 0)$

$\ell(AB) = \sqrt{16\sec^2\theta + 9\text{cosec}^2\theta}$

$= \sqrt{16 + 9 + (4\tan\theta - 3\cot\theta)^2 + 24}$

$\ell(AB)_{\min} = 7$



83. The shortest distance between the lines $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-6}{2}$ and $\frac{x-6}{3}=\frac{1-y}{2}=\frac{z+8}{0}$ is equal to

Sol. **14**

$L_1 : \bar{a} =< 2,-1,6 > \quad L_2 : \bar{b} =< 6,1,-8 >$

$\bar{p} =< 3,2,2 > \qquad \bar{q} =< 3,-2,0 >$

$$\bar{p}\times\bar{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 2 \\ 3 & -2 & 0 \end{vmatrix} \begin{matrix} =< 4,6,-12 > \\ =< 2,3,-6 > \end{matrix} \ (\text{s.f.})$$

$$b\Delta = \left|\frac{\left(\bar{b}-\bar{a}\right).|\bar{p}\times\bar{q}|}{|\bar{p}\times\bar{q}|}\right|$$

$$= \left|\frac{\left(4\hat{i}+2\hat{j}-14\hat{k}\right).\left(2\hat{i}+3\hat{j}-6\hat{k}\right)}{\sqrt{4+9+36}}\right|$$

$$= \left|\frac{8+6+84}{\sqrt{49}}\right| = \left|\frac{98}{7}\right| = \boxed{14}$$

84. Suppose $\sum_{r=0}^{2023} r^2\ {}^{2023}C_r = 2023\times\alpha\times 2^{2022}$. Then the value of α is

Sol. **(1012)**

$$\sum_{r=0}^{n} r^2\ {}^{n}C_r$$

$$= \sum_{r=0}^{n} r^2 \cdot \frac{n}{r}\ {}^{n-1}C_{r-1}$$

$$= n\sum_{r=1}^{n}\left(\left(r-1\right)^{n-1}C_{r-1} + {}^{n-1}C_{r-1}\right)$$

$$= n\sum_{r=2}^{n}\left(n-1\right)^{n-2}C_{r-2} + n\sum_{r=1}^{n}{}^{n-1}C_{r-1}$$

$$= n\left(n-1\right)\left[2^{n-2}\right] + n\left[2^{n-1}\right]$$

$= 2023 \cdot 2022 \cdot 2^{2021} + 2023 \cdot 2^{2022}$

$= 2023 \cdot 2^{2021}\,[2022+2]$

$= 2023 \cdot 2^{2021} \cdot 2024$

$= 2023 \cdot 1012 \cdot 2^{2022} \Rightarrow \alpha = 1012$

85. The value of $\frac{8}{\pi}\int_0^{\frac{\pi}{2}} \frac{(\cos x)^{2023}}{(\sin x)^{2023}+(\cos x)^{2023}}dx$ is

Sol. **(2)**

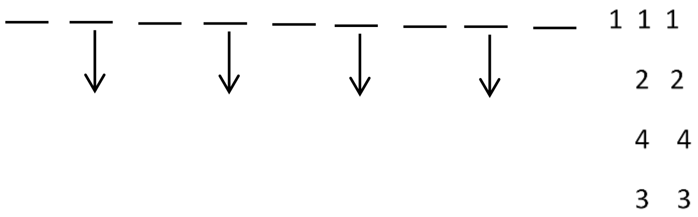
$$I = \frac{8}{\pi}\int_0^{\frac{\pi}{2}} \frac{\left(\cos x\right)^{2023}}{\underset{\downarrow KJ\&A}{\left(\sin x\right)^{2023}+\left(\cos x\right)^{2023}}}dx$$

$$2I = \frac{8}{\pi}\int_0^{\frac{\pi}{2}} 1dx$$

$$2I = \frac{8}{\pi}\cdot\frac{\pi}{2} \Rightarrow I = 2$$

86. The number of 9 digit numbers, that can be formed using all the digits of the number 123412341 so that the even digits occupy only even places, is

Sol. **(60)**



4 even place can be occupied by 4 even digits

No of always $= \frac{4:}{2!2!} = 6$

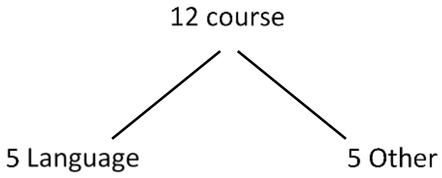
Odd place can be occupied by 5 odd digits

No of always $= \frac{5!}{3!2!} = 10$

Total no. $= 6 \times 10 = 60$

87. A boy needs to select five courses from 12 available courses, out of which 5 courses are language courses. If he can choose at most two language courses, then the number of ways he can choose five courses is

Sol. **(546)**



(0 language + 5 other) + (1 Language + 4 other) + (2 Language + 3 other)

$= 5_{C_0}.7_{C_5} + 5_{C_1}.7_{C_4} + 5_{C_2} \cdot 7_{C_3}$

$= 21 + 175 + 350$

$= 546$

88. The 4th term of GP is 500 and its common ratio is $\frac{1}{m}, m \in \mathrm{N}$. Let S_n denote the sum of the first n terms of this GP. If $S_6 > S_5 + 1$ and $S_7 < S_6 + \frac{1}{2}$, then the number of possible values of m is

Sol. **(12)**

$T_4 = 500 \Rightarrow a\left(\frac{1}{m}\right)^3 = 500 \Rightarrow a = 500\ m^3$

Now $S_n - S_{n-1} = a\left(\frac{1-r^n}{1-r}\right) - a\left(\frac{1-r^{n-1}}{1-r}\right)$

$= \frac{a}{1-r}\left[r^{n-1}(1-r)\right]$

$= a\, r^{n-1}$

$= 500\ m^3\left(\frac{1}{m}\right)^{n-1}$

$S_n - S_{n-1} = 500\ m^{4-n}$

Now $S_6 - S_5 > 1 \Rightarrow 500\ m^{-2} > 1$ … (1)

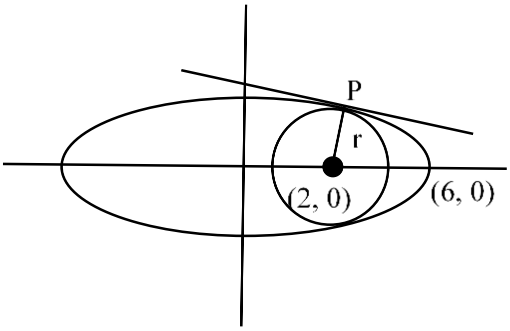
& $S_7 - S_6 < \frac{1}{2} \Rightarrow 500\ m^{-3} < \frac{1}{2}$ …(2)

$$\left.\begin{array}{ll} \text{from}(1) & m^2 < 500 \\ \text{from}(2) & m^3 > 1000 \end{array}\right] 10 < m \le 22$$

Number of possible values of m is = 12

89. Let C be the largest circle centred at $(2,0)$ and inscribed in the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$. If $(1, \alpha)$ lies on C, then $10\alpha^2$ is equal to

Sol. **(118)**



E: $\frac{x^2}{36} + \frac{y^2}{16} = 1$ & C: $(x-2)^2 + y^2 = r^2$

For largest circle r is maximum

P $(6\cos\theta, 4\sin\theta)$

N_P: $6x\sec\theta - 4y\,\text{cosec}\theta = 20$ pass (2, 0)

$12\sec\theta = 20 \Rightarrow \cos\theta = \frac{3}{5}$

Now P: $\left(6\times\frac{3}{5}, 4\times\frac{4}{5}\right) \Rightarrow$ P: $\left(\frac{18}{5}, \frac{16}{5}\right)$

$$r=\sqrt{\left(2-\frac{18}{5}\right)^2+\left(\frac{16}{5}\right)^2}$$

$$r=\frac{\sqrt{64+256}}{5}=\frac{8\sqrt{5}}{5}=\frac{8}{\sqrt{5}}$$

$$C:=(x-2)^2+y^2=\frac{64}{5}$$

Now $(1, \alpha)$ lies on C

$$\Rightarrow (1-2)^2+\alpha^2=\frac{64}{5}$$

$$\alpha^2=\frac{64}{5}-1$$

$$\alpha^2=\frac{59}{5}\Rightarrow 10\alpha^2=118$$

90. The value of $12\int_0^3|x^2-3x+2|dx$ is

Sol. **(22)**

$$I=12\int_0^3|x^2-3x+2|dx$$

$$I=12\int_0^3|(x-2)(x-1)|dx$$

$$=12\left[\int_0^1(x^2-3x+2)+\int_1^2-(x^2-3x+2)+\int_2^3(x^2-3x+2)\right]$$

$$=12\left[\left(\frac{1}{3}-\frac{3}{2}+2\right)-\left(\frac{7}{3}-\frac{9}{2}+2\right)+\left(\frac{19}{3}-\frac{15}{2}+2\right)\right]$$

$$=12\left[\frac{5}{6}+\frac{1}{6}+\frac{5}{6}\right]$$

$$=\frac{12\cdot 11}{6}$$

$$=22$$

JEE–MAIN EXAMINATION – JANUARY, 2023

(Held On Thursday 24th January, 2023) | **TIME : 3 : 00 PM to 6 : 00 PM**

Physics

SECTION - A

1. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.
Assertion A: A pendulum clock when taken to Mount Everest becomes fast.
Reason: The value of g (acceleration due to gravity) is less at Mount Everest than its value on the surface of earth.
In the light of the above statements, choose the most appropriate answer from the options given below
(1) Both **A** and **R** are correct but **R** is NOT the correct explanation of **A**
(2) A is correct but **R** is not correct
(3) Both **A** and **R** are correct and **R** is the correct explanation of **A**
(4) **A** is not correct but **R** is correct

Sol. **4**

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$T \propto \frac{1}{\sqrt{g}}$$

on Everest g decreases, so T increases, so moves slow.

2. The frequency (v) of an oscillating liquid drop may depend upon radius (r) of the drop, density (ρ) of liquid and the surface tension (s) of the liquid as : $v = r^a \rho^b s^c$. The values of a, b and c respectively are
(1) $\left(-\frac{3}{2}, \frac{1}{2}, \frac{1}{2}\right)$ (2) $\left(\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}\right)$ (3) $\left(-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}\right)$ (4) $\left(\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}\right)$

Sol. **3**

$f \, \alpha \, r^a \rho^b s^c$

$T^{-1} = [L]^a \left[M\,L^{-3}\right]^b \left[MT^{-2}\right]^c$

$T^{-1} = L^{a-3b}, M^{b+c}\ T^{-2c}$

$-2c = -1$ ……. (1)

$c = ½$

$b + c = 0$ ……….(2)

$b = -½$

$a - 3b = 0$ ………(3)

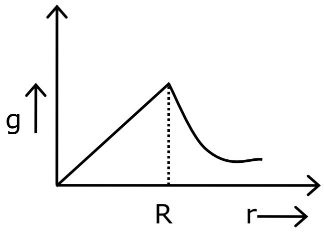
$a = 3b = \frac{-3}{2}$

3. Given below are two statements:
Statement I : Acceleration due to earth's gravity decreases as you go 'up' or 'down' from earth's surface.
Statement II : Acceleration due to earth's gravity is same at a height 'h' and depth 'd' from earth's surface, if h = d.
In the light of above statements, choose the most appropriate answer form the options given below
(1) Both Statement I and Statement II are incorrect
(2) Statement I is incorrect but statement II is correct
(3) Both Statement I and II are correct
(4) Statement I is correct but statement II is incorrect

Sol. **4**

$$g\left(1-\frac{2h}{R}\right) = g\left(1-\frac{d}{R}\right)$$

$$h = \frac{d}{2}$$



4. A long solenoid is formed by winding 70 turns cm^{-1}. If 2.0 A current flows, then the magnetic field produced inside the solenoid is ___________ $(\mu_0 = 4\pi \times 10^{-7}\ TmA^{-1})$

(1) 88×10^{-4} T (2) 352×10^{-4} T (3) 176×10^{-4} T (4) 1232×10^{-4} T

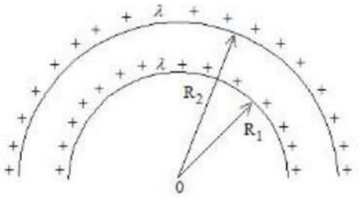
Sol. **3**

$B = \mu_0 ni$

$= 4 \times \frac{22}{7} \times 10^{-7} \times 70 \times 100 \times 2$

$= 176 \times 10^{-4}$

5. The electric potential at the centre of two concentric half rings of radii R_1 and R_2, having same linear charge density λ is :



(1) $\frac{\lambda}{2\varepsilon_0}$ (2) $\frac{\lambda}{4\varepsilon_0}$ (3) $\frac{2\lambda}{\varepsilon_0}$ (4) $\frac{\lambda}{\varepsilon_0}$

Sol. **1**

$V_C = \frac{K}{R_1} q_1 + \frac{kq_2}{R_2}$

$= \frac{1}{4\pi\varepsilon_0} \times \frac{\lambda\pi R_1}{R_1} + \frac{1}{4\pi\varepsilon_0} \times \frac{\lambda\pi R_2}{R_2}$

$= \frac{\lambda}{2\varepsilon_0}$

6. If the distance of the earth from Sun is 1.5×10^6 km. Then the distance of an imaginary planet from Sun, if its period of revolution is 2.83 years is :

(1) 6×10^6 km (2) 3×10^6 km (3) 3×10^7 km (4) 6×10^7 km

Sol. **2**

$T^2 \propto R^3$

$\left(\frac{T_E}{T_p}\right)^{\frac{2}{3}} = \left(\frac{R_E}{R_p}\right)$

$\left(\frac{1}{2.83}\right)^{\frac{2}{3}} = \frac{1.5 \times 10^6}{R}$

$R = 1.5 \times 10^6 \times (2.83)^{\frac{2}{3}}$

$1.5 \times 10^6 \times (1.41 \times 2)^{\frac{2}{3}}$

$1.5 \times 10^6 \times (2\sqrt{2})^{\frac{2}{3}}$

$1.5 \times 10^6 \times (\sqrt{8})^{\frac{2}{3}}$

3×10^6 KM

7. A photon is emitted in transition from n = 4 to n = 1 level in hydrogen atom. The corresponding wavelength for this transition is (given, $h = 4 \times 10^{-15}$eVs) :

(1) 99.3 nm (2) 941 nm (3) 974 nm (4) 94.1 nm

Sol. **4**

$\Delta E = E_4 - E_1$

$$\frac{hc}{\lambda} = -0.85 - (-13.6)$$

$$\frac{4\times10^{-15}\times3\times10^{17}}{\lambda_{(nm)}}\frac{nm}{s} = 12.75$$

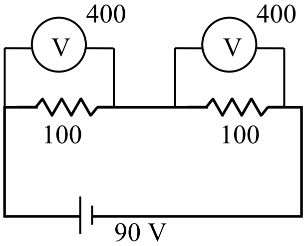
$$\lambda = \frac{1200}{12.75}\ nm$$

$= 94.1$ nm

8. A cell of emf 90 V is connected across series combination of two resistors each of 100Ω resistance. A voltmeter of resistance 400Ω is used to measure the potential difference across each resistor. The reading of the voltmeter will be:

(1) 90 V (2) 45 V (3) 80 V (4) 40 V

Sol. **2**



as Resistance are same so equal division of potential.

$$\therefore \frac{90}{2} = 45\ V$$

9. If two vectors $\vec{P} = \hat{\imath} + 2m\hat{\jmath} + m\hat{k}$ and $\vec{Q} = 4\hat{\imath} - 2\hat{\jmath} + m\hat{k}$ are perpendicular to each other. Then, the value of m will be:

(1) −1 (2) 3 (3) 2 (4) 1

Sol. **3**

$\vec{P}.\vec{Q} = 0$

$4 \times 1 + 2mx - 2 + m^2 = 0$

$m^2 - 4m + 4 = 0$

$(m - 2)^2 = 0$

$m = 2$

10. The electric field and magnetic field components of an electromagnetic wave going through vacuum is described by

$E_x = E_0\sin(kz - \omega t)$

$B_y = B_0\sin(kz - \omega t)$

Then the correct relation between E_o and B_o is given by

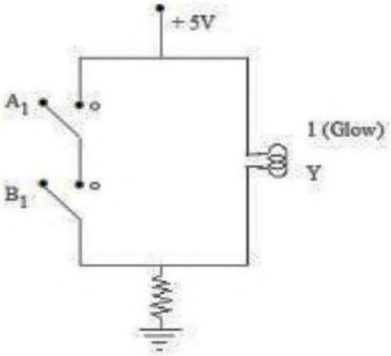
(1) $E_0 B_0 = \omega k$ (2) $E_0 = kB_0$ (3) $kE_o = \omega B_o$ (4) $\omega E_0 = kB_0$

Sol. **3**

by theory of EM wave

$$\frac{E_0}{B_0} = v = \frac{\omega}{K}$$

11. The logic gate equivalent to the given circuit diagram is :



(1) NAND (2) OR (3) AND (4) NOR

Sol. **1**

by truth table

A_1	B_1	V_1
0	0	1
0	1	1
1	0	1
1	1	0

NAND gate

12. Let γ_1 be the ratio of molar specific heat at constant pressure and molar specific heat at constant volume of a monoatomic gas and γ_2 be the similar ratio of diatomic gas. Considering the diatomic gas molecule as a rigid rotator, the ratio, $\frac{\gamma_1}{\gamma_2}$ is :

(1) $\frac{25}{21}$ (2) $\frac{35}{27}$ (3) $\frac{21}{25}$ (4) $\frac{27}{35}$

Sol. **1**

$$\frac{r_1}{r_2} = \frac{\frac{5}{3}}{\frac{7}{5}} = \frac{25}{21}$$

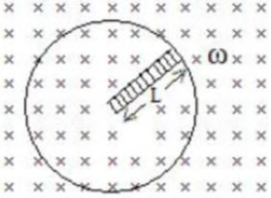
13. When a beam of white light is allowed to pass through convex lens parallel to principal axis, the different colours of light converge at different point on the principle axis after refraction. This is called:

(1) Spherical aberration (2) Polarisation

(3) Chromatic aberration (4) Scattering

Sol. Theory : Colors are due to chromatic aberration.

14. A metallic rod of length ' L ' is rotated with an angular speed of ' ω ' normal to a uniform magnetic field ' B ' about an axis passing through one end of rod as shown in figure. The induced emf will be:



(1) $\frac{1}{4}BL^2\omega$ (2) $\frac{1}{2}B^2L^2\omega$ (3) $\frac{1}{4}$ B^2 $L\omega$ (4) $\frac{1}{2}BL^2\omega$

Sol. **4**

$$\in = \int_0^L B\omega x\, dx$$

$$= \frac{1}{2}B\omega L^2$$

15. An a-particle, a proton and an electron have the same kinetic energy. Which one of the following is correct in case of their de-Broglie wavelength:

(1) $\lambda_\alpha < \lambda_p < \lambda_e$ (2) $\lambda_\alpha = \lambda_p = \lambda_e$ (3) $\lambda_\alpha > \lambda_p > \lambda_e$ (4) $\lambda_\alpha > \lambda_p < \lambda_e$

Sol. **1**

$$\lambda = \frac{h}{\sqrt{2mkE}} \propto \frac{1}{\sqrt{m}}$$

$$m_a > m_p > m_e$$

$$\therefore \ \lambda_\alpha < \lambda_p < \lambda_e$$

16. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason

Assertion A : Steel is used in the construction of buildings and bridges.

Reason R : Steel is more elastic and its elastic limit is high.

In the light of above statements, choose the most appropriate answer from the options given below

(1) Both **A** and **R** are correct and **R** is the correct explanation of **A**

(2) Both **A** and **R** are correct but **R** is NOT the correct explanation of **A**

(3) A is correct but **R** is not correct

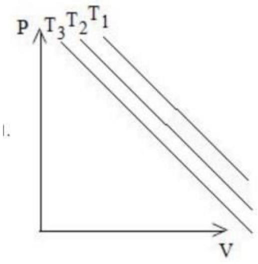
(4) A is not correct but **R** is correct

Sol. **1**

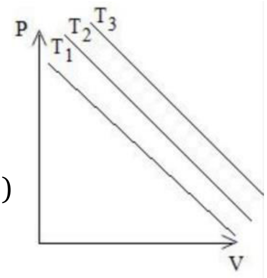
Steel is more elastic.

17. In an Isothermal change, the change in pressure and volume of a gas can be represented for three different temperature; $T_3 > T_2 > T_1$ as:

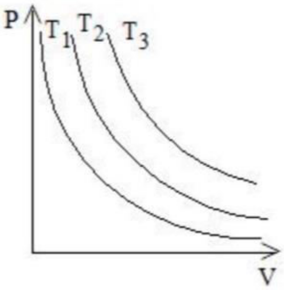
(1)



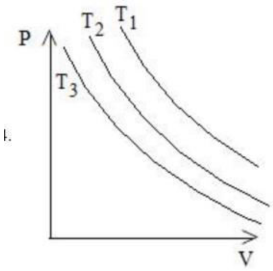
(2)



(3)

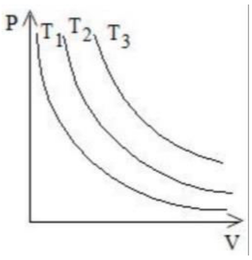


(4)



Sol. **3**

PV = nRT const.



$$P \propto \frac{1}{V}$$

18. Match List I with List II

LIST I		**LIST II**	
A.	AM Broadcast	I.	88 – 108MHz
B.	FM Broadcast	II.	540 – 1600kHz
C.	Television	III	3.7 – 4.2GHz
D.	Satellite Communication	IV.	54MHz – 890MHz

Choose the correct answer from the options given below:

(1) A-II, B-I, C-IV, D-III (2) A-I, B-III, C-II, D-IV

(3) A-IV, B-III, C-I, D-II (4) A-II, B-III, C-I, D-IV

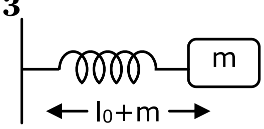
Sol. **1**

by concept of AM & FM freq. range

19. A body of mass 200 g is tied to a spring of spring constant 12.5 N/m, while the other end of spring is fixed at point O. If the body moves about O in a circular path on a smooth horizontal surface with constant angular speed 5rad/s. Then the ratio of extension in the spring to its natural length will be :

(1) 2:5 (2) 1: 1 (3) 2: 3 (4) 1: 2

Sol. **3**



$kx = m\omega^2(\ell_0 + x)$

$\frac{k}{m\omega^2} = \frac{\ell_0}{x} + 1$

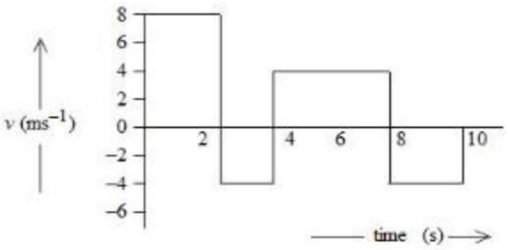
$\frac{12.5}{0.2 \times 25} = \frac{\ell_0}{x} + 1$

$\frac{125}{50} - 1 = \frac{\ell_0}{x}$

$\frac{3}{2} = \frac{\ell_0}{x}$

$\frac{x}{\ell_0} = \frac{2}{3}$

20. The velocity time graph of a body moving in a straight line is shown in figure.



The ratio of displacement to distance travelled by the body in time 0 to 10 s is :

(1) 1: 1 (2) 1: 2 (3) 1: 3 (4) 1: 4

Sol. **3**

disp. = area

$= 8 \times 2 + (4 \times 4) - 2 \times 4 - 2 \times 4$

$= 32 - 16$

$= 16$

distance = 32 + 16

= 48

SECTION - B

21. A body of mass 1 kg begins to move under the action of a time dependent force $\vec{F} = (t\hat{\imath} + 3t^2\hat{\jmath})$N, where $\hat{\imath}$ and $\hat{\jmath}$ are the unit vectors along x and y axis. The power developed by above force, at the time $t = 2s$, will be __________ W.

Sol. **100**

$$\vec{v} = \int_0^2 t\,dt\,\hat{i} + 3\int_0^2 t^2\,dt\,\hat{j}$$

$= 2\hat{i} + 8\hat{j}$

$\vec{F} = 2\hat{i} + 12\hat{j}$

$P = \vec{F}.\vec{V}$

$= 4 + 96$

$= 100$ w

22. A convex lens of refractive index 1.5 and focal length 18 cm in air is immersed in water. The change in focal length of the lens will be _________ cm

(Given refractive index of water $= \frac{4}{3}$)

Sol. **54**

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{18} = (1.5 - 1)\frac{2}{R} \quad \text{.... (1)}$$

$$\frac{1}{f} = \left(\frac{1.5}{\frac{4}{3}} - 1\right)\frac{2}{R} \quad \text{..... (2)}$$

Div eq. by eq. 2

$$\frac{f}{18} = \frac{0.5 \times 8}{1}$$

f = 72 cm

change = 72 – 18

= 54

23. The energy released per fission of nucleus of ^{240}X is 200MeV. The energy released if all the atoms in 120 g of pure ^{240}X undergo fission is _______ $\times 10^{25}$MeV (Given $N_A = 6 \times 10^{23}$)

Sol. **6**

no. of atoms $= \frac{120}{240} \times 6 \times 10^{23}$

$= 3 \times 10^{23}$

Energy rebased $= 200 \times 3 \times 10^{23}$

$= 6 \times 10^{25}$

24. A uniform solid cylinder with radius R and length L has moment of inertia I_1, about the axis of the cylinder. A concentric solid cylinder of radius $R' = \frac{R}{2}$ and length $L' = \frac{L}{2}$ is carved out of the original cylinder. If I_2 is the moment of inertia of the carved out portion of the cylinder then $\frac{I_1}{I_2} =$ ________

(Both I_1 and I_2 are about the axis of the cylinder)

Sol. **32**

$$I_1 = \frac{MR^2}{2}$$

$$\text{mass} = \rho\pi\frac{R^2}{4}.\frac{L}{2}$$

$$m_2 = \frac{M}{8}$$

$$I_2 = \frac{m_2R_2^2}{2} = \frac{MR^2}{8\times4\times2}$$

$$\frac{I_1}{I_2} = 32$$

25. A parallel plate capacitor with air between the plate has a capacitance of 15pF. The separation between the plate becomes twice and the space between them is filled with a medium of dielectric constant 3.5. Then the capacitance becomes $\frac{x}{4}$pF. The value of x is __________

Sol. **105**

$$C = \frac{A\varepsilon_0}{d}$$

$$C = \frac{KA\varepsilon_0}{2d}$$

$$= \frac{KC}{2}$$

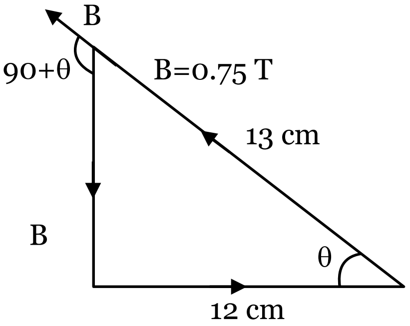
$$= \frac{3.5\times15}{2}$$

$$= \frac{105}{4}$$

= 105

26. A single turn current loop in the shape of a right angle triangle with sides 5 cm, 12 cm, 13 cm is carrying a current of 2 A. The loop is in a uniform magnetic field of magnitude 0.75 T whose direction is parallel to the current in the 13 cm side of the loop. The magnitude of the magnetic force on the 5 cm side will be $\frac{x}{130}$ N. The value of x is ___________

Sol. **9**



$$\text{Force on 5 cm length} = i\int d\vec{\ell}\times\vec{B}$$

$$= i\times\left(\frac{5}{100}\right)\times0.75\times\sin(90+\theta)$$

$$= 2\times\frac{5}{100}\times0.75\times\cos\theta$$

$$= \frac{10}{100}\times0.75\times\frac{12}{13} = \frac{x}{130}$$

$$\Rightarrow x = 9$$

27. A mass *m* attached to free end of a spring executes SHM with a period of 1 s. If the mass is increased by 3 kg the period of oscillation increases by one second, the value of mass m is ______ kg.

Sol. **1**

$2\pi\sqrt{\frac{m}{k}} = 1$ .... (1)

$2\pi\sqrt{\frac{m+3}{k}} = 2$ .... (2)

(2) ÷ (1)

$\sqrt{\frac{m+3}{m}} = \frac{2}{1}$

$\frac{m+3}{m} = 4$

4 m = m + 3

m = 1 kg.

28. If a copper wire is stretched to increase its length by 20%. The percentage increase in resistance of the wire is ___________ %

Sol. **44**

Length becomes = 1.2 times

$\ell' = 1.2\ell$

$R' = n^2R$

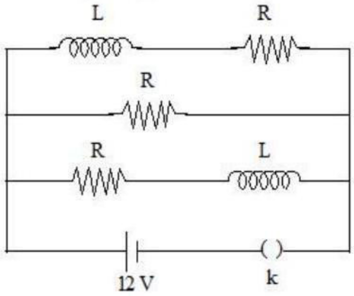
$= (1.2)^2 R$

= 1.44 R

ΔR = 0.44 R

$\frac{\Delta R}{R}\times 100\ \%$ = 44%

29. Three identical resistors with resistance R = 12Ω and two identical inductors with self inductance L = 5mH are connected to an ideal battery with emf of 12 V as shown in figure. The current through the battery long after the switch has been closed will be _________ A.



Sol. **3**

Short all inductor

Req. = $\frac{R}{3} = \frac{12}{3}$ = 4Ω

I = $\frac{12}{4}$ = 3A

30. A Spherical ball of radius 1 mm and density 10.5 g/cc is dropped in glycerine of coefficient of viscosity 9.8 poise and density 1.5 g/cc. Viscous force on the ball when it attains constant velocity is 3696×10^{-x} N. The value of x is (Given, $g = 9.8\ \mathrm{m/s^2}$ and $\pi = \frac{22}{7}$)

Sol. 7

$$V_T = \frac{2r^2g(\sigma_s - \rho_\ell)}{a_n}$$

$$\frac{2\times10^{-6}\times9.8\times(10.5-1.5)\times10^3}{9.8\times0.1\times9}$$

$= 2 \times 10^{-2}$ m/s

$F = 6\pi x\ rV_T$

$$= 6\times\frac{22}{7}\times9.8\times0.1\times10^{-3}\times18\times10^{-2}$$

$= 3696 \times 10^{-7}$

= 7

Chemistry

SECTION - A

31. Identify the correct statements about alkali metals.

A. The order of standard reduction potential $(M^+ \mid M)$ for alkali metal ions is $Na > Rb > Li$.

B. CsI is highly soluble in water.

C. Lithium carbonate is highly stable to heat.

D. Potassium dissolved in concentrated liquid ammonia is blue in colour and paramagnetic.

E. All the alkali metal hydrides are ionic solids.

Choose the correct answer from the options given below:

(1) C and E only (2) A, B and E only (3) A, B, D only (4) A and E only

Sol. **4**

(i) These standard potentionals of

Element	Li	Na	Rb
SRP	–3.237	–2.898	–3.079

(ii) All the alkali metal hydrides are ionic solids with high M.P.

32. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason **R**

Assertion A: Beryllium has less negative value of reduction potential compared to the other alkaline earth metals.

Reason : Beryllium has large hydration energy due to small size of Be^{2+} but relatively large value of atomization enthalpy

In the light of the above statements, choose the most appropriate answer from the options given below

(1) A is not correct but R is correct

(2) A is correct but *R* is not correct
(3) Both A and R are correct and R is the correct explanation of A

(4) Both A and R are correct but R is NOT the correct explanation of A

Sol. **3**

Be has least negative SRP value. Value in alkaline earth metal group as it has high hydration enthalpy and high enthalpy of atomization.

33. A student has studied the decomposition of a gas AB_3 at 25°C. He obtained the following data.

p(mmHg)	50	100	200	400
relative $t_{1/2}$(s)	4	2	1	0.5

The order of the reaction is

(1) 0 (zero) (2) 0.5 (3) 1 (4) 2

Sol. **4**

$$t½ \propto (Co)^{1-n}$$

$$= \frac{(t½)_1}{(t½)_2} = \left(\frac{P_1}{P_2}\right)^{1-n}$$

$$= \frac{4}{2} = \left(\frac{50}{100}\right)^{1-n} \Rightarrow 2\left(\frac{1}{2}\right)^{1-n}$$

$$2 = (2)^{n-1}$$

$$n = 2$$

34. $K_2Cr_2O_7$ paper acidified with dilute H_2SO_4 turns green when exposed to

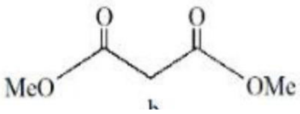
(1) Carbon dioxide (2) Sulphur trioxide (3) Sulphur dioxide (4) Hydrogen sulphide

Sol. **3**

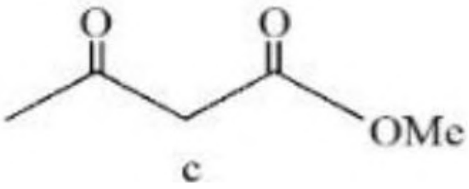
$$K_2Cr_2O_7 + 2H^+ + SO_2 \rightarrow \underset{\text{(green)}}{2Cr^{+3}} + 3SO_4^{-2} + H_2O$$

35. Which will undergo deprotonation most readily in basic medium?

a



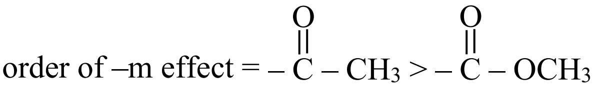
b



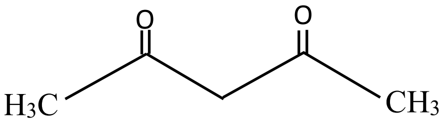
c

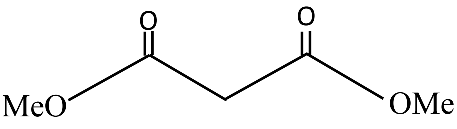
(1) c only (2) only (3) Both a and c (4) b only

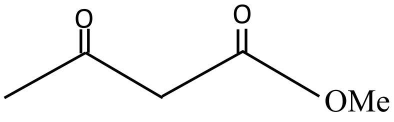
Sol. **2**



order of –m effect = $-\overset{\overset{\displaystyle O}{||}}{C}-CH_3 > -\overset{\overset{\displaystyle O}{||}}{C}-OCH_3$







strong –m effect of both ketone

36. The hybridization and magnetic behaviour of cobalt ion in $[Co(NH_3)_6]^{3+}$ complex, respectively is

(1) d^2sp^3 and paramagnetic(2) sp^3 d^2 and diamagnetic

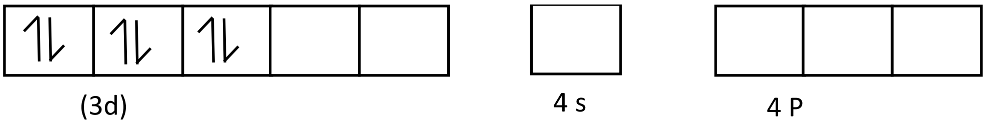
(3) d^2sp^3 and diamagnetic (4) sp^3 d^2 and paramagnetic

Sol. **3**

$[Co(NH_3)_6]^{+3}$

$Co^{+3} \rightarrow [Ar]3d^6 4S^0$

$NH_3 \rightarrow SFL$, Pairing of e^-

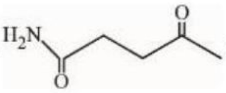


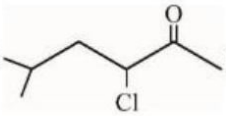
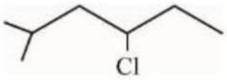
hybridisation d^2sp^3

$\mu = 0$

diamagnetic

37. Given below are two statements:

Statement I :  under Clemmensen reduction conditions will give 

Statement II :  under Wolff-Kishner reduction condition will give 

In the light of the above statements, choose the correct answer from the options given below:

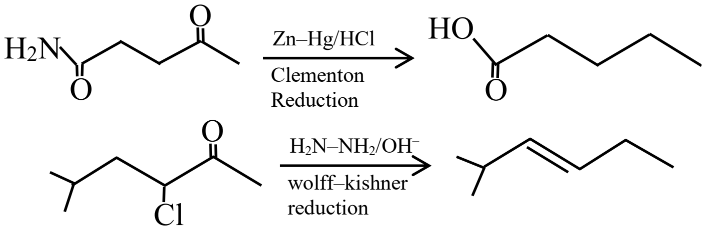
(1) Statement I is false but Statement II is true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are true

(4) Both Statement I and Statement II are false

Sol. **2**



38. Which of the following cannot be explained by crystal field theory?

(1) The order of spectrochemical series

(2) Stability of metal complexes

(3) Magnetic properties of transition metal complexes

(4) Colour of metal complexes

Sol. **1**

Crystal field theory introduce spectrochemical series based upon the experimental value of Δ but can't explain it's order. While other three points are explained by CFT. Specially when the CFSE increases thermodynamic stability of the comples increases.

39. The number of s-electrons present in an ion with 55 protons in its unipositive state is

(1) 8 (2) 10 (3) 9 (4) 12

Sol. **2**

$Cs^{+}_{(55)} = 1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 4s^2, 3d^{10}, 4p^6, 5s^2, 4d^{10}, 5p^6$

no. of s-electron = 10

40. Which one amongst the following are good oxidizing agents?

(A) Sm^{2+} (B) Ce^{2+} (C) Ce^{4+} (D) Tb^{4+}

Choose the most appropriate answer from the options given below:

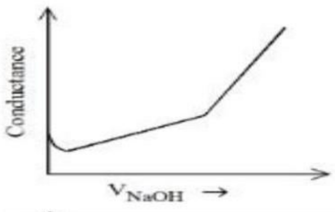
(1) D only (2) C only (3) C and *D* only (4) A and B only

Sol. **3**

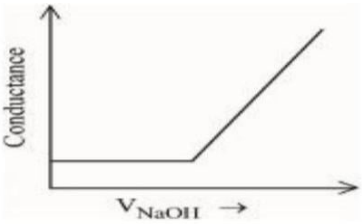
Ce^{+4} & Tb^{+4} are good oxidizing agent.

41. Choose the correct representation of conductometric titration of benzoic acid vs sodium hydroxide.

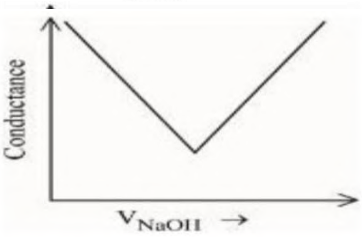
1.



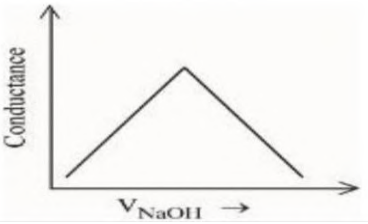
2.



3.



4.



Sol. **1**

$C_6H_5COOH + NaOH \rightarrow C_6H_5COONa + H_2O$

when weak acid C_6H_5COOH titrated against strong base NaOH in the beginning the conductance Inc. slowly and after equivalent point it increase rapidly.

42. Match List I with List II

LIST I Type		LIST II Name	
A.	Antifertility drug	I.	Norethindrone
B.	Tranquilizer	II.	Meprobomate
C.	Antihistamine	III	Seldane
D.	Antibiotic	IV.	Ampicillin

Choose the correct answer from the options given below:

(1) A-I, B-III, C-II, D-IV

(2) A-IV, B-III, C-II, D-I

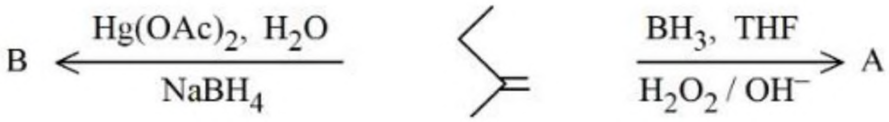
(3) A-I, B-II, C-III, D-IV

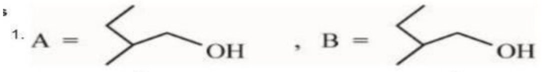
(4)A-II, B-I, C-III, D-IV

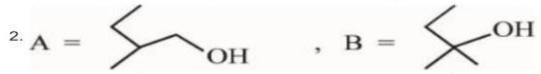
Sol. **3**

LIST I Type		LIST II Name	
A.	Antifertility drug	I.	Norethindrone
B.	Tranquilizer	II.	Meprobomate
C.	Antihistamine	III	Seldane
D.	Antibiotic	IV.	Ampicillin

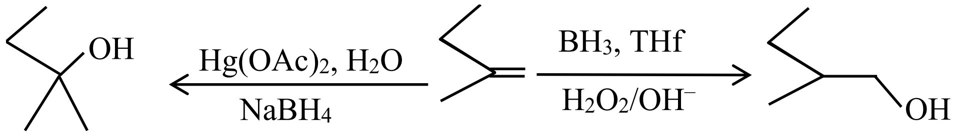
43. Find out the major products from the following reaction







Sol. **2**



(A)

44. Given below are two statements, one is labelled as Assertion **A** and the other is labelled as Reason **R**

Assertion : Benzene is more stable than hypothetical cyclohexatriene

Reason : The delocalized π electron cloud is attracted more strongly by nuclei of carbon atoms.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both A and R are correct and R is the correct explanation of A

(2) Both A and R are correct but R is NOT the correct explanation of A

(3) A is false but R is true

(4) A is true but *R* is false

Sol. **1**

Both A and R are correct and R is the correct explanation of A

45. In which of the following reactions the hydrogen peroxide acts as a reducing agent?

(1) $\mathrm{PbS}+4\mathrm{H_2O_2} \rightarrow \mathrm{PbSO_4}+4\mathrm{H_2O}$(2) $\mathrm{Mn^{2+}}+\mathrm{H_2O_2} \rightarrow \mathrm{Mn^{4+}}+2\mathrm{OH^-}$

(3) $\mathrm{HOCl}+\mathrm{H_2O_2} \rightarrow \mathrm{H_3O^+}+\mathrm{Cl^-}+\mathrm{O_2}$(4) $2\mathrm{Fe^{2+}}+\mathrm{H_2O_2} \rightarrow 2\mathrm{Fe^{3+}}+2\mathrm{OH^-}$

Sol. **3**

$\mathrm{HOCl}+\underbrace{\mathrm{H_2O_2} \rightarrow \mathrm{H_3O^+}+\mathrm{Cl^-}+\mathrm{O_2}}$ hydrogen peroxide acts as a reducing agent

46. Given below are two statements:

Statement I : Pure Aniline and other arylamines are usually colourless.

Statement II : Arylamines get coloured on storage due to atmospheric reduction

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both Statement I and Statement II are incorrect

(2) Statement I is incorrect but Statement II is correct

(3) Statement I is correct but Statement II is incorrect

(4) Both Statement I and Statement II are correct

Sol. **3**

47. Correct statement is:

(1) An average human being consumes nearly 15 times more air than food

(2) An average human being consumes 100 times more air than food

(3) An average human being consumes equal amount of food and air

(4) An average human being consumes more food than air

Sol. **1**

An average human being requires. hearly 12 –15 times more air than the food.

48. What is the number of unpaired electron(s) in the highest occupied molecular orbital of the following species : $N_2; N_2^+; O_2; O_2^+$?

(1) 2,1,0,1 (2) 0, 1, 0, 1 (3) 0,1,0,1 (4) 2,1,2,1

Sol. **2**

$$N_2 = \sigma_{1s}^{2}, \overset{*}{\sigma}_{1s}^{2}, \sigma_{2s}^{2}, \overset{*}{\sigma}_{2s}^{2}, \pi_{2Px}^{2} = \pi_{2Py}^{2}, \sigma_{2Pz}^{2}$$

no. of e⁻ present in Homo = 0

$$N_2^+ = \sigma_{1s}^{2}, \overset{*}{\sigma}_{1s}^{2}, \sigma_{2s}^{2}, \overset{*}{\sigma}_{2s}^{2}, \pi_{2Px}^{2} = \pi_{2Py}^{2}, \sigma_{2Pz}^{1}$$

no. of unpaired e⁻ present in HOMO = 1

$$O_2 = \sigma_{1s}^{2}, \overset{*}{\sigma}_{1s}^{2}, \sigma_{2s}^{2}, \overset{*}{\sigma}_{2s}^{2}, \sigma_{2Pz}^{2}, \pi_{2Px}^{2} = \pi_{2Py}^{2}, \pi^*_{2Px}{}^{1} = \pi^*_{2Py}{}^{1}$$

no. of unpaired e⁻ present in HOMO = 2

$$O_2^+ = \sigma_{1s}^{2}, \overset{*}{\sigma}_{1s}^{2}, \sigma_{2s}^{2}, \overset{*}{\sigma}_{2s}^{2}, \sigma_{2Pz}^{2}, \pi_{2Px}^{2} = \pi_{2Py}^{2}, \pi^*_{2Px}{}^{1} = \pi^*_{2Py}{}^{0}$$

no. of unpaired e⁻ present in HOMO = 1

49. The metal which is extracted by oxidation and subsequent reduction from its ore is:

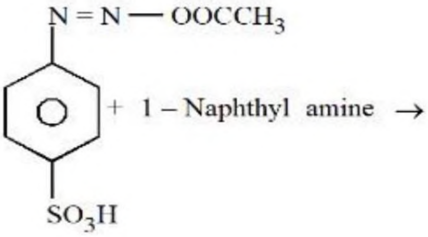
(1) Ag (2) Fe (3) Cu (4) Al

Sol. **1**

$$4Ag(s) + 8CN^-(aq) + 2H_2O(aq) + O_2(g) \rightarrow 4[Ag(CN)_2]^-_{(aq)} + 4OH^-(aq)$$

$$2[Ag(CN)_2]^-_{(aq)} + Zn(s) \rightarrow 2Ag(s) + [Zn(CN)_4]^{-2}_{(aq)}$$

50. Choose the correct colour of the product for the following reaction.



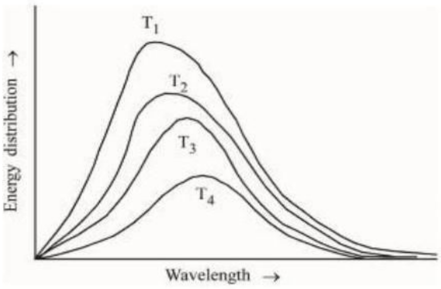
(1) White (2) Red (3) Blue (4) Yellow

Sol. **2**

Red

Section: B

51. Following figure shows spectrum of an ideal black body at four different temperatures. The number of correct statement/s from the following is________________.



A. $T_4 > T_3 > T_2 > T_1$

B. The black body consists of particles performing simple harmonic motion.

C. The peak of the spectrum shifts to shorter wavelength as temperature increases.

D. $\frac{T_1}{v_1} = \frac{T_2}{v_2} = \frac{T_3}{v_3} \neq$ constant

E. The given spectrum could be explained using quantisation of energy.

Sol. **2**

(A) $T_4 > T_3 > T_2 > T_1$

(C) The peak of the spectrum shift to shorter wavelength of temp. Inc.

52. The number of units, which are used to express concentration of solutions from the following is_______

Mass percent, Mole, Mole fraction, Molarity, ppm, Molality

Sol. **5**

Conc. Express in → mass percentage
→ mole fraction
→ molarity
→ PPM
→ molality

53. The number of statement/s which are the characteristics of physisorption is______________

A. It is highly specific in nature

B. Enthalpy of adsorption is high

C. It decreases with increase in temperature

D. It results into unimolecular layer

E. No activation energy is needed

Sol. **2**

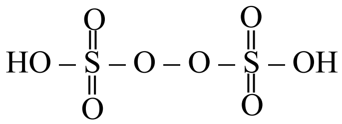
(C) It decreases with increase in temperature

(E) No activation energy is needed

54. Sum of π - bonds present in peroxodisulphuric acid and pyrosulphuric acid is:

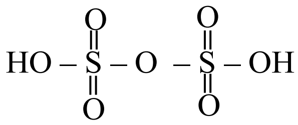
Sol. **8**

Peroxodisulphuric acid ($H_2S_2O_8$)



$$\mathrm{HO-\underset{\underset{O}{\|}}{\overset{\overset{O}{\|}}{S}}-O-O-\underset{\underset{O}{\|}}{\overset{\overset{O}{\|}}{S}}-OH}$$

Pyrosulphuric acid ($H_2S_2O_7$)



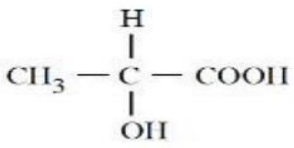
$$\mathrm{HO-\underset{\underset{O}{\|}}{\overset{\overset{O}{\|}}{S}}-O\ -\underset{\underset{O}{\|}}{\overset{\overset{O}{\|}}{S}}-OH}$$

π bond = 4

total π bond = 4 + 4 = 8

55. If the pKa of lactic acid is 5, then the pH of 0.005M calcium lactate solution at 25°C is _________ $\times 10^{-1}$ (Nearest integer)

Lactic acid



$$\mathrm{CH_3-\underset{\underset{OH}{|}}{\overset{\overset{H}{|}}{C}}-COOH}$$

Sol. **85**

$$\mathrm{Ca(Lac)_2 \longrightarrow Ca^{+2} + 2lac^{-}}$$

5×10^{-3} $\quad$ 5×10^{-3} $\quad$ 10^{-2} M

Salt of strong base weak acid salt

$$pH = 7 + \frac{1}{2}pka + \frac{1}{2}\log c$$

$$= 7 + \frac{1}{2}\times 5 + \frac{1}{2}\log 10^{-2}$$

$$= 7 + 2.5 - 1 = 8.5$$

$$= 85 \times 10^{-1}$$

56. The total pressure observed by mixing two liquids A and B is 350 mmHg when their mole fractions are 0.7 and 0.3 respectively. The total pressure become 410 mmHg if the mole fractions are changed to 0.2 and 0.8 respectively for A and B. The vapour pressure of pure A is________ mm Hg. (Nearest integer) Consider the liquids and solutions behave ideally.

Sol. **314**

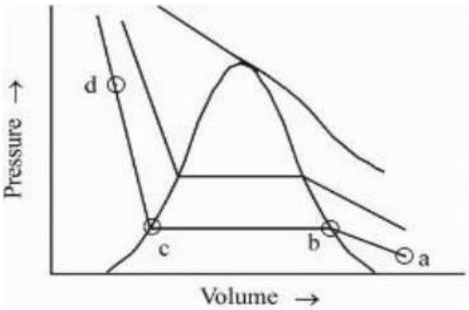
$$XaP_A^o + X_B P_B^o = P_S$$

$$0.7P_A^o + 0.3P_B^o = 350$$

$$0.2P_A^o + 0.8P_B^o = 410$$

$$\therefore P_A^o = 314\text{torr}$$

57. The number of statement/s, which are correct with respect to the compression of carbon dioxide from point (a) in the Andrews isotherm from the following is_____________



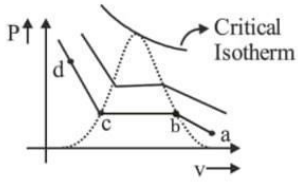
A. Carbon dioxide remains as a gas upto point (b)

B. Liquid carbon dioxide appears at point (c)

C. Liquid and gaseous carbon dioxide coexist between points (b) and (c)

D. As the volume decreases from (b) to (c), the amount of liquid decreases

Sol. **4**



At

(a) → CO_2 exist as gas

(b) → liquefaction of CO_2 starts

(c) → liquefaction ends

(d) → CO_2 exist as liquid

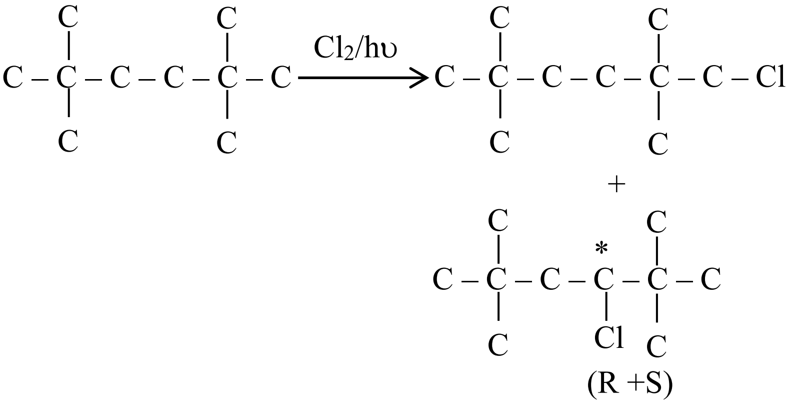
Between (b) & (c) → liquid and gaseous CO_2 co-exist.

As volume changes from (b) to (c) gas decreases and liquid increases.

(A), (C) → Correct

58. Maximum number of isomeric monochloro derivatives which can be obtained from 2, 2, 5, 5 tetramethylhexane by chlorination is____________

Sol. **3**

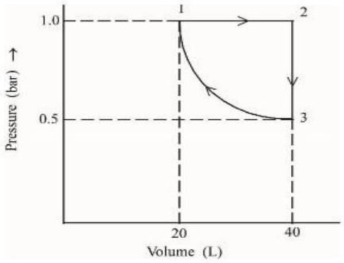


59. Total number of tripeptides possible by mixing of valine and proline is____________

Sol. **8**

(1) P–P–P (2) V–V–V (3) P–V–V (4) V–P–V (5) V–V–P (6) V–P–P
(7) P–V–P (8) P–P–V

60. One mole of an ideal monoatomic gas is subjected to changes as shown in the graph. The magnitude of the work done (by the system or on the system) is_________ J (nearest integer)



Sol. **6**

I→II → Isobaric

II → III → Isochoric

III → I → Isothermal

$$W_{I-II} = -1[40-20] = -20\,\text{Lit atm}$$

$$W_{II-III} = 0$$

$$W_{IV-I} = 2.303\,nRt\ \log\frac{V_2}{V_1}$$

$$= 2.303PV\log\frac{V_2}{V_1}$$

$$= 2.303(1\times 20)\log 2$$

$$= 2.303\times 20\times 0.3010 = 13.818$$

W total = – 20 + 13.818 = (– 6.182 lit alm) = 6.182 lit alm

Mathematics

SECTION - A

61. If, $f(x)=x^3-x^2f'(1)+xf''(2)-f'''(3), x\in\mathbb{R}$ then

(1) $f(1)+f(2)+f(3)=f(0)$ (2) $2f(0)-f(1)+f(3)=f(2)$

(3) $3f(1)+f(2)=f(3)$ (4) $f(3)-f(2)=f(1)$

Sol. **2**

$f(x) = x^3 - x^2 f'(1) + x f''(2) - f'''(3)$

$f(x) = x^3 - ax^2 + bx - c$

$f'(x) = 3x^2 - 2ax + b$

$f''(x) = 6x - 2a$

$f'''(x) = 6$

$f'''(3) = 6$

$f'(1) = 3 - 2a + b = a \Rightarrow 3a = b + 3$

$f''(2) = 12 - 2a = b \Rightarrow 2a = 12 - b$

$a = 3, b = 6$

$f'''(3) = 6 = c$

$f(x) = x^3 - 3x^2 + 6x - 6$

$f(0) = -6$ $f(2) = 2$

$f(1) = -2$ $f(3) = 12$

62. If the system of equations

$x + 2y + 3z = 3$

$4x + 3y - 4z = 4$

$8x + 4y - \lambda z = 9 + \mu$

has infinitely many solutions, then the ordered pair (λ, μ) is equal to :

(1) $\left(-\frac{72}{5}, \frac{21}{5}\right)$ (2) $\left(-\frac{72}{5} \cdot -\frac{21}{5}\right)$ (3) $\left(\frac{72}{5}, -\frac{21}{5}\right)$ (4) $\left(\frac{72}{5}, \frac{21}{5}\right)$

Sol. **3**

Planes are not parallel

$\therefore (x + 2y + 3z - 3) + a(4x + 3y - 4z - 4)$

$= 8x + 4y - \lambda z - 9 - \mu = 0$

$$\frac{1+4a}{8} = \frac{2+3a}{4} = \frac{3-4a}{-\lambda} = \frac{-3-4a}{-9-\mu}$$

(i) $1 + 4a = 4 + 6a$

$a = \frac{-3}{2}$

(ii) $\frac{2-\frac{9}{2}}{4} = \frac{3+6}{-\lambda}$

$$-\lambda = \frac{36}{-5}\times 2$$

$$\lambda = \frac{72}{5}$$

(iii) $\frac{-5}{8} = \frac{-3-4a}{-9-\mu}$

$$\frac{5}{8} = \frac{3-6}{-9-\mu}$$

$$-9-\mu = \frac{-24}{5}$$

$$\mu = \frac{-45+24}{5}$$

$$\mu = \frac{-21}{5}$$

63. If, then $f(x) = \frac{2^{2x}}{2^{2x}+2}, x \in \mathbb{R}$, then $f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + \cdots + f\left(\frac{2022}{2023}\right)$ is equal to

(1) 1011 (2) 2010 (3) 1010 (4) 2011

Sol. **1**

$$f(x) = \frac{4^x}{4^x+2}$$

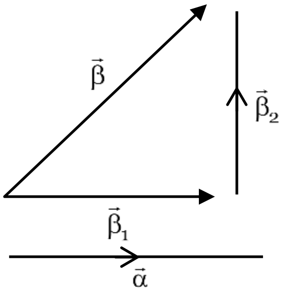
$$f(1-x) = \frac{\frac{4}{4^x}}{\frac{4}{4^x}+2} = \frac{4}{4+2.4^x} = \frac{2}{2+4^x}$$

$$f(x) + f(1-x) = 1$$

64. Let $\vec{\alpha} = 4\hat{i} + 3\hat{j} + 5\hat{k}$ and $\vec{\beta} = \hat{i} + 2\hat{j} - 4\hat{k}$. Let $\vec{\beta}_1$ be parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ be perpendicular to $\vec{\alpha}$. If $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, then the value of $5\vec{\beta}_2 \cdot (\hat{i} + \hat{j} + \hat{k})$ is

(1) 7 (2) 9 (3) 6 (4) 11

Sol. **1**



$$\overrightarrow{\beta_1} = \frac{(\vec{\alpha}\cdot\vec{\beta})}{|\vec{\alpha}|}\hat{\alpha}$$

$$=\left(\frac{4+6-20}{\sqrt{16+9+25}}\right)\frac{(4,3,5)}{\sqrt{50}}$$

$$=\frac{-10}{50}(4,3,5)$$

$$\vec{\beta}_1=\frac{(-4,-3,-5)}{5}$$

$$\vec{\beta}_1+\vec{\beta}_2-=(1,2,-4)$$

$$\beta_2=\left(1+\frac{4}{5},2+\frac{3}{5},-4+1\right)$$

$$\beta_2=\left(\frac{9}{5},\frac{13}{5},-3\right)$$

$$\therefore 5\beta_2=(9,13,-15)$$

$$\therefore 5\beta_2\cdot(1,1,1)=9+13-15$$

$$=7$$

65. Let $y=y(x)$ be the solution of the differential equation $(x^2-3y^2)dx+3xydy=0, y(1)=1$. Then $6y^2(e)$ is equal to

(1) $2e^2$ (2) $3e^2$ (3) e^2 (4) $\frac{3}{2}e^2$

Sol. **1**

$$(x^2-3y^2)dx+3xydy=0$$

$$\frac{dy}{dx}=\frac{3y^2-x^2}{3xy}$$

$$y=vx$$

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

$$v+x\frac{dv}{dx}=\frac{3v^2x^2-x^2}{3vx^2}$$

$$v+x\frac{dv}{dx}=\frac{3v^2-1}{3v}$$

$$\frac{xdy}{dx}=\frac{3v^2-1}{3v}-v\Rightarrow\frac{-1}{3v}$$

$$3vdv=-\frac{dx}{x}$$

$$\frac{3v^2}{2}=-\ln x+C\ [y(1)=1]$$

$$\frac{3y^2}{2x^2}=-\ln x+C$$

$C = \frac{3}{2}$

$\frac{3y^2}{2x^2} = \frac{3}{2}\ln e = \ln x$

$\therefore 3y^2 = 3x^2 \ln e - 2x^2 \ln x$

$x = e \quad 3y^2 = 3e^2 \ln e - 2e^2 \ln e$

$= e^2 \ln e$

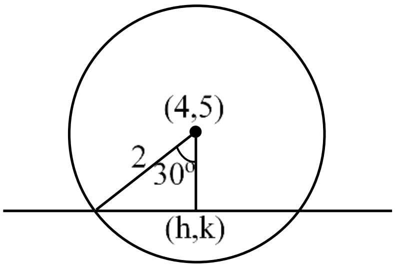
$= e^2$

$6y^2 = 2e^2$

66. The locus of the mid points of the chords of the circle C_1: $(x-4)^2 + (y-5)^2 = 4$ which subtend an angle θ_1 at the centre of the circle C_1, is a circle of radius r_i. If $\theta_1 = \frac{\pi}{3}, \theta_3 = \frac{2\pi}{3}$ and $r_1^2 = r_2^2 + r_3^2$, then θ_2 is equal to

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{6}$ (4) $\frac{3\pi}{4}$

Sol. **2**



$r_1 = 2\cos 30^\circ$

$r_3 = 2\cos 60^\circ$

$3 = r_2^2 + 1 \Rightarrow r_2 = \sqrt{2}$

$2\cos\theta = \sqrt{2}, \ \theta = \frac{\pi}{4}$

$\therefore \theta_2 = \frac{\pi}{2}$

67. The number of real solutions of the equation $3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0$, is

(1) 0 (2) 3 (3) 4 (4) 2

Sol. **1**

$3\left[\left(x + \frac{1}{x}\right)^2 - 2\right] - 2\left[x + \frac{1}{x}\right] + 5 = 0$

$3t^2 - 2t - 1 = 0$

$3t^2 - 3t + t - 1 = 0$

$(3t + 1)(t - 1) = 0$

$t = 1, t = \frac{-1}{3}$

68. Let A be a 3×3 matrix such that $|\mathrm{adj}(\mathrm{adj}(\mathrm{adj}\,A))| = 12^4$ Then $|A^{-1}\mathrm{adj}\,A|$ is equal to

(1) $\sqrt{6}$ (2) $2\sqrt{3}$ (3) 12 (4) 1

Sol. **2**

$$|\mathrm{adj}(\mathrm{adj}\ \mathrm{adj}A)| = |A|^{(n-1)^3} = 12^4$$

$$|A|^8 = (12)^4$$

$$|A| = (12)^{\frac{1}{2}}$$

$$\therefore \left|A^{-1}\cdot \mathrm{adj}(A)\right| = \left|A^{-1}\right| \times |\mathrm{adj}A|$$

$$= \frac{1}{|A|} \times |A|^{n-1}$$

$$= \frac{1}{|A|} \times |A|^2 = |A| = \sqrt{12} = 2\sqrt{3}$$

69. $\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}}dx$ is equal to

(1) 2π (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

Sol. **1**

$$48\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{dx}{\sqrt{9-4x^2}} = \frac{48}{2}\sin^{-1}\left\{\frac{2x}{3}\right\}^{\frac{3\sqrt{3}}{4}}$$

$$= 24\left[\sin^{-1}\frac{\sqrt{3}}{2} - \sin^{-1}\frac{1}{\sqrt{2}}\right]^{\frac{3\sqrt{3}}{4}}$$

$$= 24\left[\frac{\pi}{3} - \frac{\pi}{4}\right] = 2\pi$$

70. The number of square matrices of order 5 with entries form the set {0, 1}, such that the sum of all the elements in each row is 1 and the sum of all the elements in each column is also 1, is

(1) 125 (2) 225 (3) 150 (4) 120

Sol. **4**

$$\begin{bmatrix} \_\_ & \_\_ & \_\_ & \boxed{1} & \_\_ \\ \_\_ & \_\_ & \boxed{1} & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ & \_\_ \end{bmatrix} \longrightarrow {}^5C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1 \times {}^1C_1$$

${}^5C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1 \times {}^1C_1$

$= 120$

71. If $\left({}^{30}C_1\right)^2 + 2\left({}^{30}C_2\right)^2 + 3\left({}^{30}C_3\right)^2 + ... + 30\left({}^{30}C_{30}\right)^2 = \frac{\alpha 60!}{(30!)^2}$ then α is equal to :

(1) 30 (2) 10 (3) 60 (4) 15

Sol. **4**

$C_1^2 + 2C_2^2 + 3C_3^2 ... + 30C_{30}^2$

$S = 0C_0^2 + 1C_1^2 + ... + 30C_{30}^2$

$S = 30C_{30}^2 + 29C_{29}^2 + ... + 0C_0^2$

$2S = 30\left[C_0^2 + C_1^2 + ... + C_{30}^2\right]$

$S = 15 \times {}^{60}C_{30} = \frac{\alpha \cdot 60!}{(30!)^2} \Rightarrow \alpha = 15$

72. Let the plane containing the line of intersection of the planes P1: $x + (\lambda + 4)y + z = 1$ and P2: $2x + y + z = 2$ pass through the points (0,1,0) and (1,0,1). Then the distance of the point $(2\lambda, \lambda, -\lambda)$ from the plane P2 is

(1) $4\sqrt{6}$ (2) $3\sqrt{6}$ (3) $5\sqrt{6}$ (4) $2\sqrt{6}$

Sol. **2**

$[x + (\lambda+4)y + z - 1] + \mu[2x + y + z - 2] = 0$

(0,1,0)

(i) $(\lambda + 4 - 1) + \mu[-1] = 0$

$\lambda - \mu = -3$

(1,0,1) (ii) $1 + \mu[1] = 0 \Rightarrow \mu = -1, \lambda = -4$

$\therefore$ point $(-8,-4,4)$; $2x + y + z - 2 = 0$

$$d = \left|\frac{-16-4+4-2}{\sqrt{6}}\right| = \frac{18}{\sqrt{6}} = 3\sqrt{6}$$

73. Let f(x) be a funciton such that $f(x+y) = f(x) \cdot f(y)$ for all $x, y \in N$. If f(1) = 3 and $\sum_{k=1}^{n} f(k) = 3279$, then the value of n is

(1) 9 (2) 6 (3) 8 (4) 7

Sol. **4**

$f(x + y) = f(x).f(y), x,y \in N$

$f(2) = 3^2$

$f(3) = 3^3$ $\therefore 3\frac{\left[3^n - 1\right]}{2} = 3279$

$3^n - 1 = 1093 \times 2$

$3^n - 1 = 2186$

$3^n = 2187$

$n = 7$

74. Let the six numbers $a_1, a_2, a_3, a_4, a_5, a_6$, be in A.P. and $a_1 + a_3 = 10$. If the mean of these six numbers is $\frac{19}{2}$ and their variance is σ^2, then $8\sigma^2$ is equal to :

(1) 210 (2) 220 (3) 200 (4) 105

Sol. **1**

$a + (a + 2d) = 10 \Rightarrow a + d = 5$ ...(1)

$$\text{Mean} \Rightarrow \frac{\frac{6}{2}[2a+5d]}{6} = \frac{19}{2}$$

$2a + 5d = 19$ ...(2)

from (1) and (2)

$3d = 9 \Rightarrow d = 3; a = 2$

$$\therefore \sigma^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$$

$$= \frac{2^2+5^2+8^2+11^2+14^2+17^2}{6} - \left(\frac{19}{2}\right)^2$$

$$= \frac{699}{6} - \frac{361}{4}$$

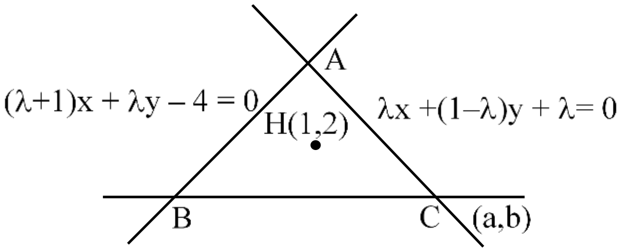
$$= \frac{233}{2} - \frac{361}{4}$$

$8\sigma^2 = 932 - 722 = 210$

75. The equations of the sides AB and AC of a triangle ABC are $(\lambda + 1)x + \lambda y = 4$ and $\lambda x + (1 - \lambda)y + \lambda = 0$ respectively. Its vertex A is on the y - axis and its orthocentre is (1,2). The length of the tangent from the point C to the part of the parabola $y^2 = 6x$ in the first quadrant is :

(1) 4 (2) 2 (3) $\sqrt{6}$ (4) $2\sqrt{2}$

Sol. **4**



$(\lambda+1)(1-\lambda)x + \lambda(1-\lambda)y = 4(1-\lambda)$

$\lambda^2 x + \lambda(1-\lambda)y = -\lambda^2$

$-$ $-$ $+$

$(1-\lambda^2 - \lambda^2)x = 4 - 4\lambda + \lambda^2$

$(1 - 2\lambda^2)x = 4 - 4\lambda + \lambda^2$

$x = 0 \Rightarrow \lambda = 2$

$$\left.\begin{array}{l} AB: 3x+2y=4 \\ AC: 2x-y+2=0 \end{array}\right\} A(0,2)$$

$CH \perp AB$

$$\left(\frac{b-2}{a-1}\right) \times \left(\frac{-3}{2}\right) = -1$$

$3b - 6 = 2a - 2$ Also $2a - b + 2 = 0$

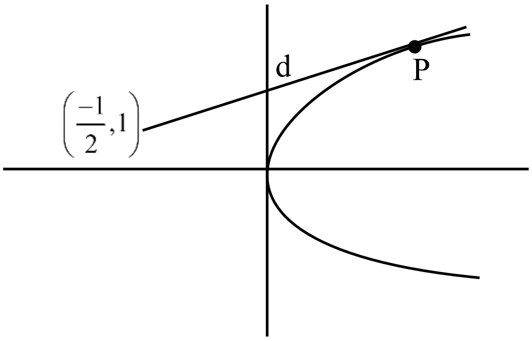
$3b - 2a = 4$ $b = 2a + 2$

$6a + 6 = 2a + 4$ $C\left(-\frac{1}{2}, 1\right)$

$4a = -2$

$a = \frac{-1}{2}, b = 1$

$\therefore y^2 = 6x$



$ty = x + \frac{3}{2}t^2$

$t = -\frac{1}{2} + \frac{3}{2}t^2$

$3t^2 - 1 = 2t$

$3t^2 - 2t - 1 = 0$

$3t^2 - 3t + t - 1 = 0$

$(3t + 1)(t-1) = 0$

$t = 1$

$P\left(\frac{3}{2}, 3\right)$ $\therefore d = \sqrt{\left(\frac{3}{2}+\frac{1}{2}\right)^2 + (3-1)^2}$

$= \sqrt{4+4} = 2\sqrt{2}$

76. Let p and q be two statements. Then $\sim (p \wedge (p \Rightarrow \sim q))$ is equivalent to

(1) $p \vee (p \wedge q)$ (2) $p \vee (p \wedge (\sim q))$ (3) $(\sim p) \vee q$ (4) $p \vee ((\sim p) \wedge q)$

Sol. **3**

$P \wedge (P \Rightarrow \sim q)$ $\qquad$ $P \to q$

$P \wedge (\sim P \vee \sim q)$

$\therefore$ Its negation will be

$\sim P \vee [P \wedge q]$

$= [\sim P \vee P] \wedge [\sim P \vee q]$

$= \sim P \vee q$

77. The set of all values of a for which $\lim_{x \to a}([x-5]-[2x+2])=0$, where $[\propto]$ denotes the greatest integer less than or equal to α is equal to

(1) $[-7.5, -6.5)$ (2) $[-7.5, -6.5]$ (3) $(-7.5, -6.5]$ (4) $(-7.5, -6.5)$

Sol. **4**

$\lim_{x \to \alpha}([x]-5-[2x]-2)=0$

$\lim_{x \to \alpha}([x]-[2x])=7$

$[\alpha]-[2\alpha]=7$

If $\alpha = -7.5$ $\quad$ $[-7.5] = -8$

$[-15] = -15$ $\quad$ $\therefore -8 + 15 = 7$

If $\alpha = -6.5$ $\quad$ $[-6.5] = -7$ $\quad$ $-7 + 13 = 6$

$[-13] = -13$

$\therefore$ $\quad$ $\alpha \in (-7.5, -6.5)$

78. If the foot of the perpendicular drawn from (1,9,7) to the line passing through the point (3,2,1) and parallel to the planes $x + 2y + z = 0$ and $3y - z = 3$ is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to

(1) 3 (2) 1 (3) -1 (4) 5

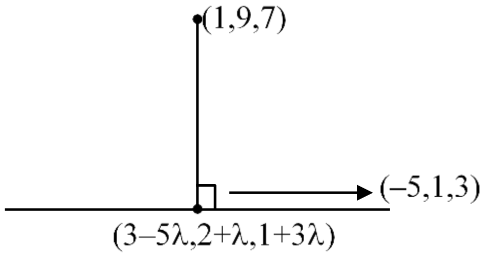
Sol. **4**

$\bar{n}_1 = (1,2,1) \bar{n}_2 = (0,3,-1)$

$$\bar{P} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{vmatrix} = \hat{i}(-5) - \hat{j}(-1) + \hat{k}(3)$$

$= (-5, 1, 3)$

$\therefore$ line : $\dfrac{x-3}{-5} = \dfrac{y-2}{1} = \dfrac{z-1}{3} = \lambda$



(2–5λ). – 5 + (λ–7).1 + (3λ–6).3 = 0

25λ – 10 + λ – 7 + 9λ – 18 = 0

35λ = 35

λ = 1

∴ Point is (–2,3,4)   α + β + γ = 5

79. The number of integers, greater than 7000 that can be formed, using the digits 3,5,6,7,8 without repetition, is

(1) 168   (2) 220   (3) 120   (4) 48

Sol. **1**

C-1   □ □ □ □

$2\times4\times3\times2=48$

C-2   5!(5 digit nos) = $\dfrac{120}{168}$

80. The value of $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is

(1) $-\frac{1}{2}(\sqrt{3}-i)$ (2) $-\frac{1}{2}(1-i\sqrt{3})$ (3) $\frac{1}{2}(1-i\sqrt{3})$ (4) $\frac{1}{2}(\sqrt{3}+i)$

Sol. **1**

$$\frac{\pi}{2}-\frac{2\pi}{9}$$

$$=\frac{95-4\pi}{18}=\frac{5\pi}{18}$$

$$\Rightarrow\frac{1+\cos\frac{5\pi}{18}+i\sin\frac{5\pi}{18}}{1+\cos\frac{5\pi}{18}-i\sin\frac{5\pi}{18}}$$

$$=\frac{2\cos^2\frac{5\pi}{36}+2i\sin\frac{5\pi}{36}.\cos\frac{5\pi}{36}}{2\cos^2\frac{5\pi}{36}-2i\sin\frac{5\pi}{36}\cos\frac{5\pi}{36}}\Rightarrow\left(\frac{e^{i\frac{5\pi}{36}}}{e^{-i\frac{5\pi}{36}}}\right)^3$$

$$=e^{i\left(\frac{5\pi}{18}\right)^3}\quad=e^{i\left(\frac{5\pi}{6}\right)}$$

$$\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)=-\frac{\sqrt{3}}{2}+\frac{i}{2}$$

SECTION B

81. If the shortest distance between the lines $\frac{x+\sqrt{6}}{2}=\frac{y-\sqrt{6}}{3}=\frac{z-\sqrt{6}}{4}$ and $\frac{x-\lambda}{3}=\frac{y-2\sqrt{6}}{4}=\frac{z+2\sqrt{6}}{5}$ is 6 , then the square of sum of all possible values of λ is

Sol. **384**

$$P\left(-\sqrt{6},\sqrt{6},\sqrt{6}\right) \qquad Q\left(\lambda,2\sqrt{6},-2\sqrt{6}\right)$$

$$\bar{n}_1=(2,3,4) \qquad \bar{n}_2=(3,4,5)$$

$$\bar{n}_1\times\bar{n}_2 \Rightarrow \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 4\\ 3 & 4 & 5\end{vmatrix}=\hat{i}(-1)-\hat{j}(-2)+\hat{k}(-1)$$

$= (-1, 2, -1)$

$$\therefore S_d\left|\frac{\overline{PQ}\cdot(-1,2,-1)}{\sqrt{6}}\right|=\frac{\left(\lambda+\sqrt{6},\sqrt{6},-3\sqrt{6}\right)\cdot(-1,2,-1)}{\sqrt{6}}$$

$$=\left|\frac{-\lambda-\sqrt{6}+2\sqrt{6}+3\sqrt{6}}{\sqrt{6}}\right|=6$$

$$\Rightarrow\left|-\lambda+4\sqrt{6}\right|=6\sqrt{6}$$

$(+)\qquad -\lambda+4\sqrt{6}=6\sqrt{6} \qquad\qquad (-)\qquad \lambda-4\sqrt{6}=6\sqrt{6}$

$\lambda=-2\sqrt{6} \qquad\qquad \lambda=10\sqrt{6}$

$$\therefore\left(8\sqrt{6}\right)^2=384$$

82. Three urns A, B and C contain 4 red, 6 black; 5 red, 5 black; and λ red, 4 black balls respectively. One of the urns is selected at random and a ball is drawn. If the ball drawn is red and the probability that it is drawn from urn C is 0.4 then the square of the length of the side of the largest equilateral triangle, inscribed in the parabola $y^2=\lambda x$ with one vertex at the vertex of the parabola, is

Sol. 432

4R 6B	5R 5B	4R 4B
A	B	C

$$\text{P(Red from C)}=\frac{\frac{1}{3}\times\frac{\lambda}{\lambda+4}}{\frac{1}{3}\cdot\frac{\lambda}{\lambda+4}+\frac{1}{3}\cdot\frac{4}{10}+\frac{1}{3}\cdot\frac{5}{10}}$$

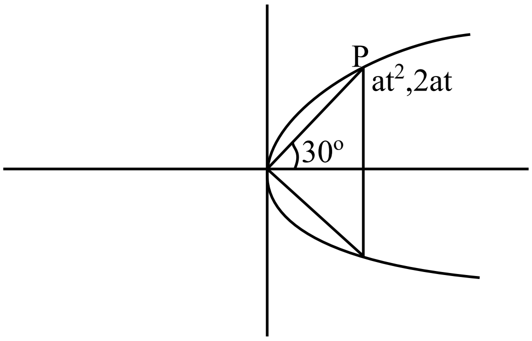
$$=\frac{\frac{\lambda}{\lambda+4}}{\frac{\lambda}{\lambda+4}+\frac{9}{10}}$$

$$\Rightarrow \frac{10\lambda}{10\lambda+9(\lambda+4)}=\frac{4}{10}$$

$$\Rightarrow 100\lambda = 40\lambda+36\lambda+144$$

$24\lambda = 144$

$\lambda = 6$



$$m = \frac{2}{t}=\frac{1}{\sqrt{3}}$$

$t = 2\sqrt{3}$

$P\left(12a, 4\sqrt{3}a\right)$

$(\text{Side})^2 = 144a^2 + 48a^2$

$$= 192\times\frac{9}{4} = 432$$

83. Let $S = \{\theta \in [0,2\pi): \tan(\pi\cos\theta) + \tan(\pi\sin\theta) = 0\}$.

Then $\sum_{\theta\in s}\sin^2\left(\theta+\frac{\pi}{4}\right)$ is equal to

Sol. **2**

$\tan(\pi\cos\theta) = \tan[-\pi\sin\theta]$

$\pi\cos\theta = n\pi - \pi\sin\theta \qquad (n\in I)$

$\cos\theta + \sin\theta = n$

$n\in\left[-\sqrt{2},\sqrt{2}\right] \qquad n\in\{-1,0,1\}$

$\cos\left(\theta-\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}, \qquad \cos\left(\theta-\frac{\pi}{4}\right)=0, \qquad \cos\left(\theta-\frac{\pi}{4}\right)=\frac{-1}{\sqrt{2}}$

$\theta-\frac{\pi}{4}=2m\pi\pm\frac{\pi}{4}, \theta-\frac{\pi}{4}=2m\pi \ +\frac{\pi}{2}, \qquad \theta-\frac{\pi}{4}=2m\pi\pm\frac{3\pi}{4}$

$\theta = 2m\pi+\frac{\pi}{2}, \qquad \theta = 2m\pi+\frac{3\pi}{4}, \qquad \theta = 2m\pi+\pi$

$\theta = 2m\pi, \qquad \theta = 2m\pi-\frac{\pi}{2}$

$$\theta = \left\{\frac{\pi}{2}, 0, \cancel{\frac{\pi}{4}}, \pi, \frac{3\pi}{2}, \frac{3\pi}{4}, \frac{7\pi}{4}\right\}$$

$$\sin\left(\theta+\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},\frac{-1}{\sqrt{2}},\frac{-1}{\sqrt{2}}$$

$$\therefore \Sigma\sin^2\left(\theta+\frac{\pi}{4}\right)=\frac{1}{2}+\frac{1}{2}+\cancel{1}+\frac{1}{2}+\frac{1}{2}=2 \text{ Ans.}$$

84. If $\frac{1^3+2^3+3^3+\cdots \text{ up to n terms}}{1\cdot3+2\cdot5+3\cdot7+\cdots \text{ up to n terms}}=\frac{9}{5}$, then the value of n is

Sol. **5**

$$\frac{\left(\frac{n(n+1)}{2}\right)^2}{\Sigma r(2r+1)}$$

$$\Rightarrow \frac{\frac{n^2(n+1)^2}{4}}{\frac{2n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}}$$

$$\Rightarrow \frac{\frac{n(n+1)}{4}}{\frac{2n+1}{3}+\frac{1}{2}} \Rightarrow \frac{\frac{n(n+1)}{4}}{\frac{(4n+5)}{6}}=\frac{9}{5}$$

$$\Rightarrow \frac{3(n+1)n}{2(4n+5)}=\frac{9}{5}$$

$\Rightarrow 5n^2 + 5n = 24n + 30$

$\Rightarrow 5n^2 - 19n - 30 = 0$

$5n^2 - 25n + 6n - 30 = 0$

$(5n + 6)(n - 5) = 0$

$n = 5$

85. Let the sum of the coefficients of the first three terms in the expansion of $\left(x-\frac{3}{x^2}\right)^n, x \neq 0. n \in \mathbb{N}$, be 376. Then the coefficient of x^4 is

Sol. **405**

${}^nC_0 - {}^nC_1 \cdot 3 + {}^nC_2 \cdot 3^2$

$1 - 3n + \frac{9n(n-1)}{2} = 376$

$2 - 6n + 9n^2 - 9n = 752$

$9n^2 - 15n - 750 = 0$

$3n^2 - 5n - 250 = 0$

$3n^2 - 30n + 25n - 250 = 0$

$(3n + 25)(n - 10) = 0$

$n = 10$

$\therefore T_{r+1} = {}^{10}C_r(x)^{10-r}\left(\frac{-3}{x^2}\right)^r$

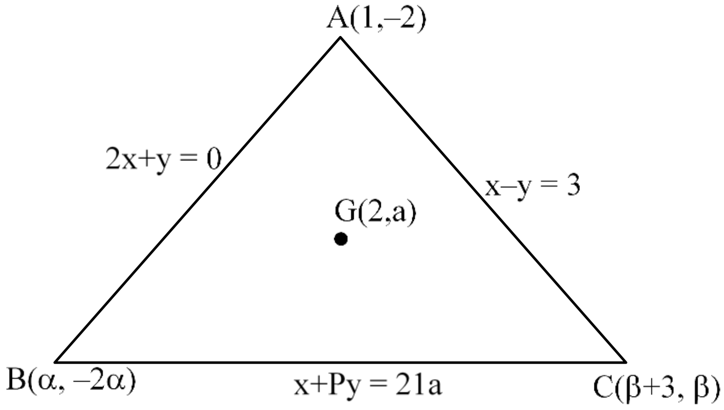
$x^{10-3r} = x^4 \Rightarrow 3r = 6$

$r = 2$

$\therefore T_3 = {}^{10}C_2 \times 3^2 \Rightarrow \frac{10\times 9}{2}\times 9 = 405$

86. The equations of the sides AB, BC and CA of a triangle ABC are : $2x + y = 0, x + py = 21a, (a \neq 0)$ and $x - y = 3$ respectively. Let $P(2, a)$ be the centroid of $\triangle ABC$. Then $(BC)^2$ is equal to

Sol. **122**



$\frac{\alpha+\beta+4}{3} = 2 \qquad \frac{-2\alpha-2+\beta}{3} = a$

$\alpha + \beta = 2 \qquad -2\alpha + \beta = 3a + 2$

$-2\alpha + 2 - \alpha = 3a + 2$

$\alpha = -a$

put 'B' in BC

$\alpha - 2p\alpha = 21a$

$\alpha.(1 - 2p) = 21a \qquad 2p - 1 = 21$

$p = 11$

put 'C' in BC

$\beta + 3 + 11\beta = 21a$

$21\alpha + 12\beta + 3 = 0$

also $\beta = 2 - \alpha \qquad$ Solving $\alpha = -3, \beta = 5$

$\therefore BC = \sqrt{122}$

$BC^2 = 122$

87. Let $\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k}, \vec{b} = 3\hat{i} - 5\hat{j} - \lambda\hat{k}, \vec{a}\cdot\vec{c} = 7, 2\vec{b}\cdot\vec{c} + 43 = 0, \vec{a}\times\vec{c} = \vec{b}\times\vec{c}$. Then $|\vec{a}\cdot\vec{b}|$ is equal to

Sol. **8**

$\bar{a} = (1, 2, \lambda) \qquad \bar{b} = (3, -5, -\lambda)$

$\bar{a}\cdot\bar{c}=7$, $\qquad \bar{b}\cdot\bar{c}=\frac{-43}{2}$,

$(\bar{a}-\bar{b})\times\bar{c}=0$

$c=x[-2,7,2\lambda]=(-2x,7x,2\lambda x)$

now, $\quad \bar{a}.\bar{c}=-2x+14x+2\lambda^2x=7$

$$2\lambda^2x+12x=7 \qquad ...(1)$$

$\bar{b}\cdot\bar{c}=-6x-35x-2\lambda^2x=\frac{-43}{2}$

$$-41x-2\lambda^2x=\frac{-43}{2} \qquad ...(2)$$

by adding (1) + (2)

$$-29x=\frac{-29}{2}\Rightarrow x=\frac{1}{2}$$

$\therefore \lambda^2+6=7 \quad \Rightarrow \lambda^2=1$

$|\bar{a}\cdot\bar{b}|=|3-10-\lambda^2|=|-8|$

88. The minimum number of elements that must be added to the relation R = {(a, b), (b, c), (b, d)} on the set {a, b, c, d} so that it is an equivalence relation, is

Sol. **13**

	a	b	c	d
a	1	✓	8	9
b	5	2	✓	✓
c	10	6	3	11
d	12	7	13	4

1, 2, 3, 4 → for reflexive

5, 6, 7 → for symmetric

8, 9, 10, 11, 12, 13 → for transitive

89. If the area of the region bounded by the curves $y^2-2y=-x, x+y=0$ is A, then 8 A is equal to

Sol. **36**

$y^2-2y+1=-x+1$	$x+y=0$
$(y-1)^2=-(x-1)$	$x+1+y+1=0$
$y^2=-4Ax$	$x+y+2=0$

$y=y-1$

$x=x-1$

$y^2=-x$

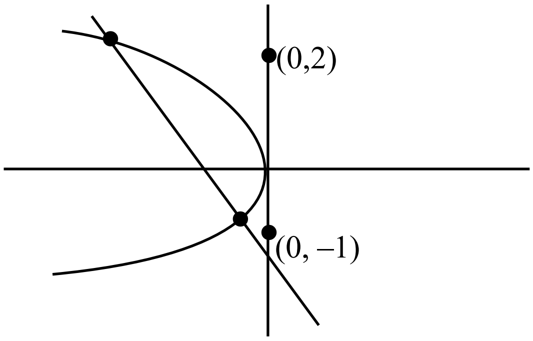
$x+y=-2$

$y^2 = y + 2$

$y^2 - y - 2 = 0$

$(y-2)\ (y + 1) = 0$

$y = 2,\ y = -1$



$$\text{Area} = \int_{-1}^{2} \left(-y^2\right) - (-2 - y)\,dy$$

$$= \left(\frac{-y^3}{3} + 2y + \frac{y^2}{2}\right)_{-1}^{2}$$

$$= \left(\frac{-8}{3} + 4 + 2\right) - \left(\frac{1}{3} - 2 + \frac{1}{2}\right)$$

$$= \frac{-8}{3} + 6 - \frac{1}{3} + 2 - \frac{1}{2}$$

$$A = -3 + 8 - \frac{1}{2} \Rightarrow \frac{9}{2}$$

$\therefore\ 8A = 36$

90. Let f be a differentiable function defined on $\left[0, \frac{\pi}{2}\right]$ such that $f(x) > 0$ and $f(x) + \int_0^x f(t)\sqrt{1 - (\log_e f(t))^2}\,dt = e, \forall x \in \left[0, \frac{\pi}{2}\right]$. Then $\left(6 \log_e f\left(\frac{\pi}{6}\right)\right)^2$ is equal to

Sol. $f'(x) + f(x) \cdot \sqrt{1 - \log^2 \cdot f(x)} = 0$

$$\frac{dy}{dx} = -y\sqrt{1 - \log^2 y}$$

$f(0) = e$

$$\frac{dy}{y\sqrt{1 - \log^2 y}} = -dx$$

$\log y = t$

$$\frac{1}{y}dy = dt$$

$$\frac{dt}{\sqrt{1 - t^2}} = -dx$$

$\text{Sin}^{-1}(t) = -x + C$

$\text{Sin}^{-1}[\log y] = -x + C$

$x = 0 \quad \text{Sin}^{-1}(1) = C \Rightarrow \frac{\pi}{2}$

$\log(y) = \text{Sin}\left(\frac{\pi}{2} - x\right)$

$x = \frac{\pi}{6} \quad \log_e\left[f\left(\frac{\pi}{6}\right)\right] = \text{Sin}\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\therefore \left(6 \times \frac{\sqrt{3}}{2}\right)^2 = 27$

JEE–MAIN EXAMINATION – JANUARY, 2023

(Held On Thursday 25th January, 2023) | **TIME : 9 : 00 AM to 12 : 00 NOON**

Physics

SECTION - A

1. Match List I with List II

List I	List II
A. Surface tension	I. $kgm^{-1}\ s^{-1}$
B. Pressure	II. $kgms^{-1}$
C. Viscosity	III. $kgm^{-1}\ s^{-2}$
D. Impulse	IV. kgs^{-2}

Choose the correct answer from the options given below:

(1) A-II , B-I , C-III , D-IV
(2) A-IV, B-III , C-I , D-II
(3) A-III , B-IV, C-I , D-II
(4) A-IV , B-III , C-II , D-I

Sol. 2

Surface tension (S) = $\frac{F}{l} \to kg\frac{M}{S^2}.\frac{1}{M} \to Kg\ s^{-2}$

Impulse (J) = $\int Fdt \quad \to N - S$

$\to Kg\ ms^{-2}\ .\ S$

$\to Kg\ ms^{-1}$

Pressure (P) = $\frac{F}{A} \to Kgms^{-2},\ m^{-2}$

$\to Kg\ ms^{-1}\ s^{-2}$

Viscocity (η) = $\frac{F}{6\pi rv}$

$\to \frac{kg\ ms^{-2}}{m.ms^{-1}}$

$\to kg\ m^{-1}\ s^{-1}$

2. The ratio of the density of oxygen nucleus (${}^{16}_{8}O$) and helium nucleus (${}^{4}_{2}He$) is

(1) 4:1 (2) 2: 1(3) 1: 1(4) 8: 1

Sol. 3

$\rho = \frac{M}{V}$ and $V = \frac{4}{3}\pi r^3$ when $r = R_0 A^{1/3}$

$\therefore\ \rho = \frac{M}{\frac{4}{3}\pi R_0^3 A}$

$\therefore\ \rho \propto \frac{M}{A}$

$\frac{\rho_o}{\rho_{He}} = \frac{M_o}{A_o} \times \frac{A_{He}}{M_{He}} = \frac{16}{8} \times \frac{2}{4} = 1$

3. The root mean square velocity of molecules of gas is

(1) Inversely proportional to square root of temperature $\left(\sqrt{\frac{1}{T}}\right)$

(2) Proportional to square of temperature (T^2)

(3) Proportional to temperature (T)

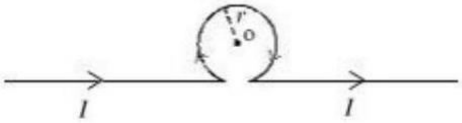
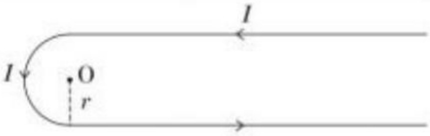
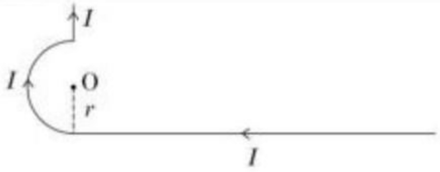
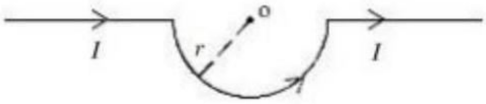
(4) Proportional to square root of temperature $(\sqrt{T})$

Sol. 4

$$V_{rms} = \sqrt{\frac{3RT}{M_0}}$$

$$\therefore\ V_{rms} \propto \sqrt{T}$$

4. Match List I with List II

List I (Current configuration)	List II (Magnitude of Magnetic Field at point O)
A. 	I. $B_0 = \frac{\mu_0 I}{4\pi r}[\pi + 2]$
B. 	II. $B_0 = \frac{\mu_0}{4}\frac{I}{r}$
C. 	III. $B_0 = \frac{\mu_0 I}{2\pi r}[\pi - 1]$
D. 	IV. $B_0 = \frac{\mu_0 I}{4\pi r}[\pi + 1]$

Choose the correct answer from the options given below :

(1) A-III, B-I, C-IV, D-II
(2) A-I, B-III, C-IV, D-II
(3) A-III, B-IV, C-I, D-II
(4) A-II, B-I, C-IV, D-III

Sol. 1

(A) $$B = \frac{\mu_0 I}{4\pi r} \times 2 - \frac{\mu_0 I}{2r}$$

$$= \frac{\mu I}{2r}\left(\frac{1}{\pi} - 1\right)$$

$$= \frac{\mu I}{2\pi r}\left(1-\pi\right) \odot$$

$$= \frac{\mu I}{2\pi r}(\pi-1) \otimes$$

(B) $B = \frac{\mu_0 I}{4\pi r}\times\pi+\frac{\mu_0 I}{4\pi r}\times 2$

$$= \frac{\mu_0 I}{4\pi r}(\pi+2) \odot$$

(C) $B = \frac{\mu_0 I}{4\pi r}.\pi+0+\frac{\mu_0 I}{4\pi r}$

$$= \frac{\mu_0 I}{4\pi r}(\pi+1) \otimes$$

(D) $B = \frac{\mu_0 I}{4r} \odot$

5. A message signal of frequency 5kHz is used to modulate a carrier signal of frequency 2MHz. The bandwidth for amplitude modulation is:

(1) 20 kHz (2) 5kHz (3) 10kHz (4) 2.5kHz

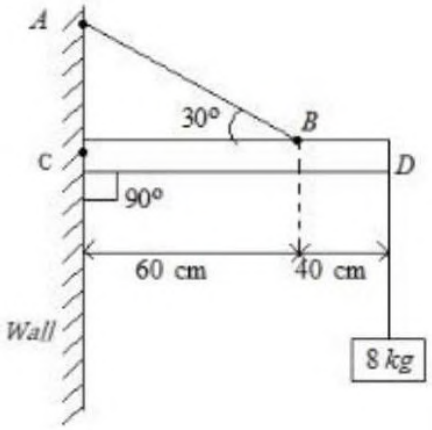
Sol. 3

Bandwidth = 2 × highest of base band frequency

= 2 × 5 = 10 kHZ

6. An object of mass 8 kg hanging from one end of a uniform rod CD of mass 2 kg and length 1m pivoted at its end C on a vertical walls as shown in figure. It is supported by a cable AB such that the system is in equilibrium. The tension in the cable is:

(Take g = 10 m/s^2)



(1) 90 N (2) 30 N (3) 300 N (4) 240 N

Sol. 3

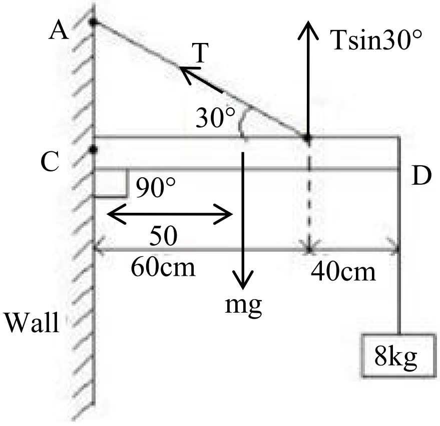
The rod is in equilibrium. So, net torque about any point will be zero.

$\tau_c = 0$

$Mg \times 50 + 80 \times 100 = T\sin 30° \times 30$

$$20 \times 50 + 80 \times 100 = \frac{T}{2} \times 60$$

$$T = \frac{900}{3} = 300 \text{ N}$$



7. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R

Assertion A: Photodiodes are used in forward bias usually for measuring the light intensity.

Reason R: For a p-n junction diode, at applied voltage

V the current in the forward bias is more than the current in the reverse bias for $|V_z| > \pm V \geq |V_0|$ where V_0 is the threshold voltage and V_z is the breakdown voltage.

In the light of the above statements, choose the correct answer from the options given below

(1) Both A and R are true and R is correct explanation A

(2) A is false but R is true

(3) Both A and R are true but R is NOT the correct explanation A

(4) A is true but R is false

Sol. 2

Photo diodes are not used in forward bias.

8. In an LC oscillator, if values of inductance and capacitance become twice and eight times, respectively, then the resonant frequency of oscillator becomes x times its initial resonant frequency ω_0. The value of x is:

(1) 4 (2) 1/16 (3) 16 (4) ¼

Sol. 4

$$\omega = \frac{1}{\sqrt{LC}}$$

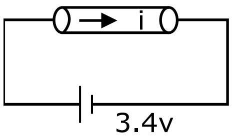
$$\frac{\omega_1}{\omega_2} = \sqrt{\frac{L_2C_2}{L_1C_1}} = \sqrt{\frac{2L.8C}{L.C}} = 4$$

$$\omega_2 = \frac{\omega_1}{4} = \frac{\omega_0}{4}$$

9. A uniform metallic wire carries a current 2A, when 3.4 V battery is connected across it. The mass of uniform metallic wires is 8.92×10^{-3} kg density is 8.92×10^{3} kg/m^3 and resistivity is 1.7×10^{-8} Ω – m. The length of wire is:

(1) $l = 10$ m (2) $l = 100$ m (3) $l = 5$ m (4) $l = 6.8$ m

Sol. 1



Given, i = 2A

v = 3.4 v

v = iR

$$R = \frac{v}{i} = \frac{3.4}{2} = 1.7\Omega$$

$$\text{volume} = \frac{\text{mass}}{\text{Density}} = \frac{8.92 \times 10^{-3}}{8.92 \times 10^{3}} \text{m}^3 = 10^{-6} \text{m}^3$$

$\Rightarrow A\ell = 10^{-6}$ m^3 ____ (i)

$$R = \frac{\rho\ell}{A}$$

$$\Rightarrow \frac{\rho}{R} = \frac{A}{\ell}$$

$$\frac{1.7 \times 10^{-8}}{1.7} = \frac{A}{\ell}$$

$\frac{A}{\ell} = 10^{-8}$ ____ (ii)

$$\frac{\text{eq(i)}}{\text{eq(ii)}}$$

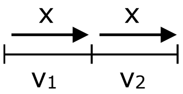
$\ell^2 = 10^2$

$\ell = 10$ m

10. A car travels a distance of ' x ' with speed v_1 and then same distance ' x ' with speed v_2 in the same direction. The average speed of the car is:

(1) $\frac{2v_1v_2}{v_1+v_2}$ (2) $\frac{2x}{v_1+v_2}$ (3) $\frac{v_1v_2}{2(v_1+v_2)}$ (4) $\frac{v_1+v_2}{2}$

Sol. 1



$$v_{avg} = \frac{\text{total Distance}}{\text{Total Time}}$$

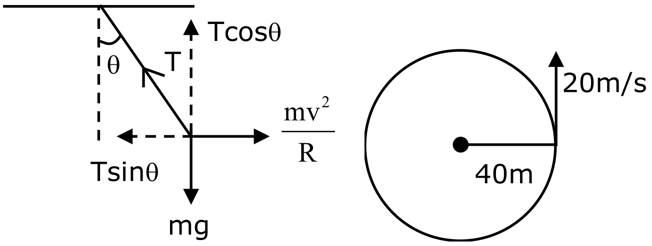
$$= \frac{2x}{\frac{x}{v_1} + \frac{x}{v_2}}$$

$$= \frac{2v_1v_2}{v_1 + v_2}$$

11. A car is moving with a constant speed of 20 m/s in a circular horizontal track of radius 40m. A bob is suspended from the roof of the car by a massless string. The angle made by the string with the vertical will be: (Take g = 10 m/s^2)

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{6}$

Sol. 3



$T\cos\theta = mg$ __ (i)

$T\sin\theta = \frac{mv^2}{R}$ __ (ii)

$$\frac{eq(i)}{eq(ii)}; \frac{\cos\theta}{\sin\theta} = \frac{gR}{v^2}$$

$$\tan\theta = \frac{v^2}{Rg} = \frac{400}{40\times10} = 1$$

$$\theta = \frac{\pi}{4}$$

12. A bowl filled with very hot soup cools from 98°C to 86°C in 2 minutes when the room temperature is 22°C. How long it will take to cool from 75°C to 69°C?

(1) 1 minute (2) 1.4 minutes (3) 0.5 minute (4) 2 minutes

Sol. 2

According to NLC,

$$-\frac{d\theta}{dt} = k\theta$$

$$\frac{12}{2} = K\left(\frac{98+86}{2} - 22\right)$$

$\Rightarrow 6 = K(92 - 22) = K \times 70$

$\Rightarrow K = \frac{6}{70}$ …. (i)

Now, $\frac{6}{t_2}=\frac{6}{70}\left(\frac{75+69}{2}-22\right)$

$=\frac{6}{70}\times(72-22)$

$t_2=\frac{6\times70}{6\times50}$

$\frac{7}{5}=1.4\,\text{min}$

13. A solenoid of 1200 turns is wound uniformly in a single layer on a glass tube 2m long and 0.2m in diameter. The magnetic intensity at the center of the solenoid when a current of 2A flows through it is?

(1) 2.4×10^{3} A m^{-1}　　(2) 1.2×10^{3} A m^{-1}

(3) 2.4×10^{-3} A m^{-1}　　(4) 1 A m^{-1}

Sol. 2

$B=\mu_0 nI$ and $n=\frac{1200}{2}=600$

Magnetic field intensity $H=\frac{B}{\mu_0}=nI=600\times2=1200=1.2\times10^{3}$ A m^{-1}

14. In Young's double slits experiment, the position of 5th bright fringe from the central maximum is 5cm. The distance between slits and screen is 1m and wavelength of used monochromatic light is 600 nm. The separation between the slits is:

(1) 48μm　　(2)36μm　　(3)12μm　　(4) 60μm

Sol. 4

$5\beta=5$ cm

$\Rightarrow\beta=1$cm

$\frac{\lambda D}{d}=1\text{ cm}=\frac{1}{100}\text{m}$

$\Rightarrow d=600\times10^{-9}\times100\times1$

$=60\times10^{-6}$ m

$=60\ \mu$m

15. An electromagnetic wave is transporting energy in the negative z direction. At a certain point and certain time the direction of electric field of the wave is along positive y direction. What will be the direction of the magnetic field of the wave at the point and instant?

(1) Negative direction of y　　(2) Positive direction of z

(3) Positive direction of x　　(4) Negative direction of x

Sol. 3

$\vec{B}\quad\perp r\quad\vec{E}$　and　Direction of propagation is given by

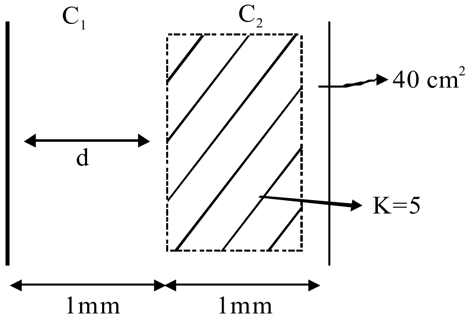
$\vec{E}\times\vec{B}$.

$\boxed{\hat{j}\times\hat{i}=-k}$

16. A parallel plate capacitor has plate area 40 cm^2 and plates separation 2mm. The space between the plates is filled with a dielectric medium of a thickness 1 mm and dielectric constant 5. The capacitance of the system is:

(1) $24\varepsilon_0$ F (2) $\frac{10}{3}\varepsilon_0$ F (3) $\frac{3}{10}\varepsilon_0$ F (4) $10\varepsilon_0$ F

Sol. 2



$$C_1 = \frac{\varepsilon_0 A}{d} = C_0$$

$$C_2 = K\frac{\varepsilon_0 A}{d} = K\varepsilon_0$$

$$Ceq = \frac{C_1C_2}{C_1+C_2} = \frac{C_0 \times KC_0}{(K+1)\varepsilon_0} = \frac{KC_0}{K+1}$$

$$= \frac{5\times\varepsilon_0\times 40\times 10^{-4}}{1\times 10^{-3}\times 6}$$

$$= \frac{10}{3}\varepsilon_0 F$$

17. Assume that the earth is a solid sphere of uniform density and a tunnel is dug along its diameter throughout the earth. It is found that when a particle is released in this tunnel, it executes a simple harmonic motion. The mass of the particle is 100 g. The time period of the motion of the particle will be (approximately)

(Take $g = 10$ m s^{-2}, radius of earth $= 6400$ km)

(1) 12 hours (2) 1 hour 40 minutes

(3) 24 hours (4) 1 hour 24 minutes

Sol. 4

Inside earth, force is given by$F = -\frac{GM_e mx}{R_e^3}$

And $g_0(on\ surface\ of\ earth) = \frac{GM_e}{R_e^2}$

$\therefore F = -\frac{g_0 m}{R_e}x$

$\Rightarrow a = -\frac{g_0}{R_e}x$

$\omega = \sqrt{\frac{g_0}{R_e}}$

$\Rightarrow T = 2\pi\sqrt{\frac{R_e}{g_0}} = 2\pi\sqrt{\frac{6400\times10^3}{10}} = 2\times3.13\times8\times10^2 sec = 5024\ sec = 1.4\ hr$

$T = 1.4\ hr = 1\ hr\ 24\ minutes$

18. Electron beam used in an electron microscope, when accelerated by a voltage of 20kV, has a de – Broglie wavelength of λ_0. If the voltage is increased to 40kV, then the de-Broglie wavelength associated with the electron beam would be:

(1) $3\lambda_0$(2) $\frac{\lambda_0}{2}$(3) $\frac{\lambda_0}{\sqrt{2}}$(4) $9\lambda_0$

Sol. 3

$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2meV}}$

$\Rightarrow \lambda\, \alpha \frac{1}{\sqrt{V}}$

$\Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{V_2}{V_1}}$

$\Rightarrow \frac{\lambda_0}{\lambda_2} = \sqrt{\frac{40}{20}} = \sqrt{2}$

$\Rightarrow \lambda_2 = \frac{\lambda_0}{\sqrt{2}}$

19. A Carnot engine with efficiency 50% takes heat from a source at 600 K. In order to increase the efficiency to 70%, keeping the temperature of sink same, the new temperature of the source will be :

(1) 300 K (2) 900 K (3) 1000 K (4) 360 K

Sol. 3

$\eta = 1 - \frac{T_L}{T_H}$

$0.5 = 1 - \frac{T_L}{600} \Rightarrow T_L = (1 - 0.5) \times 600\ K = 300\ K$

Now $0.7 = 1 - \frac{300}{T_2}$

$\frac{300}{T_2} = 0.3 \Rightarrow T_2 = \frac{300}{0.3} = 1000\ K$

20. T is the time period of simple pendulum on the earth's surface. Its time Period becomes x T when taken to a height R (equal to earth's radius) above the earth's surface. Then, the value of x will be:

(1) 4 (2) 2 (3) $\frac{1}{4}$ (4) $\frac{1}{2}$

Sol. 2

$T = 2\pi\sqrt{\frac{l}{g_{eff}}}$

And $g_{eff\ above\ earth's\ surface} = \frac{GM}{(R+h)^2} = \frac{GM}{4R^2} = \frac{g_0}{4}$

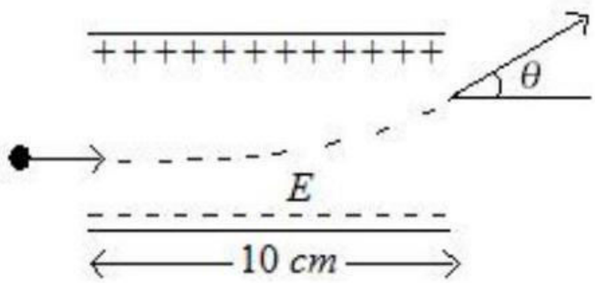
Now $\frac{T_1}{T_2} = \sqrt{\frac{\frac{g_0}{4}}{g_0}} = \frac{1}{2}$

$T_2 = 2T_1$

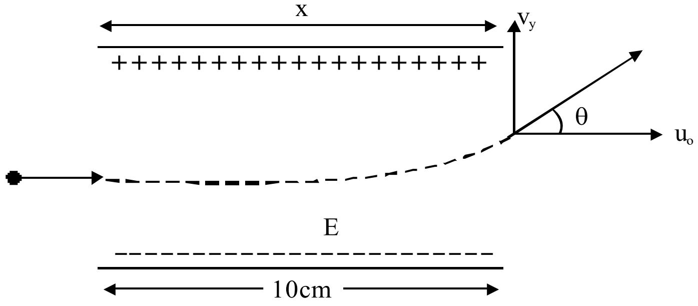
$\therefore x = 2$

SECTION – B

21. A uniform electric field of 10 N/C is created between two parallel charged pates (as shown in figure). An electron enters the field symmetrically between the plates with a kinetic energy 0.5eV. The length of each pate is 10 cm. The angle (θ) of deviation of the path of electron as it comes out of the field is ______ (in degree).



Sol. 45^0



Force due to electric field is given by $\vec{F} = q\vec{E}$

$\therefore F = eE$

$\Rightarrow a = \frac{eE}{m}$

The electron will take a parabolic path i.e., projectile motion.

Here, $s_x = 10cm = 0.1m$

$\therefore t = \frac{0.1}{u_x} - - - - (i)$

Now $v_y = u_y + a_y t$

$\Rightarrow v_y = 0 + \frac{eE}{m} \times \frac{0.1}{u_x} - - - - (ii)$

Also $KE = \frac{1}{2} mv^2 = \frac{1}{2} mu_x^2$

$mu_x^2 = 2 \times KE = 2 \times 0.5e = e - - - - (iii)$

From eq (i),(ii) and (iii), $tan\theta = \frac{v_y}{u_x} = \frac{eE}{m} \times \frac{0.1}{u_x} \times \frac{1}{u_x} = \frac{0.1eE}{mu_x^2} = \frac{0.1eE}{e} = 0.1 \times 10 = 1$

$\Rightarrow tan\theta = 1$

$\therefore \theta = 45^0$

22. The wavelength of the radiation emitted is λ_0 when an electron jumps from the second excited state to the first excited state of hydrogen atom. If the electron jumps from the third excited state to the second orbit of the hydrogen atom, the wavelength of the radiation emitted will be $\frac{20}{x}\lambda_0$. The value of x is _________.

Sol. 27

Bohr's energy is given by $E = -13.6 \times \frac{1}{n^2}$ for hydrogen atom.

And $E = \frac{hc}{\lambda}$

For 1st condition, $\frac{hc}{\lambda_0} = 13.6\left(\frac{1}{4} - \frac{1}{9}\right) = 13.6 \times \frac{5}{36} - - - - - - (i)$

For 2nd condition, $\frac{hc}{\lambda} = 13.6\left(\frac{1}{4} - \frac{1}{16}\right) = 13.6 \times \frac{3}{16} - - - - - - (ii)$

Dividing equation (i) by (ii),

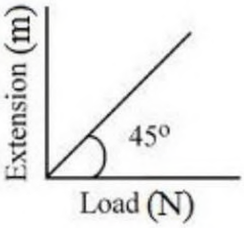
$\frac{\lambda}{\lambda_0} = \frac{5}{36} \times \frac{16}{3} = \frac{20}{27}$

$\Rightarrow \lambda = \frac{20}{27}\lambda_0$

$\Rightarrow n = 27$

23. As shown in the figure, in an experiment to determine Young's modulus of a wire, the extension-load curve is plotted. The curve is a straight line passing through the origin and makes an angle of 45° with the load axis. The length of wire is 62.8cm and its diameter is 4 mm. The Young's modulus is found to be x × 10^4 Nm^{-2}.

The value of x is __________



Sol. 5

From graph, $tan45^0 = \frac{\Delta l}{F}$

$\Rightarrow \frac{\Delta l}{F} = 1 - - - - (i)$

Also, Young's modulus is given by $Y = \frac{Fl}{A\Delta l} = \frac{l}{A} \times \frac{F}{\Delta l} = \frac{l}{A} \times 1$

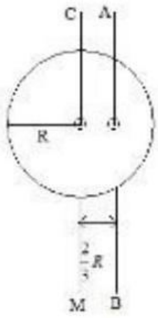
$\therefore Y = \frac{l}{A} = \frac{62.8 \times 10^{-2}}{\pi \times 4 \times 10^{-6}} = 5 \times 10^4\ Nm^{-2}$

$\therefore x = 5$

24 I_{CM} is the moment of inertia of a circular disc about an axis (CM)passing through its center and perpendicular. To the plane of disc. I_{AB} is it's moment of inertia about an axis AB perpendicular to plane and parallel to axis CM at a distance $\frac{2}{3}R$ from center.

Where R is the radius of the disc. The ratio of I_{AB} and I_{CM} is x: 9.

The value of x is __________



Sol. 17

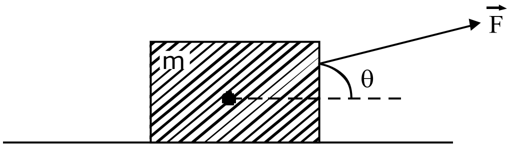
$I_{CM} = \frac{MR^2}{2}$

$I_{AB} = \frac{MR^2}{2} + M\left(\frac{2}{3}R\right)^2 = \frac{MR^2}{2} + \frac{4MR^2}{9} = \frac{17MR^2}{18}$

As per question, $\frac{I_{AB}}{I_{CM}} = \frac{17}{9}$

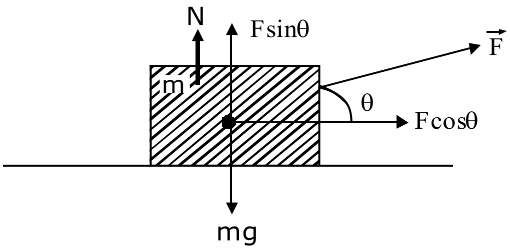
$\therefore x = 17$

25. An object of mass 'm' initially at rest on a smooth horizontal plane starts moving under the action of force F = 2N. In the process of its linear motion, the angle θ (as shown in figure) between the direction of force and horizontal varies as θ = kx, where k is constant and x is the distance covered by the object from the initial positon. The expression of kinetic energy of the object will be $E = \frac{n}{k}\sin\theta$, The value of n is ______.



Smooth horizontal surface

Sol. 2



$F_x = 2coskx$

$F_y = 2sinkx - mg$

According to Work Energy Theorem, $\Delta K = \Delta W$

Taking motion only along horizontal direction(X) i.e., linear motion as mentioned in question, $\Delta K = \int_0^x F_x\, dx$

$K_f - K_i = \int_0^x 2coskx\, dx = \frac{2sinkx}{k}$

Hence $K_i = 0, \therefore K_f = \frac{2sinkx}{k}$

$\therefore n = 2$

26. An LCR series circuit of capacitance 62.5nF and resistance of 50Ω, is connected to an A.C. source of frequency 2.0kHz. For maximum value of amplitude of current in circuit, the value of inductance is ________ mH. Take $\pi^2 = 10$)

Sol. 100

At maximum current, there will be condition of resonance.

So, $\omega = \frac{1}{\sqrt{LC}}$

$\Rightarrow L = \frac{1}{\omega^2 C} = \frac{1}{4\times\pi^2\times4\times10^6\times62.5\times10^{-9}} H = 0.1H = 100mH$

27. The distance between two consecutive points with phase difference of 60° in a wave of frequency 500 Hz is 6.0 m. The velocity with which wave is traveling is ________ km/s

Sol. 18

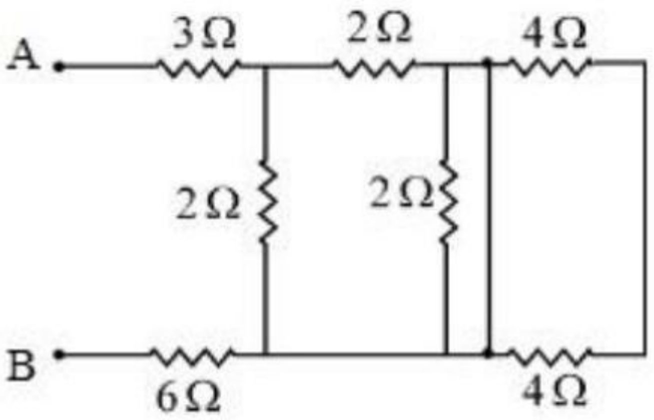
$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$

$\Rightarrow \frac{\pi}{3} = \frac{2\pi}{\lambda} \times 6$

$\Rightarrow \lambda = 36m$

Now $v = f\lambda = 500 \times 36\, m/s = 18000\, m/s = 18km/s$

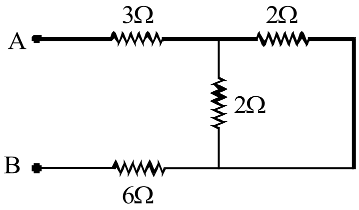
28. In the given circuit, the equivalent resistance between the terminal A and B is Ω.



Sol. 10

Due to short circuit, 3 resistances get vanished from the circuit.

The circuit is



$R_{eq} = 3 + 3 + 1 = 10\Omega$

29. If $\vec{P} = 3\hat{\imath} + \sqrt{3}\hat{\jmath} + 2\hat{k}$ and $\vec{Q} = 4\hat{\imath} + \sqrt{3}\hat{\jmath} + 2.5\hat{k}$ then, The unit vector in the direction of $\vec{P} \times \vec{Q}$ is $\frac{1}{x}(\sqrt{3}i + \hat{\jmath} - 2\sqrt{3}\hat{k})$. The value of x is

Sol. 4

Let $\vec{C} = \vec{P} \times \vec{Q} = 3\sqrt{3}\hat{k} - 7.5\hat{\jmath} - 4\sqrt{3}\hat{k} + 2.5\sqrt{3}\hat{\imath} + 8\hat{\jmath} - 2\sqrt{3}\hat{\imath}$

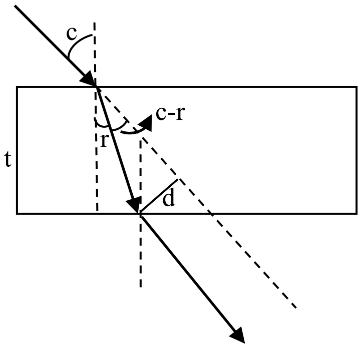
$$= \frac{1}{2}(\sqrt{3}\hat{\imath} + \hat{\jmath} - 2\sqrt{3}\hat{k})$$

$|\vec{C}| = \frac{1}{2}\sqrt{3+1+12} = \frac{1}{2} \times 4 = 2$

$\therefore \hat{C} = \frac{\vec{C}}{|\vec{C}|} = \frac{1}{4}(\sqrt{3}\hat{\imath} + \hat{\jmath} - 2\sqrt{3}\hat{k})$

30. A ray of light is incident from air on a glass plate having thickness $\sqrt{3}$ cm and refractive index $\sqrt{2}$. The angle of incidence of a ray is equal to the critical angle for glass-air interface. The lateral displacement of the ray when it passes through the plate is _______ $\times 10^{-2}$ cm. (given sin 15° = 0.26)

Sol. 52



$sinc = \frac{1}{\sqrt{2}} \Rightarrow c = 45^0$

Using Snell's law on 1st surface, $sinc = \sqrt{2}\, sinr$

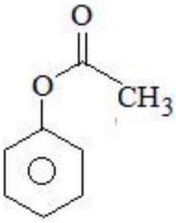
$$\Rightarrow sinr = \frac{1}{2} \Rightarrow r = 30^0$$

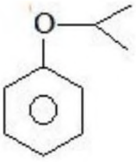
$d = tsecr \times \sin(c - r) = \sqrt{3} \times \frac{2}{\sqrt{3}} \times 0.26 = 0.52cm = 52 \times 10^{-2}cm$

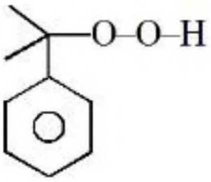
Chemistry

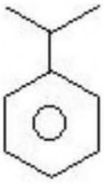
SECTION - A

31. In the cumene to phenol preparation in presence of air, the intermediate is

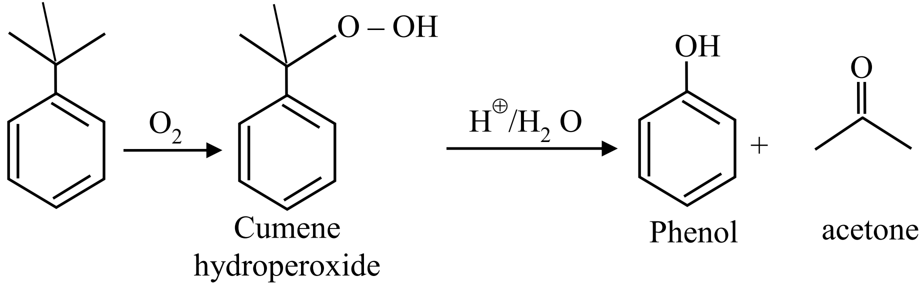
(1) 

(2) 

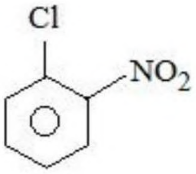
(3) 

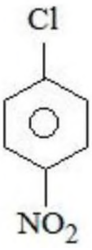
(4) 

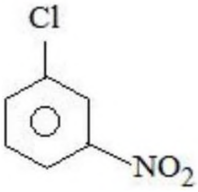
Sol. **3**

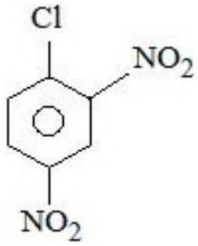


32. The compound which will have the lowest rate towards nucleophilic aromatic substitution on treatment with $OH^-$is

(1) 

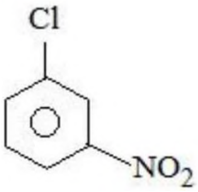
(2) 

(3) 

(4) 

Sol. **3**

Rate of nucleophilic aromatic substitution decrease by e^- withdrawing group



$-NO_2$ of meta shows –I effect which is less dominating than –M

33. Match List I with List II

LIST I Elements		LIST II Colour imparted to the flame	
A.	K	I.	Brick Red
B.	Ca	II.	Violet
C.	Sr	III	Apple Green
D.	Ba	IV.	Crimson Red

Choose the correct answer from the options given below:

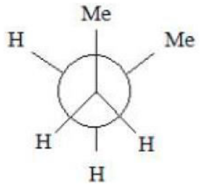
(1) A-II, B-I, C-III, D-IV
(2) A-II, B-I, C-IV, D-III
(3) A-IV, B-III, C-II, D-I
(4) A-II, B-IV, C-I, D-III

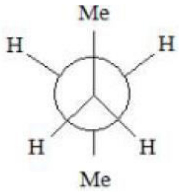
Sol. **2**

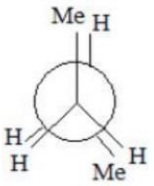
Flame Test.

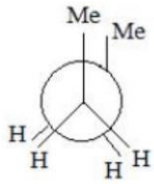
Metals	**Colour of flame test**
K	Violet
Ca	Brick Red
Sr	Crimson Red
Ba	Apple Green

34. Which of the following conformations will be the most stable ?

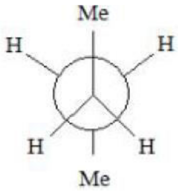
(1) 

(2) 

(3) 

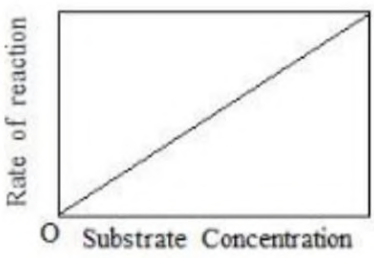
(4) 

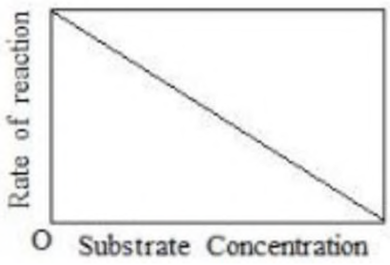
Sol. **2**

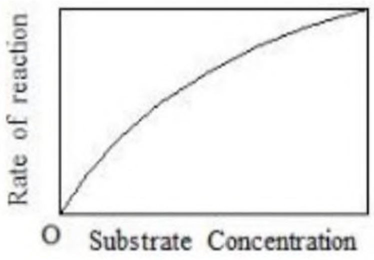


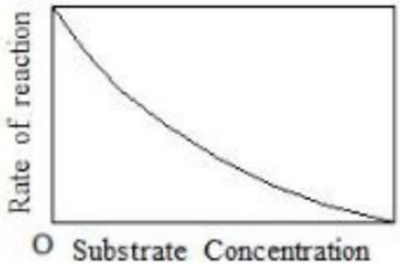
Anti position highly stable (bulky group maximum distance)

35. The variation of the rate of an enzyme catalyzed reaction with substrate concentration is correctly represented by graph

(a) 

(b) 

(c) 

(d) 

(1) (b) (2) (a) (3) (d) (4) (c)

Sol. **4**

Fact base.

36. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason **R** :

Assertion A : Acetal / Ketal is stable in basic medium.

Reason R : The high leaving tendency of alkoxide ion gives the stability to acetal/ ketal in basic medium.

In the light of the above statements, choose the correct answer from the options given below :

(1) A is true but R is false

(2) A is false but R is true

(3) Both A and R are true but R is NOT the correct explanation of A

(4) Both A and R are true and R is the correct explanation of A

Sol. **1**

Acetal and ketals are basically ether hence they must be stable in basic medium but should break down in acidic medium.

Hence assertion is correct.

Alkoxide ion (RO^-) is not considered a good leaving group hence reason must be false.

37. A cubic solid is made up of two elements X and Y. Atoms of X are present on every alternate corner and one at the center of cube. Y is at $\frac{1}{3}$rd of the total faces. The empirical formula of the compound is

(1) $XY_{2.5}$ (2) $X_2Y_{1.5}$ (3) $X_{2.5}Y$ (4) $X_{1.5}Y_2$

Sol. **4**

Number of X-atom per unit cell = $1+4\times\frac{1}{8} = \frac{3}{2}$

Number of Y-atoms per unit cell = $2\times\frac{1}{2}=1$

$\therefore$ Empirical formula of the solid is X_3Y_2.

38. Match the List-I with List-II

List-I	List-II
Cations	Group reagents
A → Pb^{2+}, Cu^{2+}	i) H_2 S gas in presence of dilute HCl
B → Al^{3+}, Fe^{3+}	ii) $(NH_4)_2CO_3$ in presence of NH_4OH
C → Co^{2+}, Ni^{2+}	iii) NH_4OH in presence of NH_4Cl
D → Ba^{2+}, Ca^{2+}	iv) H_2 S in presence of NH_4OH

Correct match is -

(1) A → iii, B → i, C → iv, D → ii

(2) A → i, B → iii, C → ii, D → iv

(3) A → iv, B → ii, C → iii, D → i

(4) A → i, B → iii, C → iv, D → ii

Sol. **4**

Cations	Group No.	Group reagents
Pb^{2+}, Cu^{2+}	II	H_2S + HCl
Al^{3+}, Fe^{3+}	III	$NH_4Cl + NH_4OH$
Co^{2+}, Ni^{2+}	IV	$NH_4OH + H_2S$
Ba^{2+}, Ca^{2+}	V	NH_4OH , Na_2CO_3

39. Which of the following statements is incorrect for antibiotics?
(1) An antibiotic must be a product of metabolism.
(2) An antibiotic should promote the growth or survival of microorganisms.
(3) An antibiotic is a synthetic substance produced as a structural analogue of naturally occurring antibiotic.
(4) An antibiotic should be effective in low concentrations.

Sol. **2**
Antibiotic kill or inhibit the growth of microorganism

40. The correct order in aqueous medium of basic strength in case of methyl substituted amines is :
(1) $Me_3\,N > Me_2NH > MeNH_2 > NH_3$
(2) $Me_2NH > MeNH_2 > Me_3\,N > NH_3$
(3) $Me_2NH > Me_3\,N > MeNH_2 > NH_3$
(4) $NH_3 > Me_3\,N > MeNH_2 > Me_2NH$

Sol. **2**
In aqueous medium basic strength is dependent on electron density on nitrogen as well as solvation of cation formed after accepting H^+.After considering all these factors overall basic strength order is $Me_2NH > MeNH_2 > Me_3\,N > NH_3$

41. '25 volume' hydrogen peroxide means
(1) 1 L marketed solution contains 25 g of H_2O_2.
(2) 1 L marketed solution contains 75 g of H_2O_2.
(3) 1 L marketed solution contains 250 g of H_2O_2.
(4) 100 mL marketed solution contains 25 g of H_2O_2.

Sol. **2**
25 V H_2O_2 means : 1 lit of H_2O_2 on decomposition give 25 lit of $O_2(g)$ at STP.

$$2H_2O_2(\ell) \rightarrow 2H_2O(\ell) + O_2(g)$$

$$2\left[\frac{25}{22.4}\right]\text{mole} \qquad \left[\frac{25}{22.4}\right]\text{mole}$$

Mass of $H_2O_2 = \frac{2\times 25}{22.4}\times 34 = 75.89\,\text{gram}$.

42. The radius of the 2^{nd} orbit of Li^{2+} is x. The expected radius of the 3^{rd} orbit of Be^{3+} is
(1) $\frac{27}{16}x$ (2) $\frac{4}{9}x$ (3) $\frac{9}{4}x$ (4) $\frac{16}{27}x$

Sol. **1**

$$R = 0.529\times\frac{n^2}{Z}$$

$$r_{Li^{2+}\ n-2} = 0.529\times\frac{(2)^2}{3} = x$$

$$r_{Be^{3+}\ n-3} = 0.529\times\frac{(3)^2}{4}$$

$$\frac{r_{Li^{2+}\ n-2}}{r_{Be^{3+}\ n-3}} = \frac{\frac{r_0\times(2)^2}{3}}{\frac{r_0\times(3)^2}{4}}$$

$$\frac{x}{r_{Be^{3+}\ n-3}} = \frac{16}{27}$$

$$\therefore \quad (r_{Be^{3+}})_{n=3} = \frac{27x}{16}$$

43. Reaction of thionyl chloride with white phosphorus forms a compound [A], which on hydrolysis gives [B], a dibasic acid. [A] and [B] are respectively
(1) P_4O_6 and H_3PO_3 (2) PCl_5 and H_3PO_4 (3) $POCl_3$ and H_3PO_4 (4) PCl_3 and H_3PO_3

Sol. **4**

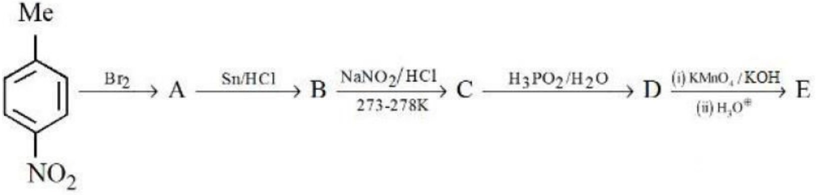
$$P_4 + 8SOCl_2 \longrightarrow \underset{(A)}{4PCl_3} + 4SO_2 + 2S_2Cl_2$$

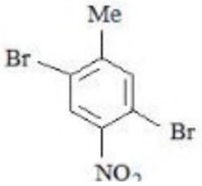
$$PCl_3 + 3H_2O \longrightarrow \underset{(B)}{H_3PO_3} + 3HCl$$

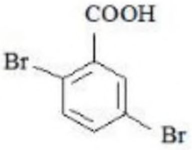
44. Inert gases have positive electron gain enthalpy. Its correct order is
(1) He < Kr < Xe < Ne (2) He < Xe < Kr < Ne
(3) He < Ne < Kr < Xe (4) Xe < Kr < Ne < He

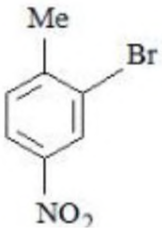
Sol. **2**
Positive electron gain enthalpy. of inert gas is in order of
Ne > Ar = Kr > Xe > He

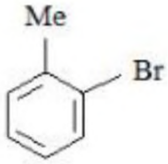
45. Identify the product formed (and *E*)

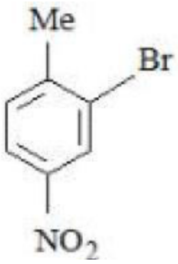


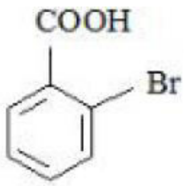
(1) A = 

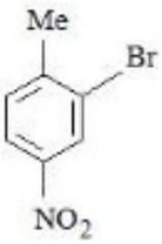
, E = 

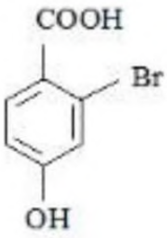
(2) 

, E = 

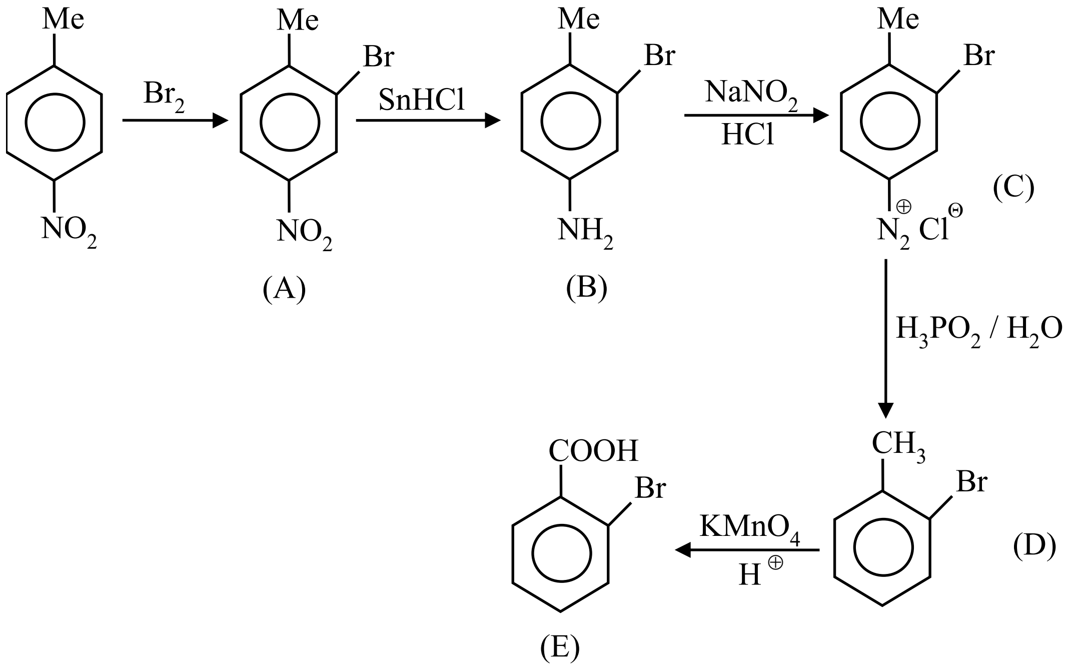
(3) 

, E = 

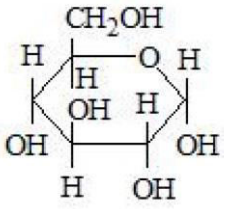
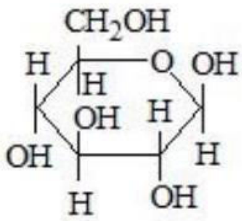
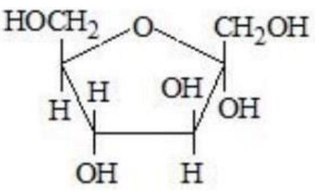
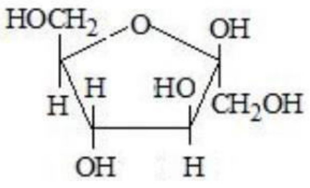
(4) 

, E = 

Sol. **3**



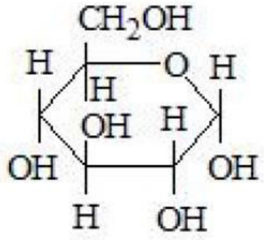
46. Match items of Row I with those of Row II.

Row I	Row II
(A) 	(i) $\alpha-\mathrm{D}-(-)$-Fructofuranose,
(B) 	(ii) $\beta-\mathrm{D}-(-)-$ Fructofuranose
(C) 	(iii) $\alpha-D-(-)$ Glucopyranose,
(D) 	(iv) $\beta-D-(-)$-Glucopyranose

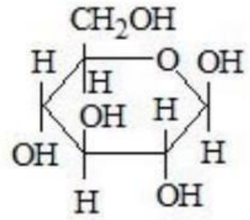
Correct match is

(1) A → i, B → ii, C → ii, D → iv (2) A → iv, B → iii, C → i, D → ii
(3) A → iii, B → iv, C → ii, D → I (4) A → iii, B → iV, C → i, D → ii

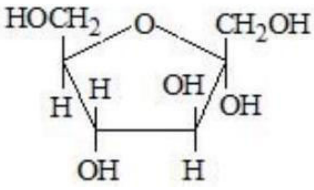
Sol. **4**



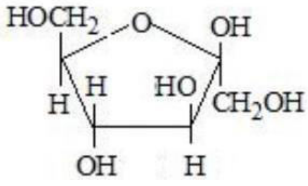
$\alpha - D - (-)$ Glucopyranose



$\beta - D - (-)$-Glucopyranose



α – D – (–)-Fructofuranose



β – D – (–) – Fructofuranose

47. Which one of the following reactions does not occur during extraction of copper ?
(1) $2Cu_2\,S + 3O_2 \rightarrow 2Cu_2O + 2SO_2$ (2) $FeO + SiO_2 \rightarrow FeSiO_3$
(3) $2FeS + 3O_2 \rightarrow 2FeO + 2SO_2$ (4) $CaO + SiO_2 \rightarrow CaSiO_3$

Sol. **4**

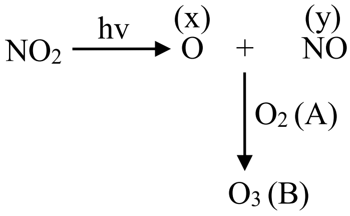
$$\underset{\text{Impmily}}{CaO} + \underset{\text{Flux}}{SiO_2} \longrightarrow \underset{\text{Slog}}{CaSiO_3}$$

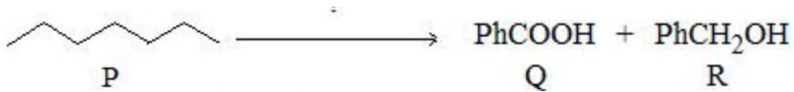
In metallurgy iron will occur not in metallurgy of Cu.

48. Some reactions of NO_2 relevant to photochemical smog formation are

Identify A, B, X and Y
(1) $X = \frac{1}{2}O_2, Y = NO_2, A = O_3, B = O_2$ (2) $X = [O], Y = NO, A = O_2, B = O_3$
(3) $X = N_2O, Y = [O], A = O_3, B = NO$ (4) $X = NO, Y = [O], A = O_2, B = N_2O_3$

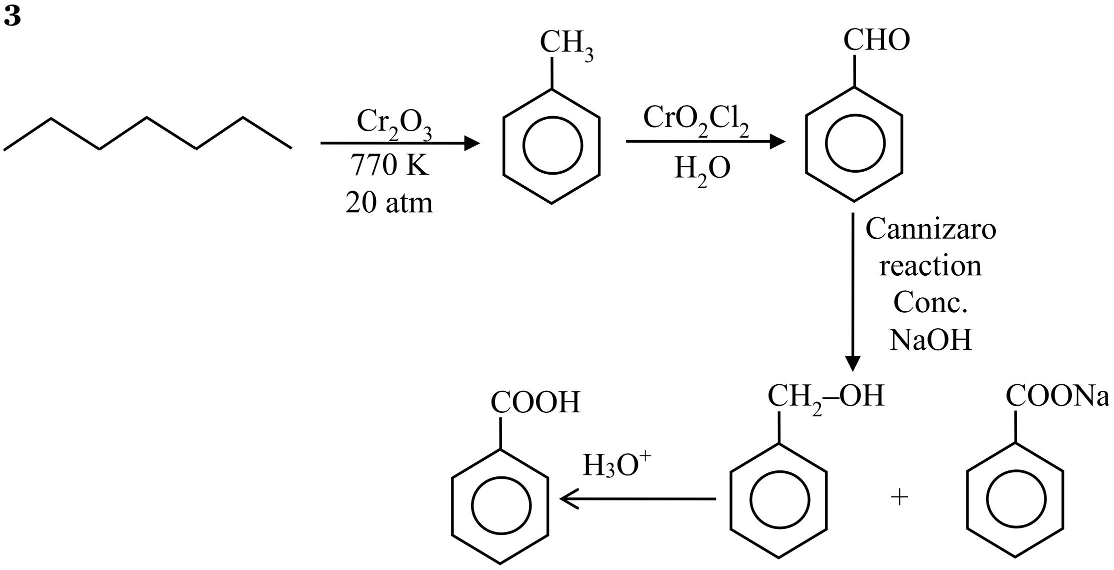
Sol. **2**



49. 

The correct sequence of reagents for the preparation of *Q* and *R* is :
(1) (i) CrO_2Cl_2, H_3O^+; (ii) Cr_2O_3, 770 K, 20 atm; (iii) NaOH; (iv) H_3O^+
(2) (i) $KMnO_4, OH^-$; (ii) Mo_2O_3, Δ; (iii) NaOH; (iv) H_3O^+
(3) (i) Cr_2O_3, 770 K, 20 atm; (ii) CrO_2Cl_2, H_3O^+; (iii) NaOH; (iv) H_3O^+
(4) (i) Mo_2O_3, Δ; (ii) CrO_2Cl_2, H_3O^+; (iii) NaOH; (iv) H_3O^+

Sol. **3**



50. Compound A reacts with NH_4Cl and forms a compound B. Compound B reacts with H_2O and excess of CO_2 to form compound C which on passing through or reaction with saturated NaCl solution forms sodium hydrogen carbonate.
Compound A, B and C, are respectively.

(1) $CaCl_2, NH_3, NH_4HCO_3$	(2) $Ca(OH)_2, NH_4^{\oplus}, (NH_4)_2CO_3$
(3) $CaCl_2, NH_4^{\oplus}, (NH_4)_2CO_3$	(4) $Ca(OH)_2, NH_3, NH_4HCO_3$

Sol. **4**

$$\begin{tikzcd}
 & \overset{(A)}{Ca(OH)_2} + 2NH_4Cl \arrow[d, "\Delta"] \\
 & \overset{(B)}{2NH_3 + CaCl_2 + 2H_2O} \arrow[d, "CO_2+\ \text{Excess}"', "H_2O"] \\
NH_4Cl + NaHCO_3 & \underset{(C)}{NH_4HCO_3} + NaCl \arrow[l]
\end{tikzcd}$$

SECTION - B

51. For the first order reaction A → B, the half life is 30 min. The time taken for 75% completion of the reaction is_____min. (Nearest integer)

Given : $\log 2 = 0.3010$

$\log 3 = 0.4771$

$\log 5 = 0.6989$

Sol. **60**

$t_{75\%} = 2t_{1/2}$ [For 1st order reaction]

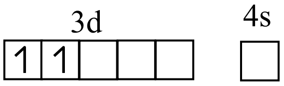
$t_{75\%} = 2 \times 30 = 60\,\text{min}$.

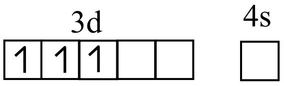
52. How many of the following metal ions have similar value of spin only magnetic moment in gaseous state?

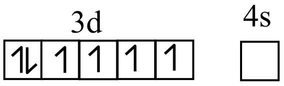
(Given: Atomic number : V, 23; Cr, 24; Fe, 26; Ni, 28)

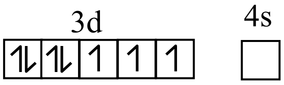
$V^{3+}, Cr^{3+}, Fe^{2+}, Ni^{3+}$

Sol. **2 (Cr^{+3} & Ni^{+3})**

V^{3+} =


Cr^{3+} =


Fe^{2+} =


Ni^{3+} =


53. In sulphur estimation, 0.471 g of an organic compound gave 1.4439 g of barium sulphate. The percentage of sulphur in the compound is_______(Nearest Integer)

(Given: Atomic mass Ba: 137u, S: 32u, O: 16u)

Sol. **42**

Organic compound → $BaSO_4$

Weight = 0.417 g   Weight = 1.44 g

Moles $BaSO_4 = \frac{1.44}{233}$ = moles of Sulphur

Weight Sulphur = $\frac{1.44}{233} \times 32\,\text{gram}$

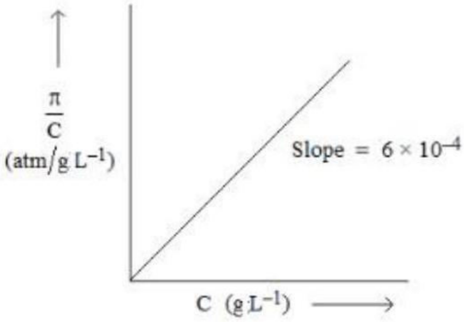
$$\% S = \frac{\text{weight of sulphur}}{\text{weight of organic}} \times 100$$

$$\Rightarrow \frac{1.44 \times 32}{233 \times 0.471} \times 100$$

$$\Rightarrow \frac{46.08}{109.743} \times 100$$

$$\Rightarrow 41.98 \simeq 42$$

54. The osmotic pressure of solutions of PVC in cyclohexanone at 300 K are plotted on the graph. The molar mass of PVC is_____gmol^{-1} (Nearest integer)



(Given : R = 0.083 L atm K^{-1} mol^{-1})

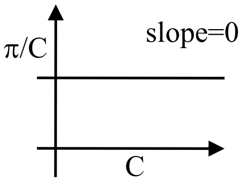
Sol. 41500

$$\pi = M'RT = \left(\frac{W/M}{V}\right)RT$$

$$\Rightarrow \pi = \left(\frac{W}{V}\right)\left(\frac{1}{M}\right)RT = C\left(\frac{RT}{M}\right)$$

$$\Rightarrow \frac{\pi}{C} = \frac{RT}{M} \neq f(c)$$

If we assume graph between $\frac{\pi}{C}$ and C



Assuming π vs C graph

$$\text{Slope} = \frac{RT}{M} = \frac{0.083 \times 300}{M} = 6 \times 10^{-4}$$

$$\therefore M = \frac{0.083 \times 300}{6 \times 10^{-4}} = \frac{830 \times 300}{6}$$

= 41,500

55. The density of a monobasic strong acid (Molar mass 24.2 g/mol) is 1.21 kg/L. The volume of its solution required for the complete neutralization of 25 mL of 0.24MNaOH is____× 10^{-2} mL (Nearest integer)

Sol. **12**

Molarity of acid = $\frac{1.2 \times 10^{3}}{24.2} = \frac{1000}{20} = 50\,M$

Neutralization reaction :

$$HA + NaOH \rightarrow NaA + H_2O$$

$$M_1V_1 = M_2V_2$$

$$[50] \times V = [0.24 \times 25]$$

V = 00. 12 ml

56. An athlete is given 100 g of glucose ($C_6H_{12}O_6$) for energy. This is equivalent to 1800 kJ of energy. The 50% of this energy gained is utilized by the athlete for sports activities at the event. In order to avoid storage of energy, the weight of extra water he would need to perspire is_____g (Nearest integer) Assume that there is no other way of consuming stored energy.
Given : The enthalpy of evaporation of water is 45 kJ mol^{-1}
Molar mass of C, H&O are 12,1 and 16 g mol^{-1}

Sol. **360**

$$C_6H_{12}O_6 + 6O_2 \rightarrow 6CO_2 + 6H_2O(\ell)$$

$$n = \frac{100}{180}$$

Energy needed to perspire water = $1800 \times \frac{1}{2}$

Moles of water evaporated = $\frac{900}{45} = 20\,\text{moles}$

Weight of water evaporated $\Rightarrow$ 20×18

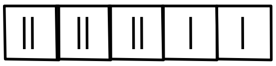
$\Rightarrow$ 360 gram

57. The number of paramagnetic species from the following is

$[Ni(CN)_4]^{2-}$, $[Ni(CO)_4]$, $[NiCl_4]^{2-}$
$[Fe(CN)_6]^{4-}$, $[Cu(NH_3)_4]^{2+}$
$[Fe(CN)_6]^{3-}$ and $[Fe(H_2O)_6]^{2+}$

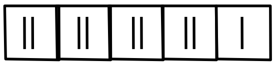
Sol. **4**

$(NiCl_4)^{-2} \rightarrow Ni^{+2} \rightarrow 3d^8$
$Cl^{+} \rightarrow$ weak field layered



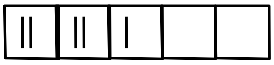
paramagnetic (n-e=2)

$(Cu(NH_3)_4)^{+2} \rightarrow Cu^{+2} \rightarrow 3d^9$



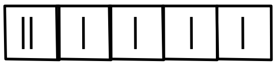
u-e = 1 paramagnetic

$(Fe(CN)_6)^{-3} \rightarrow Fe^{+3} \rightarrow 3d^5$



CN^{-} is strong field ligand so u-e=1
so paramagnetic

$(Fe(H_2O)_6)^{+2} \rightarrow Fe^{+2} \rightarrow 3d^6$
H_2O is weak field ligand



u-e=4 so paramagnetic

58. Consider the cell

Pt(s) | H_2(g) (1 atm) | H^+(aq, $[H^+]$ = 1) || Fe^{3+}(aq), Fe^{2+}(aq) | Pt(s)

Given $E^{o}_{Fe^{3+}/Fe^{2+}}$ = 0.771 V and $E^{o}_{H/1/2H_2}$ = 0 V, T= 298 K

If the potential of the cell is 0.712 V, the ratio of concentration of Fe^{2+} to Fe^{3+} is (Nearest integer)

Sol. **10**

$$\text{Anode} \Rightarrow \frac{1}{2}H_2(g) \rightarrow H^+(aq) + e^-$$

$$\underline{\text{Cathode} \Rightarrow Fe^{3+} + e^- \rightarrow Fe^{2+}}$$

$$\text{Overall} \frac{1}{2}H_2 + Fe^{3+} \xrightarrow{n-1} H^+ + Fe^{2+}$$

$$E_{cell} = E^o_{cell} - \frac{0.059}{1}\log\frac{[Fe^{2+}]}{[Fe^{3+}]} \times \frac{[H^+]}{[P_{H_2}]^{\frac{1}{2}}}$$

$$0.712 = 0.771 - 0.059\log\frac{[Fe^{2+}]}{[Fe^{3+}]}$$

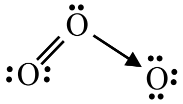
$$\log\frac{[Fe^{2+}]}{[Fe^{3+}]} = 1$$

$$\text{So } \frac{[Fe^{2+}]}{[Fe^{3+}]} = 10$$

59. The total number of lone pairs of electrons on oxygen atoms of ozone is

Sol. **6**

Not l.p. e^- in O_3 is = 6



60. A litre of buffer solution contains 0.1 mole of each of NH_3 and NH_4Cl. On the addition of 0.02 mole of HCl by dissolving gaseous HCl, the pH of the solution is found to be_____ $\times 10^{-3}$ (Nearest integer)

[Given : $pK_b(NH_3) = 4.745$
$\log 2 = 0.301$
$\log 3 = 0.477$
$T = 298$ K]

Sol. **9**

$$\begin{array}{lll} NH_3 & + & NH_4Cl \\ 0.1\,\text{mole} & & 0.1\,\text{mole} \end{array}$$

$$\begin{array}{lllll} NH_3 & + & HCl & \rightarrow & NH_4Cl \\ 0.1\,\text{mole} & & 0.02 & & 0.1 \\ 0.08\,\text{mole} & & - & & 0.12\,\text{mole} \end{array}$$

$$P_{OH} \Rightarrow P_{Kb} + \log\frac{[NH_4Cl]}{[NH_3]}$$

$$\Rightarrow 4.745 + \log\left(\frac{0.12}{0.08}\right)$$

$$\Rightarrow 4.745 + \log\left(\frac{3}{2}\right)$$

$\Rightarrow 4.745 + (0.477 - 0.301)$
$\Rightarrow 4.745 + 0.176$
$\Rightarrow 4.569$
$pH \Rightarrow 14 - 4.569$
$\Rightarrow 9.431 \simeq 9$

Mathematics

Section A

61. The points of intersection of the line $ax + by = 0, (a \neq b)$ and the circle $x^2 + y^2 - 2x = 0$ are $A(\alpha, 0)$ and $B(1, \beta)$. The image of the circle with AB as a diameter in the line $x + y + 2 = 0$ is :

(1) $x^2 + y^2 + 3x + 3y + 4 = 0$
(2) $x^2 + y^2 + 3x + 5y + 8 = 0$
(3) $x^2 + y^2 - 5x - 5y + 12 = 0$
(4) $x^2 + y^2 + 5x + 5y + 12 = 0$

Sol. 4

Only possibilities is $\alpha = 0, \beta = 1$

Equation of circle

$(x - 0)(x - 1) + (y - 0)(y - 1) = 0$

$x^2 + y^2 - x - y = 0$

Image of circle in live $x + y + 2 = 0$

$x^2 + y^2 + 5x + 5y + 12 = 0$

62. The distance of the point $(6, -2\sqrt{2})$ from the common tangent $y = mx + c, m > 0$, of the curves $x = 2y^2$ and $x = 1 + y^2$ is :

(1) $\frac{14}{3}$
(2) $5\sqrt{3}$
(3) $\frac{1}{3}$
(4) 5

Sol. 4

$$\left.\begin{array}{l} y^2 = \frac{x}{2} \\ y^2 = x - 1 \end{array}\right]$$

Tangent to $y^2 = \frac{x}{2}$ is $y = mx + \frac{1}{8m}$ ...(1)

$y^2 = x–1$ is $y = m(x–1) + \frac{1}{4m}$

$$y = mx - m + \frac{1}{4m} \qquad ...(2)$$

(1) & (2)

$$\frac{1}{8m} = -m + \frac{1}{4m}$$

$$m = \frac{1}{4m} - \frac{1}{8m}$$

$$m = \frac{1}{8m} \Rightarrow m^2 = \frac{1}{8} \Rightarrow m = \frac{1}{2\sqrt{2}} (m > 0)$$

From (1)

$$y = \frac{1}{2\sqrt{2}}x + \frac{1}{2\sqrt{2}}$$

distance from $\left(6, -2\sqrt{2}\right)$

$$\frac{\left|\frac{3}{\sqrt{2}} + 2\sqrt{2} + \frac{1}{2\sqrt{2}}\right|}{\sqrt{1 + \frac{1}{8}}} = \frac{6 + 8 + 1}{3} = \frac{15}{3} = 5$$

63. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non zero vectors such that $\vec{b} \cdot \vec{c} = 0$ and $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\vec{b}-\vec{c}}{2}$.
If $\vec{d}$ be a vector such that $\vec{b} \cdot \vec{d} = \vec{a} \cdot \vec{b}$, then $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d})$ is equal to

(1) $-\frac{1}{4}$ (2) $\frac{1}{4}$ (3) $\frac{3}{4}$ (4) $\frac{1}{2}$

Sol. **2**

$$(\bar{a}.\bar{c})\bar{b}-(\bar{a}.\bar{b})\bar{c}=\frac{\bar{b}}{2}-\frac{\bar{c}}{2}$$

$$\bar{a}.\bar{c}=\frac{1}{2}, \bar{a}.\bar{b}=\frac{1}{2}$$

$$\bar{b}.\bar{d}=\frac{1}{2}$$

$$\begin{aligned}(\bar{a}\times\bar{b}).(\bar{c}\times\bar{d}) &= \bar{a}.\left[\bar{b}\times(\bar{c}\times\bar{d})\right]\\ &= \bar{a}.\left[(\bar{b}.\bar{d})\bar{c}-(\bar{b}.\bar{c})\bar{d}\right]\\ &= \bar{a}.\left[\bar{c}/2\right]\\ &= \frac{1}{2}(\bar{a}.\bar{c})\\ &= \frac{1}{4}\end{aligned}$$

64. The vector $\vec{a} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$ is rotated through a right angle, passing through the y-axis in its way and the resulting vector is $\vec{b}$. Then the projection of $3\vec{a} + \sqrt{2}\vec{b}$ on $\vec{c} = 5\hat{\imath} + 4\hat{\jmath} + 3\hat{k}$ is :

(1) $2\sqrt{3}$ (2) 1 (3) $3\sqrt{2}$ (4) $\sqrt{6}$

Sol. **3**

$$\bar{b}=\lambda\bar{a}+\mu\hat{J}$$

$$=\lambda\left(-\hat{i}+2\hat{j}+\hat{k}\right)+\mu\hat{j}$$

$$\bar{b}=-\lambda\hat{i}+(2\lambda+\mu)\hat{j}+\lambda\hat{k}$$

$$|\bar{a}|=|\bar{b}|$$

$$|\bar{a}|^2=|\bar{b}|^2 \quad \Rightarrow 6=\lambda^2+(2\lambda+\mu)^2+\lambda^2 \text{ ......(1)}$$

$$\because \bar{a}.\bar{b}=0 \quad \Rightarrow \lambda+2(2\lambda+\mu)+(1)(\lambda)=0$$

$$\Rightarrow 6\lambda+2\mu=0$$

$$\Rightarrow \mu=-3\lambda \text{ ....(2)}$$

from (1) & (2)

$$3\lambda^2=6$$

$$\lambda^2=2 \qquad \Rightarrow \lambda=\pm\sqrt{2}$$

$$\Rightarrow \mu=\pm3\sqrt{2}$$

Projection of $3\bar{a}+2\bar{b}$ on $\bar{c}$ is $=\frac{(3\bar{a}+\sqrt{2}\bar{b}).\bar{c}}{|\bar{c}|}$

$$=\frac{3\bar{a}.\bar{c}+\sqrt{2}\bar{b}.\bar{c}}{|\bar{c}|}$$

$$= \frac{18+\sqrt{2}\left(-6\sqrt{2}\right)}{\sqrt{50}}$$

$$= \frac{6}{\sqrt{50}} = \frac{6}{5\sqrt{2}} = \frac{3\sqrt{2}}{5}$$

Case I :

$$\left(\bar{a}.\bar{c} = -5+8+3=6\right) \qquad \frac{18+\sqrt{2}\left(-6\sqrt{2}\right)}{\sqrt{50}}$$

$\lambda = \sqrt{2}$ $\quad \bar{b} = -\sqrt{2}\hat{i} + 12\sqrt{2} - 3\sqrt{2}\hat{j} + \sqrt{2}\hat{k}$

$\bar{b} = -\sqrt{2}\hat{i} - \sqrt{2}\hat{j} + \sqrt{2}\hat{k}$

$\bar{b}.\bar{c} = -5\sqrt{2} - 4\sqrt{2} + 3\sqrt{2}$

$= -6\sqrt{2}$

Case II : $\left.\begin{array}{l}\lambda = -\sqrt{2}\\ \mu = 3\sqrt{2}\end{array}\right]$ $\quad \bar{b} = \sqrt{2}\hat{i} + \left(\sqrt{2}\right)\hat{j} + \left(-\sqrt{2}\right)\hat{k}$

$$= \frac{18+\sqrt{2}\left(6\sqrt{2}\right)}{\sqrt{50}}$$

$$= \frac{30}{\sqrt{50}} = \frac{30}{5\sqrt{2}} = \frac{6}{\sqrt{2}} = 3\sqrt{2} \text{ Ans.}$$

Complex Number, Easy

65. Let $z_1 = 2 + 3i$ and $z_2 = 3 + 4i$. The set $S = \{z \in C: |z - z_1|^2 - |z - z_2|^2 = |z_1 - z_2|^2\}$ represents a

(1) hyperbola with the length of the transverse axis 7
(2) hyperbola with eccentricity 2
(3) straight line with the sum of its intercepts on the coordinate axes equals −18
(4) straight line with the sum of its intercepts on the coordinate axes equals 14

Sol. **4**

Let $z = x + iy$

$z - z_1 = (x-2) + i(y-3)$

$|z - z_1|^2 = (x-2)^2 + (y-3)^2$

$z - z_2 = (x-3) + i(y-4)$

$|z - z_2|^2 = (x-3)^2 + (y-4)^2$

$((x-2)^2 + (y-3)^2) - ((x-3)^2 + (y-4)^2) = 2$

$\Rightarrow 2x + 2y = 14$

$= x + y = 7$

straight line with sum of intercept on C.A = 14

66. The mean and variance of the marks obtained by the students in a test are 10 and 4 respectively. Later, the marks of one of the students is increased from 8 to 12 . If the new mean of the marks is 10.2, then their new variance is equal to :

(1) 3.96 (2) 4.08 (3) 4.04 (4) 3.92

Sol. **3.96**

Let Number of observations is = n

$$\frac{\sum x_i}{n} = 10$$

$$\sum x_i = 10n \_\_\_\_\_(1)$$

$$\frac{\sum x_i - 8 + 12}{n} = 10.2$$

$$\sum x_i = (10.2)n - 4 \_\_\_(2)$$

10n = (10.2)n–4

$\Rightarrow$ (.2)n = 4 $\quad\Rightarrow\quad \boxed{n = 20}$

Given $\frac{\sum x_i^2}{20} - (10)^2 = 4 \qquad \Rightarrow \sum x_i^2 = 2080$

After Change

$$\sum x_i^2 = 2080 - 8^2 + (12)^2$$
$$= 2160$$

New vanance $= \frac{\sum x_i^2}{20} - (\bar{x})^2$

$$= \frac{2160}{20} - (10.2)^2$$
$$= 108 - (10.2)^2$$
$$= 3.96$$

67. Let S_1 and S_2 be respectively the sets of all $a \in \mathbb{R} - \{0\}$ for which the system of linear equations

$$ax + 2ay - 3az = 1$$
$$(2a+1)x + (2a+3)y + (a+1)z = 2$$
$$(3a+5)x + (a+5)y + (a+2)z = 3$$

has unique solution and infinitely many solutions. Then

(1) S_1 is an infinite set and $n(S_2) = 2$ (2) $S_1 = \Phi$ and $S_2 = \mathbb{R} - \{0\}$

(3) $n(S_1) = 2$ and S_2 is an infinite set (4) $S_1 = \mathbb{R} - \{0\}$ and $S_2 = \Phi$

Sol. **4**

$$\Delta = \begin{vmatrix} a & 2a & -3a \\ 2a+1 & 2a+3 & a+1 \\ 3a+5 & a+5 & a+2 \end{vmatrix}$$

$\Delta = a(15a^2 + 31a + 36) = 0$

a = 0

$\Delta \neq 0$ for all $a \in \mathbb{R} - \{0\}$

$\therefore S_1 = \mathbb{R} - \{0\}, S_2 = \Phi$

68. The value of $\lim_{n\to\infty} \frac{1+2-3+4+5-6+\cdots\ldots+(3n-2)+(3n-1)-3n}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$ is :

(1) $\frac{3}{2}(\sqrt{2}+1)$ (2) $\frac{3}{2\sqrt{2}}$ (3) $\frac{\sqrt{2}+1}{2}$ (4) $3(\sqrt{2}+1)$

Sol. **1**

$$\lim_{n\to\infty} \frac{(1+2+4+5+\ldots+(3n-2)+(3n-1)-3+6+\ldots+3n)}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$$

Let $N^r = \sum_{n=1}^{\infty}(3n-2)+(3n-1)-3n$

$$= \sum_{n=1}^{\infty}(3n-3)$$

$$= \frac{3n(n+1)}{2} - 3n = \frac{3}{2}\left(n^2 - n\right)$$

$$\frac{3}{2}\lim_{n\to\infty} \frac{n^2\left(1-\frac{1}{n}\right)}{n^2\left(\sqrt{2+\frac{4}{n^3}+\frac{3}{n^4}}-\sqrt{1+\frac{5}{n^3}+\frac{4}{n^4}}\right)}$$

$$= \frac{3}{2\left(\sqrt{2}-1\right)} \text{ or } \frac{3}{2}\left(\sqrt{2}+1\right) \text{Ans.}$$

69. The statement $(p \wedge (\sim q)) \Rightarrow (p \Rightarrow (\sim q))$ is

(1) a tautology (2) a contradiction (3) equivalent to $p \vee q$ (4) equivalent to $(\sim p) \vee (\sim q)$

Sol. **1**

$(p \wedge (\sim q)) \Rightarrow (p \Rightarrow (\sim q))$

P	q	$\sim q$	$p \wedge \sim q$	$p \Rightarrow \sim q$	$(p \wedge \sim q) \Rightarrow (p \Rightarrow \sim q)$
T	T	F	F	F	T
T	F	T	T	T	T
F	T	F	F	T	T
F	F	T	F	T	T

Tautology

70. Consider the lines L_1 and L_2 given by

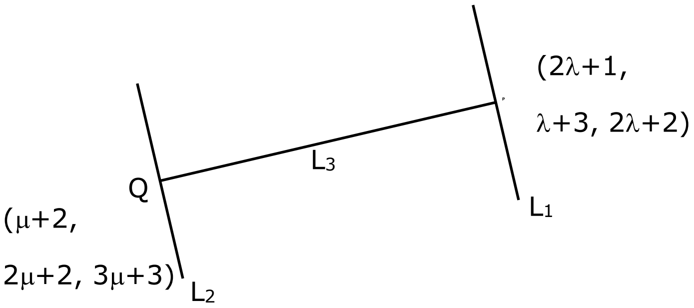
$L_1: \frac{x-1}{2} = \frac{y-3}{1} = \frac{z-2}{2}$

$L_2: \frac{x-2}{1} = \frac{y-2}{2} = \frac{z-3}{3}$.

A line L_3 having direction ratios $1, -1, -2$, intersects L_1 and L_2 at the points P and Q respectively. Then the length of line segment PQ is

(1) $3\sqrt{2}$ (2) $4\sqrt{3}$ (3) 4 (4) $2\sqrt{6}$

Sol. **4**



D.R's of PQ are $= (2\lambda-\mu-1, \lambda-2\mu+1, 2\lambda-3\mu-1)$

given D.R's are $= (1, -1, -2)$

$$\frac{2\lambda-\mu-1}{1} = \frac{\lambda-2\mu+1}{-1} = \frac{2\lambda-3\mu-1}{-2}$$

$\lambda = \mu = 3$

$P = (7, 6, 8)$

$Q = (5, 8, 12)$

$PQ = 2\sqrt{6}$

71. Let $f(x)=\int \frac{2x}{(x^2+1)(x^2+3)}dx$. If $f(3)=\frac{1}{2}(\log_e 5-\log_e 6)$, then $f(4)$ is equal to

(1) $\log_e 19-\log_e 20$ (2) $\log_e 17-\log_e 18$

(3) $\frac{1}{2}(\log_e 19-\log_e 17)$ (4) $\frac{1}{2}(\log_e 17-\log_e 19)$

Sol. **4**

Let $x^2=t$

$2xdx=dt$

$$f(x)=\int\frac{dt}{(t+1)(t+3)}$$

$$=\frac{1}{2}\int\left(\frac{1}{t+1}-\frac{1}{t+3}\right)dt$$

$$=\frac{1}{2}\ln\left|\frac{t+1}{t+3}\right|+C$$

$$f(x)=\frac{1}{2}\ln\left|\frac{x^2+1}{x^2+3}\right|+C$$

$x=3$

$$\frac{1}{2}\ln\left(\frac{5}{6}\right)=\frac{1}{2}\ln\left(\frac{5}{6}\right)+C\Rightarrow C=0$$

$$f(x)=\frac{1}{2}\ln\left(\frac{x^2+1}{x^2+3}\right)$$

$$f(x)=\frac{1}{2}\ln\left(\frac{17}{19}\right)$$

$$=\frac{1}{2}\ [\ln 17-\ln 19]$$

72. The minimum value of the function $f(x)=\int_0^2 e^{|x-t|}dt$ is :

(1) $e(e-1)$ (2) $2(e-1)$ (3) 2 (4) $2e-1$

Sol. **2**

Case I $x<0$

$$f(x)=\int_0^2 e^{-(x-t)}dt$$

$$=e^{-x}\int_0^2 e^t dt=e^{-x}\left(e^2-1\right)$$

Case II

$(0<x<2)$ $$f(x)=\int_0^x e^{x-t}dt+\int_x^2 e^{-(x-t)}dt$$

$$=e^x\left(-e^{-t}\right)_0^x+e^{-x}[e^t]_x^2$$

$$=e^x\left[-e^{-x}+1\right]+e^{-x}\left[e^2-e^x\right]$$

$$=-1+e^x+e^{2-x}-1$$

Case III

$x\ge 2$ $$f(x)=\int_0^2 e^{(x-t)}dt$$

$$=e^x\left[-e^{-t}\right]_0^2$$

$$=e^x\left[-e^{-2}+1\right]$$

$$=e^x\left(1-e^{-2}\right)$$

$$f(x)=\left[\begin{array}{lll} e^{-x}(e^2-1), & x\le 0\to & (e^2-1)\\ e^x+e^{2-x}-2, & 0\le x\le 2\to & 2(e-1)\\ ex(1-e^{-2}), & x\ge 2\to & (e^2-1)\end{array}\right.$$

Minimum value = 2(e–1)

73. Let M be the maximum value of the product of two positive integers when their sum is 66. Let the sample space $S=\left\{x\in\mathbb{Z}: x(66-x)\ge\frac{5}{9}M\right\}$ and the event $A=\{x\in S: x \text{ is a multiple of } 3\}$. Then P(A) is equal to

(1) $\frac{7}{22}$ (2) $\frac{1}{5}$ (3) $\frac{15}{44}$ (4) $\frac{1}{3}$

Sol. **4**

Let a, b → 2 positive number

$$\frac{a+b}{2}\ge\sqrt{ab}$$

$$\sqrt{ab}\le 33$$

$$\boxed{ab\le(33)^2}\qquad M=(33)^2$$

$$x(66-x)\ge\frac{5}{9}(33)^2$$

$66x - x^2 \ge 605$

$0 \ge x^2 - 66x + 605$

$(x-11)(x-55)\le 0$

$x\in[11,55]$

A = {12, 15, 18...54}

Total number in A=15

$P(A)=\frac{15}{45}=\frac{1}{3}$ Ans.

74. Let $x=2$ be a local minima of the function $f(x)=2x^4-18x^2+8x+12$, $x\in(-4,4)$. If M is local maximum value of the function f in $(-4,4)$, then M =

(1) $18\sqrt{6}-\frac{31}{2}$ (2) $18\sqrt{6}-\frac{33}{2}$ (3) $12\sqrt{6}-\frac{33}{2}$ (4) $12\sqrt{6}-\frac{31}{2}$

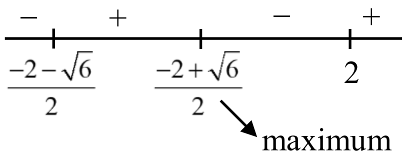
Sol. **3**

$f'(x) = 8x^3 - 36x + 8$

$= 4[2x^3 - 9x + 2]$

$= 4[(x-2)(2x^2 + 4x - 1)]$

$$=4\Bigg[(x-2)\Bigg(x-\bigg(-\frac{2-\sqrt{6}}{2}\bigg)\Bigg(x-\bigg(\frac{-2+\sqrt{6}}{2}\bigg)\bigg)\Bigg)\Bigg)\Bigg]$$



$$M=2\left(\frac{-2+\sqrt{6}}{2}\right)^4-18\left(\frac{-2+\sqrt{6}}{2}\right)^2+8\left(\frac{-2+\sqrt{6}}{2}\right)+12$$

$$=12\sqrt{6}-\frac{33}{2}$$

75. Let $f: (0,1) \to \mathbb{R}$ be a function defined by $f(x) = \frac{1}{1-e^{-x}}$, and $g(x) = (f(-x) - f(x))$. Consider two statements

(I) g is an increasing function in $(0,1)$

(II) g is one-one in $(0,1)$

Then,

(1) Both (I) and (II) are true

(2) Neither (I) nor (II) is true

(3) Only (I) is true

(4) Only (II) is true

Sol. **1**

$$f(x) = \frac{1}{1-e^{-x}}$$

$$g(x) = (f(-x) - f(x))$$

$$= \frac{1}{1-e^{x}} - \frac{1}{1-e^{-x}}$$

$$= \frac{1}{1-e^{x}} - \frac{e^{x}}{e^{x}-1}$$

$$g(x) = \frac{1+e^{x}}{1-e^{x}}$$

$$g'(x) = \frac{(1-e^{x})(e^{x})-(1+e^{x})(-e^{x})}{(1-e^{x})^{2}}$$

$$= \frac{e^{x}-e^{2x}+e^{x}+e^{2x}}{(1-e^{x})^{2}}$$

$$g'(x) = \frac{2e^{x}}{(1-e^{x})^{2}}$$

$$g'(x) > 0 \Rightarrow g(x)\uparrow$$

g(x) is one-one

76. Let $y(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)(1+x^{16})$. Then $y' - y''$ at $x = -1$ is equal to :

(1) 976

(2) 944

(3) 464

(4) 496

Sol. **4**

$$f(x) = y = \frac{(1-x)(1+x)(1+x^{2})(1+x^{4})(1+x^{8})(1+x^{16})}{(1-x)}$$

$$f(x) = y = \frac{(1-x^{32})}{1-x} \Rightarrow f(-1) = 0$$

$(1–x)y = 1 - x^{32}$

differentiate both side

$(1—x)y' + y(–1) = –32x^{31}$ $\boxed{x = -1 \Rightarrow y' = 16}$

differentiate both side

$(1–x)y' + y'(–1)–y' = –(32)(31)\ x30$

Put $x = –1$

$2y'' – 2y' = – (32)\ (31)$

$y'' – y' = – (16)(31)$

$\boxed{y' - y'' = 496}$

77. The distance of the point $P(4,6,-2)$ from the line passing through the point $(-3,2,3)$ and parallel to a line with direction ratios $3,3,-1$ is equal to :

(1) $\sqrt{14}$ (2) 3 (3) $\sqrt{6}$ (4) $2\sqrt{3}$

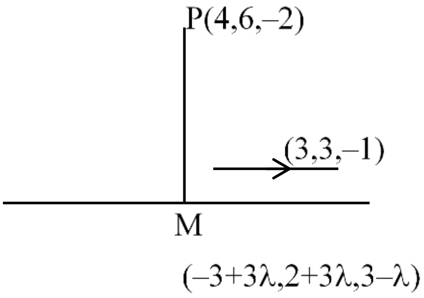
Sol. **1**

equation of line

$$\overline{r} = (-3,2,3) + \lambda(3,3,-1)$$

$\overline{PM}.(3,3,-1) = 0$

$(3\lambda - 7, 3\lambda - 4, 5 - \lambda) . (3,3,-1) = 0$



$\Rightarrow 3(3\lambda - 7) + 3(3\lambda - 4) - 1(15 - \lambda) = 0$

$\Rightarrow 19\lambda = 38 \Rightarrow \lambda = 2$

$M = (3, 8, 1)$

$PM = \sqrt{1+4+9} = \sqrt{14}$

78. Let $x, y, z > 1$ and $A = \begin{bmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 2 & \log_y z \\ \log_z x & \log_z y & 3 \end{bmatrix}$. Then $|\text{adj}(\text{adj } A^2)|$ is equal to

(1) 2^8 (2) 4^8 (3) 6^4 (4) 2^4

Sol. **1**

$|\text{adj}(\text{adj}A^2| = \left|A^2\right|^{(3-1)^2} = |A|^8$

$$|A| = \begin{vmatrix} 1 & \frac{\ln y}{\ln} & \frac{\ln z}{\ln x} \\ \frac{\ln x}{\ln y} & 2 & \frac{\ln z}{\ln y} \\ \frac{\ln x}{\ln z} & \frac{\ln y}{\ln z} & 3 \end{vmatrix}$$

$$= \frac{1}{\ln x \ln y \ln z} \begin{vmatrix} \ln x & \ln y & \ln z \\ \ln x & 2\ln y & \ln z \\ \ln x & \ln y & 3\ln z \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{vmatrix}$$

$|A| = 2$

$\therefore$ $|\text{adj}(\text{adj}A)| = 2^8$ Ans.

79. If a_r is the coefficient of x^{10-r} in the Binomial expansion of $(1+x)^{10}$, then $\sum_{r=1}^{10} r^3 \left(\frac{a_r}{a_{r-1}}\right)^2$ is equal to

(1) 5445 (2) 3025 (3) 4895 (4) 1210

Sol. **4**

$$\sum_{r=1}^{10} r^3 \left[\frac{a_r}{a_{r-1}}\right]^2$$

$\because \frac{a_r}{a_{r-1}} = \frac{10-r+1}{r} = \frac{11-r}{r}$

$\because (1+x)^{10} \Rightarrow {}^{10}C_r n^r$

$\Rightarrow {}^{10}C_{10-r} n^{10-r}$

$= {}^{10}C_{10-r}$ or $= {}^{10}C_r x^{10-r}$

$a_r = {}^{10}C_r$

$$\sum_{r=1}^{10} r^3\left[\frac{11-r}{r}\right]^2$$

$$\sum_{r=1}^{10} r(11-r)^2$$

$$\sum_{r=1}^{10}\left[r^3-22r^2+121r\right]$$

$$=\left(\frac{(10)(11)}{2}\right)^2-22\left(\frac{(10)(11)(21)}{6}\right)+\left(\frac{(10)(11)}{2}\right)(121)$$

= 1210 Ans.

80. Let $y = y(x)$ be the solution curve of the differential equation

$$\frac{dy}{dx}=\frac{y}{x}\left(1+xy^2(1+\log_e x)\right), x>0, y(1)=3. \text{ Then } \frac{y^2(x)}{9} \text{ is equal to :}$$

(1) $\frac{x^2}{2x^3(2+\log_e x^3)-3}$ (2) $\frac{x^2}{3x^3(1+\log_e x^2)-2}$

(3) $\frac{x^2}{7-3x^3(2+\log_e x^2)}$ (4) $\frac{x^2}{5-2x^3(2+\log_e x^3)}$

Sol. **(4)**

$$\frac{dy}{dx}=\frac{y}{x}\left[1+xy^2(1+\ln x)\right]$$

$$\frac{dy}{dx}-\frac{y}{x}=y^3(1+\ln x)$$

$$\frac{1}{y^3}\frac{dy}{dx}-\frac{1}{x}\cdot\frac{1}{y^2}=1+\ln x \qquad ...(1)$$

$$-\frac{1}{y^2}=t\Rightarrow\frac{2}{y^3}\frac{dy}{dx}=\frac{dt}{dx}$$

From (1)

$$\frac{1}{2}\frac{dy}{dx}+t\left(\frac{1}{x}\right)=1+\ln x$$

$$\frac{dy}{dx}+t\left(\frac{2}{x}\right)=2(1+\ln x)$$

$$\text{I.F} = e^{\int\frac{2}{x}dx}=e^{2\ln x}=x^2$$

$$t\left(x^2\right)=\int\left[2(1+\ln x)\cdot x^2\right]dx$$

$$\Rightarrow t\cdot x^2=\frac{2x^3}{3}+2\int x^2\ln x\,dx$$

$$\Rightarrow\frac{-x^2}{y^2}-\frac{2x^3}{3}+2\left[\ln x\cdot\frac{x^3}{3}-\frac{x^2}{9}\right]+C$$

$$x=1, y=3 \qquad \Rightarrow C=\frac{-5}{9}$$

$$\frac{-x^2}{y^2}=\frac{2x^3}{3}+2\left[\ln x\cdot\frac{x^3}{3}-\frac{x^3}{9}\right]\frac{-5}{9}$$

Section B

81. The constant term in the expansion of $\left(2x+\frac{1}{x^7}+3x^2\right)^5$ is

Sol. **1080**

General term $=\frac{5!}{r_1!r_2!r_3!}(2x)^{r_1}\left(\frac{1}{x}\right)^{r_2}\left(3x^2\right)^{r_3}$

$$=\frac{5!}{r_1!r_2!r_3!}2^{r_1}.3^{r_3}\left[x^{r_1-7r_2+2r_3}\right]$$

$$\left.\begin{aligned} r_1-7r_2+2r_3=0 \\ r_1+r_2+r_3=5 \end{aligned}\right]$$

$$r_1=1, r_2=1, r_3=3$$

Constant term = 1080

82. For some $a, b, c \in \mathbb{N}$, let $f(x)=ax-3$ and $g(x)=x^b+c, x\in\mathbb{R}$. If $(f\circ g)^{-1}(x)=\left(\frac{x-7}{2}\right)^{1/3}$, then $(f\circ g)(ac)+(g\circ f)(b)$ is equal to

Sol. **2039**

Let f(g(x)) = h(x)

$f(g(x)) = 2x^3 + 7$

$a(x^b + c) - 3 = 2x^3 + 7$

$$\left.\begin{aligned} a=2, b=3, ac=10 \\ c=5 \end{aligned}\right]$$

$$\left.\begin{aligned} g(f(x))(3)=32 \\ f(g(10))=2007 \end{aligned}\right]$$

Sum = 2039

83. Let $S=\{1,2,3,5,7,10,11\}$. The number of non-empty subsets of S that have the sum of all elements a multiple of 3 , is

Sol. **43**

No. of element 1 = {3}

No. of element 2 = {(3K + 1), (3k + 2)}

(3) (3) = 9

No. of element 3 = {3k, 3k+1, 3K + 2} = (1) (3) (3) = 9

= {(3k + 1), (3k + 1), (3k + 1)} = 1

= {(3K + 2), (3k + 2), (3k + 2)} = $\frac{1}{11}$

No. of element 4 = {3k, 3k+1, 3k+1, 3k+1} → 1

= {3k, 3k + 2, 3k+2, 3k+2} → 1

= (3k+1, 3k+2, 3k+2, 3k+1} → ${}^3C_2 \times {}^3C_2 = 9$

No. of element 5 = 9, no. of element 6 = 1, no. of element 7 = 1

Total = 43.

84. Let the equation of the plane passing through the line $x-2y-z-5=0=x+y+3z-5$ and parallel to the line $x+y+2z-7=0=2x+3y+z-2$ be $ax+by+cz=65$. Then the distance of the point (a,b,c) from the plane $2x+2y-z+16=0$ is

Sol. Equation of plane is

$(x - 2y - z - 5) + b\,(x + y + 3z - 5) = 0$

$$\begin{vmatrix} 1+b & -2+b & -1+3b \\ 1 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix} = 0$$

$\Rightarrow b = 12$

Plane is $13x + 10y + 35z = 65$

Distance From given point is = 9

85. If the sum of all the solutions of $\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{\pi}{3}, -1 < x < 1, x \neq 0$, is $\alpha - \frac{4}{\sqrt{3}}$, then α is equal to

Sol. $\boldsymbol{\alpha = 2}$

$x \in (-1, 1)$ $\quad \tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1}x$

$x \in (0, 1)$ $\quad \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1}x$

$x \in (-1, 0)$ $\quad \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \text{IT} + \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \text{IT} + 2\tan^{-1}x$

$x \in (0, 1)$ $\quad 2\tan^{-1}x + 2\tan^{-1}x = \pi/3$

$\tan^{-1}x = \frac{\pi}{12}$

$\boxed{x = 2 - \sqrt{3}}$

$x \in (-1, 0)$ $\quad 2\tan^{-1}x + \text{IT} + 2\tan^{-1}x = \pi/3$

$4\tan^{-1}x = \frac{-2\pi}{3}$

$\tan^{-1}x = -\pi/6$

$\boxed{x = -1/\sqrt{3}}$

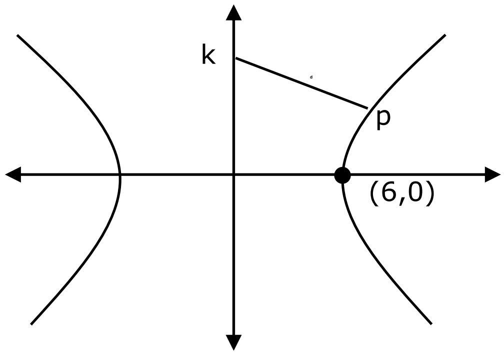
$\left(2 - \sqrt{3}\right) + \left(-\frac{1}{\sqrt{3}}\right) = \alpha - \frac{4}{\sqrt{3}}$

$2 - 4/\sqrt{3} = \alpha - 4/\sqrt{3}$

$\boxed{\alpha = 2}$

86. The vertices of a hyperbola H are $(\pm 6,0)$ and its eccentricity is $\frac{\sqrt{5}}{2}$. Let N be the normal to H at a point in the first quadrant and parallel to the line $\sqrt{2}x + y = 2\sqrt{2}$.
If d is the length of the line segment of N between H and the y-axis then d^2 is equal to

Sol. **216**



$$H : \frac{x^2}{36} - \frac{y^2}{9} = 1$$

Equation of normal is $6x\cos\theta + 3y\cot\theta = 45$

$$M = -2\sin\theta = -\sqrt{2}$$

$$\theta = \pi/4$$

Equation of normal is $\sqrt{2}x + y = 15$

$P(a\sec\theta, b\tan\theta)$

$P\left(6\sqrt{2}, 3\right)$, $k(0,15)$

$d^2 = 216$

87. Let x and y be distinct integers where $1 \le x \le 25$ and $1 \le y \le 25$. Then, the number of ways of choosing x and y, such that $x + y$ is divisible by 5 , is

Sol.

$x + y = 5\lambda$

X	y	No. of ways
5λ	5λ	20
$5\lambda + 1$	$5\lambda + 4$	25
$5\lambda + 2$	$5\lambda + 3$	25
$5\lambda + 3$	$5\lambda + 2$	25
$5\lambda + 4$	$5\lambda + 1$	25
		1200

Total Ways = 120

88. Let $S = \left\{\alpha : \log_2(9^{2\alpha-4} + 13) - \log_2\left(\frac{5}{2} \cdot 3^{2\alpha-4} + 1\right) = 2\right\}$. Then the maximum value of β for which the equation $x^2 - 2\left(\sum_{\alpha\in s}\alpha\right)^2 x + \sum_{\alpha\in s}(\alpha+1)^2\beta = 0$ has real roots, is

Sol. **25**

$$\log_2\left[\frac{9^{2\alpha-4}+13}{3^{2\alpha-4}.\frac{5}{2}+1}\right] = 2$$

$$= \frac{9^{2\alpha-4}+13}{3^{2\alpha-4}.\frac{5}{2}+1} = 4$$

$= 9^{2\alpha-4} + 13 = 10 . 3^{2\alpha-4} + 4$

$t^2 - 10\,t + 9 = 0$

$t = 1, 9$

$3^{2\alpha-4} = 3^0, 3^2$

$2\alpha - 4 = 0\,,\,2$

$= \alpha = 2, 3$

$x^2 - 2(25)\,x + 25\beta = 0$

$D \geq 0$

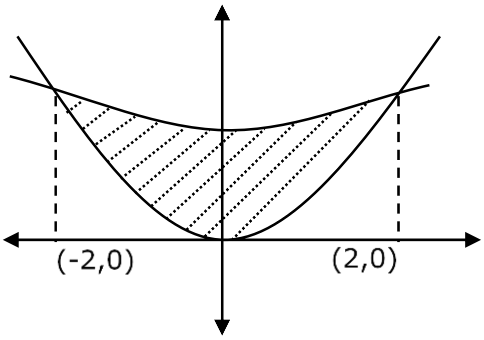
$(2)^2\,(25)^2 - 4\,(25)\,(\beta) \geq 0$

$\beta \leq 25$

$\beta_{max} = 25$

89. It the area enclosed by the parabolas $P_1: 2y = 5x^2$ and $P_2: x^2 - y + 6 = 0$ is equal to the area enclosed by P_1 and $y = \alpha x, \alpha > 0$, then α^3 is equal to

Sol. **600**



$y = \frac{5x^2}{2}$, $\qquad y = x^2 + 6$

$\frac{5x^2}{2} = x^2 + 6$

$3x^2 = 12 \Rightarrow x^2 = 4$

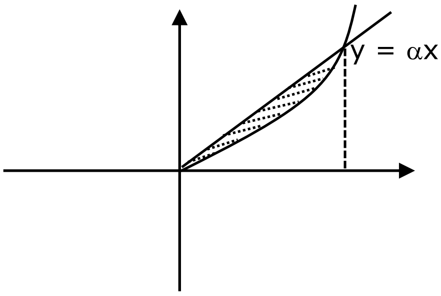
$\boxed{x = \pm 2}$

$A_1 = 2\int_0^2 \left(x^2 + 6 - \frac{5}{2}x^2 \right) dx$

$= 2\int_0^2 \left(6 - \frac{3x^2}{2} \right) dx$

$= 2\left[6x - {x^3}/{2} \right]_0^2 = 2[12 - 4]$

$= 16$



$y=\frac{5}{2}x^2, y=\alpha x(\alpha>0)$

$\text{area}=\frac{8}{3}\left[a^2m^3\right]$

$=\frac{8}{3}\left[1/10\right]^2.\alpha^3$

$=\frac{8}{300}-\alpha^3=\frac{2}{75}\alpha^3$

$\because\frac{2}{75}-\alpha^3=16 \qquad \Rightarrow\alpha^3=8\times75$

$\boxed{\alpha^3=600}$

90. Let A_1, A_2, A_3 be the three A.P. with the same common difference d and having their first terms as A, A +1, A + 2, respectively. Let a, b, c be the 7th, 9th, 17th terms of A_1, A_2, A_3, respectively such that $\begin{vmatrix} a & 7 & 1 \\ 2b & 17 & 1 \\ c & 17 & 1 \end{vmatrix}+70=0$.

If $a=29$, then the sum of first 20 terms of an AP whose first term is $c-a-b$ and common difference is $\frac{d}{12}$, is equal to

Sol. $\begin{vmatrix} A+6d & 7 & 1 \\ 21(A+1+8d) & 17 & 1 \\ A+2+16d & 17 & 1 \end{vmatrix}+70=0$

$A=-7, d=6$

$\therefore\ c-a-b=20$

$\therefore\ 5_{20}=495$

JEE–MAIN EXAMINATION – JANUARY, 20

(Held On Thursday 25th January, 2023)	TIME : 3 : 00 PM to 6 : 00 PM

Physics

SECTION - A

1. According to law of equipartition of energy the molar specific heat of a diatomic gas at constant volume where the molecule has one additional vibrational mode is:-

(1) $\frac{5}{2}R$ (2) $\frac{9}{2}R$ (3) $\frac{7}{2}R$ (4) $\frac{3}{2}R$

Sol. **3**

(degree of freedom)

$\Rightarrow f = 3 + 2 + 2 = 7$

$C_V = \frac{fR}{2} = \frac{7R}{2}$

2. A wire of length 1 m moving with velocity 8 m/s at right angles to a magnetic field of 2 T. The magnitude of induced emf, between the ends of wire will be

(1) 20 V (2) 8 V (3) 12 V (4) 16 V

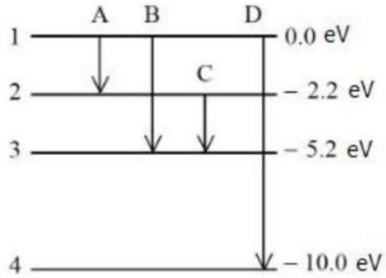
Sol. **4**

$e = B\vartheta l$

$e = 2 \times 8 \times 1$

e = 16 volt

3. The energy levels of an atom is shown in figure.



Which one of these transitions will result in the emission of a photon of wavelength 124.1 nm ?
Given ($h = 6.62 \times 10^{-34}$Js)

(1) D (2) B (3) C (4) A

Sol. **1**

$$\underset{(\text{nm})}{\underset{\downarrow}{\lambda}} = \frac{hc}{\Delta E} = \frac{1241}{\Delta E(\text{ev})} = \frac{1241}{10} = 124.1$$

4. Given below are two statements :

Statement I: Stopping potential in photoelectric effect does not depend on the power of the light source.

Statement II: For a given metal, the maximum kinetic energy of the photoelectron depends on the wavelength of the incident light.

In the light of above statements, choose the most appropriate answer from the options given below

(1) Statement I is incorrect but statement II is correct
(2) Statement **I** is correct but statement **II** is incorrect
(3) Both Statement **I** and statement II are correct
(4) Both Statement I and Statement II are incorrect

Sol. **3**

Both statement I and statement II are correct

5. The distance travelled by a particle is related to time t as $x = 4t^2$. The velocity of the particle at t = 5 s is:-

(1) 40 ms^{-1} (2) 20 ms^{-1} (3) 8 ms^{-1} (4) 25 ms^{-1}

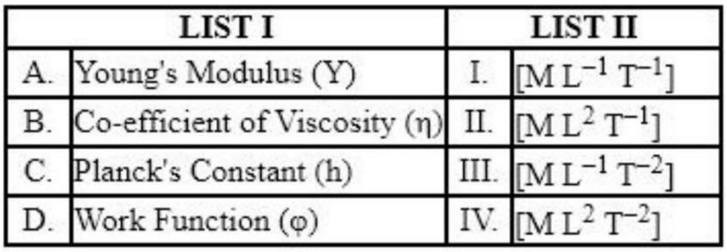
Sol. **1**

$$v = \frac{dx}{dt} = 8t$$

$v = 8 \times 5$

$v = 40$ m/s

6. Match List I with List II



LIST I		LIST II	
A.	Young's Modulus (Y)	I.	$[M L^{-1} T^{-1}]$
B.	Co-efficient of Viscosity (η)	II.	$[M L^2 T^{-1}]$
C.	Planck's Constant (h)	III.	$[M L^{-1} T^{-2}]$
D.	Work Function (φ)	IV.	$[M L^2 T^{-2}]$

Choose the correct answer from the options given below: options

(1) A-I, B-II, C-III, D-IV (2) A-II, B-III, C-IV, D-I

(3) A-I, B-III, C-IV, D-II (4) A-III, B-I, C-II, D-IV

Sol. **4**

$$[Y] = \frac{F}{A}.\frac{\Delta L}{L} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$$

$F = 6\pi\, \eta\, r\, v$

$$[\eta] = \frac{F}{6\pi rv} = \frac{MLT^{-2}}{L\ LT^{-1}}$$

$[\eta] = M\, L^{-1}\, T^{-1}$

$$[h] = \frac{E}{f} = \frac{ML^2T^{-2}}{T^{-1}} = ML^2T^{-1}$$

Work function (φ) = ML^2T^{-2}

7. Match List I with List II

LIST I		LIST II	
A.	Troposphere	I.	Approximate 65 – 75 km over Earth's surface
B.	E- Part of Stratosphere	II.	Approximate 300 km over Earth's surface
C.	F2- Part of Thermosphere	III.	Approximate 10 km over Earth's surface
D.	D- Part of Stratosphere	IV.	Approximate 100 km over Earth's surface

Choose the correct answer from the options given below:

(1) A-III, B-IV, C-II, D-I (2) A-III, B-II, C-I, D-IV

(3) A-I, B-IV, C-III, D-II (4) A-I, B-II, C-IV, D-III

Sol. **1**

By theory

8. The light rays from an object have been reflected towards an observer from a standard flat mirror, the image observed by the observer are:-

A. Real

B. Erect

C. Smaller in size then object

D. Laterally inverted

Choose the most appropriate answer from the options given below:

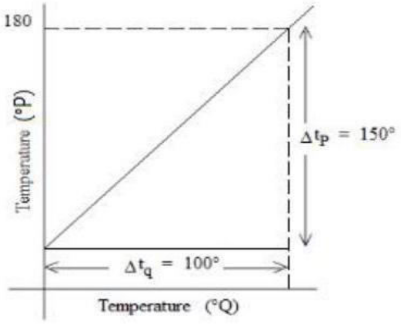
(1) A, C, and D Only (2) B and D Only

(3) A and D Only (4) B and C Only

Sol. **2**

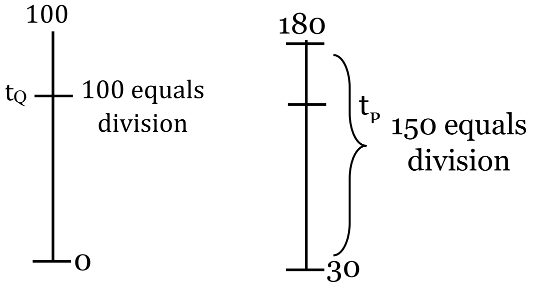
By theory

9. The graph between two temperature scales P and Q is shown in the figure. Between upper fixed point and lower fixed point there are 150 equal divisions of scale P and 100 divisions on scale Q. The relationship for conversion between the two scales is given by:-



(1) $\frac{t_P}{100} = \frac{t_Q-180}{150}$ (2) $\frac{t_Q}{150} = \frac{t_P-180}{100}$ (3) $\frac{t_p}{180} - \frac{t_Q-40}{100}$ (4) $\frac{t_Q}{100} = \frac{t_P-30}{150}$

Sol. **4**

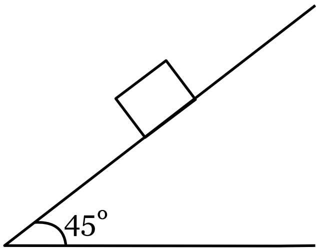


$$\frac{t_P-30}{180-30} = \frac{t_Q-0}{100-0}$$

$$\frac{t_P-30}{150} = \frac{t_Q}{100}$$

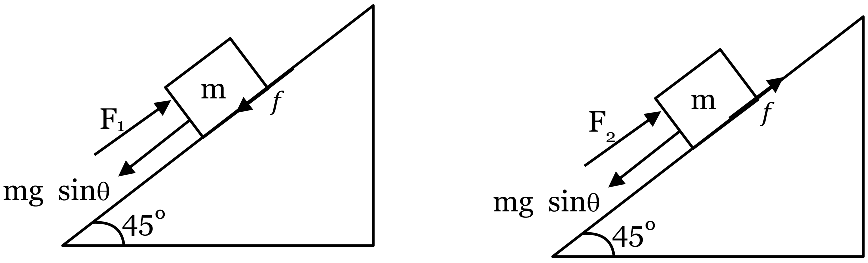
$$\frac{t_Q}{100} = \frac{t_P-30}{150}$$

10. Consider a block kept on an inclined plane (inclined at 45°) as shown in the figure. If the force required to just push it up the incline is 2 times the force required to just prevent it from sliding down, the coefficient of friction between the block and inclined plane (μ) is equal to :



(1) 0.25 (2) 0.50 (3) 0.60 (4) 0.33

Sol. 4



$F_1 = mg\sin\theta + \mu mg\cos\theta$
$F_1 = mg\sin 45 + \mu mg\cos 45$
$F_2 = mg\sin 45 - \mu mg\cos 45$

$F_1 = 2F_2$

$$mg\left(\frac{1}{\sqrt{2}}+\frac{\mu}{\sqrt{2}}\right)=2mg\left(\frac{1}{\sqrt{2}}-\frac{\mu}{\sqrt{2}}\right)$$

$1 + \mu = 2 - 2\mu$
$3\mu = 1$
$\mu = \frac{1}{3} = 0.33$

11. Every planet revolves around the sun in an elliptical orbit:-

A. The force acting on a planet is inversely proportional to square of distance from sun.
B. Force acting on planet is inversely proportional to product of the masses of the planet and the sun.
C. The Centripetal force acting on the planet is directed away from the sun.
D. The square of time period of revolution of planet around sun is directly proportional to cube of semi-major axis of elliptical orbit.

Choose the correct answer from the options given below:

(1) B and C only (2) A and C Only (3) A and D only (4) C and D only

Sol. 3

By Newton's law $F = \frac{Gm_1m_2}{r^2}$

By kepler's law $T^2 \alpha a^3$

12. For a moving coil galvanometer, the deflection in the coil is 0.05 rad when a current of 10 mA is passed through it. If the torsional constant of suspension wire is 4.0×10^{-5} N m rad $^{-1}$, the magnetic field is 0.01 T and the number of turns in the coil is 200 , the area of each turn (in cm^2) is :

(1) 1.0 (2) 2.0 (3) 1.5 (4) 0.5

Sol. **1**

$$\theta = \frac{NBA}{C} I$$

$$A = \frac{C\theta}{IBN}$$

$$= \frac{4 \times 10^{-5} \times .05}{10 \times 10^{-3} \times 0.01 \times 200}$$

$A = 10^{-4}$ m^2

$= 1$ cm^2

13. Match List I with List II

LIST I		LIST II	
A.	Gauss's Law in Electrostatics	I.	$\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$
B.	Faraday's Law	II.	$\oint \vec{B} \cdot d\vec{A} = 0$
C.	Gauss's Law in Magnetism	III.	$\oint \vec{B} \cdot d\vec{l}$ $= \mu_0 i_c + \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$
D.	Ampere-Maxwell Law	IV.	$\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$

Choose the correct answer from the options given below:

(1) A-IV, B-I, C-II, D-III (2) A-II, B-III, C-IV, D-I

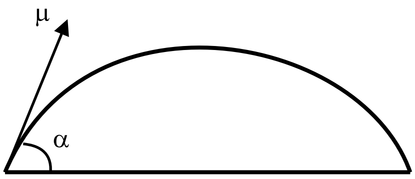
(3) A-III, B-IV, C-I, D-II (4) A-I, B-II, C-III, D-IV

Sol. **1**

14. Two objects are projected with same velocity 'u' however at different angles α and β with the horizontal. If $\alpha + \beta = 90°$, the ratio of horizontal range of the first object to the 2 nd object will be:

(1) 2: 1 (2) 1: 2 (3) 1: 1 (4) 4: 1

Sol. **3**



$$R_1 = \frac{u^2 \sin 2\alpha}{g}$$

$$R_2 = \frac{u^2 \sin 2\beta}{g} = \frac{u^2 \sin 2(90 - \alpha)}{g}$$

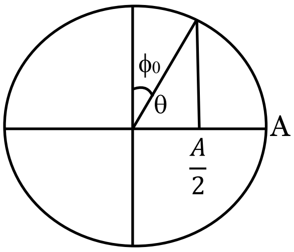
$$R_2 = \frac{u^2 \sin 2\alpha}{g} = R_1$$

$R_1 : R_2 = 1 : 1$

15. A particle executes simple harmonic motion between $x = -A$ and $x = +A$. If time taken by particle to go from $x = 0$ to $\frac{A}{2}$ is 2 s; then time taken by particle in going from $x = \frac{A}{2}$ to A is

(1) 4 S (2) 1.5 S (3) 2 S (4) 3 S

Sol. **1**



$$\cos\theta = \frac{A}{2 \times A} = \frac{1}{2} = \cos 60^\circ$$

$$\theta = 60 = \frac{\pi}{3}$$

$$\phi_0 = 30 = \frac{\pi}{6}$$

$$0 \rightarrow \frac{A}{2}, t = \frac{\frac{\pi}{6}}{\frac{2\pi}{T}} = \frac{T}{12} = 2$$

$$T = 24$$

$$\frac{A}{2} \rightarrow A, \; t = \frac{\pi / 3}{2\pi / T} = \frac{T}{6} = \frac{24}{6} = 4 \text{ sec}$$

16. Match List I with List II

LIST I		LIST II	
A.	Isothermal Process	I.	Work done by the gas decreases internal energy
B.	Adiabatic Process	II.	No change in internal energy
C.	Isochoric Process	III.	The heat absorbed goes partly to increase internal energy and partly to do work
D.	Isobaric Process	IV.	No work is done on or by the gas

Choose the correct answer from the options given below:

(1) A-I, B-II, C-III, D-IV (2) A-II, B-I, C-IV, D-III

(3) A-II, B-I, C-III, D-IV (4) A-I, B-II, C-IV, D-III

Sol. **2**

By theory

Isonormal $\rightarrow \Delta u = 0 \quad A \rightarrow II$

Adiabatic $\rightarrow \Delta Q = 0, \Delta w(+)$ so $\Delta u\ (-) \downarrow B \rightarrow I$

Isochoric $= \Delta V = 0$

$\Delta V = 0 \rightarrow \Delta w = 0$

$C \rightarrow IV$

Isobasic $\rightarrow P\Delta u \neq 0$

$\Delta v \neq 0$

$D \rightarrow III$

17. Statement I: When a Si sample is doped with Boron, it becomes P type and when doped by Arsenic it becomes N-type semi conductor such that P-type has excess holes and N-type has excess electrons.
Statement II: When such P-type and N-type semi-conductors, are fused to make a junction, a current will automatically flow which can be detected with an externally connected ammeter.
In the light of above statements, choose the most appropriate answer from the options given below
(1) Both Statement I and statement II are correct
(2) Statement **I** is incorrect but statement II is correct
(3) Both Statement I and Statement II are incorrect
(4) Statement I is correct but statement II is incorrect

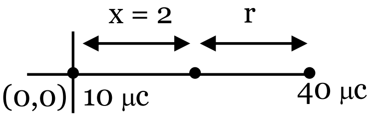
Sol. **4**

By theory

18. A point charge of $10\mu C$ is placed at the origin. At what location on the X-axis should a point charge of $40\mu C$ be placed so that the net electric field is zero at $x = 2$ cm on the X-axis?

(1) $x = -4$ cm	(2) $x = 6$ cm	(3) $x = 4$ cm	(4) $x = 8$ cm

Sol. **2**



$E_1 = E_2$

$$\frac{K \times 10}{(2)^2} = \frac{K \times 40}{4^2}$$

r = 4 cm

Distance from origin = 2 + 4 = 6 cm

19. The resistance of a wire is 5Ω. It's new resistance in ohm if stretched to 5 times of it's original length will be :

(1) 25	(2) 125	(3) 5	(4) 625

Sol. **2**

$R_{new} = n^2R$

$= (5)^2 \times 5$

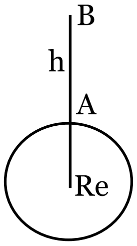
$= 125$

20. A body of mass is taken from earth surface to the height h equal to twice the radius of earth (R_e), the increase in potential energy will be:
(g = acceleration due to gravity on the surface of Earth)

(1) $3\ mgR_e$	(2) $\frac{1}{3}mgR_e$	(3) $\frac{2}{3}mgR_e$	(4) $\frac{1}{2}mgR_e$

Sol. **3**

h = 2 Re



$\Delta U = U_B - U_A$

$= \frac{-GM_e m}{(R_e + h)} - \left(\frac{-GM_e m}{R_e}\right)$

$= \frac{-GM_e m}{R_e + 2R_e} + \frac{GM_e m}{R_e} = \frac{2}{3}\frac{GM_e m}{R_e}$

$= \frac{2}{3}\frac{GM_e m}{R_e^2} R_e$

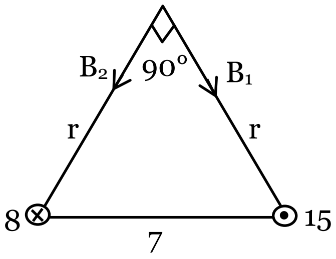
$\Delta U = = \frac{2}{3} mg R_e$

SECTION - B

21. Two long parallel wires carrying currents 8 A and 15 A in opposite directions are placed at a distance of 7 cm from each other. A point *P* is at equidistant from both the wires such that the lines joining the point *P* to the wires are perpendicular to each other. The magnitude of magnetic field at *P* is ______ × 10^{-6} T

(Given : $\sqrt{2} = 1 \cdot 4$)

Sol. **60**



$r = \frac{7}{\sqrt{2}}$ cm

$B = \sqrt{B_1^2 + B_2^2} = \sqrt{\left(\frac{\mu_0 I_1}{2\pi r}\right)^2 + \left(\frac{\mu_0 I_2}{2\pi r}\right)^2}$

$= \frac{\mu_0}{2\pi r}\sqrt{8^2 + 15^2}$

$= \frac{4\pi \times 10^{-7} \times 17}{2\pi \times \frac{7}{\sqrt{2}} \times 10^{-2}} = 68 \times 10^{-6}$

= 68

22. A spherical drop of liquid splits into 1000 identical spherical drops. If u_i is the surface energy of the original drop and u_f is the total surface energy of the resulting drops, the (ignoring evaporation), $\frac{u_f}{u_i} = \left(\frac{10}{x}\right)$. Then value of x is ________.

Sol. **1**

$U_I = T\, 4\pi R^2 = T\, 4\pi\, (10r)^2 = 100 \times T \times 4\pi r^2$

$$1000 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$R = 10r$

$$\frac{u_f}{u_i} = \frac{1000 \times T \times 4\pi r^2}{100 \times T \times 4\pi r^2} = 10$$

$\therefore$ x = 1

23. A nucleus disintegrates into two smaller parts, which have their velocities in the ratio 3: 2. The ratio of their nuclear sizes will be $\left(\frac{x}{3}\right)^{\frac{1}{3}}$. The value of ' x ' is:-

Sol. **2**

$0 = m_1\, 3v - m_2\, 2v$

$$\frac{m_1}{m_2} = \frac{2}{3}$$

$$\frac{8v_1}{8v_2} = \frac{2}{3}$$

$$\frac{\frac{4}{3}\pi R_1^3}{\frac{4}{3}\pi R_2^3} = \frac{2}{3} = \frac{R_1}{R_2} = \left(\frac{2}{3}\right)^{\frac{1}{3}}$$

$\therefore$ x = 2

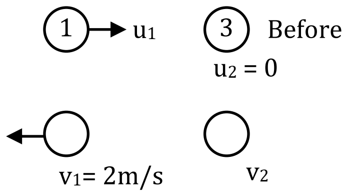
24. A train blowing a whistle of frequency 320 Hz approaches an observer standing on the platform at a speed of 66 m/s. The frequency observed by the observer will be (given speed of sound = 330 ms^{-1}) ________ Hz.

Sol. **400**

$$f = \left(\frac{v \pm v_0}{v \pm v_s}\right) f_0 = \frac{330 \times 320}{330 - 66} = \frac{330 \times 320}{264} = 400$$

25. A body of mass 1 kg collides head on elastically with a stationary body of mass 3 kg. After collision, the smaller body reverses its direction of motion and moves with a speed of 2 m/s. The initial speed of the smaller body before collision is __________ ms^{-1}

Sol. **4.00**



$p_i = p_f$

$u_1 + 0 = -1\times2 + 3v_2$

$u_1 = 3v_2 - 2$ ...(i)

$e = 1 = \frac{v_2 - (-2)}{u_1 - 0}$

$v_2 = u_1 - 2$ ...(2)

$u_1 = 3(u_1 - 2) - 2$

$2u_1 = 8, u_1 = 4$

26. A series LCR circuit is connected to an AC source of 220 V, 50 Hz. The circuit contains a resistance R = 80Ω, an inductor of inductive reactance $X_L = 70\Omega$, and a capacitor of capacitive reactance $X_C = 130\Omega$. The power factor of circuit is $\frac{x}{10}$. The value of x is:

Sol. **8.00**

$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_C - X_L)^2}}$

$= \frac{80}{\sqrt{(80)^2 + (130-70)^2}} = \frac{80}{\sqrt{(80)^2 + (60)^2}}$

$\cos\phi = \frac{80}{100} = \frac{8}{10}$

$x = 8$

27. If a solid sphere of mass 5 kg and a disc of mass 4 kg have the same radius. Then the ratio of moment of inertia of the disc about a tangent in its plane to the moment of inertia of the sphere about its tangent will be $\frac{x}{7}$. The the value of x is __________.

Sol. **5.00**

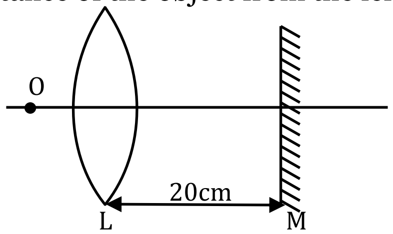
$I_{ss} = \frac{2}{5}mR^2 + mR^2 = \frac{7}{5}mR^2 = \frac{7}{5}\times5\times R^2 = 7R^2$

$I_{Disc} = \frac{mR^2}{4} + mR^2 = \frac{5mR^2}{4} = \frac{5}{4}\times4\times R^2 = 5R^2$

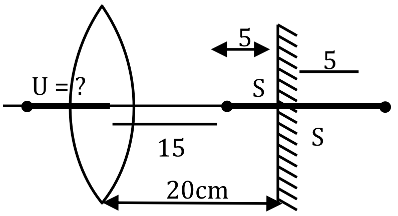
$\frac{I_{Disc}}{I_{SS}} = \frac{5R^2}{7R^2} = \frac{5}{7}$

$x = 5$

28. An object is placed on the principal axis of convex lens of focal length 10 cm as shown. A plane mirror is placed on the other side of lens at a distance of 20 cm. The image produced by the plane mirror is 5 cm inside the mirror. The distance of the object from the lens is cm



Sol. **30.00**



$\therefore$ for lens v = 20 – 5 = 15 cm

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{15}-\frac{1}{u}=\frac{1}{10}$$

$$\frac{1}{u}=\frac{1}{15}-\frac{1}{10}=\frac{2-3}{30}=\frac{1}{30}$$

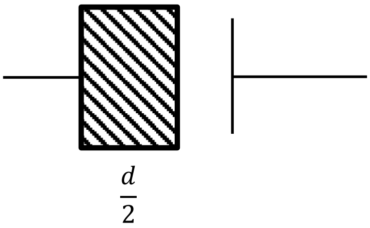
u = –30

= 30

29. A capacitor has capacitance 5μF when it's parallel plates are separated by air medium of thickness d. A slab of material of dielectric constant 1.5 having area equal to that of plates but thickness $\frac{d}{2}$ is inserted between the plates. Capacitance of the capacitor in the presence of slab will be μF.

Sol. **6**

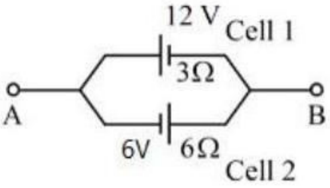
$$C_0=\frac{\in_o A}{d}=5$$



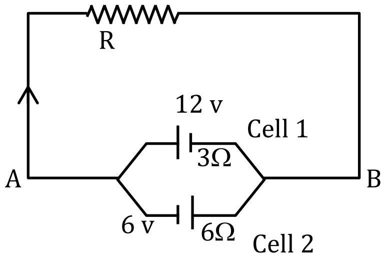
$$C_1=\frac{\in_o (1.5)A}{\frac{d}{2}}=3C_0\ ,\ C_2=\frac{\in_o A}{\frac{d}{2}}=2Co$$

$$=\frac{3C_o\times 2C_o}{5C_o}=\frac{6}{5}\times 5=6\mu f$$

30. Two cells are connected between points A and B as shown. Cell 1 has emf of 12 V and internal resistance of 3Ω. Cell 2 has emf of 6 V and internal resistance of 6Ω. An external resistor R of 4Ω is connected across A and B. The current flowing through R will be ____________ A.

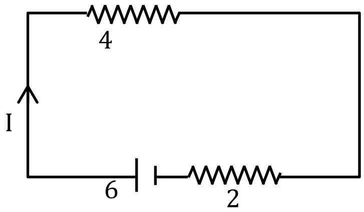


Sol. **1**



$$v = \frac{12\times6-6\times3}{6+3} = \frac{54}{9} = 6 \text{ volt}$$

$$r_{eq} = \frac{6\times3}{6+3} = 2\Omega$$



$$I = \frac{6}{4+2} = 1A$$

Chemistry

SECTION - A

31. When the hydrogen ion concentration $[H^+]$changes by a factor of 1000 , the value of pH of the solution

(1) increases by 2 units (2) increases by 1000 units

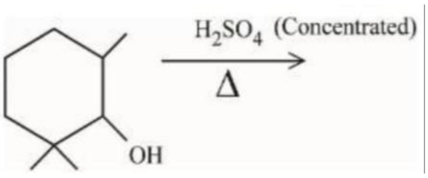
(3) decreases by 2 units (4) decreases by 3 units

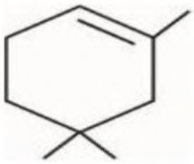
Sol. 4

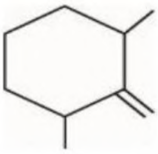
If $[H^+] \rightarrow 10^3$ times

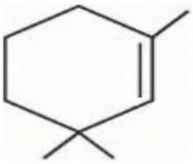
then pH decreases by 3 units.

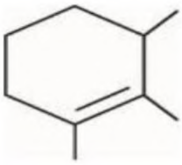
32. Find out the major product from the following reaction.



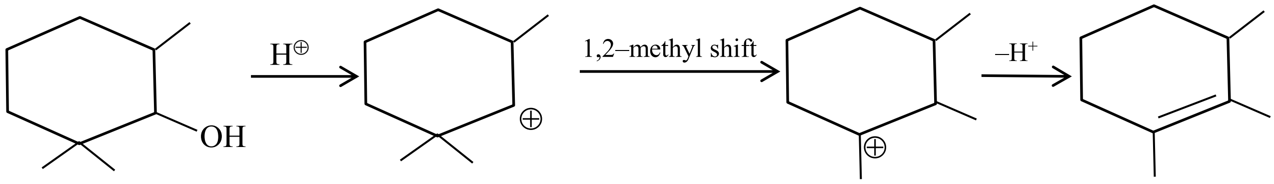
1.


2.


3.


4.


Sol. 4



33. Given below are two statements, one is labelled as Assertion **A** and the other is labelled as Reason **R**

Assertion A : Carbon forms two important oxides - CO and CO_2. CO is neutral whereas CO_2 is acidic in nature

Reason **R**: CO_2 can combine with water in a limited way to form carbonic acid, while CO is sparingly soluble in water

In the light of the above statements, choose the most appropriate answer from the options given below

(1) Both A and R are correct but R is NOT the correct explanation of A

(2) A is correct but R is not correct

(3) Both A and R are correct and R is the correct explanation of A

(4) A is not correct but R is correct

Sol. 3

(i) CO_2 is acidic as it from carbonic acid

$$CO_2 + H_2O \rightarrow H_2CO_3$$

(ii) CO is almost insoluble in water

34. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason **R**

Assertion A : The alkali metals and their salts impart characteristic colour to reducing flame.

Reason R : Alkali metals can be detected using flame tests.

In the light of the above statements, choose the most appropriate answer from the options given below

(1) A is not correct but R is correct

(2) Both A and R are correct but R is NOT the correct explanation of A

(3) A is correct but R is not correct

(4) Both A and R are correct and R is the correct explanation of A

Sol. **1**

The alkali metal and their salts impart characteristic colour to an oxidizing flame. this is because the heat from the flame excites the outemost orbital electron to a higher energy level : when the excited electron comes back to the ground state, there is emission of radiation in the visible region.

Alkali metal can therefore, be detected by the respective flame test and can be determined by flame photometry or atomic absorption spectroscopy.

35. Potassium dichromate acts as a strong oxidizing agent in acidic solution. During this process, the oxidation state changes from

(1) +2 to +1 (2) +3 to +1 (3) +6 to +2 (4) +6 to +3

Sol. **4**

$$Cr_2O_7^{-2} + 14H^+ + 6e^- \longrightarrow 2Cr^{+3} + 7H_2O$$

$$+6 \xrightarrow{\hspace{4cm}} +3$$

36. Match List I with List II

LIST I (Name of polymer)		LIST II (Uses)	
A.	Glyptal	I.	Flexible pipes
B.	Neoprene	II.	Synthetic wool
C.	Acrilan	III	Paints and Lacquers
D.	LDP	IV.	Gaskets

Choose the correct answer from the options given below:

(1) A-III, B-IV, C-I, D-II (2) A-III, B-II, C-IV, D-I

(3) A-III, B-I, C-IV, D-II (4) A-III, B-IV, C-II, D-I

Sol. **4**

(A) Glyptal → Paints and Lacquers (III)

(B) Neoprene → Gaskets (IV)

(C) Acrilan → Synthetic wool (II)

(D) LDP → Flexible pipes (I)

37. Which of the following represents the correct order of metallic character of the given elements ?

(1) Si < Be < Mg < K
(2) Be < Si < K < Mg
(3) Be < Si < Mg < K
(4) K < Mg < Be < Si

Sol. **1**

Si < Be < Mg < K

Si is having Non-metallic character.

38. Match List I with List II

LIST I		LIST II	
A.	Cobalt catalyst	I.	$(H_2 + Cl_2)$ production
B.	Syngas	II.	Water gas production
C.	Nickel catalyst	III.	Coal gasification
D.	Brine solution	IV.	Methanol production

Choose the correct answer from the options given below:

(1) A-IV, B-I, C-II, D-III
(2) A-IV, B-III, C-II, D-I
(3) A-II, B-III, C-IV, D-I
(4) A-IV, B-III, C-I, D-II

Sol. **2**

(a) Cobalt catalyst → methanol production.

(b) Syngas → coal gasification

(c) Nickel Catalyst → water gas production .

(d) Brine solution → $H_2 + Cl_2$ production.

39. Match List I with List II

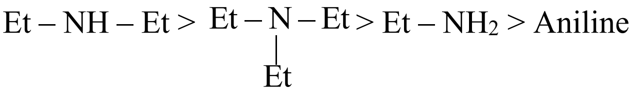
LIST I (Amines)		LIST II ($\mathbf{pK_b}$)	
A.	Aniline	I.	3.25
B.	Ethanamine	II.	3.00
C.	N-Ethylethanamine	III	9.38
D.	N. N-Diethylethanamine	IV.	3.29

Choose the correct answer from the options given below:

(1) A-III, B-IV, C-II, D-I
(2) A-III, B-II, C-I, D-IV
(3) A-I, B-IV, C-II, D-III
(4) A-III, B-II, C-IV, D-I

Sol. 1

Basicity order



$Et-NH-Et > Et-\underset{Et}{\underset{|}{N}}-Et > Et-NH_2 >$ Aniline

pKb : 3.00, pKb : 3.25 pKb : 3.29 pKb : 9.38

40. Match List I with List II

LIST I		LIST II	
	Isomeric pairs		Type of isomers
A.	Propanamine and N-Methylethanamine	I.	Metamers
B.	Hexan-2-one and Hexan-3-one	II.	Positional isomers
C.	Ethanamide and Hydroxyethanimine	III.	Functional isomers
D.	o-nitrophenol and p-nitrophenol	IV.	Tautomers

Choose the correct answer from the options given below:

(1) A-II, B-III, C-I, D-IV

(2) A-III, B-I, C-IV, D-II

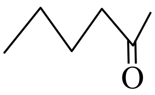
(3) A-III, B-IV, C-I, D-II

(4) A-IV, B-III, C-I, D-II

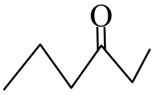
Sol. 2

(A) C–C–C–NH_2 & C – NH – C – C : functional isomer (III)

(B)

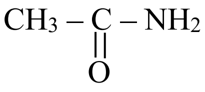


&



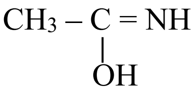
: Metamer (I)

(C)



$CH_3-\underset{O}{\underset{\|}{C}}-NH_2$

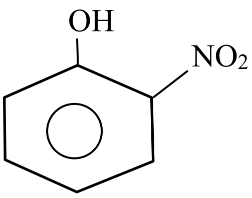
&



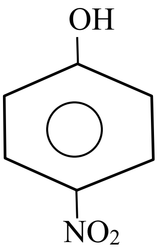
$CH_3-\underset{OH}{\underset{|}{C}}=NH$

: Tautamer (IV)

(D)



&



: Position isomer (II)

41. What is the mass ratio of ethylene glycol ($C_2H_6O_2$, molar mass = 62 g/mol) required for making 500 g of 0.25 molal aqueous solution and 250 mL of 0.25 molal aqueous solution?

(1) 1 : 1 (2) 2 : 1 (3) 1 : 2 (4) 3 : 1

Sol. 2

Case I

x gm $C_2H_6O_2$ present

$$0.25 = \frac{x/62}{500 - x} \times 1000$$

$$125 = \left(\frac{1000}{62} + 0.25\right)x \qquad \text{..........(1)}$$

Case II

y gm $C_2H_6O_2$ is present.

$$0.25 = \frac{y/62}{250 - y} \times 1000$$

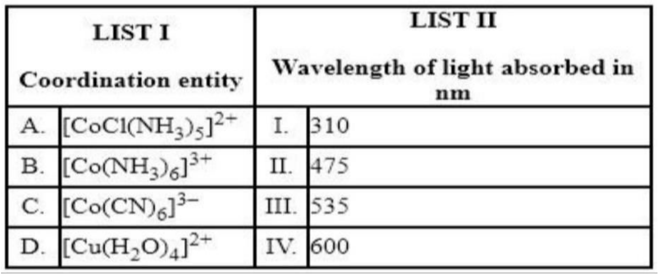
$$62.5 - 0.25y = \frac{1000}{62}y$$

$$62.5 = \left(\frac{1000}{62} + 0.25\right)y \qquad \text{.........(2)}$$

equation (1) ÷ equation (2)

$$\frac{x}{y} = \frac{125}{62.5} = \frac{2}{1}$$

42. Match list I with List II



LIST I Coordination entity		LIST II Wavelength of light absorbed in nm	
A.	$[CoCl(NH_3)_5]^{2+}$	I.	310
B.	$[Co(NH_3)_6]^{3+}$	II.	475
C.	$[Co(CN)_6]^{3-}$	III.	535
D.	$[Cu(H_2O)_4]^{2+}$	IV.	600

Choose the correct answer from the options given below:

(1) A-III, B-I, C-II, D-IV (2) A-IV, B-I, C-III, D-II

(3) A-III, B-II, C-I, D-IV (4) A-II, B-III, C-IV, D-I

Sol. **3**

$\Delta_o \uparrow \lambda \downarrow$

(splitting energy = $\frac{hc}{\lambda_{abs}}$)

43. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason **R**

Assertion A : Butylated hydroxy anisole when added to butter increases its shelf life.

Reason R : Butylated hydroxy anisole is more reactive towards oxygen than food.

In the light of the above statements, choose the most appropriate answer from the options given below

(1) A is correct but R is not correct

(2) A is not correct but R is correct

(3) Both A and R are correct and R is the correct explanation of A

(4) Both A and R are correct but R is NOT the correct explanation of A

Sol. **3**

The molecule BHA = Butylated hydroxyanisole commonly used as food preservatives which normally acts as antifungal and antiviral BHA reduces the rancidity of oil and fat which halps in retaining the nutrients (Butter contains saturated fats).

44. The isomeric deuterated bromide with molecular formula C_4H_8DBr having two chiral carbon atoms is

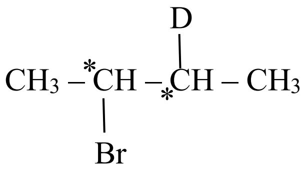
(1) 2 - Bromo - 2 - deuterobutane

(2) 2 - Bromo-1-deuterobutane

(3) 2 - Bromo - 1 - deutero - 2 - methylpropane

(4) 2 - Bromo −3 – deuterobutane

Sol. **4**



$$CH_3-\underset{Br}{\overset{*}{C}H}-\overset{D}{\underset{*}{C}H}-CH_3$$

2 - Bromo −3 – deuterobutane

45. A chloride salt solution acidified with dil. HNO_3 gives a curdy white precipitate, [A], on addition of $AgNO_3 \cdot$ [A] on treatment with NH_4OH gives a clear solution, B. A and B are respectively

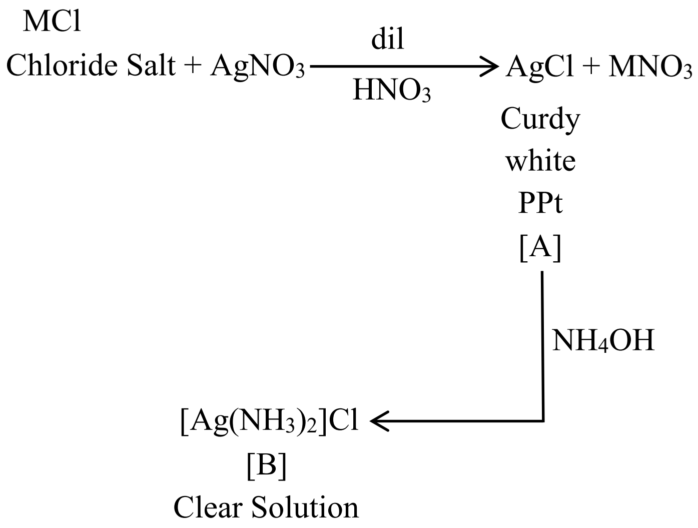
(1) $AgCl\&(NH_4)[Ag(OH)_2]$

(2) $AgCl\&[Ag(NH_3)_2]Cl$

(3) $H[AgCl_3]\&(NH_4)[Ag(OH)_2]$

(4)$H[AgCl_3]\&[Ag(NH_3)_2]Cl$

Sol. **2**



46. Statement I : Dipole moment is a vector quantity and by convention it is depicted by a small arrow with tail on the negative centre and head pointing towards the positive centre.

Statement II : The crossed arrow of the dipole moment symbolizes the direction of the shift of charges in the molecules.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Statement I is incorrect but Statement II is correct

(2) Statement I is correct but Statement II is incorrect

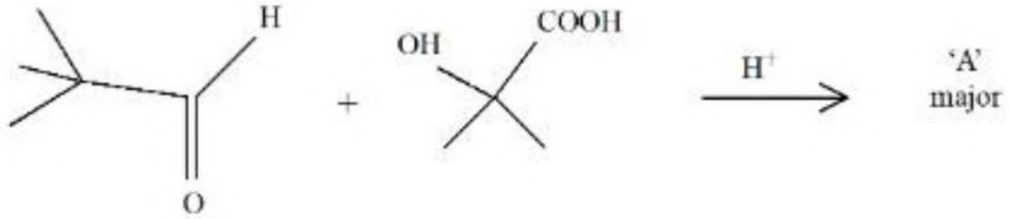
(3) Both Statement I and Statement II are incorrect

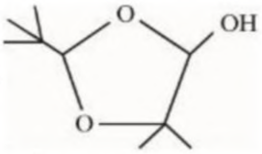
(4) Both Statement I and Statement II are correct

Sol. **2**

The crossed arrow of the dipole moment symbolizes the direction of the shift of electron density in the molecules.

47. ' A ' in the given reaction is



1.


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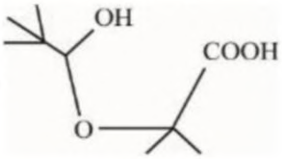

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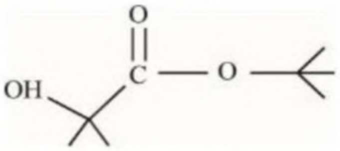
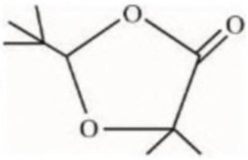
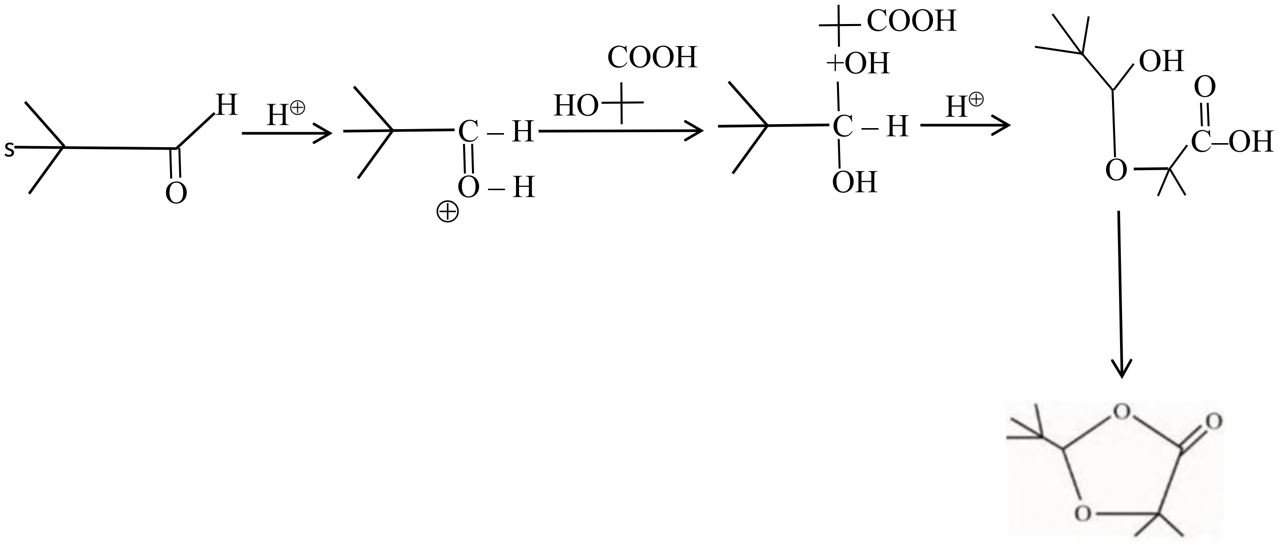


4.


Sol. **4**



48. A. Ammonium salts produce haze in atmosphere.

B. Ozone gets produced when atmospheric oxygen reacts with chlorine radicals.

C. Polychlorinated biphenyls act as cleansing solvents.

D. 'Blue baby' syndrome occurs due to the presence of excess of sulphate ions in water.

Choose the correct answer from the options given below:

(1) A and D only (2) A, B and C only (3) A and C only (4) B and C only

Sol. **3**

(i) Ammonium salt are major component of both atmospheric nitrogen aerosols and wet deposited.

(iii) PCB belongs to a broad family of man-made organic chemicals known. as chlorinated hydrocarbons.

49. Given below are two statements:

Statement I : In froth floatation method a rotating paddle agitates the mixture to drive air out of it.

Statement II : Iron pyrites are generally avoided for extraction of iron due to environmental reasons.

In the light of the above statements, choose the correct answer from the options given below:

(1) Statement I is false but Statement II is true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are true

Sol. **1**

The rotating paddle in the froth flotation process violently agitates the suspension of powdered ore in water, as well the collectors and froth stablisers, generating frothing.

50. Which one among the following metals is the weakest reducing agent?

(1) Li (2) K (3) Rb (4) Na

Sol. **4**

Na metals is the weakest Reducing agent.

Section B

51. Total number of moles of AgCl precipitated on addition of excess of $AgNO_3$ to one mole each of the following complexes $[Co(NH_3)_4Cl_2]Cl$, $[Ni(H_2O)_6]Cl_2$, $[Pt(NH_3)_2Cl_2]$ and $[Pd(NH_3)_4]Cl_2$ is

Sol. **5**

(i) $[Co(NH_3)_4Cl_2]Cl + AgNO_3 \rightarrow [Co(NH_3)_4Cl_2]^+ + AgCl$

(ii) $[Ni(H_2O)_6]Cl_2 + AgNO_3 \rightarrow [Ni(H_2O)_6]^{+2} + 2AgCl$

(iii) $[Pt(NH_3)_2Cl_2] + AgNO_3 \rightarrow$ no AgCl mole are ppt

(iv) $[Pd(NH_3)_4]Cl_2 + AgNO_3 \rightarrow [Pd(NH_3)_4]^{+2} + 2AgCl$

Total 5 mole AgCl are formed.

52. The number of incorrect statement/s from the following is/are

A. Water vapours are adsorbed by anhydrous calcium chloride.

B. There is a decrease in surface energy during adsorption.

C. As the adsorption proceeds, ΔH becomes more and more negative.

D. Adsorption is accompanied by decrease in entropy of the system.

Sol. **2**

A & C are incorrect

$CaCl_2$ absorbs water vapour.

As adsorption proceeds,

ΔH becomes less negative.

53. Number of hydrogen atoms per molecule of a hydrocarbon A having 85.8% carbon is (Given: Molar mass of A = 84 g mol^{-1})

Sol. **12**

$C \rightarrow 85.8\%$

$H \rightarrow 14.2\ \%$

mass of H in one molecule $= 84 \times \frac{14.2}{100} \approx 12$

No. of H– atoms $= \frac{12}{1}$

$= 12$

54. The number of given orbitals which have electron density along the axis is

$P_x, P_y, P_z, d_{xy}, d_{yz}, d_{xz}, d_z^2, d_{x^2} - y^2$

Sol. 5

$Px, Py, Pz, dz^2, dx^2 - y^2$ have Electron density along the axis.

55. 28.0 L of CO_2 is produced on complete combustion of 16.8 L gaseous mixture of ethene and methane at 25°C and 1 atm. Heat evolved during the combustion process is________ kJ.

Given : $\Delta H_C(CH_4) = -900\,kJ\,mol^{-1}$

$\Delta H_c(C_2H_4) = -1400$ kJ mol^{-1}

Sol. **847**

Moles of mixture $= \frac{Pv}{RT} = \frac{1 \times 16.8}{0.0821 \times 298} = 0.6866$ moles

Moles of $CO_2 = \frac{1 \times 28}{0.0821 \times 298} = 1.144$ mole

$$CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O$$

x x

$$C_2H_4 + 3O_2 \longrightarrow 2CO_2 + 2H_2O$$

(0.6866 – x) 2(0.686 – x)

Total CO_2 produced = 1.144

x + 2 (0.6866 – x) = 1.144

x = 1.3732 – 1.144

= 0.2292

Moles of CH_4 = 0.2292

Moles of C_2H_4 = 0.6866 – 0.2292

= 0.4574

Total Heat produced

= (900 × 0.2292) + (0.4574 × 1400)

= 206.28 + 640.36 = 846.64

56. $Pt(s)|H_2(g)(1bar)||H^+(aq)(1M) \parallel M^{3+}(aq), M^+(aq)|Pt(s)$

The E_{cell} for the given cell is 0.1115 V at 298 K when $\frac{[M^+(aq)]}{[M^{3+}(aq)]} = 10^a$

The value of a is

Given : $E^{\theta} M^{3+}/M^{+} = 0.2$ V

$\frac{2.303RT}{F} = 0.059$ V

Sol. **3**

Cell Reaction

$$H_2 + M^{3+} \longrightarrow 2H^{+} + M^{+}$$

$$E_{cell} = E^{o}_{cell} - \frac{2.303RT}{2F} \log \frac{\left[M^{+}\right]\left[H^{+}\right]^2}{[M^{3+}]}$$

$$0.1115 = 0.2 - \frac{0.059}{2} \log 10^{a}$$

$$\frac{0.059}{2} \log 10^{a} = 0.0885$$

a = 3

57. The number of pairs of the solutions having the same value of the osmotic pressure from the following is (Assume 100% ionization)

A. 0.500 M C_2H_5OH (aq) and 0.25 M KBr (aq)

B. 0.100 M $K_4[Fe(CN)_6]$ (aq) and 0.100 M $FeSO_4(NH_4)_2SO_4$ (aq)

C. 0.05 M $K_4[Fe(CN)_6]$ (aq) and 0.25 M NaCl (aq)

D. 0.15 M NaCl(aq) and 0.1 M $BaCl_2$(aq)

E. 0.02 M $KCl \cdot MgCl_2 \cdot 6H_2O$(aq) and 0.05 M KCl(aq)

Sol. **4**

(a) $(i\,c)_{c_2H_5OH} = 0.5$

$(i\,c)_{kBr} = 2 \times 0.25 = 0.5$

osmotic pressure will be same.

(b) $(i\ c)_{k_4[Fe(CN)_6]} = 0.1 \times 5 = 0.5$

$(i\ c)_{FeSO_4} \cdot (NH_4)_2SO_4 = 0.1 \times 5 = 0.5$

osmotic pressure will be same.

(c) $(i\ c)_{k_4[Fe(CN)_6]} = 5 \times 0.05 = 0.25$

$(i\ c)_{NaCl} = 0.25 \times 2 = 0.5$

osmotic pressure will not be same.

(d) $(i\ c)_{NaCl} = 0.15 \times 2 = 0.3$

$(i\ c)_{BaCl_2} = 0.1 \times 3 = 0.3$

osmotic pressure will be same.

(e) $(i\ c)_{Kcl.MgCl.6H_2O} = 0.02 \times 5 = 0.1$

$(i\ c)_{Kcl} = 0.05 \times 2 = 0.1$

osmotic pressure will be same.

58. A first order reaction has the rate constant, $= 4.6 \times 10^{-3}\ s^{-1}$. The number of correct statement/s from the following is/are

Given: log 3 = 0.48

A. Reaction completes in 1000 s.

B. The reaction has a half-life of 500 s.

C. The time required for 10% completion is 25 times the time required for 90% completion.

D. The degree of dissociation is equal to $(1 - e^{-kt})$

E. The rate and the rate constant have the same unit.

Sol. **1**

$k = 4.6 \times 10^{-3}\ sec^{-1}$

for Ist order :-

$$t½ = \frac{0.693}{k} = \frac{0.693}{4.6 \times 10^{-3}} = 150.65\,sec.$$

$t_{completion} = \infty$

$$\text{Degree of dissociation } (\propto) = \frac{x}{[A]_0} = \frac{[A]_0 - [A]_t}{[A]_0}$$

$$= \frac{[A]_0 - [A]_0 e^{-kt}}{[A]_0} = 1 - e^{-kt}$$

rate and rate constant have different units

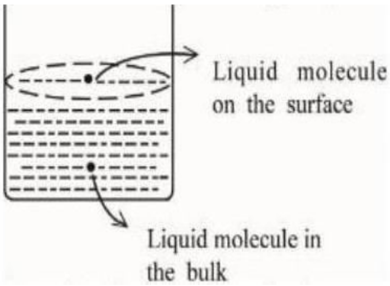
$$t_{10\%} = \frac{1}{K} \ln\frac{100}{90}$$

$$t_{90\%} = \frac{1}{K} \ln\frac{100}{10}$$

$$\frac{t_{10\%}}{t_{90\%}}=\frac{\log 10-\log 9}{\log 10}=0.045$$

$$t_{10\%}=0.045t_{90\%}$$

59. Based on the given figure, the number of correct statement/s is/are _____________



A. Surface tension is the outcome of equal attractive and repulsive forces acting on the liquid molecule in bulk.

B. Surface tension is due to uneven forces acting on the molecules present on the surface.

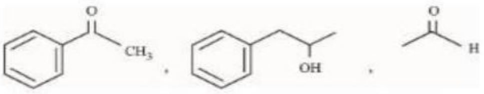
C. The molecule in the bulk can never come to the liquid surface.

D. The molecules on the surface are responsible for vapours pressure if system is a closed system.

Sol. 2

B & D option are correct.

60. Number of compounds giving (i) red colouration with ceric ammonium nitrate and also (ii) positive iodoform test from the following is



Sol. **3**

Mathematics

SECTION - A

61. Let $\Delta, \nabla \in \{\wedge, V\}$ be such that $(p \rightarrow q)\Delta(p\nabla q)$ is a tautology. Then

(1) $\Delta = V, \nabla = V$ (2) $\Delta = V, \nabla = \Lambda$ (3) $\Delta = \Lambda, \nabla = V$ (4) $\Delta = \Lambda, \nabla = \Lambda$

Sol. **(1)**

p	q	$p \rightarrow q$	$p \vee q$	$(p \rightarrow q) \vee (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	T

62. If the four points, whose position vectors are $3\hat{i} - 4\hat{j} + 2\hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$, $-2\hat{i} - \hat{j} + 3\hat{k}$ and $5\hat{i} - 2\alpha\hat{j} + 4\hat{k}$ are coplanar, then α is equal to

(1) $\frac{73}{17}$ (2) $\frac{107}{17}$ (3) $\frac{-73}{17}$ (4) $\frac{-107}{17}$

Sol. **(1)**

$$\underbrace{3\hat{i} - 4\hat{j} + 2\hat{k}}_{P},\ \underbrace{\hat{i} + 2\hat{j} - \hat{k}}_{Q},\ \underbrace{-2\hat{i} - \hat{j} + 3\hat{k}}_{R},\ \underbrace{5\hat{i} - 2\alpha\hat{j} + 4\hat{k}}_{S}$$

$$\overrightarrow{PQ} = -2\hat{i} + 6\hat{j} - 3\hat{k}$$

$$\overrightarrow{QR} = -3\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\overrightarrow{RS} = 7\hat{i} + (1 - 2\alpha)\hat{j} + \hat{k}$$

$$\left[\overrightarrow{PQ}\ \overrightarrow{QR}\ \overrightarrow{RS}\right] = 0$$

$$\begin{vmatrix} -2 & 6 & -3 \\ -3 & -3 & 4 \\ 7 & 1-2\alpha & 1 \end{vmatrix} = 0$$

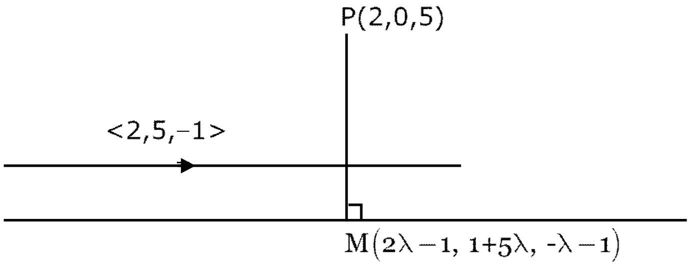
$$-2(-3 + 8\alpha - 4) - 6(-31) - 3(6\alpha - 3 + 21) = 0$$

$$\alpha = \frac{73}{17}$$

63. The foot of perpendicular of the point (2,0,5) on the line $\frac{x+1}{2} = \frac{y-1}{5} = \frac{z+1}{-1}$ is (α, β, γ) . Then, which of the following is NOT correct?

(1) $\frac{\beta}{\gamma} = -5$ (2) $\frac{\gamma}{\alpha} = \frac{5}{8}$ (3) $\frac{\alpha}{\beta} = -8$ (4) $\frac{\alpha\beta}{\gamma} = \frac{4}{15}$

Sol. **(1)**



$\overrightarrow{PM}(2,5,-1)=0$

$(2\lambda-3,\ 5\lambda+1,\ -\lambda-6)\cdot(2,5,-1)=0$

$4\lambda-6+25\lambda+5+\lambda+6=0$

$\boxed{\lambda=-\frac{1}{6}}$

Now, $\alpha=2\left(-\frac{1}{6}\right)-1=-\frac{4}{3}$

$\beta=\frac{1}{6}$

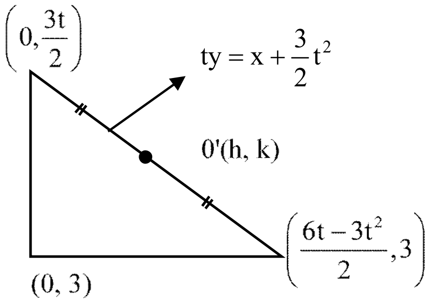
$\gamma=-\frac{5}{6}$

64. The equations of two sides of a variable triangle are x= 0 and y=3, and its third side is a tangent to parabola $y^2=6x$. The locus of its circumcentre is:

(1) $4y^2-18y-3x-18=0$ (2) $4y^2-18y-3x+18=0$

(3) $4y^2-18y+3x+18=0$ (4) $4y^2+18y+3x+18=0$

Sol. **(3)**



$2h=\frac{6t-3t^2}{2}$

$4h=6t-3t^2$ .....(i)

& $2k=\frac{3t+6}{2}$

$\boxed{\frac{4k-6}{3}=t}$

$4h=8k-12-\frac{1}{3}\left(16k^2-48k+36\right)$

$12h=24k-36-16k^2+48k-36$

$4y^2-18y+3x+18=0$

65. Let $f(x)=2x^n+\lambda,\lambda\in\mathbb{R},n\in\mathbb{N}$, and f(4) = 133, f(5)255.

Then the sum of all the positive integer divisors of (f(3) − f(2)) is

(1) 60 (2) 59 (3) 61 (4) 58

Sol. **(1)**

$133 = 2(4^n)+\lambda$

$255 = 2(5^n)+\lambda$

$122 = 2[5^n-4^n]$

$\boxed{5^n - 4^n = 61}$

$\Downarrow$

$\boxed{n = 3}$

Now,

$f(3) = 2(3)^3+ \lambda$

$f(2) = 2(2)^3+ \lambda$

$f(3) – f(2) = 38 = 2 \times 19$

$(2^0 +2^1) (19^0 + 19)$

$= 60$

66. $\sum_{k=0}^{6} {}^{51}C_3$ is equal to

(1) ${}^{51}C_4 - {}^{45}C_4$ (2) ${}^{52}C_3 - {}^{45}C_3$ (3) ${}^{52}C_4 - {}^{45}C_4$ (4) ${}^{51}C_3 - {}^{45}C_3$

Sol. **(3)**

${}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{46}C_3 + {}^{45}C_3$

add and subtract ${}^{45}C_4$

$\left({}^{45}C_4 + {}^{45}C_3\right) + {}^{46}C_3 + {}^{47}C_3 + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3 - {}^{45}C_4 \quad \left({}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r\right)$

${}^{52}C_4 - {}^{45}C_4$

$\Rightarrow$ [C]

67. Let the function $f(x) = 2x^3 + (2p - 7)x^2 + 3(2p - 9)x - 6$ have a maxima for some value of $x < 0$ and a minima for some value of $x > 0$. Then, the set of all values of p is

(1) $\left(0,\frac{9}{2}\right)$ (2) $\left(-\infty,\frac{9}{2}\right)$ (3) $\left(-\frac{9}{2},\frac{9}{2}\right)$ (4) $\left(\frac{9}{2},\infty\right)$

Sol. **(2)**

$f(x) = 2x^3 + (2p - 7)x^2 + 3(2p - 9)x - 6$

$$f'(x)=6x^2+(4p-14)x+6p-27=0\begin{smallmatrix}\nearrow \alpha\\ \searrow \beta\end{smallmatrix}$$

let $\alpha > 0$ & $\beta < 0$

Products of roots $< 0 \Rightarrow$ (2)

68. Let $A = \begin{bmatrix}\frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}}\\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}}\end{bmatrix}$ and $B = \begin{bmatrix}1 & -i\\ 0 & 1\end{bmatrix}$, where $i = \sqrt{-1}$.

If $M = A^TBA$, then the inverse of the matrix $AM^{2023}A^T$ is

(1) $\begin{bmatrix}1 & 0\\ -2023i & 1\end{bmatrix}$ (2) $\begin{bmatrix}1 & -2023i\\ 0 & 1\end{bmatrix}$ (3) $\begin{bmatrix}1 & 0\\ 2023i & 1\end{bmatrix}$ (4) $\begin{bmatrix}1 & 2023i\\ 0 & 1\end{bmatrix}$

Sol. **(4)**

Now, $M^2 = \left(A^TBA\right)\left(A^TBA\right) = A^TB^2A \qquad \boxed{AA^T = I}$

$\Rightarrow M^{2023} = A^TB^{2023}A$

Let $D = AM^{2023}A^T = A\,A^TB^{2023}AA^T \qquad \boxed{AA^T = I}$

$D = B^{2023}$

Now, $B^2 = \begin{bmatrix}1- & i\\ 0 & 1\end{bmatrix}\begin{bmatrix}1- & i\\ 0 & 1\end{bmatrix} = \begin{bmatrix}1 & -2i\\ 0 & 1\end{bmatrix}$

Now, $B^{2023} = \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$

$D^{-1} = \begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

69. Let $\vec{a} = -\hat{i} - \hat{j} + \hat{k}$, $\vec{a} \cdot \vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{i} - \hat{j}$. Then $\vec{a} - 6\vec{b}$ is equal to

(1) $3(\hat{i} - \hat{j} + \hat{k})$ (2) $(\hat{i} + \hat{j} - \hat{k})$ (3) $3(\hat{i} + \hat{j} + \hat{k})$ (4) $3(\hat{i} - \hat{j} - \hat{k})$

Sol. **(3)**

$$\vec{a} \times \left(\vec{a} \times \vec{b}\right) = \begin{vmatrix} i & j & k \\ -1 & -1 & 1 \\ 1 & -1 & 0 \end{vmatrix} = \hat{i} + \hat{j} + 2\hat{k}$$

$\vec{a} - 3\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$

$-3\vec{b} = 2\hat{i} + 2\hat{j} + \hat{k}$

$-6\vec{b} = 4\hat{i} + 4\hat{j} + 2\hat{k}$

Now, $\vec{a} - 6\vec{b} = 3\left(\hat{i} + \hat{j} + \hat{k}\right)$

70. The integral $16\int_1^2 \frac{dx}{x^3(x^2+2)^2}$ is equal to

(1) $\frac{11}{12} - \log_e 4$ (2) $\frac{11}{6} - \log_e 4$ (2) $\frac{11}{6} + \log_e 4$ (4) $\frac{11}{12} + \log_e 4$

Sol. **(2)**

$$16\int_1^2 \frac{dx}{x^3 x^4\left(1 + \frac{2}{x^2}\right)^2}$$

Let, $1 + \frac{2}{x^2} = t \Rightarrow -\frac{4}{x^3}dx = dt$

$$\frac{-4}{4}\int_3^{3/2} \frac{(t-1)^2}{t^2}dt = \int_{3/2}^{3} \frac{t^2 - 2t + 1}{t^2}dt$$

$$\Rightarrow \int_{3/2}^{3}\left(1 - \frac{2}{t} + \frac{1}{t^2}\right)dt$$

$$\Rightarrow 3 - \frac{3}{2} - 2\left(\ln 3 - \ln 3/2\right) - \frac{1}{3} + \frac{2}{3}$$

$$\Rightarrow \frac{11}{6} - 2\ln 2 \Rightarrow \frac{11}{6} - \ln 4$$

71. Let T and C respectively be the transverse and conjugate axes of the hyperbola $16x^2 - y^2 + 64x + 4y + 44 = 0$. Then the area of the region above the parabola $x^2 = y + 4$, below the transverse axis T and on the right of the conjugate axis C is:

(1) $4\sqrt{6} + \frac{28}{3}$ (2) $4\sqrt{6} - \frac{44}{3}$ (3) $4\sqrt{6} + \frac{44}{3}$ (4) $4\sqrt{6} - \frac{28}{3}$

Sol. **(1)**

$16(x^2 + 4x) - (y^2 - 4y) + 44 = 0$

$16\{(x+2)^2 - 4\} - (y-2)^2 + 4 + 44 = 0$

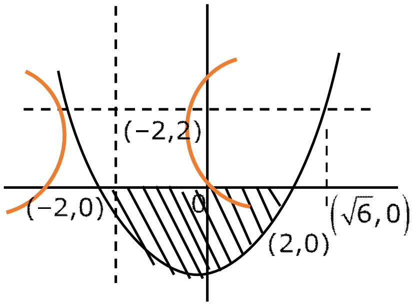
$16(x+2)^2 - (y-2)^2 = 16$

$$\frac{(x+2)^2}{1} - \frac{(y-2)^2}{16}$$

$$\text{Area} = \int_{-2}^{\sqrt{6}} (y_2 - y_1)\,dx$$

$$= \int_{-2}^{\sqrt{6}} \left(2 - \left(x^2 - 4\right)\right) dx$$

$$= 4\sqrt{6} + \frac{28}{3}$$



72. Let N be the sum of the numbers appeared when two fair dice are rolled and let the probability that $N - 2,\ \sqrt{3N},\ N + 2$ are in geometric progression be $\frac{k}{48}$. Then the value of k is

(1) 8 (2) 16 (3) 2 (4) 4

Sol. **(4)**

$3N = N^2 - 4$

$N^2 - 3N - 4 = 0$

$\boxed{N = 4}$

Sum should be equal to 4 so possible outcomes are {(1,3), (2,2), (3,1)}

$\Rightarrow \text{Prob} = \frac{3}{36} = \frac{1}{12} = \frac{k}{48}$

$\boxed{K = 4}$

73. If the function $f(x) = \begin{cases} (1 + |\cos x|)\,\frac{\lambda}{|\cos x|} & ,0 < x < \frac{\pi}{2} \\ \mu & ,\ x = \frac{\pi}{2} \\ \frac{\cot 6x}{e^{\cot 4x}} & \frac{\pi}{2} < x < \pi \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then $9\lambda + 6\log_e \mu + \mu^6 - e^{6\lambda}$ is equal to

(1) 10 (2) $2e^4 + 8$ (3) 11 (4) 8

Sol. $f\left(\frac{\pi^+}{2}\right) = e^{\lim\limits_{h\to 0} \frac{\cot 6h}{\cot 4h}} \Rightarrow {}^{2}\!/_{3}$

$f\left(\frac{\pi^-}{2}\right) = \lim\limits_{h\to 0}(1 + \sin h)\frac{\lambda}{\sin h}$

$= \frac{\lambda}{0}$

$\Rightarrow$ limit DNE (does not exist)

74. The number of functions $f:\{1,2,3,4\} \to \{a \in : \mathbb{Z}|a| \le 8\}$ satisfying $f(n) + \frac{1}{n}f(n+1) = 1, \forall n \in \{1,2,3\}$ is

(1) 1 (2) 4 (3) 2 (4) 3

Sol. **(2)**

$$f:\{1,2,3,4\}\to\{a\in z:|a|\le 8\}$$

$$f(n)+\frac{1}{n}f(n+1)=1\ \forall n\in\{1,2,3\}$$

$$f(n+1)=n(1-f(n))$$

Put n = 1, $\quad f(2)=1-f(1)$

Put n = 2, $\quad f(3)=2(1-f(2))=2f(1)$

Put n = 3, $\quad f(4)=3(1-f(3))=3(1-2f(1))$

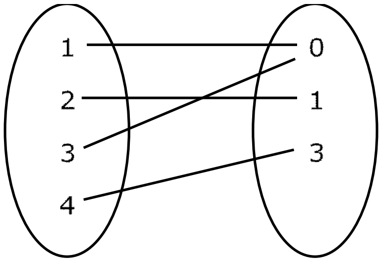
$$f(4)=3-6f(1)$$

Now : $\quad f(2)=1-f(1)$

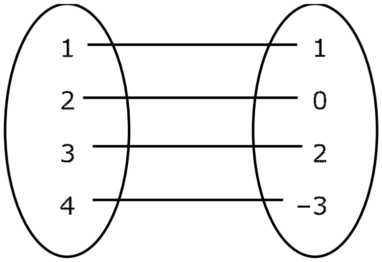
$$f(3)=2f(1)$$

$$f(4)=3-6f(1)$$

$\boxed{\text{Case – I}}$ Take f(1) = 0



$\boxed{\text{Case – II}}$ Take f(1) = 1



No. of function = 2

Ans : $\boxed{4}$

75. Let y = y(t) be a solution of the differential equation $\frac{dy}{dt}+\alpha y=\gamma e^{-\beta t}$ where, $\alpha>0,\beta>0$ and $\gamma>0$.

Then $\lim\limits_{t\to\infty} y(t)$

(1) is −1 (2) is 1 (3) does not exist (4) is 0

Sol. **(4)**

$$\frac{dy}{dt}+\alpha y=\gamma e^{-\beta t}$$

L.D.E (Linear differential equation)

I.F. = $e^{\int \alpha\cdot dt}=e^{\alpha t}$

$$y\left(e^{\alpha t}\right)=\int \gamma e^{-\beta t}\cdot e^{\alpha t}\cdot dt$$

$$\Rightarrow ye^{\alpha t} = \gamma \frac{e^{(\alpha-\beta)t}}{\alpha-\beta} + C$$

$$\Rightarrow y(t) = \frac{\gamma}{\alpha-\beta} e^{-\beta t} + C \cdot e^{-\alpha t}$$

$$\lim_{t\to\infty} y(t) = \lim_{t\to\infty}\left\{\frac{\gamma}{\alpha-\beta} e^{-\beta t} + c \cdot e^{-\alpha t}\right\}$$

= 0 + 0

$$\Rightarrow \lim_{t\to\infty} y(t) = 0$$

76. Let z be a complex number such that $\left|\frac{z-2i}{z+i}\right| = 2, z \neq -i$. Then z lies on the circle of radius 2 and centre

(1) (2,0) (2) (0,2) (3) (0, −2) (4) (0,0)

Sol. $$\left|\frac{x+i(y-2)}{x+i(y+1)}\right| = 2$$

$$x^2 + (y-2)^2 = 4\left(x^2 + (y+1)^2\right)$$

$$3x^2 + 4y^2 + 4 + 8y - y^2 - 4 + 4y = 0$$

$$3\left(x^2 + y^2\right) + 12y = 0$$

$$x^2 + y^2 + 4y = 0$$

C(0,–2)

77. Let A, B, C be 3 × 3 matrices such that A is symmetric and B and C are skew-symmetric.

Consider the statements

(S1) $A^{13}B^{26} - B^{26}A^{13}$ is symmetric

(S2) $A^{26}C^{13} - C^{13}A^{26}$ is symmetric

Then,

(1) Only S2 is true(2) Both S1 and S2 are false

(3) Only S1 is true(4) Both S1 and S2 are true

Sol. **(1)**

$A^T = A$, $B^T = -B$, $C^T = -C$

(S_1) : $(A^{13}B^{26} - B^{26}A^{13})^T$

$= (A^{13}B^{26})^T - (B^{26}A^{13})^T$

$= (B^T)^{26}(A^T)^{13} - (A^T)^{13}(B^T)^{26}$

$= (-B)^{26}(A)^{13} - (A)^{13}(-B)^{26}$

$= B^{26}A^{13} - A^{13}B^{26}$

$= -(A^{13}B^{26} - B^{26}A^{13})$

($S_1 \rightarrow$ false)

(S_2) : $(A^{26}C^{13} - C^{13}A^{26})^T$

$= (A^{26}C^{13})^T - (C^{13}A^{26})^T$

$= (C^T)^{13}(A^T)^{26} - (A^T)^{26}(C^T)^{13}$

$= - C^{13} A^{26} - A^{26} (-C)^{13}$

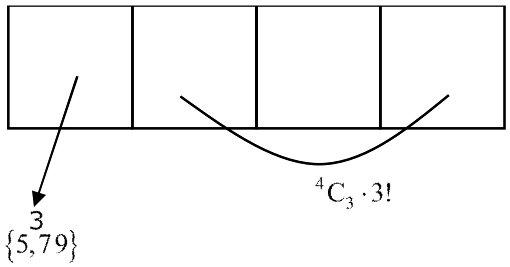
$= A^{26} C^{13} - C^{13} A^{26}$

($S_2 \rightarrow$ True)

78. The number of numbers, strictly between 5000 and 10000 can be formed using the digits 1,3,5,7,9 without repetition, is

(1) 12 (2) 120 (3) 72 (4) 6

Sol. **(3)**



No. of ways = 3.4 × 3! =3.4! = 72

79. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by

$f(x) = \log_{\sqrt{m}}\{\sqrt{2}(\sin x - \cos x) + m - 2\}$, for some m, such that the range of f is $[0,2]$. Then the value of m is

(1) 5 (2) 4 (3) 3 (4) 2

Sol. **(1)**

$\because -\sqrt{2} \le \sin x - \cos x \le \sqrt{2}$

$\Rightarrow -2 \le \sqrt{2}(\sin x - \cos x) \le 2$

$\Rightarrow m - 4 \le \sqrt{2}(\sin x - \cos x) + m - 2 \le m$

$\Rightarrow \log_{\sqrt{m}}^{(m-4)} \le \log_{\sqrt{m}}^{\{\sqrt{2}(\sin x - \cos x)+m-2\}} \le \log_{\sqrt{m}}^{m}$

$\Downarrow$

0

$\Rightarrow \log_{\sqrt{m}}^{(m-4)} = 0$

$\Rightarrow \boxed{m=5}$

80. The shortest distance between the lines $x + 1 = 2y = -12z$ and $x = y + 2 = 6z - 6$ is

(1) $\frac{3}{2}$ (2) 2 (3) $\frac{5}{2}$ (4) 3

Sol. **(2)**

$$\frac{x+1}{1} = \frac{y}{\frac{1}{2}} = \frac{z}{\frac{-1}{12}}, \qquad \frac{x}{1} = \frac{y+2}{1} = \frac{z-1}{\frac{1}{6}}$$

$$d = \left|\frac{(\vec{b} - \vec{a}) \cdot (\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|}\right|$$

$\vec{a} = (-1,0,0)$, $\vec{b} = (0,-2,1)$

$$\vec{p}=\left(1,\frac{1}{2},\frac{-1}{12}\right), \qquad \vec{q}=\left(1,1,\frac{1}{6}\right)$$

$$\vec{b}-\vec{a}=\hat{i}-2\hat{j}+\hat{k}$$

$$\vec{p}\times\vec{q}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & \frac{1}{2} & -\frac{1}{12}\\ 1 & 1 & \frac{1}{6}\end{vmatrix}$$

$$=\hat{i}\left(\frac{1}{12}+\frac{1}{12}\right)-\hat{j}\left(\frac{1}{6}+\frac{1}{12}\right)+\hat{k}\left(1-\frac{1}{2}\right)$$

$$=\frac{\hat{i}}{6}-\frac{\hat{j}}{4}+\frac{\hat{k}}{2}$$

$$\left|\vec{p}\times\vec{q}\right|=\sqrt{\frac{1}{36}+\frac{1}{16}+\frac{1}{4}}=\frac{7}{12}$$

$$d=\left|\frac{\left(\hat{i}-2\hat{j}+\hat{k}\right)\cdot\left(\frac{\hat{i}}{6}-\frac{\hat{j}}{4}+\frac{\hat{k}}{2}\right)}{\frac{7}{12}}\right|$$

$$d=\left|\frac{\frac{1}{6}+\frac{1}{2}+\frac{1}{2}}{\frac{7}{12}}\right|=\frac{\frac{7}{6}}{\frac{7}{12}}=2$$

SECTION - B

81. 25% of the population are smokers. A smoker has 27 times more chances to develop lung cancer than a non smoker. A person is diagnosed with lung cancer and the probability that this person is a smoker is $\frac{k}{10}$. Then the value of k is.

Sol. **9**

$$P(\text{smoker}) = \frac{1}{4}$$

$$P(\text{non smoker}) = \frac{3}{4}$$

Probability that a smoker has lung cancer

$$P\left(\frac{C}{S}\right) = 27\ P\left(\frac{C}{NS}\right)$$

Probability that a person is smoker when he has lung cancer

$$= \frac{P(S)\cdot P\left(\frac{C}{S}\right)}{P(S)\cdot P\left(\frac{C}{S}\right) + P(NS)\cdot P\left(\frac{C}{NS}\right)}$$

$$= \frac{\frac{1}{4}\times P\left(\frac{C}{S}\right)}{\frac{1}{4}\times P\left(\frac{C}{S}\right) + \frac{3}{4}P\left(\frac{C}{NS}\right)}$$

$$= \frac{\frac{1}{4}\times 27P\left(\frac{C}{NS}\right)}{\frac{1}{4}\times 27P\left(\frac{C}{NS}\right) + \frac{3}{4}P\left(\frac{C}{NS}\right)}$$

$$\frac{27}{30} = \frac{k}{10}$$

$$\boxed{k = 9}$$

82. The remainder when $(2023)^{2023}$ is divided by 35 is

Sol. **7**

$2023 = 289 \times 7$

2023 is a multiple of 7

$n = (2023)^{2023}$ is multiple of 7

and $(2023)^{2023} = (-2)^{2023} = -2(2^2)^{1011}$

$$= -2(5\text{-}1)^{1011}$$

$$= -2\left[{}^5C_0 5^{1011} - {}^5C_1 5^{1010} + ... - {}^{1011}C_{1011}\right]$$

$(2023)^{2023}$ when divided by 5

gives remainder 2

If n= $(2023)^{2023}$ divided by 35 = 7 × 5

n = 7k

n – 7 = 7 (k – 1) → n 7 is multiple of 7

and n = 5 m + 2

so n – 7 = 5m – 5 = multiple of 5

so n – 7 is multiple of 35 so when n is divided by 35, reminder = 7

83. Let $a \in \mathbb{R}$ and let α, β be the roots of the equation $x^2 + 60^{\frac{1}{4}}x + a = 0$

If $\alpha^4 + \beta^4 = -30$, then the product of all possible values of a is

Sol. **(45)**

$\alpha + \beta = -60^{\frac{1}{4}}$ and $\alpha\beta = a$

$\alpha^2 + \beta^2 = 60^{\frac{1}{2}} - 2a$

$\alpha^4 + \beta^4 + 2\alpha^2\beta^2 = 60\ 4a^2 - 4a \cdot 60^{\frac{1}{2}}$

$-30 + 2a^2 = 60 + 4a^2 - 4a\sqrt{60}$

$a^2 - 2a\sqrt{60} + 45 = 0$

$\boxed{\text{Product} = 45}$

84. For the two positive numbers a, b is a, b and $\frac{1}{18}$ are in a geometric progression, while $\frac{1}{a}$, 10 and $\frac{1}{b}$ are in an arithmetic progression, then 16a + b is equal to

Sol. **(3)**

$b^2 = \frac{a}{18}$

$20 = \frac{1}{a} + \frac{1}{b}$

$a = \frac{b}{20b - 1}$

$b^2 = \frac{1}{18} \times \frac{b}{20b - 1}$

$360b^2 - 18b - 1 = 0$

$360b^2 - 30b + 12b - 1 = 0$

$(12b - 1)(30b + 1) = 0$

$b = \frac{1}{12}, \frac{-1}{30}$ (rejected)

$a = \frac{1}{8}$

16 a + 12 b = 2 + 1 = 3

85. If m and n respectively are the numbers of positive and negative values of q in the interval [–p, p] that satisfy the equation $\cos 2\theta \cos\frac{\theta}{2} = \cos 3\theta \cos\frac{9\theta}{2}$, then mn is equal to

Sol. **25**

$$2\cos 2\theta\cos\frac{\theta}{2} = 2\cos 3\theta\cos\frac{9\theta}{2}$$

$$\cos\frac{5\theta}{2} + \cos\frac{3\theta}{2} = \cos\frac{15\theta}{2} + \cos\frac{3\theta}{2}$$

$$\cos\frac{5\theta}{2} - \cos\frac{15\theta}{2} = 0$$

$$\sin 5\theta = 0 \text{ or } \sin\frac{5\theta}{2} = 0$$

$$\theta = \frac{n\pi}{5} \text{ or } \frac{2n\pi}{5}$$

$$\theta = 0, \pm\frac{\pi}{5}, \pm\frac{2\pi}{5}, \pm\frac{3\pi}{5}, \pm\frac{4\pi}{5}, \pm\pi$$

$m = n = 5$

$$\boxed{mn = 25}$$

86. If the shortest distance between the line joining the points $(1,2,3)$ and $(2,3,4)$, and the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-2}{0}$ is a, then $28a^2$ is equal to

Sol. **18**

A (1, 2, 3) B (2, 3, 4)

Equation of line AB

$$\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1}$$

Given line

$$\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-2}{0}$$

$$\text{shortest distance } = \left|\frac{\left(\vec{a}_2 - \vec{a}_1\right)\cdot\left(\vec{b}_2 \times \vec{b}_1\right)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right|$$

$$= \left|\frac{\left(3\hat{j} - \hat{k}\right)\cdot\left(\hat{i} + 2\hat{j} - 3\hat{k}\right)}{\sqrt{1+4+9}}\right|$$

$$\alpha = \frac{3}{\sqrt{14}}$$

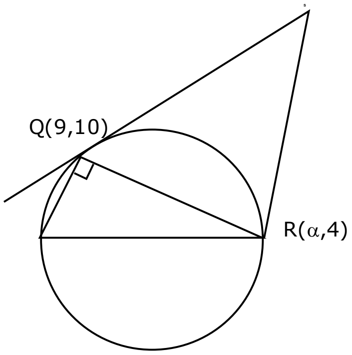
$$28\alpha^2 = 28 \times \frac{9}{14} = 18$$

87. Points P(–3,2),Q(9,10) and (a,4) lie on a circle C with PR as its diameter, The tangents to C at the points Q and R intersect at the point S. If S lies on the line 2x – ky = 1, then k is equal to

Sol. **(3)**

Equation of circle is

$(x + 3)(x - \alpha) + (y - 2)(y - 4) = 0$



Q lies on it

$12(9 - \alpha) + 8 \times 6 = 0$

$\boxed{\alpha = 13}$

$x^2 + y^2 - 10x - 6y - 31 = 0$

Equation of Tangent at Q

$x.9 + y.10 - 5(x + 9) - 3(y + 10) - 31 = 0$

$4x + 7y = 106$ …….(1)

Equation of Tangent at R

$x.13 + y.4 - 5(x + 13) - 3(y + 4) - 31 = 0$

$8x + y = 108$ …….(2)

Solution (1) and (2)

$s = \left(\frac{25}{2}, 8\right)$

which lies on 2x – ky = 1

$\boxed{k = 3}$

88. Suppose Anil's mother wants to give 5 whole fruits to Anil from a basket of 7 red apples, 5 white apples and 8 oranges. If in the selected 5 fruits, at least 2 oranges, at least one red apple and at least one white apple must be given, then the number of ways, Anil's mother can offer 5 fruits to Anil is

Sol. **6860**

Three cases are possible

1R 1W 3O + 2R 1W 2O + 1R 2W 2O

$^7C_1 \cdot {}^5C_1 \cdot {}^8C_3 + {}^7C_2 \cdot {}^5C_1 \cdot {}^8C_2 + {}^7C_1 \cdot {}^5C_2 \cdot {}^8C_2$

= 6860

89. If $\int_{\frac{1}{3}}^{3} |\log_e x| dx = \frac{m}{n} \log_e \left(\frac{n^2}{e}\right)$, where m and n are coprime natural numbers, then $m^2 + n^2 - 5$ is equal to

Sol. **20**

$$\int_{\frac{1}{3}}^{3} |\log_e x| dx$$

$$= \int_{\frac{1}{3}}^{1} (-\ln x) dx + \int_{1}^{3} (\ln x) dx$$

$$-[x \ln x - x]_{\frac{1}{3}}^{1} + [x \ln x - x]_{1}^{3}$$

$$= \frac{4}{3} \ln\left(\frac{9}{e}\right) = \frac{m}{n} \ln\left(\frac{n^2}{e}\right)$$

m = 4 and n = 3

so $m^2 + n^2 - 5 = 16 + 9 - 5 = 20$

90. A triangle is formed by X- axis, Y-axis and the line 3x + 4y = 4y = 60. Then the number of points P(a, b) which lie strictly inside the triangle, where a is an integer and b is a multiple of a, is

Sol. **31**

3x + 4y = 60

x = 1, 4y = 57, y=14.2

x = 1, y = 1, 2, 3,…….14 → 14 points

x = 2, 4y = 54, y=13.5

x = 2, y = 2, 4, 6, 8, 10, 12 → 6 points

x = 3, y = 3, 6, 9, 12 → 4 points

x = 4, y = 4, 8 → 2 points

x = 5, y = 5, 10 → 2 points

x = 6, y = 6 → 1 points

x = 7, y = 7 → 1 points

x = 8, y = 8 → 1 points

x = 9, 4y = 23, y=5.7 × no point

Total points = 14 + 6 + 4 + 2 + 2 + 1 + 1 + 1 = 31

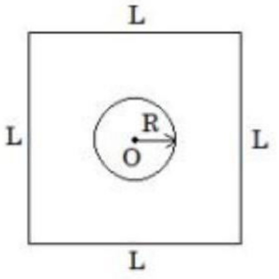
JEE–MAIN EXAMINATION – JANUARY, 2023

(Held On Thursday 29th January, 2023) | **TIME : 9 : 00 AM to 12 : 00 NOON**

Physics

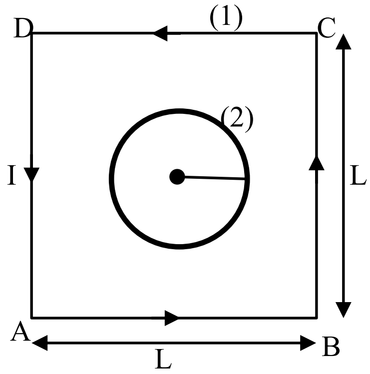
SECTION - A

1. Find the mutual inductance in the arrangement, when a small circular loop of wire of radius 'R' is placed inside a large square loop of wire of side $(L \gg R)$. The loops are coplanar and their centers coincide :



(1) $M = \frac{\sqrt{2}\mu_0 R^2}{L}$ (2) $M = \frac{2\sqrt{2}\mu_0 R}{L^2}$ (3) $M = \frac{\sqrt{2}\mu_0 R}{L^2}$ (4) $M = \frac{2\sqrt{2}\mu_0 R^2}{L}$

Sol. **(4)**



$\phi = MI$

$\phi_2 = MI_1$

$B_1A_2 = MI_1$

$M = \frac{B_1A_2}{I_1}$ ….(1)

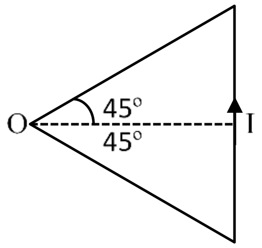
$B_1 \rightarrow$ magnetic field due to square frame

$A_2 \rightarrow$ Area of circle

$I_1 \rightarrow$ current in square frame.

$B_1 \rightarrow$

$B_1 = 4.\ B_{AB}$



$$= 4\left[\frac{\mu_0 I_1}{24\pi\frac{L}{2}}[\sin 45^{o} + \sin 45]\right]$$

$$B_1 = 2\frac{\mu_0 I_1}{\pi L}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = 2\sqrt{2}\frac{\mu_0 I_1}{\pi L}$$

$$M = \frac{B_1 \cdot A_2}{I_1}$$

$$M = \left(\frac{2\sqrt{2}\mu_0 I_1}{\pi L}\right) \times \frac{\pi R^2}{I_1} = \frac{2\sqrt{2}\mu_0 R^2}{L}$$

2. The threshold wavelength for photoelectric emission from a material is 5500A. Photoelectrons will be emitted, when this material is illuminated with monochromatic radiation from a

(A) 75 W infra –red lamp (B) 10 W infra-red lamp

(C) 75 W ultra – violet lamp (D) 10 W ultra-violet lamp

Choose the correct answer from the options given below:

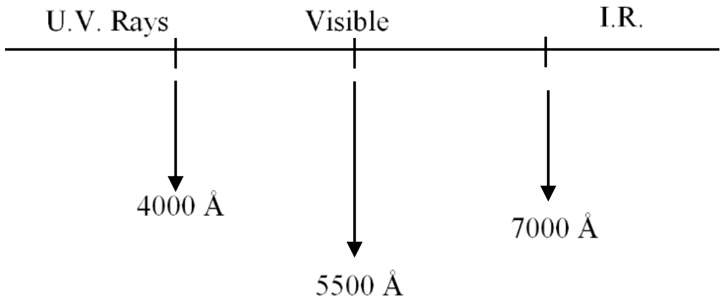
(1) B and C only (2) A and D only

(3) C only (4) C and D Only

Sol. **(4)**

$$\lambda_0 = 5500\text{Å} \rightarrow \phi_0 = \frac{12400}{5500} = 2.25\,\text{eV}$$

$$\phi = 3.6 \times 10^{-19}\,\text{J}$$



- P.E.E will occur if wavelength of incidence wave is less then threshold wavelength. So u. v. rays will be useful for emission.
  So both U.V. rays lamps can be used.

3. Match List I with List II:

List I (Physical Quantity)		List II (Dimensional Formula)	
A.	Pressure gradient	I.	$[M^0\ L^2\ T^{-2}]$
B.	Energy density	II.	$[M^1\ L^{-1}\ T^{-2}]$
C.	Electric Field	III.	$[M^1\ L^{-2}\ T^{-2}]$
D.	Latent heat	IV.	$[M^1\ L^1\ T^{-3}\ A^{-1}]$

Choose the correct answer from the options given below:

(1) A-II, B – III, C-I, D-IV (2) A-II, B – III, C-IV, D-I

(3) A-III, B – II, C-IV, D-I (4) A-III, B – II, C-I, D-IV

Sol. **(3)**

$$\text{(A) Pressure gradient} = \frac{\text{Pressure}}{\text{Length}} = \frac{\text{Force}}{\text{Area} \times \text{length}}$$

$$= \frac{MLT^{-2}}{L^2 \cdot L} = [ML^{-2}T^{-2}]$$

$$\text{(B) Energy density} = \frac{\text{Energy}}{\text{Volume}} = \frac{ML^2T^{-2}}{L^3} = [M\,L^{-1}T^{-2}]$$

$$\text{(C) Electric field} = \frac{\text{Force}}{\text{Charge}} = \frac{MLT^{-2}}{AT} = [M\,LT^{-3}\,A^{-1}]$$

$$\text{(D) Latent heat} = \frac{\text{Heat}}{\text{Mass}} = \frac{ML^2T^{-2}}{M} = [L^2\,T^{-2}]$$

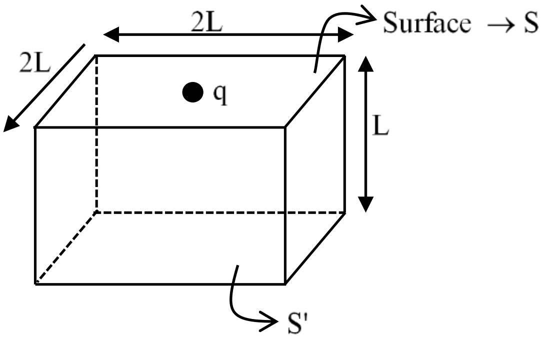
Ans : A-III, B-II, C-IV, D-I

Ans. : (3)

4. In a cuboid of dimension 2L × 2L × L, a charge q is placed at the center of the surface ' S ' having area of $4L^2$. The flux through the opposite surface to ' S ' is given by

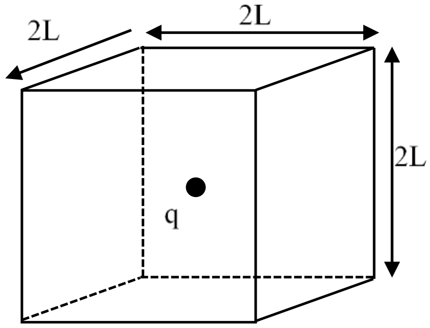
(1) $\frac{q}{12\varepsilon_0}$ (2) $\frac{q}{6\varepsilon_0}$ (3) $\frac{q}{3\varepsilon_0}$ (4) $\frac{q}{2\varepsilon_0}$

Sol. **(2)**



When smaller box is considered on the given box then charge 'q' will be at center.

So flux from surface S' = $\left(\frac{q}{\varepsilon_0}\right)\cdot\frac{1}{6}=\frac{q}{6\varepsilon_0}$



Ans : (2)

5. A person observes two moving trains, 'A' reaching the station and 'B' leaving the station with equal speed of 30 m/s. If both trains emit sounds with frequency 300 Hz, (Speed of sound: $\frac{330\ m}{s}$) approximate difference of frequencies heard by the person will be:

(1) 55 Hz (2) 80 Hz (3) 33 Hz (4) 10 Hz

Sol. **(1)**

$\boxed{A}\rightarrow 30\,m/s,$ $\boxed{\text{Observer}}$ $\boxed{B}\rightarrow 30\,m/s$

$f_0 = 300$ Hz

V = 330 m/sec.

$$f_A = f_0\left[\frac{V}{V-V_A}\right]=300\left[\frac{330}{330-30}\right]=330Hz$$

$$f_B = f_0\left[\frac{V}{V+V_A}\right]=300\left[\frac{330}{360}\right]=275Hz$$

$$\Delta f = f_A - f_B = 330-275=55Hz$$

Ans. : (1)

6. A block of mass m slides down the plane inclined at angle 30° with an acceleration $\frac{g}{4}$. The value of coefficient of kinetic friction will be:

(1) $\frac{1}{2\sqrt{3}}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{2\sqrt{3}+1}{2}$ (4) $\frac{2\sqrt{3}-1}{2}$

Sol. **(1)**

$f_k = \mu N$

$N = mg\cos\theta$

$f_k = \mu\, mg\cos\theta$

$$a = \frac{mg\sin\theta - \mu mg\cos\theta}{m}$$

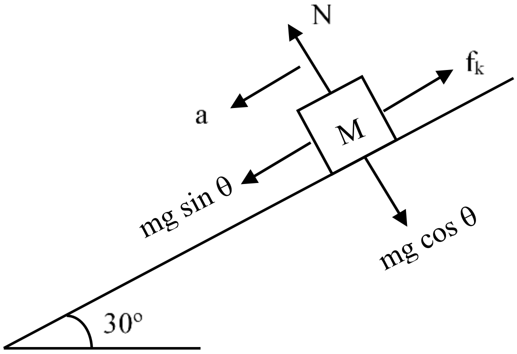
$a = g\sin 30^\circ - \mu g\cos 30^\circ$

$$\frac{g}{4} = g\left[\frac{1}{2} - \frac{\sqrt{3}\mu}{2}\right]$$

$$\frac{1}{2} = 1 - \sqrt{3}\mu$$

$$\sqrt{3}\mu = \frac{1}{2}$$

$$\boxed{\mu = \frac{1}{2\sqrt{3}}}$$



Ans. : 1

7. A bicycle tyre is filled with air having pressure of 270 kPa at 27°C. The approximate pressure of the air in the tyre when the temperature increases to 36° C is

(1) 270 kPa (2) 262 kPa (3) 360 kPa (4) 278 kPa

Sol. **(4)**

PV = nRT

n → const. V = const.

P α T,

$P_1 = 270$ kpa,

$T_1 = 27^\circ C = 300$ K

$P_2 = ?$,

$T_2 = 36^\circ = 36 + 273 = 309$ K

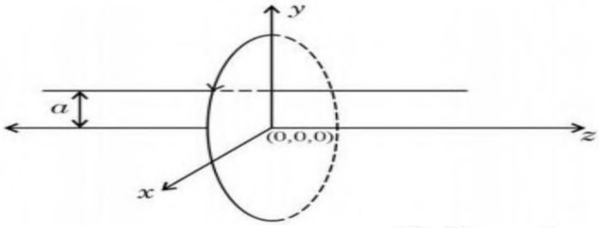
$$\frac{P_2}{P_1} = \frac{T_2}{T_1} \quad \ldots(1)$$

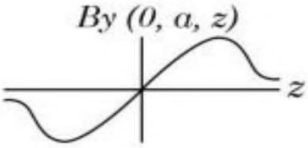
$$\frac{P_2}{270\text{ KPa}} = \frac{309}{300}$$

$$P_2 = \frac{103}{100} \times 270\text{KPa} \simeq 278\text{KPa}$$

Option : (4)

8. A single current carrying loop of wire carrying current I flowing in anticlockwise direction seen from +ve z direction and lying in xy plane is shown in figure. The plot of $\hat{j}$ component of magnetic field (By) at a distance $'a'$ (less than radius of the coil) and on yz plane vs z coordinate looks like



(1)


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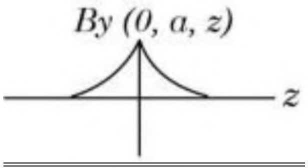
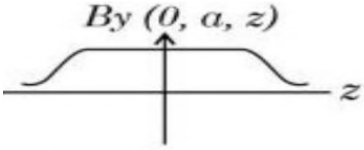
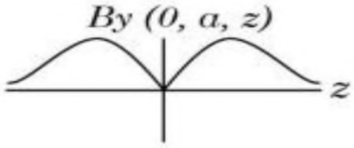
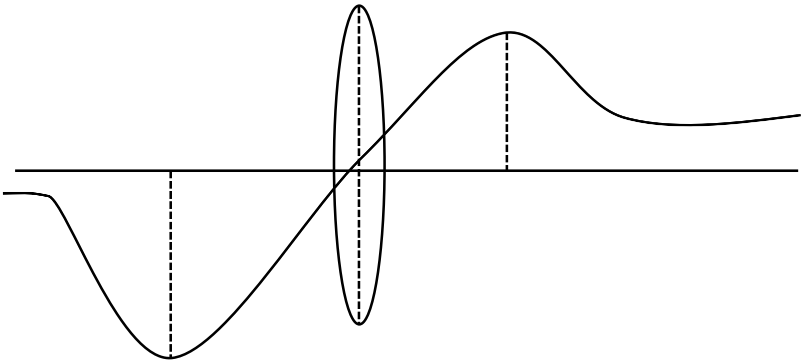


(3)


(4)


Sol. **(1)**

Theory based concept



9. Surface tension of a soap bubble is 2.0×10^{-2} Nm^{-1}. Work done to increase the radius of soap bubble from 3.5 cm to 7 cm will be:

Take $\left[\pi = \frac{22}{7}\right]$

(1) 9.24×10^{-4} J (2) 5.76×10^{-4} J (3) 0.72×10^{-4} J (4) 18.48×10^{-4} J

Sol. **(4)**

$T = 2.0 \times 10^{-2}$ Nm^{-1}

$r_1 = 3.5$ cm, $r_2 = 7$ cm

$W = T\Delta A \times$ No. of air – liquid surface

$$W = 2T.4\pi\left(r_2^2 - r_1^2\right)$$

$$W = 2 \times 2 \times 10^{-2} \times 4\pi\left[49 - \frac{49}{4}\right] \times 10^{-4}$$

$$W = 16\pi \times 10^{-6} \times 49 \times \frac{3}{4}$$

$W = 1847.26 \times 10^{-6}$

$W = 18.47 \times 10^{-4}$ J

10. Given below are two statements: One is labelled as Assertion **A** and the other is labelled as Reason **R.**

Assertion A: If

dQ and dW represent the heat supplied to the system and the work done on the system respectively. Then according to the first law of thermodynamics $dQ = dU - dW$.

Reason R: First law of thermodynamics is based on law of conservation of energy.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both A and R are correct and R is the correct explanation of A

(2) A is not correct but R is correct

(3) A is correct but R is not correct

(4) Both A and R are correct but R is not the correct explanation of A

Sol. **(1)**

First law of thermodynamics is based on energy conservation

$dQ = dU + dW$

Here dW → work done on the system so volume decreases.

So dW → –ve

$dQ = dU - dW$

11. If a radioactive element having half-life of 30 min is undergoing beta decay, the fraction of radioactive element remains undecayed after 90 min. will be

(1) $\frac{1}{8}$ (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) $\frac{1}{16}$

Sol. **(1)**

T = 30 min.

t = 90 min

$$n = \frac{t}{T} = \frac{90\,\text{min}}{30\,\text{min}} = 3$$

$$N\,(\text{active}) = \frac{N_0}{2^n} = \frac{N_0}{2^3} = \frac{N_0}{8}$$

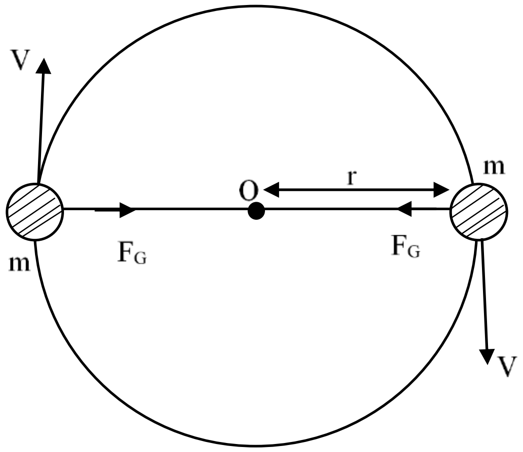
$$\boxed{\frac{N}{N_0} = \frac{1}{8}}$$

12. Two particles of equal mass 'm' move in a circle of radius 'r' under the action of their mutual gravitational attraction. The speed of each particle will be :

(1) $\sqrt{\frac{4Gm}{r}}$ (2) $\sqrt{\frac{Gm}{4r}}$

(3) $\sqrt{\frac{Gm}{r}}$ (4) $\sqrt{\frac{Gm}{2r}}$

Sol. **(2)**



$$\frac{mv^2}{r} = \frac{Gm.m}{(2r)^2}$$

$$\frac{v^2}{r} = \frac{Gm}{4r^2}$$

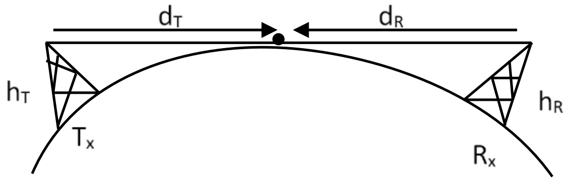
$$\boxed{V = \sqrt{\frac{Gm}{4r}}}$$

13. If the height of transmitting and receiving antennas are 80 m each, the maximum line of sight distance will be: Given: Earth's radius = 6.4×10^6 m

(1) 28 km (2) 36 km (3) 32 km (4) 64 km

Sol. **(4)**

$h_T = h_R = h = 80$ m



$d_T = \sqrt{2R.h}$ and $d_R = \sqrt{2R.h}$

Maximum line of sight = $d_T + d_R$

$= \sqrt{2Rh} + \sqrt{2Rh}$

$= 2\sqrt{2Rh} = 2\sqrt{2 \times 6.4 \times 10^6 \times 80}$

$= 2\sqrt{64 \times 16 \times 10^6}$

$= 2 \times 8 \times 4 \times 10^3$

$= 64 \times 10^3 = 64$ km

14. A car is moving on a horizontal curved road with radius 50 m. The approximate maximum speed of car will be, if friction between tyres and road is 0.34. [take $g = 10\ ms^{-2}$]

(1) 17 ms–1 (2) 13 ms^{-1} (3) 22.4 ms^{-1} (4) 3.4 ms^{-1}

Sol. **(2)**

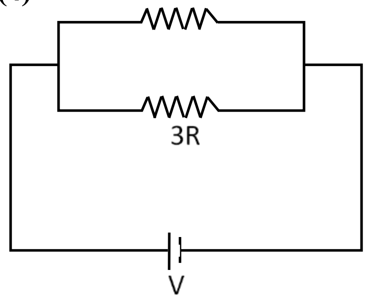
$\mu = 0.34$, R = 50 m

$V = \sqrt{\mu Rg} = \sqrt{0.34 \times 50 \times 10} = \sqrt{34 \times 5} = \sqrt{170} \simeq 13$

15. Ratio of thermal energy released in two resistors R and $3R$ connected in parallel in an electric circuit is :

(1) 1 : 27 (2) 1 : 1 (3) 1 : 3 (4) 3 : 1

Sol. **(4)**



$$H = I^2Rt = \frac{V^2}{R}.t$$

V = const.

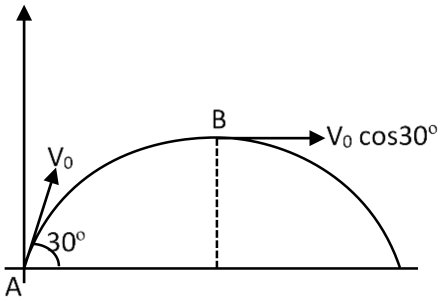
So, $H \alpha \frac{1}{R}$

$$\frac{H_1}{H_2} = \frac{3R}{R} = \frac{3}{1}$$

16. A stone is projected at angle 30° to the horizontal. The ratio of kinetic energy of the stone at point of projection to its kinetic energy at the highest point of flight will be –

(1) 1 : 2 (2) 1 : 4 (3) 4 : 1 (4) 4 : 3

Sol. **(4)**



$$K_A = \frac{1}{2}mV_A^2$$

$$K_A = \frac{1}{2}mV_0^2 \quad \ldots(1)$$

$$K_B = \frac{1}{2}m(V_0 \cos 30^\circ)^2$$

$$K_B = \frac{m}{2}.V_0^2 \frac{3}{4} = \frac{3}{8}mV_0^2 \quad \ldots(2)$$

$$\frac{K_A}{K_B} = \frac{\left(\frac{mV_0^2}{2}\right)}{\left(\frac{3mV_0^2}{8}\right)}$$

$$\frac{K_A}{K_B} = \frac{4}{3}$$

17. Which of the following are true?

A. Speed of light in vacuum is dependent on the direction of propagation.
B. Speed of light in a medium is independent of the wavelength of light.
C. The speed of light is independent of the motion of the source.
D. The speed of light in a medium is independent of intensity.

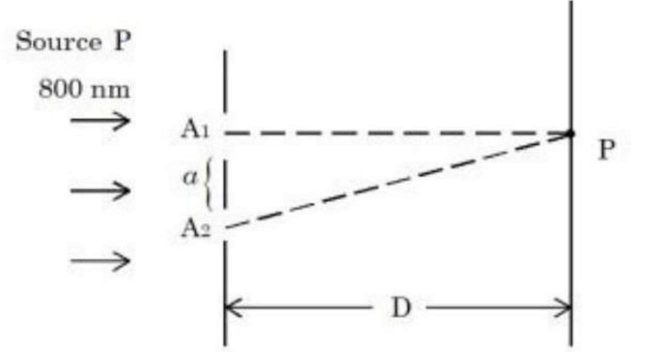
Choose the correct answer from the options given below:

(1) C and D only (2) B and C only (3) A and C only (4) B and D only

Sol. **(1)**

velocity of light depends on Refractive index of medium and independent of intensity and source.

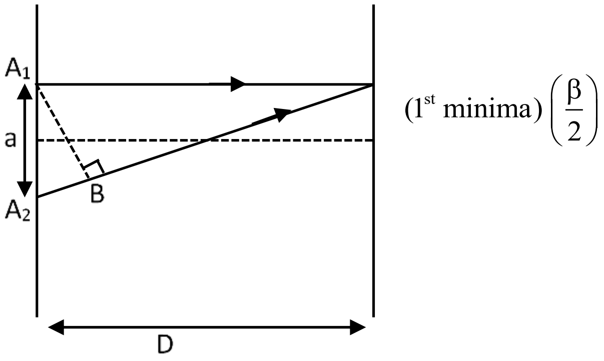
18. In a Young's double slit experiment, two slits are illuminated with a light of wavelength 800 nm. The line joining A_1P is perpendicular to A_1A_2 as shown in the figure. If the first minimum is detected at P, the value of slits separation 'a' will be:



The distance of screen from slits D = 5 cm

(1) 0.5 mm (2) 0.1 mm (3) 0.4 mm (4) 0.2 mm

Sol. **(4)**



$$\frac{\beta}{2}=\frac{a}{2}$$

$$\boxed{\beta=a}$$

$$\frac{\lambda D}{a}=a$$

$$\lambda D=a^2$$

$$a^2=800\times10^{-9}\times5\times10^{-2}$$

$$a^2=4000\times10^{-11}$$

$$a=2\times10^{-4}$$

$$\boxed{a=0.2\text{mm}}$$

19. Which one of the following statement is not correct in the case of light emitting diodes?

A. It is a heavily doped p-n junction.

B. It emits light only when it is forward biased.

C. It emits light only when it is reverse biased.

D. The energy of the light emitted is equal to or slightly less than the energy gap of the semiconductor used.

Choose the correct answer from the options given below:

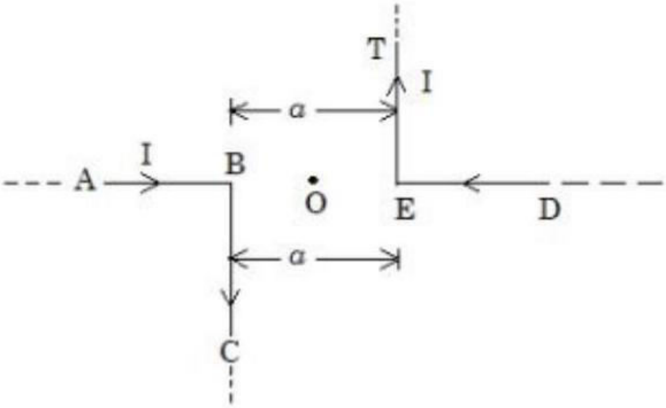
(1) A (2) C and D (3) C (4) B

Sol. **(3)**

Light emitting diode only used in forward bias

Option : 3

20. The magnitude of magnetic induction at mid point O due to current arrangement as shown in Fig will be



(1) $\frac{\mu_0 I}{\pi a}$ (2) $\frac{\mu_0 I}{2\pi a}$ (3) 0 (4) $\frac{\mu_0 I}{4\pi a}$

Sol. **(1)**

Magnetic field due to "AB" and "ED" will be zero

magnetic field due to "BC" and "ET" will be equal in amount and direction.

$$\text{'B' due BC} = \frac{\mu_0 I}{4\pi r} = \frac{\mu_0 I}{4\pi\frac{a}{2}} = \frac{\mu_0 I}{2\pi a} \odot \qquad \text{.....(1)}$$

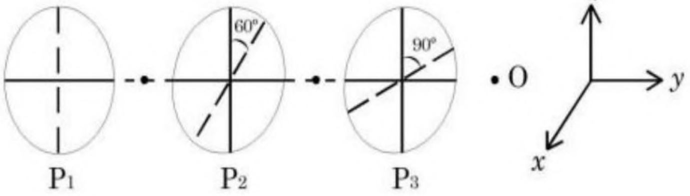
$$\text{'B' due to TE} = \frac{\mu_0 I}{2\pi a} \odot$$

$$B_{net} \text{ at point 'O'} = \left(\frac{\mu_0 I}{2\pi a} + \frac{\mu_0 I}{2\pi a}\right) = \frac{\mu_0 I}{\pi a} \odot \text{ outward}$$

SECTION – B

21. As shown in the figure, three identical polaroids P_1, P_2 and P_3 are placed one after another. The pass axis of P_2 and P_3 are inclined at angle of 60° and 90° with respect to axis of P_1.

The source S has an intensity of $256\frac{W}{m^2}$. The intensity of light at point O is $-\frac{W}{m^2}$.



Sol. **(24)**

Intensity of source $I_0 = 256\frac{W}{m^2}$

intensity after passing P₁ is $I_1 = \frac{I_0}{2} = 128\frac{W}{m^2}$

intensity after passing P₂ is $I_2 = I_1 \cos^2\theta$

$= (128).\cos^2 60^o$

$128\times\frac{1}{4} = 32\frac{W}{m^2}$

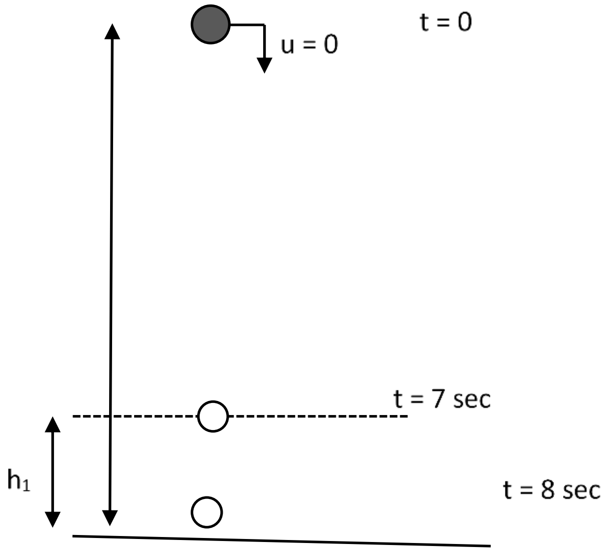
intensity after passing P₃ is $I_3 = I_2 \cos^2\theta$

angle b/w p_2 and $p_3 = 30^o$

So, $I_3 = 32\cos^2 30^o = 32\times\frac{3}{4} = 24\frac{W}{m^2}$

22. A 0.4 kg mass takes 8 s to reach ground when dropped from a certain height 'P' above surface of earth. The loss of potential energy in the last second of fall is J.
(Take $g = 10\ m/s^2$)

Sol. 300 J



$S = ut + \frac{1}{2}at^2$

$h = 0 + \frac{1}{2}.g(8)^2 = \frac{10}{2}\times 8\times 8 = 320m$

Distance covered in last second

$h_1 = u + \frac{a}{2}(2n-1)$

$= 0 + \frac{10}{2}[2(8)-1]$

$h_i = 5[15] = 75m$

$\Delta U_{loss} = mg\Delta h$

$\Delta U_{loss} = 0.4\times 10\times 75 = 300J$

Ans $\rightarrow$ 300 J

23. Two simple harmonic waves having equal amplitudes of 8 cm and equal frequency of 10 Hz are moving along the same direction. The resultant amplitude is also 8 cm. The phase difference between the individual waves is _______degree.

Sol. **120**

$A_1 = A \quad A_2 = A \quad A_{eq} = A$

$A_1^2 + A_2^2 + 2A_1A_2\cos\phi = A_{eq}^2$

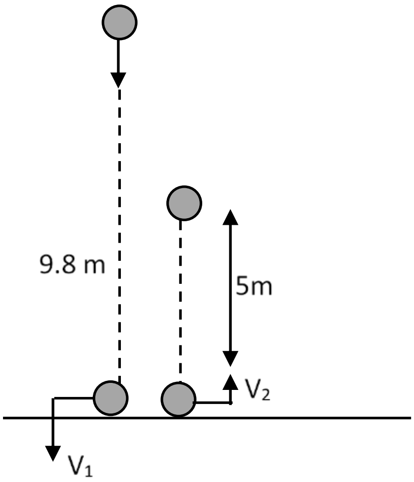
$A^2 + A^2 + 2A^2\cos\phi = A^2$

$1 + 2\cos\phi = 0 \Rightarrow \cos\phi = -\frac{1}{2}$

$\phi = 120$

24. A tennis ball is dropped on to the floor from a height of 9.8 m. It rebounds to a height 5.0 m. Ball comes in contact with the floor for 0.2 s. The average acceleration during contact is ms^{-2}
(Given $g = 10\ ms^{-2}$)

Sol. **(120m / sec^2)**



$v_1 = \sqrt{2gh} = \sqrt{2\times10\times9.8} = \sqrt{196}$

$v_1 = 14m/sec$

$v_2 = \sqrt{2gh}$

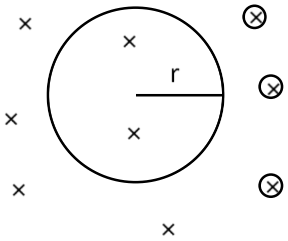
$v_2 = \sqrt{2\times10\times5} = 10m/sec.$

$a_{aug} = \frac{\Delta v}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{10-(-14)}{0.2}$

$a_{ay} = \frac{24}{0.2} = 120m/sec^2$

25. A certain elastic conducting material is stretched into a circular loop. It is placed with its plane perpendicular to a uniform magnetic field B = 0.8 T. When released the radius of the loop starts shrinking at a constant rate of $2cms^{-1}$. The induced emf in the loop at an instant when the radius of the loop is 10 cm will be ____ mV.
(Given $g = 10\ ms^{-2}$)

Sol. **(10)**



$$B = 0.8T$$

$$\frac{dr}{dt} = 2\text{cms}^{-1}$$

$$emf = \frac{d\phi}{dt} = \frac{d(BA)}{dt}$$

$$emf = B\frac{d}{dt}\pi r^2 = \pi B(2r)\frac{dr}{dt}$$

$$emf = 2\pi Br \cdot (0.02)$$

$$= 2\pi(0.8)(0.1) \times 0.02$$

$$= 32\pi \times 10^{-4}$$

$$= 100.48 \times 10^{-4}$$

$$= 10.048 \times 10^{-3}$$

$$= 10.04\text{mV} \approx 10 \text{ mV}$$

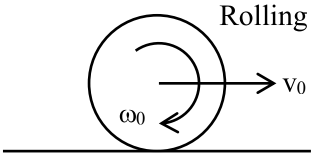
26. A solid sphere of mass 2 kg is making pure rolling on a horizontal surface with kinetic energy 2240 J. The velocity of centre of mass of the sphere will be _____ ms^{-1}

Sol. **(40)**

Mass = 2 kg

K.E = 2240 J

$$\text{K.E} = \frac{1}{2}\text{mv}_0^2 + \frac{1}{2}\text{I}\omega_0^2$$



$$= \frac{1}{2}\text{mv}_0^2 + \frac{1}{2}\cdot\frac{2}{5}\text{mR}^2 \cdot \frac{\text{v}_0^2}{\text{R}^2}$$

$$= \frac{1}{2}\text{mv}_0^2 + \frac{\text{mv}_0^2}{5}$$

$$\text{K.E} = \frac{7}{10}\text{mv}_0^2$$

$$2240 = \frac{7}{10} \times 2 \times \text{v}_0^2$$

$$\text{v}_0^2 = \frac{22400}{14} = 1600$$

$$\boxed{\text{v}_0 = 40 \text{ m / sec}}$$

27. A body cools from 60°C to 40°C in 6 minutes. If, temperature of surroundings is 10°C. Then, after the next 6 minutes, its temperature will be °C.

Sol. **(28)**

$$60°\text{C} \xrightarrow[\text{6 min}]{} 40°\text{C} \xrightarrow[\text{6 min}]{} \text{T} \qquad \text{T}_0 = 10°\text{C}$$

$$\frac{\Delta\text{T}}{\Delta\text{t}} = \text{k}(\text{T} - \text{T}_0)$$

$$\frac{(60-40)}{6\,\text{min}} = \text{k}[50-10] \qquad \ldots(1)$$

$$\text{And } \frac{(40-\text{T})}{6\,\text{min}} = \text{K}\left[\frac{40+\text{T}}{2} - 10\right] \qquad \ldots(2)$$

(1) / (2)

$$\frac{20}{40-\text{T}} = \frac{40}{\left(\frac{40+\text{T}-20}{2}\right)}$$

$$\frac{20}{40-T}=\frac{40\times 2}{20+T}$$

(20 + T) = (40 – T)4

20 + T = 160 – 4T ⇒ ST = 140

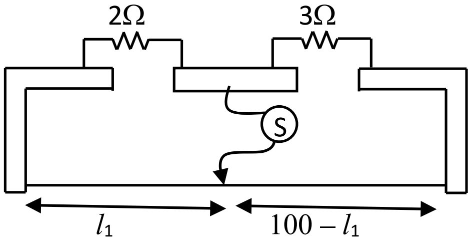
$$T=\frac{140}{5}=28°C$$

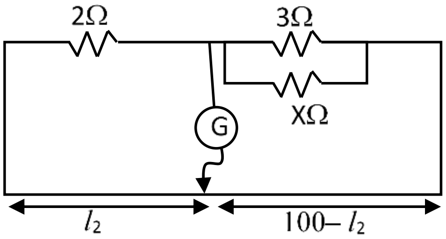
28. In a metre bridge experiment the balance point is obtained if the gaps are closed by 2Ω and 3Ω. A shunt of XΩ is added to 3Ω resistor to shift the balancing point by 22.5 cm. The value of X is -

Sol. **x = 2**

$$\frac{2}{\ell_1}=\frac{3}{100-\ell_1}$$

$200-2\ell_1=3\ell_1$





$200=5\ell_1$

$\boxed{\ell_1=40\,\text{cm}}$

Now $\ell_2=\ell_1+22.5$

$\ell_2=40+22.5=62.5$ cm

$$\text{So, }\frac{2}{62.5}=\frac{\left(\frac{3\cdot x}{3+x}\right)}{37.5}\Rightarrow(37.5)\times 2=\frac{(62.5)(3x)}{3+x}$$

$$3+x=\frac{(62.5)}{25}x$$

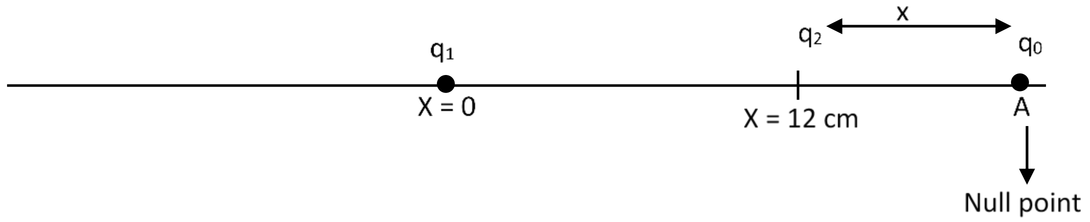
$3+x=2.5x$

$3=1.5x\Rightarrow\boxed{x=2}$

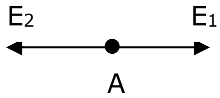
29. A point charge $q_1=4q_0$ is placed at origin. Another point charge $q_2=-q_0$ is placed at = 12 cm. Charge of proton is q_0. The proton is placed on x axis so that the electrostatic force on the proton is zero. In this situation, the position of the proton from the origin is ________ cm.

Sol. **24**

$q_1 = 4q_0 \text{and} q_2 = -q_0$



Electric field at point A will be zero.



$\left|\vec{E_1}\right| = \left|\vec{E_2}\right|$

$\frac{kq_1.q_0}{(12+x)^2} = \frac{kq_2.q_0}{x^2}$

$\frac{4q_0}{(12+x)^2} = \frac{q_0}{x^2}$

$4x^2 = (12 + x)^2$

$\pm 2x = (12 + x)$

$2x = 12 + x$	$-2x = 12 + x$
$x = 12$	$-3x = 12$
$x = 12$ cm	$x = x = -\frac{12}{3} = -4$

Position of proton from origin will be $\rightarrow 12 + 12$

$\rightarrow 24\text{cm}$

30. A radioactive element ${}^{242}_{92}X$ emits two α –articles, one electron and two positrons. The product nucleus is represented by ${}^{234}_{P}Y$. The value of P is

Sol. **(87)**

$${}_{92}X^{242} \longrightarrow {}_{P}Y^{234} + 2\,{}_{2}\alpha^4 + {}_{-1}e^0 + 2\,{}_{+1}e^0$$

Using charge conservation:

$92 = P + 2(2) + (-1) + 2(1)$

$92 = P + 5$

$\boxed{P = 87}$ Ans.

Chemistry

SECTION - A

31. "A" obtained by Ostwald's method involving air oxidation of NH_3, upon further air oxidation produces "B". "B" on hydration forms an oxoacid of Nitrogen along with evolution of "A". The oxoacid also produces "A" and gives positive brown ring test.

Identify *A* and *B*, respectively.

(1) N_2O_3, NO_2 (2) $NO_2,\ N_2O_4$ (3) $NO_2,\ N_2O_5$ (4) NO, NO_2

Sol. 4

$$4\ NH_3+5O_2 \xrightarrow{\Delta} \underset{(A)}{4NO}+6H_2O$$

$$\underset{(B)}{2NO+O_2 \rightarrow 2NO_2}$$

32. Correct statement about smog is:

(1) Classical smog also has high concentration of oxidizing agents

(2) Both NO_2 and SO_2 are present in classical smog

(3) NO_2 is present in classical smog

(4) Photochemical smog has high concentration of oxidizing agents

Sol. 4

Photochemical smog is oxidizing smog. Its high concentration of oxidizing agent like ozone and HNO_3

33. The standard electrode potential (M^{3+}/M^{2+}) for V, Cr, Mn& Co are -0.26 V, -0.41 V, $+1.57$ V and $+1.97$ V, respectively. The metal ions which can liberate H_2 from a dilute acid are

(1) Mn^{2+} and Co^{2+} (2) Cr^{2+} and Co^{2+} (3) V^{2+} and Cr^{2+} (4) V^{2+} and Mn^{2+}

Sol. 3

V^{+2} and Cr^{+2}

The metal ion for which have less value of reduction potential can release H_2 on reaction with dilute acid.

34. The shortest wavelength of hydrogen atom in Lyman series is λ. The longest wavelength in Balmer series of $He^+$is

(1) $\frac{36\lambda}{5}$ (2) $\frac{9\lambda}{5}$ (3) $\frac{5}{9\lambda}$ (4) $\frac{5\lambda}{9}$

Sol. 2

For lymen seriese → $\frac{1}{\lambda_{min}} = R \times 1\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)$

For balmer seriese → $\frac{1}{\lambda_{max}} = R \times 4\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$

$$\frac{\frac{1}{\lambda_{min}}}{\frac{1}{\lambda_{max}}} = \frac{\lambda_{max}}{\lambda_{min}} = \frac{\lambda_{max}}{\lambda} = \frac{9R}{5R}$$

$$\lambda_{max} = \frac{9\lambda}{5}$$

35. The bond dissociation energy is highest for

(1) F_2 (2) Br_2 (3) I_2 (4) Cl_2

Sol. 4

Order of B.D.E in halogen is

(E) Cl–Cl > Br-Br > F-F > I-I

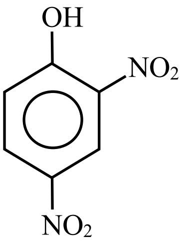
36. The increasing order of pK_a for the following phenols is

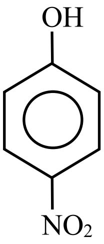
(A) 2, 4-Dinitrophenol
(B) 4-Nitrophenol
(C) 2, 4,5 – Trimethylphenol
(D) Phenol
(E) 3-Chlorophenol

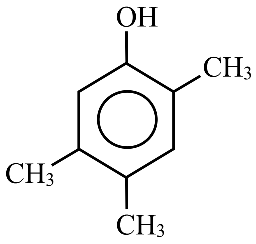
Choose the correct answer from the option given below:

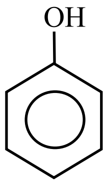
(1) (A), (B), (E), (D), (C)
(2) (C), (D), (E), (B), (A)
(3) (A), (E), (B), (D), (C)
(4) (C), (E), (D), (B), (A)

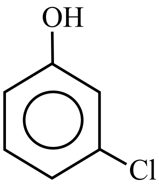
Sol. 1


(A)


(B)

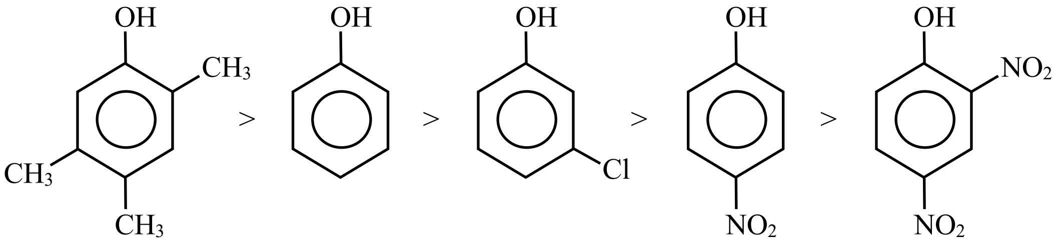

(C)


(D)

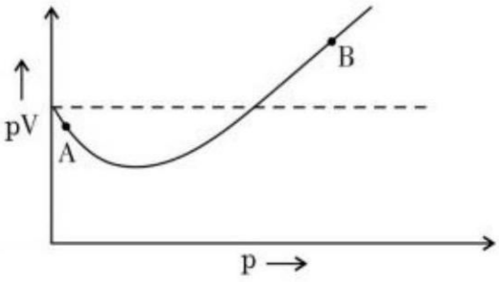

(E)

acedic strength $\propto K_a$

$$\propto \frac{1}{PK_a}$$


(C) (D) (E) (B) (A)

37. For 1 mol of gas, the plot of pV vs. p is shown below. p is the pressure and V is the volume of the gas



What is the value of compressibility factor at point ?

(1) $1 + \frac{a}{RTV}$
(2) $1 - \frac{a}{RTV}$
(3) $1 + \frac{b}{V}$
(4) $1 - \frac{b}{V}$

Sol. 2

At point A → low pressure, volume of gas very high

$\rightarrow V\text{–}b \approx V$

$$\left(p+\frac{a}{V^2}\right)\left(v-\underset{\text{neglect}}{b}\right)=RT$$

$$\left(p+\frac{a}{V^2}\right)v=RT$$

$$PV+\frac{a}{v}=RT$$

$$z+\frac{a}{RTV}=1$$

$$z=1-\frac{a}{RTV}$$

38. Match List I with List II.

List I	List II
Antimicrobials	Names
(A) Narrow Spectrum Antibiotic	(I) Furacin
(B) Antiseptic	(II) Sulphur dioxide
(C) Disinfectants	(III) Penicillin G
(D) Broad spectrum antibiotic	(IV) Chloramphenicol

Choose the correct answer from the options given below:

(1) (A) – II, (B) – I, (C) – IV, (D) – III
(2) (A) –I, (B) – II, (C) – IV, (D) – III
(3) (A) – II, (B) – I, (C) – IV, (D) – II
(4) (A) –III, (B) – I, (C) – II, (D) – IV

Sol. 4

Narrow Spectrum Antibiotic → Penicillin G (used in pathgens)

Antiseptic → Furacin

Disinfectants → Sulphur dioxide

Broad spectrum antibiotic → Chloramphenicol

39. During the borax bead test with $CuSO_4$, a blue green colour of the bead was observed in oxidising flame due to the formation of

(1) CuO
(2) $Cu(BO_2)_2$
(3) $Cu_3\ B_2$
(4) Cu

Sol. 2

Blue green colour is due to formation of $Cu(BO_2)_2$

$CuSO_4 \xrightarrow{\Delta} CuO+SO_3$

$CuO+B2O_3 \rightarrow Cu(BO_2)_2$

40. Which of the following salt solution would coagulate the colloid solution formed when FeCl3 is added to NaOH solution, at the fastest rate?

(1) 10 mL of 0.1 mol dm^{-3} Na_2SO_4 (2) 10 mL of 0.2 mol dm^{-3} $AlCl_3$

(3) 10 mL of 0.1 mol dm^{-3} $Ca_3(PO_4)_2$ (4) 10 mL of 0.15 mol dm^{-3} $CaCl_2$

Sol. 2

$FeCl_3 + NaOH \rightarrow Fe(OH)_3/OH^-$

Negative colloidal particle

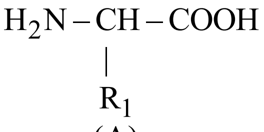
Positive ion required for coagulation of sol.

41. Number of cyclic tripeptides formed with 2 amino acids *A* and *B* is:

(1) 5 (2) 2 (3) 4 (4) 3

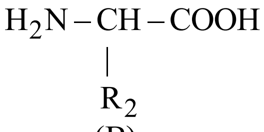
Sol. 3

To amine acid



$$\underset{R_1}{H_2N-\underset{|}{C}H-COOH}$$

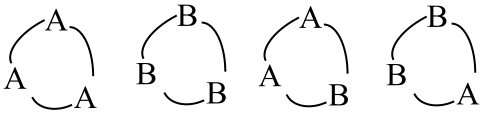
(A)



$$\underset{R_2}{H_2N-\underset{|}{C}H-COOH}$$

(B)

Tripeptide are formed →



42. The correct order of hydration enthalpies is

(A) K^+ (B) Rb^+ (C) Mg^{2+} (D) Cs^+

(E) Ca^{2+}

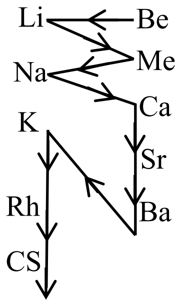
Choose the correct answer from the options given below:

(1) E > C > A > B > D (2)C > A > E > B > D

(2) C > E > A > D > B (4) C > E > A > B > D

Sol. 4

Order of hydration enthalpy is size order



$Mg^{2+} > Ca^{2+} > K^+ > Rb^+ > CS^+$

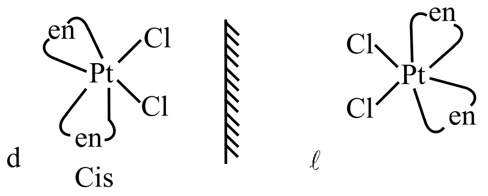
43. Chiral complex from the following is:

Here en = ethylene diamine

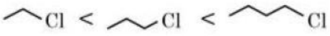
(1) cis $^-[PtCl_2(en)_2]^{2+}$ (2) trans – $[PtCl_2(en)_2]^{2+}$

(3) cis – $[PtCl_2(NH_3)_2]$ (4) trans – $[Co(NH_3)_4Cl_2]^+$

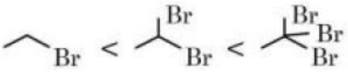
Sol. 1

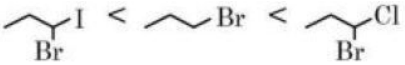


44. Identify the correct order for the given property for following compounds.

(A) Boiling Point: 

(B) Density: 

(C) Boiling Point: 

(D) Density: 

(E) Boiling Point: 

Choose the correct answer from the option given below:

(1) (B), (C) and (D) only	(2) (A), (C) and (D) only
(3) (A), (B) and (E) only	(4) (A), (C) and (E) only

Sol. 4

(i) B.P. $\propto$ Molecular mass

(ii) B.P. $\propto$ polarity↑

(iii) B.P. $\propto \frac{1}{\text{No.of Branches}}$

45. The magnetic behavior of Li_2O, Na_2O_2 and KO_2, respectively, are

(1) Paramagnetic, paramagnetic and diamagnetic

(2) diamagnetic, paramagnetic and diamagnetic

(3) paramagnetic, diamagnetic and paramagnetic

(4) diamagnetic, diamagnetic and paramagnetic

Sol. 4

Li_2O	O^{--}	Diamagnetic
Na_2O_2	O_2^{--}	Diamagnetic
KO_2	O_2^{-}	paramagnetic

46. The reaction representing the Mond process for metal refining is____________

(1) $ZnO + C \xrightarrow{\Delta} Zn + CO$

(2) $Zr + 2I_2 \xrightarrow{\Delta} ZrI_4$

(3) $2\,K[Au(CN)_2] + Zn \xrightarrow{\Delta} K_2[Zn(CN)_4] + 2Au$

(4) $Ni + 4CO \xrightarrow{\Delta} Ni(CO)_4$

Sol. 4

$$\begin{tikzcd} \underset{\text{Impure}}{\text{Ni+4CO}} \arrow[r, "50^{o}C"] & \underset{\text{vap.}}{\text{Ni(CO)}_4} \arrow[d, "250^{o}C"] \\ & \underset{\text{pure}}{\text{Ni + 4CO}} \end{tikzcd}$$

47. Which of the given compounds can enhance the efficiency of hydrogen storage tank?

(1) Di-isobutylaluminium hydride (2) $NaNi_5$

(3) Li/P_4 (4) SiH_4

Sol. 2

Ni can adsorb 800 times more hydrogen then its own volume

48. Match List I with List II.

List I		List II
Reaction		Reagents
(A)	Hoffmann Degradation	(I) Conc.KOH, Δ
(B)	Clemenson reduction	(II) $CHCl_3, NaOH/H3O^{\oplus}$
(C)	Cannizaro reaction	(III) $Br_2, NaOH$
(D)	Reimer-Tiemann Reaction	(IV) $Zn - Hg/HCl$

Choose the correct answer from the options given below:

(1) (A) –III, (B) –IV, (C) – I, (D) – II (2) (A) - II, (B) –I, (C) - III, (D) – IV

(3) (A) –III, (B) –IV, (C) – II, (D) – I (4) (A) –II, (B) – IV, (C) – I, (D) – III

Sol. 1

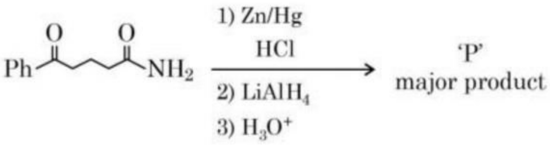
Hoffmann degradation → Br_2, NaOH

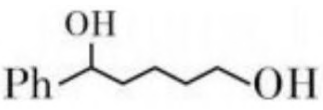
Clemenson reduction → Zn-Hg/HCl

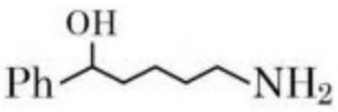
Cannizaro reaction → Conc. KOH, Δ

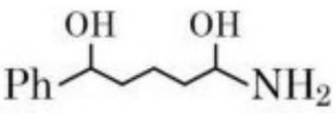
Reimer-Tiemann reaction → $CuCl_3, NaOH/H_3O^{\oplus}$

49. The major product 'P' for the following sequence of reactions is:



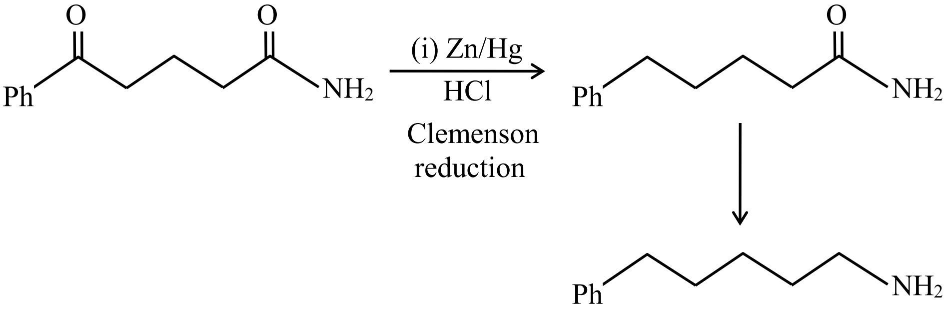
(1) 

(2) 

(3) 

(4) 

Sol. 4



50. Compound that will give positive Lassaigne's test for both nitrogen and halogen is:

(1) $NH_2OH.HCl$ (2) $CH_3NH_2.HCl$ (3) NH_4Cl (4) $N_2H_4.HCl$

Sol. 2

Lassaigne test for both N and X is given by the compound which have C, N as well X atom in compound.

51. Millimoles of calcium hydroxide required to produce 100 mL of the aqueous solution of pH 12 is $x \times 10^{-1}$. The value of x is________(Nearest integer).

Assume complete dissociation.

Sol. 5

pH=12, pOH=2 $[OH^-]=10^{-2}$ N

Molarity of $Ca(OH)_2$= $\frac{N}{2}=\frac{10^{-2}}{2}=0.005$ N

$$0.005=\frac{\text{mili moles}}{100}$$

$$=\frac{5}{1000}=\frac{\text{mili moles}}{100}$$

$= 5\times10^{-1}$ milimoles

52. Water decomposes at 2300 K

$$H_2O(g) \rightarrow H_2(g)+\frac{1}{2}O_2(g)$$

The percent of water decomposing at 2300 K and 1 bar is________(Nearest integer).

Equilibrium constant for the reaction is 2×10^{-3} at 2300 K.

Sol. 2

$H_2O(g) \rightarrow H_2(g)+1/2O_2$

$1-\propto \quad \propto \quad \propto/2$

$$k_p=\frac{\propto(\propto/2)^{1/2}}{1-\propto}=2\times10^{-3}$$

$$2\times10^{-3}=\frac{\propto^{3/2}}{\sqrt{2}(1-\propto)} \qquad \propto\ll1$$

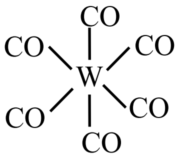
$2^{3/2}\times(10^{-2})^{3/2}=\propto^{3/2}$

$\propto=2\times10^{-2}$

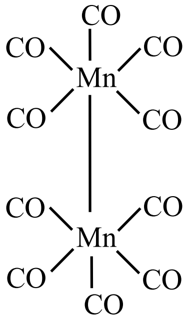
53. The sum of bridging carbonyls in $W(CO)_6$ and $Mn_2(CO)_{10}$ is________

Sol. 0

$W(CO)_6 \rightarrow 0$ Bridge CO



$Mn_2(CO)_{10} \rightarrow 0$



54. Solid Lead nitrate is dissolved in 1 litre of water. The solution was found to boil at 100.15^0C. When 0.2 mol of NaCl is added to the resulting solution, it was observed that the solution froze at -0.8^0C. The solubility product of $PbCl_2$ formed is______$\times 10^{-6}$ at 298 K. (Nearest integer)

(Given : K_b=0.5 K kgmol⁻¹ and $K_f = 1.8$ K kg mol^{-1}. Assume molality to be equal to molarity in all cases.)

Sol. 13

Let a mole Pb $(NO_3)_2$ be added

$$Pb(NO_3)_2 \rightarrow Pb^{2+} + 2NO_3^-$$

$$\begin{array}{lll} a & a & 2a \end{array}$$

$$\Delta T_b = 0.15 = 0.5[3a] \Rightarrow a = 0.1$$

$$Pb^{2+}_{(aq)} + 2Cl^-_{(aq)} \rightarrow PbCl_2(s)$$

$$\begin{array}{lll} t = 0 & 0.1 & 0.2 \\ t = \infty & (0.1 - x) & (0.2 - 2x) \end{array}$$

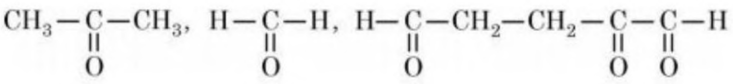
In final solution

$$\Delta T_f = 0.8 = 1.8\left[\frac{0.3 + 3x + 0.2 + 0.2}{1}\right]$$

$$\Rightarrow x = \frac{2.3}{27}$$

$$\Rightarrow K_{sp} = \left(0.1 - \frac{2.3}{27}\right)\left(0.2 - \frac{4.6}{27}\right)^2 = 13 \times 10^{-6}$$

55. 17mg of a hydrocarbon (M.F. $C_{10}H_{16}$) takes up 8.40 mL of the H_2 gas measured at 0°C and 760 mm of Hg. Ozonolysis of the same hydrocarbon yields



$$CH_3-\underset{\underset{O}{\|}}{C}-CH_3,\ H-\underset{\underset{O}{\|}}{C}-H,\ H-\underset{\underset{O}{\|}}{C}-CH_2-CH_2-\underset{\underset{O}{\|}}{C}-\underset{\underset{O}{\|}}{C}-H$$

The number of double bond/s present in the hydrocarbon is________

Sol. 3

$$\text{Moles of hydrocarbon} = \frac{17\times10^{-3}}{136}=1.25\times10^{-4}$$

$$nH_2 = 1\times\frac{8.4}{1000}=n\times0.0821\times273$$

$$\Rightarrow\ n=3.75\times10^{-4}$$

Hydrogen molecule used for 1 molecule of hydrogen is 3

$$=\frac{3.75\times10^{-4}}{1.25\times10^{-4}}=3$$

56. Consider the following reaction approaching equilibrium at 27°C and 1 atm pressure

$$A + B \underset{k_r=10^2}{\overset{k_f=10^{3}}{\rightleftharpoons}} C + D$$

The standard Gibb's energy change $(\Delta_r G^\theta)$ at 27°C is (–)_______KJ mol^{-1}

(Nearest integer).

(Given: $R = 8.3\ J\ K^{-1}\ mol^{-1}$ and $\ln 10 = 2.3$)

Sol. 6

$$K_{eq}=\frac{K_f}{K_b}=\frac{10^3}{10^2}=10$$

$$\Delta G^o = -RT \ln K_{eq}$$

$$= -8.3\times300 \ln 10$$

$$= -8.3\times300 \times2.3$$

$$= -5.72\times10^{+3}\ J$$

$$= 5.72\ KJ$$

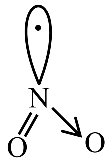
57. The number of molecules or ions from the following, which do not have odd number of electrons are_______

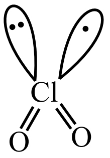
(A) NO_2 (B) ICl_4^- (C) BrF_3 (D) ClO_2

(E) NO_2^+ (F) NO

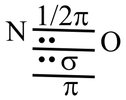
Sol. 3

odd e$^-$

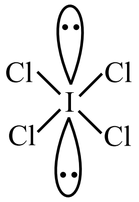


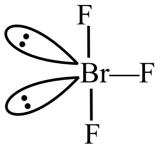


ICl_4^-, BrF_3 and NO_2^+ do not have odd number of electron.



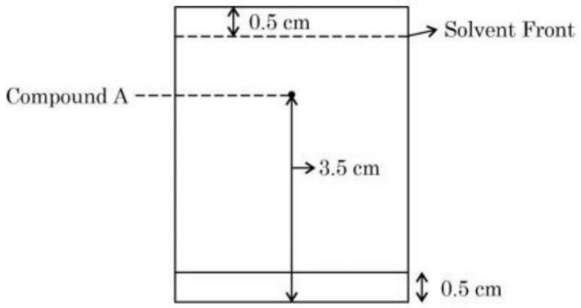
Odd e$^-$ absent





$O=\overset{+}{N}=O$

58. Following chromatogram was developed by adsorption of compound 'A' on a 6 cm TLC glass plate. Retardation factor of the compound 'A' is_______$\times 10^{-1}$

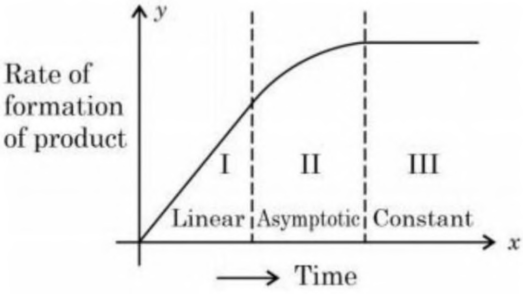


Sol. 6

$$R_f = \frac{\text{Distance moved by the substance from base line}}{\text{Distance moved by the solvent from base line}}$$

$$= \frac{3.0 \text{ cm}}{5.0 \text{ cm}} = 0.6 \text{ or } 6 \times 10^{-1}$$

59. For certain chemical reaction $X \rightarrow Y$, the rate of formation of product is plotted against the time as shown in the figure. The number of correct statement/s from the following is______

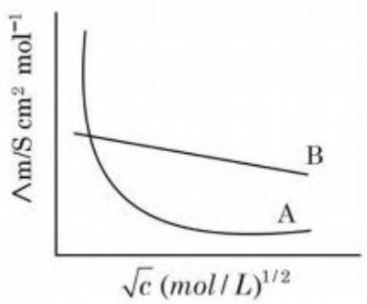


(A) Over all order of this reaction is one
(B) Order of this reaction can't be determined
(C) In region I and III, the reaction is of first and zero order respectively
(D) In region-II, the reaction is of first order
(E) In region-II, the order of reaction is in the range of 0.1 to 0.9.

Sol. 2

Only option (B) is correctr as order cannot be determined.

60. Following figure shows dependence of molar conductance of two electrolytes on concentration. $\overset{\circ}{\Lambda m}$ is the limiting molar conductivity.



The number of incorrect statement(s) from the following is_________

(A) $\overset{\circ}{\Lambda m}$ for electrolyte A is obtained by extrapolation

(B) For electrolyte B, Λm vs $\sqrt{c}$ graph is a straight line with intercept equal to $\overset{\circ}{\Lambda m}$
(C) At infinite dilution, the value of degree of dissociation approaches zero for electrolyte B.

(D) $\overset{\circ}{\Lambda m}$ for any electrolyte A or B can be calculated using $\lambda°$ for individual ions

Sol. 2

Statement (A) and Statement (C) are incorrect.

Mathematics

Section A

61. Let α and β be real numbers. Consider a 3×3 matrix A such that $A^2 = 3A + \alpha I$. If $A^4 = 21A + \beta I$, then

(1) $\beta = -8$ (2) $\beta = 8$ (3) $\alpha = 4$ (4) $\alpha = 1$

Sol. **1**

$A^2 = 3A + \alpha I$ ...... (1)

and $A^4 = 21A + \beta I$ ....(2)

Now $A^4 = A^2 . A^2$

$A^4 = (3A + \alpha I) . (3A + \alpha I)$ {from (1)}

$A^4 = 9A^2 + 6\alpha A + \alpha^2 I$ ....(3)

From (2) and (3)

$9A^2 + 6\alpha A + \alpha^2 I = 21\ A + \beta I$

putting value of A^2 from (1)

$9(3A + \alpha I) + 6\alpha A + \alpha^2 I = 21\ A + \beta I$

$(27 + 6\alpha)A + (9\alpha + \alpha^2)\ I = 21A + \beta I$

by comparison

$27 + 6\alpha = 21$ and $9\alpha + \alpha^2 = \beta$

$\Rightarrow 6\alpha = -6$ putting $\alpha = -1$

$\Rightarrow \alpha = -1$ $\therefore \beta = -8$

62. Let $x = 2$ be a root of the equation $x^2 + px + q = 0$ and

$$f(x) = \begin{cases} \dfrac{1 - \cos(x^2 - 4px + q^2 + 8q + 16)}{(x - 2p)^4} & , x \neq 2p \\ 0, & , x = 2p \end{cases}$$

$\lim_{x \to 2p^+} [f(x)]$

where $[\cdot]$ denotes greatest integer function, is

(1) 0 (2) -1 (3) 2 (4) 1

Sol. **1**

$$f(x) = \begin{cases} \dfrac{1-\cos(x^2-4px+q^2+8q+16)}{(x-2p)^4} & , x \neq 2p \\ 0 & , \ x = 2p \end{cases}$$

$\because$ x = 2 is a root of equation $x^2 + px + q = 0$

$\therefore\ 4 + 2p + q = 0$

$\Rightarrow 2p = -q - 4$

$\Rightarrow 4p^2 = (q + 4)^2 = q^2 + 8q + 16$ ....(1)

Now $\lim\limits_{x \to 2p^+} f(x) = \lim\limits_{x \to 2p^+} \dfrac{1-\cos(x^2-4px+4p^2)}{(x-2p)^4}$ (from (1))

$$= \lim_{x \to 2p^+} \left[\frac{1-\cos(x-2p)^2}{\{(x-2p)^2\}^2} \right]$$

$$= \frac{1}{2} \quad \left\{ \because \lim_{x \to 0} \frac{1-\cos\theta}{\theta^2} = \frac{1}{2} \right\}$$

$$\therefore \lim_{x \to 2p^+} [f(x)] = \left[\frac{1}{2} \right] = 0$$

63. Let B and C be the two points on the line $y + x = 0$ such that B and C are symmetric with respect to the origin. Suppose A is a point on $y - 2x = 2$ such that $\triangle ABC$ is an equilateral triangle. Then, the area of the $\triangle ABC$ is

(1) $\frac{10}{\sqrt{3}}$ (2) $3\sqrt{3}$ (3) $2\sqrt{3}$ (4) $\frac{8}{\sqrt{3}}$

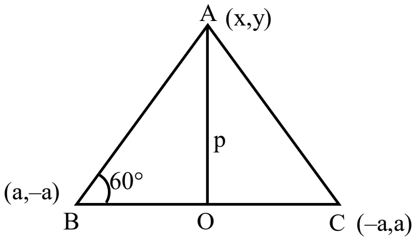
Sol. **4**

Since, A lies on perpendicular bisector of BC, whose equation is

y = x ....(1)

Now, A is the point of intersection of y = x and y – 2x = 2

$\therefore$ point A, after solving is A(–2, –2)



In ΔAOC $\tan 60° = \frac{p}{OC} \Rightarrow OC = \frac{p}{\sqrt{3}}$ $\{\because OA = p\}$

$\therefore BC = 2 \times OC = \frac{2p}{\sqrt{3}}$

Now, Area of $\Delta ABC = \frac{1}{2} \times BC \times OA$

$= \frac{1}{2} \times \frac{2p}{\sqrt{3}} \times p = \frac{p^2}{\sqrt{3}}$ sq. unit

and $p = OA = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

So, Area of $\Delta ABC = \frac{(2\sqrt{2})^2}{\sqrt{3}} = \frac{8}{\sqrt{3}}$ sq. unit

64. Consider the following system of equations

$\alpha x + 2y + z = 1$

$2\alpha x + 3y + z = 1$

$3x + \alpha y + 2z = \beta$

for some $\alpha, \beta \in \mathbb{R}$. Then which of the following is NOT correct.

(1) It has a solution if $\alpha = -1$ and $\beta \neq 2$

(2) It has a solution for all $\alpha \neq -1$ and $\beta = 2$

(3) It has no solution for $\alpha = 3$ and for all $\beta \neq 2$

(4) It has no solution for $\alpha = -1$ and for all $\beta \in \mathbb{R}$

Sol. **4**

$$\because D = \begin{vmatrix} \alpha & 2 & 1 \\ 2\alpha & 3 & 1 \\ 3 & \alpha & 2 \end{vmatrix}$$

$D = \alpha(6 - \alpha) + 2(3 - 4\alpha) + 1(2\alpha^2 - 9)$

$= 6\alpha - \alpha^2 + 6 - 8\alpha + 2\alpha^2 - 9$

$D = \alpha^2 - 2\alpha - 3$

for no solution , D = 0

$\Rightarrow \quad \alpha^2 - 2\alpha - 3 = 0$

$\quad (\alpha + 1)(\alpha - 3) = 0$

$\Rightarrow \quad \alpha = -1, \alpha = 3$

Now,

$$D_1 = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \\ \beta & \alpha & 2 \end{vmatrix}, D_2 = \begin{vmatrix} \alpha & 1 & 1 \\ 2\alpha & 1 & 1 \\ 3 & \beta & 2 \end{vmatrix} \text{ and } D_3 = \begin{vmatrix} \alpha & 2 & 1 \\ 2\alpha & 3 & 1 \\ 3 & \alpha & \beta \end{vmatrix}$$

if $\alpha = -1$ then

$$D_1 = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \\ \beta & -1 & 2 \end{vmatrix}, D_2 = \begin{vmatrix} -1 & 1 & 1 \\ -2 & 1 & 1 \\ 3 & \beta & 2 \end{vmatrix}, D_3 = \begin{vmatrix} -1 & 2 & 1 \\ -2 & 3 & 1 \\ 3 & -1 & \beta \end{vmatrix}$$

$\Rightarrow$ only for $\beta = 2$, $D_1 = 0$, $D_2 = 0$, $D_3 = 0$

$\therefore$ It has no solution if $\alpha = -1$ and $\beta \neq 2$

if $\alpha = 3$

$$D_1 = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \\ \beta & 3 & 2 \end{vmatrix}, D_2 = \begin{vmatrix} 3 & 1 & 1 \\ 6 & 1 & 1 \\ 3 & \beta & 2 \end{vmatrix}, D_3 = \begin{vmatrix} 3 & 2 & 1 \\ 6 & 3 & 1 \\ 3 & 3 & \beta \end{vmatrix}$$

$\Rightarrow$ Only for $\beta = 2$, $D_1 = D_2 = D_3 = 0$

$\Rightarrow$ It has no solution for $\beta \neq 2$

$\therefore$ It has no solution for $\alpha = 3$ and for all $\beta \neq 2$

65. Let $y = f(x)$ be the solution of the differential equation $y(x+1)dx - x^2dy = 0, y(1) = e$. Then $\lim_{x \to 0^+} f(x)$ is equal to

(1) $\frac{1}{e^2}$ (2) e^2 (3) 0 (4) $\frac{1}{e}$

Sol. **3**

$y(x + 1)dx - x^2 dy = 0, \qquad y(1) = e$

$$\Rightarrow \frac{dy}{dx} = \frac{y(x+1)}{x^2}$$

$$\Rightarrow \int \frac{dy}{y} = \int \frac{(x+1)dx}{x^2}$$

$$\ell ny = \ell nx - \frac{1}{x} + c$$

$\because y(1) = e$

$\therefore 1 = 0 - 1 + C \Rightarrow C = 2$

$$\text{Now, } \ell ny = \ell nx - \frac{1}{x} + 2$$

$\Rightarrow \ell n\left(\frac{y}{x}\right)=2-\frac{1}{x}$

$\Rightarrow \frac{y}{x}=e^{2-\frac{1}{x}}$

$\Rightarrow y=x,\ e^{2-\frac{1}{x}}$

So, $\lim\limits_{x\to 0^+} y=\lim\limits_{x\to 0^+} x e^{2-\frac{1}{x}}=0$

66. The domain of $f(x)=\frac{\log_{(x+1)}(x-2)}{e^{2\log_e x}-(2x+3)}, x\in\mathbb{R}$ is

(1) $\mathbb{R}-\{3\}$ (2) $(-1,\infty)-\{3\}$ (3) $(2,\infty)-\{3\}$ (4) $\mathbb{R}-\{-1,3\}$

Sol. **3**

$f(x)=\frac{\log_{(x+1)}(x-2)}{e^{2\ln x}-(2x+3)}$

case (i) $x-2>0\Rightarrow x>2$

$x\in(2,\infty)$

case (ii) $x+1>0$ and $x+1\neq 1$

$x>-1$, $x\neq 0$

$\therefore\ x_t(-1,0)\cup(0,\infty)$

case (iii) $x>0\ \Rightarrow\ x_t\ (0,\infty)$

case (iv) $e^{2\ell nx}-(2x+3)\neq 0$

$\Rightarrow\ x^2-2x+3\neq 0$

$(x-3)\ (x+1)\neq 0$

$\Rightarrow\ x\neq 3\ ,\ x\neq -1$

$\therefore$ from (i) n (ii) n (iii)n (iv)

$x_t\ (2,\infty)-\{3\}$

67. Fifteen football players of a club-team are given 15 T-shirts with their names written on the backside. If the players pick up the T-shirts randomly, then the probability that at least 3 players pick the correct T-shirt is

(1) $\frac{5}{24}$ (2) $\frac{1}{6}$ (3) $\frac{5}{36}$ (4) $\frac{2}{15}$

Sol. **2**

Required probability $=1-\frac{D_{(15)}+{}^{15}C_1D_{(14)}+{}^{15}C_2D_{(3)}}{15!}$

Taking $D_{(15)}$ as $\frac{15!}{e}$

$D_{(14)}$ as $\frac{14!}{e}$

$D_{(13)}$ as $\frac{13!}{e}$

We get $1-\left(\frac{\frac{15!}{e}+15\frac{14!}{e}+\frac{15\times14}{2\times1}\times\frac{13!}{e}}{15!}\right)$

$= 1-\left(\frac{1}{e}+\frac{1}{e}+\frac{1}{2e}\right)=1-\frac{5}{2e}\simeq 0.08$

68. Let $[x]$ denote the greatest integer $\le x$. Consider the function $f(x)=\max\{x^2,1+[x]\}$. Then the value of the integral $\int_0^2 f(x)dx$ is

(1) $\frac{5+4\sqrt{2}}{3}$ (2) $\frac{4+5\sqrt{2}}{3}$ (3) $\frac{1+5\sqrt{2}}{3}$ (4) $\frac{8+4\sqrt{2}}{3}$

Sol. **1**

f(x) = Max. $\{x^2, 1+[x]\}$

Now, f(x) = $\begin{cases}1+[x] & 0\le x\le\sqrt{2}\\ x^2 & \sqrt{2}<x\le 2\end{cases}$

$$\int_0^2 f(x)dx=\int_0^{\sqrt{2}}(1+[x])dx+\int_{\sqrt{2}}^2 x^2dx$$

$$=\int_0^1 1dx+\int_1^{\sqrt{2}}2dx+\int_{\sqrt{2}}^2 x^2dx$$

$$=(x)_0^1+2(x)_1^{\sqrt{2}}+\frac{1}{3}(x^3)_{\sqrt{2}}^2$$

$$=1+2(\sqrt{2}-1)+\frac{1}{3}(8-2\sqrt{2})$$

$$=\frac{4\sqrt{2}+5}{3}$$

69. For two non-zero complex numbers z_1 and z_2, if $\text{Re}(z_1z_2)=0$ and $\text{Re}(z_1+z_2)=0$, then which of the following are possible?

A. $\text{Im}(z_1)>0$ and $\text{Im}(z_2)>0$

B. $\text{Im}(z_1)<0$ and $\text{Im}(z_2)>0$

C. $\text{Im}(z_1)>0$ and $\text{Im}(z_2)<0$

D. $\text{Im}(z_1)<0$ and $\text{Im}(z_2)<0$

Choose the correct answer from the options given below:

(1) B and D (2) A and B (3) B and C (4) A and C

Sol. **3**

$\text{Re}(z_1z_2)=0$ and $\text{Re}(z_1+z_2)=0$

Let $z_1=a_1+ib_1$ and $z_2=a_2+ib_2$

$z_1z_2=(a_1a_2-b_1b_2)+i(a_1b_2+b_1a_2)$

$\because\ \text{Re}(z_1z_2)=a_1a_2-b_1b_2=0$

$\therefore\ a_1a_2=b_1b_2$ ....(1)

and $\text{Re}(z_1+z_2)=0\Rightarrow a_1+a_2=0$

$\Rightarrow a_2 = -a_1$ ....(2)

from (1) and (2)

$b_1b_2 = -a_1^2 < 0$

Product of b_1b_2 is Negative.

$\therefore$ $\text{Im}(z_1)$ and $\text{Im}(z_2)$ are also of opposite sign.

70. If the vectors $\vec{a} = \lambda\hat{i} + \mu\hat{j} + 4\hat{k}, \vec{b} = 2\hat{i} + 4\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} + 3\hat{j} + \hat{k}$ are coplanar and the projection of $\vec{a}$ on the vector $\vec{b}$ is $\sqrt{54}$ units, then the sum of all possible values of $\lambda + \mu$ is equal to

(1) 0 (2) 24 (3) 6 (4) 18

Sol. **2**

Vector $\vec{a} = \lambda\hat{i} + \mu\hat{j} + 4\hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} + 3\hat{j} + \hat{k}$

are coplanar then

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 0 \quad \Rightarrow \begin{vmatrix} \lambda & \mu & 4 \\ -2 & 4 & -2 \\ 2 & 3 & 1 \end{vmatrix} = 0$$

$\Rightarrow 10\lambda - 2\mu - 56 = 0$

$\Rightarrow 5\lambda - \mu = 28$ ....(1)

also projection of $\vec{a}$ on the $\vec{b}$ is $\sqrt{54}$ units. then

$\vec{a}.\vec{b} = \sqrt{54}$

$$\Rightarrow \frac{-2\lambda + 4\mu - 8}{\sqrt{24}} = \sqrt{54}$$

$\Rightarrow -2\lambda + 4\mu - 8 = 36$

$\Rightarrow -2\lambda + 4\mu = 44$ ....(2)

from (1) and (2)

$$\lambda = \frac{26}{3} \text{ and } \mu = \frac{46}{3}$$

$$\Rightarrow \lambda + \mu = \frac{26 + 46}{3} = \frac{72}{3} = 24$$

71. Let $f(\theta) = 3\left(\sin^4\left(\frac{3\pi}{2} - \theta\right) + \sin^4(3\pi + \theta)\right) - 2(1 - \sin^2 2\theta)$ and $S = \left\{\theta \in [0, \pi]: f'(\theta) = -\frac{\sqrt{3}}{2}\right\}$. If $4\beta = \sum_{\theta \in S} \theta$, then $f(\beta)$ is equal to

(1) $\frac{5}{4}$ (2) $\frac{3}{2}$ (3) $\frac{9}{8}$ (4) $\frac{11}{8}$

Sol. **2**

$$f(\theta) = 3\left(\sin^4\left(\frac{3\pi}{2} - \theta\right) + \sin^4(3\pi + \theta)\right) - 2(1 - \sin^2 2\theta)$$

$$= 3\left(\cos^4\theta + \sin^4\theta\right) - 2\cos^2 2\theta$$

$$= 3\left(1 - \frac{\sin^2 2\theta}{2}\right) - 2\cos^2 2\theta$$

$$=3\left(\frac{2-\sin^2 2\theta}{2}\right)-2\cos^2 2\theta$$

$$=3\left(\frac{1+\cos^2 2\theta}{2}\right)-2\cos^2 2\theta$$

$$f(\theta) = \frac{3-\cos^2 2\theta}{2}$$

$$f^1(\theta) = \frac{2}{2}\cos 2\theta \sin 2\theta \times 2$$

$$f^1(\theta) = \sin 4\theta = \frac{-\sqrt{3}}{2}$$

$\theta \in [0, \pi]$

$4\theta \in [0, 4\pi]$

$$\sin 4\theta = \frac{-\sqrt{3}}{2}$$

$$4\theta = \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3}$$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{12}, \frac{5\pi}{6}, \frac{11\pi}{12}$$

$$4\beta = \sum_{\theta \in s} \theta = \frac{\pi}{3} + \frac{5\pi}{12} + \frac{5\pi}{6} + \frac{11\pi}{12} = \frac{4\pi + 5\pi + 10\pi + 11\pi}{12} = \frac{30\pi}{12} = \frac{5\pi}{2}$$

$$\beta = \frac{5\pi}{8}$$

$$f(\beta) = f\left(\frac{5\pi}{8}\right) = \frac{3-\cos^2\left(\frac{5\pi}{4}\right)}{2} = \frac{3-\frac{1}{2}}{2} = \frac{5}{4}$$

72. If p, q and r three propositions, then which of the following combination of truth values of p, q and r makes the logical expression $\{(p \vee q) \wedge ((\sim p) \vee r)\} \rightarrow ((\sim q) \vee r)$ false?

(1) p = T, q = T, r = F
(2) p = T, q = F, r = T
(3) p = F, q = T, r = F
(4) p = T, q = F, r = F

Sol. **3**

$(p \vee q) \vee (\sim p) \vee r) \rightarrow ((\sim q) \vee r)$

$T \rightarrow F \equiv F$

$\therefore (p \vee q) \wedge ((\sim p) \vee r) \equiv T$ ....(1)

$(\sim q) \vee r \equiv F$ ....(2)

$\Rightarrow \sim q = F, r = F$

$\Rightarrow q = T$

From (1) $p \vee q \equiv T$

$\sim p \vee r \equiv T$

$\therefore r = F$

$\Rightarrow \sim p = T$

$\Rightarrow p = F$

$\therefore$ p = F, q = T, r = F

73. Let Δ be the area of the region $\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \le 21, y^2 \le 4x, x \ge 1\}$. Then $\frac{1}{2}\left(\Delta - 21\sin^{-1}\frac{2}{\sqrt{7}}\right)$ is equal to

(1) $2\sqrt{3} - \frac{2}{3}$ (2) $\sqrt{3} - \frac{4}{3}$ (3) $\sqrt{3} - \frac{2}{3}$ (4) $2\sqrt{3} - \frac{1}{3}$

Sol. **2**

Area of Required Region

$$\Delta = 2\left[\int_1^3 2\sqrt{x}\,dx + \int_3^{\sqrt{21}} \sqrt{21-x^2}\,dx\right]$$

$$= 2\left[2\frac{\left(x^{3/2}\right)_1^3}{(3/2)} + \left\{\frac{(21)}{2}\sin^{-1}\left(\frac{x}{\sqrt{21}}\right) + \frac{x}{2}\sqrt{21-x^2}\right\}_3^{\sqrt{21}}\right]$$

$$= 2\left[4\sqrt{3} - \frac{4}{3}\right] + (21\sin^{-1}1 + 0) - \left(21\sin^{-1}\left(\frac{3}{\sqrt{21}}\right) + 3\sqrt{12}\right)$$

$$\Delta = 8\sqrt{3} - \frac{8}{3} + \frac{21\pi}{2} - 6\sqrt{3} - 21\sin^{-1}\sqrt{\frac{3}{7}}$$

$$\Delta = 2\sqrt{3} + \frac{21\pi}{2} - \frac{8}{3} - 21\sin^{-1}\left(\sqrt{\frac{3}{7}}\right)$$

Now,

$$\frac{1}{2}\left(\Delta_1 - 21\sin^{-1}\frac{2}{\sqrt{7}}\right) = \frac{1}{2}\left[2\sqrt{3} + \frac{21}{2}\pi - \frac{8}{3} - 21\sin^{-1}\left(\sqrt{\frac{3}{7}}\right) - 21\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)\right]$$

$$= \frac{1}{2}\left[2\sqrt{3} + \frac{21}{2}\pi - \frac{8}{3} - 21\sin^{-1}1\right]$$

$$\left\{\text{using} \sin^{-1}x + \sin^{-1}y = \sin^{-1}\left\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\right\}\right\}$$

$$= \frac{1}{2}\left[2\sqrt{3} - \frac{8}{3}\right] = \sqrt{3} - \frac{4}{3}$$

74. A light ray emits from the origin making an angle 30° with the positive x-axis. After getting reflected by the line $x + y = 1$, if this ray intersects x-axis at Q, then the abscissa of Q is

(1) $\frac{\sqrt{3}}{2(\sqrt{3}+1)}$ (2) $\frac{2}{3+\sqrt{3}}$ (3) $\frac{2}{(\sqrt{3}-1)}$ (4) $\frac{2}{3-\sqrt{3}}$

Sol. **2**

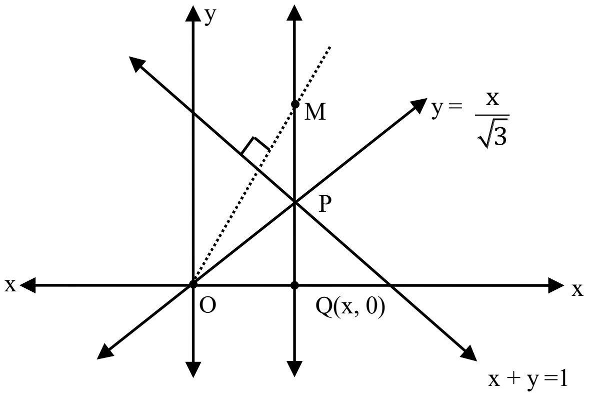
Equation of ray is

$$y = \frac{1}{\sqrt{3}}x \qquad \text{........(1)}$$

Image of 0(0, 0) in the line x + y = 1 is lies on reflected ray.

$$\frac{x-0}{1} = \frac{y-0}{1} = -2\frac{(0+0-1)}{2}$$

$\Rightarrow$ M (1, 1)



$\therefore$ Point of Intersection of lines $y = \frac{x}{\sqrt{3}}$ and x + y = 1 is p (x, y)

$\therefore p\left(\frac{3-\sqrt{3}}{2}, \frac{\sqrt{3}-1}{2}\right)$

Now Reflected Ray is same as line passing through PM.

$$\therefore \text{Slope of PM} = \frac{\frac{\sqrt{3}-1}{2}-1}{\frac{3-\sqrt{3}}{2}-1} = \frac{\sqrt{3}-3}{1-\sqrt{3}} = \sqrt{3}$$

Equation of PM whose slope is $\sqrt{3}$ and passing through M (1, 1).

$y - 1 = \sqrt{3}(x - 1)$

$y = \sqrt{3}\ x + \left(-\sqrt{3}+1\right)$

$\because$ ray, Intersects x-axis at α(x, 0)

$\therefore$ y = 0

$\Rightarrow \sqrt{3}x = -1\left(-\sqrt{3}+1\right) \Rightarrow \sqrt{3}x = \sqrt{3}-1$

$\Rightarrow x = 1 - \frac{1}{\sqrt{3}}$

$$x = \frac{\sqrt{3}-1}{\sqrt{3}} \times \frac{\sqrt{3}+1}{\left(\sqrt{3}+1\right)} = \frac{2}{3+\sqrt{3}}$$

$\therefore$ abscissa of α is $\frac{2}{3+\sqrt{3}}$

75. Let $A = \{(x,y) \in \mathbb{R}^2 : y \geq 0, 2x \leq y \leq \sqrt{4-(x-1)^2}\}$ and

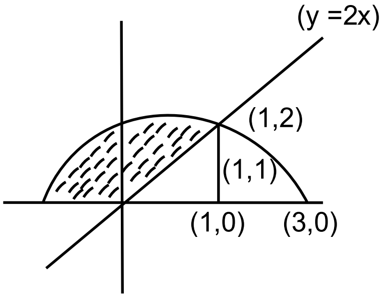
$$B = \left\{(x,y) \in \mathbb{R} \times \mathbb{R} : 0 \leq y \leq \min\left\{2x, \sqrt{4-(x-1)^2}\right\}\right\}.$$

Then the ratio of the area of A to the area of B is

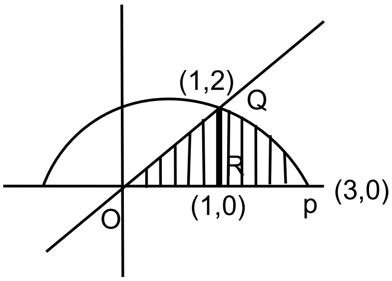
(1) $\frac{\pi+1}{\pi-1}$ (2) $\frac{\pi}{\pi-1}$ (3) $\frac{\pi-1}{\pi+1}$ (4) $\frac{\pi}{\pi+1}$

Sol. **3**

A =



B =



$y^2 = 4 - (x - 1)^2$

$(x - 1)^2 + y^2 = 2^2$

$y = 2x$

$(x - 1)^2 + 4x^2 = 4$

$x^2 + 1 - 2x + 4x^2 = 4$

$5x^2 - 2x - 3 = 0$

$5x^2 - 5x + 3x - 3 = 0$

$5x(x - 1) + 3(x - 1) = 0$

$x = 1, -3/5$

For B : req. area = ar (ΔDRQ) + ar (RPQ)

$= \frac{1}{2}\times 1\times 2 + \int_1^3 \sqrt{4-(x-1)^2}\,dx$

$= 1 + \left[\left(\frac{x-1}{2}\right)\sqrt{4-(x-1)^2} + \frac{4}{2}\sin^{-1}\left(\frac{x-1}{2}\right)\right]_1^3$

$= 1 + 2\sin^{-1} 1 = 1 + \pi$ ………(1)

For A : req. area = area of semi circle – shaded area of B

$= \frac{\pi r^2}{2} - (1+\pi)$

$= \frac{\pi\times 4}{2} - (1+\pi)$ $\{\because r = 2\}$

$A = \pi - 1$ …….. (2)

$\therefore \frac{A}{B} = \frac{\pi - 1}{\pi + 1}$

76. Let $\lambda \neq 0$ be a real number. Let α, β be the roots of the equation $14x^2 - 31x + 3\lambda = 0$ and α,γ be the roots of the equation $35x^2 - 53x + 4\lambda = 0$. Then $\frac{3\alpha}{\beta}$ and $\frac{4\alpha}{\gamma}$ are the roots of the equation

(1) $49x^2 - 245x + 250 = 0$ (2) $7x^2 + 245x - 250 = 0$

(3) $7x^2 - 245x + 250 = 0$ (4) $49x^2 + 245x + 250 = 0$

Sol. **1**

$$\begin{tikzcd} & \alpha \\ 14x^2 - 31x + 3\lambda = 0 \arrow[ur] \arrow[dr] & \\ & \beta \end{tikzcd}$$

$$\begin{tikzcd} & \alpha \\ \text{and } 35x^2 - 53x + 4\lambda = 0 \arrow[ur] \arrow[dr] & \\ & \gamma \end{tikzcd}$$

Now, one root is common then

$\therefore\ 14\alpha^2 - 31\alpha + 3\lambda = 0$ …….. (1)

$35\,\alpha^2 - 53\alpha + 4\lambda = 0$ ……… (2)

$$\frac{\alpha^2}{-124\lambda + 159\lambda} = \frac{-\alpha}{56\lambda - 105\lambda} = \frac{1}{343}$$

$$\Rightarrow \frac{\alpha^2}{35\lambda} = \frac{\alpha}{49\lambda} = \frac{1}{343}$$

$\Rightarrow \alpha = \frac{\lambda}{7}$ {from (ii) and (iii)}

and $\alpha^2 = \frac{35\lambda}{343}$

$$\Rightarrow \frac{\lambda^2}{49} = \frac{35\lambda}{343}$$

$\lambda^2 - 5\lambda = 0$

$\lambda\,(\lambda - 5) = 0$

$\lambda = 0, \lambda = 5 \quad \Rightarrow \alpha = 5/7$

not possible $\therefore$ only $\lambda = 5$ possible

Now, $\alpha + \beta = \frac{31}{14}$, $\alpha\beta = \frac{3\lambda}{14}$, $\alpha + \gamma = \frac{53}{35}$, $\alpha\gamma = \frac{4\lambda}{35}$

$\therefore\ \beta = \frac{3}{2}$ and $\gamma = \frac{4}{5}$

Now equation having roots $\left(\frac{3\alpha}{\beta}, \frac{4\alpha}{\gamma}\right) = \left(\frac{10}{7}, \frac{25}{7}\right)$ is

$$x^2 - \frac{35}{7}x + \frac{250}{49} = 0$$

$\Rightarrow 49x^2 - 245\,x + 250 = 0$

77. Let the tangents at the points $A(4, -11)$ and $B(8, -5)$ on the circle $x^2 + y^2 - 3x + 10y - 15 = 0$, intersect at the point C. Then the radius of the circle, whose centre is C and the line joining A and B is its tangent, is equal to

(1) $2\sqrt{13}$ (2) $\sqrt{13}$ (3) $\frac{3\sqrt{3}}{4}$ (4) $\frac{2\sqrt{13}}{3}$

Sol. **4**

Equation of line AB is

$$y + 5 = \left(\frac{-5+11}{8-4}\right)(x-8)$$

$$\Rightarrow y + 5 = \frac{3}{2}(x - 8) Þ 2y + 10 = 3x - 24$$

$3x - 2y - 34 = 0$ ……..(i)

Let C be (h, k) then equation of AB

$$hx + ky - \frac{3}{2}(x + h) + 5(y + k) - 15 = 0$$

$$x(h - \frac{3}{2}) + y(k + 5) - \frac{3}{2}h + 5k - 15 = 0 \qquad \text{……..(ii)}$$

Now, by comparing (i) and (ii)

$$\frac{h-\frac{3}{2}}{3} = \frac{k+5}{-2} = \frac{-\frac{3}{2}h+5k-15}{-34}$$

after solving centre C is

$$(h, k) = \left(8, \frac{-28}{3}\right)$$

and radius of circle is

$$r = \left|\frac{3(8)-2\left(\frac{-28}{3}\right)-34}{\sqrt{9+4}}\right| = \left|\frac{24+2\frac{56}{3}-34}{\sqrt{13}}\right|$$

$$r = \left|\frac{26}{3\sqrt{13}}\right| = \frac{2\sqrt{13}}{3}$$

78. Let $f(x) = x + \frac{a}{\pi^2-4}\sin x + \frac{b}{\pi^2-4}\cos x, x \in \mathbb{R}$ be a function which satisfies $f(x) = x + \int_0^{\pi/2} \sin(x + y)f(y)dy$. Then (a+b) is equal to

(1) –2π(π–2) (2) –2π(π+2) (3) –π(π–2) (4) –π(π+2)

Sol. **2**

$$f(x) = x + \int_0^{\frac{\pi}{2}}(\sin x\cos y + \cos x\sin y)f(y)dy$$

$$f(x) = x + \int_0^{\frac{\pi}{2}}(\cos y\, f(y)dy)\sin x + (\sin y\, f(y)dy)\cos x) \qquad \text{………(1)}$$

$$\text{given : } f(x) = x + \frac{a}{\pi^2-4}\sin x + \frac{b}{\pi^2-4}\cos x \qquad \text{…….(2)}$$

by comparing (1) and (2)

$$\Rightarrow \frac{a}{\pi^2-4}=\int_0^{\frac{\pi}{2}}\cos y\ f(y)dy \qquad ....(3)$$

$$\text{and } \frac{b}{\pi^2-4}=\int_0^{\frac{\pi}{2}}\sin y\ f(y)dy \qquad ....(4)$$

adding (3) and (4)

$$\frac{a+b}{\pi^2-4}=\int_0^{\frac{\pi}{2}}(\sin y+\cos y)f(y)dy \qquad ....(5)$$

$$\frac{a+b}{\pi^2-4}=\int_0^{\frac{\pi}{2}}(\sin y+\cos y)f\left(\frac{\pi}{2}-y\right)dy \qquad ....(6)$$

Additing (5) and (6)

$$\frac{2(a+b)}{\pi^2-4}=\int_0^{\frac{\pi}{2}}(\sin y+\cos y)\left(\frac{\pi}{2}+\frac{a+b}{\pi^2-4}(\sin y+\cos y)\right)dy$$

$$=\pi+\frac{a+b}{\pi^2-4}\left(\frac{\pi}{2}+1\right)$$

$\Rightarrow a + b = - 2\pi (\pi + 2)$

79. Let $f:\mathbf{R}\to\mathbf{R}$ be a function such that $f(x)=\frac{x^2+2x+1}{x^2+1}$. Then

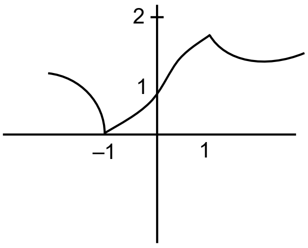
(1) $f(x)$ is one-one in $[1,\infty)$ but not in $(-\infty,\infty)$

(2) $f(x)$ is one-one in $(-\infty,\infty)$

(3) $f(x)$ is many-one in $(-\infty,-1)$

(4) $f(x)$ is many-one in $(1,\infty)$

Sol. **1**



$$f(x)=\frac{(x+1)^2}{x^2+1}=1+\frac{2x}{x^2+1}$$

$$f(x)=1+\frac{2}{x+\frac{1}{x}}$$

Clearly, f(x) is one – one in [1, ∞]

but not in (-∞, ∞)

80. Three rotten apples are mixed accidently with seven good apples and four apples are drawn one by one without replacement. Let the random variable X denote the number of rotten apples. If μ and σ^2 represent mean and variance of X, respectively, then $10(\mu^2 + \sigma^2)$ is equal to

(1) 250 (2) 25 (3) 30 (4) 20

Sol. **4**

Total Apple = 10, Rotten apple = 3, good apple = 7

Prob. of rotten apple (p) = $\frac{3}{10}$

Prob. of good apple (q) = $\frac{7}{10}$

x → Number of rotten apples

here x = 0, 1, 2, 3

$$p(x=0) = {}^4C_0\left(\frac{3}{10}\right)^0 \times \frac{7}{10}\times\frac{6}{9}\times\frac{5}{8}\times\frac{4}{7}=\frac{1}{6}$$

$$p(x=1) = {}^4C_1\left(\frac{3}{10}\right)\times\frac{7}{9}\times\frac{6}{8}\times\frac{5}{7}=\frac{1}{2}$$

$$p(x=2) = {}^4C_2\left(\frac{3}{10}\times\frac{2}{9}\right)\times\frac{7}{8}\times\frac{6}{7}=\frac{3}{10}$$

$$p(x=3) = {}^4C_3\left(\frac{3}{10}\times\frac{2}{9}\times\frac{1}{8}\right)\times\frac{7}{7}=\frac{1}{30}$$

x_i	0	1	2	3
p_i	$\frac{35}{210}$	$\frac{105}{210}$	$\frac{3}{10}$	$\frac{1}{30}$

Now,

$$\mu = \sum p_i x_i = \frac{1}{6}\times 0+\frac{1}{2}\times 1+2\times\frac{3}{10}+3\times\frac{1}{30}=\frac{6}{5}$$

$$\text{and } \sigma^2 = \sum p_i x_i^2 - \mu^2 = \frac{1}{2}+\frac{3}{10}\times 4+\frac{1}{30}\times 9-\frac{36}{25}=\frac{14}{25}$$

$$\therefore\ 10(\mu^2+\sigma^2) = 10\left(\frac{36}{25}+\frac{14}{25}\right)$$

$$= 10\times\left(\frac{50}{25}\right)=10\times 2$$

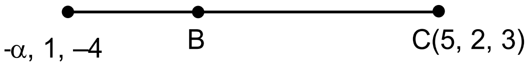
$$= 20$$

Section B

81. Let the co-ordinates of one vertex of $\triangle ABC$ be $A(0,2,\alpha)$ and the other two vertices lie on the line $\frac{x+\alpha}{5}=\frac{y-1}{2}=\frac{z+4}{3}$. For $\alpha \in \mathbb{Z}$, if the area of $\triangle ABC$ is 21 sq. units and the line segment BC has length $2\sqrt{21}$ units, then α^2 is equal to

Sol. **9**

A (0, 2, α)



$$\left|\frac{1}{2}.2\sqrt{21}\begin{vmatrix} i & j & k \\ \alpha & 1 & \alpha+4 \\ 5 & 2 & 3 \end{vmatrix}\frac{1}{\sqrt{25+4+9}}\right| = 21$$

$$\sqrt{(2\alpha+5)^2+(2\alpha+20)^2+(2\alpha-5)^2} = \sqrt{21}\sqrt{38}$$

$12\alpha^2 + 80\alpha + 450 = 798$

$12\alpha^2 + 80\alpha - 398 = 0$

$\alpha = 3 \Rightarrow \alpha^2 = 9$

82. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function that satisfies the relation $f(x+y) = f(x) + f(y) - 1, \forall x, y \in \mathbb{R}$. If $f'(0) = 2$, then $|f(-2)|$ is equal to

Sol. **3**

Given $f(x + y) = f(x) + f(y) - 1 \ \forall \ x, y \in IR$ and $f'(0) = 2$

Partial differentiate w.r.t x

$\Rightarrow f'(x + y) \ f'(x)$

for $x = 0$

$f'(y) = f'(0) = 2$

on Integrating

$\Rightarrow f(y) = 2y + c$ ………(2)

for $y = 0$

$\Rightarrow f(0) = C$ …….(3)

Put $x = y = 0$ in (1)

$\Rightarrow f(0) = f(0) + f(0) - 1$

$\Rightarrow f(0) = 1$ ……..(4)

from (3) & (4)

$c = 1$

$\Rightarrow f(y) = 2y + 1$

$\Rightarrow f(-2) = -4 + 1 = -3$

$\therefore |f(-2)| = 3$

83. Suppose f is a function satisfying $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{N}$ and $f(1) = \frac{1}{5}$. If $\sum_{n=1}^{m} \frac{f(n)}{n(n+1)(n+2)} = \frac{1}{12}$, then m is equal to

Sol. **10**

$f(x + y) = f(x) + f(y) \ \forall \ x, y \in N$ and $f(1) = \frac{1}{5}$

for $x = y = 1$

$f(2) = f(1) + f(1) = 2f(1)$

$f(3) = f(2+1) = f(2) + f(1) = 3f(1)$

In General

$f(n) = nf(1) = \frac{n}{5}$

$$\sum_{n=1}^{m}\frac{f(n)}{n(n+1)(n+2)}=\frac{1}{12}$$

$$\Rightarrow \sum_{n=1}^{m}\frac{n}{5n(n+1)(n+2)}=\frac{1}{12}$$

$$\Rightarrow \frac{1}{5}\sum_{n=1}^{m}\frac{1}{(n+1)(n+2)}=\frac{1}{12}$$

$$\Rightarrow \sum_{n=1}^{m}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)=\frac{5}{12}$$

$$\Rightarrow \left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\ldots\ldots+\left(\frac{1}{m+1}-\frac{1}{m+2}\right)=\frac{5}{12}$$

$$\Rightarrow \frac{1}{2}-\frac{1}{m+2}=\frac{5}{12}$$

$$\Rightarrow \frac{1}{m+2}=\frac{1}{2}-\frac{5}{12}=\frac{1}{12}$$

$$\Rightarrow m=10$$

84. Let the coefficients of three consecutive terms in the binomial expansion of $(1+2x)^n$ be in the ratio $2:5:8$. Then the coefficient of the term, which is in the middle of these three terms, is

Sol. **1120**

Let r + 1, r + 2 and r + 3 be three consecutive terms

$$\frac{{}^nC_r 2^r}{{}^nC_{r+1} 2^{r+1}}=\frac{2}{5}$$

$$\Rightarrow \frac{r+1}{n-r}=\frac{4}{5} \qquad \ldots\ldots(1)$$

Also,

$$\frac{{}^nC_{r+1} 2^{r+1}}{{}^4C_{r+2} 2^{r+2}}=\frac{5}{8}$$

$$\Rightarrow \frac{r+2}{n-r-1}=\frac{5}{4} \qquad \ldots\ldots.(2)$$

on solving (1) & (2), we get

n = 8, r = 3

Here n = 8 (even)

middle term = r + 2 = 3 + 2 = 5

coefficient of T_5 = ${}^8C_4 2^4=70(16)=1120$

85. Let $a_1, a_2, a_3, \ldots$ be a GP of increasing positive numbers. If the product of fourth and sixth terms is 9 and the sum of fifth and seventh terms is 24 , then $a_1a_9+a_2a_4a_9+a_5+a_7$ is equal to

Sol. **60**

Let first term of G.P be a with common ratio r

Given : $a_4 \cdot a_6 = 9$

$a_5 + a_7 = 24$

$a_4 = ar^3$, $a_5 = ar^4$, $a_6 = ar^5$, $a_7 = ar^6$

$a_4 \cdot a_6 = a^2r^8 = 9$

$\Rightarrow \quad ar^4 = 3$

$a_5 = 3$

$\therefore \quad a_7 = 24 - 3 = 21$

$\Rightarrow \quad \frac{a_7}{a_5} = r^2 = 7$

$\Rightarrow \quad r = \sqrt{7}, a = \frac{3}{49}$

$a_1 a_9 + a_2 a_4 a_9 + \quad a_5 + a_7 = a_1 a_9 + (ar)(ar^3) a_9 + 24$

$= a_1 a_9 + a_1(ar^4)a_9 + 24$

$= a_1 a_9 (1 + a_5) + 24 = (ar^4)^2 (4) + 24$

$= 36 + 24 = 60$

86. Let the equation of the plane P containing the line $x + 10 = \frac{8-y}{2} = z$ be $ax + by + 3z = 2(a + b)$ and the distance of the plane P from the point $(1,27,7)$ be c. Then $a^2 + b^2 + c^2$ is equal to

Sol. **355**

Given equation of plane is

$ax + by + 3z = 2(a + b) \quad \ldots\ldots..(1)$

It containing the line

$$\frac{x-(-10)}{1} = \frac{y-8}{-2} = \frac{z-0}{1}$$

$\therefore$ plane (1) must passes through (–10, 8, 0) and parallel to 1, –2, 1

Hence,

$a(-10) + 8b = 2a + 2b$

$\Rightarrow \quad 12a - 6b = 0 \quad \ldots\ldots..(2)$

and $\quad a - 2b + 3 = 0 \quad \ldots\ldots..(3)$

on solving (2) and (3), we get

$b = 2, a = 1$

$\therefore$ equation of the plane is

$x + 2y + 3z = 6 \quad \ldots\ldots..(4)$

c is perpendicular distance from (1, 27, 7) to the plane (4)

$$\Rightarrow c = \left|\frac{1+2\times27+3\times7-6}{\sqrt{1^2+2^2+3^2}}\right| = \left|\frac{70}{\sqrt{14}}\right| = \frac{10\sqrt{7}}{\sqrt{2}}$$

Now, $a^2 + b^2 + c^2 = 1 + 4 + \frac{700}{2} = \frac{710}{2} = 355$

87. If the co-efficient of x^9 in $\left(\alpha x^3 + \frac{1}{\beta x}\right)^{11}$ and the co-efficient of x^{-9} in $\left(\alpha x - \frac{1}{\beta x^3}\right)^{11}$ are equal, then $(\alpha\beta)^2$ is equal to

Sol. **1**

For $\left(\alpha x^3 + \frac{1}{\beta x}\right)^{11}$

$$T_{r+1} = {}^{11}C_r (\alpha x^3)^{11-r} \left(\frac{1}{\beta x}\right)^r$$

$= {}^{11}C_r \alpha^{11-r} \beta^{-r} x^{33-4r}$

Coefficient of $x^9 = {}^{11}C_6 \alpha^{11-6} \beta^{-6}$

$= {}^{11}C_6 \alpha^5 \beta^{-6}$

For $\left(\alpha x - \dfrac{1}{\beta x^3}\right)^{11}$

$T_{r+1} = {}^{11}C_r (\alpha x)^{11-r} \left(\dfrac{-1}{\beta x^3}\right)^r$

$= (-1)^r \; {}^{11}C_r \alpha^{11-r} \beta^{-r} x^{11-4r}$

coefficient of $x^{-9} = - \; {}^{11}C_5 \alpha^6 \beta^{-5}$

$\Rightarrow {}^{11}C_6 \alpha^5 \beta^{-6} = {}^{11}\, C_5 \alpha^6 \beta^{-5}$

$\Rightarrow \alpha\beta = -\dfrac{{}^{11}C_6}{{}^{11}C_5} = -1$

$\therefore (\alpha\beta)^2 = 1$

88. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero non-coplanar vectors. Let the position vectors of four points A, B, C and D be $\vec{a} - \vec{b} + \vec{c}$, $\lambda\vec{a} - 3\vec{b} + 4\vec{c}$, $-\vec{a} + 2\vec{b} - 3\vec{c}$ and $2\vec{a} - 4\vec{b} + 6\vec{c}$ respectively. If $\overrightarrow{AB}, \overrightarrow{AC}$ and $\overrightarrow{AD}$ are coplanar, then λ is equal to

Sol. **2**

$\overrightarrow{AB} = (\lambda\vec{a} - 3\vec{b} + 4\vec{c}) - (\vec{a} - \vec{b} + \vec{c})$

$= (\lambda - 1)\ \vec{a} - 2\vec{b} + 3\vec{c}$

$\overrightarrow{AC} = (-\vec{a} + 2\vec{b} - 3\vec{c}) - (\vec{a} - \vec{b} + \vec{c})$

$= -2\vec{a} + 3\vec{b} - 4\vec{c}$

$\overrightarrow{AD} = (2\vec{a} - 4\vec{b} + 6\vec{c}) - (\vec{a} - \vec{b} + \vec{c})$

$= \vec{a} - 3\vec{b} + 5\vec{c}$

For coplanar vectors

$$\begin{vmatrix} \lambda - 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix} = 0$$

$\Rightarrow 3\lambda - 6 = 0$

$\therefore \lambda = 2$

89. Five digit numbers are formed using the digits 1, 2, 3,5, 7 with repetitions and are written in descending order with serial numbers. For example, the number 77777 has serial number 1 . Then the serial number of 35337 is

Sol. **1436**

Number starting with 7 = $7\underset{\underset{5}{\uparrow}}{\_}\;\underset{\underset{5}{\uparrow}}{\_}\;\underset{\underset{5}{\uparrow}}{\_}\;\underset{\underset{5}{\uparrow}}{\_} = 625$

Number starting with 5 = $5\underset{\underset{5}{\uparrow}}{\_}\;\underset{\underset{5}{\uparrow}}{\_}\;\underset{\underset{5}{\uparrow}}{\_}\;\underset{\underset{5}{\uparrow}}{\_} = 625$

Number starting with 37 = $37\underset{\underset{5}{\uparrow}}{\_}\;\underset{\underset{5}{\uparrow}}{\_}\;\underset{\underset{5}{\uparrow}}{\_} = 125$

Number starting with 357 = $357\underset{\underset{5}{\uparrow}}{\_}\;\underset{\underset{5}{\uparrow}}{\_} = 25$

Number starting with 355 = $355\_\;\_ = 25$

Number starting with 3537 = $3537\ \_ = 5$

Number starting with 3535 = $3535\ \_ = 5$

Number starting with $\underline{35337 \qquad\qquad = 1}$

Total = 1436

Therefore, the serial number of 35337 is 1436

90. If all the six digit numbers $x_1x_2x_3x_4x_5x_6$ with $0 < x_1 < x_2 < x_3 < x_4 < x_5 < x_6$ are arranged in the increasing order, then the sum of the digits in the 72th number is

Sol. **32**

Number of six digit number starting with 1 is 1 ………….. = $^8C_5 = 56$

As remaining five digits can be selected from 8 digits that are greater than (i.e., 2, 3, 4, 5, 6, 7, 8)

Number of six digit number starting with 23 ………. = $^6C_4 = 15$

Total = 56 + 15 = 71

Now, 72nd number = 245678

∴ sum of the digits = 2 + 4 + 5 + 6 + 7 + 8 =32

(Held On Thursday 29th January, 2023)	TIME : 3 : 00 PM to 6 : 00 PM

Physics

SECTION - A

1. Substance A has atomic mass number 16 and half-life of 1 day. Another substance *B* has atomic mass number 32 and half life of $\frac{1}{2}$ day. If both *A* and *B* simultaneously start undergo radio activity at the same time with initial mass 320 g each, how many total atoms of A and B combined would be left after 2 days.

(1) 3.38×10^{24} (2) 1.69×10^{24} (3) 6.76×10^{24} (4) 6.76×10^{23}

Sol. **(1)**

$$(N_0)_A = \frac{320}{16} = 20 \text{ moles}$$

$$(N_0)_B = \frac{320}{32} = 10 \text{ moles}$$

$$N_A = \frac{(N_0)_A}{2^{n_1}} = \frac{20}{4} = 5$$

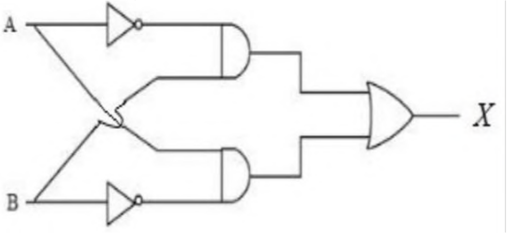
$$N_B = \frac{(N_0)_B}{2^{n_2}} = \frac{10}{(2)^{\frac{2}{0.5}}} = \frac{10}{2^4} = 0.625$$

Total N = 5.625 moles

No. of atoms $= (N)(N_A)$

$=5.625 \times 6.023 \times 10^{23} = (3.38 \times 10^{24})$

2. For the given logic gates combination, the correct truth table will be



(1)

A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

(2)

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

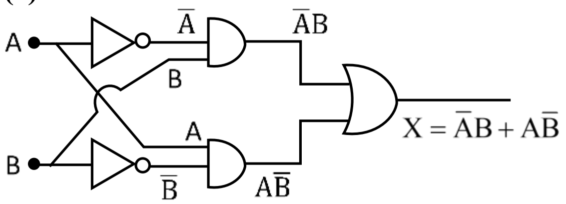
(3)

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

(4)

A	B	X
0	0	1
0	1	0
1	0	1
1	1	0

Sol. **(3)**



From Bodean Algebra :

$X = \bar{A}B + A\bar{B}$

The correct truth table will be

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

3. The time taken by an object to slide down 45° rough inclined plane is n times as it takes to slide down a perfectly smooth 45° incline plane. The coefficient of kinetic friction between the object and the incline plane is:

(1) $\sqrt{1-\frac{1}{n^2}}$ (2) $1+\frac{1}{n^2}$ (3) $1-\frac{1}{n^2}$ (4) $\sqrt{\frac{1}{1-n^2}}$

Sol. **(3)**

Acceleration on the smooth inclined plane

$$a_1 = g\sin\theta = \frac{g}{\sqrt{2}}$$

Acceleration on the rough inclined plane

$$a_2 = g\sin\theta - \mu g\cos\theta = \frac{g}{\sqrt{2}} - \frac{Kg}{\sqrt{2}}(K=\mu)$$

Given that:

$$t_2 = nt_1 \quad \text{and} \quad \frac{1}{2}a_1t_1^2 = \frac{1}{2}a_2t_2^2$$

$$a_1t_1^2 = a_2t_2^2$$

$$\frac{g}{\sqrt{2}}t_1^2 = \left(\frac{g}{\sqrt{2}} - \frac{Kg}{\sqrt{2}}\right)(n^2t_1^2)$$

$$\frac{g}{\sqrt{2}} = n^2\left(\frac{g}{\sqrt{2}} - \frac{Kg}{\sqrt{2}}\right)$$

$$K = 1 - \frac{1}{n^2}$$

4. Heat energy of 184 kJ is given to ice of mass 600 g at −12°C. Specific heat of ice is 2222.3 J $kg^{-1}C^{-1}$ and latent heat of ice in 336 $kJkg^{-1}$

A. Final temperature of system will be 0°C.
B. Final temperature of the system will be greater than 0°C.
C. The final system will have a mixture of ice and water in the ratio of 5: 1.
D. The final system will have a mixture of ice and water in the ratio of 1:5.
E. The final system will have water only.
Choose the correct answer from the options given below:

(1) A and D Only (2) A and E Only (3) A and C Only (4) B and D Only

Sol. **(1)**

Heat energy given $= 184KJ = 184\times10^3 J$

Amount of heat required to raise the temperature

$$\theta_1 = ms_{ice}\Delta T = 0.6\times2222.3\times12$$
$$=16000.56 \text{ J}$$

Remaining heat $\theta_2 = 184000 - 16000.56 = 167999.44$ J

For melting at 0°C heat required = mL_f
= 0.6 × 336000
= (201600) J needed

∴ 100% ice is not melted

Amount of ice melted

167999.44 = m × 336000

m = mass of water = 0.4999 Kg

Mass of ice = 0.1001

$$\text{Ratio } = \frac{0.1001}{0.4999} \approx 1:5$$

5. Identify the correct statements from the following:

A. Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket is negative.

B. Work done by gravitational force in lifting a bucket out of a well by a rope tied to the bucket is negative.

C. Work done by friction on a body sliding down an inclined plane is positive.

D. Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity in zero.

E. Work done by the air resistance on an oscillating pendulum in negative.

Choose the correct answer from the options given below:

(1) B, D and E only (2) A and C Only (3) B and D only (4) B and E only

Sol. **(4)**

→ Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket is positive
→ Work done by friction on a body sliding down an inclined plane is negative
→ Work done by a applied force on a body moving on a rough horizontal plane with uniform velocity is positive

6. A scientist is observing a bacteria through a compound microscope. For better analysis and to improve its resolving power he should. (Select the best option)

(1) Increase the refractive index of the medium between the object and objective lens
(2) Decrease the diameter of the objective lens
(3) Increase the wave length of the light
(4) Decrease the focal length of the eye piece.

Sol. **(1)**

$$\text{R.P} = \frac{2\mu\sin\theta}{1.22\lambda}$$

$\mu\uparrow, \text{R.P}\uparrow$

$\text{D}\downarrow, \theta\downarrow, \text{R.P}\downarrow$

$\lambda\uparrow, \text{R.P}\downarrow$

R.P is independent of focal length of eye piece

7. With the help of potentiometer, we can determine the value of emf of a given cell. The sensitivity of the potentiometer is

(A) directly proportional to the length of the potentiometer wire
(B) directly proportional to the potential gradient of the wire
(C) inversely proportional to the potential gradient of the wire
(D) inversely proportional to the length of the potentiometer wire

Choose the correct option for the above statements:

(1) A only (2) C only (3) A and C only (4) B and D only

Sol. **(3)**

If on displacing the jockey slightly from the null point position, the galvanometer shows a large deflection, than the potentiometer is said to be sensitive. The sensitivity of the potentiometer depends upon the potential gradient along the wire. The smaller potential gradient greater will be sensitivity.

Sensitivity ↑, potential gradient ↓, length ↑

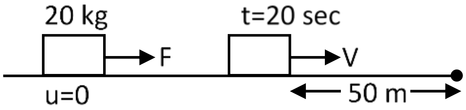
Sensitivity $\propto$ length

$$\text{Sensitivity} \propto \frac{1}{\text{Potential gradient}}$$

8. A force acts for 20 s on a body of mass 20 kg, starting from rest, after which the force ceases and then body describes 50 m in the next 10 s. The value of force will be:

(1) 40 N (2) 5 N (3) 20 N (4) 10 N

Sol. **(2)**



$$50 = V \times 10$$
$$V = 5\ ms^{-1}$$
$$V = 0 + a \times 20$$
$$5 = a \times 20$$
$$a = \frac{1}{4} ms^{-2}$$
$$F = ma = 20 \times \frac{1}{4} = 5N$$

9. The modulation index for an A.M. wave having maximum and minimum peak-to-peak voltages of 14 mV and 6 mV respectively is:

(1) 0.4 (2) 0.6 (3) 0.2 (4) 1.4

Sol. **(1)**

$$\mu = \text{Modulating index} = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$
$$= \frac{14-6}{14+6}$$
$$= 0.4$$

10. Given below are two statements:

Statement I: Electromagnetic waves are not deflected by electric and magnetic field.

Statement II: The amplitude of electric field and the magnetic field in electromagnetic waves are related to each other as $E_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} B_0$.

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is true but statement II is false

(2) Both Statement I and Statement II are false

(3) Statement I is false but statement II is true

(4) Both Statement I and Statement II are true

Sol. **(1)**

Statement -I is correct as
EMW are neutral
Statement – II is wrong

$$E_0 = \sqrt{\frac{1}{\mu_0 \in_0}} B_0$$

11. A square loop of area 25 cm^2 has a resistance of 10Ω. The loop is placed in uniform magnetic field of magnitude 40.0 T. The plane of loop is perpendicular to the magnetic field. The work done in pulling the loop out of the magnetic field slowly and uniformly in 1.0sec, will be

(1) 1.0×10^{-3} J (2) 2.5×10^{-3} J (3) 5×10^{-3} J (4) 1.0×10^{-4} J

Sol. **(1)**

$l = 5$ cm

$t = 1$ sec

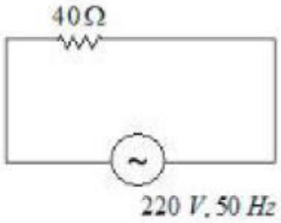
$$V = \frac{0.05}{1} = 0.05ms^{-1}$$

$$I = \frac{40 \times 0.05 \times 0.05}{10} = \frac{BLV}{R} = 0.01A$$

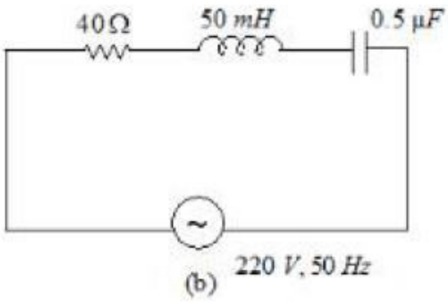
$$F = BIL = 40 \times 0.010.05 = 0.02N$$

$$W = F\,\ell = 0.02 \times 0.05 = 1 \times 10^{-3}\,J$$

12. For the given figures, choose the correct options:



(a)



(b)

(1) At resonance, current in (b) is less than that in (a)
(2) The rms current in circuit (b) can never be larger than that in (a)
(3) The rms current in figure(a) is always equal to that in figure (b)
(4) The rms current in circuit (b) can be larger than that in (a)

Sol. **(2)**

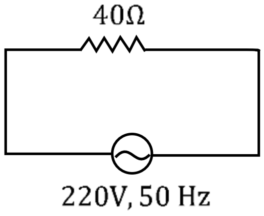


fig (a)

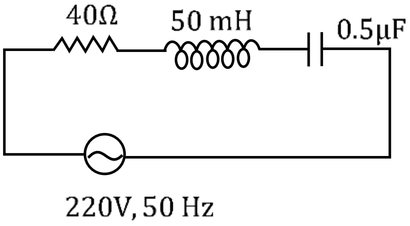


fig (a)

$$I_{rms} = \frac{220}{40} = 5.5A$$

X_L is not equal to X_C, so rms current

In (b) can never be large than (a)

13. A fully loaded boeing aircraft has a mass of 5.4×10^5 kg. Its total wing area is 500 m^2. It is in level flight with a speed of 1080 km/h. If the density of air ρ is 1.2 kg m^{-3}, the fractional increase in the speed of the air on the upper surface of the wing relative to the lower surface in percentage will be. $(g = 10 \text{ m/s}^2)$

(1) 16 (2) 10 (3) 8 (4) 6

Sol. **(2)**

$$P_2A - P_1A = 5.4 \times 10^5 \times g$$

$$P_2 - P_1 = \frac{5.4 \times 10^6}{500} = 10.8 \times 10^3$$

$$P_2 + 0 + \frac{1}{2}\rho v_2^2 = P_1 + 0 + \frac{1}{2}\rho v_1^2$$

$$P_2 - P_1 = \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) = \frac{1}{2}\rho\left(v_1 + v_2\right)\left(v_1 - v_2\right)$$

$$10.8 \times 10^3 = \frac{1}{2} \times 1.2 \times \left(v_1 - v_2\right) \times 2 \times 3 \times 10^2$$

$$v_1 - v_2 = 30$$

$$\frac{v_1 - v_2}{v} \times 100 = \frac{30}{300} \times 100 = 10\%$$

14. The ratio of de-Broglie wavelength of an α particle and a proton accelerated from rest by the same potential is $\frac{1}{\sqrt{m}}$, the value of m is-

(1) 16 (2) 4 (3) 2 (4) 8

Sol. **(4)**

$$\frac{\lambda_\alpha}{\lambda_p} = \frac{\frac{h}{\sqrt{2m_\alpha q_\alpha v}}}{\frac{h}{\sqrt{2m_p q_p v}}}$$

$$\frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{1}{8}}$$

$$M = 8$$

15. The time period of a satellite of earth is 24 hours. If the separation between the earth and the satellite is decreased to one fourth of the previous value, then its new time period will become.

(1) 4 hours (2) 6 hours (3) 3 hours (4) 12 hours

Sol. **(3)**

$$T^2 \propto R^3$$

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} \Rightarrow \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R}{\frac{R}{4}}\right)^3$$

$$\frac{T_1^2}{T_2^2} = 64$$

$$T_2^2 = \frac{T_1^2}{64}$$

$$T_2 = \frac{T_1}{8} = \frac{24}{8} = 3$$

16. The electric current in a circular coil of four turns produces a magnetic induction 32 T at its centre. The coil is unwound and is rewound into a circular coil of single turn, the magnetic induction at the centre of the coil by the same current will be :

(1) 16 T (2) 2 T (3) 8 T (4) 4 T

Sol. **(2)**

$$B=\frac{\mu_o i}{2R}\times 4$$

$$B'=\frac{\mu_o i}{2R'}$$

$$R'=4R$$

$$B'=\frac{\mu_0 i}{8R}$$

$$\frac{B'}{B}=\frac{1}{16}$$

$$B'=2T$$

17. A point charge 2×10^{-2}C is moved from P to S in a uniform electric field of $30NC^{-1}$ directed along positive x-axis. If coordinates of P and S are (1,2,0)m and (0,0,0)m respectively, the work done by electric field will be

(1) 1200 mJ (2) −1200 mJ (3) −600 mJ (4) 600 mJ

Sol. (3)

$$W_E=q\vec{E}.\vec{S}\ =2\times10^{-2}\times(-30)$$

$$=-0.6J\ =-600mJ$$

18. An object moves at a constant speed along a circular path in a horizontal plane with center at the origin. When the object is at = +2 m, its velocity is $-4\hat{j}$m/s.

The object's velocity (v) and acceleration (a) at $x=-2$ m will be

(1) $v=-4\hat{i}\frac{m}{s}, a=-8\hat{j}\ m/s^2$ (2) $v=4\hat{i}\frac{m}{s}, a=8\hat{j}\ m/s^2$

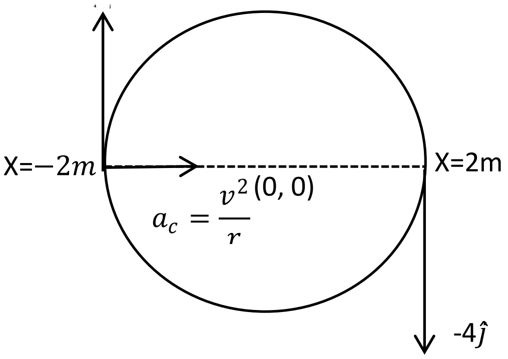
(3) $v=4\hat{j}\frac{m}{s}, a=8\hat{i}\ m/s^2$ (4) $v=-4\hat{j}\frac{m}{s}, a=8\hat{i}\ m/s^2$

Sol. (3)

$$a_c=\frac{v^2}{r}=\frac{4^2}{2}=8ms^{-2}$$

$$\vec{v}=4\hat{j}$$

$$\vec{a_c}=8i$$



19. At 300 K the rms speed of oxygen molecules is $\sqrt{\frac{\alpha+5}{\alpha}}$ times to that of its average speed in the gas. Then, the value of α will be

(used $=\frac{22}{7}$)

(1) 28 (2) 24 (3) 32 (4) 27

Sol. (1)

$$\sqrt{\frac{3RT}{M}}=\sqrt{\frac{\alpha+5}{\alpha}}\sqrt{\frac{8}{\pi}\frac{RT}{M}}$$

$$3=\left(\frac{\alpha+5}{\alpha}\right)\left(\frac{8}{\pi}\right)$$

$$\alpha=28$$

20. The equation of a circle is given by $x^2+y^2=a^2$, where a is the radius. If the equation is modified to change the origin other than (0,0), then find out the correct dimensions of A and B in a new equation : $(x-At)^2+\left(y-\frac{t}{B}\right)^2=a^2$. The dimensions of t is given as $[\mathrm{T}^{-1}]$.

(1) $\mathrm{A}=[\mathrm{LT}], \mathrm{B}=[\mathrm{L}^{-1}\ \mathrm{T}^{-1}]$
(2) $\mathrm{A}=[\mathrm{L}^{-1}\ \mathrm{T}^{-1}], \mathrm{B}=[\mathrm{LT}]$
(3) $\mathrm{A}=[\mathrm{L}^{-1}\ \mathrm{T}], \mathrm{B}=[\mathrm{LT}^{-1}]$
(4) $\mathrm{A}=[\mathrm{L}^{-1}\ \mathrm{T}^{-1}], \mathrm{B}=[\mathrm{LT}^{-1}]$

Sol. (1)

$$(x-At)^2+\left(y-\frac{t}{B}\right)^2=a^2$$

$$A=L^1T^1$$

$\frac{t}{B}$ is in meter

$$\frac{t}{B}=L$$

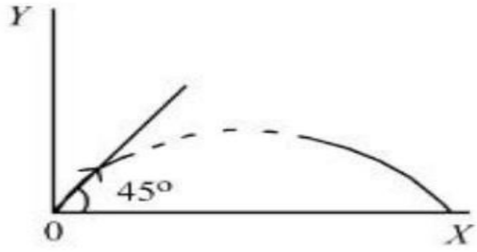
$$\frac{T^{-1}}{B}=L$$

$$B=T^{-1}L^{-1}$$

SECTION - B

21. A particle of mass 100 g is projected at time $t=0$ with a speed $20\ \mathrm{ms}^{-1}$ at an angle 45° to the horizontal as given in the figure. The magnitude of the angular momentum of the particle about the starting point at time $t=2$ s is found to be $\sqrt{\mathrm{K}}\ \mathrm{kgm}^2/\mathrm{s}$. The value of K is ________.

(Take $\mathrm{g}=10\ \mathrm{ms}^{-2}$)



Sol. **800**

Use $\Delta L = \int_0^t \tau dt$

$$L_0 = \int_0^2 (mg)(v_x t)dt$$

$$= (mgv_x)\frac{t^2}{2}$$

$$= (0.1)(10)(10)(\sqrt{2}) \times \frac{2^2}{2}$$

$$= 20\sqrt{2}$$
$$= \sqrt{800}$$

22. Unpolarised light is incident on the boundary between two dielectric media, whose dielectric constants are 2.8 (medium -1) and 6.8 (medium -2), respectively. To satisfy the condition, so that the reflected and refracted rays are perpendicular to each other, the angle of incidence should be $\tan^{-1}\left(1+\frac{10}{\theta}\right)^{\frac{1}{2}}$ the value of θ is _________.
(Given for dielectric media, $\mu_r = 1$)

Sol. 7

$$\mu_1 = \sqrt{2.8}$$
$$\mu_2 = \sqrt{6.8}$$
$$\mu \sin i = \mu_2 \cos i$$

$$\tan i = \frac{\mu_2}{\mu_1} = \sqrt{\frac{6.8}{2.8}}$$

$$\tan i = \left(\frac{2.8+4}{2.8}\right)^{\frac{1}{2}}$$

$$i = \tan^{-1}\left(1+\frac{10}{7}\right)^{\frac{1}{2}}$$

$$\theta = 7$$

23. A particle of mass 250 g executes a simple harmonic motion under a periodic force $F = (-25x)$N. The particle attains a maximum speed of 4 m/s during its oscillation. The amplitude of the motion is ________ cm.

Sol. **(40)**

$$F = ma$$

$$-25x = \frac{250}{100}a$$

$$a = -100x$$
$$\omega^2 = 100$$
$$\omega = 100$$
$$A\omega = 4$$

$$A = \frac{4}{10} = 0.4m$$

$$A = 40cm$$

24. A car is moving on a circular path of radius 600 m such that the magnitudes of the tangential acceleration and centripetal acceleration are equal. The time taken by the car to complete first quarter of revolution, if it is moving with an initial speed of 54 km/hr is $t\left(1-e^{-\pi/2}\right)s$. The value of t is ________.

Sol. **(40)**

$$\frac{dv}{dt}=\frac{v^2}{R}$$

$$\frac{vdv}{dx}=\frac{v^2}{R}$$

$$\frac{dv}{dx}=\frac{v}{R}$$

$$\int_{15}^{v}\frac{dv}{v}=\int_0^x\frac{dx}{R}$$

$$\frac{v}{15}=\frac{x}{R}$$

$$\frac{v}{15}=e^{\frac{x}{R}}$$

$$v=15e^{\frac{x}{R}}$$

$$\frac{dx}{dt}=15e^{\frac{x}{R}}$$

$$\int_0^{\frac{\pi R}{2}}e^{-\frac{x}{R}}dx=15\int_0^{to}dt$$

$$t_0=40\left(1-e^{-\frac{\pi}{2}}\right)s$$

$$\boxed{t=40}$$

25. When two resistances R_1 and R_2 connected in series and introduced into the left gap of a meter bridge and a resistance of 10Ω is introduced into the right gap, a null point is found at 60 cm from left side. When R_1 and R_2 are connected in parallel and introduced into the left gap, a resistance of 3Ω is introduced into the right-gap to get null point at 40 cm from left end. The product of R_1R_2 is ________ Ω^2

Sol. (30)

$$\frac{R_1+R_2}{10}=\frac{60}{40}$$

$$R_1+R_2=15 \qquad \text{............ (1)}$$

$$\frac{R_1R_2}{(R_1+R_2)x3}=\frac{40}{60}$$

$$R_1R_2=30$$

26. In an experiment of measuring the refractive index of a glass slab using travelling microscope in physics lab, a student measures real thickness of the glass slab as 5.25 mm and apparent thickness of the glass slab as 5.00 mm. Travelling microscope has 20 divisions in one cm on main scale and 50 divisions on vernier scale is equal to 49 divisions on main scale. The estimated uncertainty in the measurement of refractive index of the slab is $\frac{x}{10} \times 10^{-3}$, where x is ________.

Sol. **(41)**

$$\mu = \frac{h}{h^1} = \frac{\text{Real depth}}{\text{Apparent depth}}$$

Least Count = M.S.D. – V.S.D

$$= \text{M.S.D.} - \frac{49}{50}\text{M.S.D}$$

$$= \left(\frac{50-49}{50}\right)\text{M.S.D}$$

$$= \frac{1}{50}\text{M.S.D}$$

$$= \frac{1}{50} \times \frac{1}{20}\text{cm}$$

$$= \frac{1}{1000}\text{cm}$$

$$= \frac{10}{1000}\text{mm} = 0.01\text{mm}$$

$$\ln\mu = \ln h - \ln h^{'}$$

$$\frac{d\mu}{\mu} = \frac{dh}{h} + \frac{dh'}{h'}$$

$$d\mu = \mu\left[\frac{dh}{h} + \frac{dh'}{h'}\right]$$

$$d\mu = \mu\left[\frac{dh}{h} + \frac{dh'}{h}\right] = \frac{5.25}{5.00}\left[\frac{0.01}{5.25} + \frac{0.01}{5.00}\right]$$

$$= \frac{41}{10} \times 10^{-3}$$

27. An inductor of inductance 2μH is connected in series with a resistance, a variable capacitor and an AC source of frequency 7kHz. The value of capacitance for which maximum current is drawn into the circuit is $\frac{1}{x}$ F, where the value of x is ________. (Take $\pi = \frac{22}{7}$)

Sol. **(3872)**

For Maximum current is drawn

$$X_L = X_C$$

$$\omega L = \frac{1}{\omega C}$$

$$2\pi f L = \frac{1}{2\pi f c}$$

$$C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4 \times \pi^2 \times 49 \times 10^6 \times 2 \times 10^{-6}}$$

$$C = \frac{1}{3872} F$$

$$X = 3872$$

28. A null point is found at 200 cm in potentiometer when cell in secondary circuit is shunted by 5Ω. When a resistance of 15Ω is used for shunting, null point moves to 300 cm. The internal resistance of the cell is ________ Ω.

Sol. **(5)**

Potential Gradient $= \frac{\Delta V}{L}$

$$E - Ir = \left(\frac{\Delta v}{L}\right)x$$

$$\frac{ER}{R + r} = \left(\frac{\Delta V}{L}\right)x$$

$$\frac{E \times 5}{5 + r} = \frac{\Delta V}{L} \times 200 \qquad \text{.........} \ (1)$$

$$\frac{E \times 15}{15 + r} = \frac{\Delta V}{L} \times 300 \qquad \text{.........} \ (2)$$

$$= r = 5\omega$$

29. For a charged spherical ball, electrostatic potential inside the ball varies with r as $V = 2ar^2 + b$. Here, a and b are constant and r is the distance from the center. The volume charge density inside the ball is $-\lambda a\varepsilon$. The value of λ is __________.

ε = permittivity of the medium

Sol. **(12)**

$$E = -\frac{dv}{dr} = -4ar$$

By the Gauss' theorem

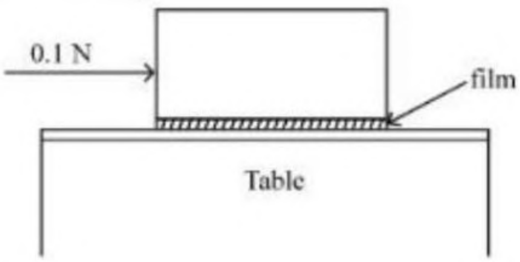
$$\oint \vec{E}.\overrightarrow{dA} = \frac{q_{\text{inside}}}{\varepsilon}$$

$$E \times 4\pi r^2 = \frac{\rho \times \frac{4}{3}\pi r^3}{\varepsilon}$$

$$E = \frac{\rho r}{3\varepsilon} = -4ar$$

$$\rho = -12a\varepsilon$$

30. A metal block of base area 0.20 m^2 is placed on a table, as shown in figure. A liquid film of thickness 0.25 mm is inserted between the block and the table. The block is pushed by a horizontal force of 0.1 N and moves with a constant speed. If the viscosity of the liquid is 5.0×10^{-3} Pl, the speed of block is ________ $\times 10^{-3}$ m/s.



Sol. **(25)**

$$|F| = \eta A \frac{\Delta v}{\Delta h}$$

$$0.1 = 5 \times 10^{-3} \times 0.2 \times \frac{v}{0.25 \times 10^{-3}}$$

$$v = 0.025 ms^{-1}$$

$$v = 25 \times 10^{-3} ms^{-1}$$

Chemistry

SECTION - A

31. According to MO theory the bond orders for $O_2{}^{2-}$, CO and NO^+ respectively, are

(1) 1, 2 and 3 (2) 1,3 and 2 (3) 2,3 and 3 (4) 1, 3 and 3

Sol. **4**

Molecules	Total No. of e^-	Bond order
O_2^{-2}	18	1
CO	14	3
NO^+	14	3

32. A doctor prescribed the drug Equanil to a patient. The patient was likely to have symptoms of which disease?

(1) Hyperacidity (2) Anxiety and stress

(3) Depression and hypertension (4) Stomach ulcers

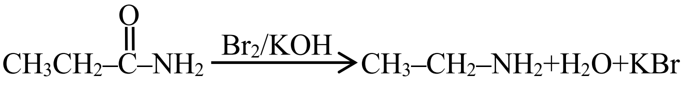
Sol. **3**

Equanil is a tranquiliger, used for treatment of depression and hypertension.

33. Reaction of propanamide with $Br_2/KOH(aq)$ produces :

(1) Propylamine (2) Ethylnitrile(3) Propanenitrile (4) Ethylamine

Sol. **4**



$$CH_3CH_2{-}\overset{\overset{O}{\|}}{C}{-}NH_2 \xrightarrow{Br_2/KOH} CH_3{-}CH_2{-}NH_2+H_2O+KBr$$

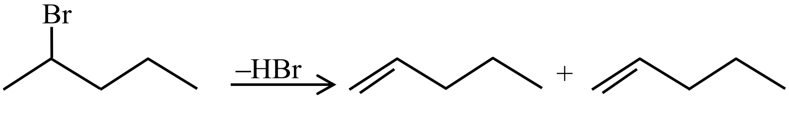
Hoffmann's bromamide reaction

34. The one giving maximum number of isomeric alkenes on dehydrohalogenation reaction is (excluding rearrangement)

(1) 2-Bromopropane (2) 2-Bromo-3,3-dimethylpentane

(3) 1-Bromo-2-methylbutane (4) 2-Bromopentane

Sol. **4**



2–bromo pentane (Cis+trons)

35. An indicator ' X ' is used for studying the effect of variation in concentration of iodide : on the rate of reaction of iodide ion with H_2O_2 at room temp. The indicator ' X ' forms blue colored complex with compound ' A ' present in the solution. The indicator ' X ' and compound 'A' respectively are

(1) Methyl orange and H_2O_2

(2) Starch and iodine

(3) Starch and H_2O_2

(4) Methyl orange and iodine

Sol. **2**

$$I^- + H_2O_2 \longrightarrow \underset{(A)}{I_2} + H_2O$$

$$I_2 + \underset{\text{(Indicator)}}{\text{Starch}} \longrightarrow \text{Blue}$$

36. The major component of which of the following ore is sulphide based mineral?

(1) Siderite (2) Sphalerite (3) Malachite (4) Calamine

Sol. **2**

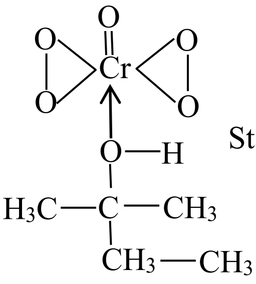
Zinc blade	Sphalerite	→	Zns
	Siderite	→	feCO_3
	Malachite	→	$CuCO_3 \cdot CuCOHl_2$
	Calamine	→	$ZnCO_3$

37. A solution of C_rO_5 in amyl alcohol has a _______ colour.

(1) Green (2) Orange-Red (3) Yellow (4) Blue

Sol. **4**

Blue



Stablize by L.P.

38. The set of correct statements is :

(i) Manganese exhibits +7 oxidation state in its oxide.

(ii) Ruthenium and Osmium exhibit +8 oxidation in their oxides.

(iii) Sc shows +4 oxidation state which is oxidizing in nature.

(iv) Cr shows oxidising nature in +6 oxidation state.

(1) (ii) and (iii) (2) (i), (ii) and (iv) (3) (ii), (iii) and (iv) (4) (i) and (iii)

Sol. **2**

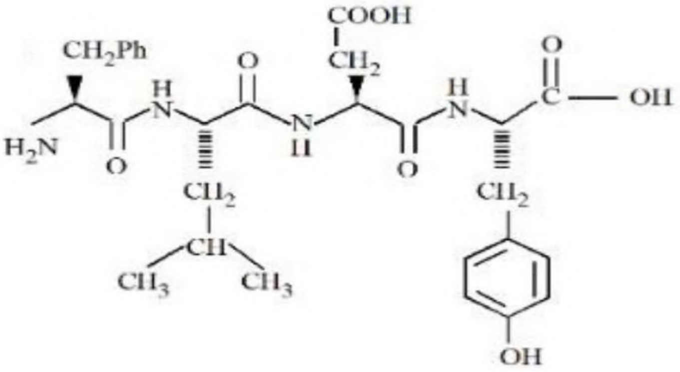
(i) Mn_2O_7

(ii) RuO_4 & OsO_4

(iii) Sc (+4) oxidation state not possible in oxidizing nature

(iv) Cr show oxidizing nature in +6 oxidation state

39. Following tetrapeptide can be represented as

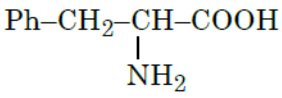


(F, L, D, Y, I, Q, P are one letter codes for amino acids)

(1) PLDY (2) FIQY (3) YQLF (4) FLDY

Sol. **4**

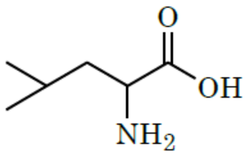
Hydrolysis of the given tetrapeptide will give the following:



$$Ph{-}CH_2{-}\underset{\displaystyle NH_2}{\underset{|}{CH}}{-}COOH$$

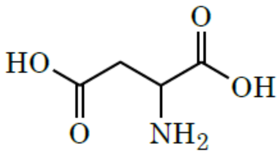
(F)

(Phenylalanine)



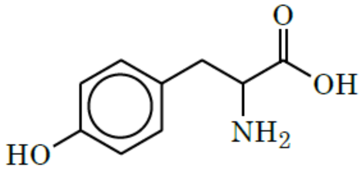
(L)

(Leucine)



(D)

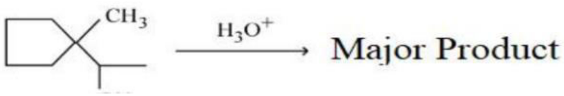
(Aspartic acid)



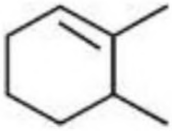
(Y)

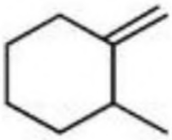
(Tyrosine)

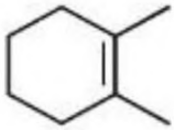
40. Find out the major product for the following reaction.



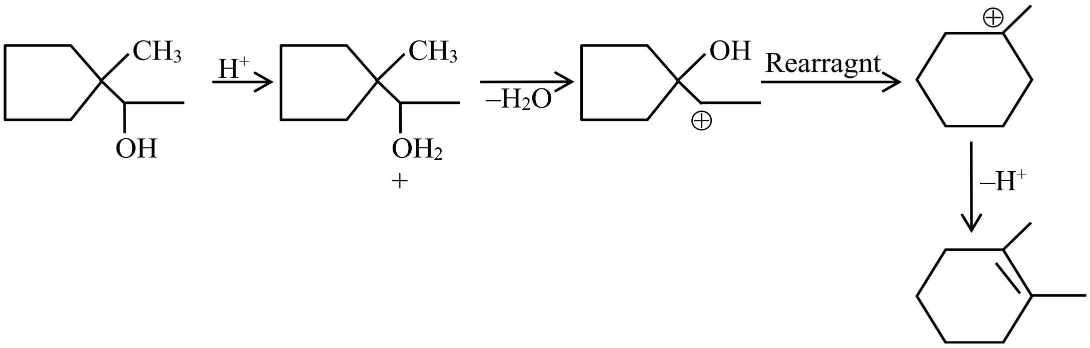
1. 

2. 

3. 

4. 

Sol. 4



41.

List I	List II
A. van't Hoff factor, i	I. Cryoscopic constant
B. k_f	II. Isotonic solutions
C. Solution with same with same osmotic pressure	III. $\frac{\text{Normal molar mass}}{\text{Abnormal molar mass}}$
D. Azeotropes	IV. Solutions with same composition of vapour above it

Choose the correct answer from the options given below :

(1) A-I, B-III, C-II, D-IV
(2) A-III, B-I, C-IV, D-II
(3) A-III, B-I, C-II, D-IV
(4) A-III, B-II, C-I, D-IV

Sol. 3

(A) van’t Hoff factor, i

$$i = \frac{\text{Normal molar mass}}{\text{Abnormal molar mass}}$$

(B) k_f = Cryoscopic constant

(C) Solutions with same osmotic pressure are known as isotonic solutions.

(D) Solutions with same composition of vapour over them are called Azeotrope.‘

42. Correct order of spin only magnetic moment of the following complex ions is:

(Given At.no. Fe: 26, Co:27)

(1) $[FeF_6]^{3-} > [Co(C_2O_4)_3]^{3-} > [CoF_6]^{3-}$
(2) $[FeF_6]^{3-} > [CoF_6]^{3-} > [Co(C_2O_4)_3]^{3-}$
(3) $[Co(C_2O_4)_3]^{3-} > [CoF_6]^{3-} > [FeF_6]^{3-}$
(4) $[CoF_6]^{3-} > [FeF_6]^{3-} > [Co(C_2O_4)_3]^{3-}$

Sol. **2**

Complex	Central Metal E.C.	No. Of unpaired e^-	$\mu = \sqrt{n(n+2)}$ B.M.
(i) $[Fef_6]^{-3}$	$Fe^{+3} \to 3d^5 \to t2g^{1,1,1}, eg^{1,1}$	5	$\sqrt{35}$ Br
(ii) $[Cof_6]^{-3}$	$CO^{+3} \to 3d^6 \to t_2g^{2,1,1}, eg^{1,1}$	4	$\sqrt{24}$ Br
(iii) $[Co(C_2O_4)_3]^{-3}$	$CO^{+3} \to 3d^6 \to t_2g^{2,2,2}, eg^{0,0}$	0	0 Br

43. Match List I with List II

List I	List II
A. Elastomeric polymer	I. Urea formaldehyde resin
B. Fibre Polymer	II. Polystyrene
C. Thermosetting Polymer	III. Polyester
D. Thermoplastic Polymer	IV. Neoprene

Choose the correct answer from the options given below :

(1) A-II, B-III, C-I, D-IV (2) A-IV, B-III, C-I, D-II

(3) A-IV, B-I, C-III, D-II (4) A-II, B-I, C-IV, D-III

Sol. **2**

Neoprene : Elastomer

Polyester Fibre

Polstyrene : THermolastic

Urea–Formaldhyde Resin: Thermosetting polymer

44. The concentration of dissolved Oxygen in water for growth of fish should be more than $\underline{X}$ppm and Biochemical Oxygen Demand in clean water should be less than $\underline{Y}$ ppm. X and Y in ppm are, respectively.

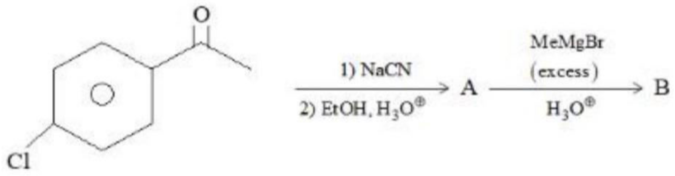
1. X Y
   4 8
2. X Y
   6 5
3. X Y
   4 15
4. X Y
   6 12

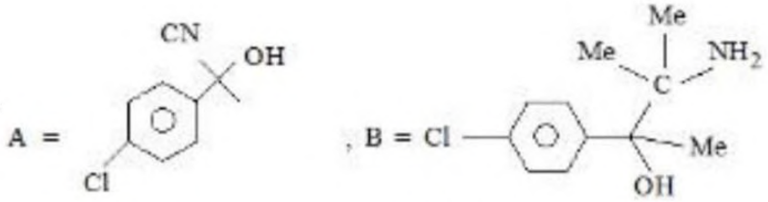
Sol. **2**

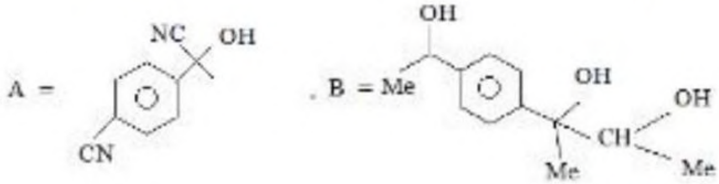
→ BOD value of water of water is in the range –3–5 (Less than 5)

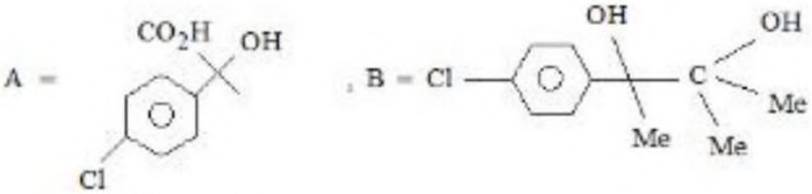
→ dissolve oxygen in water for growth of wish → Less than (6)

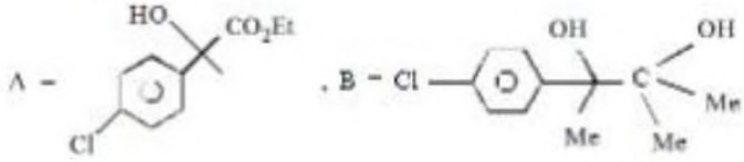
45. Find out the major products from the following reaction sequence.



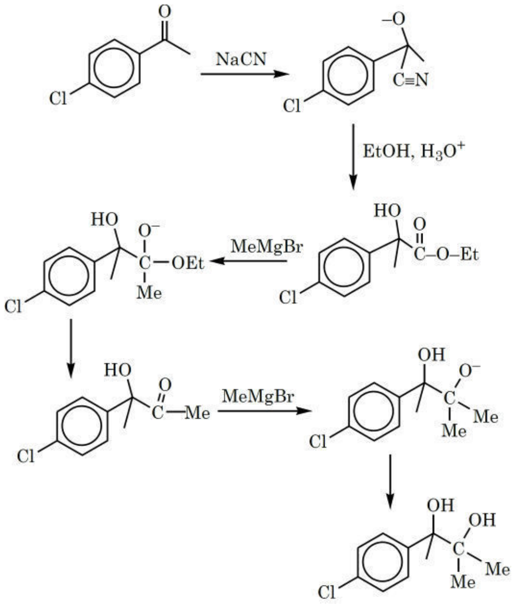
1.


2.


3.


4.


Sol. **4**



46. When a hydrocarbon A undergoes combustion in the presence of air, it requirs 9.5 equivalents of oxygen and produces 3 equivalents of water. What is the molecular formula of A ?

(1) C_9H_9 (2) C_8H_6 (3) C_9H_6 (4) C_6H_6

Sol. **2**

$$C_xH_y + \left(x + \frac{y}{4}\right)O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O$$

Number of equivalents of O_2 = Number of equivalents of H_2O

Number of equivalents of H_2O = $\frac{y}{2} = 3$

y = 6

Number of equivalents of O_2 = x + $\frac{y}{4} = 9.5$

$$x + \frac{6}{4} = 9.5$$

x = 9.5 – 1.5 = 8

$C_xH_y = C_8H_6$

47. Given below are two statements:

Statement I : Nickel is being used as the catalyst for producing syn gas and edible fats.

Statement II : Silicon forms both electron rich and electron deficient hydrides.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Statement I is correct but statement II is incorrect

(2) Both the statements I and II are incorrect

(3) Statement I is incorrect but statement II is correct

(4) Both the statements I and II are correct

Sol. **1**

(i) $\underset{(g)}{CH_4} + \underset{(g)}{H_2O} \xrightarrow[Ni]{1270K} CO + 3H_2(g)$

Ni used as a catalyst

(ii) Si neither formed e– deficient hydride nor electron rich species.

48. Which of the following relations are correct?

(A) $\Delta U = q + p\Delta V$ (B) $\Delta G = \Delta H - T\Delta S$ (C) $\Delta S = \frac{q_{rev}}{T}$ (D) $\Delta H = \Delta U - \Delta nRT$

Choose the most appropriate answer from the options given below:

(1) B and D Only (2) A and B Only

(3) B and C Only (4) C and D Only

Sol. **3**

Only (B) and (C) are correct.

(B) $G = H - TS$

At constant T

$$\Delta G = \Delta H - T\Delta S$$

(A) First law is given by

$$\Delta U = Q + W$$

If we apply constant P and reversible work.

$$\Delta U = Q - P\Delta V$$

(C) By definition of entropy change

$$dS = \frac{q_{rev}}{T}$$

At constant T

$$\Delta S = \frac{q_{rev}}{T}$$

(D) $H = U + PV$

For ideal gas

$H = U + nRT$

At constant T

$\Delta H = \Delta U + \Delta nRT$

49. Given below are two statements :

Statement I : The decrease in first ionization enthalpy from B to Al is much larger than that from Al to Ga.

Statement II : The d orbitals in Ga are completely filled.

In the light of the above statements, choose the most appropriate answer from the options given below

(1) Statement I is incorrect but statement II is correct

(2) Both the statements I and II are correct

(3) Both the statements I and II are incorrect

(4) Statement I is correct but statement II is incorrect

Sol. **1**

$$\begin{array}{lccccccccc} & B & > & Tl & > & Ga & > & Al & > & I \\ \text{Ionisation enthalpy} \rightarrow & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\ & 801 & & 589 & & 579 & & 577 & & 558 \end{array}$$

$\rightarrow {}_{31}Ga \rightarrow [Ar]4s^2, 3d^{10}, 4p^1$

Ga → completely filled d orbital.

50. Match List I and List II

List I	List II
A. Osmosis	I. Solvent molecules pass through semi permeable membrane towards solvent side.
B. Reverse osmosis	II. Movement of charged colloidal particles under the influence of applied electric potential towards oppositely charged electrodes.
C. Electro osmosis	III. Solvent molecules pass through semi permeable membrane towards solution side.
D. Electrophoresis	IV. Dispersion medium moves in an electric field.

Choose the correct answer from the options given below :

(1) A-I, B-III, C-IV, D-II

(2) A-III, B-I, C-IV, D-II

(3) A-III, B-I, C-II, D-IV

(4) A-I, B-III, C-II, D-IV

Sol. **2**

(i) Electro osmosis: When movement of colloidal particles is prevented by some suitable means (porous diaphragm or semi permeable membranes), it is observed that the D.M. begins to move in an electric field. This phenomenon is termed electrosmosis.

(ii) Solvent molecules pass through semi-permeable membrane towards solvent side is termed as reverse osmosis.

(iii) When an electric potential is applied across two platinum electrodes dipping in a colloidal solution, the colloidal particles move towards move towards one or the other electrode. The movement of colloidal particles under an applied electric potential is called electrophoresis.

(iv) Solvent molecules pass through semipermeable membrane towards the solution side is termed as osmosis.

51. Assume that the radius of the first Bohr orbit of hydrogen atom is 0.6Å. The radius of the third Bohr orbit of $He^+$is ________ picometer. (Nearest Integer)

Sol. **(270)**

$$r \propto \frac{n^2}{Z}$$

$$r_{He^+} = r_H \times \frac{n^2}{Z}$$

$$r_{He^+} = 0.6 \times \frac{(3)^2}{2}$$

$$= 2.7\ \text{Å}$$

$$r_{He^+} = 270\ \text{pm}$$

52. Total number of acidic oxides among

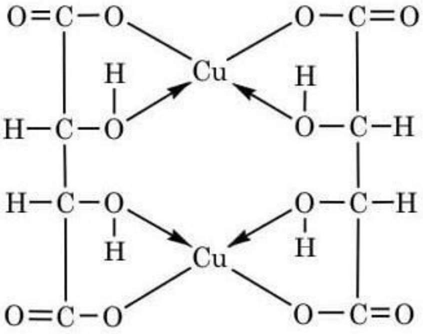
$N_2O_3, NO_2, N_2O, Cl_2O_7, SO_2, CO, CaO, Na_2O$ and NO is ________

Sol. **4**

Acidic oxide → $N_2O_3, NO_2, Cl_2O_7, SO_2$

53. The denticity of the ligand present in the Fehling's reagent is ________

Sol. **4**



Copper tartrate complex

Denticity = 2

54. The equilibrium constant for the reaction

$Zn(s) + Sn^{2+}(aq) \rightleftharpoons Zn^{2+}(aq) + Sn(s)$ is 1×10^{20} at 298 K. The magnitude of standard electrode potential of Sn/Sn^{2+} if $E^{\circ}_{Zn^{2+}/Zn} = -0.76V$ is ________ $\times 10^{-2}$ V (Nearest integer).

Given : $\frac{2.303RT}{F} = 0.059$ V

Sol. **17**

Given

$$Zn(s) + Sn^{2+}(aq.) \rightleftharpoons Zn^{2+}(aq.) + Sn(s)$$

$K_C = 1 \times 10^{20}$

$E^{\circ}_{Zn^{2+}/Zn} = -0.76V$

$$E_{cell} = E^{\circ}_{cell} - \frac{0.059}{n}\log_{10}K_c$$

$$0 = E^{\circ}_{cell} - \frac{0.059}{2} \times 20$$

$E^{\circ}_{cell} = 0.59$

$E^{\circ}_{cell} = E^{\circ}_{\underset{(RP)}{Cathode}} - E^{\circ}_{\underset{(RP)}{Anode}}$

$0.59 - E^{\circ}_{Sn^{2+}/Sn} - E^{\circ}_{Zn^{2+}/Zn}$

$0.59 = E^{\circ}_{Sn^{Z+}/Sn} - (-0.76)$

$E^{\circ}_{Sn^{2+}/Sn} = 0.17$

$E^{\circ}_{Sn/Sn^{2+}} = 17 \times 10^{-2}$

55. The volume of HCl, containing $73\ g\ L^{-1}$, required to completely neutralise NaOH obtained by reacting 0.69 g of metallic sodium with water, is _______ mL. (Nearest Integer)

(Given : molar Masses of Na, Cl, O, H, are 23,35.5,16 and $1\ g\ mol^{-1}$ respectively)

Sol. **15**

Mole of Na = $\frac{0.69}{23} = 3 \times 10^{-2}$

$Na + H_2O \longrightarrow NaOH + \frac{1}{2}H_2$

By using POAC

Moles of NaOH = 3×10^{-2}

NaOH reacts with HCl

No. of equivalent of NaOH = No. of equivalent of HCl

$$3 \times 10^{-2} \times 1 = \frac{73}{36.5} \times V(\text{in L}) \times 1$$

$V = 1.5 \times 10^{-2}$ L

Volume of HCl = 15 ml.

56. For conversion of compound A → B, the rate constant of the reaction was found to be $4.6 \times 10^{-5}\ L\ mol^{-1}\ s^{-1}$. The order of the reaction is _________.

Sol. **2**

As unit of rate constant is $(\text{conc.})^{1-n}\ \text{time}^{-1}$

Put n = 2

then $L\ mol^{-1}\ s^{-1}$

So order of the reaction is 2.

57. On heating, $LiNO_3$ gives how many compounds among the following? _______

$Li_2O, N_2, O_2, LiNO_2, NO_2$

Sol. **3**

$$4LiNO_3 \longrightarrow 2Li_2O + 4NO_2 + O_2$$

58. When 0.01 mol of an organic compound containing 60% carbon was burnt completely, 4.4 g of CO_2 was produced. The molar mass of compound is _______ $gmol^{-1}$ (Nearest integer).

Sol. **200**

Let M is the molar mass of the compound (g/mol)

mass of compound = 0.01 M gm

$$\text{mass of carbon} = 0.01\text{ M} \times \frac{60}{100}$$

$$\text{mass of carbon} = \frac{0.01\text{M}}{12} \times \frac{60}{100}$$

$$\text{moles of } CO_2 \text{ from combustion} = \frac{4.4}{44} = \text{moles of carbon}$$

$$\frac{0.01\text{M}}{12} \times \frac{60}{100} = \frac{4.4}{44}$$

$$\text{M} = \frac{4.4}{44} \times \frac{100}{60} \times \frac{12}{0.01} = 200 \text{ gm/mol}$$

59. At 298 K

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g), K_1 = 4 \times 10^5$

$N_2(g) + O_2(g) \rightleftharpoons 2NO(g), K_2 = 1.6 \times 10^{12}$

$H_2(g) + \frac{1}{2}O_2(g) \rightleftharpoons H_2O(g), K_3 = 1.0 \times 10^{-13}$

Based on above equilibria, the equilibrium constant of the reaction,

$2NH_3(g) + \frac{5}{2}O_2(g) \rightleftharpoons 2NO(g) + 3H_2O(g)$ is _______ $\times 10^{-33}$ (Nearest integer).

Sol. **4**

Reverse equation (1) So $K_1^1 = \frac{1}{K_1}$ … (a)

+

Add equation …. (2) $K_1^1 = K_2$ ….(b)

+

Multiply equation (3) by (3) $K_3^1 = k_3^3$ ….(c)

Add (a), (b) & (c)

$$K_c^1 = \frac{K_2 \times K_3^3}{K_1} = \frac{1.6 \times 10^{12} \times 1 \times 10^{-39}}{4 \times 10^5}$$

$\Rightarrow 4 \times 10^{-33}$

60. A metal M forms hexagonal close-packed structure. The total number of voids in 0.02 mol of it is _______ $\times 10^{21}$ (Nearest integer).(Given $\mathrm{N_A} = 6.02 \times 10^{23}$)

Sol. **(36)**

One unit cell of hcp contains = 18 voids

No. of voids in 0.02 mol of hcp

$\frac{18}{6} \times 6.02 \times 10^{23} \times 0.02$

$\approx 3.6 \times 10^{22}$

$\approx 36 \times 10^{21}$

Mathematics

SECTION - A

61. The statement $B \Rightarrow ((\sim A) \vee B)$ is equivalent to :

(1) $A \Rightarrow (A \Leftrightarrow B)$
(2) $A \Rightarrow ((\sim A) \Rightarrow B)$
(3) $B \Rightarrow (A \Rightarrow B)$
(4) $B \Rightarrow ((\sim A) \Rightarrow B)$

Sol. **1, 3 or 4**

$B \Rightarrow (\sim A)$ VB

A	**B**	**~ A**	**~ AVB**	**B ⇒ (~A) VB**
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

A ⇒ B	**~A ⇒ B**	**B ⇒ A ⇒ B**	**A ⇒ ((~A) ⇒ B)**	**B ⇒ ((~A) ⇒ B)**
T	T	T	T	T
F	T	T	T	T
T	T	T	T	T
T	F	T	T	T

62. The value of the integral $\int_1^2 \left(\frac{t^4+1}{t^6+1}\right) dt$ is

(1) $\tan^{-1} 2 - \frac{1}{3}\tan^{-1} 8 + \frac{\pi}{3}$
(2) $\tan^{-1}\frac{1}{2} + \frac{1}{3}\tan^{-1} 8 - \frac{\pi}{3}$
(3) $\tan^{-1}\frac{1}{2} - \frac{1}{3}\tan^{-1} 8 + \frac{\pi}{3}$
(4) $\tan^{-1} 2 + \frac{1}{3}\tan^{-1} 8 - \frac{\pi}{3}$

Sol.

$$I = \int_1^2 \left(\frac{t^4+1}{t^6+1}\right) dt$$

$$\Rightarrow \int \frac{t^4+1-t^2+t^2}{(t^2+1)(t^4-t^2+1)} dt$$

$$\Rightarrow \int \frac{(t^4-t^2+1)+t^2}{(t^2+1)(t^4-t^2+1)} dt$$

$$\Rightarrow \int_1^2 \left[\frac{t^4-t^2+1}{(t^2+1)(t^4-t^2+1)} + \frac{t^2}{t^6+1}\right] dt$$

$$\Rightarrow \int_1^2 \frac{1}{t^2+1} dt + \frac{1}{3}\int_1^2 \frac{3t^2}{(t^3)^2+1} dt$$

$$\Rightarrow \left[\tan^{-1} t + \frac{1}{3}\tan^{-1}(t^3)\right]_1^2$$

$$\Rightarrow \tan^{-1} 2 + \frac{1}{3}\tan^{-1}(8) - \tan^{-1}(1) - \frac{1}{3}\tan^{-1}(1)$$

$$\Rightarrow \tan^{-1} 2 + \frac{1}{3}\tan^{-1} 8 - \frac{\pi}{4} - \frac{\pi}{4}.\frac{1}{3}$$

$\Rightarrow \tan^{-1}2+\frac{1}{3}\tan^{-1}8-\frac{3\pi+\pi}{12}$

$\Rightarrow \tan^{-1}2+\frac{1}{3}\tan^{-1}8-\frac{\pi}{3}$

63. The set of all values of λ for which the equation $\cos^2 2x - 2\sin^4 x - 2\cos^2 x = \lambda$ has a real solution x, is

(1) $[-2,-1]$ (2) $\left[-1,-\frac{1}{2}\right]$ (3) $\left[-\frac{3}{2},-1\right]$ (4) $\left[-2,-\frac{3}{2}\right]$

Sol.

$\cos^2 2x - 2\left(\frac{1-\cos 2x}{2}\right)^2 - (1+\cos 2x) = \lambda$

$\Rightarrow \cos^2 2x - 2\left(\frac{1-\cos^2 2x - 2\cos 2x}{4}\right) - 1 - \cos 2x = \lambda$

Let cos2x = t

$\Rightarrow 2t^2 - 1 - t^2 + 2t - 2 - 2t = 2\lambda$

$\Rightarrow t^2 - 3 = 2\lambda \qquad \because 0 \le t^2 \le 1$

$\Rightarrow t^2 = 2\lambda + 3$

$0 \le 2\lambda + 3 \le 1$

$-3 \le 2\lambda \le -2$

$\frac{-3}{2} \le \lambda \le -1$

64. Let R be a relation defined on $\mathbb{N}$ as a R b if $2a + 3b$ is a multiple of 5, $a, b \in \mathbb{N}$. Then R is

(1) an equivalence relation (2)transitive but not symmetric

(3) not reflexive (4) symmetric but not transitive

Sol.

Reflexive

Let $a \in N$

a R a $\Rightarrow$ 2a + 3a is a multiple of 5

$\Rightarrow$ 5a which is a multiple of 5

$\Rightarrow$ R is reflexive

Symmetric

Let $a, b \in N$

a R b $\Rightarrow 2a + 3b = 5\lambda_1 \qquad \lambda_1 \in N$

b R a $\Rightarrow 2b + 3a = 5\lambda_2 \qquad \lambda_2 \in N$

On Adding

$(2a + 3b) + (2b + 3a) = 5(\lambda_1 + \lambda_2)$

$5a + 5b = 5(\lambda_1 + \lambda_2)$

$\Rightarrow$ Both sides are multiple of 5

$\Rightarrow$ R is symmetric

Transitive

Let a, b, c $\in$ N

a R b $\Rightarrow 2a + 3b = 5\lambda_1$ …(1)

b R c $\Rightarrow 2a + 3c = 5\lambda_2$ …(2)

$2a + 5b + 3c = 5(\lambda_1 + \lambda_2)$

$\Rightarrow (2a + 3c) = 5(\lambda_1 + \lambda_2 - b)$

2a + 3c is divisible by 5

$\Rightarrow$ a R c is true

$\Rightarrow$ R is transitive

R is Equivalence Relation

65. Consider a function $f: \mathbb{N} \to \mathbb{R}$, satisfying

$f(1) + 2f(2) + 3f(3) + \cdots + xf(x) = x(x+1)f(x); x \geq 2$ with $f(1) = 1$.

Then $\frac{1}{f(2022)} + \frac{1}{f(2028)}$ is equal to

(1) 8100 (2) 8400 (3) 8000 (4) 8200

Sol. **1**

$f(1) + 2f(2) + 3f(3) + \ldots\ldots. + xf(x) = x^2f(x) + xf(x)$

$\Rightarrow f(1) + 2f(2) + 3f(3) + \ldots\ldots + (x-1)\, f(x-1) = x^2\, f(x)$

$x = 2 \quad f(1) = 2^2\, f(2) \Rightarrow f(2) = \frac{1}{4}$

$x = 3 \quad f(1) + 2f(2) = 3^2\, f(3)$

$$\Rightarrow f(3) = \frac{1}{9}\left(1+\frac{2}{4}\right) = \frac{1}{9}\times\frac{3}{2} = \frac{1}{6}$$

$x = 4 \quad f(1) + 2f(2) + 3f(3) = 4^2f(4)$

$$\Rightarrow \; f(4) = \left(1+\frac{1}{2}+\frac{1}{2}\right).\frac{1}{16} \Rightarrow \; f(4) = \frac{1}{8}$$

$x = 5 \quad f(1) + 2f(2) + 3f(3) + 4f(4) = 5^2f(5)$

$$f(5) = \left(1+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\right)\frac{1}{25} = \frac{5}{2}.\frac{1}{25} = \frac{1}{10}$$

In general $\quad f(x) = \frac{1}{2x}$

$$\therefore \quad \frac{1}{f(2022)} + \frac{1}{f(2028)}$$

$$\frac{1}{\frac{1}{2\times 2022}} + \frac{1}{\frac{1}{2\times 2028}}$$

$\Rightarrow \quad 2[2022 + 2028]$

$\Rightarrow \quad 2 \times 4050$

$\Rightarrow \quad 8100$

66. If $\vec{a} = \hat{\imath} + 2\hat{k}, \vec{b} = \hat{\imath} + \hat{\jmath} + \hat{k}, \vec{c} = 7\hat{\imath} - 3\hat{\jmath} + 4\hat{k}, \vec{r} \times \vec{b} + \vec{b} \times \vec{c} = \vec{0}$ and $\vec{r} \cdot \vec{a} = 0$.

Then $\vec{r} \cdot \vec{c}$ is equal to

(1) 32 (2) 30 (3) 36 (4) 34

Sol. **4**

$\vec{r}\times\vec{b}+\vec{b}\times\vec{c}=0$

$\Rightarrow\ \vec{r}\times\vec{b}-\vec{c}\times\vec{b}=0$

$\Rightarrow\ \left(\vec{r}-\vec{c}\right)\times\vec{b}=0$

$\vec{r}-\vec{c}\,||\,\vec{b}$

$\vec{r}-\vec{c}=\lambda\vec{b}$

$\vec{r}=\lambda\vec{b}+\vec{c}$

$=\ \lambda(i+j+k)+(7i-3j+4k)$

$= i(\lambda + 7) + j(\lambda - 3) + k(\lambda + 4)$

$\vec{r}.\vec{a}=0$

$\Rightarrow (7 + \lambda) + 2(\lambda + 4) = 0$

$\Rightarrow 3\lambda = -15 \quad \Rightarrow \lambda = -5$

$\therefore\ \vec{r}=2i-8j-k$

$\vec{r}.\vec{c}\ = (2i - 8j - k)\ .\ (7i - 3j + 4k)$

$= 14 + 24 - 4\ = 34$

67. The shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$ and $\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}$

(1) $5\sqrt{3}$ (2) $2\sqrt{3}$ (3) $3\sqrt{3}$ (4) $4\sqrt{3}$

Sol. **4**

$L_1=\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}=\lambda$

$L_2\ \frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}=\mu$

$\text{S.D.}=\left|\frac{(\vec{b}-\vec{a}).(\vec{b}_1\times\vec{b}_2)}{\left|\vec{b}_1\times\vec{b}_2\right|}\right|$ $\qquad \vec{a}=i-8j+4k$

$\vec{b}=i+2j+6k$

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix} i & j & k \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix}$$

$= i(21\text{-}5) - j(-6 - 10) + k(2 + 14)$

$= 16i + 16j + 16\ k$

$\left|\vec{b}_1\times\vec{b}_2\right|=|16(i+j+k)\,|$

$=\ 16\times\sqrt{3}$

$\vec{b}-\vec{a}=(10j+2k)$

$$\text{S.D.}=\left|\frac{(10j+2k).16(i+j+k)}{16\sqrt{3}}\right|$$

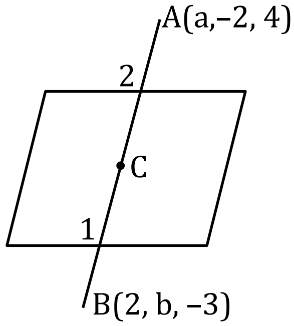
$$=\left|\frac{16(10+2)}{16\sqrt{3}}\right|=\frac{12}{\sqrt{3}}\Rightarrow\frac{12}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$

$$=\frac{12\sqrt{3}}{3}=4\sqrt{3}$$

68. The plane $2x - y + z = 4$ intersects the line segment joining the points $A(a, -2, 4)$ and $B(2, b, -3)$ at the point C in the ratio $2:1$ and the distance of the point C from the origin is $\sqrt{5}$. If $ab < 0$ and P is the point $(a - b, b, 2b - a)$ then CP^2 is equal to

(1) $\frac{97}{3}$ (2) $\frac{17}{3}$ (3) $\frac{16}{3}$ (4) $\frac{73}{3}$

Sol. **2**



C divides AB in 2 : 1

$$C\left(\frac{4+a}{3}, \frac{2b-2}{3}, \frac{-6+4}{3}\right)$$

$$C\left(\frac{a+4}{3}, \frac{2b-2}{3}, \frac{-2}{3}\right)$$

C lies in the plane

$$\therefore\ 2\left(\frac{a+4}{3}\right) - \left(\frac{2b-2}{3}\right) + \left(\frac{-2}{3}\right) = 4$$

$\Rightarrow 2a - 2b = 4$

$a - b = 2$

$\because OC = \sqrt{5}$

$OC^2 = 5$

$$\Rightarrow \left(\frac{b+6}{3}\right)^2 + \left(\frac{2b-2}{3}\right)^2 + \left(\frac{-2}{3}\right)^2 = 5$$

$\Rightarrow 5b^2 + 4b - 1 = 0$

$\Rightarrow 5b^2 + 5b - b - 1 = 0$

$\Rightarrow 5b(b + 1) - 1(b + 1) = 0$

$b = -1\ \&\ \frac{1}{5}$

$a = 1$

$\because ab < 0$

$\therefore\ a = 1, b = -1$

$$C\left(\frac{b+6}{3}, \frac{2b-2}{3}, \frac{-2}{3}\right)$$

Now,

$$C\left(\frac{5}{3}, \frac{-4}{3}, \frac{-2}{3}\right)$$

$P(a - b, b, 2b - a)$

$(2, -1, -3)$

$$CP^2 = \left(\frac{5}{3} - 2\right)^2 + \left(\frac{-4}{3} + 1\right)^2 + \left(\frac{-2}{3} + 3\right)^2 = \frac{17}{3}$$

69. The value of the integral $\int_{1/2}^{2} \frac{\tan^{-1} x}{x} dx$ is equal to

(1) $\frac{\pi}{2}\log_e 2$ (2) $\pi\log_e 2$ (3) $\frac{1}{2}\log_e 2$ (4) $\frac{\pi}{4}\log_e 2$

Sol.

Let $x = \frac{1}{t}$

$dx = -\frac{1}{t^2}dt$

$$I = \int_{2}^{1/2} \frac{\tan^{-1}\left(\frac{1}{t}\right)}{\frac{1}{t}} \times -\frac{1}{t^2}dt$$

$$\Rightarrow \int_{1/2}^{2} \frac{\cot^{-1}(t)}{t}\,dt$$

$$2I = \int_{1/2}^{2} \frac{\tan^{-1}x + \cot^{-1}x}{x}\,dx$$

$$\Rightarrow \int_{1/2}^{2} \frac{\pi/2}{x}\,dx$$

$$\Rightarrow \frac{\pi}{2}\left[\ell n\, x\right]_{1/2}^{2}$$

$$\Rightarrow \frac{\pi}{2}\left(\ell n\, 2 - \ell n \frac{1}{2}\right)$$

$$\Rightarrow \frac{\pi}{2}\left(\ell n\, 2 + \ell n\, 2\right)$$

$$2I = \pi \ell n 2$$

$$\Rightarrow I = \frac{\pi}{2}\ell n\, 2$$

70. The letters of the word OUGHT are written in all possible ways and these words are arranged as in a dictionary, in a series. Then the serial number of the word TOUGH is

(1) 84 (2) 79 (3) 89 (4) 86

Sol. **3**

G H O T U

$$\left.\begin{array}{l}\text{The words start from G}\\ \text{The words start from H}\\ \text{The words start from O}\end{array}\right\} \begin{array}{l}\lfloor\underline{4}\\ \lfloor\underline{4}\ 24\times 3\\ \lfloor\underline{4}\end{array}$$

T

$$\left.\begin{array}{l}\text{Start form TG}\\ \text{TH}\end{array}\right\} \begin{array}{l}\lfloor\underline{3}\\ \lfloor\underline{3}\ 6\times 2\end{array}$$

TO

$$\left.\begin{array}{l}\text{TOG } \lfloor\underline{2}\\ \text{TOH } \lfloor\underline{2}\end{array}\right\} 2\times 2$$

TOU

$$\left.\begin{array}{l}\text{TOUG } \lfloor\underline{1}\end{array}\right\} \quad 1$$

$$\overline{\quad 89 \quad}$$

71. The set of all values of $t \in \mathbb{R}$, for which the matrix

$$\begin{bmatrix} e^{t} & e^{-t}(\sin t - 2\cos t) & e^{-t}(-2\sin t - \cos t) \\ e^{t} & e^{-t}(2\sin t + \cos t) & e^{-t}(\sin t - 2\cos t) \\ e^{t} & e^{-t}\cos t & e^{-t}\sin t \end{bmatrix}$$ is invertible, is

(1) $\mathbb{R}$ (2) $\left\{k\pi + \frac{\pi}{4}, k \in \mathbb{Z}\right\}$ (3) $\{k\pi, k \in \mathbb{Z}\}$ (4) $\left\{(2k+1)\frac{\pi}{2}, k \in \mathbb{Z}\right\}$

Sol. **(1)**

$$|A| = \begin{vmatrix} e^t & e^{-t}(s-2c) & e^{-t}(-2s-c) \\ e^t & e^{-t}(2s+c) & e^{-t}(s-2c) \\ e^t & e^{-t}c & e^{-t}s \end{vmatrix}$$

$$\Rightarrow \ = e^t.e^{-t}.e^{-t}\begin{vmatrix} 1 & s-2c & -2s-c \\ 1 & 2s+c & s-2c \\ 1 & c & s \end{vmatrix}$$

$R_1 \rightarrow R_1 - R_2$ & $R_2 \rightarrow R_2 - R_3$

$$= e^t\begin{vmatrix} 0 & -s-3c & -3s-c \\ 0 & 2s & -2c \\ 1 & c & s \end{vmatrix}$$

$\Rightarrow e^{-t}[1(2sc + 6c^2 + 6s^2 + 2sc)]$

$\Rightarrow e^{-t}[4sc + 6(c^2 + s^2)] \ = e^{-t}(6 + 2\sin 2t)$

$\because 2\sin 2t \in [-2, 2]$

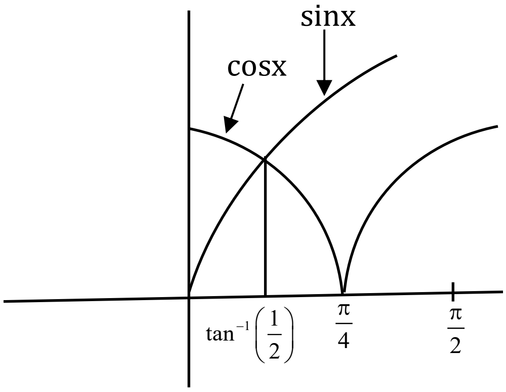
$\therefore e^{-t}(6 + 2\sin 2t) \neq 0 \quad \forall t \in R$

72. The area of the region $A = \left\{(x, y): |\cos x - \sin x| \le y \le \sin x, 0 \le x \le \frac{\pi}{2}\right\}$ is

(1) $\sqrt{5} + 2\sqrt{2} - 4.5$ (2) $1 - \frac{3}{\sqrt{2}} + \frac{4}{\sqrt{5}}$ (3) $\frac{3}{\sqrt{5}} - \frac{3}{\sqrt{2}} + 1$ (4) $\sqrt{5} - 2\sqrt{2} + 1$

Sol. **(4)**

|cosx – sinx|



A = area under the curve

y = sinx & above the curve |cosx – sinx|

$$A = \int_0^{\pi/2} (\sin x - |\cos x - \sin x|)\,dx$$

When 0 to $\frac{\pi}{4}$

|cosx – sinx| = cosx – sinx

sinx = cosx – sinx

2sinx = cosx

$\tan x = \frac{1}{2}$

$x = \tan^{-1}\left(\frac{1}{2}\right)$

when $\frac{\pi}{4}$ to $\frac{\pi}{2}$

|cosx – sinx| = sinx – cosx

sinx = sinx – cosx

cosx = 0

$x = \frac{\pi}{2}$

$$A = \int_{\tan^{-1}(1/2)}^{\pi/4} \{\sin x - (\cos x - \sin x)\}\,dx \ + \int_{\pi/4}^{\pi/2} \{\sin x + (\cos x - \sin x)\}\,dx$$

$$\Rightarrow \int_{\tan^{-1}\left(\frac{1}{2}\right)}^{\pi/4}(2\sin x-\cos x)dx \;+\; \int_{\pi/4}^{\pi/2}\cos x\,dx$$

$$\Rightarrow (-2\cos x-\sin x)_{\tan^{-1}\left(\frac{1}{2}\right)}^{\pi/4}+(\sin x)_{\pi/4}^{\pi/2}$$

$$\Rightarrow \left(-2\times\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right)-\left\{-2\cos\left(\tan^{-1}\frac{1}{2}\right)-\sin\left(\tan^{-1}\frac{1}{2}\right)\right\}+\left(1-\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow -\frac{3}{\sqrt{2}}+2\times\frac{2}{\sqrt{5}}+\frac{1}{\sqrt{5}}+1-\frac{1}{\sqrt{2}} = \frac{-4}{\sqrt{2}}+\frac{5}{\sqrt{5}}+1 = -2\sqrt{2}+\sqrt{5}+1$$

73. The number of 3 digit numbers, that are divisible by either 3 or 4 but not divisible by 48, is

(1) 507 (2) 432 (3) 472 (4) 400

Sol. (2)

Nos div. by 3

102, 105, 108 ……… 999

A.P. $a = 102 \quad d = 3 \quad \ell = 999$

$$n = \frac{\ell - a}{d}+1 = \frac{999-102}{3}+1 = 300$$

Numbers div. by 4

100, 104, 108 ……… 996

A.P. $a = 100 \quad d = 4 \quad \ell = 996$

$$n = \frac{996-100}{4}+1 = \frac{896}{4}+1 = 224 + 1 = 225$$

Numbers div. by 12

108, 120, ……… 996

A.P. $a = 108 \quad d = 12 \quad \ell = 996$

$$n = \frac{996-108}{12}+1 = \frac{888}{12}+1 = 74 + 1 = 75$$

Numbers div. by 48

144, 192, ……… 960

A.P. $a = 144 \quad d = 48 \quad \ell = 960$

$$n = \frac{996-144}{12}+1 = \frac{816}{48}+1 = 17 + 1 = 18$$

$\therefore$ No. Div. by 354 but not by 48

300 + 225 – 75 – 18

= 450 – 18 = 432

74. If the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+3}{1}$ and $\frac{x-a}{2}=\frac{y+2}{3}=\frac{z-3}{1}$ intersect at the point P, then the distance of the point P from the plane $z=a$ is :

(1) 28 (2) 16 (3) 10 (4) 22

Sol. **1**

Let $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+3}{1}=\lambda$

P(λ + 1, 2λ + 2, λ – 3)

& $\frac{x-a}{2}=\frac{y+2}{3}=\frac{z-3}{1}=\mu$

P(2μ + a, 3μ – 2, μ + 3)

$\lambda+1=2\mu+a$	$2\lambda+2=3\mu-2$	$\lambda-3=\mu+3$
$22+1=2\times16+a$	$2(\mu+6)+2=3\mu-2$	$\lambda=\mu+6$
$a=23-32$	$2\mu+12=3\mu-4$	
$a=-9$	$\mu=16$	
	$\therefore\ \lambda=22$	

$\therefore$P(23, 46, 19) Plane is z = a

z = – 9

The distance of p from z = –9 is 19 – (–9) = 28

75. Let $y=y(x)$ be the solution of the differential equation $x\log_e x\frac{dy}{dx}+y=x^2\log_e x,(x>1)$. If $y(2)=2$, then $y(e)$ is equal to

(1) $\frac{1+e^2}{2}$ (2) $\frac{4+e^2}{4}$ (3) $\frac{2+e^2}{2}$ (3) $\frac{1+e^2}{4}$

Sol. **2**

D.E. $\frac{dy}{dx}+\frac{1}{x\log x}y=x$

Linear diff. eq^n $\frac{dy}{dx}+py=Q$

$$\text{If}=e^{\int Pdx}$$

$$=e^{\int\frac{1}{x\log x}dx}\qquad \log x=t$$

$$\frac{1}{x}dx=dt$$

$$=e^{\int\frac{1}{t}dt}=e^{\ell nt}=t$$

I.F. = ℓnx

Solution of DE.

$$y.\ \text{If}=\int q(\text{If})\,dx+c$$

$$y.\ell nx=\int x.(\ell n x)dx+c$$

$$=\ell nx\ .\ \frac{x^2}{2}\ell nx-\frac{x^2}{4}+C$$

At x = 2, y = 2

$$2\ell n2=\frac{4}{2}\ell n2-\frac{4}{4}+C\ \Rightarrow\ C=1$$

$$\therefore\qquad y\ell nx=\frac{x^2}{2}\ell nx-\frac{x^2}{4}+1$$

At x = e

$$y(e)\ \ell ne = \frac{e^2}{2}\ell n\, e - \frac{e^2}{4} + 1$$

$$y(e) = \frac{e^2}{4} + 1$$

76. Let f and g be twice differentiable functions on $\mathbb{R}$ such that

$$f''(x) = g''(x) + 6x$$

$$f'(1) = 4g'(1) - 3 = 9$$

$$f(2) = 3g(2) = 12.$$

Then which of the following is NOT true?

(1) There exists $x_0 \in (1, 3/2)$ such that $f(x_0) = g(x_0)$

(2) $|f'(x) - g'(x)| < 6 \Rightarrow -1 < x < 1$

(3) If $-1 < x < 2$, then $|f(x) - g(x)| < 8$

(4) $g(-2) - f(-2) = 20$

Sol. **3**

Let $F(x) = f(x) - g(x)$

Given $f'(x) = g''(x) + 6x$

$$f'(x) = g'(x) + \frac{6x^2}{2} + c_1$$

$x = 1 \quad f'(1) = g'(1) + 3 \times (1)^2 + c_1$

$9 = 3 + 3 + c_1$

$c_1 = 3$

$\therefore \quad f'(x) = g'(x) + 3x^2 + 3$

$$f(x) = g(x) + \frac{3x^3}{3} + 3x + c_2$$

$x = 2 \quad f(2) = g(2) + (2)^3 + 3(2) + c_2$

$12 = 4 + 8 + 6 + c_2$

$c_2 = -6$

$\therefore \quad f'(x) = g(x) + x^3 + 3x - 6$

$= x^3 + 3x - 6$

Option (1)

$x_0 \in \left(1, \frac{3}{2}\right)$ such that $f(x_0) = 9(x_0)$

$\because \quad F(1) = f(1) - g(1) \qquad F\left(\frac{3}{2}\right) = f\left(\frac{3}{2}\right) - g\left(\frac{3}{2}\right)$

$= 1 + 3 - 6 = -2 \qquad = (2)^3 + 3(2) - 6$

$= 8 + 6 - 6 = 8$

$\because \quad F(1)\ F\left(\frac{3}{2}\right) < 0$

$\Rightarrow$ At least one root of F(x) = 0 lies in $\left(1, \frac{3}{2}\right)$

$\Rightarrow \quad f(x) - g(x) = 0$

$\Rightarrow \quad f(x) = g(x)$

Option (2)

$| f'(x) - g'(x)| < 6 \Rightarrow -1 < x < 1$

$F'(x) = x^3 + 3x - 6$

$F'(x) = 3x^2 + 3$

$f'(x) - g'(x) = 3x^2 + 3$

$|f'(x) - g'(x)| < 6$

$\Rightarrow \quad 3x^2 + 3 < 6$

$\Rightarrow \quad 3x^2 <x\ 3$

$x^2 < 1$

$\Rightarrow \quad x \in (-1, 1)$

Option (3)

If $-1 < x < 2$ then $|f(x) - g(x)| < 8$

$F(x) = x^3 + 3x - 6$

$F(-1) = -1 -3 - 6 = -10$ But $|f'(x) - g'(x)| < 10$

$F(2) = (2)^3 + 3(2) - 6 = 8$

Option is not true

Option (4)

$g(-2) - f(-2) = 20$

$F(-2) = f(-2) - g(-2)$

$= (-2)^3 + 3(-2) - 6$

$-8 -6 -6 = -20$

$g(-2) - f(-2) = 20$

77. If the tangent at a point P on the parabola $y^2 = 3x$ is parallel to the line $x + 2y = 1$ and the tangents at the points Q and R on the ellipse $\frac{x^2}{4} + \frac{y^2}{1} = 1$ are perpendicular to the line $x - y = 2$, then the area of the triangle PQR is :

(1) $\frac{3}{2}\sqrt{5}$ (2) $3\sqrt{5}$ (3) $\frac{9}{\sqrt{5}}$ (4) $5\sqrt{3}$

Sol. **2**

$x + 2y = 1$ $\qquad$ $y^2 = 3x$

$m = -\frac{1}{2}$ $\qquad$ $T_P : y = -\frac{1}{2}x + \frac{\frac{3}{4}}{-\frac{1}{2}}$

$y = -\frac{x}{2} - \frac{3}{2}$

$2y + x + 3 = 0 \quad \ldots(1)$

$x - y = 2$ $\qquad$ $E : \frac{x^2}{4} + \frac{y^2}{1} = 1$

$m = 1$ $\qquad$ $y = -x \pm \sqrt{(-1)^2 4 + 1}$

slope of tangent at Q & R is –1 $\qquad$ $y = -x \pm \sqrt{5}$

$x + y = \sqrt{5} \quad \ldots(2) \qquad x + y = -\sqrt{5} \quad \ldots(3)$

Point P :

T = O

$yy_1 = \frac{3}{2}(x+x_1)$

$3x - 2yy_1 + 3x_1 = 0$

Comparison with (1)

$\frac{3}{1} = \frac{-2y_1}{2} = \frac{3x_1}{3}$

$y_1 = -3$, $x_1 = 3$

Point Q :

$\frac{xx_2}{4} + \frac{yy_2}{1} = 1$

$xx_2 + 4yy_2 - 4 = 0$

$\frac{x_2}{1} = \frac{4y_2}{1} = \frac{-4}{-\sqrt{5}}$

$x_2 = \frac{4}{\sqrt{5}} - y_2 = \frac{1}{\sqrt{5}}$

Point R:

$\frac{x_2}{1} = \frac{4y_2}{1} = \frac{-4}{\sqrt{5}}$

$x_2 = \frac{-4}{\sqrt{5}}$, $y = \frac{1}{\sqrt{5}}$

Area of ΔPQR

$$= \frac{1}{2}\begin{vmatrix} 3 & -3 & 1 \\ \frac{4}{\sqrt{5}} & \frac{1}{\sqrt{5}} & 1 \\ -\frac{4}{\sqrt{5}} & -\frac{1}{\sqrt{5}} & 1 \end{vmatrix}$$

$$\Rightarrow \frac{1}{2}\left[3\left(\frac{1}{\sqrt{5}}+\frac{1}{\sqrt{5}}\right)+3\left(\frac{4}{\sqrt{5}}+\frac{4}{\sqrt{5}}\right)+1\left(-\frac{4}{5}+\frac{4}{5}\right)\right]$$

$$\Rightarrow \frac{1}{2}\left[\frac{6}{\sqrt{5}}+\frac{24}{\sqrt{5}}\right]$$

$$\Rightarrow \frac{1}{2}\times\frac{30}{\sqrt{5}} = \frac{5\times 3}{\sqrt{5}} = 3\sqrt{5}$$

78. Let $\vec{a} = 4\hat{\imath} + 3\hat{\jmath}$ and $\vec{b} = 3\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$. If $\vec{c}$ is a vector such that $\vec{c}\cdot(\vec{a}\times\vec{b}) + 25 = 0, \vec{c}\cdot(\hat{\imath}+\hat{\jmath}+\hat{k}) = 4$, and projection of $\vec{c}$ on $\vec{a}$ is 1 , then the projection of $\vec{c}$ on $\vec{b}$ equals

(1) $\frac{1}{5}$ (2) $\frac{5}{\sqrt{2}}$ (3) $\frac{3}{\sqrt{2}}$ (4) $\frac{1}{\sqrt{2}}$

Sol. **(2)**

Let $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

$\vec{c}\,.\,\vec{a}\times\vec{b}$

$$\begin{vmatrix} c_1 & c_2 & c_3 \\ 4 & 3 & 0 \\ 3 & -4 & 5 \end{vmatrix} = -25$$

$\Rightarrow c_1(15-0) - c_2(20-0) + c_3(-16-9) = -25$

$\Rightarrow 15c_1 - 20c_2 - 25c_3 = -25$

$\Rightarrow 3c_1 - 4c_2 - 5c_3 = -5$ …(2)

$\vec{c}\cdot(i+j+k) = 4$

$c_1 + c_2 + c_3 = 4$ …(i)

Proj. of $\vec{c}$ on $\vec{a} = \frac{\vec{a}.\vec{c}}{|\vec{a}|} = 1$

$\Rightarrow \frac{(4\hat{i}+3\hat{j})(c_1\hat{i}+c_2\hat{j}+c_3\hat{k})}{\sqrt{16+9}} = 1$

$\Rightarrow 4c_1 + 3c_2 = 5$

$\Rightarrow 4c_1 = 5 - 3c_2$

$\Rightarrow c_1 = \frac{5-3c_2}{4} \qquad \ldots(3)$

Eq^n. (1) & (3)

$\frac{5-3c_2}{4} + c_2 + c_3 = 4$

$5 - 3c_2 + 4c_2 + 4c_3 = 16$

$c_2 + 4c_3 = 11 \quad \ldots(4)$

Eq^n. (2) & (3)

$3\left(\frac{5-3c_2}{4}\right) - 4c_2 - 5c_3 = -5$

$15 - 9c2 - 16c2 - 20c_3 = -20$

$-25c_2 - 20c_3 = -35 \qquad \ldots(5)$

Eq^n. (4) & (5)

$c_2 = 11 - 4c_3$

$c_2 = 11 - 4 \times 3$

$= 11 - 12$

$c_2 = -1$

$c_1 = \frac{5-3c_2}{4}$

$= \frac{5-3(-1)}{4}$

$c_1 = 2$

$-25c_2 - 20c_3 = -35$

$-25(11 - 4c_3) - 20c_3 = -35$

$5(11 - 4c_3) + 4c_3 = 7$

$55 - 20c_3 + 4c_3 = 7$

$-16c_3 = -48$

$c_3 = 3$

$\vec{c} = 2\hat{i} - \hat{j} + 3\hat{k}$

Projection of $\vec{c}$ on $\vec{b}$ $= \left|\frac{\vec{c}.\vec{b}}{|\vec{b}|}\right|$

$\Rightarrow \left|\frac{(2\hat{i}-\hat{j}+3\hat{k}).(3\hat{i}-4\hat{j}+5\hat{k})}{\sqrt{9+16+25}}\right|$

$\Rightarrow \left|\frac{6+4+15}{5\sqrt{2}}\right| = \frac{25}{5\sqrt{2}} = \frac{5}{\sqrt{2}}$

79. Let $S = \{w_1, w_2, \ldots\ldots\}$ be the sample space associated to a random experiment. Let $P(w_n) = \frac{P(w_{n-1})}{2}, n \geq 2$. Let $A = \{2k + 3l : \boldsymbol{k}, l \in \mathbb{N}\}$ and $B = \{w_n : n \in A\}$. Then $P(\,B)$ is equal to

(1) $\frac{3}{64}$ (2) $\frac{1}{16}$ (3) $\frac{1}{32}$ (4) $\frac{3}{32}$

Sol. **1**

A = {5, 7, 8, 9, 10, 11 ......}

$P(W_1) + P(W_2) + P(W_3) + \ldots\ldots P(W_n) = 1$

$P(W_1) + \frac{P(W_1)}{2} + \frac{P(W_2)}{2^2} + \ldots\ldots = 1$

$\Rightarrow P(W_1).\left(\frac{1}{1-\frac{1}{2}}\right) = 1$

$\boxed{P(W_1)=\frac{1}{2}}$ $\qquad P(W_n)=\frac{1}{2}\cdot\left(\frac{1}{2}\right)^{n-1}=\frac{1}{2^n}$

$\because B = \{W_n : n \in A\}$

$= \{W_5, W_7, W_8, \ldots\ldots.\}$

$P(B) = P(W_5) + P(W_7) + P(W_8) + P(W_9) + P(W_{10}) + P(W_{11})$

$$= \frac{1}{2^5}+\frac{1}{2^7}+\frac{1}{2^8}+\ldots..$$

$$= \frac{1}{32}+\frac{\frac{1}{2^7}}{1-\frac{1}{2}}$$

$$= \frac{1}{32}+\frac{1}{2^7}\times 2$$

$$= \frac{1}{32}+\frac{1}{64}=\frac{2+1}{64}=\frac{3}{64}$$

80. Let K be the sum of the coefficients of the odd powers of x in the expansion of $(1+x)^{99}$. Let a be the middle term in the expansion of $\left(2+\frac{1}{\sqrt{2}}\right)^{200}$. If $\frac{{}^{200}C_{99}\,K}{a}=\frac{2^l\,m}{n}$, where m and n are odd numbers, then the ordered pair (l,n) is equal to

(1) $(50,51)$ (2) $(50,101)$ (3) $(51,99)$ (4) $(51,101)$

Sol. **(2)**

$$K=\frac{(1+1)^{99}}{2}=2^{98}$$

$$a={}^{200}C_{100}2^{100}\times\frac{1}{\left(\sqrt{2}\right)^{100}}$$

$$a={}^{200}C_{100}2^{50}$$

$$\frac{{}^{200}C_{99}.K}{a}=\frac{{}^{200}C_{99}.2^{98}}{{}^{200}C_{100}.2^{50}}$$

$$\because \frac{{}^{200}C_{99}}{{}^{200}C_{100}}=\frac{1200}{\underline{|99}\underline{|101}}\times\frac{\underline{|100}\underline{|100}}{\underline{|200}}$$

$$=\frac{100}{101}$$

$$\therefore \frac{{}^{200}C_{99}K}{a}=\frac{100}{101}\times 2^{48}$$

$$=\frac{25\times 2^{50}}{101}$$

$\ell = 50$, $n = 101$

$(\ell, n) = (50, 101)$

Section B

81. The total number of 4-digit numbers whose greatest common divisor with 54 is 2 , is

Sol. **3000**

$54 = 2 \times 3^3$

4 digit even numbers are

9	10	10	5

4500

4 digit even numbers which are multiple of 3

Are the numbers which are multiple of 6

$= \frac{9000}{6} = 1500$

$\therefore$ The no. which has GCD with 54 us 2 is 4500 – 1500 = 3000

82. Let $a_1 = b_1 = 1$ and $a_n = a_{n-1} + (n-1), b_n = b_{n-1} + a_{n-1}, \forall n \geq 2$. If $S = \sum_{n=1}^{10} \frac{b_n}{2^n}$ and $T = \sum_{n=1}^{8} \frac{n}{2^{n-1}}$, then $2^7(2S - T)$ is equal to

Sol. **461**

$$T = \frac{1}{2^0} + \frac{2}{2^1} + \frac{3}{2^2} + \frac{4}{2^3} + \ldots\ldots\ldots\ldots \frac{8}{2^7} \quad \ldots..(1)$$

$$S = \frac{b_1}{2} + \frac{b_2}{2^2} + \frac{b_3}{2^3} + \frac{b_4}{2^4} + \ldots\ldots\ldots\ldots \frac{b_{10}}{2^{10}}$$

$$\frac{S}{2} = \frac{b_1}{2^2} + \frac{b_2}{2^3} + \frac{b_3}{2^4} + \ldots\ldots\ldots\ldots \frac{b_9}{2^{10}} + \frac{b_{10}}{2^{11}} \text{ (Subtract)}$$

$$\frac{S}{2} = \frac{b_1}{2} + \left(\frac{b_2 - b_1}{2^2}\right) + \left(\frac{b_3 - b_2}{2^3}\right) + \left(\frac{b_4 - b_3}{2^4}\right) + \ldots\ldots + \left(\frac{b_{10} - b_9}{2^{10}}\right) - \frac{b_{10}}{2^{11}}$$

$$\frac{S}{2} = \frac{b_1}{2} + \frac{a_1}{2^2} + \frac{a_2}{2^3} + \frac{a_3}{2^4} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots \frac{a_9}{2^{10}} - \frac{b_{10}}{2^{11}}$$

$$\Rightarrow S = b_1 + \frac{a_1}{2} + \frac{a_2}{2^2} + \frac{a_3}{2^3} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots \frac{a_9}{2^{10}} - \frac{b_{10}}{2^{10}}$$

$$S = \left(b_1 - \frac{b_{10}}{2^{10}}\right) + \left(\frac{a_1}{2} + \frac{a_2}{2^2} + \ldots\ldots\ldots \frac{a_9}{2^9}\right)$$

$$\Rightarrow \frac{S}{2} = \left(\frac{b_1}{2} - \frac{b_{10}}{2^{11}}\right) + \left(\frac{a_1}{2^2} + \frac{a_2}{2^3} + \ldots\ldots\ldots \frac{a_8}{2^9} + \frac{a_9}{2^{10}}\right) \text{ (Subtract)}$$

$$\frac{S}{2} = \frac{b_1}{2} - \frac{b_{10}}{2^{11}} + \left(\frac{a_1}{2} - \frac{a_9}{2^{10}}\right) + \left(\frac{1}{2^2} + \frac{2}{2^3} + \ldots\ldots\ldots\ldots\ldots \frac{8}{2^9}\right)$$

$$= \frac{b_1}{2} + \frac{a_1}{2} - \left(\frac{b_{10} + 2a_9}{2^{11}}\right) + \frac{T}{4} \qquad \text{from } \ldots\ldots\ldots(1)$$

Now $2s = 2(a_1 + b_1) - \left(\frac{b_{10} + 2a_9}{2^9}\right) + T$

$2S - T = 2(a_1 + b_1) - \left(\frac{b_{10} + 2a_9}{2^9}\right)$

$$\boxed{2^7(2S-T)=2^8(a_1+b_1)-\frac{b_{10}+2a_9}{4}} \qquad \text{……………} \ (2)$$

$\because\ a_n - a_{n-1} = n-1$

$a_2 - a_1 = 1$

$a_3 - a_2 = 2$

$a_4 - a_3 = 3$

……

……

$a_9 - a_8 = 8$

$a_9 - a_1 = 1 + 2 + 3 + \ldots\ldots\ldots\ldots + 8$

$\boxed{a_9 = 36 + 1 = 37}$

and $b_n = b_{n-1} = a_{n-1}$

$b_{10} - b_1 = a_1 + a_2 + a_3 + \ldots\ldots + a_9$

$b_{10} - 1 = 1 + 2 + 4 + 7 + 11 + 16 + 22 + 29 + 37$

$b_{10} - 1 = 29$

$\boxed{b_{10} = 130}$

$$\boxed{\begin{aligned}&2^7(2S-T)=2^8\times(1+1)-\frac{130+2\times37}{4}\\&=2^9-\frac{102}{2}\\&=512-51=461\end{aligned}}$$

83. A triangle is formed by the tangents at the point $(2,2)$ on the curves $y^2 = 2x$ and $x^2 + y^2 = 4x$, and the line $x + y + 2 = 0$. If r is the radius of its circumcircle, then r^2 is equal to

Sol. **10**

Tangent at $y^2 = 2x$

$T\!: 2y = 2\left(\frac{x+2}{2}\right)$

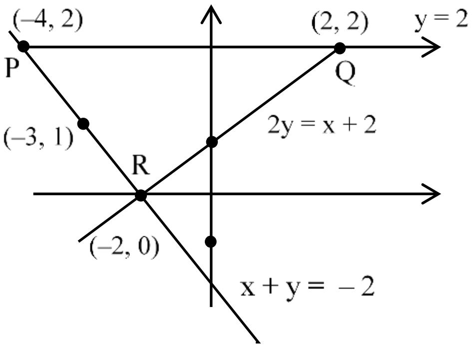
$2y = x + 2$

Tangent at $x^2 + y^2 = 4x$

$2x + 2y = \frac{4\times(x+2)}{2}$

$2x + 2y = 2x + 4$

$y = 2$



$M_{PR} = -1$

Slope of $\perp^{r\ell}$ Bisector = 1

$y - 1 = 1\ (x + 3)$

$y = x + 3 + 1$

$y = x + 4$

$\perp^{r\ell}$ Bisector of PQ

$x = -1$

$\therefore$ Centre is

$y = -1 + 4 = 3$

$(-1, 3)$

Radius: $r = \sqrt{(-1+4)^2 + (3-2)^2}$

$= \sqrt{9+1}$

$= \sqrt{10}$

$r^2 = 10$

84. Let $\alpha_1, \alpha_2, \ldots., \alpha_7$ be the roots of the equation $x^7 + 3x^5 - 13x^3 - 15x = 0$ and $|\alpha_1| \geq |\alpha_2| \geq \cdots \geq |\alpha_7|$. Then $\alpha_1\alpha_2 - \alpha_3\alpha_4 + \alpha_5\alpha_6$ is equal to

Sol. **3**

$\alpha_1, \alpha_2, \ldots\ \alpha_7$

$x^7 + 3x^5 - 13x^3 - 15x = 0$

$x(x^6 - 3x^4 - 13x^2 - 15) = 0$

$|\alpha_1| \geq |\alpha_2| \geq \ldots \geq |\alpha_7|$

$\alpha_1\alpha_2 - \alpha_3\alpha_4 + \alpha_3\alpha_6 = ?$

$\boxed{\alpha_7 = 0}$

$\Rightarrow x(x^6 + 3x^4 - 13x^2 - 15) = 0$

$x = 0\ x^6 + 3x^4 - 13x^2 - 15 = 0$

$\Rightarrow t^3 + 3t^2 - 13t - 15 = 0$

$\Rightarrow (t - 3)\ (t^2 + 6t + 5) = 0$

$t = 3, t = -5, -1$

$x = 0,\ x = \pm\sqrt{3},\ x = \pm\sqrt{5}i,\ x = \pm i$

$\alpha_1 = \sqrt{5}i$

$\alpha_2 = -\sqrt{5}i$

$\alpha_3 = \sqrt{3}$

$\alpha_4 = -\sqrt{3}$

$\alpha_5 = i$

$\alpha_6 = -i$

$\alpha_7 = 0$

$\alpha_1\alpha_2 = 5,\ \alpha_3\alpha_4 = 3,\ \alpha_5\alpha_6 = 1$

$\alpha_1\alpha_2 - \alpha_3\alpha_4 + \alpha_5\alpha_6$

$\Rightarrow 5 - 3 + 1 = 3$

85. Let $X = \{11,12,13, \dots ,40,41\}$ and $Y = \{61,62,63, \dots ,90,91\}$ be the two sets of observations. If $\bar{x}$ and $\bar{y}$ are their respective means and σ^2 is the variance of all the observations in $X \cup Y$, then $|\bar{x} + \bar{y} - \sigma^2|$ is equal to

Sol. **603**

$$\bar{x} = \frac{11+12+........+41}{31} \qquad \bar{y} = \frac{61+62+63+.....+91}{31}$$

$$= \frac{\frac{31}{2}(11+41)}{31} = \frac{52}{2} = 26 \qquad = \frac{\frac{31}{2}(61+91)}{31} = \frac{152}{2} = 76$$

$$\sigma^2 = \frac{\sum x_i^2 + \sum y_i^2}{31+31} - \bar{x}^2$$

$$= \frac{\left(\sum_{n=1}^{41} n^2 - \sum_{n=1}^{10} n^2\right) + \left(\sum_{n=1}^{91} n^2 - \sum_{n=1}^{60} n^2\right)}{62} - \left(\frac{31\times 26 + 76\times 31}{62}\right)^2$$

$$\Rightarrow \frac{\frac{41\times 42\times 83}{6} - \frac{10\times 11\times 21}{6} + \frac{91\times 92\times 183}{6} - \frac{60\times 61\times 121}{6}}{62} - (51)^2$$

$$\Rightarrow \frac{7(41\times 83 - 55) + 61(91\times 46 - 1210)}{62}$$

$$\Rightarrow \frac{7(3403 - 55) + 61(4186 - 1210)}{62}$$

$$\Rightarrow \frac{7\times 3348 + 61\times 2976}{62} - 2601$$

$\Rightarrow 3306 - 2601 = 705 \Rightarrow \sigma^2 = 705$

$\therefore |\bar{x} + \bar{y} - \sigma^2| = |26 + 76 - 705| = 603$

86. If the equation of the normal to the curve $y = \frac{x-a}{(x+b)(x-2)}$ at the point $(1,-3)$ is $x - 4y = 13$, then the value of $a + b$ is equal to

Sol. **– 6**

(1, –3) is on the curve

$$\therefore -3 = \frac{1-a}{(1+b)(1-2)} \Rightarrow -3 = \frac{1-a}{(-1)(1+b)}$$

$\Rightarrow 3 + 3b = 1 - a \Rightarrow a + 3b = -2$

$a = -2 - 3b$

$\ell n y = \ell n (x - a) - \ell n(x + b) - \ell n(x - 2)$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x-a} - \frac{1}{x+b} - \frac{1}{x-2}$$

$$\left.\frac{dy}{dx}\right|_{(1,-3)} = -3\left(\frac{1}{1-a} - \frac{1}{1+b} - \frac{1}{1-2}\right) = -4$$

$$\Rightarrow \left(\frac{1}{1+2+3b} - \frac{1}{1+b} + 1\right) = \frac{4}{12} = \frac{1}{3}$$

$\Rightarrow \dfrac{1}{3(b+1)} - \dfrac{1}{b+1} = \dfrac{1}{3} - 1$

$\Rightarrow \dfrac{1-3}{3(b+1)} = -\dfrac{2}{3}$

$\Rightarrow$ b + 1 = 3

b = 2

a = – 2– 3b

a = –2 – 3 × 2

a = – 2 – 6 = – 8

a + b

$\Rightarrow$ –8 + 2

$\Rightarrow$ – 6

87. Let A be a symmetric matrix such that $|A| = 2$ and $\begin{bmatrix} 2 & 1 \\ 3 & \frac{3}{2} \end{bmatrix} A - \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$.

If the sum of the diagonal elements of A is s, then $\frac{\beta s}{\alpha^2}$ is equal to

Sol. **5**

A be a symmetric matrix such that |A| = 2 and $\begin{bmatrix} 2 & 1 \\ 3 & 3/2 \end{bmatrix} A = \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$

$A = \begin{bmatrix} a & b \\ b & d \end{bmatrix}$ $\quad |A| = ad - b^2 = 2$

$$\begin{bmatrix} 2 & 1 \\ 3 & 3/2 \end{bmatrix} \begin{bmatrix} a & b \\ b & d \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$$

$$\begin{bmatrix} 2a+b & 2b+d \\ 3a+3/2\,b & 3b+3/2\,d \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$$

2a + b = 1

b = 1 – 2a

2b + d = 2

d = 2 – 2b

= 2 – 2(1 – 2a)

= 2 – 2 +4a

$ad - b^2 = 2$

$a.4a - (1 - 2a)^2 = 2$

$\Rightarrow 4a^2 - 1 - 4a^2 + 4a = 2$

4a = 3

$a = \dfrac{3}{4}$

$b = 1 - 2 \times \dfrac{3}{4}$

$= \dfrac{-1}{2}$

$d = 4 \times \dfrac{3}{4} = 3$

Now $\alpha = 3a + \dfrac{3}{2}b$

$= \dfrac{9}{4} + \dfrac{3}{2} \cdot \left(\dfrac{-1}{2}\right)$

$= \dfrac{9-3}{4} = \dfrac{6}{4} = \dfrac{3}{2}$

$\beta = 3b + \dfrac{3}{2}d$

$3 \times \left(\dfrac{-1}{2}\right) + \dfrac{3}{2} \times 3$

$\dfrac{-3+9}{2} = 3$

$$A=\begin{bmatrix}3/4 & -1/2\\ -1/2 & 3\end{bmatrix}\quad s=\frac{3}{4}+3=\frac{15}{4}$$

$$\frac{Bs}{\alpha^2}=\frac{3\times\frac{15}{4}}{\frac{9}{4}}=5$$

88. Let $\alpha = 8-14i, A=\left\{z\in\mathbb{C}: \frac{\alpha z-\bar{\alpha}\bar{z}}{z^2-(\bar{z})^2-112i}=1\right\}$ and $B=\{z\in\mathbb{C}: |z+3i|=4\}$.

Then $\sum_{z\in A\cap B}(Re\, z - Im\, z)$ is equal to

Sol. **14**

$$\alpha=8-14i\quad A=\left\{z\in c:\frac{\alpha z-\overline{\alpha z}}{z^2-\left(\bar{z}\right)^2-112i}=1\right\}$$

$z^2-\bar{z}^2$

$\Rightarrow\left(z+\bar{z}\right)\left(z-\bar{z}\right)\Rightarrow 2x\cdot 2yi=4xyi$

$\alpha z=(8-14i)(x+iy)$

$= 8x - 8iy + 14xi + 14y$

$= (8x + 14y) + i(8y - 14x)$

$\overline{\alpha z} = (8 + 14i)(x - iy)$

$= 8x - 8iy + 14ix + 14y$

$= (8x + 14y) + i(14x - 8y)$

$$\frac{\alpha z-\overline{\alpha z}}{z^2-\bar{z}^2-112i}=1$$

$$\Rightarrow\frac{(16y-28x)i}{4xyi-112i}=1$$

$\Rightarrow 4y - 7x = xy - 28$

$\Rightarrow 4y - 7x - xy + 28 = 0$

$\Rightarrow y(4 - x) - 7(x - 4) = 0$

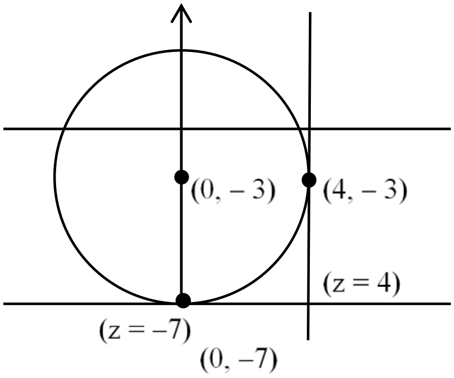
$(x - 4)(-y - 7) = 0$

$x = 4\quad y = -7$

$z = 4$ or $z = -7i$

$B \Rightarrow |z + 3i| = 4$

$x^2 + (y - 3)^2 = 16$



$\sum \text{Re}(z)-\text{Im}(z)$

$= (0 + 4) - (-7 - 3)$
$= 4 + 10 = 14$

89. A circle with centre (2,3) and radius 4 intersects the line $x + y = 3$ at the points P and Q. If the tangents at P and Q intersect at the point $S(\alpha, \beta)$, then $4\alpha - 7\beta$ is equal to

Sol. **11**

$(x-2)^2 + (y-3)^2 = 16$
$x^2 + y^2 - 4x - 6y - 3 = 0$
let the chord of contact w.r.t. to the point $S(\alpha,\beta)$ is
$T = 0$
$\alpha x + \beta y - 2(x + \alpha) - 3(y + \beta) - 3 = 0$
$x(\alpha - 2) + y(\beta - 3) - 2\alpha - 3\beta - 3 = 0$
Comparison with $x + y = 3$

$$\frac{\alpha-2}{1} = \frac{\beta-3}{1} = \frac{-2\alpha-3\beta-3}{-3}$$

$\alpha - 2 = \beta - 3 \qquad -3\beta + 9 = -2\alpha - 3\beta - 3$
$\alpha - \beta = -1 \qquad 2\alpha = -3-9$
$\beta = \alpha + 1 \qquad \alpha = \frac{-12}{2}$
$= -6 + 1 = -5 \qquad \alpha = -6$
$4\alpha - 7\beta$
$4(-6) - 7(-5)$
$\Rightarrow -24 + 35 = 11$

90. Let $\{a_k\}$ and $\{b_k\}, k \in \mathbb{N}$, be two G.P.s with common ratios r_1 and r_2 respectively such that $a_1 = b_1 = 4$ and $r_1 < r_2$. Let $c_k = a_k + b_k, k \in \mathbb{N}$. If $c_2 = 5$ and $c_3 = \frac{13}{4}$ then $\sum_{k=1}^{\infty} c_k - (12a_6 + 8b_4)$ is equal to

Sol. **9**

$a_1 = 4 \qquad$ GP $4, 4r_1, 4r_1^2$ - -
$b_1 = 4 \qquad$ GP $4, 4r_2, 4r_2^2$ - -
$C_2 = a_2 + b_2 \qquad C_3 = a_3 + b_3$
$5 = 4r_1 + 4r_2 \qquad \frac{13}{4} = 4r_1^2 + 4r_2^2$
$\frac{5}{4} = r_1 + r_2 \ldots(1) \qquad r_1^2 + r_2^2 = \frac{13}{16} \ldots (2)$

$$\frac{25}{16} = r_1^2 + r_2^2 + 2r_1r_2$$

$$\frac{25}{16} = \frac{13}{16} + 2r_1r_2$$

$$\Rightarrow r_1r_2 = \frac{12}{16 \times 2} = \frac{3}{8}$$

Now $r_1 + \frac{3}{8r_1} = \frac{5}{4}$

$8r_1^2 + 3 = 10\, r_1$

$\Rightarrow 8r_1^2 - 10r_1 + 3 = 0$

$r_1 = \frac{3}{4}, r_1 = \frac{1}{2}$

$r_2 = \frac{1}{2} \quad r_2 = \frac{3}{4}$

$\because r_1 < r_2$

$$\therefore \begin{matrix} r_1 = \frac{1}{2} \\ r_2 = \frac{3}{4} \end{matrix}$$

Now $C_k = a_k + b_k$

$$\sum_{k=1}^{\infty} C_k = \frac{4}{1-r_1} + \frac{4}{1-r_2}$$

$$= \frac{4}{1-\frac{1}{2}} + \frac{4}{1-\frac{3}{4}}$$

$= 8 + 16 = 24$

$$\sum_{k=1}^{\infty} C_k - (12a_6 + 8b_4) \Rightarrow 24 - \left\{12 \times 4\left(\frac{1}{2}\right)^5 + 8 \times 4\left(\frac{3}{4}\right)^3\right\}$$

$$= 24 - \left(12 \times \frac{1}{8} + 8 \times \frac{27}{16}\right)$$

$$= 24 - \left\{\frac{3}{2} + \frac{27}{2}\right\}$$

$= 24 - 15$

$= 9$

JEE–MAIN EXAMINATION – JANUARY, 202

(Held On Thursday 30th January, 2023) | **TIME : 9 : 00 AM to 12 : 00 NOON**

Physics

SECTION - A

1. The magnetic moments associated with two closely wound circular coils A and B of radius $r_A = 10$ cm and $r_B = 20$ cm respectively are equal if : (Where N_A, I_A and N_B, I_B are number of turn and current of A and B respectively)

(1) $4\ N_A I_A = N_B I_B$ (2) $N_A = 2\ N_B$ (3) $N_A I_A = 4\ N_B I_B$ (4) $2\ N_A I_A = N_B I_B$

Sol. **(3)**

Magnetic moment m = IAN

Magnetic moment of coil A →

$$m_A = I_A \pi r_A^2 N_A$$

$$m_A = I_A \pi N_A (10)^2 \qquad ...(1)$$

Magnetic moment of coil B →

$$m_B = I_B N_B \pi r_B^2$$

$$m_B = I_B N_B \pi (20)^2 \qquad ...(2)$$

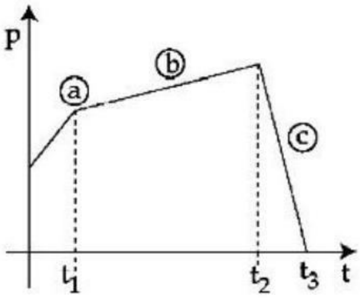
Now $m_A = m_B$

$$I_A .\pi N_A (100) = I_B N_B \pi 400$$

$$\boxed{I_A N_A = 4 I_B N_B}$$

2. The figure represents the momentum time (p-t) curve for a particle moving along an axis under the influence of the force. Identify the regions on the graph where the magnitude of the force is maximum and minimum respectively ?

If $(t_3 - t_2) < t_1$



(1) c and b (2) b and c (3) *a* and *b* (4) c and a

Sol. **(1)**

Slope of curve P-t will represent the force so

$$F = \frac{dP}{dt} = \text{slope}$$

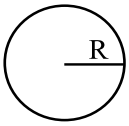
Maximum slope → (c)

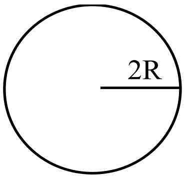
Minimum slope → (b)

3. Two isolated metallic solid spheres of radii *R* and 2R are charged such that both have same charge density σ. The spheres are then connected by a thin conducting wire. If the new charge density of the bigger sphere is σ'. The ratio $\frac{\sigma'}{\sigma}$ is :

(1) $\frac{4}{3}$ (2) $\frac{5}{3}$ (3) $\frac{5}{6}$ (4) $\frac{9}{4}$

Sol. **(3)**



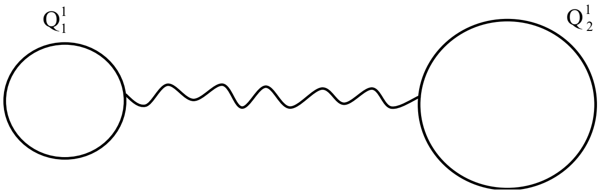


$Q_2 = \sigma 4\pi(2R)^2$

$Q_2 = \sigma 16\pi R^2$

$Q_1 = \sigma 4\pi R^2$

Now



Charge will flow until voltage of both sphere become equal so

$$c = 4\pi\varepsilon_0 R$$

$$v_1^1 = v_2^1$$

$$\frac{Q_1}{c_1} = \frac{Q_2}{c_2} \Rightarrow \frac{Q_1'}{4\pi\varepsilon_0 R} = \frac{Q_2'}{4\pi\varepsilon_0(2R)}$$

$$\Rightarrow \quad 2Q_1' = Q_2' \quad \ldots(1)$$

$$Q_1 + Q_2 = Q_1' + Q_2'$$

$$\sigma 20\pi R^2 = Q_2' + \frac{Q_2'}{2} = \frac{3}{2}Q_2' \Rightarrow Q_2' = \frac{\sigma 40\pi R^2}{3} \quad \ldots(2)$$

$$Q_2' = \frac{\sigma 40\pi R^2}{3}$$

Now $\sigma' 4\pi(2R)^2 = \frac{\sigma 40\pi R^2}{3}$

$$\sigma' 16\pi R^2 = \frac{\sigma 40\pi R^2}{3}$$

$$\frac{\sigma'}{\sigma} = \frac{40}{3} \times \frac{1}{16} = \frac{5}{6}$$

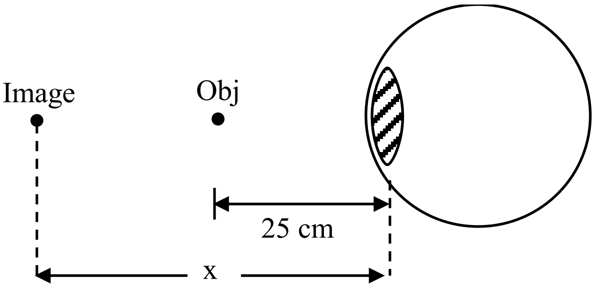
4. A person has been using spectacles of power −1.0 dioptre for distant vision and a separate reading glass of power 2.0 dioptres. What is the least distance of distinct vision for this person :

(1) 40 cm (2) 30 cm (3) 10 cm (4) 50 cm

Sol. **(4)**

Power convex lens = 2.0 D

Power of concave lens = –1.0 D



x → least distance of distinct vision

$$f = \frac{1}{2} \times 100 = 50\text{cm}$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{50} = \frac{1}{(-x)} - \frac{1}{-25} \quad \Rightarrow \frac{1}{50} - \frac{1}{25} = \frac{1}{(-x)}$$

$$\Rightarrow \quad \frac{1-2}{50} = \frac{-1}{x}$$

$$\Rightarrow \quad \boxed{x = 50\text{cm}}$$

5. A small object at rest, absorbs a light pulse of power 20 mW and duration 300 ns. Assuming speed of light as 3×10^8 m/s, the momentum of the object becomes equal to :

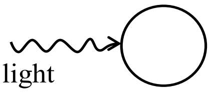
(1) 3×10^{-17} kg m/s (2) 2×10^{-17} kg m/s (3) 1×10^{-17} kg m/s (4) 0.5×10^{-17} kg m/s

Sol. **(2)**

Power = 20 mw

t = 300 nsec

energy absorbed $= 300 \times 10^{-9} \times 20 \times 10^{-3} = 6 \times 10^3 \times 10^{-12} = 6 \times 10^{-9}$ J



$$\text{Pressure} = \frac{\text{Intensity}}{C} = \frac{\text{Power}}{\text{Area} \times C}$$

$$\text{Pressure} \times \text{Area} = \frac{\text{Power}}{C}$$

$$\text{Force} = \frac{\text{Power}}{C} = \frac{20 \times 10^{-3}}{3 \times 10^8}$$

$$F = \frac{20}{3} \times 10^{-11} \text{N}$$

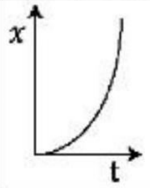
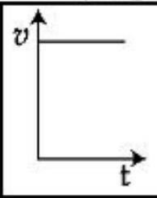
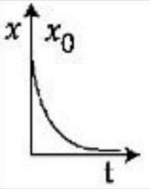
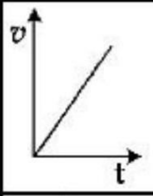
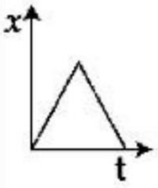
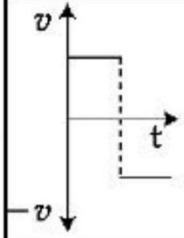
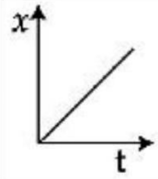
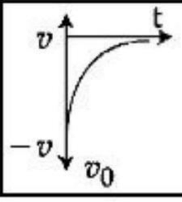
$$F\Delta t = \Delta P \ \text{(momentum)}$$

$$\frac{20}{3} \times 10^{-11} \times 300 \times 10^{-9} = P_f - P_i$$

$$20 \times 10^{-20} \times 100 = P_f$$

$$\boxed{2 \times 10^{-17} = P_f}$$

6. Match Column-I with Column-II :

Column-I (x-t graphs)		Column-II (v-t graphs)	
A.		I.	
B.		II.	
C.		III.	
D.		IV.	

Choose the correct answer from the options given below:

(1) A- I, B-II, C-III, D-IV

(2) A- II, B-III, C-IV, D-I

(3) A- I, B-III, C-IV, D-II

(4) A- II, B-IV, C-III, D-I

Sol. **(4)**

(A) $x \propto t^2$

$\frac{dx}{dt} \propto 2t \Rightarrow \boxed{V \propto t}$ $\quad$ $A \rightarrow II$

(B) $x = x_0 e^{-\alpha t}$

$\frac{dx}{dt} = x_0 e^{-\alpha t}(-\alpha) = -\alpha(x_0 e^{-\alpha t})$

$V = -\alpha x_0 e^{-\alpha t}$ $\quad$ $B \rightarrow IV$

(C) $x \propto t \;\rightarrow\; V = \text{const}$

$x \propto -t \;\rightarrow\; V = -\text{const}$ $\quad C \rightarrow III$

(D) $x \propto t \;\rightarrow\; V = \text{const}$ $\quad$ $D \rightarrow I$

7. The pressure (P) and temperature (T) relationship of an ideal gas obeys the equation PT^2 = constant. The volume expansion coefficient of the gas will be :

(1) $\frac{3}{T^3}$ $\quad$ (2) $\frac{3}{T^2}$ $\quad$ (3) $3\,T^2$ $\quad$ (4) $\frac{3}{T}$

Sol. **(4)**

$PT^2 = \text{const.}$

$dV = V\gamma dT$

$$\gamma = \frac{1}{V}\frac{dV}{dT} \qquad \ldots(1)$$

Using $PV = nRT$ and $PT^2 = \text{cont.}$

$$\frac{nRT}{V}.T^2 = \text{const}$$

$$V \propto T^3 \Rightarrow V = KT^3 \quad \ldots(2)$$

Now put in (1)

$$\gamma = \frac{1}{KT^3}\times 3KT^2 = \frac{3}{T} \quad \Rightarrow \quad \gamma = \frac{3}{T}$$

8. Heat is given to an ideal gas in an isothermal process.

A. Internal energy of the gas will decrease.
B. Internal energy of the gas will increase.
C. Internal energy of the gas will not change.
D. The gas will do positive work.
E. The gas will do negative work.

Choose the correct answer from the options given below :

(1) C and D only (2) C and E only (3) A and E only (4) B and D only

Sol. **(1)**

In isothermal process

$$\Delta T = 0$$

So $\Delta U = 0$

$$\Delta Q = \omega + \Delta U$$

$$\boxed{\Delta Q = \omega}$$

So heat will be used to do positive work

9. If the gravitational field in the space is given as $\left(-\frac{K}{r^2}\right)$. Taking the reference point to be at r = 2 cm with gravitational potential V = 10 J/kg. Find the gravitational potential at r = 3 cm in SI unit (Given, that K = 6Jcm/kg)

(1) 9 (2) 10 (3) 11 (4) 12

Sol. **(3)**

$$\Delta V = -\int_2^3 \vec{E}.d\vec{r}$$

$$V(3) - V(2) = -\int_2^3 \frac{-K}{r^2}.dr$$

$$V(3) - 10 = -K\left(\frac{1}{r}\right)_2^3$$

$$V(3) - 10 = -6\left[\frac{1}{3} - \frac{1}{2}\right]$$

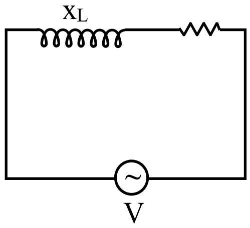
$$V - 10 = -6\left[\frac{2-3}{6}\right] = 1$$

$$\boxed{V = 11}$$

10. In a series LR circuit with $X_L = R$, power factor is P_1. If a capacitor of capacitance C with $X_C = X_L$ is added to the circuit the power factor becomes P_2. The ratio of P_1 to P_2 will be :

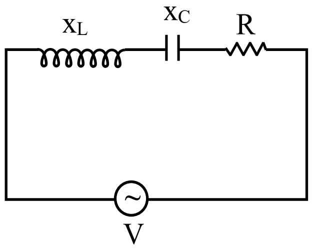
(1) 1:3 (2) 1: 2 (3) $1:\sqrt{2}$ (4) 1: 1

Sol. **(3)**



Power factor = $\cos\phi = \frac{R}{Z}$

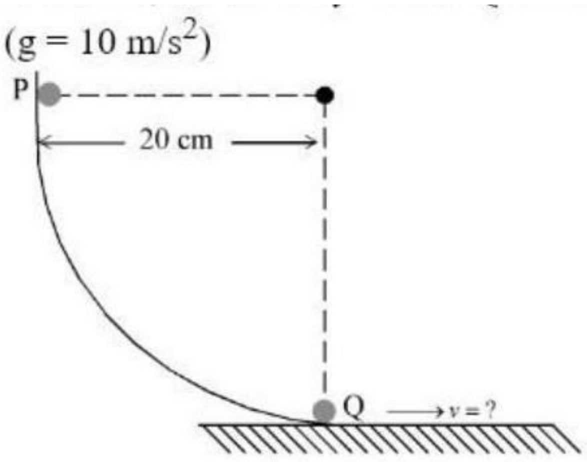
$$P_1 = \frac{R}{\sqrt{x_L^2 + R^2}} = \frac{R}{\sqrt{R^2 + R^2}} = \frac{R}{\sqrt{2}R} = \frac{1}{\sqrt{2}}$$



$$P_2 = \frac{R}{Z} = \frac{R}{\sqrt{(x_L - x_C)^2 + R^2}} = \frac{R}{R} = 1$$

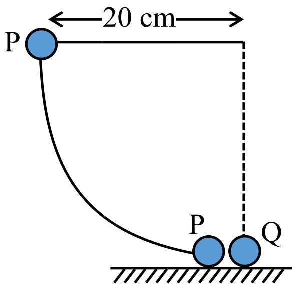
$$\frac{P_1}{P_2} = \frac{1}{\sqrt{2}}$$

11. As per the given figure, a small ball P slides down the quadrant of a circle and hits the other ball Q of equal mass which is initially at rest. Neglecting the effect of friction and assume the collision to be elastic, the velocity of ball Q after collision will be :



(1) 0 (2) 4 m/s (3) 2 m/s (4) 0.25 m/s

Sol. **(3)**



Energy conservation for 'P'

$$mgh = \frac{1}{2}mV^2$$

$$V = \sqrt{2gh}$$

$$V = \sqrt{2 \times 10 \times 0.2}$$

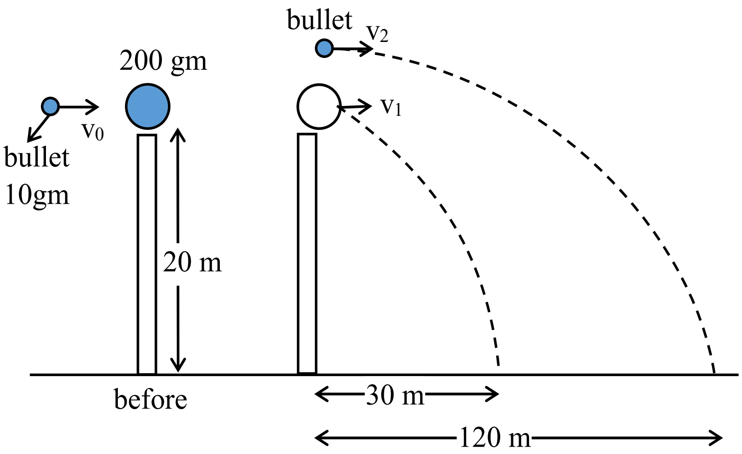
$$\boxed{V = 2m/sec}$$

Now collision between P and Q is elastic and both have same mass then P will transfer all velocity to then Q. So velocity Q will be 2 m/sec

12. A ball of mass 200 g rests on a vertical post of height 20 m. A bullet of mass 10 g, travelling in horizontal direction, hits the centre of the ball. After collision both travels independently. The ball hits the ground at a distance 30 m and the bullet at a distance of 120 m from the foot of the post. The value of initial velocity of the bullet will be (if $g = 10\ m/s^2$) :

(1)360 m/s (2) 400 m/s (3) 60 m/s (4) 120 m/s

Sol. **(1)**



Time to reach ground will be same for both

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 20}{10}} = 2sec$$

Range of bullet = 120

$$120 = v_2(2) \Rightarrow \boxed{v_2 = 60m/sec}$$

Range of ball = 30

$$30 = V_1(2) \quad \Rightarrow \quad \boxed{v_1 = 15\text{m / sec}}$$

Now apply momentum conservation

$$P_i = P_f$$

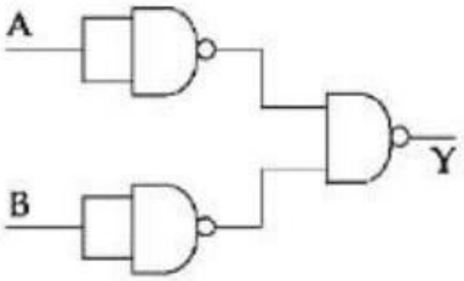
$$P_{ball} + P_{bullet} = P_{ball} + P_{bullet}$$

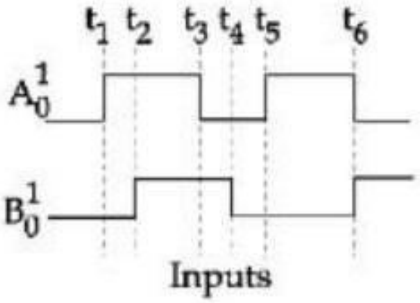
$$0 + \left(\frac{10}{1000}\right)v_0 = \left(\frac{200}{1000}\right)(15) + \left(\frac{10}{1000} \times 60\right)$$

$$10v_0 = 3000 + 600$$

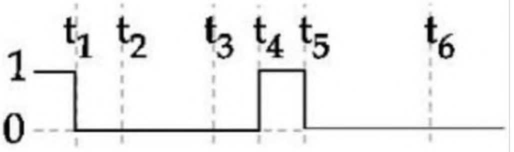
$$v_0 = \frac{3600}{10} \quad \Rightarrow \quad \boxed{v_0 = 360\text{m / sec}}$$

13. The output waveform of the given logical circuit for the following inputs A and B as shown below, is

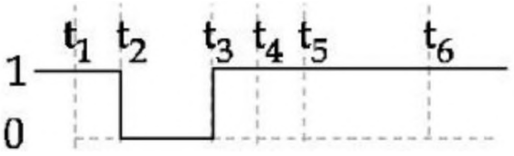




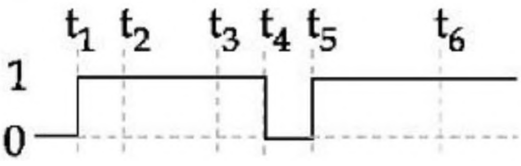
(1)



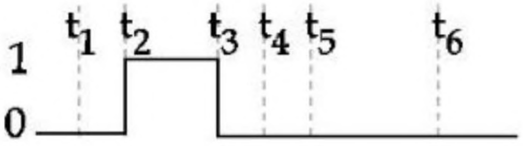
(2)



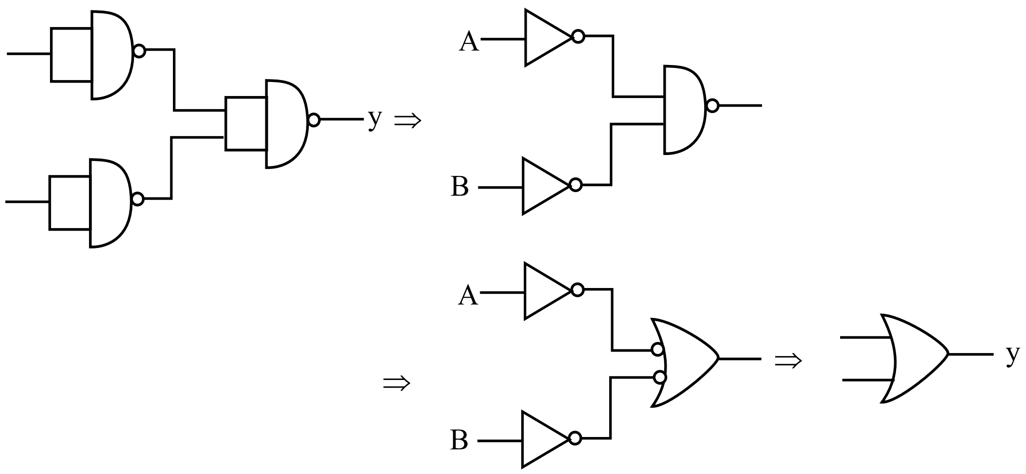
(3)



(4)

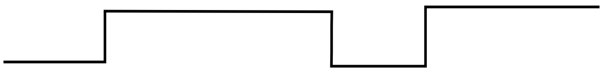


Sol. **(3)**



$y = A + B$

A	0	1	1	0	0	1	0
B	0	0	1	1	0	0	1
y	0	1	1	1	0	1	1



14. The charge flowing in a conductor changes with time as $Q(t) = \alpha t - \beta t^2 + \gamma t^3$. Where α, β and γ are constants. Minimum value of current is :

(1) $\alpha - \frac{3\beta^2}{\gamma}$ (2) $\alpha - \frac{\gamma^2}{3\beta}$ (3) $\alpha - \frac{\beta^2}{3\gamma}$ (4) $\beta - \frac{\alpha^2}{3\gamma}$

Sol. **(3)**

$Q = \alpha t - \beta t^2 + \gamma t^3$

$$I = \frac{dQ}{dt} = \alpha - 2\beta t + 3\gamma t^2$$

$$\frac{dI}{dt} = 0 = 0 - 2\beta + 6\gamma t \quad \Rightarrow \quad t = \frac{2\beta}{6\gamma} = \frac{\beta}{2\gamma}$$

$$I_{\min} = \alpha - 2\beta\left(\frac{\beta}{3\gamma}\right) + 3\gamma\left(\frac{\beta}{3\gamma}\right)^2$$

$$= \alpha - \frac{2\beta^2}{3\gamma} + \frac{\beta^2}{3\gamma}$$

$$\boxed{I_{\min} = \alpha - \frac{\beta^2}{3\gamma}}$$

15. Choose the correct relationship between Poisson ratio (σ), bulk modulus (K) and modulus of rigidity (η) of a given solid object :

(1) $\sigma = \frac{3K + 2\eta}{6K + 2\eta}$ (2) $\sigma = \frac{3K - 2\eta}{6K + 2\eta}$ (3) $\sigma = \frac{6K + 2\eta}{3K - 2\eta}$ (4) $\sigma = \frac{6K - 2\eta}{3K - 2\eta}$

Sol. **(2)**

$Y = 2\eta[1+\sigma]$

and $Y = 3K[1-2\sigma]$

Now $2\eta(1+\sigma) = 3K(1-2\sigma)$

$2\eta\sigma + 2\eta = 3K - 6K\sigma$

$(2n + 6K)\sigma = 3K - 2n$

$$\boxed{\sigma = \frac{3K - 2\eta}{2\eta + 6K}}$$

16. Speed of an electron in Bohr's 7th orbit for Hydrogen atom is 3.6×10^6 m/s. The corresponding speed of the electron in 3rd orbit, in m/s is:

(1) (1.8×10^6) (2) (3.6×10^6) (3) (7.5×10^6) (4) (8.4×10^6)

Sol. **(4)**

We now

$V \propto \frac{z}{n}$

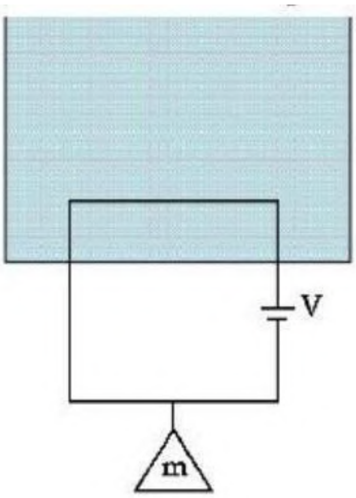
$\frac{V_3}{V_7} = \frac{7}{3}$

$V_3 = V_7 \times \frac{7}{3} = 3.6 \times 10^6 \times \frac{7}{3} = 1.2 \times 7 \times 10^6$

$$\boxed{V_3 = 8.4 \times 10^6 \mathrm{m/s}}$$

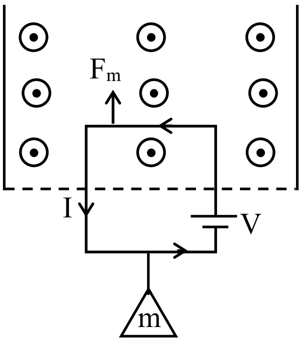
17. A massless square loop, of wire of resistance 10Ω, supporting a mass of 1 g, hangs vertically with one of its sides in a uniform magnetic field of 10^3G, directed outwards in the shaded region. A dc voltage V is applied to the loop. For what value of V, the magnetic force will exactly balance the weight of the supporting mass of 1 g ?

(If sides of the loop $= 10$ cm, $g = 10$ ms^{-2})



(1) $\frac{1}{10}$ V (2) 100 V (3) 10 V (4) 1 V

Sol. **(3)**



For balancing → $l = 10$ cm ,

$B = 10^3$ G $= 0.1$ T,

$m = 1$ g

$F_m = mg$

$I\ell B = mg$

$$\frac{V}{R}(0.1)(0.1) = \frac{1}{1000}\times 10$$

$$\frac{V}{100R} = \frac{1}{100}$$

$$\frac{V}{10} = 1 \Rightarrow \boxed{V = 10\text{Volt}}$$

18. Electric field in a certain region is given by $\vec{E} = \left(\frac{A}{x^2}\hat{i} + \frac{B}{y^2}\hat{j}\right)$. The SI unit of A and B are :

(1) Nm^3C^{-1}; Nm^2C^{-1} (2) Nm^2C^{-1}; Nm^3C^{-1} (3) Nm^3C; Nm^2C (4) Nm^2C; Nm^3C

Sol. **(2)**

$$\vec{E} = \frac{A}{x^2}\hat{i} + \frac{B}{y^3}\hat{j}$$

Unit of A → $\frac{N}{c}\times m^2 = Nm^2c^{-1}$

Unit of B → $\frac{N}{c}\times m^3 = Nm^3c^{-1}$

19. The height of liquid column raised in a capillary tube of certain radius when dipped in liquid A vertically is, 5 cm. If the tube is dipped in a similar manner in another liquid B of surface tension and density double the values of liquid A, the height of liquid column raised in liquid B would be m

(1) 0.05 (2) 0.10 (3) 0.20 (4) 0.5

Sol. **(1)**

$$h = \frac{2T\cos\theta}{r\rho g}$$

$$h \propto \frac{T}{\rho}$$

$$\frac{h_2}{h_1} = \frac{T_2}{T_1}\times\frac{\rho_1}{\rho_2}$$

$$\frac{h_2}{5cm} = \frac{2T}{T}\times\frac{\rho}{2\rho} = 1$$

$$\boxed{h_2 = 5cm = 0.05m}$$

20. A sinusoidal carrier voltage is amplitude modulated. The resultant amplitude modulated wave has maximum and minimum amplitude of 120 V and 80 V respectively. The amplitude of each sideband is :

(1) 20 V (2) 15 V (3) 10 V (4) 5 V

Sol. **(3)**

$V_{max} = V_m + V_c$

$120 = V_c + V_m$ ...(1)

$V_{min} = V_c - V_m$

$80 = V_c - V_m$ ...(2)

(1) + (2)

$200 = 2V_c \Rightarrow \boxed{V_c = 100}$

$V_M = 120 - 100 = 20 \Rightarrow \boxed{V_M = 20}$

$\mu = \frac{V_m}{V_c} = \frac{20}{100} = 0.2$

Amplitude of side bond $= \frac{\mu A_c}{2} = 0.2 \times \frac{100}{2} = 10V$

SECTION - B

21. The general displacement of a simple harmonic oscillator is $x = A\sin\omega t$. Let T be its time period. The slope of its potential energy (U) - time (t) curve will be maximum when $t = \frac{T}{\beta}$. The value of β is

Sol. **(8)**

$x = A\sin(\omega t)$

Potential energy $U = \frac{1}{2}kx^2$

$U = \frac{1}{2}.K.A^2\sin^2(\omega t)$

$\frac{dU}{dt} = \frac{KA^2}{2}.2\sin(\omega t)\cos(\omega t).\omega$

Slope $= \frac{dU}{dt} = \frac{\omega KA^2}{2}\sin(2\omega t)$

$\rightarrow$ Slope will be maximum for $\sin(2\omega t)$ will maximum

$2\omega t = \frac{\pi}{2}$

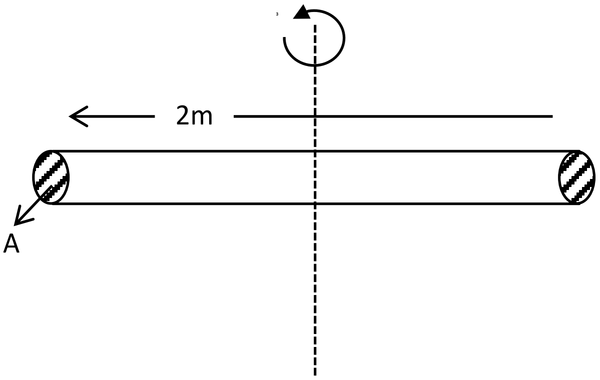
$2\omega.\frac{T}{\beta} = \frac{\pi}{2}$

$2\frac{2\pi}{T} \times \frac{T}{\beta} = \frac{\pi}{2} \Rightarrow \beta = 8$

Ans. = 8

22. A thin uniform rod of length 2 m, cross sectional area ' A ' and density ' d ' is rotated about an axis passing through the centre and perpendicular to its length with angular velocity ω. If value of ω in terms of its rotational kinetic energy E is $\sqrt{\frac{\alpha E}{Ad}}$ then value of α is

Sol. **(3)**



density $= \mathrm{d}$

Area = A

mass $m = d.A\ell$

$m = dA(2) = 2Ad$ …………… (1)

$$K.E. = \frac{1}{2} I\omega^2$$

$$E = \frac{1}{2}.\frac{m\ell^2}{12}.\omega^2$$

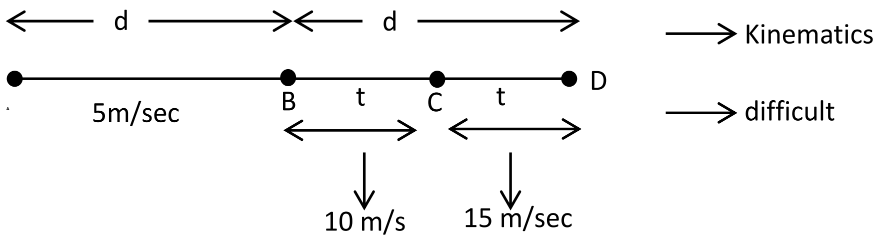
$$E = \frac{1}{24}.2Ad.(2)^2\,\omega^2$$

$$\frac{24E}{8Ad} = \omega^2 \Rightarrow \omega = \sqrt{\frac{3E}{Ad}}$$

Ans. $\alpha = 3$

23. A horse rider covers half the distance with 5 m/s speed. The remaining part of the distance was travelled with speed 10 m/s for half the time and with speed 15 m/s for other half of the time. The mean speed of the rider averaged over the whole time of motion is $\frac{x}{7}$ m/s. The value of x is

Sol. **(50)**



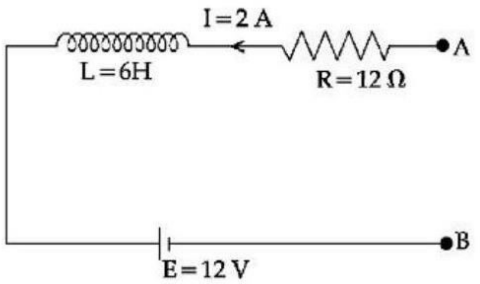
Avg. speed from B to D $\rightarrow V_{BD}=\frac{10+15}{2}=\frac{25}{2}m/sec$

Now, $\frac{2}{V_{ay}}=\frac{1}{5}+\frac{2}{25}$

$\frac{2}{V_{ag}}=\frac{7}{25}\Rightarrow V_{ag}=\frac{50}{7}$

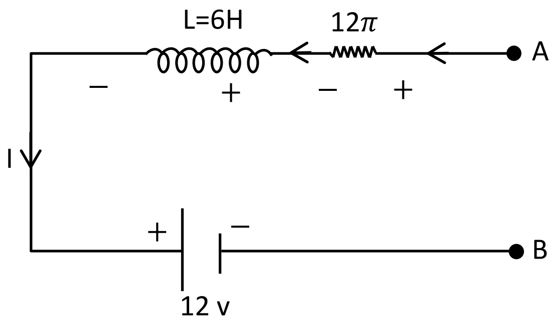
Ans. $x=50$

24.



As per the given figure, if $\frac{dI}{dt}=-1\ A/s$ then the value of V_{AB} at this instant will be V.

Sol. **(30)**



$I=2A$

$$\frac{dI}{dt} = -1A/\sec$$

$$V_A - IR - L\frac{dI}{dt} - 12 = V_B$$

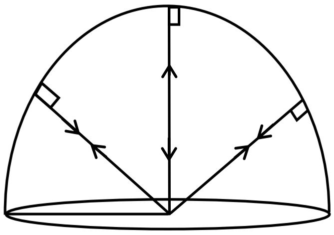
$$V_A - 2(12) + 6(1) - 12 = V_B$$

$$V_A - V_B = 24 + 12 - 6 = 24 + 6 = 30$$

Ans. 30

25. A point source of light is placed at the centre of curvature of a hemispherical surface. The source emits a power of 24 W. The radius of curvature of hemisphere is 10 cm and the inner surface is completely reflecting. The force on the hemisphere due to the light falling on it is ______ 10^{-8} N

Sol. **(4)**



θ = 0°

Presses due reflecting surface $= \frac{2I}{C}$

Net force $= \frac{2I}{C}$ Area .... (1)

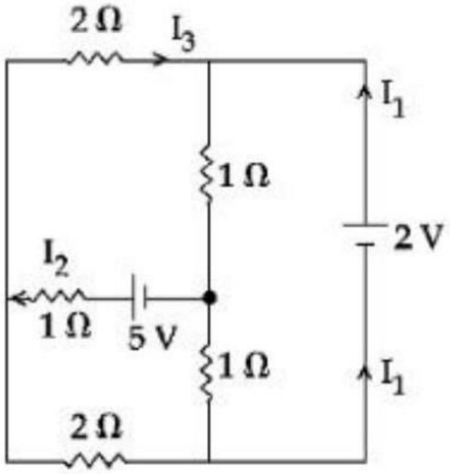
Now $I = \frac{\text{Power}}{\text{Area}} = \frac{\text{Power}}{4\pi r^2}$

From $F_{net} = \frac{2I}{C} \times$ Projected Area

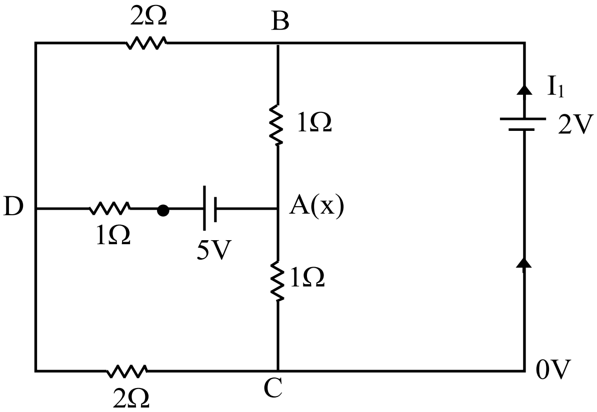
$$F_{net} = \frac{2}{C} \times \frac{\text{Power}}{4\pi r^2} \times \pi r^2$$

$$F_{net} = \frac{2 \times 24}{3 \times 10^8 \times 4} = 4 \times 10^{-8}$$

26. In the following circuit, the magnitude of current I_1, is _______ A.



Sol. **(1.5)**



Let at junction A → voltage = x

$V_A = x$

$V_D = y$

$V_C = 0$

$V_B = 2$

At junction 'A'

$$\frac{x-2}{1}+\frac{x-0}{1}+\frac{x+5-y}{1}=0$$

$$3x-y+3=0 \qquad \ldots(1)$$

At junction 'D'

$$\frac{y-0}{2}+\frac{y-2}{2}+\frac{y-x-5}{1}=0$$

$$4y-2x=12$$

$$2y-x=6 \qquad \ldots(2)$$

From (1) and (2)

$x = 0$; $y = 3$

So curent through 2V cell is

$$I=\frac{3}{2}=1.5\text{ A}$$

27. In a screw gauge, there are 100 divisions on the circular scale and the main scale moves by 0.5 mm on a complete rotation of the circular scale. The zero of circular scale lies 6 divisions below the line of graduation when two studs are brought in contact with each other. When a wire is placed between the studs, 4 linear scale divisions are clearly visible while 46th division the circular scale coincide with the reference line. The diameter of the wire is ________ $\times 10^{-2}$ mm

Sol. **(220)**

Pitch = 0.5 mm

$$\text{L.C.} = \frac{\text{pitch}}{\text{circular division}} = \frac{0.5\text{mm}}{100} = 0.005\text{mm}$$

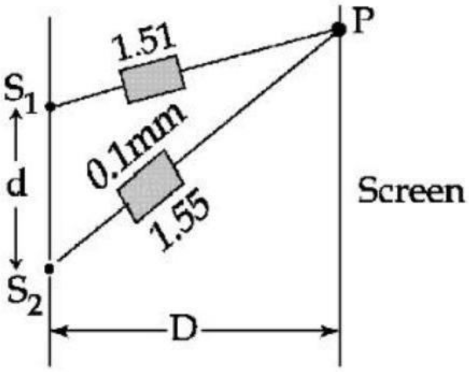
Zero error = 6 × L.C. = 6 × (0.005) mm

Reading = main linear scale reading + n(L.C.) – zero error

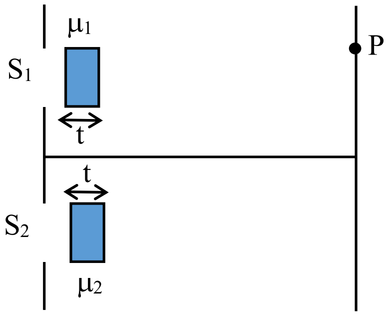
$$= 4(0.5\text{mm}) + 46\,(0.005) - 6(0.005)$$
$$= 2\text{ mm} + 40 \times 0.005\text{ mm}$$
$$= 2\text{ mm} + \frac{200}{1000}\text{ mm}$$
$$= 2.2\text{ mm}$$

Rading = 220×10^{-2} mm

28. In Young's double slit experiment, two slits S_1 and S_2 are ' d ' distance apart and the separation from slits to screen is D (as shown in figure). Now if two transparent slabs of equal thickness 0.1 mm but refractive index 1.51 and 1.55 are introduced in the path of beam ($\lambda = 4000\text{Å}$) from S_1 and S_2 respectively. The central bright fringe spot will shift by number of fringes.



Sol. **(10)**



$\mu_1 = 1.51$ $\quad$ $t = 0.1\text{mm}$

$\mu_2 = 1.55$ $\quad$ $\lambda = 4000\text{Å}$

Shifting central maxima

$$\Delta x = \left[S_1P + (\mu_1 - 1)t\right] - \left[S_2P + (\mu_2 - 1)t\right]$$

$$0 = (S_1P - S_2P) + (\mu_1 - 1)t - (\mu_2 - 1)t$$

$$0 = \frac{yd}{D} + (\mu_1 - \mu_2)t$$

$$(\mu_2 - \mu_1)t = \frac{yd}{D}$$

$$(1.55 - 1.51)(0.1\text{mm}) = y \times \frac{d}{D}$$

$$\frac{D}{d}(0.04 \times 0.1) \times 10^{-3} = y \qquad \ldots(1)$$

Now

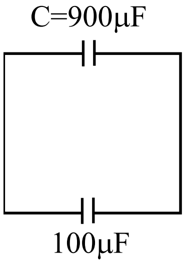
Fringe width $\Rightarrow \beta = \frac{\lambda D}{d}$

No. of fringes shifted $= \frac{y}{\beta} = \frac{4 \times 10^{-6}}{4000\text{Å}} = 10$

Ans. 10

29. A capacitor of capacitance 900μF is charged by a 100 V battery. The capacitor is disconnected from the battery and connected to another uncharged identical capacitor such that one plate of uncharged capacitor connected to positive plate and another plate of uncharged capacitor connected to negative plate of the charged capacitor. The loss of energy in this process is measured as $x \times 10^{-2}$ J. The value of x is

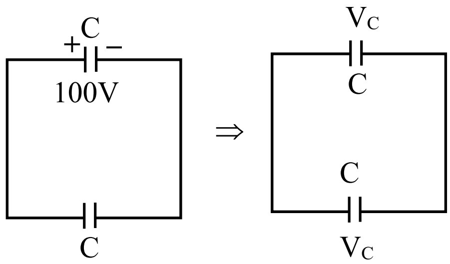
Sol. **(225)**



$Q = 900 \times 100\ \mu C$

$Q = 9 \times 10^{-2}\ C \qquad \ldots(1)$

Now



$$\Delta U = \frac{1}{2}\frac{C_1.C_2}{C_1 + C_2}(V_1 - V_2)^2$$

$$= \frac{1}{2} \times \frac{C \times C}{2C} \times (100 - 0)^2$$

$$= \frac{C}{4} \times 100 \times 100$$

$$= \frac{900}{4} \times 10^{-6} \times 10^{4}$$

$$= \frac{9}{4} = 2.25\text{J}$$

$$\boxed{\Delta U = 225 \times 10^{-2}\,\text{J}}$$

30. In an experiment for estimating the value of focal length of converging mirror, image of an object placed at 40 cm from the pole of the mirror is formed at distance 120 cm from the pole of the mirror. These distances are measured with a modified scale in which there are 20 small divisions in 1 cm. The value of error in measurement of focal length of the mirror is $\frac{1}{K}$ cm. The value of K is

Sol. **32**

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \qquad dv = du = \frac{1\text{cm}}{20} = 0.05\text{cm (given)}$$

$$f^{-1} = v^{-1} + u^{-1}$$

$$(-1)f^{-2}df = (-1)v^{-2}dv - u^{-2}du$$

$$\frac{df}{f^2} = \frac{dv}{v^2} + \frac{du}{u^2} \qquad \ldots(1)$$

$$\frac{1}{f} = \frac{1}{(-120)} + \frac{1}{-40}$$

$$\frac{1}{f} = \frac{1+3}{(-120)} = \frac{4}{-120} \Rightarrow \boxed{f = -30\text{cm}}$$

Put value of f, du, dv in (1)

$$\frac{df}{(30)^2} = \frac{0.05}{(120)^2} + \frac{0.05}{(40)^2}$$

$$df = \frac{1}{32}\text{cm} \qquad \text{so} \qquad \boxed{K = 32}$$

Chemistry

SECTION - A

31. Lithium aluminium hydride can be prepared from the reaction of

(1) LiH and Al$(OH)_3$ (2) LiH and Al_2Cl_6

(3) LiCl and Al_2H_6 (4) LiCl, Al and H_2

Sol. 2

$8\ LiH+Al_2Cl_6 \rightarrow 2\ LiAlH_4+6\ LiCl$

32. Amongst the following compounds, which one is an antacid?

(1) Terfenadine (2) Meprobamate (3) Brompheniramine (4) Ranitidine

Sol. **4**

Ranitidine is an antacid it is an antihistamine and decrease the reaction of gastric juice in stomach

33. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : In expensive scientific instruments, silica gel is kept in watch-glasses or in semipermeable membrane bags.

Reason (R) : Silica gel adsorbs moisture from air via adsorption, thus protects the instrument from water corrosion (rusting) and / or prevents malfunctioning.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(2) (A) is false but (R) is true

(3) Both (A) and (R) are true and (R) is the correct explanation of (A)

(4) (A) is true but (R) is false

Sol. **3**

Theory based

34. Match List I with List II

LIST I (Atomic number)		LIST II (Block of periodic table)	
A.	37	I.	p-block
B.	78	II.	d-block
C.	52	III.	f-block
D.	65	IV.	s-block

Choose the correct answer from the options given below:

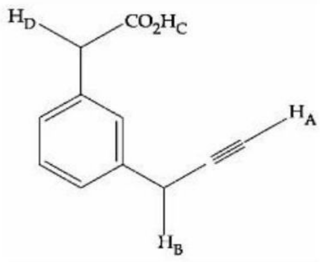
(1) A - IV, B - III, C - II, D – I (2) A - II, B - IV, C - I, D – III

(3) A - IV, B - II, C - I, D – III (4) A - I, B - III, C - IV, D - II

Sol. **3**

37 (K) s-block
78 (pt) d-block
52 (Te) p-block
65 (Tb) f-block

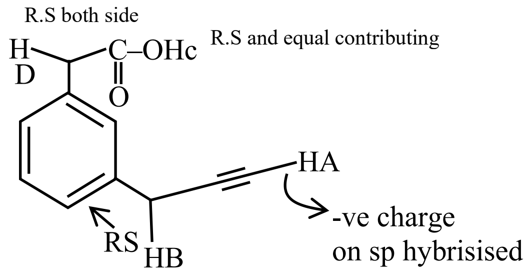
35. What is the correct order of acidity of the protons marked A-D in the given compounds ?

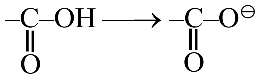


(1) $H_C > H_A > H_D > H_B$ (2) $H_D > H_C > H_B > H_A$

(3) $H_C > H_D > H_B > H_A$ (4) $H_C > H_D > H_A > H_B$

Sol. **4**





$$-\underset{\underset{O}{\|}}{C}-OH \longrightarrow -\underset{\underset{O}{\|}}{C}-O^{\ominus}$$

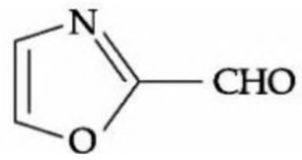
Equal contributing and resonance stablize

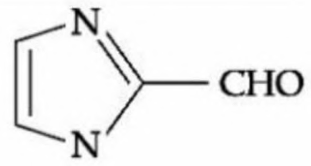
So order $H_C > H_D > H_A > H_B$

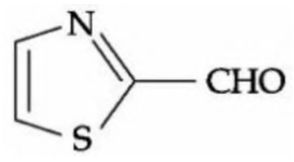
36. Which of the following compounds would give the following set of qualitative analysis?

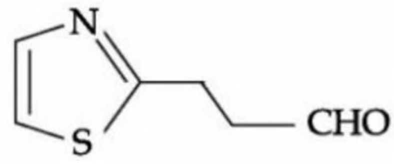
(i) Fehling's Test : Positive

(ii) Na fusion extract upon treatment with sodium nitroprusside gives a blood red colour but not prussian blue.

(1) 

(2) 

(3) 

(4) 

Sol. **4**

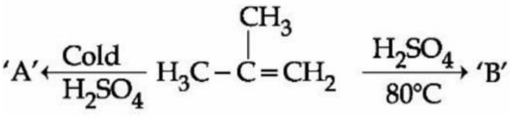
fehling test gives positive result for aliphatic aldehyde While sodium nitroprasside gives blood red color with S and N.

So Na+N+C+S → NaSCN (Sodium thiocyanate)

$SCN^- + Fe^{3+} \rightarrow [Fe(SCN)]^{2+}$ Ferric thiocyanate (Blood red color)

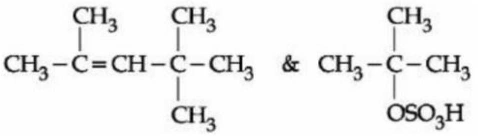
Confims presence of N and S

37. The major products ' A' and ' B ', respectively, are



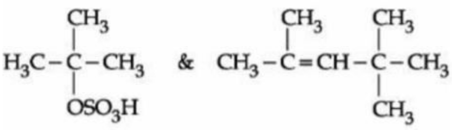
$$\text{'A'} \xleftarrow[H_2SO_4]{\text{Cold}} H_3C-\overset{CH_3}{\overset{|}{C}}=CH_2 \xrightarrow[80^\circ C]{H_2SO_4} \text{'B'}$$

(1)



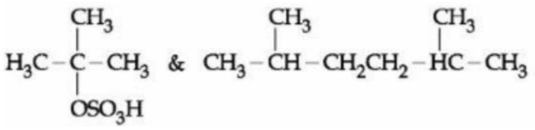
$CH_3-\overset{CH_3}{\overset{|}{C}}=CH-\underset{CH_3}{\underset{|}{\overset{CH_3}{\overset{|}{C}}}}-CH_3$ & $CH_3-\underset{OSO_3H}{\underset{|}{\overset{CH_3}{\overset{|}{C}}}}-CH_3$

(2)



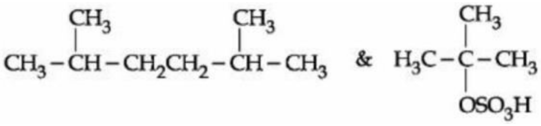
$H_3C-\underset{OSO_3H}{\underset{|}{\overset{CH_3}{\overset{|}{C}}}}-CH_3$ & $CH_3-\overset{CH_3}{\overset{|}{C}}=CH-\underset{CH_3}{\underset{|}{\overset{CH_3}{\overset{|}{C}}}}-CH_3$

(3)



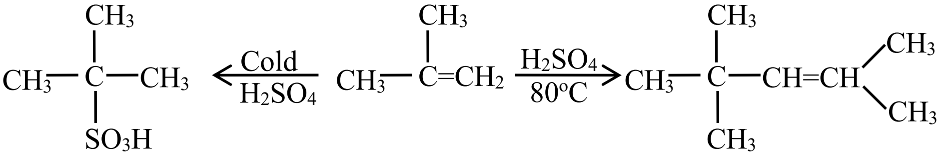
$H_3C-\underset{OSO_3H}{\underset{|}{\overset{CH_3}{\overset{|}{C}}}}-CH_3$ & $CH_3-\overset{CH_3}{\overset{|}{CH}}-CH_2CH_2-\overset{CH_3}{\overset{|}{HC}}-CH_3$

(4)



$CH_3-\overset{CH_3}{\overset{|}{CH}}-CH_2CH_2-\overset{CH_3}{\overset{|}{CH}}-CH_3$ & $H_3C-\underset{OSO_3H}{\underset{|}{\overset{CH_3}{\overset{|}{C}}}}-CH_3$

Sol. **2**



$$CH_3-\underset{SO_3H}{\underset{|}{\overset{CH_3}{\overset{|}{C}}}}-CH_3 \xleftarrow[H_2SO_4]{\text{Cold}} CH_3-\overset{CH_3}{\overset{|}{C}}=CH_2 \xrightarrow[80^\circ C]{H_2SO_4} CH_3-\underset{CH_3}{\underset{|}{\overset{CH_3}{\overset{|}{C}}}}-CH=CH\begin{matrix}\diagup CH_3\\ \diagdown CH_3\end{matrix}$$

38. During the qualitative analysis of SO_3^{2-} using dilute H_2SO_4, SO_2 gas is evolved which turns $K_2Cr_2O_7$ solution (acidified with dilute H_2SO_4) :

(1) green (2) blue (3) red (4) black

Sol. **1**

$Na_2SO_3+HCl \rightarrow NaCl+H_2O+SO_2\uparrow$

$K_2Cr_2O_7+H_2SO_4+SO_2 \rightarrow K_2SO_4+\underset{\text{green}}{Cr_2(SO_4)_3}+H_2O$

39. In the wet tests for identification of various cations by precipitation, which transition element cation doesn't belong to group IV in qualitative inorganic analysis ?

(1) Ni^{2+} (2) Zn^{2+} (3) Co^{2+} (4) Fe^{3+}

sol. **4**

Zn^{+2}, CO^{+2}, Ni^{+2}, IV^{th} group

Fe^{+3} = III^{rd} group

40. For OF_2 molecule consider the following :

A. Number of lone pairs on oxygen is 2.

B. FOF angle is less than 104.5°.

C. Oxidation state of O is −2.

D. Molecule is bent ' V ' shaped.

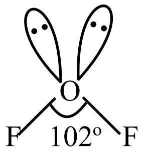
E. Molecular geometry is linear.

correct options are:

(1) A, C, D only (2) C, D, E only (3) A, B, D only (4) B, E, A only

Sol. **3**

OF$_2$

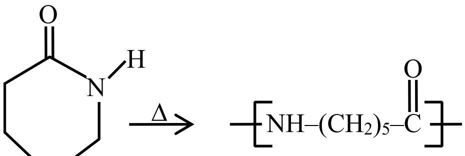


2 l.pe$^-$ in 'O'
bond angle 102°
bent/V shape

41. Caprolactam when heated at high temperature in presence of water, gives

(1) Nylon 6, 6 (2) Nylon 6 (3) Teflon (4) Dacron

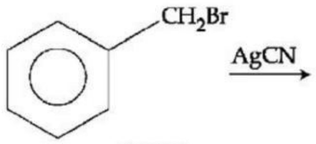
Sol. **2**



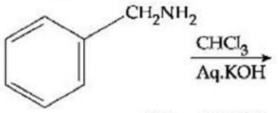
Nylon 6

42. Benzyl isocyanide can be obtained by :

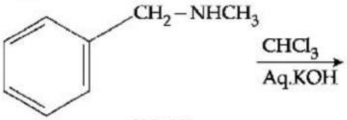
A.



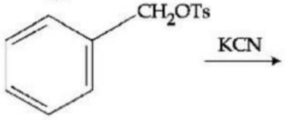
B.



C.



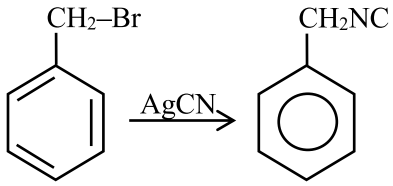
D.

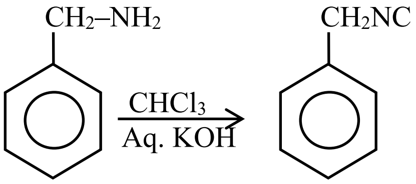


Choose the correct answer from the options given below :

(1) A and D (2) Only B (3) B and C (4) A and B

Sol. **4**





43. Formation of photochemical smog involves the following reaction in which A, B and C are respectively.

$$\text{i. } NO_2 \xrightarrow{h\upsilon} A + B$$
$$\text{ii. } B + O_2 \rightarrow C$$
$$\text{iii. } A + C \rightarrow NO_2 + O_2$$

Choose the correct answer from the options given below:

(1) $O, N_2O \& NO$ (2) $O, NO \& NO_3^-$ (3) $NO, O \& O_3$ (4) $N, O_2 \& O_3$

Sol. **3**

$$\begin{tikzcd} NO_2 \arrow[r, "h\upsilon"] & \underset{(A)}{NO} + \underset{(B)}{O} \arrow[d, "O_2"] \\ & \underset{(C)}{O_3} \end{tikzcd}$$

44. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Ketoses give Seliwanoff's test faster than Aldoses.

Reason (R) : Ketoses undergo β-elimination followed by formation of furfural.

In the light of the above statements, choose the correct answer from the options given below :

(1) (A) is false but (R) is true

(2) (A) is true but (R) is false

(3) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(4) Both (A) and (R) are true and (R) is the correct explanation of (A)

Sol. **2**

Seliwanoff's test – Test to differentiate for ketose and aldose.

In this keto hexose are more rapidly dehydrated to form 5–hydroxy methyl furfural when heated in acidic medium which on condensation with resorcinol, as result brown red colored complex is formed.

45. Match List I with List II

LIST I (molecules/ions)		LIST II (No. of lone pairs of e^- on central atom)	
A.	IF_7	I.	Three
B.	ICl_4^-	II.	One
C.	XeF_6	III.	Two
D.	XeF_2	IV.	Zero

Choose the correct answer from the options given below:

(1) A - II, B - III, C - IV, D – I (2) A - II, B - I, C - IV, D – III

(3) A - IV, B - I, C - II, D – III (4) A - IV, B - III, C - II, D – I

Sol. **4**

Molecule	l.pe⁻ of C.M.
IF_7	0
ICl_4^-	2
XeF_6	1
XeF_2	3

46. To inhibit the growth of tumours, identify the compounds used from the following :

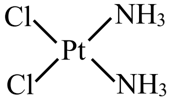
A. EDTA
B. Coordination Compounds of Pt
C. D – Penicillamine
D. Cis - Platin

Choose the correct answer from the option given below:

(1) B and D Only (2) C and D Only (3) A and C Only (4) A and B Only

Sol. **1**

Cis plating



is used as Anticancer agent

47. The alkaline earth metal sulphate(s) which are readily soluble in water is/are :

A. $BeSO_4$
B. $MgSO_4$
C. $CaSO_4$
D. $SrSO_4$
E. $BaSO_4$

Choose the correct answer from the options given below :

(1) B only (2) *A* and *B* (3) B and C (4) A only

Sol. **2**

$BeSO_4$ & $MgSO_4$ are soluble in water
$CaSO_4$ is partially soluble
$SrSO_4$ & $BaSO_4$ is insoluble

48. Which of the following is correct order of ligand field strength ?

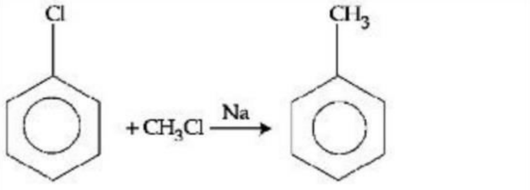
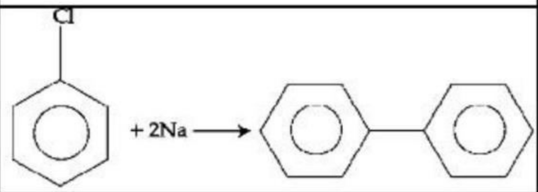
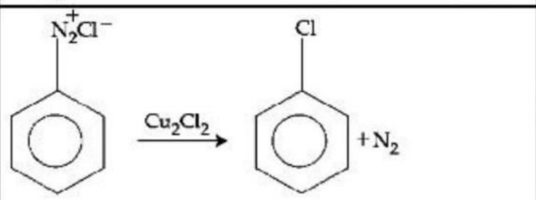
(1) $CO < \text{ en } < NH_3 < C_2O_4^{2-} < S^{2-}$
(2) $NH_3 < \text{ en } < CO < S^{2-} < C_2O_4^{2-}$
(3) $S^{2-} < C_2O_4^{2-} < NH_3 < \text{ en } < CO$
(4) $S^{2-} < NH_3 < en < CO < C_2O_4^{2}$

Sol. **3**

order of ligand strength

$S^{2-} < C_2O_4^{2-} < NH_3 < en < CO$

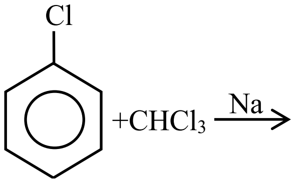
49. Match List I with List II

LIST I		LIST II	
A.		I.	Fittig reaction
B.		II.	Wurtz Fittig reaction
C.		III.	Finkelstein reaction
D.	$C_2H_5Cl + NaI \rightarrow C_2H_5I + NaCl$	IV.	Sandmeyer reaction

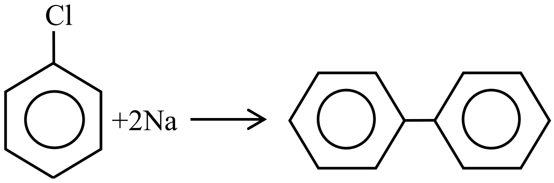
Choose the correct answer from the options given below:

(1) A - II, B - I, C - IV, D – III
(2) A - IV, B - II, C - III, D – I
(3) A - III, B - II, C - IV, D – I
(4) A - II, B - I, C - III, D – IV

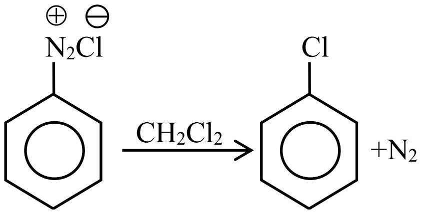
Sol. **1**

(A) 

Wurtz fittig reaction

(B) 

fittig rxn

(C) 

sandmeyer rxn

(D) $C_2H_5Cl + NaI \rightarrow C_2H_5I + NaCl$ Finkelstein rxn

50. In the extraction of copper, its sulphide ore is heated in a reverberatory furnace after mixing with silica to:

(1) remove FeO as $FeSiO_3$
(2) decrease the temperature needed for roasting of $Cu_2\ S$
(3) separate CuO as $CuSiO_3$
(4) remove calcium as $CaSiO_3$

Sol. **1**

The copper ore contains iron, it is mixed with silica before heating in reverberatory furnace, feO of slags off as $FeSiO_3$

$FeO + SiO_2 \rightarrow FeSiO_3$

SECTION - B

51. 600 mL of 0.01MHCl is mixed with 400 mL of $0.01MH_2SO_4$. The pH of the mixture is __________ $\times 10^{-2}$. (Nearest integer)

[Given log2 2 = 0.30
$\log 3 = 0.48$
$\log 5 = 0.69$
$\log 7 = 0.84$
$\log 11 = 1.04$]

Sol. **186**

$$[H^+]_{mix} = \frac{(600 \times 0.01) + (400 \times 0.01 \times 2)}{1000}$$

$$= \frac{6+8}{1000} = 14 \times 10^{-3}$$

$pH = -\log(14 \times 10^{-3})$
$= 3 - \log 2 - \log 7$
$= 3 - 0.30 - 0.84$
$pH = 1.86$

52. The energy of one mole of photons of radiation of frequency 2×10^{12} Hz in J mol⁻¹ is . (Nearest integer)

[Given : $h = 6.626 \times 10^{-34}$Js

$N_A = 6.022 \times 10^{23}$ mol^{-1}]

Sol. **789**

$E_{photon} = 6.626 \times 10^{-34} \times 2 \times 10^{12} \times 6.023 \times 10^{23}$

$= 79.81 \times 10$

$= 798.1 \approx 798$

53. Consider the cell

$Pt_{(s)}|H_2(g, 1\ atm)|H^+(aq, 1M)||Fe^{3+}(aq), Fe^{2+}(aq) \mid Pt(s)$

When the potential of the cell is 0.712 V at 298 K, the ratio $[Fe^{2+}]/[Fe^{3+}]$ is (Nearest integer)

Given : $Fe^{3+} + e^- = Fe^{2+}, E^\theta Fe^{3+}, Fe^{2+} \mid Pt = 0.771$

$$\frac{2.303RT}{F} = 0.06\ V$$

Sol. **10**

Cell reaction :-

$H_2 + 2Fe^{3+} \rightarrow 2H^+ + 2Fe^{2+}$

$$E_{cell} = 0.771 - \frac{2.303RT}{2F} \log \frac{\left[Fe^{2+}\right]^2\left[H^+\right]^2}{\left[Fe^{3+}\right]^2}$$

$0.712 = 0.771 - 0.03 \log(x)^2$

$\frac{0.059}{2} \log(x)^2 = 0.059$

$\log x = 1$

$$x = \frac{\left[Fe^{2+}\right]}{\left[Fe^{3+}\right]} = 10$$

54. The number of electrons involved in the reduction of permanganate to manganese dioxide in acidic medium is

Sol. **3**

$4H^+ + MnO_4^- + 3e^- \rightarrow MnO_2 + 2H_2O$

55. A 300 mL bottle of soft drink has $0.2MCO_2$ dissolved in it. Assuming CO_2 behaves as an ideal gas, the volume of the dissolved CO_2 at STP is______mL. (Nearest integer)

Given : At STP, molar volume of an ideal gas is 22.7 L mol^{-1}

Sol. **1362**

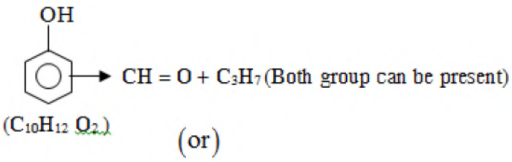
Mole of dissolved CO_2 = 0.2×300=60 mmol

$V_{CO_2} = 60 \times 10^{-3} \times 22.7$

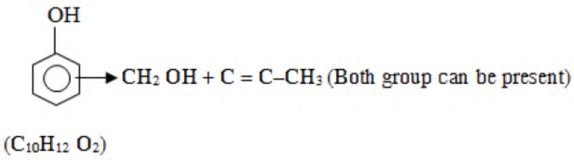
= 1362 ml

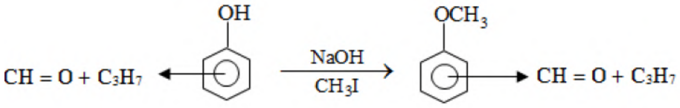
56. A trisubstituted compound ' A ', $C_{10}H_{12}O_2$ gives neutral $FeCl_3$ test positive. Treatment of compound 'A' with NaOH and CH_3Br gives $C_{11}H_{14}O_2$, with hydroiodic acid gives methyl iodide and with hot conc. NaOH gives a compound B, $C_{10}H_{12}O_2$. Compound 'A' also decolorises alkaline $KMnO_4$. The number of π bond/s present in the compound ' A ' is

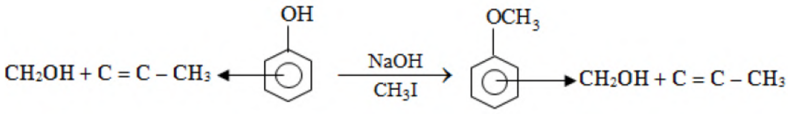
Sol. **4**



(or)







57. If compound A reacts with B following first order kinetics with rate constant $2.011 \times 10^{-3}\ s^{-1}$. The time taken by A (in seconds) to reduce from 7 g to 2 g will be (Nearest Integer)

[$\log 5 = 0.698, \log 7 = 0.845, \log 2 = 0.301$]

Sol. **623**

For Iˢᵗ order:-

$$t = \frac{1}{2.011\times 1^{-3}}\times 2.303\times \log\frac{7}{2}$$

$$= \frac{2.303\times(0.845-0.301)}{2.011\times 10^{-3}}$$

$= 622.9 \approx 623$

58. A solution containing 2 g of a non-volatile solute in 20 g of water boils at 373.52 K. The molecular mass of the solute is______________g mol^{-1}. (Nearest integer)

Given, water boils at 373 K, K_b for water $= 0.52\ K\ kg\ mol^{-1}$

Sol. **100**

$\Delta T_b = 373.52 - 373 = 0.52$

$\Delta T_b = iK_bm$ $\quad$ i=1

$$0.52 = 0.52\times\frac{2/x}{20}\times 1000$$

x = 100 gm/mol

59. When 2 litre of ideal gas expands isothermally into vacuum to a total volume of 6 litre, the change in internal energy is J. (Nearest integer)

Sol. **0**

$\Delta U = 0$

process is Isothermal

60. Some amount of dichloromethane (CH_2Cl_2) is added to 671.141 mL of chloroform ($CHCl_3$) to prepare 2.6×10^{-3}M solution of CH_2Cl_2(DCM). The concentration of DCM is ppm (by mass).

Given : atomic mass : C = 12

H = 1

Cl = 35.5

density of $CHCl_3 = 1.49 \text{ g cm}^{-3}$

Sol. **148.322**

Molar mass = 12+2+71

= 85

mmoles of DCM = $671.141 \times 2.6 \times 10^{-3}$

mass of solution = 1.49×671.141

$$PPM = \frac{671.141 \times 2.6 \times 10^{-3} \times 85 \times 10^{-3}}{1.49 \times 671.141} \times 10^{6}$$

148.322

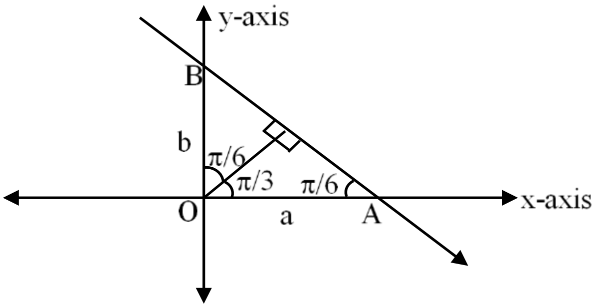
Mathematics

SECTION - A

61. A straight line cuts off the intercepts OA = a and OB = b on the positive directions of x-axis and y axis respectively. If the perpendicular from origin O to this line makes an angle of $\frac{\pi}{6}$ with positive direction of y-axis and the area of $\triangle$ OAB is $\frac{98}{3}\sqrt{3}$, then $a^2 - b^2$ is equal to:

(1) $\frac{392}{3}$ (2) $\frac{196}{3}$ (3) 98 (4) 196

Sol. **1**



In ΔAOB

$$\tan\frac{\pi}{6} = \frac{OB}{OA} = \frac{b}{a}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{b}{a}$$

$$\Rightarrow \boxed{a = \sqrt{3}b}$$

$$\because \text{ area of triangle } \Delta OAB = \frac{1}{2}\times ab = \frac{98}{3}\times\sqrt{3}$$

$$\Rightarrow \frac{\sqrt{3}b^2}{2} = \frac{98}{\sqrt{3}}$$

$$\Rightarrow b^2 = \frac{98}{3}\times 2$$

$$\Rightarrow \boxed{b = \sqrt{\frac{196}{3}}}$$

$$\boxed{a = \sqrt{196}}$$

$$a^2 - b^2 = 196 - \frac{196}{3} = \frac{588-196}{3}$$

$$\Rightarrow \boxed{a^2 - b^2 = \frac{392}{3}}$$

62. The minimum number of elements that must be added to the relation R={(a, b), (b, c)} on the set {a, b, c} so that is becomes symmetric and transitive is :

(1) 3 (2) 4 (3) 5 (4) 7

Sol. **4**

$R=\{(a,b),(b,c)\}$

For symmetric relation (b, a), (c, b) must be added in R

For transitive relation (a, c), (a, a), (b, b), (c, c), (c, a) must be added in R

So, minimum number of element = 7

63. If an unbiased die, marked with $-2,-1,0,1,2,3$ on its faces, is thrown five times, then the probability that the product of the outcomes is positive, is :

(1) $\frac{881}{2592}$ (2) $\frac{27}{288}$ (3) $\frac{440}{2592}$ (4) $\frac{521}{2592}$

Sol. **4**

Unbiased die. Marked with –2, –1, 0, 1, 2, 3

Product of outcomes is positive if

All time get positive number, 3 time positive and 2 time negative, 1 time positive and 4 time negative.

P (Product of the outcomes is positive) = $\underbrace{{}^5C_5\left(\frac{3}{6}\right)^5}_{\text{All positive}}+\underbrace{{}^5C_3\left(\frac{3}{6}\right)^3\left(\frac{2}{6}\right)^2}_{3\text{ positive, }2\text{ negative}}+\underbrace{{}^5C_1\left(\frac{3}{6}\right)\left(\frac{2}{6}\right)^4}_{1\text{ positive, }4\text{ negative}}$

$$=\frac{3^5}{6^5}+\frac{10\times3^3\times2^2}{6^5}+\frac{5\times3\times2^4}{6^5}$$

$$=\frac{1563}{6^5}=\frac{521}{2592}$$

64. If $\vec{a},\vec{b},\vec{c}$ are three non-zero vectors and $\hat{n}$ is a unit vector perpendicular to $\vec{c}$ such that $\vec{a}=\alpha\vec{b}-\hat{n}$, $(\alpha\neq0)$ and $\vec{b}\cdot\vec{c}=12$, then $|\vec{c}\times(\vec{a}\times\vec{b})|$ is equal to :

(1) 9 (2) 15 (3) 6 (4) 12

Sol. **4**

$\vec{a}=\alpha\vec{b}-\hat{n},\vec{b}.\vec{c}=12$

$\vec{c}\times(\vec{a}\times\vec{b})=(\vec{c}.\vec{b})\vec{a}-(\vec{c}.\vec{a})\vec{b}$

$\vec{c}\times(\vec{a}\times\vec{b})=12\vec{a}-(\vec{c}.\vec{a})\vec{b}$ ….(1)

$\because\ \vec{a}=\alpha\vec{b}-n$

$\vec{c}.\vec{a}=\alpha\vec{c}.\vec{b}-\vec{c}.\vec{n}$

$\boxed{\vec{c}.\vec{a}=12\alpha}$ ….(2)

Equation (2) put in equation (1)

$\vec{c}\times(\vec{a}\times\vec{b})=12\vec{a}-12\alpha\vec{b}$

$\left|\vec{c}\times(\vec{a}\times\vec{b})\right|=12\left|\vec{a}-\alpha\vec{b}\right|$ $\left[\because\vec{a}-\alpha\vec{b}=-n\text{ then }|\vec{a}-\alpha\vec{b}|=1\right]$

$\Rightarrow\boxed{\left|\vec{c}\times(\vec{a}\times\vec{b})\right|=12}$

65. Among the statements :

(S1) $((p \vee q) \Rightarrow r) \Leftrightarrow (p \Rightarrow r)$

(S2) $((p \vee q) \Rightarrow r) \Leftrightarrow ((p \Rightarrow r) \vee (q \Rightarrow r))$

(1) only (S2) is a tautology (2) only (S1) is a tautology

(3) neither (S1) nor (S2) is a tautology (4) both (S1) and (S2) are tautologies

Sol. **3**

$S_1 : \left((p \vee q) \Rightarrow r\right) \Leftrightarrow (p \Rightarrow r)$

p	q	r	$(p\vee q) \Rightarrow r$ $(\sim p \wedge \sim q) \vee r$	$\sim p \vee r$	$\left((p \vee q) \Rightarrow r\right) \Leftrightarrow (p \Rightarrow r)$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	F	T	T	T
F	T	F	F	T	F
F	F	T	T	T	T

S_1 is not a tautology

$S_2 = \left((p \vee q) \Rightarrow r\right) \Leftrightarrow \left((p \Rightarrow r) \vee (q \Rightarrow r)\right)$

p	q	r	$(p \vee q) \Rightarrow r$	$(p \Rightarrow r) \vee (q \Rightarrow r)$	$\left((p \vee q) \Rightarrow r\right) \Leftrightarrow \left((p \Rightarrow r) \vee (q \Rightarrow r)\right)$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	F	F	T	T	T
F	T	F	F	T	F
F	F	T	T	T	T

S_2 is not a tautology

So, neither S_1 nor S_2 is a tautology.

66. If $P(h, k)$ be a point on the parabola $x = 4y^2$, which is nearest to the point $Q(0,33)$, then the distance of P from the directrix of the parabola $y^2 = 4(x + y)$ is equal to :

(1) 2 (2) 6 (3) 8 (4) 4

Sol. **2**

Equation of normal of the parabola $x = 4y^2$

At a point $P\left(\frac{t^2}{16}, \frac{2t}{16}\right)$ is

$$y + tx = \frac{2t}{16} + \frac{1}{16}t^3$$

$\because$ Normal pass through $Q(0,33)$ then

$$33=\frac{t}{8}+\frac{t^3}{16}$$

$$\Rightarrow t^3+2t-528=0$$

$$\Rightarrow (t-8)(t^2+8+166)=0$$

$$\Rightarrow t=8$$

Point P is (4, 1)

Given parabola is $y^2=4(x+y)$

$$y^2-4y=4x$$

$$(y-2)^2=4(x+1)$$

directrix is x + 1 = – 1

$$\boxed{x=-2}$$

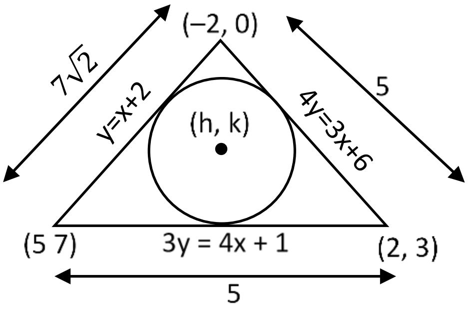
Distance of P(4, 1) from the directrix x = –2 is 6.

67. Let $y=x+2, 4y=3x+6$ and $3y=4x+1$ be three tangent lines to the circle $(x-h)^2+(y-k)^2=r^2$.

Then $h+k$ is equal to :

(1) $5(1+\sqrt{2})$ (2) $5\sqrt{2}$ (3) 6 (4) 5

Sol. **4**



In centre of triangle is (h, k)

$$=\left(\frac{5(-2)+2\times7\sqrt{2}+5\times5}{5+5+7\sqrt{2}},\frac{3\times(7\sqrt{2})+0\times5+7\times5}{5+5+7\sqrt{2}}\right)$$

$$=\left(\frac{14\sqrt{2}+15}{10+7\sqrt{2}},\frac{21\sqrt{2}+35}{10+7\sqrt{2}}\right)$$

So, $h+k=\frac{14\sqrt{2}+15}{10+7\sqrt{2}}+\frac{21\sqrt{2}+35}{10+7\sqrt{2}}$

$$h+k=\frac{35\sqrt{2}+50}{7\sqrt{2}+10}=\frac{5(7\sqrt{2}+10)}{7\sqrt{2}+10}=5$$

$$\Rightarrow \boxed{h+k=5}$$

68. The number of points on the curve $y = 54x^5 - 135x^4 - 70x^3 + 180x^2 + 210x$ at which the normal lines are parallel to $x + 90y + 2 = 0$ is :

(1) 4 (2) 2 (3) 0 (4) 3

Sol. **1**

Given curve is $y = 54x^5 - 135x^4 - 70x^3 + 180x^2 + 210x$

$\frac{dy}{dx} = 270x^4 - 540x^3 - 210x^2 + 360x + 210$

$\because$ Normal is parallel to $x + 90y + 2 = 0$

Then tangent is $\perp^r$ to $x + 90\ y + 2 = 0$

Then $(270x^4 - 540x^3 - 210x^2 + 360x + 210)\left(\frac{-1}{90}\right) = -1$

$270x^4 - 540x^3 - 210x^2 + 360x + 120 = 0$

$\Rightarrow 9x^4 - 18x^3 - 7x^2 + 12x + 4 = 0$

$\Rightarrow (x-1)(x-2)(3x+1)(3x+2) = 0$

$\Rightarrow x = 1, 2, -\frac{1}{3}, -\frac{2}{3}$

Number of points are 4

69. If $a_n = \frac{-2}{4n^2-16n+15}$, then $a_1 + a_2 + \cdots \ldots + a_{25}$ is equal to:

(1) $\frac{52}{147}$ (2) $\frac{49}{138}$ (3) $\frac{50}{141}$ (4) $\frac{51}{144}$

Sol. **3**

given that $a_n = \frac{-2}{4n^2 - 16n + 15}$

$a_1 + a_2 + a_3 + \ldots . a_{25} = \sum_{n=1}^{25} \frac{-2}{(2n-3)(2n-5)}$

$= \sum_{n=1}^{25} \frac{(2n-5)-(2n-3)}{(2n-3)(2n-5)}$

$= \sum_{n=1}^{25} \left(\frac{1}{2n-3} - \frac{1}{(2n-5)}\right)$

$= \frac{1}{-1} - \frac{1}{-3}$

$+ \frac{1}{1} - \frac{1}{-1}$

$+ \frac{1}{3} - \frac{1}{1}$

$\vdots \quad \vdots$

$\frac{1}{47} - \frac{1}{45}$

$= \frac{1}{47} + \frac{1}{3}$

$= \frac{3+47}{141} = \frac{50}{141}$

70. If $\tan 15^\circ + \frac{1}{\tan 75^\circ} + \frac{1}{\tan 105^\circ} + \tan 195^\circ = 2a$, then the value of $\left(a + \frac{1}{a}\right)$ is :

(1) 2 (2) $4 - 2\sqrt{3}$ (3) $5 - \frac{3}{2}\sqrt{3}$ (4) 4

Sol. **4**

$$\tan 15^\circ + \frac{1}{\tan 75^\circ} + \frac{1}{\tan 105^\circ} + \tan 195^\circ = 2a$$

$$\Rightarrow \tan 15^\circ + \frac{1}{\cot 15^\circ} - \frac{1}{\cot 15^\circ} + \tan 15^\circ = 2a$$

$$\Rightarrow \tan 15^\circ + \tan 15^\circ - \tan 15^\circ + \tan 15^\circ = 2a$$

$$\Rightarrow 2\tan 15^\circ = 2a$$

$$\Rightarrow a = \tan 15^\circ$$

$$a + \frac{1}{a} = \tan 15^\circ + \frac{1}{\tan 15^\circ}$$

$$= \tan 15^\circ + \cot 15^\circ$$

$$= 2 - \sqrt{3} + 2 + \sqrt{3}$$

$$\Rightarrow a + \frac{1}{a} = 4$$

71. If the solution of the equation $\log_{\cos x} \cot x + 4\log_{\sin x} \tan x = 1, x \in \left(0, \frac{\pi}{2}\right)$, is $\sin^{-1}\left(\frac{\alpha + \sqrt{\beta}}{2}\right)$, where α and β are integers, then α + β is equal to :

(1) 5 (2) 6 (3) 4 (4) 3

Sol. **3**

$$\log_{\cos x} \cot x + 4\log_{\sin x} \tan x = 1, x \in \left(0, \frac{\pi}{2}\right)$$

$$\Rightarrow \log_{\cos x} \frac{\cos x}{\sin x} + 4\log_{\sin x} \frac{\sin x}{\cos x} = 1$$

$$\Rightarrow 1 - \log_{\cos x} \sin x + 4 - 4\log_{\sin x} \cos x = 1$$

$$\Rightarrow 4 = \log_{\cos x} \sin x + 4\log_{\sin x} \cos x$$

Let $\log_{\cos x} \sin x = t$

$$\Rightarrow 4 = t + \frac{4}{t}$$

$$\Rightarrow t^2 - 4t + 4 = 0$$

$$\Rightarrow (t-2)^2 = 0$$

$$\Rightarrow t = 2$$

$$\Rightarrow \log_{\cos x} \sin x = 2$$

$$\Rightarrow \sin x = \cos^2 x$$

$$\Rightarrow \sin x = 1 - \sin^2 x$$

$$\Rightarrow \sin^2 x + \sin x - 1 = 0$$

$\Rightarrow \sin x = \dfrac{-1 \pm \sqrt{1+4}}{2}$

$\Rightarrow \sin x = \dfrac{-1+\sqrt{5}}{2}, \dfrac{-1-\sqrt{5}}{2} \quad \because x \in \left(0, \dfrac{\pi}{2}\right)$ then $\dfrac{-1-\sqrt{5}}{2}$ not possible

$\Rightarrow x = \sin^{-1}\left(\dfrac{-1+\sqrt{5}}{2}\right)$

$\because \alpha = -1, \beta = 5$ then

$\boxed{\alpha + \beta = 4}$

72. Let the system of linear equations

$x + y + kz = 2$

$2x + 3y - z = 1$

$3x + 4y + 2z = k$

have infinitely many solutions. Then the system

$(k + 1)x + (2k - 1)y = 7$

$(2k + 1)x + (k + 5)y = 10$

has:

(1) infinitely many solutions

(2) unique solution satisfying $x - y = 1$

(3) unique solution satisfying $x + y = 1$

(4) no solution

Sol. **3**

$x + y + kz = 2$

$2x + 3y - z = 1$

$3x + 4y + 2z = k$

Have Infinitely many solution then

$$\begin{vmatrix} 1 & 1 & k \\ 2 & 3 & -1 \\ 3 & 4 & 2 \end{vmatrix} = 0$$

$1(10) - 1(7) + k(8 - 9) = 0$

$\Rightarrow 10 - 7 - k = 0$

$\Rightarrow \boxed{k = 3}$

For $k = 3$

$4x + 5y = 7$

$7x + 8y = 10$

has unique solution and solution is $(-2, 3)$.

Hence solution is unique and satisfying $x + y = 1$

73. The line l_1 passes through the point (2,6,2) and is perpendicular to the plane $2x + y - 2z = 10$. Then the shortest distance between the line l_1 and the line $\frac{x+1}{2} = \frac{y+4}{-3} = \frac{z}{2}$ is :

(1) $\frac{13}{3}$ (2) $\frac{19}{3}$ (3) 7 (4) 9

Sol. **9**

equation of l_1 is $\frac{x-2}{2} = \frac{y-6}{1} = \frac{z-2}{-2}$

Let l_2 is $\frac{x+1}{2} = \frac{y+4}{-3} = \frac{z}{2}$

Point on l_1 is a = (2, 6, 2), direction $\vec{p} = <2,1,-2>$

Point on l_2 is b = (–1, –4, 0) direction $\vec{q} = <2,-3,2>$

Shortest distance between l_1 and l_2 = $\left|\frac{(\vec{a}-\vec{b}).(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right|$

$$\because \quad \vec{p}\times\vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 2 & 1 & -2 \\ 2 & -3 & 2 \end{vmatrix} = \hat{i}(-4) - \hat{j}(8) + k(-8)$$

$$= \left|\frac{\langle 3,10,2\rangle\cdot\langle -4,-8,-8\rangle}{\sqrt{16+64+64}}\right|$$

$$= \left|\frac{-12-80-16}{\sqrt{144}}\right|$$

$$= \frac{108}{12}$$

$= 9$

Shortest distance between the lines is 9.

74. Let $A = \begin{pmatrix} m & n \\ p & q \end{pmatrix}$, $d = |A| \neq 0$ and $|A - d(\text{Adj}A)| = 0$. Then

(1) $1 + d^2 = m^2 + q^2$ (2) $1 + d^2 = (m + q)^2$

(3) $(1 + d)^2 = m^2 + q^2$ (4) $(1 + d)^2 = (m + q)^2$

Sol. **4**

$A = \begin{bmatrix} m & n \\ p & q \end{bmatrix}$, d =|A| = mq – np

$$A - d(\text{Adj. }A) = \begin{bmatrix} m & n \\ p & q \end{bmatrix} - d\begin{bmatrix} q & -n \\ -p & m \end{bmatrix}$$

$$= \begin{bmatrix} m - dq & n + dn \\ p + pd & q - dm \end{bmatrix}$$

$|A - d(\text{Adj }A)| = (m - dq)(q - dm) - (n + dn)(p + pd) = 0$

$\Rightarrow mq - m^2d - dq^2 + d^2qm = np(1+d)^2$

$\Rightarrow (mq - m^2d - dq^2 + d^2qm) = (mq - d)(1 + d)^2$

$\Rightarrow mq - m^2d - dq^2 + d^2qm = mq + mqd^2 + 2mqd - d(1 + d)^2$

$\Rightarrow d(1+d)^2 = m^2d + dq^2 + 2mqd$

$\Rightarrow \boxed{(1+d)^2 = (m+q)^2}$

75. If [t] denotes the greatest integer $\leq$ t, then the value of $\frac{3(e-1)}{e}\int_1^2 x^2 e^{[x]+[x^3]} dx$ is :

(1) $e^8 - 1$ (2) $e^7 - 1$ (3) $e^8 - e$ (4) $e^9 - e$

Sol. **3**

$$\frac{3(e-1)^2}{e}\int_1^2 x^2 e^{[x]+[x^3]}\,dx$$

$$\text{Let } I = \int_1^2 x^2 e^{[x]+[x^3]}\,dx$$

$$I = \int_1^2 x^2 e^{1+[x^3]}\,dx\,dx$$

$$\Rightarrow I = e\int_1^2 x^2 e^{[x^3]}\,dx$$

Let $x^3 = t$

$3x^2\,dx = dt$

$$I = \frac{e}{3}\int_1^8 e^{[t]}\,dt$$

$$\Rightarrow I = \frac{e}{3}\left[\int_1^2 e\,dt + \int_2^3 e^2 dt + \int_3^4 e^3 dt + \ldots.. + \int_7^8 e^7 dt\right]$$

$$\Rightarrow I = \frac{e}{3}\left[e + e^2 + e^3 + \ldots.. + e^7\right]$$

$$\Rightarrow I = \frac{e}{3}\left[\frac{e(e^7 - 1)}{e - 1}\right]$$

$$\text{Therefore } \frac{3(e-1)}{e}\int_1^2 x^2 e^{[x]+[x^3]}\,dx = \frac{3(e-1)}{e} \times \frac{e^2}{3}\,\frac{(e^7 - 1)}{e - 1}$$

$$\Rightarrow \frac{3(e-1)}{e}\int_1^2 x^2 e^{[x]+[x^3]}dx = e^8 - e$$

76. Let a unit vector $\widehat{OP}$ make angles α, β, γ with the positive directions of the co-ordinate axes OX, OY, OZ respectively, where $\beta \in \left(0, \frac{\pi}{2}\right)$. If $\widehat{OP}$ is perpendicular to the plane through points (1,2,3), (2,3,4) and (1,5,7), then which one of the following is true ?

(1) $\alpha \in \left(0, \frac{\pi}{2}\right)$ and $\gamma \in \left(0, \frac{\pi}{2}\right)$ (2) $\alpha \in \left(0, \frac{\pi}{2}\right)$ and $\gamma \in \left(\frac{\pi}{2}, \pi\right)$

(3) $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\gamma \in \left(\frac{\pi}{2}, \pi\right)$ (4) $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\gamma \in \left(0, \frac{\pi}{2}\right)$

Sol. **3**

∵ $\overrightarrow{OP}$ makes angle α, β, γ with positive directions of the co-ordinate axes then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

Point on planes are a(1, 2, 3), b(2, 3, 4) and c(1, 5, 7).

∵ $\overrightarrow{ab} = <1, 1, 1>$

$\overrightarrow{ac} = <0,3, 4>$

normal vector of plane = $\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 0 & 3 & 4 \end{bmatrix}$

$= \hat{i}(1) - \hat{j}(4) + \hat{k}(3)$

$= <1, -4, 3>$

direction cosine of normal is $= \left\langle \pm\frac{1}{\sqrt{26}}, \pm\frac{4}{\sqrt{26}}, \pm\frac{3}{\sqrt{26}} \right\rangle$

then direction cosine of $\overrightarrow{op}$ is $\left\langle -\frac{1}{\sqrt{26}}, \frac{4}{\sqrt{26}}, -\frac{3}{\sqrt{26}} \right\rangle$

$\left(\because \beta \in \left(0, \frac{\pi}{2}\right)\right)$

Hence $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\gamma \in \left(\frac{\pi}{2}, \pi\right)$

77. The coefficient of x^{301} in $(1+x)^{500} + x(1+x)^{499} + x^2(1+x)^{498} + \cdots \ldots x^{500}$ is :

(1) $^{500}C_{300}$ (2) $^{501}C_{200}$ (3) $^{501}C_{302}$ (4) $^{500}C_{301}$

Sol. **2**

$x^0(1+x)^{500} + x(1+x)^{499} + x^2(1+x)^{498} + \ldots + x^{500}$

$$= (1+x)^{500} \frac{\left[\left(\frac{x}{1+x}\right)^{501} - 1\right]}{\frac{x}{1+x} - 1}$$

$$= \frac{(1+x)^{500}(x^{501} - (1+x)^{501})}{(1+x)^{501}\left(\frac{-1}{x+x}\right)}$$

$= (1+x)^{501} - x^{501}$

Coefficient of x^{301} in above expression is $^{501}C_{301}$ or $^{501}C_{200}$.

78. Let the solution curve $y = y(x)$ of the differential equation $\frac{dy}{dx}-\frac{3x^5\tan^{-1}(x^3)}{(1+x^6)^{\frac{3}{2}}}y = 2x\exp\left\{\frac{x^3-\tan^{-1}x^3}{\sqrt{(1+x^6)}}\right\}$ pass through the origin. Then $y(1)$ is equal to :

(1) $\exp\left(\frac{4+\pi}{4\sqrt{2}}\right)$ (2) $\exp\left(\frac{1-\pi}{4\sqrt{2}}\right)$ (3) $\exp\left(\frac{\pi-4}{4\sqrt{2}}\right)$ (4) $\exp\left(\frac{4-\pi}{4\sqrt{2}}\right)$

Sol. **4**

$$\left(\frac{dy}{dx}\right)-\frac{3x^5\tan^{-1}(x^3)}{(1+x^6)^{3/2}}y = 2x\exp\left(\frac{x^3-\tan^{-1}x^3}{\sqrt{(1+x^6)}}\right)$$

above equation is linear differential equation.

$$\text{I.F.} = e^{\int\frac{-3x^5\tan^{-1}(x^3)}{(1+x^6)^{3/2}}dx}$$

$$= e^{-\int\frac{3x^2\cdot x^3\tan^{-1}(x^3)}{(1+x^6)^{3/2}}dx}$$

Let $\tan^{-1}(x^3) = t$ then

$$\frac{3x^2\cdot dx}{1+x^6} = dt$$

$$= e^{-\int\frac{t\tan t}{\sqrt{1+\tan^2 t}}dt}$$

$$= e^{-\int\frac{t\tan t}{\sec t}dt}$$

$$= e^{-\int t\sin t\,dt}$$

$$= e^{-[-t\cos t+\sin t]}$$

$$= e^{t\cos t-\sin t}$$

$$\text{I.F.} = e^{\frac{\tan^{-1}x^3}{\sqrt{1+x^6}}-\frac{x^3}{\sqrt{1+x^6}}}$$

Solution is

$$y\left(e^{\frac{\tan^{-1}x^3}{\sqrt{1+x^6}}-\frac{x^3}{\sqrt{1+x^6}}}\right) = \int 2x\,e^{\left(\frac{x^3-\tan^{-1}x^3}{\sqrt{1+x^6}}\right)}\cdot e^{\frac{\tan^{-1}x^3-x^3}{\sqrt{1+x^6}}}dx$$

$$y\left(e^{\frac{\tan^{-1}x^3-x^3}{\sqrt{1+x^6}}}\right) = \int 2x\,dx = x^2+c$$

above eq. is passing through (0, 0) then $c = 0$

$$y = x^2\,e^{\frac{x^3-\tan^{-1}x^3}{\sqrt{1+x^6}}}$$

Put $x = 1$ then

$$y(1) = e^{\frac{1-\frac{\pi}{4}}{\sqrt{2}}} = e^{\frac{4-\pi}{4\sqrt{2}}}$$

$$\Rightarrow y(1) = \exp\left(\frac{4-\pi}{4\sqrt{2}}\right)$$

79. If the coefficient of x^{15} in the expansion of $\left(ax^3 + \frac{1}{bx^{1/3}}\right)^{15}$ is equal to the coefficient of x^{-15} in the expansion of $\left(ax^{1/3} - \frac{1}{bx^3}\right)^{15}$, where a and b are positive real numbers, then for each such ordered pair (a, b) :

(1) $ab = 3$ (2) $ab = 1$ (3) $a = b$ (4) $a = 3b$

Sol. **2**

$$\left(ax^3 + \frac{1}{bx^{1/3}}\right)^{15}$$

general term is $T_{r+1} = {}^{15}C_r\,(ax^3)^{15-r}\left(\frac{1}{bx^{1/3}}\right)^r$

$\Rightarrow\ T_{r+1} = {}^{15}C_r\,\frac{a^{15-r}}{b^r}\,x^{45-3r-\frac{r}{3}}$

For coefficient of $x^{15} \Rightarrow 45 - 3r - \frac{r}{3} = 15$

$$30 = \frac{10r}{3}$$

$$r = 9$$

Coefficient of x^{15} is $= {}^{15}C_9\,a^6\,b^{-9}$ … (1)

$\because$ general term of $\left(ax^{1/3} - \frac{1}{bx^3}\right)^{15}$ is

$T_{r+1} = {}^{15}C_r\,(ax^{1/3})^{15-r}\left(\frac{-1}{bx^3}\right)^r$

For coefficient of $x^{-15} \Rightarrow \frac{15-r}{3} - 3r = -15$

$\Rightarrow 15 - r - 9r = -45$

$\Rightarrow 60 = 10\ r$

$\boxed{r = 6}$

Coefficient of x^{-15} is $= {}^{15}C_6\,a^9\,b^{-b}$ … (2)

$\because$ both coefficient are equal then

${}^{15}C_9\,a^6\,b^{-9} = {}^{15}C_6\,a^9\,b^{-6}$

$\Rightarrow a^6\,b^{-9} = a^9\,b^{-6}$

$\Rightarrow a^3\,b^3 = 1$

$\Rightarrow \boxed{ab = 1}$

80. Suppose $f: \mathbb{R} \to (0, \infty)$ be a differentiable function such that $5f(x+y) = f(x) \cdot f(y), \forall x, y \in \mathbb{R}$. If $f(3) = 320$, then $\sum_{n=0}^{5} f(n)$ is equal to :

(1) 6875 (2) 6525 (3) 6825 (4) 6575

Sol. **3**

$f : \mathbb{R} \to (0, \infty)$

$5\, f(x+y) = f(x) \cdot f(y)$

Put $x = 3,\ y = 0$ then

$5\, f(3) = f(3)\, f(0)$

$\Rightarrow \boxed{f(0) = 5}$

Put $x = 1,\ y = 1$ then $\quad 5\, f(2) = f^2(1)$

Put $x = 1,\ y = 2$ then $\quad 5\, f(3) = f(1)\, f(2)$

$$5 \times 320 = \frac{f^3(1)}{5} = f(1) = 20$$

$$\Rightarrow f(2) = 80$$

Put $x = 2,\ y = 2$ then $\quad 5\, f(4) = f(2)\, f(2)$

$$f(4) = \frac{80 \times 80}{5} = 1280$$

Put $x = 2,\ y = 3$ then $\quad 5\, f(5) = f(2) \cdot f(2)$

$$f(5) = \frac{80 \times 320}{5} = 5120$$

$$\sum_{n=0}^{5} F(n) = f(0) + f(1) + \ldots. + f(5)$$

$$= 5 + 20 + 80 + 320 + 1280 + 5120$$

$$= 5(1 + 2^2 + 2^4 + 2^6 + 2^8 + 2^{10})$$

$$= 6825$$

Section B

81. Let $z = 1 + i$ and $z_1 = \frac{1+i\bar{z}}{\bar{z}(1-z)+\frac{1}{z}}$. Then $\frac{12}{\pi}\arg(z_1)$ is equal to

Sol. **9**

$z = 1 + i, \quad \bar{z} = 1 - i, \quad i\,\bar{z} = 1 + i$

$$z_1 = \frac{1 + i\bar{z}}{\bar{z}(1 - z) + \frac{1}{z}}$$

$$z_1 = \frac{i + z}{\bar{z} - z\bar{z}) + \frac{1}{z}}$$

$$z_1 = \frac{i + 2}{1 - i - 2 + \frac{1 - i}{2}}$$

$$z_1 = \frac{i + 2}{-\frac{1}{2} - \frac{3i}{2}}$$

$$z_1 = \frac{-2(i+2)}{(1+3i)} \times \frac{(1-3i)}{(1-3i)}$$

$$z_1 = \frac{-2(5-5i)}{10}$$

$$z_1 = -1+i$$

$$\arg.(z_1) = \pi - \tan^{-1}\left(\frac{1}{1}\right) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

$$\therefore \arg(z_1) = \frac{12}{\pi} \times \frac{3\pi}{4} = 9$$

82. If $\lambda_1 < \lambda_2$ are two values of λ such that the angle between the planes $P_1 : \vec{r} \cdot (3\hat{i} - 5\hat{j} + \hat{k}) = 7$ and $P_2 : \vec{r} \cdot (\lambda\hat{i} + \hat{j} - 3\hat{k}) = 9$ is $\sin^{-1}\left(\frac{2\sqrt{6}}{5}\right)$, then the square of the length of perpendicular from the point $(38\lambda_1, 10\lambda_2, 2)$ to the plane P_1 is

Sol. **315s**

Plane $P_1 : \vec{r}.(3\hat{i} - 5\hat{j} + \hat{k}) = 7$

$P_2 : \vec{r}.(\lambda\hat{i} + \hat{j} - 3k) = 9$

angle between plane is same as angle between their normal.

angle between normal θ then

$$\text{Cos } \theta = \frac{\langle 3, -5, 1 \rangle \cdot \langle \lambda, 1, -3 \rangle}{\sqrt{9+25+1}\sqrt{\lambda^2+1+9}}$$

$$\text{Cos } \theta = \frac{3\lambda - 5 - 3}{\sqrt{35}\sqrt{\lambda^2+10}} \qquad \dots (1)$$

$$\because \quad \theta = \sin^{-1}\left(\frac{2\sqrt{6}}{5}\right) \text{ then}$$

$$\sin\theta = \frac{2\sqrt{6}}{5}$$

$$\cos\theta = \frac{1}{5}$$

from equation (1)

$$\frac{3\lambda - 8}{\sqrt{35}\sqrt{\lambda^2+10}} = \frac{1}{5}$$

$$\Rightarrow \frac{(3\lambda-8)^2}{35(\lambda^2+10)} = \frac{1}{25}$$

$$\Rightarrow 5(3\lambda - 8)^2 = 7(\lambda^2 + 10)$$

$$\Rightarrow 5(9\lambda^2 - 48\lambda + 64) = 7\lambda^2 + 70$$

$$\Rightarrow 38\lambda^2 - 240\lambda + 250 = 0$$

$$\Rightarrow 19\lambda^2 - 120\lambda + 125 = 0$$

$$\Rightarrow \lambda = 5, \frac{25}{19}$$

$$\lambda_1 = \frac{25}{19}, \lambda_2 = 5$$

Point (38 λ_1, 10 λ_2, 2) ≡ (50, 50, 2)

distance of (50, 50, 2) from plane P_1 is

$$d = \left|\frac{3 \times 50 - 5 \times 50 + 2 - 7}{\sqrt{9 + 25 + 1}}\right|$$

$$d = \left|\frac{150 - 250 + 2 - 7}{\sqrt{35}}\right|$$

$$d = \left|\frac{105}{\sqrt{35}}\right|$$

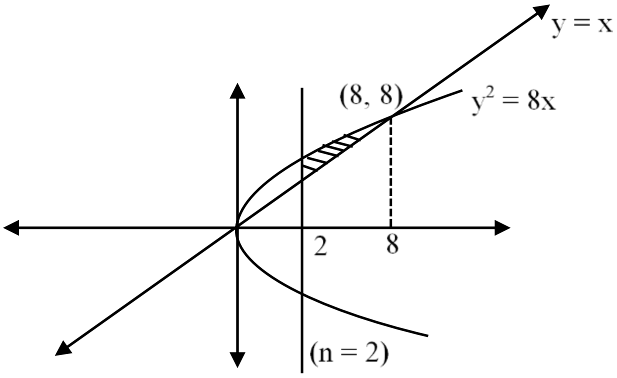
$$d = 3\sqrt{35}$$

$$\boxed{d^2 = 315}$$

83. Let α be the area of the larger region bounded by the curve $y^2 = 8x$ and the lines $y = x$ and $x = 2$, which lies in the first quadrant. Then the value of 3α is equal to

Sol. **22**

$$\text{area } (\alpha) = \int_2^8 (2\sqrt{2}\sqrt{x} - x)dx$$



$$= \left[2\sqrt{2}\cdot\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right]_2^8$$

$$= \frac{4\sqrt{2}}{3}[8 \times 2\sqrt{2} - 2\sqrt{2}] - 30$$

$$= \frac{28 \times 4}{3} - 30$$

$$= \frac{112}{3} - 30$$

$$\alpha = \frac{22}{3}$$

$$\boxed{3\alpha = 22}$$

84. Let $\sum_{n=0}^{\infty} \frac{n^3((2n)!)+(2n-1)(n!)}{(n!)((2n)!)} = ae + \frac{b}{e} + c$, where $a, b, c \in \mathbb{Z}$ and $e = \sum_{n=0}^{\infty} \frac{1}{n!}$ Then $a^2 - b + c$ is equal to

Sol. **26**

Let $\sum_{n=0}^{\infty} \frac{n^3((2n)!) + (2n-1)n!}{(n!)((2n)!)}$

$= \sum_{n=0}^{\infty} \frac{n^3(2n)!}{n!(2n)!} + \frac{(2n-1)n!}{n!(2n)!}$

$= \quad S_1 \quad + \quad S_2$

Let $S_1 = \sum_{n=0}^{\infty} \frac{n^3(2n)!}{n!(2n)!} = \sum_{n=0}^{\infty} \frac{n^3}{n!} = \sum_{n=1}^{\infty} \frac{n^2}{(n-1)!}$

$= \sum_{n=1}^{\infty} \frac{n^2 - 1 + 1}{(n-1)!}$

$= \sum_{n=2}^{\infty} \frac{(n+1)}{(n-2)!} + \sum_{n=1}^{\infty} \frac{1}{(n-1)!}$

$= \sum_{n=2}^{\infty} \frac{(n-2)+3}{(n-2)!} + \sum_{n=1}^{\infty} \frac{1}{(n-1)!}$

$= \sum_{n=3}^{\infty} \frac{1}{(n-3)!} + 3\sum_{n=2}^{\infty} \frac{1}{(n-2)!} + \sum_{n=1}^{\infty} \frac{1}{(n-1)!}$

$S_1 = e + 3e + e = 5e$

$\because \quad S_2 = \sum_{n=0}^{\infty} \frac{(2n-1)n!}{n!(2n)!}$

$= \sum_{n=0}^{\infty} \frac{2n-1}{(2n)!}$

$= \sum_{n=1}^{\infty} \frac{1}{(2n-1)!} - \sum_{n=0}^{\infty} \frac{1}{(2n)!}$

$= \left(\frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \ldots.\right) - \left(1 + \frac{1}{2!} + \frac{1}{4!} + \ldots..\right)$

$= -1 + \frac{1}{1!} - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \frac{1}{5!} - \ldots.$

$= -\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} + \ldots.\right)$

$= -e^{-1}$

$S_1 + S_2 = 5e - \frac{1}{e} = ae + \frac{b}{e} + c$

Compare both side

$a = 5, b = -1, c = 0$

$a^2 - b + c = 25 + 1 + 0 = 26$

85. If the equation of the plane passing through the point (1,1,2) and perpendicular to the line $x - 3y + 2z - 1 = 0 = 4x - y + z$ is $Ax + By + Cz = 1$, then $140(C - B + A)$ is equal to

Sol. **15**

give line is $x - 3y + 2z - 1 = 0 = 4x - y + z$

Direction of line $\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & -3 & 2 \\ 4 & -1 & 1 \end{vmatrix} = \hat{i}(-1) - \hat{j}(-7) + k(11)$

$\Rightarrow \quad \vec{a} = <-1, 7, 11>$

$\because$ Line is $\perp^r$ to the plane then direction of line is parallel to normal of plane.

$\vec{n} = <-1, 7, 11>$

Equation of plane is

$-1(x-1) + 7(y-1) + 11(z-2) = 0$

$-x + 7y + 11z + 1 - 7 - 22 = 0$

$\Rightarrow \quad -x + 7y + 11z = 28$

$\Rightarrow \quad -\frac{1}{28}x + \frac{7}{28}y + \frac{11}{28}z = 1$

$A = -\frac{1}{28}, B = \frac{7}{28}, C = \frac{11}{28}$

$140(C - B + A) = 140\left(\frac{11}{28} - \frac{7}{28} - \frac{1}{28}\right)$

$= \frac{140 \times 3}{28} = 15$

86. Number of 4-digit numbers (the repeation of digits is allowed) which are made using the digits 1, 2, 3 and 5 , and are divisible by 15 , is equal to

Sol. **21**

			5

Last digit must be 5 and sum of digits is divisible by 3 for divisible by 15

Remaining 3 digits	Arrange
(1, 1, 2)	$\frac{3!}{2} = 3$
(1, 3, 3)	$\frac{3!}{2} = 3$
(1, 5, 1)	$\frac{3!}{2} = 3$
(2, 2, 3)	$\frac{3!}{2} = 3$
(2, 3, 5)	$3! = 6$
(3, 5, 5)	$\frac{3!}{2} = 3$

Total numbers = 21

87. Let $f^1(x) = \frac{3x+2}{2x+3}, x \in \mathbf{R} - \left\{\frac{-3}{2}\right\}$

For $n \geq 2$, define $f^n(x) = f^1$ of $f^{n-1}(x)$

If $f^5(x) = \frac{ax+b}{bx+a}, \gcd(a, b) = 1$, then $a + b$ is equal to

Sol. **3125**

$$f^1(x) = \frac{3x+2}{2x+3}, x \in R - \left\{-\frac{3}{2}\right\}$$

$$f^2(x) = f^1 0\ f^1(x) = f^1\left(\frac{3x+2}{2x+3}\right)$$

$$= \frac{3\left(\frac{3x+2)}{2x+3)}\right)+2}{2\left(\frac{3x+2}{2x+3}\right)+3}$$

$$= \frac{9x+6+4x+6}{6x+4+6x+9}$$

$$f^2(x) = \frac{13x+12}{12x+13}$$

$$f^3(x) = f^1 \text{ o } f^2(x)$$

$$= f^1\left(\frac{13x+12}{12x+13}\right)$$

$$= \frac{3\left(\frac{13x+12}{12x+13}\right)+2}{2\left(\frac{13x+12}{12x+13}\right)+3}$$

$$= \frac{39x+36+24x+26}{26x+24+36x+39}$$

$$f^3(x) = \frac{63x+62}{62x+63}$$

$$f^4(x) = f^1\left(\frac{63x+62}{62x+63}\right)$$

$$= \frac{3\left(\frac{63x+62}{62x+63}\right)+2}{2\left(\frac{63x}{62x}+\frac{62}{63}\right)+2}$$

$$f^4(x) = \frac{313x+312}{312x+313}$$

$$f^5(x) = f^1\left(\frac{313x+312}{312x+313}\right)$$

$$=\frac{3\left(\frac{313x+312}{312x+313}\right)+2}{2\left(\frac{313x+312}{312x+313}\right)+3}$$

$$f^5(x)=\frac{1563x+1562}{1562x+1563}$$

$\because a=1563, b=1562$

$\boxed{a+b=3125}$

88. The mean and variance of 7 observations are 8 and 16 respectively. If one observation 14 is omitted and a and b are respectively mean and variance of remaining 6 observation, then a + 3b − 5 is equal to

Sol. **37**

Mean of 7 observations = $\frac{\sum_{i=1}^{7} x_i}{7}$

$\Rightarrow \quad \sum_{i=1}^{7} x_i = 7\times 8 = 56$

Variance = $\frac{\Sigma x_i^2}{n} - (\bar{x})^2$

$\Sigma x_i^2 = 7(16+64) = 560$

If 14 is removed then

Mean = $a = \frac{\sum_{i=1}^{7} x_i - 14}{6} \Rightarrow 6a = 56 - 14$

$\Rightarrow a = 7$

Variance = $b = \frac{\sum_{i=1}^{7} x_i^2 - (14)^2}{6} - 49$

$\Rightarrow \quad 6b = 560 - 196 - 294$

$\Rightarrow \quad 6b = 70$

$\Rightarrow \quad 3b = 35$

$\therefore \quad a + 3b - 5 = 7 + 35 - 5 = 37$

89. Let S = {1,2,3,4,5,6}. Then the number of one-one functions $f: S \to P(S)$, where P(S) denote the power set of S, such that $f(n) \subset f(m)$ where $n < m$ is

Sol. **3240**

Case – I

f(6) = S i.e. 1 option

f(5) = any 5 element subset A of S i.e. ${}^{6}C_{5} = 6$ options

f(4) = any 4 element subset B of A i.e. ${}^{5}C_{4} = 5$ options

f(3) = any 3 element subset C of B i.e. ${}^{4}C_{3} = 4$ options

f(2) = any 2 element subset D of C i.e. ${}^{3}C_{2} = 3$ options

f(1) = any 1 element subset E of D or empty subset i.e. 3 options

Total function = $6\times5\times4\times3\times2\times3=1080$

Case – II

f(6) = S

f(5) = any 4 element subset A of S i.e. ${}^{6}C_{4} = 15$ options

f(4) = any 3 element subset B of A i.e. ${}^{4}C_{3} = 4$ options

f(3) = any 2 element subset C of B i.e. ${}^{3}C_{2} = 2$ options

f(2) = any 1 element subset D of C i.e. ${}^{2}C_{1} = 2$ options

f(1) = empty subset i.e. 1 options

Total function = $15\times4\times3\times2\times1=360$

Case – III

f(6) = S

f(5) = any 5 element subset A of S i.e. ${}^{6}C_{5} = 6$ options

f(4) = any 3 element subset B of A i.e. ${}^{5}C_{3} = 10$ options

f(3) = any 2 element subset C of B i.e. ${}^{3}C_{2} = 3$ options

f(2) = any 1 element subset D of C i.e. ${}^{2}C_{1} = 2$ options

f(1) = empty subset i.e. 1 options

Total function = $6\times10\times3\times2\times1=360$

Case – IV

f(6) = S

f(5) = any 5 element subset A of S i.e. ${}^{6}C_{5} = 6$ options

f(4) = any 4 element subset B of A i.e. ${}^{5}C_{4} = 5$ options

f(3) = any 2 element subset C of B i.e. ${}^{4}C_{2} = 6$ options

f(2) = any 1 element subset D of C i.e. ${}^{2}C_{1} = 2$ options

f(1) = empty subset i.e. 1 options

Total function = $6\times5\times6\times2\times1=360$

Case – V

f(6) = S

f(5) = any 5 element subset A of S i.e. ${}^6C_5 = 6$ options

f(4) = any 4 element subset B of A i.e. ${}^5C_4 = 5$ options

f(3) = any 3 element subset C of B i.e. ${}^4C_3 = 4$ options

f(2) = any 1 element subset D of C i.e. ${}^3C_1 = 3$ options

f(1) = empty subset i.e. 1 options

Total function = 6×5×4×3×1=360

Case – VI

f(6) = any 5 element subset A of S i.e. ${}^6C_5 = 6$ options

f(5) = any 4 element subset B of A i.e. ${}^5C_4 = 5$ options

f(4) = any 3 element subset C of B i.e. ${}^4C_3 = 4$ options

f(3) = any 2 element subset D of C i.e. ${}^3C_2 = 3$ options

f(2) = any 1 element subset E of D i.e. ${}^2C_1 = 2$ options

f(1) = empty subset i.e. 1 options

Total function = 6×5×4×3×2×1=720

Total number of such functions = 1080 + (4× 360) +720 = **3240**

90. $\lim_{x\to 0}\frac{48}{x^4}\int_0^x \frac{t^3}{t^6+1}$ dt is equal to

Sol. **12**

$$\lim_{x\to 0}\frac{48}{x^4}\int_0^x \frac{t^3}{t^6+1} = \left(\frac{0}{0}\right) \text{ form}$$

Using L Hopital Rule

$$= \lim_{x\to 0}\frac{48\times\frac{x^3}{x^6+1}}{4x^3}$$

$$= \lim_{x\to 0}\frac{48}{4(x^6+1)}$$

$$= \frac{48}{4}$$

= 12

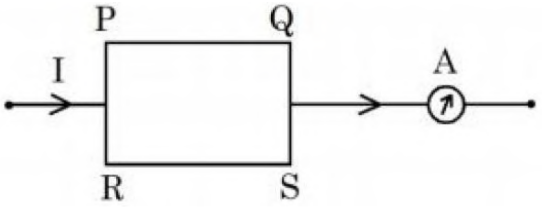
JEE–MAIN EXAMINATION – JANUARY, 2023

(Held On Thursday 30th January, 2023) | TIME : 3 : 00 PM to 6 : 00 PM

Physics

SECTION - A

1. A current carrying rectangular loop $PQRS$ is made of uniform wire. The length $PR = QS = 5$ cm and $= RS = 100$ cm. If ammeter current reading changes from I to $2I$, the ratio of magnetic forces per unit length on the wire PQ due to wire RS in the two cases respectively $\left(f_{PQ}^{I} : f_{PQ}^{2I}\right)$ is:



(1) 1: 2 (2) 1: 3 (3) 1: 4 (4) 1: 5

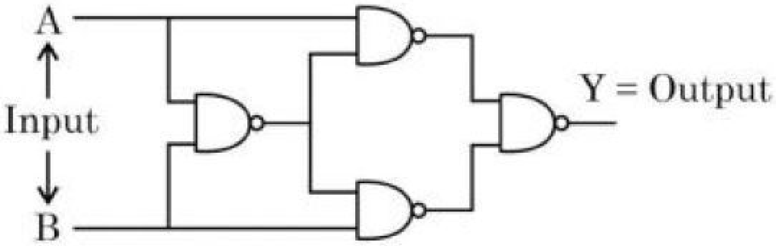
Sol. **(3)**

$F \propto I_1 I_2$

$$\frac{F_I}{F_{2I}} = \frac{1}{4}$$

Ans. (3)

2. The output Y for the inputs A and B of circuit is given by



Truth table of the shown circuit is:

(1)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

(2)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

(3)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(4)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Sol. **(3)**

3. Given below are two statements: one is labelled as Assertion **A** and the other is labelled as Reason R

Assertion A: Efficiency of a reversible heat engine will be highest at $-273^\circ\mathrm{C}$ temperature of cold reservoir.

Reason R: The efficiency of Carnot's engine depends not only on temperature of cold reservoir but it depends on the temperature of hot reservoir too and is given as $\eta = \left(1 - \frac{T_2}{T_1}\right)$.

In the light of the above statements, choose the correct answer from the options given below

(1) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

(2) Both **A** and **R** are true and **R** is the correct explanation of **A**

(3) A is false but **R** is true

(4) A is true but **R** is false

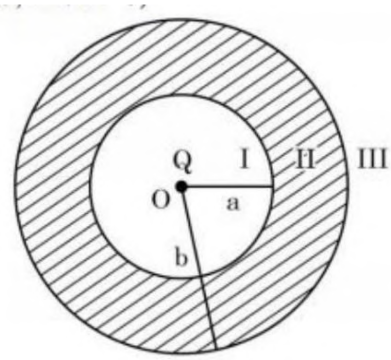
Sol. (2)

$$\eta = 1 - \frac{T_L}{T_H} = \frac{T_H - T_L}{T_H}$$

Efficiently of Carnot's engine will be highest at – 273° = 0K

Ans. (2)

4. As shown in the figure, a point charge Q is placed at the centre of conducting spherical shell of inner radius a and outer radius b. The electric field due to charge Q in three different regions I, II and III is given by: (I: $r < a$, II: $a < r < b$, III: $r > b$)



(1) $E_I = 0, E_{II} = 0, E_{III} = 0$

(2) $E_I = 0, E_{II} = 0, E_{III} \neq 0$

(3) $E_I \neq 0, E_{II} = 0, E_{III} \neq 0$

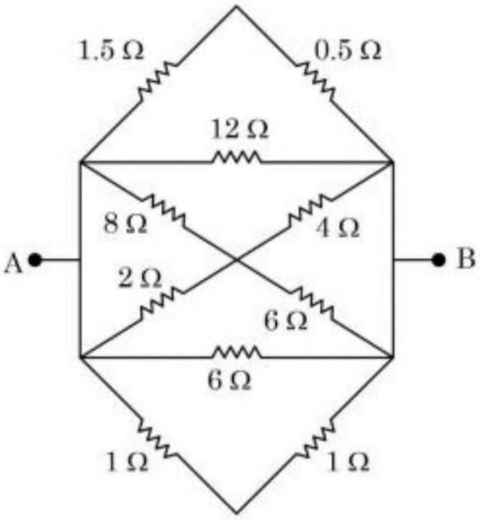
(4) $E_I \neq 0, E_{II} = 0, E_{III} = 0$

Sol. **Sol. (3)**

Electric field inside material of conductor is zero

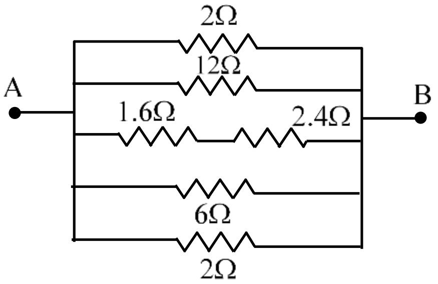
Ans. (3)

5. The equivalent resistance between *A* and *B* is



(1) $\frac{1}{3}\Omega$ (2) $\frac{1}{2}\Omega$ (3) $\frac{3}{2}\Omega$ (4) $\frac{2}{3}\Omega$

Sol. **(4)**



$$\frac{1}{R_{eq}} = \frac{1}{2}+\frac{1}{12}+\frac{1}{4}+\frac{1}{6}+\frac{1}{2}$$

$$=\frac{18}{12}=\frac{3}{2}$$

$$R_{eq}=\frac{2}{3}\ \Omega$$

Ans. (4)

6. A vehicle travels 4 km with speed of 3 km/h and another 4 km with sped of 5 km/h, then its average speed is

(1) 3.50 km/h (2) 4.25 km/h (3) 4.00 km/h (4) 3.75 km/h

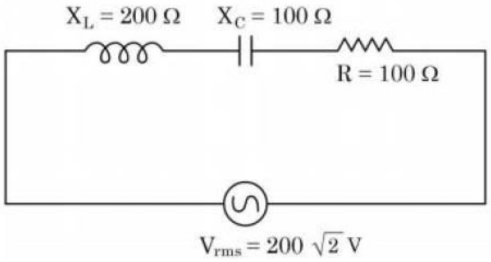
Sol. **(4)**

$$\frac{2}{V_{av}}=\frac{1}{3}+\frac{1}{5}=\frac{8}{15}$$

$$V_{av}=\frac{15}{4}=3.75\ \text{km hr}^{-1}$$

Ans. (4)

7. In the given circuit, rms value of current (I_{rms}) through the resistor R is:



(1) $2\sqrt{2}$ A (2) 2 A (3) 20 A (4) $\frac{1}{2}$ A

Sol. **(2)**

$$Z = \sqrt{R^2 + (X_L - X_e)^2}$$

$$Z = \sqrt{(100)^2 + (200-100)^2}$$

$$Z = 100\sqrt{2}\ \ \Omega$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{200\sqrt{2}}{100\sqrt{2}} = 2\text{ A}$$

8. A point source of 100 W emits light with 5% efficiency. At a distance of 5 m from the source, the intensity produced by the electric field component is:

(1) $\frac{1}{2\pi}\frac{W}{m^2}$ (2) $\frac{1}{20\pi}\frac{W}{m^2}$ (3) $\frac{1}{10\pi}\frac{W}{m^2}$ (4) $\frac{1}{40\pi}\frac{W}{m^2}$

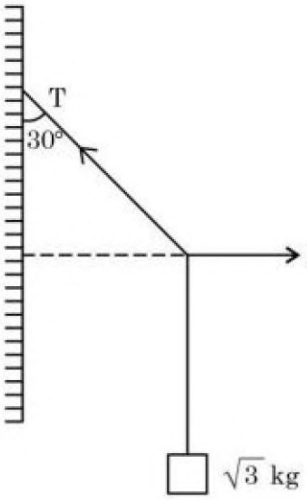
Sol. **(4)**

$$I_{EF} = \frac{1}{2} \times \frac{5}{4\pi(5)^2}$$

$$= \frac{1}{40\pi}\text{ w/m}^2$$

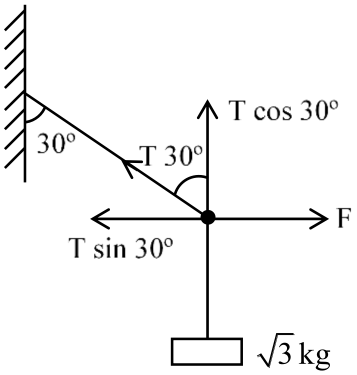
Ans: (4)

9. A block of $\sqrt{3}$ kg is attached to a string whose other end is attached to the wall. An unknown force F is applied so that the string makes an angle of 30° with the wall. The tension T is: (Given $g = 10\ ms^{-2}$)



(1) 20 N (2) 10 N (3) 15 N (4) 25 N

Sol. **(1)**



$F = T\sin 30°$

$\sqrt{3}g = T\cos 30°$

$\tan 30° = \frac{F}{\sqrt{3}}g$

$\frac{1}{\sqrt{3}} = \frac{F}{\sqrt{3}g}$

F = 10 N

$T = \frac{F}{\sin 30°} = 10 \times 2$

T = 10 × 2 = 20 N

Ans: (1)

10. Match List I with List II:

List I	List II
A. Attenuation	I. Combination of a receiver and transmitter.
B. Transducer	II. process of retrieval of information from the carrier wave at receiver
C. Demodulation	III. converts one form of energy into another
D. Repeater	IV. Loss of strength of a signal while propogating through a medium.

Choose the correct answer from the options given below:

(1) A-IV, B-III, C-I, D-II (2) A-I, B-II, C-III, D-IV

(3) A-IV, B-III, C-II, D-I (4) A-II, B-III, C-IV, D-I

Sol. **(3)**

Theory

11. An electron accelerated through a potential difference V_1 has a de-Broglie wavelength of λ. When the potential is changed to V_2, its de-Broglie wavelength increases by 50%. The value of $\left(\frac{V_1}{V_2}\right)$ is equal to

(1) 3 (2) $\frac{3}{2}$ (3) 4 (4) $\frac{9}{4}$

Sol. **(4)**

$KE = \frac{P^2}{2m}$

$P = \frac{h}{\lambda}$

$eV_1 = \frac{\left(\frac{h}{\lambda}\right)^2}{2m}$

$eV_2 = \frac{\left(\frac{h}{1.5\lambda}\right)^2}{2m}$

$\frac{V_1}{V_2} = (1.5)^2 = \frac{9}{4}$

Ans: (4)

12. A flask contains hydrogen and oxygen in the ratio of 2:1 by mass at temperature 27°C. The ratio of average kinetic energy per molecule of hydrogen and oxygen respectively is:

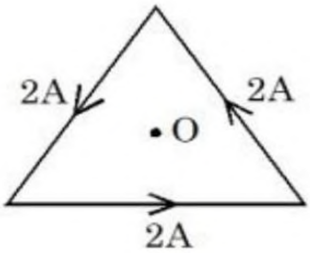
(1) 2 : 1 (2) 1 : 1 (3) 1 : 4 (4) 4 : 1

Sol. **(2)**

Average kinetic energy per molecule = $\frac{5}{2}KT$

Ratio = $\frac{1}{1}$

13. As shown in the figure, a current of 2 A flowing in an equilateral triangle of side $4\sqrt{3}$ cm. The magnetic field at the centroid O of the triangle is



(Neglect the effect of earth's magnetic field)

(1) $1.4\sqrt{3} \times 10^{-5}$ T (2) $4\sqrt{3} \times 10^{-4}$ T (3) $3\sqrt{3} \times 10^{-5}$ T (4) $\sqrt{3} \times 10^{-4}$ T

Sol. **(3)**

$d \tan 60° = 2\sqrt{3}$

$d = 2$ cm

$B = 3\left(\frac{\mu_0 I}{2\pi d}\right)\sin 60°$

$B = \frac{3 \times 2 \times 10^{-7} \times 2}{2 \times 10^{-2}} \times \frac{\sqrt{3}}{2}$

$B = 3\sqrt{3} \times 10^{-5}$ T

14. An object is allowed to fall from a height R above the earth, where R is the radius of earth. Its velocity when it strikes the earth's surface, ignoring air resistance, will be

(1) $\sqrt{2gR}$ (2) $\sqrt{\frac{gR}{2}}$ (3) $2\sqrt{gR}$ (4) $\sqrt{gR}$

Sol. **(4)**

Use work energy theorem

$\Delta KE = w_g$

$$\frac{1}{2}mv^2 - 0 = -[u_f - u_i]$$

$$\frac{1}{2}mv^2 = -\left[-\frac{GMm}{R} - \left(-\frac{GMm}{2R}\right)\right]$$

$$\frac{1}{2}mv^2 = \frac{GMm}{R} - \frac{GMm}{2R}$$

$$= \frac{GMm}{R}\left(\frac{2-1}{2}\right)$$

$$\frac{1}{2}mv^2 = \frac{GMm}{2R}$$

$$V = \sqrt{\frac{GM}{R}}$$

$$V = \sqrt{gR} \quad (GM = gR^2)$$

15. Match List I with List II:

List I	**List II**
A. Torque	I. $\text{kg m}^{-1}\ \text{s}^{-2}$
B. Energy density	II. kg ms^{-1}
C. Pressure gradient	III. $\text{kg m}^{-2}\ \text{s}^{-2}$
D. Impulse	IV. $\text{kg m}^2\ \text{s}^{-2}$

Choose the correct answer from the options given below:

(1) A – IV, B – I, C – III, D – II (2) A – IV, B – III, C – I, D – II

(3) A – IV, B – I, C – II, D – III (4) A – I, B – IV, C – III, D – II

Sol. **(1)**

Torque = N – m

$$= \text{kg}\ \frac{\text{m}}{\text{sec}^2}\ \text{m}$$

$$= \frac{\text{kg}\ \text{m}^2}{\text{sec}^2}$$

$$\text{Energy Density} = \frac{\text{N}-\text{m}}{\text{m}^3} = \frac{\text{N}}{\text{m}^2}$$

$$= \text{kg}\ \frac{\text{m}}{\text{sec}^2} \times \frac{1}{\text{m}^2}$$

$$\text{Pressure gradient} = \frac{\text{Pressure}}{\text{length}} = \frac{\text{F}}{\text{A}-\text{length}}$$

$= \text{kg m}^{-2}\ \text{sec}^{-2}$

Impulse = $\Delta P = \text{kg m} - \text{s}^{-1}$

16. Given below are two statements: one is labelled as Assertion **A** and the other is labelled as Reason **R**

Assertion A: The nuclear density of nuclides ${}_{5}^{10}\mathrm{B}$, ${}_{3}^{6}\mathrm{Li}$, ${}_{26}^{56}\mathrm{Fe}$, ${}_{10}^{20}\mathrm{Ne}$ and ${}_{83}^{209}\mathrm{Bi}$ can be arranged as $\rho_{\mathrm{Bi}}^{\mathrm{N}} > \rho_{\mathrm{Fe}}^{\mathrm{N}} > \rho_{\mathrm{Ne}}^{\mathrm{N}} > \rho_{\mathrm{B}}^{\mathrm{N}} > \rho_{\mathrm{Li}}^{N}$

Reason R: The radius R of nucleus is related to its mass number A as $R = R_0 A^{1/3}$, where R_0 is a constant.

In the light of the above statements, choose the correct answer from the options given below

(1) A is false but **R** is true

(2) A is true but **R** is false

(3) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

(4) Both **A** and **R** are true and **R** is the correct explanation of **A**

Sol. **(1)**

Nuclear density is independent of A

Ans: (1)

17. A force is applied to a steel wire 'A', rigidly clamped at one end. As a result elongation in the wire is 0.2 mm. If same force is applied to another steel wire 'B' of double the length and a diameter 2.4 times that of the wire 'A', the elongation in the wire ' B ' will be (wires having uniform circular cross sections)

(1) 6.06×10^{-2} mm (2) 2.77×10^{-2} mm

(3) 3.0×10^{-2} mm (4) 6.9×10^{-2} mm

Sol. **(4)**

$$Y = \frac{F\ell}{A\Delta\ell}$$

$$F = \frac{YA\Delta\ell}{\ell}$$

$$\left(\frac{A\Delta\ell}{\ell}\right)_1 = \left(\frac{A\Delta\ell}{\ell}\right)_2$$

$$\frac{\Delta\ell_2}{\Delta\ell_1} = \frac{A_1}{A_2} \times \frac{\ell_2}{\ell_1}$$

$$\frac{(\Delta\ell)_2}{0.2} = \frac{1}{2.4 \times 2.4} \times \frac{2}{1}$$

$$(\Delta\ell)_2 = 6.9 \times 10^{-2} \text{ mm}$$

Ans: (4)

18. A thin prism, P_1 with an angle 6th and made of glass of refractive index 1.54 is combined with another prism P_2 made from glass of refractive index 1.72 to produce dispersion without average deviation. The angle of prism P_2 is

(1) 1.3° (2) 6° (3) 4.5° (4) 7.8°

Sol. **(3)**

$\delta_1 = \delta_2$ [For no deviation]

$6(1.54 - 1) = A(1.72 - 1)$

$$A = \frac{18}{4} = 4.5^{\circ}$$

Ans: (3)

19. A machine gun of mass 10 kg fires 20 g bullets at the rate of 180 bullets per minute with a speed of 100 m s^{-1} each. The recoil velocity of the gun is

(1) 1.5 m/s (2) 0.6 m/s (3) 2.5 m/s (4) 0.02 m/s

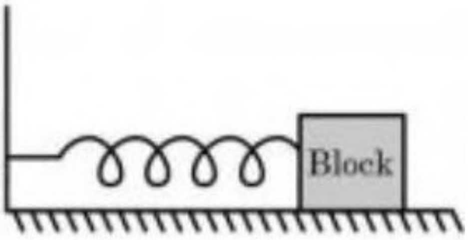
Sol. **(2)**

$$20 \times 10^{-3} \times \frac{180}{60} \times 100 = 10\text{ V}$$

$\text{V} = 0.6\text{ ms}^{-1}$

Ans: (2)

20. For a simple harmonic motion in a mass spring system shown, the surface is frictionless. When the mass of the block is 1 kg, the angular frequency is ω_1. When the mass block is 2 kg the angular frequency is ω_2. The ratio ω_2/ω_1 is



(1) $1/\sqrt{2}$ (2) $\sqrt{2}$ (3) 2 (4) 1/2

Sol. **(1)**

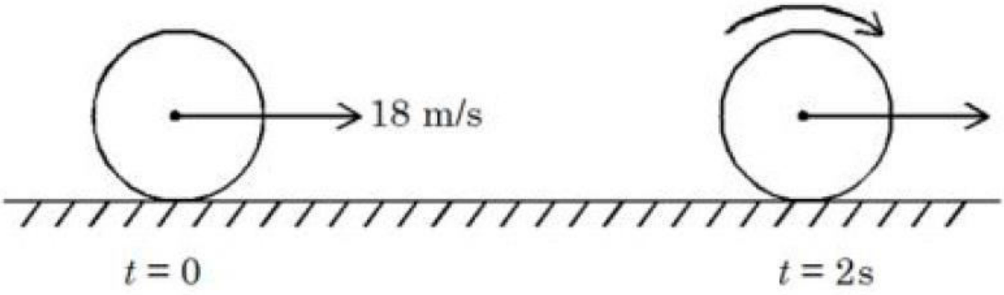
$$\omega = \sqrt{\frac{k}{m}}$$

$$\frac{\omega_2}{\omega_1} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{2}}$$

Ans: (1)

SECTION - B

21. A uniform disc of mass 0.5 kg and radius r is projected with velocity 18 m/s at t = 0 s on a rough horizontal surface. It starts off with a purely sliding motion at t = 0 s. After 2 s it acquires a purely rolling motion (see figure). The total kinetic energy of the disc after 2 s will be ________ J (given, coefficient of friction is 0.3 and $g = 10\text{ m/s}^2$).



Sol. **(54)**

$a = -\mu_k g = -3$

$v = u + at$

$v = 18 - 3 \times 2 = 12\ ms^{-1}$

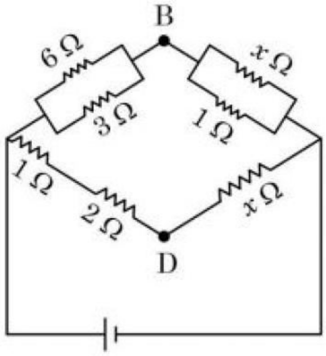
$$KE = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{mr^2}{2}\right)\left(\frac{v}{r}\right)^2$$

$$KE = \frac{3}{4}mv^2$$

$KE = 3 \times 18 = 54\ J$

Ans: (54)

22. If the potential difference between B and D is zero, the value of x is $\frac{1}{n}\Omega$. The value of n is ________.



Sol. **(2)**

$$\frac{2}{3} = \frac{\frac{x}{x+1}}{x}$$

$$\frac{2}{3} = \frac{1}{x+1}$$

$$x = 0.5 = \frac{1}{2}$$

$n = 2$

Ans: (2)

23. A stone tied to 180 cm long string at its end is making 28 revolutions in horizontal circle in every minute. The magnitude of acceleration of stone is $\frac{1936}{x}ms^{-2}$. The value of x ________. (Take $\pi = \frac{22}{7}$)

Sol. **(125)**

$a = \omega^2 r$

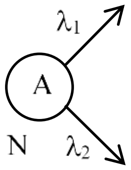
$$a = \left(\frac{28 \times 2\pi}{60}\right)^2 \times 1.8$$

$$a = \frac{1936 \times 1.8}{225} = \frac{1936}{125}ms^{-2}$$

x = 125

24. A radioactive nucleus decays by two different process. The half life of the first process is 5 minutes and that of the second process is 30 s. The effective half-life of the nucleus is calculated to be $\frac{\alpha}{11}$ s. The value of α is ________.

Sol. **(300)**



$$\frac{dN}{dt} = -(\lambda_1 + \lambda_2)N$$

$$\lambda_{eq} = \lambda_1 + \lambda_2$$

$$\frac{1}{t_{1/2}} = \frac{1}{300} + \frac{1}{30} = \frac{11}{300}$$

$$t_{1/2} = \left(\frac{300}{11}\right) \text{ sec}$$

Ans: (300)

25. A faulty thermometer reads 5°C in melting ice and 95°C in stream. The correct temperature on absolute scale will be ________ K when the faulty thermometer reads 41°C.

Sol. **(313)**

Ans: $\frac{41^\circ - 5^\circ}{95^\circ - 5^\circ} = \frac{R - 0}{100 - 0}$

$R = 40^\circ C$

$R = 313$ K

26. In an ac generator, a rectangular coil of 100 turns each having area 14×10^{-2} m^2 is rotated at 360rev/min about an axis perpendicular to a uniform magnetic field of magnitude 3.0 T. The maximum value of the emf produced will be ________ V.

$\left(\text{Take } \pi = \frac{22}{7}\right)$

Sol. **(1584)**

$E_{max} = NAB\omega$

$= 100 \times 14 \times 10^{-2} \times 3 \times \frac{360 \times 2\pi}{60}$

$= 1584$ V

Ans: (1584)

27. A body of mass 2 kg is initially at rest. It starts moving unidirectionally under the influence of a source of constant power P. Its displacement in 4 s is $\frac{1}{3}\alpha^2\sqrt{Pm}$. The value of α will be ________.

Sol. **(4)**

$$\frac{1}{2}mv^2 = pt$$

$$V = \sqrt{\frac{2pt}{m}}$$

$$\frac{dx}{dt} = \sqrt{\frac{2pt}{m}}$$

$$\int dx = \sqrt{\frac{2p}{m}}\int \sqrt{t}dt$$

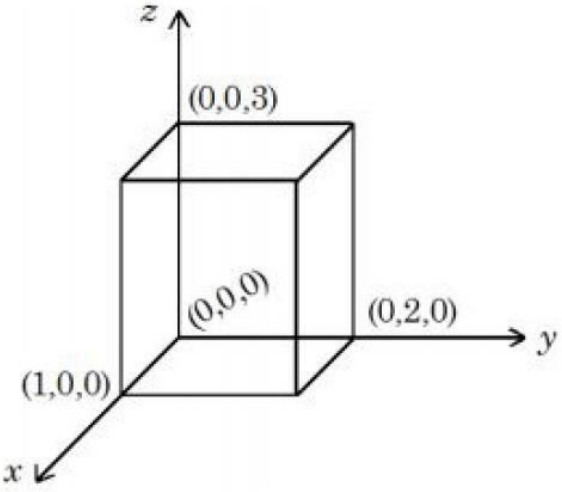
$$x = \sqrt{\frac{2p}{m}}\left[t^{3/2}\right]_0^4$$

$$x = \frac{1}{3}\times 16\sqrt{p}$$

$$\propto = 4$$

Ans: (4)

28. As shown in figure, a cuboid lies in a region with electric field $= 2x^2\hat{\imath} - 4y\hat{\jmath} + 6\hat{k}$ N/C. The magnitude of charge within the cuboid is $n \in_0 C$.
The value of n is ________ (if dimension of cuboid is $1 \times 2 \times 3$ m^3).



Sol. **(12)**

$$\phi_{net} = -8 \times 3 + 2 \times 6$$

$$= -12$$

$$\phi_{net} = \frac{q_{inside}}{\in_0}$$

$$q_{inside} = -12 \in_0$$

Ans: (12)

29. In a Young's double slit experiment, the intensities at two points, for the path differences $\frac{\lambda}{4}$ and $\frac{\lambda}{3}$ (λ being the wavelength of light used) are I_1 and I_2 respectively. If I_0 denotes the intensity produced by each one of the individual slits, then $\frac{I_1+I_2}{I_0}$ = ________.

Sol. **(3)**

$$I = 4I_0 \cos^2\frac{\phi}{2}$$

$$\Delta\phi = \frac{2\pi}{\lambda}\times\Delta x$$

$$I_1 = 4I_0 \cos^2\frac{\pi}{4} = 2I_0$$

$$I_2 = 4I_0 \cos^2\frac{2\pi}{3} = I_0$$

$$\Rightarrow \frac{I_1 + I_2}{I_0} = 3$$

Ans: (3)

30. The velocity of a particle executing SHM varies with displacement (x) as $4v^2 = 50 - x^2$. The time period of oscillations is $\frac{x}{7}s$. The value of x is ________.

$$\left(\text{Take } \pi = \frac{22}{7}\right)$$

Sol. **(88)**

$$4v^2 = 50 - x^2$$

$$V = \frac{1}{2}\sqrt{50 - x^2}$$

$$\omega = \frac{1}{2}$$

$$T = \frac{2\pi}{\omega} = 4\pi = \frac{88}{7}$$

$x = 88$

Ans: (88)

Chemistry

SECTION - A

31. The Cl – Co – Cl bond angle values in a fac- $[Co(NH_3)_3Cl_3]$ complex is/are:

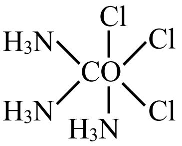
(1) 90°

(2) 90°&120°

(3) 180°

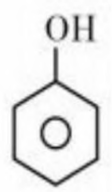
(4) 90°&180°

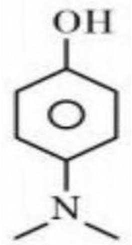
Sol. 1

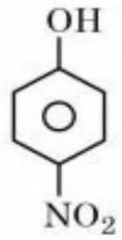


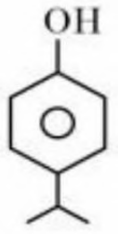
(90°)

32. The correct order of pK_a values for the following compounds is:

a 

b 

c 

d 

(1) c > a > d > b

(2) b > a > d > c

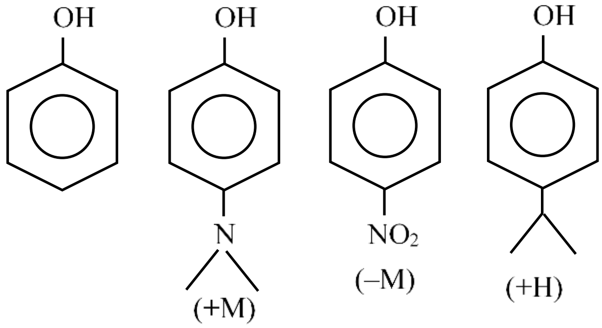
(3) $b > d > a > c$

(4) $a > b > c > d$

Sol. **3**

Acidic strength $\propto$ (–M, –H, – I)

$\propto \frac{1}{(+M,+H,+I)}$



PKa $\propto \frac{1}{\text{Acidic}}$ strength

A order of acidic strength: c > a > d > b

Order of PKa : c < a < d < b

33. Given below are two statements:

Statement I : During Electrolytic refining, the pure metal is made to act as anode and its impure metallic form is used as cathode.

Statement II : During the Hall-Heroult electrolysis process, purified Al_2O_3 is mixed with Na_3AlF_6 to lower the melting point of the mixture.

In the light of the above statements, choose the most appropriate answer from the options given below:

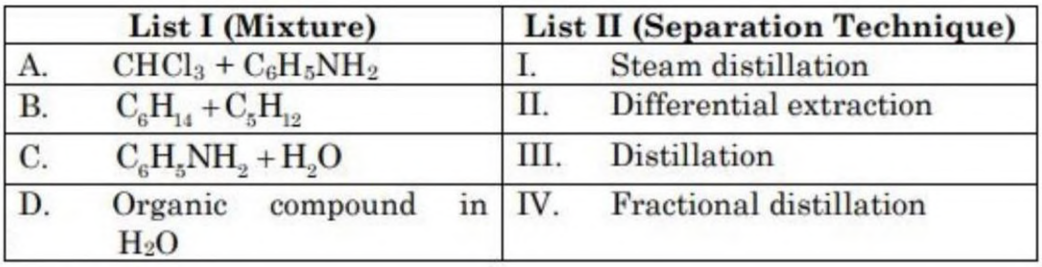
(1) Statement I is correct but Statement II is incorrect
(2) Both Statement I and Statement II are incorrect
(3) Both Statement I and Statement II are correct
(4) Statement I is incorrect but Statement II is correct

Sol. 4

Mixture of CaF_2 & Na_3AlF_6 decreasing the M.P. of Al_2O_3.

In electrolytic definning, pure metal is always deposited at the cathode

34. Match List I with List II:



List I (Mixture)		List II (Separation Technique)	
A.	$CHCl_3 + C_6H_5NH_2$	I.	Steam distillation
B.	$C_6H_{14} + C_5H_{12}$	II.	Differential extraction
C.	$C_6H_5NH_2 + H_2O$	III.	Distillation
D.	Organic compound in H_2O	IV.	Fractional distillation

(1) A-IV, B-I, C-III, D-II
(2) A-III, B-IV, C-I, D-II
(3) A-III, B-I, C-IV, D-II
(4) A-II, B-I, C-III, D-IV

Sol. 2

A. $CHCl_3 + C_6H_5NH_2 \rightarrow$ Distillation (III)
B. $C_6H_{14} + C_5H_{12} \rightarrow$ fractional distillation (IV)
C. $C_6H_5NH_2 \rightarrow H_2O \rightarrow$ Steam distillation (I)
D. Organic compound in $H_2O \rightarrow$ Differential extraction (II)

35. 1 L, 0.02M solution of $[Co(NH_3)_5SO_4]Br$ is mixed with 1 L, 0.02M solution of $[Co(NH_3)_5Br]SO_4$. The resulting solution is divided into two equal parts (X) and treated with excess of $AgNO_3$ solution and $BaCl_2$ solution respectively as shown below:

1 L solution (X) + $AgNO_3$ solution (excess) $\rightarrow$ Y
1 L Solution (X)+$BaCl_2$ solution (excess) $\rightarrow$ Z

The number of moles of Y and Z respectively are

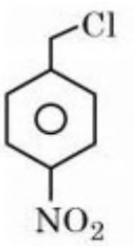
(1) 0.02, 0.01 (2) 0.01, 0.01
(3) 0.01, 0.02 (4) 0.02, 0.02

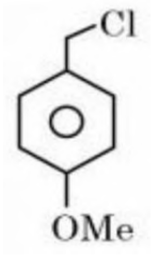
Sol. **2**

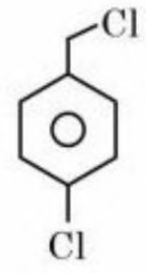
$$\left[\underset{0.01\,\text{mol}}{Co(NH_3)_5SO_4}\right]Br + \underset{\text{excess}}{AgNO_3} \rightarrow \underset{0.01\,\text{mol}}{AgBr}\downarrow$$

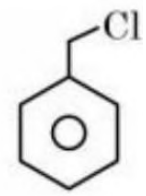
$$\left[\underset{0.02\,\text{mol}}{Co(NH_3)_5Br}\right]SO_4 + \underset{\text{excess}}{BaCl_2} \rightarrow \underset{0.01\,\text{mol}}{BaSO_4}\downarrow$$

36. Decreasing order towards SN 1 reaction for the following compounds is:


a


b


c


d

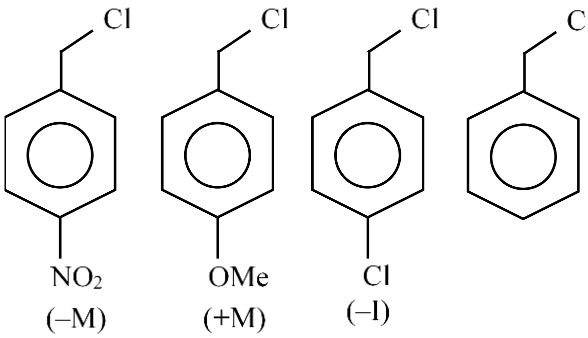
(1) a > c > d > b

(2) b > d > c > a

(3) *a* > *b* > *c* > *d*

(4) d > b > c > a

Sol. **2**



b > d > c > a

37. Which of the following reaction is correct?

(1) $4LiNO_3 \xrightarrow{\Delta} 2Li_2O + 2\ N_2O_4 + O_2$

(2) $2LiNO_3 \xrightarrow{\Delta} 2NaNO_2 + O_2$

(3) $2LiNO_3 \longrightarrow 2Li + 2NO_2 + O_2$

(4) $4LiNO_3 \xrightarrow{\Delta} 2Li_2O + 4NO_2 + O_2$

Sol. **4**

4 LiNO$_3$ $\xrightarrow{\Delta}$ 2 Li$_2$O + 4 NO$_2$+O$_2$

38. Boric acid is solid, whereas BF$_3$ is gas at room emperature because of

(1) Strong van der Waal's interaction in Boric acid

(2) Strong covalent bond in BF$_3$

(3) Strong ionic bond in Boric acid

(4) Strong hydrogen bond in Boric acid

Sol. **4**

Due to strong hydrogen bonding present in boric acid, boric acid present in solid form.

39. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason **R**.

Assertion A: Antihistamines do not affect the secretion of acid in stomach.

Reason : Antiallergic and antacid drugs work on different receptors.

In the light of the above statements, choose the correct answer from the options given below:

(1) A is false but R is true

(2) Both A and R are true but R is not the correct explanation of A

(3) Both A and R are true and R is the correct explanation of A

(4) A is true but R is false

Sol. **3**

40. Formulae for Nessler's reagent is:

(1) HgI$_2$

(2) K$_2$HgI$_4$

(3) KHgI$_3$

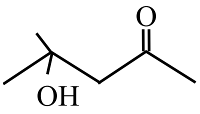
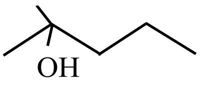
(4) KHg$_2$I$_2$

Sol. **2**

Nessler's reagent

K$_2$HgI$_4$ + KOH

41. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A:  can be easily reduced using Zn-Hg/HCl to 

Reason R: Zn − Hg/HCl is used to reduce carbonyl group to −CH$_2$ − group.

In the light of the above statements, choose the correct answer from the options given below:

(1) A is true but R is false

(2) Both A and R are true and R is the correct explanation of A

(3) A is false but R is true

(4) Both A and R are true but R is not the correct explanation of A

Sol. **2**

42. Maximum number of electrons that can be accommodated in shell with n=4

(1) 16 (2) 32 (C) 72 (D) 50

Sol. **2**

Max e^- that can be accommodated in shell = $2n^2$

(n=4)

$2(4)^2=32$

43. The wave function (Ψ) of 2 s is given by

$$\Psi_{2\,s}=\frac{1}{2\sqrt{2\pi}}\left(\frac{1}{a_0}\right)^{1/2}\left(2-\frac{r}{a_0}\right)e^{-r/2a_0}$$

At $r=r_0$, radial node is formed. Thus, r_0 in terms of a_0

(1) $r_0=4a_0$

(2) $r_0=\frac{a_0}{2}$

(3) $r_0=a_0$

(4) $r_0=2a_0$

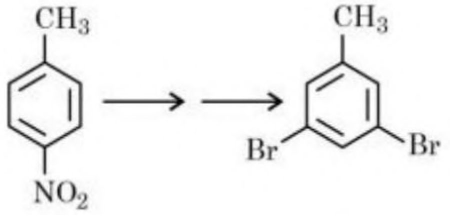
Sol. **4**

At node $\psi_{2s}=0$

$2-\frac{r_0}{a_0}=0$

$r_0=2a_0$

44.



(X) (Y)

In the above conversion of compound (X) to product (Y), the sequence of reagents to be used will be:

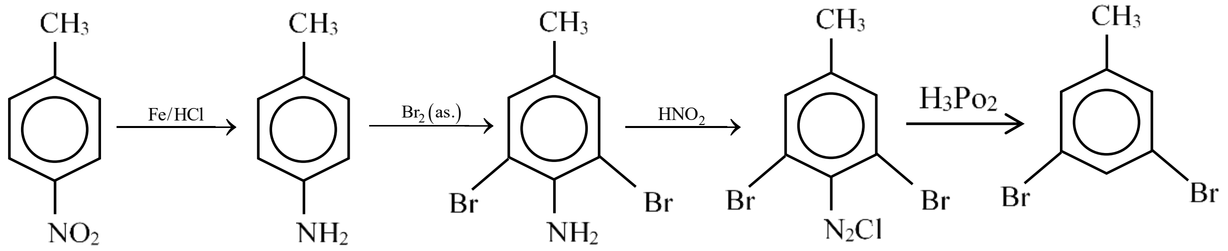
(1) (i) $Br_2(aq)$ (ii) $LiAIH_4$ (iii) H_3O^+

(2) (i) Br_2, Fe (ii) Fe, H^+ (iii) $LiAIH_4$

(3) (i) Fe, H^+ (ii) Br_2 (aq) (iii) HNO_2 (iv) H_3PO_2

(4) (i) Fe, H^+ (ii) Br_2 (aq) (iii) HNO_2 (iv) CuBr

Sol. **3**



45. Match List I with List II:

List I (Complexes)	List II (Hybridisation)
A. $[Ni(CO)_4]$	I. sp^3
B. $[Cu(NH_3)_4]^{2+}$	II. dsp^2
C. $[Fe(NH_3)_6]^{2+}$	III. sp^3d^2
D. $[Fe(H_2O)_6]^{2+}$	IV. d^2sp^3

(1) A-I, B-II, C-IV, D-III

(2) A-II, B-I, C-III, D-IV

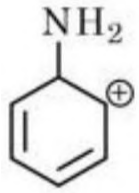
(3) A-II, B-I, C-IV, D-III

(4) A-I, B-II, C-III, D-IV

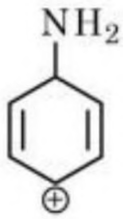
Sol. **1**

Complex	Hyloridisation
(A) $Ni(CO)_4$	sp^3
(B) $[Cu(NH_3)_4]^{+2}$	dsp^2
(C) $[Fe(NH_3)_6]^{+2}$	d^2sp^3
(D) $[Fe(H_2O)_6]^{+2}$	sp^3d^2

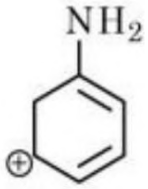
46. The most stable carbocation for the following is:



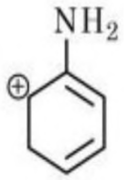
a



b



c



d

(1) a (2) c (3) d (4) b

Sol. **3**

47. Chlorides of which metal are soluble in organic solvents:

(1) K (2) Be (3) Mg (4) Ca

Sol. **2**

Due to smaller size, Be^{+2}. will show more polarising power, hence, Be will have maximum covalent character & most soluble in organic solvent.

48. $KMnO_4$ oxidises I^- in acidic and neutral/faintly alkaline solution, respectively, to

(1) IO_3^- & IO_3^-

(2) I_2 & IO_3^-

(3) I_2 & I_2

(4) IO_3^- & I_2

Sol. **2**

$2\ KMnO_4 + 10I^- + 16H^+ \rightarrow 2Mn^{+2} + 8H_2O + 5I_2$

neutral/faintly alkaline sol^n.

$2\ MnO_4^- + I^- + H_2O \rightarrow 2\ MnO_2 + 2OH^- + IO_3^-$

49. Bond dissociation energy of "E-H" bond of the "H_2E" hydrides of group 16 elements (given below), follows order.

A. O

B. S

C. Se

D. Te

Choose the correct from the options given below:

(1) B > A > C > D

(2) A > B > D > C

(3) A > B > C > D

(4) D > C > B > A

Sol. **3**

$H_2O > H_2S > H_2Se > H_2Te$

50. The water quality of a pond was analysed and its BOD was found to be 4. The pond has

(1) Highly polluted water

(2) Slightly polluted water

(3) Water has high amount of fluoride compounds

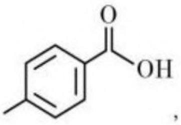
(4) Very clean water

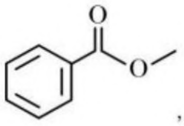
Sol. **4**

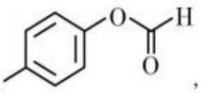
Clean water have BOD value less than 5 ppm while highly polluted water have. BOD value of 17 ppm or more.

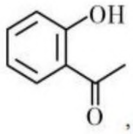
SECTION B

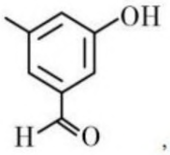
51. Number of compounds from the following which will not dissolve in cold $NaHCO_3$ and NaOH solutions but will dissolve in hot NaOH solution is

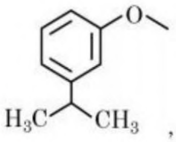


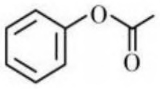












Sol. **3**

52. 1 mole of ideal gas is allowed to expand reversibly and adiabatically from a temperature of 27°C. The work done is 3 kJ mol^{-1}. The final temperature of the gas is_______ K (Nearest integer). Given C_V = 20 J mol^{-1} K^{-1}

Sol. **150**

$q = 0$

$\Delta U = W = nCv\Delta T$

$= 1 \times 20 \times [T_2 - 300] = -3000$

$= T_2 - 300 = -150$

$= T_2 = 150$ K

53. A short peptide on complete hydrolysis produces 3 moles of glycine (G), two moles of leucine (L) and two moles of valine (V) per mole of peptide. The number of peptide linkages in it are

Sol. **6**

54. Lead storage battery contains 38% by weight solution of H_2SO_4. The van't Hoff factor is 2.67 at this concentration. The temperature in Kelvin at which the solution in the battery will freeze is __ (Nearest integer). Given $K_f = 1.8$ K kg mol^{-1}

Sol. **243**

$\Delta T_f = i \cdot kf \cdot m$

$$m = \frac{38}{98} \times \frac{1000}{62}$$

$$\Delta T_f = 2.67 \times 1.8 \times \frac{38}{98} \times \frac{1000}{62}$$

$\Delta T_f = 30.05$

F.P. = 273 – 30 = 243 K

55. The strength of 50 volume solution of hydrogen peroxide is__________ g/L (Nearest integer).

Given:

Molar mass of H_2O_2 is 34 g mol^{-1}

Molar volume of gas at STP $= 22.7$ L.

Sol. **150**

$$\text{Molarity} = \frac{\text{Volume Strength}}{11.35}$$

Strength (g/lit) = Molarity × mol. Wt

$$= \frac{50}{11.35} \times 34 == 150 \text{ gm/lit}$$

56. The electrode potential of the following half cell at 298 K

$X|X^{2+}(0.001\mathrm{M}) \parallel Y^{2+}(0.01\mathrm{M})|Y$ is___________ $\times 10^{-2}$ V (Nearest integer).

Given: $E^0_{X^{2+}|X} = -2.36$ V

$E^0_{Y^{+2}|Y}$

$E^0Y^{2+1Y} = +0.36$ V

$$\frac{2.303RT}{F} = 0.06 \text{ V}$$

Sol. **275**

$x + y^{+2} \rightarrow y + x^{+2}$

E° Cell $= E^\circ_{\text{Cathode}} - E^\circ_{\text{Anode}}$

E° Cell $= 0.36 - (-2.36) = 2.72$ V

$$E_{\text{Cell}} = 2.72 - \frac{0.06}{2} \log \frac{x^{+2}}{y^{+2}}$$

$$E_{\text{Cell}} = 2.72 - \frac{0.06}{2} \log \frac{0.001}{0.01}$$

$= 2.72 + 0.03 = 2.75$ V

$= 275 \times 10^{-2}$ V

57. An organic compound undergoes first order decomposition. If the time taken for the 60% decomposition is 540 s, then the time required for 90% decomposition will be is______ s. (Nearest integer).

Given: $\ln 10 = 2.3; \log 2 = 0.3$

Sol. **1350**

$$K = \frac{2.303}{540} \log \frac{100}{40}$$

$$K = \frac{2.303}{540} \times 0.4$$

$$t_{90} = \frac{2.303 \times 540}{2.303 \times 0.4} \log \frac{100}{10}$$

$$t_{90} = 1350$$

58. Consider the following equation:

$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g), \Delta H = -190 \text{ kJ}$

The number of factors which will increase the yield of SO_3 at equilibrium from the following is

A. Increasing temperature

B. Increasing pressure

C. Adding more SO_2

D. Adding more O_2

E. Addition of catalyst

Sol. **3**

The yield of SO_3 at equilibrium will be due to:

B. Increasing pressure

C. Adding more SO_2

D. Adding more O_2

59. Iron oxide FeO, crystallises in a cubic lattice with a unit cell edge length of 5.0Å. If density of the FeO in the crystal is 4.0 g cm^{-3}, then the number of FeO units present per unit cell is______ (Nearest integer)

Given: Molar mass of Fe and O is 56 and 16 g mol^{-1} respectively. $N_A = 6.0 \times 10^{23} \text{ mol}^{-1}$

Sol. **4**

$$d = \frac{z \times M}{N_0 \times a^3}$$

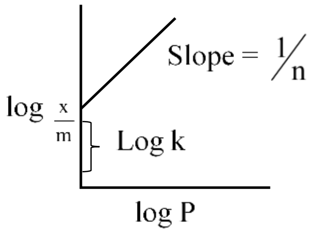
$$4 = \frac{z \times 72}{6 \times 10^{23} \times 125 \times 10^{-24}}$$

$$Z = 4.166 \cong 4$$

60. The graph of $\log \frac{x}{m}$ vs $\log p$ for an adsorption process is a straight line inclined at an angle of 45° with intercept equal to 0.6020. The mass of gas adsorbed per unit mass of adsorbent at the pressure of 0.4 atm is_______ $\times 10^{-1}$ (Nearest integer)

Given: $\log 2 = 0.3010$

Sol. **16**



Slope = tan 45° = 1

logK = 0.6020 = log 4

K = 4

$$\frac{x}{m} = KP^{1/n}$$

$$\frac{x}{m} = 4(0.4)1 = 16 \ \times 10^{-1}$$

Mathematics

SECTION - A

61. A vector $\vec{v}$ in the first octant is inclined to the x-axis at 60°, to the y-axis at 45 and to the z-axis at an acute angle. If a plane passing through the points $(\sqrt{2},-1,1)$ and (a,b,c), is normal to $\vec{v}$, then

(1) $\sqrt{2}a+b+c=1$ (2) $a+\sqrt{2}b+c=1$

(3) $a+b+\sqrt{2}c=1$ (4) $\sqrt{2}a-b+c=1$

Sol. **2**

$\cos\alpha=\cos 60$ $\cos\beta=\cos 45$

$\ell=\frac{1}{2}$ $m=\frac{1}{\sqrt{2}}$

$1^2+m^2+n^2=1$

$\Rightarrow \frac{1}{4}+\frac{1}{2}+n^2=1$ $n^2=1-\frac{3}{4}=\frac{1}{4}$

$\boxed{n=\frac{1}{2}}$

Direction of $\vec{v}$ is $\frac{1}{2}\hat{i}+\frac{1}{\sqrt{2}}\hat{j}+\frac{1}{2}\hat{k}$

Equation of plane through ($\sqrt{2}$, –1, 1) & Normla to $\vec{v}$ is

$\frac{1}{2}(x-\sqrt{2})+\frac{1}{\sqrt{2}}(y+1)+\frac{1}{2}(z-1)=0$

It passes through (a,b,c)

$\left(a-\sqrt{2}\right)+\sqrt{2}\left(b+1\right)+\left(c-1\right)=0$

$\Rightarrow a+\sqrt{2}b+c=\sqrt{2}-\sqrt{2}+1$

$\Rightarrow \boxed{a+\sqrt{2}b+c=1}$

62. Let $a,b,c>1, a^3, b^3$ and c^3 be in A.P., and $\log_a b, \log_c a$ and $\log_b c$ be in G.P. If the sum of first 20 terms of an A.P., whose first term is $\frac{a+4b+c}{3}$ and the common difference is $\frac{a-8b+c}{10}$ is -444, then abc is equal to:

(1) $\frac{125}{8}$ (2) 216 (3) 343 (4) $\frac{343}{8}$

Sol. **2**

If $\log_a b, \log_c a, \log_b c \rightarrow$ G.P.

$(\log_c a)^2=\log_a b\times\log_b c$

$(\log_c a)^2=\log_a c$

$\Rightarrow (\log_c a)^2=\frac{1}{\log_c a}$

$\Rightarrow (\log_c a)^3=1$

$\Rightarrow \log_c a=1$

$\boxed{a=c}$

If $a^3b^3c^3 \to$ A.P

$2b^3 = a^3 + c^3$

If $a = c$

$\Rightarrow \boxed{a = b = c}$

For AP

$A = \frac{a + 4a + a}{3}$ $D = \frac{a - 8a + a}{10}$

$A = 2a$ $D = \frac{-3a}{5}$

$$S_{20} = \frac{20}{2}\left[2 \times 2a + (20-1)\left(\frac{-3a}{5}\right)\right]$$

$$= 10\left[4a - \frac{57a}{5}\right]$$

$$= 10\left[-\frac{37a}{5}\right] = -444$$

$$\Rightarrow \ a = \frac{444 \times 5}{37 \times 10}$$

$$\boxed{a = 6}$$

$$\Rightarrow \boxed{abc = 6 \times 6 \times 6 = 216}$$

63. Let $a_1 = 1, a_2, a_3, a_4, ...$ be consecutive natural numbers.

Then $\tan^{-1}\left(\frac{1}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{1}{1+a_2a_3}\right) + \cdots . + \tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$ is equal to

(1) $\cot^{-1}(2022) - \frac{\pi}{4}$

(2) $\frac{\pi}{4} - \cot^{-1}(2022)$

(3) $\tan^{-1}(2022) - \frac{\pi}{4}$

(4) $\frac{\pi}{4} - \tan^{-1}(2022)$

Sol. **3**

$a_1 = 1, a_2, a_3, ...a_n$ be consecative natural numbers.

$$\tan^{-1}\left(\frac{1}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{1}{1+a_2a_3}\right) + ... + \tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$$

$$\Rightarrow \quad T_K = \tan^{-1}\left(\frac{1}{1+K(K+1)}\right)$$

$$= \tan^{-1}\left(\frac{K+1-K}{1+K(K+1)}\right)$$

$$= \tan^{-1}(K+1) - \tan^{-1} K$$

$T_1 = \tan^{-1} 2 - \tan^{-1} 1$

$T_2 = \tan^{-1} 3 - \tan^{-1} 2$

$T_3 = \tan^{-1} 4 - \tan^{-1} 3$

$\vdots$

$\underline{T_{2021}=\tan^{-1}(2022)-\tan^{-1}(2021)}$

On adding

$\Sigma T_n=\tan^{-1}(2022)-\tan^{-1}(1)$

$$\boxed{\sum_{n=1}^{2021}T_n=\tan^{-1}(2022)-\frac{\pi}{4}}$$

64. Let $\lambda\in\mathbb{R},\vec{a}=\lambda\hat{\imath}+2\hat{\jmath}-3\hat{k},\vec{b}=\hat{\imath}-\lambda\hat{\jmath}+2\hat{k}$

If $((\vec{a}+\vec{b})\times(\vec{a}\times\vec{b}))\times(\vec{a}-\vec{b})=8\hat{\imath}-40\hat{\jmath}-24\hat{k}$, then $|\lambda(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})|^2$ is equal to

(1) 132 (2)136 (3) 140 (4) 144

Sol. **3**

$\left((\vec{a}+\vec{b})\times(\vec{a}\times\vec{b})\right)\times\left(\vec{a}-\vec{b}\right)-=8\hat{i}-40\hat{j}-24\hat{k}$

$\Rightarrow\left(\vec{a}\times\left(\vec{a}\times\vec{b}\right)+\vec{b}\times\left(\vec{a}\times\vec{b}\right)\right)\times(\vec{a}-\vec{b})$

$\Rightarrow\left(\left(\vec{a}\cdot\vec{b}\right)\vec{a}-(\vec{a}\cdot\vec{a})\vec{b}+(\vec{b}\cdot\vec{b})\vec{a}-(\vec{b}\cdot\vec{a})\vec{b}\right)\times(\vec{a}-\vec{b})$

$\Rightarrow\ 0-(\vec{a}\cdot\vec{b})(\vec{a}\times\vec{b})-a^2(\vec{b}\times\vec{a})+0-b^2(\vec{a}\times\vec{b})-\ (\vec{a}\cdot\vec{b})\vec{b}\times\vec{a}=8\hat{i}-40\hat{j}-24\hat{k}$

$\Rightarrow\ \left(a^2-b^2\right)(\vec{a}\times\vec{b})=8\hat{i}-40\hat{j}-24\hat{k}$

$\left(\left(\lambda^2+4+9\right)-\left(1+\lambda^2+4\right)\right)\left(\vec{a}\times\vec{b}\right)$

$8\left(\vec{a}\times\vec{b}\right)=8(\hat{i}-5\hat{j}-3\hat{k})$

$$\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\\lambda&2&-3\\1&-\lambda&2\end{vmatrix}=\hat{i}-5\hat{j}-3\hat{k}$$

$\hat{i}(4-3\lambda)-\hat{j}(2\lambda+3)+\hat{k}(-\lambda^2-2)=\hat{i}-5\hat{j}-3\hat{k}$

$\Rightarrow\quad 4-3\lambda=1\qquad 2\lambda+3=5\qquad -\lambda^2-2=-3$

$3\lambda=3\qquad\qquad \lambda^2=1$

$\boxed{\lambda=1}\qquad\qquad \boxed{\lambda=1}$

$\left|\lambda(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})\right|=\left|(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})\right|^2$

$\Rightarrow\qquad \left|-\vec{a}\times\vec{b}+\vec{b}\times\vec{a}\right|^2=\left|2(\vec{a}\times\vec{b})\right|^2=4(1+25+9)=140$

65. Let q be the maximum integral value of p in [0,10] for which the roots of the equation $x^2-px+\frac{5}{4}p=0$ are rational. Then the area of the region $\{(x,y):0\le y\le(x-q)^2,0\le x\le q\}$ is

(1) 243 (2) 164 (3) $\frac{125}{3}$ (4) 25

Sol. **1**

$x^2 - px + \frac{5}{4}p = 0$

Roots are rational

D = A perfect square

$p^2 – 4(1)\frac{5}{4}p$

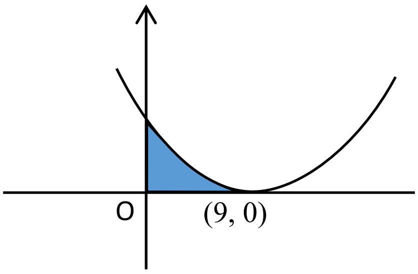
$p^2 – 5p$ = A perfect square

for p = 0, p = 5, p = 9 the D is a perfect square

∴ maximum integral of p is 9.

$\boxed{q = 9}$

$\{(x, y); 0 \le y \le (x-9)^2, 0 \le x \le 9\}$



Area = $\int_0^9 (x-9)^2 dx$

$\Rightarrow \left.\frac{(x-9)^3}{3}\right]_0^3$

$\Rightarrow 0 - \frac{(0-9)^3}{3}$

$\Rightarrow \frac{9 \times 9 \times 9}{3}$

= 243

66. Let f, g and h be the real valued functions defined on ℝ as

$$f(x) = \begin{cases} \frac{x}{|x|}, & x \ne 0 \\ 1, & x = 0 \end{cases}, g(x) = \begin{cases} \frac{\sin(x+1)}{(x+1)}, & x \ne -1 \\ 1, & x = -1 \end{cases}$$

and $h(x) = 2[x] - f(x)$, where $[x]$ is the greatest integer ≤ x.

Then the value of $\lim_{x \to 1} g(h(x-1))$ is

(1) -1 (2) 0 (3) $\sin(1)$ (4) 1

Sol. **4**

LHL

$\lim_{\delta \to 0} g(h(–\delta)) \quad \delta > 0$

$\lim_{\delta \to 0} g(–2+1)$

$\Rightarrow g(-1) = 1$

RHL

$\lim\limits_{\delta\to 0} g(h(\delta))$

$\lim\limits_{\delta\to 0} g(2\times 0\text{-}1)$

$\lim\limits_{\delta\to 0} g(-1)$

$\lim\limits_{x\to 1} g(h(x-1) = 1$

67. Let S be the set of all values of a_1 for which the mean deviation about the mean of 100 consecutive positive integers $a_1, a_2, a_3, \ldots . a_{100}$ is 25 . Then S is

(1) N (2) ϕ (3) {99} (4) {9}

Sol. **1**

Let $a_1 = n \qquad a_2 = n+1 \qquad a_3 = n+2 \qquad \ldots\ldots$

$$\bar{x} = \frac{n+(n+1)+(n+2)+\ldots\ldots n+99}{100}$$

$$= \frac{100n + \frac{100\times 99}{2}}{100} = n + \frac{99}{2}$$

Mean deviation about the mean

$$\frac{1}{100}\sum|x_i - \bar{x}|$$

$$\Rightarrow \frac{1}{100}\left(\frac{99}{2}+\frac{97}{2}+\frac{95}{2}+\ldots\ldots+\frac{97}{2}+\frac{99}{2}\right)$$

$$\Rightarrow \frac{2}{100}\left(\frac{99}{2}+\frac{97}{2}+\frac{95}{2}+\ldots\ldots 50\text{ terms}\right)$$

$$\Rightarrow \frac{2}{100}\times\frac{1}{2}\times(50)^2 = \frac{50\times 50}{100} = 25$$

It is 25 irrespective of the value of

$\therefore n \in N$

$\Rightarrow \boxed{S = N}$

68. For $\alpha, \beta \in \mathbb{R}$, suppose the system of linear equations

$x - y + z = 5$

$2x + 2y + \alpha z = 8$

$3x - y + 4z = \beta$

has infinitely many solutions. Then α and β are the roots of

(1) $x^2 + 14x + 24 = 0$ (2) $x^2 + 18x + 56 = 0$

(3) $x^2 - 18x + 56 = 0$ (4) $x^2 - 10x + 16 = 0$

Sol. **3**

$$\begin{vmatrix} 1 & -1 & 1 \\ 2 & 2 & \alpha \\ 3 & -1 & 4 \end{vmatrix} = 0$$

$1(8+\alpha)+1(8-3\alpha)+1(-2-6)=0$

$\Rightarrow\ 8+\alpha+8-3\alpha-8=0$

$-2\alpha=-8$

$\boxed{\alpha=4}$

$D_1 = 0$

$$\Rightarrow \begin{vmatrix} 5 & -1 & 1 \\ 8 & 2 & 4 \\ \beta & -1 & 4 \end{vmatrix} = 0$$

$5(8+4)+1(32-4\beta)+1(-8-2\beta)=0$

$60+32-4\beta-8-2\beta=0$

$\Rightarrow\ -6\beta=-84$

$\boxed{\beta=14}$

Equation having roots as $\alpha \,\&\, \beta$

$\boxed{x^2-18x+56=0}$

69. Let $\vec{a}$ and $\vec{b}$ be two vectors, Let $|\vec{a}| = 1, |\vec{b}| = 4$ and $\vec{a}\cdot\vec{b} = 2$. If $\vec{c} = (2\vec{a}\times\vec{b}) - 3\vec{b}$, then the value of $\vec{b}\cdot\vec{c}$ is

(1) -24 (2) -84 (3) -48 (4) -60

Sol. **3**

$\vec{b}.\vec{c}=(2\vec{a}\times\vec{b}).\vec{b}-3\vec{b}.\vec{b}$

$= 0-3b^2$

$=-3 \times 16=-48$

$\boxed{\vec{b}.\vec{c}=-48}$

70. If the functions $f(x) = \frac{x^3}{3} + 2bx + \frac{ax^2}{2}$ and $g(x) = \frac{x^3}{3} + ax + bx^2, a \neq 2b$ have a common extreme point, then $a + 2b + 7$ is equal to:

(1) $\frac{3}{2}$ (2) 3 (3) 4 (4) 6

Sol. **4**

$f'(x)=x^2+2b+ax=0$

$g'(x)=x^2+a+2bx=0$

$x = 1$ is the common root

$1 + 2b + a = 0$

$2b + a + 1 + 6 = 6$

$\boxed{2b+a+7=6}$

71. If P is a 3×3 real matrix such that $P^T = aP + (a - 1)I$, where $a > 1$, then

(1) $|\text{AdjP}| = \frac{1}{2}$ (2) $|\text{AdjP}| = 1$

(3) P is a singular matrix (4) $|\text{AdjP}| > 1$

Sol. **2**

$$\left(P^T\right)^T = aP^T(a-1)I$$

$$P = a(aP + (a-1)I) + (a-1)I$$

$$= a^2P + (a^2 - a)I + (a-1)I$$

$$= a^2P + (a^2 - a + a - 1)I$$

$$P = a^2P + (a^2 - 1)I \qquad \Rightarrow \qquad P = (1 - a^2) = (a^2 - 1)I$$

$$\boxed{P = -I}$$

$$|\text{AdjP}| = |P|^{3-1} = (-1)^2$$

$$= 1$$

72. The number of ways of selecting two numbers a and b, $a \in \{2,4,6,\ldots,100\}$ and $b \in \{1,3,5,\ldots,99\}$ such that 2 is the remainder when $a + b$ is divided by 23 is

(1) 268 (2) 108 (3) 54 (4) 186

Sol. **2**

a + b = 25,		a + b = 71		a + b = 117		a + b = 163	
a	b	a	b	a	b	a	b
2	23	2	69	18	99	64	99
4	21	4	67	20	97	66	97
⋮		⋮		⋮		⋮	
24	1	70	1	100	17	100	63
12 cases		35 cases		42 cases		19 cases	

Total ways = 12 + 35 + 42 + 19

= 108

73. $\lim\limits_{n\to\infty} \frac{3}{n}\left\{4 + \left(2 + \frac{1}{n}\right)^2 + \left(2 + \frac{2}{n}\right)^2 + \cdots + \left(3 - \frac{1}{n}\right)^2\right\}$ is equal to

(1) 12 (2) $\frac{19}{3}$ (3) 0 (4) 19

Sol. **4**

$$\lim_{x\to\infty} \frac{3}{n}\left\{\underset{\overset{\uparrow}{\left(2+0/n\right)^2}}{4}\ \left(2 + 1/n\right)^2 + \left(2 + \frac{2}{n}\right)^2 + \ldots. + \left(3 - \frac{1}{n}\right)^2\right\}$$

$$\Rightarrow \sum_{r=0}^{n-1} \frac{3}{n}\left(2+\frac{r}{n}\right)^2$$

$$\Rightarrow \int_0^1 3(2+x)^2 dx$$

$$\Rightarrow \left[(2+x)^3\right]_0^1$$

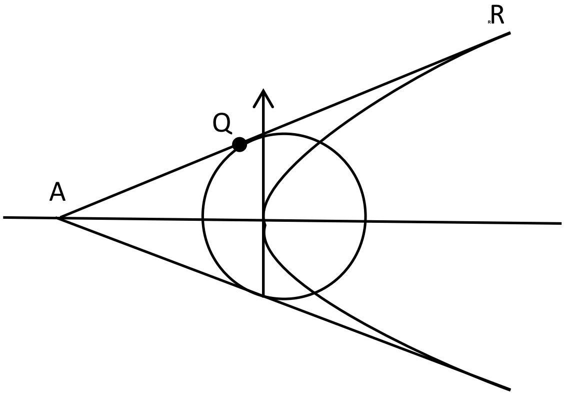
$$\Rightarrow (2+1)^3 - (2+0)^3$$

$$\Rightarrow 27-8=19$$

74. Let A be a point on the x-axis. Common tangents are drawn from A to the curves $x^2 + y^2 = 8$ and $y^2 = 16x$. If one of these tangents touches the two curves at Q and R, then $(QR)^2$ is equal to

(1) 81 (2) 72 (3) 76 (4) 64

Sol. **2**



$y^2 = 16x$ circle

Tangent $x^2 + y^2 = 8$

$y = mx + \frac{4}{m}$ Tagent

$y = mx \pm 2\sqrt{2}\sqrt{1+m^2}$

$\frac{4}{m} = \pm 2\sqrt{2}\sqrt{1+m^2}$

$\frac{16}{m^2} = 8 + 8m^2$

$8m^4 + 8m^2 = 16$

$m^4 + m^2 = 2$

$m^2 = 1, -2$

Let $m = 1$

$\underline{m > 1}$

$\therefore y = x + 4$

Point of tangecy at parabola

$Q\left(\frac{4}{m^2}, \frac{8}{m}\right)$

$Q(4,8)$

Point of tangency at circle

eq^h at tangent at $R(x_1, y_1)$

is $\boxed{T=0}$

$xx_1 + yy_1 = 8$

Comparison with

$x-y+4=0$

$\frac{x_1}{1} = \frac{y_1}{-1} = -\frac{8}{4}$

$x_1 = -2 \qquad y_1 = 2$

$R(-2,2)$

Now $QR^2 = \sqrt{(4+2)^2 + (8-2)^2}$

$\boxed{QR^2 = 72}$

75. If a plane passes through the points $(-1, k, 0), (2, k, -1), (1,1,2)$ and is parallel to the line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$, then the value of $\frac{k^2+1}{(k-1)(k-2)}$ is

(1) $\frac{17}{5}$ (2) $\frac{13}{6}$ (3) $\frac{6}{13}$ (4) $\frac{5}{17}$

Sol. **2**

Eq^n of plane

$$\begin{vmatrix} x-2 & y-k & 3+1 \\ 1-2 & 1-k & 2+1 \\ -1-2 & k-k & 0+1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-2 & y-k & 3+1 \\ -1 & 1-k & 3 \\ -3 & 0 & 1 \end{vmatrix} = 0$$

$(x-2)(1-k-0)-(y-k)(-1+9)+(3+1)(0+3-3k)=0$

$\Rightarrow (1-k)x-8y+(3-3k)z-2+2k+8k+3-3k=0$

$(1-K)x-8y+(3-3K)z+7K+1=0$

Plane is parallel to the line L:

$\therefore (1-k)1-8.1+(3-3K)(-1)=0$

$\Rightarrow 1-k-8-3+3k=0$

$2k=10$

$\boxed{k=5}$

$\frac{k^2+1}{(k-1)(k-2)}$

$= \frac{25+1}{(5-1)(5-2)} = \frac{26}{4\times 3} = \frac{13}{6}$

76. The range of the function $f(x) = \sqrt{3-x} + \sqrt{2+x}$ is:

(1) $[2\sqrt{2}, \sqrt{11}]$ (2) $[\sqrt{5}, \sqrt{13}]$ (3) $[\sqrt{2}, \sqrt{7}]$ (4) $[\sqrt{5}, \sqrt{10}]$

Sol. **4**

$3-x \geq 0$ $2+x \geq 0$

$x \leq 3$ $x \geq -2$

$x \in [-2,3]$

Now, $f(-2) = \sqrt{3+2} = \sqrt{5}$

$f(3) = \sqrt{2+3} = \sqrt{5}$

$f(x) = \sqrt{3-x} + \sqrt{2+x}$

$$f'(x) = \frac{1}{2\sqrt{3-x}} x - 1 + \frac{1}{2\sqrt{2+x}} = 0$$

$$\Rightarrow \frac{1}{\sqrt{3-x}} = \frac{1}{\sqrt{2+x}}$$

$\Rightarrow 3-x = 2+x$

$\Rightarrow 2x = 1$

$x = ½$

$$f\left(\frac{1}{2}\right) = \sqrt{3-1/2} + \sqrt{2+1/2}$$

$$= \sqrt{\frac{5}{2}} + \sqrt{\frac{5}{2}} \Rightarrow 2\sqrt{\frac{5}{2}} \Rightarrow \sqrt{10}$$

Range $= \left[\sqrt{5}, \sqrt{10}\right]$

77. The solution of the differential equation $\frac{dy}{dx} = -\left(\frac{x^2+3y^2}{3x^2+y^2}\right), y(1) = 0$ is

(1) $\log_e|x+y| - \frac{xy}{(x+y)^2} = 0$ (2) $\log_e|x+y| + \frac{2xy}{(x+y)^2} = 0$

(3) $\log_e|x+y| - \frac{2xy}{(x+y)^2} = 0$ (4) $\log_e|x+y| + \frac{xy}{(x+y)^2} = 0$

Sol. **2**

$y = Vx$

$$\frac{dy}{dx} = V + \frac{xdv}{dx}$$

$$V + \frac{xdv}{dx} = -\frac{x^2 + 3v^2x^2}{3x^2 + v^2x^2}$$

$$x\frac{dv}{dx} = -\frac{1+3v^2}{3+v^2} - v$$

$$x\frac{dv}{dx} = -\frac{1+3v^2+3v+v^3}{3+v^2}$$

$$\int \frac{3+v^2}{1+3v^2+3v+v^3} dv = -\int \frac{dx}{x}$$

$$\Rightarrow \int \frac{3+v^2}{(1+v)^3} dv = -\ln x + C$$

Let $v+1=t$

$dv = dt$

$$\int \frac{3+(t-1)^2}{t^3} dt = -\ln x + c$$

$$\Rightarrow \int \frac{t^2-2t+4}{t^3} dt$$

$$\Rightarrow \int \left(\frac{1}{t}-\frac{2}{t^2}+\frac{4}{t^3}\right) dt = -\ln x + c$$

$$\Rightarrow \ln t + \frac{2}{t} - \frac{4}{2t^2} = -\ln x + C$$

$$\Rightarrow \ln\left(\frac{y}{x}+1\right)^{-1} \frac{2}{\frac{y}{x}+1} - \frac{4}{2\left(y/x+1\right)^2} = -\ln x + C$$

$$\Rightarrow \ln\left(\frac{y+x}{x}\right) + \frac{2x}{y+x} - \frac{2x^2}{(x+y)^2} = -\ln x + c$$

$$\Rightarrow \ln|x+y| + \frac{2x}{(x+y)^2}(x+y-x) = C$$

$$\Rightarrow \boxed{\ln|x+y| + \frac{2xy}{(x+y)^2} = C}$$

78. The parabolas : $ax^2 + 2bx + cy = 0$ and $dx^2 + 2ex + fy = 0$ intersect on the line $y = 1$. If a, b, c, d, e, f are positive real numbers and a, b, c are in G.P., then

(1) d, e, f are in G.P.	(2) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.
(3) d, e, f are in A.P.	(4) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.

Sol. **2**

at $y = 1$, Both curve intersect

$$\Rightarrow \left.\begin{matrix} ax^2+2bx+c=0 \\ dx^2+2ex+f=0 \end{matrix}\right\} \text{Common Root}$$

Given a, b, c are in G.P

$\boxed{b^2 = ac}$

$\Rightarrow D = 4b^2 - 4ac = 0$ for the first equation

$\Rightarrow$ Both the Root are equal

$\because$ sum of the roots $= -2\frac{b}{a}$

$$\alpha + \alpha = -2\frac{b}{a}$$

$$\boxed{\alpha = -\frac{b}{a}}$$

It satisfies the second equation also

$$d\left(-\frac{b}{a}\right)^2 + 2e\left(-\frac{b}{a}\right) + f = 0$$

$$d\left(\frac{b^2}{a^2}\right) - \frac{2eb}{a} + f = 0$$

$$d\left(\frac{ac}{a^2}\right) - 2e\frac{b}{a} + f = 0$$

$$\frac{d}{a} - \frac{2eb}{ac} + \frac{f}{c} = 0$$

$$\frac{d}{a} - \frac{2eb}{b^2} + \frac{f}{c} = 0$$

$$\Rightarrow \boxed{2\frac{e}{b} = \frac{d}{a} + \frac{f}{c}} \Rightarrow \boxed{\frac{d}{a}, \frac{e}{b}, \frac{f}{c} \text{are in AP}}$$

79. Consider the following statements:

P : I have fever

Q: I will not take medicine

R : I will take rest.

The statement "If I have fever, then I will take medicine and I will take rest" is equivalent to:

(1) $((\sim P) \vee \sim Q) \wedge ((\sim P) \vee R)$ (2) $(P \vee Q) \wedge ((\sim P) \vee R)$

(3) $((\sim P) \vee \sim Q) \wedge ((\sim P) \vee \sim R)$ (4) $(P \vee \sim Q) \wedge (P \vee \sim R)$

Sol. **1**

$$P \to (\sim Q \wedge R)$$

$$\sim P V(\sim Q \wedge R)$$

$$\Rightarrow ((\sim P) V(\sim Q)) \wedge ((\sim P) vR)$$

80. $x = (8\sqrt{3} + 13)^{13}$ and $y = (7\sqrt{2} + 9)^{9}$. If $[t]$ denotes the greatest integer $\leq t$, then

(1) $[x]$ is odd but $[y]$ is even (2) $[x] + [y]$ is even

(3) $[x]$ and $[y]$ are both odd (4) $[x]$ is even but $[y]$ is odd

Sol. **2**

Let $x = I_1 + f_1$

$$\left(8\sqrt{3} + 13\right)^{13} = I_1 + f_1$$

$$\left(8\sqrt{3} - 13\right)^{13} = f_1'(\text{let})$$

On Subtaction

$\left(8\sqrt{3}+13\right)^{13}-\left(8\sqrt{b}-13\right)^{13}=I+f_1-f_1'$

$2\left[{}^{13}C_1\left(8\sqrt{3}\right)^{12}\right]13+{}^{13}C_3\left(8\sqrt{3}\right)^{10}13^3$

$+----]=I+0$

$\because I=\text{Even Number}$

$[x]=\text{Even}$

similarly,

Let $y=I_2+f_2$

$\left(7\sqrt{2}+9\right)^9=I_2+f_2$

$\left(7\sqrt{2}-9\right)^9=f_2'$

On Subtaction

$\left(7\sqrt{2}+9\right)^9-\left(7\sqrt{2}-9\right)^9=I_2+f_2-f_2^1$

$2\left[{}^9C_1\left(7\sqrt{2}\right)^8.9+{}^9C_3\left(7\sqrt{2}\right)^7 9^2---\right]=I_2+0$

$I_2=\text{Even}$

$$\boxed{\begin{aligned}\because [x]+[y]&=\text{Even}+\text{Even}\\&=\text{Even}\end{aligned}}$$

SECTION - B

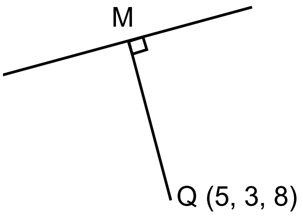
81. Let a line L pass through the point P(2,3,1) and be parallel to the line $x+3y-2z-2=0=x-y+2z$. If the distance of L from the point (5,3,8) is α, then $3\alpha^2$ is equal to ______.

Sol. **158**

The Direction ratio of line

$$\begin{vmatrix} i & j & k \\ 1 & 3 & -2 \\ 1 & -1 & 2 \end{vmatrix}\begin{aligned}&=i(6-2)-j(2+2)+k(-1-3)\\&=4i-4j-4k\end{aligned}$$

Equation of line L



$\frac{x-2}{1}=\frac{y-3}{-1}=\frac{z-1}{-1}=\lambda(\alpha d)$

Let $M(\lambda+2, -\lambda+3, -\lambda+1)$

DR's of MQ is $<\lambda+2-5, -\lambda+3-3, -\lambda+1-8>$

$$< \lambda - 3, -\lambda . -\lambda > 7>$$

$\because$ L$\perp$MQ

$\Rightarrow (\lambda - 3)(1) + (-\lambda)(-1) + (-\lambda - 7)(-1) = 0$

$\Rightarrow \lambda - 3 + \lambda + \lambda + 7 = 0$

$\Rightarrow 3\lambda = -4 \Rightarrow \lambda = -\frac{4}{3}$

$\therefore\ M\left(-\frac{4}{3}+2, \frac{+4}{3}+3, \frac{4}{3}+1\right) = \left(\frac{2}{3}, \frac{13}{3}, \frac{7}{3}\right)$

MQ = α

$\therefore\ 3\alpha^2 = 3 \times \left(\left(5-\frac{2}{3}\right)^2 + \left(3-\frac{13}{3}\right)^2 + \left(8-\frac{7}{5}\right)^2\right)$

$= 3\left(\frac{169}{9} + \frac{16}{9} + \frac{289}{9}\right) \Rightarrow \frac{474}{9} = 158$

82. A bag contains six balls of different colours. Two balls are drawn in succession with replacement. The probability that both the balls are of the same colour is p. Next four balls are drawn in succession with replacement and the probability that exactly three balls are of the same colour is q. If $p : q = m : n$, where m and n are coprime, then $m + n$ is equal to ______.

Sol. **14**

$p = 1.\frac{1}{6}$

$q = \left({}^6C_1.\frac{1}{6}.\frac{1}{6}.\frac{1}{6}.\frac{5}{6}\right)\frac{4!}{3!} = \frac{5}{216} \times 4 = \frac{5}{54}$

$\frac{p}{q} = \frac{1/6}{5/54} = \frac{9}{5}$

m = 9

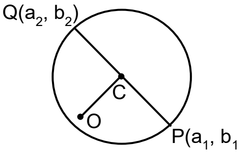
n = 5

m + n = 9 + 5 = 14

83. Let $P(a_1, b_1)$ and $Q(a_2, b_2)$ be two distinct points on a circle with center $C(\sqrt{2}, \sqrt{3})$. Let O be the origin and OC be perpendicular to both CP and CQ.

If the area of the triangle OCP is $\frac{\sqrt{35}}{2}$, then $a_1^2 + a_2^2 + b_1^2 + b_2^2$ is equal to ______.

Sol. **24**



OC is $\perp^r$ to both CP & CQ

⇒ PQ is a Diameter

Area of ΔOCP = $\frac{\sqrt{35}}{2}$

$\frac{1}{2}\times CP\times OC=\frac{\sqrt{35}}{2}$

CP × $\sqrt{2+3}=\sqrt{35}$

CP = $\sqrt{7}$ ⇒ radius = $\sqrt{7}$

Now $OP^2 = OC^2 + PC^2$

$a_1^2+b_1^2=2+3+7=12$

Similarly

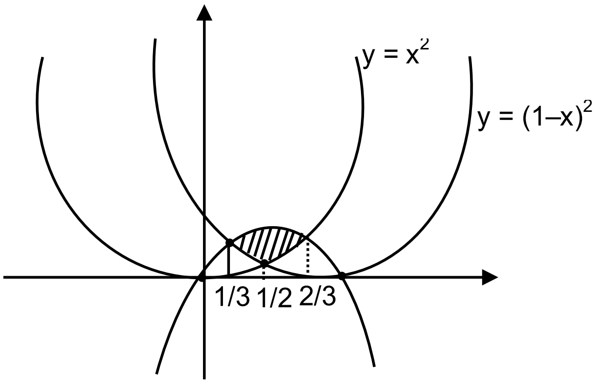
$OQ^2 = OC^2 + CQ^2$

$a_1^2+b_1^2=2+3+7=12$

$\therefore\ a_1^2+a_2^2+b_1^2+b_2^2=24$

84. Let A be the area of the region $\{(x,y): y \geq x^2, y \geq (1-x)^2, y \leq 2x(1-x)\}$. Then 540 A is equal to ______.

Sol. **25**



$x^2 = (1-x)^2$

$x^3 = 1 + x^2 - 2x$

$x = \frac{1}{2}$

$x^2 = 2x - 2x^2$

$3x^1 = 2x$

$x(3x - 2) = 0$

$x = 0, \frac{2}{3}$

$(1-x)^2 + 2x - 2x^2$

$1 + x^2 - 2x = 2x - 2x^2$

$\Rightarrow 3x^2 - 4x + 1 = 0$

$\Rightarrow 3x^2 - 3x - x + 1 = 0$

$\Rightarrow 3x(x-1) - 1(x-1) = 0$

$x = 1, \frac{1}{3}$

Required Area

$$A = \int_{\frac{1}{3}}^{\frac{1}{2}}\left\{(2x-2x^2)-(1-x)^2\right\}dx+\int_{\frac{1}{2}}^{\frac{2}{3}}\left((2x-2x^2)-x^2\right)dx$$

$$\Rightarrow \left[x^2 - \frac{2x^3}{3} + \frac{(1-x)^3}{3}\right]_{\frac{1}{3}}^{\frac{1}{2}} + (x^2 - x^3)_{1/2}^{2/3}$$

$$\Rightarrow \left(\frac{1}{4} - \frac{2}{3}.\frac{1}{8} + \frac{1}{8.3}\right) - \left(\frac{1}{9} - \frac{2}{3}.\frac{1}{27} + \frac{8}{27.3}\right) + \left(\frac{4}{9} - \frac{8}{27}\right) - \left(\frac{1}{4} - \frac{1}{8}\right)$$

$$\Rightarrow -\frac{1}{24} - \frac{1}{9} - \frac{6}{3\times 27} + \frac{4}{9} - \frac{8}{27} + \frac{1}{8}$$

$$\Rightarrow -\frac{1}{24} + \frac{3}{9} - \frac{10}{27} + \frac{3}{24} = \frac{-27+216-240+81}{24\times 27} = \frac{297-267}{24\times 27} = A$$

$$540\ A = 540 \times \frac{30}{24\times 27} = 25$$

85. The 8th common term of the series

$$S_1 = 3 + 7 + 11 + 15 + 19 + \cdots..$$
$$S_2 = 1 + 6 + 11 + 16 + 21 + \cdots.$$

is ______.

Sol. **151**

8th common term of the series

$S_1 = 3 + 7 + 11 + 15 + 19 + \ldots\ldots\ldots$

$S_2 = 1 + 6 + 11 + 16 + 21 + \ldots\ldots\ldots$

First common term = 11

common diff of the AP of common terms

= L.C.M of {4, 5}

= 20

∴ AP

11, 31, 51, …..

$T_8 = 11 + (8 - 1)20$

$= 11 + 140$

$T_8 = 151$

86. Let $A = \{1,2,3,5,8,9\}$. Then the number of possible functions $f: A \rightarrow A$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in A$ with $m \cdot n \in A$ is equal to ______.

Sol. **1**

LHL	RHL
$\lim\limits_{h\to 0} g(H(1-h-1))$	$\lim\limits_{h\to 0} g(H(1+h-1))$
$\lim\limits_{h\to 0} g(2(-1)-f(-h))$	$\lim\limits_{h\to 0} g(H(h))$
$\lim\limits_{h\to 0} g\left(-2-\frac{1-h}{\lvert -h\rvert}\right)$	$\Rightarrow \lim\limits_{h\to 0} g(2(0)+(h))$
$\Rightarrow g\left(-2-\frac{-1}{1}\right)$	$g(0-1)$
$\Rightarrow 2(-1) = +1$	$\Rightarrow 1$

$\therefore \lim\limits_{h\to 0} g(H(x-1) = 1$

87. If $\int \sqrt{\sec 2x - 1}dx = \alpha \log_e \left|\cos 2x + \beta + \sqrt{\cos 2x\left(1 + \cos\frac{1}{\beta}x\right)}\right|$ + constant, then $\beta - \alpha$ is equal to ______.

Sol. **1**

$$I = \int \sqrt{\sec 2x - 1}dx$$

$$\Rightarrow \int \sqrt{\frac{1-\cos 2x}{\cos 2x}}dx$$

$$\Rightarrow \int \frac{\sqrt{2}\sin x}{\sqrt{2\cos^2 x - 1}}dx$$

Let $\sqrt{2}\cos x = t$

$-\sqrt{\alpha}\sin x \ dx = dt$

$$I = \int \frac{-dt}{\sqrt{t^2-1}} = \ln\left|t + \sqrt{t'-1}\right| + c$$

$$\Rightarrow -\ln\left|\sqrt{2}\cos x + \sqrt{2\cos^2 x - 1}\right| + c$$

$$\Rightarrow \frac{-1}{2}\ln\left|\left(\sqrt{2}\cos x + \sqrt{\cos 2x}\right)^2\right| + c$$

$$\Rightarrow \frac{-1}{2}\ln\left|2\cos^2 x + \cos 2x + 2\sqrt{2}\cos x\sqrt{\cos 2x}\right| + c$$

$$\Rightarrow \frac{-1}{2}\ln\left|1 + \cos 2x + \cos 2x + 2\sqrt{2}\sqrt{\cos 2x} \times \sqrt{\frac{1+\cos x}{2}}\right| + c$$

$$\Rightarrow \frac{-1}{2}\ln\left|2\cos 2x + 1 + 2\sqrt{\cos 2x\left(1 + \cos 2x\right)}\right| + c$$

$$\Rightarrow \frac{-1}{2}\ln\left|\cos 2x + \frac{1}{2}\sqrt{\cos 2x\left(1 + \cos 2x\right)}\right| + c$$

$\alpha = \frac{-1}{2}$ $\qquad$ $\beta = \frac{1}{2}$

$$\therefore \boxed{\beta - \alpha = \frac{1}{2} - \left(\frac{-1}{2}\right) = 1}$$

88. If the value of real number $a > 0$ for which $x^2 - 5ax + 1 = 0$ and $x^2 - ax - 5 = 0$ have a common real root is $\frac{3}{\sqrt{2\beta}}$ then β is equal to ______.

Sol. **13**

$$\begin{array}{l} x^2 - 5ax + 1 = 0 \\ x^2 - ax - 5 = 0 \\ - \quad + \quad + \\ \hline -4ax + 6 = 0 \end{array}$$

$x = \frac{6}{4a} = \frac{3}{2a}$ (common root)

$\therefore \left(\frac{3}{2a}\right)^2 - 5a\left(\frac{3}{2a}\right) + 1 = 0$

$\Rightarrow 9 - 30a^2 + 4a^2 = 0$

$\Rightarrow 26a^2 = 9$

$a^2 = \frac{9}{26} \Rightarrow a = \frac{3}{\sqrt{26}} = \frac{3}{\sqrt{2\beta}}$

$\beta = 13$

89. 50th root of a number x is 12 and 50th root of another number y is 18 . Then the remainder obtained on dividing $(x + y)$ by 25 is ______.

Sol. **23**

$x^{\frac{1}{50}} = 12 \qquad y^{\frac{1}{50}} = 18$

Remainder when x + y is division by 25.

$x = 12^{50} \qquad y = 18^{50}$

$x + y = 12^{50} + 18^{50}$

$= 6^{50}(2^{50} + 3^{50})$

$= (5 + 1)^{50}\ ((2^2)^{25} + (3^2)^{25})$

$= (25\lambda_1 + 1)\ ((5–1)^{25} + (10–1)^{25})$

$= (25\lambda_1 + 1)\ (25\ (\lambda_2 + \lambda_3)–2)$

$= (25\lambda_1 + 1)\ (25\ K – 2)$

$\Rightarrow 25\lambda_1 \,.\, 25K – 50\lambda_1 + 25K – 2$

$\Rightarrow 25n_1 – 2$

$\Rightarrow 25n_2 + 23$

Remainder = 23

90. The number of seven digits odd numbers, that can be formed using all the seven digits 1,2,2,2,3,3,5 is ______.

Sol. **240**

The no. of 7 digit odd Numbers that can be formed using

1, 2, 2, 2, 3, 3, 5

						1

$\frac{\lfloor 6}{\lfloor 3 \lfloor 2} = \frac{720}{12} = 60$

						3

$\frac{\lfloor 6}{\lfloor 3} = \frac{720}{6} = 120$

						5

$\frac{\lfloor 6}{\lfloor 3 \lfloor 2} = \frac{720}{12} = 60$

= 240

JEE–MAIN EXAMINATION – JANUARY, 2023

(Held On Thursday 31st January, 2023) | **TIME : 9 : 00 AM to 12 : 00 NOON**

Physics

SECTION - A

1. The maximum potential energy of a block executing simple harmonic motion is 25 J. A is amplitude of oscillation. At $A/2$, the kinetic energy of the block is :

(1) 18.75 J (2) 9.75 J (3) 37.5 J (4) 12.5 J

Sol. **(1)**

Total Energy in SHM, $E = \frac{1}{2}m\omega^2 A^2 = 25J$

$$\text{at}\,\frac{A}{2}, U = PE = \frac{1}{2}m\omega^2 x^2$$

$$U = \frac{1}{2}m\omega^2\left(\frac{A}{2}\right)^2$$

$$k + U = E$$

$$k = \frac{1}{2}m\omega^2 A^2\left(1 - \frac{1}{4}\right)$$

$$k = 25 \times \frac{3}{4} = 18.75J$$

2. The drift velocity of electrons for a conductor connected in an electrical circuit is V_d. The conductor in now replaced by another conductor with same material and same length but double the area of cross section. The applied voltage remains same. The new drift velocity of electrons will be

(1) V_d (2) $\frac{V_d}{4}$ (3) $2\ V_d$ (4) $\frac{V_d}{2}$

Sol. **(1)**

$$V = IR = I\left(\frac{\rho l}{A}\right)$$

$$A \rightarrow 2A$$

$$I \rightarrow 2I$$

$$I = AneV_d$$

$$V_d \propto \frac{I}{A}$$

3. The initial speed of a projectile fired from ground is u. At the highest point during its motion, the speed of projectile is $\frac{\sqrt{3}}{2}u$. The time of flight of the projectile is :

(1) $\frac{2u}{g}$ (2) $\frac{u}{2g}$ (3) $\frac{\sqrt{3}u}{g}$ (4) $\frac{u}{g}$

Sol. **(4)**

At highest point -

$$u\cos\theta = \frac{\sqrt{3}u}{2}$$

$$\theta = 30^\circ$$

$$T = \frac{2u\sin\theta}{g} = \frac{u}{g}$$

4. The correct relation between $\gamma = \frac{c_p}{c_v}$ and temperature T is :

(1) $\gamma \alpha T^0$ (2) $\gamma \alpha \mathrm{T}$ (3) $\gamma \alpha \frac{1}{\sqrt{\mathrm{T}}}$ (4) $\gamma \alpha \frac{1}{\mathrm{T}}$

Sol. **(1)**

$\gamma = \frac{C_p}{C_v}$, Independent on T

5. The effect of increase in temperature on the number of electrons in conduction band ($\mathrm{n_e}$) and resistance of a semiconductor will be as:

(1) Both $\mathrm{n_e}$ and resistance increase
(2) Both $\mathrm{n_e}$ and resistance decrease
(3) $\mathrm{n_e}$ decreases, resistance increases
(4) $\mathrm{n_e}$ increases, resistance decreases

Sol. **(4)**

In semi conductors,

$T\uparrow, n_e$ in Conduction Band increases

$T\uparrow, R\downarrow$

6. The amplitude of $15\sin(1000\pi t)$ is modulated by $10\sin(4\pi t)$ signal. The amplitude modulated signal contains frequency (ies) of

A. 500 Hz B. 2 Hz C. 250 Hz D. 498 Hz E. 502 Hz

Choose the correct answer from the options given below:

(1) A Only (2) B Only (3) A and B Only (4) A, D and E Only

Sol. **(4)**

$$f_c = \frac{1000\pi}{2\pi} = 500\mathrm{Hz}$$

$$f_m = \frac{4\pi}{2\pi} = 2\mathrm{Hz}$$

Upper side Band, $\mathrm{USB} = \mathrm{f_c} + \mathrm{f_m}$

$\mathrm{USB} = 502\mathrm{HZ}$

Lower side Band, $LSB = f_c - f_m$

$LSB = 498\mathrm{Hz}$

7. Two polaroide A and B are placed in such a way that the pass-axis of polaroids are perpendicular to each other. Now, another polaroid C is placed between A and B bisecting angle between them. If intensity of unpolarized light is $\mathrm{I_0}$ then intensity of transmitted light after passing through polaroid B will be:

(1) $\frac{I_0}{4}$ (2) $\frac{I_0}{2}$ (3) Zero (4) $\frac{I_0}{8}$

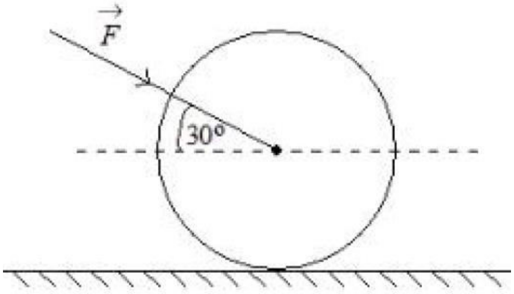
Sol. **(4)**

After A, $\mathrm{I} = \frac{\mathrm{I}}{2}$

After C, $\mathrm{I} = \frac{\mathrm{I_0}}{2}\cos^2 45 = \frac{\mathrm{I_0}}{4}$

After B, $\mathrm{I} = \frac{\mathrm{I_0}}{4}\cos^2 45 = \frac{\mathrm{I_0}}{8}$

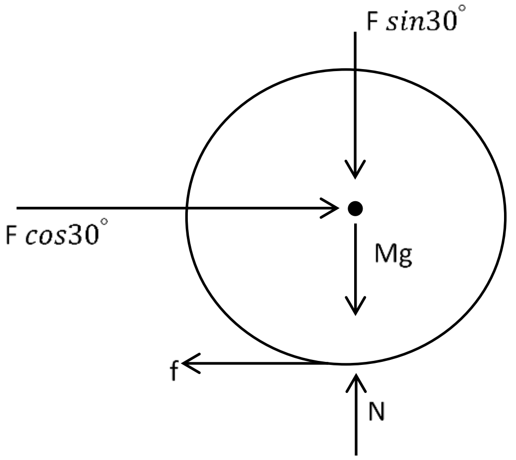
8. As shown in figure, a 70 kg garden roller is pushed with a force of $\vec{F} = 200$ N at an angle of 30° with horizontal. The normal reaction on the roller is
(Given $g = 10 \text{ m s}^{-2}$)



(1) $800\sqrt{2}$ N (2) $200\sqrt{3}$ N (3) 600 N (4) 800 N

Sol. **(4)**

FBD of Sphere ⇒



N= Mg + F$sin30^\circ$

N= 700 +200 $sin30^\circ$

N= 800 N

9. If 1000 droplets of water of surface tension 0.07 N/m, having same radius 1 mm each, combine to from a single drop. In the process the released surface energy is-
(Take $\pi = \frac{22}{7}$)

(1) 8.8×10^{-5} J (2) 7.92×10^{-4} J (3) 7.92×10^{-6} J (4) 9.68×10^{-4} J

Sol. **(2)**

$V_1 = V_2$

$1000 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$R = 10r$

$E = U_1 - U_2$

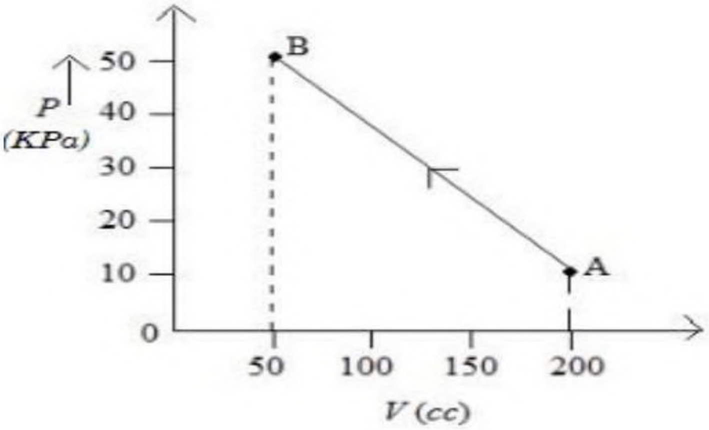
$= 1000\left(T \times 4\pi r^2\right) - T \times 4\pi R^2$

$E = 4\pi T\left(1000 \times r^2 - 100r^2\right)$

$E = 4 \times \frac{22}{7} \times 0.07 \times 900 \times 10^{-6}$

$E = 7.92 \times 10^{-4}$ J

10. The pressure of a gas changes linearly with volume from *A* to *B* as shown in figure. If no heat is supplied to or extracted from the gas then change in the internal energy of the gas will be



(1) −4.5 J (2) zero (3) 4.5 J (4) 6 J

Sol. **C**

W = Area of *PV* Graph

$W = -\frac{1}{2} \times [50 + 10] \times 10^3 \times 150 \times 10^{-6}$

$W = -4.5J$

$Q = \Delta U + W$

$0 = \Delta U - 4.5$

$= \Delta U = 4.5J$

11. Given below are two statements: One is labelled as Assertion **A** and the other is labelled as Reason **R**

Assertion A: The beam of electrons show wave nature and exhibit interference and diffraction.

Reason R: Davisson Germer Experimentally verified the wave nature of electrons.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both A and R are correct and R is the correct explanation of A
(2) A is not correct but R is correct
(3) A is correct but R is not correct
(4) Both A and R are correct but R is Not the correct explanation of A

Sol. **(1)**

Theoritical

12. A free neutron decays into a proton but a free proton does not decay into neutron. This is because

(1) proton is a charged particle
(2) neutron is an uncharged particle
(3) neutron is a composite particle made of a proton and an electron
(4) neutron has larger rest mass than proton

Sol. **(4)**

Rest mass of neutron is greater than proton.

13. Spherical insulating ball and a spherical metallic ball of same size and mass are dropped from the same height. Choose the correct statement out of the following Assume negligible air friction}

(1) Insulating ball will reach the earth's surface earlier than the metal ball
(2) Metal ball will reach the earth's surface earlier than the insulating ball
(3) Both will reach the earth's surface simultaneously.
(4) Time taken by them to reach the earth's surface will be independent of the properties of their materials

Sol. **(1)**

In Conductor, A portion of the Gravitational Potential Energy goes into generating eddy current.

14. If R, X_L, and X_C represent resistance, inductive reactance and capacitive reactance. Then which of the following is dimensionless :

(1) $\frac{R}{X_L X_C}$ (2) $\frac{R}{\sqrt{X_L X_C}}$ (3) $R\frac{X_L}{X_C}$ (4) $RX_L\, X_C$

Sol. **(2)**

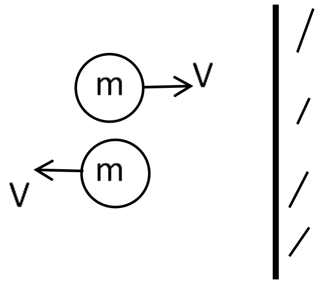
R, X_L, X_C have same unit i.e. ohm

$$\frac{R}{\sqrt{X_L X_C}} \to \frac{\text{ohm}}{\sqrt{\text{ohm}^2}} \to \text{Dimensionless}$$

15. 100 balls each of mass m moving with speed v simultaneously strike a wall normally and reflected back with same speed, in time t sec. The total force exerted by the balls on the wall is

(1) $\frac{100mv}{t}$ (2) $200mvt$ (3) $\frac{mv}{100t}$ (4) $\frac{200mv}{t}$

Sol. **(4)**



Change in momentum,

$|\overrightarrow{\Delta p}| = 2mV$

Average force,

$$F_{avg} = N\frac{|\overrightarrow{\Delta p}|}{t}$$

$$F_{avg} = 100\left(\frac{2mV}{t}\right)$$

$$F_{avg} = \frac{200mV}{t}$$

16. If a source of electromagnetic radiation having power 15 kW produces 10^{16} photons per second, the radiation belongs to a part of spectrum is.
(Take Planck constant $h = 6 \times 10^{-34}$Js)
(1) Micro waves (2) Ultraviolet rays (3) Gamma rays (4) Radio waves

Sol. **(3)**

$$P = \frac{N}{t}\left(\frac{hc}{\lambda}\right)$$

$$15 \times 10^{3} = 10^{16} \times \frac{6 \times 10^{-34} \times 3 \times 10^{8}}{\lambda}$$

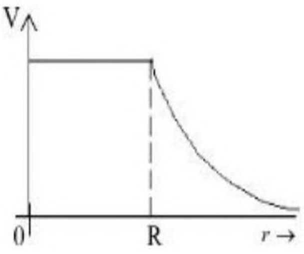
$$\lambda = 1.2 \times 10^{-13} \text{m}$$

$$\lambda = 0.0012 A^{0}$$

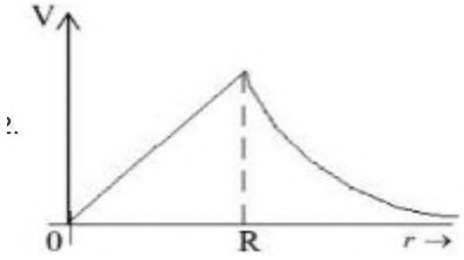
Corresponds to Gamma rays

17. Which of the following correctly represents the variation of electric potential (V) of a charged spherical conductor of radius (R) with radial distance (r) from the center?

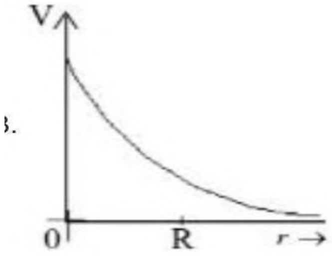
(1)



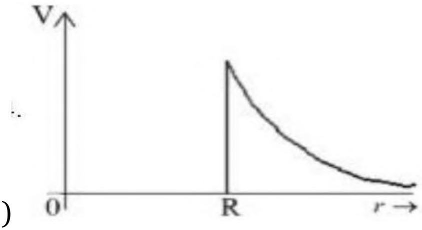
(2)



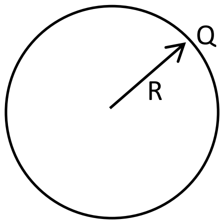
(3)



(4)

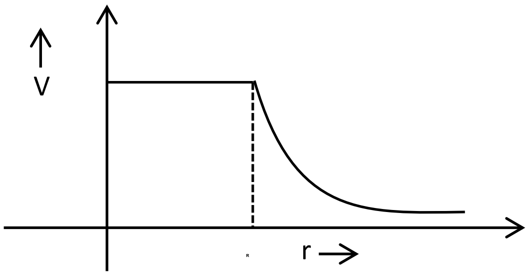


Sol. **(1)**



$$V_{in} = \frac{kQ}{R} \rightarrow \text{Constant}$$

$$V_{out} = \frac{kQ}{r} \propto \frac{1}{r}$$



18. A bar magnet with a magnetic moment $5.0 \mathrm{Am}^2$ is placed in parallel position relative to a magnetic field of 0.4 T. The amount of required work done in turning the magnet from parallel to antiparallel position relative to the field direction is ________.

(1) 1 J (2) 4 J (3) 2 J (4) zero

Sol. **(2)**

$$W = MB(\cos\theta_1 - \cos\theta_2)$$

$$W = MB(\cos 0^\circ - \cos 180^\circ)$$

$$W = 2MB$$

$$W = 2\times 5\times 0.4$$

$$W = 4\,\mathrm{J}$$

19. At a certain depth "d " below surface of earth, value of acceleration due to gravity becomes four times that of its value at a height $3R$ above earth surface. Where R is Radius of earth (Take $R = 6400$ km). The depth d is equal to

(1) 4800 km (2) 2560 km (3) 640 km (4) 5260 km

Sol. **(A)**

Given

$$g\left(1-\frac{d}{R}\right) = 4\frac{g}{\left(1+\frac{h}{R}\right)^2}$$

$$1-\frac{d}{R} = \frac{4}{(1+3)^2} = \frac{1}{4}$$

$$\frac{d}{R} = \frac{3}{4}$$

$$d = \frac{3R}{4} = \frac{3}{4}\times 6400$$

$$d = 4800\mathrm{km}$$

20. A rod with circular cross-section area $2\ \mathrm{cm}^2$ and length 40 cm is wound uniformly with 400 turns of an insulated wire. If a current of 0.4 A flows in the wire windings, the total magnetic flux produced inside windings is $4\pi \times 10^{-6}$ Wb. The relative permeability of the rod is

(Given : Permeability of vacuum $\mu_0 = 4\pi \times 10^{-7} \mathrm{NA}^{-2}$)

(1) $\frac{5}{16}$ (2) 12.5 (3) 125 (4) $\frac{32}{5}$

Sol. **(1)**

NTA Ans. (3)

Magnetic field in the Solenoid,

$$\mathrm{B} = \mu_0\mu_r \mathrm{nI}$$

Magnetic flux, $\phi = \mathrm{N(BA)}$

$$\phi = N(\mu_0\mu_r nIA)$$

$$4\pi\times 10^{-6} = 400\left(4\pi\times 10^{-7}\mu_y \times \frac{400}{0.4}\times 0.4\times 2\times 10^{-4}\right)$$

$$\frac{1}{40} = \mu_r \times 8\times 10^{-2}$$

$$\mu_r = \frac{100}{320} = \frac{5}{16}$$

SECTION - B

21. In a medium the speed of light wave decreases to 0.2 times to its speed in free space The ratio of relative permittivity to the refractive index of the medium is $x:1$. The value of x is (Given speed of light in free space $= 3 \times 10^8$ m s^{-1} and for the given medium $\mu_r = 1$)

Sol. **(5)**

$$V = \frac{c}{n}$$

$n \rightarrow$ refractive index

$$n = \frac{c}{0.2c} = 5$$

$$n = \sqrt{\mu_r \varepsilon_r}$$

$$\varepsilon_r = n^2 = 25$$

$$\frac{\varepsilon_r}{n} = \frac{25}{5} = \frac{5}{1}$$

22. A solid sphere of mass 1 kg rolls without slipping on a plane surface. Its kinetic energy is 7×10^{-3} J. The speed of the centre of mass of the sphere is ___________ cms^{-1}

Sol. **(10)**

On Rolling,

$$KE = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2$$

$$KE = \frac{1}{2}MV^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2$$

$$KE = \frac{7}{10}MV^2 = 7 \times 10^{-3}$$

$$V^2 = 10^{-2}$$

$$V = 10^{-1}\text{m/s}$$

$$V = 10\text{cm/s}$$

23. A lift of mass $M = 500$ kg is descending with speed of 2 ms^{-1}. Its supporting cable begins to slip thus allowing it to fall with a constant acceleration of 2 ms^{-2}. The kinetic energy of the lift at the end of fall through to a distance of 6 m will be _________ kJ.

Sol. **(7)**

Acceleration is constant,

$$\text{v}^2 = u^2 + 2as$$

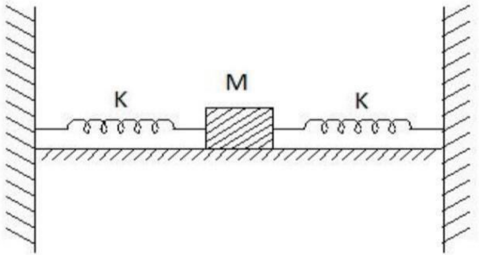
$$\text{v}^2 = 2^2 + 2(2)(6)$$

$$\text{v}^2 = 28$$

$$\frac{1}{2}\text{Mv}^2 = \frac{1}{2} \times 500 \times 28$$

$$\text{KE} = 7\text{kJ}$$

24. In the figure given below, a block of mass $M = 490$ g placed on a frictionless table is connected with two springs having same spring constant ($\mathrm{K} = 2\ \mathrm{N\ m^{-1}}$). If the block is horizontally displaced through ' X ' m then the number of complete oscillations it will make in 14π seconds will be ________.



Sol. **(20)**

$$T = 2\pi\sqrt{\frac{m}{k_{eq}}}$$

$$T = 2\pi\sqrt{\frac{m}{2k}}$$

$$T = 2\pi\sqrt{\frac{0.49}{2\times 2}}$$

$$T = 2\pi\times\frac{0.7}{2} = 0.7\pi$$

in $14\pi\,\text{sec}, \frac{14\pi}{0.7\pi} = 20$

25. An inductor of 0.5mH, a capacitor of 20μF and resistance of 20Ω are connected in series with a 220 V ac source. If the current is in phase with the emf, the amplitude of current of the circuit is $\sqrt{x}$ A. The value of x is-

Sol. **(242)**

Current is in phase with EMF. Hence, Circuit is at Resonance.

$$I_{rms} = \frac{V_{rms}}{R} = \frac{220}{20}$$

$$I_{rms} = 11\text{A}$$

$$I_0 = \sqrt{2}\,I_{rms} = \sqrt{242}\text{A}$$

26. The speed of a swimmer is 4 km $\mathrm{h^{-1}}$ in still water. If the swimmer makes his strokes normal to the flow of river of width 1 km, he reaches a point 750 m down the stream on the opposite bank. The speed of the river water is ________ $\mathrm{kmh^{-1}}$.

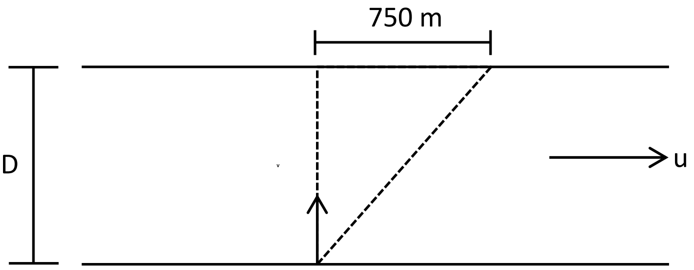
Sol. **(3)**

$$T = \frac{D}{V} = \frac{1}{4}hr$$

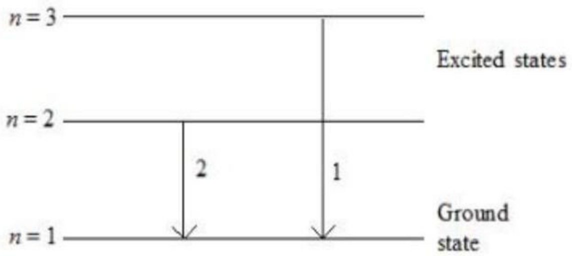
$$\text{Drift} = uT$$

$$\frac{750}{1000}\text{kM} = u\times\frac{1}{4}hr$$

$$u = 3\text{km/hr}$$



27. For hydrogen atom, λ_1 and λ_2 are the wavelengths corresponding to the transitions 1 and 2 respectively as shown in figure. The ratio of λ_1 and λ_2 is $\frac{x}{32}$. The value of x is ________



Sol. **(27)**

$$\frac{1}{\lambda_1}=R\left(\frac{1}{1^2}-\frac{1}{3^2}\right)$$

$$\lambda_1=\frac{9}{8R}$$

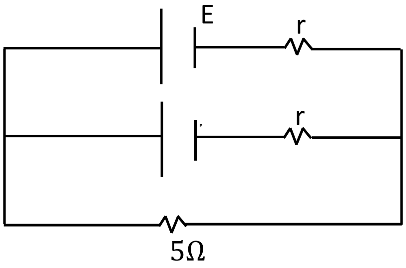
$$\frac{1}{\lambda_2}=R\left(\frac{1}{1^2}-\frac{1}{2^2}\right)$$

$$\lambda_2=\frac{4}{3R}$$

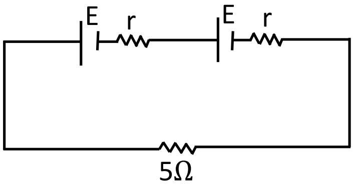
$$\frac{\lambda_1}{\lambda_2}=\frac{27}{32}$$

28. Two identical cells, when connected either in parallel or in series gives same current in an external resistance 5Ω. The internal resistance of each cell will be ________ Ω.

Sol. **(5)**



(I)



(II)

$$r_{eq}=\frac{r}{2}\ ,r_{eq}=2r$$

$$E_{eq}=\frac{r}{2}\left(\frac{E}{r}+\frac{E}{r}\right)=E\ ,\ E_{eq}=2E$$

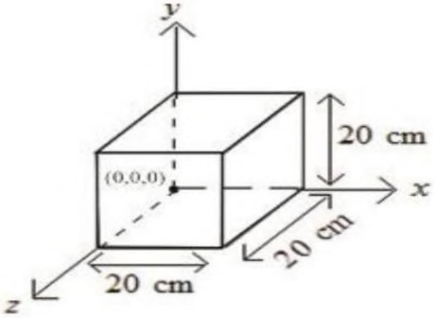
$$I_1=\frac{E}{5+\frac{r}{2}},\ I_2=\frac{2E}{2r+5}$$

$$I_1=I_2$$

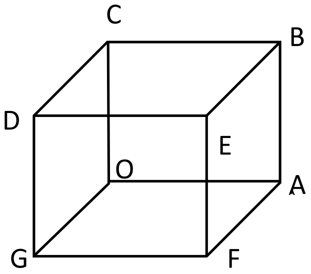
$$2r+5\ =2\left(5+\frac{r}{2}\right)$$

$$r\ =5\Omega$$

29. Expression for an electric field is given by $\vec{E} = 4000x^2\hat{\imath}\frac{V}{m}$. The electric flux through the cube of side 20 cm when placed in electric field (as shown in the figure) is ______ V cm



Sol. **(640)**



$\vec{E} \perp \vec{A}, \phi_{Top} = \phi_{Bottom} = \phi_{front} = \phi_{Back} = 0$

for $OCDG, x = 0, E = 0, \phi = 0$

for $ABEF, x = 0.2\text{m}$

$E = 4000 \times (0.2)^2$

$E = 160\text{V}/\text{m}$

$\phi = E(a^2) = 160\text{V}/\text{m} \times (0.2)^2\text{m}^2$

$\phi = 6.4\text{V}-\text{m}$

$\phi = 640\text{V}-\text{cm}$

30. A thin rod having a length of 1 m and area of cross-section 3×10^{-6} m^2 is suspended vertically from one end. The rod is cooled from 210°C to 160°C. After cooling, a mass M is attached at the lower end of the rod such that the length of rod again becomes 1 m. Young's modulus and coefficient of linear expansion of the rod are 2×10^{11} N m^{-2} and 2×10^{-5} K^{-1}, respectively. The value of M is ______ kg. (Take $g = 10$ m s^{-2})

Sol. **(60)**

$Y = \frac{FL}{A\Delta L}$

$F = YA\left(\frac{\Delta L}{L}\right)$

$F = YA(\alpha\Delta T)$

$Mg = YA(\alpha\Delta T)$

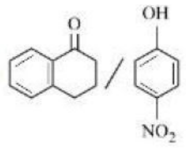
$M \times 10 = 2 \times 10^{11} \times 3 \times 10^{-6} \times 2 \times 10^{-5} \times 50$

$M = 60\text{kg}$

Chemistry

SECTION - A

31. Match items of column I and II

Column I (Mixture of compounds)	Column II (Separation Technique)
A. H_2O/CH_2Cl_2	i. Crystallization
B. 	ii. Differential solvent extraction
C. Kerosene /Naphthalene	iii. Column chromatography
D. $C_6H_{12}O_6/NaCl$	iv. Fractional Distillation

Correct match is

(1) A-(ii), B-(iii), C-(iv), D-(i) (2) A-(i), B-(iii), C-(ii), D-(iv)

(3) A-(ii), B-(iv), C-(i), D-(iii) (4) A-(iii), B-(iv), C-(ii), D-(i)

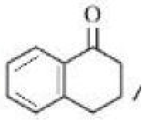
Sol. **1**

A-(ii),

Density of CH_2Cl_2 > Density of H2O

(Can separated by differential solvent extraction

B-(iii),

Having intermolecular H–Bond so can be separated from  through column chromatography

C-(iv),

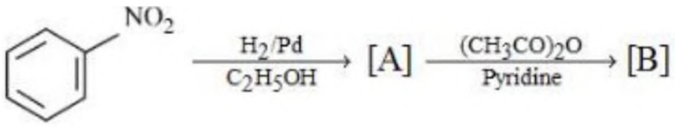
Due to difference in B.P. of kerosene and Naphthalene, it can be separated by fractional distillation

D-(i)

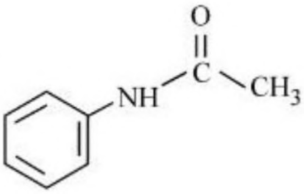
NaCl → ionic compound

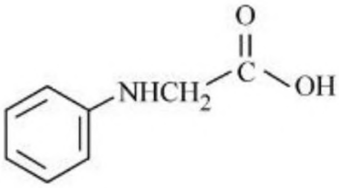
$C_6H_{12}O_6$ → Non ionic compound

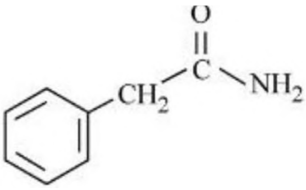
so NaCl can by crystallized.

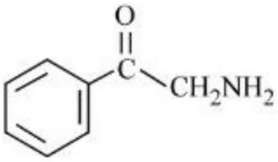


32. Consider the above reaction and identify the product B. Options

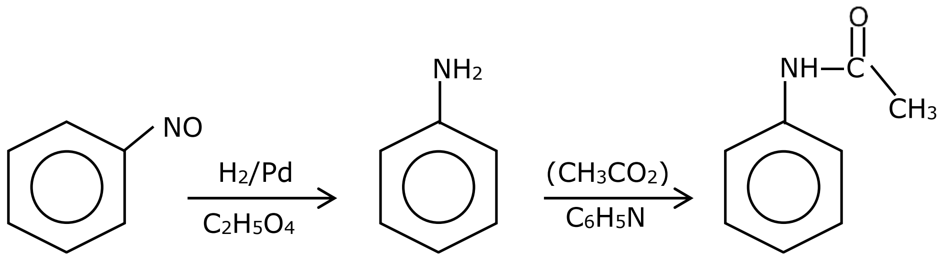
(1) 

(2) 

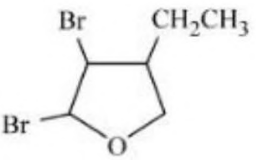
(2) 

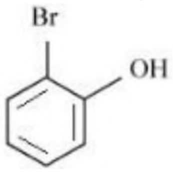
(4) 

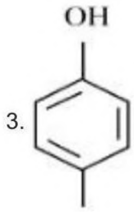
Sol. **1**

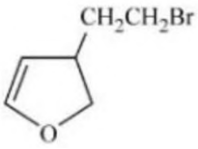


33. An organic compound 'A' with emperical formula C_6H_6O gives sooty flame on burning. Its reaction with bromine solution in low polarity solvent results in high yield of $B.B$ is

(1) 

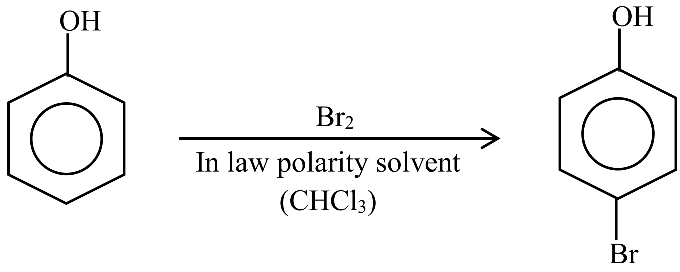
(2) 

(3) 

(4) 

Sol. **3**

Phenol will give sooty flame while burning (aromatic compound)



(Major)

34. When Cu^{2+} ion is treated with KI, a white precipitate, X appears in solution. The solution is titrated with sodium thiosulphate, the compound Y is formed. X and Y respectively are

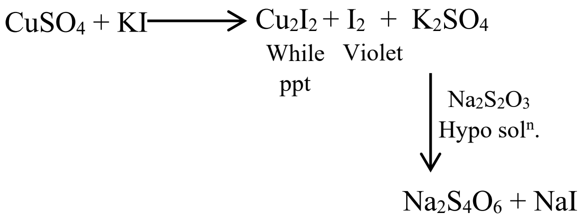
(1) $X = CuI_2$ $Y = Na_2\ S_4O_6$

(2) $X = CuI_2$ $Y = Na_2\ S_2O_3$

(3) $X = Cu_2I_2$ $Y = Na_2\ S_4O_5$

(4) $X = Cu_2I_2$ $Y = Na_2\ S_4O_6$

Sol. **4**



$$CuSO_4 + KI \longrightarrow \underset{\substack{\text{While}\\ \text{ppt}}}{Cu_2I_2} + \underset{\text{Violet}}{I_2} + K_2SO_4$$

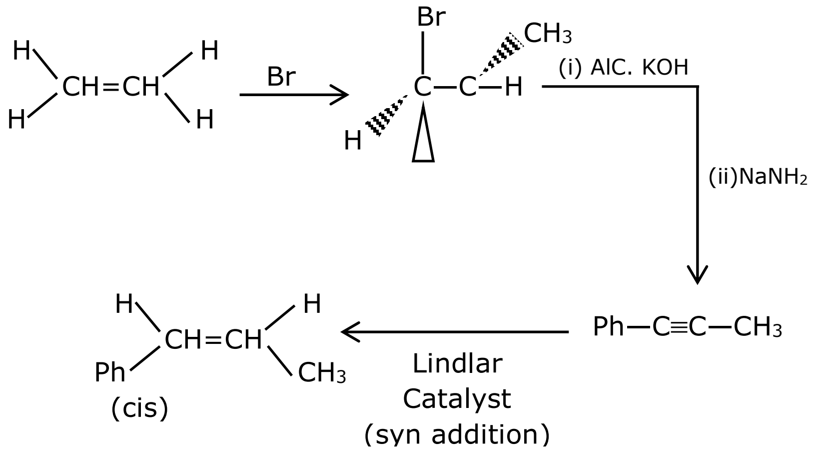
$$\Big\downarrow \begin{matrix} Na_2S_2O_3 \\ \text{Hypo sol}^n. \end{matrix}$$

$$Na_2S_4O_6 + NaI$$

'M' Electrolysis & liquation is method of purification where as hydraulic washing, leading, froth flotation are method of can conbration.

35. Choose the correct set of reagents for the following conversion.
$\text{trans}(Ph - CH = CH - CH_3) \rightarrow \text{cis}(Ph - CH = CH - CH_3)$
(1) $Br_2, aq \cdot KOH, NaNH_2, Na(LiqNH_3)$
(2) Br_2, alc $\cdot$ $KOH, NaNH_2, H_2$ Lindlar Catalyst
(3) $Br_2, aq \cdot KOH, NaNH_2, H_2$ Lindlar Catalyst
(4) Br_2, alc $\cdot$ $KOH, NaNH_2, Na(LiqNH_3)$

Sol. 2

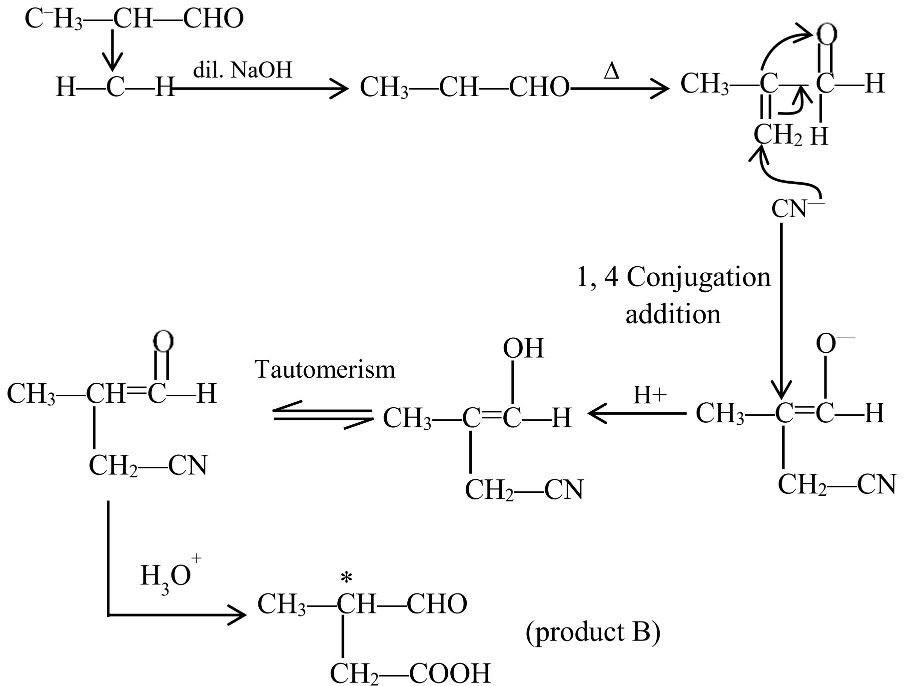


36. Consider the following reaction

$$\text{Propanal} + \text{Methanal} \xrightarrow[\substack{\text{(ii) } \Delta \\ \text{(iii) NaCN} \\ \text{(iv) } H_3O^+}]{\text{(i) dil.NaOH}} \text{Product B } (C_5H_8O_3)$$

The correct statement for product B is. It is
(1) optically active alcohol and is neutral
(2) racemic mixture and gives a gas with saturated $NaHCO_3$ solution
(3) optically active and adds one mole of bromine
(4) racemic mixture and is neutral

Sol. 2



Carboxylic acid will give CO_2 gas with $NaHCO_3$ solutions

37. The methods NOT involved in concentration of ore are

A. Liquation B. Leaching C. Electrolysis D. Hydraulic washing

E. Froth floatation

Choose the correct answer from the options given below :

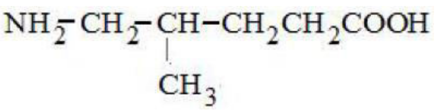
(1) C, D and E only (2) B, D and C only (3) A and C only (4) B, D and E only

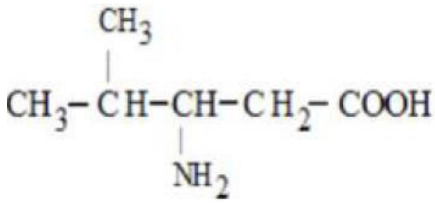
Sol. **3**

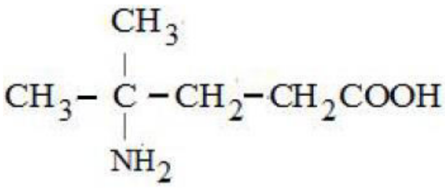
Methods involved in concentration of one are

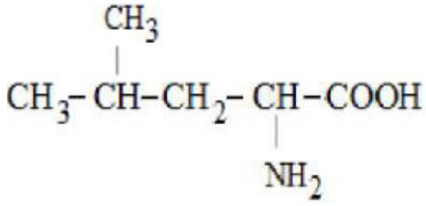
(i) Hydraulic Washing
(ii) Froth Flotation
(iii) Magnetic Separation
(iv) Leaching

38. A protein 'X' with molecular weight of 70,000u, on hydrolysis gives amino acids. One of these amino acid is

(1)

$NH_2-CH_2-CH(CH_3)-CH_2CH_2COOH$

(2)

$CH_3-CH(CH_3)-CH(NH_2)-CH_2-COOH$

(3)

$CH_3-C(CH_3)(NH_2)-CH_2-CH_2COOH$

(4)

$CH_3-CH(CH_3)-CH_2-CH(NH_2)-COOH$

Sol. **4**

From protein, only ∝–Amino acid is possible so answer is (4).

39. Nd^{2+} =

(1) $4f^3$ (2) $4f^4 6\ s^2$ (3) $4f^4$ (4) $4f^2 6\ s^2$

Sol. **3**

Na = $4f^4\ 5d^0\ 6s^2$
Na^{+2} = $4f^4\ 5d^0\ 6s^0$

40. Match List I with List II

List I	List II
A. XeF_4	I. See-saw
B. SF_4	II. Square planar
C. NH_4^+	III. Bent T-shaped
D. BrF_3	IV. Tetrahedral

Choose the correct answer from the options given below :

(1) A-IV, B-III, C-II, D-I (2) A-IV, B-I, C-II, D-III
(3) A-II, B-I, C-III, D-IV (4) A-II, B-I, C-IV, D-III

Sol. 4

XeF_4	Sq. planar
SF_4	see saw
NH_4^+	Tetrahedral
BrF_3	Bent 'T' shaped.

41. Identify X, Y and Z in the following reaction. (Equation not balanced)

$$ClO + NO_2 \rightarrow \underline{X} \xrightarrow{H_2O} \underline{Y} + \underline{Z}$$

(1) $X = ClONO_2, Y = HOCl, Z = HNO_3$
(2) $X = ClONO_2, Y = HOCl, Z = NO_2$
(3) $X = ClNO_2, Y = HCl, Z = HNO_3$
(4) $X = ClNO_3, Y = Cl_2, Z = NO_2$

Sol. 1

$$Cl\overset{\circ}{O} + NO_2 \longrightarrow \underset{(x)}{ClO.NO_2} \xrightarrow{H_2O} \underset{(y)}{HOCl} + \underset{(z)}{HNO_3}$$

42. The correct increasing order of the ionic radii is

(1) $S^{2-} < Cl^- < Ca^{2+} < K^+$ (2) $K^+ < S^{2-} < Ca^{2+} < Cl^-$
(3) $Ca^{2+} < K^+ < Cl^- < S^{2-}$ (4) $Cl^- < Ca^{2+} < K^+ < S^{2-}$

Sol. 3

For isoelectronic species size $\propto \frac{1}{z}$

$Ca^{+2} < K^+ < Cl^- < S^{-2}$: size
Z : 20 19 17 18

43. Cobalt chloride when dissolved in water forms pink colored complex $\underline{X}$ which has octahedral geometry. This solution on treating with conc HCl forms deep blue complex, $\underline{Y}$ which has a $\underline{Z}$ geometry. X, Y and Z, respectively, are

(1) $X = [Co(H_2O)_6]^{2+}, Y = [CoCl_4]^{2-}, Z =$ Tetrahedral
(2) $X = [Co(H_2O)_6]^{2+}, Y = [CoCl_6]^{3-}, Z =$ Octahedral
(3) $X = [Co(H_2O)_4Cl_2]^+, Y = [CoCl_4]^{2-}, Z =$ Tetrahedral
(4) $X = [Co(H_2O)_6]^{3+}, Y = [CoCl_6]^{3-}, Z =$ Octahedral

Sol. 1

$$CoCl_2 + H_2O \longrightarrow \underset{\text{Pink}}{[Co(H_2O)_6]^{2+}} \xrightarrow{\text{conc. HCl}} \underset{\text{Blue Tetrahderal}}{[CoCl_4]^{2-}}$$

44. H_2O_2 acts as a reducing agent in

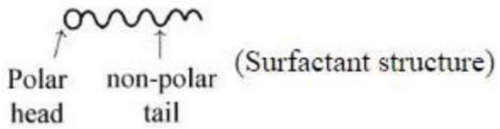
(1) $2NaOCl + H_2O_2 \rightarrow 2NaCl + H_2O + O_2$ (2) $Na_2S + 4H_2O_2 \rightarrow Na_2SO_4 + 4H_2O$
(3) $2Fe^{2+} + 2H^+ + H_2O_2 \rightarrow 2Fe^{3+} + 2H_2O$ (4) $Mn^{2+} + 2H_2O_2 \rightarrow MnO_2 + 2H_2O$

Sol. 1

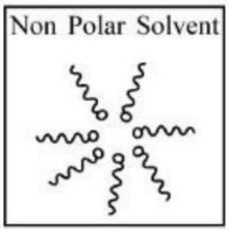
$$2NaO\overset{+1}{Cl} + H_2O_2 \longrightarrow 2Na\overset{-1}{Cl} + H_2O + O_2$$

H_2O_2 acts as reducing agent.

45. Adding surfactants in non polar solvent, the micelles structure will look like



(a)



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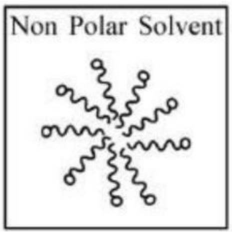
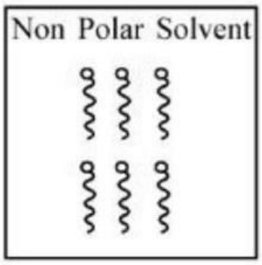
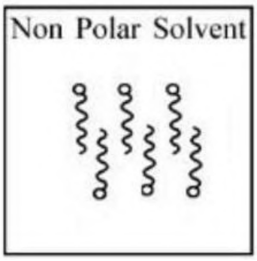


(c)



(d)



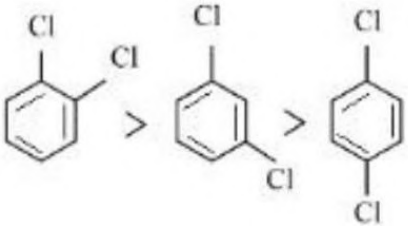
(1) a (2) d (3) b (4) c

Sol. **1**

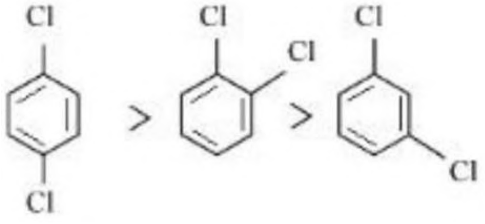
Non polar end will be towards non polar solvent

46. The correct order of melting points of dichlorobenzenes is

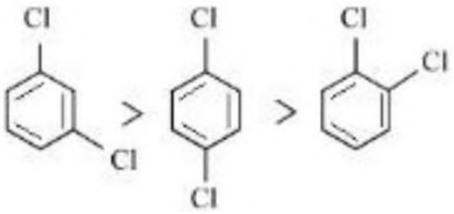
(1)



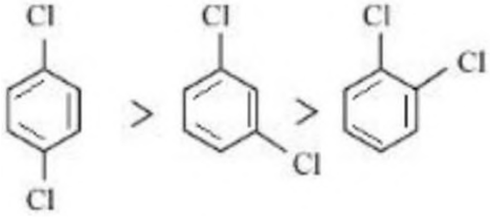
(2)



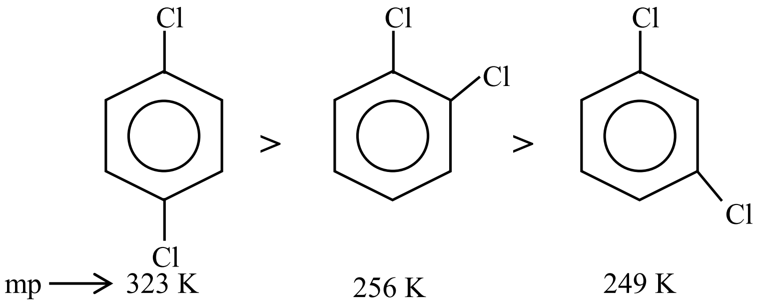
(3)



(4)



Sol. **2**



47. The correct order of basicity of oxides of vanadium is

(1) $V_2O_5 > V_2O_4 > V_2O_3$ (2) $V_2O_4 > V_2O_3 > V_2O_5$

(3) $V_2O_3 > V_2O_5 > V_2O_4$ (4) $V_2O_3 > V_2O_4 > V_2O_5$

Sol. **4**

Leaser is charge on canter atom more will be the basicity.

48. Which of the following artificial sweeteners has the highest sweetness value in comparison to cane sugar ?

(1) Sucralose (2) Aspartame (3) Alitame (4) Saccharin

Sol. **3**

Alitame has 2000 has times more sweetner as compare to cane sugar.

49. Which one of the following statements is correct for electrolysis of brine solution?

(1) Cl_2 is formed at cathode (2) O_2 is formed at cathode

(3) H_2 is formed at anode (4) $OH^-$is formed at cathode

Sol. **4**

Brine sol^n gives H_2/OH^- at cathode & Cl_2 at anode.

50. Which transition in the hydrogen spectrum would have the same wavelength as the Balmer type transition from $n = 4$ to $n = 2$ of He^+spectrum

(1) $n = 2$ to $n = 1$ (2) $n = 1$ to $n = 2$ (3) $n = 3$ to $n = 4$ (4) $n = 1$ to $n = 3$

Sol. **1**

$$\lambda_H = \lambda_{He^+}$$

$$R_H \times (1)^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = R_H \times (2)^2 \left(\frac{1}{(2)^2} - \frac{1}{(4)^2}\right)$$

$$\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = \left(\frac{4}{4}\right) - \left(\frac{4}{16}\right)$$

$$\frac{1}{n_1^2} - \frac{1}{n_2^2} = \frac{1}{1} - \frac{1}{4}$$

$n_1 = 1 : n_2 = 2$ for H-atom

SECTION B

51. The oxidation state of phosphorus in hypophosphoric acid is +

Sol. Hypophosphoric acid is $H_4P_2O_6$ oxidation state of P is +4.

52. The enthalpy change for the conversion of $\frac{1}{2}Cl_2(g)$ to Cl^-(aq) is (-) $kJmol^{-1}$ (Nearest integer)

Given : $\Delta_{dis} H^{\ominus}_{Cl_{2}(g)} = 240 \text{ kJ mol}^{-1}, \Delta_{eg} H^{\ominus}_{Cl(g)} = -350 \text{ kJ mol}^{-1}$, $\Delta_{hyd} H^{\ominus}_{Cl^-_{(g)}} = -380 \text{ kJ mol}^{-1}$

Sol. **610**

$$\begin{tikzcd}
\frac{1}{2}Cl_2 \arrow[r] \arrow[d, "\frac{1}{2}\ BE"'] & Cl^-\ aq. \\
Cl_{(g)} \arrow[r, "\Delta H_{eg}.\ Of\ Cl_{(g)}"'] & Cl_{(g)} \arrow[u, "\Delta H_{Hyd.}\ Of\ Cl_{(g)}"']
\end{tikzcd}$$

$$\Delta H^{\circ}_{\gamma} = \frac{1}{2} \times BE + \Delta H_{eg} + \Delta H_{Hyd}$$

$$= \frac{1}{2} \times 240 + (-350) + (-380)$$

$\Rightarrow$ 120 – 350 – 380

$\Rightarrow$ –610

53. The logarithm of equilibrium constant for the reaction $Pd^{2+} + 4Cl^- \rightleftharpoons PdCl_4^{2-}$ is (Nearest integer)

Given : $\frac{2.303RT}{F} = 0.06\ V$

$Pd^{2+}_{(aq)} + 2e^- \rightleftharpoons Pd(s)\ E^{\ominus} = 0.83\ V$

$PdCl_4^{2-}(aq) + 2e^- \rightleftharpoons Pd(s) + 4Cl^-(aq)\ E^{\theta} = 0.65\ V$

Sol. **6**

$\Delta G° = -RT\ell nK$

$-nFE°_{cell} = -RT \times 2.303\ (\log_{10}K)$ ....(1)

Net reaction → Pd^{2+} (aq.) + $4Cl^-$ (aq.) $\rightleftharpoons$ $PdCl_4^{2-}$ (aq.)

$E°_{cell} = E°_{cathod} - E°_{anode}$

$E°_{cell} = 0.83 - 0.65$

From equation (1)

Also n = 2

logK = 6

54. On complete combustion, 0.492 g of an organic compound gave 0.792 g of CO_2. The % of carbon in the organic compound is (Nearest integer)

Sol. **44**

44 gm of CO_2 contains 12 g carbon.

0.792 gm of CO_2 contains $\frac{0.792 \times 12}{44}$ g of carbon

% of carbon = $\frac{0.216}{0.492} \times 100$

= 43.9% = 44%

55. Zinc reacts with hydrochloric acid to give hydrogen and zinc chloride. The volume of hydrogen gas produced at STP from the reaction of 11.5 g of zinc with excess HCl is L (Nearest integer) (Given : Molar mass of Zn is 65.4 g mol^{-1} and Molar volume of H_2 at STP = 22.7 L)

Sol. **4**

$$Zn + 2HCl \longrightarrow ZnCl_2 + H_{2.}$$

No. of moles of Zn = $\frac{11.5}{65.3}$ = No. of moles of H_2

No. of H_2 liberated = 0.176 × 22.7 Lt.

= 3.99 L = 4 Lt.

56. $A \rightarrow B$

The rate constants of the above reaction at $200\ \mathrm{K}$ and $300\ \mathrm{K}$ are $0.03\ \mathrm{min}^{-1}$ and $0.05\ \mathrm{min}^{-1}$ respectively. The activation energy for the reaction is J (Nearest integer) (Given : $\ln 10 = 2.3$

$R = 8.3\ \mathrm{J\ K^{-1}\ mol^{-1}}$

$\log 5 = 0.70$

$\log 3 = 0.48$

$\log 2 = 0.30)$

Sol. 2520

$$\text{In}\left(\frac{K_2}{K_1}\right)=\frac{Ea}{R}\left[\frac{1}{T_1}-\frac{1}{T_2}\right]$$

$$\text{Log}\left(\frac{0.05}{0.03}\right)=\frac{Ea}{2.3\times 8.3}\left[\frac{1}{200}-\frac{1}{300}\right]$$

$$[0.70-0.48]=\frac{Ea}{2.3\times 8.3}\left[\frac{300-200}{300\times 200}\right]$$

$$0.22=\frac{Ea}{2.3\times 8.3}\left[\frac{1}{600}\right]$$

Ea = 0.22 × 2.3 × 8.3 × 600

= 2519.88 J

≈ 2520

57. For reaction: $SO_2(g)+\frac{1}{2}O_2(g) \rightleftharpoons SO_3(g)$

$K_p = 2 \times 10^{12}$ at $27^{\circ}C$ and 1 atm pressure. The K_c for the same reaction is $\times 10^{13}$. (Nearest integer)

(Given $R = 0.082\ \mathrm{L\ atm\ K^{-1}\ mol^{-1}}$)

Sol. **1**

$K_C = 1 \times 10^{13}$

$SO_2(g)+\frac{1}{2}O_2 \rightleftharpoons SO_3(g)$

$\Delta n = \frac{-1}{2}$

$K_P = 2 \times 10^{12}$

$K_P = K_C\,(RT)^{\Delta ng}$

$2 \times 10^{12} = K_C\,(0.082 \times 300)^{-1/2}$

$K_C = 1 \times 10^{13}$

P = 1 atm

T = 27°C

58. The total pressure of a mixture of non-reacting gases $X(0.6\ g)$ and $Y(0.45\ g)$ in a vessel is $740\ \mathrm{mm}$ of Hg. The partial pressure of the gas X is mm of Hg.

(Nearest Integer)

(Given : molar mass $X = 20$ and $Y = 45\ \mathrm{g\ mol^{-1}}$)

Sol. **555**

Number of moles of gas $X=\frac{0.6}{20}=0.03$

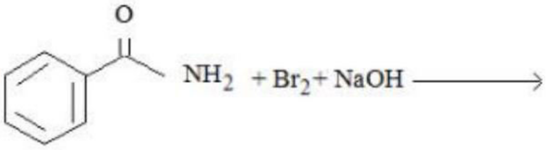
Number of moles of gas $Y=\frac{0.45}{45}=0.01$

Total number of moles = 0.03 + 0.01 = 0.04 mole

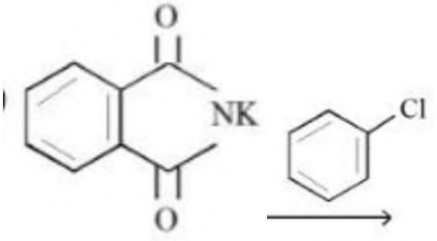
Partial pressure of gas X = Mole fraction × Total pressure

$$=\frac{0.03}{0.04}\times 740=555$$

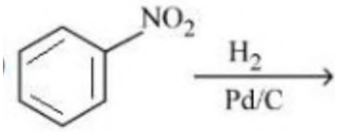
59. How many of the transformations given below would result in aromatic amines ?



(1)



(2)



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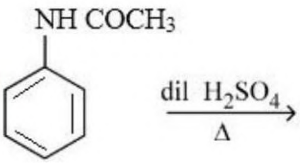
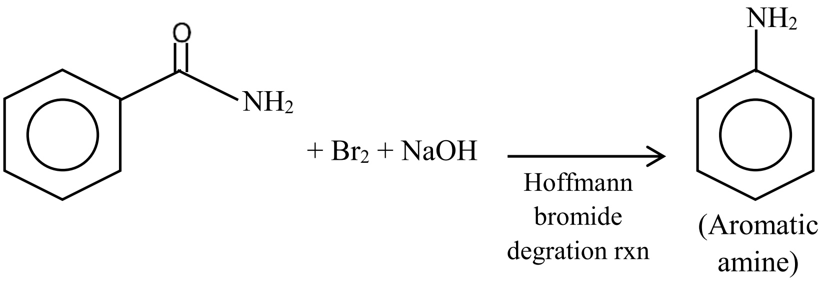


(4)

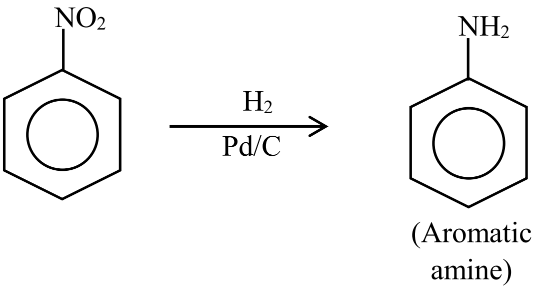
Sol. **3**

(1)

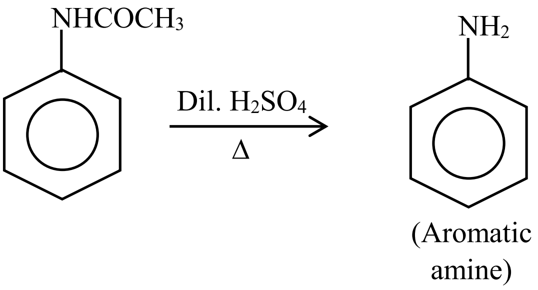


(2) In Gabriel phthalimide synthesis chloro-benzene is poor substrok for S_{N_2} , Hence reaction will not observed.

(3)



(4)



60. At $27^{\circ}C$, a solution containing $2.5\ g$ of solute in $250.0\ mL$ of solution exerts an osmotic pressure of $400\ Pa$. The molar mass of the solute is $gmol^{-1}$ (Nearest integer)

(Given : $R = 0.083\ L_{bar}\ K^{-1}\ mol^{-1}$)

Sol. **62250**

π = CRT

$$\frac{400Pa}{10^5} = \frac{\frac{2.5g}{M_\circ}}{250/1000} \times 0.083 \frac{L-bar}{Kmol} \times 300\,K$$

M∘ = 62250

Mathematics

SECTION - A

61. If the maximum distance of normal to the ellipse $\frac{x^2}{4}+\frac{y^2}{b^2}=1, b<2$, from the origin is 1 , then the eccentricity of the ellipse is :

(1) $\frac{1}{2}$ (2) $\frac{\sqrt{3}}{4}$ (3) $\frac{\sqrt{3}}{2}$ (4) $\frac{1}{\sqrt{2}}$

Sol.

Normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at point (a cosθ, b sinθ) is $\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$

Its distance from origin is

$$d=\frac{|a^2-b^2|}{\sqrt{a^2\sec^2\theta+b^2\operatorname{cosec}^2\theta}}$$

$$d=\frac{|a^2-b^2|}{\sqrt{a^2+b^2+2ab+(a\ \tan\theta-b\ \cot\theta)^2}}$$

$$d\frac{|(a-b)(a+b)|}{\sqrt{a^2+b^2+2ab+(a\ \tan\theta-b\ \tan\theta)^2}}$$

$$d_{max}=\frac{|(a-b)(a+b)|}{a+b}=|a-b|$$

$\because$ $d_{max} = 1$

$|2 - b| = 1$

$2 - b = 1$ [$\because$ $b < 2$]

$\boxed{b=1}$

$$\text{Eccentricity}=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{1}{4}}=\frac{\sqrt{3}}{2}$$

$$\Rightarrow \boxed{e=\frac{\sqrt{3}}{2}}$$

62. Let a differentiable function f satisfy $f(x)+\int_3^x \frac{f(t)}{t}dt=\sqrt{x+1}, x\ge 3$. Then $12f(8)$ is equal to :

(1) 34 (2) 1 (3) 17 (4) 19

Sol.

$$f(x)+\int_3^x \frac{f(t)}{t}dt=\sqrt{x+1},\ x\ge 3$$

Differentiate both side w.r.t. x

$$f^1(x)+\frac{f(x)}{x}=\frac{1}{2\sqrt{x+1}}$$

Above eqn. is linear differential equation

$$\text{I.f.} = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$$

Solution is

$$f(x)\cdot x = \int \frac{x}{2\sqrt{x+1}}dx + C$$

$$f(x)\cdot x = \frac{1}{2}\int\left(\frac{x+1}{\sqrt{x+1}} - \frac{1}{\sqrt{x+1}}\right)dx + C$$

$$f(x)\cdot x = \frac{1}{2}\int\left(\sqrt{x+1} - \frac{1}{\sqrt{x+1}}\right)dx + C$$

$$f(x)\cdot x = \frac{1}{2}\left[\frac{2}{3}(x+1)^{3/2} - 2\sqrt{x+1}\right] + C$$

$\because$ f(3) = 2

than

$$2.3 = \frac{1}{2}\left[\frac{2}{3}\times 8 - 2\times 2\right] + C$$

$$6 = \frac{1}{2}\left[\frac{16}{3} - 4\right] + C$$

$$6 = \frac{2}{3} + C$$

$$\boxed{C = \frac{16}{3}}$$

$$f(x).\ x = \frac{1}{2}\left[\frac{2}{3}(x+1)^{3/2} - 2\sqrt{x+1}\right] + \frac{16}{3}$$

Put x = 8

$$f(8)\cdot 8 = \frac{1}{2}\left[\frac{2}{3}\times 27 - 2\times 3\right] + \frac{16}{3}$$

$$f(8)\cdot 8 = \frac{1}{2}[12] + \frac{16}{3}$$

$$f(8)\cdot 8 = 6 + \frac{16}{3} = \frac{34}{3}$$

$$\boxed{12f(8) = 17}$$

63. For all $z \in C$ on the curve $C_1: |z| = 4$, let the locus of the point $z + \frac{1}{z}$ be the curve C_2. Then :

(1) the curve C_1 lies inside C_2

(2) the curve C_2 lies inside C_1

(3) the curves C_1 and C_2 intersect at 4 points

(4) the curves C_1 and C_2 intersect at 2 points

Sol.

$C_1 : |z| = 4$ then $z\bar{z} = 16$

$$z + \frac{1}{z} = z + \frac{\bar{z}}{16}$$

$$= x + iy + \frac{x - iy}{16}$$

$$z + \frac{1}{z} = \frac{17x}{16} + i\frac{15y}{16}$$

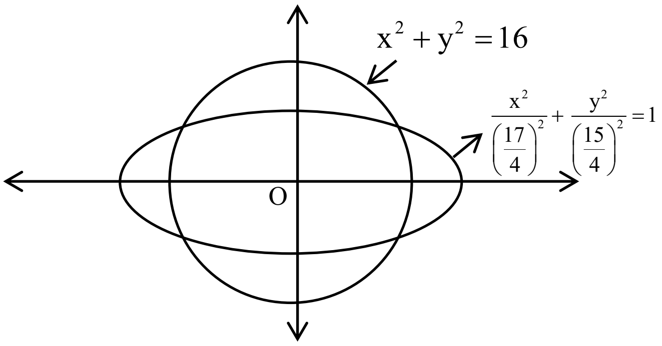
Let $X = \frac{17x}{16}, \quad Y = \frac{15}{16}y$

$$\frac{X}{\left(\frac{17}{16}\right)} = x, \quad \frac{Y}{\left(\frac{15}{16}\right)} = y$$

$\because x^2 + y^2 = 16$

$$\frac{X^2}{\left(\frac{17}{16}\right)^2} + \frac{Y^2}{\left(\frac{15}{16}\right)^2} = 16$$

$$\Rightarrow C_2 : \frac{x^2}{\left(\frac{17}{4}\right)^2} + \frac{y^2}{\left(\frac{15}{4}\right)^2} = 1 \qquad \text{(Ellipse)}$$



Curve C_1 and C_2 intersect at 4 point.

64. $y = f(x) = \sin^3\left(\frac{\pi}{3}\left(\cos\left(\frac{\pi}{3\sqrt{2}}(-4x^3 + 5x^2 + 1)^{\frac{3}{2}}\right)\right)\right)$. Then, at $x = 1$,

(1) $\sqrt{2}y' - 3\pi^2 y = 0$ (2) $y' + 3\pi^2 y = 0$ (3) $2y' + 3\pi^2 y = 0$ (4) $2y' + \sqrt{3}\pi^2 y = 0$

Sol.

$$y = f(x) = \sin^3\left(\frac{\pi}{3}\left(\cos\left(\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{3/2}\right)\right)\right)$$

Let $g(x) = \frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{3/2}$

$g(1) = \frac{2\pi}{3}$

$y = \sin^3\left(\frac{\pi}{3}\cos(g(x))\right)$

Differentiate w.r.t. x

$$y' = 3\sin^2\left(\frac{\pi}{3}\cos(g(x))\right)\times\cos\left(\frac{\pi}{3}\cos(g(x))\right)\times\frac{\pi}{3}\left(-\sin g(x)\right)g'(x)$$

$$\because g^1(x) = \frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{1/2}\left(-12x^2+10x\right)$$

$$g^1(1) = \frac{\pi}{2\sqrt{2}}\left(\sqrt{2}\right)(-2) = -\pi$$

$$y^1(1) = \frac{3}{4}\times\frac{\sqrt{3}}{2}\times\frac{\pi}{3}\left(\frac{-\sqrt{3}}{2}\right)(-\pi) = \frac{3\pi^2}{16}$$

$$\boxed{y^1(1) = \frac{3\pi^2}{16}}$$

$$y(1) = \sin^{3.}\left(\frac{\pi}{3}\cos\frac{2\pi}{3}\right) = \frac{-1}{8}$$

$$\boxed{2y^1(1)+3\pi^2 y(1) = 0}$$

65. A wire of length 20 m is to be cut into two pieces. A piece of length l_1 is bent to make a square of area A_1 and the other piece of length l_2 is made into a circle of area A_2. If $2A_1 + 3A_2$ is minimum then $(\pi l_1): l_2$ is equal to :

(1) 1:6 (2) 6: 1 (3) 3: 1 (4) 4: 1

Sol.

Total length of wire = 20 m

area of square (A_1) = $\left(\frac{\ell_1}{4}\right)^2$

area of circle (A_2) = $\pi\left(\frac{\ell_2}{2\pi}\right)^2$

Let S = $2A_1 + 3A_2$

$$S = \frac{\ell_1^2}{8} + \frac{3\ell_2^2}{4\pi}$$

$\because \ \ell_1 + \ell_2 = 20$ then

$$1 + \frac{d\ell_2}{d\ell_1} = 0$$

$$\frac{d\ell_2}{d\ell_1} = -1$$

$$\frac{ds}{d\ell_1} = \frac{\ell_1}{4} + \frac{6\ell_2}{4\pi} \cdot \frac{d\ell_2}{d\ell_1} = 0$$

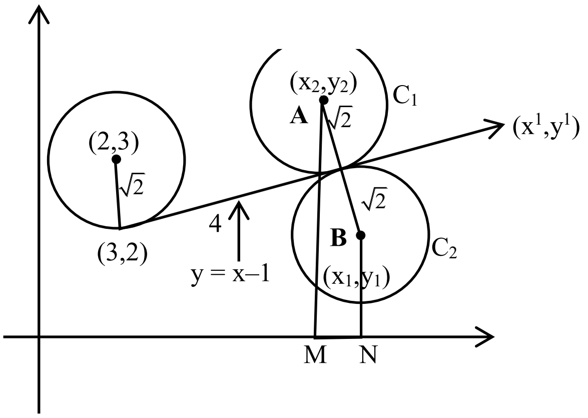
$$= \frac{\ell_1}{4} = \frac{6\ell_2}{4\pi}$$

$$= \frac{\pi\ell_1}{\ell_2} = \frac{6}{1}$$

66. Let a circle C_1 be obtained on rolling the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ upwards 4 units on the tangent T to it at the point $(3,2)$. Let C_2 be the image of C_1 in T. Let A and B be the centers of circles C_1 and C_2 respectively, and M and N be respectively the feet of perpendiculars drawn from A and B on the x-axis. Then the area of the trapezium AMNB is :

(1) $4(1+\sqrt{2})$ (2) $3+2\sqrt{2}$ (3) $2(1+\sqrt{2})$ (4) $2(2+\sqrt{2})$

Sol.



(x', y') point lies on line y = x – 1 have distance 4 unit from (3, 2).

$$x' = \frac{4}{\sqrt{2}} + 3 \quad = 2\sqrt{2} + 3$$

$$y' = \frac{4}{\sqrt{2}} + 2 = 2\sqrt{2} + 2$$

Slope of line AB is –1. i.e. $= \tan\theta = -1$ then $\sin\theta = \frac{1}{\sqrt{2}}$, $\cos\theta = -\frac{1}{\sqrt{2}}$

for point A and B

$$x = \pm\sqrt{2}\left(\frac{-1}{\sqrt{2}}\right)+\left(2\sqrt{2}+3\right)$$

$$y = \pm\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)+\left(2\sqrt{2}+2\right)$$

for point A we take +ve sign

$$(x_2, y_2) = \left(2\sqrt{2}+2,\ 2\sqrt{2}+3\right)$$

for point B we take –ve sign

$$(x_1, y_1) = \left(2\sqrt{2}+4,\ 2\sqrt{2}+1\right)$$

$MN = |x_2 - x_1| = 2$

$AM + BN = 2\sqrt{2}+3+2\sqrt{2}+1=4+4\sqrt{2}$

area of trapezium $= \frac{1}{2}\times 2\times\left(4+4\sqrt{2}\right)$

$$= 4\left(1+\sqrt{2}\right)$$

67. A bag contains 6 balls. Two balls are drawn from it at random and both are found to be black. The probability that the bag contains at least 5 black balls is

(1) $\frac{3}{7}$ (2) $\frac{5}{7}$ (3) $\frac{5}{6}$ (4) $\frac{2}{7}$

Sol. $\text{Probability} = \dfrac{{}^3C_2 + {}^6C_2}{{}^2C_2 + {}^3C_2 + {}^4C_2 + {}^5C_2 + {}^6C_2}$

$$= \frac{10+15}{1+3+6+10+15}$$

$$= \frac{5}{7}$$

68. Let $y = f(x)$ represent a parabola with focus $\left(-\frac{1}{2}, 0\right)$ and directrix $y = -\frac{1}{2}$.

Then $S = \left\{x \in \mathbb{R}: \tan^{-1}(\sqrt{f(x)}) + \sin^{-1}(\sqrt{f(x)+1}) = \frac{\pi}{2}\right\}$:

(1) contains exactly two elements (2) contains exactly one element

(3) is an empty set (4) is an infinite set

Sol. equation of parabola which have focus $\left(-\frac{1}{2},\ 0\right)$ and directrix $y = -\frac{1}{2}$ is

$$\left(x+\frac{1}{2}\right)^2 = \left(y+\frac{1}{4}\right)$$

$y = f(x) = (x^2 + x)$

$$\because\ S = \left\{x \in R : \tan^{-1}\left(\left(\sqrt{f(x)}\right) + \sin^{-1}\left(\sqrt{f(x)+1}\right) = \frac{\pi}{2}\right)\right\}$$

$$\tan^{-1}\left(\sqrt{f(x)}\right) + \sin^{-1}\left(\sqrt{f(x)+1}\right) = \frac{\pi}{2}$$

$f(x) \geq 0$ & $\sqrt{f(x)+1}$ can not greater then 1, so f(x) must be 0

i.e. f(x) = 0

$\Rightarrow\ x^2 + x = 0$

$x(x+1) = 0$

$x = 0,\ x = -1$

S contain 2 element.

69. Let $\vec{a} = 2\hat{\imath} + \hat{\jmath} + \hat{k}$, and $\vec{b}$ and $\vec{c}$ be two nonzero vectors such that $|\vec{a} + \vec{b} + \vec{c}| = |\vec{a} + \vec{b} - \vec{c}|$ and $\vec{b} \cdot \vec{c} = 0$. Consider the following two statements:

(A) $|\vec{a} + \lambda\vec{c}| \geq |\vec{a}|$ for all $\lambda \in \mathbb{R}$.

(B) $\vec{a}$ and $\vec{c}$ are always parallel.

Then.

(1) both (A) and (B) are correct

(2) only (A) is correct

(3) neither (A) nor (B) is correct

(4) only (B) is correct

Sol.

$\left|\vec{a} + \vec{b} + \vec{c}\right| = \left|\vec{a} + \vec{b} - \vec{c}\right|,\ \vec{b}.\vec{c} = 0$

$\left|\vec{a} + \vec{b} + \vec{c}\right|^2 = \left|\vec{a} + \vec{b} - \vec{c}\right|^2$

$\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + \left|\vec{c}\right|^2 + 2\vec{a}.\vec{b} + 2\vec{b}.\vec{c} + 2\vec{a}.\vec{c}$

$= \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + \left|\vec{c}\right|^2 + 2\vec{a}.\vec{b} - 2\vec{b}.\vec{c} - 2\vec{a}.\vec{c}$

$2\vec{a}.\vec{b} + 2\vec{b}.\vec{c} + 2\vec{a}.\vec{c} = 2\vec{a}.\vec{b} - 2\vec{b}.\vec{c} - 2\vec{a}.\vec{c}$

$\vec{a}.\vec{b} + \vec{a}.\vec{c} = \vec{a}.\vec{b} - \vec{a}.\vec{c}$

$\vec{a}.\vec{c} = 0$ (B is incorrect)

$\left|\vec{a} + \lambda\vec{c}\right|^2 \geq \left|\vec{a}\right|^2$

$\left|\vec{a}\right|^2 + \lambda^2\left|\vec{c}\right|^2 + 2\lambda\vec{a}\cdot\vec{c} \geq \left|\vec{a}\right|^2$

$= \lambda^2 c^2 \geq 0$

True $\forall \lambda \in R$ (A is correct)

70. The value of $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{(2+3\sin x)}{\sin x(1+\cos x)}dx$ is equal to

(1) $\frac{10}{3} - \sqrt{3} - \log_e \sqrt{3}$ (2) $\frac{7}{2} - \sqrt{3} - \log_e \sqrt{3}$

(3) $-2 + 3\sqrt{3} + \log_e \sqrt{3}$ (4) $\frac{10}{3} - \sqrt{3} + \log_e \sqrt{3}$

Sol. **(4)**

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{(2+3\sin x)}{\sin x(1+\cos x)}dx$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{2}{\sin x + \sin x \cos x}dx + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{3}{1+\cos x}dx$$

$= I_1 + I_2$

$$I_1 = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{2dx}{\sin x(1+\cos x)} = 2\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\left(1+\tan^2\frac{x}{2}\right)dx}{2\tan\frac{x}{2} \times \left(1+\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}$$

$$= 2\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\left(1+\tan^2\frac{x}{2}\right)\left(1+\tan^2\frac{x}{2}\right)dx}{2\tan\frac{x}{2} \times 2}$$

$$= 2\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sec^2\frac{x}{2}\left(1+\tan^2\frac{x}{2}\right)}{4\tan\frac{x}{2}}dx$$

Let, $\tan\frac{x}{2} = t$ then $\sec^2\frac{x}{2} \times \frac{1}{2}dx = dt$

$$= 2\int_{\frac{1}{\sqrt{3}}}^{1} \frac{1+t^2}{2t}dt$$

$$= \left[\ell nt + \frac{t^2}{2}\right]_{\frac{1}{\sqrt{3}}}^{1}$$

$$= \left[\frac{1}{2} - \ell n\frac{1}{\sqrt{3}} - \frac{1}{6}\right]$$

$$I_1 = \left[\ell n\sqrt{3} + \frac{1}{3}\right]$$

$$I_2 = 3\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{dx}{1+\cos x} = 3\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1-\cos x}{\sin^2 x}dx$$

$$I_2 = 3\left[\operatorname{cosec}x - \cot x\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} = 3-\sqrt{3}$$

$$I_1 + I_2 = \ln\sqrt{3} + \frac{1}{3} + 3 - \sqrt{3}$$

$$= \frac{10}{3} + \ln\sqrt{3} - \sqrt{3}$$

71. Let the shortest distance between the lines $L: \frac{x-5}{-2} = \frac{y-\lambda}{0} = \frac{z+\lambda}{1}, \lambda \geq 0$ and $L_1: x+1 = y-1 = 4-z$ be $2\sqrt{6}$. If (α, β, γ) lies on L, then which of the following is NOT possible ?

(1) $\alpha - 2\gamma = 19$ (2) $2\alpha + \gamma = 7$ (3) $2\alpha - \gamma = 9$ (4) $\alpha + 2\gamma = 24$

Sol. **(4)**

Let $\vec{b}_1 = <-2, 0, 1> \quad \vec{a}_1 = (5, \lambda, -\lambda)$

$\vec{b}_2 = <1, 1, -1 \quad > \vec{a}_2 = (-1, 1, 4)$

Normal vector of both line is $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix}$

$\hat{i}(-1) - \hat{j}(1) + \hat{k}(-2)$

$\vec{b}_1 \times \vec{b}_2 = <-1, -1, -2>$

$\vec{a}_1 - \vec{a}_2 = <6, \lambda-1, -\lambda-4>$

Shortest distance $d = \left|\frac{(\vec{a}_2 - \vec{a}_1) \times (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$

$$2\sqrt{6} = \left|\frac{<6, \lambda-1, -\lambda-4> \times <-1, -1, -2>}{\sqrt{(1)^2 + (1)^2 + (2)^2}}\right|$$

$$12 = |-6 - \lambda + 1 + 2\lambda + 8|$$

$$|\lambda + 3| = 12$$

$$\lambda = 9, \text{-}15$$

$$\lambda = 9 (\because \lambda \geq 0)$$

$\because$ (α, β, γ) lies on line L then

$$\frac{\alpha - 5}{-2} = \frac{\beta - 9}{0} = \frac{\gamma + 9}{1} = K$$

$\alpha = 5 - 2K$, $\beta = 9K$, $\gamma = -9 + K$

$\alpha + 2\gamma, = 5 - 2K - 18 + 2K = -13 \neq 24$

Therefore $\alpha + 2\gamma = 24$ is not possible.

72. For the system of linear equations

$x + y + z = 6$

$\alpha x + \beta y + 7z = 3$

$x + 2y + 3z = 14$

which of the following is NOT true ?

(1) If $\alpha = \beta$ and $\alpha \neq 7$, then the system has a unique solution

(2) If $\alpha = \beta = 7$, then the system has no solution

(3) For every point $(\alpha, \beta) \neq (7,7)$ on the line $x - 2y + 7 = 0$, the system has infinitely many solutions

(4) There is a unique point (α, β) on the line $x + 2y + 18 = 0$ for which the system has infinitely many solutions

Sol. **(3)**

$x + y + z = 6$ … (1)

$\alpha x + \beta y + 7z = 3$ … (2)

$x + 2y + 3z = 14$ … (3)

equation (3) – equation (1)

$y + 2z = 8$

$y = 8 - 2z$

From (1) $x = -2 + z$

Value of x and y put in equation (2)

$\alpha(-2 + z) + \beta(8 - 2z) + 7z = 3$

$-2\alpha + \alpha z + 8\beta - 2\beta z + 7z = 3$

$(\alpha - 2\beta + 7) z = 2\alpha - 8\beta + 3$

if $\alpha - 2\beta + 7 \neq 0$ then system has unique solution

if $(\alpha - 2\beta + 7 = 0)$ and $2\alpha - 8\beta + 3 \neq 0$ then system has no solution

if $(\alpha - 2\beta + 7 = 0)$ and $2\alpha - 8\beta + 3 = 0$ then system has infinite solution

73. If the domain of the function $f(x) = \frac{[x]}{1+x^2}$, where $[x]$ is greatest integer $\leq x$, is $[2,6)$, then its range is

(1) $\left(\frac{5}{26}, \frac{2}{5}\right]$

(2) $\left(\frac{5}{37}, \frac{2}{5}\right] - \left\{\frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53}\right\}$

(3) $\left(\frac{5}{37}, \frac{2}{5}\right]$

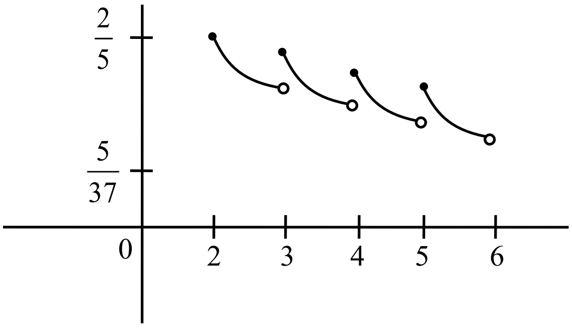
(4) $\left(\frac{5}{26}, \frac{2}{5}\right] - \left\{\frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53}\right\}$

Sol. **(3)**

$f(x) = \frac{[x]}{1+x^2}$, $x = \in [2, 6]$

$$f(x)=\begin{cases}\dfrac{2}{1+x^2} & x\in[2,3)\\ \dfrac{3}{1+x^2} & x\in[3,4)\\ \dfrac{4}{1+x^2} & x\in[4,5)\\ \dfrac{5}{1+x^2} & x\in[5,6)\end{cases}$$

$\because$ $f(x)$ is $\downarrow$ in $x\in[2, 6)$



range is $\left(\frac{5}{37},\frac{2}{5}\right]$

74. Let R be a relation on N×N defined by $(a,b)R(c,d)$ if and only if $ad(b-c)=bc(a-d)$. Then R is
(1) transitive but neither reflexive nor symmetric
(2) symmetric but neither reflexive nor transitive
(3) symmetric and transitive but not reflexive
(4) reflexive and symmetric but not transitive

Sol. **(2)**

(a, b) R (c, d) $\Leftrightarrow$ ad(b – c) = bc(a – d)

For reflexive

(a, b) R (a, b)

$\Rightarrow$ ab (b – a) $\neq$ ba(a – b)

R is not reflexive

For symmetric:

(a,b) R(c,d) $\Rightarrow$ ad(b –c) = bc (a –d)

then we check

(c, d) R (a, b) $\Rightarrow$ cb(d – a) = ad(c – b)

$\Rightarrow$ cb(a – d) = ad(b – c)

R is symmetric :

For transitive:

$\because$ (2,3) R (3,2) and (3,2) R (5,30)

But (2,3) is not related to (5,30)

R is not transitive.

75. $(S1)(p \Rightarrow q) \vee (p \wedge (\sim q))$ is a tautology

$(S2)((\sim p) \Rightarrow (\sim q)) \wedge ((\sim p) \vee q)$ is a contradiction.

Then

(1) both (S1) and (S2) are correct
(2) only (S1) is correct
(3) only (S2) is correct
(4) both (S1) and (S2) are wrong

Sol. **(2)**

$S_1 : (P \Rightarrow q) \vee (P \wedge (\sim q))$

P	q	$P \Rightarrow q$	~q	$P \wedge \sim q$	$(P \Rightarrow q) \vee (P \wedge \sim q)$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	T	F	T

S_1 is a tautology

$S_2 : ((\sim P) \Rightarrow (\sim q)) \wedge ((\sim P) \vee q)$

~P	~q	$\sim P \Rightarrow \sim q$	~P v q	$((\sim P) \Rightarrow (\sim q)) \wedge (\sim P)\, vq)$
F	F	T	T	T
F	T	T	F	F
T	F	F	T	F
T	T	T	T	T

S_2 is not a contradiction

76. If the sum and product of four positive consecutive terms of a G.P., are 126 and 1296 , respectively, then the sum of common ratios of all such GPs is

(1) 7
(2) 3
(3) $\frac{9}{2}$
(4) 14

Sol. **(1)**

Four term of G.P. $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$

$$\frac{a}{r^3} + \frac{a}{r} + ar + ar^3 = 126$$

$$\frac{a}{r^3} \cdot \frac{a}{r} \cdot ar \cdot ar^3 = 1296$$

$a^4 = 1296$

$a = 6$

$$\frac{6}{r^3} + \frac{6}{r} + 6r + 6r^3 = 126$$

$$\left(r+\frac{1}{r}\right)+r^3+\frac{1}{r^3}=21$$

$$\left(r+\frac{1}{r}\right)+\left(r+\frac{1}{r}\right)^3-3\left(r+\frac{1}{r}\right)=21$$

Let $r+\frac{1}{r}=t$

$t^3-2t=21$

$\Rightarrow t=3$

$r+\frac{1}{r}=3$

$r^2-3r+1=0$

$r=\frac{3\pm\sqrt{9-4}}{2}$

$r=\frac{3\pm\sqrt{5}}{2}$

Sum of common ratio = $\frac{9}{4}+\frac{5}{4}+\frac{3\sqrt{5}}{2}+\frac{9}{4}+\frac{5}{4}-\frac{3\sqrt{5}}{2}$

= 7

77. Let $A=\begin{pmatrix}1 & 0 & 0\\ 0 & 4 & -1\\ 0 & 12 & -3\end{pmatrix}$. Then the sum of the diagonal elements of the matrix $(A+I)^{11}$ is equal to

(1) 6144 (2) 2050 (3) 4097 (4) 4094

Sol. **(3)**

$$A=\begin{bmatrix}1 & 0 & 0\\ 0 & 4 & -1\\ 0 & 12 & -3\end{bmatrix}$$

$$A^2=\begin{bmatrix}1 & 0 & 0\\ 0 & 4 & -1\\ 0 & 12 & -3\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 4 & -1\\ 0 & 12 & -3\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 4 & -1\\ 0 & 12 & -3\end{bmatrix}$$

$A^3=A^4=A^5\ldots=A$

$(A+I)^{11}={}^{11}C_0A^{11}+{}^{11}C_1A^{10}+{}^{11}C_2A^9+\ldots{}^{11}C_{11}I$

$= =\left({}^{11}C_0+{}^{11}C_1+{}^{11}C_2+\ldots{}^{11}C_{10}\right)A+I$

$=(2^{11}-1)A+I$

= 2047 A + I

Sum of diagonal element = 2047(1 + 4– 3) +3

= 4097

78. The number of real roots of the equation $\sqrt{x^2-4x+3}+\sqrt{x^2-9}=\sqrt{4x^2-14x+6}$, is :

(1) 3 (2) 1 (3) 2 (4) 0

Sol. **(2)**

$$\sqrt{x^2-4x+3}+\sqrt{x^2-9}=\sqrt{4x^2-14x+6}$$

$$\sqrt{(x-3)(x-1)}+\sqrt{(x-3)(x+3)}=\sqrt{4x^2-12x-2x+6}$$

$$\Rightarrow\sqrt{x-3}\left(\sqrt{x-1}+\sqrt{x+3}\right)=\sqrt{4x(x-3)-2(x-3)}$$

$$\Rightarrow\sqrt{x-3}\left(\sqrt{x-1}+\sqrt{x+3}\right)=\sqrt{(4x-2)(x-3)}$$

$$\Rightarrow\sqrt{x-3}\left(\sqrt{x-1}+\sqrt{x+3}-\sqrt{2(2x-1)}\right)=0$$

$$\sqrt{x-3}=0 \text{ or } \sqrt{x-1}+\sqrt{x+3}-\sqrt{2(2x-1)}=0$$

$$x=3 \text{ or } \sqrt{x-1}+\sqrt{x+3}=\sqrt{2(2x-1)}$$

$$x-1+x+3+2\sqrt{(x-1)(x+3)}=4x-2$$

$$\Rightarrow 2\sqrt{(x-1)(x+3)}=2x-4$$

$$\Rightarrow(x-1)(x+3)=(x-2)^2$$

$$\Rightarrow x^2+2x-3=x^2+4-4x$$

$$\Rightarrow 6x=7$$

$$x=\frac{7}{6} \text{ (not possible)}$$

Number of real root = 1

79. If $\sin^{-1}\frac{\alpha}{17}+\cos^{-1}\frac{4}{5}-\tan^{-1}\frac{77}{36}=0, 0<\alpha<13$, then $\sin^{-1}(\sin\alpha)+\cos^{-1}(\cos\alpha)$ is equal to

(1) 16 (2) 0 (3) π (4) $16-5\pi$

Sol. **(3)**

$$\sin^{-1}\frac{\alpha}{17}+\cos^{-1}\frac{4}{5}-\tan^{-1}\frac{77}{36}=0,\ 0<\alpha<13$$

$$\sin^{-1}\frac{\alpha}{17}=\tan^{-1}\frac{77}{36}-\tan^{-1}\frac{3}{4}$$

$$=\tan^{-1}\left(\frac{\frac{77}{36}-\frac{3}{4}}{1+\frac{77}{36}\cdot\frac{3}{4}}\right)$$

$$\sin^{-1}\frac{\alpha}{17} = \tan^{-1}\left(\frac{8}{15}\right) = \sin^{-1}\left(\frac{8}{17}\right)$$

$$\frac{\alpha}{17} = \frac{8}{17}$$

$\alpha = 8$

$\sin^{-1}(\sin 8) + \cos^{-1}(\cos 8)$

$= 3\pi - 8 + 8 - 2\pi$

$= \pi$

80. Let $\alpha \in (0,1)$ and $\beta = \log_e(1-\alpha)$. Let $P_n(x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots + \frac{x^n}{n}, x \in (0,1)$.

Then the integral $\int_0^\alpha \frac{t^{50}}{1-t} dt$ is equal to

(1) $\beta + P_{50}(\alpha)$ (2) $P_{50}(\alpha) - \beta$ (3) $\beta - P_{50}(\alpha)$ (4) $-(\beta + P_{50}(\alpha))$

Sol. **4**

$\alpha \in (0,1), \beta = \log_e(1-\alpha)$

$P_n(x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \ldots\ldots + \frac{x^n}{n}$, $x \in (0, 1)$

$$\int_0^\alpha \frac{t^{50} - 1 + 1}{1-t} dt$$

$$-\int_0^\alpha \frac{1-t^{50}}{1-t} dt + \int_0^\alpha \frac{1}{1-t} dt$$

$$-\int_0^\alpha \left(1 + t + t^2 + \ldots\ldots t^{49}\right) dt - \left[\ln(1-t)\right]_0^\alpha$$

$$-\left[t + \frac{t^2}{2} + \frac{t^3}{3} + \ldots\ldots \frac{t^{50}}{50}\right]_0^\alpha - \ln(1-\alpha)$$

$$-\left[\alpha + \frac{\alpha^2}{2} + \frac{\alpha^3}{3} + \ldots\ldots + \frac{\alpha^{50}}{50}\right] - \ln(1-\alpha)$$

$- P_{50}(\alpha) - \ln(1-\alpha)$

$- (\beta + P_{50}(\alpha))$

Section : Mathematics Section B

81. Let $\alpha > 0$, be the smallest number such that the expansion of $\left(x^{\frac{2}{3}} + \frac{2}{x^3}\right)^{30}$ has a term

$\beta x^{-\alpha}, \beta \in \mathbb{N}$. Then α is equal to

Sol. 2

$$T_{r+1} = {}^{30}C_r\left(x^{\frac{2}{3}}\right)^{30-r}\left(\frac{2}{x^3}\right)^4$$

$$= {}^{30}C_r 2^r x^{\frac{60-11r}{3}}$$

$$\frac{60-11r}{3} < 0$$

$11\ r > 60$

$r > \frac{60}{11}$

$r = 6$

$T_7 = {}^{30}C_6\ 2^6 x^{-2}$ then

$\beta = {}^{30}C_6 \times 2^6 \in N$

$\alpha = 2$

82. Let for $x \in \mathbb{R}$,

$$f(x) = \frac{x+|x|}{2} \text{ and } g(x) = \begin{cases} x, & x < 0 \\ x^2, & x \geq 0 \end{cases}.$$

Then area bounded by the curve $y = (f \circ g)(x)$ and the lines $y = 0, 2y - x = 15$ is equal to

Sol. 72

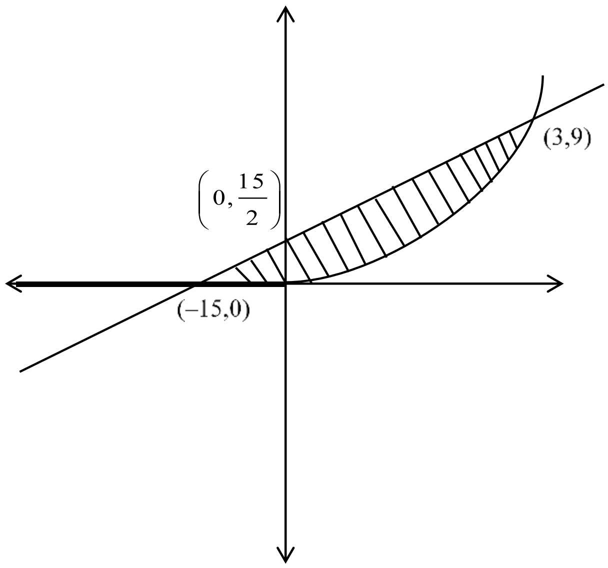
$$f(x) = \frac{x+|x|}{2} = \begin{cases} x & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$g(x) = \begin{cases} x^2 & x \geq 0 \\ x & x < 0 \end{cases}$$

$$Fog(x) = f\{g(x)\} = \begin{cases} g(x) & g(x) \geq 0 \\ 0 & g(x) < 0 \end{cases}$$

$$fog(x) = \begin{cases} x^2 & x \geq 0 \\ 0 & x < 0 \end{cases}$$

given lines are $2y - x = 15$ and $y = 0$



Area = $\int_0^3 \left(\frac{x+15}{2} - x^2\right)dx + \frac{1}{2}\times\frac{15}{2}\times 15$

$= \left.\frac{x^2}{4} + \frac{15x}{2} - \frac{x^3}{3}\right]_0^3 + \frac{225}{4}$

$= \frac{9}{4} + \frac{45}{2} - 9 + \frac{225}{4}$

Area = 72

83. Number of 4-digit numbers that are less than or equal to 2800 and either divisible by 3 or by 11 , is equal to

Sol. **710**

4 digit number which are less then 2800 are 1000 – 2799

Number which are divisible by 3

$2799 = 1002 + (n - 1)\,3$

$n = 600$

Number which are divisible by 11 in 1000 – 2799

= (Number which are divisible by 11 in 1 – 2799)

– (Number which are divisible by 11 in 1 – 999)

$= \left[\frac{2799}{11}\right] - \left[\frac{999}{11}\right]$

= 254 – 90

= 164

Number which are divisible by 33 in 1000 – 2799

= (Number which are divisible by 33 in 1 – 2799) – (Number which are divisible by 33 in 1 – 999)

$= \left[\frac{2799}{33}\right] - \left[\frac{999}{33}\right]$

= 84 – 30 = 54

total number = n(3) + n(11) – n(33)

= 600 + 164 – 54 = 710

84. If the variance of the frequency distribution

x_i	2	3	4	5	6	7	8
Frequency f_i	3	6	16	α	9	5	6

is 3 , then α is equal to

Sol. **5**

x_i	f_i	$d_i = x_i - 5$	$(f_i\,d_i)^2$	f_id_i
2	3	–3	27	–9
3	6	–2	24	–12
4	16	–1	16	–16
5	α	0	0	0
6	9	1	9	9
7	5	2	20	10
8	6	3	54	18

$$\sigma^2 = \frac{\sum f_i d_i^2}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i}\right)^2$$

$$= \frac{150}{45+\alpha} - 0 = 3$$

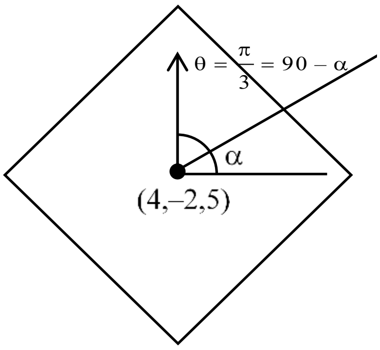
$$\Rightarrow 150 = 135 + 3\alpha$$

$$\Rightarrow 3\alpha = 15$$

$$\Rightarrow \alpha = 5$$

85. Let θ be the angle between the planes $P_1 : \vec{r}\cdot(\hat{\imath} + \hat{\jmath} + 2\hat{k}) = 9$ and $P_2 : \vec{r}\cdot(2\hat{\imath} - \hat{\jmath} + \hat{k}) = 15$. Let L be the line that meets P_2 at the point $(4, -2, 5)$ and makes an angle θ with the normal of P_2. If α is the angle between L and P_2, then $(\tan^2\theta)(\cot^2\alpha)$ is equal to

Sol. **9**



$$\cos\theta = \frac{\langle 1, 1, 2\rangle . \langle 2, -1, 1\rangle}{6}$$

$$= \frac{2-1+2}{6} = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

then $\frac{\pi}{2} - \alpha = \frac{\pi}{3}$

$$\alpha = \frac{\pi}{6}$$

$$\left(\tan^2\theta\right)\left(\cot^2\alpha\right) = \left(\sqrt{3}\right)^2 \times \left(\sqrt{3}\right)^2 = 9$$

86. Let 5 digit numbers be constructed using the digits 0,2,3,4,7,9 with repetition allowed, and are arranged in ascending order with serial numbers. Then the serial number of the number 42923 is

Sol. **2997**

2	$\underset{6}{-}$	$\underset{6}{-}$	$\underset{6}{-}$	$\underset{6}{-} = 1296$
3	$\underset{6}{-}$	$\underset{6}{-}$	$\underset{6}{-}$	$\underset{6}{-} = 1296$
4	0	$\underset{6}{-}$	$\underset{6}{-}$	$\underset{6}{-} =216$
4	2	0	$\underset{6}{-}$	$\underset{6}{-} = 36$
4	2	2	$\underset{6}{-}$	$\underset{6}{-} = 36$
4	2	3	$\underset{6}{-}$	$\underset{6}{-} = 36$
4	2	4	$\underset{6}{-}$	$\underset{6}{-} = 36$
4	2	7	$\underset{6}{-}$	$\underset{6}{-} = 36$
4	2	9	0	$\underset{6}{-} = 6$
4	2	9	2	$\underline{0} = 1$
4	2	9	2	$\underline{2} = 1$
4	2	9	2	$\underline{3} = \frac{1}{2997}$

87. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}| = \sqrt{14}, |\vec{b}| = \sqrt{6}$ and $|\vec{a} \times \vec{b}| = \sqrt{48}$.

Then $(\vec{a} \cdot \vec{b})^2$ is equal to

Sol. **36**

$$\left|\vec{a} \times \vec{b}\right|^2 = \left|\vec{a}\right|^2 \left|\vec{b}\right|^2 - \left(\vec{a} \cdot \vec{b}\right)^2$$

$$48 = 14 \times 6 - \left(\vec{a} \cdot \vec{b}\right)^2$$

$$\left(\vec{a} \cdot \vec{b}\right)^2 = 84 - 48$$

$$\left(\vec{a} \cdot \vec{b}\right)^2 = 36$$

88. Let the line $L: \frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{1}$ intersect the plane $2x + y + 3z = 16$ at the point P. Let the point Q be the foot of perpendicular from the point $R(1,-1,-3)$ on the line L. If α is the area of triangle PQR, then α^2 is equal to

Sol. **180**

Point on line L is $(2\lambda + 1, -\lambda - 1, \lambda + 3)$

If above point is intersection point of line L and plane then

$2(2\lambda + 1) + (-\lambda - 1) + 3(\lambda + 3) = 16$

$\lambda = 1$

Point P = (3, –2, 4)

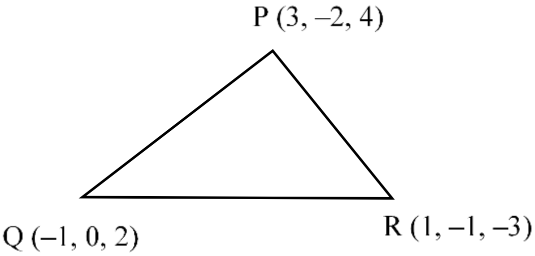
Dr of QR = $< 2\lambda, -\lambda, \lambda + 6 >$

Dr of L = $< 2, -1, 1 >$

$4\lambda + \lambda + \lambda + 6 = 0$

$\lambda = -1$

Q = (–1, 0, 2)



$\overrightarrow{QR} = 2\hat{i} - \hat{j} - 5\hat{k}$

$\overrightarrow{QP} = 4\hat{i} - 2\hat{j} + 2\hat{k}$

$$\overrightarrow{QR} \times \overrightarrow{QP} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & -5 \\ 4 & -2 & 2 \end{vmatrix} = -12\hat{i} - 24\hat{j}$$

$$\alpha = \frac{1}{2} \times \sqrt{144 + 576}$$

$$\alpha^2 = \frac{720}{4} = 180$$

$$\alpha^2 = 180$$

89. Let $a_1, a_2, \dots, a_n$ be in A.P. If $a_5 = 2a_7$ and $a_{11} = 18$, then $12\left(\frac{1}{\sqrt{a_{10}}+\sqrt{a_{11}}} + \frac{1}{\sqrt{a_{11}}+\sqrt{a_{12}}} + \cdots + \frac{1}{\sqrt{a_{17}}+\sqrt{a_{18}}}\right)$ is equal to

Sol. **8**

Given that

$a_5 = 2a_7$

$a_1 + 4d = 2(a_1 + 6d)$

$a_1 + 8d = 0$

$a_1 + 10d = 18$

$a_1 = -72, d = 9$

$a_{18} = a_1 + 17d = -72 + 153 = 81$

$a_{10} = a_1 + 9d = 9$

$$12\left(\frac{\sqrt{a_{11}}-\sqrt{a_{10}}}{d}+\frac{\sqrt{a_{12}}-\sqrt{a_{11}}}{d}+\ldots\ldots\frac{\sqrt{a_{18}}-\sqrt{a_{17}}}{d}\right)$$

$$= 12\left(\frac{\sqrt{a_{18}}-\sqrt{a_{10}}}{d}\right)$$

$$= \frac{12\times(9-3)}{9} = 8$$

90. The remainder on dividing 5^{99} by 11 is :

Sol. **9**

$5^{99} = 5^4\ 5^{95}$

$= 625\ (5^5)^{19}$

$= 625\ (3125)^{19}$

$= 625(3124 + 1)^{19}$

$= 625(11\lambda + 1)$

$= 11\ \lambda \times 625 + 625$

$= 11\ \lambda \times 625 + 616 + 9$

$= 11 \times k + 9$

Remainder = 9

JEE–MAIN EXAMINATION – JANUARY, 2023	
(Held On Thursday 31st January, 2023)	TIME : 3 : 00 PM to 6 : 00 PM

Physics

SECTION - A

1. Given below are two statements:

Statement I: In a typical transistor, all three regions emitter, base and collector have same doping level.

Statement II: In a transistor, collector is the thickest and base is the thinnest segment.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Both Statement I and Statement II are correct
(2) Statement I is incorrect but Statement II is correct
(3) Statement I is correct but Statement II is incorrect
(4) Both Statement I and Statement II are incorrect

Sol. **(2)**

Emitter	Base	Collector
Moderate Size	Thin	Thick
Maximum Doping	Minimum Doping	Moderate Doping

2. If the two metals *A* and *B* are exposed to radiation of wavelength 350 nm. The work functions of metals A and B are 4.8eV and 2.2eV. Then choose the correct option.

(1) Both metals A and B will emit photo-electrons
(2) Metal *A* will not emit photo-electrons
(3) Metal B will not emit photo-electrons
(4) Both metals A and B will not emit photo-electrons

Sol. **(2)**

$$E = \frac{hc}{\lambda} = \frac{1240}{350} = 3.54 eV$$

If $E > \phi$, photo electrons will emit.
A will not emit and B will emit.

3. Heat energy of 735 J is given to a diatomic gas allowing the gas to expand at constant pressure. Each gas molecule rotates around an internal axis but do not oscillate. The increase in the internal energy of the gas will be :

(1) 525 J (2) 441 J (3) 572 J (4) 735 J

Sol. **(1)**

At constant Pressure,
$Q = nC_pdT = 735J$

$$\Delta U = nC_vdT = \frac{735}{\left(C_p/C_v\right)} = \frac{735}{8}$$

$$\Delta U = \frac{735}{\left(7/5\right)} = 525J$$

4. Match List I with List II

LIST I		LIST II	
A.	Angular momentum	I.	$[ML^2\ T^{-2}]$
B.	Torque	II.	$[ML^{-2}\ T^{-2}]$
C.	Stress	III.	$[ML^2\ T^{-1}]$
D.	Pressure gradient	IV.	$[ML^{-1}\ T^{-2}]$

Choose the correct answer from the options given below:

(1) A - III, B - I, C - IV, D – II
(2) A - II, B - III, C - IV, D – I
(3) A - IV, B - II, C - I, D – III
(4) A - I, B - IV, C - III, D – II

Sol. **(1)**

$L = mvr = [M^1L^2T^{-1}]$

$\tau = rF = [M^1L^2T^{-2}]$

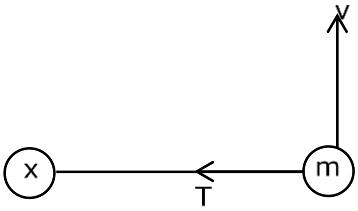
$\text{Stress} = \dfrac{F}{A} = [M^1L^{-1}T^{-2}]$

$\text{Pressure Gradient} = \dfrac{dp}{dx} = [M^1L^{-2}T^{-2}]$

5. A stone of mass 1 kg is tied to end of a massless string of length 1 m. If the breaking tension of the string is 400 N, then maximum linear velocity, the stone can have without breaking the string, while rotating in horizontal plane, is :

(1) 40 ms^{-1}
(2) 400 ms^{-1}
(3) 20 ms^{-1}
(4) 10 ms^{-1}

Sol. **(3)**



$T = \dfrac{mv^2}{\ell}$

$400 = \dfrac{1 \times v^2}{1}$

$V = 20\ m/s$

6. A microscope is focused on an object at the bottom of a bucket. If liquid with refractive index $\frac{5}{3}$ is poured inside the bucket, then microscope have to be raised by 30 cm to focus the object again. The height of the liquid in the bucket is :

(1) 12 cm
(2) 50 cm
(3) 18 cm
(4) 75 cm

Sol. **(4)**

$d_{app} = \dfrac{d}{\mu} = \dfrac{h}{\left(5/3\right)}$

$$\text{Shift} = h\ \frac{-3h}{5} = 30$$

h = 75 cm

7. The number of turns of the coil of a moving coil galvanometer is increased in order to increase current sensitivity by 50%. The percentage change in voltage sensitivity of the galvanometer will be :

(1) 0% (2) 75% (3) 50% (4) 100%

Sol. **(1)**

$$\alpha_V = \frac{NAB}{KR} \alpha \frac{N}{R}$$

$$\alpha_I = \frac{NAB}{K} \alpha\ N$$

$$N\uparrow, \alpha_I\uparrow, \frac{N}{R} \rightarrow \text{Constant}$$

$$\Delta\alpha_V = 0$$

8. A body is moving with constant speed, in a circle of radius 10 m. The body completes one revolution in 4s. At the end of 3rd second, the displacement of body (in m) from its starting point is:

(1) 15π (2) $10\sqrt{2}$ (3) 30 (4) 5π

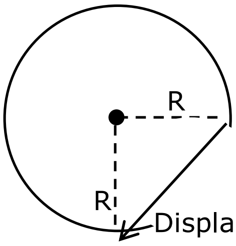
Sol. **(2)**

$$w = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}\ \text{rad}/s$$

$$\theta = wt$$

$$\theta = \frac{\pi}{2} \times 3$$

$$\theta = \frac{3\pi}{2}\ \text{rad}$$



Displacement = $\sqrt{2}R = 10\sqrt{2}m$

9. The H amount of thermal energy is developed by a resistor in 10 s when a current of 4 A is passed through it. If the current is increased to 16 A, the thermal energy developed by the resistor in 10 s will be :

(1) $\frac{H}{4}$ (2) 16H (3) 4H (4) H

Sol. **(2)**

$$H = I^2Rt$$

$$\frac{H_1}{H_2} = \left(\frac{I_1}{I_2}\right)^2 = \left(\frac{4}{16}\right)^2$$

$$H_2 = 16H_1$$

10. A long conducting wire having a current I flowing through it, is bent into a circular coil of N turns. Then it is bent into a circular coil of n turns. The magnetic field is calculated at the centre of coils in both the cases. The ratio of the magnetic field in first case to that of second case is:

(1) n: N (2) n^2: N^2 (3) N^2: n^2 (4) N: n

Sol. **(3)**

Length Remains Same.

$$\ell = N(2\pi r_1) = n(2\pi r_2)$$

$$\frac{B_1}{B_2} = \frac{\left(N\frac{\mu_0 I}{2r_1}\right)}{\left(n\frac{\mu_0 I}{2r_2}\right)} = \frac{N}{n}\left(\frac{r_2}{r_1}\right) = \frac{N}{n}\left(\frac{N}{n}\right)$$

$$\frac{B_1}{B_2} = \left(\frac{N}{n}\right)^2$$

11. A body weight W, is projected vertically upwards from earth's surface to reach a height above the earth which is equal to nine times the radius of earth. The weight of the body at that height will be :

(1) $\frac{W}{100}$ (2) $\frac{W}{91}$ (3) $\frac{W}{3}$ (4) $\frac{W}{9}$

Sol. **(1)**

$$g_h = \frac{g}{\left(1+\frac{h}{R}\right)^2}$$

h = 9R

$$g_h = \frac{g}{(1+9)^2} = \frac{g}{100}$$

$$w_h = \frac{mg}{100} = \frac{w}{100}$$

12. The radius of electron's second stationary orbit in Bohr's atom is R. The radius of 3rd orbit will be

(1) $\frac{R}{3}$ (2) 3R (3) 2.25R (4) 9R

Sol. **(3)**

$$R \alpha \frac{n^2}{z}$$

$$\frac{R_1}{R_2} = \left(\frac{n_1}{n_2}\right)^2 = \left(\frac{2}{3}\right)^2$$

$$R_2 = \frac{9R}{4} = 2.25R$$

13. A hypothetical gas expands adiabatically such that its volume changes from 08 litres to 27 litres. If the ratio of final pressure of the gas to initial pressure of the gas is $\frac{16}{81}$. Then the ratio of $\frac{Cp}{Cv}$ will be.

(1) $\frac{1}{2}$ (2) $\frac{4}{3}$ (3) $\frac{3}{2}$ (4) $\frac{3}{1}$

Sol. **(2)**

For Adiabatic process,

$$P_1V_1^\gamma = P_2V_2^\gamma$$

$$\left(\frac{8}{27}\right)^\gamma = \frac{16}{81}$$

$$\left(\frac{2}{3}\right)^{3\gamma} = \left(\frac{2}{3}\right)^4$$

$$3\gamma = 4$$

$$\gamma = \frac{4}{3} = \frac{Cp}{Cv}$$

14. For a solid rod, the Young's modulus of elasticity is $3.2 \times 10^{11}\text{Nm}^{-2}$ and density is 8×10^3 kg m^{-3}. The velocity of longitudinal wave in the rod will be.

(1) $145.75 \times 10^3\ \text{ms}^{-1}$

(2) $18.96 \times 10^3\ \text{ms}^{-1}$

(3) $3.65 \times 10^3\ \text{ms}^{-1}$

(4) $6.32 \times 10^3\ \text{ms}^{-1}$

Sol. **(4)**

$$V = \sqrt{\frac{Y}{\rho}}$$

$$V = \sqrt{\frac{3.2\times10^{11}}{8\times10^3}} = \sqrt{0.4\times10^8}$$

$$V = \sqrt{40\times10^6}$$

$$V = 6.32\times10^3\ \text{m}/\text{s}$$

15. A body of mass 10 kg is moving with an initial speed of 20 m/s. The body stops after 5 s due to friction between body and the floor. The value of the coefficient of friction is: (Take acceleration due to gravity $g = 10\ \text{ms}^{-2}$)

(1) 0.3 (2) 0.5 (3) 0.2 (4) 0.4

Sol. **(4)**

$$v = u + at$$

$$0 = 20 - \mu g(5)$$

$$\mu = \frac{2}{5} = 0.4$$

16. Given below are two statements :

Statement I : For transmitting a signal, size of antenna (l) should be comparable to wavelength of signal (at least $l = \frac{\lambda}{4}$ in dimension)

Statement II : In amplitude modulation, amplitude of carrier wave remains constant (unchanged).

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Statement **I** is correct but Statement II is incorrect

(2) Both Statement I and Statement II are correct

(3) Statement I is incorrect but Statement II is correct

(4) Both Statement I and Statement II are incorrect

Sol. **(1)**

Statement –1 is correct.

In Modulation Amplitude of carrier wave is increased.

17. An alternating voltage source V = 260sin(628t) is connected across a pure inductor of 5mH. Inductive reactance in the circuit is :

(1) 0.318Ω (2) 6.28Ω (3) 3.14Ω (4) 0.5Ω

Sol. **(3)**

$\omega = 628 \text{ rad/s}$

$X_L = \omega L = 628 \times 5 \times 10^{-3}$

$X_L = 3.14\Omega$

18. Under the same load, wire A having length 5.0 m and cross section 2.5×10^{-5} m^2 stretches uniformly by the same amount as another wire B of length 6.0 m and a cross section of 3.0×10^{-5} m^2 stretches. The ratio of the Young's modulus of wire A to that of wire B will be :

(1) 1: 1 (2) 1: 10 (3) 1: 2 (4) 1: 4

Sol. **(1)**

By Hooke's Law,

$$Y = \frac{FL}{A\Delta L}$$

F, $\Delta L \rightarrow$ Same

$$\frac{Y_1A_1}{L_1} = \frac{Y_2A_2}{L_2}$$

$$\frac{Y_1}{Y_2} = \frac{3\times10^{-5}}{2.5\times10^{-5}} \times \frac{5}{6} = \frac{1}{1}$$

19. Match List I with List II

LIST I		LIST II	
A.	Microwaves	I.	Physiotherapy
B.	UV rays	II.	Treatment of cancer
C.	Infra-red light	III.	Lasik eye surgery
D.	X-ray	IV.	Aircraft navigation

Choose the correct answer from the options given below:

(1) A – IV, B - III, C - I, D – II (2) A – IV, B – I, C - II, D – III

(3) A - III, B - II, C - I, D – IV (4) A - II, B - IV, C - III, D – I

Sol. **(1)**

Theoritical

20. Considering a group of positive charges, which of the following statements is correct?

(1) Both the net potential and the net electric field cannot be zero at a point.

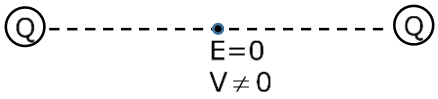
(2) Net potential of the system at a point can be zero but net electric field can't be zero at that point.

(3) Net potential of the system cannot be zero at a point but net electric field can be zero at that point.

(4) Both the net potential and the net field can be zero at a point.

Sol. **(3)**

Electric field is a Vector Quantity.
Electric Potential is a Scalar Quantity.
Eg.



SECTION - B

21. A series LCR circuit consists of $R = 80\Omega, X_L = 100\Omega$, and $X_C = 40\Omega$. The input voltage is 2500 $\cos(100\pi t)$V. The amplitude of current, in the circuit, is ______A.

Sol. **(25)**

$R = 80\Omega$, $X_L = 100\ \Omega$, $X_c = 40\ \Omega$

$$Z = \sqrt{R^2 + (x_L - X_C)^2}$$

$$Z = \sqrt{80^2 + 60^2} = 100\Omega$$

$$I_0 = \frac{V_0}{Z} = \frac{2500}{100} = 25A$$

22. Two bodies are projected from ground with same speeds 40 ms^{-1} at two different angles with respect to horizontal. The bodies were found to have same range. If one of the body was projected at an angle of 60°, with horizontal then sum of the maximum heights, attained by the two projectiles, is ______m. (Given $g = 10\ ms^{-2}$)

Sol. **(80)**

In Range is same.

$$\alpha + \beta = 90^\circ$$

$$\alpha = 60^\circ$$

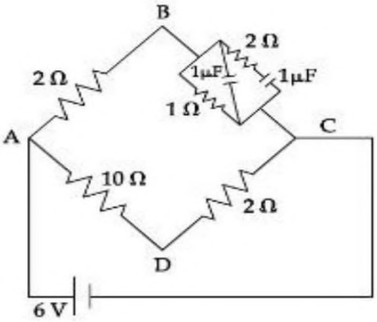
$$\beta = 30^\circ$$

$$H_1 + H_2 = \frac{u_1^2 \sin^2 60^\circ}{2g} + \frac{u_2^2 \sin^2 30^\circ}{2g}$$

$$= \frac{u^2}{2g}\left(\frac{3}{4} + \frac{1}{4}\right) \qquad [u_1 = u_2]$$

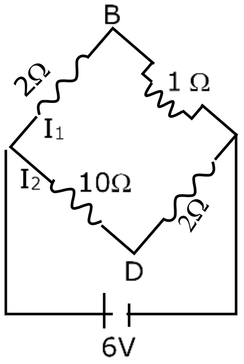
$$H_1 + H_2 = \frac{(40)^2}{20} = 80m$$

23. For the given circuit, in the steady state, $|V_B - V_D| =$ ______V.



Sol. **(1)**

At steady state, C → open Circuit



$$I_1 = \frac{6}{3} = 2A$$

$$I_2 = \frac{6}{12} = \frac{1}{2}A$$

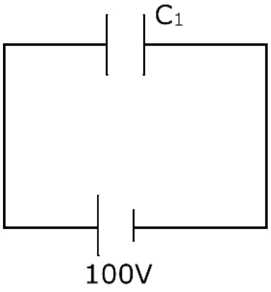
$$V_B + 2I_1 - 10I_2 = V_D$$

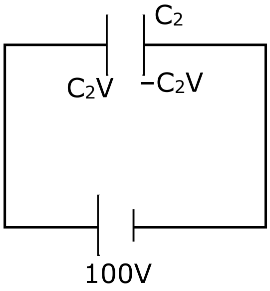
$$V_B - V_D = 5 - 4 = 1V$$

24. Two parallel plate capacitors C_1 and C_2 each having capacitance of 10μF are individually charged by a 100 V D.C. source. Capacitor C_1 is kept connected to the source and a dielectric slab is inserted between it plates. Capacitor C_2 is disconnected from the source and then a dielectric slab is inserted in it. Afterwards the capacitor C_1 is also disconnected from the source and the two capacitors are finally connected in parallel combination. The common potential of the combination will be _____V. (Assuming Dielectric constant = 10)

Sol. **(55)**

1.





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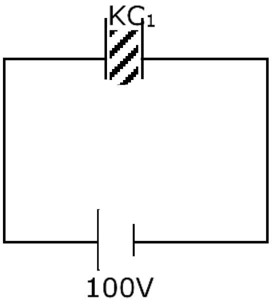

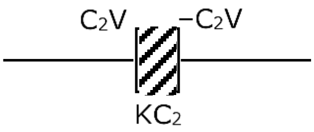

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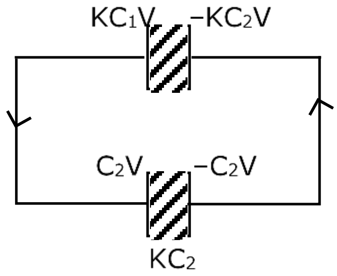


By charge conservation

$Q_1 = Q_2$

$KC_1V + C_2V = (KC_1+KC_2)\ V_{common}$

$$V_{common} = \frac{(K+1)CV}{2KC} = \frac{K+1}{2K}V$$

$$V_{common} = \frac{11}{20}\times 100 = 55V$$

25. Two light waves of wavelengths 800 and 600 nm are used in Young's double slit experiment to obtain interference fringes on a screen placed 7 m away from plane of slits. If the two slits are separated by 0.35 mm, then shortest distance from the central bright maximum to the point where the bright fringes of the two wavelength coincide will be _______ mm.

Sol. **(48)**

$d = 0.35$ mm, $D = 7$m

To Coincide, $$n_1\left(\frac{\lambda_1 D}{d}\right) = n_2\left(\frac{\lambda_2 D}{d}\right)$$

$$\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{6}{8} = \frac{3}{4}$$

3rd Maxima of λ_1 and 4th Maxima of λ_2 will coincide.

$$Y = \frac{3\lambda_1 D}{d} = \frac{3\times 800\times 10^{-9}\times 7}{35\times 10^{-5}}$$

$Y = 3 \times 160 \times 10^{-4}$ m

$Y = 48$mm

26. A ball is dropped from a height of 20 m. If the coefficient of restitution for the collision between ball and floor is 0.5, after hitting the floor, the ball rebounds to a height of ______ m

Sol. **(5)**

$$V = \sqrt{2g(20)}$$

$$eV = \sqrt{2gh}$$

$$\frac{1}{e} = \sqrt{\frac{20}{h}}$$

$$h = 20e^2 = 20\left(\frac{1}{2}\right)^2$$

$h = 5$m

27. If the binding energy of ground state electron in a hydrogen atom is 13.6eV, then, the energy required to remove the electron from the second excited state of Li^{2+} will be : $x \times 10^{-1}$eV. The value of x is____.

Sol. (136)

$$BE = 13.6 \times \frac{z^2}{n^2}$$

$$BE = 13.6 \times \left(\frac{3}{3}\right)^2 = 13.6eV$$

$$BE = 136 \times 10^{-1}\ eV$$

$$x = 136$$

28. A water heater of power 2000 W is used to heat water. The specific heat capacity of water is 4200 J $\mathrm{kg}^{-1}\ \mathrm{K}^{-1}$. The efficiency of heater is 70%. Time required to heat 2 kg of water from 10°C to 60°C is ______s.

(Assume that the specific heat capacity of water remains constant over the temperature range of the water).

Sol. (300)

$$P_{used} = 0.7 \times 2000 = 1400W$$

$$P = \frac{ms\Delta T}{t}$$

$$t = \frac{2 \times 4200 \times 50}{1400}$$

$$t = 300\ sec$$

29. Two discs of same mass and different radii are made of different materials such that their thicknesses are 1 cm and 0.5 cm respectively. The densities of materials are in the ratio 3: 5. The moment of inertia of these discs respectively about their diameters will be in the ratio of $\frac{x}{6}$. The value of x is _______.

Sol. (5)

$$M_1 = M_2$$

$$S_1\left(\pi R_1^2 t_1\right) = S_2\left(\pi R_2^2 t_2\right)$$

$$\frac{R_1^2}{R_2^2} = \frac{5}{3} \times \frac{0.5}{1} = \frac{5}{6}$$

$$I = \frac{MR^2}{4}$$

$$\frac{I_1}{I_2} = \left(\frac{R_1}{R_2}\right)^2 = \frac{5}{6}$$

30. The displacement equations of two interfering waves are given by $y_1 = 10\sin\left(\omega t + \frac{\pi}{3}\right)$ cm, $y_2 = 5[\sin\omega t + \sqrt{3}\cos\omega t]$cm respectively. The amplitude of the resultant wave is ______ cm.

Sol. (20)

$$y_1 = 10\sin\left(\omega t + \frac{\pi}{3}\right)$$

$$y_2 = 10\left[\sin\omega t \times \frac{1}{2} + \frac{\sqrt{3}}{2}\cos\omega t\right]$$

$$y_2 = 10\sin\left(\omega t + \frac{\pi}{3}\right)$$

y_1 and y_2 are in same phase

$A_r = A_1 + A_2 = 20$ cm

Chemistry

SECTION - A

31. Which one of the following statements is incorrect ?

(1) van Arkel method is used to purify tungsten.

(2) The malleable iron is prepared from cast iron by oxidising impurities in a reverberatory furnace.

(3) Cast iron is obtained by melting pig iron with scrap iron and coke using hot air blast.

(4) Boron and Indium can be purified by zone refining method.

Sol. **1**

Van Arkel method is used for refining of Ti, Zr, Hf, Bi, B

32. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : The first ionization enthalpy of 3 d series elements is more than that of group 2 metals

Reason (R) : In 3d series of elements successive filling of d-orbitals takes place.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(2) Both (A) and (R) are true and (R) is the correct explanation of (A)

(3) (A) is true but (R) is false

(4) (A) is false but (R) is true

Sol. **2**

d-block elements have more first I.E. than group 2 elements due to poor shielding of d-orbitals

33. Given below are two statements :

Statement I : H_2O_2 is used in the synthesis of Cephalosporin

Statement II : H_2O_2 is used for the restoration of aerobic conditions to sewage wastes.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both Statement I and Statement II are incorrect

(2) Statement I is incorrect but Statement II is correct

(3) Statement I is correct but Statement II is incorrect

(4) Both Statement I and Statement II are correct

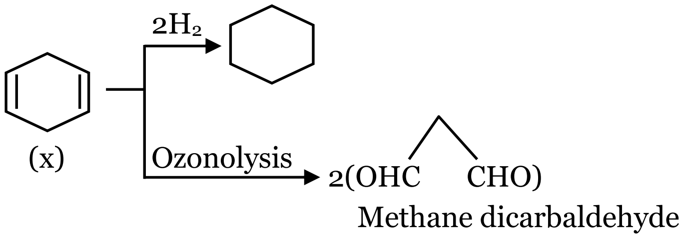
Sol. **4**

Fact (NCERT based)

34. A hydrocarbon 'X' with formula C_6H_8 uses two moles H_2 on catalytic hydrogenation of its one mole. On ozonolysis, 'X' yields two moles of methane dicarbaldehyde. The hydrocarbon 'X' is :

(1) cyclohexa-1, 4-diene
(2) cyclohexa - 1, 3 – diene
(3) 1-methylcyclopenta-1, 4-diene
(4) hexa-1, 3, 5-triene

Sol. **1**



35. Evaluate the following statements for their correctness.

A. The elevation in boiling point temperature of water will be same for 0.1MNaCl and 0.1M urea.

B. Azeotropic mixtures boil without change in their composition.

C. Osmosis always takes place from hypertonic to hypotonic solution.

D. The density of $32\%H_2SO_4$ solution having molarity 4.09M is approximately $1.26\ g\ mL^{-1}$.

E. A negatively charged sol is obtained when KI solution is added to silver nitrate solution.

Choose the correct answer from the options given below :

(1) A, B and D only (2) B and D only

(3) B, D and E only (4) A and C only

Sol. **2**

(A) Value of i is different for both the solutions.

(B) True

(C) Osmosic takes place from hypotonic to hypertonic solution.

(D) $d = \dfrac{100}{\dfrac{1000}{4.09} \times \dfrac{32}{98}} \cong$ 1.26 gm/ml

(E) Positively charged sol will be form.

36. The Lewis acid character of boron tri halides follows the order :

(1) $BI_3 > BBr_3 > BCl_3 > BF_3$ (2) $BBr_3 > BI_3 > BCl_3 > BF_3$

(3) $BCl_3 > BF_3 > BBr_3 > BI_3$ (4) $BF_3 > BCl_3 > BBr_3 > BI_3$

Sol. **1**

Due to back bonding Lewis acidic strength of Boron halides is $BI_3 > BBr_3 > BCl_3 > BF_3$

37. When a hydrocarbon A undergoes complete combustion it requires 11 equivalents of oxygen and produces 4 equivalents of water. What is the molecular formula of A ?

(1) C_5H_8(2) $C_{11}H_4$(3) C_9H_8(4) $C_{11}H_8$

Sol. **3**

$$C_xH_y + \left(x + \frac{y}{4}\right)O_2 \to xCO_2 + \frac{y}{2}H_2O$$

$x + \frac{y}{4} = 11$ $\frac{y}{2} = 4$

x = 9 y = 8 (C_9H_8)

38. Arrange the following orbitals in decreasing order of energy.

A. n = 3, l = 0, m = 0 B. n = 4, l = 0, m = 0

C. n = 3, l = 1, m = 0 D. n = 3, l = 2, m = 1

The correct option for the order is :

(1) D > B > C > A (2)D > B > A > C (3)A > C > B > D (4) B > D > C > A

Sol. **1**

According to (n+l) rule Orbital has more value of (n+l) has more energy. If value of some then orbital has more value of n has more energy

39. The element playing significant role in neuromuscular function and interneuronal transmission is :

(1) Li (2) Mg (3)Be (4) Ca

Sol. **4**

Fact (NCERT based)

40. Given below are two statements :

Statement I : Upon heating a borax bead dipped in cupric sulphate in a luminous flame, the colour of the bead becomes green

Statement II : The green colour observerd is due to the formation of copper(I) metaborate

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both Statement I and Statement II are true
(2) Statement I is true but Statement II is false
(3) Statement I is false but Statement II is true
(4) Both Statement I and Statement II are false

Sol. **4**

Due to formation of Cu (II) met borate it gives blue colour

41. Which of the following compounds are not used as disinfectants ?

A. Chloroxylenol B. Bithional C. Veronal D. Prontosil
E. Terpineol

Choose the correct answer from the options given below :

(1) C, D (2) B, D, E (3)A, B (4) A, B, E

Sol. **1**

* Vernonal is a tranquilizer
* Prontosil is a antibiotic drug.

42. Incorrect statement for the use of indicators in acid-base titration is :

(1) Methyl orange may be used for a weak acid vs weak base titration.
(2) Phenolphthalein is a suitable indicator for a weak acid vs strong base titration.
(3) Methyl orange is a suitable indicator for a strong acid vs weak base titration.
(4) Phenolphthalein may be used for a strong acid vs strong base titration.

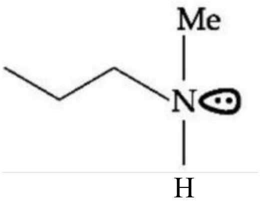
Sol. **1**

Weak acid – weak base :-
Neither phenolphthalein nor methyl orange is suitable.

43. An organic compound [A]($C_4H_{11}N$), shows optical activity and gives N_2 gas on treatment with HNO_2. The compound [A] reacts with $PhSO_2Cl$ producing a compound which is soluble in KOH.

(1)

(2)



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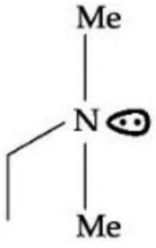

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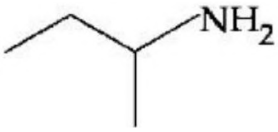
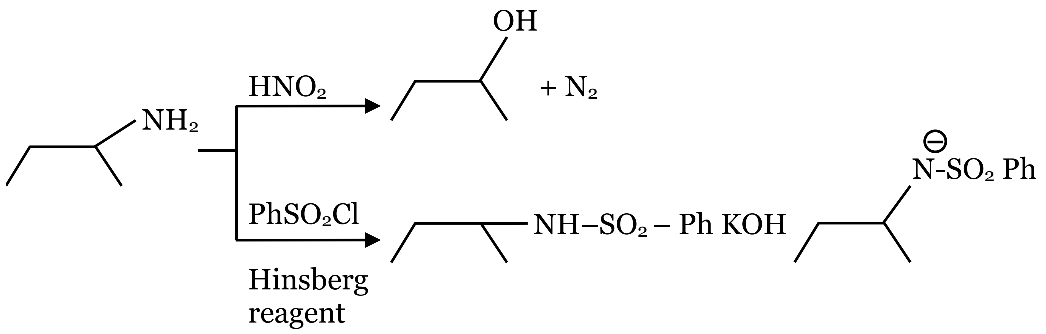


Sol. **4**



44. The normal rain water is slightly acidic and its pH value is 5.6 because of which one of the following?

(1) $CO_2 + H_2O \rightarrow H_2CO_3$
(2) $2SO_2 + O_2 + 2H_2O \rightarrow 2H_2SO_4$
(3) $4NO_2 + O_2 + 2H_2O \rightarrow 4HNO_3$
(4) $N_2O_5 + H_2O \rightarrow 2HNO_3$

Sol. **1**

Due to presence of CO_2 in air normal rain water is slightly acidic

45. Match List I with List II

LIST I		LIST II	
A.	Physisorption	I.	Single Layer Adsorption
B.	Chemisorption	II.	$20-40 \text{ kJ mol}^{-1}$
C.	$N_2(g) + 3H_2(g) \xrightarrow{Fe(s)} 2NH_3(g)$	III.	Chromatography
D.	Analytical Application or Adsorption	IV.	Heterogeneous catalysis

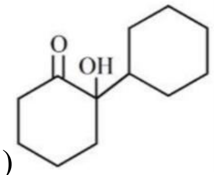
Choose the correct answer from the options given below:

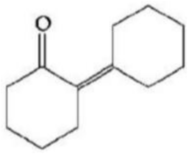
(1) A - II, B - I, C - IV, D – III
(2) A - IV, B - II, C - III, D – I
(3) A - II, B - III, C - I, D – IV
(4) A - III, B - IV, C - I, D – II

Sol. **1**

Theory based

46. Cyclohexylamine when treated with nitrous acid yields (P). On treating (P) with PCC results in (Q). When (Q) is heated with dil. NaOH we get (R) The final product (R) is :

(1)


(2)


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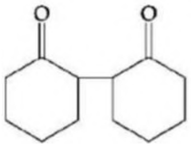
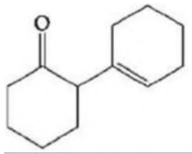
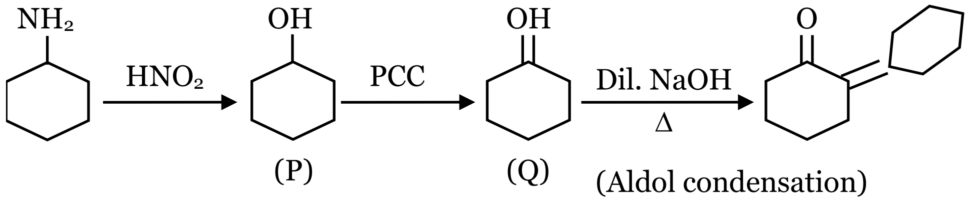


(4)


Sol. **2**

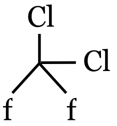


47. In the following halogenated organic compounds the one with maximum number of chlorine atoms in its structure is :

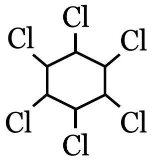
(1) Freon-12 (2) Gammaxene (3) Chloropicrin (4) Chloral

Sol. **2**

1. Freon – 12

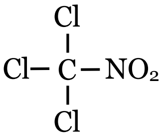


2. Gammaxene



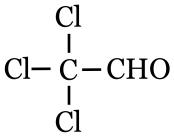
or $C_6H_6Cl_6$

3. Chloropicrin



$$\mathrm{Cl-\overset{\overset{\Large Cl}{|}}{\underset{\underset{\Large Cl}{|}}{C}}-NO_2}$$

4. Chloral



$$\mathrm{Cl-\overset{\overset{\Large Cl}{|}}{\underset{\underset{\Large Cl}{|}}{C}}-CHO}$$

48. In Dumas method for the estimation of N_2, the sample is heated with copper oxide and the gas evolved is passed over :

(1) Copper oxide (2) Ni (3) Pd (4) Copper gauze

Sol. **2**

Duma's method. The nitrogen containing organic compound, when heated with CuO in a atmosphere of CO_2, yields free N_2 in addition to CO_2 and H_2O.

$$C_xH_yN_z + (2x + \frac{y}{2})CuO \rightarrow$$

$$xCO_2 + \frac{y}{2}H_2O + \frac{z}{2}N_2 + (2x + \frac{y}{2})Cu$$

Traces of nitrogen oxides formed, if any, are reduced to nitrogen by passing the gaseous mixture over heated copper gauze.

49. Which of the following elements have half-filled f-orbitals in their ground state ?

(Given : atomic number $Sm = 62; Eu = 63; Tb = 65; Gd = 64, Pm = 61$)

A. Sm B. B. EuC. Tb D. Gd E. Pm

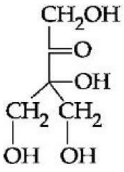
Choose the correct answer from the options given below :

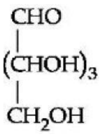
(1) A and B only (2) A and E only (3) C and D only (4) B and D only

Sol. **4**

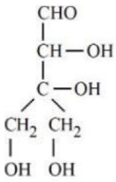
Fact (NCERT based)

50. Compound A, $C_5H_{10}O_5$, given a tetraacetate with AC_2O and oxidation of A with $Br_2 - H_2O$ gives an acid, $C_5H_{10}O_6$. Reduction of A with HI gives isopentane. The possible structure of A is :

(1)


(2)


$CHO-(CHOH)_3-CH_2OH$

(3)


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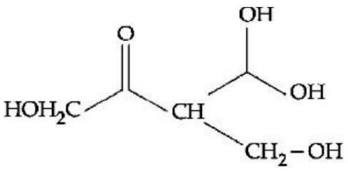
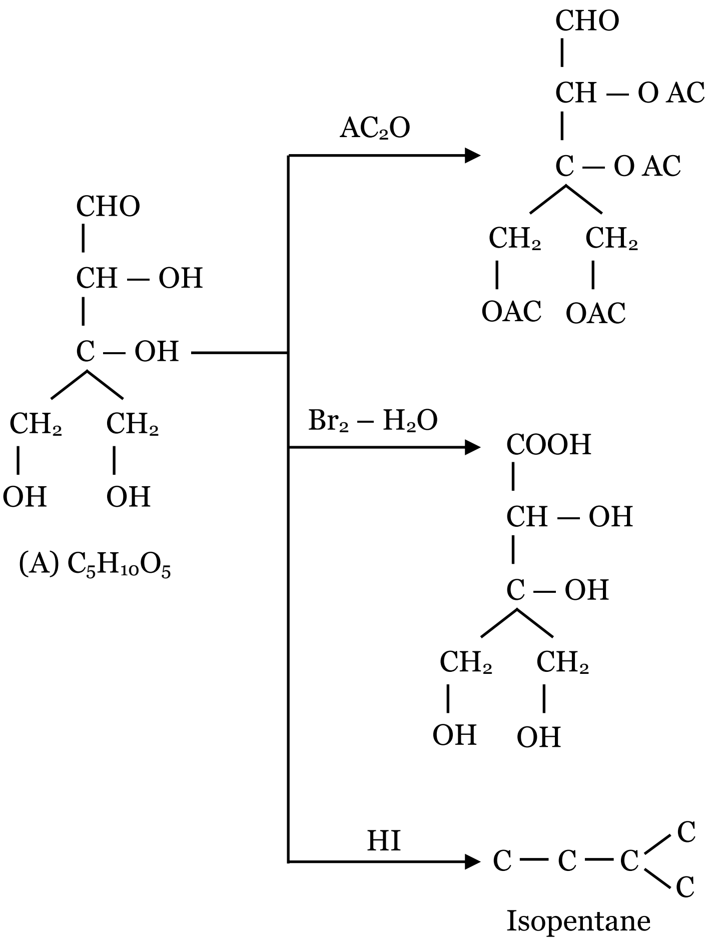


Sol. 3



SECTION B

51. The rate constant for a first order reaction is $20\ \text{min}^{-1}$. The time required for the initial concentration of the reactant to reduce to its $\frac{1}{32}$ level is ______ 10^{-2} min. (Nearest integer)

(Given : ln 10 = 2.303
log 2 = 0.3010)

Sol. **17**

$$t = \frac{1}{20}\ln 32$$

$$= \frac{2.303 \times 5 \times 0.3010}{20} = 17.33 \times 10^{-2}$$

$\approx 17 \times 10^{-2}$

52. Enthalpies of formation of $CCl_4(g), H_2O(g), CO_2(g)$ and $HCl(g)$ are $-105, -242, -394$ and $-92\ \text{kJ mol}^{-1}$ respectively. The magnitude of enthalpy of the reaction given below is kJmol^{-1}. (nearest integer)

$$CCl_4(g) + 2H_2O(g) \rightarrow CO_2(g) + 4HCl(g)$$

Sol. **173**

$$\Delta H_r = (\Delta H_f)CO_2 + (\Delta H_f)_{HCl} - (\Delta H_f)_{CCl_4} - 2(\Delta H_f)H_2O$$

$= -173$

53. A sample of a metal oxide has formula $M_{0.83}O_{1.00}$. The metal M can exist in two oxidation states $+2$ and $+3$. In the sample of $M_{0.83}O_{1.00}$, the percentage of metal ions existing in $+2$ oxidation state is %. (nearest integer)

Sol. **59**

$M^{2+} \rightarrow x \quad M^{3+} \rightarrow (0.83 - x)$

$2x + 3(0.83 - x) = 2$

$x = 2.49 - 2 = 0.49$

% of $M^{2+} = \frac{0.49}{0.83} \times 100 = 59\%$

54. The resistivity of a 0.8M solution of an electrolyte is $5 \times 10^{-3}\ \Omega\text{cm}$. Its molar conductivity is $\times 10^4\ \Omega^{-1}\ \text{cm}^2\ \text{mol}^{-1}$

(Nearest integer)

Sol. **25**

$$K = \frac{1}{5 \times 10^{-3}}$$

$$\wedge_m = K \times \frac{1000}{M} = \frac{1}{5 \times 10^{-3}} \times \frac{1000}{0.8}$$

$$= \frac{1000}{40} \times 10^4 = 25 \times 10^4$$

55. At 298 K, the solubility of silver chloride in water is 1.434×10^{-3} g L^{-1}. The value of $-\log K_{sp}$ for silver chloride is (Given mass of Ag is 107.9 g mol^{-1} and mass of Cl is 35.5 g mol^{-1})

Sol. 10

1.434×10^{-3} gm/L

$$= \frac{1.434 \times 10^{-3}}{107.9 + 35.5} M = 10^{-5} m$$

$Ksp = S^2 = 10^{-10} \Rightarrow -\log Ksp = +10$

56. If the CFSE of $[Ti(H_2O)_6]^{3+}$ is -96.0 kJ/mol, this complex will absorb maximum at wavelength nm. (nearest integer)

Assume Planck's constant $(h) = 6.4 \times 10^{-34}$Js, Speed of light $(c) = 3.0 \times 10^8$ m/s and Avogadro's Constant $(N_A) = 6 \times 10^{23}$/mol

Sol. **480**

$$CFSE = \left(-\frac{2}{5}x + \frac{3}{5}y\right)\Delta_0$$

$$-96 = \frac{-2}{5} \times 1 \times \Delta_0$$

$$\Delta_0 = 240\,kJ / mole = \frac{240 \times 10^3}{NA / molecule}$$

$$\Delta_0 = \frac{hc}{\lambda abs}$$

$$\frac{240 \times 10^3}{6 \times 10^{23}} = \frac{6.4 \times 10^{-34} \times 3 \times 10^8}{\lambda abs}$$

$$\lambda ab = \frac{6.4 \times 3 \times 6 \times 10^{-3}}{240 \times 10^3} m$$

$= 4.8 \times 10^{-7}$ m

$= 4.8 \times 10^{-7} \times 10^9$ nm

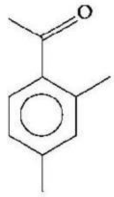
$= 480$ nm

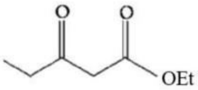
57. The number of alkali metal(s), from Li, K, Cs, Rb having ionization enthalpy greater than 400 kJ mol^{-1} and forming stable super oxide is

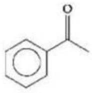
Sol. **2**

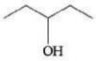
K, Rb and Cs form stable super oxides but Cs has ionisation enthalpy less than 400 kJ.

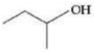
58. The number of molecules which gives haloform test among the following molecules is

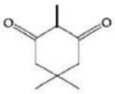












Sol. **3**

59. Assume carbon burns according to following equation :

$2C_{(s)} + O_{2(g)} \rightarrow 2CO(g)$

when 12 g carbon is burnt in 48 g of oxygen, the volume of carbon monoxide produced is $\times 10^{-1}$ L at STP [nearest integer]

[Given: Assume co as ideal gas, Mass of c is 12 g mol^{-1}, Mass of O is 16 g mol^{-1} and molar volume of an ideal gas STP is 22.7 L mol^{-1}]

Sol. **227**

$2C + O_2 \rightarrow 2CO$

12g 48 gm

1 mole 1.5 mole

"C" is LR.

Moles of CO formed = 1

Volume of CO = 1 × 22.7

= 227 × 10^{-1} L

60. Amongst the following, the number of species having the linear shape is

$XeF_2, I_3^+, C_3O_2, I_3^-, CO_2, SO_2, BeCl_2$ and $BCl_2^{\ominus}$

Sol. **5**

XeF_2, I_3^-, C_3O_2, CO_2, $BeCl_2$

Mathematics

Section A

61. The equation $e^{4x} + 8e^{3x} + 13e^{2x} - 8e^{x} + 1 = 0, x \in \mathbb{R}$ has :
(1) four solutions two of which are negative
(2) two solutions and only one of them is negative
(3) two solutions and both are negative
(4) no solution

Sol. **3**

$e^{4x} + 8e^{3x} + 13e^{2x} + 13e^{2x} - 8e^{x} + 1 = 0, x \in R$

Let $e^{x} = t > 0$ & $x = \ln t$

$t^4 + 8t^3 + 13t^2 - 8t + 1 = 0$

Dividing by t^2,

$$t^2 + 8t + 13 - \frac{8}{t} + \frac{1}{t^2} = 0$$

$$t^2 + \frac{1}{t^2} + 8\left(t - \frac{1}{t}\right) + 13 = 0$$

Let $t - \frac{1}{t} = u \Rightarrow t^2 + \frac{1}{t^2} - 2\ u^2$

$\Rightarrow t^2 + \frac{1}{t^2} = u^2 + 2$

$u^2 + 2 + 8u + 13 = 0$

$(u + 3)(u + 5) = 0$

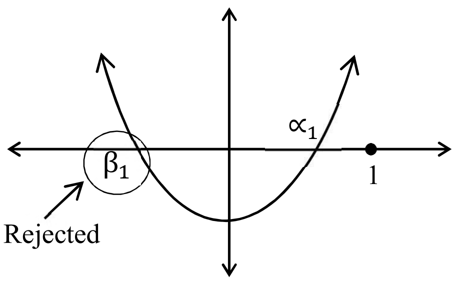
$u = -3$ & $u = -5$

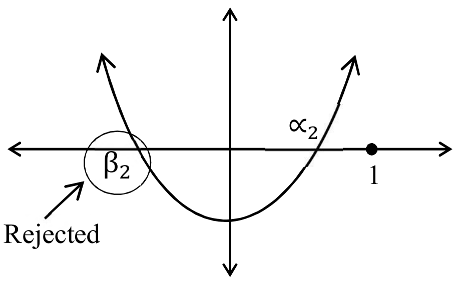
$t - \frac{1}{t} = -3$

$t^2 + 3t - 1 = 0$

$t - \frac{1}{t} = -5$

$t^2 + 5t - 1 = 0$





$0 < \alpha_1 < 1$

$0 < \alpha_2 < 1$

$\Rightarrow x_1 = \ell n\ \alpha_1 < 0$ $\Rightarrow x_2 = \ln \alpha_2 < 0$

62. Among the relations

$S = \left\{(a, b): a, b \in \mathbb{R} - \{0\}, 2 + \frac{a}{b} > 0\right\}$ and $T = \{(a, b): a, b \in \mathbb{R}, a^2 - b^2 \in \mathbb{Z}\}$,

(1) neither S nor T is transitive (2) S is transitive but T is not

(3) T is symmetric but S is not (4) both S and T are symmetric

Sol. **3**

$S = \{(a, b) \mid a, b \in R - \{0\}, 2 + \frac{a}{b} > 0\}$ &

$T = \left\{(a, b) \mid a, b \in R, a^2 - b^2 \in Z\right\}$

For S, $2 + \frac{a}{b} > 0 \Rightarrow \frac{a}{b} > -2$

Let $(-1, 2) \in S\left(\therefore -\frac{1}{2} > -2\right)$

$\&(2, -1) \notin s\left(\therefore \frac{2}{-1} \text{ not greater than } -2\right)$

So, S is not symmetric

For T,

If $(a, b) \in T \Rightarrow a^2 - b^2 \in Z$

$\Rightarrow -\left(a^2 - b^2\right) \in Z$

$\Rightarrow b^2 - a^2 \in Z$

$\Rightarrow (b, a) \in T$

So, T is symmetric

63. Let $\alpha > 0$. If $\int_0^\alpha \frac{x}{\sqrt{x+\alpha} - \sqrt{x}} dx = \frac{16 + 20\sqrt{2}}{15}$, then α is equal to :

(1) 4 (2) $2\sqrt{2}$ (3) $\sqrt{2}$ (4) 2

Sol. **3**

$\alpha > 0$

$$I = \int_0^\alpha \frac{x}{\sqrt{x+2} - \sqrt{x}} dx = \frac{16 + 2\sqrt{2}}{15}$$

$$I = \int_0^\alpha \frac{x\left(\sqrt{x+\alpha} + \sqrt{x}\right)}{\alpha} dx$$

$$= \frac{1}{\alpha}\left[\int_0^\alpha x\sqrt{x+\alpha}\sqrt{x+\alpha}\, dx + \int_0^\alpha x^{3/2} dx\right]$$

$$I_1 = \int_0^{\alpha} (x+\alpha-\alpha)\sqrt{x+\alpha}\,dx$$

$$= \int_0^{\alpha} (x+\alpha)^{3/2} - \alpha\int_0^{\alpha} (x+\alpha)^{1/2} dx$$

$$= \frac{2}{5}\left[(x+\alpha)^{5/2}\right]^{\alpha} - \frac{\alpha(2)}{3}\left[(x+\alpha)^{3/2}\right]_0^{\alpha}$$

$$= \frac{2}{5}\left[(2\alpha)^{5/2} - \alpha^{5/2}\right] - \frac{2\alpha}{3}\left[(2\alpha)^{3/2} - \alpha^{3/2}\right]$$

$$= \frac{2}{5}(2\alpha)^{5/2} - \frac{2}{5}\alpha^{5/2} - \frac{(2\alpha)^{5/2}}{3} + \frac{2\alpha^{5/2}}{3}$$

$$= (2\alpha)^{5/2}\left[\frac{2}{5} - \frac{1}{3}\right] + 2\alpha^{5/2}\left[\frac{1}{3} - \frac{1}{5}\right]$$

$$= (2\alpha)^{5/2}\left[\frac{1}{15}\right] + 2\alpha^{5/2}\left[\frac{2}{15}\right]$$

$$= \frac{(2\alpha)^{5/2}}{15} + \frac{4\alpha^{5/2}}{15}$$

$$= \frac{4\alpha^{5/2}}{15}\left[\sqrt{2}+1\right]$$

$$I_2 = \int_0^{\alpha} x^{3/2} dx = \frac{2}{5}\left[x^{5/2}\right]_0^{\alpha} = \frac{2}{5}\alpha^{5/2}$$

$$I = \frac{1}{\alpha}(I_1 + I_2)$$

$$I = \frac{1}{\alpha}\left[\frac{4\alpha^{5/2}\left(\sqrt{2}+1\right)}{15} + \frac{2}{5}\alpha^{5/2}\right]$$

$$= \frac{2\alpha^{5/2}}{15\alpha}\left[2\left(\sqrt{2}+1\right)+3\right]$$

$$= \frac{2}{15}\alpha^{3/2}\left[2\sqrt{2}+5\right]$$

$$\frac{16+20\sqrt{2}}{15} = \frac{2}{15}\alpha^{3/2}\left[2\sqrt{2}+5\right]$$

$$\alpha^{3/2} = 2\sqrt{2}$$

$$\alpha^3 = 8$$

$$\alpha = 2$$

64. The complex number $z=\frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$ is equal to :

(1) $\sqrt{2}i\left(\cos\frac{5\pi}{12}-i\sin\frac{5\pi}{12}\right)$ (2) $\sqrt{2}\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$

(3) $\sqrt{2}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$ (4) $\cos\frac{\pi}{12}-i\sin\frac{\pi}{12}$

Sol. **3**

$$Z=\frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$$

$$i-1=\sqrt{2}\left(\frac{-1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)=\sqrt{2}\cdot e^{i\cdot\frac{3\pi}{4}}$$

$$z=\frac{\sqrt{2}\cdot e^{i\cdot\frac{3\pi}{4}}}{e^{i\cdot\frac{3\pi}{4}}}$$

$$=\sqrt{2}\cdot e^{i\left(\frac{3\pi}{4}-\frac{\pi}{3}\right)}$$

$$=\sqrt{2}\,e^{\frac{5\pi}{12}i}$$

$$=\sqrt{2}\left(\cos\left(\frac{5\pi}{12}\right)+i\sin\left(\frac{5\pi}{12}\right)\right)$$

65. Let $y=y(x)$ be the solution of the differential equation $(3y^2-5x^2)y\,dx+2x(x^2-y^2)dy=0$ such that $y(1)=1$. Then $|(y(2))^3-12y(2)|$ is equal to :

(1) $16\sqrt{2}$ (2) $32\sqrt{2}$ (3) 32 (4) 64

Sol. **2**

$$\left(3y^2-5x^2\right)ydx+2x\left(x^2-y^2\right)dy=0$$

$$2x\left(x^2-y^2\right)dy=\left(5x^2-3y^2\right)y\,dx$$

$$\Rightarrow\frac{dy}{dx}=\frac{\left(5x^2-3y^2\right)y}{2x\left(x^2-y^2\right)}$$

Let y = tx

$$\frac{dy}{dx}=t+x\frac{dt}{dx}$$

$$t+x\frac{dt}{dx}=\frac{\left(5-3t^2\right)t}{2\left(1-t^2\right)}$$

$$x\frac{dt}{dx}=t\left[\frac{5-3t^2-2+2t^2}{2\left(1-t^2\right)}\right]$$

$$=\frac{t}{2}\left[\frac{3-t^2}{1-t^2}\right]$$

$$\frac{1}{3}\int\frac{3\left(1-t^2\right)dt}{3t-t^3}=\int\frac{dx}{2x}$$

$$\frac{1}{3}\ell n\left|3t-t^3\right|=\frac{1}{2}\ln|x|+c$$

$$2\ell\, n\left|3t-t^3\right|=3\ln|x|+6c$$

$$\left(3t-t^3\right)^2=x^3\lambda$$

$$\left(\frac{3y}{x}-\frac{y^3}{x^3}\right)^2=x^3\lambda$$

$$\left(3y\,x^2-y^3\right)^2=x^9\lambda$$

$$x=1\Rightarrow y=1$$

$$\left(3-1\right)^2=1\times\lambda$$

$$\lambda=4$$

$$\left(3yx^2-y^3\right)^2=4x^9$$

Let x =2

$$\left(3\,y(2)\times4-\left(y(2)^3\right)^2\right)=4(2)^9$$

Taking square root both the sides,

$$\left|\left(y(2)\right)^3-12y(2)\right|=2(2)^{9/2}$$

$$=2(2)^4\sqrt{2}=32\sqrt{2}$$

66. $\lim\limits_{x\to\infty}\frac{(\sqrt{3x+1}+\sqrt{3x-1})^6+(\sqrt{3x+1}-\sqrt{3x-1})^6}{\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6}x^3$

(1) does not exist (2) is equal to 27 (3) is equal to $\frac{27}{2}$ (4) is equal to 9

Sol. **2**

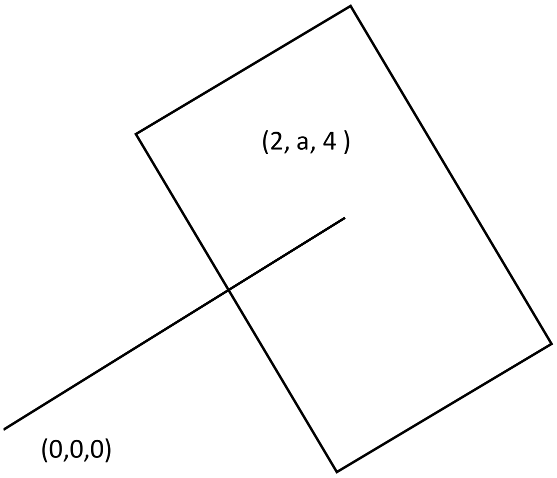
$$\lim_{x\to\infty}\frac{\left(\sqrt{3x+1}+\sqrt{3x-1}\right)^6+\left(\sqrt{3x+1}-\sqrt{3x-1}\right)^6}{\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6}$$

Taking height power common

$$\lim_{x\to\infty}\frac{x^6\left[\left(\sqrt{3+\frac{1}{x}}+\sqrt{3-\frac{1}{x}}\right)^6+\left(\sqrt{3+\frac{1}{x}}-\sqrt{3-\frac{1}{x}}\right)^6\right]}{x^6\left[\left(1+\sqrt{1-\frac{1}{x^2}}\right)^6+\left(1-\sqrt{1-\frac{1}{x^2}}\right)^6\right]}$$

$$=\frac{\left(\sqrt{3}+\sqrt{3}\right)^6+\left(\sqrt{3}-\sqrt{3}\right)^6}{(1+1)^6+(1-1)^6}$$

$$=\frac{\left(2\sqrt{3}\right)^6}{2^6}=\left(\sqrt{3}\right)^6=27$$



67. The foot of perpendicular from the origin O to a plane P which meets the co-ordinate axes at the points A, B, C is $(2, a, 4), a \in N$. If the volume of the tetrahedron OABC is 144unit^3, then which of the following points is NOT on P ?

(1) (0,6,3) (2) (0,4,4) (3) (2,2,4) (4) (3,0,4)

Sol. **4**

$\vec{n}=(2,a,4)$

Plane is

$2x+ay+4z=4+a^2+16$

$=20+a^2$

$A\left(\frac{20+a^2}{2},0,0\right)$

$B\left(0,\frac{20+a^2}{a},0\right)$

$$C\left(0,0,\frac{20+a^2}{4}\right)$$

$$\frac{1}{6}\times\frac{\left(20+a^2\right)^3}{8a}=144=2^4\times3^2$$

$$\left(20+a^2\right)^3=2^8 3^3 a$$

$$20+a^2=\left(4a\right)^{\frac{1}{3}}\left(12\right)$$

a = 2 satisfies above equation

So, 2x + 2y + 4z = 24

X + Y + 2z = 12

(A) (0, 6, 3)

(B) (0, 4, 4)

(C) (2,2,4)

(D) (3,0,4)

68. Let $(a,b)\subset(0,2\pi)$ be the largest interval for which $\sin^{-1}(\sin\theta)-\cos^{-1}(\sin\theta)>0, \theta\in(0,2\pi)$, holds. If $\alpha x^2+\beta x+\sin^{-1}(x^2-6x+10)+\cos^{-1}(x^2-6x+10)=0$ and $\alpha-\beta=b-a$, then α is equal to :

(1) $\frac{\pi}{16}$ (2) $\frac{\pi}{48}$ (3) $\frac{\pi}{12}$ (4) $\frac{\pi}{8}$

Sol. **3**

$$x^2-6x+10=\left(x-3\right)^2+1\geq1$$

So, x = 3 is the only element in the Domain

So, $\alpha x^2+\beta x+\sin^{-1}\left(x^2-6x+10\right)+\cos^{-1}\left(x^2-6x+10\right)=0$

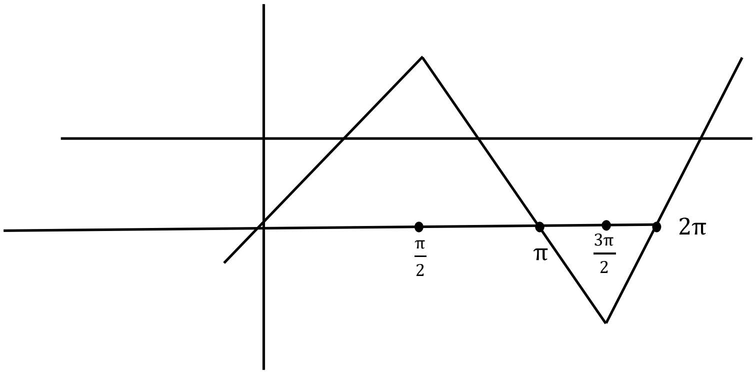
$$9\alpha+3\beta+\frac{\pi}{2}=0$$

$$\text{Sin}^{-1}\left(\sin\theta\right)-\cos^{-1}\left(\sin\theta\right)>0$$

$$\sin^{-1}\left(\sin\theta\right)-\left(\frac{\pi}{2}-\sin^{-1}\left(\sin\theta\right)\right)>0$$

$$2\sin^{-1}\left(\sin\theta\right)>\frac{\pi}{2}$$

$$\sin^{-1}\left(\sin\theta\right)>\frac{\pi}{4}$$



So, $\theta \in \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

$\alpha - \beta = \frac{3\pi}{4} - \frac{\pi}{4} = \frac{\pi}{2}$ ……(1)

& $9\alpha + 3\beta = \frac{-\pi}{2}$

$3\alpha + \beta = \frac{-\pi}{6}$ ……(2)

Adding (1) & (2)

$4\alpha = \frac{\pi}{2} - \frac{\pi}{6} = \frac{2\pi}{6}$

$\alpha = \frac{\pi}{12}$

69. Let the mean and standard deviation of marks of class A of 100 students be respectively 40 and α ($>$ 0), and the mean and standard deviation of marks of class B of n students be respectively 55 and 30 $-\alpha$. If the mean and variance of the marks of the combined class of $100 +$ n studants are respectively 50 and 350 , then the sum of variances of classes A and B is :

(1) 650 (2) 450 (3) 900 (4) 500

Sol. **4**

$m_A = 40$ $S.d_A = \alpha > 0$ $n_A = 100$

$m_B = 55$ $S.d_B = 30 - \alpha$ $n_B = n$

$m_{AVB} = 50$ $\text{Variance}_{AVB} = 350$ $n_{AVB} = 100 + n$

$A = \{x_1, ..., x_{100}\}$ $B = \{y_1 ...., y_n\}$

$\sum x_i = 4000$

$\sum y_i = 55n$

$\sum(x_i + y_i) = 50(100 + n)$

$4000 + 55n = 5000 + 50n$

Using formula of standard deviation

$5n = 1000\ n = 200$

$\alpha^2 = \frac{\sum x_i^2}{100} - (40)^2$ | $(30-\alpha)^2 \frac{\sum y_i^2}{200} - (55)^2$

$\sum x_i^2 = 100(1000 + \alpha^2)$

$\sum y_i^2 = 200\left((55)^2 + (30-\alpha)^2\right)$

$350 = \frac{\sum(x_i^2 + y_i^2)}{300} - (50)^2$

$\sum x_i^2 + \sum y_i^2 = \left((50)^2 + 350\right)300$

$160000 + 100\alpha^2 + 200(55)^2 + 200(30-\alpha)^2$

$(50)^2 300 + 350 \times 300$

$1600 + \alpha^2 + 6050 + 2(30-\alpha)^2 = 7500 + 1050$

$\alpha^2 + 1800 - 120\alpha + 2\alpha^2 - 900 = 0$

$3\alpha^2 - 120\alpha + 900 = 0$

$\alpha^2 - 40\alpha + 300 = 0$

$(\alpha - 10)(\alpha - 30) = 0$

$\alpha = 10 \text{ or } \alpha = 30$

if $\alpha = 10$ VarA = 100 & VarB = 400

$\text{Var}_A + \text{V}_{arB} = 500$

70. The absolute minimum value, of the function $f(x) = |x^2 - x + 1| + [x^2 - x + 1]$, where $[t]$ denotes the greatest integer function, in the interval $[-1,2]$, is:

(1) $\frac{1}{4}$ (2) $\frac{3}{2}$ (3) $\frac{5}{4}$ (4) $\frac{3}{4}$

Sol. **4**

$f(x) = |x^2 - x + 1| + [x^2 - x + 1]$

$x \in [-1,2]$ Here $x^2 - x + 1 > 0, \forall x \in R$

Minimum value of $x^2 - x + 1$ occurs at $a = \frac{1}{2} \in [-1,2]$

So, $\text{Min} f(x) = f\left(\frac{1}{2}\right)$

$= \frac{3}{4} + \left[\frac{3}{4}\right] = \frac{3}{4}$

71. Let H be the hyperbola, whose foci are $(1 \pm \sqrt{2}, 0)$ and eccentricity is $\sqrt{2}$. Then the length of its latus rectum is

(1) $\frac{3}{2}$ (2) 2 (3) 3 (4) $\frac{5}{2}$

Sol. **2**

$$F_1F_2 = 2ae = \left(1+\sqrt{2}\right)-\left(1-\sqrt{2}\right) = 2\sqrt{2}$$

$$ae = \sqrt{2}$$

$$e = \sqrt{2}$$

$$\Rightarrow a = 1 \quad \Rightarrow b = 1 \left(\because e = \sqrt{2}\right)$$

$$\text{L.L.R.} = \frac{2b^2}{a} = \frac{2(1)^2}{1} = 2$$

72. Let $a_1, a_2, a_3, \ldots$ be an A.P. If $a_7 = 3$, the product $a_1 a_4$ is minimum and the sum of its first n terms is zero, then $n! - 4a_{n(n+2)}$ is equal to :

(1) 9 (2) $\frac{33}{4}$ (3) $\frac{381}{4}$ (4) 24

Sol. **4**

$$a_7 = 3 \ a_1a_4 \text{ minimum}$$

$$a + 6d = 3$$

$$a\left(a+3d\right) \to \text{minimum}$$

$$S_n = 0 \quad \Rightarrow \frac{n}{2}\left[na_1 + \left(n-1\right)d\right] = 0$$

$$2a_1 + \left(n-1\right)d = 0 \qquad \ldots..(1)$$

Let $a\left(a+3d\right)$ is minimum

$$f(d) = \left(3-6d\right)\left(3-6d+3d\right)$$

$$f\left(d\right) = \left(3-6d\right)\left(3-3d\right)$$

$$= 18d^2 - 27d + 9 \text{ is minimum at } d = \frac{27}{2\times18} = \frac{9\times3}{2\times9\times2} = \frac{3}{4}$$

So, $d = \frac{3}{4}$

$$a_1 + 6d = 3$$

$$a_1 = 3 - 6\left(\frac{3}{4}\right) = 3 - \frac{9}{2} = -\frac{3}{2}$$

Putting $a_1 = \frac{-3}{2} \& d = \frac{3}{4}$ in (1)

$$2\left(\frac{-3}{2}\right)+(n-1)\left(\frac{3}{4}\right)=0$$

$$\frac{3}{4}(n-1)=3$$

$n-1=4$

$n=5$

$ni-4a_{n(n+2)}$

$n=5$ so $n!=5!=120$

& $a_{5(7)}=a_{35}=\frac{-3}{2}+(34)\left(\frac{3}{4}\right)$

$$=\frac{-3}{2}+\frac{51}{2}$$

$$=\frac{48}{2}=24$$

$5!-4(24)=24$

73. If a point P(α,β,γ) satisfying

$$(\alpha\beta\gamma)\begin{pmatrix}2 & 10 & 8\\ 9 & 3 & 8\\ 8 & 4 & 8\end{pmatrix}=(000)$$

lies on the plane $2x+4y+3z=5$, then $6\alpha+9\beta+7\gamma$ is equal to :

(1) –1 (2) $\frac{11}{5}$ (3) $\frac{5}{4}$ (4) 11

Sol. **4**

$2\alpha+9\beta+8\gamma=0$ …….(1)

$10\alpha+3\beta+4\gamma=0$ …….(2)

$8\alpha+8\beta+8\gamma=0$ …….(3)

$\alpha+\beta+\gamma=0$

$\gamma=-\alpha-\beta$

$2\alpha+9\beta-8\alpha-8\beta=0$

$\beta=6\alpha$

$\gamma=-\alpha-6\alpha=-7\alpha$

$(\alpha,6\alpha,-7\alpha)$ Satisfies the above system of equation

$2\alpha+4(6\alpha)+3(-7\alpha)=5$

$5\alpha=5$

$\alpha=1$

$\beta=6$

$\gamma = -7$

$6\alpha + 9\beta + 7\gamma = 6 + 54 - 49 = 11$

74. Let : $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}, \vec{b} = \hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $\vec{c} = 5\hat{\imath} - 3\hat{\jmath} + 3\hat{k}$ be there vectors. If $\vec{r}$ is a vector such that, $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r} \cdot \vec{a} = 0$, then $25|\vec{r}|^2$ is equal to

(1) 560 (2) 449 (3) 339 (4) 336

Sol. **3**

$\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$

$\vec{r} \times \vec{b} - \vec{c} \times \vec{b} = \vec{0}$

$(\vec{r} - \vec{c}) \times \vec{b} = \vec{0}$

$\vec{r} - \vec{c} = \lambda\vec{b}$

$\vec{r} = \lambda\vec{b} + \vec{c} = (\lambda + 5)\hat{i} - (\lambda + 3)\hat{j} + (2\lambda + 3)\hat{k}$

$\vec{r}.\vec{a} = 0$

$1(\lambda + 5) - 2(\lambda + 3) + 3(2\lambda + 3) = 0$

$5\lambda + 8 = 0 \Rightarrow \lambda = \frac{-8}{5}$

$\vec{r} = \frac{17}{5}\hat{i} - \frac{7}{5}\hat{j} + \frac{1}{5}\hat{k}$

$25|\vec{r}|^2 = 17^2 + 7^2 + 1^2 = 339$

75. Let the plane $P: 8x + \alpha_1 y + \alpha_2 z + 12 = 0$ be parallel to the line L: $\frac{x+2}{2} = \frac{y-3}{3} = \frac{z+4}{5}$. If the intercept of P on the y-axis is 1 , then the distance between P and L is :

(1) $\sqrt{\frac{7}{2}}$ (2) $\sqrt{\frac{2}{7}}$ (3) $\frac{6}{\sqrt{14}}$ (4) $\sqrt{14}$

Sol. **4**

$y - \text{intercept} = \frac{-12}{\alpha_1} = 1$

$\alpha_1 = -12$ & $\vec{n} = (8, \alpha_1, \alpha_2)$

$\vec{\ell} = (2,3,5)$

$\vec{n}.\vec{\ell} = 0$ ($\therefore$ plane P & line L are parallel)

$16 + 3\alpha_1 + 5\alpha_2 = 0$

$16 - 36 + 5\alpha_2 = 0$

$5\alpha_2 = 20$

$\alpha_2 = 4$

$8x-12y+4z+12=0$

$\Rightarrow 2x-3y+z+3=0$

(–2, 3, – 4) is a point on the line L distance bet^n the point (–2, 3, –4) and the plane P is :-

$$d=\frac{|-4-9-4+3|}{\sqrt{2^2+3^2+1^2}}$$

$$=\frac{14}{\sqrt{14}}=\sqrt{14}$$

76. Let P be the plane, passing through the point $(1,-1,-5)$ and perpendicular to the line joining the points $(4,1,-3)$ and $(2,4,3)$. Then the distance of P from the point $(3,-2,2)$ is

(1) 5 (2) 4 (3) 7 (4) 6

Sol. **1**

Let A(4, 1, –3) & B(2, 4, 3)

$\vec{n}=\overrightarrow{AB}=(-2,3,6)$

Plane P is :

$-2(x-1)+3(y+1)+6(z+5)=0$

$-2x+2+3y+3+6z+30=0$

$2x-3y-6z=35$

Distance of P from the point (3, –2, 2) is

$$=\frac{|6+6-12-35|}{\sqrt{2^2+3^2+6^2}}$$

$=\frac{35}{7}=5$ **Ans. (1)**

77. The number of values of $r\in\{p, q, \sim p, \sim q\}$ for which $((p\wedge q)\Rightarrow(r\vee q))\wedge((p\wedge r)\Rightarrow q)$ is a tautology, is:

(1) 3 (2) 4 (3) 1 (4) 2

Sol. **4**

$((p\wedge q)\Rightarrow(r\vee q))\wedge((p\wedge r)\Rightarrow q)$

$(p\wedge q)\Rightarrow(r\vee q)$

$\sim(p\wedge q)\vee(r\vee q)$

$\sim p\vee\sim q\vee r\vee q$

& $(p\wedge r)\Rightarrow q$

$\sim(p\wedge r)\vee q$

$\sim p\vee\sim r\vee q$

$(\sim p\vee\sim q\vee r\vee q)\wedge(\sim p\vee\sim r\vee q)$

$\equiv\sim p\vee\sim r\vee q$

If $r=p$

$\sim p\vee\sim p\vee q$ → Not tautology

If $r=\sim p$

$\sim p \vee p \vee q$ → tautology
If $r = q$
$\sim p \vee \sim q \vee q$ → tautology
If $r = \sim q$
$\sim p \vee q \vee q$ → Not tautology

Ans. 2 (D)

78. The set of all values of a^2 for which the line $x + y = 0$ bisects two distinct chords drawn from a point $\mathrm{P}\left(\frac{1+a}{2}, \frac{1-a}{2}\right)$ on the circle $2x^2 + 2y^2 - (1 + a)x - (1 - a)y = 0$, is equal to :

(1) $(0,4]$ (2) $(4, \infty)$ (3) $(2,12]$ (4) $(8, \infty)$

Sol. **4**

$$x^2 + y^2 - \left(\frac{1+a}{2}\right) x - \left(\frac{1-a}{2}\right) y = 0$$

$$x\left(x - \left(\frac{1+a}{2}\right)\right) + y\left(y - \left(\frac{1-a}{2}\right)\right) = 0$$

$$y - y_1 = m(x - x_1) \qquad (x_1, y_1) = \left(\frac{1+a}{2}, \frac{1-a}{2}\right)$$

$\&\ x + y = 0$

$-\ x - y_1 = mx - mx_1$

$mx_1 - y_1 = (1 + m)x$

$$x = \frac{mx_1 - y_1}{1+m} \quad \& \qquad y = \frac{y_1 - mx_1}{1+m}$$

$$\frac{\frac{y_1}{2} - y}{\frac{x_1}{2} - x} = -\frac{1}{m}$$

$$\frac{\frac{y_1}{2} - \left(\frac{y_1 - mx_1}{1+m}\right)}{\frac{x_1}{2} - \left(\frac{mx_1 - y_1}{1+m}\right)} = -\frac{1}{m}$$

$$= \frac{(1+m)y_1 - 2y_1 + 2mx_1}{(1+m)x_1 - 2mx_1 + 2y_1} = -\frac{1}{m}$$

$$\frac{m(y_1 + 2x_1) - y_1}{-mx_1 + x_1 + 2y_1} = -\frac{1}{m}$$

$m^2(y_1+2x_1) - my_1 = mx_1 - x_1 - 2y_1$

$m^2(y_1+2x_1) - (y_1 + x_2)m + x_1 + 2y_1 = 0$

$D > 0$

$(y_1+x_1)^2 - 4(y_1 + 2x_1)(x_2 + 2y_1) > 0$

$x_1 = \frac{1+a}{2}$, $y_1 = \frac{1-a}{2}$

$x_1 + y_1 = 1$

$y_1 + 2x_1 = \frac{1-a}{2} + 1 + a = \frac{3}{2} - \frac{a}{2} = \frac{3-a}{2}$

$x_1 + 2y_1 = \frac{1+a}{2} + 1 - a = \frac{3}{2} + \frac{a}{2} = \frac{3+a}{2}$

$1 - 4\left(\frac{3-a}{2}\right)\left(\frac{3+a}{2}\right) > 0$

$1 - (9 - a^2) > 0$

$a^2 - 8 > 0$

$a^2 > 8 \qquad \rightarrow (8, \infty)$

Ans. 4

79. If $\phi(x) = \frac{1}{\sqrt{x}}\int_{\frac{\pi}{4}}^{x}\left(4\sqrt{2}\sin t - 3\phi'(t)\right)dt, x > 0$, then $\emptyset'\left(\frac{\pi}{4}\right)$ is equal to :

(1) $\frac{8}{6+\sqrt{\pi}}$ (2) $\frac{4}{6+\sqrt{\pi}}$ (3) $\frac{8}{\sqrt{\pi}}$ (4) $\frac{4}{6-\sqrt{\pi}}$

Sol. **1**

$$\sqrt{x}\ \phi(x) = \int_{\frac{\pi}{4}}^{x}(4\sqrt{2}\sin t - 3\phi'(t))dt$$

Differentiating w.r.t. x,

$$\frac{1}{2\sqrt{x}}\phi(x) + \sqrt{x}\,\phi'(x) = 4\sqrt{2}\sin x - 3\phi'(x)$$

$$(\sqrt{x}+3)\,\phi'(x) + \frac{1}{2\sqrt{x}}\phi(x) = 4\sqrt{2}\sin x$$

$$\phi'(x) + \frac{1}{2\sqrt{x}(\sqrt{x}+3)}\ \phi(x) = \frac{4\sqrt{2}\sin x}{\sqrt{x}+3}$$

Put $x = \left(\frac{\pi}{4}\right)$

$$\phi'\left(\frac{\pi}{4}\right) + 0 = \frac{4\sqrt{2}\times\frac{1}{\sqrt{2}}}{\sqrt{\frac{\pi}{4}}+3} \quad \left(\because \phi\left(\frac{\pi}{4}\right) = 0\right)$$

$$\phi'\left(\frac{\pi}{4}\right) = \frac{4\times 2}{\sqrt{\pi}+6} = \frac{8}{\sqrt{\pi}+6}$$ **Ans. 1**

80. Let $f:\mathbb{R} - \{2,6\} \rightarrow \mathbb{R}$ be real valued function defined as $f(x) = \frac{x^2+2x+1}{x^2-8x+12}$. Then range of f is

(1) $\left(-\infty,-\frac{21}{4}\right)\cup(0,\infty)$ (2) $\left(-\infty,-\frac{21}{4}\right]\cup[1,\infty)$

(3) $\left(-\infty,-\frac{21}{4}\right]\cup\left[\frac{21}{4},\infty\right)$ (4) $\left(-\infty,-\frac{21}{4}\right]\cup[0,\infty)$

Sol. **4**

$y=\dfrac{x^2+2x+1}{x^2-8x+12}$

$x^2y-8xy+12y=x^2+2x+1$

$x^2(y-1)-(8y+2)x+12y-1=0$

$D\ge 0$

$(8y+2)^2-4(y-1)(12y-1)\ge 0$

$4(4y+1)^2-4(y-1)(12y-1)\ge 0$

$16y^2+8y+1-(12y^2-13y+1)\ge 0$

$4y^2+21y\ge 0$

$4y\left[y+\dfrac{21}{4}\right]\ge 0$

$\left(-\infty,\dfrac{-21}{4}\right]\cup[0,\infty)$ **Ans. 4**

Section B

81. Let $A=[a_{ij}], a_{ij}\in Z\cap[0,4], 1\le i,j\le 2$. The number of matrices A such that the sum of all entries is a prime number $p\in(2,13)$ is

Sol. **204**

$A=[a_{ij}]$, aij $\in Z\cap[0,4]$, so $a_{ij}=\{0,1,2,3,4\}$

$A=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}_{2\times2}$

$a_{11}+a_{12}+a_{21}+a_{22}=P$ where P is a prime number

$a_{11}+a_{12}+a_{21}+a_{22}=3, 5, 7, 11$

$(1+x+x^2+x^3+x^4)^4$

$(1-x^5)^4(1-x)^{-4}$

$(1-4x^5+6x^{10}-4x^{15}+x^{20})(1+{}^4C_1x+{}^5C_2x^2+{}^6C_3x^3+{}^7C_4x^7\ldots.)$

For 3 ${}^6C_3=\dfrac{6\times5\times4}{6}=20.$

For 5 $(-4)+{}^8C_5=-4+56=52.$

For 7 $(-4)\,{}^5C_2+{}^{10}C_7=-40+120=80.$

For 11 ${}^{14}C_{11}+(-4)\,{}^9C_6+6({}^4C_1)=364-336+24=52$

Total number of matrices = 204

$a_{11}+a_{12}+a_{22}+a_{12}=3$

$0 \quad 0 \quad 0 \quad 3 \quad = \frac{4!}{3!} = 4$

$0 \quad 0 \quad 2 \quad 1 \quad = \frac{4!}{2!} = 12$

$0 \quad 1 \quad 1 \quad 1 \quad = \frac{4!}{3!} = 4$

20

$a_{11} + a_{12} + a_{21} + a_{22} = 5$

$0 \quad 0 \quad 1 \quad 4 \quad = \frac{4!}{2!} = 12$

$0 \quad 0 \quad 2 \quad 3 \quad = \frac{4!}{2!} = 12$

$0 \quad 1 \quad 1 \quad 3 \quad = \frac{4!}{2!} = 12$

$0 \quad 1 \quad 2 \quad 2 \quad = \frac{4!}{2!} = 12$

$1 \quad 1 \quad 1 \quad 2 \quad = \frac{4!}{3!} = 4$

52

$a_{11} + a_{12} + a_{21} + a_{22} = 7$

$0 \quad 0 \quad 3 \quad 4 \quad = \frac{4!}{2!} = 12$

$0 \quad 1 \quad 3 \quad 3 \quad = \frac{4!}{2!} = 12$

$0 \quad 1 \quad 2 \quad 4 \quad = 4! = 24$

$1 \quad 1 \quad 1 \quad 4 \quad = \frac{4!}{3!} = 4$

$0 \quad 2 \quad 2 \quad 3 \quad = \frac{4!}{3!} = 12$

$1 \quad 1 \quad 2 \quad 3 \quad = \frac{4!}{2!} = 12$

$1 \quad 2 \quad 2 \quad 2 \quad = \frac{4!}{3!} = 4$

80

$a_{11} + a_{12} + a_{21} + a_{22} = 11$

$0 \quad 3 \quad 4 \quad 4 \quad = \frac{4!}{2!} = 12$

$1 \quad 2 \quad 4 \quad 4 \quad = \frac{4!}{2!} = 12$

$1 \quad 3 \quad 3 \quad 4 \quad = \frac{4!}{2!} = 12$

$2 \quad 3 \quad 3 \quad 4 \quad = \frac{4!}{3!} = 12$

$2 \quad 3 \quad 3 \quad 3 \quad = \frac{4!}{3!} = 4$

$\overline{\quad 52 \quad}$

total matrix is = 20 + 52 + 80 + 52 = 204.

82. Let A be a $n \times n$ matrix such that $|A| = 2$. If the determinant of the matrix $\mathrm{Adj}(2 \cdot \mathrm{Adj}(2\,A^{-1})) \cdot$ is 2^{84}, then n is equal to

Sol. **84**

$|A| = 2$

$|\mathrm{Adj}\ (2\ \mathrm{Adj}(2A^{-1}))| = 2^{84}$

$|2\ \mathrm{Adj}\ (2\ A^{-1})|^{n-1} = 2^{84}$

$(2^n\ |\mathrm{Adj}\ (2\ A^{-1})|)^{n-1} = 2^{84}$

$(2^n\ |\ 2^{n-1}\ \mathrm{Adj}\ (A^{-1})|)^{n-1} = 2^{84}$

$(2^n \times (2^{n-1})^n\ |\mathrm{Adj}\ (A^{-1})|)^{n-1} = 2^{84}$

$(2^n \times 2^{n(n-1)} \times |A^{-1}|)^{n-1} = 2^{84}$

$(2^n \times 2^{n(n-1)} \times \left(\frac{1}{2}\right)^{n-1})^{n-1} = 2^{84}$

$(2^{n+n^2-n-n+1})^{n-1} = 2^{84}$

$(2^{n^2-n+1})^{n-1} = 2^{84}$

$(n^2 - n + 1)(n + 1) = 84$

83. If the constant term in the binomial expansion of $\left(\frac{x^{\frac{5}{2}}}{2} - \frac{4}{x^l}\right)^9$ is -84 and the coefficient of x^{-3l} is $2^\alpha\beta$, where $\beta < 0$ is an odd number, then $|\alpha l - \beta|$ is equal to

Sol. **98**

$$\left(\frac{x^{5/2}}{2} - \frac{4}{x^\ell}\right)^9$$

$$T_{r+1} = {}^9C_r \left(\frac{x^{5/2}}{2}\right)^{9-r} \left(\frac{-4}{x^\ell}\right)^r$$

$$= {}^9C_r \left(\frac{1}{2}\right)^{9-r} (-4)^r \quad x^{\frac{45-5r}{2}-\ell r}$$

For constant term

$= {}^9C_r \left(\frac{1}{2}\right)^{9-r} (-4)^r = -84$

$= {}^9C_r \left(\frac{1}{2}\right)^{9-r} (-1)^r\, 2^{2r} = -84$

$= {}^9C_r\, 2^{r-9}\, 2^{2r}\, (-1)^r = -84$

$= {}^9C_r\, 2^{3r-9}\, (-1)^r = -84$

$\Rightarrow \boxed{r = 3}$

$\frac{45 - 5r}{2} - \ell r = 0$

$\frac{45 - 15}{2} - 3\ell = 0$

$15 = 3\,\ell$

$\boxed{\ell = 5}$

For coefficient of x^{-15} is

$\frac{45 - 5r}{2} - 5r = -15$

$45 - 5r - 10r = -30$

$75 = 15r$

$\boxed{r = 5}$

For coefficient of x^{-15} is ${}^9C_5 \left(\frac{1}{2}\right)^4 (-4)^5$

$\frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} \times \frac{1}{2^4} \times 2^{10} \times (-1)$

$= 9 \times 2 \times 7 \times 2^6 \times (-1)$

$= 2^7(-63) = 2^\alpha \beta$

$\alpha = 7, \beta = -63$

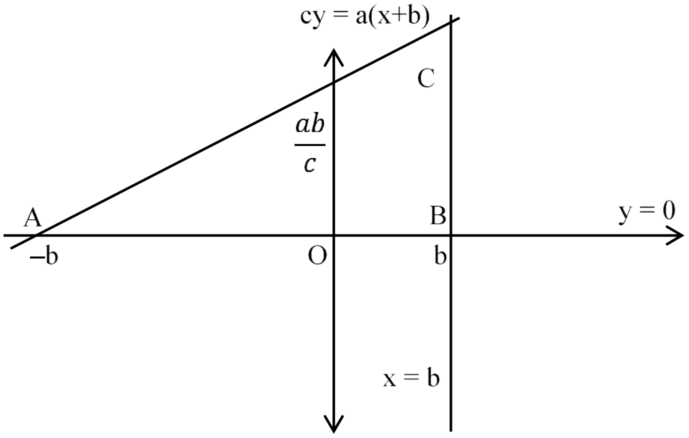
$|\alpha\ell - \beta| = |7(5) + 63| = |35 + 63|$

$|\alpha\ell - \beta| = 98.$

84. Let S be the set of all a ∈ N such that the area of the triangle formed by the tangent at the point P(b, c), b, c ∈ ℕ, on the parabola $y^2 = 2ax$ and the lines $x = b, y = 0$ is 16 unit^2, then $\sum_{a \in s} a$ is equal to

Sol. **146**

tangent at P(b, c) $my^2 = 2ax$ is



area $= \left|\frac{1}{2} \times 2b \times \frac{2ba}{c}\right| = 16$

$\frac{2b^2a}{C} = 16$

$\frac{b^2a}{C} = 8$

$\because$ P(b, c) lies on $y^2 = 2ax$

$C^2 = 2ab$

$\Rightarrow \quad \frac{b^4a^2}{c^2} = 64$

$\Rightarrow \quad \frac{b^4a^2}{2ab} = 64$

$\Rightarrow \quad b^3a = 128$

$\Rightarrow \quad a = \frac{128}{b^3}$

a can be 128, 16, 2 then

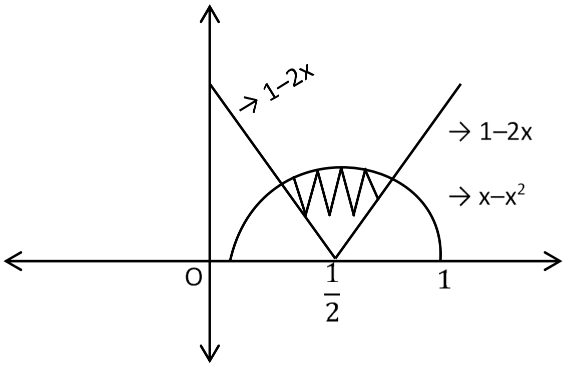
S = {2, 16, 128}

$\sum_{a \in s} a = 146$

85. Let the area of the region $\{(x, y): |2x - 1| \le y \le |x^2 - x|, 0 \le x \le 1\}$ be A. Then $(6\text{ A} + 11)^2$ is equal to

Sol. **125**

$|2x - 1| \le y \le |x^2 - x|, 0 \le x \le 1$



$$x - x^2 = 1 - 2x$$

$$x^2 - 3x + 1 = 0$$

$$x = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2}$$

$$x = \frac{3-\sqrt{5}}{2} \quad \text{as } 0 < x < \frac{1}{2}$$

$$\text{Area} = 2\int_{\frac{3-\sqrt{5}}{2}}^{1/2} \left[\left(x - x^2\right) - \left(1 - 2x\right)\right] dx$$

$$2\int_{\frac{3-\sqrt{5}}{2}}^{1/2} \left[3x - x^2 - 1\right] dx$$

$$= 2\left[\frac{3x^2}{2} - \frac{x^3}{3} - x\right]_{\frac{3-\sqrt{5}}{2}}^{1/2}$$

For $x = \frac{3-\sqrt{5}}{2}$,

$$x^2 = 3x - 1$$

$$x^3 = 3x^2 - x$$

$$= 3\left(3x - 1\right) - x$$

$$= 8x - 3$$

$$\frac{3}{2}x^2 - \frac{1}{3}x^3 - x = \frac{3}{2}\left(3x - 1\right) - \frac{1}{3}\left(8x - 3\right) - x$$

$$= \frac{9x - 3}{2} - \frac{\left(8x - 3\right)}{3} - x$$

$$= \frac{27x - 9 - \left(16x - 6\right)}{6} - x$$

$$= \frac{11x - 3}{6} - x$$

$$=\frac{5x-3}{6}$$

For $x=\frac{1}{2}$,

$$\frac{3}{2}x^2-\frac{1}{3}x^3-x=\frac{3}{2}\left(\frac{1}{4}\right)-\frac{1}{3}\left(\frac{1}{8}\right)-\frac{1}{2}$$

$$=\frac{9-1-12}{24}$$

$$=\frac{-4}{24}=-\frac{1}{6}$$

$$\text{Area}=2\left[-\frac{1}{6}-\left(\frac{\frac{5\left(3-\sqrt{5}\right)}{2}-3}{6}\right)\right]$$

$$=2\left[-\frac{1}{6}-\left(\frac{15-5\sqrt{5}-6}{12}\right)\right]$$

$$=2\left[-\frac{1}{6}-\left(\frac{9-5\sqrt{5}}{12}\right)\right]$$

$$=2\left[\frac{-2-9+5\sqrt{5}}{12}\right]$$

$$=\frac{5\sqrt{5}-11}{6}$$

$$\left(6A+11\right)^2=\left(5\sqrt{5}\right)^2=125$$

86. The coefficient of x^{-6}, in the expansion of $\left(\frac{4x}{5}+\frac{5}{2x^2}\right)^9$, is

Sol. **5040**

$$\left(\frac{4x}{5}\times\frac{5}{2x^2}\right)^9$$

General term is

$$T_{r+1}={}^9C_r\left(\frac{4n}{r}\right)^{9-r}\left(\frac{5}{2n^2}\right)^r$$

$$={}^9C_r\left(\frac{4}{5}\right)^{9-r}\left(\frac{5}{2}\right)^r x^{9-r-2r}$$

For coefficient of term x^{-6} $\quad 9-r-2r=-6$

$15=3r$

$\boxed{r = 5}$

Coefficient of term x^{-6} $\quad {}^9C_5 \left(\frac{4}{5}\right)^4 \left(\frac{5}{2}\right)^5$

$= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} \times 5 \times 2^3$

$= 9 \times 2 \times 7 \times 5 \times 8$

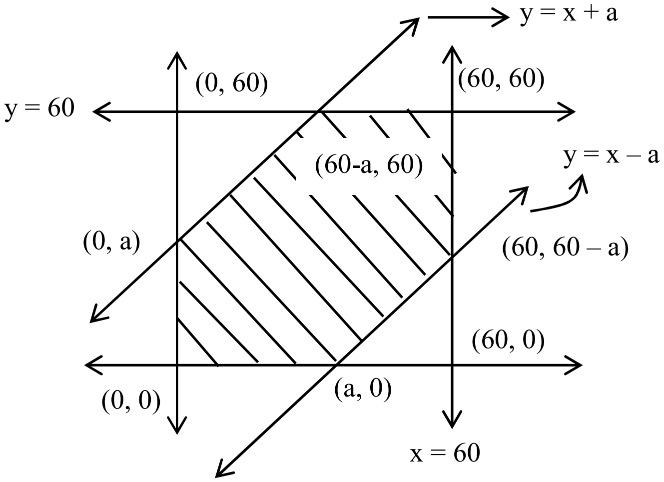
$= 5040$

87. Let A be the event that the absolute difference between two randomly choosen real numbers in the sample space [0,60] is less than or equal to a . If $P(A) = \frac{11}{36}$, then a is equal to

Sol. **10**

$|x - y| < a \rightarrow -a < x - y \quad \& \quad x - y < a$

$x, y \in [0, 60]$



$$P(A) = \frac{\text{Shaded area}}{\text{Total area}} = \frac{(60)^2 - \left[\frac{1}{2}(60 - a)^2 + \frac{1}{2} \times (60 - a)^2\right]}{(60)^2}$$

$$P(A) = \frac{(60)^2 - (60 - a)^2}{(60)^2}$$

$$\frac{11}{36} = \frac{120a - a^2}{3600}$$

$1100 = 120a - a^2$

$a^2 - 120a + 1100 = 0$

$a^2 - 110a - 10a + 1100 = 0$

$a(a - 110) - 10(a - 110) = 0 =$

$(a - 10)(a - 110) = 0$

Ans. $\boxed{a = 10}$

($\because$ for a = 110, P(A) = 1)

88. If ${}^{2n+1}\mathrm{P}_{n-1}:{}^{2n-1}\mathrm{P}_{n}=11:21$, then n^2+n+15 is equal to :

Sol. **45**

$${}^{2n+1}P_{n-1} : {}^{2n-1}P_{n} = 11 : 21$$

$$\frac{(2n+1)!}{(n+2)!}\times\frac{(n-1)!}{(2n-1)!}=\frac{11}{21}$$

$$\Rightarrow \frac{(2n+1)(2n)}{(n+2)(n+1)n}=\frac{11}{21}$$

$$\Rightarrow \frac{2(2n+1)}{(n+2)(n+1)}=\frac{11}{21}$$

$\Rightarrow 42(2n+1) = 11(n^2+3n+2)$

$\Rightarrow 84n + 42 = 11n^2+33n+22$

$\Rightarrow 11n^2 - 51n - 20 = 0$

$\Rightarrow n = 5$

$n^2 + n + 15 = 25 + 5 + 15 = 45$

89. Let $\vec{a},\vec{b},\vec{c}$ be three vectors such that $|\vec{a}|=\sqrt{31}, 4|\vec{b}|=|\vec{c}|=2$ and $2(\vec{a}\times\vec{b})=3(\vec{c}\times\vec{a})$. If the angle between $\vec{b}$ and $\vec{c}$ is $\frac{2\pi}{3}$, then $\left(\frac{\vec{a}\times\vec{c}}{\vec{a}\cdot\vec{b}}\right)^2$ is equal to

Sol. **3**

$|\vec{a}| = \sqrt{31} \qquad 4|\vec{b}| = |\vec{c}| = 2$

$2(\vec{a}\times\vec{b}) = 3(\vec{c}\times\vec{a})$

$$\vec{b} \wedge \vec{c} = \frac{2\pi}{3}$$

$\vec{a}\times 2\vec{b} = 3\vec{c}\times\vec{a} = -\vec{a}\times 3\vec{c}$

$\vec{a}\times(2\vec{b}+3\vec{c}) = \vec{o}$

$\vec{a} = \lambda(2\vec{b}+3\vec{c})$

$|\vec{a}|^2 = \lambda^2 |2\vec{b}+3\vec{c}|^2$

$$\left|\vec{a}\right|^2=\lambda^2\left(4\left|\vec{b}\right|^2+9\left|\vec{c}\right|^2+12\left|\vec{b}\right|\left|\vec{c}\right|\cos\theta\right)$$

$$31 = \lambda^2\left(1 + 9(2)^2 + 12|\vec{b}||\vec{c}|\cos\frac{2\pi}{3}\right)$$

$$31 = \lambda^2\left(1 + 36 - 6\times\frac{1}{2}\times 2\right)$$

$31 = \lambda^2(31)$

$\boxed{\lambda^2 = 1}$

$\Rightarrow \lambda = \pm 1$

$\vec{a} = \pm\left(2\vec{b}+3\vec{c}\right)$

$$\vec{a}\times\vec{c}=\pm\left(2\vec{b}+3\vec{c}\right)\times\vec{c}$$

$$=\pm 2\left(\vec{b}\times\vec{c}\right)$$

$$\left|\vec{a}\times\vec{c}\right|^2=4\left|\vec{b}\times\vec{c}\right|^2=3$$

$$\vec{a}\cdot\vec{b}=\mp 1$$

$$\left(\frac{\left|\vec{a}\times\vec{c}\right|}{\vec{a}\cdot\vec{b}}\right)^2=3$$

90. The sum $1^2-2\cdot 3^2+3\cdot 5^2-4\cdot 7^2+5\cdot 9^2-\cdots+15\cdot 29^2$ is

Sol. **6952**

$1^2 - 2.3^2 + 3.5^2 - 4.7^2 + 5.9^2 - \ldots + 15.29^2$

$S = \underbrace{15.29^2 - 14.27^2} + \ldots. + \underbrace{3.5^2 - 2.3^2} + 1^2$

$(n+1)(2n+1)^2 - n(2n-1)^2$

$n(4n^2+4n+1)+ 4n^2+4n+1 - n(4n^2-4n+1)$

$= 12n^2 + 4n + 1$

$S = [\Sigma\ 12n^2+4n+1$ for $n = 2, 4, 6, 8, 10, 12, 14] + 1$

$$S_1 = \sum_{k=1}^{7} 12(2k)^2 + 4(2k) + 1$$

$$= \sum_{k=1}^{7} [48k^2 + 8k + 1]$$

$$= 48\sum_{k=1}^{7} k^2 + 8\sum_{k=1}^{7} k + \sum_{k=1}^{7} 1$$

$$= \frac{48(7)(8)(15)}{6} + \frac{8(7)(8)}{2} + 7 = 6951$$

$S = \boxed{6952}$

JEE–MAIN EXAMINATION – JANUARY, 2023

(Held On Thursday 1st February, 2023) | TIME : 9 : 00 AM to 12 : 00 NOON

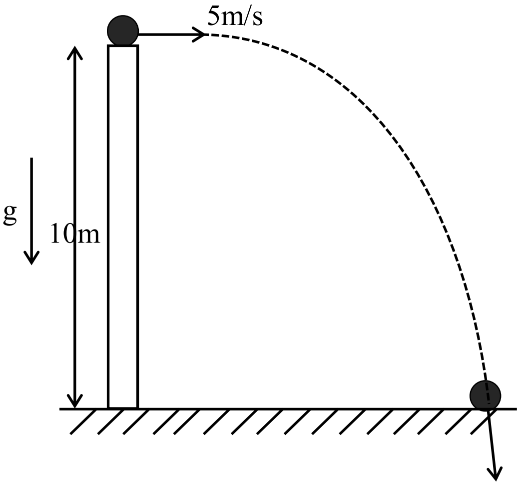
Physics

SECTION - A

1. A child stands on the edge of the cliff 10 m above the ground and throws a stone horizontally with an initial speed of 5 ms^{-1}. Neglecting the air resistance, the speed with which the stone hits the ground will be ___ ms^{-1} (given, $g = 10\ \mathrm{ms}^{-2}$).

(1) 15 (2) 20 (3) 30 (4) 25

Sol. **(1)**



Along vertical direction

$u_y = 0$ $\qquad v_y^2 = u_y^2 + 2a_y g_y$

$a_y = +g$ $\qquad = (0)^2 + 2\times10\times10$

$v_y = ?$ $\qquad v_y^2 = 200$

$s_y = 10$ m

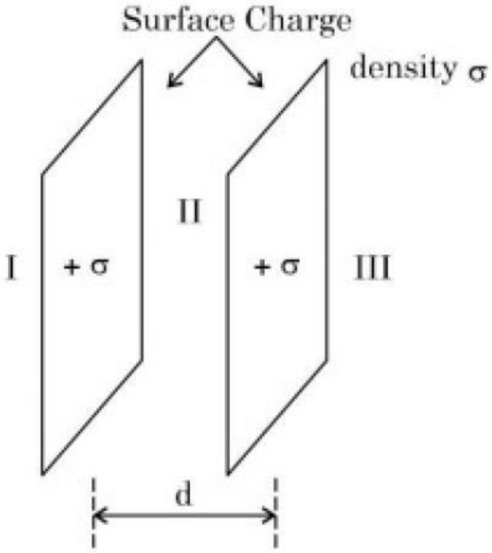
$\boxed{v_y^2 = 200}$

$\therefore \quad v = \sqrt{v_x^2 + v_y^2}$

$= \sqrt{25+200} \quad = \sqrt{225}$

= 15 m/s

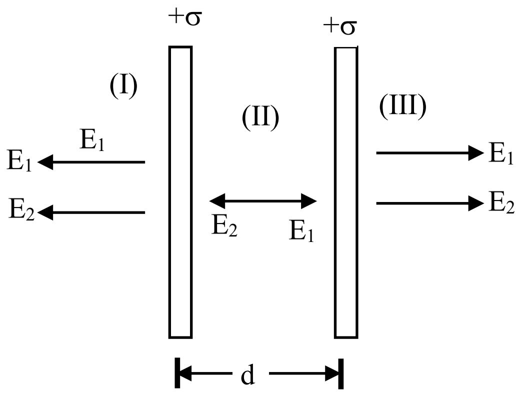
2. Let σ be the uniform surface charge density of two infinite thin plane sheets shown in figure. Then the electric fields in three different region E_I, E_{II} and E_{III} are:



(1) $\vec{E}_I = \frac{2\sigma}{\epsilon_0}\hat{n}, \vec{E}_{II} = 0, \vec{E}_{III} = \frac{2\sigma}{\epsilon_0}\hat{n}$ (2) $\vec{E}_I = \frac{\sigma}{2\epsilon_0}\hat{n}, \vec{E}_{II} = 0, \vec{E}_{III} = \frac{\sigma}{2\epsilon_0}\hat{n}$

(3) $\vec{E}_I = -\frac{\sigma}{\epsilon_0}\hat{n}, \vec{E}_{II} = 0, \vec{E}_{III} = \frac{\sigma}{\epsilon_0}\hat{n}$ (4) $\vec{E}_I = 0, \vec{E}_{II} = \frac{\sigma}{\epsilon_0}\hat{n}, E_{III} = 0$

Sol. **(3)**



$\therefore E_I = -\frac{\sigma}{E_0}\hat{n}$

$\therefore E_{II} = 0$

$\therefore E_{III} = -\frac{\sigma}{E_0}\hat{n}$

3. A mercury drop of radius 10^{-3} m is broken into 125 equal size droplets. Surface tension of mercury is 0.45Nm^{-1}. The gain in surface energy is:

(1) 28×10^{-5} J (2) 17.5×10^{-5} J (3) 5×10^{-5} J (4) 2.26×10^{-5} J

Sol. **(4)**

[Volume of bigger drop] = [volume of smaller drop] × 125

$\frac{4}{3}\pi R^3 = 125 \times \frac{4}{3}\pi r^3$

$R^3 = 125 r^3$

$\boxed{\therefore R = 5 \times r}$

⇒ Gain in sinface energy = TdA

$$= 0.45 \times [A_2 - A_1]$$

$$= 0.45 \times \left[125 \times 4\pi r^2 - 4\pi R^2\right]$$

$$= 0.45 \times \left[125 \times 4\pi\left(\frac{R}{5}\right)^2 - 4\pi R^2\right]$$

$$= 0.45 \times \left[20\pi R^2 - 4\pi R^2\right]$$

$$= 0.45 \times 16\pi R^2$$

$$= 0.45 \times 16 \times 3.14 \times \left(10^{-3}\right)^2$$

$$= 2.26 \times 10^{-5}\,\text{J}$$

4. If earth has a mass nine times and radius twice to that of a planet P. Then $\frac{v_e}{3}\sqrt{x}\ \text{ms}^{-1}$ will be the minimum velocity required by a rocket to pull out of gravitational force of P, where v_e is escape velocity on earth. The value of x is

(1) 1 (2) 3 (3) 18 (4) 2

Sol. **(4)**

$$M_E = 9M_P$$

$$R_E = 2R_P$$

$$V_c^1 = \sqrt{\frac{2GM_P}{R_P}} = \sqrt{\frac{2G\frac{M_E}{9}}{\frac{R_E}{2}}}$$

$$= \sqrt{\frac{2GM_E}{R_E}} \times \sqrt{\frac{2}{9}}$$

$$\boxed{V_c^1 = \frac{V_e}{3}\sqrt{2}}$$

5. A sample of gas at temperature T is adiabatically expanded to double its volume. The work done by the gas in the process is $\left(\text{given, } \gamma = \frac{3}{2}\right)$:

(1) $W = \frac{T}{R}[\sqrt{2} - 2]$ (2) $W = RT[2 - \sqrt{2}]$ (3) $W = TR[\sqrt{2} - 2]$ (4) $W = \frac{R}{T}[2 - \sqrt{2}]$

Sol. **(2)**

Work done in the process is given by

$$W = \frac{R}{\gamma - 1}(T_1 - T_2)$$

For adiabatic process:

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$TV^{\frac{3}{2}-1} = T_2(2V)^{\frac{3}{2}-1}$$

$$TV^{\frac{1}{2}} = T_2(2V)^{\frac{1}{2}}$$

$$T^2V = T_2^2 \times 2V$$

$$\therefore T_2 = \frac{T}{\sqrt{2}}$$

$$\therefore W = \frac{R}{\gamma - 1} \times \left(T - \frac{T}{\sqrt{2}}\right)$$

$$= 2RT\left[1 - \frac{1}{\sqrt{2}}\right]$$

$$= RT\left[2 - \frac{2}{\sqrt{2}}\right]$$

$$= RT[2 - \sqrt{2}]$$

$$\boxed{W = RT\left[2 - \sqrt{2}\right]}$$

6. $\left(P+\frac{a}{V^2}\right)(V-b)=RT$ represents the equation of state of some gases. Where P is the pressure, V is the volume, T is the temperature and a, b, R are the constants. The physical quantity, which has dimensional formula as that of $\frac{b^2}{a}$, will be:

(1) Compressibility
(2) Energy density
(3) Modulus of rigidity
(4) Bulk modulus

Sol. **(1)**

$$[b]=[L^3]$$

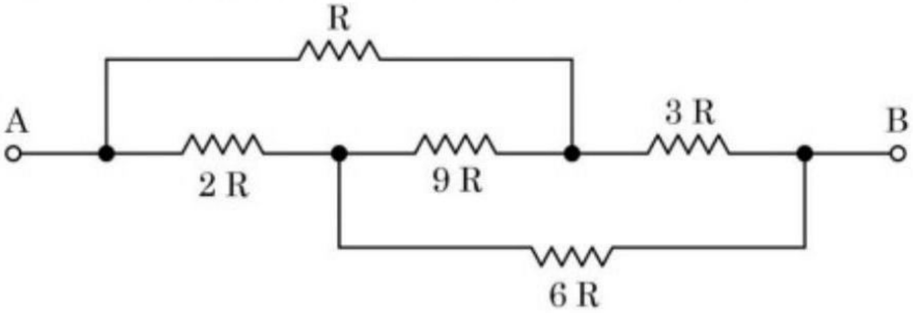
$$[a]=[PV^2]$$

$$=[ML^{-1}T^{-2}][L^6]$$

$$=[ML^5T^{-2}]$$

$$\frac{[b^2]}{[a]}=\frac{[L^6]}{[ML^5T^{-2}]}=[M^{-1}L^1T^2]$$

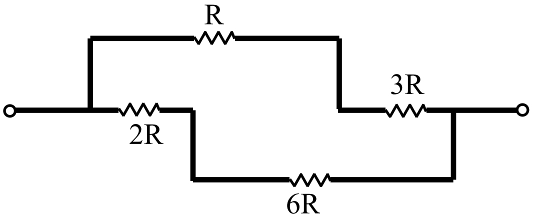
7. The equivalent resistance between A and B of the network shown in figure:



(1) $\frac{8}{3}$R
(2) 21R
(3) 14R
(4) $11\frac{2}{3}$R

Sol. **(1)**

$\because$ The given network is wheat-stone network



$$\therefore \quad R_{eq}=\frac{4R\times 8R}{4R+8R}$$

$$=\frac{4R\times 8R}{12R}$$

$$\boxed{R_{eq}=\frac{8}{3}R}$$

8. Match List I with List II:

List I	**List II**
A. AC generator	I. Presence of both L and C
B. Transformer	II. Electromagnetic Induction
C. Resonance phenomenon to occur	III. Quality factor
D. Sharpness of resonance	IV. Mutual Induction

Choose the correct answer from the options given below:

(1) A-IV, B-III, C-I, D-II
(2) A-IV, B-II, C-I, D-III
(3) A-II, B-IV, C-I, D-III
(4) A-II, B-I, C-III, D-IV

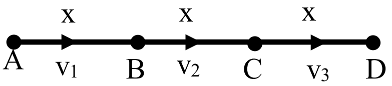
Sol. **(3)**

(A) A.C. generator → II. Electro-magnetic induction
(B) transformer → IV Mutual induction
(C) Resonance phenomenon to occur → (I) presence of both L and C
(D) Sharpness of resonance → (III) Quality factor

9. An object moves with speed v_1, v_2 and v_3 along a line segment AB, BC and CD respectively as shown in figure. Where AB = BC and AD = 3AB, then average speed of the object will be:

(1) $\frac{(v_1+v_2+v_3)}{3v_1v_2v_3}$
(2) $\frac{(v_1+v_2+v_3)}{3}$
(3) $\frac{3v_1v_2v_3}{(v_1v_2+v_2v_3+v_3v_1)}$
(4) $\frac{v_1v_2v_3}{3(v_1v_2+v_2v_3+v_3v_1)}$

Sol. **(3)**



$$<v> = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{3x}{\frac{x}{v_1}+\frac{x}{v_2}+\frac{x}{v_3}}$$

$$= \frac{3}{\left[\frac{1}{v_1}+\frac{1}{v_2}+\frac{1}{v_3}\right]} = \frac{3}{\left[\frac{v_2v_3+v_1v_3+v_1v_2}{v_1v_2v_3}\right]}$$

$$<v> = \frac{3v_1v_2v_3}{[v_3v_2+v_1v_2+v_1v_2]}$$

10. 'n' polarizing sheets are arranged such that each makes an angle 45° with the preceeding sheet. An unpolarized light of intensity I is incident into this arrangement. The output intensity is found to be $I/64$. The value of n will be:

(1) 4 (2) 3 (3) 5 (4) 6

Sol. **(D)**

According to Malus law:

$$I = \frac{I_0}{2}\left[\cos^2 45 \times \cos^2 45 \times \cos^2 45 \times ...(n-1)\,\text{times}\right]$$

$$\frac{I_0}{64} = \frac{I_0}{2} \times \left(\frac{1}{2}\right)^{n-1}$$

$$\frac{1}{32} = \frac{1}{2^{n-1}} \Rightarrow \frac{1}{(2)^5} = \frac{1}{2^{n-1}}$$

$$\therefore\ n - 1 = 5$$

$$\boxed{\therefore n = 6}$$

11. Match List I with List II:

List I	List II
A. Microwaves	I. Radio active decay of the nucleus
B. Gamma rays	II. Rapid acceleration and deceleration of electron in aerials
C. Radio waves	III. Inner shell electrons
D. X-rays	IV. Klystron valve

Choose the correct answer from the options given below:

(1) A-I, B-III, C-IV, D-II
(2) A-IV, B-I, C-II, D-III
(3) A-IV, B-III, C-II, D-I
(4) A-I, B-II, C-III, D-IV

Sol. **(B)**

(A) Micro-wave	(IV) Klystron valve
(B) Gamma rays	(I) Radio-active decay of nucleus
(C) Radio-waves	(II) Rapid acceleration and deceleration of electrons in aerials
(D) X-rays	(III) Inner shell electron

12. A proton moving with one tenth of velocity of light has a certain de Broglie wavelength of λ. An alpha particle having certain kinetic energy has the same de-Brogle wavelength λ. The ratio of kinetic energy of proton and that of alpha particle is:

(1) 2: 1 (2) 1: 2 (3) 1: 4 (4) 4: 1

Sol. **(C)**

The wavelength of matter is given by

$$\boxed{\lambda = \frac{h}{p}}$$

$$\frac{\lambda_p}{\lambda_\alpha} = \frac{p_\alpha}{p_p} = \frac{\sqrt{2k_\alpha m_\alpha}}{\sqrt{2k_p m_p}} = 1$$

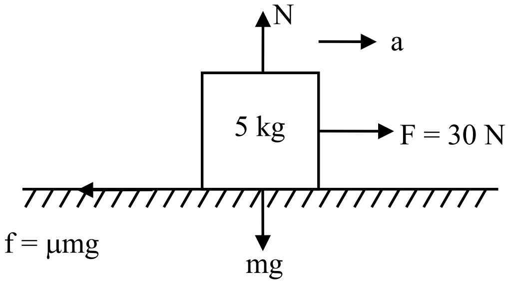
$$\therefore \frac{k_\alpha}{k_p} \times \frac{m_\alpha}{m_p} = 1 \Rightarrow \frac{k_\alpha}{k_p} = \frac{m_p}{m_\alpha}$$

$$\boxed{\frac{k_\alpha}{k_p} = \frac{1}{4}}$$

13. A block of mass 5 kg is placed at rest on a table of rough surface. Now, if a force of 30 N is applied in the direction parallel to surface of the table, the block slides through a distance of 50 m in an interval of time 10 s. Coefficient of kinetic friction is (given, $g = 10\ ms^{-2}$):

(1) 0.60 (2) 0.25 (3) 0.75 (4) 0.50

Sol. **(D)**



$$S = ut + \frac{1}{2}at^2$$

$$50 = 0 \times t + \frac{1}{2} \times a \times (10)^2$$

$$50 = \frac{1}{2} \times a \times 100$$

$$a = \frac{100}{100} \Rightarrow \boxed{a = 1\ m/s^2}$$

$$\sum F_x = ma_x$$

$$30 - \mu mg = ma$$

$$30 - \mu \times 50 = 5$$

$$50\mu = 25$$

$$\mu = \frac{25}{50}$$

$$= \frac{1}{2}$$

$$\Rightarrow \boxed{\mu = 0.5}$$

14. Given below are two statements:

Statement I: Acceleration due to gravity is different at different places on the surface of earth.

Statement II: Acceleration due to gravity increases as we go down below the earth's surface.

In the light of the above statements, choose the correct answer from the options given below

(1) Statement I is false but Statement II is true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are false

(4) Both Statement I and Statement II are true

Sol. **(B)**
Statement (I) is true But
Statement (II) is false

15. Which of the following frequencies does not belong to FM broadcast.

(1) 64MHz (2) 89MHz (3) 99MHz (4) 106MHz

Sol. **(A)**
The Frequencies for FM Broadcast is between 87.5 MHz to 108 MHz.

16. The mass of proton, neutron and helium nucleus are respectively $1.0073u$, $1.0087u$ and $4.0015u$. The binding energy of helium nucleus is:

(1) 28.4MeV (2) 56.8MeV (3) 14.2MeV (4) 7.1MeV

Sol. **(A)**

$2P + 2n = {}_2^4He + E$

$\therefore$ B.E $= [2 \times (1.0073 + 1.0087) - 4.0015] \times 931$

$= 0.0305 \times 931$

$= 28.3955$ MeV

17. A steel wire with mass per unit length 7.0×10^{-3} kg m^{-1} is under tension of 70 N. The speed of transverse waves in the wire will be:

(1) 100 m/s (2) 10 m/s (3) 50 m/s (4) 200πm/s

Sol. **(A)**
The velocity of Transverse wave on string is given by

$$V = \sqrt{\frac{T}{\mu}}$$

$$= \sqrt{\frac{70}{7 \times 10^{-3}}} = \sqrt{\frac{70 \times 10^3}{7}}$$

$$= \sqrt{10^4} = 100 \text{ m/s}$$

18. Match List I with List II:

List I	**List II**
A. Intrinsic semiconductor	I. Fermi-level near the valence band
B. n-type semiconductor	II. . Fermi-level in the middle of valence and conduction band
C. p-type semiconductor	III. Fermi-level near the conduction band
D. Metals	IV. Fermi-level inside the conduction band

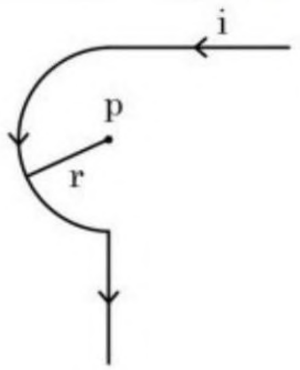
Choose the correct answer from the options given below:

(1) A-II, B-III, C-I, D-IV (2) A-I, B-II, C-III, D-IV
(3) A-II, B-I, C-III, D-IV (4) A-III, B-I, C-II, D-IV

Sol. **(A)**

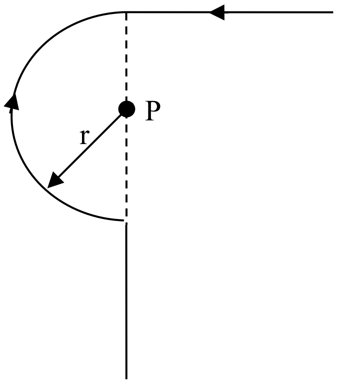
(A) Intrinsic	(II) Fermi-level in the middle of valence and conduction band
(B) n-type semiconductor	(III) Fermi-level near conduction band
(C) p-type semiconductor	(I) Fermi-level near valence band
(D) Metals	(IV) Fermi-level inside the conduction band

19. Find the magnetic field at the point P in figure. The curved portion is a semicircle connected to two long straight wires.



(1) $\frac{\mu_0 i}{2r}\left(1+\frac{2}{\pi}\right)$ (2) $\frac{\mu_0 i}{2r}\left(\frac{1}{2}+\frac{1}{2\pi}\right)$ (3) $\frac{\mu_0 i}{2r}\left(1+\frac{1}{\pi}\right)$ (4) $\frac{\mu_0 i}{2r}\left(\frac{1}{2}+\frac{1}{\pi}\right)$

Sol. **(B)**



$B_P = B_1 + B_2$

$$= \frac{\mu_0 i}{4\pi r} + \frac{\mu_0 i}{4r}$$

$$= \frac{\mu_0 i}{4r}\left[\frac{1}{\pi}+1\right]$$

$$\boxed{B_P = \frac{\mu_0 i}{2r}\left[\frac{1}{2\pi}+\frac{1}{2}\right]}$$

20. The average kinetic energy of a molecule of the gas is
(1) proportional to absolute temperature (2) proportional to pressure
(3) proportional to volume (4) dependent on the nature of the gas

Sol. **(A)**
The average kinetic energy of gas molecule is given by,

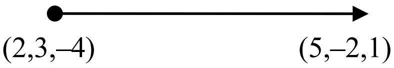
$$K.E_{avg} = \frac{3}{2}KT$$

$$\boxed{\therefore K.E_{avg} \propto T}$$

SECTION - B

21. A small particle moves to position $5\hat{\imath} - 2\hat{\jmath} + \hat{k}$ from its initial position $2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$ under the action of force $5\hat{\imath} + 2\hat{\jmath} + 7\hat{k}$ N. The value of work done will be __________ J.

Sol. **40**



$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$

$$= \left(5\hat{i} - 2\hat{j} + \hat{k}\right) - \left(2\hat{i} + 3\hat{j} - 4\hat{k}\right)$$

$$\overrightarrow{\Delta r} = 3\hat{i} - 5\hat{j} + 5\hat{k}$$

$$\therefore\ W = \vec{F}\cdot\overrightarrow{\Delta r}$$

$$= \left(5\hat{i} + 2\hat{j} + 7\hat{k}\right)\cdot\left(3\hat{i} - 5\hat{j} + 5\hat{k}\right)$$

$$= 15 - 10 + 35$$

$$= 5 + 35$$

$$\boxed{W = 40J}$$

22. A certain pressure 'P' is applied to 1 litre of water and 2 litre of a liquid separately. Water gets compressed to 0.01% whereas the liquid gets compressed to 0.03%. The ratio of Bulk modulus of water to that of the liquid is $\frac{3}{x}$.

The value of x is __________.

Sol. **1**

$$\text{Bulk Modulus} = V\frac{dP}{dV}$$

$$\frac{(B)_{\text{water}}}{(B)_{\text{liquid}}} = \frac{V\ dP/dV}{V\ dP/dV} = \frac{dP/0.01}{dP/0.03}$$

$$\therefore \frac{(B)_{\text{water}}}{(B)_{\text{liquid}}} = \frac{0.03}{0.01} = \frac{3}{1}$$

$$\boxed{\frac{(B)_{\text{water}}}{(B)_{\text{liquid}}} = \frac{3}{1}}$$

$\therefore$ On comparing with $\frac{3}{x}$, The value of "x" will be "1'.

23. A light of energy 12.75eV is incident on a hydrogen atom in its ground state. The atom absorbs the radiation and reaches to one of its excited states. The angular momentum of the atom in the excited state is $\frac{x}{\pi} \times 10^{-17}$eVs. The value of x is __________ (use $h = 4.14 \times 10^{-15}$eVs, $c = 3 \times 10^{8}$ ms^{-1}).

Sol. **x = 828**

The energy of electron in ground state = –13.6 eV

$E_n - E_1 = 12.75$

$\therefore\ E_n = 12.75 - 13.6$

$E_n = -0.85$

So "n" is given by

$$E_n = -\frac{13.6}{n^2}$$

$$n^2 = \frac{-13.6}{-0.85}$$

$n^2 = 16 \Rightarrow \boxed{n = 4}$

$\Rightarrow L = \frac{nh}{2\pi} = \frac{x}{\pi} \times 10^{-17}$

$\Rightarrow 4 \times \frac{h}{2\pi} = \frac{x}{\pi} \times 10^{-17}$

$4 \times \frac{4.14 \times 10^{-15}}{2\pi} = \frac{x}{\pi} \times 10^{-17} \Rightarrow \frac{2 \times 4.14 \times 10^{-15}}{10^{-17}} = x$

$x = 8.28 \times 10^2 \Rightarrow \boxed{x = 828}$

24. A charge particle of $2\mu C$ accelerated by a potential difference of 100 V enters a region of uniform magnetic field of magnitude 4mT at right angle to the direction of field. The charge particle completes semicircle of radius 3 cm inside magnetic field. The mass of the charge particle is ________ $\times 10^{-18}$ kg.

Sol. **144**

$R = \frac{mv}{qB} = \frac{p}{qB}$

$R = \frac{\sqrt{2mq\Delta V}}{qB}$

$3 \times 10^{-2} = \frac{\sqrt{2m \times 2 \times 10^{-6} \times 10^2}}{2 \times 10^{-6} \times 4 \times 10^{-3}}$

$3 \times 10^{-2} \times 2 \times 10^{-6} \times 4 \times 10^{-3} = \sqrt{4m \times 10^{-4}}$

$24 \times 10^{-11} = \sqrt{4m \times 10^{-4}}$

$m = \frac{24 \times 24 \times 10^{-22}}{4 \times 10^{-4}}$

$\boxed{m = 144 \times 10^{-18} Kg}$

25. The amplitude of a particle executing SHM is 3 cm. The displacement at which its kinetic energy will be 25% more than the potential energy is: __________ cm.

Sol. **2**

$K.E = P.E + \frac{25}{100} \times P.E.$

$K.E = P.E + \frac{1}{4} P.E$

$K.E = \frac{5}{4} P.E$

$\frac{1}{2} K\left(A^2 - x^2\right) = \frac{5}{4} \times \frac{1}{2} Kx^2$

$4(A^2 - x^2) = 5x^2$

$4A^2 - 4x^2 = 5x^2$

$9x^2 = 4A^2$

$x^2 = \frac{4}{9} \times (3)^2$

$\boxed{\therefore x = \pm 2}$

26. In an experiment to find emf of a cell using potentiometer, the length of null point for a cell of emf 1.5 V is found to be 60 cm. If this cell is replaced by another cell of emf E, the length-of null point increases by 40 cm. The value of E is $\frac{x}{10}V$. The value of x is ________.

Sol. **25**

$E_1 = K\ell_1$ ...(i)

$E_2 = K\ell_2$ ...(ii)

$$\therefore \frac{E_2}{E_1} = \frac{\ell_2}{\ell_1}$$

$$\frac{E}{1.5} = \frac{100}{60}$$

$$\therefore E = 1.5 \times \frac{10}{6}$$

$$= \frac{3}{2} \times \frac{10}{6}$$

$$= \frac{5}{2}$$

$$= 2.5$$

$$= \frac{25}{10}$$

$$\boxed{\therefore x = 25}$$

27. A thin cylindrical rod of length 10 cm is placed horizontally on the principle axis of a concave mirror of focal length 20 cm. The rod is placed in a such a way that mid point of the rod is at 40 cm from the pole of mirror. The length of the image formed by the mirror will be $\frac{x}{3}$ cm. The value of x is ________.

Sol. **32**

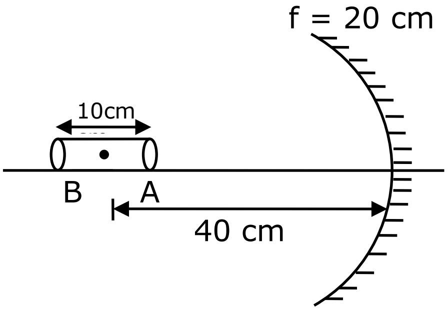


Image of end A:

u = –35 cm

f = –20 cm

v = ?

$$v = \frac{uf}{u - f}$$

$$= \frac{-35 \times -20}{-35 + 20}$$

$$= \frac{-35 \times -20}{-15}$$

$$\boxed{v = -\frac{140}{3}}$$

Image of end B:

u = –45 cm

v = ?

f = – 20 cm

$$v = \frac{uf}{u - f}$$

$$= \frac{-45 \times -20}{-45 + 20}$$

$$= \frac{-45 \times -20}{-25}$$

v = -36

$$\therefore \text{ length of image} = \left| -36 + \frac{140}{3} \right|$$

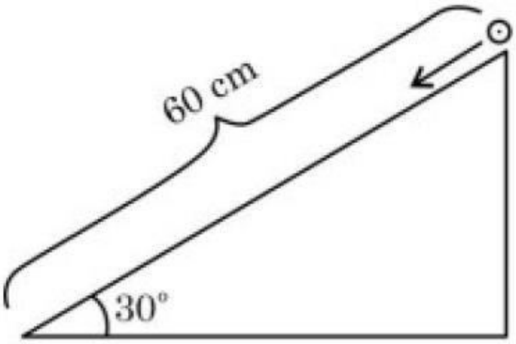
$$= \left| -\frac{108 + 140}{3} \right|$$

$$= \frac{32}{3}$$

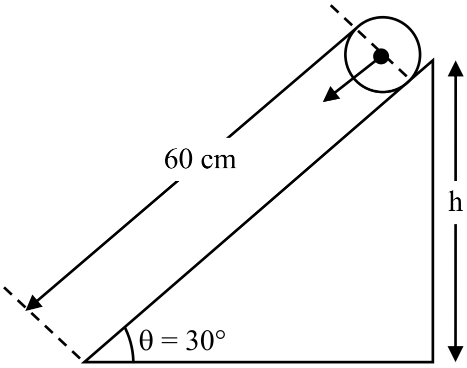
∴ The value of x = 32

28. A solid cylinder is released from rest from the top of an inclined plane of inclination 30° and length 60 cm. If the cylinder rolls without slipping, its speed upon reaching the bottom of the inclined plane is __________ ms^{-1}.

(Given g = 10 ms^{-2})



Sol.



$h = 60 \sin 30$

$\boxed{\therefore\ h = 30\text{ cm}}$

The velocity of by linder upon reaching the ground is given by

$$V = \sqrt{\frac{2gh}{1+\frac{K^2}{R^2}}}$$

$$\therefore V = \sqrt{\frac{2\times10\times30\times10^{-2}}{1+\frac{1}{2}}}$$

$$= \sqrt{\frac{6\times2}{3}}$$

$$\boxed{V = 2\text{ m/s}}$$

29. A series LCR circuit is connected to an ac source of 220 V, 50 Hz. The circuit contain a resistance $R = 100\Omega$ and an inductor of inductive reactance $X_L = 79.6\Omega$. The capacitance of the capacitor needed to maximize the average rate at which energy is supplied will be __________ μF.

Sol. **40**

For maximum power, the LCR must be in resonance.

$\therefore X_L = X_C$

$$79.6 = \frac{1}{\omega C}$$

$$C = \frac{1}{\omega\times79.6}$$

$$= \frac{1}{2\pi\times50\times79.6}$$

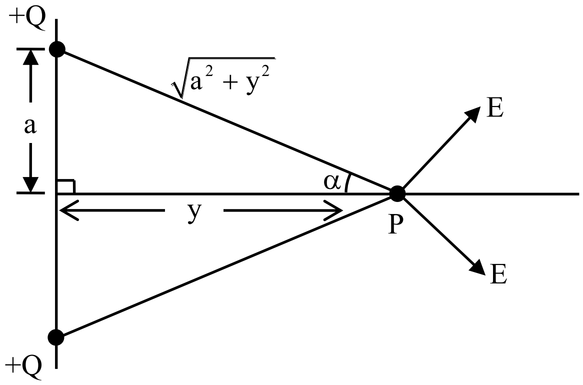
$$= \frac{1}{100\pi\times79.6}$$

$= 40 \times 10^{-6}$

$$\boxed{C = 40\mu F}$$

30. Two equal positive point charges are separated by a distance $2a$. The distance of a point from the centre of the line joining two charges on the equatorial line (perpendicular bisector) at which force experienced by a test charge q_0 becomes maximum is $\frac{a}{\sqrt{x}}$. The value of x is _________.

Sol. **2**



Electric field at point "P" due to any one change = $\frac{KQ}{a^2+y^2}$

∴ Net electric field at point "P" will be

$E_{net} = 2E\cos\alpha$

$$= \frac{2KQ}{a^2+y^2} \times \frac{y}{\sqrt{a^2+y^2}}$$

$$E_{net} = \frac{2KQy}{(a^2+y^2)^{3/2}}$$

⇒ Electric force (F) = $E_{net}\, q_0$

$$= \frac{2K\,Qq_0y}{\left(a^2+y^2\right)^{3/2}}$$

For F = max ⇒ $\frac{dF}{dy} = 0$

By solving, we get $\boxed{y = \frac{a}{\sqrt{2}}}$

∴ the value of $\boxed{x = 2}$

Chemistry

SECTION - A

31. A solution of $FeCl_3$ when treated with $K_4[Fe(CN)_6]$ gives a prussiun blue precipitate due to the formation of

(1) $K[Fe2(CN)_6]$(2) $Fe_4[Fe(CN)_6]_3$(3) $Fe[Fe(CN)_6]$(4) $Fe_3[Fe(CN)_6]_2$

Sol. **2**

$4Fecl_3 + 3K_4\,[Fe(CN)_6]$

$\longrightarrow 12KCl + Fe_4[Fe(CN)_6]_3$

Pursianblue ppt

32. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason **R**

Assertion A: Hydrogen is an environment friendly fuel.

Reason R: Atomic number of hydrogen is 1 and it is a very light element.

In the light of the above statements, choose the correct answer from the options given below

(1) A is true but **R** is false

(2) **A** is false but **R** is true

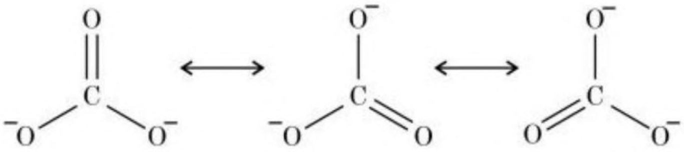
(3) Both **A** and **R** are true and **R** is the correct explanation of **A**

(4) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

Sol. **4**

No pollution occurs by combustion of hydrogen and very low density of hydrogen.

33. Resonance in carbonate ion (CO_3^{2-}) is



Which of the following is true?

(1) All these structures are in dynamic equilibrium with each other.

(2) It is possible to identify each structure individually by some physical or chemical method.

(3) Each structure exists for equal amount of time.

(4) $CO_3{}^{2-}$ has a single structure i.e., resonance hybrid of the above three structures.

Sol. **4**

Resonating structure are hypothtical and resonance hybrid is a real structure which is weighted average of all the resonating struture.

34. Match List I with List II

List I		List II
(A)	Tranquilizers	(I) Anti blood clotting
(B)	Aspirin	(II) Salvarsan
(C)	Antibiotic	(III) antidepressant drugs
(D)	Antiseptic	(IV) soframicine

Choose the correct answer from the options given below:

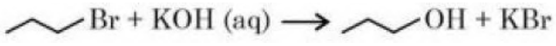
(1) (A) –IV, (B) – II, (C) – I, (D) – III(2) (A) – II, (B) – I, (C) – III, (D) – IV

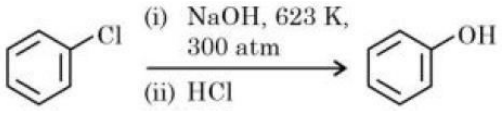
(3) (A) - III, (B) - I, (C) - II, (D) – IV (4) (A) - II, (B) –IV, (C) – I, (D) – III

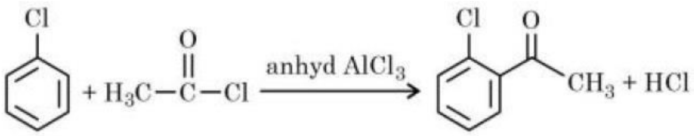
Sol. **3**

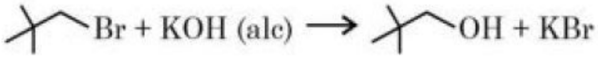
A → (iii)
B → (i)
C → (ii)
D → (iv)

35. Identify the incorrect option from the following:

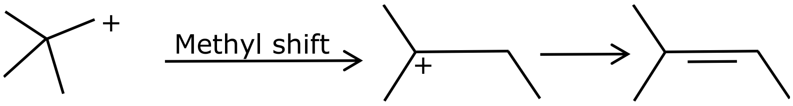
(1) 

(2) 

(3) 

(4) 

Sol. **4**



In question given option reaction is incorrect so right answer is (4)

36. But-2-yne is reacted separately with one mole of Hydrogen as shown below:

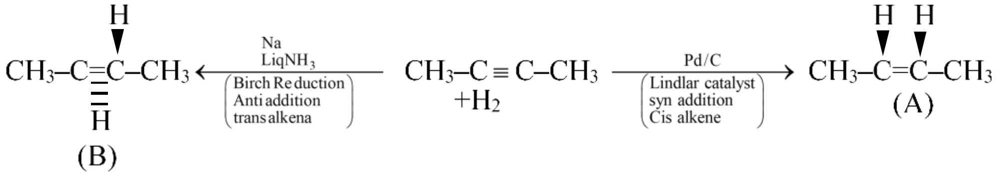
$$\underline{B} \xleftarrow[\text{liq } NH_3]{Na} \underset{+H_2}{CH_3-C\equiv C-CH_3} \xrightarrow[\Delta]{Pd/C} \underline{A}$$

A. A is more soluble than B.
B. The boiling point & melting point of A are higher and lower than B respectively.
C. A is more polar than B because dipole moment of A is zero.
D. Br_2 adds easily to B than A.

Identify the incorrect statements from the options given below:

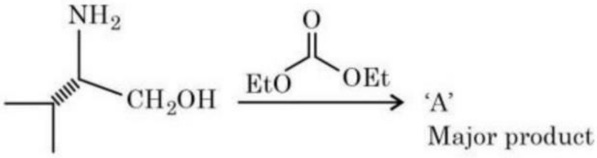
(1) B, C & D only (2) A and B only (3) A, C & D only (4) B and C only

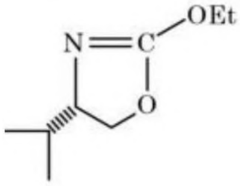
Sol. **2**

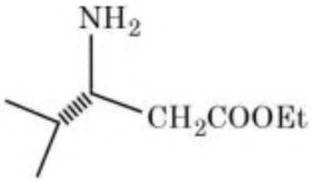


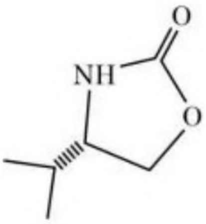
A) Cis has dipole monent, more soluble than trans (B)
B) B.P.(cis > trans), M.P. (trans > cis)
C) Dipole moment (A > B) but $\mu_A \neq 0$
D) Br_2 add easily to A not B

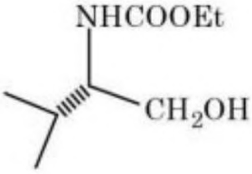
37. In the following reaction, ' A ' is



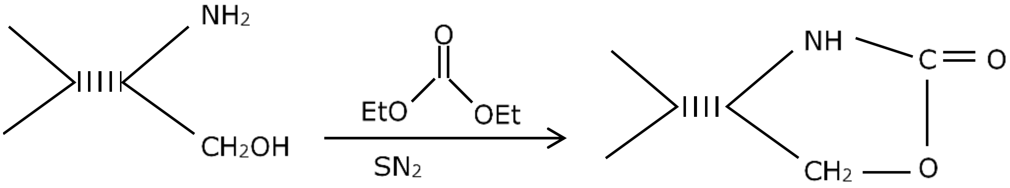
(1)


(2)


(3)


(4)


Sol. **3**



1

38. Highest oxidation state of Mn is exhibited in Mn_2O_7. The correct statements about Mn_2O_7 are

(A) Mn is tetrahedrally surrounded by oxygen atoms.

(B) Mn is octahedrally surrounded by oxygen atoms.

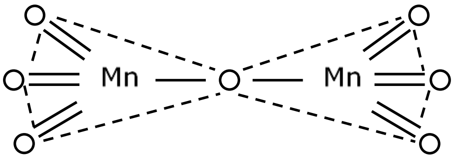
(C) Contains Mn-O-Mn bridge.

(D) Contains Mn-Mn bond.

Choose the correct answer from the options given below:

(1) A and Conly (2) A and D only (3) B and C only (4) B and D only

Sol. **1 (A & C)**



39. Match List I with List II

	List I	List II
(A)	Slaked lime	(I) NaOH
(B)	Dead burnt plaster	(II) $Ca(OH)_2$
(C)	Caustic soda	(III) $Na_2CO_3 \cdot 10H_2O$
(D)	Washing soda	(IV) $CaSO_4$

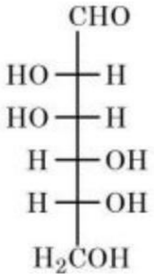
Choose the correct answer from the options given below:

(1) (A) - III, (B) - IV, (C) - II, (D) – I
(2) (A) - III, (B) - II, (C) - IV, (D) – I
(3) (A) - I, (B) - IV, (C) - II, (D) – III
(4) (A) –II, (B) – IV, (C) - I, (D) - III

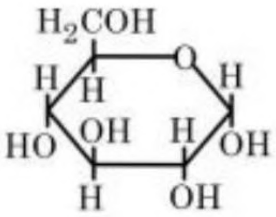
Sol. **4**

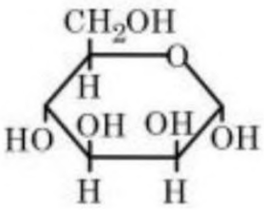
Slaked Lime → $Ca(OH)_2$
Dead burnt plaster → $CaSO_4$
Caustic Soda → $NaOH$
Washing Soda → $Na_2CO_3.10H_2O$

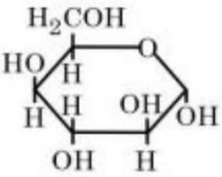
40. The correct representation in six membered pyranose form for the following sugar [X] is

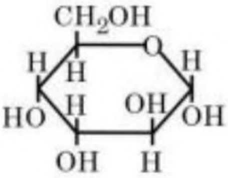


Sugar [X]

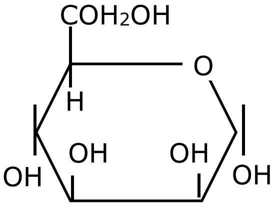
(1)


(2)


(3)


(4)


Sol. **2**



Haworth structure of mannose

41. Which of the following complex will show largest splitting of d-orbitals ?

(1) $[F_eF_6]^{3-}$ (2)$[F_e(C_2O_4)_3]^{3-}$ (3)$[F_e(CN)_6]^{3-}$ (4) $[F_e(NH_3)_6]^{3+}$

Sol. **3**

(M) Strong field ligands will split 'd' orbital largely.

CN^- is SF.L Where as $F^-, C_2O_4^{2-}$ & NH_3

Are comparatively weal feld ligand as common to CN^-

42. Which of the following are the example of double salt?

(A) $FeSO_4 \cdot (NH_4)_2SO_4 \cdot 6H_2O$ (B) $CuSO_4, 4NH_3H_2O$
(C) $K_2SO_4 \cdot Al_2(SO_4)_3 \cdot 24H_2O$ (D) $Fe(CN)_2.4KCN$

Choose the correct answer

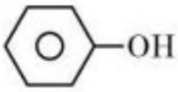
(1) *B* and D only (2) A and C only
(3) A and B only (4) A, B and D only

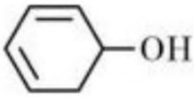
Sol. **1**

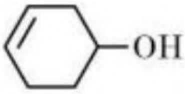
Double salt contain's two or more types of salts.

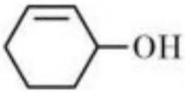
$CuSO_4.4NH_3.H_2O$ and $Fe(CN)_2.4KCN$ are complex compounds.

43. Decreasing order of dehydration of the following alcohols is

(a) 

(b) 

(c) 

(d) 

(1) $b > a > d > c$ (2) $a > d > b > c$ (3) $d > b > c > a$ (4) $b > d > c > a$

Sol. **4**

Ease of hydration α stability of carbocation

$b > d > c > a$

44. Given below are two statements:

Statement I: Chlorine can easily combine with oxygen to form oxides; and the product has a tendency to explode.

Statement II: Chemical reactivity of an element can be determined by its reaction with oxygen and halogens.

In the light of the above statements, choose the correct answer from the options given below

(1) Both the Statements I and II are true

(2) Both the Statements I and II are false

(3) Statement I is false but Statement II is true

(4) Statement I is true but Statement II is false

Sol. **1**

Chlorine oxides, Cl_2O, ClO_2, Cl_2O_6 and Cl_2O_7 are heighly Reactive oxidising Agents and tend to explode.

45. Choose the correct statement(s):

A. Beryllium oxide is purely acidic in nature.

B. Beryllium carbonate is kept in the atmosphere of CO_2.

C. Beryllium sulphate is readily soluble in water.

D. Beryllium shows anomalous behavior.

Choose the correct answer from the options given below:

(1) B, C and D only (2) A only (3) A, B and C only (4) A and B only

Sol. **1**

BeO is Amphoteric

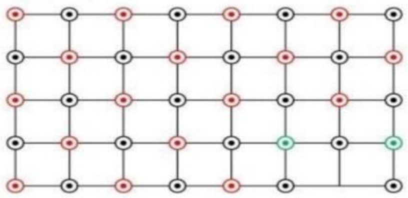
$$\begin{tikzcd} BeCO_3 \arrow[r] & BeO + CO_2 \arrow[l, bend left, "CO_2"] \end{tikzcd}$$

BeSO4 is solube in water

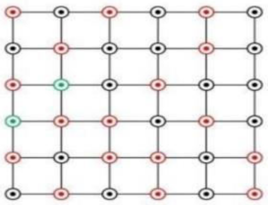
Due to small size Be shows anomalous behaviour.

46. Which of the following represents the lattice structure of $A_{0.95}O$ containing A^{2+}, A^{3+} and O^{2-} ions?
⊙ A^{2+} ⊙ A^{3+} ⊙ O^{2-}

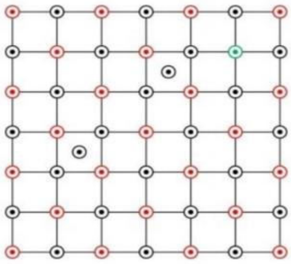
A.



B.



C.



(1) A only (2) B and C only (3) A and B only (4) B only

Sol. **1**

Some vacancy generated by this type defect.

47. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason **R**

Assertion A: In an Ellingham diagram, the oxidation of carbon to carbon monoxide shows a negative slope with respect to temperature.

Reason R: CO tends to get decomposed at higher temperature.

In the light of the above statements, choose the correct answer from the options given below

(1) Both **A** and **R** are correct but **R** is NOT the correct explanation of **A**
(2) Both **A** and **R** are correct and **R** is the correct explanation of **A**
(3) A is correct but **R** is not correct
(4) A is not correct but **R** is correct

Sol. **3**

$2C_{(S)} + O_{2(g)} \longrightarrow 2CO_{(g)}$

$\Delta S°$ is the, $\Delta G° = \Delta H^{e} - T\ \Delta S$

Thus slope is Negative.

As temperature Increase ΔC becomes more Negative thus it has loner tendency to get decomposed.

48. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason **R**

Assertion A: Amongst He, Ne, Ar and Kr; 1 g of activated charcoal adsorbs more of Kr.

Reason R: The critical volume $V_c(\text{ cm}^3\text{ mol}^{-1})$ and critical pressure P_c (atm) is highest for Krypton but the compressibility factor at critical point Z_c is lowest for Krypton.

In the light of the above statements, choose the correct answer from the options given below

(1) **A** is true but **R** is false
(2) Both **A** and **R** are true and **R** is the correct explanation of **A**
(3) A is false but **R** is true
(4) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

Sol. **1**

Assertion A correct but Reason is wrong.

49. Match List I with List II

List I	List II
Test	Functional group / Class of Compound
(A) Molisch's Test	(I) Peptide
(B) Biuret Test	(II) Carbohydrate
(C) Carbylamine Test	(III) Primary amine
(D) Schiff's Test	(IV) Aldehyde

Choose the correct answer from the options given below:

(1) (A) – III, (B) – IV, (C) – I, (D) – II

(2) (A) –II, (B) – I, (C) – III, (D) - IV

(3) (A) –III, (B) – IV, (C) – II, (D) – I

(4) (A) –I, (B) – II, (C) – III, (D) – IV

Sol. **2**

A → (II) C → (III)

B → (I) D → (IV)

50. How can photochemical smog be controlled?

(1) By using catalytic convertors in the automobiles/industry.

(2) By complete combustion of fuel.

(3) By using tall chimneys.

(4) By using catalyst.

Sol. **1**

1) By using catalytic convertors in the automobiles / industry.

51. (i) $X(g) \rightleftharpoons Y(g) + Z(g)\ K_{p1} = 3$

(ii) $A(g) \rightleftharpoons 2\ B(\ g)\ K_{p2} = 1$

If the degree of dissociation and initial concentration of both the reactants X(g) and A(g) are equal, then the ratio of the total pressure at equilibrium $\left(\frac{p_1}{p_2}\right)$ is equal to x : 1. The value of x is___ (Nearest integer)

Sol. $x(g) \rightleftharpoons y(g) + z(g) \qquad Kp_1 = 3$

$t = 0 \quad 1 \quad 0 \quad 0$

$teq \quad 1 - x \quad x \quad x$

Partial Pressure $\quad \frac{(1-x)}{1+x} P_1 \quad \frac{xP_1}{1+x} \quad \frac{xP_1}{1+x}$

$A(g) \rightleftharpoons 2B(g)$

$t = 0 \quad 1 \quad 0$

$teq \quad 1 - x \quad 2x$

Partial Pressure $\quad \frac{1-x}{1+x} \times P_2 \quad \frac{2x}{1+x} \times P_2$

$$KP_1 = \frac{\left(\frac{xP_1}{1+x}\right)\left(\frac{xP_1}{1+x}\right)}{\left(\frac{1-x}{1+x}P_1\right)}$$

$$Kp_2 = \frac{(2x)^2 \times P_2^{\,2}}{\left(\frac{1-x}{1+x}\right)P_2}$$

$$\frac{KP_1}{KP_2} = \frac{3}{1} = \frac{P_1}{4P_2}$$

$$\frac{P_1}{P_2} = \frac{12}{1}$$

52. Electrons in a cathode ray tube have been emitted with a velocity of 1000 m s^{-1}. The number of following statements which is/are true about the emitted radiation is

Given : $h = 6 \times 10^{-34}\text{Js}, m_e = 9 \times 10^{-31}$ kg.

(A) The deBroglie wavelength of the electron emitted is 666.67 nm.

(B) The characteristic of electrons emitted depend upon the material of the electrodes of the cathode ray tube.

(C) The cathode rays start from cathode and move towards anode.

(D) The nature of the emitted electrons depends on the nature of the gas present in cathode ray tube.

Sol. **2**

(A) $\lambda = \frac{h}{mv} = \frac{6 \times 10^{-34}}{9 \times 10^{-31} \times 1000}$

$= 666.67 \times 10^{-9}\text{m}$

(C) The cathode ray start from Cathode and move towards anode.

53. A and B are two substances undergoing radioactive decay in a container.

The half life of A is 15 min and that of B is 5 min. If the initial concentration of B is 4 times that of A and they both start decaying at the same time, how much time will it take for the concentration of both of them to be same?_____min.

Sol. **15**

Condition $\Rightarrow$ [B] = 4[A]

For A $\quad A \xrightarrow[15\,\text{min}]{t_{1/2}} \frac{A}{2}$

For B $\quad 4A \xrightarrow[15\,\text{min}]{t_{1/2}} 2A \frac{t_{1/2}}{5\,\text{min}} \; A \frac{t_{1/2}}{5\,\text{min}} \; A/2$

54. Sum of oxidation states of bromine in bromic acid and perbromic acid is

Sol. **12**

Bromic Acid → $HBrO_5$ → + 5

Perbromic Acid → $HBrO_7$ → + 7

Sum of oxidation state = 5 + 7 = 12

55. 25 mL of an aqueous solution of KCl was found to require 20 mL of 1M $AgNO_3$ solution when titrated using K_2CrO_4 as an indicator. What is the depression in freezing point of KCl solutions of the given concentration?______(Nearest integer).

(Given: $K_f = 2.0\ K\ kg\ mol^{-1}$)

Assume 1) 100% ionization and

2) density of the aqueous solution as $1\ g\ mL^{-1}$

Sol. **3**

$$\begin{tikzcd} KCl \arrow[d] & + & AgNO_3 \arrow[d] \arrow[r] & AgCl + KNO_3 \\ V = 25ml & & V = 20ml, M = 1M & \end{tikzcd}$$

At equivalence point,

Mmole of KCl = mmole of $AgNO_3$ = 20 mmole

Volume of solution = 25 ml

Mass of solution = 25 gm

Mass of solvent = 25 – mass of solute

$= 25 - [20 \times 10^{-3} \times 74.5]$

$= 23.51$ gm

$$\text{Molality of KCl} = \frac{\text{mole of KCl}}{\text{mass of solvent in kg}}$$

$$= \frac{20 \times 10^{-3}}{23.51 \times 10^{-3}} = 0.85$$

i of KCl = 2 (100% ionisation)

$\Delta T_f = i \times K_f \times m$

$= 2 \times 2 \times 0.85$

$= 3.4$

$\simeq 3$

56. At 25°C, the enthalpy of the following processes are given:

$H_2(g) + O_2(g) \rightarrow 2OH(g) \quad \Delta H^\circ = 78\ kJ\ mol^{-1}$

$H_2(g) + 1/2O_2(g) \rightarrow H_2O(g) \quad \Delta H^\circ = -242\ kJ\ mol^{-1}$

$H_2(g) \rightarrow 2H(g) \quad \Delta H^\circ = 436\ kJ\ mol^{-1}$

$1/2O_2(g) \rightarrow O(g) \quad \Delta H^\circ = 249\ kJ\ mol^{-1}$

What would be the value of X for the following reaction? (Nearest integer)

$H_2O(g) \rightarrow H(g) + OH(g) \Delta H^\circ = XkJmol^{-1}$

Sol. **499**

$2H_2O(g) \rightarrow H_2(g) + 2(g) \quad +(242 \times 2)$

$H_2(g) + O_2(g) \rightarrow 2OH \quad +78$

$H_2(g) \rightarrow H_2 \quad +436$

$2H_2O \rightarrow 2H + 2OH \quad +998KJ/mole$

$H_2O \rightarrow H + OH \quad 998 \times \frac{1}{2} = +499KJ/mole$

57. At what pH, given half cell $MnO_4^-(0.1M) \mid Mn^{2+}(0.001M)$ will have electrode potential of $1.282\ V$? (Nearest Integer)

Given $E^o_{MnO_4^-|Mn^{+2}} = 1.54\ V, \frac{2.303RT}{F} = 0.059\ V$

Sol. **3**

$$MnO_4^- + 84^{\oplus} + 5e^{\ominus} \rightleftharpoons Mn^{+2} + 4H_2O$$

$$E = E^{\circ} - \frac{0.059}{5}\log\frac{\left[mn^{+2}\right]}{\left[mnO_4^-\right]\left[H^+\right]^8}$$

$$1.282 = 1.54 - \frac{0.059}{5}\log\frac{10^{-3}}{10^{-1}\times\left[H^+\right]}$$

$$\frac{0.258\times 5}{0.059} = \log\frac{10^{-2}}{\left[H^+\right]^8}$$

$$21.86 = -2 + 8pH$$

$$pH = 2.98 = 3$$

58. The density of $3M$ solution of $NaCl$ is $1.0\ g\ mL^{-1}$. Molality of the solution is $\times 10^{-2}\ m$. (Nearest integer).

Given: Molar mass of Na and Cl is 23 and $35.5\ g\ mol^{-1}$ respectively.

Sol. **364**

$$m = \frac{1000\times M}{1000d - M\times M.wt} = \frac{1000\times 3}{1000\times 1 - (3\times 58.5)} = 3.64$$

$$= 364 \times 10^{-2}$$

59. Number of isomeric compounds with molecular formula $C_9H_{10}O$ which (i)do not dissolve in $NaOH$ (ii)do not dissolve in HCl. (iii)do not give orange precipitate with 2,4DNP (iv)on hydrogenation give identical compound with molecular formula $C_9H_{12}O$ is

Sol. **2**

$$\underset{D.O.U. = 5}{C_9H_{10}O} \xrightarrow{\qquad} \underset{D.O.U. = 4}{C_9H_{12}O}$$

Do not dissolve in NaOH, So no acidic group

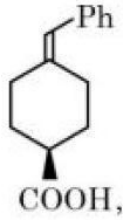
Do not dissolve in HCl, So no basic group, no alkene

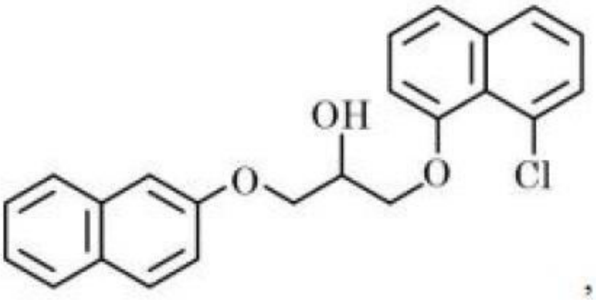
Do not give orange PPT with 2, 4-DNP so no carbonyl group

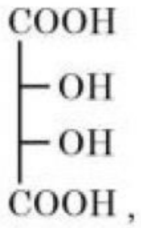
Possible compounds – cis and trans of $Ph - CH = CH - O - CH_3$

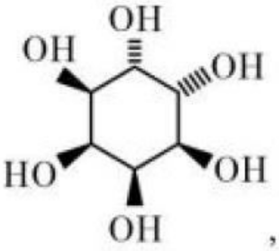
(Also Many possible products are there)

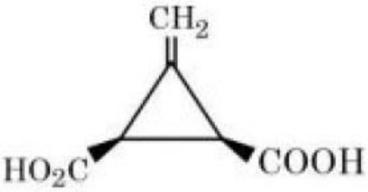
60. The total number of chiral compound/s from the following is



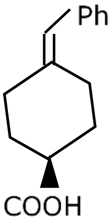




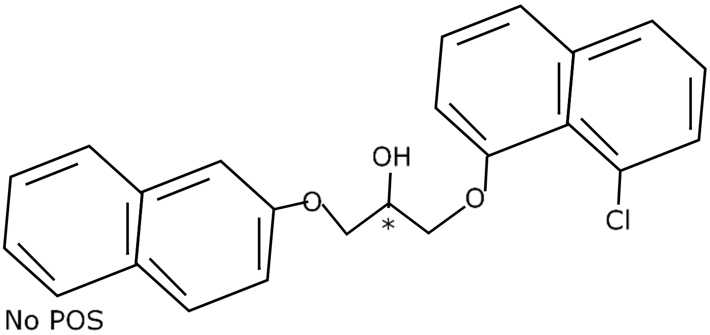




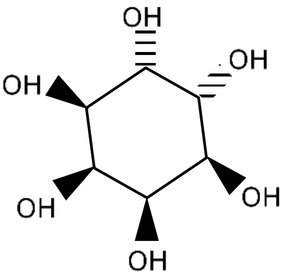
Sol.



(1) No POS
No COS



(2) No POS
No COS



No POS
No COS

(3)

Note :- Take note from gammaxene structure

Mathematics

Section A

61. If $y = y(x)$ is the solution curve of the differential equation $\frac{dy}{dx} + y\tan x = x\sec x, 0 \le x \le \frac{\pi}{3}, y(0) = 1$, then $y\left(\frac{\pi}{6}\right)$ is equal to

(1) $\frac{\pi}{12} - \frac{\sqrt{3}}{2}\log_e\left(\frac{2\sqrt{3}}{e}\right)$ (2) $\frac{\pi}{12} - \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$

(3) $\frac{\pi}{12} + \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$ (4) $\frac{\pi}{12} + \frac{\sqrt{3}}{2}\log_e\left(\frac{2\sqrt{3}}{e}\right)$

Sol. **2**

Given D.E. is linear D.E.

I.F. = $e^{\int \tan x dx}$

= $e^{\ell n \sec x}$ = secx

Solution is –

y secx = $\int x\sec^2 x\,dx$

= x tanx – $\int \tan x\,dx$

⇒ y secx = xtanx – ℓn secx + c

Put y(0) = 1

1 = 0 – 0 + c ⇒ c = 1

$$Y(x) = \frac{x\tan x}{\sec x} - \frac{\ell n \sec x}{\sec x} + \frac{1}{\sec x}$$

$$y\left(\frac{\pi}{6}\right) = \frac{\left(\frac{\pi}{6}\right)\left(\frac{1}{\sqrt{3}}\right)}{\left(\frac{2}{\sqrt{3}}\right)} - \frac{\ell n\left(\frac{2}{\sqrt{3}}\right)}{\left(\frac{2}{\sqrt{3}}\right)} + \frac{\sqrt{3}}{2}$$

$$= \frac{\pi}{12} - \frac{\sqrt{3}}{2}\ell n\left(\frac{2}{\sqrt{3}}\right) + \frac{\sqrt{3}}{2}\ell ne$$

$$= \frac{\pi}{12} - \frac{\sqrt{3}}{2}\ell n\left(\frac{2}{e\sqrt{3}}\right)$$

62. Let R be a relation on ℝ, given by

$$R = \{(a, b): 3a - 3b + \sqrt{7} \text{ is an irrational number }\}.$$

Then R is

(1) an equivalence relation
(2) reflexive and symmetric but not transitive
(3) reflexive but neither symmetric nor transitive
(4) reflexive and transitive but not symmetric

Sol. **3**

(a, a) ∈ R ⇒ 3a – 3a + $\sqrt{7}$

= $\sqrt{7}$ (irrational)

⇒ R is reflexive

Let a = $\frac{2\sqrt{7}}{3}$ and b = $\frac{\sqrt{7}}{3}$

(a, b) ∈ R ⇒ 2$\sqrt{7}$ – $\sqrt{7}$ + $\sqrt{7}$

$= 2\sqrt{7}$ (irration)

(b, a) $\in$ R $\Rightarrow \sqrt{7} - 2\sqrt{7} + \sqrt{7}$

$= 0$ (rational)

$\Rightarrow$ R is no symmetric

Let a $= \dfrac{2\sqrt{7}}{3}$, b $= \dfrac{\sqrt{7}}{3}$, C $= \dfrac{3\sqrt{7}}{3}$

(a ; b) $\in$ R $\Rightarrow 2\sqrt{7}$ (irrational)

(b ; c) $\in$ R $\Rightarrow \sqrt{7}$ (irrational)

(a, c) $\in$ R $\Rightarrow 2\sqrt{7} - 3\sqrt{7} + \sqrt{7}$

$= 0$ (rational)

R is not transitive

$\Rightarrow$ R is reflexive but neither symmetric nor transitive

63. For a triangle ABC, the value of $\cos 2A + \cos 2B + \cos 2C$ is least. If its inradius is 3 and incentre is M, then which of the following is NOT correct?

(1) perimeter of $\triangle ABC$ is $18\sqrt{3}$

(2) $\sin 2A + \sin 2B + \sin 2C = \sin A + \sin B + \sin C$

(3) $\overrightarrow{MA} \cdot \overrightarrow{MB} = -18$

(4) area of $\triangle ABC$ is $\frac{27\sqrt{3}}{2}$

Sol. **4**

Let P $= \cos 2A + \cos 2B + \cos 2C$

$= 2\cos(A + B)\cos(A - B) + 2\cos^2 C - 1$

$= 2\cos(\pi - C)\cos(A - B) + 2\cos^2 C - 1$

$= -2\cos C[\cos(A - B) + \cos(A + B)] - 1$

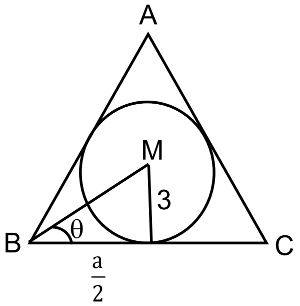
$= -1 - 4\cos A \cos B \cos C$

for P to be minimum

cosA cosB cos C must be maximum

$\Rightarrow \Delta ABC$ is equilateral triangle.

Let side length of triangle is a



$$\tan\theta = \frac{3}{a/2}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{6}{a} \quad \Rightarrow a = 6\sqrt{3}$$

area of triangle $= \dfrac{\sqrt{3}}{4}a^2$

$= \dfrac{\sqrt{3}}{4}(108) = 27\sqrt{3}$

64. Let S be the set of all solutions of the equation $\cos^{-1}(2x) - 2\cos^{-1}(\sqrt{1-x^2}) = \pi, x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$. Then $\sum_{x \in S} 2\sin^{-1}(x^2-1)$ is equal to

(1) $\pi - 2\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (2) $\pi - \sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

(3) $\frac{-2\pi}{3}$ (4) 0

Sol. **Bonus**

$\cos^{-1}(2x) = \pi + 2\cos^{-1}\sqrt{1-x^2}$

Since $\cos^{-1}(2x) \in [0,\pi]$

R.H.S. $\geq \pi$

$\pi + 2\cos^{-1}\sqrt{1-x^2} = \pi$

$\Rightarrow \cos^{-1}\sqrt{1-x^2} = 0$

$\Rightarrow \sqrt{1-x^2} = 1$

$\Rightarrow x = 0$

but at x = 0

$\cos^{-1}(2x) = \cos^{-1}(0) = \frac{\pi}{2}$

no solution possible for given equation.

$x \in \phi$

65. Let S denote the set of all real values of λ such that the system of equations

$\lambda x + y + z = 1$

$x + \lambda y + z = 1$

$x + y + \lambda z = 1$

is inconsistent, then $\sum_{\lambda \in S} (|\lambda|^2 + |\lambda|)$ is equal to

(1) 4 (2) 12 (3) 6 (4) 2

Sol. **3**

Given system of equation is inconsistent

$\Rightarrow \Delta = 0$

$$\begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda^3 - 3\lambda + 2 = 0$

$\Rightarrow (\lambda-1)^2 (\lambda+2) = 0$

$\Rightarrow \lambda = 1, -2$

But for $\lambda = 1$ all planes are same

Then $\lambda = -2$

$\sum_{\lambda \in s} (|\lambda|^2 + |\lambda|) = 4 + 2 = 6$

66. In a binomial distribution B(n,p), the sum and the product of the mean and the variance are 5 and 6 respectively, then 6(n+p–q) is equal to

(1) 52 (2) 50 (3) 51 (4) 53

Sol. **1**

Given

$np + npq = 5$

$\Rightarrow np(1 + q) = 5$ ….(i)

and $(np)(npq) = 6$

$\Rightarrow n^2p^2q = 6$ ….(ii)

$(i)^2 \div (ii)$

$$\frac{(1+9)^2}{9} = \frac{25}{6}$$

$\Rightarrow 6q^2 - 13q + 6 = 0$

$\Rightarrow q = \frac{2}{3}, \frac{3}{2}$ (reject)

$P = 1 - \frac{2}{3} = \frac{1}{3}$

$$\frac{n}{3}\left(1+\frac{2}{3}\right) = 5$$

$\Rightarrow n = 9$

$6(n + p - q) = 52$

67. The combined equation of the two lines $ax + by + c = 0$ and $a'x + b'y + c' = 0$ can be written as $(ax + by + c)(a'x + b'y + c') = 0$.

The equation of the angle bisectors of the lines represented by the equation

$2x^2 + xy - 3y^2 = 0$ is

(1) $x^2 - y^2 - 10xy = 0$ (2) $x^2 - y^2 + 10xy = 0$

(3) $3x^2 + 5xy + 2y^2 = 0$ (4) $3x^2 + xy - 2y^2 = 0$

Sol. **1**

For pair of st. liens in form

$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$

equation of angle bisector is

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

for $2x^2 + xy - 3y^2 = 0$

$a = 2, b = -3, h = \frac{1}{2}$

equation of angle bisector is

$$\frac{x^2 - y^2}{5} = \frac{xy}{1/2}$$

$\Rightarrow x^2 - y^2 - 10xy = 0$

68. The area enclosed by the closed curve C given by the differential equation $\frac{dy}{dx}+\frac{x+a}{y-2}=0, y(1)=0$ is 4π.

Let P and Q be the points of intersection of the curve C and the y-axis. If normals at P and Q on the curve C intersect x-axis at points R and S respectively, then the length of the line segment RS is

(1) 2 (2) $\frac{4\sqrt{3}}{3}$ (3) $2\sqrt{3}$ (4) $\frac{2\sqrt{3}}{3}$

Sol. **2**

$$\frac{dy}{dx}+\frac{x+\alpha}{y-2}=0, y(1)=0$$

$$\frac{dy}{dx}=\frac{-(x+\alpha)}{y-2}$$

$$\int(y-2)dy=-\int(x+\alpha)dx$$

$$\frac{y^2}{2}-2y=-\left[\frac{x^2}{2}+\alpha x\right]+\lambda$$

$y(1) = 0$

$x = 1 \Rightarrow y = 0$

$$0-0=-\left[\frac{1}{2}+\alpha\right]+\lambda$$

$$\frac{y^2}{2}-2y=-\left[\frac{x^2}{2}+\alpha x\right]+\frac{1}{2}+\alpha$$

$$\frac{x^2+y^2}{2}=2y-\alpha x+\frac{1}{2}+\alpha$$

$x^2 + y^2 + 2\alpha x - 4y - 1 - 2\alpha = 0$

Area $= 4\pi$

$\pi r^2 = 4\pi$

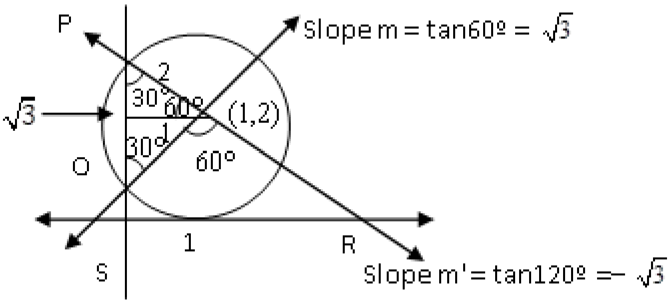
$r^2 = 4$

$\alpha^2 + 4 + 1 + 2\alpha = 4$

$\alpha^2 + 2\alpha + 1 = 0$

$(\alpha + 1)^2 = 0 \Rightarrow [\alpha = -1]$

$x^2 + y^2 - 2x - 4y + 1 = 0$



$y - 2 = \sqrt{3}(x-1)$ $y - 2 = -\sqrt{3}(x-1)$

$y = 0$ $y = 0$

$\frac{-2}{\sqrt{3}}=x-1$ $\qquad$ $1+\frac{2}{\sqrt{3}}=x$

$1-\frac{2}{\sqrt{3}}=x$ $\qquad$ $R\left(1+\frac{2}{\sqrt{3}},0\right)$

$S\left(1-\frac{2}{\sqrt{3}},0\right)$

$$\text{RS} = \left(1+\frac{2}{\sqrt{3}}\right)-\left(1-\frac{2}{\sqrt{3}}\right) = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$$

69. The value of

$\frac{1}{1!50!}+\frac{1}{3!48!}+\frac{1}{5!46!}+\cdots.+\frac{1}{49!2!}+\frac{1}{51!1!}$ is :

(1) $\frac{2^{50}}{51!}$ (2) $\frac{2^{51}}{50!}$ (3) $\frac{2^{50}}{50!}$ (4) $\frac{2^{51}}{51!}$

Sol. **1**

$$S = \frac{1}{1!50!}+\frac{1}{3!48!}+\frac{1}{5!46!}+\ldots\ldots+\frac{1}{49!2!}+\frac{1}{51!1!}$$

$$= \frac{1}{51!1!}\left(\frac{51!}{1!50!}+\frac{51!}{3!48!}+\frac{51!}{5!46!}+\ldots\ldots+\frac{51!}{49!2!}+\frac{51!}{51!0!}\right)$$

$$= \frac{1}{51!1!}\left({}^{51}C_{50}+{}^{51}C_{48}+{}^{51}C_{46}+\ldots\ldots.+{}^{51}C_2+{}^{51}C_0\right)$$

$$\because {}^nC_0+{}^nC_2+{}^nC_4+\ldots\ldots.=2^{n-1}$$

$$S = \frac{2^{50}}{51!}$$

70. The mean and variance of 5 observations are 5 and 8 respectively. If 3 observations are 1, 3, 5 then the sum of cubes of the remaining two observations is

(1) 1216 (2) 1072 (3) 1456 (4) 1792

Sol. **2**

Let remaining two observations are a and b

$$5=\frac{1+3+5+a+b}{5}$$

$\Rightarrow$ a + b = 16 …(i)

$$8=\frac{1^2+3^2+5^2+a^2+b^2}{5}-25$$

$\Rightarrow$ $a^2 + b^2 = 130$

$a^2 + b^2 = 130$ …(ii)

$(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow$ 256 = 130 + 2ab

ab = 63

$a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

$= (16)^3 - 3(63)(16)$

$= 4096 - 3024$

$\Rightarrow$ $a^3 + b^3 = 1072$

71. The sum to 10 terms of the series $\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+\cdots$. is

(1) $\frac{55}{111}$ (2) $\frac{56}{111}$ (3) $\frac{58}{111}$ (4) $\frac{59}{111}$

Sol. **1**

$$T_n=\frac{n}{1+n^2+n^4}$$

$$=\frac{n}{(n^2-n+1)(n^2+n+1)}$$

$$=\frac{1}{2}\left[\frac{(n^2+n+1)-(n^2-n+1)}{(n^2-n+1)(n^2+n+1)}\right]$$

$$\Rightarrow T_n=\frac{1}{2}\left[\frac{1}{(n^2-n+1)}-\frac{1}{(n^2+n+1)}\right]$$

$$S_n=\sum_{n=1}^{10}T_n$$

$$=\frac{1}{2}\sum\left(\frac{1}{n^2-n+1}-\frac{1}{n^2+n+1}\right)$$

$$=\frac{1}{2}\left[\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{13}\right)\right.$$

$$\left.\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+\left(\frac{1}{91}-\frac{1}{111}\right)\right]$$

$$=\frac{1}{2}\left[1-\frac{1}{111}\right]=\frac{55}{111}$$

72. The shortest distance between the lines $\frac{x-5}{1}=\frac{y-2}{2}=\frac{z-4}{-3}$ and $\frac{x+3}{1}=\frac{y+5}{4}=\frac{z-1}{-5}$ is

(1) $5\sqrt{3}$ (2) $7\sqrt{3}$ (3) $6\sqrt{3}$ (4) $4\sqrt{3}$

Sol. **3**

$$L_1:\frac{x-5}{1}=\frac{y-2}{2}=\frac{z-4}{-3}$$

$$\vec{a_1}=5\hat{i}+2\hat{j}+4\hat{k}$$

$$\vec{r_1}=\hat{i}+2\hat{j}-3\hat{k}$$

$$L_2:\frac{x+3}{1}=\frac{y+5}{4}=\frac{z-1}{-5}$$

$$\vec{a_2}=-3\hat{i}-5\hat{j}+\hat{k}$$

$$\vec{r_2}=\hat{i}+4\hat{j}-5\hat{k}$$

$$\vec{r_1}\times\vec{r_2}=\begin{vmatrix}\hat{i} & j & k\\ 1 & 2 & -3\\ 1 & 4 & -5\end{vmatrix}$$

$$=2\hat{i}+2\hat{j}+2\hat{k}$$

Shortest distance (d) = $\left|\dfrac{(\vec{r_1}\times\vec{r_2}).(\vec{a_1}-\vec{a_2})}{|\vec{r_1}\times\vec{r_2}|}\right|$

$= \dfrac{36}{2\sqrt{3}} = 6\sqrt{3}$

73. $\lim\limits_{n\to\infty}\left[\frac{1}{1+n}+\frac{1}{2+n}+\frac{1}{3+n}+\cdots+\frac{1}{2n}\right]$ is equal to

(1) $\log_e 2$ (2) $\log_e\left(\frac{3}{2}\right)$ (3) $\log_e\left(\frac{2}{3}\right)$ (4) 0

Sol. **1**

$\lim\limits_{n\to\infty}\left[\dfrac{1}{1+n}+\dfrac{1}{2+n}+\dfrac{1}{3+n}+......+\dfrac{1}{2n}\right]$

$= \lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\dfrac{1}{r+n}$

$= \lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\dfrac{1}{n}\left(\dfrac{1}{\frac{r}{n}+1}\right)$

$= \int_0^1 \dfrac{dx}{x+1}$

$= \log_e(1+x)\Big|_0^1$

$= \log_e^2$

74. Let the image of the point $P(2,-1,3)$ in the plane $x+2y-z=0$ be Q. Then the distance of the plane $3x+2y+z+29=0$ from the point Q is

(1) $\frac{24\sqrt{2}}{7}$ (2) $2\sqrt{14}$ (3) $3\sqrt{14}$ (4) $\frac{22\sqrt{2}}{7}$

Sol. **3**

let Q(α, β, γ) is image of P(2, –1, 3) in the plane x + 2y – z = 0

$\dfrac{\alpha-2}{1} = \dfrac{\beta+1}{2} = \dfrac{\gamma-3}{-1} = \dfrac{-2(2-2-3)}{1^2+2^2+(-1)^2} = 1$

α = 3, β = 1, γ = 2

Distance of Q(3, 1, 2) from

3x + 2y + z + 29 = 0

$D = \dfrac{|3(3)+2(1)+2+29|}{\sqrt{3^2+2^2+1^2}}$

$= \dfrac{42}{\sqrt{14}} = 3\sqrt{14}$

75. Let $f(x) = 2x + \tan^{-1} x$ and $g(x) = \log_e(\sqrt{1+x^2} + x), x \in [0,3]$.
Then
(1) $\min f'(x) = 1 + \max g'(x)$
(2) $\max f(x) > \max g(x)$
(3) there exist $0 < x_1 < x_2 < 3$ such that $f(x) < g(x), \forall x \in (x_1, x_2)$
(4) there exists $\hat{x} \in [0,3]$ such that $f'(\hat{x}) < g'(\hat{x})$

Sol. **2**

$$f'(x) = 2 + \frac{1}{1+x^2} > 0 \quad \text{for } x \in [0, 3]$$

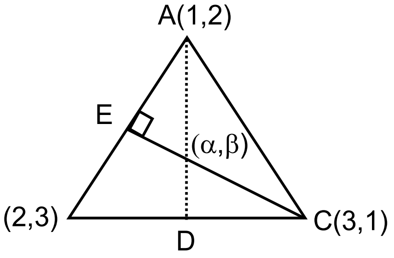
$f(x)\uparrow$ for $x \in [0, 3]$
$f(0) = 0, f(3) = 6 + \tan^{-1}(3)$

$$g'(x) = \frac{\frac{x}{\sqrt{x^2+1}} + 1}{x + \sqrt{1+x^2}} = \frac{1}{\sqrt{1+x^2}} > 0 \qquad \text{for } x \in [0, 3]$$

$g(x) \uparrow$ for $x \in [0, 3]$
$g(0) = 0, g(3) = log_e\left(\sqrt{10} + 3\right)$
max $f(x)$ > max $g(x)$
Option (2) correct

76. If the orthocentre of the triangle, whose vertices are (1,2) (2,3) and (3,1) is (α,β), then the quadratic equation whose roots are $\alpha + 4\beta$ and $4\alpha + \beta$, is
(1) $x^2 - 20x + 99 = 0$
(2) $x^2 - 19x + 90 = 0$
(3) $x^2 - 22x + 120 = 0$
(4) $x^2 - 18x + 80 = 0$

Sol. **1**



equation of AD : $x - 2y + 3 = 0$
equation of CE : $x + y - 4 = 0$
orthocenter (α, β) is $\left(\frac{5}{3}, \frac{7}{3}\right)$
$\alpha + 4\beta = 11$ and $4\alpha + \beta = 9$
Quadratic equation is
$x^2 - (11 + 9)x + (11 \times 9) = 0$
$\Rightarrow x^2 - 20x + 99 = 0$

77. Let $S = \{x: x \in \mathbb{R} \text{ and } (\sqrt{3}+\sqrt{2})^{x^2-4} + (\sqrt{3}-\sqrt{2})^{x^2-4} = 10\}$

Then $n(S)$ is equal to

(1) 4 (2) 0 (3) 6 (4) 2

Sol. **1**

$$\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4} + \left(\sqrt{3}-\sqrt{2}\right)^{x^2-4} = 10$$

$$\Rightarrow \left(\sqrt{3}+\sqrt{2}\right)^{x^2-4} + \frac{1}{\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}} = 10$$

Let $\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4} = t$

$$t + \frac{1}{t} = 10$$

$\Rightarrow t^2 - 10t + 1 = 0$

$t = 5+2\sqrt{6},\ 5-2\sqrt{6}$

If $t = 5+2\sqrt{6}$

$\left(\sqrt{3}+\sqrt{2}\right)^{x^{2-4}} = \left(\sqrt{3}+\sqrt{2}\right)^2$

$\Rightarrow x^2 - 4 = 2$

$\Rightarrow x = \pm\sqrt{6}$

If $t = 5-2\sqrt{6}$

$\left(\sqrt{3}+\sqrt{2}\right)^{x^{2-4}} = \left(\sqrt{3}+\sqrt{2}\right)^{-2}$

$\Rightarrow x^2 - 4 = -2$

$\Rightarrow x = \pm\sqrt{2}$

$S = \left\{\sqrt{6}, -\sqrt{6}, \sqrt{2}, -\sqrt{2}\right\}$

n (s) = 4

78. If the center and radius of the circle $\left|\frac{z-2}{z-3}\right| = 2$ are respectively (α,β) and γ.

then $3(\alpha + \beta + \gamma)$ is equal to

(1) 11 (2) 12 (3) 9 (4) 10

Sol. **2**

Put $z = x + iy$

$$\frac{|(x-2)+iy|}{|(x-3)+iy|} = 2$$

$\Rightarrow (x-2)^2 + y^2 = 4((x-3)^2 + y^2)$

$\Rightarrow x^2 - 4x + 4 + y^2 = 4x^2 - 24x + 36 + 4y^2$

$\Rightarrow 3x^2 + 3y^2 - 20x + 32 = 0$

$$\Rightarrow x^2 + y^2 - \frac{20}{3}x + \frac{32}{3} = 0$$

Center $(\alpha, \beta) = \left(\frac{10}{3}, 0\right)$

Radius $(\gamma) = \sqrt{\left(-\frac{10}{3}\right)^2 - \frac{32}{3}} = \frac{2}{3}$

$$3\left(\frac{10}{3} + 0 + \frac{2}{3}\right) = 12$$

79. Let $f(x)=\begin{vmatrix}1+\sin^2 x & \cos^2 x & \sin 2x\\ \sin^2 x & 1+\cos^2 x & \sin 2x\\ \sin^2 x & \cos^2 x & 1+\sin 2x\end{vmatrix}$, $x\in\left[\frac{\pi}{6},\frac{\pi}{3}\right]$. If α and β respectively are the maximum and the minimum values of f, then

(1) $\alpha^2+\beta^2=\frac{9}{2}$ (2) $\beta^2-2\sqrt{\alpha}=\frac{19}{4}$ (3) $\alpha^2-\beta^2=4\sqrt{3}$ (4) $\beta^2+2\sqrt{\alpha}=\frac{19}{4}$

Sol. **2**

$$f(\mathrm{x})=\begin{vmatrix}1+\sin^2 x & \cos^2 x & \sin 2x\\ \sin^2\mathrm{x} & 1+\cos^2 x & \sin 2x\\ \sin^2 x & \cos^2 x & 1+\sin x\end{vmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$=\begin{vmatrix}1+\sin^2 x & \cos^2 x & \sin 2x\\ -1 & 1 & 0\\ -1 & 0 & 1\end{vmatrix}$$

$= (1 + \sin^2\mathrm{x}) - \cos^2\mathrm{x}(-1) + \sin 2\mathrm{x}$

$f(\mathrm{x}) = 2 + \sin 2\mathrm{x}$

$$2x\in\left[\frac{\pi}{3},\frac{2\pi}{3}\right]\Rightarrow\frac{\sqrt{3}}{2}\le\sin 2x\le 1$$

$\alpha = 2 + 1 = 3$

$$\beta = 2+\frac{\sqrt{3}}{2}$$

$$\beta^2-2\sqrt{\alpha}=\left(2+\frac{\sqrt{3}}{2}\right)^2-2\sqrt{3}$$

$$=4+\frac{3}{4}+2\sqrt{3}-2\sqrt{3}$$

$$=\frac{19}{4}$$

80. The negation of the expression $q \vee ((\sim q) \wedge p)$ is equivalent to

(1) $(\sim p) \vee (\sim q)$ (2) $p \wedge (\sim q)$ (3) $(\sim p) \vee q$ (4) $(\sim p) \wedge (\sim q)$

Sol. **4**

$\sim(\mathrm{q}\vee(\sim q)\wedge p)$

$=\ \sim q\wedge(q\vee\sim p)$

$=\ (\sim q\wedge q)\vee(\sim q\wedge\sim p)$

$=\mathrm{F}\vee(\sim q\wedge\sim p)\ =\ (\sim q)\wedge(\sim p)$

Section B

81. Let $\vec{v}=\alpha\hat{\imath}+2\hat{\jmath}-3\hat{k}, \vec{w}=2\alpha\hat{\imath}+\hat{\jmath}-\hat{k}$ and $\vec{u}$ be a vector such that $|\vec{u}|=\alpha>0$. If the minimum value of the scalar triple product $[\vec{u}\vec{v}\vec{w}]$ is $-\alpha\sqrt{3401}$, and $|\vec{u}\cdot\hat{\imath}|^2=\frac{m}{n}$ where m and n are coprime natural numbers, then $m+n$ is equal to

Sol. **3501**

$$\vec{v}\times\vec{w}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \alpha & 2 & -3\\ 2\alpha & 1 & -1\end{vmatrix}$$

$= \hat{i} - 5\alpha\,\hat{j} - 3\alpha\,\hat{k}$

$[u\ v\ w] = \vec{u}.(\vec{v}\times\vec{w})$

$= |\vec{u}|\,|\vec{v}\times\vec{w}|\cos\theta$

since $[u\ v\ w]$ is Least $\Rightarrow \cos\theta = -1$

$[u\ v\ w] = \left(|\vec{u}|\ \sqrt{1+25\alpha^2+9\alpha^2}\right)(-1)$

$\Rightarrow -\alpha\sqrt{1+34\alpha^2} = -\alpha\sqrt{3401}$

$\Rightarrow\ \alpha^2 = 100$

$\Rightarrow\ \alpha = 10 \qquad \{\because \alpha > 0\}$

$\vec{u}$ is parallel to $\vec{v}\times\vec{w}$

$\vec{u} = \lambda(\vec{v}\times\vec{w})$

$\vec{u} = \lambda(\hat{i}-50\hat{j}-30\hat{k})$

$|\vec{u}| = 10$

$|\lambda|\ \sqrt{3401} = 10$

$|\lambda| = \dfrac{10}{\sqrt{3401}} \qquad \vec{u} = \pm\dfrac{10}{\sqrt{3401}}\left(\hat{i}-50\hat{j}-30\hat{k}\right)$

$|\vec{u}.\hat{i}|^2 = \dfrac{100}{3401} = \dfrac{m}{n}$

$m + n = 100 + 3401 = 3501$

82. The number of words, with or without meaning, that can be formed using all the letters of the word ASSASSINATION so that the vowels occur together, is

Sol. **50400**

A – 3, I – 2, S – 4, N – 2, O –1, T–1

As vowels are together

Total words formed = $\left(\dfrac{8!}{4!2!}\right)\left(\dfrac{6!}{3!2!}\right)$

$= \left(\dfrac{8\times7\times6\times5}{2}\right)\left(\dfrac{6\times5\times4}{2}\right) = 50400$

83. The remainder, when $19^{200} + 23^{200}$ is divided by 49 , is

Sol. **29**

$19^{200} + 23^{200} \quad a^{n} + b^{n}$

$19^3 = 6859 = 140 \times 49 - 1$

$= 49\lambda - 1$

$(19^3)^{66} = (49\lambda - 1)^{66}$

So, Remainder of 19^{198} divided by 49

is $(-1)^{66} = 1$

$19^2 = 361$ gives remainder 18
So, 19^{200} gives remainder 18
23^2 gives remainder 39
$(23)^3$ gives remainder 15
$(23)^4$ gives remainder 2
$((23)^4)^6$ gives remainder $(2)^6 = 64$
& 64 gives remainder 15
$(23)^{24} \longrightarrow 15$
$(23)^{25} \longrightarrow 2$
$((23)^{25})^8 \longrightarrow (2)^8 = 256 \longrightarrow 11$
So, Total remainder = 18 + 11 = 29

84. The number of 3-digit numbers, that are divisible by either 2 or 3 but not divisible by 7 , is

Sol. **514**

3 digit numbers divisible by either 2 or 3
P = n(divisible by 2) + n(divisible by 3) – n(divisible by 6)
P = 450 + 300 – 150
P = 600
Q = n(divisible by 14) + n(divisible by 21) – n(divisible by 42)
= 64 + 43 – 21 = 86
3 digit number divisible by either 2 or 3
But not divisible by –1 so P – Q = 600 – 86 = 514

85. Let $f: \mathbb{R} \to \mathbb{R}$ be a differentiable function such that $f'(x) + f(x) = \int_0^2 f(t)dt$.
If $f(0) = e^{-2}$, then $2f(0) - f(2)$ is equal to

Sol. **1**

Let $\int_0^2 f(t)dt = \lambda$

$f'(x) + f(x) = \lambda$
is linear Differential equation

I.f. $= e^{\int dx} = e^x$

$f(x).e^x = \int e^x \lambda dx$

$\Rightarrow f(x).e^x = \lambda e^x + C$
$\Rightarrow f(x) = \lambda + Ce^{-x}$
put $f(0) = e^{-2}$
$e^{-2} = \lambda + C \Rightarrow C = e^{-2} - \lambda$
$f(x) = \lambda + (e^{-2} - \lambda)\, e^{-x}$

$\lambda = \int_0^2 f(t)dt$

$= \int_0^2 (\lambda + (e^{-2} - \lambda)e^{-t})dt$

$\Rightarrow \lambda = \lambda + \lambda e^{-2} - e^{-4} + e^{-2}$
$\Rightarrow \lambda = e^{-2} - 1$
$f(x) = e^{-2} - 1 + e^{-x}$

$f(0) = e^{-2}$
$f(2) = 2e^{-2} - 1$
$2f(0) - f(2) = 1$

86. If $f(x) = x^2 + g'(1)x + g''(2)$ and $g(x) = f(1)x^2 + xf'(x) + f''(x)$, then the value of $f(4) - g(4)$ is equal to

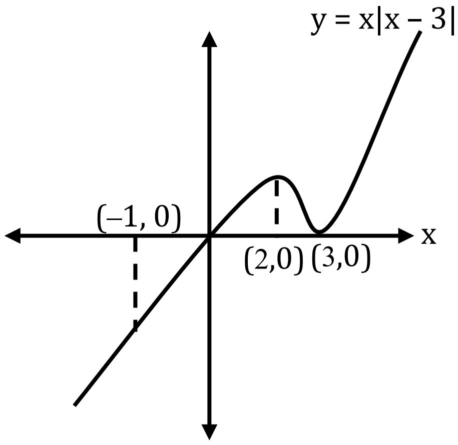
Sol. **14**

let $g'(1) = A$
$g''(2) = B$
$f(x) = x^2 + Ax + B$
$f(1) = A + B + 1$
$f'(x) = 2x + A$
$f''(x) = 2$
$g(x) = (A + B + 1)\, x^2 + x(2x + A\,) + 2$
$\Rightarrow g(x) = x^2(A + B + 2) + Ax + 2$
$g'(x) = 2x(A + B + 2) + A$
$g'(1) = A$
$\Rightarrow 2(A + B + 2) + A = A$
$A + B = -2$ ….(i)
$g''(x) = 2(A + B + 2)$
$g''(2) = B$
$\Rightarrow 2(A + B + 2) = B$
$\Rightarrow 2A + B = -4$ …(ii)
From (i) and (ii)
$A = -2$ and $B = 0$
$f(x) = x^2 - 2x$
$f(4) = 16 - 8 = 8$
$g(x) = -2x + 2$
$g(4) = -8 + 2 = -6$
$f(4) - g(4) = 8 - (-6) = 14$

87. Let A be the area bounded by the curve $y = x|x - 3|$, the x-axis and the ordinates $x = -1$ and $x = 2$. Then $12A$ is equal to

Sol. **62**

$$y = x|x-3| = \begin{cases} x(x-3); x \geq 3 \\ x(3-x); x < 3 \end{cases}$$



$$A = -\int_{-1}^{0} x(3-\mathrm{x})\,\mathrm{dx} + \int_{0}^{2} x(3-\mathrm{x})\,\mathrm{dx}$$

$$= \int_{-1}^{0} (x^2 - 3\mathrm{x})\,\mathrm{dx} + \int_{0}^{2} (3x - x^2)\,\mathrm{dx}$$

$$= \left[\frac{x^3}{3} - \frac{3x^2}{2}\right]_{-1}^{0} + \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_{0}^{2}$$

$$A = 0 - \left(\frac{-1}{3} - \frac{3}{2}\right) + 6 - \frac{8}{3} = \frac{31}{6}$$

$$A = 12\left(\frac{31}{6}\right) = 62$$

88. If $\int_0^1 (x^{21} + x^{14} + x^7)(2x^{14} + 3x^7 + 6)^{1/7} dx = \frac{1}{l}(11)^{m/n}$ where $l, m, n \in \mathbb{N}, m$ and n are coprime then $l + m + n$ is equal to

Sol. **63**

$$\mathrm{I} = \int_0^1 (\mathrm{x}^{21} + \mathrm{x}^{14} + \mathrm{x}^7)\ (2\mathrm{x}^{14} + 3\mathrm{x}^7 + 6)^{\frac{1}{7}}\,\mathrm{dx}$$

$$= \int_0^1 (\mathrm{x}^{20} + \mathrm{x}^{13} + \mathrm{x}^6)\ \ (2\mathrm{x}^{21} + 3\mathrm{x}^{14} + 6x^7)^{\frac{1}{7}}\ \mathrm{dx}$$

Put $2\mathrm{x}^{21} + 3\mathrm{x}^{14} + 6\mathrm{x}^7 = \mathrm{t}$

$\Rightarrow 42(\mathrm{x}^{20} + \mathrm{x}^{13} + \mathrm{x}^6)\mathrm{dx} = \mathrm{dt}$

$\Rightarrow (\mathrm{x}^{20} + \mathrm{x}^{13} + \mathrm{x}^6)\mathrm{dx} = \frac{dt}{42}$

$$\mathrm{I} = \int_0^{11} \frac{t^{\frac{1}{7}}}{42} dt$$

$$= \frac{1}{42}\left[\frac{t^{\frac{8}{7}}}{\frac{8}{7}}\right]_0^{11}$$

$$= \left(\frac{7}{8}\right)\left(\frac{1}{42}\right)(11)^{8/7}$$

$$= \frac{1}{48}(11)^{8/7} = \frac{1}{\ell}(11)^{\mathrm{m/n}}$$

$\ell + \mathrm{m} + \mathrm{n} = 48 + 8 + 7 = 63$

89. Let $a_1 = 8, a_2, a_3, \ldots, a_n$ be an A.P. If the sum of its first four terms is 50 and the sum of its last four terms is 170 , then the product of its middle two terms is

Sol. **754**

$\mathrm{a}_1 = 8$

d = common difference

$\frac{4}{2}$ [16 + 3d] = 50

$\Rightarrow$ d = 3

$\frac{4}{2}$ $[2a_n + 3(-d)] = 170$

$\Rightarrow 2(a_1 + (n-1)d) - 3d = 85$

$\Rightarrow 16 + 6(n-1) - 9 = 85$

$n - 1 = 13$

$n = 14$

Product of middle two terms = $T_7 \times T_8$

$= (a_1 + 6d)(a_1 + 7d)$

$= (8 + 18)(8 + 21)$

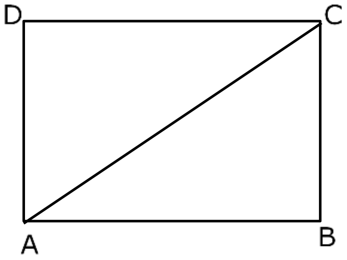
$= (26)(29) = 754$

90. $A(2,6,2), B(-4,0,\lambda), C(2,3,-1)$ and $D(4,5,0), |\lambda| \leq 5$ are the vertices of a quadrilateral $ABCD$. If its area is 18 square units, then $5 - 6\lambda$ is equal to

Sol. **11**

$\overrightarrow{AD} = 2\hat{i} - \hat{j} - 2\hat{k}$

$\overrightarrow{AC} = -3j - 3\hat{k}$



$$\overrightarrow{AD} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & -2 \\ 0 & -3 & -3 \end{vmatrix}$$

$= -3\hat{i} + 6\hat{j} - 6\hat{k}$

Area (ΔADC) = $\frac{1}{2}\left|\overrightarrow{AD} \times \overrightarrow{AC}\right|$

$= \frac{1}{2}\sqrt{9+36+36} = \frac{9}{2}$

$\overrightarrow{AB} = -6\hat{i} - 6\hat{j} + (\lambda - 2)\hat{k}$

$\overrightarrow{AC} = -3\hat{j} - 3\hat{k}$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -6 & -6 & \lambda - 2 \\ 0 & -3 & -3 \end{vmatrix}$$

$= (12 + 3\lambda)\ \hat{i} - 18\hat{j} + 18\hat{k}$

area (ΔABC) = $\frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$

$= \frac{3}{2}\sqrt{(4+\lambda)^2 + 36 + 36}$

Area(ΔABCD) = ar(ΔADC) + ar(ΔABC)

$\Rightarrow 18 = \frac{9}{2} + \frac{3}{2}\sqrt{(4+\lambda)^2 + 72}$

$\Rightarrow (4 + \lambda)^2 = 9$

$4 + \lambda = 3$ or $4 + \lambda = -3$

$\Rightarrow \lambda = -1$ or $\lambda = -7$ (reject)

$5 - 6\lambda = 5 + 6 = 11$

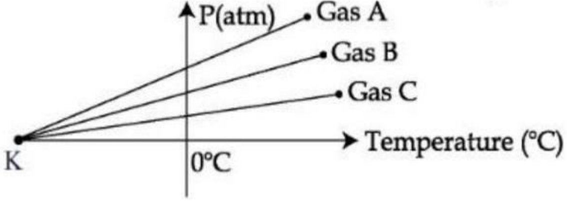
JEE–MAIN EXAMINATION – JANUARY, 2023

(Held On Thursday 1st February, 2023) | TIME : 3 : 00 PM to 6 : 00 PM

Physics

SECTION - A

1. For three low density gases A, B, C pressure versus temperature graphs are plotted while keeping them at constant volume, as shown in the figure.



The temperature corresponding to the point ' K ' is :

(1) −273°C (2) −100°C (3) −40°C (4) −373°C

Sol. **(1)**

From ideal gas equation

PV = nRT

∵ volume is constant

$P \propto T$

It is clear from graph that for all the gases lines of graphs meet at same value.

At x-axis (temperature axis) P is zero but temperature is negative and it will be equal to 0 K or –273°C

2. Given below are two statements : One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : For measuring the potential difference across a resistance of 600Ω, the voltmeter with resistance 1000Ω will be preferred over voltmeter with resistance 4000Ω.

Reason R : Voltmeter with higher resistance will draw smaller current than voltmeter with lower resistance.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Both **A** and **R** are correct and **R** is the correct explanation of **A**

(2) Both **A** and **R** are correct but **R** is not the correct explanation of **A**

(3) **A** is not correct but **R** is correct

(4) **A** is correct but **R** is not correct

Sol. **(3)**

To measure the potential difference between two point, voltmeter is used. But this voltmeter should be with higher resistance so that it cannot draw any current.

Now to measure the potential difference across 600 Ω voltmeter of 4000 Ω is much better than 1000 Ω voltmeter.

3. Figures (a), (b), (c) and (d) show variation of force with time.

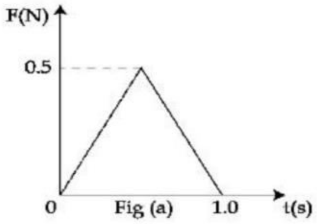


Fig (a)

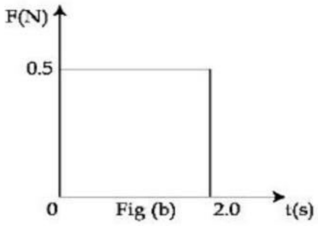


Fig (b)

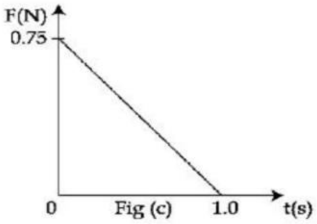


Fig (c)

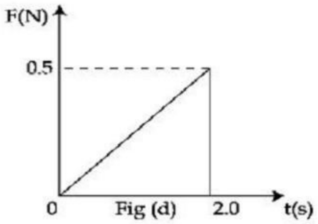


Fig (d)

The impulse is highest in figure.

(1) Fig (c) (2) Fig (b) (3) Fig (d) (4) Fig (a)

Sol. **(2)**

As we know that impulse is given by

$$I = \Delta P = F \times \Delta t$$

or $I =$ Area of f-t graph

For fig (a) → $I = \frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times 0.5 \times 1 = 0.25\,\text{N} - \text{sec}.$$

For fig (b) I = length × width

= 2 × 0.5 = 1 N-sec

For fig (c) $I = \frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times 1 \times 0.75 = 0.375 \text{ N-sec.}$$

For fig (d) $I = \frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times 2 \times 0.5 = 0.5 \text{ N-sec.}$$

Impulse is highest for that figure, whose area under F-t is maximum and i.e. figure(b)

Option (2) is correct.

4. An electron of a hydrogen like atom, having $Z = 4$, jumps from 4th energy state to 2nd energy state. The energy released in this process, will be :

(Given Rch = 13.6eV)

Where R = Rydberg constant

c = Speed of light in vacuum

h = Planck's constant

(1) 40.8eV
(2) 3.4eV
(3) 10.5eV
(4) 13.6eV

Sol. **(1)**

$$\Delta E = 13.6 Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

Z = 4 (hydrogen like atom)

$n_1 = 2, n_2 = 4$

$$\Delta E = 13.6(4)^2 \left(\frac{1}{4} - \frac{1}{16} \right)$$

$$= 13.6 \times \left(\frac{16-4}{64} \right) \times 16$$

$$\Delta E = 13.6 \times \frac{12}{64} \times 16$$

$$\boxed{\Delta E = 40.8\text{eV}}$$

5. The ratio of average electric energy density and total average energy density of electromagnetic wave is :

(1) 3
(2) $\frac{1}{2}$
(3) 1
(4) 2

Sol. **(2)**

Ratio of average electric energy density and total Avg energy density.

Avg electric energy density = $\frac{1}{4}\varepsilon_0 E_0^2$

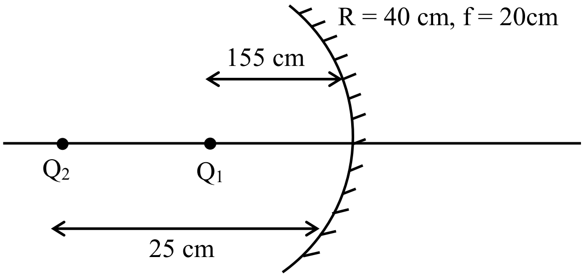
Total Avg energy density = $\frac{1}{2}\varepsilon_0 E_0^2$

$$\Rightarrow \frac{\frac{1}{4}\varepsilon_0 E_0^2}{\frac{1}{2}\varepsilon_0 E_0^2} = \frac{2}{4} = \frac{1}{2}$$

6. Two objects A and B are placed at 15 cm and 25 cm from the pole in front of a concave mirror having radius of curvature 40 cm. The distance between images formed by the mirror is ________.

(1) 100 cm (2) 60 cm (3) 160 cm (4) 40 cm

Sol. **(3)**



Using Mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{f}$$

$$\boxed{v = \frac{4f}{u - f}}$$

For object A(O$_1$) u$_i$ = –15 cm, f = –20 cm, V$_1$ = ?

$$v_1 = \frac{u_1 f}{u_1 - f} = \frac{(-15)(-20)}{(-15) - (20)} = \frac{+300}{5}$$

$$\boxed{v_1 = +60\text{cm}}$$

For object B(O$_2$) u$_2$ = –25 cm, f = –20cm v$_2$ = ?

$$v_2 = \frac{u_2 f}{u_2 - f} = \frac{(-25)(-20)}{(-25) - (-20)} = \frac{500}{-5}$$

$$\boxed{v_2 = -100\text{cm}}$$

Hence, the distance between images formed by the mirror is

$$\boxed{d = 160\text{cm}}$$

7. Equivalent resistance between the adjacent corners of a regular n-sided polygon of uniform wire of resistance R would be:

(1) $\frac{n^2R}{n-1}$ (2) $\frac{(n-1)R}{n}$ (3) $\frac{(n-1)R}{n^2}$ (4) $\frac{(n-1)R}{(2n-1)}$

Sol. **(3)**

When, a uniform wire of resistance R is shaped into a regular n-sided polygon, the resistance of each side will be

$$\frac{R}{n} = R_1$$

Let R_1 & R_2 be the resistance between adjacent corners of a regular polygon

$$\therefore \text{ The resistance of (n–1) side, } R_2 = \frac{(n-1)R}{n}$$

Since two parts are parallel, therefore R_{eq}

$$R_{eq} = \frac{R_1R_2}{R_1 + R_2} = \frac{\left(\frac{R}{n}\right)\left(\frac{n-1}{n}\right)R}{\left(\frac{R}{n}\right) + \left(\frac{n-1}{n}\right)R}$$

$$R_{eq} = \frac{(n-1)R^2}{n^2} \times \frac{n}{R + nR - R}$$

$$\boxed{R_{eq} = \frac{(n-1)R}{n^2}}$$

8. A Carnot engine operating between two reservoirs has efficiency $\frac{1}{3}$. When the temperature of cold reservoir raised by x, its efficiency decreases to $\frac{1}{6}$. The value of x, if the temperature of hot reservoir is 99°C, will be :

(1) 66 K (2) 62 K (3) 33 K (4) 16.5 K

Sol. **(2)**

Given $\eta = \frac{1}{3}$

When $T_2 \rightarrow (T_2 + x)$ i.e., temp. of cold reservior

$$\eta' = \frac{1}{6}$$

Temp. of hot reservior (T_1) = 99°C

= 99 + 273 = 372ºK

As we know,

$$\eta = 1 - \frac{T_2}{T_1} = \frac{1}{3} \qquad \ldots(1)$$

$$\eta' = 1 - \frac{(T_2 - x)}{T_1} = \frac{1}{6} \qquad \ldots(2)$$

$$\eta' = \frac{T_1 - (T_2 + x)}{T_1} = \frac{1}{6}$$

From equqation (1)

$$\frac{1}{3} = 1 - \frac{T_2}{372}$$

$$\frac{1}{3}=\frac{372-T_2}{372}$$

$$372-\frac{372}{3}=T_2$$

$$T_2=248K$$

By putting the value of T_2 in equation (2)

$$\frac{T_1-(T_2-x)}{T_1}=\frac{1}{6}$$

$$\frac{372-(248+x)}{372}=\frac{1}{6}$$

$$372-24-x=\frac{372}{6}$$

$$124-x=62$$

$$124-62=x$$

$$\boxed{x=62K}$$

9. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R. Assertion A: Two metallic spheres are charged to the same potential. One of them is hollow and another is solid, and both have the same radii. Solid sphere will have lower charge than the hollow one.

Reason R: Capacitance of metallic spheres depend on the radii of spheres.

In the light of the above statements, choose the correct answer from the options given below.

(1) Both **A** and **R** are true and **R** is the correct explanation of **A**

(2) **A** is true but **R** is false

(3) **A** is false but **R** is true 4.

(4) Both **A** and **R** are true but **R** is not the correct explanation of **A**

Sol. **(3)**

As we know, capacitance of spherical conductor

$$C=4\pi\varepsilon_0R$$

So, capacitance does not depend on its charge, it depends only on the radius of the conductor (R).

Therefore, assertion is false, R is true.

10. If the velocity of light c, universal gravitational constant G and Planck's constant h are chosen as fundamental quantities. The dimensions of mass in the new system is :

(1) $[h^{1/2}c^{-1/2}G^{1}]$ (2) $[h^{-1/2}c^{1/2}G^{1/2}]$ (3) $[n^{1/2}c^{1/2}G^{-1/2}]$ (4) $[h^{1}c^{1}G^{-1}]$

Sol. **(3)**

$$[M]=[G]^x[h]^y[c]^z$$

$$[M]=[M^{-1}L^3T^{-2}]^x[ML^2T^{-1}]^4[LT^{-1}]^z$$

$$[M^2L^0T^o]=[M^{-x+y}][L^{3x+2y+z}][T^{-2x-y-z}]$$

$y-x=1$ ...(1)

$3x+2y+z=0$ ...(2)

$-2x-y-z=0$ ...(3)

On solving, $x=-\frac{1}{2}$, $y=\frac{1}{2}$, $z=\frac{1}{2}$

So $m=\sqrt{\frac{hc}{G}}$

11. Choose the correct statement about Zener diode :

(1) It works as a voltage regulator in forward bias and behaves like simple pn junction diode in reverse bias.
(2) It works as a voltage regulator only in forward bias.
(3) It works as a voltage regulator in both forward and reverse bias.
(4) It works as a voltage regulator in reverse bias and behaves like simple pn junction diode in forward bias.

Sol. **(4)**

Zener diode act as a voltage regulator & it is used in reverse bias.
Similarly it behaves as a pn juction diode in forward bias.

12. The Young's modulus of a steel wire of length 6 m and cross-sectional area 3 mm^2, is 2×10^{11} N/m^2. The wire is suspended from its support on a given planet. A block of mass 4 kg is attached to the free end of the wire. The acceleration due to gravity on the planet is $\frac{1}{4}$ of its value on the earth. The elongation of wire is (Take g on the earth = 10 m/s^2) :

(1) 0.1 cm (2) 0.1 mm (3) 1 cm (4) 1 mm

Sol. **(2)**

As we know,

$$Y = \frac{\text{stress}}{\text{strain}}$$

$$Y = \frac{FL}{A\Delta L}$$

Given : $Y = 2 \times 10^{11}$ N/m^2

$L = 6$ m $\quad g_p = \frac{g}{4}$

$A = 3mm^2$

$M = 4$ kg

$F = mg_p$

$$F = 4 \times \frac{10}{4} = 10 \text{ N}$$

Hence $2 \times 10^{11} = \frac{10 \times 6}{3 \times 10^{-6} \times \Delta L}$

$$\boxed{\Delta L = 0.1mm}$$

13. In an amplitude modulation, a modulating signal having amplitude of X V is superimposed with a carrier signal of amplitude Y V in first case. Then, in second case, the same modulating signal is superimposed with different carrier signal of amplitude 2Y V. The ratio of modulation index in the two cases respectively will be :

(1) 2: 1 (2) 1: 2 (3) 4: 1 (4) 1: 1

Sol. **(1)**

μ = ratio of modulation index

$A_m = X, A_c = y$

$A_m = X, A_c = 2y$

$$\mu_1 = \frac{A_m}{A_c} = \frac{x}{y} \quad \ldots(1)$$

$$\mu_2 = \frac{A_m}{A_c} = \frac{x}{2y} \quad \ldots(2)$$

Hence $\frac{eq^n(1)}{eq^n(2)} = \frac{\mu_1}{\mu_2} = \frac{x/y}{x/2y} = \frac{2y}{y}$

$$\boxed{\frac{\mu_1}{\mu_2} = \frac{2}{1}}$$

14. The threshold frequency of a metal is f_0. When the light of frequency $2f_0$ is incident on the metal plate, the maximum velocity of photoelectrons is v_1. When the frequency of incident radiation is increased to $5f_0$, the maximum velocity of photoelectrons emitted is v_2. The ratio of v_1 to v_2 is:

(1) $\frac{v_1}{v_2} = \frac{1}{8}$ (2) $\frac{v_1}{v_2} = \frac{1}{4}$ (3) $\frac{v_1}{v_2} = \frac{1}{16}$ (4) $\frac{v_1}{v_2} = \frac{1}{2}$

Sol. **(4)**

Using photoelectric equation

$$hf - hf_0 = eV_0$$

As per question

$$h(2f_0) - h(f_0) = eV_1$$

$$h(2f_0 - f_0) = eV_1$$

$$hf_0 = eV_1 \qquad \ldots(1)$$

$$h(5f_0) - hf_0 = eV_2$$

$$h(5f_0 - f_0) = eV_2$$

$$4hf_0 = eV_2 \qquad \ldots(2)$$

Equation $\frac{2}{1} \Rightarrow \frac{4hf_0}{hf_0} = \frac{eV_2}{eV_1}$

$$\boxed{\frac{V_2}{V_1} = 4}$$

As we know

$$KE_{max} = eV = \frac{1}{2}mv_{max}^2$$

$$v_{max} \propto \sqrt{V}$$

$$\therefore \frac{v_2}{v_1} = \sqrt{\frac{V_2}{V_1}} = \sqrt{4} = 2$$

$$\boxed{\frac{v_1}{v_2} = \frac{1}{2}}$$

15. A coil is placed in magnetic field such that plane of coil is perpendicular to the direction of magnetic field. The magnetic flux through a coil can be changed:

A. By changing the magnitude of the magnetic field within the coil.

B. By changing the area of coil within the magnetic field.

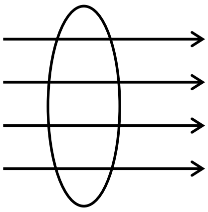
C. By changing the angle between the direction of magnetic field and the plane of the coil.

D. By reversing the magnetic field direction abruptly without changing its magnitude.

Choose the most appropriate answer from the options given below :

(1) A and B only (2) A, B and D only (3) A, B and C only (4) A and C only

Sol. **(3)**



$\boxed{\phi = BA\cos\theta}$

This show

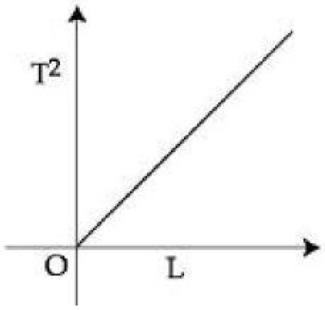
(1) by changing B

(2) by changing A

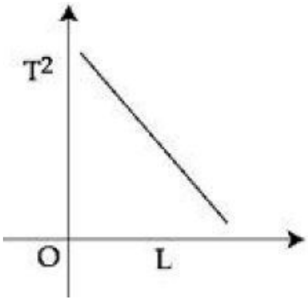
(3) Angle (θ) between B and plane of coil.

16. Choose the correct length (L) versus square of time period (T^2) graph for a simple pendulum executing simple harmonic motion.

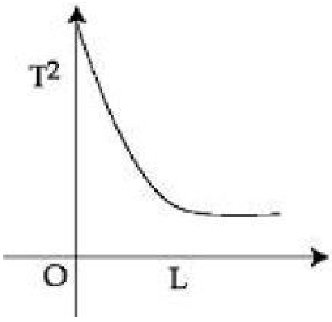
(1)



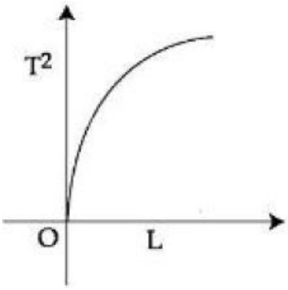
(2)



(3)



(4)



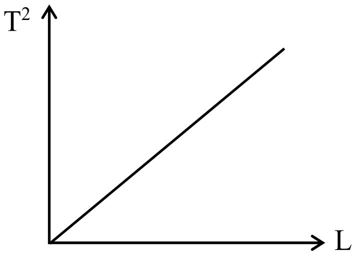
Sol. **(1)**

As we know, time period of simple pendulum is

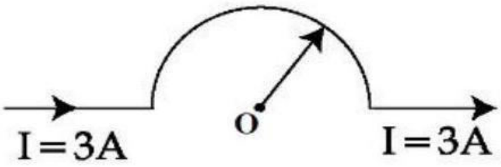
$$T = 2\pi\sqrt{\frac{L}{g}}$$

or $$T^2 = \frac{4\pi^2}{g}L \Rightarrow T^2 \propto L$$

Thus the graph between T^2 & L is a straight line

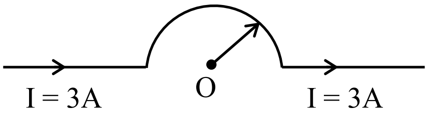


17. As shown in the figure, a long straight conductor with semicircular arc of radius $\frac{\pi}{10}$ m is carrying current I = 3 A. The magnitude of the magnetic field. at the center O of the arc is : (The permeability of the vacuum = $4\pi \times 10^{-7} NA^{-2}$)



(1) 1 μT (2) 3μT (3) 4μT (4) 6μT

Sol. (2)



Given: $R = \frac{\pi}{10} m$, $I = 3A$, $\mu_0 = 4\pi \times 10^{-7} N/A^2$

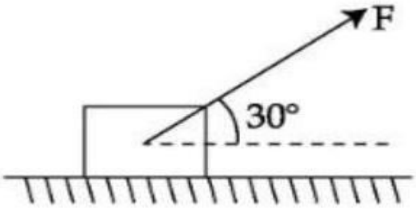
Magnetic field due to semi-circular arc is

$$B = \frac{\mu_0 I}{4R}$$

$$B = \frac{\mu_0 \times 3}{4 \times \left(\frac{\pi}{10}\right)} = \frac{4\pi \times 10^{-7} \times 3}{4 \times \frac{\pi}{10}}$$

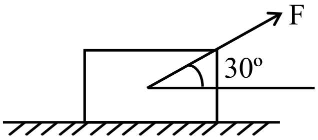
$$\boxed{B = 3\mu T}$$

18. As shown in the figure a block of mass 10 kg lying on a horizontal surface is pulled by a force F acting at an angle 30°, with horizontal. For $\mu_s = 0.25$, the block will just start to move for the value of F : [Given $g = 10\ ms^{-2}$]



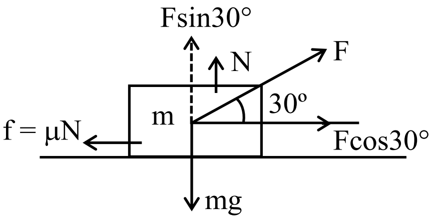
(1) 20 N (2) 33.3 N (3) 25.2 N (4) 35.7 N

Sol. (3)



Givne m = 10 kg, $\mu_s = 0.25$, $\theta = 30°$, g = 10m/sec.

F.B.D. (Free Body Diagram)



$F\cos30° = f$ ...(1)

$\text{F}\sin30^o + \text{N} = \text{mg} \Rightarrow \text{N} = \text{Mg} - \text{F}\sin30° \quad \ldots(2)$

From equation (1)

$\text{F}\sin30^o = \mu_s\text{N}$

$\text{F}\cos30^o = \mu_s(\text{mg} - \text{F}\sin30°)$

$\text{F}\cos30^o = \mu_s\text{mg} - \mu_s\text{F}\sin30°$

$\text{F}(\cos30° + \mu_s\sin30°) = \mu_s\text{mg}$

$$F = \frac{\mu_s mg}{\cos30^o + \mu_s \sin 30^o} = \frac{0.25\times10\times10}{\sqrt{3}/2\times0.25\times1/2}$$

$$F = \frac{25}{\sqrt{3}/2+\frac{0.25}{2}} = \frac{50}{1.73+0.25} = \frac{50}{1.98} = 25.2\text{N}$$

19. The escape velocities of two planets A and B are in the ratio 1: 2. If the ratio of their radii respectively is 1: 3, then the ratio of acceleration due to gravity of planet A to the acceleration of gravity of planet *B* will be :

(1) $\frac{3}{2}$ (2) $\frac{2}{3}$ (3) $\frac{3}{4}$ (4) $\frac{4}{3}$

Sol. **(3)**

Given :

$$\frac{v_A}{v_B} = \frac{1}{2}$$

$$\frac{r_A}{r_B} = \frac{1}{3}$$

$$\frac{g_A}{g_B} = ?$$

As we know,

$$v = \sqrt{\frac{2GM}{R}}$$

Hence,

$$\frac{v_A}{v_B} = \frac{\sqrt{\frac{2GM_A}{R_A}}}{\sqrt{\frac{2GM_B}{R_B}}} = \sqrt{\frac{M_A R_B}{M_B R_A}} = \frac{1}{2} \quad \ldots(1)$$

$$\text{Given : } \frac{R_A}{R_B} = \frac{1}{3} \quad \ldots(2)$$

Therefore,

$$\frac{g_A}{g_B} = \frac{M_A R_A^2}{M_B R_B^2}$$

$$= \frac{1}{4}\times\frac{1}{3}\times9$$

$$= \frac{3}{4}$$

20. For a body projected at an angle with the horizontal from the ground, choose the correct statement.
(1) The vertical component of momentum is maximum at the highest point.
(2) The Kinetic Energy (K.E.) is zero at the highest point of projectile motion.
(3) The horizontal component of velocity is zero at the highest point.
(4) Gravitational potential energy is maximum at the highest point.

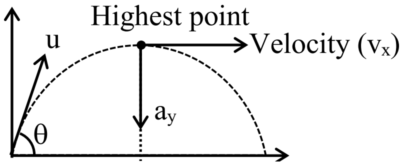
Sol. **(4)**

At highest point height is maximum and vertical component of velocity is zero.

So momentum is zero.

At highest point horizontal component of velocity will not be zero but vertical component of velocity is equal to zero and because of this K.E. will not be equal to zero.

Gravitational potential energy is maximum at highest point and equal to $mgH = mg\left(\frac{u^2 \sin^2\theta}{2g}\right)$



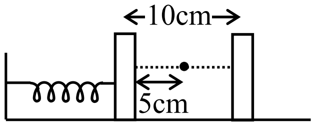
Therefore the correct option is (4).

SECTION - B

21. A block is fastened to a horizontal spring. The block is pulled to a distance $x = 10$ cm from its equilibrium position (at x = 0) on a frictionless surface from rest. The energy of the block at $x = 5$ cm is 0.25 J. The spring constant of the spring is _________ Nm^{-1}

Sol. **(50)**

Given



A= 10cm

At any instant total energy for free oscillation remains constant = $\frac{1}{2}kA^2$

$$\Rightarrow \frac{1}{2}kA^2 = 0.25J$$

$$\Rightarrow \frac{1}{2}kA^2 = 0.25J \quad \Rightarrow K = \frac{0.25 \times 2}{A^2}$$

$$\Rightarrow k = \frac{0.50}{(10cm)^2} = \frac{0.50}{\left(10 \times 10^{-2}\right)} = \frac{0.50 \times 10^4}{100}$$

k = 0.50 × 100 = 50 N/m

22. A square shaped coil of area 70 cm^2 having 600 turns rotates in a magnetic field of 0.4 wbm^{-2}, about an axis which is parallel to one of the side of the coil and perpendicular to the direction of field. If the coil completes 500 revolution in a minute, the instantaneous emf when the plane of the coil is inclined at 60° with the field, will be _________ V. (Take $\pi = \frac{22}{7}$)

Sol. **(44)**

Area (A) = 70 cm^2 = $70 \times 10^{-4}\text{m}^2$

B = 0.4 T

$$f = \frac{500 \text{ revolution}}{60 \text{ minute}} = \frac{500 \text{ rev.}}{60 \text{ sec.}}$$

Induced emf in rotating coil is given by

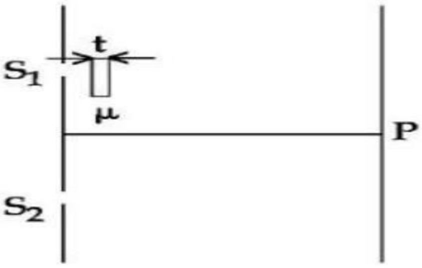
$$e = N\omega BA\sin\theta$$

$$= 600 \times 2 \times \frac{22}{7} \times \frac{500}{60} \times 0.4 \times 70 \times 10^{-4}\sin 30°$$

$$= 600 \times 2 \times \frac{22}{7} \times \frac{500}{6} \times 0.4 \times 70 \times 10^{-4} \times \frac{1}{2}$$

$$= 44\text{Volt}$$

23. As shown in the figure, in Young's double slit experiment, a thin plate of thickness $t = 10\mu$m and refractive index $\mu = 1.2$ is inserted infront of slit S_1. The experiment is conducted in air ($\mu = 1$) and uses a monochromatic light of wavelength $\lambda = 500$ nm. Due to the insertion of the plate, central maxima is shifted by a distance of $x\beta_0$. β_0 is the fringe-width befor the insertion of the plate. The value of the x is __________.



Sol. **(4)**

Given $t = 10 \times 10^{-6}$m

$\mu = 1.2$

$\lambda = 500 \times 10^{-9}$m

When the glass slab inserted infront of one slit then the shift of central fringe is obtained by

$$t = \frac{n\lambda}{(\mu - 1)}$$

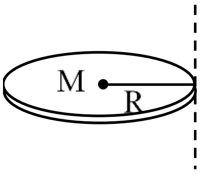
$$\Rightarrow \qquad 10 \times 10^{-6} = \frac{n \times 500 \times 10^{-9}}{(1.2 - 1)}$$

$$10 \times 10^{-6} = \frac{n \times 500 \times 10^{-9}}{0.2}$$

$$\boxed{n = 4}$$

24. Moment of inertia of a disc of mass M and radius ' R ' about any of its diameter is $\frac{MR^2}{4}$. The moment of inertia of this disc about an axis normal to the disc and passing through a point on its edge will be, $\frac{x}{2}\text{MR}^2$. The value of x is __________.

Sol. **(3)**



By using parallel axis theorem

$$I' = I_0 + MR^2$$

$$I' = \frac{MR^2}{2} + MR^2$$

$$I' = \frac{3MR^2}{2} \text{ sssss}$$

Given $I' = \frac{x}{2}MR^2$

$$\therefore \frac{3MR^2}{2} = \frac{x}{2}MR^2$$

$x = 3$

25. For a train engine moving with speed of 20 ms^{-1}, the driver must apply brakes at a distance of 500 m before the station for the train to come to rest at the station. If the brakes were applied at half of this distance, the train engine would cross the station with speed $\sqrt{x}$ ms^{-1}. The value of x is __________. (Assuming same retardation is produced by brakes)

Sol. **(200)**

By using 3rd equation of motion

$v^2 = u^2 + 2as$

$(0)^2 = u^2 + 2as$

$u^2 = -2as$

$$S = \frac{u^2}{2a} - \frac{(20)^2}{2 \times a} = 500$$

acceleration of the train, $a = -\frac{400}{1000} = -0.4 m / sec$

Now, if the brakes are applied at S = 250 m i.e. half of the distance

$v^2 = u^2 + 2as$

$v^2 = (20)2 + 2 (-0.4) \times 250$

$v^2 = 400 - 2 \times \frac{4}{10} \times 250$

$v^2 = 200$

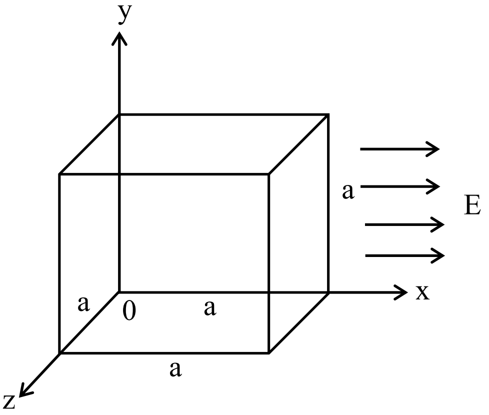
$v = \sqrt{200}$

Given $\Rightarrow v = \sqrt{x}$

$\boxed{x = 200}$

26. A cubical volume is bounded by the surfaces $x = 0, x = a, y = 0, y = a, z = 0, z = a$. The electric field in the region is given by $\vec{E} = E_0 x\hat{i}$. Where $E_0 = 4 \times 10^4 NC^{-1}\ m^{-1}$. If a = 2 cm, the charge contained in the cubical volume is $Q \times 10^{-14}C$. The value of Q is __________. (Take $\epsilon_0 = 9 \times 10^{-12} C^2/Nm^2$)

Sol. **288**



$\phi = \vec{E} \cdot \vec{A}$

$E = E_0 a\, i$

$\phi = E_0 a.a^2 = E_0 a^3$

$q_{enc.} = \phi\varepsilon_0$

$q_{enc.} = E_0 a^3 \varepsilon_0$

$= 4\times10^4 \times 8\times10^{-6} \times 9\times10^{-12}$

$q_{enc.} = 288\times10^{-14}\,C$

Hence the value of Q is 288.

27. A force $F = (5 + 3y^2)$ acts on a particle in the y-direction, where F is in newton and y is in meter. The work done by the force during a displacement from $y = 2$ m to $y = 5$ m is _________ J.

Sol. **132 J**

Given :

$F = (5 + 3y^2)$ in the y direction

Work done is given by

$$W = \int_{y_1}^{y_2} F.dy$$

$y_1 = 2m\,,\ \ y_2 = 5m$

$$W = \int_2^5 (5 + 3y^2)dy$$

$$W = \int_2^5 5dy + \int_2^5 3y^2 dy$$

$$W = [5y]_2^5 + \left[\frac{3y^3}{3}\right]_2^5$$

$$W = (5\times5 - 5\times2) + (125 - 8)$$

$$W = (25 - 10) + 117$$

$$\boxed{W = 132\text{Joule}}$$

28. The surface of water in a water tank of cross section area 750 cm^2 on the top of a house is h m above the tap level. The speed of water coming out through the tap of cross section area 500 mm^2 is 30 cm/s. At that instant, $\frac{dh}{dt}$ is $x \times 10^{-3}$ m/s. The value of x will be _________.

Sol. **(2)**

By using equation of continuity

$A_1 v_1 = A_2 v_2$

$750\times10^{-4} \times v_1 = 500\times10^{-6} \times 30\times10^{-2}$

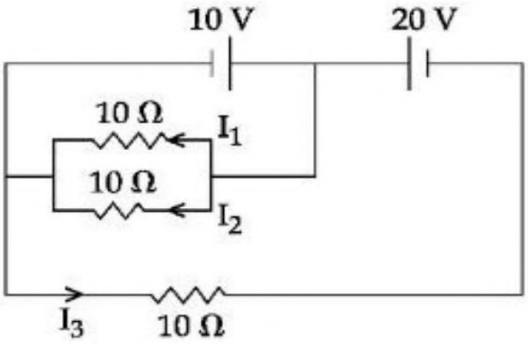
$v_1 = 20\times10^{-4}$ m/sec

$v_1 = 2\times10^{-3}$ m / sec

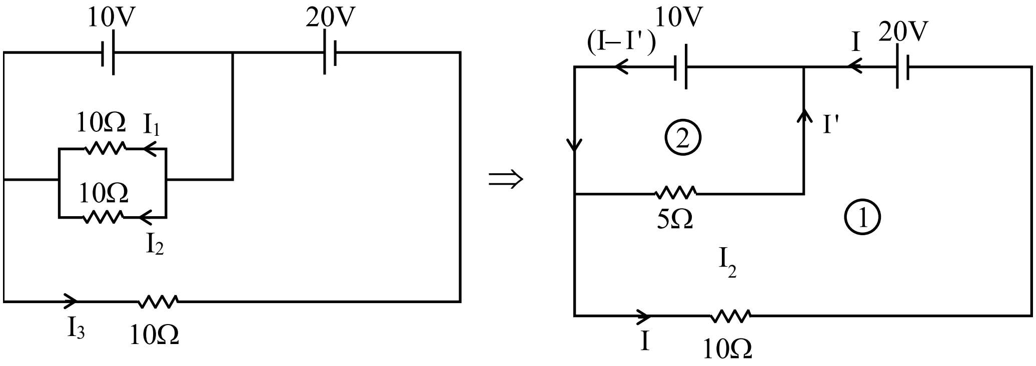
Given : $\frac{dh}{dt} = v = x \times 10^{-3}$ m/sec.

Therefore $\boxed{x = 2}$

29. In the given circuit, the value of $\left|\frac{I_1+I_3}{I_2}\right|$ is ________.



Sol. **(2)**



Apply KVL in loop (1)

$20 - 10 - 10\,I = 0$

Or $\boxed{I = 1\,\text{Amp}}$

Apply KVL in loop (2)

$-10 + 5I' = 0$

Or $\boxed{I' = 2\,\text{Amp}}$

On comparing $I_3 = 1$ A

$$I_2 = I_1 = \frac{I'}{2} = 1 \text{ Amp}$$

So, the value of $\left|\frac{I_1 + I_3}{I_2}\right| = \left|\frac{1+1}{1}\right| = 2$ Amp.

30. Nucleus A having $Z = 17$ and equal number of protons and neutrons has 1.2MeV binding energy per nucleon. Another nucleus B of $Z = 12$ has total 26 nucleons and 1.8MeV binding energy per nucleons. The difference of binding energy of B and A will be ________ MeV.

Sol. **6 MeV**

<u>For Nucleus A</u>

$Z = 17 =$ Nummber of protons Given $(Z = N)$ $\therefore$ $N = 17$

$A = 34 = Z + N$

$E_{bn} = 1.2$ MeV

$$\frac{(E_B)_1}{A} = 1.2\text{MeV}$$

$$(E_B)_1 = (1.2\text{MeV}) \times A$$

$$(E_B)_1 = (1.2\text{MeV}) \times 34$$

$\boxed{(E_B)_1 = 40.8\text{MeV}}$ → Binding energy of Nucleus A.

For Nucleus B

$Z = 12, \;\; A = 26$

$E_{bn} = 1.8\text{MeV}$

$\frac{(E_b)_2}{A} = 1.8\text{MeV}$

$(E_b)_2 = (1.8\text{MeV}) \times A$

$(E_b)_2 = (1.8\text{MeV}) \times 26$

$\boxed{(E_b)_2 = 46.8\text{MeV}}$ → Binding energy of nucleus B

Therefore, difference in binding energy of B and A is

$$\Delta E_b = (E_b)_2 - (E_b)_2$$
$$= 46.8\text{MeV} - 40.8\text{MeV} = 6\text{MeV}$$

Chemistry

SECTION - A

31. For electron gain enthalpies of the elements denoted as $\Delta_{eg}H$, the incorrect option is :

(1) $\Delta_{eg}\,H(Te) < \Delta_{eg}H(PO)$

(2) 2. $\Delta_{eg}H(Se) < \Delta_{eg}H(S)$

(3) $\Delta_{eg}H(Cl) < \Delta_{eg}H(F)$

(4) $\Delta_{eg}H(I) < \Delta_{eg}H(At)$

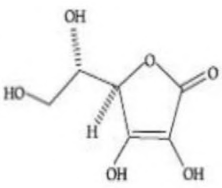
Sol. **2**

Electron gain enthalpies →

→ S > Se > Te > 0

→ Cl > F > Br > I

32. All structures given below are of vitamin C. Most stable of them is :

1.


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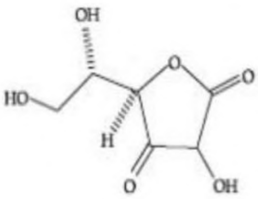
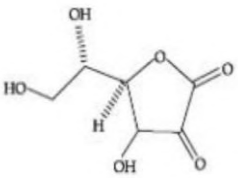


3.


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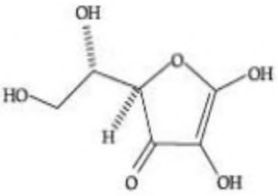
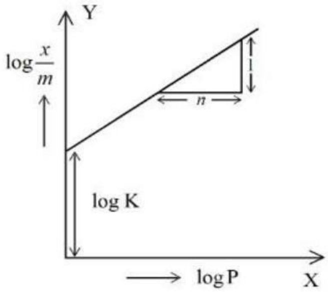


Sol. **1**

33. In figure, a straight line is given for Freundrich Adsorption $(y = 3x + 2.505)$. The value of $\frac{1}{n}$ and log K are respectively.



(1) 0.3 and 0.7033

(2) 0.3 and log 2.505

(3) 3 and 0.7033

(4) 3 and 2.505

Sol. **4**

$$\frac{x}{m} = Kp^{1/n}$$

$$\log\frac{x}{m} = \log k + \frac{1}{n}\log P$$

$$Y = 3x + 2.505,\ \frac{1}{n} = 3,\ \log K = 2.505)$$

34. The correct order of bond enthalpy $(\mathrm{kJmol^{-1}})$ is :

(1) $C-C > Si-Si > Sn-Sn > Ge-Ge$ (2) $C-C > Si-Si > Ge-Ge > Sn-Sn$

(3) $Si-Si > C-C > Sn-Sn > Ge-Ge$ (4) $Si-Si > C-C > Ge-Ge > Sn-Sn$

Sol. **2**

Bond length ↑ Bond energy ↓

35. Given below are two **statements** : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : An aqueous solution of KOH when used for volumetric analysis, its concentration should be checked before the use.

Reason (R) : On aging, KOH solution absorbs atmospheric CO_2.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(2) (A) is correct but (R) is not correct

(3) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(4) (A) is not correct but (R) is correct

Sol. **3**

KOH absorb CO_2

So its concentration should be checked.

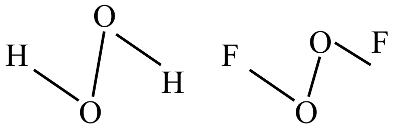
36. $O-O$ bond length in H_2O_2 is $\underline{X}$ than the $O-O$ bond length in F_2O_2. The $O-H$ bond length in H_2O_2 is $\underline{Y}$ than that of the $O-F$ bond in F_2O_2.

Choose the correct option for X and Y from those given below

(1) X-shorter, Y - longer (2) X-shorter, Y-shorter

(3) X - longer, Y-shorter (4) X-longer, Y – longer

Sol. **3**



→ (0 – 0) BL in H_2O_2 in longer then (O–O) BL in O_2F_2

→ (O–H) BL in H_2O_2 in shorter than (O–F) BL in O_2F_2

37. Given below are two **statements** : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A): Cu^{2+} in water is more stable than Cu^{+}.

Reason (R) : Enthalpy of hydration for Cu^{2+} is much less than that of Cu^{+}.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(2) (A) is not correct but (R) is correct

(3) (A) is correct but (R) is not correct

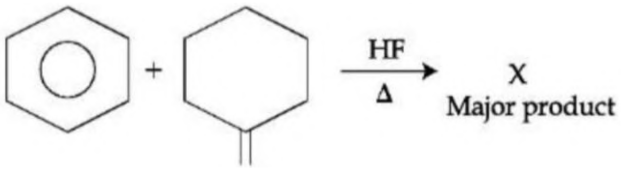
(4) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

Sol. **1**

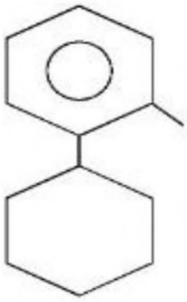
$2Cu^{+} \rightarrow Cu^{2+} + Cu$

The stability of Cu^{2+}(aq) rather than Cu^{+}(aq), is due to the much more negative $\Delta_{hyd}H$ of Cu^{2+}(aq) than Cu^{+}(aq), which more than compensates for the second ionisation enthalpy of Cu.

38. 'X' is :



1



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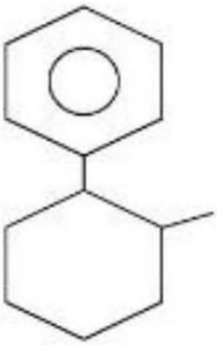

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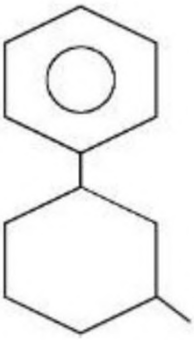

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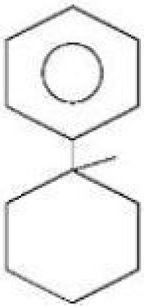
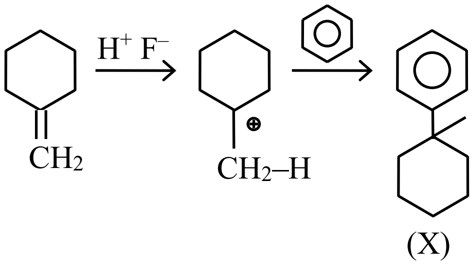


Sol. **4**



39. The complex cation which has two isomers is :

(1) $[Co(NH_3)_5NO_2]^{2+}$

(2) $[Co(H_2O)_6]^{3+}$

(3) $[Co(NH_3)_5Cl]^{+}$

(4) $[Co(NH_3)_5Cl]^{2+}$

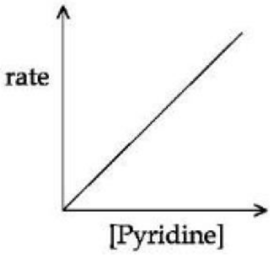
Sol. **1**

NO_2^- is ambidentante ligand, so. $[Co(NH_3)_5\ No_2]^{+2}$ will show 2 Isomer.

40. The graph which represents the following reaction is :

$$(C_6H_5)_3C - Cl \xrightarrow[\text{Pyridine}]{OH^-} (C_6H_5)_3C - OH$$

1.



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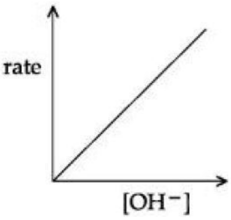

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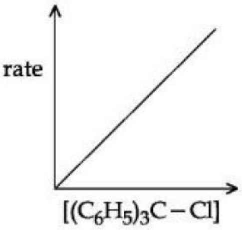

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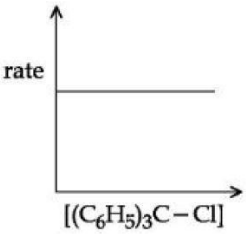


Sol. **3**

41. Given below are two **statements** : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : α-halocarboxylic acid on reaction with dil NH_3 gives good yield of α-amino carboxylic acid whereas the yield of amines is very low when prepared from alkyl halides.

Reason (R) : Amino acids exist in zwitter ion form in aqueous medium.

In the light of the above statements, choose the correct answer from the options given below :

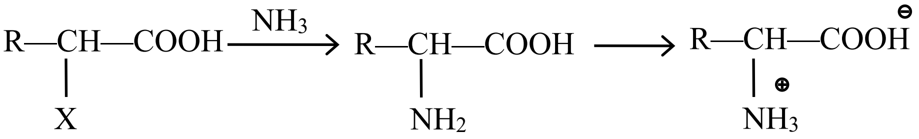
(1) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(2) (A) is not correct but (R) is correct

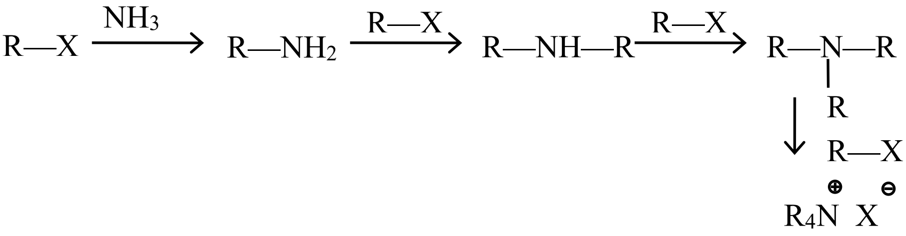
(3) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(4) (A) is correct but (R) is not correct

Sol. 1



$$R-\underset{X}{\underset{|}{CH}}-COOH \xrightarrow{NH_3} R-\underset{NH_2}{\underset{|}{CH}}-COOH \longrightarrow R-\underset{\overset{\oplus}{N}H_3}{\underset{|}{CH}}-CO\overset{\ominus}{O}$$



$$R-X \xrightarrow{NH_3} R-NH_2 \xrightarrow{R-X} R-NH-R \xrightarrow{R-X} R-\underset{R}{\underset{|}{N}}-R \xrightarrow{R-X} R_4\overset{\oplus}{N}\ \overset{\ominus}{X}$$

42. The industrial activity held least responsible for global warming is :

(1) Industrial production of urea

(2) Electricity generation in thermal power plants

(3) steel manufacturing

(4) manufacturing of cement

Sol. **1**

43. Given below are two **statements** : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Gypsum is used for making fireproof wall boards.

Reason (R): Gypsum is unstable at high temperatures.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(2) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(3) (A) is correct but (R) is not correct

(4) (A) is not correct but (R) is correct

Sol. **2**

Gypsum is used for making fireproof wall board.

44. The starting material for convenient preparation of deuterated hydrogen peroxide (D_2O_2) in laboratory is :

(1) BaO (2) $K_2 S_2O_8$ (3) BaO_2 (4) 2-ethylanthraquinol

Sol. **2**

$$2HSO_4^-(aq) \xrightarrow{\text{Electrolysis}} HO_3SOOSO_3H_{(aq)} \xrightarrow{\text{Hydrolysis}} 2HSO_4^-(aq) + 2H^+(aq) + H_2O_2(aq)$$

This method is now used for the laboratory preparation of D_2O_2.

$$K_2S_2O_8(s) + 2D_2O(cl) \longrightarrow 2KDSO_4(aq) + D_2O_2(l)$$

45. The effect of addition of helium gas to the following reaction in equilibrium state, is :

$PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$

(1) helium will deactivate PCl_5 and reaction will stop.

(2) the equilibrium will shift in the forward direction and more of Cl_2 and PCl_3 gases will be produced.

(3) the equilibrium will go backward due to suppression of dissociation of PCl_5.

(4) addition of helium will not affect the equilibrium.

Sol. **2**

PCI (g) PCl (g) + Cl (g)

(**Case 1 :** At constant P – volume will increase so reaction will shift in forward direction then answer will be A

Case 2 : At constant volume no change in active mass so reaction will not shift in any direction then answer will be D.

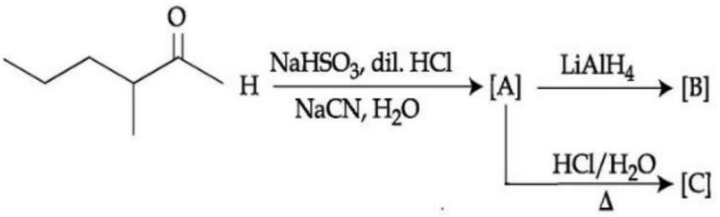
46. Which element is not present in Nessler's reagent ?

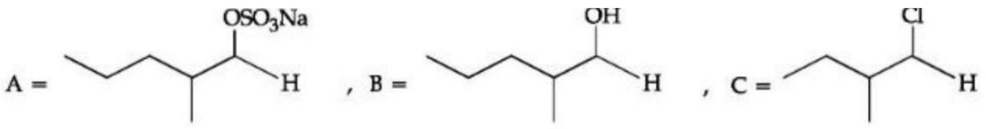
(1) Oxygen (2) Potassium (3) Mercury (4) Iodine

Sol. 1

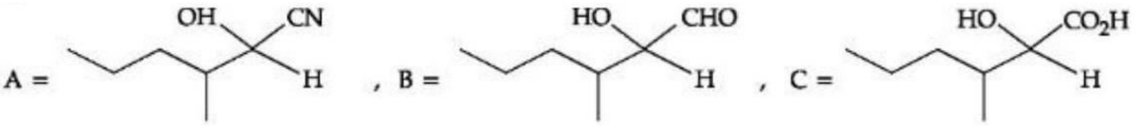
Nessler's Reagent → K_2HgI_4

47. The structures of major products A, B and C in the following reaction are sequence.

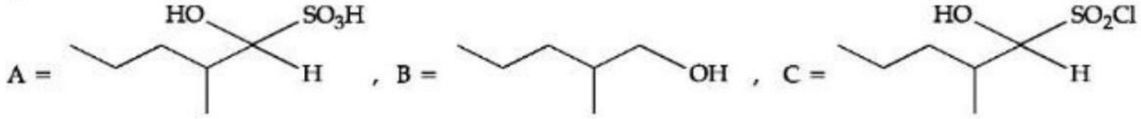




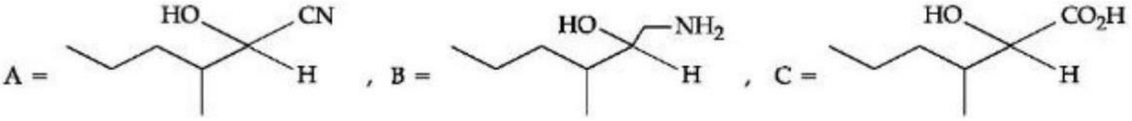
(1)



(2)

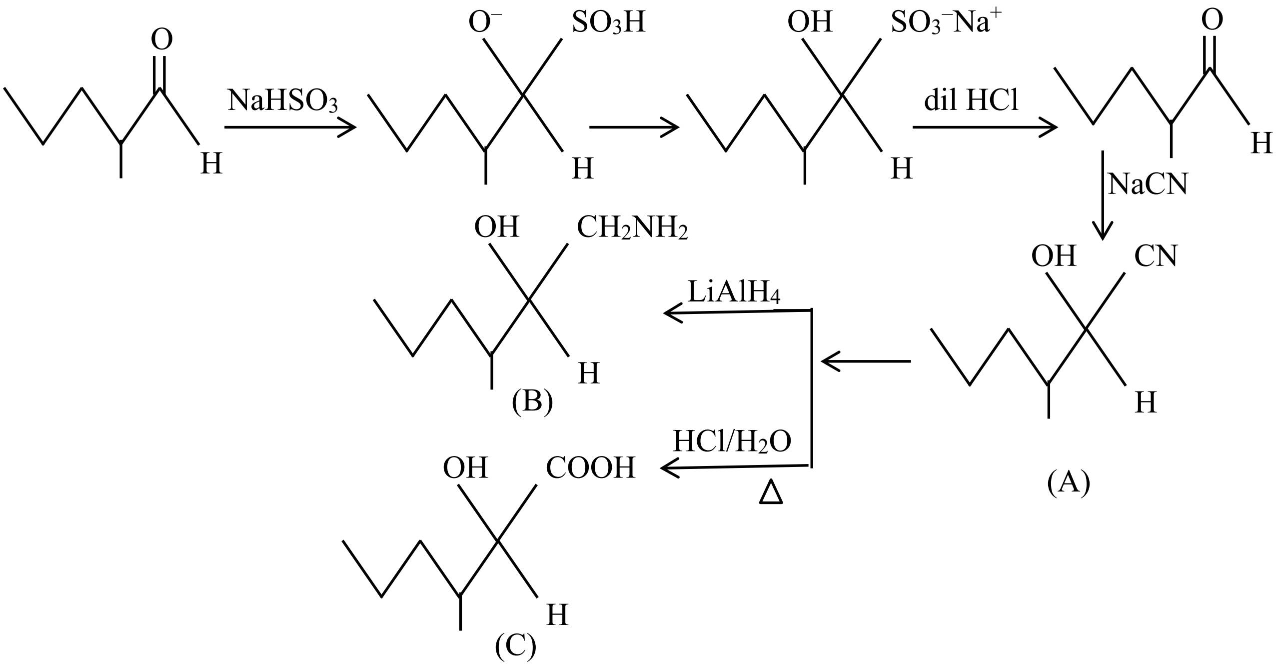


(3)

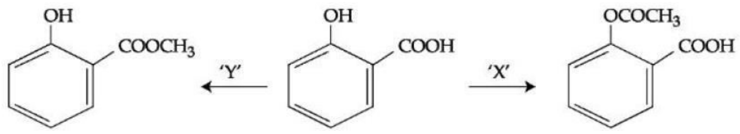


(4)

Sol. **4**



48. In a reaction,



reagents ' X ' and ' Y ' respectively are :

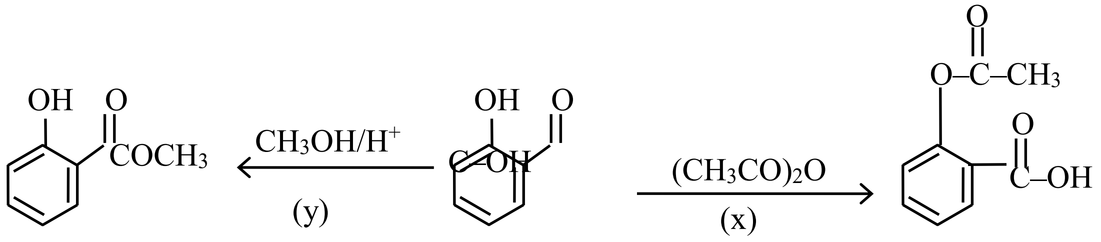
(1) $(CH_3CO)_2O/H^+$ and $(CH_3CO)_2O/H^+$

(2) $CH_3OH/H^+,\Delta$ and $(CH_3CO)_2O/H^+$

(3) $CH_3OH/H^+,\Delta$ and $CH_3OH/H^+,\Delta$

(4) $(CH_3CO)_2O/H^+$ and $CH_3OH/H^+,\Delta$

Sol. 4



Oil of wintergreen

Aspirin

49. Which one of the following sets of ions represents a collection of isoelectronic species ?
(Given: Atomic Number : $\mathrm{F: 9, Cl: 17, Na = 11, Mg = 12, Al = 13, K = 19, Ca = 20, Sc = 21}$)

(1) $Ba^{2+}, Sr^{2+}, K^+, Ca^{2+}$

(2) $Li^+, Na^+, Mg^{2+}, Ca^{2+}$

(3) $N^{3-}, O^{2-}, F^-, S^{2-}$

(4) $K^+, Cl^-, Ca^{2+}, Sc^{3+}$

Sol. 4

$K^+ = 18$

$Cl^- = 18$

$Ca^{+2} = 18$

$Sc^{+3} = 18$

50. Given below are two statements :

Statement I : Sulphanilic acid gives esterification test for carboxyl group.

Statement II : Sulphanilic acid gives red colour in Lassigne's test for extra element detection.

In the light of the above statements, choose the most appropriate answer from the options given below:

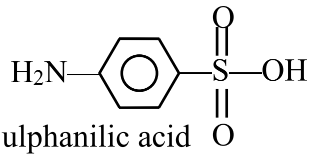
(1) Statement I is incorrect but Statement II is correct

(2) Both Statement I and Statement II are incorrect

(3) Statement I is correct but Statement II is incorrect

(4) Both Statement I and Statement II are correct

Sol. 1



Sulphanilic acid

Does not show esterification test. Presence of both sulphur and nitrogen give red colour in Lassigne's test.

SECTION B

51. 0.3 g of ethane undergoes combustion at 27°C in a bomb calorimeter. The temperature of calorimeter system (including the water) is found to rise by 0.5°C. The heat evolved during combustion of ethane at constant pressure is_____________ kJmol^{-1}. (Nearest integer)

[Given : The heat capacity of the calorimeter system is 20 kJ K^{-1}, $\text{R} = 8.3\text{JK}^{-1}\ \text{mol}^{-1}$.

Assume ideal gas behaviour.

Atomic mass of C and H are 12 and 1 g mol^{-1} respectively]

Sol. **1006**

(Bomb calorimeter → const volume Heat released By combustion of 1 mole

$$C_2H_6(\Delta U) = -\frac{20 \times .05}{0.3} \times 30 = -1000 \text{ kJ}$$

$C_2H_6(g) + 7/2\ O_2(g) \rightarrow 2CO_2(g) + 3H_2O(l)$

$\Delta ng = 2 - (2 + 7/2) = -(7/2)$

$\Delta H = \Delta U + \Delta nRT$

$= -1000 - 7/2 \times 8.3 \times 300$ kJ

$= -1000 - 6.225$

$= -1006$ kJ

So heat released = 1006 kJ mol^{-1}

52. Among the following, the number of tranquilizer/s is/are______________

A. Chloroliazepoxide B. Veronal

C. Valium D. Salvarsan

Sol. **3**

A. Chloroliazepoxide	(Tranquilizer)
B. Veronal	(Tranquilizer)
C. Valium	(Tranquilizer)
D. Salvarsan	(Antibiotic)

53. Among following compounds, the number of those present in copper matte is

A. $CuCO_3$ B. $Cu_2\ S$ C. Cu_2O D. FeO

Sol. **1**

Copper mate → Cu_2S

54. A metal M crystallizes into two lattices :- face centred cubic (fcc) and body centred cubic (bcc) with unit cell edge length of 2.0 and 2.5Å respectively. The ratio of densities of lattices fcc to bcc for the metal M is___________ (Nearest integer)

Sol. **4**

$$d = \frac{Z \times M}{N_A a^3}$$

$$\frac{d_{FCC}}{d_{BCC}} = \frac{\frac{4 \times M_w}{N_A \times (2)^3}}{\frac{2 \times M_w}{N_A \times (2.5)^3}} = 3.90$$

55. The spin only magnetic moment of $[Mn(H_2O)_6]^{2+}$ complexes is__________ B.M. (Nearest integer) (Given: Atomic no. of Mn is 25)

Sol. $[Mn(H_2O)_6]^{+2}$

$Mn^{+2} = [Ar]\ 4S°, 3d^5$

$\rightarrow t_{2g}{}^{1,1,1}\ \ eg^{1,\,1}$

$\mu = \sqrt{n(n+2)}$

$\sqrt{5 \times 7} = \sqrt{35} = 6$

56. $1 \times 10^{-5} M AgNO_3$ is added to 1 L of saturated solution of AgBr. The conductivity of this solution at 298 K is__________ $\times 10^{-8}\ S\ m^{-1}$

[Given : $K_{SP}(AgBr) = 4.9 \times 10^{-13}$ at 298 K

$\lambda^0_{Ag^+} = 6 \times 10^{-3}\ S\ m^2\ mol^{-1}$

$\lambda^0_{Br^-} = 8 \times 10^{-3}\ S\ m^2\ mol^{-1}$

$\lambda^0_{NO_3^-} = 7 \times 10^{-3}\ S\ m^2\ mol^{-1}$]

Sol. **14**

$[Ag^+] = 10^{-5}$

$\left[NO_3^-\right] = 10^{-5}$

$$\left[Br^-\right] = \frac{Ksp}{\left[Ag^+\right]} = 4.9 \times 10^{-8}$$

$$\wedge_m = \frac{k}{1000 \times M}$$

For Ag^+

$$6 \times 10^{-3} = \frac{K_{Ag^+}}{1000 \times 10^{-5}}$$

$K_{Ag^+} = 6 \times 10^{-8}$

$= 6000 \times 10^{-8}$

for Br^-

$8 \times 10^{-3} = \dfrac{K_{Br^-}}{1000 \times 4.9 \times 10^{-8}}$

$K_{Br^-} = 39.2 \times 10^{-8}$

for NO_3^-

$7 \times 10^{-3} = \dfrac{K_{NO_3^-}}{1000 \times 10^{-5}}$

$K_{NO_3^-} = 7 \times 10^{-5}$

$= 7000 \times 10^{-8}$

Conductivity of solution

$= (6000 + 7000 + 39.2) \times 10^{-8}$

$= 13039.2 \times 10^{-8}\ Sm^{-1}$

57. 20% of acetic acid is dissociated when its 5 g is added to 500 mL of water. The depression in freezing point of such water is___________ $\times 10^{-3}$ °C

Atomic mass of C, H and O are 12,1 and 16 a.m.u. respectively.

[Given : Molal depression constant and density of water are $1.86\ K\ kg\ mol^{-1}$ and $1\ g\ cm^{-3}$ respectively.

Sol. 372

$i = 1 + (n - 1)\alpha$

$(i = 1 + 0.2\ (2 - 1) = 1.2$

$\Delta T_f = i\ K_f m$

$\Delta T_f = 1.2 \times 1.86 \times \dfrac{5 \times 1000}{60 \times 500}$

$\Delta t_f = 3.72$

$\Delta T_f = 372 \times 10^{-2}$

58. $A \rightarrow B$

The above reaction is of zero order. Half life of this reaction is 50 min. The time taken for the concentration of A to reduce to one-fourth of its initial value is__________ (Nearest integer) min.

Sol. **75**

Assume reaction starts with 1 mole A

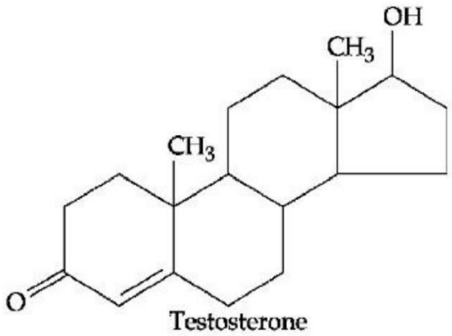
$\left(t_{1/2} = \dfrac{a}{2k}, K = \dfrac{1}{2 \times 50}\right)$

For 75% completion

$a - \dfrac{a}{4} = kt$

$t = \dfrac{3}{4}\dfrac{a}{k} = \dfrac{3}{4} \times \dfrac{100}{a} = 75$

59. Testosterone, which is a steroidal hormone, has the following structure.



Testosterone

The total number of asymmetric carbon atom /s in testosterone is___________

Sol. **6**

60. The molality of a $10\%(\mathrm{v/v})$ solution of di-bromine solution in CCl_4 (carbon tetrachloride) is ' x '.

$x =$_________ $\times 10^{-2}$M. (Nearest integer)

[Given : molar mass of $Br_2 = 160 \text{ g mol}^{-1}$

atomic mass of $\mathrm{C} = 12 \text{ g mol}^{-1}$

atomic mass of $\mathrm{Cl} = 35.5 \text{ g mol}^{-1}$

density of dibromine $= 3.2 \text{ g cm}^{-3}$

density of $CCl_4 = 1.6 \text{ g cm}^{-3}$]

Sol. **139**

(10 ml solute in 90 ml solvent

mass of solute = 10 × 3.2 = 32g

mass of solvent = 90 × 1.6g

$$m = \frac{32 \times 1000}{160 \times 90 \times 1.6} = 1.388$$

$m = 138.8 \times 10^{-2} = 139$

Mathematics

SECTION - A

61. Let $\alpha x = \exp(x^{\beta}y^{\gamma})$ be the solution of the differential equation $2x^2y\,dy - \left(1 - xy^2\right)dx = 0$, $x > 0$, $y(2) = \sqrt{\log_e 2}$. Then α + β + γ equals :

(1) 1 (2) –1 (3) 3 (4) 0

Sol. **1**

$2x^2ydy - \left(1 - xy^2\right)dx = 0$

$\Rightarrow 2x^2y\frac{dy}{dx} - 1 + xy^2 = 0$

$\Rightarrow 2y\frac{dy}{dx} - \frac{1}{x^2} + \frac{y^2}{x} = 0$

$\Rightarrow 2y\frac{dy}{dx} + \frac{y^2}{x} = \frac{1}{x^2}(\text{L.D.E})$

$y^2 = t \Rightarrow 2y\frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx} + \frac{t}{x} = \frac{1}{x^2}$

$f = e^{\int \frac{1}{x}dx} = x$

$\Rightarrow t \times x = \int x \cdot \frac{1}{x^2}dx$

$\Rightarrow y^2 \cdot x = \ln x + c$

$\text{Also}, y(2) = \sqrt{\log_e 2}$

$\log_e^2 2 = \log_e 2 + c \Rightarrow c = \log_e 2$

$\Rightarrow y^2x = \ln x + \ln 2$

$\Rightarrow y^2x = \ln 2x$

$2x = \exp\left(x^1y^2\right)$

Compare it with given solution we get,

$\alpha = 2, \beta = 1, \gamma = 2$

$\Rightarrow \alpha + \beta - \gamma = 2 + 1 - 2 = 1$

62. The sum $\sum_{n=1}^{\infty}\frac{2n^2 + 3n + 4}{(2n)!}$ is equal to :

(1) $\frac{13e}{4} + \frac{5}{4e}$ (2) $\frac{11e}{2} + \frac{7}{2e} - 4$ (3) $\frac{11e}{2} + \frac{7}{2e}$ (4) $\frac{13e}{4} + \frac{5}{4e} - 4$

Sol. **4**

$$\sum_{n=1}^{\infty}\frac{2n^2+3n+4}{(2n)!}$$

$$\frac{1}{2}\sum_{n=1}^{\infty}\frac{2n(2n-1)+8n+8}{(2n)!}$$

$$\frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{(2n-2)!}+2\sum_{n=1}^{\infty}\frac{1}{(2n-1)!}+4\sum_{n=1}^{\infty}\frac{1}{(2n)!}$$

$$e=1+1+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\ldots.$$

$$e^{-1}=1-1+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}+\ldots.$$

$$\left(e+\frac{1}{e}\right)=2\left(1+\frac{1}{2!}+\frac{1}{4!}+\ldots\ldots\right)$$

$$e-\frac{1}{e}=\left(1+\frac{1}{3!}+\frac{1}{5!}+\ldots..\right)$$

Now

$$\frac{1}{2}\left(\sum_{n=1}^{\infty}\frac{1}{(2n-2)!}\right)+2\sum_{n=1}^{\infty}\frac{1}{(2n-1)!}+4\sum_{n=1}^{\infty}\frac{1}{(2n)!}$$

$$=\frac{1}{2}\left[\frac{e+\frac{1}{e}}{2}\right]+2\left[\frac{e-\frac{1}{e}}{2}\right]+4\left(\frac{e+\frac{1}{e}-2}{2}\right)$$

$$=\frac{\left(e+\frac{1}{e}\right)}{4}+e-\frac{1}{e}+2e+\frac{2}{e}-4$$

$$=\frac{13}{4}e+\frac{5}{4e}-4$$

63. Let $\vec{a}=5\hat{i}-\hat{j}-3k$ and $\vec{b}=\hat{i}+3\hat{j}+5k$ be two vectors. Then which one of the following statements is TRUE ?

(1) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the projection vector is same as of $\vec{b}$.

(2) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$.

(3) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-17}{\sqrt{35}}$ and the direction of the projection vector is same as of $\vec{b}$.

(4) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-17}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$.

Sol. **Bonus**

Projection of $\vec{a}$ on $\vec{b}=\dfrac{\vec{a}.\vec{b}}{|\vec{b}|}$

$$\Rightarrow \frac{(5\hat{i}-\hat{j}-3\hat{k})\cdot(\hat{i}+3\hat{j}+5\hat{k})}{\sqrt{1^2+3^2+5^2}}=\frac{5-3-15}{\sqrt{35}}$$

$$\Rightarrow \frac{-13}{\sqrt{35}}$$

Ans. (4)

64. Let $\vec{a}=2\hat{i}-7\hat{j}+5k$, $\vec{b}=\hat{i}+k$ and $\vec{c}=\hat{i}+2\hat{j}-3k$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r}\times\vec{a}=\vec{c}\times\vec{a}$ and $\vec{r}\cdot\vec{b}=0$, then $|\vec{r}|$ is equal to :

(1) $\frac{11}{5}\sqrt{2}$ (2) $\frac{\sqrt{914}}{7}$ (3) $\frac{11}{7}\sqrt{2}$ (4) $\frac{11}{7}$

Sol. **3**

$$\vec{r}\times\vec{a}=\vec{c}\times\vec{a}$$

$$\Rightarrow(\vec{r}-\vec{c})\times\vec{a}=0\Rightarrow\vec{r}-\vec{c}=\lambda\vec{a}(\vec{r}-\vec{c}\&\vec{a}\text{are parallel})$$

$$\Rightarrow\vec{r}=\vec{c}+\lambda\vec{a}$$

$$\Rightarrow\vec{r}\cdot\vec{b}=\vec{c}\cdot\vec{b}+\lambda\vec{a}\cdot\vec{b}$$

$$0=(1-3)+\lambda(2+5)\Rightarrow\lambda=\frac{2}{7}$$

Hence, $\vec{r}=\vec{c}+\dfrac{2\vec{a}}{7}$

$$\vec{r}\Rightarrow\frac{11}{7}\hat{i}-\frac{11}{7}\hat{k}$$

$$|\vec{r}|=\sqrt{\left(\frac{11}{7}\right)^2+\left(-\frac{11}{7}\right)^2}\Rightarrow r=\frac{11\sqrt{2}}{7}$$

65. Let $f:\mathbb{R}-\ 0,1\ \rightarrow\mathbb{R}$ be a function such that $f(x)+f\left(\dfrac{1}{1-x}\right)=1+x$. Then $f(2)$ is equal to

(1) $\frac{9}{2}$ (2) $\frac{7}{4}$ (3) $\frac{9}{4}$ (4) $\frac{7}{3}$

Sol. **3**

For $x=2,\Rightarrow f(2)+f(-1)=3$ .... (1)

For $x=\frac{1}{2}\Rightarrow f\left(\frac{1}{2}\right)+f(-1)=\frac{3}{2}$ .... (2)

For $x=-1\Rightarrow f(-1)+f\left(\frac{1}{2}\right)=0$ .... (3)

$(2)-(3) \Rightarrow f(2)-f(-1)=\frac{3}{2}$ …….. (4)

$(1)+(4) \Rightarrow 2f(2)=\frac{9}{2} \Rightarrow f(2)=\frac{9}{4}$

Ans. 3

66. Let P(S) denote the power set of S={1, 2, 3, ………, 10}. Define the relations R_1 and R_2 on P(S) as AR_1B if $\left(A \cap B^C\right) \cup \left(B \cap A^C\right) = \varnothing$ and AR_2B if $A \cup B^C = B \cup A^C, \forall A, B \in P(S)$. Then :

(1) only R_1 is an equivalence relation (2) only R_2 is an equivalence relation

(3) both R_1 and R_2 are equivalence relations (4) both R_1 and R_2 are not equivalence relations

Sol. **3**

$S=\{1,2,3,\ldots\ldots10\}$

P(S) = power set of S

$AR,B \Rightarrow (A \cap \vec{B}) \cup (\vec{A} \cap B) = \phi$

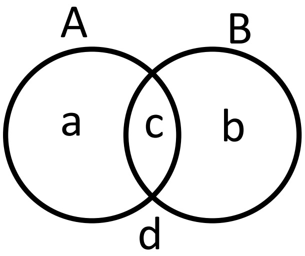
R1 is reflexive, symmetric

For transitive

$(A \cap \vec{B}) \cup (\vec{A} \cap B) = \phi; \{a\} = \phi = \{b\}\ A = B$

$(B \cap \vec{C}) \cup (\vec{B} \cap C) = \phi \therefore B = C$

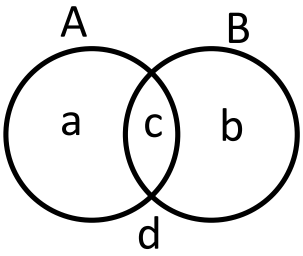
$\therefore A = C$ equivalence



$R_2 \equiv A \cup \vec{B} = \vec{A} \cup B$

$R_2 \rightarrow$ reflexive, symmetric

for transitive



$A \cup \vec{B} = \vec{A} \cup B \Rightarrow \{a,c,d\} = \{b,c,d\}$

$\{a\} = \{b\} \therefore A = B$

$B \cup \vec{C} = \vec{B} \cup C \Rightarrow B = C$

$\therefore A = C \quad \therefore A \cup \vec{C} = \vec{A} \cup C \therefore$ Equivalence

67. The area of the region given by $\left\{(x,y)\ :\ xy\ \leq 8,\ 1 \leq y \leq x^2\right\}$ is :

(1) $16\log_e 2+\frac{7}{3}$ (2) $16\log_e 2-\frac{14}{3}$ (3) $8\log_e 2-\frac{13}{3}$ (4) $8\log_e 2+\frac{7}{6}$

Sol. **2**

$$A=\int_1^4\left(\frac{8}{y}-\sqrt{y}\right)dy$$

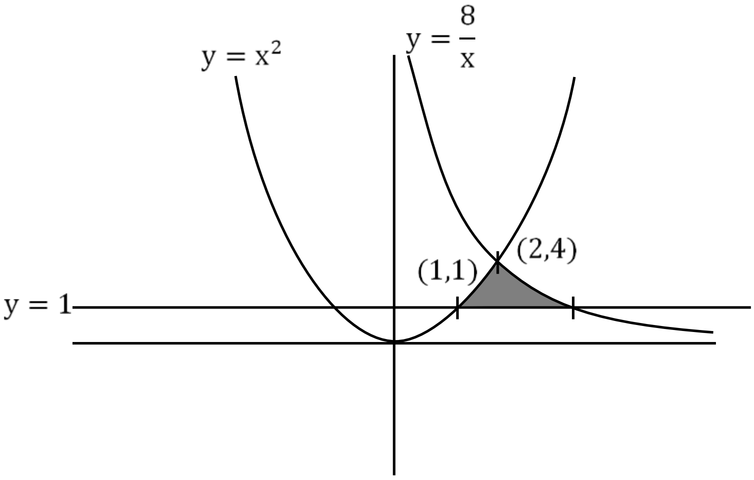
$$A=8[\ln y]_1^4-\frac{2}{3}\left(y^{\frac{3}{2}}\right)_1^4$$

$$\Rightarrow 8\ln 4-\frac{2}{3}\left(4^{3/2}-1\right)$$

$$\Rightarrow 8\ln 4-\frac{2}{3}(8-1)$$

$$\Rightarrow 16\ln 2-\frac{14}{3}$$

Ans. (2)



68. If $A=\frac{1}{2}\begin{bmatrix}1 & \sqrt{3}\\ -\sqrt{3} & 1\end{bmatrix}$, then :

(1) $A^{30}+A^{25}+A=I$ (2) $A^{30}=A^{25}$ (3) $A^{30}+A^{25}-A=I$ (4) $A^{30}-A^{25}=2I$

Sol. **3**

$$4=\frac{1}{2}\begin{bmatrix}1 & f_3\\ -f^3 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & \frac{f_3}{2}\\ -\frac{f_3}{2} & \frac{1}{2}\end{bmatrix}$$

$$|4-\lambda I|=0$$

$$\begin{vmatrix} \frac{1}{2}-\lambda & \frac{f_3}{2} \\ -\frac{f_3}{2} & \frac{1}{2}-\lambda \end{vmatrix} = 0 \qquad \Rightarrow \lambda^2 + \frac{1}{4} - \lambda + \frac{3}{4} = 0$$

$$\lambda^2 - \lambda + 1 = 0 \qquad \Rightarrow A^2 - A + 1 = 0$$

$$\Rightarrow A^3 - A^2 + A = 0$$

and $A^4 = (A-I)^2$ $\qquad \Rightarrow A^4 = A^2 + I - 2A$

$$\Rightarrow A^4 = A - I + I - 2A = -A$$

$$\boxed{A^4 = -A}$$

$$\Rightarrow A^{30} = \left(A^4\right)^7 A^2 = -A^4 = -\left(A^4\right)A = -A^3 = A - A^2$$

$$A^{25} = \left(A^4\right)^6 A = A^6 A = A^7 = A^4 A^3 = -AA^3 = -A^4 = A$$

Put these values on all options, we, get,

$$\Rightarrow A^{30} + A^{25} - A = I$$

So, option (3) is correct.

69. Which of the following statements is a tautology ?

(1) $p \vee (p \wedge q)$ (2) $(p \wedge (p \rightarrow q)) \rightarrow \sim q$

(3) $(p \wedge q) \rightarrow (\sim(p) \rightarrow q)$ (4) $p \rightarrow (p \wedge (p \rightarrow q))$

Sol. **3**

(i) $p \rightarrow (p \wedge (p \rightarrow q))$

$(\sim p) \vee (p \wedge (\sim p \vee q))$

$(\sim p) \vee (f \vee (p \wedge q))$

$\sim p \vee (p \wedge q) = (\sim p \vee p) \wedge (\sim p \vee q)$

$= \sim p \vee q$

(ii) $(p \wedge q) \rightarrow (\sim p \rightarrow q)$

$\sim (p \wedge q) \vee (p \vee q) = t$

$\{a, b, d\} \vee \{a, b, c\} = V$

Tautology

(iii) $(p \wedge (p \rightarrow q)) \rightarrow \sim q$

$\sim (p \wedge (\sim p \vee q)) \vee \sim q = \sim (p \wedge q) \vee \sim q = \sim p \vee \sim q$

Not tautology

(iv) $p \vee (p \wedge q) = p$

Not tautology.

So, option (2) is correct.

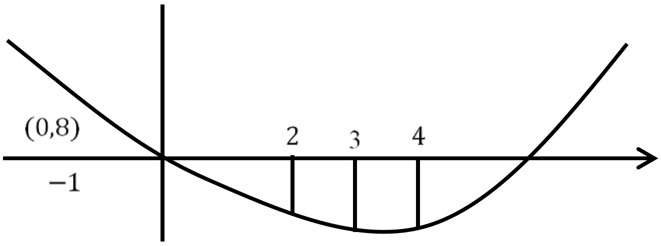
70. The sum of the absolute maximum and minimum values of the function $f(x)=|x^2-5x+6|-3x+2$ in the interval [–1,3] is equal to :

(1) 12 (2) 13 (3) 10 (4) 24

Sol. **4**

$$f(x)=\begin{cases} x^2-5x+6-3x+2 . x\varepsilon(-\infty,2)\cup[2,\infty] \\ -(x^3-5x+6)-3x-2 & . x\varepsilon[2,3] \end{cases}$$

$$f(x)=\begin{cases} x^2-8x+8 & . x\varepsilon(-\infty,2)\cup[3,\infty] \\ -x^2+2x-4 & . x\varepsilon[2,3] \end{cases}$$



Absolute maximum $=|f(-1)|=\left|(-1)^2-8(-1)+8\right|=17$

Absolute minimum $=\left|f(3)\right|=7$

Sum=17+7=24

71. Let the plane P pass through the intersection of the planes 2x + 3y – z = 2 and x + 2y + 3z = 6 and be perpendicular to the plane 2x + y – z = 0. If d is the distance of P form the point (–7,1,1,) then d^2 is equal to :

(1) $\frac{250}{83}$ (2) $\frac{250}{82}$ (3) $\frac{15}{53}$ (4) $\frac{25}{83}$

Sol. **1**

Plane P, is passing through intersection of the two planes, so,

$$2x+3y-z-2+\lambda(x+2y+3z-6)=0$$

$$x(2+\lambda)+y(3+2\lambda)+z(3\lambda-1)-2-6\lambda=0$$

It is perpendicular with plane, $2x+y-2+1=0$

So, $(2+\lambda)2+(3+2\lambda)1+(3\lambda-1)(-1)=0$

$$\boxed{\lambda=-8}$$

So, plane $p_1-6x-13y-25z+46=0$

distance of plane p from the point $(-7,1,1)$

$$d=\frac{|+42-13-25+46|}{\sqrt{36+169+625}}=\frac{50}{\sqrt{30}}=$$

$$d^2=\frac{2500}{830}=\frac{250}{83}$$

Ans. (1)

72. The number of integral values of k, for which one root of the equation $2x^2-8x+k=0$ lies in the interval (1,2) and its other root lies in the interval (2,3), is :

(A) 3 (2) 0 (3) 2 (4) 1

Sol. **4**

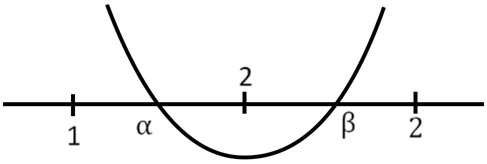
$\Rightarrow f(1)\cdot f(2)<0$

$(k-6)(k-8)<0$

$\text{Also,} f(2)\cdot f(3)<0$

$(k-8)(k-6)<0$

$k\varepsilon(6,8)\Rightarrow$ Integral value of k is 7. Ans: (4)



73. Let $P(x_0,y_0)$ be the point on the hyperbola $3x^2-4y^2=36$, which is nearest to the line 3x + 2y = 1. Then $\sqrt{2}(y_0-x_0)$ is equal to :

(A) –9 (2) –3 (3) 3 (4) 9

Sol. **1**

$3x^2-4y^2=36 \quad 3x+2y=1$

$$m=-\frac{3}{2}$$

$$m=+\frac{3\sec\theta}{\sqrt{12}\cdot\tan\theta}$$

$$\Rightarrow\frac{3}{\sqrt{12}}\times\frac{1}{\sin\theta}=\frac{-3}{2}$$

$$\sin\theta=-\frac{1}{\sqrt{3}}$$

$(\sqrt{12}\cdot\sec\theta,3\tan\theta)$

$$\left(\sqrt{12}\cdot\frac{\sqrt{3}}{\sqrt{2}},-3\times\frac{1}{\sqrt{2}}\right)\Rightarrow\left(\frac{6}{\sqrt{2}},\frac{-3}{\sqrt{2}}\right)=(x_0,y_0)$$

$$\Rightarrow\sqrt{2}(y_0-x_0)=\sqrt{2}\left(\frac{-3}{\sqrt{2}}-\frac{6}{\sqrt{2}}\right)=-9$$

74. Two dice are thrown independently. Let A be the event that the number appeared on the 1st die is less than the number appeared on the 2nd die, B be the event that the number appeared on the 1st die is even and that on the second die is odd, and C be the event that the number appeared on the 1st die is odd and that on the 2nd is even. Then :

(1) the number of favourable cases of the events A, B and C are 15,6 and 6 respectively

(2) the number of favourable cases of the event (A ∪ B) ∩ C is 6

(3) B and C are independent

(4) A and B are mutually exclusive

Sol. **2**

$A=\{(1,2),(1,3),(1,4),(1,5),(1,6),(2,3),(2,4),(2,5),(2,6)(3,4),(3,5),(3,6)(4,5),(4,6),(5,6)\}$

$n(A)=15$

$B=\{(2,1),(2,3),(2,5),(4,1),(4,3),(4,5),(6,1),(6,3),(6,5)\}$

$n(B)=9$

$C=\{(1,2),(1,4),(1,6),(3,2),(3,4),(3,6),(5,2),(5,4),(5,6)\}$

$n(C)=9$

$((A\cup B)\cap C)=\{(1,2),(1,4),(1,6),(3,4),(3,6),(5,6)\}$

$\Rightarrow n((A\cup B)\cap C)=6$

75. If $y(x)=x^x, x>0$, then $y''(2)-2y'(2)$ is equal to :

(1) $4\log_e 2+2$ (2) $8\log_e 2-2$ (3) $4(\log_e 2)^2+2$ (4) $4(\log_e 2)^2-2$

Sol. **4**

$y=x^x$

$\ln y=x\ln x$

$\frac{1}{y}y'=1+\ln x$

$y'=y(1+\ln x)$

$y'=x^x(1+\ln x)$

$\text{At} x=2 \text{we have} y=4$

So $y'(2)=4(1+\ln 2)$ .... (2)

$$\text{And} y'' = y'(1+\ln x) + \frac{y}{x}$$

$$y'(2) = y'(1+\ln 2) + 2$$

$$y'(2) = y'(2) = y'(\ln 2) + 2$$

$$y''(2) = 2y'(2) = (\ln 2 - 1)y'(2) + 2$$

$$= 4(\ln 2 - 1)(\ln 2 + 2) + 2$$

$$= 4(\ln 2)^2 - 2$$

76. Let $S = \left\{ x \in \sqcup\ : 0 < x < 1 \text{ and } 2\tan^{-1}\left(\frac{1-x}{1+x}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) \right\}$.

If n(S) denotes the number of elements in S then :

(1) n(S) = 2 and only one element in S is less then $\frac{1}{2}$.

(2) n(S) = 1 and the element in S is more then $\frac{1}{2}$.

(3) n(S)=0

(4) n(S)=1 and the element in S is less than $\frac{1}{2}$.

Sol. **1**

Put $x = \tan\theta \quad \theta \in \left(0, \frac{\pi}{4}\right)$

$$2\tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right) = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$2\tan^{-1}\left[\tan\left(\frac{\pi}{4} - \theta\right)\right] = \cos^{-1}[\cos(2\theta)]$$

$$\Rightarrow 2\left(\frac{\pi}{4} - \theta\right) = 2\theta \Rightarrow \theta = \frac{\pi}{8}$$

$$\Rightarrow x = \tan\frac{\pi}{8} = \sqrt{2} - 1 \simeq 0.414$$

77. The value of the integral $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x + \frac{\pi}{4}}{2 - \cos 2x} dx$ is :

(1) $\frac{\pi^2}{12\sqrt{3}}$ (2) $\frac{\pi^2}{6\sqrt{3}}$ (3) $\frac{\pi^2}{6}$ (4) $\frac{\pi^2}{3\sqrt{3}}$

Sol. **2**

$$I=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{xdx}{2-\cos 2x}+\frac{\pi}{4}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{xdx}{2-\cos 2x}$$

$$=0+\frac{\pi}{4}\cdot 2\int_{0}^{\frac{\pi}{4}}\frac{xdx}{2-\cos 2x}$$

$$=\pi/2\int_{0}^{\frac{\pi}{4}}\frac{\sec^2 xdx}{1+3\tan^2 x}$$

Now,

$\tan x=t$

$$=\frac{\pi}{2}\int_{0}^{1}\frac{dt}{1+3t^2}$$

$$=\frac{\pi}{2}\left[\frac{\tan^{-1}(\sqrt{3}t)}{\sqrt{3}}\right]_0^1$$

$$=\frac{\pi}{2\sqrt{3}}\left(\frac{\pi}{3}\right)=\frac{\pi^2}{6\sqrt{3}}$$

78. For the system of linear equations $\alpha x+y+z=1$, $x+\alpha y+z=1$, $x+y+\alpha z=\beta$, which one of the following statements is NOT correct ?

(1) It has infinitely many solutions if $\alpha=2$ and $\beta=-1$

(2) It has no solution if $\alpha=-2$ and $\beta=1$

(3) $x+y+z=\frac{3}{4}$ if $\alpha=2$ and $\beta=1$

(4) It has infinitely many solutions if $\alpha=1$ and $\beta=1$

Sol. 1

$$\begin{vmatrix}\alpha & 1 & 1\\ 1 & \alpha & 1\\ 1 & 1 & \alpha\end{vmatrix}=0$$

$$\alpha\left(\alpha^2-1\right)-1(\alpha-1)+1(1-\alpha)=0$$

$$\alpha^3-3\alpha+2=0$$

$$\alpha^2(\alpha-1)+\alpha(\alpha-1)-2(\alpha-1)=0$$

$$(\alpha-1)\left(\alpha^2+\alpha-2\right)=0$$

$$\alpha=1,\alpha=-2,1$$

For $\alpha=1, \beta=1$

$$\left.\begin{matrix}x+y+z=1\\ x+y+z=b\end{matrix}\right\}\text{infinite solution}$$

For $\alpha = 2, \beta = 1$

$\Delta = 4$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 3-1-1 \qquad \Rightarrow x = \frac{1}{4}$$

$$\Delta_2 = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 2-1 = 1 \qquad \Rightarrow y = \frac{1}{4}$$

$$\Delta_3 = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 2-1 = 1 \qquad \Rightarrow z = \frac{1}{4}$$

For $\alpha = 2 \Rightarrow$ unique solution

79. Let $9 = x_1 < x_2 < ..... < x_7 ......, x_7$ be in an A.P. with common difference d. If the standard deviation of $x_1 \cdot x_2 ....., x_7$ is 4 and mean is $\overline{x}$, then $\overline{x} + x_6$ is equal to :

(1) $2\left(9 + \frac{8}{\sqrt{7}}\right)$ (2) $18\left(1 + \frac{1}{\sqrt{3}}\right)$ (3) 25 (4) 34

Sol. **4**

Mean $\Rightarrow \overline{x} = \frac{\sum_{i=1}^{7} x_i}{7} = \frac{\frac{7}{2}[2a + 6d]}{7} = a + 3d = x_4$

Variance $= \frac{\sum_{i=1}^{7} (x_i - \overline{x})^2}{7} = (4)^2 \Rightarrow \frac{\sum_{i=1}^{7} (x_i - x_4)^2}{7} = 16$

$\Rightarrow \frac{(3d)^2 + (2d)^2 + d^2 + 0 + d^2 + (2d)^2 + (3d)^2}{7} = 16$

$= 4d^2 = 16 \Rightarrow d = 2$

$\Rightarrow \overline{x} = 9 + 3(2) = 15$

$\& x_0 = a + 5d = 9 + 5(2) = 19 \Rightarrow \overline{x} + x_0 = 34$

80. Let a, b be two real numbers such that ab < 0. IF the complex number $\frac{1 + ai}{b + i}$ is of unit modulus and a + ib lies on the circle $|z-1| = |2z|$, then a possible value of $\frac{1 + [a]}{4b}$, where [t] is greatest integer function, is :

(1) $-\frac{1}{2}$ (2) -1 (3) 1 (4) $\frac{1}{2}$

Sol. **Bonus**

$ab < 0 \left|\frac{1+ai}{b+i}\right| = 1$

$|1+ia| = |b+i|$

$a^2+1 = b^2+1 \Rightarrow a = \pm b \Rightarrow b = -a \quad \text{as } ab < 0$

(a,b) lies on $|z-1| = |2z|$

$|a+ib-1| = 2|a+ib|$

$(a-1)^2+b^2 = 4\left(a^2+b^2\right)$

$(a-1)^2 = a^2 = 4\left(2a^2\right)$

$1-2a = 6a^2 \Rightarrow 6a^2+2a-1 = 0$

$a = \frac{-2\pm\sqrt{28}}{12} = \frac{-1\pm\sqrt{7}}{6}$

$a = \frac{\sqrt{7}-1}{6} \& b = \frac{1-\sqrt{7}}{6}$

$[a] = 0$

$\therefore \frac{1+[a]}{4b} = \frac{6}{4(1-\sqrt{7})} = -\left(\frac{1+\sqrt{7}}{4}\right)$

or $[a] = 0$

Similarly it is not matching with $a = \frac{-1-\sqrt{7}}{6}$

No answer is matching.

SECTION - B

81. Let αx + βy + yz = 1 be the equation of a plane through the point (3,–2,5)and perpendicular to the line joning the points (1, 2, 3) and (–2, 3, 5). Then the value of αβy is equal to

Sol. **6**

Plane is perp. To the line joining the

(1,2,3) & (-2, 3, 5)

So, line will be along normal of plane.

$\vec{n} = (3,-1,-2)$ or $(-3,1,2)$

Compare it with eq. of plane, $\alpha x+\beta y+r_2 = 1$

$\alpha = 3, \beta = -1, r = -2$ or $\alpha = -3, \beta = 1, r = 2$

So, $\alpha\beta r = 6$ (in both cases)

Ans. 6

82. If the term without x in the expansion of $\left(x^{\frac{2}{3}}+\frac{a}{x^{3}}\right)^{22}$ is 7315, then |α| is equal to

Sol. **1**

$\Rightarrow$General Term, $T_{r+1}={}^{22}C_r\left(x^{\frac{2}{3}}\right)^{22-r}\cdot\left(\frac{\alpha}{x^{3}}\right)^{r}$

$T_{r+1}={}^{22}C_r\cdot x^{\frac{2(22-r)}{3}-3r}\alpha^{r}$

For term independent of x,

$\frac{2(22-r)}{3}-3r=0$

$44-2r=9r\Rightarrow 11r=44$

$T_{4+1}={}^{22}C_4\cdot\alpha^{4}=7315$

$7315.\alpha^{4}=7315\Rightarrow\alpha^{4}=1\Rightarrow|\alpha|=1$

Ans. 1

83. If the x – intercept of a focal chord of the parabola $y^2=8x+4y+4$ is 3 , then the length of this chord is equal to

Sol. **16**

$y^{2}=8x+4y+4$

$(y-2)^{2}=8(x+1)$

$y^{2}=4ax$

$a=2, X=x+1, Y=y-2$

focus $(1,2)$

$y-2=m(x-1)$

Put (3, 0) in the above line

$m=-1$

Length of focal chord = 16

84. Let the sixth term in the binomial expansion of $\left(\sqrt{\log_2\left(10-3^{3}\right)}+\sqrt[5]{2^{x\log_2 3}}\right)^{m}$, in the increasing powers of $2^{(x-2)\log_2 3}$, be 21. If the binomial coefficients of the second, third and fourth terms in the expansion are respectively the first, third and fifth terms of A.P., then the sum of the squares of all possible values of x is

Sol. **4**

$\Rightarrow$ Sixth Term, $T_{5+1} = {}^{m}C_5\left(10-3^x\right)^{\frac{m-5}{2}} 3^{x-2} = 21$

So, $2{}^{m}C_2 = {}^{m}C_1 + {}^{m}C_3$

$2\frac{m(m-1)}{2} = m + \frac{m(m-1)(m-2)}{6}$

$\Rightarrow m = 2,7$ (But $m=2$ is inadmissible)

$\Rightarrow m = 7$

Now, $T_{5+1} = {}^{7}C_5\left(10-3^x\right)^{\frac{7-5}{2}} 3^{x-2} = 21$

$\Rightarrow \frac{10.3^x - \left(3^x\right)^2}{3^2} = 1$

$\left(3^x\right)^2 - 10.3^x + 9 = 0$

$3^x = 9,1$

$\Rightarrow x = 0,2$

Sum of squeals of values of $x = 0^2 + 2^2 = 4$

Ans. (4)

85. The point of intersection C of the plane 8x + y + 2z = 0 and the line joining the point A(–3,–61) and B(2,–4,–3) divides the line segment AB internally in the ratio k:. If a, b, c (|a|, |b|, |c|) are coprime are the direction ratios of the perpendicular form the point C on the line $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$, then |a + b + c|is equal to

Sol. **10**

Plane : $8x+y+2z=0$

Given line $AB: \frac{x-2}{5} = \frac{y-4}{10} = \frac{z+3}{-4} = \lambda$

Any point on line $(5\lambda+2, 10\lambda+4, -4\lambda-3)$

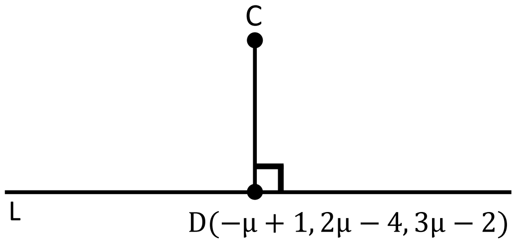
Point of intersection of line and plane

$8(5\lambda+2)+10\lambda+4-8\lambda-6=0$

$\lambda = -\frac{1}{3}$

$C\left(\frac{1}{3}, \frac{2}{3}, -\frac{5}{3}\right)$

$L: \frac{x-1}{-1} = \frac{y+4}{2} = \frac{z+2}{3} = \mu$



$$\overline{CD}=\left(-\mu+\frac{2}{3}\right)\hat{i}+\left(2\mu-\frac{14}{3}\right)\hat{j}+\left(3\mu-\frac{1}{3}\right)\hat{k}$$

$$\left(-\mu+\frac{2}{3}\right)(-1)+\left(2\mu-\frac{14}{3}\right)2+\left(3\mu-\frac{1}{3}\right)3=0$$

$$\mu=\frac{11}{14}$$

$$\overrightarrow{CD}=\frac{-5}{42},\frac{-130}{42},\frac{85}{42}$$

Direction ratio $\rightarrow(-1,-26,17)$

$|a+b+c|=10$

86. The line x = 8 is the directrix of the ellipse ellipse $E:\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with the corresponding focus (2,0). If the tangent to E at the point P in the first quadrant passes through the point $\left(0,4\sqrt{3}\right)$ and intersects that x-axis at Q then $(3PQ)^2$ equl to

Sol. **39**

$$\frac{a}{e}=8 \quad .....(1)$$

$$ae=2 \quad .....(2)$$

$$8e=\frac{2}{e}$$

$$e^2=\frac{1}{4}\Rightarrow e=\frac{1}{2}$$

$$a=4$$

$$b^2=a^2\left(1-e^2\right)$$

$$=16\left(\frac{3}{4}\right)=12$$

$$\frac{x\cos\theta}{4}+\frac{y\sin\theta}{2\sqrt{3}}=1$$

$$\sin\theta=\frac{1}{2}$$

$$\theta=30^{\circ}$$

$P(2\sqrt{3},\sqrt{3})$

$Q\left(\frac{8}{\sqrt{3}},0\right)$

$(3PQ)^2 = 39$

87. The total number of six digit numbers, formed using the digits 4, 5, 9 only and divisible by 6 , is

Sol. **81**

We have,

For this, 4 will be fixed as unit place digit

		Total number
Case I:	$4's \rightarrow 6$ times	1
Case II:	$4's \rightarrow 4$times	
	$5's \rightarrow 1$times	$\frac{5!}{3!} = 20$
	$9's \rightarrow 1$times	
Case III:	$4's \rightarrow 3$times	
	$5's \rightarrow 3$times	$\frac{5!}{2!3!} = 10$
Case IV:	$4's \rightarrow 3$times	
	$9's \rightarrow 3$times	$\frac{5!}{2!3!} = 10$
Case V:	$4's \rightarrow 2$times	
	$5's \rightarrow 2$times	$\frac{5!}{2!2!} = 30$
	$9's \rightarrow 2$times	
Case VI:	$4's \rightarrow 1$times	
	$5's \rightarrow 1$times	$\frac{5!}{4!} = 5$
	$9's \rightarrow 4$times	
Case VII:	$4's \rightarrow 1$times	
	$5's \rightarrow 4$times	$\frac{5!}{4!} = 5$
	$9's \rightarrow 1$times	

Total numbers =81

88. Number of integral solutions to the equation $x + y + z = 21$, where $x \geq 1,\ \ y \geq 3,\ \ z \geq 4$, is equal to

Sol. **105**

$$^{15}C_2 = \frac{15 \times 14}{2} = 105$$

89. The sum of the common terms of the following three arithmetic progressions.

3, 7, 11, 15,…..,399

2, 5, 8, 11,……,359 and

2, 7, 12, 17,……, 197

is equal to

Sol. **321**

3,7,11,15,………..,399 $d_1 = 4$

2,5,8,11,…………..,359 $d_2 = 3$

2,7,12,17,……..,197 $d_3 = 5$

$\mathrm{LCM}(d_1, d_2, d_3) = 60$

Common terms are 47, 107, 167

Sum = 321

90. If $\int \frac{5\cos^{x}\left(1+\cos x\cos 3x+\cos^{2} x+\cos^{3} x+\cos 3x\right)dx}{1+5^{\cos x}} = \frac{k\pi}{16}$, then k is equal to

Sol. **13**

$$I = \int_0^{\pi} \frac{5^{\cos x}\left(1+\cos x\cos 3x+\cos^2 x+\cos^3 x\cos 3x\right)}{1+5^{\cos x}}dx$$

$$I = \int_0^{\pi} \frac{5^{-\cos x}\left(1+\cos x\cos 3x+\cos^2 x+\cos^3 x\cos 3x\right)}{1+5^{-\cos x}}dx$$

$$2I = \int_0^{\pi}\left(1+\cos x\cos 3x+\cos^2 x+\cos^3 x\cos 3x\right)dx$$

$$2\mathrm{I} = 2\int_0^{\frac{\pi}{2}}\left(1+\cos x\cos 3x+\cos^2 x+\cos^3 x\cos 3x\right)dx$$

$$I = \int_0^{\frac{\pi}{2}}\left(1+\sin x(-\sin 3x)+\sin^2 x-\sin^3 x\sin 3x\right)dx$$

$$2I = \int_0^{\frac{\pi}{2}}\left(3+\cos 4x+\cos^3 x\cos 3x-\sin^3 x\sin 3x\right)dx$$

$$2I=\int_0^{\frac{\pi}{2}}3+\cos 4x+\left(\frac{\cos 3x+3\cos x}{4}\right)\cos 3x-\sin 3x\left(\frac{3\sin x-\sin 3x}{4}\right)dx$$

$$2I=\int_0^{\frac{\pi}{2}}\left(3+\cos 4x+\frac{1}{4}+\frac{3}{4}\cos 4x\right)dx$$

$$2I=\frac{13}{4}\times\frac{\pi}{2}+\frac{7}{4}\left(\frac{\sin 4x}{4}\right)_0^{\frac{\pi}{2}}\Rightarrow I=\frac{13\pi}{16}$$