JEE Main 2024 January Question Paper with Answer

27th, 29th, 30th, 31st January & 1st February (Shift 1 & Shift 2)

FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

(Held On Saturday 27th January, 2024) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS | **TEST PAPER WITH SOLUTION**

SECTION-A

1. ${}^{n-1}C_r = \left(k^2-8\right){}^{n}C_{r+1}$ if and only if :

(1) $2\sqrt{2} < k \le 3$ (2) $2\sqrt{3} < k \le 3\sqrt{2}$

(3) $2\sqrt{3} < k < 3\sqrt{3}$ (4) $2\sqrt{2} < k < 2\sqrt{3}$

Ans. (1)

Sol. ${}^{n-1}C_r = (k^2 - 8)\, {}^{n}C_{r+1}$

$$\underbrace{r+1 \ge 0,\ \ r \ge 0}_{r\ge 0}$$

$$\frac{{}^{n-1}C_r}{{}^{n}C_{r+1}} = k^2 - 8$$

$$\frac{r+1}{n} = k^2 - 8$$

$$\Rightarrow k^2 - 8 > 0$$

$$\left(k-2\sqrt{2}\right)\left(k+2\sqrt{2}\right) > 0$$

$$k \in \left(-\infty, -2\sqrt{2}\right) \cup \left(2\sqrt{2}, \infty\right) \qquad \ldots(\text{I})$$

$$\therefore\ n \ge r+1,\ \frac{r+1}{n} \le 1$$

$$\Rightarrow k^2 - 8 \le 1$$

$$k^2 - 9 \le 0$$

$$-3 \le k \le 3 \qquad \ldots(\text{II})$$

From equation (I) and (II) we get

$$k \in \left[-3, -2\sqrt{2}\right) \cup \left(2\sqrt{2}, 3\right]$$

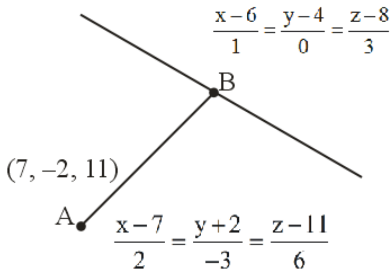
2. The distance, of the point (7, –2, 11) from the line $\frac{x-6}{1} = \frac{y-4}{0} = \frac{z-8}{3}$ along the line $\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-5}{6}$, is :

(1) 12 (2) 14

(3) 18 (4) 21

Ans. (2)

Sol. $B = (2\lambda + 7, -3\lambda - 2, 6\lambda + 11)$



Point B lies on $\frac{x-6}{1} = \frac{y-4}{0} = \frac{z-8}{3}$

$$\frac{2\lambda+7-6}{1} = \frac{-3\lambda-2-4}{0} = \frac{6\lambda+11-8}{3}$$

$$-3\lambda - 6 = 0$$

$$\lambda = -2$$

$$B \Rightarrow (3, 4, -1)$$

$$AB = \sqrt{(7-3)^2 + (4+2)^2 + (11+1)^2}$$

$$= \sqrt{16+36+144}$$

$$= \sqrt{196} = 14$$

3. Let x = x(t) and y = y(t) be solutions of the differential equations $\frac{dx}{dt} + ax = 0$ and $\frac{dy}{dt} + by = 0$ respectively, a, b ∈ R. Given that x(0) = 2; y(0) = 1 and 3y(1) = 2x(1), the value of t, for which x(t) = y(t), is :

(1) $\log_{\frac{2}{3}} 2$ (2) $\log_4 3$

(3) $\log_3 4$ (4) $\log_{\frac{4}{3}} 2$

Ans. (4)

Sol. $\frac{dx}{dt} + ax = 0$

$\frac{dx}{x} = -adt$

$\int \frac{dx}{x} = -a\int dt$

$\ln|x| = -at + c$

at $t = 0$, $x = 2$

$\ln 2 = 0 + c$

$\ln x = -at + \ln 2$

$\frac{x}{2} = e^{-at}$

$x = 2e^{-at}$ ....(i)

$\frac{dy}{dt} + by = 0$

$\frac{dy}{y} = -bdt$

$\ln|y| = -bt + \lambda$

$t = 0$, $y = 1$

$0 = 0 + \lambda$

$y = e^{-bt}$ ....(ii)

According to question

$3y(1) = 2x(1)$

$3e^{-b} = 2(2e^{-a})$

$e^{a-b} = \frac{4}{3}$

For $x(t) = y(t)$

$\Rightarrow 2e^{-at} = e^{-bt}$

$2 = e^{(a-b)t}$

$2 = \left(\frac{4}{3}\right)^t$

$\log_{\frac{4}{3}} 2 = t$

4. If (a, b) be the orthocentre of the triangle whose vertices are (1, 2), (2, 3) and (3, 1), and $I_1 = \int_a^b x\sin(4x - x^2)dx$, $I_2 = \int_a^b \sin(4x - x^2)dx$, then $36\frac{I_1}{I_2}$ is equal to :

(1) 72 (2) 88

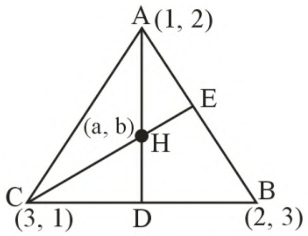
(3) 80 (4) 66

Ans. (1)

Sol. Equation of CE

$y - 1 = -(x - 3)$

$x + y = 4$



orthocentre lies on the line $x + y = 4$

so, $a + b = 4$

$I_1 = \int_a^b x\sin(x(4-x))dx$ ...(i)

Using king rule

$I_1 = \int_a^b (4-x)\sin(x(4-x))dx$ ...(ii)

(i) + (ii)

$2I_1 = \int_a^b 4\sin(x(4-x))dx$

$2I_1 = 4I_2$

$I_1 = 2I_2$

$\frac{I_1}{I_2} = 2$

$\frac{36I_1}{I_2} = 72$

5. If A denotes the sum of all the coefficients in the expansion of $(1 - 3x + 10x^2)^n$ and B denotes the sum of all the coefficients in the expansion of $(1 + x^2)^n$, then :

(1) $A = B^3$ (2) $3A = B$

(3) $B = A^3$ (4) $A = 3B$

Ans. (1)

Sol. Sum of coefficients in the expansion of

$(1 - 3x + 10x^2)^n = A$

then $A = (1 - 3 + 10)^n = 8^n$ (put $x = 1$)

and sum of coefficients in the expansion of

$(1 + x^2)^n = B$

then $B = (1 + 1)^n = 2^n$

$A = B^3$

6. The number of common terms in the progressions 4, 9, 14, 19, ... , up to 25^{th} term and 3, 6, 9, 12, ..., up to 37^{th} term is :

(1) 9 (2) 5
(3) 7 (4) 8

Ans. (3)

Sol. 4, 9, 14, 19, …., up to 25^{th} term

$T_{25} = 4 + (25 – 1) 5 = 4 + 120 = 124$

3, 6, 9, 12, …, up to 37^{th} term

$T_{37} = 3 + (37 – 1)3 = 3 + 108 = 111$

Common difference of I^{st} series $d_1 = 5$

Common difference of II^{nd} series $d_2 = 3$

First common term = 9, and

their common difference = 15 (LCM of d_1 and d_2)

then common terms are

9, 24, 39, 54, 69, 84, 99

7. If the shortest distance of the parabola $y^2 = 4x$ from the centre of the circle $x^2 + y^2 – 4x – 16y + 64 = 0$ is d, then d^2 is equal to :

(1) 16 (2) 24
(3) 20 (4) 36

Ans. (3)

Sol. Equation of normal to parabola

$y = mx – 2m – m^3$

this normal passing through center of circle (2, 8)

$8 = 2m – 2m – m^3$

$m = –2$

So point P on parabola $\Rightarrow (am^2, –2am) = (4, 4)$

And C = (2, 8)

$PC = \sqrt{4+16} = \sqrt{20}$

$d^2 = 20$

8. If the shortest distance between the lines $\frac{x-4}{1} = \frac{y+1}{2} = \frac{z}{-3}$ and $\frac{x-\lambda}{2} = \frac{y+1}{4} = \frac{z-2}{-5}$ is $\frac{6}{\sqrt{5}}$, then the sum of all possible values of λ is :

(1) 5 (2) 8
(3) 7 (4) 10

Ans. (2)

Sol. $\frac{x-4}{1} = \frac{y+1}{2} = \frac{z}{-3}$

$\frac{x-\lambda}{2} = \frac{y+1}{4} = \frac{z-2}{-5}$

the shortest distance between the lines

$$= \left|\frac{(\vec{a}-\vec{b})\cdot(\vec{d_1}\times\vec{d_2})}{|\vec{d_1}\times\vec{d_2}|}\right|$$

$$= \left|\frac{\begin{vmatrix} \lambda-4 & 0 & 2 \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}}\right|$$

$$= \left|\frac{(\lambda-4)(-10+12)-0+2(4-4)}{|2\hat{i}-1\hat{j}+0\hat{k}|}\right|$$

$$\frac{6}{\sqrt{5}} = \left|\frac{2(\lambda-4)}{\sqrt{5}}\right|$$

$3 = |\lambda – 4|$

$\lambda – 4 = \pm 3$

$\lambda = 7, 1$

Sum of all possible values of λ is = 8

9. If $\int_0^1 \frac{1}{\sqrt{3+x}+\sqrt{1+x}}dx = a + b\sqrt{2} + c\sqrt{3}$, where a, b, c are rational numbers, then 2a + 3b – 4c is equal to :

(1) 4 (2) 10
(3) 7 (4) 8

Ans. (4)

Sol. $\int_0^1 \frac{1}{\sqrt{3+x}+\sqrt{1+x}}dx = \int_0^1 \frac{\sqrt{3+x}-\sqrt{1+x}}{(3+x)-(1+x)}dx$

$$\frac{1}{2}\left[\int_0^1 \sqrt{3+x}\,dx - \int_0^1 \left(\sqrt{1+x}\right)dx\right]$$

$$\frac{1}{2}\left[2\frac{(3+x)^{\frac{3}{2}}}{3}-\frac{2(1+x)^{\frac{3}{2}}}{3}\right]_0^1$$

$$\frac{1}{2}\left[\frac{2}{3}\left(8-3\sqrt{3}\right)-\frac{2}{3}\left(2^{\frac{3}{2}}-1\right)\right]$$

$$\frac{1}{3}\left[8-3\sqrt{3}-2\sqrt{2}+1\right]$$

$$=3-\sqrt{3}-\frac{2}{3}\sqrt{2}=a+b\sqrt{2}+c\sqrt{3}$$

$$a=3,\ b=-\frac{2}{3},\ c=-1$$

$2a + 3b - 4c = 6 - 2 + 4 = 8$

10. Let S = {l, 2, 3, ... , 10}. Suppose M is the set of all the subsets of S, then the relation

R = {(A, B): $A \cap B \neq \phi$; A, B $\in$ M} is :

(1) symmetric and reflexive only

(2) reflexive only

(3) symmetric and transitive only

(4) symmetric only

Ans. (4)

Sol. Let S = {1, 2, 3, …, 10}

R = {(A, B): $A \cap B \neq \phi$; A, B $\in$ M}

For Reflexive,

M is subset of ‘S’

So $\phi \in M$

for $\phi \cap \phi = \phi$

$\Rightarrow$ but relation is $A \cap B \neq \phi$

So it is not reflexive.

For symmetric,

ARB $\quad A \cap B \neq \phi$,

$\Rightarrow$ BRA $\quad \Rightarrow B \cap A \neq \phi$,

So it is symmetric.

For transitive,

If A = {(1, 2), (2, 3)}

B = {(2, 3), (3, 4)}

C = {(3, 4), (5, 6)}

ARB & BRC but A does not relate to C

So it not transitive

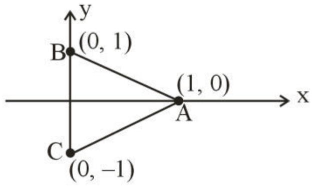
11. If S = {z $\in$ C : |z – i| = |z + i| = |z–1|}, then, n(S) is:

(1) 1 (2) 0

(3) 3 (4) 2

Ans. (1)

Sol. |z – i| = |z + i| = |z – 1|



ABC is a triangle. Hence its circum-centre will be the only point whose distance from A, B, C will be same.

So n(S) = 1

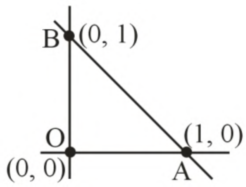
12. Four distinct points (2k, 3k), (1, 0), (0, 1) and (0, 0) lie on a circle for k equal to :

(1) $\frac{2}{13}$ (2) $\frac{3}{13}$

(3) $\frac{5}{13}$ (4) $\frac{1}{13}$

Ans. (3)

Sol. (2k, 3k) will lie on circle whose diameter is AB.



(x – 1) (x) + (y – 1) (y) = 0

$x^2 + y^2 - x - y = 0$ …(i)

Satisfy (2k, 3k) in (i)

$(2k)^2 + (3k)^2 - 2k - 3k = 0$

$13k^2 - 5k = 0$

$$k=0,\ k=\frac{5}{13}$$

$$\text{hence } k=\frac{5}{13}$$

13. Consider the function.

$$f(x)=\begin{cases}\dfrac{a(7x-12-x^2)}{b|x^2-7x+12|} & , \quad x<3\\ 2^{\frac{\sin(x-3)}{x-[x]}} & , \quad x>3\\ b & , \quad x=3\end{cases}$$

Where [x] denotes the greatest integer less than or equal to x. If S denotes the set of all ordered pairs (a, b) such that f(x) is continuous at x = 3, then the number of elements in S is :

(1) 2 (2) Infinitely many

(3) 4 (4) 1

Ans. (4)

Sol. $f(3^-)=\dfrac{a}{b}\dfrac{(7x-12-x^2)}{|x^2-7x+12|}$ (for f(x) to be cont.)

$\Rightarrow f(3^-)=\dfrac{-a}{b}\dfrac{(x-3)(x-4)}{(x-3)(x-4)};x<3\Rightarrow\dfrac{-a}{b}$

Hence $f(3^-)=\dfrac{-a}{b}$

Then $f(3^+)=2^{\lim_{x\to3^+}\left(\frac{\sin(x-3)}{x-3}\right)}=2$ and

f(3) = b.

Hence $f(3)=f(3^+)=f(3^-)$

$\Rightarrow b=2=-\dfrac{a}{b}$

b = 2, a = –4

Hence only 1 ordered pair (–4, 2).

14. Let $a_1, a_2, \ldots, a_{10}$ be 10 observations such that $\sum_{k=1}^{10} a_k = 50$ and $\sum_{\forall k<j} a_k \cdot a_j = 1100$. Then the standard deviation of $a_1, a_2, .., a_{10}$ is equal to :

(1) 5 (2) $\sqrt{5}$

(3) 10 (4) $\sqrt{115}$

Ans. (2)

Sol. $\sum_{k=1}^{10} a_k = 50$

$a_1 + a_2 + \ldots + a_{10} = 50$ ….(i)

$\sum_{\forall k<j} a_k a_j = 1100$ ....(ii)

If $a_1 + a_2 + \ldots + a_{10} = 50$.

$(a_1 + a_2 + \ldots + a_{10})^2 = 2500$

$\Rightarrow \sum_{i=1}^{10} a_i^2 + 2\sum_{k<j} a_k a_j = 2500$

$\Rightarrow \sum_{i=1}^{10} a_i^2 = 2500-2(1100)$

$\sum_{i=1}^{10} a_i^2 = 300$, Standard deviation 'σ'

$=\sqrt{\dfrac{\sum a_i^2}{10}-\left(\dfrac{\sum a_i}{10}\right)^2}=\sqrt{\dfrac{300}{10}-\left(\dfrac{50}{10}\right)^2}$

$=\sqrt{30-25}=\sqrt{5}$

15. The length of the chord of the ellipse $\dfrac{x^2}{25}+\dfrac{y^2}{16}=1$, whose mid point is $\left(1, \dfrac{2}{5}\right)$, is equal to :

(1) $\dfrac{\sqrt{1691}}{5}$ (2) $\dfrac{\sqrt{2009}}{5}$

(3) $\dfrac{\sqrt{1741}}{5}$ (4) $\dfrac{\sqrt{1541}}{5}$

Ans. (1)

Sol. Equation of chord with given middle point.

$T = S_1$

$\dfrac{x}{25}+\dfrac{y}{40}=\dfrac{1}{25}+\dfrac{1}{100}$

$\dfrac{8x+5y}{200}=\dfrac{8+2}{200}$

$y=\dfrac{10-8x}{5}$ …(i)

$$\frac{x^2}{25}+\frac{(10-8x)^2}{400}=1 \quad \text{(put in original equation)}$$

$$\frac{16x^2+100+64x^2-160x}{400}=1$$

$4x^2 - 8x - 15 = 0$

$$x=\frac{8\pm\sqrt{304}}{8}$$

$$x_1=\frac{8+\sqrt{304}}{8};\ x_2=\frac{8-\sqrt{304}}{8}$$

Similarly, $y=\frac{10-18\pm\sqrt{304}}{5}=\frac{2\pm\sqrt{304}}{5}$

$$y_1=\frac{2-\sqrt{304}}{5};\ y_2=\frac{2+\sqrt{304}}{5}$$

Distance $=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$$=\sqrt{\frac{4\times304}{64}+\frac{4\times304}{25}}=\frac{\sqrt{1691}}{5}$$

16. The portion of the line 4x + 5y = 20 in the first quadrant is trisected by the lines L_1 and L_2 passing through the origin. The tangent of an angle between the lines L_1 and L_2 is :

(1) $\frac{8}{5}$ (2) $\frac{25}{41}$

(3) $\frac{2}{5}$ (4) $\frac{30}{41}$

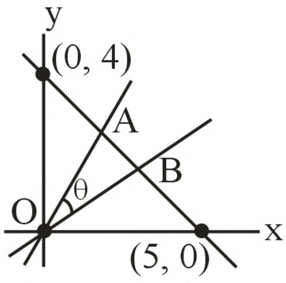
Ans. (4)

Sol. Co-ordinates of $A=\left(\frac{5}{3},\frac{8}{3}\right)$

Co-ordinates of $B=\left(\frac{10}{3},\frac{4}{3}\right)$

Slope of OA $= m_1 = \frac{8}{5}$

Slope of OB $= m_2 = \frac{2}{5}$



$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

$$\tan\theta=\frac{\frac{6}{5}}{1+\frac{16}{25}}=\frac{30}{41}$$

$$\tan\theta=\frac{30}{41}$$

17. Let $\vec{a}=\hat{i}+2\hat{j}+\hat{k}$, $\vec{b}=3(\hat{i}-\hat{j}+\hat{k})$. Let $\vec{c}$ be the vector such that $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}\cdot\vec{c}=3$. Then $\vec{a}\cdot\left(\left(\vec{c}\times\vec{b}\right)-\vec{b}-\vec{c}\right)$ is equal to :

(1) 32 (2) 24

(3) 20 (4) 36

Ans. (2)

Sol. $\vec{a}\cdot\left[\left(\vec{c}\times\vec{b}\right)-\vec{b}-\vec{c}\right]$

$\vec{a}\cdot\left(\vec{c}\times\vec{b}\right)-\vec{a}\cdot\vec{b}-\vec{a}\cdot\vec{c}$ ....(i)

given $\vec{a}\times\vec{c}=\vec{b}$

$\Rightarrow \left(\vec{a}\times\vec{c}\right)\cdot\vec{b}=\vec{b}\cdot\vec{b}=\left|\vec{b}\right|^2=27$

$\Rightarrow \vec{a}\cdot\left(\vec{c}\times\vec{b}\right)=\left[\vec{a}\ \vec{c}\ \vec{b}\right]=\left(\vec{a}\times\vec{c}\right)\cdot\vec{b}=27$ ...(ii)

Now $\vec{a}\cdot\vec{b}=3-6+3=0$ ...(iii)

$\vec{a}\cdot\vec{c}=3$ ...(iv) (given)

By (i), (ii), (iii) & (iv)

27 – 0 – 3 = 24

18. If $a = \lim_{x\to 0} \frac{\sqrt{1+\sqrt{1+x^4}} - \sqrt{2}}{x^4}$ and

$b = \lim_{x\to 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{1+\cos x}}$, then the value of ab^3 is :

(1) 36 (2) 32 (3) 25 (4) 30

Ans. (2)

Sol. $a = \lim_{x\to 0} \frac{\sqrt{1+\sqrt{1+x^4}} - \sqrt{2}}{x^4}$

$$= \lim_{x\to 0} \frac{\sqrt{1+x^4} - 1}{x^4\left(\sqrt{1+\sqrt{1+x^4}} + \sqrt{2}\right)}$$

$$= \lim_{x\to 0} \frac{x^4}{x^4\left(\sqrt{1+\sqrt{1+x^4}} + \sqrt{2}\right)\left(\sqrt{1+x^4}+1\right)}$$

Applying limit $a = \frac{1}{4\sqrt{2}}$

$$b = \lim_{x\to 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{1+\cos x}}$$

$$= \lim_{x\to 0} \frac{\left(1-\cos^2 x\right)\left(\sqrt{2}+\sqrt{1+\cos x}\right)}{2-(1+\cos x)}$$

$$b = \lim_{x\to 0}(1+\cos x)\left(\sqrt{2}+\sqrt{1+\cos x}\right)$$

Applying limits $b = 2\left(\sqrt{2}+\sqrt{2}\right) = 4\sqrt{2}$

Now, $ab^3 = \frac{1}{4\sqrt{2}} \times \left(4\sqrt{2}\right)^3 = 32$

19. Consider the matrix $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Given below are two statements :

Statement I: f(–x) is the inverse of the matrix f(x).

Statement II: f(x) f(y) = f(x + y).

In the light of the above statements, choose the correct answer from the options given below

(1) Statement I is false but Statement II is true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are true

Ans. (4)

Sol. $f(-x) = \begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$f(x)\cdot f(-x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Hence statement- I is correct

Now, checking statement II

$$f(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f(x)\cdot f(y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow f(x)\cdot f(y) = f(x+y)$

Hence statement-II is also correct.

20. The function $f : N - \{1\} \to N$; defined by f(n) = the highest prime factor of n, is :

(1) both one-one and onto

(2) one-one only

(3) onto only

(4) neither one-one nor onto

Ans. (4)

Sol. $f : N - \{1\} \to N$

f(n) = The highest prime factor of n.

f(2) = 2

f(4) = 2

$\Rightarrow$ many one

4 is not image of any element

$\Rightarrow$ into

Hence many one and into

Neither one-one nor onto.

SECTION-B

21. The least positive integral value of α, for which the angle between the vectors $\alpha\hat{i}-2\hat{j}+2\hat{k}$ and $\alpha\hat{i}+2\alpha\hat{j}-2\hat{k}$ is acute, is _____.

Ans. (5)

Sol. $\cos\theta=\dfrac{\left(\alpha\hat{i}-2\hat{j}+2\hat{k}\right)\cdot\left(\alpha\hat{i}+2\alpha\hat{j}-2\hat{k}\right)}{\sqrt{\alpha^2+4+4}\ \sqrt{\alpha^2+4\alpha^2+4}}$

$\cos\theta=\dfrac{\alpha^2-4\alpha-4}{\sqrt{\alpha^2+8}\ \sqrt{5\alpha^2+4}}$

$\Rightarrow \alpha^2-4\alpha-4>0$

$\Rightarrow \alpha^2-4\alpha+4>8 \qquad \Rightarrow (\alpha-2)^2>8$

$\Rightarrow \alpha-2>2\sqrt{2}$ or $\alpha-2<-2\sqrt{2}$

$\alpha>2+2\sqrt{2}$ or $\alpha<2-2\sqrt{2}$

$\alpha\in(-\infty,-0.82)\cup(4.82,\infty)$

Least positive integral value of $\alpha \Rightarrow 5$

22. Let for a differentiable function $f:(0,\infty)\to R$,

$f(x)-f(y)\ge \log_e\left(\dfrac{x}{y}\right)+x-y,\ \forall\ x,y\in(0,\infty)$.

Then $\sum\limits_{n=1}^{20} f'\left(\dfrac{1}{n^2}\right)$ is equal to _____.

Ans. (2890)

Sol. $f(x)-f(y)\ge \ln x-\ln y+x-y$

$\dfrac{f(x)-f(y)}{x-y}\ge\dfrac{\ln x-\ln y}{x-y}+1$

Let $x>y$

$\lim\limits_{y\to x} f'\left(x^-\right)\ge\dfrac{1}{x}+1 \qquad \ldots..(1)$

Let $x<y$

$\lim\limits_{y\to x} f'\left(x^+\right)\le\dfrac{1}{x}+1 \qquad \ldots..\ (2)$

$f^1(x^-)=f^1(x^+)$

$f^1(x)=\dfrac{1}{x}+1$

$f'\left(\dfrac{1}{x^2}\right)=x^2+1$

$\sum\limits_{x=1}^{20}\left(x^2+1\right)=\sum\limits_{x-1}^{20}x^2+20$

$=\dfrac{20\times21\times41}{6}+20$

$=2890$

23. If the solution of the differential equation $(2x+3y-2)\,dx+(4x+6y-7)\,dy=0$, $y(0)=3$, is $\alpha x+\beta y+3\log_e|2x+3y-\gamma|=6$, then $\alpha+2\beta+3\gamma$ is equal to _____.

Ans. (29)

Sol. $2x+3y-2=t \qquad\qquad 4x+6y-4=2t$

$2+3\dfrac{dy}{dx}=\dfrac{dt}{dx} \qquad\qquad 4x+6y-7=2t-3$

$\dfrac{dy}{dx}=\dfrac{-(2x+3y-2)}{4x+6y-7}$

$\dfrac{dt}{dx}=\dfrac{-3t+4t-6}{2t-3}=\dfrac{t-6}{2t-3}$

$\int\dfrac{2t-3}{t-6}\,dt=\int dx$

$\int\left(\dfrac{2t-12}{t-6}+\dfrac{9}{t-6}\right)\cdot dt=x$

$2t+9\ln(t-6)=x+c$

$2(2x+3y-2)+9\ln(2x+3y-8)=x+c$

$x=0,\ y=3$

$c=14$

$4x+6y-4+9\ln(2x+3y-8)=x+14$

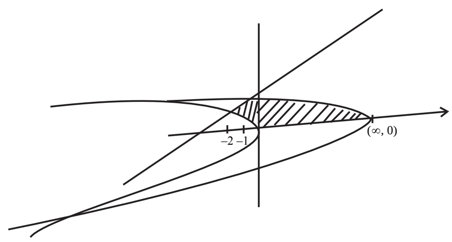
$x+2y+3\ln(2x+3y-8)=6$

$\alpha=1,\ \beta=2,\ \gamma=8$

$\alpha+2\beta+3\gamma=1+4+24=29$

24. Let the area of the region $\{(x,y):\ x-2y+4\ge0,\ x+2y^2\ge0,\ x+4y^2\le8,\ y\ge0\}$ be $\dfrac{m}{n}$, where m and n are coprime numbers. Then m + n is equal to _____.

Ans. (119)



Sol.

$A=\int\limits_0^1\left[\left(8-4y^2\right)-\left(-2y^2\right)\right]dy+$

$\int\limits_1^{3/2}\left[\left(8-4y^2\right)-(2y-4)\right]dy$

$=\left[8y-\dfrac{2y^3}{3}\right]_0^1+\left[12y-y^2-\dfrac{4y^3}{3}\right]_1^{3/2}=\dfrac{107}{12}=\dfrac{m}{n}$

$\therefore\ m+n=119$

25. If

$$8=3+\frac{1}{4}(3+p)+\frac{1}{4^2}(3+2p)+\frac{1}{4^3}(3+3p)+....\infty,$$

then the value of p is _____.

Ans. (9)

Sol. $8=\frac{3}{1-\frac{1}{4}}+\frac{p\cdot\frac{1}{4}}{\left(1-\frac{1}{4}\right)^2}$

(sum of infinite terms of A.G.P = $\frac{a}{1-r}+\frac{dr}{(1-r)^2}$)

$\Rightarrow \frac{4p}{9}=4 \;\Rightarrow p=9$

26. A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required and let a = P(X = 3), b = P(X ≥ 3) and c = P(X ≥ 6 |X > 3). Then $\frac{b+c}{a}$ is equal to _____.

Ans. (12)

Sol. $a=P(X=3)=\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}=\frac{25}{216}$

$b=P(X\geq3)=\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}+\left(\frac{5}{6}\right)^3\cdot\frac{1}{6}+\left(\frac{5}{6}\right)^4\cdot\frac{1}{6}+......$

$=\frac{\frac{25}{216}}{1-\frac{5}{6}}=\frac{25}{216}\times\frac{6}{1}=\frac{25}{36}$

$P(X\geq6)=\left(\frac{5}{6}\right)^5\cdot\frac{1}{6}+\left(\frac{5}{6}\right)^6\cdot\frac{1}{6}+.......$

$=\frac{\left(\frac{5}{6}\right)^5\cdot\frac{1}{6}}{1-\frac{5}{6}}=\left(\frac{5}{6}\right)^5$

$c=\frac{\left(\frac{5}{6}\right)^5}{\left(\frac{5}{6}\right)^3}=\frac{25}{36}$

$\frac{b+c}{a}=\frac{\left(\frac{5}{6}\right)^2+\left(\frac{5}{6}\right)^2}{\left(\frac{5}{6}\right)^2\cdot\frac{1}{6}}=12$

27. Let the set of all a ∈ R such that the equation $\cos 2x+a\sin x=2a-7$ has a solution be [p, q] and $r=\tan 9°-\tan 27°-\frac{1}{\cot 63°}+\tan 81°$, then pqr is equal to _____.

Ans. (48)

Sol. cos2x + a·sinx = 2a – 7

$a(\sin x-2)=2(\sin x-2)(\sin x+2)$

$\sin x=2,\quad a=2(\sin x+2)$

⇒ a ∈ [2, 6]

p = 2 q = 6

r = tan 9° + cot 9° – tan27 – cot27

$r=\frac{1}{\sin 9\cdot\cos 9}-\frac{1}{\sin 27\cdot\cos 27}$

$=2\left[\frac{4}{\sqrt{5}-1}-\frac{4}{\sqrt{5}+1}\right]$

r = 4

p . q . r = 2 × 6 × 4 = 48

28. Let $f(x)=x^3+x^2f'(1)+xf''(2)+f'''(3)$, x ∈ R. Then $f'(10)$ is equal to _____.

Ans. (202)

Sol. $f(x)=x^3+x^2\cdot f'(1)+x\cdot f''(2)+f'''(3)$

$f'(x)=3x^2+2xf'(1)+f''(2)$

$f''(x)=6x+2f'(1)$

$f'''(x)=6$

$f'(1)=-5,\; f''(2)=2,\; f'''(3)=6$

$f(x)=x^3+x^2\cdot(-5)+x\cdot(2)+6$

$f'(x)=3x^2-10x+2$

$f'(10)=300-100+2=202$

29. Let $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, $B = [B_1, B_2, B_3]$, where B_1, B_2, B_3 are column matrices, and $AB_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$,

$AB_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$, $AB_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

If $\alpha = |B|$ and β is the sum of all the diagonal elements of B, then $\alpha^3 + \beta^3$ is equal to _____.

Ans. (28)

Sol. $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ $\quad B = [B_1, B_2, B_3]$

$B_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$, $B_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$, $B_3 = \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix}$

$$AB_1 = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 = 1, y_1 = -1, z_1 = -1$

$$AB_2 = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$

$x_2 = 2, y_2 = 1, z_2 = -2$

$$AB_3 = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$x_3 = 2, y_3 = 0, z_3 = -1$

$$B = \begin{bmatrix} 1 & 2 & 2 \\ -1 & 1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

$\alpha = |B| = 3$

$\beta = 1$

$\alpha^3 + \beta^3 = 27 + 1 = 28$

30. If α satisfies the equation $x^2 + x + 1 = 0$ and $(1 + \alpha)^7 = A + B\alpha + C\alpha^2$, $A, B, C \geq 0$, then $5(3A - 2B - C)$ is equal to _____.

Ans. (5)

Sol. $x^2 + x + 1 = 0 \Rightarrow x = \omega, \omega^2 = \alpha$

Let $\alpha = \omega$

Now $(1 + \alpha)^7 = -\omega^{14} = -\omega^2 = 1 + \omega$

$A = 1, B = 1, C = 0$

$\therefore 5(3A - 2B - C) = 5(3 - 2 - 0) = 5$

SECTION-A

31. Position of an ant (S in metres) moving in Y-Z plane is given by $S = 2t^2\hat{j} + 5\hat{k}$ (where t is in second). The magnitude and direction of velocity of the ant at t = 1 s will be :

(1) 16 m/s in y-direction

(2) 4 m/s in x-direction

(3) 9 m/s in z-direction

(4) 4 m/s in y-direction

Ans. (4)

Sol. $\vec{v} = \frac{d\vec{s}}{dt} = 4t\hat{j}$

At t = 1 sec $\vec{v} = 4\hat{j}$

32. Given below are two statements :

Statement (I) : Viscosity of gases is greater than that of liquids.

Statement (II) : Surface tension of a liquid decreases due to the presence of insoluble impurities.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Statement I is correct but statement II is incorrect

(2) Statement I is incorrect but Statement II is correct

(3) Both Statement I and Statement II are incorrect

(4) Both Statement I and Statement II are correct

Ans. (2)

Sol. Gases have less viscosity.

Due to insoluble impurities like detergent surface tension decreases

33. If the refractive index of the material of a prism is $\cot\left(\frac{A}{2}\right)$, where A is the angle of prism then the angle of minimum deviation will be

(1) $\pi - 2A$ (2) $\frac{\pi}{2} - 2A$

(3) $\pi - A$ (4) $\frac{\pi}{2} - A$

Ans. (1)

Sol. $\cot\frac{A}{2} = \frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin\frac{A}{2}}$

$\Rightarrow \cos\frac{A}{2} = \sin\left(\frac{A+\delta_{min}}{2}\right)$

$\frac{A+\delta_{min}}{2} = \frac{\pi}{2} - \frac{A}{2}$

$\delta_{min} = \pi - 2A$

34. A proton moving with a constant velocity passes through a region of space without any change in its velocity. If $\vec{E}$ and $\vec{B}$ represent the electric and magnetic fields respectively, then the region of space may have :

(A) $E = 0, B = 0$ (B) $E = 0, B \neq 0$

(C) $E \neq 0, B = 0$ (D) $E \neq 0, B \neq 0$

Choose the most appropriate answer from the options given below :

(1)(A), (B) and (C) only

(2) (A), (C) and (D) only

(3) (A), (B) and (D) only

(4) (B), (C) and (D) only

Ans. (3)

Sol. Net force on particle must be zero i.e. $q\vec{E} + q\vec{V} \times \vec{B} = 0$

Possible cases are

(i) $\vec{E} \,\&\, \vec{B} = 0$

(ii) $\vec{V} \times \vec{B} = 0, \vec{E} = 0$

(iii) $q\vec{E} = -q\vec{V} \times \vec{B}$

$\vec{E} \neq 0 \,\&\, \vec{B} \neq 0$

35. The acceleration due to gravity on the surface of earth is g. If the diameter of earth reduces to half of its original value and mass remains constant, then acceleration due to gravity on the surface of earth would be :

(1) g/4 (2) 2g
(3) g/2 (4) 4g

Ans. **(4)**

Sol. $g = \frac{GM}{R^2} \Rightarrow g \propto \frac{1}{R^2}$

$\frac{g_2}{g_1} = \frac{R_1^2}{R_2^2}$

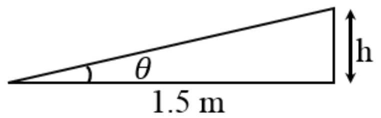
$g_2 = 4g_1 \left(R_2 = \frac{R_1}{2} \right)$

36. A train is moving with a speed of 12 m/s on rails which are 1.5 m apart. To negotiate a curve radius 400 m, the height by which the outer rail should be raised with respect to the inner rail is (Given, g = 10 m/s^2) :

(1) 6.0 cm (2) 5.4 cm
(3) 4.8 cm (4) 4.2 cm

Ans. **(2)**

Sol. $\tan\theta = \frac{v^2}{Rg} = \frac{12 \times 12}{10 \times 400}$

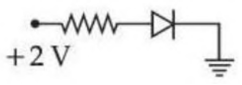


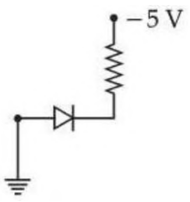
$\tan\theta = \frac{h}{1.5}$

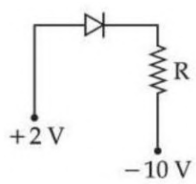
$\Rightarrow \frac{h}{1.5} = \frac{144}{4000}$

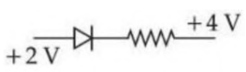
h = 5.4 cm

37. Which of the following circuits is reverse - biased ?

(1) 

(2) 

(3) 

(4) 

Ans. **(4)**

Sol. P end should be at higher potential for forward biasing.

38. Identify the physical quantity that cannot be measured using spherometer :

(1) Radius of curvature of concave surface
(2) Specific rotation of liquids
(3) Thickness of thin plates
(4) Radius of curvature of convex surface

Ans. **(2)**

Sol. Spherometer can be used to measure curvature of surface.

39. Two bodies of mass 4 g and 25 g are moving with equal kinetic energies. The ratio of magnitude of their linear momentum is :

(1) 3 : 5 (2) 5 : 4
(3) 2 : 5 (4) 4 : 5

Ans. **(3)**

Sol. $\frac{P_1^2}{2m_1} = \frac{P_2^2}{2m_2}$

$\frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}} = \frac{2}{5}$

40. 0.08 kg air is heated at constant volume through 5°C. The specific heat of air at constant volume is 0.17 kcal/kg°C and J = 4.18 joule/cal. The change in its internal energy is approximately.

(1) 318 J (2) 298 J
(3) 284 J (4) 142 J

Ans. **(3)**

Sol. $Q = \Delta U$ as work done is zero [constant volume]

$\Delta U = ms\,\Delta T$

$= 0.08 \times (170 \times 4.18) \times 5$

$\simeq 284$ J

41. The radius of third stationary orbit of electron for Bohr's atom is R. The radius of fourth stationary orbit will be:

(1) $\frac{4}{3}R$ (2) $\frac{16}{9}R$

(3) $\frac{3}{4}R$ (4) $\frac{9}{16}R$

Ans. **(2)**

Sol. $r \propto \frac{n^2}{Z}$

$\frac{r_4}{r_3} = \frac{4^2}{3^2}$

$r_4 = \frac{16}{9}R$

42. A rectangular loop of length 2.5 m and width 2 m is placed at 60° to a magnetic field of 4 T. The loop is removed from the field in 10 sec. The average emf induced in the loop during this time is

(1) – 2V (2) + 2V

(3) + 1V (4) – 1V

Ans. **(3)**

Sol. Average emf= $\frac{\text{Change in flux}}{\text{Time}} = -\frac{\Delta\phi}{\Delta t}$

$= -\frac{0-(4\times(2.5\times2)\cos 60°)}{10}$

$= +1V$

43. An electric charge $10^{-6}\mu C$ is placed at origin (0, 0) m of X –Y co-ordinate system. Two points P and Q are situated at $(\sqrt{3},\sqrt{3})$m and $(\sqrt{6},0)$m respectively. The potential difference between the points P and Q will be :

(1) $\sqrt{3}V$

(2) $\sqrt{6}V$

(3) 0 V

(4) 3 V

Ans. **(3)**

Sol. Potential difference $= \frac{KQ}{r_1} - \frac{KQ}{r_2}$

$r_1 = \sqrt{(\sqrt{3})^2 + (\sqrt{3})^2}$

$r_2 = \sqrt{(\sqrt{6})^2 + 0}$

As $r_1 = r_2 = \sqrt{6}m$

So potential difference = 0

44. A convex lens of focal length 40 cm forms an image of an extended source of light on a photo-electric cell. A current I is produced. The lens is replaced by another convex lens having the same diameter but focal length 20 cm. The photoelectric current now is :

(1) $\frac{I}{2}$ (2) 4 I

(3) 2 I (4) I

Ans. **(4)**

Sol. As amount of energy incident on cell is same so current will remain same.

45. A body of mass 1000 kg is moving horizontally with a velocity 6 m/s. If 200 kg extra mass is added, the final velocity (in m/s) is:

(1) 6 (2) 2

(3) 3 (4) 5

Ans. **(4)**

Sol. Momentum will remain conserve

$1000 \times 6 = 1200 \times v$

$v = 5$ m/s

46. A plane electromagnetic wave propagating in x-direction is described by

$E_y = (200\ Vm^{-1}) \sin[1.5 \times 10^7 t - 0.05\ x]$;

The intensity of the wave is :

(Use $\epsilon_0 = 8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}$)

(1) 35.4 Wm^{-2} (2) 53.1 Wm^{-2}

(3) 26.6 Wm^{-2} (4) 106.2 Wm^{-2}

Ans. **(2)**

Sol. $I = \frac{1}{2}\varepsilon_0 E_0^2 \times c$

$I = \frac{1}{2} \times 8.85 \times 10^{-12} \times 4 \times 10^4 \times 3 \times 10^8$

$I = 53.1\ W/m^2$

47. Given below are two statements :

Statement (I) : Planck's constant and angular momentum have same dimensions.

Statement (II) : Linear momentum and moment of force have same dimensions.

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are false

(3) Both Statement I and Statement II are true

(4) Statement I is false but Statement II is true

Ans. **(1)**

Sol. $[h] = ML^2T^{-1}$

$[L] = ML^2T^{-1}$

$[P] = MLT^{-1}$

$[\tau] = ML^2T^{-2}$

(Here h is Planck's constant, L is angular momentum, P is linear momentum and τ is moment of force)

48. A wire of length 10 cm and radius $\sqrt{7}\times10^{-4}$m connected across the right gap of a meter bridge. When a resistance of 4.5 Ω is connected on the left gap by using a resistance box, the balance length is found to be at 60 cm from the left end. If the resistivity of the wire is R × 10^{-7}Ωm, then value of R is :

(1) 63 (2) 70
(3) 66 (4) 35

Ans. **(3)**

Sol. For null point,

$$\frac{4.5}{60}=\frac{R}{40}$$

Also, $R=\frac{\rho\ell}{A}=\frac{\rho\ell}{\pi r^2}$

$$4.5\times40=\rho\times\frac{0.1}{\pi\times7\times10^{-8}}\times60$$

$$\rho=66\times10^{-7}\Omega\times m$$

49. A wire of resistance R and length L is cut into 5 equal parts. If these parts are joined parallely, then resultant resistance will be :

(1) $\frac{1}{25}R$ (2) $\frac{1}{5}R$
(3) 25 R (4) 5 R

Ans. **(1)**

Sol. Resistance of each part = $\frac{R}{5}$

Total resistance = $\frac{1}{5}\times\frac{R}{5}=\frac{R}{25}$

50. The average kinetic energy of a monatomic molecule is 0.414 eV at temperature :
(Use K_B = 1.38 × 10^{-23} J/mol-K)
(1) 3000 K
(2) 3200 K
(3) 1600 K
(4) 1500 K

Ans. **(2)**

Sol. For monoatomic molecule degree of freedom = 3.

$$\therefore\ K_{avg}=\frac{3}{2}K_BT$$

$$T=\frac{0.414\times1.6\times10^{-19}\times2}{3\times1.38\times10^{-23}}$$

= 3200 K

SECTION-B

51. A particle starts from origin at t = 0 with a velocity $5\hat{i}$ m/s and moves in x-y plane under action of a force which produces a constant acceleration of $(3\hat{i}+2\hat{j})m/s^2$. If the x-coordinate of the particle at that instant is 84 m, then the speed of the particle at this time is $\sqrt{\alpha}$ m/s. The value of α is ______.

Ans. **(673)**

Sol u_x = 5 m/s a_x = 3 m/s^2 x = 84 m

$$v_x^2-u_x^2=2ax$$

$$v_x^2-25=2(3)(84)$$

V_x = 23 m/s

$$v_x-u_x=a_xt$$

$$t=\frac{23-5}{3}=6s$$

$v_y=0+a_yt$ = 0 + 2 × (6) = 12 m/s

$$v^2=v_x^2+v_y^2=23^2+12^2=673$$

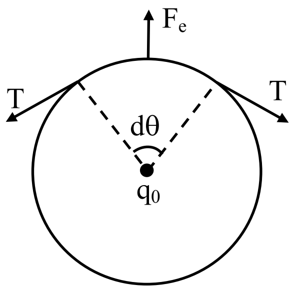
$$v=\sqrt{673}\ m/s$$

52. A thin metallic wire having cross sectional area of 10^{-4} m^2 is used to make a ring of radius 30 cm. A positive charge of 2π C is uniformly distributed over the ring, while another positive charge of 30 pC is kept at the centre of the ring. The tension in the ring is ______ N ; provided that the ring does not get deformed (neglect the influence of gravity).

(given, $\frac{1}{4\pi\epsilon_0}=9\times10^9$ SI units)

Ans. **(3)**

Sol.



$$2T\sin\frac{d\theta}{2}=\frac{kq_0}{R^2}\cdot\lambda Rd\theta$$

$$\left[\lambda=\frac{Q}{2\pi R}\right]$$

$$\Rightarrow T = \frac{Kq_0Q}{(R^2)\times 2\pi}$$

$$= \frac{(9\times10^9)(2\pi\times30\times10^{-12})}{(0.30)^2\times2\pi}$$

$$= \frac{9\times10^{-3}\times30}{9\times10^{-2}} = 3N$$

53. Two coils have mutual inductance 0.002 H. The current changes in the first coil according to the relation $i = i_0 \sin \omega t$, where $i_0 = 5A$ and $\omega = 50\pi$ rad/s. The maximum value of emf in the second coil is $\frac{\pi}{\alpha}$ V. The value of α is _____.

Ans. **(2)**

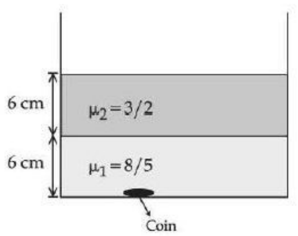
Sol. $\phi = Mi = Mi_0 \sin\omega t$

$$\text{EMF} = -M\frac{di}{dt} = -0.002(i_0\omega\cos\omega t)$$

$$\text{EMF}_{max} = i_0\omega\ (0.002) = (5)(50\pi)(0.002)$$

$$\text{EMF}_{max} = \frac{\pi}{2}\ V$$

54. Two immiscible liquids of refractive indices $\frac{8}{5}$ and $\frac{3}{2}$ respectively are put in a beaker as shown in the figure. The height of each column is 6 cm. A coin is placed at the bottom of the beaker. For near normal vision, the apparent depth of the coin is $\frac{\alpha}{4}$ cm. The value of α is______.



Ans. **(31)**

Sol. $$h_{app} = \frac{h_1}{\mu_1} + \frac{h_2}{\mu_2} = \frac{6}{3/2} + \frac{6}{8/5} = 4 + \frac{15}{4} = \frac{31}{4} cm$$

55. In a nuclear fission process, a high mass nuclide $(A \approx 236)$ with binding energy 7.6 MeV/Nucleon dissociated into middle mass nuclides $(A \approx 118)$, having binding energy of 8.6 MeV/Nucleon. The energy released in the process would be ____ MeV.

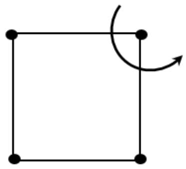
Ans. **(236)**

Sol. $Q = BE_{Product} - BE_{Rectant}$

$= 2(118)(8.6) - 236(7.6)$

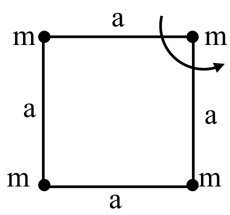
$= 236 \times 1 = 236$ MeV

56. Four particles each of mass 1 kg are placed at four corners of a square of side 2 m. Moment of inertia of system about an axis perpendicular to its plane and passing through one of its vertex is _____ kgm^2.



Ans. **(16)**

Sol.



$$I = ma^2 + ma^2 + m(\sqrt{2}a)^2$$

$= 4ma^2$

$= 4 \times 1 \times (2)^2 = 16$

57. A particle executes simple harmonic motion with an amplitude of 4 cm. At the mean position, velocity of the particle is 10 cm/s. The distance of the particle from the mean position when its speed becomes 5 cm/s is $\sqrt{\alpha}$ cm, where $\alpha =$ ______.

Ans. **(12)**

Sol. $V_{\text{at mean position}} = A\omega \Rightarrow 10 = 4\omega$

$$\omega = \frac{5}{2}$$

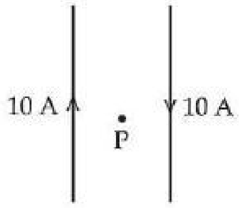
$$v = \omega\sqrt{A^2 - x^2}$$

$$5 = \frac{5}{2}\sqrt{4^2 - x^2} \Rightarrow x^2 = 16 - 4$$

$$x = \sqrt{12}\ cm$$

58. Two long, straight wires carry equal currents in opposite directions as shown in figure. The separation between the wires is 5.0 cm. The magnitude of the magnetic field at a point P midway between the wires is ____ μT

(Given : $\mu_0 = 4\pi \times 10^{-7}$ TmA^{-1})



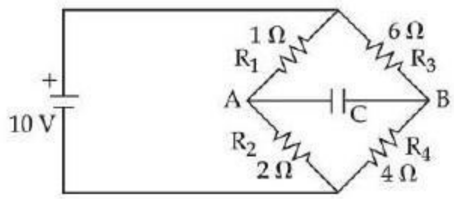
Ans. **(160)**

Sol. $B = \left(\frac{\mu_0 i}{2\pi a}\right) \times 2 = \frac{4\pi \times 10^{-7} \times 10}{\pi \times \left(\frac{5}{2} \times 10^{-2}\right)}$

$= 16 \times 10^{-5} = 160\mu T$

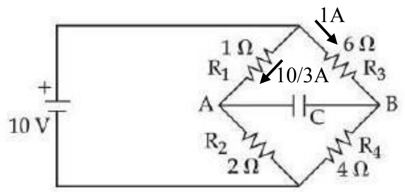
59. The charge accumulated on the capacitor connected in the following circuit is ____ μC

(Given C = 150 μF)



Ans. **(400)**

Sol.



$V_A + \frac{10}{3}(1) - 6(1) = V_B$

$V_A - V_B = 6 - \frac{10}{3} = \frac{8}{3}$ volt

$Q = C(V_A - V_B)$

$= 150 \times \frac{8}{3} = 400\mu C$

60. If average depth of an ocean is 4000 m and the bulk modulus of water is 2×10^9 Nm^{-2}, then fractional compression $\frac{\Delta V}{V}$ of water at the bottom of ocean is $\alpha \times 10^{-2}$. The value of α is _____

(Given, $g = 10\ ms^{-2}$, $\rho = 1000\ kg\ m^{-3}$)

Ans. **(2)**

Sol. $B = -\frac{\Delta P}{\left(\frac{\Delta V}{V}\right)}$

$-\left(\frac{\Delta V}{V}\right) = \frac{\rho g h}{B} = \frac{1000 \times 10 \times 4000}{2 \times 10^9}$

$= 2 \times 10^{-2}$ [–ve sign represent compression]

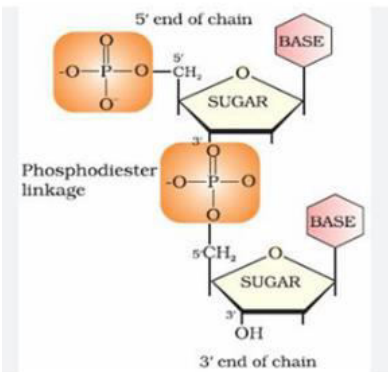
SECTION-A

61. Two nucleotides are joined together by a linkage known as :

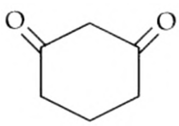
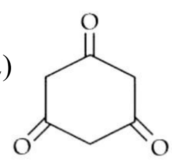
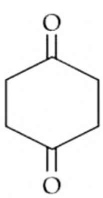
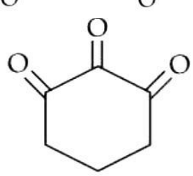
(1) Phosphodiester linkage
(2) Glycosidic linkage
(3) Disulphide linkage
(4) Peptide linkage

Ans. **(1)**

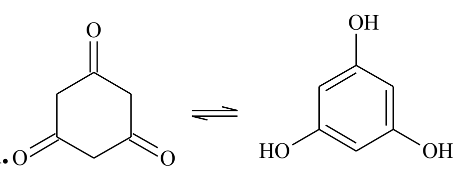
Sol. Phosphodiester linkage



62. Highest enol content will be shown by :

(1) 
(2) 
(3) 
(4) 

Ans. **(2)**

Sol. 

Aromatic

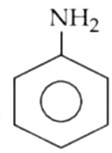
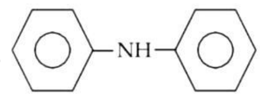
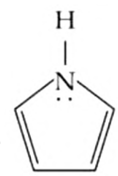
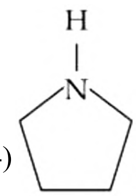
63. Element not showing variable oxidation state is :

(1)Bromine (2)Iodine
(3)Chlorine (4)Fluorine

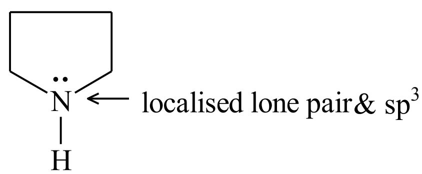
Ans. **(4)**

Sol. Fluorine does not show variable oxidation state.

64. Which of the following is strongest Bronsted base?

(1) 
(2) 
(3) 
(4) 

Ans. **(4)**

Sol. 

65. Which of the following electronic configuration would be associated with the highest magnetic moment ?

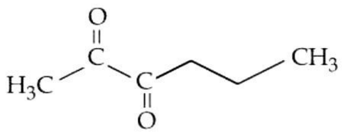
(1) [Ar] $3d^7$ (2) [Ar] $3d^8$
(3) [Ar] $3d^3$ (4) [Ar] $3d^6$

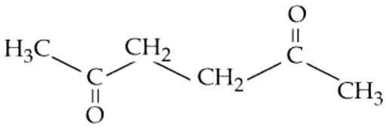
Ans. **(4)**

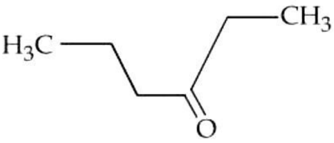
Sol.

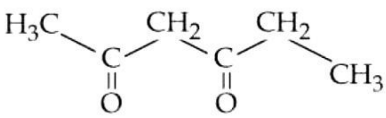
	$3d^7$	$3d^8$	$3d^3$	$3d^6$
No.of. unpaired e^-	3	2	3	4
Spin only Magnetic moment	$\sqrt{15}$ BM	$\sqrt{8}$ BM	$\sqrt{15}$ BM	$\sqrt{24}$ BM

66. Which of the following has highly acidic hydrogen?

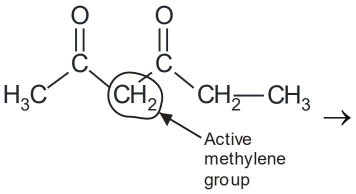
(1) 

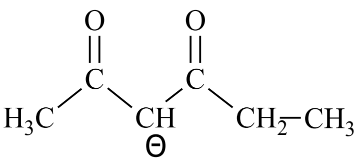
(2) 

(3) 

(4) 

Ans. (4)

Sol. 



Conjugate base is more stable due to more resonance of negative charge.

67. A solution of two miscible liquids showing negative deviation from Raoult's law will have :

(1) increased vapour pressure, increased boiling point

(2) increased vapour pressure, decreased boiling point

(3) decreased vapour pressure, decreased boiling point

(4) decreased vapour pressure, increased boiling point

Ans. (4)

Sol. Solution with negative deviation has

$P_T < P_A^0 X_A + P_B^0 X_B$

$P_A < P_A^0 X_A$

$P_B < P_B^0 X_B$

If vapour pressure decreases so boiling point increases.

68. Consider the following complex ions

$P = [FeF_6]^{3-}$

$Q = [V(H_2O)_6]^{2+}$

$R = [Fe(H_2O)_6]^{2+}$

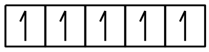
The correct order of the complex ions, according to their spin only magnetic moment values (in B.M.) is :

(1) R < Q < P (2) R < P < Q

(3) Q < R < P (4) Q < P < R

Ans. **(3)**

Sol. $[FeF_6]^{3-}$: Fe^{+3} : $[Ar]\ 3d^5$

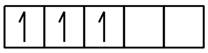
F : Weak field Ligand 

No. of unpaired electron's = 5

$\mu = \sqrt{5(5+2)}$

$\mu = \sqrt{35}$ BM

$[V(H_2O)_6]^{+2}$: V^{+2} : $3d^3$

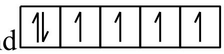


No. of unpaired electron's = 3

$\mu = \sqrt{3(3+2)}$

$\mu = \sqrt{15}$ BM

$[Fe(H_2O)_6]^{+2}$: Fe^{+2} : $3d^6$

H_2O : Weak field Ligand 

No. of unpaired electron's = 4

$\mu = \sqrt{4(4+2)}$

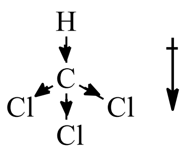
$\mu = \sqrt{24}$ BM

69. Choose the polar molecule from the following :

(1) CCl_4 (2) CO_2

(3) $CH_2 = CH_2$ (4) $CHCl_3$

Ans. **(4)**

Sol. 

$\mu \neq 0$

$CHCl_3$ is polar molecule and rest all molecules are non-polar.

70. Given below are two statements :

Statement (I) : The 4f and 5f - series of elements are placed separately in the Periodic table to preserve the principle of classification.

Statement (II) : S-block elements can be found in pure form in nature. In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Statement I is false but Statement II is true

(2) Both Statement I and Statement II are true

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are false

Ans. **(3)**

Sol. s-block elements are highly reactive and found in combined state.

71. Given below are two statements :

Statement (I) : p-nitrophenol is more acidic than m-nitrophenol and o-nitrophenol.

Statement (II) : Ethanol will give immediate turbidity with Lucas reagent.

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is true but Statement II is false

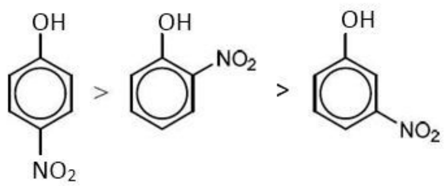
(2) Both Statement I and Statement II are true

(3) Both Statement I and Statement II are false

(4) Statement I is false but Statement II is true

Ans. **(1)**

Sol. Acidic strength



Ethanol give lucas test after long time

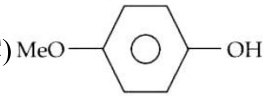
Statement (I)→correct

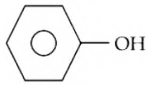
Statement (II) → incorrect

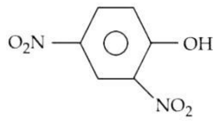
72. The ascending order of acidity of –OH group in the following compounds is :

(A) Bu – OH

(B) 

(C) 

(D) 

(E) 

Choose the correct answer from the options given below :

(1) (A) < (D) < (C) < (B) < (E)

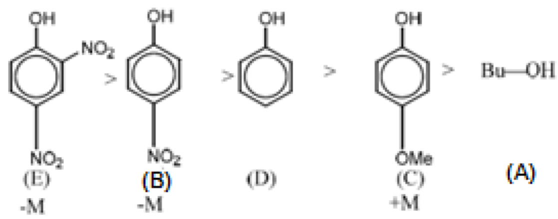
(2) (C) < (A) < (D) < (B) < (E)

(3) (C) < (D) < (B) < (A) < (E)

(4) (A) < (C) < (D) < (B) < (E)

Ans. **(4)**

Sol.



73. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Melting point of Boron (2453 K) is unusually high in group 13 elements.

Reason (R) : Solid Boron has very strong crystalline lattice.

In the light of the above statements, choose the most appropriate answer from the options given below ;

(1) Both (A) and (R) are correct but (R) Is not the correct explanation of (A)

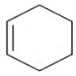
(2) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(3) (A) is true but (R) is false

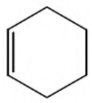
(4) (A) is false but (R) is true

Ans. **(2)**

Sol. Solid Boron has very strong crystalline lattice so its melting point unusually high in group 13 elements

74. Cyclohexene  is __________ type of an organic compound.
(1) Benzenoid aromatic
(2) Benzenoid non-aromatic
(3) Acyclic
(4) Alicyclic

Ans. (4)

Sol.  is Alicyclic

75. Yellow compound of lead chromate gets dissolved on treatment with hot NaOH solution. The product of lead formed is a :
(1) Tetraanionic complex with coordination number six
(2) Neutral complex with coordination number four
(3) Dianionic complex with coordination number six
(4) Dianionic complex with coordination number four

Ans. (4)

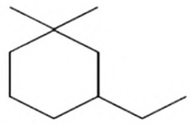
Sol. $PbCrO_4 + NaOH$ (hot excess) $\rightarrow [Pb(OH)_4]^{-2} + Na_2CrO_4$
Dianionic complex with coordination number four

76. Given below are two statements :
Statement (I) : Aqueous solution of ammonium carbonate is basic.
Statement (II) : Acidic/basic nature of salt solution of a salt of weak acid and weak base depends on K_a and K_b value of acid and the base forming it.
In the light of the above statements, choose the most appropriate answer from the options given below :
(1) Both Statement I and Statement II are correct
(2) Statement I is correct but Statement II is incorrect
(3) Both Statement I and Statement II are incorrect
(4) Statement I is incorrect but Statement II is correct

Ans. (1)

Sol. Aqueous solution of $(NH_4)_2CO_3$ is Basic
pH of salt of weak acid and weak base depends on Ka and Kb value of acid and the base forming it

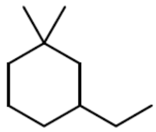
77. IUPAC name of following compound (P) is :



(P)

(1) l-Ethyl-5, 5-dimethylcyclohexane
(2) 3-Ethyl-1,1-dimethylcyclohexane
(3) l-Ethyl-3, 3-dimethylcyclohexane
(4) l,l-Dimethyl-3-ethylcyclohexane

Ans. (2)

Sol. 

3-ethy 1, 1 -dimethylcyclohexane

78. NaCl reacts with conc. H_2SO_4 and $K_2Cr_2O_7$ to give reddish fumes (B), which react with NaOH to give yellow solution (C). (B) and (C) respectively are ;
(1) CrO_2Cl_2, Na_2CrO_4 (2) Na_2CrO_4, CrO_2Cl_2
(3) CrO_2Cl_2, $KHSO_4$ (4) CrO_2Cl_2, $Na_2Cr_2O_7$

Ans. (1)

Sol. $NaCl + \text{conc. } H_2SO_4 + K_2Cr_2O_7$
$\rightarrow CrO_2Cl_2 + KHSO_4 + NaHSO_4 + H_2O$
(B)
Reddish brown
$CrO_2Cl_2 + NaOH \rightarrow Na_2CrO_4 + NaCl + H_2O$
(C)
Yellow colour

79. The correct statement regarding nucleophilic substitution reaction in a chiral alkyl halide is ;
(1) Retention occurs in S_N1 reaction and inversion occurs in S_N2 reaction.
(2) Racemisation occurs in S_N1 reaction and retention occurs in S_N2 reaction.
(3) Racemisation occurs in both S_N1 and S_N2 reactions.
(4) Racemisation occurs in S_N1 reaction and inversion occurs in S_N2 reaction.

Ans. (4)

Sol. SN^1 – Racemisation
SN^2 – Inversion

80. The electronic configuration for Neodymium is:

[Atomic Number for Neodymium 60]

(1)[Xe] $4f^4\ 6s^2$ (2) [Xe] $5f^4 7s^2$

(3) [Xe] $4f^6\ 6s^2$ (4) [Xe] $4f^1 5d^1 6s^2$

Ans. **(1)**

Sol. Electronic configuration of Nd(Z = 60) is;

[Xe] $4f^4\ 6s^2$

SECTION-B

81. The mass of silver (Molar mass of Ag : 108 $gmol^{-1}$) displaced by a quantity of electricity which displaces 5600 mL of O_2 at S.T.P. will be ______ g.

Ans. **107 gm or 108**

Sol. Eq. of Ag = Eq. of O_2

Let x gm silver displaced,

$$\frac{x \times 1}{108} = \frac{5.6}{22.7} \times 4$$

(Molar volume of gas at STP = 22.7 lit)

x = 106.57 gm

Ans. 107

OR,

as per old STP data, molar volume=22.4 lit

$$\frac{x \times 1}{108} = \frac{5.6}{22.4} \times 4 \text{, x= 108 gm.}$$

Ans. 108

82. Consider the following data for the given reaction

$2HI_{(g)} \rightarrow H_{2(g)} + I_{2(g)}$

	1	2	3
HI (mol L^{-1})	0.005	0.01	0.02
Rate (mol L^{-1}s–1)	7.5×10^{-4}	3.0×10^{-3}	1.2×10^{-2}

The order of the reaction is __________.

Ans. **(2)**

Sol. Let, R = $k[HI]^n$

using any two of given data,

$$\frac{3 \times 10^{-3}}{7.5 \times 10^{-4}} = \left(\frac{0.01}{0.005}\right)^n$$

n = 2

83. Mass of methane required to produce 22 g of CO_2 after complete combustion is _______g.

(Given Molar mass in g mol–1 C = 12.0

H = 1.0

O = 16.0)

Ans. **(8)**

Sol. $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$

Moles of $CO_2 = \frac{22}{44} = 0.5$

So, required moles of CH_4 = 0.5

Mass = 0.5 × 16 = 8gm

84. If three moles of an ideal gas at 300 K expand isotherrnally from 30 dm^3 to 45 dm^3 against a constant opposing pressure of 80 kPa, then the amount of heat transferred is________ J.

Ans. **(1200)**

Sol. Using, first law of thermodynamics,

$\Delta U = Q + W$,

$\Delta U = 0$: Process is isothermal

$Q = -W$

$W = -P_{ext}\Delta V$: Irreversible

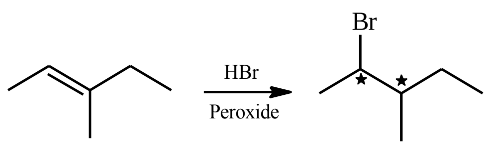
$= -80 \times 10^3 (45 - 30) \times 10^{-3}$

$= -1200$ J

85. 3-Methylhex-2-ene on reaction with HBr in presence of peroxide forms an addition product (A). The number of possible stereoisomers for 'A' is__________.

Ans. **(4)**

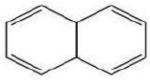
Sol.

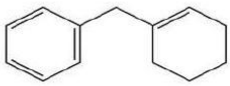


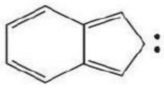
2chiral centres

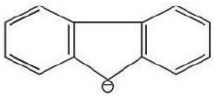
No. of stereoisomers = 4

86. Among the given organic compounds, the total number of aromatic compounds is

(A) 

(B) 

(C) 

(D) 

Ans. (3)

Sol. B,C and D are Aromatic

87. Among the following, total number of meta directing functional groups is (Integer based)

$-OCH_3$, $-NO_2$, $-CN$, $-CH_3$ $-NHCOCH_3$,

$-COR$, $-OH$, $-COOH$, $-Cl$

Ans. (4)

Sol. $-NO_2$, $-C\equiv N$, $-COR$, $-COOH$

are meta directing.

88. The number of electrons present in all the completely filled subshells having n=4 and $s = +\frac{1}{2}$ is ______.

(Where n = principal quantum number and

s = spin quantum number)

Ans. (16)

Sol. $n = 4$ can have,

	4s	4p	4d	4f
Total e^-	2	6	10	14
Total e^- with $S = +\frac{1}{2}$	1	3	5	7

So, Ans.16

89. Sum of bond order of CO and NO^+ is _____.

Ans. (6)

Sol. $CO \Rightarrow \overset{-}{C}\equiv\overset{+}{O}$: BO = 3

$NO^+ \Rightarrow N\equiv O^+$: BO = 3

90. From the given list, the number of compounds with + 4 oxidation state of Sulphur _________.

SO_3, H_2SO_3, $SOCl_2$, SF_4, $BaSO_4$, $H_2S_2O_7$

Ans. (3)

Sol.

Compounds	SO_3	H_2SO_3	$SOCl_2$	SF_4	$BaSO_4$	$H_2S_2O_7$
O.S.of Sulphur:	+6	+4	+4	+4	+6	+6

FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

(Held On Saturday 27th January, 2024) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS | **TEST PAPER WITH SOLUTION**

SECTION-A

1. Considering only the principal values of inverse trigonometric functions, the number of positive real values of x satisfying $\tan^{-1}(x)+\tan^{-1}(2x)=\frac{\pi}{4}$ is :

(1) More than 2

(2) 1

(3) 2

(4) 0

Ans. (2)

Sol. $\tan^{-1}x+\tan^{-1}2x=\frac{\pi}{4}$; $x>0$

$\Rightarrow \tan^{-1}2x=\frac{\pi}{4}-\tan^{-1}x$

Taking tan both sides

$\Rightarrow 2x=\frac{1-x}{1+x}$

$\Rightarrow 2x^2+3x-1=0$

$x=\frac{-3\pm\sqrt{9+8}}{8}=\frac{-3\pm\sqrt{17}}{8}$

Only possible $x=\frac{-3+\sqrt{17}}{8}$

2. Consider the function $f:(0,2)\to R$ defined by $f(x)=\frac{x}{2}+\frac{2}{x}$ and the function g(x) defined by

$$g(x)=\begin{cases}\min\{f(t)\}, & 0<t\le x \text{ and } 0<x\le 1\\ \frac{3}{2}+x, & 1<x<2\end{cases}$$

. Then

(1) g is continuous but not differentiable at x = 1

(2) g is not continuous for all $x\in(0,2)$

(3) g is neither continuous nor differentiable at x = 1

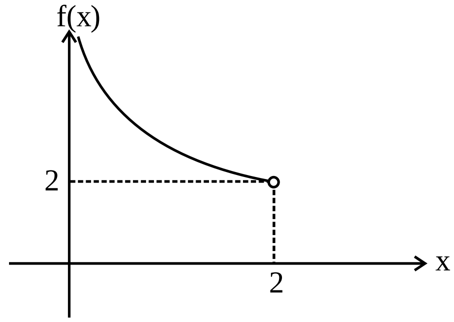
(4) g is continuous and differentiable for all $x\in(0,2)$

Ans. (1)

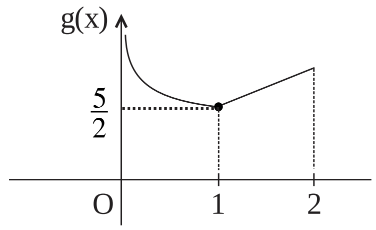
Sol. $f:(0,2)\to R$; $f(x)=\frac{x}{2}+\frac{2}{x}$

$f'(x)=\frac{1}{2}-\frac{2}{x^2}$

$\therefore f(x)$ is decreasing in domain.



$$g(x)=\begin{cases}\frac{x}{2}+\frac{2}{x} & 0<x\le 1\\ \frac{3}{2}+x & 1<x<2\end{cases}$$



3. Let the image of the point (1, 0, 7) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ be the point (α, β, γ). Then which one of the following points lies on the line passing through (α, β, γ) and making angles $\frac{2\pi}{3}$ and $\frac{3\pi}{4}$ with y-axis and z-axis respectively and an acute angle with x-axis ?

(1) $\left(1,-2,1+\sqrt{2}\right)$

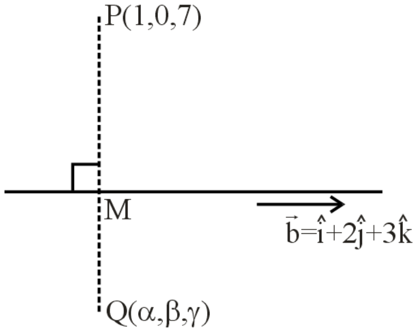
(2) $\left(1,2,1-\sqrt{2}\right)$

(3) $\left(3,4,3-2\sqrt{2}\right)$

(4) $\left(3,-4,3+2\sqrt{2}\right)$

Ans. (3)

Sol. $L_1 = \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$



$M(\lambda, 1+2\lambda, 2+3\lambda)$

$\overrightarrow{PM} = (\lambda-1)\hat{i} + (1+2\lambda)\hat{j} + (3\lambda-5)\hat{k}$

$\overrightarrow{PM}$ is perpendicular to line L_1

$\overrightarrow{PM}.\vec{b} = 0 \quad (\vec{b} = \hat{i}+2\hat{j}+3\hat{k})$

$\Rightarrow \lambda - 1 + 4\lambda + 2 + 9\lambda - 15 = 0$

$14\lambda = 14 \Rightarrow \lambda = 1$

$\therefore M = (1,3,5)$

$\vec{Q} = 2\vec{M} - \vec{P}$ [M is midpoint of $\vec{P}$ & $\vec{Q}$]

$\vec{Q} = 2\hat{i} + 6\hat{j} + 10\hat{k} - \hat{i} - 7\hat{k}$

$\vec{Q} = \hat{i} + 6\hat{j} + 3\hat{k}$

$\therefore (\alpha, \beta, \gamma) = (1, 6, 3)$

Required line having direction cosine (l, m, n)

$l^2 + m^2 + n^2 = 1$

$\Rightarrow l^2 + \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2 = 1$

$l^2 = \frac{1}{4}$

$\therefore l = \frac{1}{2}$ [Line make acute angle with x-axis]

Equation of line passing through (1, 6, 3) will be

$\vec{r} = (\hat{i} + 6\hat{j} + 3\hat{k}) + \mu\left(\frac{1}{2}\hat{i} - \frac{1}{2}\hat{j} - \frac{1}{\sqrt{2}}\hat{k}\right)$

Option (3) satisfying for $\mu = 4$

4. Let R be the interior region between the lines $3x - y + 1 = 0$ and $x + 2y - 5 = 0$ containing the origin. The set of all values of a, for which the points $(a^2, a+1)$ lie in R, is :

(1) $(-3,-1) \cup \left(-\frac{1}{3}, 1\right)$

(2) $(-3,0) \cup \left(\frac{1}{3}, 1\right)$

(3) $(-3,0) \cup \left(\frac{2}{3}, 1\right)$

(4) $(-3,-1) \cup \left(\frac{1}{3}, 1\right)$

Ans. **(2)**

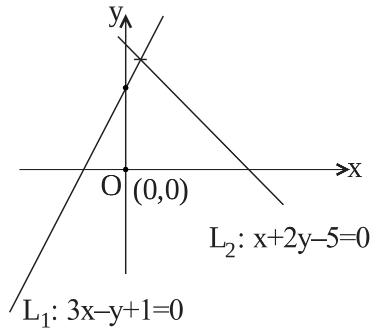
Sol. $P(a^2, a+1)$

$L_1 = 3x - y + 1 = 0$

Origin and P lies same side w.r.t. L_1

$\Rightarrow L_1(0) . L_1(P) > 0$

$\therefore 3(a^2) - (a+1) + 1 > 0$



$\Rightarrow 3a^2 - a > 0$

$a \in (-\infty, 0) \cup \left(\frac{1}{3}, \infty\right)$ ....(1)

Let $L_2 : x + 2y - 5 = 0$

Origin and P lies same side w.r.t. L_2

$\Rightarrow L_2(0).L_2(P) > 0$

$\Rightarrow a^2 + 2(a+1) - 5 < 0$

$\Rightarrow a^2 + 2a - 3 < 0$

$\Rightarrow (a+3)(a-1) < 0$

$\therefore a \in (-3, 1)$ ....(2)

Intersection of (1) and (2)

$a \in (-3, 0) \cup \left(\frac{1}{3}, 1\right)$

5. The 20th term from the end of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots, -129\frac{1}{4}$ is :-

(1) -118

(2) -110

(3) -115

(4) -100

Ans. **(3)**

Sol. $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots\ldots, -129\frac{1}{4}$

This is A.P. with common difference

$$d_1 = -1 + \frac{1}{4} = -\frac{3}{4}$$

$$-129\frac{1}{4}, \ldots\ldots\ldots\ldots, 19\frac{1}{4}, 20$$

This is also A.P. $a = -129\frac{1}{4}$ and $d = \frac{3}{4}$

Required term =

$$-129\frac{1}{4} + (20-1)\left(\frac{3}{4}\right)$$

$$= -129 - \frac{1}{4} + 15 - \frac{3}{4} = -115$$

6. Let $f : R - \left\{\frac{-1}{2}\right\} \to R$ and $g : R - \left\{\frac{-5}{2}\right\} \to R$ be defined as $f(x) = \frac{2x+3}{2x+1}$ and $g(x) = \frac{|x|+1}{2x+5}$. Then the domain of the function fog is :

(1) $R - \left\{-\frac{5}{2}\right\}$

(2) R

(3) $R - \left\{-\frac{7}{4}\right\}$

(4) $R - \left\{-\frac{5}{2}, -\frac{7}{4}\right\}$

Ans. **(1)**

Sol. $f(x) = \frac{2x+3}{2x+1}; x \neq -\frac{1}{2}$

$$g(x) = \frac{|x|+1}{2x+5}, x \neq -\frac{5}{2}$$

Domain of f(g(x))

$$f(g(x)) = \frac{2g(x)+3}{2g(x)+1}$$

$$x \neq -\frac{5}{2} \text{ and } \frac{|x|+1}{2x+5} \neq -\frac{1}{2}$$

$$x \in R - \left\{-\frac{5}{2}\right\} \text{ and } x \in R$$

$\therefore$ Domain will be $R - \left\{-\frac{5}{2}\right\}$

7. For $0 < a < 1$, the value of the integral $\int_0^{\pi} \frac{dx}{1 - 2a\cos x + a^2}$ is :

(1) $\frac{\pi^2}{\pi + a^2}$

(2) $\frac{\pi^2}{\pi - a^2}$

(3) $\frac{\pi}{1 - a^2}$

(4) $\frac{\pi}{1 + a^2}$

Ans. **(3)**

Sol. $I = \int_0^{\pi} \frac{dx}{1 - 2a\cos x + a^2}; 0 < a < 1$

$$I = \int_0^{\pi} \frac{dx}{1 + 2a\cos x + a^2}$$

$$2I = 2\int_0^{\pi/2} \frac{2(1+a^2)}{(1+a^2)^2 - 4a^2\cos^2 x}dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{2(1+a^2).\sec^2 x}{(1+a^2)^2.\sec^2 x - 4a^2}dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{2.(1+a^2).\sec^2 x}{(1+a^2)^2.\tan^2 x + (1-a^2)^2}dx$$

$$\Rightarrow I=\int_0^{\pi/2}\frac{\frac{2.\sec^2 x}{1+a^2}.dx}{\tan^2 x+\left(\frac{1-a^2}{1+a^2}\right)^2}$$

$$\Rightarrow I=\frac{2}{(1-a^2)}\left[\frac{\pi}{2}-0\right]$$

$$I=\frac{\pi}{1-a^2}$$

8. Let $g(x)=3f\left(\frac{x}{3}\right)+f(3-x)$ and $f''(x)>0$ for all $x\in(0,3)$. If g is decreasing in (0, α) and increasing in (α, 3), then 8α is

(1) 24

(2) 0

(3) 18

(4) 20

Ans. (3)

Sol. $g(x)=3f\left(\frac{x}{3}\right)+f(3-x)$ and $f''(x)>0\ \forall\ x\in(0,3)$

$\Rightarrow f'(x)$ is increasing function

$$g'(x)=3\times\frac{1}{3}.f'\left(\frac{x}{3}\right)-f'(3-x)$$

$$=f'\left(\frac{x}{3}\right)-f'(3-x)$$

If g is decreasing in (0, α)

$g'(x)<0$

$$f'\left(\frac{x}{3}\right)-f'(3-x)<0$$

$$f'\left(\frac{x}{3}\right)<f'(3-x)$$

$$\Rightarrow\frac{x}{3}<3-x$$

$$\Rightarrow x<\frac{9}{4}$$

Therefore $\alpha=\frac{9}{4}$

Then $8\alpha=8\times\frac{9}{4}=18$

9. If $\lim\limits_{x\to0}\frac{3+\alpha\sin x+\beta\cos x+\log_e(1-x)}{3\tan^2 x}=\frac{1}{3}$, then 2α–β is equal to :

(1) 2

(2) 7

(3) 5

(4) 1

Ans. (3)

Sol. $\lim\limits_{x\to0}\frac{3+\alpha\sin x+\beta\cos x+\log_e(1-x)}{3\tan^2 x}=\frac{1}{3}$

$$\Rightarrow\lim_{x\to0}\frac{3+\alpha\left[x-\frac{x^3}{3!}+....\right]+\beta\left[1-\frac{x^2}{2!}+\frac{x^4}{4!}....\right]+\left(-x-\frac{x^2}{2}-\frac{x^3}{3}...\right)}{3\tan^2 x}=\frac{1}{3}$$

$$\Rightarrow\lim_{x\to0}\frac{(3+\beta)+(\alpha-1)x+\left(-\frac{1}{2}-\frac{\beta}{2}\right)x^2+....}{3x^2}\times\frac{x^2}{\tan^2 x}=\frac{1}{3}$$

$$\Rightarrow\beta+3=0,\ \alpha-1=0\text{ and }\frac{-\frac{1}{2}-\frac{\beta}{2}}{3}=\frac{1}{3}$$

$$\Rightarrow\beta=-3,\alpha=1$$

$$\Rightarrow2\alpha-\beta=2+3=5$$

10. If α, β are the roots of the equation, $x^2-x-1=0$ and $S_n=2023\alpha^n+2024\beta^n$, then

(1) $2S_{12}=S_{11}+S_{10}$

(2) $S_{12}=S_{11}+S_{10}$

(3) $2S_{11}=S_{12}+S_{10}$

(4) $S_{11}=S_{10}+S_{12}$

Ans. (2)

Sol. $x^2-x-1=0$

$$S_n=2023\alpha^n+2024\beta^n$$

$$S_{n-1}+S_{n-2}=2023\alpha^{n-1}+2024\beta^{n-1}+2023\alpha^{n-2}+2024\beta^{n-2}$$

$$=2023\alpha^{n-2}[1+\alpha]+2024\beta^{n-2}[1+\beta]$$

$$=2023\alpha^{n-2}[\alpha^2]+2024\beta^{n-2}[\beta^2]$$

$$=2023\alpha^n+2024\beta^n$$

$$S_{n-1}+S_{n-2}=S_n$$

Put n = 12

$$S_{11}+S_{10}=S_{12}$$

11. Let A and B be two finite sets with m and n elements respectively. The total number of subsets of the set A is 56 more than the total number of subsets of B. Then the distance of the point P(m, n) from the point Q(–2, –3) is

(1) 10

(2) 6

(3) 4

(4) 8

Ans. **(1)**

Sol. $2^m - 2^n = 56$

$2^n(2^{m-n} - 1) = 2^3 \times 7$

$2^n = 2^3$ and $2^{m-n} - 1 = 7$

$\Rightarrow n = 3$ and $2^{m-n} = 8$

$\Rightarrow n = 3$ and $m - n = 3$

$\Rightarrow n = 3$ and $m = 6$

P(6,3) and Q(–2, –3)

$PQ = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$

Hence option (1) is correct

12. The values of α, for which

$$\begin{vmatrix} 1 & \frac{3}{2} & \alpha + \frac{3}{2} \\ 1 & \frac{1}{3} & \alpha + \frac{1}{3} \\ 2\alpha + 3 & 3\alpha + 1 & 0 \end{vmatrix} = 0$$, lie in the interval

(1) (–2, 1)

(2) (–3, 0)

(3) $\left(-\frac{3}{2}, \frac{3}{2}\right)$

(4) (0, 3)

Ans. **(2)**

Sol. $$\begin{vmatrix} 1 & \frac{3}{2} & \alpha + \frac{3}{2} \\ 1 & \frac{1}{3} & \alpha + \frac{1}{3} \\ 2\alpha + 3 & 3\alpha + 1 & 0 \end{vmatrix} = 0$$

$\Rightarrow (2\alpha + 3)\left\{\frac{7\alpha}{6}\right\} - (3\alpha + 1)\left\{\frac{-7}{6}\right\} = 0$

$\Rightarrow (2\alpha + 3).\frac{7\alpha}{6} + (3\alpha + 1).\frac{7}{6} = 0$

$\Rightarrow 2\alpha^2 + 3\alpha + 3\alpha + 1 = 0$

$\Rightarrow 2\alpha^2 + 6\alpha + 1 = 0$

$\Rightarrow \alpha = \frac{-3 + \sqrt{7}}{2}, \frac{-3 - \sqrt{7}}{2}$

Hence option (2) is correct.

13. An urn contains 6 white and 9 black balls. Two successive draws of 4 balls are made without replacement. The probability, that the first draw gives all white balls and the second draw gives all black balls, is :

(1) $\frac{5}{256}$ (2) $\frac{5}{715}$

(3) $\frac{3}{715}$ (4) $\frac{3}{256}$

Ans. **(3)**

Sol. $\frac{{}^6C_4}{{}^{15}C_4} \times \frac{{}^9C_4}{{}^{11}C_4} = \frac{3}{715}$

Hence option (3) is correct.

14. The integral $\int \frac{(x^8 - x^2)dx}{(x^{12} + 3x^6 + 1)\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)}$ is equal to :

(1) $\log_e\left(\left|\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right|\right)^{1/3} + C$

(2) $\log_e\left(\left|\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right|\right)^{1/2} + C$

(3) $\log_e\left(\left|\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right|\right) + C$

(4) $\log_e\left(\left|\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right|\right)^3 + C$

Ans. **(1)**

Sol. $I=\int \frac{x^8-x^2}{(x^{12}+3x^6+1)\tan^{-1}\left(x^3+\frac{1}{x^3}\right)}dx$

Let $\tan^{-1}\left(x^3+\frac{1}{x^3}\right)=t$

$\Rightarrow \frac{1}{1+\left(x^3+\frac{1}{x^3}\right)^2}\cdot\left(3x^2-\frac{3}{x^4}\right)dx=dt$

$\Rightarrow \frac{x^6}{x^{12}+3x^6+1}\cdot\frac{3x^6-3}{x^4}dx=dt$

$I=\frac{1}{3}\int\frac{dt}{t}=\frac{1}{3}\ln|t|+C$

$I=\frac{1}{3}\ln\left|\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right|+C$

$I=\ln\left|\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right|^{1/3}+C$

Hence option (1) is correct

15. If $2\tan^2\theta-5\sec\theta=1$ has exactly 7 solutions in the interval $\left[0,\frac{n\pi}{2}\right]$, for the least value of $n\in N$ then $\sum_{k=1}^{n}\frac{k}{2^k}$ is equal to :

(1) $\frac{1}{2^{15}}(2^{14}-14)$

(2) $\frac{1}{2^{14}}(2^{15}-15)$

(3) $1-\frac{15}{2^{13}}$

(4) $\frac{1}{2^{13}}(2^{14}-15)$

Ans. **(4)**

Sol. $2\tan^2\theta-5\sec\theta-1=0$

$\Rightarrow 2\sec^2\theta-5\sec\theta-3=0$

$\Rightarrow (2\sec\theta+1)(\sec\theta-3)=0$

$\Rightarrow \sec\theta=-\frac{1}{2},3$

$\Rightarrow \cos\theta=-2,\frac{1}{3}$

$\Rightarrow \cos\theta=\frac{1}{3}$

For 7 solutions n = 13

So, $\sum_{k=1}^{13}\frac{k}{2^k}=S$ (say)

$S=\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+....+\frac{13}{2^{13}}$

$\frac{1}{2}S=\frac{1}{2^2}+\frac{1}{2^3}+.....+\frac{12}{2^{13}}+\frac{13}{2^{14}}$

$\Rightarrow \frac{S}{2}=\frac{1}{2}\cdot\frac{1-\frac{1}{2^{13}}}{1-\frac{1}{2}}-\frac{13}{2^{14}}\Rightarrow S=2\cdot\left(\frac{2^{13}-1}{2^{13}}\right)-\frac{13}{2^{13}}$

16. The position vectors of the vertices A, B and C of a triangle are $2\hat{i}-3\hat{j}+3\hat{k}$, $2\hat{i}+2\hat{j}+3\hat{k}$ and $-\hat{i}+\hat{j}+3\hat{k}$ respectively. Let l denotes the length of the angle bisector AD of $\angle BAC$ where D is on the line segment BC, then $2l^2$ equals :

(1) 49

(2) 42

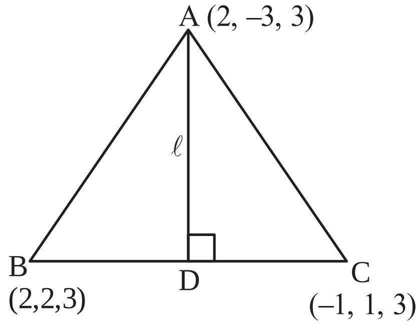
(3) 50

(4) 45

Ans. **(4)**

Sol. AB = 5

AC = 5



$\therefore$ D is midpoint of BC

$D\left(\frac{1}{2},\frac{3}{2},3\right)$

$\therefore l=\sqrt{\left(2-\frac{1}{2}\right)^2+\left(-3-\frac{3}{2}\right)^2+(3-3)^2}$

$l=\sqrt{\frac{45}{2}}$

$\therefore 2l^2=45$

17. If y = y(x) is the solution curve of the differential equation $(x^2-4)dy-(y^2-3y)dx=0$, $x>2, y(4)=\frac{3}{2}$ and the slope of the curve is never zero, then the value of y(10) equals :

(1) $\frac{3}{1+(8)^{1/4}}$

(2) $\frac{3}{1+2\sqrt{2}}$

(3) $\frac{3}{1-2\sqrt{2}}$

(4) $\frac{3}{1-(8)^{1/4}}$

Ans. (1)

Sol. $(x^2-4)dy-(y^2-3y)dx=0$

$$\Rightarrow \int\frac{dy}{y^2-3y}=\int\frac{dx}{x^2-4}$$

$$\Rightarrow \frac{1}{3}\int\frac{y-(y-3)}{y(y-3)}dy=\int\frac{dx}{x^2-4}$$

$$\Rightarrow \frac{1}{3}(\ln|y-3|-\ln|y|)=\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+C$$

$$\Rightarrow \frac{1}{3}\ln\left|\frac{y-3}{y}\right|=\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+C$$

At $x=4,\ y=\frac{3}{2}$

$$\therefore C=\frac{1}{4}\ln 3$$

$$\therefore \frac{1}{3}\ln\left|\frac{y-3}{y}\right|=\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+\frac{1}{4}\ln(3)$$

At x = 10

$$\frac{1}{3}\ln\left|\frac{y-3}{y}\right|=\frac{1}{4}\ln\left|\frac{2}{3}\right|+\frac{1}{4}\ln(3)$$

$$\ln\left|\frac{y-3}{y}\right|=\ln 2^{3/4},\ \forall x>2, \frac{dy}{dx}<0$$

as $y(4)=\frac{3}{2}\Rightarrow y\in(0,3)$

$$-y+3=8^{1/4}.y$$

$$y=\frac{3}{1+8^{1/4}}$$

18. Let e_1 be the eccentricity of the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ and e_2 be the eccentricity of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, a > b, which passes through the foci of the hyperbola. If $e_1e_2=1$, then the length of the chord of the ellipse parallel to the x-axis and passing through (0, 2) is :

(1) $4\sqrt{5}$

(2) $\frac{8\sqrt{5}}{3}$

(3) $\frac{10\sqrt{5}}{3}$

(4) $3\sqrt{5}$

Ans. (3)

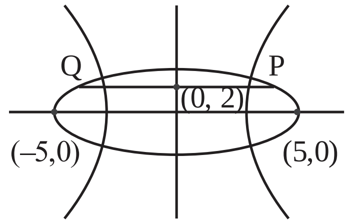
Sol. $H:\frac{x^2}{16}-\frac{y^2}{9}=1 \qquad e_1=\frac{5}{4}$

$$\therefore e_1e_2=1\Rightarrow e_2=\frac{4}{5}$$

Also, ellipse is passing through $(\pm5,0)$

$\therefore$ a = 5 and b = 3

$$E:\frac{x^2}{25}+\frac{y^2}{9}=1$$



End point of chord are $\left(\pm\frac{5\sqrt{5}}{3},2\right)$

$$\therefore L_{PQ}=\frac{10\sqrt{5}}{3}$$

19. Let $\alpha=\frac{(4!)!}{(4!)^{3!}}$ and $\beta=\frac{(5!)!}{(5!)^{4!}}$. Then :

(1) $\alpha\in N$ and $\beta\notin N$

(2) $\alpha\notin N$ and $\beta\in N$

(3) $\alpha\in N$ and $\beta\in N$

(4) $\alpha\notin N$ and $\beta\notin N$

Ans. (3)

Sol. $\alpha = \frac{(4!)!}{(4!)^{3!}}, \beta = \frac{(5!)!}{(5!)^{4!}}$

$\alpha = \frac{(24)!}{(4!)^{6}}, \beta = \frac{(120)!}{(5!)^{24}}$

Let 24 distinct objects are divided into 6 groups of 4 objects in each group.

No. of ways of formation of group = $\frac{24!}{(4!)^6.6!} \in N$

Similarly,
Let 120 distinct objects are divided into 24 groups of 5 objects in each group.
No. of ways of formation of groups

$= \frac{(120)!}{(5!)^{24}.24!} \in N$

20. Let the position vectors of the vertices A, B and C of a triangle be $2\hat{i}+2\hat{j}+\hat{k}$, $\hat{i}+2\hat{j}+2\hat{k}$ and $2\hat{i}+\hat{j}+2\hat{k}$ respectively. Let l_1, l_2 and l_3 be the lengths of perpendiculars drawn from the ortho center of the triangle on the sides AB, BC and CA respectively, then $l_1^2+l_2^2+l_3^2$ equals :

(1) $\frac{1}{5}$ (2) $\frac{1}{2}$

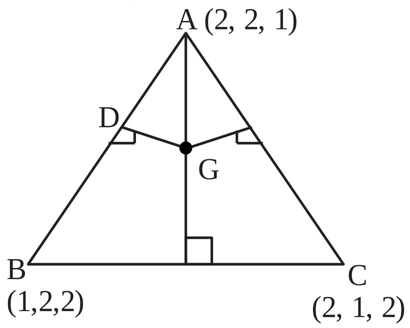
(3) $\frac{1}{4}$ (4) $\frac{1}{3}$

Ans. **(2)**

Sol. ΔABC is equilateral

Orthocentre and centroid will be same

$G\left(\frac{5}{3},\frac{5}{3},\frac{5}{3}\right)$



Mid-point of AB is $D\left(\frac{3}{2},2,\frac{3}{2}\right)$

$\therefore \ell_1 = \sqrt{\frac{1}{36}+\frac{1}{9}+\frac{1}{36}}$

$\ell_1 = \sqrt{\frac{1}{6}} = \ell_2 = \ell_3$

$\therefore \ell_1^2+\ell_2^2+\ell_3^2 = \frac{1}{2}$

SECTION-B

21. The mean and standard deviation of 15 observations were found to be 12 and 3 respectively. On rechecking it was found that an observation was read as 10 in place of 12. If μ and σ^2 denote the mean and variance of the correct observations respectively, then $15(\mu+\mu^2+\sigma^2)$ is equal to ……………………..

Ans. **(2521)**

Sol. **Let the incorrect mean be** μ' and standard deviation be σ'

We have

$\mu' = \frac{\Sigma x_i}{15} = 12 \Rightarrow \Sigma x_i = 180$

As per given information correct Σx_i = 180–10+12

$\Rightarrow \mu$ (correct mean) = $\frac{182}{15}$

Also

$\sigma' = \sqrt{\frac{\Sigma x_i^2}{15} - 144} = 3 \Rightarrow \Sigma x_i^2 = 2295$

Correct Σx_i^2 = 2295 – 100 + 144 = 2339

σ^2 (correct variance) = $\frac{2339}{15} - \frac{182\times182}{15\times15}$

Required value

$= 15(\mu+\mu^2+\sigma^2)$

$= 15\left(\frac{182}{15} + \frac{182\times182}{15\times15} + \frac{2339}{15} - \frac{182\times182}{15\times15}\right)$

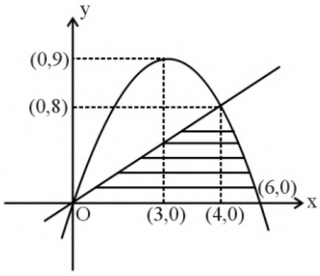
$= 15\left(\frac{182}{15} + \frac{2339}{15}\right)$

= 2521

22. If the area of the region $\{(x,y):0\le y\le \min\{2x,6x-x^2\}\}$ is A, then 12 A is equal to………………

Ans. (304)

Sol. We have



$$A=\frac{1}{2}\times 4\times 8+\int_4^6 (6x-x^2)dx$$

$$A=\frac{76}{3}$$

12A = 304

23. Let A be a 2 × 2 real matrix and I be the identity matrix of order 2. If the roots of the equation $|A-xI|=0$ be –1 and 3, then the sum of the diagonal elements of the matrix A^2 is……………

Ans. (10)

Sol. |A – xI| = 0

Roots are –1 and 3

Sum of roots = tr(A) = 2

Product of roots = |A| = –3

Let $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

We have a + d = 2

ad – bc = –3

$$A^2=\begin{bmatrix} a & b \\ c & d \end{bmatrix}\times\begin{bmatrix} a & b \\ c & d \end{bmatrix}=\begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix}$$

We need $a^2+bc+bc+d^2$

$=a^2+2bc+d^2$

$=(a+d)^2-2ad+2bc$

$=4-2(ad-bc)$

$=4-2(-3)$

$=4+6$

= 10

24. If the sum of squares of all real values of α, for which the lines 2x – y + 3 = 0, 6x + 3y + 1 = 0 and αx + 2y –2 = 0 do not form a triangle is p, then the greatest integer less than or equal to p is ………

Ans. (32)

Sol. $2x-y+3=0$

$6x+3y+1=0$

$\alpha x+2y-2=0$

Will not form a Δ if $\alpha x+2y-2=0$ is concurrent with $2x-y+3=0$ and $6x+3y+1=0$ or parallel to either of them so

Case-1: Concurrent lines

$$\begin{vmatrix} 2 & -1 & 3 \\ 6 & 3 & 1 \\ \alpha & 2 & -2 \end{vmatrix}=0\Rightarrow \alpha=\frac{4}{5}$$

Case-2 : Parallel lines

$$-\frac{\alpha}{2}=\frac{-6}{3} \text{ or } -\frac{\alpha}{2}=2$$

$\Rightarrow \alpha=4$ or $\alpha=-4$

$$P=16+16+\frac{16}{25}$$

$$[P]=\left[32+\frac{16}{25}\right]=32$$

25. The coefficient of x^{2012} in the expansion of $(1-x)^{2008}(1+x+x^2)^{2007}$ is equal to

Ans. (0)

Sol. $(1-x)(1-x)^{2007}(1+x+x^2)^{2007}$

$(1-x)(1-x^3)^{2007}$

$(1-x)({}^{2007}C_0 - {}^{2007}C_1(x^3) + ...)$

General term

$(1-x)((-1)^r\,{}^{2007}C_r x^{3r})$

$(-1)^r\,{}^{2007}C_r x^{3r} - (-1)^r\,{}^{2007}C_r x^{3r+1}$

3r = 2012

$r \neq \frac{2012}{3}$

3r + 1 = 2012

3r = 2011

$r \neq \frac{2011}{3}$

Hence there is no term containing x^{2012}.

So coefficient of $x^{2012} = 0$

26. If the solution curve, of the differential equation $\frac{dy}{dx} = \frac{x+y-2}{x-y}$ passing through the point (2, 1) is $\tan^{-1}\left(\frac{y-1}{x-1}\right) - \frac{1}{\beta}\log_e\left(\alpha + \left(\frac{y-1}{x-1}\right)^2\right) = \log_e|x-1|$, then $5\beta+\alpha$ is equal to

Ans. **(11)**

Sol. $\frac{dy}{dx} = \frac{x+y-2}{x-y}$

$x = X+h, y = Y+k$

$\frac{dY}{dX} = \frac{X+Y}{X-Y}$

$\left.\begin{matrix} h+k-2=0 \\ h-k=0 \end{matrix}\right\} h = k = 1$

$Y = vX$

$v + \frac{dv}{dX} = \frac{1+v}{1-v} \Rightarrow X - \frac{dv}{dX} = \frac{1+v^2}{1-v}$

$\frac{1-v}{1+v^2}dv = \frac{dX}{X}$

$\tan^{-1}v - \frac{1}{2}\ln(1+v^2) = \ln|X| + C$

As curve is passing through (2, 1)

$\tan^{-1}\left(\frac{y-1}{x-1}\right) - \frac{1}{2}\ln\left(1+\left(\frac{y-1}{x-1}\right)^2\right) = \ln|x-1|$

$\therefore \alpha = 1$ and $\beta = 2$

$\Rightarrow 5\beta + \alpha = 11$

27. Let $f(x) = \int_0^x g(t)\log_e\left(\frac{1-t}{1+t}\right)dt$, where g is a continuous odd function.

If $\int_{-\pi/2}^{\pi/2}\left(f(x) + \frac{x^2\cos x}{1+e^x}\right)dx = \left(\frac{\pi}{\alpha}\right)^2 - \alpha$, then α is equal to............

Ans. **(2)**

Sol. $f(x) = \int_0^x g(t)\ln\left(\frac{1-t}{1+t}\right)dt$

$f(-x) = \int_0^{-x} g(t)\ln\left(\frac{1-t}{1+t}\right)dt$

$f(-x) = -\int_0^x g(-y)\ln\left(\frac{1+y}{1-y}\right)dy$

$= -\int_0^x g(y)\ln\left(\frac{1-y}{1+y}\right)dy$ (g is odd)

$f(-x) = -f(x) \Rightarrow f$ is also odd

Now,

$I = \int_{-\pi/2}^{\pi/2}\left(f(x) + \frac{x^2\cos x}{1+e^x}\right)dx$ .....(1)

$I = \int_{-\pi/2}^{\pi/2}\left(f(-x) + \frac{x^2e^x\cos x}{1+e^x}\right)dx$ ......(2)

$2I = \int_{-\pi/2}^{\pi/2} x^2\cos x\,dx = 2\int_0^{\pi/2} x^2\cos x\,dx$

$I=\left(x^2\sin x\right)_0^{\pi/2}-\int_0^{\pi/2}2x\sin x dx$

$=\frac{\pi^2}{4}-2\left(-x\cos x+\int\cos x dx\right)_0^{\pi/2}$

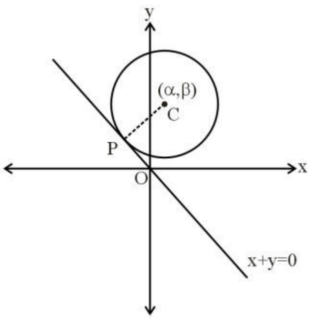
$=\frac{\pi^2}{4}-2(0+1)=\frac{\pi^2}{4}-2\Rightarrow\left(\frac{\pi}{2}\right)^2-2$

$\therefore \alpha=2$

28. Consider a circle $(x-\alpha)^2+(y-\beta)^2=50$, where $\alpha,\beta>0$. If the circle touches the line y + x = 0 at the point P, whose distance from the origin is $4\sqrt{2}$, then $(\alpha+\beta)^2$ is equal to................

Ans. **(100)**

Sol.



$S:(x-\alpha)^2+(y-\beta)^2=50$

CP = r

$\left|\frac{\alpha+\beta}{\sqrt{2}}\right|=5\sqrt{2}$

$\Rightarrow(\alpha+\beta)^2=100$

29. The lines $\frac{x-2}{2}=\frac{y}{-2}=\frac{z-7}{16}$ and $\frac{x+3}{4}=\frac{y+2}{3}=\frac{z+2}{1}$ intersect at the point P. If the distance of P from the line $\frac{x+1}{2}=\frac{y-1}{3}=\frac{z-1}{1}$ is l, then $14l^2$ is equal to................

Ans. **(108)**

Sol. $\frac{x-2}{1}=\frac{y}{-1}=\frac{z-7}{8}=\lambda$

$\frac{x+3}{4}=\frac{y+2}{3}=\frac{z+2}{1}=k$

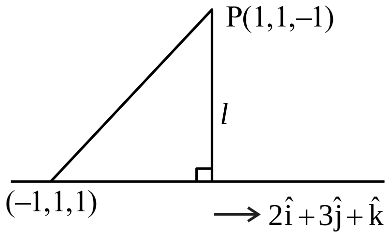
$\Rightarrow\lambda+2=4k-3$

$-\lambda=3k-2$

$\Rightarrow k=1,\lambda=-1$

$8\lambda+7=k-2$

$\therefore P=(1,1,-1)$



Projection of $2\hat{i}-2\hat{k}$ on $2\hat{i}+3\hat{j}+\hat{k}$ is

$=\frac{4-2}{\sqrt{4+9+1}}=\frac{2}{\sqrt{14}}$

$\therefore l^2=8-\frac{4}{14}=\frac{108}{14}$

$\Rightarrow 14l^2=108$

30. Let the complex numbers α and $\frac{1}{\bar{\alpha}}$ lie on the circles $|z-z_0|^2=4$ and $|z-z_0|^2=16$ respectively, where $z_0=1+i$. Then, the value of $100|\alpha|^2$ is..............

Ans. **(20)**

Sol. $|z-z_0|^2=4$

$\Rightarrow(\alpha-z_0)(\bar{\alpha}-\bar{z}_0)=4$

$\Rightarrow\alpha\bar{\alpha}-\alpha\bar{z}_0-z_0\bar{\alpha}+|z_0|^2=4$

$\Rightarrow|\alpha|^2-\alpha\bar{z}_0-z_0\bar{\alpha}=2$ ....(1)

$|z-z_0|^2=16$

$$\Rightarrow \left(\frac{1}{\bar{\alpha}} - z_0\right)\left(\frac{1}{\alpha} - \bar{z}_0\right) = 16$$

$$\Rightarrow (1 - \bar{\alpha} z_0)(1 - \alpha \bar{z}_0) = 16 |\alpha|^2$$

$$\Rightarrow 1 - \bar{\alpha} z_0 - \alpha \bar{z}_0 + |\alpha|^2 |z_0|^2 = 16 |\alpha|^2$$

$$\Rightarrow 1 - \bar{\alpha} z_0 - \alpha \bar{z}_0 = 14 |\alpha|^2 \ldots\ldots\ldots\ldots(2)$$

From (1) and (2)

$$\Rightarrow 5|\alpha|^2 = 1$$

$$\Rightarrow 100|\alpha|^2 = 20$$

SECTION-A

31. The equation of state of a real gas is given by $\left(P+\frac{a}{V^2}\right)(V-b)=RT$, where P, V and T are pressure. volume and temperature respectively and R is the universal gas constant. The dimensions of $\frac{a}{b^2}$ is similar to that of :

(1) PV

(2) P

(3) RT

(4) R

Ans. **(2)**

Sol. $[P]=\left[\frac{a}{V^2}\right]\Rightarrow[a]=\left[PV^2\right]$

And [V] = [b]

$$\frac{[a]}{\left[b^2\right]}=\frac{\left[PV^2\right]}{\left[V^2\right]}=[P]$$

32. Wheatstone bridge principle is used to measure the specific resistance (S_1) of given wire, having length L, radius r. If X is the resistance of wire, then specific resistance is : $S_1=X\left(\frac{\pi r^2}{L}\right)$. If the length of the wire gets doubled then the value of specific resistance will be :

(1) $\frac{S_1}{4}$

(2) $2S_1$

(3) $\frac{S_1}{2}$

(4) S_1

Ans. **(4)**

Sol. As specific resistance does not depends on dimension of wire so, it will not change.

33. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : The angular speed of the moon in its orbit about the earth is more than the angular speed of the earth in its orbit about the sun.

Reason (R) : The moon takes less time to move around the earth than the time taken by the earth to move around the sun.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) (A) is correct but (R) is not correct

(2) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(3) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(4) (A) is not correct but (R) is correct

Ans. **(2)**

Sol. $\omega=\frac{2\pi}{T}\Rightarrow\omega\propto\frac{1}{T}$

$T_{moon}=27$ days

$T_{earth}=365$ days 4 hour

$\Rightarrow\omega_{moon}>\omega_{earth}$

34. Given below are two statements :

Statement (I) : The limiting force of static friction depends on the area of contact and independent of materials.

Statement (II) : The limiting force of kinetic friction is independent of the area of contact and depends on materials.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Statement I is correct but Statement II is incorrect

(2) Statement I is incorrect but Statement II is correct

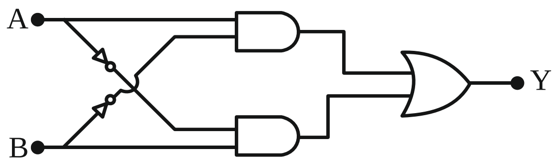
(3) Both Statement I and Statement II are incorrect

(4) Both Statement I and Statement II are correct

Ans. **(2)**

Sol. Co-efficient of friction depends on surface in contact So, depends on material of object.

35. The truth table of the given circuit diagram is :



(1)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(2)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(3)

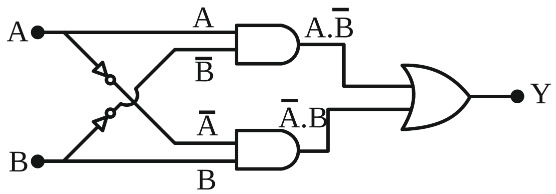
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

(4)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

Ans. **(2)**

Sol.



$Y = A\cdot\bar{B} + \bar{A}\cdot B$

This is XOR GATE

36. A current of 200 μA deflects the coil of a moving coil galvanometer through 60^0. The current to cause deflection through $\frac{\pi}{10}$ radian is :

(1) 30 μA (2) 120 μA

(3) 60 μA (4) 180 μA

Ans. **(3)**

Sol. $i \propto \theta$ (angle of deflection)

$$\therefore \frac{i_2}{i_1} = \frac{\theta_2}{\theta_1} \Rightarrow \frac{i_2}{200\ \mu A} = \frac{\pi/10}{\pi/3} = \frac{3}{10}$$

$$\Rightarrow i_2 = 60\ \mu A$$

37. The atomic mass of $_6C^{12}$ is 12.000000 u and that of $_6C^{13}$ is 13.003354 u. The required energy to remove a neutron from $_6C^{13}$, if mass of neutron is 1.008665 u, will be :

(1) 62. 5 MeV (2) 6.25 MeV

(3) 4.95 MeV (4) 49.5 MeV

Ans. **(3)**

Sol. $_6C^{13} + \text{Energy} \rightarrow {}_6C^{12} + {}_0n^1$

$\Delta m = (12.000000 + 1.008665) - 13.003354$

$= -0.00531$ u

$\therefore$ Energy required $= 0.00531 \times 931.5$ MeV

$= 4.95$ MeV

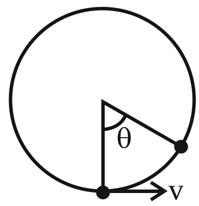
38. A ball suspended by a thread swings in a vertical plane so that its magnitude of acceleration in the extreme position and lowest position are equal. The angle (θ) of thread deflection in the extreme position will be :

(1) $\tan^{-1}\left(\sqrt{2}\right)$ (2) $2\tan^{-1}\left(\frac{1}{2}\right)$

(3) $\tan^{-1}\left(\frac{1}{2}\right)$ (4) $2\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)$

Ans. **(2)**

Sol.



Loss in kinetic energy = Gain in potential energy

$$\Rightarrow \frac{1}{2}mv^2 = mg\ell(1-\cos\theta)$$

$$\Rightarrow \frac{v^2}{\ell} = 2g(1-\cos\theta)$$

Acceleration at lowest point $= \frac{v^2}{\ell}$

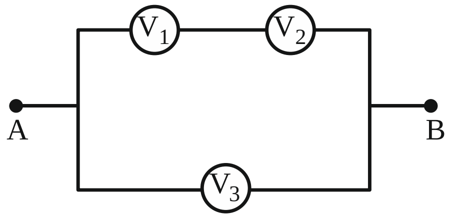
Acceleration at extreme point $= g\sin\theta$

Hence, $\frac{v^2}{\ell} = g\sin\theta$

$\therefore \sin\theta = 2(1-\cos\theta)$

$$\Rightarrow \tan\frac{\theta}{2} = \frac{1}{2} \Rightarrow \theta = 2\tan^{-1}\left(\frac{1}{2}\right)$$

39. Three voltmeters, all having different internal resistances are joined as shown in figure. When some potential difference is applied across A and B, their readings are V_1, V_2 and V_3. Choose the correct option.



(1) $V_1 = V_2$ (2) $V_1 \neq V_3 - V_2$

(3) $V_1 + V_2 > V_3$ (4) $V_1 + V_2 = V_3$

Ans. **(4)**

Sol. From KVL,

$\mathbf{V_1 + V_2 - V_3 = 0} \Rightarrow V_1 + V_2 = V_3$

40. The total kinetic energy of 1 mole of oxygen at 27°C is :

[Use universal gas constant (R)= 8.31 J/mole K]

(1) 6845.5 J (2) 5942.0 J

(3) 6232.5 J (4) 5670.5J

Ans. **(3)**

Sol. Kinetic energy $= \frac{f}{2} nRT$

$= \frac{5}{2} \times 1 \times 8.31 \times 300$ J

= 6232.5 J

41. Given below are two statements : one is labelled as Assertion(A) and the other is labelled as Reason (R).

Assertion (A) : In Vernier calliper if positive zero error exists, then while taking measurements, the reading taken will be more than the actual reading.

Reason (R) : The zero error in Vernier Calliper might have happened due to manufacturing defect or due to rough handling.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(2) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(3) (A) is true but (R) is false

(4) (A) is false but (R) is true

Ans. **(2)**

Sol. Assertion & Reason both are correct

Theory

42. Primary side of a transformer is connected to 230 V, 50 Hz supply. Turns ratio of primary to secondary winding is 10 : 1. Load resistance connected to secondary side is 46 Ω. The power consumed in it is :

(1) 12.5 W (2) 10.0 W

(3) 11.5 W (4) 12.0 W

Ans. **(3)**

Sol. $\frac{V_1}{V_2} = \frac{N_1}{N_2}$

$\frac{230}{V_2} = \frac{10}{1}$

$V_2 = 23V$

Power consumed $= \frac{V_2^2}{R}$

$= \frac{23 \times 23}{46} = 11.5$ W

43. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio of $\frac{C_P}{C_V}$ for the gas is :

(1) $\frac{5}{3}$ (2) $\frac{3}{2}$

(3) $\frac{7}{5}$ (4) $\frac{9}{7}$

Ans. **(2)**

Sol. $P \propto T^3 \Rightarrow PT^{-3} = \text{constant}$

$PV^{\gamma} = \text{const}$

$P\left(\frac{nRT}{P}\right)^{\gamma} = \text{const}$

$P^{1-\gamma}T^{\gamma} = \text{const}$

$PT^{\frac{\gamma}{1-\gamma}} = \text{const}$

$\frac{\gamma}{1-\gamma} = -3$

$\gamma = -3 + 3\gamma$

$3 = 2\gamma$

$\gamma = \frac{3}{2}$

44. The threshold frequency of a metal with work function 6.63 eV is :

(1) 16×10^{15} Hz

(2) 16×10^{12} Hz

(3) 1.6×10^{12} Hz

(4) 1.6×10^{15} Hz

Ans. **(4)**

Sol. $\phi_0 = h\nu_0$

$$6.63\times1.6\times10^{-19} = 6.63\times10^{-34}\nu_0$$

$$\nu_0 = \frac{1.6\times10^{-19}}{10^{-34}}$$

$$\nu_0 = 1.6\times10^{15}\,\text{Hz}$$

45. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R)

Assertion (A) : The property of body, by virtue of which it tends to regain its original shape when the external force is removed, is Elasticity.

Reason (R) : The restoring force depends upon the bonded inter atomic and inter molecular force of solid.

In the light of the above statements, choose the correct answer from the options given below :

(1) (A) is false but (R) is true

(2) (A) is true but (R) is false

(3) Both (A) and (R) are true and (R) is the correct explanation (A)

(4) Both (A) and (R) are true but (R) is not the correct explanation of (A)

Ans. **(3 or 4)**

Sol. Theory

46. When a polaroid sheet is rotated between two crossed polaroids then the transmitted light intensity will be maximum for a rotation of :

(1) 60° (2) 30°

(3) 90° (4) 45°

Ans. **(4)**

Sol. Let I_0 be intensity of unpolarised light incident on first polaroid.

I_1 = Intensity of light transmitted from 1st polaroid

$$= \frac{I_0}{2}$$

θ be the angle between 1st and 2nd polaroid

ϕ be the angle between 2nd and 3rd polaroid

$\theta + \phi = 90^0$ (as 1st and 3rd polaroid are crossed)

$$\phi = 90^0 - \theta$$

I_2 = Intensity from 2nd polaroid

$$I_2 = I_1\cos^2\theta = \frac{I_0}{2}\cos^2\theta$$

I_3 = Intensity from 3rd polaroid

$$I_3 = I_2\cos^2\phi$$

$$I_3 = I_1\cos^2\theta\cos^2\phi$$

$$I_3 = \frac{I_0}{2}\cos^2\theta\cos^2\phi$$

$$\phi = 90 - \theta$$

$$I_3 = \frac{I_0}{2}\cos^2\theta\sin^2\theta$$

$$I_3 = \frac{I_0}{2}\left[\frac{2\sin\theta\cos\theta}{2}\right]^2$$

$$I_3 = \frac{I_0}{8}\sin^2 2\theta$$

I_3 will be maximum when $\sin 2\theta = 1$

$2\theta = 90°$

$\theta = 45°$

47. An object is placed in a medium of refractive index 3. An electromagnetic wave of intensity 6×10^8 W/m^2 falls normally on the object and it is absorbed completely. The radiation pressure on the object would be (speed of light in free space $= 3\times10^8$ m/s) :

(1) 36 Nm^{-2} (2) 18 Nm^{-2}

(3) 6 Nm^{-2} (4) 2 Nm^{-2}

Ans. **(3)**

Sol. Radiation pressure $= \frac{I}{v}$

$$= \frac{I\cdot\mu}{c}$$

$$= \frac{6\times10^8\times3}{3\times10^8}$$

$= 6$ N/m^2

48. Given below are two statements : one is labelled a Assertion (A) and the other is labelled as Reason(R)

Assertion (A) : Work done by electric field on moving a positive charge on an equipotential surface is always zero.

Reason (R) : Electric lines of forces are always perpendicular to equipotential surfaces.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(2) ((A) is correct but (R) is not correct

(3) (A) is not correct but (R) is correct

(4) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Ans. **(4)**

Sol. Electric line of force are always perpendicular to equipotential surface so angle between farce and displacement will always be 90°. So work done equal to 0.

49. A heavy iron bar of weight 12 kg is having its one end on the ground and the other on the shoulder of a man. The rod makes an angle 60° with the horizontal, the weight experienced by the man is :

(1) 6 kg

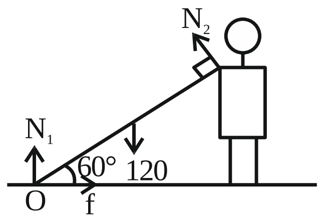
(2) 12 kg

(3) 3 kg

(4) $6\sqrt{3}$ kg

Ans. **(3)**

Sol.



Torque about O = 0

$$120\left(\frac{L}{2}\cos 60°\right) - N_2 L = 0$$

$N_2 = 30$ N

50. A bullet is fired into a fixed target looses one third of its velocity after travelling 4 cm. It penetrates further D × 10^{-3} m before coming to rest. The value of D is :

(1) 2

(2) 5

(3) 3

(4) 4

Ans. **(Bonus)**

Sol. $v^2 - u^2 = 2aS$

$$\left(\frac{2u}{3}\right)^2 = u^2 + 2(-a)\left(4\times10^{-2}\right)$$

$$\frac{4u^2}{9} = u^2 - 2a\left(4\times10^{-2}\right)$$

$$-\frac{5u^2}{9} = -2a\left(4\times10^{-2}\right) \quad \ldots(1)$$

$$0 = \left(\frac{2u}{3}\right)^2 + 2(-a)(x)$$

$$-\frac{4u^2}{9} = -2ax \quad \ldots(2)$$

(1) /**(2)**

$$\frac{5}{4} = \frac{4\times10^{-2}}{x}$$

$$x = \frac{16}{5}\times10^{-2}$$

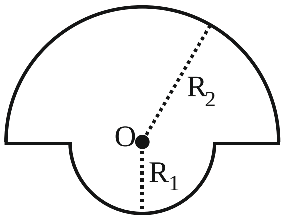
$$x = 3\cdot2\times10^{-2}\,m$$

$$x = 32\times10^{-3}\,m$$

Note : Since no option is matching, Question should be bonus.

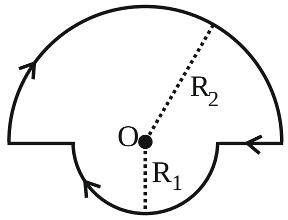
SECTION-B

51. The magnetic field at the centre of a wire loop formed by two semicircular wires of radii $R_1 = 2\pi$ m and $R_2 = 4\pi$ m carrying current I = 4A as per figure given below is $\alpha \times 10^{-7}$ T. The value of α is ______. (Centre O is common for all segments)



Ans. **(3.00)**

Sol.



$$\frac{\mu_0 i}{2R_2}\left(\frac{\pi}{2\pi}\right)\otimes + \frac{\mu_0 i}{2R_1}\left(\frac{\pi}{2\pi}\right)\otimes$$

$$\left(\frac{\mu_0 i}{4R_2} + \frac{\mu_0 i}{4R_1}\right)\otimes$$

$$\frac{4\pi\times10^{-7}\times4}{4\times4\pi} + \frac{4\pi\times10^{-7}\times4}{4\times2\pi}$$

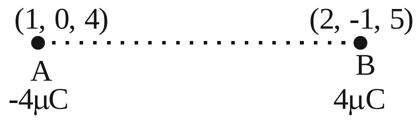
$$= 3\times10^{-7} = \alpha\times10^{-7}$$

$$\alpha = 3$$

52. Two charges of –4 μC and +4 μC are placed at the points A(1, 0, 4)m and B(2, –1, 5) m located in an electric field $\vec{E} = 0.20\,\hat{i}$ V / cm. The magnitude of the torque acting on the dipole is $8\sqrt{\alpha}\times10^{-5}$ Nm, Where α = _____.

Ans. **(2.00)**

Sol.



$$\vec{\tau} = \vec{p}\times\vec{E}$$

$$\vec{p} = q\vec{\ell}$$

$$\vec{E} = 0.2\frac{V}{cm} = 20\frac{V}{m}$$

$$\vec{p} = 4\times\left(\hat{i} - \hat{j} + \hat{k}\right)$$

$$= \left(4\hat{i} - 4\hat{j} + 4\hat{k}\right)\mu C - m$$

$$\vec{\tau} = \left(4\hat{i} - 4\hat{j} + 4\hat{k}\right)\times\left(20\hat{i}\right)\times10^{-6}\text{ Nm}$$

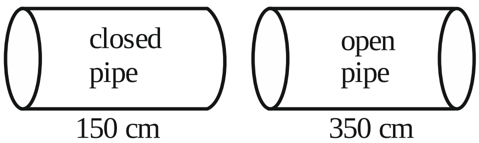
$$= \left(8\hat{k} + 8\hat{j}\right)\times10^{-5} = 8\sqrt{2}\times10^{-5}$$

$$\alpha = 2$$

53. A closed organ pipe 150 cm long gives 7 beats per second with an open organ pipe of length 350 cm, both vibrating in fundamental mode. The velocity of sound is ________ m/s.

Ans. **(294.00)**

Sol.



$$f_c = \frac{v}{4\ell_1} \qquad f_o = \frac{v}{2\ell_2}$$

$$|f_c - f_0| = 7$$

$$\frac{v}{4\times150} - \frac{v}{2\times350} = 7$$

$$\frac{v}{600\text{ cm}} - \frac{v}{700\text{ cm}} = 7$$

$$\frac{v}{6m} - \frac{v}{7m} = 7$$

$$v\left(\frac{1}{42}\right) = 7$$

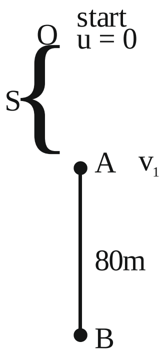
$$v = 42\times7$$

=294 m/s

54. A body falling under gravity covers two points A and B separated by 80 m in 2s. The distance of upper point A from the starting point is ________ m (use g = 10 ms^{-2})

Ans. **(45.00)**

Sol.



From $A \to B$

$$-80 = -v_1 t - \frac{1}{2} \times 10 t^2$$

$$-80 = -2v_1 - \frac{1}{2} \times 10 \times 2^2$$

$$-80 = -2v_1 - 20$$

$$-60 = -2v_1$$

$v_1 = 30$ m/s

From O to A

$$v^2 = u^2 + 2gS$$

$$30^2 = 0 + 2 \times (-10)(-S)$$

900 = 20 S

S = 45 m

55. The reading of pressure metre attached with a closed pipe is 4.5 × 10^4 N/m^2. On opening the valve, water starts flowing and the reading of pressure metre falls to 2.0 × 10^4 N/m^2. The velocity of water is found to be $\sqrt{V}$ m/s. The value of V is ________

Ans. **(50)**

Sol. Change in pressure $= \frac{1}{2}\rho v^2$

$$4.5 \times 10^4 - 2.0 \times 10^4 = \frac{1}{2} \times 10^3 \times v^2$$

$$2.5 \times 10^4 = \frac{1}{2} \times 10^3 \times v^2$$

$v^2 = 50$

$v = \sqrt{50}$

Velocity of water = $\sqrt{V} = \sqrt{50}$

= V = 50

56. A ring and a solid sphere roll down the same inclined plane without slipping. They start from rest. The radii of both bodies are identical and the ratio of their kinetic energies is $\frac{7}{x}$ where x is ________.

Ans. **(7.00)**

Sol. In pure rolling work done by friction is zero. Hence potential energy is converted into kinetic energy. Since initially the ring and the sphere have same potential energy, finally they will have same kinetic energy too.

$\therefore$ Ratio of kinetic energies = 1

$$\Rightarrow \frac{7}{x} = 1 \Rightarrow x = 7$$

57. A parallel beam of monochromatic light of wavelength 5000 Å is incident normally on a single narrow slit of width 0.001 mm. The light is focused by convex lens on screen, placed on its focal plane. The first minima will be formed for the angle of diffraction of ________ (degree).

Ans. **(30.00)**

Sol. For first minima

$$a \sin\theta = \lambda$$

$$\Rightarrow \sin\theta = \frac{\lambda}{a} = \frac{5000 \times 10^{-10}}{1 \times 10^{-6}} = \frac{1}{2}$$

$$\Rightarrow \theta = 30°$$

58. The electric potential at the surface of an atomic nucleus (z = 50) of radius 9 × 10^{-13} cm is ________ × 10^{6} V.

Ans. **(8.00)**

Sol. Potential $=\frac{kQ}{R}=\frac{k.Ze}{R}$

$$=\frac{9\times10^{9}\times50\times1.6\times10^{-19}}{9\times10^{-13}\times10^{-2}}$$

$$=8\times10^{6}\text{V}$$

59. If Rydberg's constant is R, the longest wavelength of radiation in Paschen series will be $\frac{\alpha}{7R}$, where α = ________.

Ans. **(144.00)**

Sol. Longest wavelength corresponds to transition between n = 3 and n = 4

$$\frac{1}{\lambda}=RZ^{2}\left(\frac{1}{3^{2}}-\frac{1}{4^{2}}\right)=RZ^{2}\left(\frac{1}{9}-\frac{1}{16}\right)$$

$$=\frac{7RZ^{2}}{9\times16}$$

$$\Rightarrow\lambda=\frac{144}{7R}\text{ for }Z=1\qquad\therefore\alpha=144$$

60. A series LCR circuit with $L=\frac{100}{\pi}\text{mH}$, $C=\frac{10^{-3}}{\pi}\text{F}$ and R = 10 Ω, is connected across an ac source of 220 V, 50 Hz supply. The power factor of the circuit would be ________.

Ans. **(1.00)**

Sol. $X_{c}=\frac{1}{\omega C}=\frac{\pi}{2\pi\times50\times10^{-3}}=10\Omega$

$$X_{L}=\omega L=2\pi\times50\times\frac{100}{\pi}\times10^{-3}$$

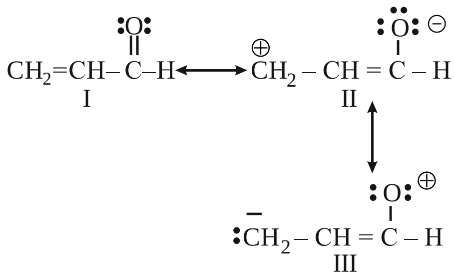
$$=10\Omega$$

$\because X_{C}=X_{L}$,Hence, circuit is in resonance

$\therefore$ power factor $=\frac{R}{Z}=\frac{R}{R}=1$

SECTION-A

61. The order of relative stability of the contributing structure is:



Choose the **correct** answer from the options given below:

(1) I > II > III

(2) II > I > III

(3) I = II = III

(4) III > II > I

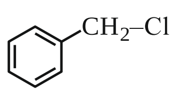
Ans. **(1)**

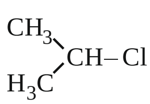
Sol. I > II > III, since neutral resonating structures are more stable than charged resonating structure. II > III, since stability of structure with –ve charge on more electronegative atom is higher.

62. Which among the following halide/s will not show S_N1 reaction:

(A) $H_2C = CH - CH_2Cl$

(B) $CH_3 - CH = CH - Cl$

(C) 

(D) 

Choose the **most appropriate** answer from the options given below:

(1) (A), (B) and (D) only
(2) (A) and (B) only
(3) (B) and (C) only
(4) (B) only

Ans. **(4)**

Sol. Since $CH_3 - CH = \overset{+}{C}H$ is very unstable, $CH_3–CH = CH – Cl$ cannot give S_{N1} reaction.

63. Which of the following statements is not correct about rusting of iron?

(1) Coating of iron surface by tin prevents rusting, even if the tin coating is peeling off.

(2) When pH lies above 9 or 10, rusting of iron does not take place.

(3) Dissolved acidic oxides SO_2, NO_2 in water act as catalyst in the process of rusting.

(4) Rusting of iron is envisaged as setting up of electrochemical cell on the surface of iron object.

Ans. **(1)**

Sol. As tin coating is peeled off, then iron is exposed to atmosphere.

64. Given below are two statements:

Statement (I) : In the Lanthanoids, the formation of Ce^{+4} is favoured by its noble gas configuration.

Statement (II) : Ce^{+4} is a strong oxidant reverting to the common +3 state.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Statement I is false but Statement II is true
(2) Both Statement I and Statement II are true
(3) Statement I is true but Statement II is false
(4) Both Statement I and Statement II are false

Ans. **(2)**

Sol. Statement (1) is true, Ce^{+4} has noble gas electronic configuration.

Statement (2) is also true due to high reduction potential for Ce^{4+}/Ce^{3+} (+1.74V), and stability of Ce^{3+}, Ce^{4+} acts as strong oxidizing agent.

65. Choose the correct option having all the elements with d^{10} electronic configuration from the following:

(1) ^{27}Co, ^{28}Ni, ^{26}Fe, ^{24}Cr
(2) ^{29}Cu, ^{30}Zn, ^{48}Cd, ^{47}Ag
(3) ^{46}Pd, ^{28}Ni, ^{26}Fe, ^{24}Cr
(4) ^{28}Ni, ^{24}Cr, ^{26}Fe, ^{29}Cu

Ans. **(2)**

Sol. $[Cr] = [Ar]4s^1 3d^5$
$[Cd] = [Kr]5s^2 4d^{10}$
$[Cu] = [Ar]4s^1 3d^{10}$
$[Ag] = [Kr]5s^1 4d^{10}$
$[Zn] = [Ar]4s^2 3d^{10}$

66. Phenolic group can be identified by a positive:
(1) Phthalein dye test
(2) Lucas test
(3) Tollen's test
(4) Carbylamine test

Ans. (1)

Sol. Carbylamine Test-Identification of primary amines
Lucas Test - Differentiation between 1°, 2° and 3° alcohols
Tollen's Test - Identification of Aldehydes
Phthalein Dye Test - Identification of phenols

67. The molecular formula of second homologue in the homologous series of mono carboxylic acids is ________.
(1) $C_3H_6O_2$
(2) $C_2H_4O_2$
(3) CH_2O
(4) $C_2H_2O_2$

Ans. (2)

Sol. HCOOH, CH_3COOH
↑
Second homologue

68. The technique used for purification of steam volatile water immiscible substance is:
(1) Fractional distillation
(2) Fractional distillation under reduced pressure
(3) Distillation
(4) Steam distillation

Ans. (4)

Sol. Steam distillation is used for those liquids which are insoluble in water, containing non-volatile impurities and are steam volatile.

69. The final product A, formed in the following reaction sequence is:

$$Ph{-}CH{=}CH_2 \xrightarrow[\text{(iii) HBr; (iv) Mg, ether, then } HCHO/H_3O^+]{\text{(i) } BH_3 \text{; (ii) } H_2O_2,\ {}^{\ominus}OH} A$$

(1) $Ph-CH_2-CH_2-CH_3$
(2) $Ph-CH(CH_3)-CH_3$
(3) $Ph-CH(CH_2OH)-CH_3$
(4) $Ph-CH_2-CH_2-CH_2-OH$

Ans. (4)

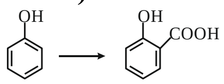
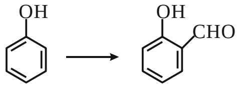
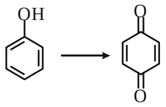
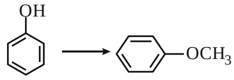
Sol. $PhCH=CH_2 \xrightarrow{B_2H_6/H_2O_2,OH^-} PhCH_2CH_2OH$

$PhCH_2CH_2OH + HBr \longrightarrow PhCH_2CH_2Br + H_2O\ (SN^{NGP})$

$PhCH_2CH_2Br \xrightarrow{\text{Mg/dry ether}} PhCH_2CH_2MgBr$

$PhCH_2CH_2MgBr \xrightarrow[\text{(II) } H_3O^+]{\text{(I) } H-C(=O)-H} PhCH_2CH_2CH_2OH$

70. Match List-I with List-II.

List – I (Reaction)	List – II (Reagent(s))
(A) 	(I) $Na_2Cr_2O_7$, H_2SO_4
(B) 	(II) (i) NaOH (ii) CH_3Cl
(C) 	(III) (i) NaOH, $CHCl_3$ (ii) NaOH (iii) HCl
(D) 	(IV) (i) NaOH (ii) CO_2 (iii) HCl

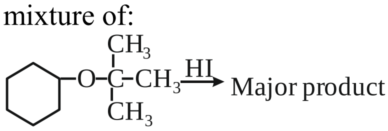
Choose the correct answer from the options given below:
(1) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)
(2) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)
(3) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)
(4) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

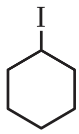
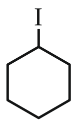
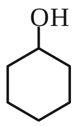
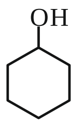
Ans. (4)

Sol. (A) → Kolbe Schmidt Reaction
(B) → Reimer Tiemann Reaction
(C) → Oxidation of phenol to p-benzoquinone
(D) → $PhOH + NaOH \rightarrow H_2O + PhO^-$

$PhO^- + CH_3-Cl \longrightarrow PhOCH_3 + Cl^-$

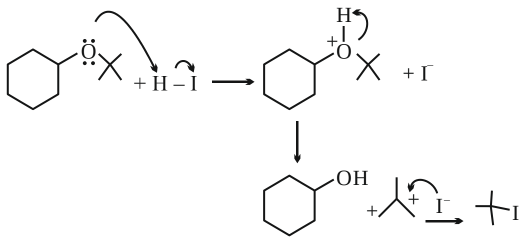
71. Major product formed in the following reaction is a mixture of:



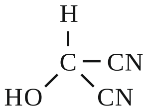
(1)  and $(CH_3)_3CI$
(2)  and $(CH_3)_3COH$
(3)  and $(CH_3)_3COH$
(4)  and $CH_3-C(CH_3)_2-I$

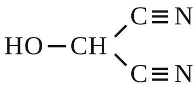
Ans. (4)

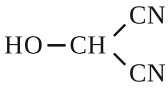
Sol.

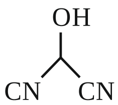


72. Bond line formula of $HOCH(CN)_2$ is:

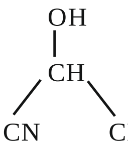
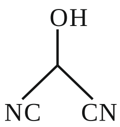
(1) 

(2) 

(3) 

(4) 

Ans. **(4)**

Sol. $CH(OH)(CN)_2$ is  or 

73. Given below are two statements:

Statement (I) : Oxygen being the first member of group 16 exhibits only –2 oxidation state.

Statement (II) : Down the group 16 stability of +4 oxidation state decreases and +6 oxidation state increases.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) Statement I is correct but Statement II is incorrect

(2) Both Statement I and Statement II are correct

(3) Both Statement I and Statement II are incorrect

(4) Statement I is incorrect but Statement II is correct

Ans. **(3)**

Sol. Statement-I: Oxygen can have oxidation state from –2 to +2, so statement I is incorrect

Statement- II: On moving down the group stability of +4 oxidation state increases whereas stability of +6 oxidation state decreases down the group, according to inert pair effect.

So both statements are wrong.

74. Identify from the following species in which d^2sp^3 hybridization is shown by central atom:

(1) $[Co(NH_3)_6]^{3+}$

(2) BrF_5

(3) $[Pt(Cl)_4]^{2-}$

(4) SF_6

Ans. **(1)**

Sol. $[Co(NH_3)_6]^{+3}$ – d^2sp^3 hybridization

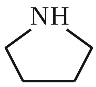
BrF_5 – sp^3d^2 hybridization

$[PtCl_4]^{-2}$ – dsp^2 hybridization

SF_6 – sp^3d^2 hybridization

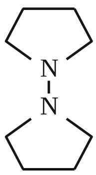
75. Identify B formed in the reaction.

$$Cl-(CH_2)_4-Cl \xrightarrow{\text{excess } NH_3} A \xrightarrow{NaOH} B + H_2O + NaCl$$

(1) 

(2) $H_2N-(CH_2)_4-NH_2$

(3) $Cl\overset{+}{N}H_3-(CH_2)_4-\overset{+}{N}H_3Cl^-$

(4) 

Ans. **(2)**

Sol.

$$Cl-(CH_2)_4-Cl \xrightarrow[NH_3]{excess} \underset{(A)}{Cl^- \overset{\oplus}{N}H_3-(CH_2)_4-\overset{\oplus}{N}H_3Cl^-}$$

$$\downarrow NaOH$$

$$2NaCl + 2H_2O + NH_2-(CH_2)_4-NH_2$$

76. The quantity which changes with temperature is:

(1) Molarity

(2) Mass percentage

(3) Molality

(4) Mole fraction

Ans. **(1)**

Sol. $$\text{Molarity} = \frac{\text{Moles of solute}}{\text{Volume of solution}}$$

Since volume depends on temperature, molarity will change upon change in temperature.

77. Which structure of protein remains intact after coagulation of egg white on boiling?
(1) Primary
(2) Tertiary
(3) Secondary
(4) Quaternary

Ans. (1)

Sol. Boiling an egg causes denaturation of its protein resulting in loss of its quarternary, tertiary and secondary structures.

78. Which of the following cannot function as an oxidising agent?
(1) N^{3-}
(2) SO_4^{2-}
(3) BrO_3^-
(4) MnO_4^-

Ans. (1)

Sol. In N^{3-} ion 'N' is present in its lowest possible oxidation state, hence it cannot be reduced further because of which it cannot act as an oxidizing agent.

79. The incorrect statement regarding conformations of ethane is:
(1) Ethane has infinite number of conformations
(2) The dihedral angle in staggered conformation is 60°
(3) Eclipsed conformation is the most stable conformation.
(4) The conformations of ethane are inter-convertible to one-another.

Ans. (3)

Sol. Eclipsed conformation is the least stable conformation of ethane.

80. Identity the incorrect pair from the following:
(1) Photography - AgBr
(2) Polythene preparation – $TiCl_4$, $Al(CH_3)_3$
(3) Haber process - Iron
(4) Wacker process – Pt Cl_2

Ans. (4)

Sol. The catalyst used in Wacker's process is $PdCl_2$

SECTION-B

81. Total number of ions from the following with noble gas configuration is _________.

Sr^{2+} (Z = 38), Cs^+ (Z = 55), La^{2+} (Z = 57) Pb^{2+} (Z = 82), Yb^{2+} (Z = 70) and Fe^{2+} (Z = 26)

Ans. (2)

Sol. Noble gas configuration = $ns^2\ np^6$
$[Sr^{2+}]$ = [Kr]
$[Cs^+]$ = [Xe]
$[Yb^{2+}]$ = [Xe] $4f^{14}$
$[La^{2+}]$ = [Xe] $5d^1$
$[Pb^{2+}]$ = [Xe] $4f^{14}\ 5d^{10}\ 6s^2$
$[Fe^{2+}]$ = [Ar] $3d^6$

82. The number of non-polar molecules from the following is ___________

HF, H_2O, SO_2, H_2, CO_2, CH_4, NH_3, HCl, $CHCl_3$, BF_3

Ans. (4)

Sol. The non-polar molecules are CO_2, H_2, CH_4 and BF_3

83. Time required for completion of 99.9% of a First order reaction is ________ times of half life ($t_{1/2}$) of the reaction.

Ans. (10)

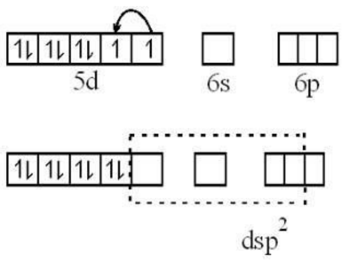
Sol.

$$\frac{t_{99.9\%}}{t_{1/2}} = \frac{\frac{2.303}{k}\left(\frac{a}{a-x}\right)}{\frac{2.303}{k}\log 2} = \frac{\log\left(\frac{100}{100-99.9}\right)}{\log 2} = \frac{\log 10^3}{\log 2} = \frac{3}{0.3} = 10$$

84. The Spin only magnetic moment value of square planar complex $[Pt(NH_3)_2Cl(NH_2CH_3)]Cl$ is ________ B.M. (Nearest integer)
(Given atomic number for Pt = 78)

Ans. (0)

Sol. Pt^{2+} (d^8)



$Pt^{2+} \rightarrow dsp^2$ hybridization and have no unpaired e^-s.
$\therefore$ Magnetic moment = 0

85. For a certain thermochemical reaction $M \rightarrow N$ at T = 400 K, $\Delta H^\ominus = 77.2$ kJ mol^{-1}, ΔS = 122 JK^{-1}, log equilibrium constant (logK) is –_______ × 10^{-1}.

Ans. (37)

Sol. $\Delta G° = \Delta H° - T\Delta S°$
$= 77.2 \times 10^3 - 400 \times 122 = 28400\,J$
$\Delta G° = -2.303\,RT\log K$
$\Rightarrow 28400 = -2.303 \times 8.314 \times 400\ \log K$
$\Rightarrow \log K = -3.708 = -37.08 \times 10^{-1}$

86. Volume of 3 M NaOH (formula weight 40 g mol^{-1}) which can be prepared from 84 g of NaOH is ________ $\times 10^{-1}$ dm^3.

Ans. **(7)**

Sol. $M = \frac{n_{NaOH}}{V_{sol}(\text{in L})} \Rightarrow 3 = \frac{(84/40)}{V} \Rightarrow V = 0.7L = 7\times10^{-1}L$

87. 1 mole of PbS is oxidised by "X" moles of O_3 to get "Y" moles of O_2. X + Y = __________

Ans. **(8)**

Sol. $PbS + 4O_3 \rightarrow PbSO_4 + 4O_2$

$x = 4, y = 4$

88. The hydrogen electrode is dipped in a solution of pH = 3 at 25°C. The potential of the electrode will be –________ $\times 10^{-2}$ V.

$\left(\frac{2.303RT}{F} = 0.059\,V\right)$

Ans. **(18)**

Sol. $2H^+_{(aq.)} + 2e^- \rightarrow H_2(g)$

$$E_{cell} = E^0_{cell} - \frac{0.059}{2}\log\frac{P_{H_2}}{\left[H^+\right]^2}$$

= 0- 0.059 × 3 = - 0.177 volts. = -17.7 × 10^{-2} V.

89. 9.3 g of aniline is subjected to reaction with excess of acetic anhydride to prepare acetanilide. The mass of acetanilide produced if the reaction is 100% completed is _________ $\times 10^{-1}$ g.
(Given molar mass in g mol^{-1} N : 14, O : 16, C : 12, H : 1)

Ans. **(135)**

Sol. $C_6H_5NH_2 + CH_3-\overset{\overset{O}{||}}{C}-O-\overset{\overset{O}{||}}{C}-CH_3 \rightarrow$
(Aniline MM=93)

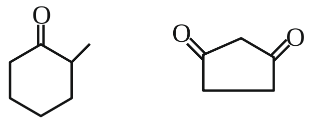
$C_6H_5NH-\overset{\overset{O}{||}}{C}-CH_3 + CH_3COOH$
(Acetanilide MM=135)

$n_{Acetanilide} = n_{Aniline}$

$\Rightarrow \frac{m}{135} = \frac{9.3}{93}$

$\Rightarrow m = 13.5\,g$

90. Total number of compounds with Chiral carbon atoms from following is ________.



$CH_3-CH_2-CH(NO_2)-COOH$

$CH_3-CH_2-CHBr-CH_2-CH_3$

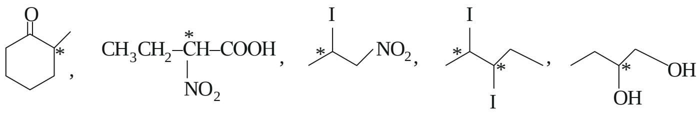
$CH_3-CH(I)-CH_2-NO_2$

$CH_3-CH_2-CH(OH)-CH_2OH$

$CH_3-\underset{\underset{I}{|}}{CH}-CH(I)-C_2H_5$

Ans. **(5)**

Sol. Chiral carbons are marked by.



MATHEMATICS | **TEST PAPER WITH SOLUTION**

SECTION-A

1. If in a G.P. of 64 terms, the sum of all the terms is 7 times the sum of the odd terms of the G.P, then the common ratio of the G.P. is equal to

(1) 7 (2) 4

(3) 5 (4) 6

Ans. (4)

Sol. $a + ar + ar^2 + ar^3 + ... + ar^{63}$

$= 7(a + ar^2 + ar^4 ..... + ar^{62})$

$\Rightarrow \frac{a(1-r^{64})}{1-r} = \frac{7a(1-r^{64})}{1-r^2}$

$r = 6$

2. In an A.P., the sixth terms $a_6 = 2$. If the $a_1 a_4 a_5$ is the greatest, then the common difference of the A.P., is equal to

(1) $\frac{3}{2}$ (2) $\frac{8}{5}$ (3) $\frac{2}{3}$ (4) $\frac{5}{8}$

Ans. (2)

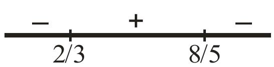
Sol. $a_6 = 2 \Rightarrow a + 5d = 2$

$a_1 a_4 a_5 = a(a+3d)(a+4d)$

$= (2-5d)(2-2d)(2-d)$

$f(d) = 8 - 32d + 34d^2 - 20d + 30d^2 - 10d^3$

$f'(d) = -2(5d-8)(3d-2)$



$d = \frac{8}{5}$

3. If $f(x) = \begin{cases} 2+2x, & -1 \le x < 0 \\ 1-\frac{x}{3}, & 0 \le x \le 3 \end{cases}$; $g(x) = \begin{cases} -x, & -3 \le x \le 0 \\ x, & 0 < x \le 1 \end{cases}$,

then range of (fog(x)) is

(1) (0, 1] (2) [0, 3)

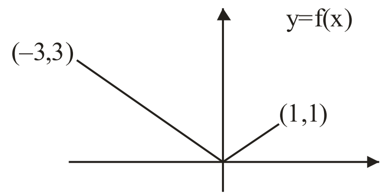
(3) [0, 1] (4) [0, 1)

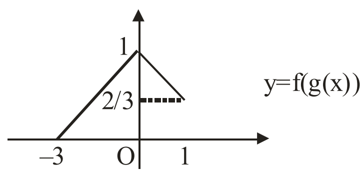
Ans. (3)

Sol. $f(g(x)) = \begin{cases} 2+2g(x), & -1 \le g(x) < 0 & .....(1) \\ 1-\frac{g(x)}{3}, & 0 \le g(x) \le 3 & .....(2) \end{cases}$

By (1) $x \in \phi$

And by (2) $x \in [-3,0]$ and $x \in [0,1]$





Range of $f(g(x))$ is [0, 1]

4. A fair die is thrown until 2 appears. Then the probability, that 2 appears in even number of throws , is

(1) $\frac{5}{6}$ (2) $\frac{1}{6}$ (3) $\frac{5}{11}$ (4) $\frac{6}{11}$

Ans. (3)

Sol. Required probability =

$\frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^3 \times \frac{1}{6} + \left(\frac{5}{6}\right)^5 \times \frac{1}{6} + ...$

$= \frac{1}{6} \times \frac{\frac{5}{6}}{1 - \frac{25}{36}} = \frac{5}{11}$

5. If $z=\frac{1}{2}-2i$, is such that $|z+1|=\alpha z+\beta(1+i), i=\sqrt{-1}$ and $\alpha,\beta\in R$, then $\alpha+\beta$ is equal to

(1) –4 (2) 3

(3) 2 (4) –1

Ans. (2)

Sol. $z=\frac{1}{2}-2i$

$$\left|z+1\right|=\alpha z+\beta(1+i)$$

$$\left|\frac{3}{2}-2i\right|=\frac{\alpha}{2}-2\alpha i+\beta+\beta i$$

$$\left|\frac{3}{2}-2i\right|=\left(\frac{\alpha}{2}+\beta\right)+\left(\beta-2\alpha\right)i$$

$$\beta=2\alpha \text{ and } \frac{\alpha}{2}+\beta=\sqrt{\frac{9}{4}+4}$$

$$\alpha+\beta=3$$

6. $\lim\limits_{x\to\frac{\pi}{2}}\left(\frac{1}{\left(x-\frac{\pi}{2}\right)^2}\int_{x^3}^{\left(\frac{\pi}{2}\right)^3}\cos\left(\frac{1}{t^3}\right)dt\right)$ is equal to

(1) $\frac{3\pi}{8}$ (2) $\frac{3\pi^2}{4}$

(3) $\frac{3\pi^2}{8}$ (4) $\frac{3\pi}{4}$

Ans. (3)

Sol. Using L'hopital rule

$$=\lim_{x\to\frac{\pi}{2}^-}\frac{0-\cos x\times 3x^2}{2\left(x-\frac{\pi}{2}\right)}$$

$$=\lim_{x\to\frac{\pi}{2}^-}\frac{\sin\left(x-\frac{\pi}{2}\right)}{2\left(x-\frac{\pi}{2}\right)}\times\frac{3\pi^2}{4}$$

$$=\frac{3\pi^2}{8}$$

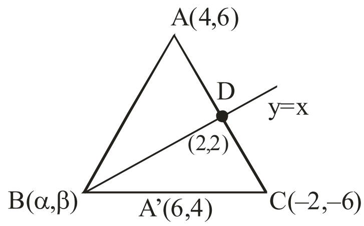
7. In a ΔABC, suppose $y = x$ is the equation of the bisector of the angle B and the equation of the side AC is $2x - y = 2$. If $2AB = BC$ and the point A and B are respectively (4, 6) and (α,β), then $\alpha+2\beta$ is equal to

(1) 42 (2) 39

(3) 48 (4) 45

Ans. (1)

Sol.



AD : DC = 1 : 2

$$\frac{4-\alpha}{6-\alpha}=\frac{10}{8}$$

$$\alpha=\beta$$

$$\alpha=14 \text{ and } \beta=14$$

8. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three non-zero vectors such that $\vec{b}$ and $\vec{c}$ are non-collinear .if $\vec{a}+5\vec{b}$ is collinear with $\vec{c}$, $\vec{b}+6\vec{c}$ is collinear with $\vec{a}$ and $\vec{a}+\alpha\vec{b}+\beta\vec{c}=\vec{0}$, then $\alpha+\beta$ is equal to

(1) 35 (2) 30

(3) – 30 (4) –25

Ans. (1)

Sol. $\vec{a}+5\vec{b}=\lambda\vec{c}$

$\vec{b}+6\vec{c}=\mu\vec{a}$

Eliminating $\vec{a}$

$$\lambda\vec{c}-5\vec{b}=\frac{6}{\mu}\vec{c}+\frac{1}{\mu}\vec{b}$$

$$\therefore \mu=\frac{-1}{5},\lambda=-30$$

$$\alpha=5,\beta=30$$

9. Let $\left(5, \frac{a}{4}\right)$, be the circumcenter of a triangle with vertices $A(a,-2), B(a, 6)$ and $C\left(\frac{a}{4},-2\right)$. Let α denote the circumradius, β denote the area and γ denote the perimeter of the triangle. Then $\alpha+\beta+\gamma$ is

(1) 60 (2) 53

(3) 62 (4) 30

Ans. (2)

Sol. $A(a, -2), B(a, 6), C\left(\frac{a}{4},-2\right)$, $O\left(5,\frac{a}{4}\right)$

AO = BO

$$(a-5)^2 + \left(\frac{a}{4}+2\right)^2 = (a-5)^2 + \left(\frac{a}{4}-6\right)^2$$

a = 8

AB = 8, AC = 6, BC = 10

$\alpha = 5, \beta = 24, \gamma = 24$

10. For $x \in \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, if

$$y(x)=\int \frac{\operatorname{cosec} x + \sin x}{\operatorname{cosec} x \sec x + \tan x \sin^2 x} dx \text{ and}$$

$\lim_{x \to \left(\frac{\pi}{2}\right)^-} y(x) = 0$ then $y\left(\frac{\pi}{4}\right)$ is equal to

(1) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (2) $\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(3) $-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (4) $\frac{1}{\sqrt{2}}\tan^{-1}\left(-\frac{1}{2}\right)$

Ans. (4)

Sol. $$y(x) = \int \frac{(1+\sin^2 x)\cos x}{1+\sin^4 x} dx$$

Put sinx = t

$$= \int \frac{1+t^2}{t^4+1} dt = \frac{1}{\sqrt{2}} \tan^{-1} \frac{\left(t-\frac{1}{t}\right)}{\sqrt{2}} + C$$

$x = \frac{\pi}{2}, t = 1$ $\therefore C = 0$

$$y\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \tan^{-1}\left(-\frac{1}{2}\right)$$

11. If $\alpha, -\frac{\pi}{2} < \alpha < \frac{\pi}{2}$ is the solution of $4\cos\theta + 5\sin\theta = 1$, then the value of $\tan\alpha$ is

(1) $\frac{10-\sqrt{10}}{6}$ (2) $\frac{10-\sqrt{10}}{12}$

(3) $\frac{\sqrt{10}-10}{12}$ (4) $\frac{\sqrt{10}-10}{6}$

Ans. (3)

Sol. $4 + 5\tan\theta = \sec\theta$

Squaring : $24\tan^2\theta + 40\tan\theta + 15 = 0$

$$\tan\theta = \frac{-10 \pm \sqrt{10}}{12}$$

and $\tan\theta = -\left(\frac{10+\sqrt{10}}{12}\right)$ is Rejected.

(3) is correct.

12. A function y = f(x) satisfies

$f(x)\sin 2x + \sin x - (1+\cos^2 x) f'(x) = 0$ with condition f(0) = 0 . Then $f\left(\frac{\pi}{2}\right)$ is equal to

(1) 1 (2) 0 (3) –1 (4) 2

Ans. (1)

Sol. $$\frac{dy}{dx} - \left(\frac{\sin 2x}{1+\cos^2 x}\right) y = \sin x$$

I.F. = $1 + \cos^2 x$

$$y \cdot (1+\cos^2 x) = \int (\sin x) dx$$

= – cosx + C

x = 0, C = 1

$$y\left(\frac{\pi}{2}\right) = 1$$

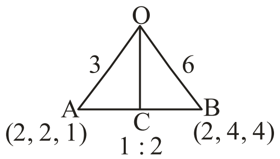
13. Let O be the origin and the position vector of A and B be $2\hat{i}+2\hat{j}+\hat{k}$ and $2\hat{i}+4\hat{j}+4\hat{k}$ respectively. If the internal bisector of $\angle AOB$ meets the line AB at C, then the length of OC is

(1) $\frac{2}{3}\sqrt{31}$ (2) $\frac{2}{3}\sqrt{34}$

(3) $\frac{3}{4}\sqrt{34}$ (4) $\frac{3}{2}\sqrt{31}$

Ans. (2)

Sol.



length of $OC = \frac{\sqrt{136}}{3} = \frac{2\sqrt{34}}{3}$

14. Consider the function $f:\left[\frac{1}{2},1\right]\to R$ defined by $f(x)=4\sqrt{2}x^3-3\sqrt{2}x-1$. Consider the statements

(I) The curve y = f(x) intersects the x-axis exactly at one point

(II) The curve y = f(x) intersects the x-axis at $x=\cos\frac{\pi}{12}$

Then

(1) Only (II) is correct

(2) Both (I) and (II) are incorrect

(3) Only (I) is correct

(4) Both (I) and (II) are correct

Ans. (4)

Sol. $f'(x)=12\sqrt{2}x^2-3\sqrt{2}\ge 0$ for $\left[\frac{1}{2},1\right]$

$f\left(\frac{1}{2}\right)<0$

f(1) > 0 ⇒ (A) is correct.

$f(x)=\sqrt{2}\left(4x^3-3x\right)-1=0$

Let cosα = x,

$\cos 3\alpha = \cos\frac{\pi}{4} \Rightarrow \alpha = \frac{\pi}{12}$

$x=\cos\frac{\pi}{12}$

(4) is correct.

15. Let $A=\begin{bmatrix}1&0&0\\0&\alpha&\beta\\0&\beta&\alpha\end{bmatrix}$ and $|2A|^3=2^{21}$ where $\alpha,\beta\in Z$,

Then a value of α is

(1) 3 (2) 5

(3) 17 (4) 9

Ans. (2)

Sol. $|A|=\alpha^2-\beta^2$

$|2A|^3=2^{21}\Rightarrow |A|=2^4$

$\alpha^2-\beta^2=16$

$(\alpha+\beta)(\alpha-\beta)=16\Rightarrow\alpha=4$ or 5

16. Let PQR be a triangle with $R(-1,4,2)$. Suppose M(2, 1, 2) is the mid point of PQ. The distance of the centroid of ΔPQR from the point of intersection of the line $\frac{x-2}{0}=\frac{y}{2}=\frac{z+3}{-1}$ and $\frac{x-1}{1}=\frac{y+3}{-3}=\frac{z+1}{1}$ is

(1) 69 (2) 9

(3) $\sqrt{69}$ (4) $\sqrt{99}$

Ans. (3)

Sol. Centroid G divides MR in 1 : 2

G(1, 2, 2)

Point of intersection A of given lines is (2,–6, 0)

$AG=\sqrt{69}$

17. Let R be a relation on Z × Z defined by

(a, b)R(c, d) if and only if ad – bc is divisible by 5.

Then R is

(1) Reflexive and symmetric but not transitive

(2) Reflexive but neither symmetric not transitive

(3) Reflexive, symmetric and transitive

(4) Reflexive and transitive but not symmetric

Ans. (1)

Sol. (a, b)R(a, b) as ab – ab = 0

Therefore reflexive

Let (a,b)R(c,d)⇒ad – bc is divisible by 5

⇒ bc – ad is divisible by 5 ⇒(c,d)R(a,b)

Therefore symmetric

Relation not transitive as (3,1)R(10,5) and (10,5)R(1,1) but (3,1) is not related to (1,1)

18. If the value of the integral

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{x^2\cos x}{1+\pi^x}+\frac{1+\sin^2 x}{1+e^{\sin x^{2023}}}\right)dx=\frac{\pi}{4}(\pi+a)-2,$$

then the value of a is

(1) 3 (2) $-\frac{3}{2}$ (3) 2 (4) $\frac{3}{2}$

Ans. (1)

Sol. $$I=\int_{-\pi/2}^{\pi/2}\left(\frac{x^2\cos x}{1+\pi^x}+\frac{1+\sin^2 x}{1+e^{\sin x^{2023}}}\right)dx$$

$$I=\int_{-\pi/2}^{\pi/2}\left(\frac{x^2\cos x}{1+\pi^{-x}}+\frac{1+\sin^2 x}{1+e^{\sin(-x)^{2023}}}\right)dx$$

On Adding, we get

$$2I=\int_{-\pi/2}^{\pi/2}\left(x^2\cos x+1+\sin^2 x\right)dx$$

On solving

$$I=\frac{\pi^2}{4}+\frac{3\pi}{4}-2$$

a = 3

19. Suppose

$$f(x)=\frac{\left(2^x+2^{-x}\right)\tan x\sqrt{\tan^{-1}\left(x^2-x+1\right)}}{\left(7x^2+3x+1\right)^3},$$

Then the value of f '(0) is equal to

(1) π (2) 0

(3) $\sqrt{\pi}$ (4) $\frac{\pi}{2}$

Ans. (3)

Sol. $$f'(0)=\lim_{h\to 0}\frac{f(h)-f(0)}{h}$$

$$=\lim_{h\to 0}\frac{(2^h+2^{-h})\tan h\sqrt{\tan^{-1}(h^2-h+1)}-0}{(7h^2+3h+1)^3 h}$$

$$=\sqrt{\pi}$$

20. Let A be a square matrix such that $AA^T=I$. Then $\frac{1}{2}A\left[\left(A+A^T\right)^2+\left(A-A^T\right)^2\right]$ is equal to

(1) A^2+I (2) A^3+I

(3) A^2+A^T (4) A^3+A^T

Ans. (4)

Sol. $AA^T=I=A^TA$

On solving given expression, we get

$$\frac{1}{2}A\left[A^2+(A^T)^2+2AA^T+A^2+(A^T)^2-2AA^T\right]$$

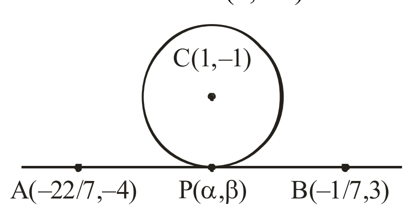
$$=A[A^2+(A^T)^2]=A^3+A^T$$

SECTION-B

21. Equation of two diameters of a circle are $2x-3y=5$ and $3x-4y=7$. The line joining the points $\left(-\frac{22}{7},-4\right)$ and $\left(-\frac{1}{7},3\right)$ intersects the circle at only one point $P(\alpha,\beta)$. Then $17\beta-\alpha$ is equal to

Ans. (2)

Sol. Centre of circle is (1, –1)



Equation of AB is 7x – 3y + 10 = 0 …(i)

Equation of CP is 3x + 7y + 4 = 0 …(ii)

Solving (i) and (ii)

$$\alpha=\frac{-41}{29},\beta=\frac{1}{29} \qquad \therefore 17\beta-\alpha=2$$

22. All the letters of the word "GTWENTY" are written in all possible ways with or without meaning and these words are written as in a dictionary. The serial number of the word "GTWENTY" IS

Ans. (553)

Sol. Words starting with E = 360

Words starting with GE = 60

Words starting with GN = 60

Words starting with GTE = 24

Words starting with GTN = 24

Words starting with GTT = 24

GTWENTY = 1

Total = 553

23. Let α,β be the roots of the equation $x^2-x+2=0$ with $\text{Im}(\alpha)>\text{Im}(\beta)$. Then $\alpha^6+\alpha^4+\beta^4-5\alpha^2$ is equal to

Ans. (13)

Sol. $\alpha^6+\alpha^4+\beta^4-5\alpha^2$

$= \alpha^4(\alpha-2)+\alpha^4-5\alpha^2+(\beta-2)^2$

$= \alpha^5-\alpha^4-5\alpha^2+\beta^2-4\beta+4$

$= \alpha^3(\alpha-2)-\alpha^4-5\alpha^2+\beta-2-4\beta+4$

$= -2\alpha^3-5\alpha^2-3\beta+2$

$= -2\alpha(\alpha-2)-5\alpha^2-3\beta+2$

$= -7\alpha^2+4\alpha-3\beta+2$

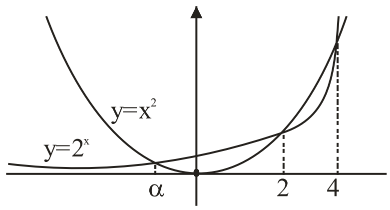
$= -7(\alpha-2)+4\alpha-3\beta+2$

$= -3\alpha-3\beta+16=-3(1)+16=13$

24. Let $f(x)=2^x-x^2, x\in R$. If m and n are respectively the number of points at which the curves y = f(x) and y = f '(x) intersects the x-axis, then the value of m + n is

Ans. (5)

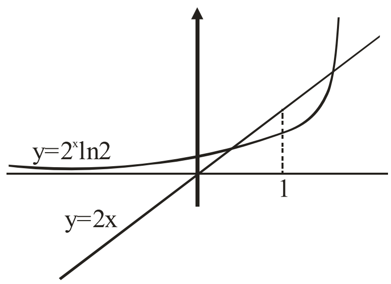
Sol.



$\therefore$ m = 3

$f'(x)=2^x\ln 2-2x=0$

$2^x\ln 2=2x$



$\therefore$ n = 2

$\Rightarrow$ m + n = 5

25. If the points of intersection of two distinct conics $x^2+y^2=4b$ and $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ lie on the curve $y^2=3x^2$, then $3\sqrt{3}$ times the area of the rectangle formed by the intersection points is __

Ans. (432)

Sol. Putting $y^2=3x^2$ in both the conics

We get $x^2=b$ and $\frac{b}{16}+\frac{3}{b}=1$

$\Rightarrow b=4,12$ (b = 4 is rejected because curves coincide)

$\therefore b=12$

Hence points of intersection are

$\left(\pm\sqrt{12},\pm 6\right)$ $\Rightarrow$ area of rectangle = 432

26. If the solution curve $y = y(x)$ of the differential equation $(1+y^2)(1+\log_e x)dx + x\,dy = 0$, $x > 0$ passes through the point (1, 1) and $y(e) = \dfrac{\alpha - \tan\left(\frac{3}{2}\right)}{\beta + \tan\left(\frac{3}{2}\right)}$, then $\alpha + 2\beta$ is

Ans. (3)

Sol. $\int\left(\dfrac{1}{x} + \dfrac{\ln x}{x}\right)dx + \int\dfrac{dy}{1+y^2} = 0$

$$\ln x + \frac{(\ln x)^2}{2} + \tan^{-1} y = C$$

Put x = y = 1

$$\therefore C = \frac{\pi}{4}$$

$$\Rightarrow \ln x + \frac{(\ln x)^2}{2} + \tan^{-1} y = \frac{\pi}{4}$$

Put x = e

$$\Rightarrow y = \tan\left(\frac{\pi}{4} - \frac{3}{2}\right) = \frac{1 - \tan\frac{3}{2}}{1 + \tan\frac{3}{2}}$$

$$\therefore \alpha = 1, \beta = 1$$

$$\Rightarrow \alpha + 2\beta = 3$$

27. If the mean and variance of the data 65, 68, 58, 44, 48, 45, 60, $\alpha, \beta, 60$ where $\alpha > \beta$ are 56 and 66.2 respectively, then $\alpha^2 + \beta^2$ is equal to

Ans. (6344)

Sol. $\bar{x} = 56$

$$\sigma^2 = 66.2$$

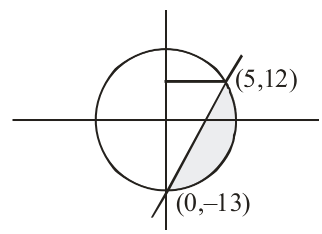
$$\Rightarrow \frac{\alpha^2 + \beta^2 + 25678}{10} - (56)^2 = 66.2$$

$$\therefore \alpha^2 + \beta^2 = 6344$$

28. The area (in sq. units) of the part of circle $x^2 + y^2 = 169$ which is below the line $5x - y = 13$ is $\dfrac{\pi\alpha}{2\beta} - \dfrac{65}{2} + \dfrac{\alpha}{\beta}\sin^{-1}\left(\dfrac{12}{13}\right)$ where α, β are coprime numbers. Then $\alpha + \beta$ is equal to

Ans. (171)

Sol.



$$\text{Area} = \int_{-13}^{12} \sqrt{169 - y^2}\,dy - \frac{1}{2} \times 25 \times 5$$

$$= \frac{\pi}{2} \times \frac{169}{2} - \frac{65}{2} + \frac{169}{2}\sin^{-1}\frac{12}{13}$$

$$\therefore \alpha + \beta = 171$$

29. If $\dfrac{{}^{11}C_1}{2} + \dfrac{{}^{11}C_2}{3} + ... + \dfrac{{}^{11}C_9}{10} = \dfrac{n}{m}$ with gcd(n, m) = 1, then n +m is equal to

Ans. (2041)

Sol. $\displaystyle\sum_{r=1}^{9} \frac{{}^{11}C_r}{r+1}$

$$= \frac{1}{12}\sum_{r=1}^{9} {}^{12}C_{r+1}$$

$$= \frac{1}{12}\left[2^{12} - 26\right] = \frac{2035}{6}$$

$$\therefore m + n = 2041$$

30. A line with direction ratios 2, 1, 2 meets the lines x = y +2 = z and x + 2 = 2y = 2z respectively at the point P and Q. if the length of the perpendicular from the point (1, 2, 12) to the line PQ is l, then l^2 is

Ans. (65)

Sol. Let $P(t, t-2, t)$ and $Q(2s-2, s, s)$

D.R's of PQ are 2, 1, 2

$$\frac{2s-2-t}{2} = \frac{s-t+2}{1} = \frac{s-t}{2}$$

$\Rightarrow t = 6$ and $s = 2$

$\Rightarrow P(6,4,6)$ and $Q(2,2,2)$

$$PQ: \frac{x-2}{2} = \frac{y-2}{1} = \frac{z-2}{2} = \lambda$$

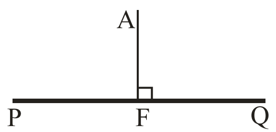
Let $F(2\lambda+2, \lambda+2, 2\lambda+2)$

$A(1,2,12)$

$\overrightarrow{AF} \cdot \overrightarrow{PQ} = 0$

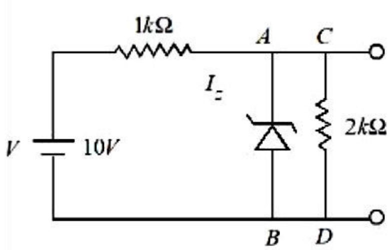
$\therefore \lambda = 2$

So F(6,4, 6) and $AF = \sqrt{65}$



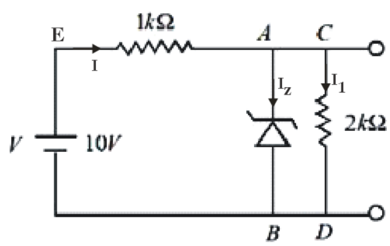
SECTION-A

31. In the given circuit, the breakdown voltage of the Zener diode is 3.0 V. What is the value of I_z?



(1) 3.3 mA (2) 5.5 mA
(3) 10 mA (4) 7 mA

Ans. (2)



Sol.

$V_z = 3V$

Let potential at B = 0 V

Potential at E(V_E) = 10 V

$V_C = V_A = 3$ V

$I_z + I_1 = I$

$$I = \frac{10-3}{1000} = \frac{7}{1000}A$$

$$I_1 = \frac{3}{2000}A$$

Therefore $I_z = \dfrac{7-1.5}{1000} = 5.5mA$

32. The electric current through a wire varies with time as $I = I_0 + \beta t$. where $I_0 = 20$ A and $\beta = 3$ A/s. The amount of electric charge crossed through a section of the wire in 20 s is :

(1) 80 C (2) 1000 C
(3) 800 C (4) 1600 C

Ans. (2)

Sol. Given that

Current $I = I_0 + \beta t$

$I_0 = 20A$

$\beta = 3A/s$

$I = 20 + 3t$

$$\frac{dq}{dt} = 20 + 3t$$

$$\int_0^q dq = \int_0^{20} (20+3t)dt$$

$$q = \int_0^{20} 20dt + \int_0^{20} 3tdt$$

$$q = \left[20t + \frac{3t^2}{2}\right]_0^{20} = 1000\ C$$

33. Given below are two statements:

Statement I : If a capillary tube is immersed first in cold water and then in hot water, the height of capillary rise will be smaller in hot water.

Statement II : If a capillary tube is immersed first in cold water and then in hot water, the height of capillary rise will be smaller in cold water.

In the light of the above statements, choose the ***most appropriate*** from the options given below

(1) Both **Statement I** and **Statement II** are true

(2) Both **Statement I** and **Statement II** are false

(3) **Statement I** is true but **Statement II** is false

(4) **Statement I** is false but **Statement II** is true

Ans. (3)

Sol. Surface tension will be less as temperature increases

$$h = \frac{2T\cos\theta}{\rho gr}$$

Height of capillary rise will be smaller in hot water and larger in cold water.

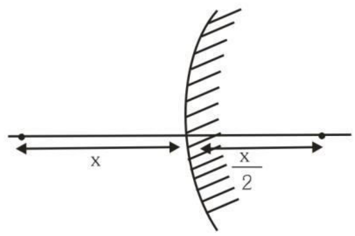
34. A convex mirror of radius of curvature 30 cm forms an image that is half the size of the object. The object distance is :

(1) –15 cm (2) 45 cm

(3) –45cm (4) 15 cm

Ans. **(1)**

Sol.



Given R = 30 cm

f = R/2 = +15 cm

Magnification $(m) = \pm\frac{1}{2}$

For convex mirror, virtual image is formed for real object.

Therefore, m is +ve

$$\frac{1}{2} = \frac{f}{f-u}$$

u = – 15 cm

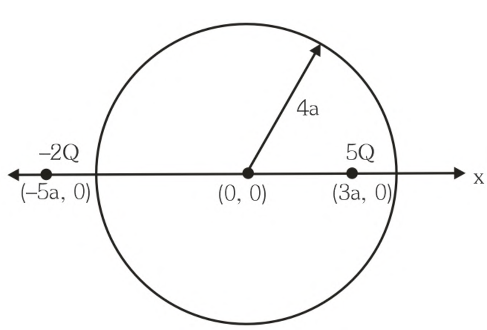
35. Two charges of 5Q and –2Q are situated at the points (3a, 0) and (–5a, 0) respectively. The electric flux through a sphere of radius '4a' having center at origin is :

(1) $\frac{2Q}{\varepsilon_0}$ (2) $\frac{5Q}{\varepsilon_0}$

(3) $\frac{7Q}{\varepsilon_0}$ (4) $\frac{3Q}{\varepsilon_0}$

Ans. **(2)**

Sol.



5Q charge is inside the spherical region

flux through sphere = $\frac{5Q}{\varepsilon_0}$

36. A body starts moving from rest with constant acceleration covers displacement S_1 in first (p – 1) seconds and S_2 in first p seconds. The displacement $S_1 + S_2$ will be made in time :

(1) $(2p+1)s$

(2) $\sqrt{(2p^2-2p+1)}s$

(3) $(2p-1)s$

(4) $(2p^2-2p+1)s$

Ans. **(2)**

Sol. S_1 in first (p – 1) sec

S_2 in first p sec

$$S_1 = \frac{1}{2}a(p-1)^2$$

$$S_2 = \frac{1}{2}a(p)^2$$

$$S_1 + S_2 = \frac{1}{2}at^2$$

$(p-1)^2 + p^2 = t^2$

$$t = \sqrt{2p^2+1-2p}$$

37. The potential energy function (in J) of a particle in a region of space is given as $U = (2x^2 + 3y^3 + 2z)$. Here x, y and z are in meter. The magnitude of x - component of force (in N) acting on the particle at point P (1, 2, 3) m is :

(1) 2 (2) 6

(3) 4 (4) 8

Ans. **(3)**

Sol. Given $U = 2x^2 + 3y^3 + 2z$

$$F_x = -\frac{\partial U}{\partial x} = -4x$$

At x = 1 magnitude of F_x is 4N

38. The resistance $R=\frac{V}{I}$ where $V=\left(200\pm 5\right)V$ and $I=\left(20\pm 0.2\right)A$, the percentage error in the measurement of R is :

(1) 3.5%

(2) 7%

(3) 3%

(4) 5.5%

Ans. **(1)**

Sol. $R=\frac{V}{I}$

According to error analysis

$$\frac{dR}{R}=\frac{dV}{V}+\frac{dI}{I}$$

$$\frac{dR}{R}=\frac{5}{200}+\frac{0.2}{20}$$

$$\frac{dR}{R}=\frac{7}{200}$$

% error $\frac{dR}{R}\times 100=\frac{7}{200}\times 100=3.5\%$

39. A block of mass 100 kg slides over a distance of 10 m on a horizontal surface. If the co-efficient of friction between the surfaces is 0.4, then the work done against friction (in J) is :

(1) 4200

(2) 3900

(3) 4000

(4) 4500

Ans. **(3)**

Sol. Given m = 100 kg

s = 10 m

μ = 0.4

As f = μmg = 0.4 × 100 × 10 = 400 N

Now W = f.s = 400 ×10 = 4000 J

40. Match List I with List II

	List I		List II
A.	$\oint \vec{B}.\vec{dl}=\mu_0 i_c+\mu_0\varepsilon_0\frac{d\phi_E}{dt}$	I.	Gauss' law for electricity
B.	$\oint \vec{E}.\vec{dl}=\frac{d\phi_B}{dt}$	II.	Gauss' law for magnetism
C.	$\oint \vec{E}.\vec{dA}=\frac{Q}{\varepsilon_0}$	III.	Faraday law
D.	$\oint \vec{B}.\vec{dA}=0$	IV.	Ampere – Maxwell law

Chose the correct answer from the options given below

(1) A-IV, B-I, C-III, D-II

(2) A-II, B-III, C-I, D-IV

(3) A-IV, B-III, C-I, D-II

(4) A-I, B-II, C-III, D-IV

Ans. **(3)**

Sol. Ampere – Maxwell law

$\rightarrow \oint \vec{B}.\vec{dl}=\mu_0 i_c+\mu_0\varepsilon_0\frac{d\phi_E}{dt}$

Faraday law $\rightarrow \oint \vec{E}.\vec{dl}=\frac{d\phi_B}{dt}$

Gauss' law for electricity $\rightarrow \oint \vec{E}.\vec{dA}=\frac{Q}{\varepsilon_0}$

Gauss ' law for magnetism $\rightarrow \oint \vec{B}.\vec{dA}=0$

41. If the radius of curvature of the path of two particles of same mass are in the ratio 3:4, then in order to have constant centripetal force, their velocities will be in the ratio of:

(1) $\sqrt{3}:2$ (2) $1:\sqrt{3}$

(3) $\sqrt{3}:1$ (4) $2:\sqrt{3}$

Ans. **(1)**

Sol. Given $m_1 = m_2$

and $\frac{r_1}{r_2} = \frac{3}{4}$

As centripetal force $F = \frac{mv^2}{r}$

In order to have constant (same in this question) centripetal force

$F_1 = F_2$

$$\frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2}$$

$$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{r_1}{r_2}} = \frac{\sqrt{3}}{2}$$

42. A galvanometer having coil resistance 10 Ω shows a full scale deflection for a current of 3mA. For it to measure a current of 8A, the value of the shunt should be:

(1) $3 \times 10^{-3}\Omega$ (2) $4.85 \times 10^{-3}\Omega$

(3) $3.75 \times 10^{-3}\Omega$ (4) $2.75 \times 10^{-3}\Omega$

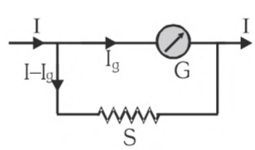
Ans. (3)

Sol. Given G = 10 Ω

$I_g = 3mA$

I = 8A

In case of conversion of galvanometer into ammeter.



We have $I_g G = (I - I_g)S$

$$S = \frac{I_g G}{I - I_g}$$

$$S = \frac{(3 \times 10^{-3})10}{8 - 0.003} = 3.75 \times 10^{-3}\Omega$$

43. The de-Broglie wavelength of an electron is the same as that of a photon. If velocity of electron is 25% of the velocity of light, then the ratio of K.E. of electron and K.E. of photon will be:

(1) $\frac{1}{1}$ (2) $\frac{1}{8}$

(3) $\frac{8}{1}$ (4) $\frac{1}{4}$

Ans. (2)

Sol. For photon

$$E_P = \frac{hc}{\lambda_p} \Rightarrow \lambda_p = \frac{hc}{E_P}$$

For electron

$$\lambda_e = \frac{h}{m_e v_e} = \frac{hv_e}{2K_e}$$

Given $v_e = 0.25$ c

$$\lambda_e = \frac{h \times 0.25c}{2K_e} = \frac{hc}{8K_e}$$

Also $\lambda_p = \lambda_e$

$$\frac{hc}{E_p} = \frac{hc}{8K_e}$$

$$\frac{K_e}{E_p} = \frac{1}{8}$$

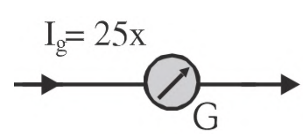
44. The deflection in moving coil galvanometer falls from 25 divisions to 5 division when a shunt of 24Ω is applied. The resistance of galvanometer coil will be :

(1) 12Ω (2) 96Ω

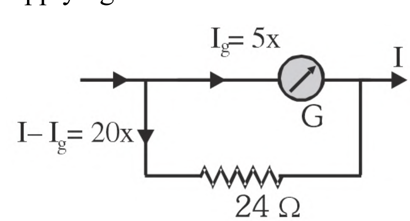
(3) 48Ω (4) 100Ω

Ans. (2)

Sol. Let x = current/division



After applying shunt



Now 5x × G = 20x × 24

G = 4 × 24

G = 96Ω

45. A biconvex lens of refractive index 1.5 has a focal length of 20 cm in air. Its focal length when immersed in a liquid of refractive index 1.6 will be:

(1) – 16 cm

(2) – 160 cm

(3) + 160 cm

(4) + 16 cm

Ans. **(2)**

Sol. $\mu_l = 1.5$

$\mu_m = 1.6$

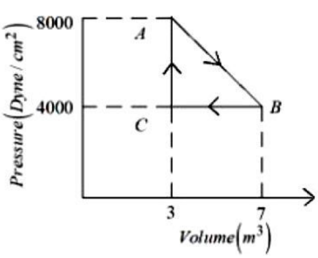
$f_a = 20$ cm

As $\frac{f_m}{f_a} = \frac{(\mu_l - 1)\mu_m}{(\mu_l - \mu_m)}$

$\frac{f_m}{20} = \frac{(1.5-1)1.6}{(1.5-1.6)}$

$f_m = -160$ cm

46. A thermodynamic system is taken from an original state A to an intermediate state B by a linear process as shown in the figure. It's volume is then reduced to the original value from B to C by an isobaric process. The total work done by the gas from A to B and B to C would be :

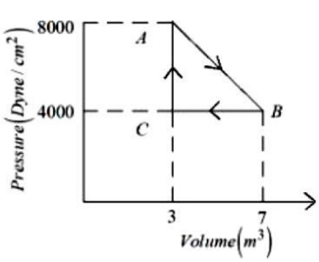


(1) 33800 J (2) 2200 J

(3) 600 J (4) 1200 J

Ans. **(BONUS)**

Sol.



Work done AB = $\frac{1}{2}$ (8000 + 6000) Dyne/cm^2 × 4m^3 = (6000Dyne/cm^2) × 4m^3

Work done BC = –(4000 Dyne/cm^2) × 4m^3

Total work done = 2000 Dyne/cm^2 × 4m^3

$$= 2\times 10^3 \times \frac{1}{10^5}\frac{N}{cm^2}\times 4m^3$$

$$= 2\times 10^{-2} \times \frac{N}{10^{-4}m^2}\times 4m^3$$

= 2 × 10^2 × 4 Nm = 800 J

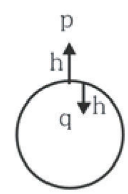
47. At what distance above and below the surface of the earth a body will have same weight, (take radius of earth as R.)

(1) $\sqrt{5}R - R$ (2) $\frac{\sqrt{3}R - R}{2}$

(3) $\frac{R}{2}$ (4) $\frac{\sqrt{5}R - R}{2}$

Ans. **(4)**

Sol. $g_p = \frac{gR^2}{(R+h)^2}$



$g_q = g\left(1 - \frac{h}{R}\right)$

$g_p = g_q$

$$\frac{g}{\left(1+\frac{h}{R}\right)^2} = g\left(1 - \frac{h}{R}\right)$$

$$\left(1 - \frac{h^2}{R^2}\right)\left(1 + \frac{h}{R}\right) = 1$$

Take $\frac{h}{R} = x$

So

$x^3 - x + x^2 = 0$

$x = \frac{\sqrt{5}-1}{2}$

$h = \frac{R}{2}\left(\sqrt{5}-1\right)$

48. A capacitor of capacitance 100 μF is charged to a potential of 12 V and connected to a 6.4 mH inductor to produce oscillations. The maximum current in the circuit would be :

(1) 3.2 A (2) 1.5 A

(3) 2.0 A (4) 1.2 A

Ans. **(2)**

Sol. By energy conservation

$$\frac{1}{2}CV^2 = \frac{1}{2}LI_{max}^2$$

$$I_{max} = \sqrt{\frac{C}{L}}\ V$$

$$= \sqrt{\frac{100\times10^{-6}}{6.4\times10^{-3}}}\times12$$

$$= \frac{12}{8} = \frac{3}{2} = 1.5\ \text{A}$$

49. The explosive in a Hydrogen bomb is a mixture of ${}_1H^2$, ${}_1H^3$ and ${}_3Li^6$ in some condensed form. The chain reaction is given by

${}_3Li^6 + {}_0n^1 \rightarrow {}_2He^4 + {}_1H^3$

${}_1H^2 + {}_1H^3 \rightarrow {}_2He^4 + {}_0n^1$

During the explosion the energy released is approximately

[Given : M(Li) = 6.01690 amu. M (${}_1H^2$) = 2.01471 amu. M (${}_2He^4$) = 4.00388amu, and 1 amu = 931.5 MeV]

(1) 28.12 MeV (2) 12.64 MeV

(3) 16.48 MeV (4) 22.22 MeV

Ans. **(4)**

Sol. ${}_3Li^6 + {}_0n^1 \rightarrow {}_2He^4 + {}_1H^3$

${}_1H^2 + {}_1H^3 \rightarrow {}_2He^4 + {}_0n^1$

$${}_3Li^6 + {}_1H^2 \rightarrow 2\left({}_2He^4\right)$$

Energy released in process

$Q = \Delta mc^2$

Q = [M(Li)+ M (${}_1H^2$) –2 × M(${}_2He^4$)] × 931.5 MeV

Q = [6.01690+2.01471–2 × 4.00388] × 931.5 MeV

Q = 22.216 MeV

Q = 22.22 MeV

50. Two vessels A and B are of the same size and are at same temperature. A contains 1g of hydrogen and B contains 1g of oxygen. P_A and P_B are the pressures of the gases in A and B respectively, then $\frac{P_A}{P_B}$ is :

(1) 16 (2) 8 (3) 4 (4) 32

Ans. **(1)**

Sol. $$\frac{P_AV_A}{P_BV_B} = \frac{n_ART_A}{n_BRT_B}$$

Given $V_A = V_B$

And $T_A = T_B$

$$\frac{P_A}{P_B} = \frac{n_A}{n_B}$$

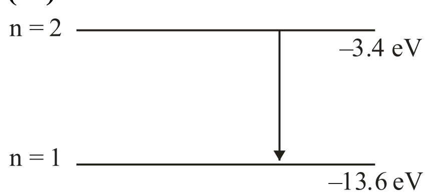
$$\frac{P_A}{P_B} = \frac{1/2}{1/32} = 16$$

SECTION-B

51. When a hydrogen atom going from n = 2 to n = 1 emits a photon, its recoil speed is $\frac{x}{5}$ *m/s*. Where x = ______ . (Use : mass of hydrogen atom = 1.6×10^{-27} kg)

Ans. **(17)**

Sol.



ΔE = 10.2 eV

Recoil speed(v) = $\frac{\Delta E}{mc}$

$$= \frac{10.2\text{eV}}{1.6\times10^{-27}\times3\times10^8}$$

$$= \frac{10.2\times1.6\times10^{-19}}{1.6\times10^{-27}\times3\times10^8}$$

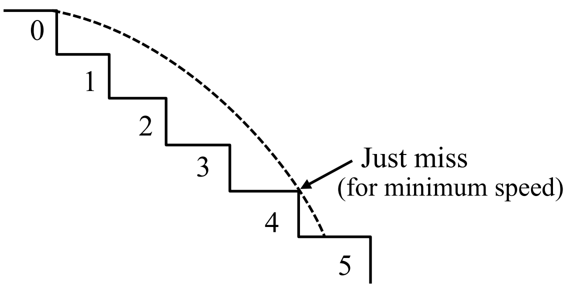
v = 3.4 m/s = $\frac{17}{5}$ m/s

Therefore, x = 17

52. A ball rolls off the top of a stairway with horizontal velocity u. The steps are 0.1 m high and 0.1 m wide. The minimum velocity u with which that ball just hits the step 5 of the stairway will be $\sqrt{x}\ ms^{-1}$ where x = ___________[use g = 10 m/s^2].

Ans. **(2)**

Sol.



The ball needs to just cross 4 steps to just hit 5^{th} step

Therefore, horizontal range (R) = 0.4 m

R = u.t

Similarly, in vertical direction

$$h = \frac{1}{2}gt^2$$

$$0.4 = \frac{1}{2}gt^2$$

$$0.4 = \frac{1}{2}g\left(\frac{0.4}{u}\right)^2$$

$u^2 = 2$

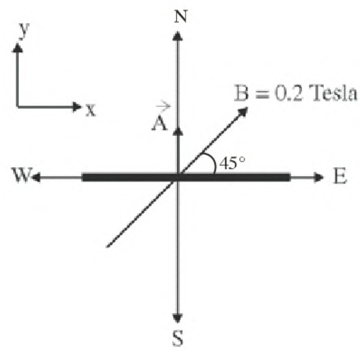
$u = \sqrt{2}$ m/s

Therefore, x = 2

53. A square loop of side 10 cm and resistance 0.7Ω is placed vertically in east-west plane. A uniform magnetic field of 0.20 T is set up across the plane in north east direction. The magnetic field is decreased to zero in 1 s at a steady rate. Then, magnitude of induced emf is $\sqrt{x} \times 10^{-3}$V. The value of x is ________.

Ans. **(2)**

Sol.



$$\vec{A} = (0.1)^2\,\hat{j}$$

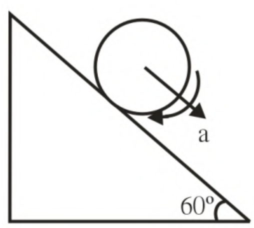
$$\vec{B} = \frac{0.2}{\sqrt{2}}\hat{i} + \frac{0.2}{\sqrt{2}}\hat{j}$$

Magnitude of induced emf

$$e = \frac{\Delta\phi}{\Delta t} = \frac{\vec{B}\cdot\vec{A} - 0}{1} = \sqrt{2}\times 10^{-3}\,V$$

54. A cylinder is rolling down on an inclined plane of inclination 60°. It's acceleration during rolling down will be $\frac{x}{\sqrt{3}}\,m/s^2$, where x = _________. (use g = 10 m/s^2).

Ans. **(10)**



Sol.

For rolling motion, $a = \dfrac{g\sin\theta}{1 + \dfrac{I_{cm}}{MR^2}}$

$$a = \frac{g\sin\theta}{1 + \frac{1}{2}}$$

$$= \frac{2\times 10\times\frac{\sqrt{3}}{2}}{3}$$

$$= \frac{10}{\sqrt{3}}$$

Therefore x = 10

55. The magnetic potential due to a magnetic dipole at a point on its axis situated at a distance of 20 cm from its center is 1.5×10^{-5}Tm. The magnetic moment of the dipole is__________Am^2.

(Given : $\frac{\mu_0}{4\pi} = 10^{-7} TmA^{-1}$)

Ans. **(6)**

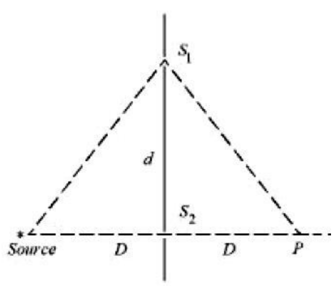
Sol. $V = \frac{\mu_0}{4\pi}\frac{M}{r^2}$

$$\Rightarrow 1.5\times10^{-5} = 10^{-7}\times\frac{M}{\left(20\times10^{-2}\right)^2}$$

$$\Rightarrow M = \frac{1.5\times10^{-5}\times20\times20\times10^{-4}}{10^{-7}}$$

$M = 1.5 \times 4 = 6$

56. In a double slit experiment shown in figure, when light of wavelength 400 nm is used, dark fringe is observed at P. If D = 0.2 m. the minimum distance between the slits S_1 and S_2 is ______ mm.



Ans. **(0.20)**

Sol. Path difference for minima at P

$$2\sqrt{D^2+d^2} - 2D = \frac{\lambda}{2}$$

$$\therefore \sqrt{D^2+d^2} - D = \frac{\lambda}{4}$$

$$\therefore \sqrt{D^2+d^2} = \frac{\lambda}{4} + D$$

$$\Rightarrow D^2 + d^2 = D^2 + \frac{\lambda^2}{16} + \frac{D\lambda}{2}$$

$$\Rightarrow d^2 = \frac{D\lambda}{2} + \frac{\lambda^2}{16}$$

$$\Rightarrow d^2 = \frac{0.2\times400\times10^{-9}}{2} + \frac{4\times10^{-14}}{4}$$

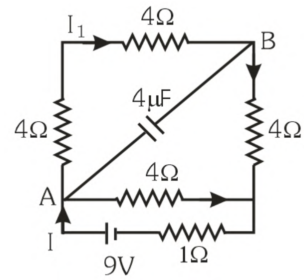
$$\Rightarrow d^2 \approx 400\times10^{-10}$$

$$\therefore d = 20\times10^{-5}$$

$$\Rightarrow d = 0.20\,mm$$

57. A 16Ω wire is bend to form a square loop. A 9V battery with internal resistance 1Ω is connected across one of its sides. If a 4μF capacitor is connected across one of its diagonals, the energy stored by the capacitor will be $\frac{x}{2}$ μJ. where x = _________.

Ans. **(81)**



Sol.

$$I = \frac{V}{R_{eq}} \quad I = \frac{V}{R_{eq}} = \frac{9}{1+\frac{12\times4}{12+4}} = \frac{9}{4}$$

$$I_1 = \frac{9}{4}\times\frac{4}{16} = \frac{9}{16}$$

$$V_A - V_B = I_1\times8 = \frac{9}{16}\times8 = \frac{9}{2}\ \mathbf{V}$$

$$\therefore U = \frac{1}{2}\times4\times\frac{81}{4}\mu J$$

$$\therefore U = \frac{81}{2}\mu J$$

$$\therefore x = 81$$

58. When the displacement of a simple harmonic oscillator is one third of its amplitude, the ratio of total energy to the kinetic energy is $\frac{x}{8}$, where x = _________.

Ans. **(9)**

Sol. Let total energy = E = $\frac{1}{2}KA^2$

$$U = \frac{1}{2}K\left(\frac{A}{3}\right)^2 = \frac{KA^2}{2\times9} = \frac{E}{9}$$

$$KE = E - \frac{E}{9} = \frac{8E}{9}$$

$$\text{Ratio } \frac{\text{Total}}{KE} = \frac{E}{\frac{8E}{9}} = \frac{9}{8}$$

x = 9

59. An electron is moving under the influence of the electric field of a uniformly charged infinite plane sheet S having surface charge density $+\sigma$. The electron at t = 0 is at a distance of 1 m from S and has a speed of 1 m/s. The maximum value of σ if the electron strikes S at t = 1 s is $\alpha\left[\frac{m\in_0}{e}\right]\frac{C}{m^2}$ the value of α is

Ans. **(8)**

Sol. $u = 1\ m/s;\ a = -\frac{\sigma e}{2\varepsilon_0 m}$

$t = 1\ s$

$S = -1\ m$

Using $S = ut + \frac{1}{2}at^2$

$-1 = 1\times1 - \frac{1}{2}\times\frac{\sigma e}{2\varepsilon_0 m}\times(1)^2$

$\therefore \sigma = 8\frac{\varepsilon_0 m}{e}$

$\therefore \alpha = 8$

60. In a test experiment on a model aeroplane in wind tunnel, the flow speeds on the upper and lower surfaces of the wings are 70 ms^{-1} and 65 ms^{-1} respectively. If the wing area is 2 m^2 the lift of the wing is _______ N.

(Given density of air = 1.2 kg m^{-3})

Ans. **(810)**

Sol. $F = \frac{1}{2}\rho\left(v_1^2 - v_2^2\right)A$

$F = \frac{1}{2}\times1.2\times\left(70^2 - 65^2\right)\times2$

= 810 N

SECTION-A

61. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**:

Assertion A: The first ionisation enthalpy decreases across a period.

Reason R: The increasing nuclear charge outweighs the shielding across the period.

In the light of the above statements, choose the most appropriate from the options given below:

(1) Both A and R are true and R is the correct explanation of A

(2) A is true but R is false

(3) A is false but R is true

(4) Both A and R are true but R is NOT the correct explanation of A

Ans. (3)

Sol. First ionisation energy **increases** along the period. Along the period Z increases which outweighs the shielding effect

62. Match List I with List II

LIST-I (Substances)	**LIST-II (Element Present)**
A.Ziegler catalyst	I.Rhodium
B.Blood Pigment	II. Cobalt
C.Wilkinson catalyst	III.Iron
D.Vitamin B_{12}	IV.Titanium

Choose the correct answer from the options given below:

(1) A-II, B-IV, C-I, D-III

(2) A-II, B-III, C-IV, D-I

(3) A-III, B-II, C-IV, D-I

(4) A-IV, B-III, C-I, D-II

Ans. (4)

Sol. Ziegler catalyst $\rightarrow$ Titanium

Blood pigment $\rightarrow$ Iron

Wilkinson catalyst $\rightarrow$ Rhodium

Vitamin $B_{12} \rightarrow$ Cobalt

63. In chromyl chloride test for confirmation of Cl^- ion, a yellow solution is obtained. Acidification of the solution and addition of amyl alcohol and 10% H_2O_2 turns organic layer blue indicating formation of chromium pentoxide. The oxidation state of chromium in that is

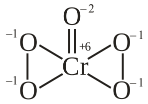
(1)+6 (2)+5

(3)+10 (4)+3

Ans. (1)

Sol. $Cl^- + K_2Cr_2O_7 + H_2SO_4 \rightarrow CrO_2Cl_2 \xrightarrow{\text{Basic medium}} \underset{\text{yellow solution}}{CrO_4^{2-}} + Cl^-$

$$\underset{\text{yellow solution}}{CrO_4^{2-}} \xrightarrow[\text{3. 10\% } H_2O_2]{\text{1. Acidification, 2. Amyl alcohol}} \underset{\text{blue compound}}{CrO_5}$$



64. The difference in energy between the actual structure and the lowest energy resonance structure for the given compound is

(1) electromeric energy

(2) resonance energy

(3) ionization energy

(4) hyperconjugation energy

Ans. (2)

Sol. The difference in energy between the actual structure and the lowest energy resonance structure for the given compound is known as resonance energy.

65. Given below are two statements :

Statement I : The electronegativity of group 14 elements from Si to Pb gradually decreases.

Statement II : Group 14 contains non-metallic, metallic, as well as metalloid elements.

In the light of the above statements, choose the most appropriate from the options given below :

(1) Statement I is false but Statement II is true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are true

(4) Both Statement I and Statement II are false

Ans. (1)

Sol.

Gr-14	EN
C	2.5
Si	1.8
Ge	1.8
Sn	1.8
Pb	1.9

The electronegativity values for elements from Si to Pb are almost same. So Statement I is false.

66. The correct set of four quantum numbers for the valence electron of rubidium atom (Z = 37) is:

(1) $5, 0, 0, +\frac{1}{2}$ (2) $5, 0, 1, +\frac{1}{2}$

(3) $5, 1, 0, +\frac{1}{2}$ (4) $5, 1, 1, +\frac{1}{2}$

Ans. (1)

Sol. Rb = $[Kr]5s^1$

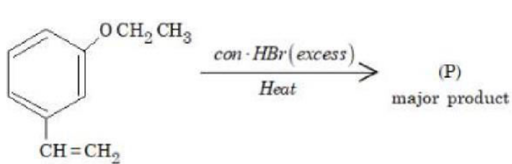
n = 5

l = 0

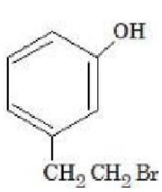
m = 0

s = +½ or –½

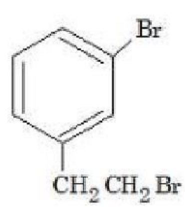
67. The major product(P) in the following reaction is



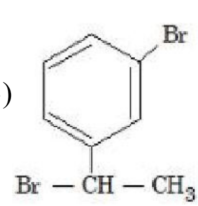
(1)



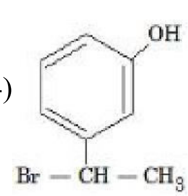
(2)



(3)

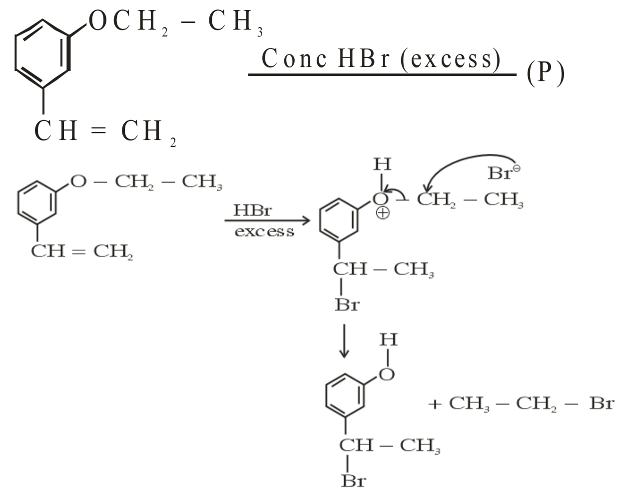


(4)



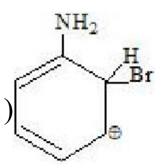
Ans. (4)

Sol.

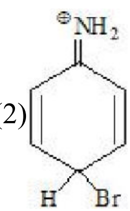


68. The arenium ion which is not involved in the bromination of Aniline is .

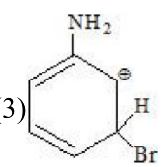
(1)



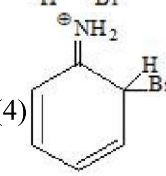
(2)



(3)

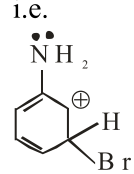


(4)



Ans. (3)

Sol. Since $-\ddot{N}H_2$ group is o/p directing hence arenium ion will not be formed by attack at meta position i.e.



Hence Answer is (3)

69. Appearance of blood red colour, on treatment of the sodium fusion extract of an organic compound with $FeSO_4$ in presence of concentrated H_2SO_4 indicates the presence of element/s

(1) Br (2) N

(3) N and S (4) S

Ans. (3)

Sol. $Fe^{2+} \xrightarrow[\text{Conc.}H_2SO_4]{H^+} Fe^{+3}$

$Fe^{+3} \xrightarrow{^{-}SCN} Fe(SCN)_3$ (blood red colour)

Appearance of blood red colour indicates presence of both nitrogen and sulphur.

70. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R :

Assertion A : Aryl halides cannot be prepared by replacement of hydroxyl group of phenol by halogen atom.

Reason R : Phenols react with halogen acids violently.

In the light of the above statements, choose the most appropriate from the options given below:

(1) Both A and R are true but R is NOT the correct explanation of A

(2) A is false but R is true

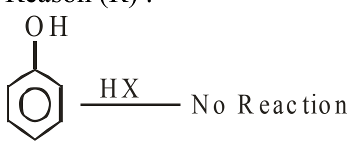
(3) A is true but R is false

(4) Both A and R are true and R is the correct explanation of A

Ans. (3)

Sol. Assertion (A): Given statement is correct because in phenol hydroxyl group cannot be replaced by halogen atom.

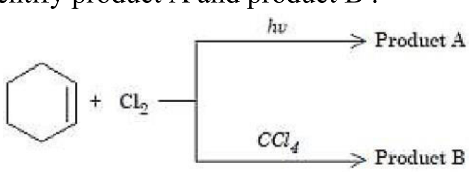
Reason (R) :

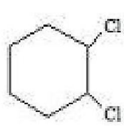
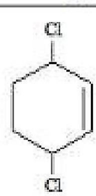


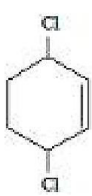
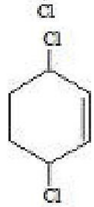
Given reason is false

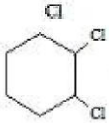
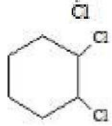
Hence Assertion (A) is correct but Reason (R) is false

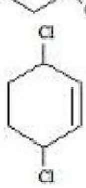
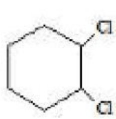
71. Identify product A and product B :



(1) A :  B : 

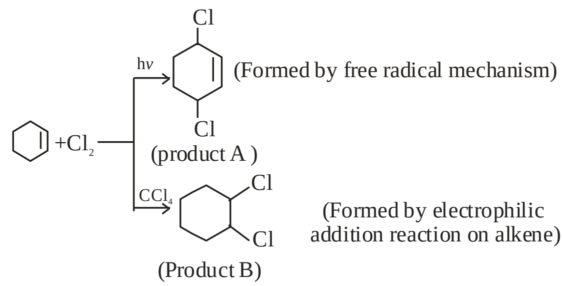
(2) A :  B : 

(3) A :  B : 

(4) A :  B : 

Ans. (4)

Sol.



Hence correct Ans. (4)

72. Identify the incorrect pair from the following :

(1) Fluorspar- BF_3

(2) Cryolite-Na_3AlF_6

(3) Fluoroapatite-$3Ca_3(PO_4)_2.CaF_2$

(4) Carnallite-$KCl.MgCl_2.6H_2O$

Ans. (1)

Sol. (1) Fluorspar is CaF_2

73. The interaction between π bond and lone pair of electrons present on an adjacent atom is responsible for

(1) Hyperconjugation

(2) Inductive effect

(3) Electromeric effect

(4) Resonance effect

Ans. (4)

Sol. It is a type of conjugation responsible for resonance.

74. $KMnO_4$ decomposes on heating at 513K to form O_2 along with

(1) MnO_2 & K_2O_2

(2) K_2MnO_4 & Mn

(3) Mn & KO_2

(4) K_2MnO_4 & MnO_2

Ans. (4)

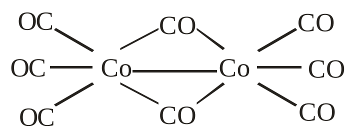
Sol. $KMnO_4 \xrightarrow{\Delta} K_2MnO_4 + MnO_2 + O_2$

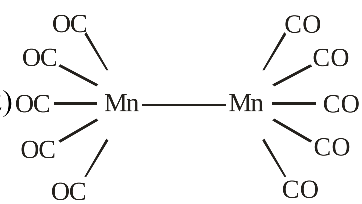
75. In which one of the following metal carbonyls, CO forms a bridge between metal atoms?

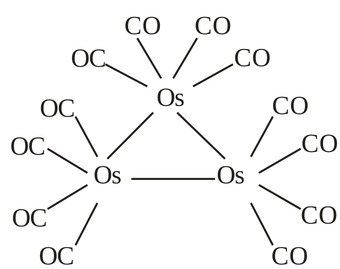
(1) $[Co_2(CO)_8]$ (2) $[Mn_2(CO)_{10}]$

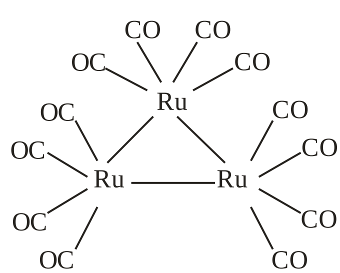
(3) $[Os_3(CO)_{12}]$ (4) $[Ru_3(CO)_{12}]$

Ans. (1)

Sol. (1) 

(2) 

(3) 

(4) 

76. Type of amino acids obtained by hydrolysis of proteins is :

(1) β (2) α

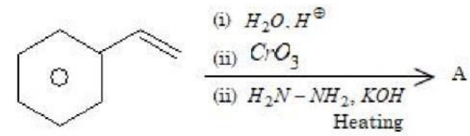
(3) δ (4) γ

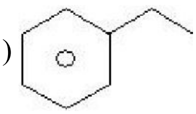
Ans. (2)

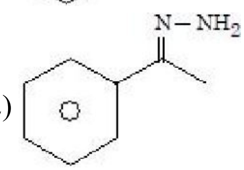
Sol. Proteins are natural polymers composed of α-amino acids which are connected by peptide linkages.

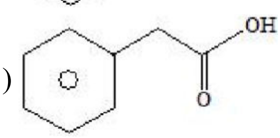
Hence proteins upon acidic hydrolysis produce α-amino acids.

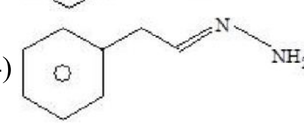
77. The final product A formed in the following multistep reaction sequence is



(1) 

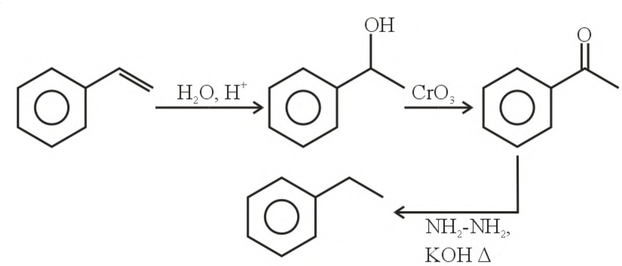
(2) 

(3) 

(4) 

Ans. (1)

Sol.



78. Which of the following is **not** correct?

(1) ΔG is negative for a spontaneous reaction

(2) ΔG is positive for a spontaneous reaction

(3) ΔG is zero for a reversible reaction

(4) ΔG is positive for a non-spontaneous reaction

Ans. (2)

Sol. $(\Delta G)_{P,T} = (+)$ve for non-spontaneous process

79. Chlorine undergoes disproportionation in alkaline medium as shown below :

a $Cl_2(g)$ + b $OH^-(aq)$ $\rightarrow$ c $ClO^-(aq)$ + d $Cl^-(aq)$ + e $H_2O(l)$

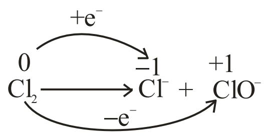
The values of a, b, c and d in a balanced redox reaction are respectively :

(1) 1, 2, 1 and 1 (2) 2, 2, 1 and 3

(3) 3, 4, 4 and 2 (4) 2, 4, 1 and 3

Ans. (1)

Sol.



$\Rightarrow Cl_2 + 2\bar{O}H \longrightarrow Cl^- + ClO^- + H_2O$

80. In alkaline medium. MnO_4^- oxidises I^- to

(1) IO_4^- (2) IO^-

(3) I_2 (4) IO_3^-

Ans. (4)

Sol. $2MnO_4^- + H_2O + I^- \xrightarrow{\text{alkaline medium}} 2MnO_2 + 2OH^- + IO_3^-$

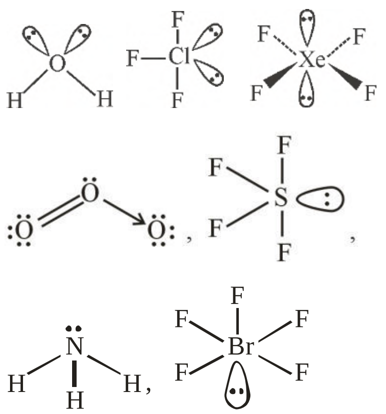
SECTION-B

81. Number of compounds with one lone pair of electrons on central atom amongst following is _

O_3, H_2O, SF_4, ClF_3, NH_3, BrF_5, XeF_4

Ans. (4)

Sol.



82. The mass of zinc produced by the electrolysis of zinc sulphate solution with a steady current of 0.015 A for 15 minutes is____ × 10^{-4} g.

(Atomic mass of zinc = 65.4 amu)

Ans. (45.75) or (46)

Sol. $Zn^{+2} + 2e^- \longrightarrow Zn$

$W = Z \times i \times t$

$= \dfrac{65.4}{2 \times 96500} \times 0.015 \times 15 \times 60$

$= 45.75 \times 10^{-4}$ gm

83. For a reaction taking place in three steps at same temperature, overall rate constant $K = \dfrac{K_1K_2}{K_3}$. If Ea_1, Ea_2 and Ea_3 are 40, 50 and 60 kJ/mol respectively, the overall Ea is____kJ/mol.

Ans. (30)

Sol. $K = \dfrac{K_1 \cdot K_2}{K_3} = \dfrac{A_1 \cdot A_2}{A_3} \cdot e^{-\frac{(E_{a_1} + E_{a_2} - E_{a_3})}{RT}}$

$A \cdot e^{-E_a/RT} = \dfrac{A_1A_2}{A_3} \cdot e^{-\frac{(E_{a_1} + E_{a_2} - E_{a_3})}{RT}}$

$E_a = E_{a_1} + E_{a_2} - E_{a_3} = 40 + 50 - 60 = 30$ kJ / mole.

84. For the reaction $N_2O_4(g) \rightleftharpoons 2NO_2(g)$, K_p= 0.492 atm at 300K. K_c for the reaction at same temperature is___ × 10^{-2}.

(Given : R = 0.082 L atm mol^{-1} K^{-1})

Ans. (2)

Sol. $K_P = K_C \cdot (RT)^{\Delta n_g}$

$\Delta n_g = 1$

$\Rightarrow K_c = \dfrac{K_P}{RT} = \dfrac{0.492}{0.082 \times 300} = 2 \times 10^{-2}$

85. A solution of H_2SO_4 is 31.4% H_2SO_4 by mass and has a density of 1.25g/mL.The molarity of the H_2SO_4 solution is____M (nearest integer)

[Given molar mass of H_2SO_4 = 98g mol^{-1}]

Ans. (4)

Sol. $M = \dfrac{n_{solute}}{V} \times 1000$

$= \dfrac{\left(\dfrac{31.4}{98}\right)}{\left(\dfrac{100}{1.25}\right)} \times 1000$

$= 4.005 \approx 4$

86. The osmotic pressure of a dilute solution is 7 × 10^5 Pa at 273K. Osmotic pressure of the same solution at 283K is____× 10^4 Nm^{-2}.

Ans. (72.56) or (73)

Sol. $\pi = CRT$

$\Rightarrow \dfrac{\pi_1}{\pi_2} = \dfrac{T_1}{T_2}$

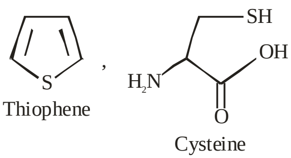
$\Rightarrow \pi_2 = \dfrac{\pi_1 T_2}{T_1} = \dfrac{7 \times 10^5 \times 283}{273}$

$= 72.56 \times 10^4$ Nm^{-2}

87. Number of compounds among the following which contain sulphur as heteroatom is____.

Furan,Thiophene, Pyridine, Pyrrole, Cysteine, Tyrosine

Ans. (2)

Sol. 

88. The number of species from the following which are paramagnetic and with bond order equal to one is____.

$$H_2, He_2^+, O_2^+, N_2^{2-}, O_2^{2-}, F_2, Ne_2^+, B_2$$

Ans. (1)

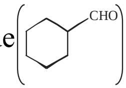
Sol.

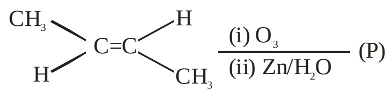
	Magnetic behaviour	Bond order
H_2	Diamagnetic	1
He_2^+	Paramagnetic	0.5
O_2^+	Paramagnetic	2.5
N_2^{2-}	Paramagnetic	2
O_2^{2-}	Diamagnetic	1
F_2	Diamagnetic	1
Ne_2^+	Paramagnetic	0.5
B_2	Paramagnetic	1

89. From the compounds given below, number of compounds which give positive Fehling's test is____.

Benzaldehyde, Acetaldehyde, Acetone, Acetophenone,Methanal, 4-nitrobenzaldehyde, cyclohexane carbaldehyde.

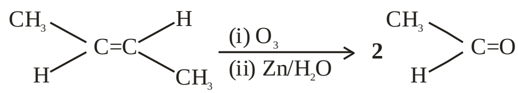
Ans. (3)

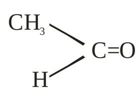
Sol. Acetaldehyde (CH_3CHO),Methanal(HCHO), and cyclohexane carbaldehyde .

90. 

Consider the given reaction. The total number of oxygen atoms present per molecule of the product (P) is____.

Ans. (1)

Sol. 

Hence total number of oxygen atom present per molecule  is 1.

FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

(Held On Monday 29th January, 2024) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS | **TEST PAPER WITH SOLUTION**

SECTION-A

1. Let $A = \begin{bmatrix} 2 & 1 & 2 \\ 6 & 2 & 11 \\ 3 & 3 & 2 \end{bmatrix}$ and $P = \begin{bmatrix} 1 & 2 & 0 \\ 5 & 0 & 2 \\ 7 & 1 & 5 \end{bmatrix}$. The sum of the prime factors of $\left|P^{-1}AP - 2I\right|$ is equal to

(1) 26 (2) 27 (3) 66 (4) 23

Ans. (1)

Sol. $\left|P^{-1}AP - 2I\right| = \left|P^{-1}AP - 2P^{-1}P\right|$

$= \left|P^{-1}(A-2I)P\right|$

$= \left|P^{-1}\right|\left|A-2I\right|\left|P\right|$

$= |A-2I|$

$= \begin{vmatrix} 0 & 1 & 2 \\ 6 & 0 & 11 \\ 3 & 3 & 0 \end{vmatrix} = 69$

So, Prime factor of 69 is 3 & 23

So, sum = 26

2. Number of ways of arranging 8 identical books into 4 identical shelves where any number of shelves may remain empty is equal to

(1) 18 (2) 16 (3) 12 (4) 15

Ans. (4)

Sol. 3 Shelf empty : $(8, 0, 0, 0) \rightarrow 1\text{way}$

2 shelf empty : $(7,1,0,0)$, $(6,2,0,0)$, $(5,3,0,0)$, $(4,4,0,0)$ $\rightarrow 4\text{ways}$

1 shelf empty : $(6,1,1,0)$, $(3,3,2,0)$, $(5,2,1,0)$, $(4,2,2,0)$, $(4,3,1,0)$ $\rightarrow 5\text{ways}$

0 Shelf empty : $(1,2,3,2)$, $(5,1,1,1)$, $(2,2,2,2)$, $(3,3,1,1)$, $(4,2,1,1)$ $\rightarrow 5\text{ways}$

Total = 15 ways

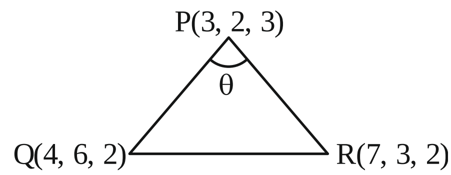
3. Let P(3, 2, 3), Q (4, 6, 2) and R (7, 3, 2) be the vertices of ΔPQR. Then, the angle $\angle QPR$ is

(1) $\frac{\pi}{6}$ (2) $\cos^{-1}\left(\frac{7}{18}\right)$

(3) $\cos^{-1}\left(\frac{1}{18}\right)$ (4) $\frac{\pi}{3}$

Ans. (4)

Sol.



Direction ratio of PR = (4, 1, -1)

Direction ratio of PQ = (1, 4, -1)

Now, $\cos\theta = \left|\frac{4+4+1}{\sqrt{18}\cdot\sqrt{18}}\right|$

$\theta = \frac{\pi}{3}$

4. If the mean and variance of five observations are $\frac{24}{5}$ and $\frac{194}{25}$ respectively and the mean of first four observations is $\frac{7}{2}$, then the variance of the first four observations in equal to

(1) $\frac{4}{5}$ (2) $\frac{77}{12}$ (3) $\frac{5}{4}$ (4) $\frac{105}{4}$

Ans. (3)

Sol. $\bar{X} = \frac{24}{5}; \sigma^2 = \frac{194}{25}$

Let first four observation be x_1, x_2, x_3, x_4

Here, $\frac{x_1+x_2+x_3+x_4+x_5}{5} = \frac{24}{5}$ ......(1)

Also, $\frac{x_1+x_2+x_3+x_4}{4} = \frac{7}{2}$

$\Rightarrow \boxed{x_1+x_2+x_3+x_4 = 14}$

Now from eqn -1

$x_5 = 10$

Now, $\sigma^2 = \frac{194}{25}$

$\frac{x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2}{5} - \frac{576}{25} = \frac{194}{25}$

$\Rightarrow x_1^2 + x_2^2 + x_3^2 + x_4^2 = 54$

Now, variance of first 4 observations

$$\text{Var} = \frac{\sum_{i=1}^{4} x_i^2}{4} - \left(\frac{\sum_{i=1}^{4} x_i}{4}\right)^2$$

$$= \frac{54}{4} - \frac{49}{4} = \frac{5}{4}$$

5. The function $f(x) = 2x + 3(x)^{\frac{2}{3}}, x \in \mathbb{R}$, has

(1) exactly one point of local minima and no point of local maxima

(2) exactly one point of local maxima and no point of local minima

(3) exactly one point of local maxima and exactly one point of local minima

(4) exactly two points of local maxima and exactly one point of local minima

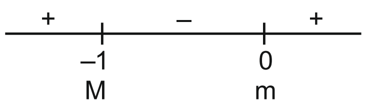
Ans. (3)

Sol. $f(x) = 2x + 3(x)^{\frac{2}{3}}$

$f'(x) = 2 + 2x^{\frac{-1}{3}}$

$= 2\left(1 + \frac{1}{x^{\frac{1}{3}}}\right)$

$= 2\left(\frac{x^{\frac{1}{3}} + 1}{x^{\frac{1}{3}}}\right)$



So, maxima (M) at x = -1 & minima(m) at x = 0

6. Let r and θ respectively be the modulus and amplitude of the complex number $z = 2 - i\left(2\tan\frac{5\pi}{8}\right)$, then (r, θ) is equal to

(1) $\left(2\sec\frac{3\pi}{8}, \frac{3\pi}{8}\right)$

(2) $\left(2\sec\frac{3\pi}{8}, \frac{5\pi}{8}\right)$

(3) $\left(2\sec\frac{5\pi}{8}, \frac{3\pi}{8}\right)$

(4) $\left(2\sec\frac{11\pi}{8}, \frac{11\pi}{8}\right)$

Ans. (1)

Sol. $z = 2 - i\left(2\tan\frac{5\pi}{8}\right) = x + iy$ (let)

$r = \sqrt{x^2 + y^2}$ & $\theta = \tan^{-1}\frac{y}{x}$

$r = \sqrt{(2)^2 + \left(2\tan\frac{5\pi}{8}\right)^2}$

$= \left|2\sec\frac{5\pi}{8}\right| = \left|2\sec\left(\pi - \frac{3\pi}{8}\right)\right|$

$= 2\sec\frac{3\pi}{8}$

& $\theta = \tan^{-1}\left(\frac{-2\tan\frac{5\pi}{8}}{2}\right)$

$= \tan^{-1}\left(\tan\left(\pi - \frac{5\pi}{8}\right)\right)$

$= \frac{3\pi}{8}$

7. The sum of the solutions $x \in \mathbb{R}$ of the equation $\frac{3\cos 2x + \cos^3 2x}{\cos^6 x - \sin^6 x} = x^3 - x^2 + 6$ is

(1) 0 (2) 1

(3) –1 (4) 3

Ans. **(3)**

Sol. $\dfrac{3\cos 2x + \cos^3 2x}{\cos^6 x - \sin^6 x} = x^3 - x^2 + 6$

$\Rightarrow \dfrac{\cos 2x\,(3 + \cos^2 2x)}{\cos 2x\,(1 - \sin^2 x \cos^2 x)} = x^3 - x^2 + 6$

$\Rightarrow \dfrac{4(3 + \cos^2 2x)}{(4 - \sin^2 2x)} = x^3 - x^2 + 6$

$\Rightarrow \dfrac{4(3 + \cos^2 2x)}{(3 + \cos^2 2x)} = x^3 - x^2 + 6$

$x^3 - x^2 + 2 = 0 \Rightarrow (x + 1)(x^2 - 2x + 2) = 0$

so, sum of real solutions = –1

8. Let $\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = 12\vec{a} + 4\vec{b}$ and $\overrightarrow{OC} = \vec{b}$, where O is the origin. If S is the parallelogram with adjacent sides OA and OC, then

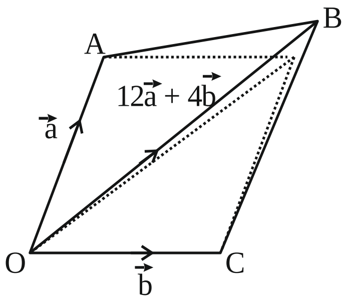
$\dfrac{\text{area of the quadrilateral OABC}}{\text{area of S}}$ is equal to ___

(1) 6 (2) 10

(3) 7 (4) 8

Ans. **(4)**

Sol.



Area of parallelogram, $S = |\vec{a} \times \vec{b}|$

Area of quadrilateral =Area(ΔOAB)+Area (ΔOBC)

$= \dfrac{1}{2}\left\{\left|\vec{a} \times (12\vec{a} + 4\vec{b})\right| + \left|\vec{b} \times (12\vec{a} + 4\vec{b})\right|\right\}$

$= 8\left|(\vec{a} \times \vec{b})\right|$

Ratio $= \dfrac{8\left|(\vec{a} \times \vec{b})\right|}{\left|(\vec{a} \times \vec{b})\right|} = 8$

9. If $\log_e a$, $\log_e b$, $\log_e c$ are in an A.P. and $\log_e a - \log_e 2b$, $\log_e 2b - \log_e 3c$, $\log_e 3c - \log_e a$ are also in an A.P, then a : b : c is equal to

(1) 9 : 6 : 4 (2) 16 : 4 : 1

(3) 25 : 10 : 4 (4) 6 : 3 : 2

Ans. **(1)**

Sol. $\log_e a, \log_e b, \log_e c$ are in A.P.

$\therefore b^2 = ac$ …..(i)

Also

$\log_e\left(\dfrac{a}{2b}\right), \log_e\left(\dfrac{2b}{3c}\right), \log_e\left(\dfrac{3c}{a}\right)$ are in A.P.

$\left(\dfrac{2b}{3c}\right)^2 = \dfrac{a}{2b} \times \dfrac{3c}{a}$

$\dfrac{b}{c} = \dfrac{3}{2}$

Putting in eq. (i) $b^2 = a \times \dfrac{2b}{3}$

$\dfrac{a}{b} = \dfrac{3}{2}$

$a : b : c = 9 : 6 : 4$

10. If

$\int \dfrac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}{\sqrt{\sin^3 x \cos^3 x \sin(x - \theta)}} dx = A\sqrt{\cos\theta \tan x - \sin\theta} + B\sqrt{\cos\theta - \sin\theta \cot x} + C,$

where C is the integration constant, then AB is equal to

(1) $4\operatorname{cosec}(2\theta)$ (2) $4\sec\theta$

(3) $2\sec\theta$ (4) $8\operatorname{cosec}(2\theta)$

Ans. **(4)**

Sol. $\int \dfrac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}{\sqrt{\sin^3 x \cos^3 x \sin(x - \theta)}} dx$

$I = \int \dfrac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}{\sqrt{\sin^3 x \cos^3 x\,(\sin x \cos\theta - \cos x \sin\theta)}} dx$

$= \int \dfrac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x \cos^2 x \sqrt{\tan x \cos\theta - \sin\theta}} dx + \int \dfrac{\cos^{\frac{3}{2}} x}{\sin^2 x \cos^{\frac{3}{2}} x \sqrt{\cos\theta - \cot x \sin\theta}} dx =$

$\int \dfrac{\sec^2 x}{\sqrt{\tan x \cos\theta - \sin\theta}} dx + \int \dfrac{\operatorname{cosec}^2 x}{\sqrt{\cos\theta - \cot x \sin\theta}} dx$

$I = I_1 + I_2$ …… {Let}

For I_1, let $\tan x \cos\theta - \sin\theta = t^2$

$\sec^2 x\, dx = \dfrac{2t\, dt}{\cos\theta}$

For I_2, let $\cos\theta - \cot x \sin\theta = z^2$

$\operatorname{cosec}^2 x\, dx = \dfrac{2z\, dz}{\sin\theta}$

$I = I_1 + I_2$

$= \int \frac{2t\,dt}{\cos\theta\ t} + \int \frac{2z\,dz}{\sin\theta\ z}$

$= \frac{2t}{\cos\theta} + \frac{2z}{\sin\theta}$

$= 2\sec\theta\sqrt{\tan x\cos\theta - \sin\theta} + 2\operatorname{cosec}\theta\sqrt{\cos\theta - \cot x\sin\theta}$

Comparing

$AB = 8\operatorname{cosec}2\theta$

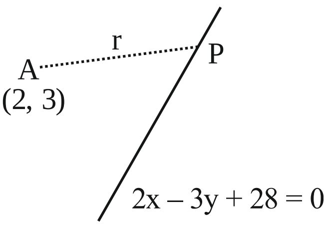
11. The distance of the point (2, 3) from the line 2x – 3y + 28 = 0, measured parallel to the line $\sqrt{3}x - y + 1 = 0$, is equal to

(1) $4\sqrt{2}$ (2) $6\sqrt{3}$

(3) $3+4\sqrt{2}$ (4) $4+6\sqrt{3}$

Ans. **(4)**

Sol.



Writing P in terms of parametric co-ordinates 2 + r cos θ, 3 + r sin θ as tan θ = $\sqrt{3}$

$P(2+\frac{r}{2}, 3+\frac{\sqrt{3}r}{2})$

P must satisfy 2x – 3y + 28 = 0

So, $2(2+\frac{r}{2}) - 3(3+\frac{\sqrt{3}r}{2}) + 28 = 0$

We find r = $4+6\sqrt{3}$

12. If $\sin\left(\frac{y}{x}\right) = \log_e|x| + \frac{\alpha}{2}$ is the solution of the differential equation $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$ and $y(1) = \frac{\pi}{3}$, then α^2 is equal to

(1) 3 (2) 12

(3) 4 (4) 9

Ans. **(1)**

Sol. Differential equation :–

$x\cos\frac{y}{x}\frac{dy}{dx} = y\cos\frac{y}{x} + x$

$\cos\frac{y}{x}\left[x\frac{dy}{dx} - y\right] = x$

Divide both sides by x^2

$\cos\frac{y}{x}\left(\frac{x\frac{dy}{dx} - y}{x^2}\right) = \frac{1}{x}$

Let $\frac{y}{x} = t$

$\cos\ t\left(\frac{dt}{dx}\right) = \frac{1}{x}$

$\cos t\ dt = \frac{1}{x}dx$

Integrating both sides

$\sin t = \ln|x| + c$

$\sin\frac{y}{x} = \ln|x| + c$

Using $y(1) = \frac{\pi}{3}$, we get $c = \frac{\sqrt{3}}{2}$

So, $\alpha = \sqrt{3} \Rightarrow \alpha^2 = 3$

13. If each term of a geometric progression $a_1, a_2, a_3, \ldots$ with $a_1 = \frac{1}{8}$ and $a_2 \neq a_1$, is the arithmetic mean of the next two terms and $S_n = a_1 + a_2 + \ldots + a_n$, then $S_{20} - S_{18}$ is equal to

(1) 2^{15} (2) -2^{18}

(3) 2^{18} (4) -2^{15}

Ans. **(4)**

Sol. Let r'th term of the GP be ar^{n-1}. Given,

$2a_r = a_{r+1} + a_{r+2}$

$2ar^{n-1} = ar^n + ar^{n+1}$

$\frac{2}{r} = 1 + r$

$r^2 + r - 2 = 0$

Hence, we get, $r = -2$ (as $r \neq 1$)

So, $S_{20} - S_{18}$ = (Sum upto 20 terms) – (Sum upto 18 terms) = $T_{19} + T_{20}$

$T_{19} + T_{20} = ar^{18}(1 + r)$

Putting the values $a = \frac{1}{8}$ and $r = -2$;

we get $T_{19} + T_{20} = -2^{15}$

14. Let A be the point of intersection of the lines $3x + 2y = 14$, $5x - y = 6$ and B be the point of intersection of the lines $4x + 3y = 8$, $6x + y = 5$. The distance of the point P(5, –2) from the line AB is

(1) $\frac{13}{2}$ (2) 8 (3) $\frac{5}{2}$ (4) 6

Ans. (4)

Sol. Solving lines L_1 ($3x + 2y = 14$) and L_2 ($5x - y = 6$) to get A(2, 4) and solving lines L_3 ($4x + 3y = 8$) and L_4 ($6x + y = 5$) to get $B\left(\frac{1}{2}, 2\right)$.

Finding Eqn. of AB : $4x - 3y + 4 = 0$

Calculate distance PM

$$\Rightarrow \left|\frac{4(5) - 3(-2) + 4}{5}\right| = 6$$

15. Let $x = \frac{m}{n}$ (m, n are co-prime natural numbers) be a solution of the equation $\cos\left(2\sin^{-1}x\right) = \frac{1}{9}$ and let $\alpha, \beta (\alpha > \beta)$ be the roots of the equation $mx^2 - nx - m + n = 0$. Then the point (α, β) lies on the line

(1) $3x + 2y = 2$ (2) $5x - 8y = -9$

(3) $3x - 2y = -2$ (4) $5x + 8y = 9$

Ans. (4)

Sol. Assume $\sin^{-1}x = \theta$

$$\cos(2\theta) = \frac{1}{9}$$

$$\sin\theta = \pm\frac{2}{3}$$

as m and n are co-prime natural numbers,

$$x = \frac{2}{3}$$

i.e. m = 2, n = 3

So, the quadratic equation becomes $2x^2 - 3x + 1 = 0$ whose roots are $\alpha = 1, \beta = \frac{1}{2}$

$\left(1, \frac{1}{2}\right)$ lies on $5x + 8y = 9$

16. The function $f(x) = \frac{x}{x^2 - 6x - 16}, x \in \mathbb{R} - \{-2, 8\}$

(1) decreases in (–2, 8) and increases in $(-\infty, -2) \cup (8, \infty)$

(2) decreases in $(-\infty, -2) \cup (-2, 8) \cup (8, \infty)$

(3) decreases in $(-\infty, -2)$ and increases in $(8, \infty)$

(4) increases in $(-\infty, -2) \cup (-2, 8) \cup (8, \infty)$

Ans. (2)

Sol. $f(x) = \frac{x}{x^2 - 6x - 16}$

Now,

$$f'(x) = \frac{-(x^2 + 16)}{(x^2 - 6x - 16)^2}$$

$$f'(x) < 0$$

Thus f(x) is decreasing in

$(-\infty, -2) \cup (-2, 8) \cup (8, \infty)$

17. Let $y = \log_e\left(\frac{1 - x^2}{1 + x^2}\right)$, $-1 < x < 1$. Then at $x = \frac{1}{2}$, the value of $225(y' - y'')$ is equal to

(1) 732 (2) 746

(3) 742 (4) 736

Ans. (4)

Sol. $y = \log_e\left(\frac{1 - x^2}{1 + x^2}\right)$

$$\frac{dy}{dx} = y' = \frac{-4x}{1 - x^4}$$

Again,

$$\frac{d^2y}{dx^2} = y'' = \frac{-4(1 + 3x^4)}{(1 - x^4)^2}$$

Again

$$y' - y'' = \frac{-4x}{1 - x^4} + \frac{4(1 + 3x^4)}{(1 - x^4)^2}$$

at $x = \frac{1}{2}$,

$$y'-y''=\frac{736}{225}$$

Thus $225\,(y'-y'') = 225\times\frac{736}{225} = 736$

18. If R is the smallest equivalence relation on the set {1, 2, 3, 4} such that {(1,2), (1,3)} $\subset$ R, then the number of elements in R is _______

(1) 10 (2) 12

(3) 8 (4) 15

Ans. **(1)**

Sol. Given set {1, 2, 3, 4}

Minimum order pairs are

(1, 1), (2, 2), (3, 3), (4, 4), (3, 1),(2, 1), (2, 3),(3,2), (1, 3), (1, 2)

Thus no. of elements = 10

19. An integer is chosen at random from the integers 1, 2, 3, …, 50. The probability that the chosen integer is a multiple of atleast one of 4, 6 and 7 is

(1) $\frac{8}{25}$ (2) $\frac{21}{50}$

(3) $\frac{9}{50}$ (4) $\frac{14}{25}$

Ans. **(2)**

Sol. Given set = {1, 2, 3, …….. 50}

P(A) = Probability that number is multiple of 4

P(B) = Probability that number is multiple of 6

P(C) = Probability that number is multiple of 7

Now,

$$P(A)=\frac{12}{50},\ P(B)=\frac{8}{50},\ P(C)=\frac{7}{50}$$

again

$$P(A\cap B)=\frac{4}{50},P(B\cap C)=\frac{1}{50},P(A\cap C)=\frac{1}{50}$$

$$P(A\cap B\cap C)=0$$

Thus

$$P(A\cup B\cup C)=\frac{12}{50}+\frac{8}{50}+\frac{7}{50}-\frac{4}{50}-\frac{1}{50}-\frac{1}{50}+0$$

$$=\frac{21}{50}$$

20. Let a unit vector $\hat{u}=x\hat{i}+y\hat{j}+z\hat{k}$ make angles $\frac{\pi}{2},\frac{\pi}{3}$ and $\frac{2\pi}{3}$ with the vectors $\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{k}$, $\frac{1}{\sqrt{2}}\hat{j}+\frac{1}{\sqrt{2}}\hat{k}$ and $\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{j}$ respectively. If $\vec{v}=\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{j}+\frac{1}{\sqrt{2}}\hat{k}$, then $|\hat{u}-\vec{v}|^2$ is equal to

(1) $\frac{11}{2}$ (2) $\frac{5}{2}$

(3) 9 (4) 7

Ans. **(2)**

Sol. Unit vector $\hat{u}=x\hat{i}+y\hat{j}+z\hat{k}$

$$\vec{p}_1=\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{k},\vec{p}_2=\frac{1}{\sqrt{2}}\hat{j}+\frac{1}{\sqrt{2}}\hat{k}$$

$$\vec{p}_3=\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{j}$$

Now angle between $\hat{u}$ and $\vec{p}_1=\frac{\pi}{2}$

$$\hat{u}\cdot\vec{p}_1=0\Rightarrow\frac{x}{\sqrt{2}}+\frac{z}{\sqrt{2}}=0$$

$$\Rightarrow x+z=0\ \ ....(i)$$

Angle between $\hat{u}$ and $\vec{p}_2=\frac{\pi}{3}$

$$\hat{u}\cdot\vec{p}_2=|\hat{u}|\cdot|\vec{p}_2|\cos\frac{\pi}{3}$$

$$\Rightarrow\frac{y}{\sqrt{2}}+\frac{z}{\sqrt{2}}=\frac{1}{2}\Rightarrow y+z=\frac{1}{\sqrt{2}}\quad ....(ii)$$

Angle between $\hat{u}$ and $\vec{p}_3=\frac{2\pi}{3}$

$$\hat{u}\cdot\vec{p}_3=|\hat{u}|\cdot|\vec{p}_3|\cos\frac{2\pi}{3}$$

$$\Rightarrow\frac{x}{\sqrt{2}}+\frac{4}{\sqrt{2}}=\frac{-1}{2}\Rightarrow x+y=\frac{-1}{\sqrt{2}}\quad ....(iii)$$

from equation (i), (ii) and (iii) we get

$$x=\frac{-1}{\sqrt{2}}\quad y=0\quad z=\frac{1}{\sqrt{2}}$$

Thus $\hat{u}-\vec{v}=\frac{-1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{k}-\frac{1}{\sqrt{2}}\hat{i}-\frac{1}{\sqrt{2}}\hat{j}-\frac{1}{\sqrt{2}}\hat{k}$

$$\hat{u}-\vec{v}=\frac{-2}{\sqrt{2}}\hat{i}-\frac{1}{\sqrt{2}}\hat{j}$$

$$\therefore |\hat{u}-\vec{v}|^2=\left(\sqrt{\frac{4}{2}+\frac{1}{2}}\right)^2=\frac{5}{2}$$

SECTION-B

21. Let α, β be the roots of the equation $x^2-\sqrt{6}x+3=0$ such that Im (α) > Im (β). Let a, b be integers not divisible by 3 and n be a natural number such that $\frac{\alpha^{99}}{\beta}+\alpha^{98}=3^n(a+ib), i=\sqrt{-1}$. Then n + a + b is equal to _______.

Ans. **49**

Sol. $x^2-\sqrt{6}x+6=0 \begin{smallmatrix}\nearrow \alpha\\ \searrow \beta\end{smallmatrix}$

$$x=\frac{\sqrt{6}\pm i\sqrt{6}}{2}=\frac{\sqrt{6}}{2}(1\pm i)$$

$$\alpha=\sqrt{3}(e^{i\frac{\pi}{4}}),\ \beta=\sqrt{3}(e^{-i\frac{\pi}{4}})$$

$$\therefore \frac{\alpha^{99}}{\beta}+\alpha^{98}=\alpha^{98}\left(\frac{\alpha}{\beta}+1\right)$$

$$=\frac{\alpha^{98}(\alpha+\beta)}{\beta}=3^{49}\left(e^{i99\frac{\pi}{4}}\right)\times\sqrt{2}$$

$= 3^{49}$ (-1+i)

$= 3^n$ (a+ib)

∴ n = 49, a = -1, b = 1

∴ n + a + b = 49 – 1 + 1 = 49

22. Let for any three distinct consecutive terms a, b, c of an A.P, the lines ax + by + c = 0 be concurrent at the point P and Q (α,β) be a point such that the system of equations

x + y + z = 6,

2x + 5y + αz = β and

x + 2y + 3z = 4, has infinitely many solutions. Then $(PQ)^2$ is equal to ________.

Ans. **113**

Sol. ∵ a, b, c and in A.P

⇒ 2b = a + c ⇒ a – 2b + c = 0

∴ ax + by + c passes through fixed point (1, -2)

∴ P = (1, - 2)

For infinite solution,

$D = D_1 = D_2 = D_3 = 0$

$$D:\begin{vmatrix}1 & 1 & 1\\ 2 & 5 & \alpha\\ 1 & 2 & 3\end{vmatrix}=0$$

⇒ α = 8

$$D_1:\begin{vmatrix}6 & 1 & 1\\ \beta & 5 & \alpha\\ 4 & 2 & 3\end{vmatrix}=0 \Rightarrow \beta=6$$

∴ Q = (8,6)

∴ $PQ^2 = 113$

23. Let P(α,β) be a point on the parabola $y^2 = 4x$. If P also lies on the chord of the parabola $x^2 = 8y$ whose mid point is $\left(1,\frac{5}{4}\right)$. Then (α-28) (β-8) is equal to ___________.

Ans. **192**

Sol. Parabola is $x^2 = 8y$

Chord with mid point (x_1,y_1) is $T = S_1$

∴ $xx_1 – 4(y+y_1) = x_1^2-8y_1$

$$\therefore (x_1, y_1)=\left(1,\frac{5}{4}\right)$$

$$\Rightarrow x-4\left(y+\frac{5}{4}\right)=1-8\times\frac{5}{4}=-9$$

∴ x-4y + 4 = 0 ...... (i)

(α,β) lies on (i) & also on $y^2 = 4x$

∴ α - 4β + 4 = 0 ..... (ii)

& $\beta^2 = 4\alpha$ ..... (iii)

Solving (ii) & (iii)

$\beta^2 = 4(4\beta-4) \Rightarrow \beta^2-16\beta + 16 = 0$

$\therefore\ \beta = 8 \pm 4\sqrt{3}$ and $\alpha = 4\beta - 4 = 28 \pm 16\sqrt{3}$

$\therefore (\alpha, \beta) = (28+16\sqrt{3}, 8+4\sqrt{3})$ & $(28-16\sqrt{3}, 8-4\sqrt{3})$

$\therefore (\alpha-28)(\beta-8) = (\pm16\sqrt{3})(\pm4\sqrt{3})$

$= 192$

24. If $\int_{\pi/6}^{\pi/3} \sqrt{1-\sin 2x}\,dx = \alpha + \beta\sqrt{2} + \gamma\sqrt{3}$, where α, β and γ are rational numbers, then $3\alpha+4\beta-\gamma$ is equal to ________.

Ans. 6

Sol. $= \int_{\pi/6}^{\pi/3} \sqrt{1-\sin 2x}\,dx$

$= \int_{\pi/6}^{\pi/3} |\sin x - \cos x|\,dx$

$= \int_{\pi/6}^{\pi/4} (\cos x - \sin x)\,dx + \int_{\pi/4}^{\pi/3} (\sin x - \cos x)\,dx$

$= -1 + 2\sqrt{2} - \sqrt{3}$

$= \alpha + \beta\sqrt{2} + \gamma\sqrt{3}$

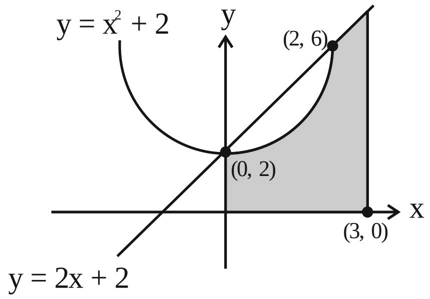
$\alpha = -1, \beta = 2, \gamma = -1$

$3\alpha + 4\beta - \gamma = 6$

25. Let the area of the region $\{(x, y): 0 \le x \le 3, 0 \le y \le \min\{x^2 + 2, 2x + 2\}\}$ be A. Then 12A is equal to _____.

Ans. 164

Sol.



$A = \int_0^2 (x^2+2)dx + \int_2^3 (2x+2)dx$

$A = \frac{41}{3}$

$12A = 41 \times 4 = 164$

26. Let O be the origin, and M and N be the points on the lines $\frac{x-5}{4} = \frac{y-4}{1} = \frac{z-5}{3}$ and $\frac{x+8}{12} = \frac{y+2}{5} = \frac{z+11}{9}$ respectively such that MN is the shortest distance between the given lines. Then $\overrightarrow{OM}\cdot\overrightarrow{ON}$ is equal to _____.

Ans. 9

Sol. $L_1 : \frac{x-5}{4} = \frac{y-4}{1} = \frac{z-5}{3} = \lambda$ drs $(4,1,3) = b_1$

$M(4\lambda+5, \lambda + 4, 3\lambda + 5)$

$L_2 : \frac{x+8}{12} = \frac{y+2}{5} = \frac{z+11}{9} = \mu$

$N(12\mu - 8, 5\mu - 2, 9\mu - 11)$

$\overrightarrow{MN} = (4\lambda - 12\mu + 13, \lambda - 5\mu + 6, 3\lambda - 9\mu + 16)$ ..(1)

Now

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 1 & 3 \\ 12 & 5 & 9 \end{vmatrix} = -6\hat{i} + 8\hat{k}$ …(2)

Equation (1) and (2)

$\therefore \frac{4\lambda - 12\mu + 13}{-6} = \frac{\lambda - 5\mu + 6}{0} = \frac{3\lambda - 9\mu + 16}{8}$

I and II

$\lambda - 5\mu + 6 = 0$ ….(3)

I and III

$\lambda - 3\mu + 4 = 0$ ….(4)

Solve (3) and (4) we get

$\lambda = -1, \mu = 1$

$\therefore$ M (1, 3, 2)

N (4, 3, –2)

$\therefore \overrightarrow{OM}\cdot\overrightarrow{ON} = 4 + 9 - 4 = 9$

27. Let $f(x)=\sqrt{\lim_{r\to x}\left\{\frac{2r^2\left[(f(r))^2-f(x)f(r)\right]}{r^2-x^2}-r^3e^{\frac{f(r)}{r}}\right\}}$ be differentiable in $(-\infty,0)\cup(0,\infty)$ and $f(1) = 1$. Then the value of ea, such that f(a) = 0, is equal to _____.

Ans. **2**

Sol. $f(1)=1,\ f(a)=0$

$$f^2(x)=\lim_{r\to x}\left(\frac{2r^2(f^2(r)-f(x)f(r))}{r^2-x^2}-r^3e^{\frac{f(r)}{r}}\right)$$

$$=\lim_{r\to x}\left(\frac{2r^2f(r)}{r+x}\frac{(f(r)-f(x))}{r-x}-r^3e^{\frac{f(r)}{r}}\right)$$

$$f^2(x)=\frac{2x^2f(x)}{2x}f'(x)-x^3e^{\frac{f(x)}{x}}$$

$$y^2=xy\frac{dy}{dx}-x^3e^{\frac{y}{x}}$$

$$\frac{y}{x}=\frac{dy}{dx}-\frac{x^2}{y}e^{\frac{y}{x}}$$

Put $\boxed{y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}}$

$$v=v+x\frac{dv}{dx}-\frac{x}{v}e^{v}$$

$$\frac{dv}{dx}=\frac{e^v}{v}\Rightarrow e^{-v}\,v\,dv=dx$$

Integrating both side

$e^v(x + c) + 1 + v = 0$

$f(1)=1\Rightarrow x=1, y=1$

$$\Rightarrow c=-1-\frac{2}{e}$$

$$e^v\left(-1-\frac{2}{e}+x\right)+1+v=0$$

$$e^{\frac{y}{x}}\left(-1-\frac{2}{e}+x\right)+1+\frac{y}{x}=0$$

$$x=a,\ y=0\Rightarrow a=\frac{2}{e}$$

$ae = 2$

28. Remainder when $64^{32^{32}}$ is divided by 9 is equal to ____.

Ans. **1**

Sol. Let $32^{32} = t$

$64^{32^{32}} = 64^t = 8^{2t} = (9-1)^{2t}$

$= 9k + 1$

Hence remainder = 1

29. Let the set $C=\{(x,y)\,|\,x^2-2^y=2023, x,y\in\mathbb{N}\}$. Then $\sum_{(x,y)\in C}(x+y)$ is equal to _______.

Ans. **46**

Sol. $x^2 - 2^y = 2023$

$\Rightarrow \boxed{x=45, y=1}$

$$\sum_{(x,y)\in C}(x+y)=46.$$

30. Let the slope of the line 45x + 5y + 3 = 0 be $27r_1+\frac{9r_2}{2}$ for some r_1, $r_2\in\mathbb{R}$. Then $\lim_{x\to3}\left(\int_3^x\frac{8t^2}{\frac{3r_2x}{2}-r_2x^2-r_1x^3-3x}dt\right)$ is equal to ____.

Ans. **12**

Sol. According to the question ,

$$27r_1 + \frac{9r_2}{2} = -9$$

$$\lim_{x\to 3} \frac{\int_3^x 8t^2 dt}{\frac{3r_2 x}{2} - r_2 x^2 - r_1 x^3 - 3x}$$

$$= \lim_{x\to 3} \frac{8x^2}{\frac{3r_2^2}{2} - 2r_2 x - 3r_1 x^2 - 3} \text{ (using LH' Rule)}$$

$$= \frac{72}{\frac{3r_2}{2} - 6r_2 - 27r_1 - 3}$$

$$= \frac{72}{-\frac{9r_2}{2} - 27r_1 - 3}$$

$$= \frac{72}{9-3} = 12$$

SECTION-A

31. Two sources of light emit with a power of 200 W. The ratio of number of photons of visible light emitted by each source having wavelengths 300 nm and 500 nm respectively, will be :

(1) 1 : 5 (2) 1 : 3

(3) 5 : 3 (4) 3 : 5

Ans. (4)

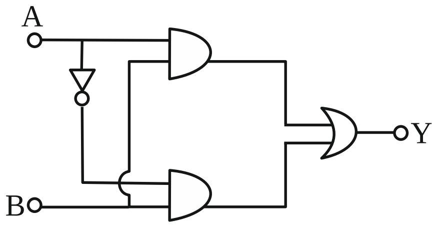
Sol. $n_1 \times \frac{hc}{\lambda_1} = 200$

$n_2 \times \frac{hc}{\lambda_2} = 200$

$\frac{n_1}{n_2} = \frac{\lambda_1}{\lambda_2} = \frac{300}{500}$

$\frac{n_1}{n_2} = \frac{3}{5}$

32. The truth table for this given circuit is :



(1)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

(2)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

(3)

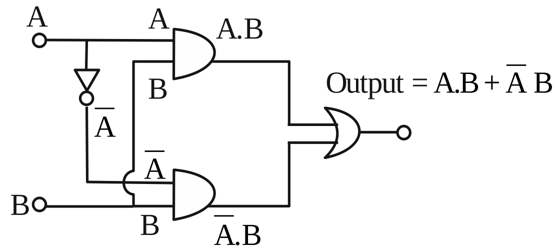
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

(4)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

Ans. (2)

Sol.



$Y = A.B + \overline{A}.B$

$= \left(A + \overline{A}\right).B$

$Y = 1.B$

$Y = B$

33. A physical quantity Q is found to depend on quantities a, b, c by the relation $Q = \frac{a^4b^3}{c^2}$. The percentage error in a, b and c are 3%, 4% and 5% respectively. Then, the percentage error in Q is :

(1) 66% (2) 43%

(3) 34% (4) 14%

Ans. (3)

Sol. $Q = \frac{a^4b^3}{c^2}$

$\frac{\Delta Q}{Q} = 4\frac{\Delta a}{a} + 3\frac{\Delta b}{b} + 2\frac{\Delta c}{c}$

$\frac{\Delta Q}{Q} \times 100 = 4\left(\frac{\Delta a}{a} \times 100\right) + 3\left(\frac{\Delta b}{b} \times 100\right) + 2\left(\frac{\Delta c}{c} \times 100\right)$

% error in Q = 4 × 3% + 3 × 4% + 2 × 5%

= 12% + 12% + 10%

= 34%

34. In an a.c. circuit, voltage and current are given by :

V = 100 sin (100 t) V and

$I = 100 \sin (100 t + \frac{\pi}{3})$ mA respectively.

The average power dissipated in one cycle is :

(1) 5 W (2) 10 W

(3) 2.5 W (4) 25 W

Ans. (3)

Sol. $P_{avg} = V_{rms}\, I_{rms}\, \cos(\Delta\phi)$

$$= \frac{100}{\sqrt{2}} \times \frac{100 \times 10^{-3}}{\sqrt{2}} \times \cos\left(\frac{\pi}{3}\right)$$

$$= \frac{10^4}{2} \times \frac{1}{2} \times 10^{-3}$$

$$= \frac{10}{4} = 2.5\,W$$

35. The temperature of a gas having 2.0×10^{25} molecules per cubic meter at 1.38 atm (Given, $k = 1.38 \times 10^{-23}\ JK^{-1}$) is :

(1) 500 K (2) 200 K

(3) 100 K (4) 300 K

Ans. **(1)**

Sol. $PV = nRT$

$$PV = \frac{N}{N_A} RT$$

N = Total no. of molecules

$$P = \frac{N}{V} kT$$

$1.38 \times 1.01 \times 10^5 = 2 \times 10^{25} \times 1.38 \times 10^{-23} \times T$

$1.01 \times 10^5 = 2 \times 10^2 \times T$

$$T = \frac{1.01 \times 10^3}{2} \approx 500\ K$$

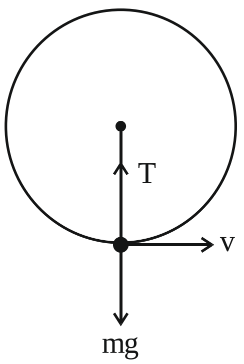
36. A stone of mass 900g is tied to a string and moved in a vertical circle of radius 1m making 10 rpm. The tension in the string, when the stone is at the lowest point is (if $\pi^2 = 9.8$ and $g = 9.8\ m/s^2$)

(1) 97 N (2) 9.8 N

(3) 8.82 N (4) 17.8 N

Ans. **(2)**

Sol. Given that



$$m = 900\ gm = \frac{900}{1000} kg = \frac{9}{10} kg$$

$r = 1m$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi(10)}{60} = \frac{\pi}{3} rad/sec$$

$T - mg = mr\omega^2$

$T = mg + mr\omega^2$

$$= \frac{9}{10} \times 9.8 + \frac{9}{10} \times 1\left(\frac{\pi}{3}\right)^2$$

$$= 8.82 + \frac{9}{10} \times \frac{\pi^2}{9}$$

$= 8.82 + 0.98$

$= 9.80\ N$

37. The bob of a pendulum was released from a horizontal position. The length of the pendulum is 10m. If it dissipates 10% of its initial energy against air resistance, the speed with which the bob arrives at the lowest point is : [Use, $g : 10\ ms^{-2}$]

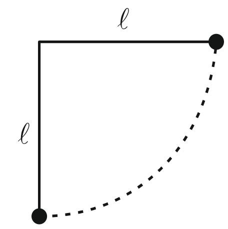
(1) $6\sqrt{5}\ ms^{-1}$ (2) $5\sqrt{6}\ ms^{-1}$

(3) $5\sqrt{5}\ ms^{-1}$ (4) $2\sqrt{5}\ ms^{-1}$

Ans. **(1)**

Sol. $\ell = 10$ m,

Initial energy = $mg\ell$



So, $\frac{9}{10} mg\ell = \frac{1}{2} mv^2$

$$\Rightarrow \frac{9}{10} \times 10 \times 10 = \frac{1}{2} v^2$$

$v^2 = 180$

$$v = \sqrt{180} = 6\sqrt{5}\, m/s$$

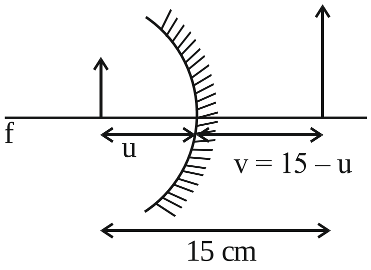
38. If the distance between object and its two times magnified virtual image produced by a curved mirror is 15 cm, the focal length of the mirror must be :

(1) 15 cm (2) –12 cm

(3) –10 cm (4) 10/3 cm

Ans. **(3)**

Sol.



$m = 2 = \frac{-v}{u}$

$2 = \frac{-(15-u)}{-u}$

$2u = 15 - u$

$3u = 15 \Rightarrow u = 5$ cm

$v = 15 - u = 15 - 5 = 10$ cm

$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

$= \frac{1}{10} + \frac{1}{(-5)} = \frac{1-2}{10} = \frac{-1}{10}$

$f = -10$ cm

39. Two particles X and Y having equal charges are being accelerated through the same potential difference. Thereafter they enter normally in a region of uniform magnetic field and describes circular paths of radii R_1 and R_2 respectively. The mass ratio of X and Y is :

(1) $\left(\frac{R_2}{R_1}\right)^2$ (2) $\left(\frac{R_1}{R_2}\right)^2$

(3) $\left(\frac{R_1}{R_2}\right)$ (4) $\left(\frac{R_2}{R_1}\right)$

Ans. **(2)**

Sol. $R = \frac{mv}{qB} = \frac{p}{qB} = \frac{\sqrt{2m(KE)}}{qB} = \frac{\sqrt{2mqV}}{qB}$

$R \propto \sqrt{m}$

$m \propto R^2$

$\frac{m_1}{m_2} = \left(\frac{R_1}{R_2}\right)^2$

40. In Young's double slit experiment, light from two identical sources are superimposing on a screen. The path difference between the two lights reaching at a point on the screen is $\frac{7\lambda}{4}$. The ratio of intensity of fringe at this point with respect to the maximum intensity of the fringe is :

(1) 1/2 (2) 3/4 (3) 1/3 (4) 1/4

Ans. **(1)**

Sol. $\Delta x = \frac{7\lambda}{4}$

$\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda} \times \frac{7\lambda}{4} = \frac{7\pi}{2}$

$I = I_{max}\cos^2\left(\frac{\phi}{2}\right)$

$\frac{I}{I_{max}} = \cos^2\left(\frac{\phi}{2}\right) = \cos^2\left(\frac{7\pi}{2\times 2}\right) = \cos^2\left(\frac{7\pi}{4}\right)$

$= \cos^2\left(2\pi - \frac{\pi}{4}\right)$

$= \cos^2\frac{\pi}{4}$

$= \frac{1}{2}$

41. A small liquid drop of radius R is divided into 27 identical liquid drops. If the surface tension is T, then the work done in the process will be :

(1) $8\pi R^2T$ (2) $3\pi R^2T$

(3) $\frac{1}{8}\pi R^2T$ (4) $4\pi R^2T$

Ans. **(1)**

Sol. Volume constant

$\frac{4}{3}\pi R^3 = 27 \times \frac{4}{3} \times \pi r^3$

$R^3 = 27r^3$

$R = 3r$

$r = \frac{R}{3}$

$r^2 = \frac{R^2}{9}$

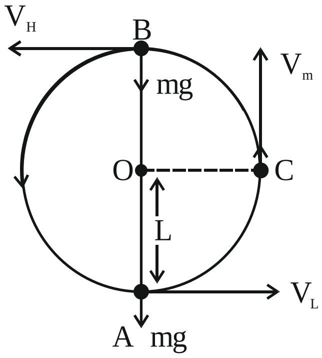
Work done $= T.\Delta A$

$= 27\,T(4\pi r^2) - T\,4\pi R^2$

$= 27T4\pi\frac{R^2}{9} - 4\pi R^2T$

$= 8\pi R^2T$

42. A bob of mass 'm' is suspended by a light string of length 'L'. It is imparted a minimum horizontal velocity at the lowest point A such that it just completes half circle reaching the top most position B. The ratio of kinetic energies $\frac{(K.E.)_A}{(K.E.)_B}$ is :



(1) 3 : 2 (2) 5 : 1

(3) 2 : 5 (4) 1 : 5

Ans. (2)

Sol. Apply energy conservation between A & B

$$\frac{1}{2}mV_L^2 = \frac{1}{2}mV_H^2 + mg(2L)$$

$\because V_L = \sqrt{5gL}$

So, $V_H = \sqrt{gL}$

$$\frac{(K.E)_A}{(K.E)_B} = \frac{\frac{1}{2}m\left(\sqrt{5gL}\right)^2}{\frac{1}{2}m\left(\sqrt{gL}\right)^2} = \frac{5}{1}$$

43. A wire of length L and radius r is clamped at one end. If its other end is pulled by a force F, its length increases by *l*. If the radius of the wire and the applied force both are reduced to half of their original values keeping original length constant, the increase in length will become.

(1) 3 times (2) 3/2 times

(3) 4 times (4) 2 times

Ans. (4)

Sol. $Y = \frac{\text{stress}}{\text{strain}}$

$$Y = \frac{\frac{F}{\pi r^2}}{\frac{\ell}{L}}$$

$$F = Y\pi r^2 \times \frac{\ell}{L} \quad \ldots.(i)$$

$$Y = \frac{\frac{F/2}{\pi r^2/4}}{\frac{\Delta\ell}{L}}$$

$$F = Y\frac{\Delta\ell}{L} \times 2 \times \frac{\pi r^2}{4}$$

From (i)

$$Y\pi r^2\frac{\ell}{L} = Y\frac{\Delta\ell}{L}\frac{\pi r^2}{2}$$

$\Delta\ell = 2\ell$

44. A planet takes 200 days to complete one revolution around the Sun. If the distance of the planet from Sun is reduced to one fourth of the original distance, how many days will it take to complete one revolution ?

(1) 25 (2) 50

(3) 100 (4) 20

Ans. (1)

Sol. $T^2 \propto r^3$

$$\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3}$$

$$\frac{(200)^2}{r^3} = \frac{T_2^2}{\left(\frac{r}{4}\right)^3}$$

$$\frac{200 \times 200}{4 \times 4 \times 4} = T_2^2$$

$$T_2 = \frac{200}{4 \times 2}$$

$T_2 = 25$ days

45. A plane electromagnetic wave of frequency 35 MHz travels in free space along the X-direction. At a particular point (in space and time) $\vec{E} = 9.6\hat{j}\,V/m$. The value of magnetic field at this point is :

(1) $3.2\times10^{-8}\hat{k}T$ (2) $3.2\times10^{-8}\hat{i}T$

(3) $9.6\hat{j}T$ (4) $9.6\times10^{-8}\hat{k}T$

Ans. **(1)**

Sol. $\frac{E}{B} = C$

$\frac{E}{B} = 3\times10^{8}$

$B = \frac{E}{3\times10^{8}} = \frac{9.6}{3\times10^{8}}$

$B = 3.2\times10^{-8}$ T

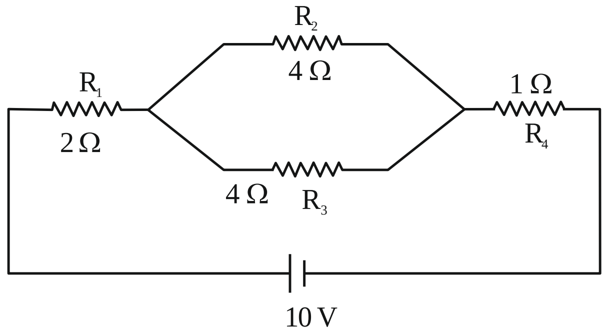
$\hat{B} = \hat{v}\times\hat{E}$

$= \hat{i}\times\hat{j} = \hat{k}$

So,

$\vec{B} = 3.2\times10^{-8}\hat{k}$ T

46. In the given circuit, the current in resistance R_3 is :

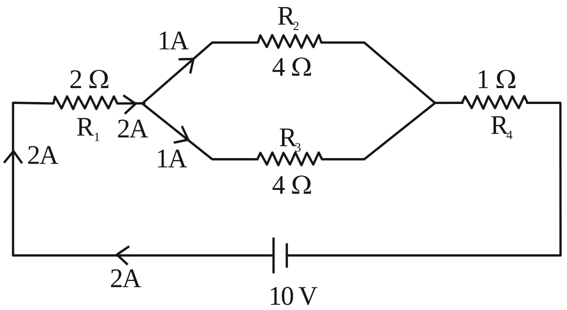


(1) 1 A (2) 1.5 A

(3) 2 A (4) 2.5 A

Ans. **(1)**

Sol.



$R_{eq} = 2\Omega + 2\,\Omega + 1\,\Omega = 5\,\Omega$

$i = \frac{V}{R_{eq}} = \frac{10}{5} = 2A$

Current in resistance $R_3 = 2\times\left(\frac{4}{4+4}\right)$

$= 2\times\frac{4}{8}$

$= 1A$

47. A particle is moving in a straight line. The variation of position 'x' as a function of time 't' is given as $x = (t^3 - 6t^2 + 20t + 15)$ m. The velocity of the body when its acceleration becomes zero is :

(1) 4 m/s (2) 8 m/s

(3) 10 m/s (4) 6 m/s

Ans. **(2)**

Sol. $x = t^3 - 6t^2 + 20t + 15$

$\frac{dx}{dt} = v = 3t^2 - 12t + 20$

$\frac{dv}{dt} = a = 6t - 12$

When a = 0

$6t - 12 = 0$; t = 2 sec

At t = 2 sec

$v = 3(2)^2 - 12(2) + 20$

v = 8 m/s

48. N moles of a polyatomic gas (f = 6) must be mixed with two moles of a monoatomic gas so that the mixture behaves as a diatomic gas. The value of N is :

(1) 6 (2) 3 (3) 4 (4) 2

Ans. **(3)**

Sol. $f_{eq} = \frac{n_1f_1 + n_2f_2}{n_1 + n_2}$

For diatomic gas $f_{eq} = 5$

$5 = \frac{(N)(6) + (2)(3)}{N+2}$

5N + 10 = 6N + 6

N = 4

49. Given below are two statements :

Statement I : Most of the mass of the atom and all its positive charge are concentrated in a tiny nucleus and the electrons revolve around it, is Rutherford's model.

Statement II : An atom is a spherical cloud of positive charges with electrons embedded in it, is a special case of Rutherford's model.

In the light of the above statements, choose the most appropriate from the options given below.

(1) Both statement I and statement II are false
(2) Statement I is false but statement II is true
(3) Statement I is true but statement II is false
(4) Both statement I and statement II are true

Ans. (3)

Sol. According to Rutherford atomic model, most of mass of atom and all its positive charge is concentrated in tiny nucleus & electron revolve around it.

According to Thomson atomic model, atom is spherical cloud of positive charge with electron embedded in it.

Hence,

Statement I is true but statement II false.

50. An electric field is given by $(6\hat{i}+5\hat{j}+3\hat{k})$ N/C.

The electric flux through a surface area $30\hat{i}\ m^2$ lying in YZ-plane (in SI unit) is :

(1) 90 (2) 150
(3) 180 (4) 60

Ans. (3)

Sol. $\vec{E}=6\hat{i}+5\hat{j}+3\hat{k}$

$\vec{A}=30\hat{i}$

$\phi=\vec{E}.\vec{A}$

$\phi=(6\hat{i}+5\hat{j}+3\hat{k}).(30\hat{i})$

$\phi=6\times30=180$

SECTION-B

51. Two metallic wires P and Q have same volume and are made up of same material. If their area of cross sections are in the ratio 4 : 1 and force F_1 is applied to P, an extension of Δl is produced. The force which is required to produce same extension in Q is F_2.

The value of $\frac{F_1}{F_2}$ is_______ .

Ans. (16)

Sol. $Y=\frac{\text{Stress}}{\text{Strain}}=\frac{F/A}{\Delta\ell/\ell}=\frac{F\ell}{A\Delta\ell}$

$\Delta\ell=\frac{F\ell}{AY}$

$V=A\ell\Rightarrow\ell=\frac{V}{A}$

$\Delta\ell=\frac{FV}{A^2Y}$

Y & V is same for both the wires

$\Delta\ell\propto\frac{F}{A^2}$

$\frac{\Delta\ell_1}{\Delta\ell_2}=\frac{F_1}{A_1^2}\times\frac{A_2^2}{F_2}$

$\Delta\ell_1=\Delta\ell_2$

$F_1A_2^2=F_2A_1^2$

$\frac{F_1}{F_2}=\frac{A_1^2}{A_2^2}=\left(\frac{4}{1}\right)^2=16$

52. A horizontal straight wire 5 m long extending from east to west falling freely at right angle to horizontal component of earth's magnetic field 0.60×10^{-4} Wbm^{-2}. The instantaneous value of emf induced in the wire when its velocity is 10 ms^{-1} is _________ $\times10^{-3}$ V.

Ans. (3)

Sol. $B_H=0.60\times10^{-4}$ Wb/m^2

Induced emf $e=B_Hv\ell$

$=0.60\times10^{-4}\times10\times5$

$=3\times10^{-3}$ V

53. Hydrogen atom is bombarded with electrons accelerated through a potential different of V, which causes excitation of hydrogen atoms. If the experiment is being formed at T = 0 K. The minimum potential difference needed to observe any Balmer series lines in the emission spectra will be $\frac{\alpha}{10}$V , where $\alpha=$________.

Ans. (121)

Sol. For minimum potential difference electron has to make transition from n = 3 to n = 2 state but first electron has to reach to n = 3 state from ground state. So, energy of bombarding electron should be equal to energy difference of n = 3 and n = 1 state.

$$\Delta E = 13.6\left[1-\frac{1}{3^2}\right]e = eV$$

$$\frac{13.6\times 8}{9} = V$$

$$V = 12.09\ V \approx 12.1\ V$$

So, $\alpha = 121$

54. A charge of 4.0 μC is moving with a velocity of $4.0 \times 10^6\ ms^{-1}$ along the positive y-axis under a magnetic field $\vec{B}$ of strength $\left(2\hat{k}\right)$ T. The force acting on the charge is $x\hat{i}N$. The value of x is __.

Ans. **(32)**

Sol. $q = 4\ \mu C,\ \vec{v} = 4\times 10^6 \hat{j}\ m/s$

$$\vec{B} = 2\hat{k}T$$

$$\vec{F} = q\left(\vec{v}\times\vec{B}\right)$$

$$= 4\times 10^{-6}\left(4\times 10^6\hat{j}\times 2\hat{k}\right)$$

$$= 4\times 10^{-6}\times 8\times 10^6\hat{i}$$

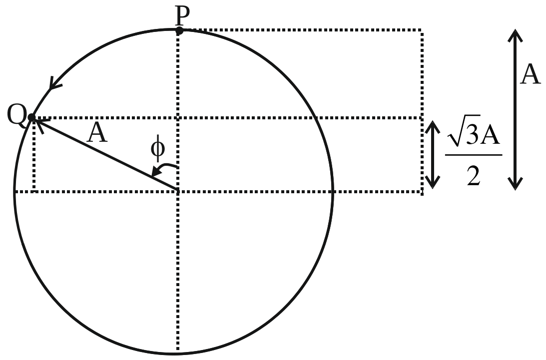
$$\vec{F} = 32\hat{i}\ N$$

$x = 32$

55. A simple harmonic oscillator has an amplitude A and time period 6π second. Assuming the oscillation starts from its mean position, the time required by it to travel from x = A to $x = \frac{\sqrt{3}}{2}A$ will be $\frac{\pi}{x}$ s, where x = _______ :

Ans. **(2)**

Sol.



From phasor diagram particle has to move from P to Q in a circle of radius equal to amplitude of SHM.

$$\cos\phi = \frac{\frac{\sqrt{3}A}{2}}{A} = \frac{\sqrt{3}}{2}$$

$$\phi = \frac{\pi}{6}$$

Now, $\frac{\pi}{6} = \omega t$

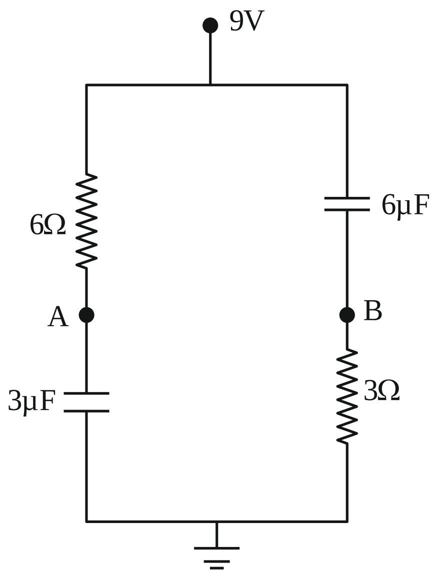
$$\frac{\pi}{6} = \frac{2\pi}{T}t$$

$$\frac{\pi}{6} = \frac{2\pi}{6\pi}t$$

$$t = \frac{\pi}{2}$$

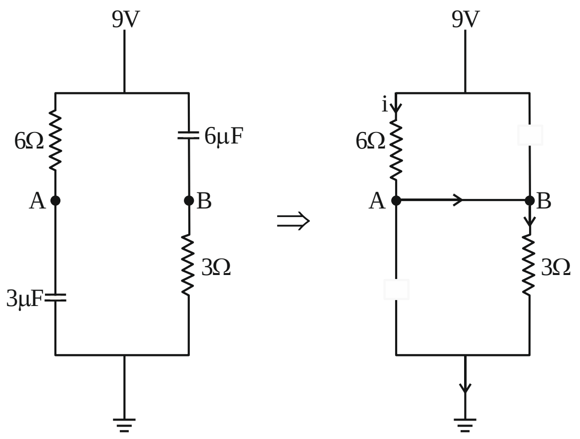
So, x = 2

56. In the given figure, the charge stored in 6μF capacitor, when points A and B are joined by a connecting wire is _______μC.



Ans. **(36)**

Sol. At steady state, capacitor behaves as an open circuit and current flows in circuit as shown in the diagram.



$R_{eq} = 9\ \Omega$

$i = \frac{9\ V}{9\ \Omega} = 1A$

$\Delta V_{6\Omega} = 1 \times 6 = 6\ V$

$V_A = 3\ V$

So, potential difference across 6μF is 6 V.

Hence Q = $C\Delta V$

$= 6 \times 6 \times 10^{-6}$ C

$= 36\ \mu C$

57. In a single slit diffraction pattern, a light of wavelength 6000 Å is used. The distance between the first and third minima in the diffraction pattern is found to be 3 mm when the screen in placed 50 cm away from slits. The width of the slit is _______ $\times 10^{-4}$ m.

Ans. **(2)**

Sol. For n^{th} minima

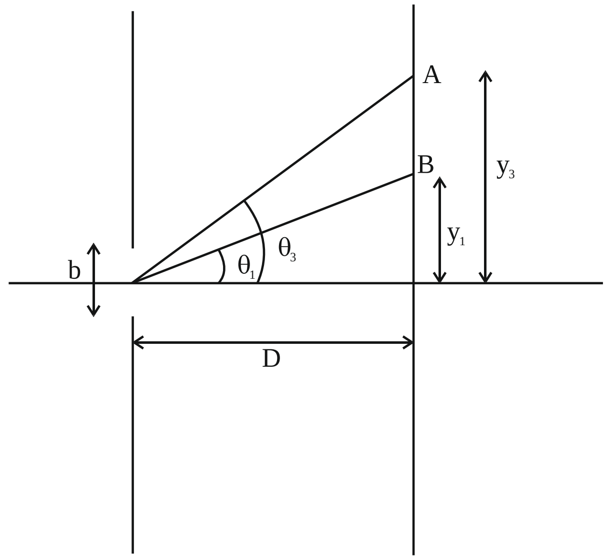
$b\sin\theta = n\lambda$

(λ is small so $\sin\theta$ is small, hence $\sin\theta \simeq \tan\theta$)

$b\tan\theta = n\lambda$

$b\frac{y}{D} = n\lambda$

$\Rightarrow y_n = \frac{n\lambda D}{b}$ (Position of n^{th} minima)



B → 1^{st} minima, A → 3^{rd} minima

$y_3 = \frac{3\lambda D}{b},\ \ y_1 = \frac{\lambda D}{b}$

$\Delta y = y_3 - y_1 = \frac{2\lambda D}{b}$

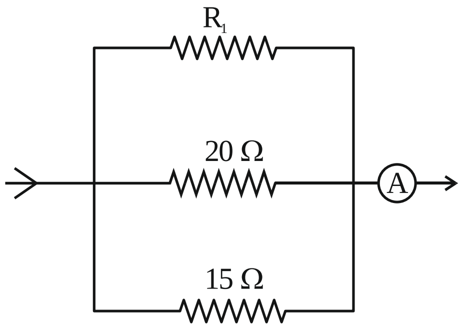
$3 \times 10^{-3} = \frac{2 \times 6000 \times 10^{-10} \times 0.5}{b}$

$b = \frac{2 \times 6000 \times 10^{-10} \times 0.5}{3 \times 10^{-3}}$

$b = 2 \times 10^{-4}$ m

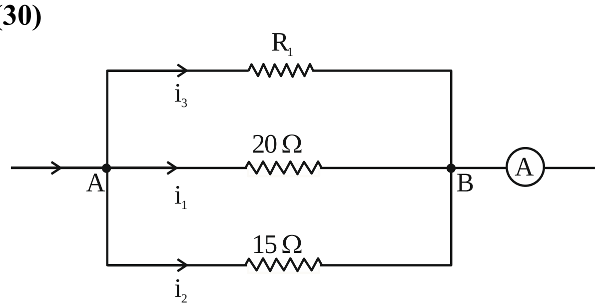
x = 2

58. In the given circuit, the current flowing through the resistance 20Ω is 0.3 A, while the ammeter reads 0.9 A. The value of R_1 is _______ Ω.



Ans. **(30)**

Sol.



Given, $i_1 = 0.3$ A, $i_1 + i_2 + i_3 = 0.9$ A

So, $V_{AB} = i_1 \times 20\Omega = 20 \times 0.3$ V = 6 V

$$i_2 = \frac{6V}{15\Omega} = \frac{2}{5}A$$

$$i_1 + i_2 + i_3 = \frac{9}{10}A$$

$$\frac{3}{10} + \frac{2}{5} + i_3 = \frac{9}{10}$$

$$\frac{7}{10} + i_3 = \frac{9}{10}$$

$i_3 = 0.2$ A

So, $i_3 \times R_1 = 6$ V

$(0.2)R_1 = 6$

$$R_1 = \frac{6}{0.2} = 30\ \Omega$$

59. A particle is moving in a circle of radius 50 cm in such a way that at any instant the normal and tangential components of its acceleration are equal. If its speed at t = 0 is 4 m/s, the time taken to complete the first revolution will be $\frac{1}{\alpha}\left[1-e^{-2\pi}\right]$ s, where α = ______.

Ans. **(8)**

Sol. $|\vec{a}_C| = |\vec{a}_t|$

$$\frac{v^2}{r} = \frac{dv}{dt}$$

$$\Rightarrow \int_4^v \frac{dv}{v^2} = \int_0^t \frac{dt}{r}$$

$$\Rightarrow \left[\frac{-1}{v}\right]_4^v = \frac{t}{r}$$

$$\Rightarrow \frac{-1}{v} + \frac{1}{4} = 2t$$

$$\Rightarrow v = \frac{4}{1-8t} = \frac{ds}{dt}$$

$$4\int_0^t \frac{dt}{1-8t} = \int_0^s ds$$

($r = 0.5$ m

$s = 2\pi r = \pi$)

$$4 \times \frac{\left[\ell n(1-8t)\right]_0^t}{-8} = \pi$$

$$\ell n(1-8t) = -2\pi$$

$1 - 8t = e^{-2\pi}$

$$t = \left(1-e^{-2\pi}\right)\frac{1}{8}\ s$$

So, $\alpha = 8$

60. A body of mass 5 kg moving with a uniform speed $3\sqrt{2}\ ms^{-1}$ in X – Y plane along the line y = x + 4. The angular momentum of the particle about the origin will be ______ kg m^2s^{-1}.

Ans. **(60)**

Sol. $y - x - 4 = 0$

d_1 is perpendicular distance of given line from origin.

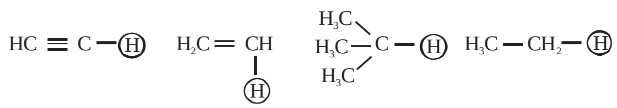
$$d_1 = \left|\frac{-4}{\sqrt{1^2+1^2}}\right| \Rightarrow 2\sqrt{2}\ m$$

So, $|\vec{L}| = mvd_1 = 5 \times 3\sqrt{2} \times 2\sqrt{2}$ kg m^2/s

$= 60$ kg m^2/s

SECTION-A

61. The ascending acidity order of the following H atoms is



A B C D

(1) C < D < B < A

(2) A < B < C < D

(3) A < B < D < C

(4) D < C < B < A

Ans. (1)

Sol. $CH \equiv C^{\ominus} > CH_2 = \overset{\ominus}{CH} > H_3C - \overset{\ominus}{CH_2} > (CH_3)_3C^{\ominus}$

Stability of conjugate base α acidic strength

C < D < B < A

62. Match List I with List II

List I (Bio Polymer)		List II (Monomer)	
A.	Starch	I.	nucleotide
B.	Cellulose	II.	α-glucose
C.	Nucleic acid	III.	β-glucose
D.	Protein	IV.	α-amino acid

Choose the correct answer from the options given below :-

(1) A-II, B-I, C-III, D-IV

(2) A-IV, B-II, C-I, D-III

(3) A-I, B-III, C-IV, D-II

(4) A-II, B-III, C-I, D-IV

Ans. (4)

Sol. A-II, B-III, C-I, D-IV

Fact based.

63. Match List I with List II

List I (Compound)		List II (pK_a value)	
A.	Ethanol	I.	10.0
B.	Phenol	II.	15.9
C.	m-Nitrophenol	III.	7.1
D.	p-Nitrophenol	IV.	8.3

Choose the correct answer from the options given below :-

(1) A-I, B-II, C-III, D-IV

(2) A-IV, B-I, C-II, D-III

(3) A-III, B-IV, C-I, D-II

(4) A-II, B-I, C-IV, D-III

Ans. (4)

Sol. Ethanol → 15.9

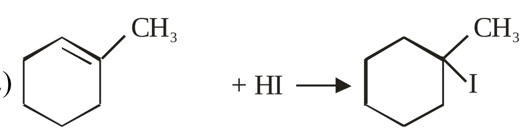
Phenol → 10

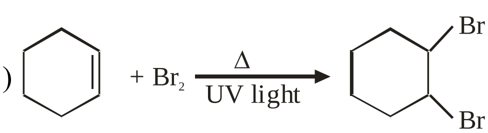
M-Nitrophenol → 8.3

P-Nitrophenol → 7.1

64. Which of the following reaction is correct ?

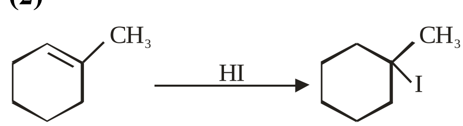
(1) $CH_3CH_2CH_2NH_2 \xrightarrow[H_2O]{HNO_2, 0^\circ C} CH_3CH_2OH + N_2 + HCl$

(2) 

(3) 

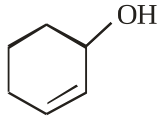
(4) $C_2H_5CONH_2 + Br_2 + NaOH \rightarrow C_2H_5CH_2NH_2 + Na_2CO_3 + NaBr + H_2O$

Ans. (2)

Sol. 

Markovnikov addition

65. According to IUPAC system, the compound



is named as

(1) Cyclohex-1-en-2-ol (2) 1-Hydroxyhex-2-ene
(3) Cyclohex-1-en-3-ol (4) Cyclohex-2-en-1-ol

Ans. (4)

Cyclohex-2-en-1-ol

66. The correct IUPAC name of K_2MnO_4 is
(1) Potassium tetraoxopermanganate (VI)
(2) Potassium tetraoxidomanganate (VI)
(3) Dipotassium tetraoxidomanganate (VII)
(4) Potassium tetraoxidomanganese (VI)

Ans. (2)

Sol. K_2MnO_4
$2 + x - 8 = 0$
$\Rightarrow x = +6$
O.S. of Mn = +6
IUPAC Name =
Potassium tetraoxidomanganate(VI)

67. A reagent which gives brilliant red precipitate with Nickel ions in basic medium is
(1) sodium nitroprusside
(2) neutral $FeCl_3$
(3) meta-dinitrobenzene
(4) dimethyl glyoxime

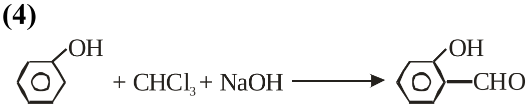
Ans. (4)

Sol. $Ni^{2+} + 2dmg^- \rightarrow [Ni(dmg)_2]$
Rosy red/Bright Red precipitate

68. Phenol treated with chloroform in presence of sodium hydroxide, which further hydrolysed in presence of an acid results
(1) Salicyclic acid
(2) Benzene-1,2-diol
(3) Benzene-1, 3-diol
(4) 2-Hydroxybenzaldehyde

Ans. (4)

Sol.



α-hydroxy-benzaldehyde

It is Reimer Tiemann Reaction

69. Match List I with List II

List I (Spectral Series for Hydrogen)		List II (Spectral Region/Higher Energy State)	
A.	Lyman	I.	Infrared region
B.	Balmer	II.	UV region
C.	Paschen	III.	Infrared region
D.	Pfund	IV.	Visible region

Choose the correct answer from the options given below :-
(1) A-II, B-III, C-I, D-IV
(2) A-I, B-III, C-II, D-IV
(3) A-II, B-IV, C-III, D-I
(4) A-I, B-II, C-III, D-IV

Ans. (3)

Sol. A – II, B – IV, C – III, D – I
Fact based.

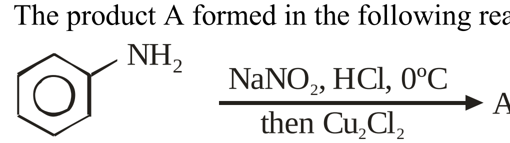
70. On passing a gas, 'X', through Nessler's reagent, a brown precipitate is obtained. The gas 'X' is
(1) H_2S (2) CO_2
(3) NH_3 (4) Cl_2

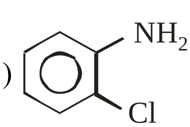
Ans. (3)

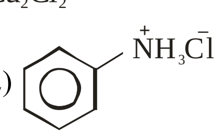
Sol. Nessler's Reagent Reaction :

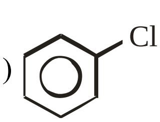
$$\underset{\text{(Nessler's Reagent)}}{2K_2HgI_4} + NH_3 + 3KOH \rightarrow \underset{\left(\substack{\text{Iodine of Millon's base}\\ \text{Brown precipitate}}\right)}{HgO.Hg(NH_2)I} + 7KI + 2H_2O$$

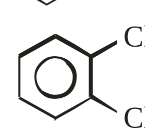
71. The product A formed in the following reaction is:



(1) 

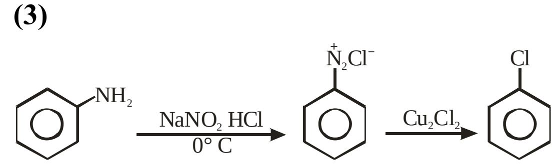
(2) 

(3) 

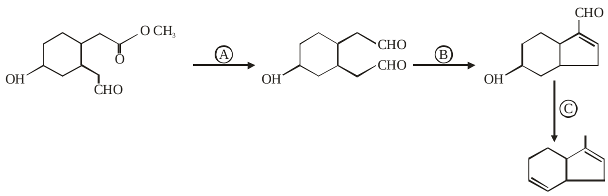
(4) 

Ans. (3)

Sol.



72. Identify the reagents used for the following conversion



(1) A = $LiAlH_4$, B = $NaOH_{(aq)}$, C = $NH_2–NH_2$/KOH, ethylene glycol

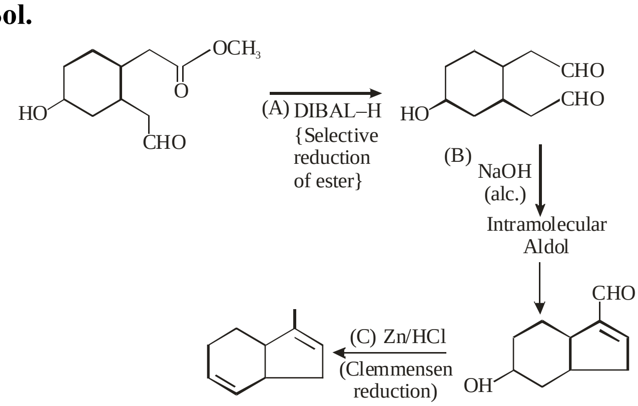
(2) A = $LiAlH_4$, B = $NaOH_{(alc)}$, C =Zn/HCl

(3) A = DIBAL-H, B= $NaOH_{(aq)}$, C = $NH_2–NH_2$/KOH, ethylene glycol

(4) A = DIBAL-H, B = $NaOH_{(alc)}$, C = Zn/HCl

Ans. **(4)**

Sol.



73. Which of the following acts as a strong reducing agent? (Atomic number : Ce = 58, Eu = 63, Gd = 64, Lu = 71)

(1) Lu^{3+} (2) Gd^{3+}

(3) Eu^{2+} (4) Ce^{4+}

Ans. **(3)**

Sol. $Eu^{+2} \longrightarrow Eu^{+3} + 1e^-$

$[Xe]4f^76s^0$ $[Xe]\ 4f^66s^0$

74. Chromatographic technique/s based on the principle of differential adsorption is/are

A. Column chromatography

B. Thin layer chromatography

C. Paper chromatography

Choose the most appropriate answer from the options given below:

(1) B only (2) A only

(3) A & B only (4) C only

Ans. **(3)**

Sol. Memory Based

75. Which of the following statements are correct about Zn, Cd and Hg?

A. They exhibit high enthalpy of atomization as the d-subshell is full.

B. Zn and Cd do not show variable oxidation state while Hg shows +I and +II.

C. Compounds of Zn, Cd and Hg are paramagnetic in nature.

D. Zn, Cd and Hg are called soft metals.

Choose the ***most appropriate*** from the options given below:

(1) B, D only (2) B, C only

(3) A, D only (4) C, D only

Sol. **Ans. (1)**

(A) Zn, Cd, Hg exhibit lowest enthalpy of atomization in respective transition series.

(C) Compounds of Zn, Cd and Hg are diamagnetic in nature.

76. The element having the highest first ionization enthalpy is

(1) Si (2) Al

(3) N (4) C

Ans. **(3)**

Sol. Al < Si < C < N ; IE_1 order.

77. Alkyl halide is converted into alkyl isocyanide by reaction with

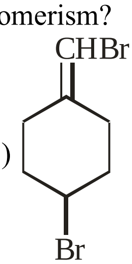
(1) NaCN (2) NH_4CN

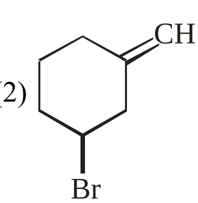
(3) KCN (4) AgCN

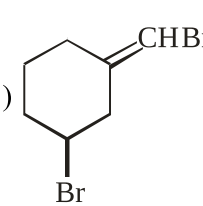
Ans. (4)

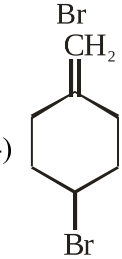
Sol. Covalent character of AgCN.

78. Which one of the following will show geometrical isomerism?

(1) 

(2) 

(3) 

(4) 

Ans. **(3)**

Sol. Due to unsymmetrical.

79. Given below are two statements:

Statement I: Fluorine has most negative electron gain enthalpy in its group.

Statement II: Oxygen has least negative electron gain enthalpy in its group.

In the light of the above statements, choose the most appropriate from the options given below.

(1) Both Statement I and Statement II are true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are false

(4) Statement I is false but Statement II is true

Ans. (4)

Sol. Statement-1 is false because chlorine has most negative electron gain enthalpy in its group.

80. Anomalous behaviour of oxygen is due to its

(1) Large size and high electronegativity

(2) Small size and low electronegativity

(3) Small size and high electronegativity

(4) Large size and low electronegativity

Ans. (3)

Sol. Fact Based.

SECTION-B

81. The total number of anti bonding molecular orbitals, formed from 2s and 2p atomic orbitals in a diatomic molecule is _____________.

Ans. (4)

Sol. Antibonding molecular orbital from 2s = 1

Antibonding molecular orbital from 2p = 3

Total = 4

82. The oxidation number of iron in the compound formed during brown ring test for NO_3^- ion is _____.

Ans. (1)

Sol. $[Fe(H_2O)_5(NO)]^{2+}$,

Oxidation no. of Fe = +1

83. The following concentrations were observed at 500 K for the formation of NH_3 from N_2 and H_2. At equilibrium :$[N_2] = 2 \times 10^{-2}$ M, $[H_2] = 3 \times 10^{-2}$ M and $[NH_3] = 1.5 \times 10^{-2}$M. Equilibrium constant for the reaction is _______.

Ans. (417)

Sol. $$K_C = \frac{[NH_3]^2}{[N_2][H_2]^3}$$

$$K_C = \frac{(1.5\times10^{-2})^2}{(2\times10^{-2})\times(3\times10^{-2})^3}$$

$K_C = 417$

84. Molality of 0.8 M H_2SO_4 solution (density 1.06 g cm^{-3}) is _____$\times10^{-3}$ m.

Ans. (815)

Sol. $$m = \frac{M\times1000}{d_{sol}\times1000 - M\times \text{Molar mass}_{solute}}$$

815×10^{-3} m

85. If 50 mL of 0.5 M oxalic acid is required to neutralise 25 mL of NaOH solution, the amount of NaOH in 50 mL of given NaOH solution is _____g.

Ans. (4)

Sol. Equivalent of Oxalic acid = Equivalents of NaOH

$50 \times 0.5 \times 2 = 25 \times M \times 1$

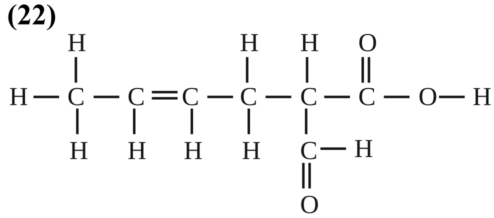
$M_{NaOH} = 2M$

W_{NaOH} in 50ml $= 2 \times 50 \times 40 \times 10^{-3}$ g = 4g

86. The total number of 'Sigma' and Pi bonds in 2-formylhex-4-enoic acid is ____.

Ans. (22)

Sol.



22 bonds

87. The half-life of radioisotopic bromine - 82 is 36 hours. The fraction which remains after one day is ___________ $\times10^{-2}$.

(Given antilog 0.2006 = 1.587)

Ans. (63)

Sol. Half life of bromine – 82 = 36 hours

$$t_{1/2} = \frac{0.693}{K}$$

$$K = \frac{0.693}{36} = 0.01925\ hr^{-1}$$

1st order rxn kinetic equation

$$t = \frac{2.303}{K}\log\frac{a}{a-x}$$

$$\log\frac{a}{a-x} = \frac{t\times K}{2.303} \quad (t = 1 day = 24hr)$$

$$\log\frac{a}{a-x} = \frac{24hr\times0.01925\,hr^{-1}}{2.303}$$

$$\log\frac{a}{a-x} = 0.2006$$

$$\frac{a}{a-x} = \text{anti}\log(0.2006)$$

$$\frac{a}{a-x} = 1.587$$

If a = 1

$$\frac{1}{1-x} = 1.587 \Rightarrow 1 - x = 0.6301 = \text{Fraction remain}$$

after one day

88. Standard enthalpy of vapourisation for CCl_4 is 30.5 kJ mol^{-1}. Heat required for vapourisation of 284g of CCl_4 at constant temperature is ____kJ.

(Given molar mass in g mol^{-1} ; C = 12, Cl = 35.5)

Ans. **(56)**

Sol. $\Delta H^0_{vap}\ CCl_4 = 30.5\ kJ / mol$

Mass of CCl_4 = 284 gm

Molar mass of CCl_4 = 154 g/mol

Moles of $CCl_4 = \dfrac{284}{154} = 1.844$ mol

$\Delta H_{vap}{}^o$ for 1 mole = 30.5 kJ/mol

$\Delta H_{vap}{}^o$ for 1.844 mol = 30.5×1.844

= 56.242 kJ

89. A constant current was passed through a solution of $AuCl_4^-$ ion between gold electrodes. After a period of 10.0 minutes, the increase in mass of cathode was 1.314 g. The total charge passed through the solution is __________ $\times 10^{-2}$ F.

(Given atomic mass of Au = 197)

Ans. **(2)**

Sol. $\dfrac{W}{E} = \dfrac{charge}{1F}$

$$\frac{1.314}{\frac{197}{3}} = \frac{Q}{1F}$$

$Q = 2 \times 10^{-2}$ F

90. The total number of molecules with zero dipole moment among CH_4, BF_3, H_2O, HF, NH_3, CO_2 and SO_2 is __________.

Ans. **(3)**

Sol. Molecules with zero dipole moment = CO_2, CH_4, BF_3

FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

(Held On Tuesday 30th January, 2024) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

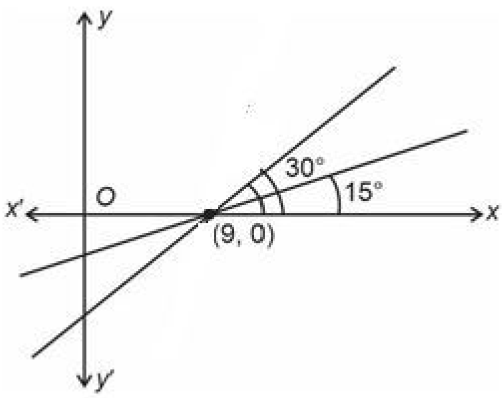
1. A line passing through the point A(9, 0) makes an angle of 30º with the positive direction of x-axis. If this line is rotated about A through an angle of 15º in the clockwise direction, then its equation in the new position is

(1) $\frac{y}{\sqrt{3}-2}+x=9$ (2) $\frac{x}{\sqrt{3}-2}+y=9$

(3) $\frac{x}{\sqrt{3}+2}+y=9$ (4) $\frac{y}{\sqrt{3}+2}+x=9$

Ans. (1)

Sol.



Eqⁿ : $y-0=\tan 15°\,(x-9)\Rightarrow y=(2-\sqrt{3})(x-9)$

2. Let S_a denote the sum of first n terms an arithmetic progression. If S_{20} = 790 and S_{10} = 145, then S_{15} – S_5 is :

(1) 395
(2) 390
(3) 405
(4) 410

Ans. (1)

Sol. $S_{20}=\frac{20}{2}[2a+19d]=790$

2a + 19d = 79 …..(1)

$S_{10}=\frac{10}{2}[2a+9d]=145$

2a + 9d = 29 …..(2)

From (1) and (2) a = -8, d = 5

$S_{15}-S_5=\frac{15}{2}[2a+14d]-\frac{5}{2}[2a+4d]$

$=\frac{15}{2}[-16+70]-\frac{5}{2}[-16+20]$

= 405-10

= 395

3. If z = x + iy, xy $\neq 0$, satisfies the equation $z^2+i\bar{z}=0$, then $|z^2|$ is equal to :

(1) 9
(2) 1
(3) 4
(4) $\frac{1}{4}$

Ans. (2)

Sol. $z^2=-i\bar{z}$

$|z^2|=|i\bar{z}|$

$|z^2| = |z|$

$|z|^2 - |z| = 0$

$|z|(|z| - 1) = 0$

$|z|$ = 0 (not acceptable)

$\therefore |z| = 1$

$\therefore |z|^2 = 1$

4. Let $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ be two vectors such that $|\vec{a}|=1;\ \vec{a}.\vec{b}=2$ and $|\vec{b}|=4$. If $\vec{c}=2(\vec{a}\times\vec{b})-3\vec{b}$, then the angle between $\vec{b}$ and $\vec{c}$ is equal to :

(1) $\cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$

(2) $\cos^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

(3) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

(4) $\cos^{-1}\left(\frac{2}{3}\right)$

Ans. (3)

Sol. Given $|\vec{a}| = 1, |\vec{b}| = 4, \vec{a}.\vec{b} = 2$

$\vec{c} = 2(\vec{a} \times \vec{b}) - 3\vec{b}$

Dot product with $\vec{a}$ on both sides

$\vec{c}.\vec{a} = -6$ …..(1)

Dot product with $\vec{b}$ on both sides

$\vec{b}.\vec{c} = -48$ …..(2)

$\vec{c}.\vec{c} = 4|\vec{a} \times \vec{b}|^2 + 9|\vec{b}|^2$

$|\vec{c}|^2 = 4\left[|a|^2|b|^2 - (a.b)^2\right] + 9|\vec{b}|^2$

$|\vec{c}|^2 = 4\left[(1)(4)^2 - (4)\right] + 9(16)$

$|\vec{c}|^2 = 4[12] + 144$

$|\vec{c}|^2 = 48 + 144$

$|\vec{c}|^2 = 192$

$\therefore \cos\theta = \dfrac{\vec{b}.\vec{c}}{|\vec{b}||\vec{c}|}$

$\therefore \cos\theta = \dfrac{-48}{\sqrt{192}.4}$

$\therefore \cos\theta = \dfrac{-48}{8\sqrt{3}.4}$

$\therefore \cos\theta = \dfrac{-3}{2\sqrt{3}}$

$\therefore \cos\theta = \dfrac{-\sqrt{3}}{2} \Rightarrow \theta = \cos^{-1}\left(\dfrac{-\sqrt{3}}{2}\right)$

5. The maximum area of a triangle whose one vertex is at (0, 0) and the other two vertices lie on the curve y = $-2x^2$ + 54 at points (x, y) and (-x, y) where y > 0 is :

(1) 88

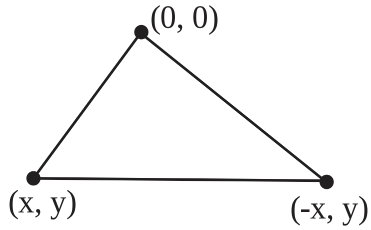
(2) 122

(3) 92

(4) 108

Ans. **(4)**

Sol.



Area of Δ

$= \dfrac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ x & y & 1 \\ -x & y & 1 \end{vmatrix}$

$\Rightarrow \left|\dfrac{1}{2}(xy + xy)\right| = |xy|$

$\text{Area}(\Delta) = |xy| = \left|x(-2x^2 + 54)\right|$

$\dfrac{d(\Delta)}{dx} = \left|(-6x^2 + 54)\right| \Rightarrow \dfrac{d\Delta}{dx} = 0$ at $x = 3$

Area = 3 (-2 × 9 + 54) = 108

6. The value of $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\dfrac{n^3}{(n^2+k^2)(n^2+3k^2)}$ is :

(1) $\dfrac{(2\sqrt{3}+3)\pi}{24}$

(2) $\dfrac{13\pi}{8(4\sqrt{3}+3)}$

(3) $\dfrac{13(2\sqrt{3}-3)\pi}{8}$

(4) $\dfrac{\pi}{8(2\sqrt{3}+3)}$

Ans. **(2)**

Sol. $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\dfrac{n^3}{n^4\left(1+\dfrac{k^2}{n^2}\right)\left(1+\dfrac{3k^2}{n^2}\right)}$

$= \lim\limits_{n\to\infty}\dfrac{1}{n}\sum\limits_{k=1}^{n}\dfrac{n^3}{\left(1+\dfrac{k^2}{n^2}\right)\left(1+\dfrac{3k^2}{n^2}\right)}$

$= \displaystyle\int_0^1 \dfrac{dx}{3(1+x^2)\left(\dfrac{1}{3}+x^2\right)}$

$$=\int_0^1 \frac{1}{3}\times\frac{3}{2}\frac{\left(x^2+1\right)-\left(x^2+\frac{1}{3}\right)}{\left(1+x^2\right)\left(x^2+\frac{1}{3}\right)}dx$$

$$=\frac{1}{2}\int_0^1\left[\frac{1}{x^2+\left(\frac{1}{\sqrt{3}}\right)^2}-\frac{1}{1+x^2}\right]dx$$

$$=\frac{1}{2}\left[\sqrt{3}\tan^{-1}\left(\sqrt{3}x\right)\right]_0^1-\frac{1}{2}\left(\tan^{-1}x\right)_0^1$$

$$=\frac{\sqrt{3}}{2}\left(\frac{\pi}{3}\right)-\frac{1}{2}\left(\frac{\pi}{4}\right)=\frac{\pi}{2\sqrt{3}}-\frac{\pi}{8}$$

$$=\frac{13\pi}{8.\left(4\sqrt{3}+3\right)}$$

7. Let $g : R\to R$ be a non constant twice differentiable such that $g'\left(\frac{1}{2}\right)=g'\left(\frac{3}{2}\right)$. If a real valued function f is defined as $f(x)=\frac{1}{2}\left[g(x)+g(2-x)\right]$, then

(1) f'(x) = 0 for atleast two x in (0, 2)

(2) f'(x) = 0 for exactly one x in (0, 1)

(3) f'(x) = 0 for no x in (0, 1)

(4) $f'\left(\frac{3}{2}\right)+f'\left(\frac{1}{2}\right)=1$

Ans. (1)

Sol. $f'(x)=\frac{g'(x)-g'(2-x)}{2}, f'\left(\frac{3}{2}\right)=\frac{g'\left(\frac{3}{2}\right)-g'\left(\frac{1}{2}\right)}{2}=0$

Also $f'\left(\frac{1}{2}\right)=\frac{g'\left(\frac{1}{2}\right)-g'\left(\frac{3}{2}\right)}{2}=0, f'\left(\frac{1}{2}\right)=0$

$$\Rightarrow f'\left(\frac{3}{2}\right)=f'\left(\frac{1}{2}\right)=0$$

$$\Rightarrow \text{roots in}\left(\frac{1}{2},1\right)\text{and}\left(1,\frac{3}{2}\right)$$

$$\Rightarrow f''(x)\ \text{is zero at least twice in}\left(\frac{1}{2},\frac{3}{2}\right)$$

8. The area (in square units) of the region bounded by the parabola $y^2 = 4(x - 2)$ and the line $y = 2x - 8$

(1) 8

(2) 9

(3) 6

(4) 7

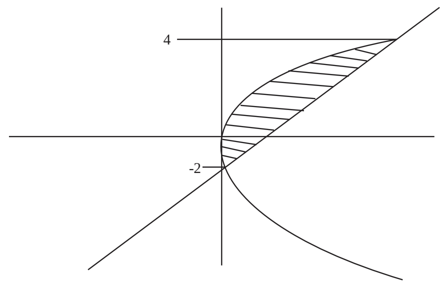
Ans. (2)

Sol. Let X = x – 2

$y^2 = 4x$, $\quad y = 2(x + 2) - 8$

$y^2 = 4x$, $\quad y = 2x - 4$

$$A=\int_{-2}^{4}\frac{y^2}{4}-\frac{y+4}{2}$$



= 9

9. Let y = y (x) be the solution of the differential equation sec x dy + {2(1 – x) tan x + x(2 – x)} dx = 0 such that y(0) = 2.Then y(2) is equal to :

(1) 2

(2) 2{1 – sin (2)}

(3) 2{sin (2) + 1}

(4) 1

Ans. (1)

Sol. $\frac{dy}{dx}=2(x-1)\sin x+\left(x^2-2x\right)\cos x$

Now both side integrate

$$y(x)=\int 2(x-1)\sin x\,dx+\left[\left(x^2-2x\right)(\sin x)-\int(2x-2)\sin x\ dx\right]$$

$$y(x)=\left(x^2-2x\right)\sin x+\lambda$$

$$y(0)=0+\lambda\Rightarrow 2=\lambda$$

$$y(x)=\left(x^2-2x\right)\sin x+2$$

$$y(2)=2$$

$\beta\ \gamma$ be the foot of perpendicular from the point (1, 2, 3) on the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$. then $19(\alpha+\beta+\gamma)$ is equal to :

(1) 102

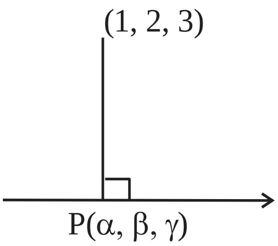
(2) 101

(3) 99

(4) 100

Ans. **(2)**

Sol.



Let foot P $(5k-3, 2k+1, 3k-4)$

DR's $\rightarrow$ AP : $5k-4,\ 2k-1, 3k-7$

DR's $\rightarrow$ Line: 5, 2, 3

Condition of perpendicular lines $(25k\text{-}20)+(4k\text{-}2)+(9k-21)=0$

Then $k=\frac{43}{38}$

Then $19(\alpha+\beta+\gamma)$**=101**

11. Two integers x and y are chosen with replacement from the set {0, 1, 2, 3, ….., 10}. Then the probability that $|x-y|>5$ is :

(1) $\frac{30}{121}$

(2) $\frac{62}{121}$

(3) $\frac{60}{121}$

(4) $\frac{31}{121}$

Ans. **(1)**

Sol. If $x=0, y=6, 7, 8, 9, 10$

If $x=1, y=7, 8, 9, 10$

If $x=2, y=8, 9, 10$

If $x=3, y=9, 10$

If $x=4, y=10$

If $x=5, y=$ no possible value

Total possible ways $=(5+4+3+2+1)\times 2$

$=30$

Required probability $=\frac{30}{11\times 11}=\frac{30}{121}$

12. If the domain of the function $f(x)=\cos^{-1}\left(\frac{2-|x|}{4}\right)+\left(\log_e(3-x)\right)^{-1}$ is $[-\alpha,\beta)-\{y\}$, then $\alpha+\beta+\gamma$ is equal to :

(1) 12

(2) 9

(3) 11

(4) 8

Ans. **(3)**

Sol. $-1\le\left|\frac{2-|x|}{4}\right|\le 1$

$\Rightarrow\left|\frac{2-|x|}{4}\right|\le 1$

$-4\le 2-|x|\le 4$

$-6\le -|x|\le 2$

$-2\le |x|\le 6$

$|x|\le 6$

$\Rightarrow\ x\in[-6, 6]$ …(1)

Now, $3-x\neq 1$

And $x\neq 2$ …(2)

and $3-x>0$

$x<3$ …(3)

From (1), (2) and (3)

$\Rightarrow\ x\in[-6, 3)-\{2\}$

$\alpha=6$

$\beta=3$

$\gamma=2$

$\alpha+\beta+\gamma=11$

13. Consider the system of linear equation $x+y+z=4\mu$, $x+2y+2\lambda z=10\mu$, $x+3y+4\lambda^2 z=\mu^2+15$, where $\lambda,\ \mu\in R$. Which one of the following statements is NOT correct ?

(1) The system has unique solution if $\lambda\neq\frac{1}{2}$ and $\mu\neq 1, 15$

(2) The system is inconsistent if $\lambda = \frac{1}{2}$ and $\mu \neq 1$

(3) The system has infinite number of solutions if $\lambda = \frac{1}{2}$ and $\mu = 15$

(4) The system is consistent if $\lambda \neq \frac{1}{2}$

Ans. **(2)**

Sol. $x + y + z = 4\mu$, $x + 2y + 2\lambda z = 10\mu$, $x + 3y + 4\lambda^2 z = \mu^2 + 15$,

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2\lambda \\ 1 & 3 & 4\lambda^2 \end{vmatrix} = (2\lambda - 1)^2$$

For unique solution $\Delta \neq 0$, $2\lambda - 1 \neq 0$, $\left(\lambda \neq \frac{1}{2}\right)$

Let $\Delta = 0$, $\lambda = \frac{1}{2}$

$$\Delta_y = 0,\ \Delta_x = \Delta_z = \begin{vmatrix} 4\mu & 1 & 1 \\ 10\mu & 2 & 1 \\ \mu^2 + 15 & 3 & 1 \end{vmatrix}$$

$= (\mu - 15)(\mu - 1)$

For infinite solution $\lambda = \frac{1}{2}$, $\mu = 1$ or 15

14. If the circles $(x+1)^2 + (y+2)^2 = r^2$ and $x^2 + y^2 - 4x - 4y + 4 = 0$ intersect at exactly two distinct points, then

(1) $5 < r < 9$

(2) $0 < r < 7$

(3) $3 < r < 7$

(4) $\frac{1}{2} < r < 7$

Ans. **(3)**

Sol. If two circles intersect at two distinct points

$\Rightarrow |r_1 - r_2| < C_1C_2 < r_1 + r_2$

$|r - 2| < \sqrt{9 + 16} < r + 2$

$|r - 2| < 5$ and $r + 2 > 5$

$-5 < r - 2 < 5$ $\quad\quad$ $r > 3$ ……….(2)

$-3 < r < 7$ $\quad\quad$ ………(1)

From (1) and (2)

$3 < r < 7$

15. If the length of the minor axis of ellipse is equal to half of the distance between the foci, then the eccentricity of the ellipse is :

(1) $\frac{\sqrt{5}}{3}$

(2) $\frac{\sqrt{3}}{2}$

(3) $\frac{1}{\sqrt{3}}$

(4) $\frac{2}{\sqrt{5}}$

Ans. **(4)**

Sol. $2b = ae$

$\frac{b}{a} = \frac{e}{2}$

$e = \sqrt{1 - \frac{e^2}{4}}$

$e = \frac{2}{\sqrt{5}}$

16. Let M denote the median of the following frequency distribution.

Class	0-4	4-8	8-12	12-16	16-20
Frequency	3	9	10	8	6

Then 20 M is equal to :

(1) 416

(2) 104

(3) 52

(4) 208

Ans. **(4)**

Sol.

Class	Frequency	Cumulative frequency
0-4	3	**3**
4-8	9	**12**
8-12	10	**22**
12-16	8	**30**
16-20	6	**36**

$$M = l + \left(\frac{\frac{N}{2} - C}{f}\right)h$$

$$M = 8 + \frac{18-12}{10} \times 4$$

M = 10.4

20M = 208

17. If $f(x) = \begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3+\sin^2 2x \\ 3+2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3+2\sin^4 x & \sin^2 2x \end{vmatrix}$ then $\frac{1}{5} f'(0)$ is equal to ______

(1) 0

(2) 1

(3) 2

(4) 6

Ans. **(1)**

Sol. $\begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3+\sin^2 2x \\ 3+2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3+2\sin^2 4x & \sin^2 2x \end{vmatrix}$

$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$\begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3+\sin^2 2x \\ 3 & 0 & -3 \\ 0 & 3 & -3 \end{vmatrix}$

f(x) = 45

$f'(x) = 0$

18. Let A (2, 3, 5) and C(-3, 4, -2) be opposite vertices of a parallelogram ABCD if the diagonal $\overrightarrow{BD} = \hat{i} + 2\hat{j} + 3\hat{k}$ then the area of the parallelogram is equal to

(1) $\frac{1}{2}\sqrt{410}$

(2) $\frac{1}{2}\sqrt{474}$

(3) $\frac{1}{2}\sqrt{586}$

(4) $\frac{1}{2}\sqrt{306}$

Ans. **(2)**

Sol. Area = $|AC \times BD|$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -1 & 7 \\ 1 & 2 & 3 \end{vmatrix}$

$= \frac{1}{2}\left|-17\hat{i} - 8\hat{j} + 11\hat{k}\right| = \frac{1}{2}\sqrt{474}$

19. If $2\sin^3 x + \sin 2x \cos x + 4\sin x - 4 = 0$ has exactly 3 solutions in the interval $\left[0, \frac{n\pi}{2}\right]$, $n \in N$, then the roots of the equation $x^2 + nx + (n-3) = 0$ belong to :

(1) $(0, \infty)$

(2) $(-\infty, 0)$

(3) $\left(-\frac{\sqrt{17}}{2}, \frac{\sqrt{17}}{2}\right)$

(4) Z

Ans. **(2)**

Sol. $2\sin^3 x + 2\sin x.\cos^2 x + 4\sin x - 4 = 0$

$2\sin^3 x + 2\sin x.(1-\sin^2 x) + 4\sin x - 4 = 0$

$6\sin x - 4 = 0$

$\sin x = \frac{2}{3}$

n = 5 (in the given interval)

$x^2 + 5x + 2 = 0$

$x = \frac{-5 \pm \sqrt{17}}{2}$

Required interval $(-\infty, 0)$

20. Let $f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to R$ be a differentiable function such that $f(0) = \frac{1}{2}$, If the $\lim_{x \to 0} \frac{x\int_0^x f(t)dt}{e^{x^2} - 1} = \alpha$, then $8\alpha^2$ is equal to :

(1) 16

(2) 2

(3) 1

(4) 4

Ans. **(2)**

Sol. $\lim\limits_{x\to 0}\dfrac{x\int_0^x f(t)dt}{\left(\dfrac{e^{x^2}-1}{x^2}\right)\times x^2}$

$\lim\limits_{x\to 0}\dfrac{\int_0^x f(t)dt}{x}$ $\left(\lim\limits_{x\to 0}\dfrac{e^{x^2}-1}{x^2}=1\right)$

$=\lim\limits_{x\to 0}\dfrac{f(x)}{1}$ (using L Hospital)

$f(0)=\dfrac{1}{2}$

$\alpha=\dfrac{1}{2}$

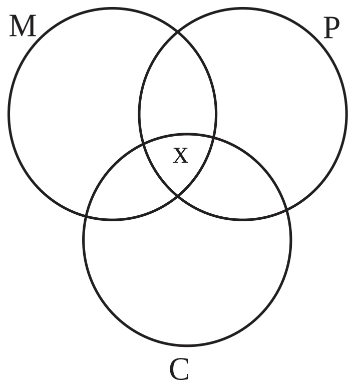
$8\alpha^2=2$

SECTION-B

21. A group of 40 students appeared in an examination of 3 subjects – Mathematics, Physics & Chemistry. It was found that all students passed in at least one of the subjects, 20 students passed in Mathematics, 25 students passed in Physics, 16 students passed in Chemistry, at most 11 students passed in both Mathematics and Physics, at most 15 students passed in both Physics and Chemistry, at most 15 students passed in both Mathematics and Chemistry. The maximum number of students passed in all the three subjects is ________.

Ans. **(10)**

Sol.

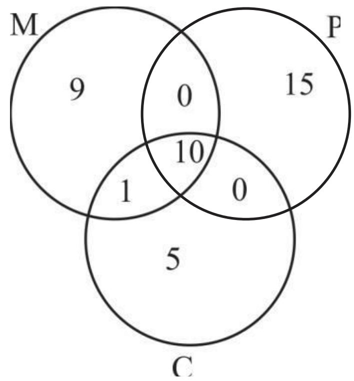


$11-x\geq 0$ (Maths and Physics)

$x\leq 11$

x =11 does not satisfy the data.

For x = 10



Hence maximum number of students passed in all the three subjects is 10.

22. If d_1 is the shortest distance between the lines $x+1=2y=-12z$, $x=y+2=6z-6$ and d_2 is the shortest distance between the lines $\dfrac{x-1}{2}=\dfrac{y+8}{-7}=\dfrac{z-4}{5}$, $\dfrac{x-1}{2}=\dfrac{y-2}{1}=\dfrac{z-6}{-3}$, then the value of $\dfrac{32\sqrt{3}d_1}{d_2}$ is :

Ans. **(16)**

Sol. $L_1:\dfrac{x+1}{1}=\dfrac{y}{1/2}=\dfrac{z}{-1/12}$, $L_2:\dfrac{x}{1}=\dfrac{y+2}{1}=\dfrac{z-1}{\frac{1}{6}}$

d_1 = shortest distance between L_1 & L_2

$=\left|\dfrac{(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)}{|(\vec{b}_1\times\vec{b}_2)|}\right|$

$d_1=2$

$L_3:\dfrac{x-1}{2}=\dfrac{y+8}{-7}=\dfrac{z-4}{5}$, $L_4:\dfrac{x-1}{2}=\dfrac{y-2}{1}=\dfrac{z-6}{-3}$

d_2 = shortest distance between L_3 & L_4

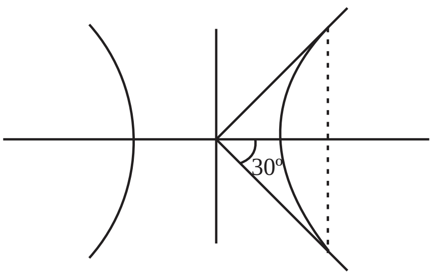
$d_2=\dfrac{12}{\sqrt{3}}$ Hence

$=\dfrac{32\sqrt{3}d_1}{d_2}=\dfrac{32\sqrt{3}\times 2}{\frac{12}{\sqrt{3}}}=16$

23. Let the latus rectum of the hyperbola $\frac{x^2}{9}-\frac{y^2}{b^2}=1$ subtend an angle of $\frac{\pi}{3}$ at the centre of the hyperbola. If b^2 is equal to $\frac{l}{m}\left(1+\sqrt{n}\right)$, where l and m are co-prime numbers, then $l^2 + m^2 + n^2$ is equal to __________

Ans. (182)

Sol. LR subtends 60º at centre



$$\Rightarrow \tan 30° = \frac{b^2/a}{ae} = \frac{b^2}{a^2e} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow e = \frac{\sqrt{3}b^2}{9}$$

$$\text{Also, } e^2 = 1+\frac{b^2}{9} \Rightarrow 1+\frac{b^2}{9} = \frac{3b^4}{81}$$

$$\Rightarrow b^4 = 3b^2 + 27$$

$$\Rightarrow b^4 - 3b^2 - 27 = 0$$

$$\Rightarrow b^2 = \frac{3}{2}(1+\sqrt{13})$$

$$\Rightarrow \ell = 3, m = 2, n = 13$$

$$\Rightarrow \ell^2 + m^2 + n^2 = 182$$

24. Let A = {1, 2, 3,….7} and let P(1) denote the power set of A. If the number of functions $f: A \rightarrow P(A)$ such that $a \in f(a), \forall a \in A$ is m^n, m and $n \in N$ and m is least, then m + n is equal to _______.

Ans. (44)

Sol. $f : A \rightarrow P(A)$

$a \in f(a)$

That means 'a' will connect with subset which contain element 'a'.

Total options for 1 will be 2^6. (Because 2^6 subsets contains 1)

Similarly, for every other element

Hence, total is $2^6 \times 2^6 \times 2^6 \times 2^6 \times 2^6 \times 2^6 \times 2^6 = 2^{42}$

Ans. 2+42 = 44

25. The value $9\int_0^9\left[\sqrt{\frac{10x}{x+1}}\right]dx$, where [t] denotes the greatest integer less than or equal to t, is _____.

Ans. (155)

Sol. $\frac{10x}{x+1}=1 \qquad \Rightarrow x = \frac{1}{9}$

$\frac{10x}{x+1}=4 \qquad \Rightarrow x = \frac{2}{3}$

$\frac{10x}{x+1}=9 \qquad \Rightarrow x = 9$

$$I = 9\left(\int_0^{1/9} 0dx + \int_{1/9}^{2/3} 1.dx + \int_{2/3}^{9} 2dx\right)$$

= 155

26. Number of integral terms in the expansion of $\left\{7^{\left(\frac{1}{2}\right)} + 11^{\left(\frac{1}{6}\right)}\right\}^{824}$ is equal to ________.

Ans. (138)

Sol. General term in expansion of $\left((7)^{1/2}+(11)^{1/6}\right)^{824}$ is

$t_{r+1} = {}^{824}C_r\ (7)^{\frac{824-r}{2}}\ (11)^{r/6}$

For integral term, r must be multiple of 6.

Hence r = 0, 6, 12, ……….822

27. Let y = y(x) be the solution of the differential equation $(1 - x^2)\, dy = \left[xy + \left(x^3+2\right)\sqrt{3\left(1-x^2\right)}\right]dx$, $-1 < x < 1, y(0) = 0$. If $y\left(\frac{1}{2}\right) = \frac{m}{n}$, m and n are co-prime numbers, then m + n is equal to ________.,

Ans. **(97)**

Sol. $\frac{dy}{dx} - \frac{xy}{1-x^2} = \frac{(x^3+2)\sqrt{3(1-x^2)}}{1-x^2}$

IF = $e^{-\int \frac{x}{1-x^2}dx} = e^{+\frac{1}{2}\ln(1-x^2)} = \sqrt{1-x^2}$

$y\sqrt{1-x^2} = \sqrt{3}\int (x^3+2)\,dx$

$y\sqrt{1-x^2} = \sqrt{3}\left(\frac{x^4}{4} + 2x\right) + c$

$\Rightarrow y(0) = 0 \qquad \therefore c = 0$

$y\left(\frac{1}{2}\right) = \frac{65}{32} = \frac{m}{n}$

$m + n = 97$

28. Let $\alpha, \beta \in N$ be roots of equation $x^2 - 70x + \lambda = 0$, where $\frac{\lambda}{2}, \frac{\lambda}{3} \notin N$. If λ assumes the minimum possible value, then $\frac{\left(\sqrt{\alpha-1}+\sqrt{\beta-1}\right)(\lambda+35)}{|\alpha-\beta|}$ is equal to :

Ans. **(60)**

Sol. $x^2 - 70x + \lambda = 0$

$\alpha + \beta = 70$

$\alpha\beta = \lambda$

$\therefore \alpha(70 - \alpha) = \lambda$

Since, 2 and 3 does not divide λ

$\therefore \alpha = 5, \beta = 65, \lambda = 325$

By putting value of α, β, λ we get the required value 60.

29. If the function $f(x) = \begin{cases} \frac{1}{|x|}, & |x| \geq 2 \\ ax^2 + 2b, & |x| < 2 \end{cases}$ is differentiable on R, then 48 (a + b) is equal to _____.

Ans. **(15)**

Sol. $f(x) \begin{cases} \frac{1}{x};\ x \geq 2 \\ ax^2 + 2b;\ -2 < x < 2 \\ -\frac{1}{x};\ x \leq -2 \end{cases}$

Continuous at x = 2 $\Rightarrow \frac{1}{2} = \frac{a}{4} + 2b$

Continuous at x = -2 $\Rightarrow \frac{1}{2} = \frac{a}{4} + 2b$

Since, it is differentiable at x = 2

$-\frac{1}{x^2} = 2ax$

Differentiable at x = 2 $\Rightarrow \frac{-1}{4} = 4a \Rightarrow a = \frac{-1}{16}$, b $= \frac{3}{8}$

30. Let $\alpha = 1^2 + 4^2 + 8^2 + 13^2 + 19^2 + 26^2 + \ldots\ldots$ upto 10 terms and $\beta = \sum_{n=1}^{10} n^4$. If $4\alpha - \beta = 55k + 40$, then k is equal to __________.

Ans. **(353)**

Sol. $\alpha = 1^2 + 4^2 + 8^2 \ldots.$

$t_n = an^2 + bn + c$

$1 = a + b + c$

$4 = 4a + 2b + c$

$8 = 9a + 3b + c$

On solving we get, $a = \frac{1}{2}$, $b = \frac{3}{2}$, $c = -1$

$$\alpha = \sum_{n=1}^{10}\left(\frac{n^2}{2} + \frac{3n}{2} - 1\right)^2$$

$$4\alpha = \sum_{n=1}^{10}\left(n^2 + 3n - 2\right)^2 \ , \ \beta = \sum_{n=1}^{10} n^4$$

$$4\alpha - \beta = \sum_{n=1}^{10}(6n^3 + 5n^2 - 12n + 4) = 55(353) + 40$$

SECTION-A

31. Match List-I with List-II.

	List-I		List-II
A.	Coefficient of viscosity	I.	$[M L^2 T^{-2}]$
B.	Surface Tension	II.	$[M L^2 T^{-1}]$
C.	Angular momentum	III.	$[M L^{-1} T^{-1}]$
D.	Rotational kinetic energy	IV.	$[M L^0 T^{-2}]$

(1) A–II, B–I, C–IV, D–III
(2) A–I, B–II, C–III, D–IV
(3) A–III, B–IV, C–II, D–I
(4) A–IV, B–III, C–II, D–I

Ans. (3)

Sol. $F = \eta A \frac{dv}{dy}$

$$\left[MLT^{-2}\right] = \eta\left[L^2\right]\left[T^{-1}\right]$$

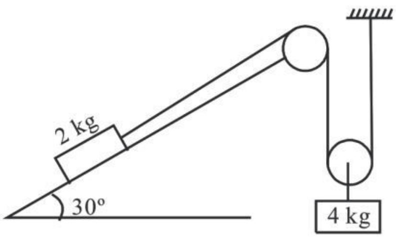
$$\eta = \left[ML^{-1}T^{-1}\right]$$

$$S.T = \frac{F}{\ell} = \frac{\left[MLT^{-2}\right]}{[L]} = \left[ML^0T^{-2}\right]$$

$$L = mvr = \left[ML^2T^{-1}\right]$$

$$K.E = \frac{1}{2}I\omega^2 = \left[ML^2T^{-2}\right]$$

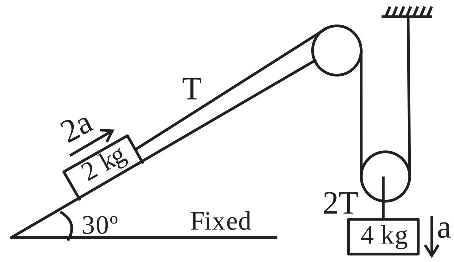
32. All surfaces shown in figure are assumed to be frictionless and the pulleys and the string are light. The acceleration of the block of mass 2 kg is :



(1) g
(2) $\frac{g}{3}$
(3) $\frac{g}{2}$
(4) $\frac{g}{4}$

Ans. (2)

Sol.

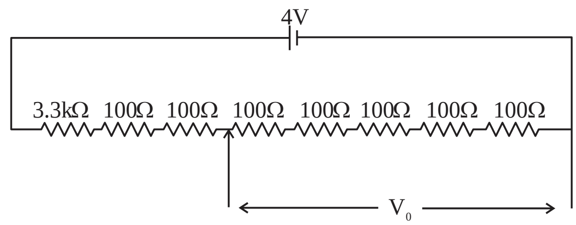


$40 - 2T = 4a$

$T - 10 = 4a \Rightarrow 20 = 12a$

$\Rightarrow a = 5/3 \Rightarrow 2a = \frac{g}{3}$

33. A potential divider circuit is shown in figure. The output voltage V_0 is



(1) 4V
(2) 2 mV
(3) 0.5 V
(4) 12 mV

Ans. (3)

Sol. $R_{eq} = 4000\,\Omega$

$$i = \frac{4}{4000} = \frac{1}{1000}A$$

$$V_0 = i.R = \frac{1}{1000}\times 500 = 0.5V$$

34. Young's modules of material of a wire of length 'L' and cross-sectional area A is Y. If the length of the wire is doubled and cross-sectional area is halved then Young's **modules** will be :

(1) $\frac{Y}{4}$
(2) $4Y$
(3) Y
(4) 2Y

Ans. (3)

Sol. Young's modulus depends on the material not length and cross sectional area. So young's modulus remains same.

35. The work function of a substance is 3.0 eV. The longest wavelength of light that can cause the emission of photoelectrons from this substance is approximately:

(1) 215 nm (2) 414 nm
(3) 400 nm (4) 200 nm

Ans. **(2)**

Sol. For P.E.E. : $\lambda \le \frac{hc}{W_e}$

$$\lambda \le \frac{1240\,nm - eV}{3eV}$$

$$\lambda \le 413.33\,nm$$

$\lambda_{max} \approx 414\,nm$ for P.E.E.

36. The ratio of the magnitude of the kinetic energy to the potential energy of an electron in the 5^{th} excited state of a hydrogen atom is :

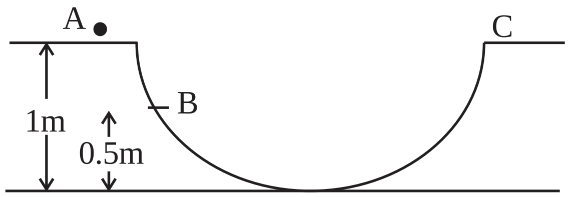
(1) 4 (2) $\frac{1}{4}$
(3) $\frac{1}{2}$ (4) 1

Ans. **(3)**

Sol. $\frac{1}{2}|PE| = KE$ for each value of n (orbit)

$$\therefore \frac{KE}{|PE|} = \frac{1}{2}$$

37. A particle is placed at the point A of a frictionless track ABC as shown in figure. It is gently pushed toward right. The speed of the particle when it reaches the point B is : (Take g = 10 m/s^2).



(1) 20 m/s (2) $\sqrt{10}\,m/s$
(3) $2\sqrt{10}\,m/s$ (4) 10 m/s

Ans. **(2)**

Sol. By COME

$KE_A + U_A = KE_B + U_B$

$$0 + mg(1) = \frac{1}{2}mv^2 + mg \times 0.5$$

$$v = \sqrt{g} = \sqrt{10}\,m/s$$

38. The electric field of an electromagnetic wave in free space is represented as $\vec{E} = E_0 \cos(\omega t - kz)\hat{i}$. The corresponding magnetic induction vector will be :

(1) $\vec{B} = E_0 C \cos(\omega t - kz)\hat{j}$

(2) $\vec{B} = \frac{E_0}{C}\cos(\omega t - kz)\hat{j}$

(3) $\vec{B} = E_0 C \cos(\omega t + kz)\hat{j}$

(4) $\vec{B} = \frac{E_0}{C}\cos(\omega t + kz)\hat{j}$

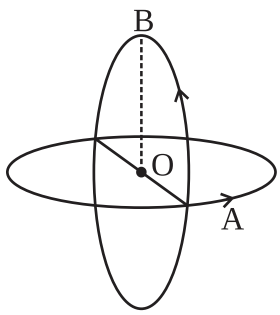
Ans. **(2)**

Sol. Given $\vec{E} = E_0 \cos(\omega t - kz)\hat{i}$

$$\vec{B} = \frac{E_0}{C}\cos(\omega t - kz)\hat{j}$$

$$\hat{C} = \hat{E} \times \hat{B}$$

39. Two insulated circular loop A and B radius 'a' carrying a current of 'I' in the anti clockwise direction as shown in figure. The magnitude of the magnetic induction at the centre will be :

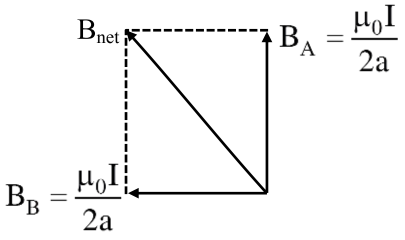


(1) $\frac{\sqrt{2}\mu_0 I}{a}$ (2) $\frac{\mu_0 I}{2a}$

(3) $\frac{\mu_0 I}{\sqrt{2}a}$ (4) $\frac{2\mu_0 I}{a}$

Ans. **(3)**

Sol.



$\because B_{net} = \dfrac{\sqrt{2}\mu_0 I}{2a}$

40. The diffraction pattern of a light of wavelength 400 nm diffracting from a slit of width 0.2 mm is focused on the focal plane of a convex lens of focal length 100 cm. The width of the 1st secondary maxima will be :

(1) 2 mm (2) 2 cm

(3) 0.02 mm (4) 0.2 mm

Ans. **(1)**

Sol. Width of 1st secondary maxima = $\dfrac{\lambda}{a}.D$

Here

$a = 0.2\times10^{-3} m$

$\lambda = 400\times10^{-9} m$

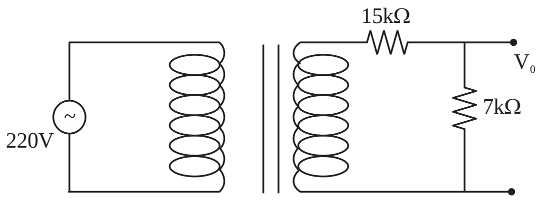
$D = 100\times10^{-2}$

Width of 1st secondary maxima

$= \dfrac{400\times10^{-9}}{0.2\times10^{-3}}\times100\times10^{-2}$

$= 2\ mm$

41. Primary coil of a transformer is connected to 220 V ac. Primary and secondary turns of the transforms are 100 and 10 respectively. Secondary coil of transformer is connected to two series resistance shown in shown in figure. The output voltage (V_0) is :



(1) 7 V (2) 15 V

(3) 44 V (4) 22 V

Ans. **(1)**

Sol. $\dfrac{\varepsilon_1}{\varepsilon_2} = \dfrac{N_1}{N_2} = \dfrac{100}{10} \Rightarrow \varepsilon_2 = 22V$

$I = \dfrac{22}{22\times10^3} = 1mA,\ V_0 = 7V$

42. The gravitational potential at a point above the surface of earth is $-5.12\times10^7\ J/kg$ and the acceleration due to gravity at that point is 6.4 m/s^2. Assume that the mean radius of earth to be 6400 km. The height of this point above the earth's surface is :

(1) 1600 km

(2) 540 km

(3) 1200 km

(4) 1000 km

Ans. **(1)**

Sol. $-\dfrac{GM_E}{R_E+h} = -5.12\times10^{-7}$ .... (i)

$\dfrac{GM_E}{(R_E+h)^2} = 6.4$ .... (ii)

By (i) and (ii)

$\Rightarrow h = 16\times10^5\ m = 1600\,km$

43. An electric toaster has resistance of 60 Ω at room temperature (27ºC). The toaster is connected to a 220 V supply. If the current flowing through it reaches 2.75 A, the temperature attained by toaster is around : (if $\alpha = 2\times10^{-4}/^oC$)

(1) 694ºC

(2) 1235ºC

(3) 1694ºC

(4) 1667ºC

Ans. **(3)**

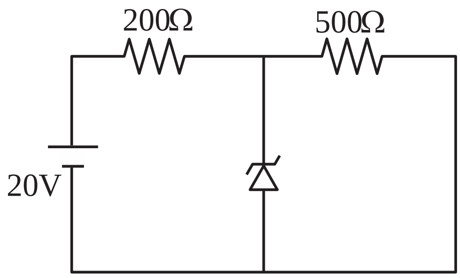
Sol. $R_{T=27} = 60\Omega,\ R_T = \dfrac{220}{2.75} = 80\Omega$

$R = R_0\,(1+\alpha\Delta T)$

$80 = 60\,[1+2\times10^{-4}(T-27)]$

$T \approx 1694^oC$

44. A Zener diode of breakdown voltage 10V is used as a voltage regulator as shown in the figure. The current through the Zener diode is

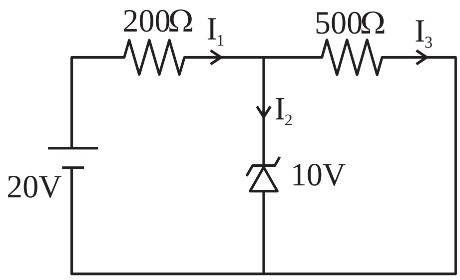


(1) 50 mA (2) 0

(3) 30 mA (4) 20 mA

Ans. (3)

Sol.



Zener is in breakdown region.

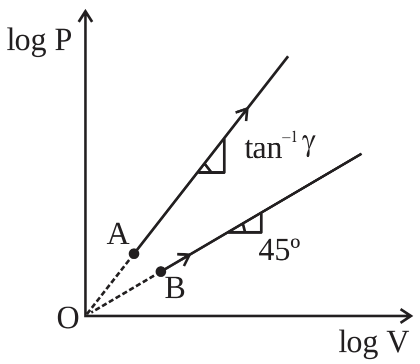
$$I_3 = \frac{10}{500} = \frac{1}{50}$$

$$I_1 = \frac{10}{200} = \frac{1}{20}$$

$$I_2 = I_1 - I_3$$

$$I_2 = \left(\frac{1}{20} - \frac{1}{50}\right) = \left(\frac{3}{100}\right) = 30\,mA$$

45. Two thermodynamical process are shown in the figure. The molar heat capacity for process A and B are C_A and C_B. The molar heat capacity at constant pressure and constant volume are represented by C_P and C_V, respectively. Choose the correct statement.



(1) $C_B = \infty, C_A = 0$

(2) $C_A = 0$ and $C_B = \infty$

(3) $C_P > C_V > C_A = C_B$

(4) $C_A > C_P > C_V$

Ans. (Bonus)

Sol. For process A

$$\log P = \gamma \log V \Rightarrow P = V^{\gamma}, (\gamma > 1)$$

$$PV^{-\gamma} = \text{Constant}$$

$$C_A = C_V + \frac{R}{1+\gamma} \quad \text{.... (i)}$$

Likewise for process B $\rightarrow$ $PV^{-1} = Constant$

$$C_B = C_v + \frac{R}{1+1}$$

$$C_B = C_v + \frac{R}{2} \quad \text{... (ii)}$$

$$C_P = C_v + R \quad \text{.... (iii)}$$

By (i), (ii) & (iii)

$C_P > C_B > C_A > C_v$ **[No answer matching]**

46. The electrostatic potential due to an electric dipole at a distance 'r' varies as :

(1) r (2) $\frac{1}{r^2}$

(3) $\frac{1}{r^3}$ (4) $\frac{1}{r}$

Ans. (2)

Sol. $V = \frac{kP\cos\theta}{r^2}$

& can also checked dimensionally

47. A spherical body of mass 100 g is dropped from a height of 10 m from the ground. After hitting the ground, the body rebounds to a height of 5m. The impulse of force imparted by the ground to the body is given by : (given g = 9.8 m/s^2)

(1) 4.32 kg ms^{-1} (2) 43.2 kg ms^{-1}

(3) 23.9 kg ms^{-1} (4) 2.39 kg ms^{-1}

Ans. (4)

Sol. $\vec{I} = \Delta\vec{P} = \vec{P}_f - \vec{P}_i$

M = 0.1 kg

$$I = \Delta P = 0.1\left(\sqrt{2\times9.8\times5} - \left(-\sqrt{2\times9.8\times10}\right)\right)$$

$$= 0.1\left(14 + 7\sqrt{2}\right) \approx 2.39 \text{ kg ms}^{-1}$$

48. A particle of mass m projected with a velocity 'u' making an angle of 30º with the horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle is at its maximum height h is :

(1) $\frac{\sqrt{3}}{16}\frac{mu^3}{g}$ (2) $\frac{\sqrt{3}}{2}\frac{mu^2}{g}$

(3) $\frac{mu^3}{\sqrt{2}g}$ (4) zero

Ans. (1)

Sol. L = $mu\cos\theta H$

$$= mu\cos\theta\times\frac{u^2\sin^2\theta}{2g}$$

$$= \frac{mu^3}{2g}\times\frac{\sqrt{3}}{2}\times\left(\frac{1}{2}\right)^2 = \frac{\sqrt{3}mu^3}{16g}$$

49. At which temperature the r.m.s. velocity of a hydrogen molecule equal to that of an oxygen molecule at 47ºC?

(1) 80 K (2) –73 K

(3) 4 K (4) 20 K

Ans. (4)

Sol. $\sqrt{\frac{3RT}{2}} = \sqrt{\frac{3R(320)}{32}}$

$$T = \frac{320}{16} = 20\,K$$

50. A series L,R circuit connected with an ac source E = (25 sin 1000 t) V has a power factor of $\frac{1}{\sqrt{2}}$. If the source of emf is changed to E = (20 sin 2000 t)V, the new power factor of the circuit will be :

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{\sqrt{3}}$

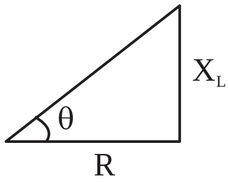
(3) $\frac{1}{\sqrt{5}}$ (4) $\frac{1}{\sqrt{7}}$

Ans. (3)

Sol. E = 25 sin (1000 t)

$$\cos\theta = \frac{1}{\sqrt{2}}$$

LR circuit



Initially $\frac{R}{\omega_1 L} = \frac{1}{\tan\theta} = \frac{1}{\tan 45^o} = 1$

$$X_L = \omega_1 L$$

$\omega_2 = 2\omega_1$, given

$$\tan\theta' = \frac{\omega_2 L}{R} = \frac{2\omega_1 L}{R}$$

$$tan\theta' = 2$$

$$\cos\theta' = \frac{1}{\sqrt{5}}$$

SECTION-B

51. The horizontal component of earth's magnetic field at a place is $3.5\times10^{-5}T$. A very long straight conductor carrying current of $\sqrt{2}A$ in the direction from South east to North West is placed. The force per unit length experienced by the conductor is ……… × 10^{-6} N/m.

Ans. (35)

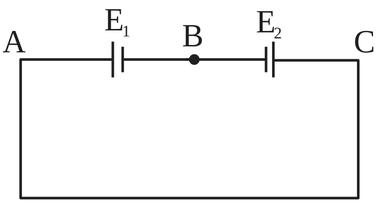
Sol. $B_H = 3.5\times10^{-5}T$

$$F = i\ell B\sin\theta,\quad \text{i} = \sqrt{2}A$$

$$\frac{F}{\ell} = iB\sin\theta = \sqrt{2}\times3.5\times10^{-5}\times\frac{1}{\sqrt{2}}$$

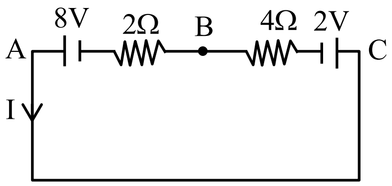
$$= 35\times10^{-6}\,N/m$$

52. Two cells are connected in opposition as shown. Cell E_1 is of 8 V emf and $2\,\Omega$ internal resistance; the cell E_2 is of 2 V emf and $4\,\Omega$ internal resistance. The terminal potential difference of cell E_2 is:



Ans. **(6)**

Sol.



$$I=\frac{8-2}{2+4}=\frac{6}{6}=1A$$

Applying Kirchhoff from C to B

$$V_C-2-4\times 1=V_B$$
$$V_C-V_B=6V$$

= 6V

53. A electron of hydrogen atom on an excited state is having energy E_n = – 0.85 eV. The maximum number of allowed transitions to lower energy level is …... .

Ans. **(6)**

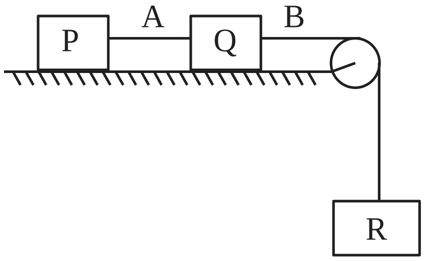
Sol. $E_n=-\frac{13.6}{n^2}=-0.85$

$\Rightarrow n=4$

No of transition

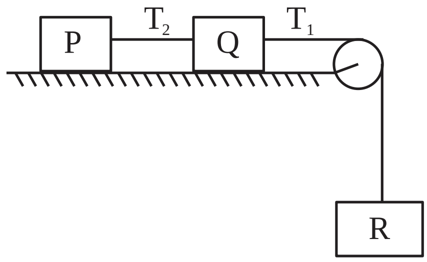
$$=\frac{n(n-1)}{2}=\frac{4(4-1)}{2}=6$$

54. Each of three blocks P, Q and R shown in figure has a mass of 3 kg. Each of the wire A and B has cross-sectional area 0.005 cm^2 and Young's modulus 2×10^{11} N m^{-2}. Neglecting friction, the longitudinal strain on wire B is _____ $\times 10^{-4}$. (Take g = 10 m/s^2)



Ans. **(2)**

Sol.



$$a=\frac{10}{3}m/s^2$$
$$30-T_1=3\times a$$
$$T_1=20\,N$$
$$strain=\frac{stress}{Y}$$
$$=2\times 10^{-4}$$

55. The distance between object and its two times magnified real image as produced by a convex lens is 45 cm. The focal length of the lens used is ______ cm.

Ans. **(10)**

Sol. $\frac{v}{u}=-2$

$v=-2u$ **…(i)**

$v-u=45$ $\quad$ …(ii)

$\Rightarrow u=-15\,cm$

$v=30\,cm$

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

$f=+10\ cm$

56. The displacement and the increase in the velocity of a moving particle in the time interval of t to (t + 1) s are 125 m and 50 m/s, respectively. The distance travelled by the particle in $(t + 2)^{th}$ s is ____ m.

Ans. (175)

Sol. Considering acceleration is constant

$$v = u + at$$
$$u + 50 = u + a \Rightarrow a = 50\ m/s^2$$
$$125 = ut + \frac{1}{2}at^2$$
$$125 = u + \frac{a}{2}$$
$$\Rightarrow u = 100\ \ m/s$$

$$\therefore S_{n^{th}} = u + \frac{a}{2}[2n - 1]$$

= 175 m

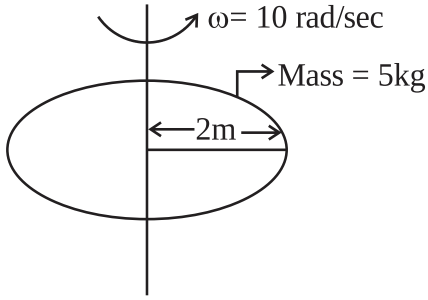
57. A capacitor of capacitance C and potential V has energy E. It is connected to another capacitor of capacitance 2 C and potential 2V. Then the loss of energy is $\frac{x}{3}E$, where x is _____ .

Ans. (2)

Sol. Energy loss = $\frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$

$$= \frac{2}{3}.E$$
$$\therefore x = 2$$

58. Consider a Disc of mass 5 kg, radius 2m, rotating with angular velocity of 10 rad/s about an axis perpendicular to the plane of rotation. An identical disc is kept gently over the rotating disc along the same axis. The energy dissipated so that both the discs continue to rotate together without slipping is _____ J.



Ans. (250)

Sol. $\vec{L}_i = I\omega_i = \frac{MR^2}{2}.\omega = 100\ kgm^2/s$

$$E_i = \frac{1}{2}.\frac{MR^2}{2}.\omega^2 = 500\,\mathbf{J}$$

$$\vec{L}_i = \vec{L}_f \Rightarrow 100 = 2I\omega_f$$

ω_f = 5 rad/sec

$$E_f = 2 \times \frac{1}{2}.\frac{5(2)^2}{2}.(5)^2 = 250\,J$$

$$\Delta E = 250\,J$$

59. In a closed organ pipe, the frequency of fundamental note is 30 Hz. A certain amount of water is now poured in the organ pipe so that the fundamental frequency is increased to 110 Hz. If the organ pipe has a cross-sectional area of 2 cm^2, the amount of water poured in the organ tube is _______ g. (Take speed of sound in air is 330 m/s)

Ans. (400)

Sol. $\frac{V}{4\ell_1} = 30 \Rightarrow \ell_1 = \frac{11}{4}m$

$\frac{V}{4\ell_2} = 110 \Rightarrow \ell_2 = \frac{3}{4}m$

$\Delta\ell = 2m,$

Change in volume = $A\Delta\ell = 400\,cm^3$

M = 400 g ; $\left(\because \rho = 1g/cm^3\right)$

60. A ceiling fan having 3 blades of length 80 cm each is rotating with an angular velocity of 1200 rpm. The magnetic field of earth in that region is 0.5 G and angle of dip is 30º. The emf induced across the blades is $N\pi \times 10^{-5} V$. The value of N is _____ .

Ans. (32)

Sol. $B_v = B\sin 30 = \frac{1}{4} \times 10^{-4}$

$\omega = 2\pi \times f = \frac{2\pi}{60} \times 1200 \ \ rad/s$

$\varepsilon = \frac{1}{2} B_V \omega \ell^2$

$= 32\pi \times 10^{-5}$ V

SECTION-A

61. Given below are two statements:

Statement-I: The gas liberated on warming a salt with dil H_2SO_4, turns a piece of paper dipped in lead acetate into black, it is a confirmatory test for sulphide ion.

Statement-II: In statement-I the colour of paper turns black because of formation of lead sulphite.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both Statement-I and Statement-II are false
(2) Statement-I is false but Statement-II is true
(3) Statement-I is true but Statement-II is false
(4) Both Statement-I and Statement-II are true.

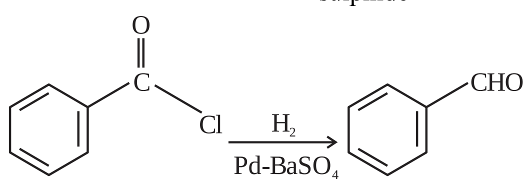
Ans. **(3)**

Sol. $Na_2S + H_2SO_4 \rightarrow Na_2SO_4 + H_2S$

$(CH_3COO)_2Pb + H_2S \rightarrow PbS + 2CH_3COOH$

Black lead sulphide

62.

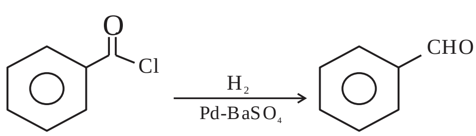


This reduction reaction is known as:

(1) Rosenmund reduction
(2) Wolff-Kishner reduction
(3) Stephen reduction
(4) Etard reduction

Ans. (1)

Sol.



It is known as rosenmund reduction that is the partial reduction of acid chloride to aldehyde

63. Sugar which does not give reddish brown precipitate with Fehling's reagent is:

(1) Sucrose (2) Lactose
(3) Glucose (4) Maltose

Ans. **(1)**

Sol. Sucrose do not contain hemiacetal group.
Hence it does not give test with Fehling solution.
While all other give positive test with Fehling solution

64. Given below are the two statements: one is labeled as Assertion (A) and the other is labeled as Reason (R).

Assertion (A): There is a considerable increase in covalent radius from N to P. However from As to Bi only a small increase in covalent radius is observed.

Reason (R): covalent and ionic radii in a particular oxidation state increases down the group.

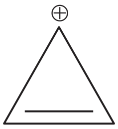
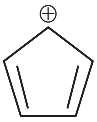
In the light of the above statement, choose the most appropriate answer from the options given below:

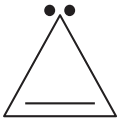
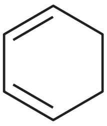
(1) (A) is false but (R) is true
(2) Both (A) and (R) are true but (R) is not the correct explanation of (A)
(3) (A) is true but (R) is false
(4) Both (A) and (R) are true and (R) is the correct explanation of (A)

Ans. **(2)**

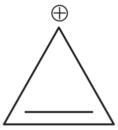
Sol. According to NCERT,
Statement-I : Factual data,
Statement-II is true.
But correct explanation is presence of completely filled d and f-orbitals of heavier members

65. Which of the following molecule/species is most stable?

(1)  (2) 

(3)  (4) 

Ans. (1)

Sol.  it is aromatic species

66. Diamagnetic Lanthanoid ions are:

(1) Nd^{3+} and Eu^{3+} (2) La^{3+} and Ce^{4+}

(3) Nd^{3+} and Ce^{4+} (4) Lu^{3+} and Eu^{3+}

Ans. (2)

Sol. Ce : [Xe] $4f^1 5d^1 6s^2$; Ce^{4+} diamagnetic

La : [Xe] $4f^0 5d^1 6s^2$; La^{3+} diamagnetic

67. Aluminium chloride in acidified aqueous solution forms an ion having geometry

(1) Octahedral

(2) Square Planar

(3) Tetrahedral

(4) Trigonal bipyramidal

Ans. (1)

Sol. $AlCl_3$ in acidified aqueous solution forms octahedral geometry $[Al(H_2O)_6]^{3+}$

68. Given below are two statements:

Statement-I: The orbitals having same energy are called as degenerate orbitals.

Statement-II: In hydrogen atom, 3p and 3d orbitals are not degenerate orbitals.

In the light of the above statements, choose the **most appropriate** answer from the options given

(1) Statement-I is true but Statement-II is false

(2) Both Statement-I and Statement-II are true.

(3) Both Statement-I and Statement-II are false

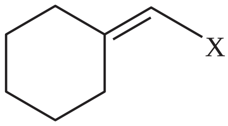
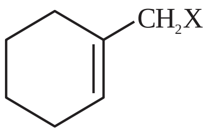
(4) Statement-I is false but Statement-II is true

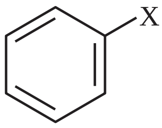
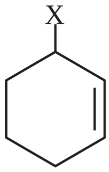
Ans. (1)

Sol. For single electron species the energy depends upon principal quantum number 'n' only. So, statement II is false.

Statement I is correct definition of degenerate orbitals.

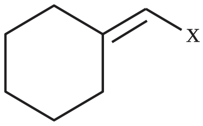
69. Example of vinylic halide is

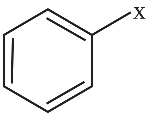
(1)  (2) 

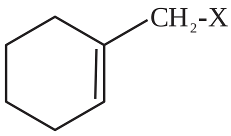
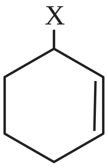
(3)  (4) 

Ans. (1)

Sol. Vinyl carbon is sp^2 hybridized aliphatic carbon

 is vinyl halide

While  is aryl halide

and   are allyl halide

70. Structure of 4-Methylpent-2-enal is

(1) $H_2C=C(H)-C(H)(CH_3)-CH_2-C(=O)-H$

(2) $CH_3-CH_2-C(CH_3)=CH-C(=O)-H$

(3) $CH_3-CH_2-CH=C(CH_3)-C(=O)-H$

(4) $CH_3-CH(CH_3)-CH=CH-C(=O)-H$

Ans. (4)

Sol. $\overset{5}{CH_3}-\overset{4}{CH}(CH_3)-\underset{3}{CH}=\underset{2}{CH}-\underset{1}{C}(=O)-H$

4-Methylpent-2-enal

71. Match List-I with List-II

List-I	**List-II**
Molecule	**Shape**
(A) BrF_5	(I) T-shape
(B) H_2O	(II) See saw
(C) ClF_3	(III) Bent
(D) SF_4	(IV) Square pyramidal

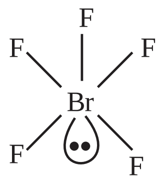
(1) (A)-I, (B)-II, (C)-IV, (D)-III

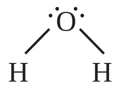
(2) (A) –II, (B)-I, (C)-III, (D)-IV

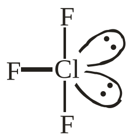
(3) (A)-III, (B)-IV, (C)-I, (D)-II

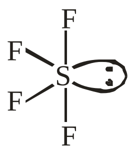
(4) (A)-IV, (B)-III, (C)-I, (D)-II

Ans. (4)

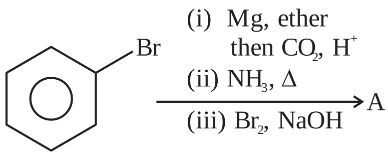
Sol. BrF_5  Square pyramidal

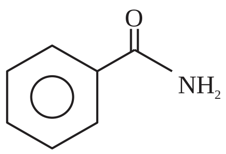
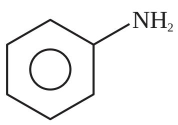
H_2O  Bent

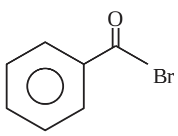
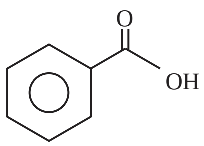
ClF_3  T-shape

SF_4,  See-saw

72. The final product A, formed in the following multistep reaction sequence is:

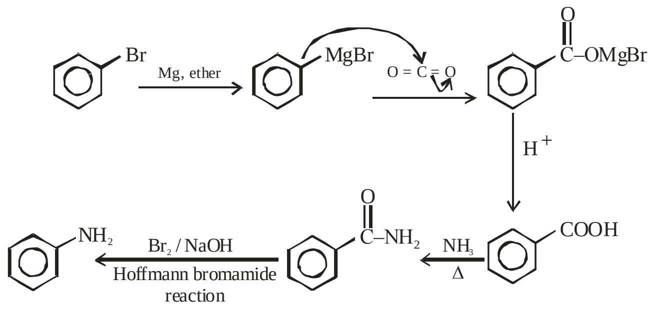


(1)  (2) 

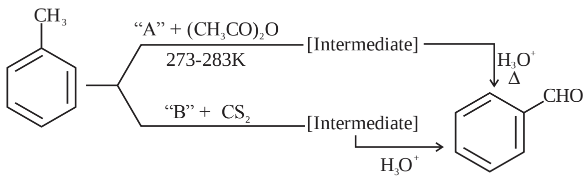
(3)  (4) 

Ans. (2)

Sol.



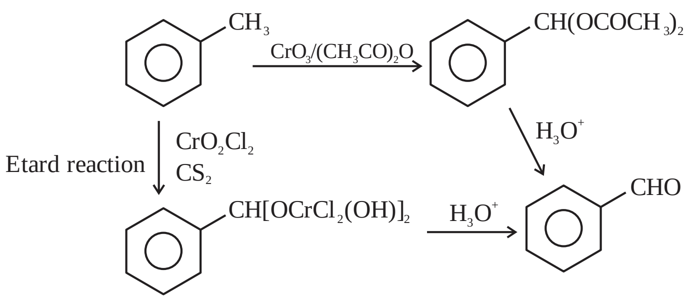
73. In the given reactions identify the reagent A and reagent B



(1) A-CrO_3	B-CrO_3
(2) A-CrO_3	B-CrO_2Cl_2
(3) A-CrO_2Cl_2	B-CrO_2Cl_2
(4) A-CrO_2Cl_2	B-CrO_3

Ans. (2)

Sol.



74. Given below are two statement one is labeled as **Assertion (A)** and the other is labeled as **Reason (R).**

Assertion (A): $CH_2 = CH - CH_2 - Cl$ is an example of allyl halide

Reason (R): Allyl halides are the compounds in which the halogen atom is attached to sp^2 hybridised carbon atom.

In the light of the two above statements, choose the **most appropriate** answer from the options given below:

(1) **(A)** is true but **(R)** is false

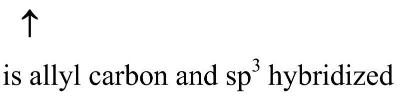
(2) Both **(A)** and **(R)** are true but **(R)** is **not** the correct explanation of **(A)**

(3) **(A)** is false but **(R)** is true

(4) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

Ans. (1)

Sol. $CH_2 = CH - CH_2 - Cl$



75. What happens to freezing point of benzene when small quantity of napthalene is added to benzene?

(1) Increases

(2) Remains unchanged

(3) First decreases and then increases

(4) Decreases

Ans. (4)

Sol. On addition of naphthalene to benzene there is depression in freezing point of benzene.

76. Match List-I with List-II

List-I	**List-II**
Species	**Electronic distribution**
(A) Cr^{+2}	(I) $3d^8$
(B) Mn^+	(II) $3d^3 4s^1$
(C) Ni^{+2}	(III) $3d^4$
(D) V^+	(IV) $3d^5 4s^1$

Choose the correct answer from the options given below:

(1) (A)-I, (B)-II, (C)-III, (D)-IV

(2) (A)-III, (B) – IV, (C) – I, (D)-II

(3) (A)-IV, (B)-III, (C)-I, (D)-II

(4) (A)-II, (B)-I, (C)-IV, (D)-III

Ans. (2)

Sol. ${}_{24}Cr \rightarrow [Ar]\ 3d^5 4s^1;\ Cr^{2+} \rightarrow [Ar]\ 3d^4$

${}_{25}Mn \rightarrow [Ar]\ 3d^5 4s^2;\ Mn^+ \rightarrow [Ar]\ 3d^5 4s^1$

${}_{28}Ni \rightarrow [Ar]\ 3d^8 4s^2;\ Ni^{2+} \rightarrow [Ar]\ 3d^8$

${}_{23}V \rightarrow [Ar]\ 3d^3 4s^2;\ V^+ \rightarrow [Ar]\ 3d^3 4s^1$

77. Compound A formed in the following reaction reacts with B gives the product C. Find out A and B.

$$CH_3 - C \equiv CH + Na \rightarrow A \xrightarrow{B} \underset{(C)}{CH_3 - C \equiv C - CH_2 - \underset{\underset{CH_3}{|}}{CH_2}} + NaBr$$

(1) $A = CH_3 - C \equiv \bar{C}\overset{+}{Na}$, $B = CH_3 - CH_2 - CH_2 - Br$

(2) $A = CH_3 - CH = CH_2$, $B = CH_3 - CH_2 - CH_2 - Br$

(3) $A = CH_3 - CH_2 - CH_3$, $B = CH_3 - C \equiv CH$

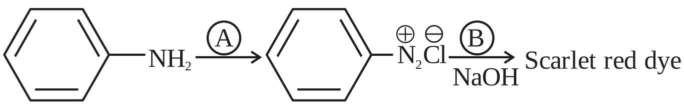
(4) $A = CH_3 - C \equiv \bar{C}\overset{+}{Na}$, $B = CH_3 - CH_2 - CH_3$

Ans. (1)

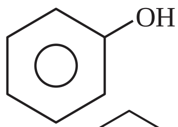
Sol.

$$CH_3 - C \equiv CH \xrightarrow{Na} CH_3 - C \equiv C^- Na^+ \xrightarrow{CH_3CH_2CH_2 - Br} NaBr + CH_3 - C \equiv C - CH_2CH_2CH_3$$

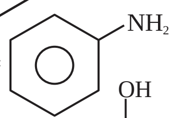
78. Following is a confirmatory test for aromatic primary amines. Identify reagent (A) and (B)



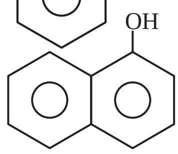
(1) $A = HNO_3/H_2SO_4$; B =



(2) $A = NaNO_2 + HCl,\ 0 - 5°C$; B =

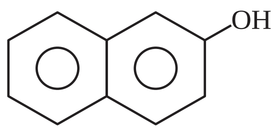


(3) $A = NaNO_2 + HCl,\ 0 - 5°C$; B =



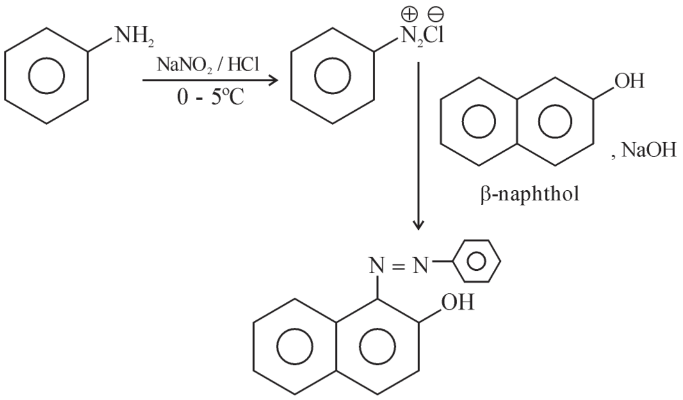
(4) $A = NaNO_2 + HCl,\ 0 - 5°C$;

B =



Ans. (4)

Sol.



Scarlet red dye

79. The Lassiagne's extract is boiled with dil HNO_3 before testing for halogens because,

(1) AgCN is soluble in HNO_3

(2) Silver halides are soluble in HNO_3

(3) Ag_2S is soluble in HNO_3

(4) Na_2S and NaCN are decomposed by HNO_3

Ans. **(4)**

Sol. If nitrogen or sulphur is also present in the compound, the sodium fusion extract is first boiled with concentrated nitric acid to decompose cyanide or sulphide of sodium during Lassaigne's test

80. Choose the correct Statements from the following:

(A) Ethane-1 2-diamine is a chelating ligand.

(B) Metallic aluminium is produced by elecrtrolysis of aluminium oxide in presence of cryolite.

(C) Cyanide ion is used as ligand for leaching of silver.

(D) Phosphine act as a ligand in Wilkinson catalyst.

(E) The stability constants of Ca^{2+} and Mg^{2+} are similar with EDTA complexes.

Choose the correct answer from the options given below:

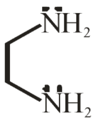
(1) (B), (C), (E) only

(2) (C), (D), (E) only

(3) (A), (B), (C) only

(4) (A), (D), (E) only

Ans. **(3)**

Sol. 

Bidentate, chelating

Based on Hall-Heroults process

$[Rh(PPh_3)_3Cl]$ Wilkinson's catalyst

$$Ag_2S + NaCN \xrightleftharpoons{Air} Na[Ag(CN)_2] + Na_2S$$

Ca^{++} ion forms more stable complex with EDTA

SECTION-B

81. The rate of first order reaction is 0.04 mol L^{-1} s^{-1} at 10 minutes and 0.03 mol L^{-1} s^{-1} at 20 minutes after initiation. Half life of the reaction is _______ minutes. (Given log2=0.3010, log3=0.4771)

Ans. **(24)**

Sol. $0.04 = k[A]_0 e^{-k \times 10 \times 60}$ ...(1)

$0.03 = k[A]_0 e^{-k \times 20 \times 60}$ ...(2)

(1)/(2)

$$\frac{4}{3} = e^{600k(2-1)}$$

$$\frac{4}{3} = e^{600k}$$

$$\ln\frac{4}{3} = 600k$$

$$\ln\frac{4}{3} = 600 \times \frac{\ln 2}{t_{1/2}}$$

$$t_{1/2} = 600\frac{\ln 2}{\ln\frac{4}{3}} \text{sec}$$

$$t_{1/2} = 600 \times \frac{\log 2}{\log 4 - \log 3} \text{ sec.} = 10 \times \frac{0.3010}{0.6020 - 0.477} \text{ min}$$

$t_{1/2} = 24.08$ min

Ans. 24

82. The pH at which $Mg(OH)_2$ [$K_{sp} = 1 \times 10^{-11}$] begins to precipitate from a solution containing 0.10 M Mg^{2+} ions is ______

Ans. **(09)**

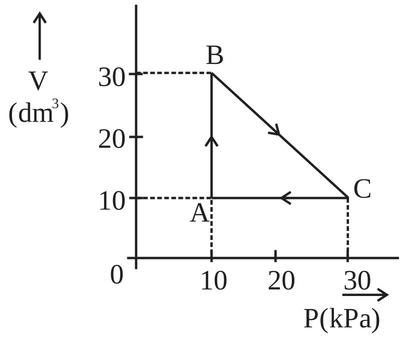
Sol. Precipitation when $Q_{sp} = K_{sp}$

$[Mg^{2+}][OH^-]^2 = 10^{-11}$

$0.1 \times [OH^-]^2 = 10^{-11} \Rightarrow [OH^-] = 10^{-5}$

$\Rightarrow pOH = 5 \qquad \Rightarrow pH = 9$

83.



An ideal gas undergoes a cyclic transformation starting from the point A and coming back to the same point by tracing the path A → B → C → A as shown in the diagram. The total work done in the process is _____ J.

Ans. (200)

Sol. Work done is given by area enclosed in the P vs V cyclic graph or V vs P cyclic graph.

Sign of work is positive for clockwise cyclic process for V vs P graph.

$$W = \frac{1}{2} \times (30 - 10) \times (30 - 10) = 200 \text{ kPa} - \text{dm}^3$$

$= 200 \times 1000$ Pa – L = 2 L-bar = 200 J

84. if IUPAC name of an element is "Unununnium" then the element belongs to nth group of periodic table. The value of n is______

Ans. (11)

Sol. 111 belongs to 11^{th} group

85. The total number of molecular orbitals formed from 2s and 2p atomic orbitals of a diatomic molecule

Ans. (08)

Sol. Two molecular orbitals σ 2s and σ*2s.

Six molecular orbitals σ $2p_z$ and σ*$2p_z$.

$\pi 2p_x$, $\pi 2p_y$ and $\pi^* 2p_x$, $\pi^* 2p_y$

86. On a thin layer chromatographic plate, an organic compound moved by 3.5 cm, while the solvent moved by 5 cm. The retardation factor of the organic compound is _____ $\times 10^{-1}$

Ans. (07)

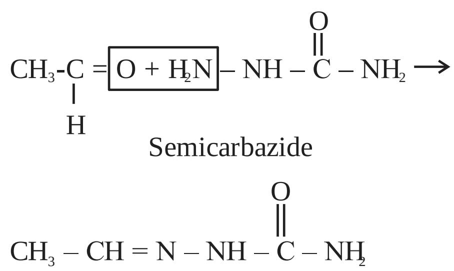
Sol. Retardation factor = $\dfrac{\text{Distance travelled by sample/organic compound}}{\text{Distance travelled by solvent}}$

$$= \frac{3.5}{5} = 7 \times 10^{-1}$$

87. The compound formed by the reaction of ethanal with semicarbazide contains _____number of nitrogen atoms.

Ans. (03)

Sol.



88. 0.05 cm thick coating of silver is deposited on a plate of 0.05 m^2 area. The number of silver atoms deposited on plate are _____ $\times 10^{23}$. (At mass Ag = 108, d = 7.9 g cm^{-3})

Ans. (11)

Sol. Volume of silver coating = 0.05 × 0.05 × 10000 = 25 cm^3

Mass of silver deposited = 25 × 7.9 g

Moles of silver atoms = $\dfrac{25 \times 7.9}{108}$

Number of silver atoms = $\dfrac{25 \times 7.9}{108} \times 6.023 \times 10^{23}$

$= 11.01 \times 10^{23}$

Ans. 11

89. $2MnO_4^- + bI^- + cH_2O \rightarrow xI_2 + yMnO_2 + zOH^-$

If the above equation is balanced with integer coefficients, the value of z is _______

Ans. **(08)**

Sol. **Reduction Half** **Oxidation Half**

$2MnO_4^- \rightarrow 2MnO_2$ $2I^- \rightarrow I_2 + 2e^-$

$2MnO_4^- + 4H_2O + 6e^- \rightarrow 2MnO_2 + 8OH^-$ $6I^- \rightarrow 3I_2 + 6e^-$

Adding oxidation half and reduction half, net reaction is

$2MnO_4^- + 6I^- + 4H_2O \rightarrow 3I_2 + 2MnO_2 + 8OH^-$

$\Rightarrow z = 8$

$\Rightarrow$ Ans 8

90. The mass of sodium acetate (CH_3COONa) required to prepare 250 mL of 0.35 M aqueous solution is ______ g. (Molar mass of CH_3COONa is 82.02 g mol^{-1})

Ans. **(7)**

Sol. Moles = Molarity × Volume in litres

= 0.35 × 0.25

Mass = moles × molar mass

= 0.35 × 0.25 × 82.02 = 7.18 g

Ans. 7

FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

(Held On Tuesday 30th January, 2024) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS	TEST PAPER WITH SOLUTION

SECTION-A

1. Consider the system of linear equations

$x + y + z = 5$, $x + 2y + \lambda^2 z = 9$,

$x + 3y + \lambda z = \mu$, where $\lambda, \mu \in \mathbb{R}$. Then, which of the following statement is NOT correct?

(1) System has infinite number of solution if $\lambda = 1$ and $\mu = 13$

(2) System is inconsistent if $\lambda = 1$ and $\mu \neq 13$

(3) System is consistent if $\lambda \neq 1$ and $\mu = 13$

(4) System has unique solution if $\lambda \neq 1$ and $\mu \neq 13$

Ans. (4)

Sol. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \lambda^2 \\ 1 & 3 & \lambda \end{vmatrix} = 0$

$\Rightarrow 2\lambda^2 - \lambda - 1 = 0$

$\lambda = 1, -\frac{1}{2}$

$\begin{vmatrix} 1 & 1 & 5 \\ 2 & \lambda^2 & 9 \\ 3 & \lambda & \mu \end{vmatrix} = 0 \Rightarrow \mu = 13$

Infinite solution $\lambda = 1$ & $\mu = 13$

For unique sol^n $\lambda \neq 1$

For no sol^n $\lambda = 1$ & $\mu \neq 13$

If $\lambda \neq 1$ and $\mu \neq 13$

Considering the case when $\lambda = -\frac{1}{2}$ and $\mu \neq 13$ this will generate no solution case

2. For $\alpha, \beta \in \left(0, \frac{\pi}{2}\right)$, let $3\sin(\alpha+\beta) = 2\sin(\alpha-\beta)$ and a real number k be such that $\tan\alpha = k\tan\beta$. Then the value of k is equal to :

(1) $-\frac{2}{3}$ (2) -5

(3) $\frac{2}{3}$ (4) 5

Ans. (2)

Sol. $3\sin\alpha\cos\beta + 3\sin\beta\cos\alpha$

$= 2\sin\alpha\cos\beta - 2\sin\beta\cos\alpha$

$5\sin\beta\cos\alpha = -\sin\alpha\cos\beta$

$\tan\beta = -\frac{1}{5}\tan\alpha$

$\tan\alpha = -5\tan\beta$

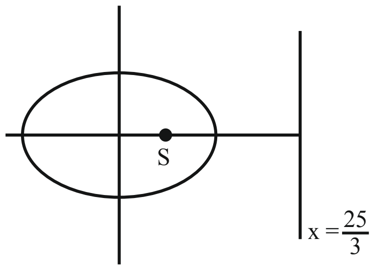
3. Let $A(\alpha, 0)$ and $B(0, \beta)$ be the points on the line $5x + 7y = 50$. Let the point P divide the line segment AB internally in the ratio 7 : 3. Let $3x - 25 = 0$ be a directrix of the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the corresponding focus be S. If from S, the perpendicular on the x-axis passes through P, then the length of the latus rectum of E is equal to

(1) $\frac{25}{3}$ (2) $\frac{32}{9}$

(3) $\frac{25}{9}$ (4) $\frac{32}{5}$

Ans. (4)

Sol. $\left.\begin{matrix} A = (10, 0) \\ B = \left(0, \frac{50}{7}\right) \end{matrix}\right\} P = (3, 5)$



$ae = 3$

$\frac{a}{e} = \frac{25}{3}$

$a = 5$

$b = 4$

Length of $LR = \frac{2b^2}{a} = \frac{32}{5}$

4. Let $\vec{a}=\hat{i}+\alpha\hat{j}+\beta\hat{k},\ \alpha,\beta\in R$. Let a vector $\vec{b}$ be such that the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$ and $\left|\vec{b}\right|^2=6$, If $\vec{a}.\vec{b}=3\sqrt{2}$, then the value of $\left(\alpha^2+\beta^2\right)\left|\vec{a}\times\vec{b}\right|^2$ is equal to

(1) 90 (2) 75

(3) 95 (4) 85

Ans. (1)

Sol. $|\vec{b}|^2=6\ ;\ |\vec{a}||\vec{b}|\cos\theta=3\sqrt{2}$

$|\vec{a}|^2\ |\vec{b}|^2\ \cos^2\theta=18$

$|\vec{a}|^2=6$

Also $1+\alpha^2+\beta^2=6$

$\alpha^2+\beta^2=5$

to find

$(\alpha^2+\beta^2)\ |\vec{a}|^2|\vec{b}|^2\sin^2\theta$

$=(5)(6)(6)\left(\frac{1}{2}\right)$

$=\ 90$

5. Let $f(x)=(x+3)^2(x-2)^3, x\in[-4, 4]$. If M and m are the maximum and minimum values of f, respectively in [–4, 4], then the value of M – m is :

(1) 600 (2) 392

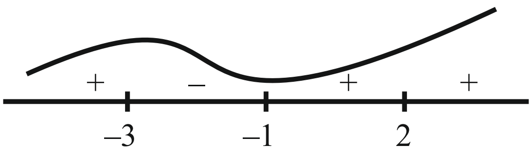
(3) 608 (4) 108

Ans. (3)

Sol. $f'(x) = (x + 3)^2 . 3(x - 2)^2 + (x -2)^3\, 2(x + 3)$

$= 5(x + 3)\ (x - 2)^2\ (x + 1)$

$f'(x) = 0,\ x = -3, -1, 2$



$f(-4) = -216$

$f(-3) = 0,\ f(4) = 49 \times 8 = 392$

$M = 392,\ m = -216$

$M - m = 392 + 216 = 608$

Ans = '3'

6. Let a and b be be two distinct positive real numbers. Let 11th term of a GP, whose first term is a and third term is b, is equal to pth term of another GP, whose first term is a and fifth term is b. Then p is equal to

(1) 20 (2) 25

(3) 21 (4) 24

Ans. (3)

Sol. 1st GP $\Rightarrow t_1 = a,\ t_3 = b = ar^2 \Rightarrow r^2 = \frac{b}{a}$

$t_{11} = ar^{10} = a(r^2)^5 = a\cdot\left(\frac{b}{a}\right)^5$

2nd G.P. $\Rightarrow T_1 = a,\ T_5 = ar^4 = b$

$\Rightarrow r^4 = \left(\frac{b}{a}\right) \Rightarrow r = \left(\frac{b}{a}\right)^{1/4}$

$T_p = ar^{p-1} = a\left(\frac{b}{a}\right)^{\frac{p-1}{4}}$

$t_{11} = T_p \Rightarrow a\left(\frac{b}{a}\right)^5 = a\left(\frac{b}{a}\right)^{\frac{p-1}{4}}$

$\Rightarrow\ 5=\frac{p-1}{4} \Rightarrow p=21$

7. If $x^2 - y^2 + 2hxy + 2gx + 2fy + c = 0$ is the locus of a point, which moves such that it is always equidistant from the lines $x + 2y + 7 = 0$ and $2x - y + 8 = 0$, then the value of $g + c + h - f$ equals

(1) 14 (2) 6

(3) 8 (4) 29

Ans. (1)

Sol. Cocus of point P(x, y) whose distance from Gives

X + 2y + 7 = 0 & 2x – y + 8 = 0 are equal is

$\frac{x+2y+7}{\sqrt{5}}=\pm\frac{2x-y+8}{\sqrt{5}}$

$(x + 2y + 7)^2 - (2x - y + 8)^2 = 0$

Combined equation of lines

$(x - 3y + 1)(3x + y + 15) = 0$

$3x^2 - 3y^2 - 8xy + 18x - 44y + 15 = 0$

$x^2 - y^2 - \frac{8}{3}xy + 6x - \frac{44}{3}y + 5 = 0$

$x^2 - y^2 + 2h\,xy + 2gx\,2 + 2fy + c = 0$

$h = \frac{4}{3},\ g = 3,\ f = -\frac{22}{3},\ c = 5$

$g + c + h - f = 3 + 5 - \frac{4}{3} + \frac{22}{3} = 8 + 6 = 14$

8. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{b}| = 1$ and $|\vec{b} \times \vec{a}| = 2$. Then $\left|(\vec{b} \times \vec{a}) - \vec{b}\right|^2$ is equal to

(1) 3

(2) 5

(3) 1

(4) 4

Ans. (2)

Sol. $|\vec{b}| = 1$ & $|\vec{b} \times \vec{a}| = 2$

$\left(\vec{b} \times \vec{a}\right) \cdot \vec{b} = \vec{b} \cdot \left(\vec{b} \times \vec{a}\right) = 0$

$\left|(\vec{b} \times \vec{a}) - \vec{b}\right|^2 = \left|\vec{b} \times \vec{a}\right|^2 + \left|\vec{b}\right|^2$

$= 4 + 1 = 5$

9. Let $y = f(x)$ be a thrice differentiable function in $(-5, 5)$. Let the tangents to the curve $y = f(x)$ at $(1, f(1))$ and $(3, f(3))$ make angles $\frac{\pi}{6}$ and $\frac{\pi}{4}$, respectively with positive x-axis. If $27\int_1^3 \left(\left(f'(t)\right)^2 + 1\right) f''(t)dt = \alpha + \beta\sqrt{3}$ where α, β are integers, then the value of $\alpha + \beta$ equals

(1) –14

(2) 26

(3) –16

(4) 36

Ans. (2)

Sol. $y = f(x) \Rightarrow \frac{dy}{dx} = f'(x)$

$\left.\frac{dy}{dx}\right)_{(1,f(1))} = f'(1) = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}} \Rightarrow f'(1) = \frac{1}{\sqrt{3}}$

$\left.\frac{dy}{dx}\right)_{(3,f(3))} = f'(3) = \tan\frac{\pi}{4} = 1 \Rightarrow f'(3) = 1$

$27\int_1^3 \left(\left(f'(t)\right)^2 + 1\right) f''(t)dt = \alpha + \beta\sqrt{3}$

$I = \int_1^3 \left(\left(f'(t)\right)^2 + 1\right) f''(t)dt$

$f'(t) = z \Rightarrow f''(t)\,dt = dz$

$z = f'(3) = 1$

$z = f'(1) = \frac{1}{\sqrt{3}}$

$I = \int_{1/\sqrt{3}}^{1} (z^2 + 1)dz = \left(\frac{z^3}{3} + z\right)_{1/\sqrt{3}}^{1}$

$= \left(\frac{1}{3} + 1\right) - \left(\frac{1}{3} \cdot \frac{1}{3\sqrt{3}} + \frac{1}{\sqrt{3}}\right)$

$= \frac{4}{3} - \frac{10}{9\sqrt{3}} = \frac{4}{3} - \frac{10}{27}\sqrt{3}$

$\alpha + \beta\sqrt{3} = 27\left(\frac{4}{3} - \frac{10}{27}\sqrt{3}\right) = 36 - 10\sqrt{3}$

$\alpha = 36,\ \beta = -10$

$\alpha + \beta = 36 - 10 = 26$

10. Let P be a point on the hyperbola $H: \frac{x^2}{9} - \frac{y^2}{4} = 1$, in the first quadrant such that the area of triangle formed by P and the two foci of H is $2\sqrt{13}$. Then, the square of the distance of P from the origin is

(1) 18

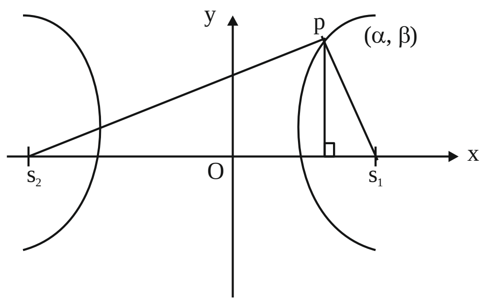
(2) 26

(3) 22

(4) 20

Ans. (3)

Sol.



$\frac{x^2}{9}-\frac{y^2}{4}=1$

$a^2 = 9, b^2 = 4$

$b^2=a^2(e^2-1) \Rightarrow e^2=1+\frac{b^2}{a^2}$

$e^2=1+\frac{4}{9}=\frac{13}{9}$

$e=\frac{\sqrt{13}}{3} \Rightarrow s_1s_2=2ae=2\times 3\times\sqrt{\frac{13}{3}}=2\sqrt{13}$

Area of $\Delta PS_1S_2=\frac{1}{2}\times\beta\times s_1s_2=2\sqrt{13}$

$\Rightarrow\frac{1}{2}\times\beta\times(2\sqrt{13})=2\sqrt{13}\Rightarrow\beta=2$

$\frac{\alpha^2}{9}-\frac{\beta^2}{4}=1\Rightarrow\frac{\alpha^2}{9}-1=1\Rightarrow\alpha^2=18\Rightarrow\alpha=3\sqrt{2}$

Distance of P from origin = $\sqrt{\alpha^2+\beta^2}$

$=\sqrt{18+4}=\sqrt{22}$

11. Bag A contains 3 white, 7 red balls and bag B contains 3 white, 2 red balls. One bag is selected at random and a ball is drawn from it. The probability of drawing the ball from the bag A, if the ball drawn in white, is :

(1) $\frac{1}{4}$ (2) $\frac{1}{9}$

(3) $\frac{1}{3}$ (4) $\frac{3}{10}$

Ans. (3)

Sol. E_1 : A is selected

A	B
3W 7R	3W 2R

E_2 : B is selected

E : white ball is drawn

P (E_1/E) =

$$\frac{P(E).P(E/E_1)}{P(E_1).P(E/E_1)+P(E_2).P(E/E_2)}=\frac{\frac{1}{2}\times\frac{3}{10}}{\frac{1}{2}\times\frac{3}{10}+\frac{1}{2}\times\frac{3}{5}}$$

$=\frac{3}{3+6}=\frac{1}{3}$

12. Let f : R→ R be defined $f(x)=ae^{2x}+be^{x}+cx$. If $f(0)=-1$, $f'(\log_e 2)=21$ and $\int_0^{\log_e 4}(f(x)-cx)dx=\frac{39}{2}$, then the value of $|a+b+c|$ equals :

(1) 16 (2) 10

(3) 12 (4) 8

Ans. (4)

Sol. $f(x) = ae^{2x} + be^x + cx$ $\quad f(0) = -1$

$a + b = -1$

$f'(x) = 2ae^{2x} + be^x + c$ $\quad f'(\ln 2) = 21$

$8a + 2b + c = 21$

$\int_0^{\ln 4}(ae^{2x}+be^x)dx=\frac{39}{2}$

$\left[\frac{ae^{2x}}{2}+be^x\right]_0^{\ln 4}=\frac{39}{2}\Rightarrow 8a+4b-\frac{a}{2}-b=\frac{39}{2}$

$15a + 6b = 39$

$15a - 6a - 6 = 39$

$9a = 45 \Rightarrow a = 5$

$b = -6$

$c = 21 - 40 + 12 = -7$

$a + b + c - 8$

$|a + b + c| = 8$

13. Let $L_1 : \vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \lambda(\hat{i} - \hat{j} + 2\hat{k}),\ \lambda \in R$

$L_2 : \vec{r} = (\hat{j} - \hat{k}) + \mu(3\hat{i} + \hat{j} + p\hat{k}),\ \mu \in R$ and

$L_3 : \vec{r} = \delta(\ell\hat{i} + m\hat{j} + n\hat{k})\ \delta \in R$

Be three lines such that L_1 is perpendicular to L_2 and L_3 is perpendicular to both L_1 and L_2. Then the point which lies on L_3 is

(1) (–1, 7, 4) (2) (–1, –7, 4)

(3) (1, 7, –4) (4) (1, –7, 4)

Ans. (1)

Sol. $L_1 \perp L_2$ $\qquad$ $L_3 \perp L_1,\ L_2$

$3 - 1 + 2P = 0$

$P = -1$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 3 & 1 & -1 \end{vmatrix} = -\hat{i} + 7\hat{j} + 4\hat{k}$$

$\therefore$ $(-\delta, 7\delta, 4\delta)$ will lie on L_3

For $\delta = 1$ the point will be (-1, 7, 4)

14. Let a and b be real constants such that the function f defined by $f(x) = \begin{cases} x^2 + 3x + a, & x \le 1 \\ bx + 2, & x > 1 \end{cases}$ be differentiable on R. Then, the value of $\int_{-2}^{2} f(x)\,dx$ equals

(1) $\frac{15}{6}$ (2) $\frac{19}{6}$

(3) 21 (4) 17

Ans. (4)

Sol. f is continuous $\qquad$ $f'(x) = 2x + 3\ ,\ k < 1$

$\therefore$ $4 + a = b + 2$ $\qquad$ $b\ ,\ x > 1$

$a = b - 2$ $\qquad$ f is differentiable

$\therefore$ $b = 5$

$\therefore$ $a = 3$

$$\int_{-2}^{1}(x^2 + 3x + 3)dx + \int_{1}^{2}(5x + 2)dx$$

$$= \left[\frac{x^3}{3} + \frac{3x^2}{2} + 3x\right]_{-2}^{1} + \left[\frac{5x^2}{2} + 2x\right]_{1}^{2}$$

$$= \left(\frac{1}{3} + \frac{3}{2} + 3\right) - \left(\frac{-8}{3} + 6 - 6\right) + \left(10 + 4 - \frac{5}{2} - 2\right)$$

$$= 6 + \frac{3}{2} + 12 - \frac{5}{2} = 17$$

15. Let $f : \mathbb{R} - \{0\} \to \mathbb{R}$ be a function satisfying $f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}$ for all x, y, $f(y) \neq 0$. If $f'(1) = 2024$, then

(1) $xf'(x) - 2024\,f(x) = 0$

(2) $xf'(x) + 2024f(x) = 0$

(3) $xf'(x) + f(x) = 2024$

(4) $xf'(x) - 2023f(x) = 0$

Ans. (1)

Sol. $f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}$ $\qquad$ $f'(1) = 2024$

$f(1) = 1$

Partially differentiating w. r. t. x

$$f'\left(\frac{x}{y}\right).\frac{1}{y} = \frac{1}{f(y)}f'(x)$$

$y \to x$

$$f'(1).\frac{1}{x} = \frac{f'(x)}{f(x)}$$

$2024f(x) = xf'(x) \Rightarrow xf'(x) - 2024\,f(x) = 0$

16. If z is a complex number, then the number of common roots of the equation $z^{1985} + z^{100} + 1 = 0$ and $z^3 + 2z^2 + 2z + 1 = 0$, is equal to :

(1) 1 (2) 2

(3) 0 (4) 3

Ans. (2)

Sol. $z^{1985} + z^{100} + 1 = 0$ & $z^3 + 2z^2 + 2z + 1 = 0$

$(z + 1)(z^2 - z + 1) + 2z(z + 1) = 0$

$(z + 1)(z^2 + z + 1) = 0$

$\Rightarrow$ $z = -1\ ,\ z = w, w^2$

Now putting $z = -1$ not satisfy

Now put $z = w$

$\Rightarrow$ $w^{1985} + w^{100} + 1$

$\Rightarrow$ $w^2 + w + 1 = 0$

Also, $z = w^2$

$\Rightarrow$ $w^{3970} + w^{200} + 1$

$\Rightarrow$ $w + w^2 + 1 = 0$

Two common root

17. Suppose $2-p, p, 2-\alpha, \alpha$ are the coefficient of four consecutive terms in the expansion of $(1+x)^n$. Then the value of $p^2-\alpha^2+6\alpha+2p$ equals

(1) 4 (2) 10
(3) 8 (4) 6

Ans. (Bonus)

Sol. $2-p, p, 2-\alpha, \alpha$

Binomial coefficients are

${}^nC_r$, ${}^nC_{r+1}$, ${}^nC_{r+2}$, ${}^nC_{r+3}$ respectively

$\Rightarrow$ ${}^nC_r + {}^nC_{r+1} = 2$

$\Rightarrow$ ${}^{n+1}C_{r+1} = 2$ ……(1)

Also, ${}^nC_{r+2} + {}^nC_{r+3} = 2$

$\Rightarrow$ ${}^{n+1}C_{r+3} = 2$ …….(2)

From (1) and (2)

${}^{n+1}C_{r+1} = {}^{n+1}C_{r+3}$

$\Rightarrow$ $2r+4 = n+1$

$n = 2r+3$

${}^{2r+4}C_{r+1} = 2$

Data Inconsistent

18. If the domain of the function $f(x) = \log_e\left(\frac{2x+3}{4x^2+x-3}\right)+\cos^{-1}\left(\frac{2x-1}{x+2}\right)$ is $(\alpha, \beta]$, then the value of $5\beta-4\alpha$ is equal to

(1) 10 (2) 12
(3) 11 (4) 9

Ans. (2)

Sol. $\frac{2x+3}{4x^2+x-3} > 0$ and $-1 \le \frac{2x-1}{x+2} \le 1$

$\frac{2x+3}{(4x-3)(x+1)} > 0$ $\quad \frac{3x+1}{x+2} \ge 0$ & $\frac{x-3}{x+2} \le 0$

– + – +
–3/2 –1 3/4

$(-\infty,-2)\cup\left[\frac{-1}{3},\infty\right)$ …..(1)

$(-2,3]$ …….(2)

$\left[\frac{-1}{3},3\right]$ ……(3) (1) ∩ (2) ∩ (3)

$\left(\frac{3}{4},3\right]$

$\alpha = \frac{3}{4}$ $\beta = 3$

$5\beta-4\alpha = 15-3 = 12$

19. Let $f : R \to R$ be a function defined $f(x)=\frac{x}{(1+x^4)^{1/4}}$ and $g(x)=f(f(f(f(x))))$ then

$18\int_0^{\sqrt{2\sqrt5}} x^2 g(x)dx$

(1) 33 (2) 36
(3) 42 (4) 39

Ans. (4)

Sol. $f(x) = \frac{x}{(1+x^4)^{1/4}}$

$$fof(x) = \frac{f(x)}{(1+f(x)^4)^{1/4}} = \frac{\frac{x}{(1+x^4)^{1/4}}}{\left(1+\frac{x^4}{1+x^4}\right)^{1/4}} = \frac{x}{(1+2x^4)^{1/4}}$$

$$f(f(f(f(x)))) = \frac{x}{(1+4x^4)^{1/4}}$$

$$18\int_0^{\sqrt{2\sqrt5}} \frac{x^3}{(1+4x^4)^{1/4}}dx$$

Let $1+4x^4 = t^4$

$16x^3\,dx = 4t^3\,dt$

$$\frac{18}{4}\int_1^3 \frac{t^3dt}{t}$$

$$= \frac{9}{2}\left(\frac{t^3}{3}\right)_1^3$$

$$= \frac{3}{2}[26] = 39$$

20. Let $R = \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix}$ be a non-zero 3 × 3 matrix, where $x\sin\theta = y\sin\left(\theta+\frac{2\pi}{3}\right) = z\sin\left(\theta+\frac{4\pi}{3}\right)$ $\ne 0, \theta \in (0,2\pi)$. For a square matrix M, let trace (M) denote the sum of all the diagonal entries of M. Then, among the statements:

(I) Trace (R) = 0

(II) If trace (adj(adj(R)) = 0, then R has exactly one non-zero entry.

(1) Both (I) and (II) are true
(2) Neither (I) nor (II) is true
(3) Only (II) is true
(4) Only (I) is true

Ans. (2)

Sol. $x\sin\theta = y\sin\left(\theta+\frac{2\pi}{3}\right) = z\sin\left(\theta+\frac{4\pi}{3}\right) \neq 0$

$\Rightarrow x, y, z \neq 0$

Also,

$\sin\theta+\sin\left(\theta+\frac{2\pi}{3}\right)+\sin\left(\theta+\frac{4\pi}{3}\right)=0 \ \forall\ \theta\in R$

$\Rightarrow \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0$

$\Rightarrow xy + yz + zx = 0$

(i) Trace (R) = x + y + z

If x + y + z = 0 and x y + yz + zx = 0

$\Rightarrow x = y = z = 0$

Statement (i) is False

(ii) Adj(Adj(R)) = |R| R

Trace (Adj(Adj(R)))

= xyz (x + y + z) ≠ 0

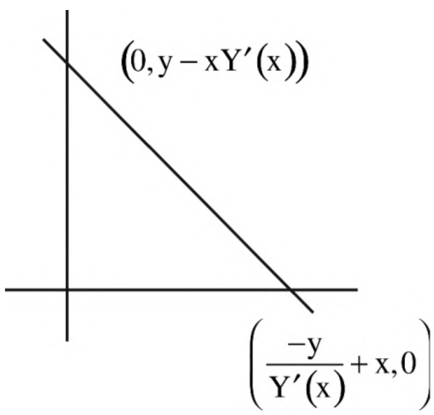
Statement (ii) is also False

SECTION-B

21. Let Y = Y(X) be a curve lying in the first quadrant such that the area enclosed by the line Y – y = Y′(x) (X – x) and the co-ordinate axes, where (x, y) is any point on the curve, is always $\frac{-y^2}{2Y'(x)}+1$, Y′(x) ≠ 0. If Y(1) = 1, then 12Y(2) equals _______ .

Ans. (20)

Sol. $A=\frac{1}{2}\left(\frac{-y}{Y'(x)}+x\right)(y-xY/x) = \frac{-y^2}{2Y'(x)}+1$



$\Rightarrow (-y+xY'(x))(y-xY'(x)) = -y^2+2Y'(x)$

$-y^2+xyY'(x)+xyY'(x)-x^2[Y'(x)]^2$

$= -y^2+2Y'(x)$

$2xy - x^2\,Y'(x) = 2$

$\frac{dy}{dx}=\frac{2xy-2}{x^2}$

$\frac{dy}{dx}-\frac{2}{x}y=\frac{-2}{x^2}$

$\text{I.F.} = e^{-2\ln x}=\frac{1}{x^2}$

$y\cdot\frac{1}{x^2}=\frac{2}{3}x^{-3}+c$

Put x = 1, y = 1

$1=\frac{2}{3}+c \Rightarrow c=\frac{1}{3}$

$Y=\frac{2}{3}\cdot\frac{1}{X}+\frac{1}{3}X^2$

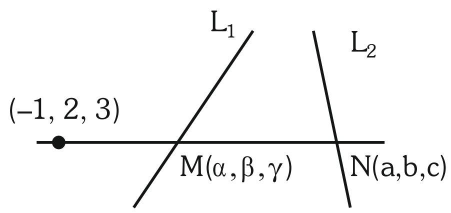
$\Rightarrow 12Y(2)=\frac{5}{3}\times 12 = 20$

22. Let a line passing through the point (–1, 2, 3) intersect the lines $L_1 : \frac{x-1}{3}=\frac{y-2}{2}=\frac{z+1}{-2}$ at $M(\alpha,\beta,\gamma)$ and $L_2 : \frac{x+2}{-3}=\frac{y-2}{-2}=\frac{z-1}{4}$ at N(a, b, c). Then the value of $\frac{(\alpha+\beta+\gamma)^2}{(a+b+c)^2}$ equals ____.

Ans. (196)

Sol. $M(3\lambda+1, 2\lambda+2, -2\lambda-1) \ \therefore\ \alpha+\beta+\gamma = 3\lambda+2$

$N(-3\mu-2, -2\mu+2, 4\mu+1) \ \therefore\ a+b+c=-\mu+1$



$\frac{3\lambda+2}{-3\mu-1}=\frac{2\lambda}{-2\mu}=\frac{-2\lambda-4}{4\mu-2}$

$3\lambda\mu+2\mu = 3\lambda\mu+\lambda$

$2\mu=\lambda$

$2\lambda\mu-\lambda=\lambda\mu+2\mu$

$\lambda\mu=\lambda+2\mu$

$\Rightarrow \lambda\mu = 2\lambda$

$\Rightarrow \quad \mu = 2 \qquad (\lambda \neq 0)$

$\therefore \quad \lambda = 4$

$\alpha + \beta + \gamma = 14$

$a + b + c = -1$

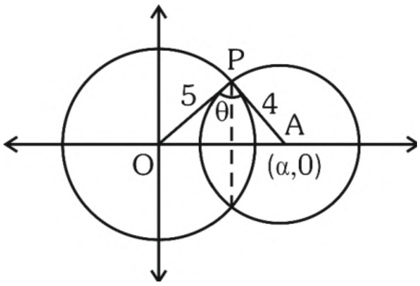
$$\frac{(\alpha+\beta+\gamma)^2}{(a+b+c)^2} = 196$$

23. Consider two circles $C_1 : x^2 + y^2 = 25$ and $C_2 : (x - \alpha)^2 + y^2 = 16$, where $\alpha \in (5, 9)$. Let the angle between the two radii (one to each circle) drawn from one of the intersection points of C_1 and C_2 be $\sin^{-1}\left(\frac{\sqrt{63}}{8}\right)$. If the length of common chord of C_1 and C_2 is β, then the value of $(\alpha\beta)^2$ equals ______ .

Ans. (1575)

Sol. $C_1 : x^2 + y^2 = 25$, $C_2 : (x-\alpha)^2 + y^2 = 16$

$5 < \alpha < 9$



$$\theta = \sin^{-1}\left(\frac{\sqrt{63}}{8}\right)$$

$$\sin\theta = \frac{\sqrt{63}}{8}$$

Area of $\Delta OAP = \frac{1}{2} \times \alpha\left(\frac{\beta}{2}\right) = \frac{1}{2} \times 5 \times 4 \sin\theta$

$$\Rightarrow \quad \alpha\beta = 40 \times \frac{\sqrt{63}}{8}$$

$$\alpha\beta = 5 \times \sqrt{63}$$

$$(\alpha\beta)^2 = 25 \times 63 = 1575$$

24. Let $\alpha = \sum_{k=0}^{n}\left(\frac{\left({}^nC_k\right)^2}{k+1}\right)$ and $\beta = \sum_{k=0}^{n-1}\left(\frac{{}^nC_k\, {}^nC_{k+1}}{k+2}\right)$.

If $5\alpha = 6\beta$, then n equals ____________ .

Ans. (10)

Sol. $\alpha = \sum_{k=0}^{n} \frac{{}^nC_k \cdot {}^nC_k}{k+1} \cdot \frac{n+1}{n+1}$

$$= \frac{1}{n+1}\sum_{k=0}^{n} {}^{n+1}C_{k+1} \cdot {}^nC_{n-k}$$

$$\alpha = \frac{1}{n+1} \cdot {}^{2n+1}C_{n+1}$$

$$\beta = \sum_{k=0}^{n-1} {}^nC_k \cdot \frac{{}^nC_{k+1}}{k+2} \frac{n+1}{n+1}$$

$$\frac{1}{n+1}\sum_{k=0}^{n-1} {}^nC_{n-k} \cdot {}^{n+1}C_{k+2}$$

$$= \frac{1}{n+1} \cdot {}^{2n+1}C_{n+2}$$

$$\frac{\beta}{\alpha} = \frac{{}^{2n+1}C_{n+2}}{{}^{2n+1}C_{n+1}} = \frac{2n+1-(n+2)+1}{n+2}$$

$$\frac{\beta}{\alpha} = \frac{n}{n+2} = \frac{5}{6}$$

$n = 10$

25. Let S_n be the sum to n-terms of an arithmetic progression 3, 7, 11, ...

If $40 < \left(\frac{6}{n(n+1)}\sum_{k=1}^{n} S_k\right) < 42$, then n equals ____ .

Ans. (9)

Sol. $S_n = 3 + 7 + 11 + \ldots\ldots$ n terms

$$= \frac{n}{2}(6+(n-1)4) = 3n + 2n^2 - 2n$$

$$= 2n^2 + n$$

$$\sum_{k=1}^{n} S_k = 2\sum_{k=1}^{n} K^2 + \sum_{k=1}^{n} K$$

$$= 2 \cdot \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= n(n+1)\left[\frac{2n+1}{3} + \frac{1}{2}\right]$$

$$= \frac{n(n+1)(4n+5)}{6}$$

$$\Rightarrow \quad 40 < \frac{6}{n(n+1)}\sum_{k=1}^{n} S_k < 42$$

$40 < 4n + 5 < 42$

$35 < 4n < 37$

$n = 9$

26. In an examination of Mathematics paper, there are 20 questions of equal marks and the question paper is divided into three sections : A, B and C . A student is required to attempt total 15 questions taking at least 4 questions from each section. If section A has 8 questions, section B has 6 questions and section C has 6 questions, then the total number of ways a student can select 15 questions is _______ .

Ans. (11376)

Sol. If 4 questions from each section are selected

Remaining 3 questions can be selected either in (1, 1, 1) or (3, 0, 0) or (2, 1, 0)

$\therefore$ Total ways = ${}^{8}c_5 \cdot {}^{6}c_5 \cdot {}^{6}c_5 + {}^{8}c_6\, {}^{6}c_5 \cdot {}^{6}c_4 \times 2$ + ${}^{8}c_5 \cdot {}^{6}c_6 \cdot {}^{6}c_4 \times 2 + {}^{8}c_4 \cdot {}^{6}c_6 \cdot {}^{6}c_5 \times 2 + {}^{8}c_7 \cdot {}^{6}c_4 \cdot {}^{6}c_4$

= 56 . 6 . 6 + 28 . 6 . 15 . 2 + 56 . 15 . 2 + 70 . 6 . 2 + 8 . 15 . 15

= 2016 + 5040 + 1680 + 840 + 1800 = 11376

27. The number of symmetric relations defined on the set {1, 2, 3, 4} which are not reflexive is _____ .

Ans. (960)

Sol. Total number of relation both symmetric and reflexive = $2^{\frac{n^2-n}{2}}$

Total number of symmetric relation = $2^{\left(\frac{n^2+n}{2}\right)}$

$\Rightarrow$ Then number of symmetric relation which are not reflexive

$\Rightarrow 2^{\frac{n(n+1)}{2}} - 2^{\frac{n(n-1)}{2}}$

$\Rightarrow 2^{10} - 2^{6}$

$\Rightarrow 1024 - 64$

$= 960$

28. The number of real solutions of the equation $x\left(x^2+3|x|+5|x-1|+6|x-2|\right)=0$ is ______ .

Ans. (1)

Sol. $x = 0$ and $x^2+3|x|+5|x-1|+6|x-2|=0$

Here all terms are +ve except at x = 0

So there is no value of x

Satisfies this equation

Only solution x = 0

No of solution 1.

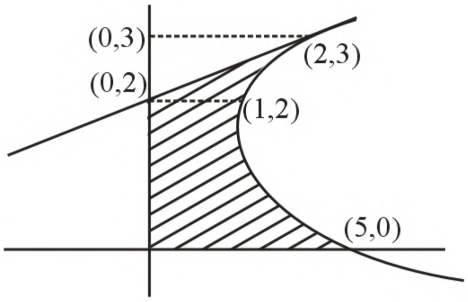
29. The area of the region enclosed by the parabola $(y-2)^2 = x - 1$, the line $x - 2y + 4 = 0$ and the positive coordinate axes is ______.

Ans. (5)

Sol. Solving the equations

$(y-2)^2 = x - 1$ and $x - 2y + 4 = 0$

$X = 2(y-2)$



$\frac{x^2}{4} = x - 1$

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0$

$x = 2$

Exclose area (w.r.t. y-axis) = $\int_0^3 x\,dy$ – Area of Δ.

$= \int_0^3 \left((y-2)^2+1\right)dy - \frac{1}{2}\times 1 \times 2$

$= \int_0^3 \left(y^2-4y+5\right)dy - 1$

$= \left[\frac{y^3}{3} - 2y^2 + 5y\right]_0^3 - 1$

$= 9 - 18 + 15 - 1 = 5$

30. The variance σ^2 of the data

x_i	0	1	5	6	10	12	17
f_i	3	2	3	2	6	3	3

Is ________ .

Ans. (29)

Sol.

x_i	f_i	f_ix_i	$f_ix_i^2$
0	3	0	0
1	2	2	2
5	3	15	75
6	2	12	72
10	6	60	600
12	3	36	432
17	3	51	867
	$\Sigma f_i = 22$		$\Sigma f_ix_i^2 = 2048$

$\therefore \quad \Sigma f_ix_i = 176$

So $\bar{x} = \dfrac{\sum f_ix_i}{\sum f_i} = \dfrac{176}{22} = 8$

for $\sigma^2 = \dfrac{1}{N}\sum f_ix_i^2 - (\bar{x})^2$

$= \dfrac{1}{22} \times 2048 - (8)^2$

$= 93.090964$

$= 29.0909$

SECTION-A

31. If 50 Vernier divisions are equal to 49 main scale divisions of a travelling microscope and one smallest reading of main scale is 0.5 mm, the Vernier constant of travelling microscope is:

(1) 0.1 mm

(2) 0.1 cm

(3) 0.01 cm

(4) 0.01 mm

Ans. **(4)**

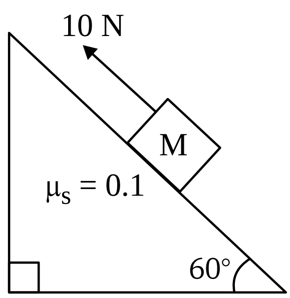
Sol. 50 V+S = 49S + S

S = 50 (S – V)

.5 = 50 (S – V)

$$S - V = \frac{0.5}{50} = \frac{1}{100} = 0.01 \text{ mm}$$

32. A block of mass 1 kg is pushed up a surface inclined to horizontal at an angle of 60° by a force of 10 N parallel to the inclined surface as shown in figure. When the block is pushed up by 10 m along inclined surface, the work done against frictional force is : [g = 10 m/s^2]



(1) $5\sqrt{3}$ J (2) 5 J

(3) 5×10^3 J (4) 10 J

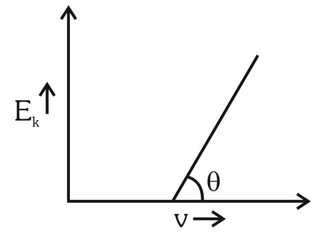
Ans. **(2)**

Sol. Work done again frictional force

$= \mu N \times 10$

$= 0.1 \times 5 \times 10 = 5J$

33. For the photoelectric effect, the maximum kinetic energy (E_k) of the photoelectrons is plotted against the frequency (v) of the incident photons as shown in figure. The slope of the graph gives



(1) Ratio of Planck's constant to electric charge

(2) Work function of the metal

(3) Charge of electron

(4) Planck's constant

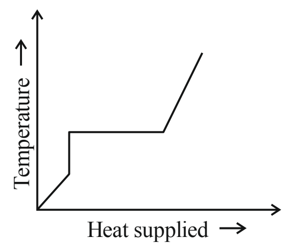
Ans. **(4)**

Sol. K.E. = hf – ϕ

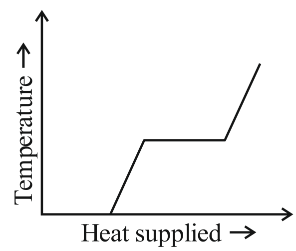
tan θ = h

34. A block of ice at –10°C is slowly heated and converted to steam at 100°C. Which of the following curves represent the phenomenon qualitatively:

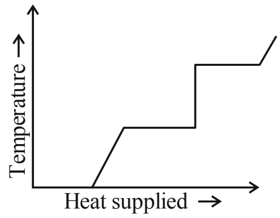
(1)



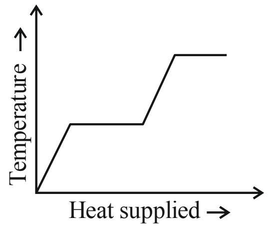
(2)



(3)



(4)



Ans. **(4)**

35. In a nuclear fission reaction of an isotope of mass M, three similar daughter nuclei of same mass are formed. The speed of a daughter nuclei in terms of mass defect ΔM will be :

(1) $\sqrt{\frac{2c\Delta M}{M}}$ (2) $\frac{\Delta Mc^2}{3}$

(3) $c\sqrt{\frac{2\Delta M}{M}}$ (4) $c\sqrt{\frac{3\Delta M}{M}}$

Ans. **(3)**

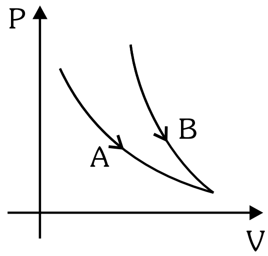
Sol. (X) → (Y) + (Z) + (P)

M M/3 M/3 M/3

$$\Delta Mc^2 = \frac{1}{2}\frac{M}{3}V^2 + \frac{1}{2}\frac{M}{3}V^2 + \frac{1}{2}\frac{M}{3}V^2$$

$$V = c\sqrt{\frac{2\Delta M}{M}}$$

36. Choose the correct statement for processes A & B shown in figure.



(1) $PV^{\gamma} = k$ for process B and $PV = k$ for process A.

(2) $PV = k$ for process B and A.

(3) $\frac{P^{\gamma-1}}{T^{\gamma}} = k$ for process B and $T = k$ for process A.

(4) $\frac{T^{\gamma}}{P^{\gamma-1}} = k$ for process A and $PV = k$ for process B.

Ans. **(1 & 3)**

Sol. Steeper curve (B) is adiabatic

Adiabatic ⇒ $PV^{\nu} = \text{const.}$

Or $P\left(\frac{T}{P}\right)^{\nu} = \text{const.}$

$$\frac{T^{\nu}}{P^{\nu-1}} = \text{const.}$$

Curve (A) is isothermal

T = const.

PV = const.

37. An electron revolving in n^{th} Bohr orbit has magnetic moment μ_n. If $\mu_n \alpha n^x$, the value of x is:

(1) 2 (2) 1

(3) 3 (4) 0

Ans. **(2)**

Sol. Magnetic moment = $i\pi r^2$

$$\mu = \frac{evr}{2}$$

$$\mu\ \alpha \left(\frac{1}{n}\right) n^2$$

$\mu\ \alpha\ n$

x = 1

38. An alternating voltage V(t) = 220 sin 100 πt volt is applied to a purely resistive load of 50 Ω. The time taken for the current to rise from half of the peak value to the peak value is:

(1) 5 ms

(2) 3.3 ms

(3) 7.2 ms

(4) 2.2 ms

Ans. **(2)**

Sol. Rising half to peak

t = T/6

$$t = \frac{2\pi}{6\omega} = \frac{\pi}{3\omega} = \frac{\pi}{300\pi} = \frac{1}{300} = 3.33\,\text{ms}$$

39. A block of mass m is placed on a surface having vertical cross section given by $y = x^2/4$. If coefficient of friction is 0.5, the maximum height above the ground at which block can be placed without slipping is:

(1) 1/4 m (2) 1/2 m

(3) 1/6 m (4) 1/3 m

Ans. **(1)**

Sol. $\frac{dy}{dx} = \tan\theta = \frac{x}{2} = \mu = \frac{1}{2}$

x = 1 , y = 1/4

40. If the total energy transferred to a surface in time t is 6.48×10^5 J, then the magnitude of the total momentum delivered to this surface for complete absorption will be :

(1) 2.46×10^{-3} kg m/s

(2) 2.16×10^{-3} kg m/s

(3) 1.58×10^{-3} kg m/s

(4) 4.32×10^{-3} kg m/s

Ans. **(2)**

Sol. $p = \frac{E}{C} = \frac{6.48\times10^5}{3\times10^8} = 2.16\times10^{-3}$

41. A beam of unpolarised light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A. The intensity of emergent light is :

(1) $I_0/4$ (2) I_0

(3) $I_0/2$ (4) $I_0/8$

Ans. **(1)**

Sol. Intensity of emergent light

$= \frac{I_0}{2}\cos^2 45° = \frac{I_0}{4}$

42. Escape velocity of a body from earth is 11.2 km/s. If the radius of a planet be one-third the radius of earth and mass be one-sixth that of earth, the escape velocity from the plate is:

(1) 11.2 km/s (2) 8.4 km/s

(3) 4.2 km/s (4) 7.9 km/s

Ans. **(4)**

Sol. $R_P = \frac{R_E}{3},\ M_P = \frac{M_E}{6}$

$V_e = \sqrt{\frac{2GM_e}{R_e}}$ ...(i)

$V_P = \sqrt{\frac{2GM_P}{R_P}}$ ...(ii)

$\frac{V_e}{V_p} = \sqrt{2}$

$V_P = \frac{V_e}{\sqrt{2}} = \frac{11.2}{\sqrt{2}} = 7.9$ km/sec

43. A particle of charge '–q' and mass 'm' moves in a circle of radius 'r' around an infinitely long line charge of linear density '+λ'. Then time period will be given as:

(Consider k as Coulomb's constant)

(1) $T^2 = \frac{4\pi^2 m}{2k\lambda q} r^3$ (2) $T = 2\pi r\sqrt{\frac{m}{2k\lambda q}}$

(3) $T = \frac{1}{2\pi r}\sqrt{\frac{m}{2k\lambda q}}$ (4) $T = \frac{1}{2\pi}\sqrt{\frac{2k\lambda q}{m}}$

Ans. **(2)**

Sol. $\frac{2k\lambda q}{r} = m\omega^2 r$

$\omega^2 = \frac{2k\lambda q}{mr^2}$

$\left(\frac{2\pi}{T}\right)^2 = \frac{2k\lambda q}{mr^2}$

$T = 2\pi r\sqrt{\frac{m}{2k\lambda q}}$

44. If mass is written as $m = k\, c^P\, G^{-1/2}\, h^{1/2}$ then the value of P will be : (Constants have their usual meaning with k a dimensionless constant)

(1) 1/2

(2) 1/3

(3) 2

(4) –1/3

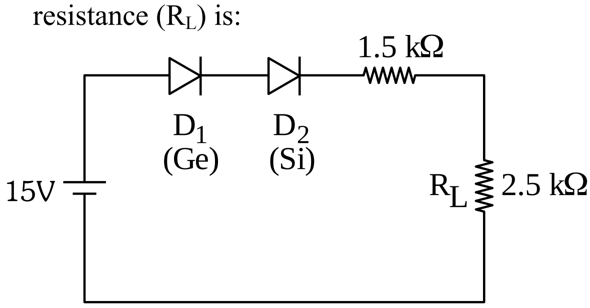
Ans. **(1)**

Sol. $m = k\, c^P\, G^{-1/2}\, h^{1/2}$

$M^1L^0T^0 = [LT^{-1}]^P[M^{-1}L^3T^{-2}]^{-1/2}[ML^2T^{-1}]^{1/2}$

By comparing P = 1/2

45. In the given circuit, the voltage across load resistance (R_L) is:



(1) 8.75 V

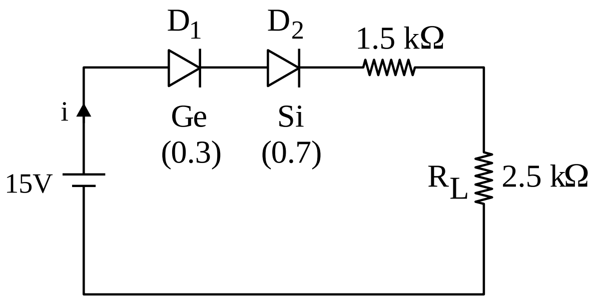
(2) 9.00 V

(3) 8.50 V

(4) 14.00 V

Ans. **(1)**

Sol.



$i = \frac{14}{4} = 3.5\,mA$

$V_L = iR_L = 3.5 \times 2.5$ volt

$= 8.75\,volt$

46. If three moles of monoatomic gas $\left(\gamma = \frac{5}{3}\right)$ is mixed with two moles of a diatomic gas $\left(\gamma = \frac{7}{5}\right)$, the value of adiabatic exponent γ for the mixture is:

(1) 1.75 (2) 1.40

(3) 1.52 (3) 1.35

Ans. (3)

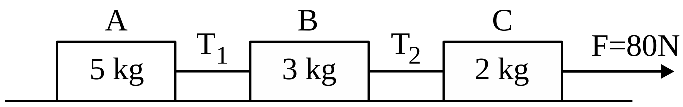
Sol. $f_1 = 3,\quad f_2 = 5$

$n_1 = 3,\quad n_2 = 2$

$$f_{mixture} = \frac{n_1 f_1 + n_2 f_2}{n_1 + n_2} = \frac{9+10}{f} = \frac{19}{5}$$

$$\gamma_{mixture} = 1 + \frac{2 \times 5}{19} = \frac{29}{19} = 1.52$$

47. Three blocks A, B and C are pulled on a horizontal smooth surface by a force of 80 N as shown in figure



The tensions T_1 and T_2 in the string are respectively:

(1) 40N, 64N

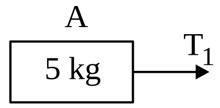
(2) 60N, 80N

(3) 88N, 96N

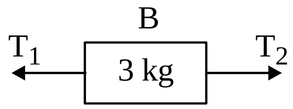
(4) 80N, 100N

Ans. (1)

Sol. $a_A = a_B = a_C = \frac{F}{5+3+2} = \frac{80}{10} = 8\,m/s^2$



$T_1 = 5 \times 8 = 40$



$T_2 - T_1 = 3 \times 8 \Rightarrow T_2 = 64$

48. When a potential difference V is applied across a wire of resistance R, it dissipates energy at a rate W. If the wire is cut into two halves and these halves are connected mutually parallel across the same supply, the same supply, the energy dissipation rate will become:

(1) 1/4W (2) 1/2W

(3) 2W (4) 4W

Ans. (4)

Sol. $\frac{v^2}{R} = W$ ....(i)

$\frac{v^2}{\frac{1}{2}\left(\frac{R}{2}\right)} = W'$ ....(ii)

From (i) & (ii), we get

$W' = 4W$

49. Match List I with List II

	List-I		List-II
A.	Gauss's law of magnetostatics	I.	$\oint \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0}\int \rho dV$
B.	Faraday's law of electro magnetic induction	II.	$\oint \vec{B}\cdot d\vec{a} = -0$
C.	Ampere's law	III.	$\oint \vec{E}\cdot d\vec{l} = \frac{-d}{dt}\int \vec{B}\cdot d\vec{a}$
D.	Gauss's law of electrostatics	IV.	$\oint \vec{B}\cdot d\vec{l} = -\mu_0 I$

Choose the correct answer from the options given below:

(1) A-I, B-III, C-IV, D-II

(2) A-III, B-IV, C-I, D-II

(3) A-IV, B-II, C-III, D-I

(4) A-II, B-III, C-IV, D-I

Ans. (4)

Sol. Maxwell's equation

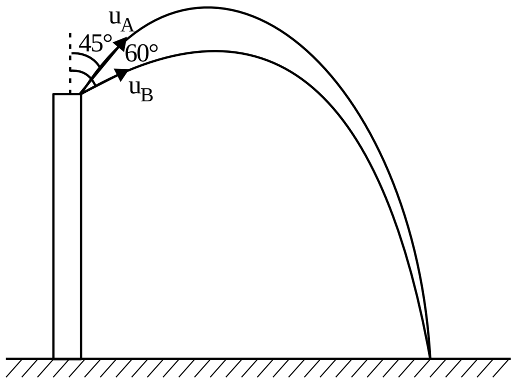
50. Projectiles A and B are thrown at angles of 45° and 60° with vertical respectively from top of a 400 m high tower. If their ranges and times of flight are same, the ratio of their speeds of projection $v_A : v_B$ is :

(1) $1:\sqrt{3}$ (2) $\sqrt{2}:1$

(3) 1 : 2 (4) $1:\sqrt{2}$

Ans. **(Bonus)**

Sol.



For u_A & u_B time of flight and range can not be same. So above options are incorrect.

SECTION-B

51. A power transmission line feeds input power at 2.3 kV to a step down transformer with its primary winding having 3000 turns. The output power is delivered at 230 V by the transformer. The current in the primary of the transformer is 5A and its efficiency is 90%. The winding of transformer is made of copper. The output current of transformer is_____A.

Ans. **(45)**

Sol. $P_i = 2300 \times 5$ watt

$P_0 = 2300 \times 5 \times 0.9 = 230 \times I_2$

$I_2 = 45A$

52. A big drop is formed by coalescing 1000 small identical drops of water. If E_1 be the total surface energy of 1000 small drops of water and E_2 be the surface energy of single big drop of water, the E_1 : E_2 is x : 1 where x = ________.

Ans. **(10)**

Sol. $\rho\left(\frac{4}{3}\pi r^3\right)1000 = \frac{4}{3}\pi R^3 \rho$

$R = 10r$

$E_1 = 1000 \times 4\pi r^2 \times S$

$E_2 = 4\pi\ (10r)^2\ S$

$\frac{E_1}{E_2} = \frac{10}{1}, \quad x = 10$

53. Two discs of moment of inertia $I_1 = 4$ kg m^2 and $I_2 = 2$ kg m^2 about their central axes & normal to their planes, rotating with angular speeds 10 rad/s & 4 rad/s respectively are brought into contact face to face with their axe of rotation coincident. The loss in kinetic energy of the system in the process is_____J.

Ans. **(24)**

Sol. $I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega_0$ (C.O.A.M.)

gives $\omega_0 = 8$ rad/s

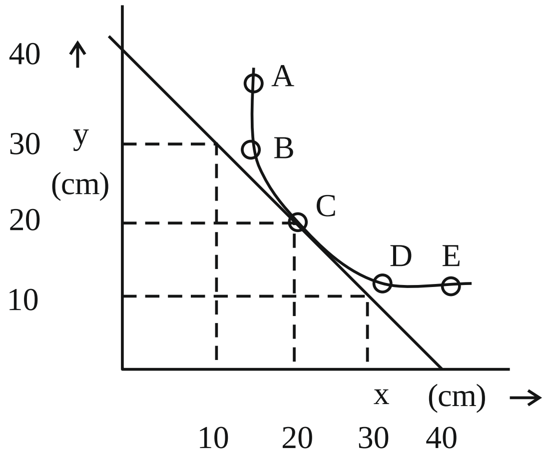
$$E_1 = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 = 216J$$

$$E_2 = \frac{1}{2}(I_1 + I_2)\omega_0^2 = 192J$$

$\therefore\ \Delta E = 24J$

54. In an experiment to measure the focal length (f) of a convex lens, the magnitude of object distance (x) and the image distance (y) are measured with reference to the focal point of the lens. The y-x plot is shown in figure.

The focal length of the lens is_____cm.



Ans. **(20)**

Sol. $$\frac{1}{f+20} - \frac{1}{-(f+20)} = \frac{1}{f}$$

$$\frac{2}{f+20} = \frac{1}{f} \quad f = 20cm$$

Or $x_1x_2 = f^2$ gives f = 20 cm

55. A vector has magnitude same as that of $\vec{A} - = 3\hat{j} + 4\hat{j}$ and is parallel to $\vec{B} = 4\hat{i} + 3\hat{j}$. The x and y components of this vector in first quadrant are x and 3 respectively where x = _______.

Ans. **(4)**

Sol. $\bar{N} = |\bar{A}|\hat{B} = \frac{5(4\hat{i} + 3\hat{j})}{5} = 4\hat{i} + 3\hat{j}$

$\therefore x = 4$

56. The current of 5A flows in a square loop of sides 1 m is placed in air. The magnetic field at the centre of the loop is $X\sqrt{2} \times 10^{-7}\,T$. The value of X is_____.

Ans. **(40)**

Sol. $B = 4 \times \frac{\mu_0 i}{4\pi\left(\frac{1}{2}\right)}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$

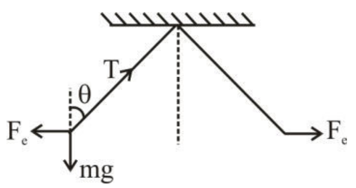
$= 4 \times 10^{-7} \times 5 \times 2 \times \sqrt{2}$

$= 40\sqrt{2} \times 10^{-7}\,T$

57. Two identical charged spheres are suspended by string of equal lengths. The string make an angle of 37° with each other. When suspended in a liquid of density 0.7 g/cm^3, the angle remains same. If density of material of the sphere is 1.4 g/cm^3, the dielectric constant of the liquid is _________ $\left(\tan 37° = \frac{3}{4}\right)$.

Ans. **(2)**

Sol.



$T\cos\theta = mg$

$T\sin\theta = F_e$

$\tan\theta = \frac{F_e}{mg}$

$\tan\theta = \frac{F_e}{\rho_B V g}$ ….(i)

$\tan\theta = \frac{\frac{F_e}{k}}{(\rho_B - \rho_L)Vg}$ ….(ii)

From Eq. (i) & (ii)

$\rho_B V g = (\rho_B - \rho_L) k V g$

1.4 = 0.7 k

k = 2

58. A simple pendulum is placed at a place where its distance from the earth's surface is equal to the radius of the earth. If the length of the string is 4m, then the time period of small oscillations will be _______s. [take g = π^2 ms^{-2}]

Ans. **(8)**

Sol. Acceleration due to gravity g' = $\frac{g}{4}$

$T = 2\pi\sqrt{\frac{4\ell}{g}}$

$T = 2\pi\sqrt{\frac{4 \times 4}{g}}$

$T = 2\pi\frac{4}{\pi} = 8s$

59. A point source is emitting sound waves of intensity 16 × 10^{-8} Wm^{-2} at the origin. The difference in intensity (magnitude only) at two points located at a distances of 2m and 4m from the origin respectively will be _______ × 10^{-8} Wm^{-2}.

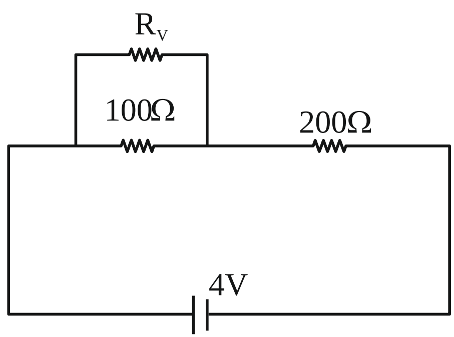
Ans. **(Bonus)**

Sol. **Question is wrong as data is incomplete.**

60. Two resistance of 100 Ω and 200 Ω are connected in series with a battery of 4 V and negligible internal resistance. A voltmeter is used to measure voltage across 100 Ω resistance, which gives reading as 1 V. The resistance of voltmeter must be______Ω.

Ans. **(200)**

Sol.



$$\frac{R_v 100}{R_v + 100} = \frac{200}{3}$$

$3R_v = 2R_v + 200$

$R_v = 200$

SECTION-A

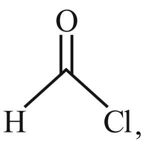
61. Which among the following purification methods is based on the principle of "Solubility" in two different solvents?

(1) Column Chromatography
(2) Sublimation
(3) Distillation
(4) Differential Extraction

Ans. **(4)**

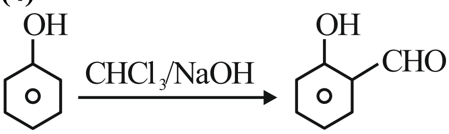
Sol. Different Extraction
Different layers are formed which can be separated in funnel. (Theory based).

62. Salicylaldehyde is synthesized from phenol, when reacted with

(1) 

, NaOH
(2) CO_2, NaOH
(3) CCl_4, NaOH
(4) $HCCl_3$, NaOH

Ans. **(4)**

Sol. 

63. Given below are two statements:

Statement – I: High concentration of strong nucleophilic reagent with secondary alkyl halides which do not have bulky substituents will follow S_N2 mechanism.

Statement – II: A secondary alkyl halide when treated with a large excess of ethanol follows S_N1 mechanism.

In the the light of the above statements, choose the most appropriate from the questions given below:

(1) Statement I is true but Statement II is false.
(3) Statement I is false but Statement II is true.
(3) Both statement I and Statement II are false.
(4) Both statement I and Statement II are true.

Ans. **(4)**

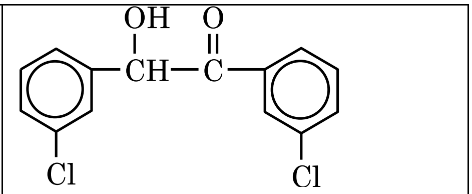
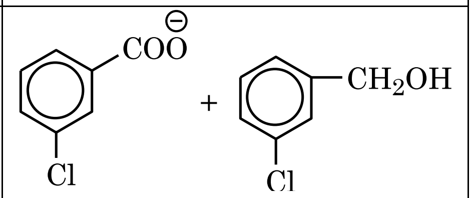
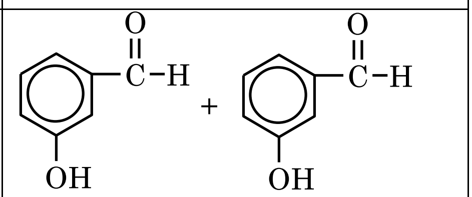
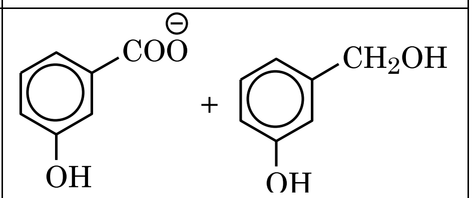
Sol. **Statement – I:** Rate of $S_N2 \propto [R\text{–}X][Nu^-]$

S_N2 reaction is favoured by high concentration of nucleophile (Nu^-) & less crowding in the substrate molecule.

Statement – II: Solvolysis follows S_N1 path.

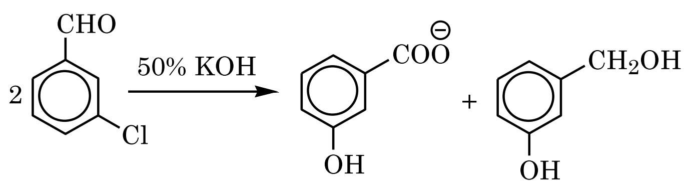
Both are correct Statements.

64. m–chlorobenzaldehyde on treatment with 50% KOH solution yields

(1)	
(2)	
(3)	
(4)	

Ans. **(2)**

Sol. Meta–chlorobenzaldehyde will undergo Cannizzaro reaction with 50% KOH to give m–chlorobenzoate ion and m–chlorobenzyl alcohol.



65. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : H_2Te is more acidic than H_2S.

Reason R: Bond dissociation enthalpy of H_2Te is lower than H_2S.

In the light of the above statements. Choose the most appropriate from the options given below.

(1) Both A and R are true but R is NOT the correct explanation of A.

(2) Both A and R are true and R is the correct explanation of A.

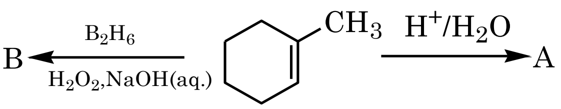
(3) A is false but R is true.

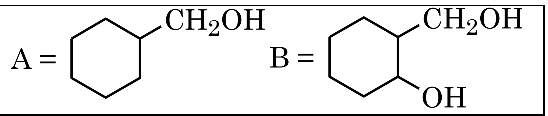
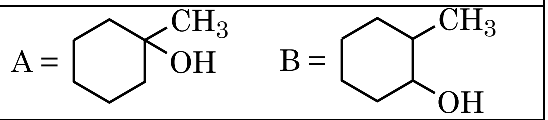
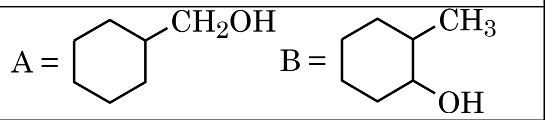
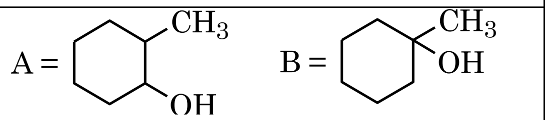
(4) A is true but R is false.

Ans. (2)

Sol. Due to lower Bond dissociation enthalpy of H_2Te it ionizes to give H^+ more easily than H_2S.

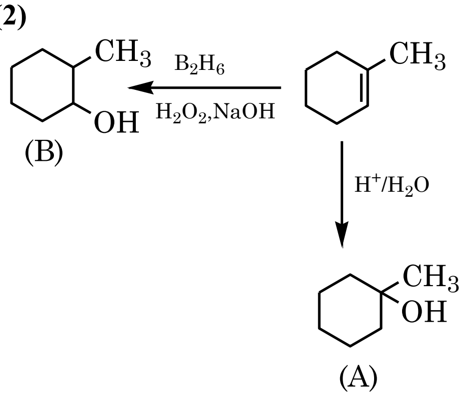
66. Product A and B formed in the following set of reactions are:



(1)	
(2)	
(3)	
(4)	

Ans. (2)

Sol.



67. IUPAC name of following compound is

$$CH_3-\underset{\displaystyle NH_2}{\underset{|}{C}H}-CH_2-CN$$

(1) 2–Aminopentanenitrile

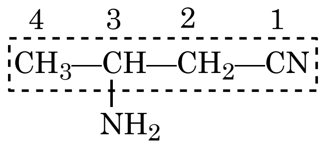
(2) 2–Aminobutanenitrile

(3) 3–Aminobutanenitrile

(4) 3–Aminopropanenitrile

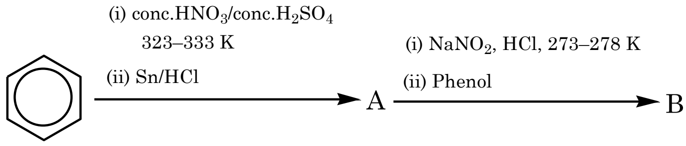
Ans. (3)

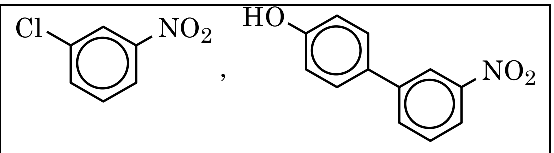
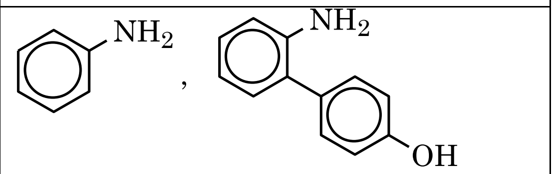
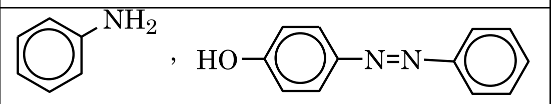
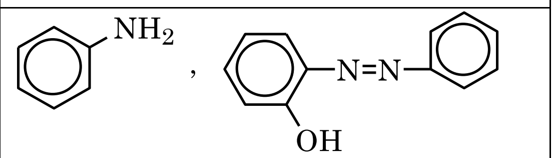
Sol.



3–Aminobutanenitrile

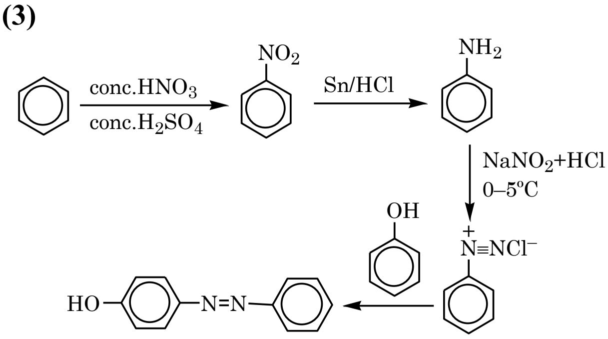
68. The products A and B formed in the following reaction scheme are respectively



(1)	
(2)	
(3)	
(4)	

Ans. (3)

Sol.



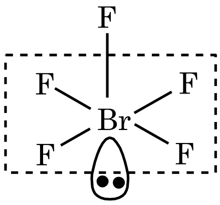
69. The molecule/ion with square pyramidal shape is:

(1) $[Ni(CN)_4]^{2-}$ (2) PCl_5

(3) BrF_5 (4) PF_5

Ans. (3)

Sol. BrF_5



Square Pyramidal.

70. The orange colour of $K_2Cr_2O_7$ and purple colour of $KMnO_4$ is due to

(1) Charge transfer transition in both.

(2) d→d transition in $KMnO_4$ and charge transfer transitions in $K_2Cr_2O_7$.

(3) d→d transition in $K_2Cr_2O_7$ and charge transfer transitions in $KMnO_4$.

(4) d→d transition in both.

Ans. (1)

Sol. $\left.\begin{array}{l} K_2Cr_2O_7 \rightarrow Cr^{+6} \rightarrow \text{No d} - \text{d transition} \\ KMnO_4 \rightarrow Mn^{7+} \rightarrow \text{No d} - \text{d transition} \end{array}\right\} \text{Charge transfer}$

71. Alkaline oxidative fusion of MnO_2 gives "A" which on electrolytic oxidation in alkaline solution produces B. A and B respectively are:

(1) Mn_2O_7 and MnO_4^-

(2) MnO_4^{2-} and MnO_4^-

(3) Mn_2O_3 and MnO_4^{2-}

(4) MnO_4^{2-} and Mn_2O_7

Ans. (2)

Sol. Alkaline oxidative fusion of MnO_2:

$$2MnO_2 + 4OH^- + O_2 \rightarrow 2MnO_4^{2-} + 2H_2O$$

Electrolytic oxidation of MnO_4^{2-} in alkaline medium.

$$MnO_4^{2-} \rightarrow MnO_4^- + e^-$$

72. If a substance 'A' dissolves in solution of a mixture of 'B' and 'C' with their respective number of moles as n_A, n_B and n_C, mole fraction of C in the solution is:

(1) $\frac{n_C}{n_A \times n_B \times n_C}$ (2) $\frac{n_C}{n_A + n_B + n_C}$

(3) $\frac{n_C}{n_A - n_B - n_C}$ (4) $\frac{n_B}{n_A + n_B}$

Ans. (2)

Sol. Mole fraction of C = $\frac{n_C}{n_A + n_B + n_C}$

73. Given below are two statements:

Statement – I: Along the period, the chemical reactivity of the element gradually increases from group 1 to group 18.

Statement – II: The nature of oxides formed by group 1 element is basic while that of group 17 elements is acidic.

In the the light above statements, choose the most appropriate from the questions given below:

(1) Both statement I and Statement II are true.

(2) Statement I is true but Statement II is False.

(3) Statement I is false but Statement II is true.

(4) Both Statement I and Statement II is false.

Ans. (3)

Sol. Chemical reactivity of elements decreases along the period therefore statement – I is false.

Group – 1 elements from basic nature oxides while group – 17 elements form acidic oxides therefore statement – II is true.

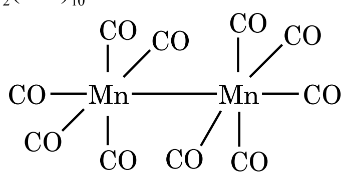
74. The coordination geometry around the manganese in decacarbonyldimanganese(0)

(1) Octahedral (2) Trigonal bipyramidal

(3) Square pyramidal (4) Square planar

Ans. (1)

Sol. $Mn_2(CO)_{10}$



Octahedral around Mn

75. Given below are two statements:

Statement-I: Since fluorine is more electronegative than nitrogen, the net dipole moment of NF_3 is greater than NH_3.

Statement-II: In NH_3, the orbital dipole due to lone pair and the dipole moment of NH bonds are in opposite direction, but in NF_3 the orbital dipole due to lone pair and dipole moments of N-F bonds are in same direction.

In the light of the above statements. Choose the most appropriate from the options given below.

(1) Statement I is true but Statement II is false.

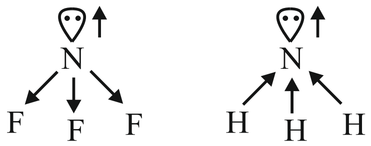
(2) Both Statement I and Statement II are false.

(3) Both statement I and Statement II is are true.

(4) Statement I is false but Statement II is are true.

Ans. **(2)**

Sol.



76. The correct stability order of carbocations is

(1) $(CH_3)_3C^+ > CH_3-\overset{+}{C}H_2 > (CH_3)_2\overset{+}{C}H > \overset{+}{C}H_3$

(2) $\overset{+}{C}H_3 > (CH_3)_2\overset{+}{C}H > CH_3-\overset{+}{C}H_2 > (CH_3)_3\overset{+}{C}$

(3) $(CH_3)_3\overset{+}{C} > (CH_3)_2\overset{+}{C}H > CH_3-\overset{+}{C}H_2 > \overset{+}{C}H_3$

(4) $\overset{+}{C}H_3 > CH_3-\overset{+}{C}H_2 > CH_3-\underset{\displaystyle CH_3}{\underset{|}{\overset{+}{C}}}H > (CH_3)C^+$

Ans. **(3)**

Sol. More no. of hyperconjugable Hydrogens, more stable is the carbocations.

77. The solution from the following with highest depression in freezing point/lowest freezing point is

(1) 180 g of acetic acid dissolved in water

(2) 180 g of acetic acid dissolved in benzene

(3) 180 g of benzoic acid dissolved in benzene

(4) 180 g of glucose dissolved in water

Ans. **(1)**

Sol. ΔT_f is maximum when i × m is maximum.

1) $m_1 = \frac{180}{60} = 3,\ i = 1 + \alpha$

Hence

$\Delta T_f = (1+\alpha)\cdot k_f = 3\times 1.86 = 5.58\,°C\ (\alpha << 1)$

2) $m_2 = \frac{180}{60} = 3,\ i = 0.5,\ \Delta T_f = \frac{3}{2}\times k_f' = 7.68°C$

3) $m_3 = \frac{180}{122} = 1.48,\ i = 0.5,\ \Delta T_f = \frac{1.48}{2}\times k_f' = 3.8°C$

4) $m_4 = \frac{180}{180} = 1,\ i = 1,\ \Delta T_f = 1\cdot k_f' = 1.86°C$

As per NCERT, $k_f'(H_2O) = 1.86\ k\cdot kg\ mol^{-1}$

$k_f'(\text{Benzene}) = 5.12\ k\cdot kg\ mol^{-1}$

78. A and B formed in the following reactions are:

$CrO_2Cl_2 + 4NaOH \rightarrow A + 2NaCl + 2H_2O$

$A + 2HCl + 2H_2O_2 \rightarrow B + 3H_2O$

(1) $A = Na_2CrO_4$, $B = CrO_5$

(2) $A = Na_2Cr_2O_4$, $B = CrO_4$

(3) $A = Na_2Cr_2O_7$, $B = CrO_3$

(4) $A = Na_2Cr_2O_7$, $B = CrO_5$

Ans. **(1)**

Sol. $CrO_2Cl_2 + 4NaOH \rightarrow \underset{(A)}{Na_2CrO_4} + 2NaCl + 2H_2O$

$Na_2CrO_4 + 2H_2O_2 + 2HCl \rightarrow \underset{(B)}{CrO_5} + \underset{\text{Missing from balanced eqaution}}{2NaCl} + 3H_2O$

79. Choose the correct statements about the hydrides of group 15 elements.

A. The stability of the hydrides decreases in the order $NH_3 > PH_3 > AsH_3 > SbH_3 > BiH_3$

B. The reducing ability of the hydrides increases in the order $NH_3 < PH_3 < AsH_3 < SbH_3 < BiH_3$

C. Among the hydrides, NH_3 is strong reducing agent while BiH_3 is mild reducing agent.

D. The basicity of the hydrides increases in the order $NH_3 < PH_3 < AsH_3 < SbH_3 < BiH_3$

Choose the most appropriate from the option given below:

(1) B and C only
(2) C and D only
(3) A and B only
(4) A and D only

Ans. **(3)**

Sol. On moving down the group, bond strength of M–H bond decreases, which reduces the thermal stability but increases reducing nature of hydrides, hence A and B are correct statements.

80. Reduction potential of ions are given below:

ClO_4^-	IO_4^-	BrO_4^-
$E° = 1.19V$	$E° = 1.65V$	$E° = 1.74V$

The correct order of their oxidising power is:

(1) $ClO_4^- > IO_4^- > BrO_4^-$

(2) $BrO_4^- > IO_4^- > ClO_4^-$

(3) $BrO_4^- > ClO_4^- > IO_4^-$

(4) $IO_4^- > BrO_4^- > ClO_4^-$

Ans. (2)

Sol. Higher the value of ⊕ve SRP (Std. reduction potential) more is tendency to undergo reduction, so better is oxidising power of reactant.

Hence, ox. Power:- $BrO_4^- > IO_4^- > ClO_4^-$

SECTION-B

81. Number of complexes which show optical isomerism among the following is ________.

$cis-[Cr(ox)_2Cl_2]^{3-}$, $[Co(en)_3]^{3+}$,

$cis-[Pt(en)_2Cl_2]^{2+}$, $cis-[Co(en)_2Cl_2]^{+}$,

$trans-[Pt(en)_2Cl_2]^{2+}$, $trans-[Cr(ox)_2Cl_2]^{3-}$

Ans. (4)

Sol. $cis-[Cr(ox)_2Cl_2]^{3-} \rightarrow$ can show optical isomerism (no POS & COS)

$[Co(en)_3]^{3+} \rightarrow$ can show (no POS & COS)

$cis-[Pt(en)_2Cl_2]^{2+} \rightarrow$ can show (no POS & COS)

$cis-[Co(en)_2Cl_2]^{+} \rightarrow$ can show (no POS & COS)

$trans-[Pt(en)_2Cl_2]^{2+} \rightarrow$ can't show (contains POS & COS)

$trans-[Cr(ox)_2Cl_2]^{3-} \rightarrow$ can't show (contains POS & COS)

82. NO_2 required for a reaction is produced by decomposition of N_2O_5 in CCl_4 as by equation

$2N_2O_{5(g)} \rightarrow 4NO_{2(g)} + O_{2(g)}$

The initial concentration of N_2O_5 is 3 mol L^{-1} and it is 2.75 mol L^{-1} after 30 minutes.

The rate of formation of NO_2 is x × 10^{-3} mol L^{-1} min^{-1}, value of x is ________.

Ans. (17)

Sol. Rate of reaction (ROR)

$$= -\frac{1}{2}\frac{\Delta[N_2O_5]}{\Delta t} = \frac{1}{4}\frac{[NO_2]}{\Delta t} = \frac{\Delta[O_2]}{\Delta t}$$

$$ROR = -\frac{1}{2}\frac{\Delta[N_2O_5]}{\Delta t} = -\frac{1}{2}\frac{(2.75-3)}{30} mol\,L^{-1}\,min^{-1}$$

$$ROR = -\frac{1}{2}\frac{(-0.25)}{30} mol\,L^{-1}\,min^{-1}$$

$$ROR = \frac{1}{240} mol\,L^{-1}\,min^{-1}$$

Rate of formation of NO_2 $= \frac{\Delta[NO_2]}{\Delta t} = 4 \times ROR$

$$= \frac{4}{240} = 16.66 \times 10^{-3}\,molL^{-1}\,min^{-1} \simeq 17 \times 10^{-3}.$$

83. Two reactions are given below:

$$2Fe_{(s)} + \frac{3}{2}O_{2(g)} \rightarrow Fe_2O_{3(s)}, \Delta H° = -822\,kJ/mol$$

$$C_{(s)} + \frac{1}{2}O_{2(g)} \rightarrow CO_{(g)}, \Delta H° = -110\,kJ/mol$$

Then enthalpy change for following reaction

$3C_{(s)} + Fe_2O_{3(s)} \rightarrow 2Fe_{(s)} + 3CO_{(g)}$

Ans. (492)

Sol. $2Fe_{(s)} + \frac{3}{2}O_{2(g)} \rightarrow Fe_2O_{3(s)}, \Delta H° = -822\,kJ/mol$ **……(1)**

$C_{(s)} + \frac{1}{2}O_{2(g)} \rightarrow CO_{(g)}, \Delta H° = -110\,kJ/mol$ **……(2)**

$3C_{(s)} + Fe_2O_{3(s)} \rightarrow 2Fe_{(s)} + 3CO_{(g)}$, $\Delta H_3 = ?$

(3) = 3 × (2) – (1)

$\Delta H_3 = 3 \times \Delta H_2 - \Delta H_1$

= 3(–110) + 822

= 492 kJ/mole

84. The total number of correct statements, regarding the nucleic acids is_________.

A. RNA is regarded as the reserve of genetic information.

B. DNA molecule self-duplicates during cell division

C. DNA synthesizes proteins in the cell.

D. The message for the synthesis of particular proteins is present in DNA

E. Identical DNA strands are transferred to daughter cells.

Ans. (3)

Sol. A. RNA is regarded as the reserve of genetic information. (False)

B. DNA molecule self-duplicates during cell division. (True)

C. DNA synthesizes proteins in the cell. (False)

D. The message for the synthesis of particular proteins is present in DNA. (True)

E. Identical DNA strands are transferred to daughter cells. (True)

85. The pH of an aqueous solution containing 1M benzoic acid ($pK_a = 4.20$) and 1M sodium benzoate is 4.5. The volume of benzoic acid solution in 300 mL of this buffer solution is _______ mL.

Ans. (100)

Sol.

	1M Benzoic acid	+	1M Sodium Benzoate
	($V_a$ml)		($V_s$ml)
Millimole	$V_a \times 1$		$V_s \times 1$

$pH = 4.5$

$$pH = pka + \log\frac{[salt]}{[acid]}$$

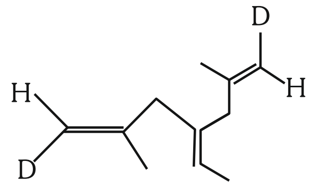
$$4.5 = 4.2 + \log\left(\frac{V_s}{V_a}\right)$$

$$\frac{V_s}{V_a} = 2 \qquad \ldots\ldots(1)$$

$$V_s + V_a = 300 \qquad \ldots\ldots(2)$$

$V_a = 100$ ml

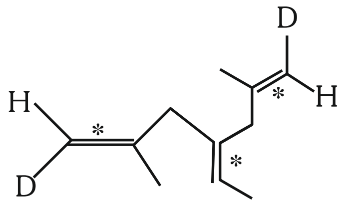
86. Number of geometrical isomers possible for the given structure is/are ___________.



Ans. (4)

Sol. 3 stereocenteres, symmetrical

Total Geometrical isomers → 4. EE, ZZ, EZ (two isomers)



87. Total number of species from the following which can undergo disproportionation reaction _______.

$H_2O_2, ClO_3^-, P_4, Cl_2, Ag, Cu^{+1}, F_2, NO_2, K^+$

Ans. (6)

Sol. Intermediate oxidation state of element can undergo disproportionation.

$H_2O_2, ClO_3^-, P_4, Cl_2, Cu^{+1}, NO_2$

88. Number of metal ions characterized by flame test among the following is _______.

$Sr^{2+}, Ba^{2+}, Ca^{2+}, Cu^{2+}, Zn^{2+}, Co^{2+}, Fe^{2+}$

Ans. (4)

Sol. All the following metal ions will respond to flame test.

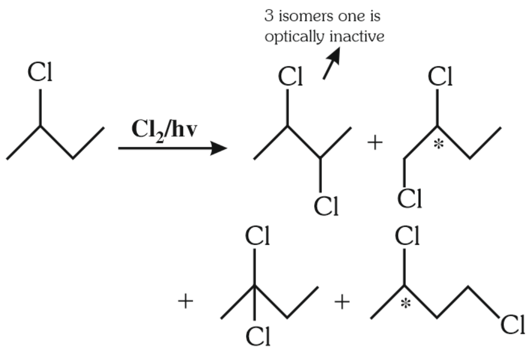
$Sr^{2+}, Ba^{2+}, Ca^{2+}, Cu^{2+}$

89. 2-chlorobutane + $Cl_2 \rightarrow C_4H_8Cl_2$(isomers)

Total number of optically active isomers shown by $C_4H_8Cl_2$, obtained in the above reaction is_______.

Ans. **(6)**

Sol.



90. Number of spectral lines obtained in He^+ spectra, when an electron makes transition from fifth excited state to first excited state will be

Ans. **(10)**

Sol. 5^{th} excited state $\Rightarrow n_1 = 6$

1^{st} excited state $\Rightarrow n_2 = 2$

$\Delta n = n_1 - n_2 = 6 - 2 = 4$

Maximum number of spectral lines

$$= \frac{\Delta n(\Delta n + 1)}{2} = \frac{4(4+1)}{2} = 10$$

FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

(Held On Wednesday 31st January, 2024) TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS	TEST PAPER WITH SOLUTION

SECTION-A

1. For 0 < c < b < a, let $(a + b - 2c)x^2 + (b + c - 2a)x + (c + a - 2b) = 0$ and $\alpha \neq 1$ be one of its root. Then, among the two statements

(I) If $\alpha \in (-1, 0)$, then b cannot be the geometric mean of a and c

(II) If $\alpha \in (0, 1)$, then b may be the geometric mean of a and c

(1) Both (I) and (II) are true

(2) Neither (I) nor (II) is true

(3) Only (II) is true

(4) Only (I) is true

Ans. (1)

Sol. $f(x) = (a + b - 2c)\,x^2 + (b + c - 2a)\,x + (c + a - 2b)$

$f(x) = a + b - 2c + b + c - 2a + c + a - 2b = 0$

$f(1) = 0$

$\therefore \alpha \cdot 1 = \dfrac{c+a-2b}{a+b-2c}$

$\alpha = \dfrac{c+a-2b}{a+b-2c}$

If, $-1 < \alpha < 0$

$-1 < \dfrac{c+a-2b}{a+b-2c} < 0$

$b + c < 2a$ and $b > \dfrac{a+c}{2}$

therefore, b cannot be G.M. between a and c.

If, $0 < \alpha < 1$

$0 < \dfrac{c+a-2b}{a+b-2c} < 1$

$b > c$ and $b < \dfrac{a+c}{2}$

Therefore, b may be the G.M. between a and c.

2. Let a be the sum of all coefficients in the expansion of $(1 - 2x + 2x^2)^{2023}\,(3 - 4x^2 + 2x^3)^{2024}$ and $b = \lim\limits_{x\to 0}\left(\dfrac{\int_0^x \frac{\log(1+t)}{t^{2024}+1}dt}{x^2}\right)$. If the equations $cx^2 + dx + e = 0$ and $2bx^2 + ax + 4 = 0$ have a common root, where c, d, e ∈ R, then d : c : e equals

(1) 2 : 1 : 4 (2) 4 : 1 : 4

(3) 1 : 2 : 4 (4) 1 : 1 : 4

Ans. (4)

Sol. Put x = 1

$\therefore a = 1$

$b = \lim\limits_{x\to 0}\dfrac{\int_0^x \frac{\ln(1+t)}{1+t^{2024}}dt}{x^2}$

Using L' HOPITAL Rule

$b = \lim\limits_{x\to 0}\dfrac{\ln(1+x)}{(1+x^{2024})}\times\dfrac{1}{2x} = \dfrac{1}{2}$

Now, $cx^2 + dx + e = 0,\quad x^2 + x + 4 = 0$

(D < 0)

$\therefore \dfrac{c}{1} = \dfrac{d}{1} = \dfrac{e}{4}$

3. If the foci of a hyperbola are same as that of the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{25} = 1$ and the eccentricity of the hyperbola is $\dfrac{15}{8}$ times the eccentricity of the ellipse, then the smaller focal distance of the point $\left(\sqrt{2}, \dfrac{14}{3}\sqrt{\dfrac{2}{5}}\right)$ on the hyperbola, is equal to

(1) $7\sqrt{\dfrac{2}{5}} - \dfrac{8}{3}$ (2) $14\sqrt{\dfrac{2}{5}} - \dfrac{4}{3}$

(3) $14\sqrt{\dfrac{2}{5}} - \dfrac{16}{3}$ (4) $7\sqrt{\dfrac{2}{5}} + \dfrac{8}{3}$

Ans. (1)

Sol. $\frac{x^2}{9}+\frac{y^2}{25}=1$

$a = 3,\ b = 5$

$e=\sqrt{1-\frac{9}{25}}=\frac{4}{5}\quad \therefore \text{foci}=(0,\pm be)=(0,\pm 4)$

$\therefore e_H=\frac{4}{5}\times\frac{15}{8}=\frac{3}{2}$

Let equation hyperbola

$\frac{x^2}{A^2}-\frac{y^2}{B^2}=-1$

$\therefore B\cdot e_H=4\quad \therefore B=\frac{8}{3}$

$\therefore A^2=B^2\left(e_H^2-1\right)=\frac{64}{9}\left(\frac{9}{4}-1\right)\quad \therefore A^2=\frac{80}{9}$

$\therefore \frac{x^2}{\frac{80}{9}}-\frac{y^2}{\frac{64}{9}}=-1$

Directrix : $y=\pm\frac{B}{e_H}=\pm\frac{16}{9}$

PS = $e\cdot PM=\frac{3}{2}\left|\frac{14}{3}\cdot\sqrt{\frac{2}{5}}-\frac{16}{9}\right|$

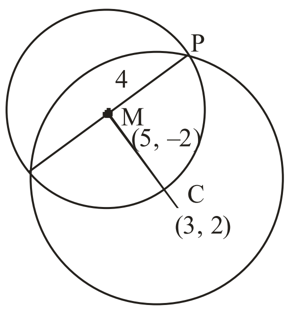
$=7\sqrt{\frac{2}{5}}-\frac{8}{3}$

4. If one of the diameters of the circle $x^2 + y^2 - 10x + 4y + 13 = 0$ is a chord of another circle C, whose center is the point of intersection of the lines $2x + 3y = 12$ and $3x - 2y = 5$, then the radius of the circle C is

(1) $\sqrt{20}$ (2) 4

(3) 6 (4) $3\sqrt{2}$

Ans. **(3)**



Sol.

$2x + 3y = 12$

$3x - 2y = 5$

$13x = 39$

$x = 3,\ y = 2$

Center of given circle is (5, –2)

Radius $\sqrt{25+4-13}=4$

$\therefore CM=\sqrt{4+16}=5\sqrt{2}$

$\therefore CP=\sqrt{16+20}=6$

5. The area of the region

$\left\{(x,y): y^2\le 4x, x<4, \frac{xy(x-1)(x-2)}{(x-3)(x-4)}>0, x\ne 3\right\}$

is

(1) $\frac{16}{3}$ (2) $\frac{64}{3}$

(3) $\frac{8}{3}$ (4) $\frac{32}{3}$

Ans. (4)

Sol. $y^2\le 4x, x<4$

$\frac{xy(x-1)(x-2)}{(x-3)(x-4)}>0$

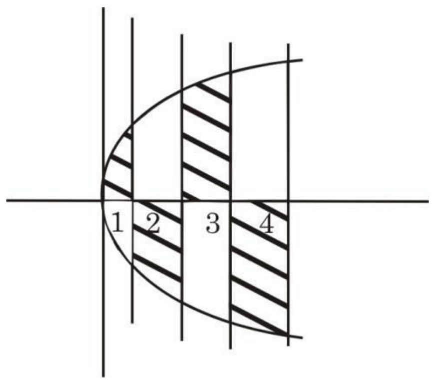
Case – I : $y>0$

$\frac{x(x-1)(x-2)}{(x-3)(x-4)}>0$

$x\in(0,1)\cup(2,3)$

Case – II : $y<0$

$\frac{x(x-1)(x-2)}{(x-3)(x-4)}<0,\ x\in(1,2)\cup(3,4)$



$\text{Area}=2\int_0^4\sqrt{x}\,dx$

$=2\cdot\frac{2}{3}\left[x^{3/2}\right]_0^4=\frac{32}{3}$

6. If $f(x)=\frac{4x+3}{6x-4}, x\neq\frac{2}{3}$ and (fof) (x) = g(x), where $g:\mathbb{R}-\left\{\frac{2}{3}\right\}\to\mathbb{R}-\left\{\frac{2}{3}\right\}$, then (gogog) (4) is equal to

(1) $-\frac{19}{20}$ (2) $\frac{19}{20}$

(3) – 4 (4) 4

Ans. (4)

Sol. $f(x)=\frac{4x+3}{6x-4}$

$$g(x)=\frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4}=\frac{34x}{34}=x$$

$g(x)=x \therefore g(g(g(4)))=4$

7. $\lim_{x\to 0}\frac{e^{2|\sin x|}-2|\sin x|-1}{x^2}$

(1) is equal to – 1 (2) does not exist

(3) is equal to 1 (4) is equal to 2

Ans. (4)

Sol. $\lim_{x\to 0}\frac{e^{2|\sin x|}-2|\sin x|-1}{x^2}$

$\lim_{x\to 0}\frac{e^{2|\sin x|}-2|\sin x|-1}{|\sin x|^2}\times\frac{\sin^2 x}{x^2}$

Let |sinx| = t

$\lim_{t\to 0}\frac{e^{2t}-2t-1}{t^2}\times\lim_{x\to 0}\frac{\sin^2 x}{x^2}$

$=\lim_{t\to 0}\frac{2e^{2t}-2}{2t}\times 1=2\times 1=2$

8. If the system of linear equations

$x-2y+z=-4$

$2x+\alpha y+3z=5$

$3x-y+\beta z=3$

has infinitely many solutions, then 12α + 13β is equal to

(1) 60 (2) 64

(3) 54 (4) 58

Ans. (4)

Sol. $D=\begin{vmatrix}1 & -2 & 1\\ 2 & \alpha & 3\\ 3 & -1 & \beta\end{vmatrix}$

$= 1(\alpha\beta + 3) + 2(2\beta - 9) + 1(-2 - 3\alpha)$

$= \alpha\beta + 3 + 4\beta - 18 - 2 - 3\alpha$

For infinite solutions $D = 0$, $D_1 = 0$, $D_2 = 0$ and $D_3 = 0$

$D = 0$

$\alpha\beta - 3\alpha + 4\beta = 17$ ….(1)

$D_1=\begin{vmatrix}-4 & -2 & 1\\ 5 & \alpha & 3\\ 3 & -1 & \beta\end{vmatrix}=0$

$D_2=\begin{vmatrix}1 & -4 & 1\\ 2 & 5 & 3\\ 3 & 3 & \beta\end{vmatrix}=0$

$\Rightarrow 1(5\beta-9)+4(2\beta-9)+1(6-15)=0$

$13\beta-9-36-9=0$

$13\beta=54, \beta=\frac{54}{13}$ put in (1)

$\frac{54}{13}\alpha-3\alpha+4\left(\frac{54}{13}\right)=17$

$54\alpha-39\alpha+216=221$

$15\alpha=5 \quad \alpha=\frac{1}{3}$

Now, $12\alpha+13\beta=12.\frac{1}{3}+13.\frac{54}{13}$

$= 4 + 54 = 58$

9. The solution curve of the differential equation $y\frac{dx}{dy}=x(\log_e x-\log_e y+1)$, x > 0, y > 0 passing through the point (e, 1) is

(1) $\left|\log_e\frac{y}{x}\right|=x$ (2) $\left|\log_e\frac{y}{x}\right|=y^2$

(3) $\left|\log_e\frac{x}{y}\right|=y$ (4) $2\left|\log_e\frac{x}{y}\right|=y+1$

Ans. (3)

Sol. $\frac{dx}{dy}=\frac{x}{y}\left(\ln\left(\frac{x}{y}\right)+1\right)$

Let $\frac{x}{y}=t\Rightarrow x=ty$

$\frac{dx}{dy}=t+y\frac{dt}{dy}$

$t+y\frac{dt}{dy}=t\left(\ln(t)+1\right)$

$y\frac{dt}{dy}=t\ln(t)\Rightarrow\frac{dt}{t\ln(t)}=\frac{dy}{y}$

$\Rightarrow\int\frac{dt}{t.\ln(t)}=\int\frac{dy}{y}$

$\Rightarrow\int\frac{dp}{p}=\int\frac{dy}{y}$ let $\ln t=p$

$\frac{1}{t}dt=dp$

$\Rightarrow \ln p=\ln y+c$

$\ln(\ln t)=\ln y+c$

$\ln\left(\ln\left(\frac{x}{y}\right)\right)=\ln y+c$

at $x=e,\ y=1$

$\ln\left(\ln\left(\frac{e}{1}\right)\right)=\ln(1)+c\Rightarrow c=0$

$\ln\left|\ln\left(\frac{x}{y}\right)\right|=\ln y$

$\left|\ln\left(\frac{x}{y}\right)\right|=e^{\ln y}$

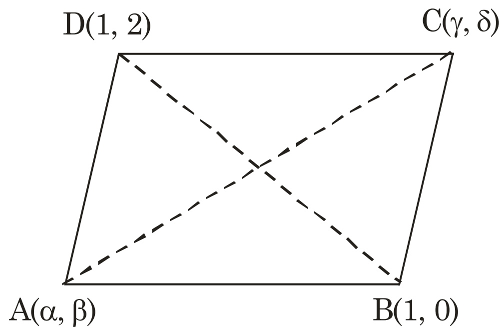
$\left|\ln\left(\frac{x}{y}\right)\right|=y$

10. Let $\alpha,\beta,\gamma,\delta\in Z$ and let A (α,β), B $(1,0)$, C (γ,δ) and D $(1,2)$ be the vertices of a parallelogram ABCD. If AB $=\sqrt{10}$ and the points A and C lie on the line $3y=2x+1$, then $2(\alpha+\beta+\gamma+\delta)$ is equal to

(1) 10 (2) 5

(3) 12 (4) 8

Ans. (4)



Sol. Let E is mid point of diagonals

$\frac{\alpha+\gamma}{2}=\frac{1+1}{2}$ & $\frac{\beta+\delta}{2}=\frac{2+0}{2}$

$\alpha+\gamma=2$ $\beta+\delta=2$

$2(\alpha+\beta+\gamma+\delta)=2(2+2)=8$

11. Let $y=y(x)$ be the solution of the differential equation $\frac{dy}{dx}=\frac{(\tan x)+y}{\sin x(\sec x-\sin x\tan x)}$, $x\in\left(0,\frac{\pi}{2}\right)$ satisfying the condition $y\left(\frac{\pi}{4}\right)=2$. Then, $y\left(\frac{\pi}{3}\right)$ is

(1) $\sqrt{3}\left(2+\log_e\sqrt{3}\right)$

(2) $\frac{\sqrt{3}}{2}\left(2+\log_e 3\right)$

(3) $\sqrt{3}\left(1+2\log_e 3\right)$

(4) $\sqrt{3}\left(2+\log_e 3\right)$

Ans. (1)

Sol. $\frac{dy}{dx}=\frac{\sin x+y\cos x}{\sin x.\cos x\left(\frac{1}{\cos x}-\sin x.\frac{\sin x}{\cos x}\right)}$

$=\frac{\sin x+y\cos x}{\sin x\left(1-\sin^2 x\right)}$

$\frac{dy}{dx}=\sec^2 x+y.2(\operatorname{cosec}2x)$

$\frac{dy}{dx}-2\operatorname{cosec}(2x).y=\sec^2 x$

$\frac{dy}{dx}+p.y=Q$

$$\text{I.F.} = e^{\int p\,dx} = e^{\int -2\,\text{cosec}(2x)dx}$$

Let 2x = t

$$2\frac{dx}{dt} = 1$$

$$dx = \frac{dt}{2}$$

$$= e^{-\int \text{cosec}(t)dt}$$

$$= e^{-\ln\left|\tan\frac{t}{2}\right|}$$

$$= e^{-\ln|\tan x|} = \frac{1}{|\tan x|}$$

$$y(\text{IF}) = \int Q(\text{IF})dx + c$$

$$\Rightarrow y\frac{1}{|\tan x|} = \int \sec^2 x \cdot \frac{1}{|\tan x|} + c$$

$$y.\frac{1}{|\tan x|} = \int \frac{dt}{|t|} + c \qquad \text{for } \tan x = t$$

$$y.\frac{1}{|\tan x|} = \ln|t| + c$$

$$y = |\tan x|(\ln|\tan x| + c)$$

Put $x = \frac{\pi}{4}$, $y = 2$

$2 = \ln 1 + c \ \Rightarrow\ c = 2$

$$y = |\tan x|(\ln|\tan x| + 2)$$

$$y\left(\frac{\pi}{3}\right) = \sqrt{3}\left(\ln\sqrt{3} + 2\right)$$

12. Let $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = 4\hat{i} + \hat{j} + 7\hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} + 4\hat{k}$ be three vectors. If a vectors $\vec{p}$ satisfies $\vec{p} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{p} \cdot \vec{a} = 0$, then $\vec{p} \cdot (\hat{i} - \hat{j} - \hat{k})$ is equal to

(1) 24
(2) 36
(3) 28
(4) 32

Ans. **(4)**

Sol. $\vec{p} \times \vec{b} - \vec{c} \times \vec{b} = \vec{0}$

$(\vec{p} - \vec{c}) \times \vec{b} = \vec{0}$

$\vec{p} - \vec{c} = \lambda\vec{b} \Rightarrow \vec{p} = \vec{c} + \lambda\vec{b}$

Now, $\vec{p}.\vec{a} = 0$ (given)

So, $\vec{c}.\vec{a} + \lambda\vec{a}.\vec{b} = 0$

$(3 - 3 - 8) + \lambda(12 + 1 - 14) = 0$

$\lambda = -8$

$\vec{p} = \vec{c} - 8\vec{b}$

$\vec{p} = -31\hat{i} - 11\hat{j} - 52\hat{k}$

So, $\vec{p}.(\hat{i} - \hat{j} - \hat{k})$

$= -31 + 11 + 52$

$= 32$

13. The sum of the series $\frac{1}{1 - 3\cdot 1^2 + 1^4} + \frac{2}{1 - 3\cdot 2^2 + 2^4} + \frac{3}{1 - 3\cdot 3^2 + 3^4} + \ldots$.. up to 10 terms is

(1) $\frac{45}{109}$ (2) $-\frac{45}{109}$

(3) $\frac{55}{109}$ (4) $-\frac{55}{109}$

Ans. **(4)**

Sol. General term of the sequence,

$$T_r = \frac{r}{1 - 3r^2 + r^4}$$

$$T_r = \frac{r}{r^4 - 2r^2 + 1 - r^2}$$

$$T_r = \frac{r}{(r^2 - 1)^2 - r^2}$$

$$T_r = \frac{r}{(r^2 - r - 1)(r^2 + r - 1)}$$

$$T_r = \frac{\frac{1}{2}\left[(r^2 + r - 1) - (r^2 - r - 1)\right]}{(r^2 - r - 1)(r^2 + r - 1)}$$

$$= \frac{1}{2}\left[\frac{1}{r^2 - r - 1} - \frac{1}{r^2 + r - 1}\right]$$

Sum of 10 terms,

$$\sum_{r=1}^{10} T_r = \frac{1}{2}\left[\frac{1}{-1} - \frac{1}{109}\right] = \frac{-55}{109}$$

14. The distance of the point Q(0, 2, –2) form the line passing through the point P(5, –4, 3) and perpendicular to the lines $\vec{r} = \left(-3\hat{i} + 2\hat{k}\right)$ + $\lambda\left(2\hat{i} + 3\hat{j} + 5\hat{k}\right)$, $\lambda \in \mathbb{R}$ and $\vec{r} = \left(\hat{i} - 2\hat{j} + \hat{k}\right)$ + $\mu\left(-\hat{i} + 3\hat{j} + 2\hat{k}\right)$, $\mu \in \mathbb{R}$ is

(1) $\sqrt{86}$

(2) $\sqrt{20}$

(3) $\sqrt{54}$

(4) $\sqrt{74}$

Ans. **(4)**

Sol. A vector in the direction of the required line can be obtained by cross product of

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 5 \\ -1 & 3 & 2 \end{vmatrix}$$

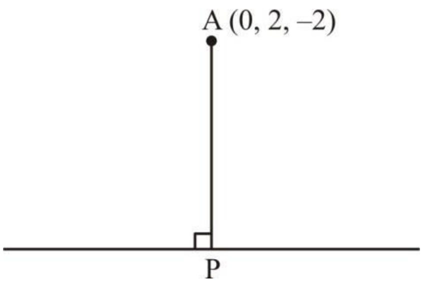
$= -9\hat{i} - 9\hat{j} + 9\hat{k}$

Required line,

$\vec{r} = \left(5\hat{i} - 4\hat{j} + 3\hat{k}\right) + \lambda'\left(-9\hat{i} - 9\hat{j} + 9\hat{k}\right)$

$\vec{r} = \left(5\hat{i} - 4\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} + \hat{j} - \hat{k}\right)$

Now distance of (0, 2, –2)



P.V. of $P \equiv (5+\lambda)\hat{i} + (\lambda - 4)\hat{j} + (3-\lambda)\hat{k}$

$\overrightarrow{AP} = (5+\lambda)\hat{i} + (\lambda - 6)\hat{j} + (5-\lambda)\hat{k}$

$\overrightarrow{AP} \cdot \left(\hat{i} + \hat{j} - \hat{k}\right) = 0$

$5 + \lambda + \lambda - 6 - 5 + \lambda = 0$

$\lambda = 2$

$\left|\overrightarrow{AP}\right| = \sqrt{49 + 16 + 9}$

$\left|\overrightarrow{AP}\right| = \sqrt{74}$

15. For $\alpha, \beta, \gamma \neq 0$. If $\sin^{-1}\alpha + \sin^{-1}\beta + \sin^{-1}\gamma = \pi$ and $(\alpha + \beta + \gamma)(\alpha - \gamma + \beta) = 3\alpha\beta$, then γ equal to

(1) $\frac{\sqrt{3}}{2}$

(2) $\frac{1}{\sqrt{2}}$

(3) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

(4) $\sqrt{3}$

Ans. **(1)**

Sol. Let $\sin^{-1}\alpha = A, \sin^{-1}\beta = B, \sin^{-1}\gamma = C$

$A + B + C = \pi$

$(\alpha + \beta)^2 - \gamma^2 = 3\alpha\beta$

$\alpha^2 + \beta^2 - \gamma^2 = \alpha\beta$

$\frac{\alpha^2 + \beta^2 - \gamma^2}{2\alpha\beta} = \frac{1}{2}$

$\Rightarrow \cos C = \frac{1}{2}$

$\sin C = \gamma$

$\cos C = \sqrt{1-\gamma^2} = \frac{1}{2}$

$\gamma = \frac{\sqrt{3}}{2}$

16. Two marbles are drawn in succession from a box containing 10 red, 30 white, 20 blue and 15 orange marbles, with replacement being made after each drawing. Then the probability, that first drawn marble is red and second drawn marble is white, is

(1) $\frac{2}{25}$

(2) $\frac{4}{25}$

(3) $\frac{2}{3}$

(4) $\frac{4}{75}$

Ans. **(4)**

Sol. Probability of drawing first red and then white

$= \frac{10}{75} \times \frac{30}{75} = \frac{4}{75}$

17. Let g(x) be a linear function and

$$f(x)=\begin{cases} g(x) & ,x\le 0 \\ \left(\dfrac{1+x}{2+x}\right)^{\frac{1}{x}} & ,x>0 \end{cases}$$, is continuous at x = 0.

If $f'(1)=f(-1)$, then the value of g(3) is

(1) $\frac{1}{3}\log_e\left(\frac{4}{9e^{1/3}}\right)$

(2) $\frac{1}{3}\log_e\left(\frac{4}{9}\right)+1$

(3) $\log_e\left(\frac{4}{9}\right)-1$

(4) $\log_e\left(\frac{4}{9e^{1/3}}\right)$

Ans. (4)

Sol. Let g(x) = ax + b

Now function f(x) in continuous at x = 0

$\therefore \lim\limits_{x\to 0^+} f(x)=f(0)$

$\Rightarrow \lim\limits_{x\to 0}\left(\frac{1+x}{2+x}\right)^{\frac{1}{x}}=b$

$\Rightarrow 0=b$

$\therefore g(x)=ax$

Now, for x > 0

$$f'(x)=\frac{1}{x}\cdot\left(\frac{1+x}{2+x}\right)^{\frac{1}{x}-1}\cdot\frac{1}{(2+x)^2}$$

$$+\left(\frac{1+x}{2+x}\right)^{\frac{1}{x}}\cdot\ln\left(\frac{1+x}{2+x}\right)\cdot\left(-\frac{1}{x^2}\right)$$

$\therefore f'(1)=\frac{1}{9}-\frac{2}{3}\cdot\ln\left(\frac{2}{3}\right)$

And f(–1) = g(–1) = –a

$\therefore a=\frac{2}{3}\ln\left(\frac{2}{3}\right)-\frac{1}{9}$

$\therefore g(3)=2\ln\left(\frac{2}{3}\right)-\frac{1}{3}$

$=\ln\left(\frac{4}{9\cdot e^{1/3}}\right)$

18. If $f(x)=\begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{vmatrix}$

for all $x\in\mathbb{R}$, then 2f(0) + $f'(0)$ is equal to

(1) 48 (2) 24

(3) 42 (4) 18

Ans. (3)

Sol. $f(0)=\begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ 0 & 4 & -2 \end{vmatrix}=12$

$$f'(x)=\begin{vmatrix} 3x^2 & 4x & 3 \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{vmatrix}+$$

$$\begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 6x & 2 & 3x^2 \\ x^3-x & 4 & x^2-2 \end{vmatrix}+$$

$$\begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ 3x^2-1 & 0 & 2x \end{vmatrix}$$

$$\therefore f'(0)=\begin{vmatrix} 0 & 0 & 3 \\ 2 & 0 & 6 \\ 0 & 4 & -2 \end{vmatrix}+\begin{vmatrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 4 & -2 \end{vmatrix}+\begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ -1 & 0 & 0 \end{vmatrix}$$

= 24 – 6 = 18

$\therefore 2f(0)+f'(0)=42$

19. Three rotten apples are accidently mixed with fifteen good apples. Assuming the random variable x to be the number of rotten apples in a draw of two apples, the variance of x is

(1) $\frac{37}{153}$

(2) $\frac{57}{153}$

(3) $\frac{47}{153}$

(4) $\frac{40}{153}$

Ans. (4)

Sol. 3 bad apples, 15 good apples.

Let X be no of bad apples

Then P(X = 0) $= \frac{^{15}C_2}{^{18}C_2} = \frac{105}{153}$

$P(X=1) = \frac{^{3}C_1 \times ^{15}C_1}{^{18}C_2} = \frac{45}{153}$

$P(X=2) = \frac{^{3}C_2}{^{18}C_2} = \frac{3}{153}$

$E(X) = 0 \times \frac{105}{153} + 1 \times \frac{45}{153} + 2 \times \frac{3}{153} = \frac{51}{153}$

$= \frac{1}{3}$

$Var(X) = E(X^2) - (E(X))^2$

$= 0 \times \frac{105}{153} + 1 \times \frac{45}{153} + 4 \times \frac{3}{153} - \left(\frac{1}{3}\right)^2$

$= \frac{57}{153} - \frac{1}{9} = \frac{40}{153}$

20. Let S be the set of positive integral values of a for which $\frac{ax^2 + 2(a+1)x + 9a + 4}{x^2 - 8x + 32} < 0, \forall x \in \mathbb{R}$.

Then, the number of elements in S is :

(1) 1

(2) 0

(3) ∞

(4) 3

Ans. **(2)**

Sol. $ax^2 + 2(a + 1)x + 9a + 4 < 0 \ \forall x \in R$

$\therefore a < 0$

SECTION-B

21. If the integral

$$525\int_0^{\frac{\pi}{2}} \sin 2x \cos^{\frac{11}{2}} x \left(1 + \cos^{\frac{5}{2}} x\right)^{\frac{1}{2}} dx \text{ is equal to}$$

$(n\sqrt{2} - 64)$, then n is equal to ______

Ans. (176)

Sol. $I = \int_0^{\frac{\pi}{2}} \sin 2x \cdot (\cos x)^{\frac{11}{2}} \left(1 + (\cos x)^{\frac{5}{2}}\right)^{\frac{1}{2}} dx$

Put $\cos x = t^2 \Rightarrow \sin x\, dx = -2\, t\, dt$

$\therefore I = 4\int_0^1 t^2 \cdot t^{11} \sqrt{(1+t^5)}(t)dt$

$I = 4\int_0^1 t^{14}\sqrt{1+t^5}dt$

Put $1 + t^5 = = k^2$

$\Rightarrow 5t^4 dt = 2k\, dk$

$\therefore I = 4 \cdot \int_1^{\sqrt{2}} (k^2 - 1)^2 \cdot k \frac{2k}{5} dk$

$I = \frac{8}{5}\int_1^{\sqrt{2}} k^6 - 2k^4 + k^2 \ dk$

$I = \frac{8}{5}\left[\frac{k^7}{7} - \frac{2k^5}{5} + \frac{k^3}{3}\right]_1^{\sqrt{2}}$

$I = \frac{8}{5}\left[\frac{8\sqrt{2}}{7} - \frac{8\sqrt{2}}{5} + \frac{2\sqrt{2}}{3} - \frac{1}{7} + \frac{2}{5} - \frac{1}{3}\right]$

$I = \frac{8}{5}\left[\frac{22\sqrt{2}}{105} - \frac{8}{105}\right]$

$\therefore 525 \cdot I = 176\sqrt{2} - 64$

22. Let $S = (-1, \infty)$ and $f : S \to \mathbb{R}$ be defined as

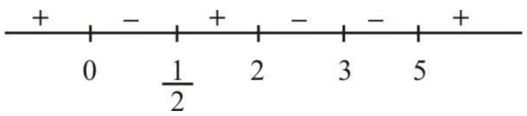
$f(x) = \int_{-1}^{x} (e^t - 1)^{11} (2t-1)^5 (t-2)^7 (t-3)^{12} (2t-10)^{61} dt$

Let p = Sum of square of the values of x, where f(x) attains local maxima on S. and q = Sum of the values of x, where f(x) attains local minima on S. Then, the value of $p^2 + 2q$ is ________

Ans. (27)

Sol.

$$f'(x)=(e^x-1)^{11}(2x-1)^5(x-2)^7(x-3)^{12}(2x-10)^{61}$$



Local minima at $x=\frac{1}{2},\ x=5$

Local maxima at $x = 0, x = 2$

$\therefore p=0+4=4,\ q=\frac{1}{2}+5=\frac{11}{2}$

Then $p^2 + 2q = 16 + 11 = 27$

23. The total number of words (with or without meaning) that can be formed out of the letters of the word 'DISTRIBUTION' taken four at a time, is equal to _____

Ans. (3734)

Sol. We have III, TT, D, S, R, B, U, O, N

Number of words with selection (a, a, a, b)

$={}^8C_1\times\frac{4!}{3!}=32$

Number of words with selection (a, a, b, b)

$=\frac{4!}{2!2!}=6$

Number of words with selection (a, a, b, c)

$={}^2C_1\times{}^8C_2\times\frac{4!}{2!}=672$

Number of words with selection (a, b, c, d)

$={}^9C_4\times 4!=3024$

$\therefore$ total = 3024 + 672 + 6 + 32

= 3734

24. Let Q and R be the feet of perpendiculars from the point P(a, a, a) on the lines x = y, z = 1 and x = –y, z = –1 respectively. If $\angle QPR$ is a right angle, then $12a^2$ is equal to _____

Ans. (12)

Sol. $\frac{x}{1}=\frac{y}{1}=\frac{z-1}{0}=r\rightarrow Q(r,r,1)$

$\frac{x}{1}=\frac{y}{-1}=\frac{z+1}{0}=k\rightarrow R(k,-k,-1)$

$\overline{PQ}=(a-r)\hat{i}+(a-r)\hat{j}+(a-1)\hat{k}$

$a = r + a – r = 0.$

$2a = 2r \rightarrow a = r$

$\overrightarrow{PR}=(a-k)i+(a+k)\hat{j}+(a+1)\hat{k}$

$a-k-a-k=0\ \Rightarrow k=0$

As, $PQ\perp PR$

$(a-r)(a-k)+(a-r)(a+k)+(a-1)(a+1)=0$

$a = 1$ or -1

$12a^2 = 12$

25. In the expansion of

$(1+x)(1-x^2)\left(1+\frac{3}{x}+\frac{3}{x^2}+\frac{1}{x^3}\right)^5$, $x\neq 0$, the sum of the coefficient of x^3 and x^{-13} is equal to ___

Ans. (118)

Sol. $(1+x)(1-x^2)\left(1+\frac{3}{x}+\frac{3}{x^2}+\frac{1}{x^3}\right)^5$

$=(1+x)(1-x^2)\left(\left(1+\frac{1}{x}\right)^3\right)^5$

$=\frac{(1+x)^2(1-x)(1+x)^{15}}{x^{15}}$

$=\frac{(1+x)^{17}-x(1+x)^{17}}{x^{15}}$

$=\text{coeff}(x^3)$ in the expansion $\approx\text{coeff}(x^{18})$ in $(1+x)^{17}-x(1+x)^{17}$

= 0 – 1

= –1

$\text{coeff}(x^{-13})$ in the expansion $\approx\text{coeff}(x^2)$ in $(1 + x)^{17} – x(1 + x)^{17}$

$=\binom{17}{2}-\binom{17}{1}$

= 17 × 8 – 17

= 17 × 7

= 119

Hence Answer = 119 – 1 = 118

26. If α denotes the number of solutions of $|1 - i|^x = 2^x$

and $\beta = \left(\dfrac{|z|}{\arg(z)}\right)$, where

$$z = \frac{\pi}{4}(1+i)^4\left(\frac{1-\sqrt{\pi}i}{\sqrt{\pi}+i}+\frac{\sqrt{\pi}-i}{1+\sqrt{\pi}i}\right),\ i=\sqrt{-1}\text{, then}$$

the distance of the point (α, β) from the line $4x - 3y = 7$ is _____

Ans. (3)

Sol. $\left(\sqrt{2}\right)^x = 2^x \Rightarrow x = 0 \Rightarrow \alpha = 1$

$$z = \frac{\pi}{4}(1+i)^4\left[\frac{\sqrt{\pi}-\pi i-i-\sqrt{\pi}}{\pi+1}+\frac{\sqrt{\pi}-i-\pi i-\sqrt{\pi}}{1+\pi}\right]$$

$$= -\frac{\pi i}{2}\left(1+4i+6i^2+4i^3+1\right)$$

$$= 2\pi i$$

$$\beta = \frac{2\pi}{\frac{\pi}{2}} = 4$$

Distance from (1, 4) to $4x - 3y = 7$

Will be $\dfrac{15}{5} = 3$

27. Let the foci and length of the latus rectum of an ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$, a > b be $(\pm 5, 0)$ and $\sqrt{50}$, respectively. Then, the square of the eccentricity of the hyperbola $\dfrac{x^2}{b^2}-\dfrac{y^2}{a^2b^2}=1$ equals

Ans. (51)

Sol. $\text{focii} \equiv (\pm 5, 0);\ \dfrac{2b^2}{a} = \sqrt{50}$

ae = 5 $\qquad b^2 = \dfrac{5\sqrt{2}a}{2}$

$$b^2 = a^2\left(1-e^2\right) = \frac{5\sqrt{2}a}{2}$$

$$\Rightarrow a\left(1-e^2\right) = \frac{5\sqrt{2}}{2}$$

$$\Rightarrow \frac{5}{e}\left(1-e^2\right) = \frac{5\sqrt{2}}{2}$$

$$\Rightarrow \sqrt{2}-\sqrt{2}e^2 = e$$

$$\Rightarrow \sqrt{2}e^2+e-\sqrt{2} = 0$$

$$\Rightarrow \sqrt{2}e^2+2e-e-\sqrt{2} = 0$$

$$\Rightarrow \sqrt{2}e\left(e+\sqrt{2}\right)-1\left(1+\sqrt{2}\right) = 0$$

$$\Rightarrow \left(e+\sqrt{2}\right)\left(\sqrt{2}e-1\right) = 0$$

$$\therefore e \neq -\sqrt{2}\ ;\ e = \frac{1}{\sqrt{2}}$$

$\dfrac{x^2}{b^2}-\dfrac{y^2}{a^2b^2}=1 \qquad a = 5\sqrt{2}$

$b = 5$

$$a^2b^2 = b^2\left(e_1^2-1\right) \Rightarrow e_1^2 = 51$$

28. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}| = 1, |\vec{b}| = 4$ and $\vec{a}\cdot\vec{b} = 2$. If $\vec{c} = \left(2\vec{a}\times\vec{b}\right)-3\vec{b}$ and the angle between $\vec{b}$ and $\vec{c}$ is α, then $192\sin^2\alpha$ is equal to______

Ans. (48)

Sol. $\vec{b}\cdot\vec{c} = \left(2\vec{a}\times\vec{b}\right)\cdot\vec{b}-3|b|^2$

$|b||c|\cos\alpha = -3|b|^2$

$|c|\cos\alpha = -12$, as $|b| = 4$

$\vec{a}\cdot\vec{b} = 2$

$$\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

$$|c|^2 = \left|\left(2\vec{a}\times\vec{b}\right)-3\vec{b}\right|^2$$

$$= 64\times\frac{3}{4}+144 = 192$$

$$|c|^2\cos^2\alpha = 144$$

$$192\cos^2\alpha = 144$$

$$192\sin^2\alpha = 48$$

29. Let A = {1, 2, 3, 4} and R = {(1, 2), (2, 3), (1, 4)} be a relation on A. Let S be the equivalence relation on A such that $R \subset S$ and the number of elements in S is n. Then, the minimum value of n is _______

Ans. (16)

Sol. All elements are included

Answer is 16

30. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by

$$f(x) = \frac{4^x}{4^x + 2} \text{ and}$$

$$M = \int_{f(a)}^{f(1-a)} x \sin^4\left(x(1-x)\right)dx,$$

$$N = \int_{f(a)}^{f(1-a)} \sin^4\left(x(1-x)\right)dx; a \neq \frac{1}{2}. \text{ If}$$

$\alpha M = \beta N, \alpha, \beta \in \mathbb{N}$, then the least value of $\alpha^2 + \beta^2$ is equal to ______

Ans. (5)

Sol. f(a) + f(1 – a) = 1.

$$M = \int_{f(a)}^{f(1-a)} (1-x).\sin^4 x(1-x)dx$$

M = N – M 2M = N

α = 2; β = 1;

Ans. 5

PHYSICS	TEST PAPER WITH SOLUTION

SECTION-A

31. The parameter that remains the same for molecules of all gases at a given temperature is :

(1) kinetic energy (2) momentum

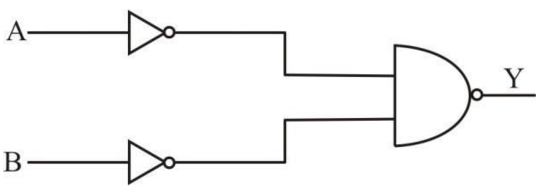
(3) mass (4) speed

Ans. (1)

Sol. $KE = \frac{f}{2}kT$

Conceptual

32. Identify the logic operation performed by the given circuit.



(1) NAND (2) NOR

(3) OR (4) AND

Ans. (3)

Sol. $Y = \overline{\overline{A}\cdot\overline{B}} = \overline{\overline{A}} + \overline{\overline{B}} = A + B$

(De-Morgan's law)

33. The relation between time 't' and distance 'x' is $t = \alpha x^2 + \beta x$, where α and β are constants. The relation between acceleration (a) and velocity (v) is:

(1) $a = -2\alpha v^3$ (2) $a = -5\alpha v^5$

(3) $a = -3\alpha v^2$ (4) $a = -4\alpha v^4$

Ans. (1)

Sol. $t = \alpha x^2 + \beta x$ (differentiating wrt time)

$$\frac{dt}{dx} = 2\alpha x + \beta$$

$$\frac{1}{v} = 2\alpha x + \beta$$

(differentiating wrt time)

$$-\frac{1}{v^2}\frac{dv}{dt} = 2\alpha\frac{dx}{dt}$$

$$\frac{dv}{dt} = -2\alpha v^3$$

34. The refractive index of a prism with apex angle A is cot A/2. The angle of minimum deviation is :

(1) $\delta_m = 180° - A$

(2) $\delta_m = 180° - 3A$

(3) $\delta_m = 180° - 4A$

(4) $\delta_m = 180° - 2A$

Ans. (4)

Sol. $$\mu = \frac{\sin\left(\frac{A+\delta m}{2}\right)}{\sin\frac{A}{2}}$$

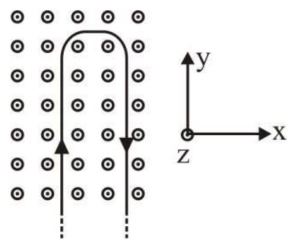
$$\frac{\cos\frac{A}{2}}{\sin\frac{A}{2}} = \frac{\sin\left(\frac{A+\delta m}{2}\right)}{\sin\frac{A}{2}}$$

$$\sin\left(\frac{\pi}{2} - \frac{A}{2}\right) = \sin\left(\frac{A+\delta_m}{2}\right)$$

$$\frac{\pi}{2} - \frac{A}{2} = \frac{A}{2} + \frac{\delta m}{2}$$

$$\delta_m = \pi - 2A$$

35. A rigid wire consists of a semicircular portion of radius R and two straight sections. The wire is partially immerged in a perpendicular magnetic field $B = B_0\,\hat{j}$ as shown in figure. The magnetic force on the wire if it has a current i is :

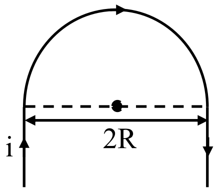


(1) $-iBR\,\hat{j}$ (2) $2iBR\,\hat{j}$

(3) $iBR\,\hat{j}$ (4) $-2iBR\,\hat{j}$

Ans. (4)

Sol.



Note : Direction of magnetic field is in $+\hat{k}$

$\vec{F} = i\vec{\ell} \times \vec{B}$

$\ell = 2R$

$\vec{F} = -2iRB\hat{j}$

36. If the wavelength of the first member of Lyman series of hydrogen is λ. The wavelength of the second member will be

(1) $\frac{27}{32}\lambda$ (2) $\frac{32}{27}\lambda$

(3) $\frac{27}{5}\lambda$ (4) $\frac{5}{27}\lambda$

Ans. **(1)**

Sol. $\frac{1}{\lambda} = \frac{13.6z^2}{hc}\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$ ….. (i)

$\frac{1}{\lambda'} = \frac{13.6z^2}{hc}\left[\frac{1}{1^2} - \frac{1}{3^2}\right]$ ….. (ii)

On dividing (i) & (ii)

$\lambda' = \frac{27}{32}\lambda$

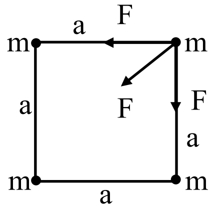
37. Four identical particles of mass m are kept at the four corners of a square. If the gravitational force exerted on one of the masses by the other masses is $\left(\frac{2\sqrt{2}+1}{32}\right)\frac{Gm^2}{L^2}$, the length of the sides of the square is

(1) $\frac{L}{2}$ (2) 4L

(3) 3L (4) 2L

Ans. **(2)**

Sol.



$F_{net} = \sqrt{2}F + F'$

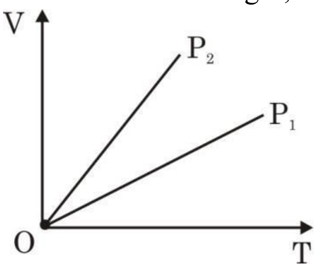
$F = \frac{Gm^2}{a^2}$ and $F' = \frac{Gm^2}{\left(\sqrt{2}a\right)^2}$

$F_{net} = \sqrt{2}\frac{Gm^2}{a^2} + \frac{Gm^2}{2a^2}$

$\left(\frac{2\sqrt{2}+1}{32}\right)\frac{Gm^2}{L^2} = \frac{Gm^2}{a^2}\left(\frac{2\sqrt{2}+1}{2}\right)$

$a = 4L$

38. The given figure represents two isobaric processes for the same mass of an ideal gas, then



(1) $P_2 \geq P_1$ (2) $P_2 > P_1$

(3) $P_1 = P_2$ (4) $P_1 > P_2$

Ans. **(4)**

Sol. $PV = nRT$

$V = \left(\frac{nR}{P}\right)T$

Slope $= \frac{nR}{P}$

Slope $\propto \frac{1}{P}$

$(\text{Slope})_2 > (\text{Slope})_1$

$P_2 < P_1$

39. If the percentage errors in measuring the length and the diameter of a wire are 0.1% each. The percentage error in measuring its resistance will be:

(1) 0.2% (2) 0.3%

(3) 0.1% (4) 0.144%

Ans. **(2)**

Sol. $R = \dfrac{\rho L}{\pi \dfrac{d^2}{4}}$

$\dfrac{\Delta R}{R} = \dfrac{\Delta L}{L} + \dfrac{2\Delta d}{d}$

$\dfrac{\Delta L}{L} = 0.1\%$ and $\dfrac{\Delta d}{d} = 0.1\%$

$\dfrac{\Delta R}{R} = 0.3\%$

40. In a plane EM wave, the electric field oscillates sinusoidally at a frequency of 5×10^{10} Hz and an amplitude of 50 Vm^{-1}. The total average energy density of the electromagnetic field of the wave is :

[Use $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2 / Nm^2$]

(1) $1.106 \times 10^{-8}\ Jm^{-3}$

(2) $4.425 \times 10^{-8}\ Jm^{-3}$

(3) $2.212 \times 10^{-8}\ Jm^{-3}$

(4) $2.212 \times 10^{-10}\ Jm^{-3}$

Ans. **(1)**

Sol. $U_E = \dfrac{1}{2} \epsilon_0 E^2$

$U_E = \dfrac{1}{2} \times 8.85 \times 10^{-12} \times (50)^2$

$= 1.106 \times 10^{-8}\ J/m^3$

41. A force is represented by $F = ax^2 + bt^{1/2}$

Where x = distance and t = time. The dimensions of b^2/a are :

(1) $[ML^3T^{-3}]$ (2) $[MLT^{-2}]$

(3) $[ML^{-1}T^{-1}]$ (4) $[ML^2T^{-3}]$

Ans. **(1)**

Sol. $F = ax^2 + bt^{1/2}$

$[a] = \dfrac{[F]}{[x^2]} = [M^1L^{-1}T^{-2}]$

$[b] = \dfrac{[F]}{[t^{1/2}]} = [M^1L^1T^{-5/2}]$

$\left[\dfrac{b^2}{a}\right] = \dfrac{[M^2L^2T^{-5}]}{[M^1L^{-1}T^{-2}]} = [M^1L^3T^{-3}]$

42. Two charges q and 3q are separated by a distance 'r' in air. At a distance x from charge q, the resultant electric field is zero. The value of x is :

(1) $\dfrac{(1+\sqrt{3})}{r}$

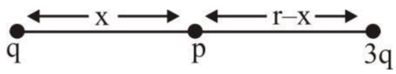
(2) $\dfrac{r}{3(1+\sqrt{3})}$

(3) $\dfrac{r}{(1+\sqrt{3})}$

(4) $r(1+\sqrt{3})$

Ans. **(3)**

Sol.



$(\vec{E}_{net})_P = 0$

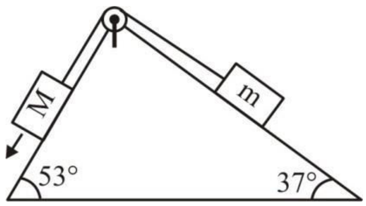
$\dfrac{kq}{x^2} = \dfrac{k \cdot 3q}{(r-x)^2}$

$(r - x)^2 = 3x^2$

$r - x = \sqrt{3}x$

$x = \dfrac{r}{\sqrt{3}+1}$

43. In the given arrangement of a doubly inclined plane two blocks of masses M and m are placed. The blocks are connected by a light string passing over an ideal pulley as shown. The coefficient of friction between the surface of the plane and the blocks is 0.25. The value of m, for which M = 10 kg will move down with an acceleration of 2 m/s^2, is : (take g = 10 m/s^2 and tan 37° = 3/4)



(1) 9 kg

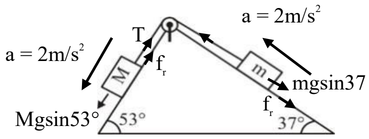
(2) 4.5 kg

(3) 6.5 kg

(4) 2.25 kg

Ans. **(2)**

Sol.



For M block

$10g\sin 53° - \mu\,(10g)\cos 53° - T = 10 \times 2$

$T = 80 - 15 - 20$

$T = 45$ N

For m block

$T - mg\sin 37° - \mu mg\cos 37° = m \times 2$

$45 = 10\,m$

$m = 4.5$ kg

44. A coil is placed perpendicular to a magnetic field of 5000 T. When the field is changed to 3000 T in 2s, an induced emf of 22 V is produced in the coil. If the diameter of the coil is 0.02 m, then the number of turns in the coil is :

(1) 7 (2) 70

(3) 35 (4) 140

Ans. **(2)**

Sol. $\varepsilon = N\left(\dfrac{\Delta\phi}{t}\right)$

$\Delta\phi = (\Delta B)A$

$B_i = 5000$ T,

$B_f = 3000$ T

$d = 0.02$ m

$r = 0.01$ m

$\Delta\phi = (\Delta B)A$

$= (2000)\pi(0.01)^2 = 0.2\pi$

$\varepsilon = N\left(\dfrac{\Delta\phi}{t}\right) \Rightarrow 22 = N\left(\dfrac{0.2\pi}{2}\right)$

$N = 70$

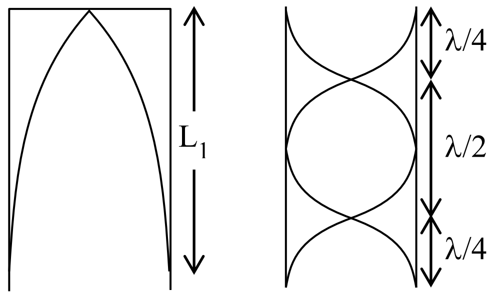
45. The fundamental frequency of a closed organ pipe is equal to the first overtone frequency of an open organ pipe. If length of the open pipe is 60 cm, the length of the closed pipe will be :

(1) 60 cm (2) 45 cm

(3) 30 cm (4) 15 cm

Ans. **(4)**

Sol.



$\dfrac{\lambda}{4} = L_1$ $\qquad$ $2\left(\dfrac{\lambda}{2}\right) = \lambda$

$v = f\lambda$ $\qquad$ $f_2 = \dfrac{2v}{2L_2}$

$v = f_1(4L_1)$ $\qquad$ $f_2 = \dfrac{v}{L_2}$

$f_1 = \dfrac{v}{4L_1}$

$f_1 = f_2$

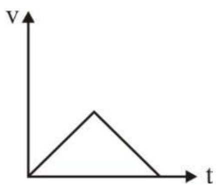
$\dfrac{v}{4L_1} = \dfrac{v}{L_2}$

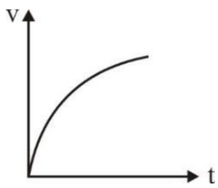
$\Rightarrow L_2 = 4L_1$

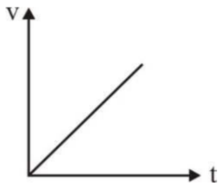
$60 = 4 \times L_1$

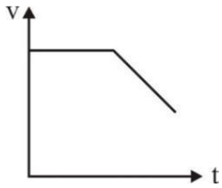
$L_1 = 15$ cm

46. A small steel ball is dropped into a long cylinder containing glycerine. Which one of the following is the correct representation of the velocity time graph for the transit of the ball?

(1)


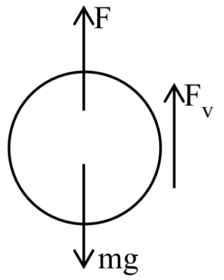
(2)


(3)


(4)


Ans. **(2)**

Sol.



$mg - F_B - F_v = ma$

$$\left(\rho\frac{4}{3}\pi r^3\right)g - \left(\rho_L\frac{4}{3}\pi r^3\right)g - 6\pi\eta rv = m\frac{dv}{dt}$$

Let $\frac{4}{3m}\pi R^3 g(\rho - \rho_L) = K_1$ and $\frac{6\pi\eta r}{m} = K_2$

$$\frac{dv}{dt} = K_1 - K_2 v$$

$$\int_0^v \frac{dv}{K_1 - K_2 v} = \int_0^t dt$$

$$-\frac{1}{K_2}\ell n[K_1 - K_2 v]_0^v = t$$

$$\ell n\left(\frac{K_1 - K_2 v}{K_1}\right) = -K_2 t$$

$$K_1 - K_2 v = K_1 e^{-K_2 t}$$

$$v = \frac{K_1}{K_2}\left[1 - e^{-K_2 t}\right]$$

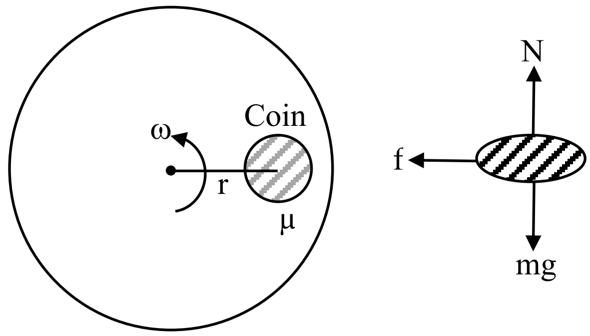
47. A coin is placed on a disc. The coefficient of friction between the coin and the disc is μ. If the distance of the coin from the center of the disc is r, the maximum angular velocity which can be given to the disc, so that the coin does not slip away, is :

(1) $\frac{\mu g}{r}$ (2) $\sqrt{\frac{r}{\mu g}}$

(3) $\sqrt{\frac{\mu g}{r}}$ (4) $\frac{\mu}{\sqrt{rg}}$

Ans. **(3)**

Sol.



$N = mg$

$f = m\omega^2 r$

$f = \mu N$

$\mu mg = mr\omega^2$

$$\omega = \sqrt{\frac{\mu g}{r}}$$

48. Two conductors have the same resistances at 0°C but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coefficients for their series and parallel combinations are :

(1) $\alpha_1 + \alpha_2, \frac{\alpha_1 + \alpha_2}{2}$

(2) $\frac{\alpha_1 + \alpha_2}{2}, \frac{\alpha_1 + \alpha_2}{2}$

(3) $\alpha_1 + \alpha_2, \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2}$

(4) $\frac{\alpha_1 + \alpha_2}{2}, \alpha_1 + \alpha_2$

Ans. **(2)**

Sol. **Series :**

$R_{eq} = R_1 + R_2$

$$2R(1 + \alpha_{eq}\Delta\theta) = R(1 + \alpha_1\Delta\theta) + R(1 + \alpha_2\Delta\theta)$$

$$2R(1 + \alpha_{eq}\Delta\theta) = 2R + (\alpha_1 + \alpha_2)R\Delta\theta$$

$$\alpha_{eq} = \frac{\alpha_1 + \alpha_2}{2}$$

Parallel :

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{\frac{R}{2}(1 + \alpha_{eq}\Delta\theta)} = \frac{1}{R(1 + \alpha_1\Delta\theta)} + \frac{1}{R(1 + \alpha_2\Delta\theta)}$$

$$\frac{2}{1+\alpha_{eq}\Delta\theta}=\frac{1}{1+\alpha_1\Delta\theta}+\frac{1}{1+\alpha_2\Delta\theta}$$

$$\frac{2}{1+\alpha_{eq}\Delta\theta}=\frac{1+\alpha_2\Delta\theta+1+\alpha_1\Delta\theta}{(1+\alpha_1\Delta\theta)(1+\alpha_2\Delta\theta)}$$

$$2\left[(1+\alpha_1\Delta\theta)(1+\alpha_2\Delta\theta)\right]$$

$$=\left[2+(\alpha_1+\alpha_2)\Delta\theta\right]\left[1+\alpha_{eq}\Delta\theta\right]$$

$$2[1+\alpha_1\Delta\theta+\alpha_2\Delta\theta+\alpha_1\alpha_2\Delta\theta]$$

$=$

$$2+2\alpha_{eq}\Delta\theta+(\alpha_1+\alpha_2)\Delta\theta+\alpha_{eq}(\alpha_1+\alpha_2)\Delta\theta^2$$

Neglecting small terms

$$2+2(\alpha_1+\alpha_2)\Delta\theta=2+2\alpha_{eq}\Delta\theta+(\alpha_1+\alpha_2)\Delta\theta$$

$$(\alpha_1+\alpha_2)\Delta\theta=2\alpha_{eq}\Delta\theta$$

$$\alpha_{eq}=\frac{\alpha_1+\alpha_2}{2}$$

49. An artillery piece of mass M_1 fires a shell of mass M_2 horizontally. Instantaneously after the firing, the ratio of kinetic energy of the artillery and that of the shell is :

(1) $M_1 / (M_1 + M_2)$ (2) $\frac{M_2}{M_1}$

(3) $M_2 / (M_1 + M_2)$ (4) $\frac{M_1}{M_2}$

Ans. **(2)**

Sol. $|\vec{p_1}|=|\vec{p_2}|$

$KE=\frac{p^2}{2M}$; p same

$$KE\propto\frac{1}{m}$$

$$\frac{KE_1}{KE_2}=\frac{p^2/2M_1}{p^2/2M_2}=\frac{M_2}{M_1}$$

50. When a metal surface is illuminated by light of wavelength λ, the stopping potential is 8V. When the same surface is illuminated by light of wavelength 3λ, stopping potential is 2V. The threshold wavelength for this surface is :

(1) 5λ

(2) 3λ

(3) 9λ

(4) 4.5λ

Ans. **(3)**

Sol. $E=\phi+K_{max}$

$$\phi=\frac{hc}{\lambda_0}$$

$$K_{max}=eV_0$$

$$8e=\frac{hc}{\lambda}-\frac{hc}{\lambda_0}\quad......(i)$$

$$2e=\frac{hc}{3\lambda}-\frac{hc}{\lambda_0}\quad.......(ii)$$

on solving (i) & (ii)

$$\lambda_0=9\lambda$$

SECTION-B

51. An electron moves through a uniform magnetic field $\vec{B}=B_0\hat{i}+2B_0\hat{j}$ T. At a particular instant of time, the velocity of electron is $\vec{u}=3\hat{i}+5\hat{j}$ m/s. If the magnetic force acting on electron is $\vec{F}=5e\hat{k}$ N, where e is the charge of electron, then the value of B_0 is ____ T.

Ans. **(5)**

Sol. $\vec{F}=q(\vec{v}\times\vec{B})$

$$5e\hat{k}=e(3\hat{i}+5\hat{j})\times(B_0\hat{i}+2B_0\hat{j})$$

$$5e\hat{k}=e(6B_0\hat{k}-5B_0\hat{k})$$

$$\Rightarrow B_0=5T$$

52. A parallel plate capacitor with plate separation 5 mm is charged up by a battery. It is found that on introducing a dielectric sheet of thickness 2 mm, while keeping the battery connections intact, the capacitor draws 25% more charge from the battery than before. The dielectric constant of the sheet is ____.

Ans. **(2)**

Sol. Without dielectric

$$Q = \frac{A \in_0}{d} V$$

with dielectric

$$Q = \frac{A \in_0 V}{d - t + \frac{t}{K}}$$

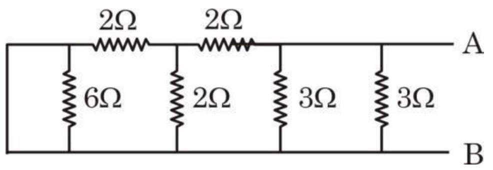
given

$$\frac{A \in_0 V}{d - t + \frac{t}{K}} = (1.25)\frac{A \in_0 V}{d}$$

$$\Rightarrow 1.25\left(3 + \frac{2}{K}\right) = 5$$

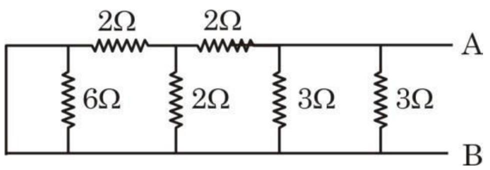
$$\Rightarrow K = 2$$

53. Equivalent resistance of the following network is _____ Ω.

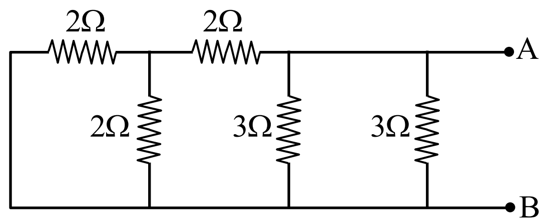


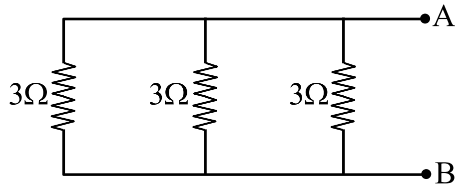
Ans. **(1)**

Sol.



6Ω is short circuit





$$R_{eq} = 3 \times \frac{1}{3} = 1\Omega$$

54. A solid circular disc of mass 50 kg rolls along a horizontal floor so that its center of mass has a speed of 0.4 m/s. The absolute value of work done on the disc to stop it is ______ J.

Ans. **(6)**

Sol. Using work energy theorem

$$W = \Delta KE = 0 - \left(\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2\right)$$

$$W = 0 - \frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right)$$

$$= -\frac{1}{2} \times 50 \times 0.4^2\left(1 + \frac{1}{2}\right) = -6J$$

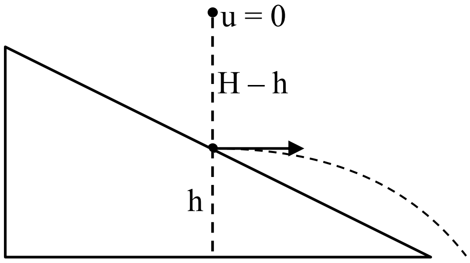
Absolute work = +6J

$W = -6J \quad |W| = 6J$

55. A body starts falling freely from height H hits an inclined plane in its path at height h. As a result of this perfectly elastic impact, the direction of the velocity of the body becomes horizontal. The value of $\frac{H}{h}$ for which the body will take the maximum time to reach the ground is _____.

Ans. **(2)**

Sol.



Total time of flight = T

$$T = \sqrt{\frac{2h}{g}} + \sqrt{\frac{2(H-h)}{g}}$$

For max. time = $\frac{dT}{dh} = 0$

$$\sqrt{\frac{2}{g}}\left(\frac{-1}{2\sqrt{H-h}} + \frac{1}{2\sqrt{h}}\right) = 0$$

$$\sqrt{H-h} = \sqrt{h}$$

$$h = \frac{H}{2} \Rightarrow \frac{H}{h} = 2$$

56. Two waves of intensity ratio 1 : 9 cross each other at a point. The resultant intensities at the point, when (a) Waves are incoherent is I_1(b) Waves are coherent is I_2 and differ in phase by 60°. If $\frac{I_1}{I_2} = \frac{10}{x}$ then x = _____.

Ans. **(13)**

Sol. For incoherent wave $I_1 = I_A + I_B \Rightarrow I_1 = I_0 + 9I_0$

$I_1 = 10I_0$

For coherent wave $I_2 = I_A + I_B + 2\sqrt{I_A I_B}\cos 60°$

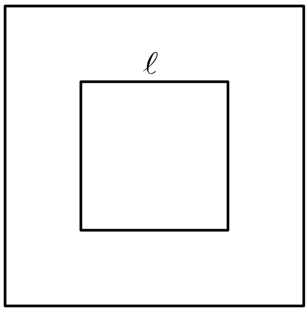
$$I_2 = I_0 + 9I_0 + 2\sqrt{9I_0^2}.\frac{1}{2} = 13\ I_0$$

$$\frac{I_1}{I_2} = \frac{10}{13}$$

57. A small square loop of wire of side ℓ is placed inside a large square loop of wire of side L ($L = \ell^2$). The loops are coplanar and their centers coinside. The value of the mutual inductance of the system is $\sqrt{x} \times 10^{-7}$ H, where x = _____.

Ans. **(128)**

Sol.



Flux linkage for inner loop.

$\phi = B_{center} \cdot \ell^2$

$$= 4 \times \frac{\mu_0 i}{4\pi\frac{L}{2}}(\sin 45 + \sin 45)\ell^2$$

$$\phi = 2\sqrt{2}\frac{\mu_0 i}{\pi L}\ell^2$$

$$M = \frac{\phi}{i} = \frac{2\sqrt{2}\mu_0\ell^2}{\pi L} = 2\sqrt{2}\frac{\mu_0}{\pi}$$

$$= 2\sqrt{2}\frac{4\pi}{\pi} \times 10^{-7}$$

$$= 8\sqrt{2} \times 10^{-7}\text{H}$$

$$= \sqrt{128} \times 10^{-7}\text{H}$$

x = 128

58. The depth below the surface of sea to which a rubber ball be taken so as to decrease its volume by 0.02% is _____ m.

(Take density of sea water = 10^3 kgm^{-3}, Bulk modulus of rubber = 9×10^8 Nm^{-2}, and $g = 10$ ms^{-2})

Ans. **(18)**

Sol. $\beta = \frac{-\Delta P}{\frac{\Delta V}{V}}$

$$\Delta P = -\beta\frac{\Delta V}{V}$$

$$\rho g h = -\beta\frac{\Delta V}{V}$$

$$10^3 \times 10 \times h = -9 \times 10^8 \times \left(-\frac{0.02}{100}\right)$$

$\Rightarrow h = 18$ m

59. A particle performs simple harmonic motion with amplitude A. Its speed is increased to three times at an instant when its displacement is $\frac{2A}{3}$. The new amplitude of motion is $\frac{nA}{3}$. The value of n is ___.

Ans. **(7)**

Sol. $v = \omega\sqrt{A^2 - x^2}$

at $x = \frac{2A}{3}$

$v = \omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2} = \frac{\sqrt{5}A\omega}{3}$

New amplitude = A'

$v' = 3v = \sqrt{5}A\omega = \omega\sqrt{(A')^2 - \left(\frac{2A}{3}\right)^2}$

$A' = \frac{7A}{3}$

60. The mass defect in a particular reaction is 0.4g. The amount of energy liberated is n × 10^7 kWh, where n = _____.

(speed of light = 3 × 10^8 m/s)

Ans. **(1)**

Sol. $E = \Delta mc^2$

$= 0.4 \times 10^{-3} \times (3 \times 10^8)^2$

$= 3600 \times 10^7$ kWs

$= \frac{3600 \times 10^7}{3600}\text{kWh} = 1 \times 10^7\text{kWh}$

CHEMISTRY | **TEST PAPER WITH SOLUTION**

SECTION-A

61. Give below are two statements:

Statement-I : Noble gases have very high boiling points.

Statement-II: Noble gases are monoatomic gases. They are held together by strong dispersion forces. Because of this they are liquefied at very low temperature. Hence, they have very high boiling points.

In the light of the above statements. choose the ***correct answer*** from the options given below:

(1) Statement I is false but Statement II is true.

(2) Both Statement I and Statement II are true.

(3) Statement I is true but Statement II is false.

(4) Both Statement I and Statement II are false.

Ans. **(4)**

Sol. Statement I and II are False

Noble gases have low boiling points

Noble gases are held together by weak dispersion forces.

62. For the given reaction, choose the correct expression of K_C from the following :-

$$Fe^{3+}_{(aq)} + SCN^{-}_{(aq)} \rightleftharpoons (FeSCN)^{2+}_{(aq)}$$

(1) $K_C = \dfrac{[FeSCN^{2+}]}{[Fe^{3+}][SCN^{-}]}$

(2) $K_C = \dfrac{[Fe^{3+}][SCN^{-}]}{[FeSCN^{2+}]}$

(3) $K_C = \dfrac{[FeSCN^{2+}]}{[Fe^{3+}]^2[SCN^{-}]^2}$

(4) $K_C = \dfrac{[FeSCN^{2+}]^2}{[Fe^{3+}][SCN^{-}]}$

Ans. **(1)**

Sol. $K_C = \dfrac{\text{Products ion conc.}}{\text{Reactants ion conc.}}$

$$K_C = \frac{[FeSCN^{2+}]}{[Fe^{3+}][SCN^{-}]}$$

63. Identify the mixture that shows positive deviations from Raoult's Law

(1) $(CH_3)_2CO + C_6H_5NH_2$

(2) $CHCl_3 + C_6H_6$

(3) $CHCl_3 + (CH_3)_2CO$

(4) $(CH_3)_2CO + CS_2$

Ans. **(4)**

Sol. $(CH_3)_2CO + CS_2$ Exibits positive deviations from Raoult’s Law

64. The compound that is white in color is

(1) ammonium sulphide

(2) lead sulphate

(3) lead iodide

(4) ammonium arsinomolybdate

Ans. **(2)**

Sol. Lead sulphate-white

Ammonium sulphide-soluble

Lead iodide-Bright yellow

Ammonium arsinomolybdate-yellow

65. The metals that are employed in the battery industries are

A. Fe

B. Mn

C. Ni

D. Cr

E. Cd

Choose the correct answer from the options given below:

(1) B, C and E only

(2) A, B, C, D and E

(3) A, B, C and D only

(4) B, D and E only

Ans. **(1)**

Sol. Mn, Ni and Cd metals used in battery industries.

66. A species having carbon with sextet of electrons and can act as electrophile is called

(1) carbon free radical

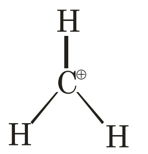
(2) carbanion

(3) carbocation

(4) pentavalent carbon

Ans. (3)

Sol.



Six electron species

67. Identify the factor from the following that does not affect electrolytic conductance of a solution.

(1) The nature of the electrolyte added.

(2) The nature of the electrode used.

(3) Concentration of the electrolyte.

(4) The nature of solvent used.

Ans. (2)

Sol. Conductivity of electrolytic cell is affected by concentration of electrolyte, nature of electrolyte and nature of solvent.

68. The product (C) in the below mentioned reaction is:

$$CH_3-CH_2-CH_2-Br \xrightarrow[\Delta]{KOH_{(alc)}} A \xrightarrow{HBr} B \xrightarrow[KOH_{(aq)}]{\Delta} C$$

(1) Propan-1-ol

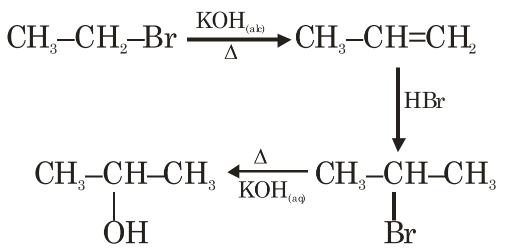
(2) Propene

(3) Propyne

(4) Propan-2-ol

Ans. (4)

Sol.



69. Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R**:

Assertion A: Alcohols react both as nucleophiles and electrophiles.

Reason R: Alcohols react with active metals such as sodium, potassium and aluminum to yield corresponding alkoxides and liberate hydrogen.

In the light of the above statements, choose the ***correct answer*** from the options given below:

(1) A is false but R is true.

(2) A is true but R is false.

(3) Both A and R are true and R is the correct explanation of A.

(4) Both A and R are true but R is NOT the correct explanation of A

Ans. (4)

Sol. As per NCERT, Assertion (A) and Reason (R) is correct but Reason (R) is not the correct explanation.

70. The correct sequence of electron gain enthalpy of the elements listed below is

A. Ar

B. Br

C. F

D. S

Choose the ***most appropriate*** from the options given below:

(1) C > B > D > A

(2) A > D > B > C

(3) A > D > C > B

(4) D > C > B > A

Ans. (2)

Sol.

Element	$\Delta_{eg}H$(kJ/mol)
F	–333
S	–200
Br	–325
Ar	+96

71. Identify correct statements from below:

A. The chromate ion is square planar.

B. Dichromates are generally prepared from chromates.

C. The green manganate ion is diamagnetic.

D. Dark green coloured K_2MnO_4 disproportionates in a neutral or acidic medium to give permanganate.

E. With increasing oxidation number of transition metal, ionic character of the oxides decreases.

Choose the correct answer from the options given below:

(1) B, C, D only

(2) A, D, E only

(3) A, B, C only

(4) B, D, E only

Ans. **(4)**

Sol. A. CrO_4^{2-} is tetrahedral

B. $2Na_2CrO_4 + 2H^+ \rightarrow Na_2Cr_2O_7 + 2Na^+ + H_2O$

C. As per NCERT, green manganate is paramagnetic with 1 unpaired electron.

D. Statement is correct

E. Statement is correct

72. 'Adsorption' principle is used for which of the following purification method?

(1) Extraction

(2) Chromatography

(3) Distillation

(4) Sublimation

Ans. **(2)**

Sol. Principle used in chromotography is adsorption.

73. Integrated rate law equation for a first order gas phase reaction is given by (where P_i is initial pressure and P_t is total pressure at time t)

(1) $k = \frac{2.303}{t} \times \log \frac{P_i}{(2P_i - P_t)}$

(2) $k = \frac{2.303}{t} \times \log \frac{2P_i}{(2P_i - P_t)}$

(3) $k = \frac{2.303}{t} \times \log \frac{(2P_i - P_t)}{P_i}$

(4) $k = \frac{2.303}{t} \times \frac{P_i}{(2P_i - P_t)}$

Ans. **(1)**

Sol.

A	$\rightarrow$	B	+	C
P_i		0		0
$P_i - x$		x		x

$P_t = P_i + x$

$P_i - x = P_i - P_t + P_i$

$= 2P_i - P_t$

$$K = \frac{2.303}{t} \log \frac{P_i}{2P_i - P_t}$$

74. Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R:**

Assertion A: pK_a value of phenol is 10.0 while that of ethanol is 15.9.

Reason R: Ethanol is stronger acid than phenol.

In the light of the above statements, choose the ***correct answer*** from the options given below:

(1) A is true but R is false.

(2) A is false but R is true.

(3) Both A and R are true and R is the correct explanation of A.

(4) Both A and R are true but R is NOT the correct explanation of A.

Ans. **(1)**

Sol. Phenol is more acidic than ethanol because conjugate base of phenoxide is more stable than ethoxide.

75. Given below are two statements:

Statement I: IUPAC name of $HO–CH_2–(CH_2)_3–CH_2– COCH_3$ is 7-hydroxyheptan-2-one.

Statement II: 2-oxoheptan-7-ol is the correct IUPAC name for above compound.

In the light of the above statements. choose the **most appropriate answer** from the options given below:

(1) Statement I is correct but Statement II is incorrect.

(2) Both Statement I and Statement II are incorrect.

(3) Both Statement I and Statement II are correct.

(4) Statement I is incorrect but Statement II is correct.

Ans. **(1)**

Sol. 7-Hydroxyheptan-2-one is correct IUPAC name

76. The correct statements from following are:

A. The strength of anionic ligands can be explained by crystal field theory.

B. Valence bond theory does not give a quantitative interpretation of kinetic stability of coordination compounds.

C. The hybridization involved in formation of $[Ni(CN)_4]^{2-}$complex is dsp^2.

D. The number of possible isomer(s) of cis-$[PtC1_2 (en)_2]^{2+}$ is one

Choose the correct answer from the options given below:

(1) A, D only

(2) A, C only

(3) B, D only

(4) B, C only

Ans. (4)

Sol. B. VBT does not explain stability of complex

C. Hybridisation of $[Ni(CN)_4]^{-2}$ is dsp^2.

77. The linear combination of atomic orbitals to form molecular orbitals takes place only when the combining atomic orbitals

A. have the same energy

B. have the minimum overlap

C. have same symmetry about the molecular axis

D. have different symmetry about the molecular axis

Choose the ***most appropriate*** from the options given below:

(1) A, B, C only

(2) A and C only

(3) B, C, D only

(4) B and D only

Ans. (2)

Sol. * Molecular orbital should have maximum overlap

* Symmetry about the molecular axis should be similar

78. Match List I with List II

LIST-I		LIST-II	
A.	Glucose/$NaHCO_3$/Δ	I.	Gluconic acid
B.	Glucose/HNO_3	II.	No reaction
C.	Glucose/HI/ Δ	III.	n-hexane
D.	Glucose/Bromine water]	IV.	Saccharic acid

Choose the correct answer from the options given below:

(1) A-IV, B-I, C-III, D-II

(2) A-II, B-IV, C-III, D-I

(3) A-III, B-II, C-I, D-IV

(4) A-I, B-IV, C-III, D-II

Ans. (2)

Sol. Glucose $\xrightarrow[\Delta]{NaHCO_3}$ no reaction

Glucose $\xrightarrow[\Delta]{HNO_3}$ saccharic acid

Glucose $\xrightarrow[\Delta]{HI}$ n-hexane

Glucose $\xrightarrow[\Delta]{Br_2}$ Gluconic acid

79. Consider the oxides of group 14 elements SiO_2, GeO_2, SnO_2, PbO_2, CO and GeO. The amphoteric oxides are

(1) GeO, GeO_2

(2) SiO_2, GeO_2

(3) SnO_2, PbO_2

(4) SnO_2, CO

Ans. (3)

Sol. SnO_2 and PbO_2 are amphoteric

80. Match List I with List II

LIST I (Technique)		LIST II (Application)	
A.	Distillation	I.	Separation of glycerol from spent-lye
B.	Fractional distillation	II.	Aniline - Water mixture
C.	Steam distillation	III.	Separation of crude oil fractions
D.	Distillation under reduced pressure	IV.	Chloroform-Aniline

Choose the correct answer from the options given below:

(1) A-IV, B-I, C-II, D-III

(2) A-IV, B-III, C-II. D-I

(3) A-I. B-II, C-IV, D-III

(4) A-II, B-III. C-I, D-IV

Ans. (2)

Sol. Fact (NCERT)

SECTION-B

81. Molar mass of the salt from NaBr, $NaNO_3$, KI and CaF_2 which does not evolve coloured vapours on heating with concentrated H_2SO_4 is _____ g mol^{-1}, (Molar mass in g mol^{-1} : Na : 23, N : 14, K : 39,

O : 16, Br : 80, I : 127, F : 19, Ca : 40

Ans. (78)

Sol. **CaF_2** does not evolve any gas with concentrated H_2SO_4.

$NaBr \rightarrow$ evolve Br_2

$NaNO_3 \rightarrow$ evolve NO_2

$KI \rightarrow$ evolve I_2

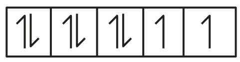
82. The 'Spin only' Magnetic moment for $[Ni(NH_3)_6]^{2+}$ is______$\times 10^{-1}$ BM.

(given = Atomic number of Ni : 28)

Ans. (28)

Sol. NH_3 act as WFL with Ni^{2+}

$Ni^{2+} = 3d^8$



No. of unpaired electron = 2

$$\mu = \sqrt{n(n+2)} = \sqrt{8} = 2.82 \text{ BM}$$

$$= 28.2 \times 10^{-1} \text{ BM}$$

$$x = 28$$

83. Number of moles of methane required to produce 22g $CO_{2(g)}$ after combustion is $x \times 10^{-2}$ moles. The value of x is

Ans. (50)

Sol. $CH_{4(g)} + 2O_{2(g)} \rightarrow CO_{2(g)} + 2H_2O_{(\ell)}$

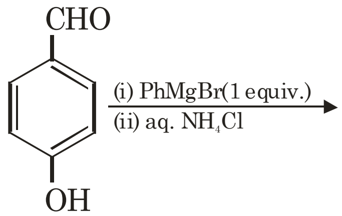
$$n_{CO_2} = \frac{22}{44} = 0.5 \text{ moles}$$

So moles of CH_4 required = 0.5 moles

i.e. 50×10^{-2} mole

x = 50

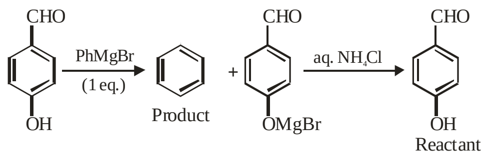
84. The product of the following reaction is P.



The number of hydroxyl groups present in the product P is________.

Ans. (0)

Sol. Product benzene has zero hydroxyl group



85. The number of species from the following in which the central atom uses sp^3 hybrid orbitals in its bonding is__________.

NH_3, SO_2, SiO_2, $BeCl_2$, CO_2, H_2O, CH_4, BF_3

Ans. (4)

Sol. $NH_3 \rightarrow sp^3$

$SO_2 \rightarrow sp^2$

$SiO_2 \rightarrow sp^3$

$BeCl_2 \rightarrow sp$

$CO_2 \rightarrow sp$

$H_2O \rightarrow sp^3$

$CH_4 \rightarrow sp^3$

$BF_3 \rightarrow sp^2$

86. $CH_3CH_2Br + NaOH \xrightarrow{C_2H_5OH}$ Product A; $\xrightarrow{H_2O}$ Product B

The total number of hydrogen atoms in product A and product B is__________.

Ans. (10)

Sol. $CH_3CH_2Br + NaOH \xrightarrow{C_2H_5OH} CH_2{=}CH_2$; $\xrightarrow{H_2O} CH_3{-}CH_2{-}OH$

Total number of hydrogen atom in A and B is 10

87. Number of alkanes obtained on electrolysis of a mixture of CH_3COONa and C_2H_5COONa is_____.

Ans. (3)

Sol. $CH_3COONa \rightarrow \dot{C}H_3$

$C_2H_5COONa \rightarrow \dot{C}_2H_5$

$2\dot{C}_2H_5 \rightarrow CH_3 - CH_2 - CH_2 - CH_3$

$2\dot{C}H_3 \rightarrow CH_3 - CH_3$

$\dot{C}H_3 + \dot{C}_2H_5 \rightarrow CH_3 - CH_2 - CH_3$

88. Consider the following reaction at 298 K. $\frac{3}{2}O_{2(g)} \rightleftharpoons O_{3(g)}.K_P = 2.47\times10^{-29}$.

$\Delta_rG^{\ominus}$ for the reaction is _________ kJ. (Given R = $8.314\ JK^{-1}\ mol^{-1}$)

Ans. **(163)**

Sol. $\frac{3}{2}O_{2(g)} \rightleftharpoons O_{3(g)}.K_P = 2.47\times10^{-29}$.

$\Delta_rG^{\ominus} = -RT\ln K_P$

$= -8.314\times10^{-3}\times298\times\ln(2.47\times10^{-29})$

$= -8.314\times10^{-3}\times298\times(-65.87)$

$= 163.19$ kJ

89. The ionization energy of sodium in kJ mol^{-1}. If electromagnetic radiation of wavelength 242 nm is just sufficient to ionize sodium atom is______.

Ans. **(494)**

Sol. $E = \frac{1240}{\lambda(nm)}eV$

$= \frac{1240}{242}eV$

$= 5.12$ eV

$= 5.12\times1.6\times10^{-19}$

$= 8.198\times10^{-19}$ J/atom

$= 494$ kJ/mol

90. One Faraday of electricity liberates $x\times10^{-1}$ gram atom of copper from copper sulphate, x is______.

Ans. **(5)**

Sol. $Cu^{2+} + 2e^- \rightarrow Cu$

2 Faraday $\rightarrow$ 1 mol Cu

1 Faraday $\rightarrow$ 0.5 mol Cu deposit

0.5 mol = 0.5 g atom = 5×10^{-1}

x = 5

FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

(Held On Wednesday 31st January, 2024) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. The number of ways in which 21 identical apples can be distributed among three children such that each child gets at least 2 apples, is

(1) 406

(2) 130

(3) 142

(4) 136

Ans. (4)

Sol. After giving 2 apples to each child 15 apples left now 15 apples can be distributed in $^{15+3-1}C_2 = {}^{17}C_2$ ways

$= \frac{17 \times 16}{2} = 136$

2. Let A (a, b), B(3, 4) and (–6, –8) respectively denote the centroid, circumcentre and orthocentre of a triangle. Then, the distance of the point P(2a + 3, 7b + 5) from the line 2x + 3y – 4 = 0 measured parallel to the line x – 2y – 1 = 0 is

(1) $\frac{15\sqrt{5}}{7}$

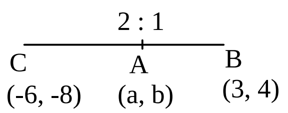
(2) $\frac{17\sqrt{5}}{6}$

(3) $\frac{17\sqrt{5}}{7}$

(4) $\frac{\sqrt{5}}{17}$

Ans. (3)

Sol. A(a,b), B(3,4), C(-6, -8)



$\Rightarrow a = 0,\ b = 0 \quad \Rightarrow P(3,5)$

Distance from P measured along x – 2y – 1 = 0

$\Rightarrow x = 3 + r\cos\theta, \quad y = 5 + r\sin\theta$

Where $\tan\theta = \frac{1}{2}$

$r(2\cos\theta + 3\sin\theta) = -17$

$\Rightarrow r = \left|\frac{-17\sqrt{5}}{7}\right| = \frac{17\sqrt{5}}{7}$

3. Let z_1 and z_2 be two complex number such that $z_1 + z_2 = 5$ and $z_1^3 + z_2^3 = 20 + 15i$. Then $|z_1^4 + z_2^4|$ equals-

(1) $30\sqrt{3}$

(2) 75

(3) $15\sqrt{15}$

(4) $25\sqrt{3}$

Ans. (2)

Sol.- $z_1 + z_2 = 5$

$z_1^3 + z_2^3 = 20 + 15i$

$z_1^3 + z_2^3 = (z_1 + z_2)^3 - 3z_1z_2(z_1 + z_2)$

$z_1^3 + z_2^3 = 125 - 3z_1.z_2(5)$

$\Rightarrow 20 + 15i = 125 - 15z_1z_2$

$\Rightarrow 3z_1z_2 = 25 - 4 - 3i$

$\Rightarrow 3z_1z_2 = 21 - 3i$

$\Rightarrow z_1.z_2 = 7 - i$

$\Rightarrow (z_1 + z_2)^2 = 25$

$\Rightarrow z_1^2 + z_2^2 = 25 - 2(7 - i)$

$\Rightarrow 11 + 2i$

$(z_1^2 + z_2^2)^2 = 121 - 4 + 44i$

$\Rightarrow \quad z_1^4 + z_2^4 + 2(7 - i)^2 = 117 + 44i$

$\Rightarrow \quad z_1^4 + z_2^4 = 117 + 44i - 2(49 - 1 - 14i)$

$\Rightarrow \quad |z_1^4 + z_2^4| = 75$

4. Let a variable line passing through the centre of the circle $x^2 + y^2 - 16x - 4y = 0$, meet the positive co-ordinate axes at the point A and B. Then the minimum value of OA + OB, where O is the origin, is equal to

(1) 12

(2) 18

(3) 20

(4) 24

Ans. (2)

Sol.- $(y-2) = m(x-8)$

$\Rightarrow$ x-intercept

$\Rightarrow \left(\frac{-2}{m}+8\right)$

$\Rightarrow$ y-intercept

$\Rightarrow (-8m+2)$

$\Rightarrow OA + OB = \frac{-2}{m} + 8 - 8m + 2$

$f'(m) = \frac{2}{m^2} - 8 = 0$

$\Rightarrow m^2 = \frac{1}{4}$

$\Rightarrow m = \frac{-1}{2}$

$\Rightarrow f\left(\frac{-1}{2}\right) = 18$

$\Rightarrow$ Minimum = 18

5. Let $f, g : (0,\infty) \to R$ be two functions defined by $f(x) = \int_{-x}^{x} \left(|t| - t^2\right) e^{-t^2} dt$ and $g(x) = \int_0^{x^2} t^{1/2} e^{-t} dt$. Then the value of $\left(f\left(\sqrt{\log_e 9}\right) + g\left(\sqrt{\log_e 9}\right)\right)$ is equal to

(1) 6

(2) 9

(3) 8

(4) 10

Ans. (3)

Sol.-

$$f(x) = \int_{-x}^{x} \left(|t| - t^2\right) e^{-t^2} dt$$

$$\Rightarrow f'(x) = 2.\left(|x| - x^2\right) e^{-x^2} \text{............}(1)$$

$$g(x) = \int_0^{x^2} t^{\frac{1}{2}} e^{-t} dt$$

$$g'(x) = xe^{-x^2}(2x) - 0$$

$$f'(x) + g'(x) = 2xe^{-x^2} - 2x^2e^{-x^2} + 2x^2e^{-x^2}$$

Integrating both sides w.r.t.x

$$f(x) + g(x) = \int_0^{\alpha} 2xe^{-x^2} dx$$

$$x^2 = t$$

$$\Rightarrow \int_0^{\sqrt{\alpha}} e^{-t} dt = \left[-e^{-t}\right]_0^{\sqrt{\alpha}}$$

$$= -e^{\left(\log_e(9)^{-1}\right)+1}$$

$$\Rightarrow 9\left(f(x) + g(x)\right) = \left(1 - \frac{1}{9}\right)9 = 8$$

6. Let (α, β, γ) be mirror image of the point (2, 3, 5) in the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$.

Then $2\alpha + 3\beta + 4\gamma$ is equal to

(1) 32

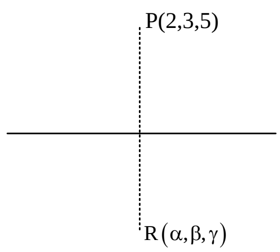
(2) 33

(3) 31

(4) 34

Ans. (2)

Sol.



$\because \overrightarrow{PR} \perp (2,3,4)$

$\therefore \overrightarrow{PR}.(2,3,4) = 0$

$(\alpha-2, \beta-3, \gamma-5).(2,3,4) = 0$

$\Rightarrow 2\alpha + 3\beta + 4\gamma = 4 + 9 + 20 = 33$

7. Let P be a parabola with vertex (2, 3) and directrix 2x + y = 6. Let an ellipse $E:\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$ of eccentricity $\frac{1}{\sqrt{2}}$ pass through the focus of the parabola P. Then the square of the length of the latus rectum of E, is

(1) $\frac{385}{8}$

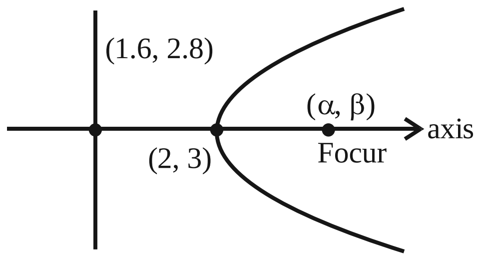
(2) $\frac{347}{8}$

(3) $\frac{512}{25}$

(4) $\frac{656}{25}$

Ans. **(4)**

Sol.-



Slope of axis $=\frac{1}{2}$

$$y-3=\frac{1}{2}(x-2)$$

$$\Rightarrow 2y-6=x-2$$

$$\Rightarrow 2y-x-4=0$$

$$2x+y-6=0$$

$$4x+2y-12=0$$

$$\alpha+1.6=4\Rightarrow\alpha=2.4$$

$$\beta+2.8=6\Rightarrow\beta=3.2$$

Ellipse passes through (2.4, 3.2)

$$\Rightarrow\frac{\left(\frac{24}{10}\right)^2}{a^2}+\frac{\left(\frac{32}{10}\right)^2}{b^2}=1 \quad \ldots\ldots\ldots(1)$$

Also $1-\frac{b^2}{a^2}=\frac{1}{2}=\frac{b^2}{a^2}=\frac{1}{2}$

$$\Rightarrow a^2=2b^2$$

Put in (1) $\Rightarrow b^2=\frac{328}{25}$

$$\Rightarrow\left(\frac{2b^2}{a}\right)^2=\frac{4b^2}{a^2}\times b^2=4\times\frac{1}{2}\times\frac{328}{25}=\frac{656}{25}$$

8. The temperature T(t) of a body at time t = 0 is 160° F and it decreases continuously as per the differential equation $\frac{dT}{dt}=-K(T-80)$, where K is positive constant. If T(15) = 120°F, then T(45) is equal to

(1) 85° F

(2) 95° F

(3) 90° F

(4) 80° F

Ans. **(3)**

Sol.-

$$\frac{dT}{dt}=-k(T-80)$$

$$\int_{160}^{T}\frac{dT}{(T-80)}=\int_{0}^{t}-Kdt$$

$$\left[\ln|T-80|\right]_{160}^{T}=-kt$$

$$\ln|T-80|-\ln 80=-kt$$

$$\ln\left|\frac{T-80}{80}\right|=-kt$$

$$T=80+80e^{-kt}$$

$$120=80+80e^{-k.15}$$

$$\frac{40}{80}=e^{-k15}=\frac{1}{2}$$

$$\therefore T(45)=80+80e^{-k.45}$$

$$=80+80\left(e^{-k.15}\right)^3$$

$$=80+80\times\frac{1}{8}$$

$$=90$$

9. Let 2^{nd}, 8^{th} and 44^{th}, terms of a non-constant A.P. be respectively the 1^{st}, 2^{nd} and 3^{rd} terms of G.P. If the first term of A.P. is 1 then the sum of first 20 terms is equal to-

(1) 980 (2) 960

(3) 990 (4) 970

Ans. **(4)**

Sol.- $1 + d,\ \ 1 + 7d,\ \ 1 + 43d$ are in GP

$(1 + 7d)^2 = (1 + d)(1 + 43d)$

$1 + 49d^2 + 14d = 1 + 44d + 43d^2$

$6d^2 - 30d = 0$

$d = 5$

$$S_{20} = \frac{20}{2}\left[2\times1+(20-1)\times5\right]$$

$$=10\left[2+95\right]$$

$$=970$$

10. Let $f :\to R \to (0,\infty)$ be strictly increasing function such that $\lim_{x\to\infty}\frac{f(7x)}{f(x)}=1$. Then, the value of $\lim_{x\to\infty}\left[\frac{f(5x)}{f(x)}-1\right]$ is equal to

(1) 4

(2) 0

(3) 7/5

(4) 1

Ans. **(2)**

Sol.- $f : R \to (0,\infty)$

$$\lim_{x\to\infty}\frac{f(7x)}{f(x)}=1$$

$\because$ f is increasing

$$\therefore f(x)<f(5x)<f(7x)$$

$$\because \frac{f(x)}{f(x)}<\frac{f(5x)}{f(x)}<\frac{f(7x)}{f(x)}$$

$$1<\lim_{x\to\infty}\frac{f(5x)}{f(x)}<1$$

$$\therefore \left[\frac{f(5x)}{f(x)}-1\right]$$

$$\Rightarrow 1-1=0$$

11. The area of the region enclosed by the parabola $y = 4x - x^2$ and $3y = (x - 4)^2$ is equal to

(1) $\frac{32}{9}$

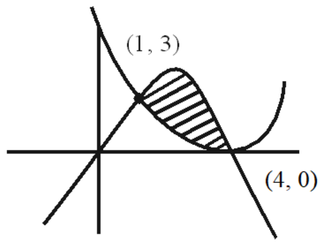
(2) 4

(3) 6

(4) $\frac{14}{3}$

Ans. **(3)**

Sol.-



$$\text{Area } =\left|\int_1^4\left[(4x-x^2)-\frac{(x-4)^2}{3}\right]\right|dx$$

$$\text{Area } =\left|\frac{4x^2}{2}-\frac{x^3}{3}-\frac{(x-4)^3}{9}\right|_1^4$$

$$=\left|\left(\frac{64}{2}-\frac{64}{3}-\frac{4}{2}+\frac{1}{3}-\frac{27}{9}\right)\right|$$

$$\Rightarrow(27-21)=6$$

12. Let the mean and the variance of 6 observation a, b, 68, 44, 48, 60 be 55 and 194, respectively if a > b, then a + 3b is

(1) 200

(2) 190

(3) 180

(4) 210

Ans. **(3)**

Sol.- a, b, 68, 44, 48, 60

Mean = 55 a > b

Variance = 194 a + 3b

$$\frac{a+b+68+44+48+60}{6}=55$$

$$\Rightarrow 220+a+b=330$$

$$\therefore\ a+b=110......(1)$$

Also,

$$\sum\frac{(x_i-\bar{x})^2}{n}=194$$

$$\Rightarrow(a-55)^2+(b-55)^2+(68-55)^2+(44-55)^2$$
$$+(48-55)^2+(60-55)^2=194\times6$$
$$\Rightarrow(a-55)^2+(b-55)^2+169+121+49+25=1164$$
$$\Rightarrow(a-55)^2+(b-55)^2=1164-364=800$$
$$a^2+3025-110a+b^2+3025-110b=800$$
$$\Rightarrow a^2+b^2=800-6050+12100$$
$$a^2+b^2=6850.......(2)$$

Solve (1) & (2);

a=75,b=35

$\therefore a+3b=75+3(35)=75+105=180$

13. If the function $f:(-\infty,-1]\to(a,b]$ defined by $f(x)=e^{x^3-3x+1}$ is one-one and onto, then the distance of the point P(2b + 4, a + 2) from the line $x+e^{-3}y=4$ is :

(1) $2\sqrt{1+e^6}$
(2) $4\sqrt{1+e^6}$
(3) $3\sqrt{1+e^6}$
(4) $\sqrt{1+e^6}$

Ans. **(1)**

Sol.- $f(x)=e^{x^3-3x+1}$

$f'(x)=e^{x^3-3x+1}.(3x^2-3)$

$=e^{x^3-3x+1}.3(x-1)(x+1)$

For $f'(x)\geq0$

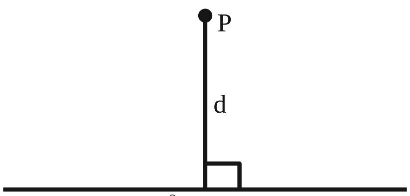
$\therefore f(x)$ is increasing function

$\therefore a=e^{-\infty}=0=f(-\infty)$

$b=e^{-1+3+1}=e^3=f(-1)$

P(2b + 4, a + 2)

$\therefore P(2e^3+4,2)$



$$d=\frac{(2e^3+4)+2e^{-3}-4}{\sqrt{1+e^{-6}}}=2\sqrt{1+e^6}$$

14. Consider the function $f:(0,\infty)\to R$ defined by $f(x)=e^{-|\log_e x|}$. If m and n be respectively the number of points at which f is not continuous and f is not differentiable, then m + n is

(1) 0
(2) 3
(3) 1
(4) 2

Ans. **(3)**

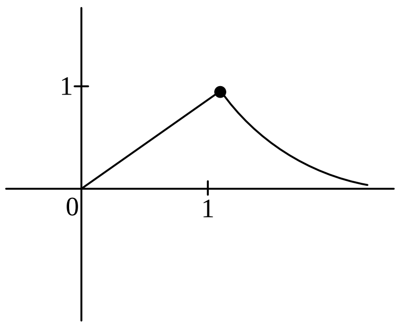
Sol.-

$f:(0,\infty)\to R$

$f(x)=e^{-|\log_e x|}$

$$f(x)=\frac{1}{e^{|\ln x|}}=\begin{cases}\frac{1}{e^{-\ln x}};0<x<1\\ \frac{1}{e^{\ln x}};x\geq1\end{cases}$$

$$\begin{cases}\frac{1}{\frac{1}{x}}=x;0<x<1\\ \frac{1}{x},x\geq1\end{cases}$$



m = 0 (No point at which function is not continuous)

n = 1 (Not differentiable)

$\therefore$ m + n = 1

15. The number of solutions, of the equation $e^{\sin x}-2e^{-\sin x}=2$ is

(1) 2
(2) more than 2
(3) 1
(4) 0

Ans. **(4)**

Sol.- Take $e^{\sin x} = t\left(t > 0\right)$

$\Rightarrow t - \frac{2}{t} = 2$

$\Rightarrow \frac{t^2 - 2}{t} = 2$

$\Rightarrow t^2 - 2t - 2 = 0$

$\Rightarrow t^2 - 2t + 1 = 3$

$\Rightarrow \left(t - 1\right)^2 = 3$

$\Rightarrow t = 1 \pm \sqrt{3}$

$\Rightarrow t = 1 \pm 1.73$

$\Rightarrow t = 2.73$ or -0.73 (rejected as $t > 0$)

$\Rightarrow e^{\sin x} = 2.73$

$\Rightarrow \log_e e^{\sin x} = \log_e 2.73$

$\Rightarrow \sin x = \log_e 2.73 > 1$

So no solution.

16. If $a = \sin^{-1}\left(\sin\left(5\right)\right)$ and $b = \cos^{-1}\left(\cos\left(5\right)\right)$, then $a^2 + b^2$ is equal to

(1) $4\pi^2 + 25$

(2) $8\pi^2 - 40\pi + 50$

(3) $4\pi^2 - 20\pi + 50$

(4) 25

Ans. (2)

Sol. $a = \sin^{-1}\left(\sin 5\right) = 5 - 2\pi$

and $b = \cos^{-1}\left(\cos 5\right) = 2\pi - 5$

$\therefore a^2 + b^2 = \left(5 - 2\pi\right)^2 + \left(2\pi - 5\right)^2$

$= 8\pi^2 - 40\pi + 50$

17. If for some m, n; ${}^6C_m + 2\left({}^6C_{m+1}\right) + {}^6C_{m+2} > {}^8C_3$ and ${}^{n-1}P_3 : {}^nP_4 = 1:8$, then ${}^nP_{m+1} + {}^{n+1}C_m$ is equal to

(1) 380

(2) 376

(3) 384

(4) 372

Ans. (4)

Sol.- ${}^6C_m + 2\left({}^6C_{m+1}\right) + {}^6C_{m+2} > {}^8C_3$

${}^7C_{m+1} + {}^7C_{m+2} > {}^8C_3$

${}^8C_{m+2} > {}^8C_3$

$\therefore m = 2$

And ${}^{n-1}P_3 : {}^nP_4 = 1:8$

$\frac{\left(n-1\right)\left(n-2\right)\left(n-3\right)}{n\left(n-1\right)\left(n-2\right)\left(n-3\right)} = \frac{1}{8}$

$\therefore n = 8$

$\therefore {}^nP_{m+1} + {}^{n+1}C_m = {}^8P_3 + {}^9C_2$

$= 8 \times 7 \times 6 + \frac{9 \times 8}{2}$

$= 372$

18. A coin is based so that a head is twice as likely to occur as a tail. If the coin is tossed 3 times, then the probability of getting two tails and one head is-

(1) $\frac{2}{9}$

(2) $\frac{1}{9}$

(3) $\frac{2}{27}$

(4) $\frac{1}{27}$

Ans. (1)

Sol. Let probability of tail is $\frac{1}{3}$

$\Rightarrow$ Probability of getting head $= \frac{2}{3}$

$\therefore$ Probability of getting 2 tails and 1 head

$= \left(\frac{1}{3} \times \frac{2}{3} \times \frac{1}{3}\right) \times 3$

$= \frac{2}{27} \times 3$

$= \frac{2}{9}$

19. Let A be a 3 3 real matrix such that

$$A\begin{pmatrix}1\\0\\1\end{pmatrix}=2\begin{pmatrix}1\\0\\1\end{pmatrix}, A\begin{pmatrix}-1\\0\\1\end{pmatrix}=4\begin{pmatrix}-1\\0\\1\end{pmatrix}, A\begin{pmatrix}0\\1\\0\end{pmatrix}=2\begin{pmatrix}0\\1\\0\end{pmatrix}.$$

Then, the system $(A-3I)\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\2\\3\end{pmatrix}$ has

(1) unique solution

(2) exactly two solutions

(3) no solution

(4) infinitely many solutions

Ans. (1)

Sol.- Let $A=\begin{bmatrix}x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\\ x_3 & y_3 & z_3\end{bmatrix}$

Given $A\begin{bmatrix}1\\0\\1\end{bmatrix}=\begin{bmatrix}2\\0\\2\end{bmatrix}$ .... (1)

$\therefore \begin{bmatrix}x_1+z_1\\x_2+z_2\\x_3+z_3\end{bmatrix}=\begin{bmatrix}2\\0\\2\end{bmatrix}$

$\therefore x_1+z_1=2$ .... (2)

$x_2+z_2=0$ .... (3)

$x_3+z_3=0$ .... (4)

Given $A\begin{bmatrix}-1\\0\\1\end{bmatrix}=\begin{bmatrix}-4\\0\\4\end{bmatrix}$

$\therefore \begin{bmatrix}-x_1+z_1\\-x_2+z_2\\-x_3+z_3\end{bmatrix}=\begin{bmatrix}4\\0\\4\end{bmatrix}$

$\Rightarrow -x_1+z_1=-4$ .... (5)

$-x_2+x_2=0$ .... (6)

$-x_3+z_3=4$

Given $A\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}0\\2\\0\end{bmatrix}$

$\therefore \begin{bmatrix}y_1\\y_2\\y_3\end{bmatrix}=\begin{bmatrix}0\\2\\0\end{bmatrix}$

$\therefore y_1=0, y_2=2, y_3=0$

$\therefore$ from (2), (3), (4), (5), (6) and (7)

$x_1=3x, x_2=0, x_3=-1$

$y_1=0, y_2=2, y_3=0$

$z_1=-1, z_2=0, z_3=3$

$\therefore A=\begin{bmatrix}3 & 0 & -1\\ 0 & 2 & 0\\ -1 & 0 & 3\end{bmatrix}$

$\therefore$ Now $(A-3I)\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-1\\2\\3\end{bmatrix}$

$\therefore \begin{bmatrix}0 & 0 & -1\\ 0 & -1 & 0\\ -1 & 0 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-1\\2\\3\end{bmatrix}$

$\begin{bmatrix}-z\\-y\\-x\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$

$[z=-1],[y=-2],[x=-3]$

20. The shortest distance between lines L_1 and L_2, where $L_1:\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+4}{2}$ and L_2 is the line passing through the points $A(-4,4,3).B(-1,6,3)$ and perpendicular to the line $\frac{x-3}{-2}=\frac{y}{3}=\frac{z-1}{1}$, is

(1) $\frac{121}{\sqrt{221}}$

(2) $\frac{24}{\sqrt{117}}$

(3) $\frac{141}{\sqrt{221}}$

(4) $\frac{42}{\sqrt{117}}$

Ans. (3)

Sol.-

$$L_2 = \frac{x+4}{3} = \frac{y-4}{2} = \frac{z-3}{0}$$

$$\therefore S.D = \frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ 2 & -3 & 2 \\ 3 & 2 & 0 \end{vmatrix}}{|\vec{n_1}\times\vec{n_2}|}$$

$$= \frac{\begin{vmatrix} 5 & -5 & -7 \\ 2 & -3 & 2 \\ 3 & 2 & 0 \end{vmatrix}}{|\vec{n_1}\times\vec{n_2}|}$$

$$= \frac{141}{|-4\hat{i}+6\hat{j}+13\hat{k}|}$$

$$= \frac{141}{\sqrt{16+36+169}}$$

$$= \frac{141}{\sqrt{221}}$$

SECTION-B

21. $\left|\frac{120}{\pi^3}\int_0^{\pi}\frac{x^2\sin x\cos x}{\sin^4 x+\cos^4 x}dx\right|$ is equal to ________.

Ans. (15)

Sol.- $\int_0^{\pi}\frac{x^2\sin x.\cos x}{\sin^4 x+\cos^4 x}dx$

$$= \int_0^{\frac{\pi}{2}}\frac{\sin x.\cos x}{\sin^4 x+\cos^4 x}\left(x^2-(\pi-x)^2\right)dx$$

$$= \int_0^{\frac{\pi}{2}}\frac{\sin x\cdot\cos x\left(2\pi x-\pi^2\right)}{\sin^4 x+\cos^4 x}$$

$$= 2\pi\int_0^{\frac{\pi}{2}}\frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}dx - \pi^2\int_0^{\frac{\pi}{2}}\frac{\sin x\cos x}{\sin^4 x+\cos^4 x}dx$$

$$= 2\pi\cdot\frac{\pi}{4}\int_0^{\frac{\pi}{2}}\frac{\sin x\cos x}{\sin^4 x+\cos^4 x}dx - \pi^2\int_0^{\frac{\pi}{2}}\frac{\sin x\cos x}{\sin^4 x+\cos^4 x}dx$$

$$= -\frac{\pi^2}{2}\int_0^{\frac{\pi}{2}}\frac{\sin x\cos x}{\sin^4 x+\cos^4 x}dx$$

$$= -\frac{\pi^2}{2}\int_0^{\frac{\pi}{2}}\frac{\sin x\cos xdx}{1-2\sin^2 x\times\cos^2 x}$$

$$= -\frac{\pi^2}{2}\int_0^{\frac{\pi}{2}}\frac{\sin 2x}{2-\sin^2 2x}dx$$

$$= -\frac{\pi^2}{2}\int_0^{\frac{\pi}{2}}\frac{\sin 2x}{1+\cos^2 2x}dx$$

Let $\cos 2x = t$

22. Let a, b, c be the length of three sides of a triangle satisfying the condition $(a^2 + b^2)x^2 - 2b(a + c).x + (b^2 + c^2) = 0$. If the set of all possible values of x is the interval (α,β), then $12\left(\alpha^2+\beta^2\right)$ is equal to ______.

Ans. (36)

Sol.- $\left(a^2+b^2\right)x^2-2b(a+c)x+b^2+c^2=0$

$$\Rightarrow a^2x^2-2abx+b^2+b^2x^2-2bcx+c^2=0$$

$$\Rightarrow (ax-b)^2+(bx-c)^2=0$$

$$\Rightarrow ax-b=0,\quad bx-c=0$$

$$\Rightarrow a+b>c \quad b+c>a \quad c+a>b$$

$a+ax>bx$	$ax+bx>a$	$ax^2+a>ax$
$a+ax>ax^2$	$ax+ax^2>a$	$x^2-x+1>0$
$x^2-x-1<0$	$x^2+x-1>0$	always true

$$\frac{1-\sqrt{5}}{2}<x<\frac{1+\sqrt{5}}{2}$$

$$x<\frac{-1-\sqrt{5}}{2},\quad \text{or } x>\frac{-1+\sqrt{5}}{2}$$

$$\Rightarrow \frac{\sqrt{5}-1}{2} < x < \frac{\sqrt{5}+1}{2}$$

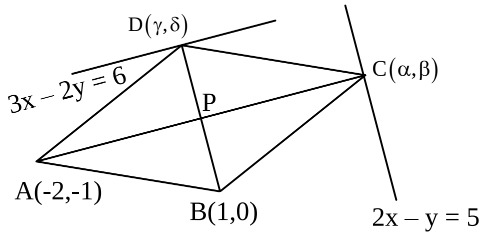
$$\Rightarrow \alpha = \frac{\sqrt{5}-1}{2}, \beta = \frac{\sqrt{5}+1}{2}$$

$$12\left(\alpha^2+\beta^2\right)=12\left(\frac{\left(\sqrt{5}-1\right)^2+\left(\sqrt{5}+1\right)^2}{4}\right)=36$$

23. Let A(–2, –1), B(1, 0), $C(\alpha,\beta)$ and $D(\gamma,\delta)$ be the vertices of a parallelogram ABCD. If the point C lies on 2x – y = 5 and the point D lies on 3x – 2y = 6, then the value of $|\alpha+\beta+\gamma+\delta|$ is equal to ________.

Ans. **(32)**

Sol.-



$$P \equiv \left(\frac{\alpha-2}{2}, \frac{\beta-1}{2}\right) \equiv \left(\frac{\gamma+1}{2}, \frac{\delta}{2}\right)$$

$$\frac{\alpha-2}{2} = \frac{\gamma+1}{2} \text{ and } \frac{\beta-1}{2} = \frac{\delta}{2}$$

$$\Rightarrow \alpha-\gamma=3.....(1),\quad \beta-\delta=1......(2)$$

Also, (γ,δ) lies on 3x – 2y = 6

$$3\gamma-2\delta=6 \text{ ......(3)}$$

and (α,β) lies on 2x – y = 5

$$\Rightarrow 2\alpha-\beta=5.........(4)$$

Solving (1), (2), (3), (4)

$$\alpha=-3,\ \beta=-11, \gamma=-6, \delta=-12$$

$$|\alpha+\beta+\gamma+\delta|=32$$

24. Let the coefficient of x^r in the expansion of

$$(x+3)^{n-1}+(x+3)^{n-2}(x+2)+$$
$$(x+3)^{n-3}(x+2)^2+.......+(x+2)^{n-1}$$

be α_r. If $\sum_{r=0}^{n}\alpha_r=\beta^n-\gamma^n, \beta,\gamma\in N$, then the value of $\beta^2+\gamma^2$ equals __________.

Ans. **(25)**

Sol.-

$$(x+3)^{n-1}+(x+3)^{n-2}(x+2)+(x+3)^{n-3}$$
$$(x+2)^2+........+(x+2)^{n-1}$$

$$\sum\alpha_r=4^{n-1}+4^{n-2}\times3+4^{n-3}\times3^2.......+3^{n-1}$$

$$=4^{n-1}\left[1+\frac{3}{4}+\left(\frac{3}{4}\right)^2.....+\left(\frac{3}{4}\right)^{n-1}\right]$$

$$=4^{n-1}\times\frac{1-\left(\frac{3}{4}\right)^n}{1-\frac{3}{4}}$$

$$=4^n-3^n=\beta^n-\gamma^n$$

$$\beta=4, \gamma=3$$

$$\beta^2+\gamma^2=16+9=25$$

25. Let A be a 3×3 matrix and det (A) = 2. If

$$n=\det\underbrace{\left(\text{adj}\left(\text{adj}\left(......\left(\text{adj}A\right)\right)\right)\right)}_{\text{2024-times}}$$

Then the remainder when n is divided by 9 is equal to ___________.

Ans. **(7)**

Sol.- $|A|=2$

$$\underbrace{\text{adj}\left(\text{adj}\left(\text{adj}.....(a)\right)\right)}_{\text{2024 times}}=|A|^{(n-1)^{2024}}$$

$$=|A|^{2^{2024}}$$

$$=2^{2^{2024}}$$

$$2^{2024}=(2^2)2^{2022}=4(8)^{674}=4(9-1)^{674}$$

$$\Rightarrow 2^{2024}\equiv 4(\text{mod } 9)$$

$$\Rightarrow 2^{2024}\equiv 9m+4,\ m\leftarrow \text{even}$$

$$2^{9m+4}\equiv 16\cdot(2^3)^{3m}\equiv 16(\text{mod } 9)$$

$$\equiv 7$$

26. Let $\vec{a}=3\hat{i}+2\hat{j}+\hat{k},\vec{b}=2\hat{i}-\hat{j}+3\hat{k}$ and $\vec{c}$ be a vector such that $(\vec{a}+\vec{b})\times\vec{c}=2(\vec{a}\times\vec{b})+24\hat{j}-6\hat{k}$ and $(\vec{a}-\vec{b}+\hat{i}).\vec{c}=-3$. Then $|\vec{c}|^2$ is equal to _____.

Ans. (38)

Sol.- $(\vec{a}+\vec{b})\times\vec{c}=2(\vec{a}\times\vec{b})+24\hat{j}-6\hat{k}$

$$(5\hat{i}+\hat{j}+4\hat{k})\times\vec{c}=2(7\hat{i}-7\hat{j}-7\hat{k})+24\hat{j}-6\hat{k}$$

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 5 & 1 & 4\\ x & y & z\end{vmatrix}=14\hat{i}+10\hat{j}-20\hat{k}$$

$$\Rightarrow \hat{i}(z-4y)-\hat{j}(5z-4x)+\hat{k}(5y-x)=14\hat{i}+10\hat{j}-20\hat{k}$$

$$z-4y=14, 4x-5z=10, 5y-x=-20$$

$$(a-b+i).\vec{c}=-3$$

$$(2\hat{i}+3\hat{j}-2\hat{k}).\vec{c}=-3$$

$$2x+3y-2z=-3$$

$$\therefore x=5, y=-3, z=2$$

$$|\vec{c}|^2=25+9+4=38$$

27. If $\lim\limits_{x\to 0}\dfrac{ax^2e^x-b\log_e(1+x)+cxe^{-x}}{x^2\sin x}=1,$ then $16(a^2+b^2+c^2)$ is equal to _______.

Ans. (81)

Sol.-

$$\lim_{x\to 0}\frac{ax^2\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+.....\right)-b\left(x-\frac{x^2}{2}+\frac{x^3}{3}-.......\right)+cx\left(1-x+\frac{x^2}{x!}-\frac{x^3}{3!}+.......\right)}{x^3.\frac{\sin x}{x}}$$

$$=\lim_{x\to\infty}\frac{(c-b)x+\left(\frac{b}{2}-c+a\right)x^2+\left(a-\frac{b}{3}+\frac{c}{2}\right)x^3+......}{x^3}=1$$

$$c-b=0,\quad \frac{b}{2}-c+a=0$$

$$a-\frac{b}{3}+\frac{c}{2}=1\quad a=\frac{3}{4}\quad b=c=\frac{3}{2}$$

$$a^2+b^2+c^2=\frac{9}{16}+\frac{9}{4}+\frac{9}{4}$$

$$16(a^2+b^2+c^2)=81$$

28. A line passes through A(4, –6, –2) and B(16, –2,4). The point P(a, b, c) where a, b, c are non-negative integers, on the line AB lies at a distance of 21 units, from the point A. The distance between the points P(a, b, c) and Q(4, –12, 3) is equal to ____.

Ans. (22)

Sol.-

$$\frac{x-4}{12}=\frac{x+6}{4}=\frac{z+2}{6}$$

$$\frac{x-4}{\frac{6}{7}}=\frac{y+6}{\frac{2}{7}}=\frac{z+2}{\frac{3}{7}}=21$$

$$\left(21\times\frac{6}{7}+4,\frac{2}{7}\times 21-6,\frac{3}{7}\times 21-2\right)$$

$$=(22,0,7)=(a,b,c)$$

$$\therefore \sqrt{324+144+16}=22$$

29. Let y = y(x) be the solution of the differential equation

$$\sec^2 x dx+(e^{2y}\tan^2 x+\tan x)dy=0,$$

$$0<x<\frac{\pi}{2}, y\left(\frac{\pi}{4}\right)=0. \text{ If } y\left(\frac{\pi}{6}\right)=\alpha,$$

Then $e^{8\alpha}$ is equal to ______.

Ans. (9)

Sol.-

$$\sec^2 x\frac{dx}{dy}+e^{2y}\tan^2 x+\tan x=0$$

$$\left(\text{Put } \tan x=t\Rightarrow \sec^2 x\frac{dx}{dy}=\frac{dt}{dy}\right)$$

$$\frac{dt}{dy}+e^{2y}\times t^2+t=0$$

$$\frac{dt}{dy}+t=-t^2.e^{2y}$$

$$\frac{1}{t^2}\frac{dt}{dy}+\frac{1}{t}=-e^{2y}$$

$$\left(\text{Put } \frac{1}{t}=u \quad \frac{-1}{t^2}\frac{dt}{dy}=\frac{du}{dy}\right)$$

$$\frac{-du}{dy}+u=-e^{2y}$$

$$\frac{du}{dy}-u=e^{2y}$$

$$\text{I.F.}=e^{-\int dy}=e^{-y}$$

$$ue^{-y}=\int e^{-y}\times e^{2y}dy$$

$$\frac{1}{\tan x}\times e^{-y}=e^{y}+c$$

$$x=\frac{\pi}{4}, y=0, c=0$$

$$x=\frac{\pi}{6},\quad y=\alpha$$

$$\sqrt{3}e^{-\alpha}=e^{\alpha}+0$$

$$e^{2\alpha}=\sqrt{3}$$

$$e^{8\alpha}=9$$

30. Let A = {1, 2, 3, ...100}. Let R be a relation on A defined by (x, y) $\in$ R if and only if 2x = 3y. Let R_1 be a symmetric relation on A such that $R\subset R_1$ and the number of elements in R_1 is n. Then, the minimum value of n is ___________.

Ans. (66)

Sol.-

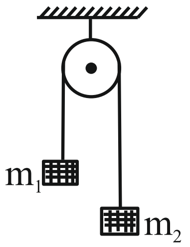
$$R=\{(3,2),(6,4),(9,6),(12,8),.........(99,66)\}$$

$$n(R)=33$$

$$\therefore 66$$

SECTION-A

31. A light string passing over a smooth light fixed pulley connects two blocks of masses m_1 and m_2. If the acceleration of the system is g/8, then the ratio of masses is



(1) $\frac{9}{7}$ (2) $\frac{8}{1}$

(3) $\frac{4}{3}$ (4) $\frac{5}{3}$

Ans. (1)

Sol. $a = \frac{(m_1 - m_2)g}{(m_1 + m_2)} = \frac{g}{8}$

$8m_1 - 8m_2 = m_1 + m_2$

$7m_1 = 9m_2$

$\frac{m_1}{m_2} = \frac{9}{7}$

32. A uniform magnetic field of 2×10^{-3}T acts along positive Y-direction. A rectangular loop of sides 20 cm and 10 cm with current of 5 A is Y-Z plane. The current is in anticlockwise sense with reference to negative X axis. Magnitude and direction of the torque is :

(1) 2×10^{-4}N – m along positive Z –direction

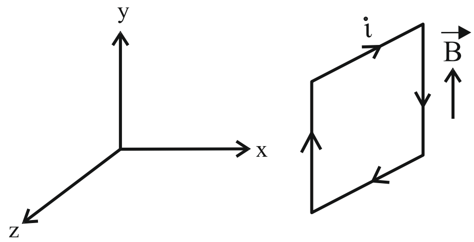
(2) 2×10^{-4}N – m along negative Z-direction

(3) 2×10^{-4}N – m along positive X-direction

(4) 2×10^{-4}N – m along positive Y-direction

Ans. (2)

Sol.



$\vec{M} = i\vec{A}$

$= 5 \times (0.2) \times (0.1)(-\hat{i})$

$= 0.1(-\hat{i})$

$\vec{\tau} = \vec{M} \times \vec{B} = 0.1(-\hat{i}) \times (2 \times 10^{-3})(\hat{j})$

$= 2 \times 10^{-4}(-\hat{k})\ N - m$

33. The measured value of the length of a simple pendulum is 20 cm with 2 mm accuracy. The time for 50 oscillations was measured to be 40 seconds with 1 second resolution. From these measurements, the accuracy in the measurement of acceleration due to gravity is N%. The value of N is:

(1) 4 (2) 8

(3) 6 (4) 5

Ans. (3)

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$g = \frac{4\pi^2 \ell}{T^2}$

$\frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + \frac{2\Delta T}{T}$

$= \frac{0.2}{20} + 2\left(\frac{1}{40}\right)$

$= \frac{0.3}{20}$

Percentage change $= \frac{0.3}{20} \times 100 = 6\%$

34. Force between two point charges q_1 and q_2 placed in vacuum at 'r' cm apart is F. Force between them when placed in a medium having dielectric K = 5 at 'r/5' cm apart will be:

(1) F/25 (2) 5F
(3) F/5 (4) 25F

Ans. **(2)**

Sol. In air $F = \frac{1}{4\pi \in_0} \frac{q_1 q_2}{r_2}$

In medium

$$F' = \frac{1}{4\pi(K \in_0)} \frac{q_1 q_2}{(r')^2} = \frac{25}{4\pi(5 \in_0)} \frac{q_1 q_2}{(r)^2} = 5F$$

35. An AC voltage $V = 20 \sin 200\pi t$ is applied to a series LCR circuit which drives a current $I = 10 \sin\left(200\pi t + \frac{\pi}{3}\right)$. The average power dissipated is:

(1) 21.6 W (2) 200 W
(3) 173.2 W (4) 50 W

Ans. **(4)**

Sol. $< P >= IV \cos\phi$

$= \frac{20}{\sqrt{2}} \times \frac{10}{\sqrt{2}} \times \cos 60^o$

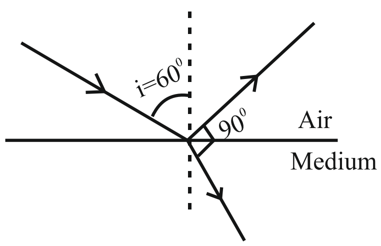
$= 50$ W

36. When unpolarized light is incident at an angle of 60° on a transparent medium from air. The reflected ray is completely polarized. The angle of refraction in the medium is

(1) 30^0 (2) 60^0
(3) 90^0 (4) 45^0

Ans. **(1)**

Sol. By Brewster's law



At complete reflection refracted ray and reflected ray are perpendicular.

37. The speed of sound in oxygen at S.T.P. will be approximately:

(Given, $R = 8.3\ JK^{-1}, \gamma = 1.4$)

(1) $310\ m/s$

(2) 333 m/s

(3) 341 m/s

(4) 325 m/s

Ans. **(1)**

Sol. $v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{1.4 \times 8.3 \times 273}{32 \times 10^{-3}}}$

$= 314.8541 \simeq 315\ m/s$

38. **A** gas mixture consists of 8 moles of argon and 6 moles of oxygen at temperature T. Neglecting all vibrational modes, the total internal energy of the system is

(1) 29 RT

(2) 20 RT

(3) 27 RT

(4) 21 RT

Ans. **(3)**

Sol. $U = nC_V T$

$\Rightarrow U = n_1 C_{V_1} T + n_2 C_{V_2} T$

$\Rightarrow 8 \times \frac{3R}{2} \times T + 6 \times \frac{5R}{2} \times T$

$= 27RT$

39. The resistance per centimeter of a meter bridge wire is r, with $X\,\Omega$ resistance in left gap. Balancing length from left end is at 40 cm with 25 Ω resistance in right gap. Now the wire is replaced by another wire of 2r resistance per centimeter. The new balancing length for same settings will be at

(1) 20 cm

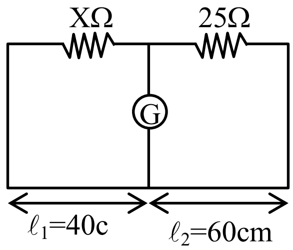
(2) 10 cm

(3) 80 cm

(4) 40 cm

Ans. **(4)**

Sol.



$\frac{25}{r\ell_1} = \frac{X}{r\ell_2}$ .....(i)

$\frac{25}{2r\ell'_1} = \frac{X}{2r\ell'_2}$ ..... (ii)

From (i) and (ii)

$\ell'_2 = \ell_2 = 40$ cm

40. Given below are two statements:

Statement I: Electromagnetic waves carry energy as they travel through space and this energy is equally shared by the electric and magnetic fields.

Statement II: When electromagnetic waves strike a surface, a pressure is exerted on the surface.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Statement I is incorrect but Statement II is correct

(2) Both Statement I and Statement II are correct.

(3) Both Statement I and Statement II are incorrect.

(4) Statement I is correct but Statement II is incorrect.

Ans. (2)

Sol. $\frac{1}{2}\varepsilon_0 E^2 = \frac{B^2}{2\mu_0}$

$\because E = CB$ and $C = \frac{1}{\mu_0\varepsilon_0}$

41. In a photoelectric effect experiment a light of frequency 1.5 times the threshold frequency is made to fall on the surface of photosensitive material. Now if the frequency is halved and intensity is doubled, the number of photo electrons emitted will be:

(1) Doubled (2) Quadrupled

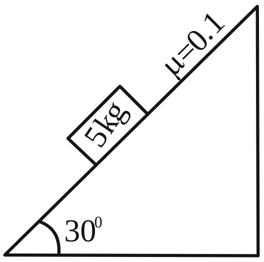
(3) Zero (4) Halved

Ans. (3)

Sol. Since $\frac{f}{2} < f_0$ i.e. the incident frequency is less than threshold frequency. Hence there will be no emission of photoelectrons.

$\Rightarrow$ current = 0

42. A block of mass 5 kg is placed on a rough inclined surface as shown in the figure.



If $\vec{F}_1$ is the force required to just move the block up the inclined plane and $\vec{F}_2$ is the force required to just prevent the block from sliding down, then the value of $|\vec{F}_1| - |\vec{F}_2|$ is : [Use $g = 10 m/s^2$]

(1) $25\sqrt{3}$ N (2) $50\sqrt{3}$ N

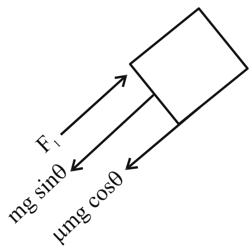
(3) $\frac{5\sqrt{3}}{2}$N (4) 10 N

Ans. ($5\sqrt{3}$ N) BONUS

Sol. $f_K = \mu mg\cos\theta$

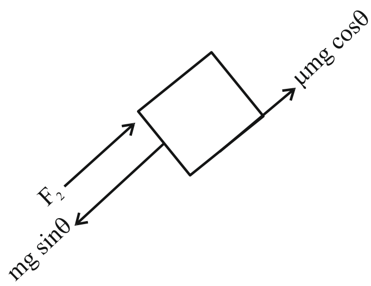
$= 0.1 \times \frac{50\times\sqrt{3}}{2}$

$= 2.5\sqrt{3}$ N



$F_1 = mg\sin\theta + f_K$

$= 25 + 2.5\sqrt{3}$



$F_2 = mg\sin\theta - f_K$

$= 25 - 2.5\sqrt{3}$

$\therefore F_1 - F_2 = 5\sqrt{3}$ N

43. By what percentage will the illumination of the lamp decrease if the current drops by 20%?

(1) 46% (2) 26%

(3) 36% (4) 56%

Ans. (3)

Sol. $P = i^2 R$

$P_{int} = I^2_{int} R$

$P_{final} = (0.8\, I_{int})^2 R$

% change in power =

$\frac{P_{final} - P_{int}}{P_{int}} \times 100 = (0.64 - 1) \times 100 = -36\%$

44. If two vectors $\vec{A}$ and $\vec{B}$ having equal magnitude R are inclined at an angle θ, then

(1) $|\vec{A} - \vec{B}| = \sqrt{2}\, R \sin\left(\frac{\theta}{2}\right)$

(2) $|\vec{A} + \vec{B}| = 2\, R \sin\left(\frac{\theta}{2}\right)$

(3) $|\vec{A} + \vec{B}| = 2\, R \cos\left(\frac{\theta}{2}\right)$

(4) $|\vec{A} - \vec{B}| = 2\, R \cos\left(\frac{\theta}{2}\right)$

Ans. (3)

Sol. The magnitude of resultant vector

$R' = \sqrt{a^2 + b^2 + 2ab\cos\theta}$

Here $a = b = R$

Then $R' = \sqrt{R^2 + R^2 + 2R^2 \cos\theta}$

$= R\sqrt{2}\,\sqrt{1 + \cos\theta}$

$= \sqrt{2}R\sqrt{2\cos^2\frac{\theta}{2}}$

$= 2R\cos\frac{\theta}{2}$

45. The mass number of nucleus having radius equal to half of the radius of nucleus with mass number 192 is:

(1) 24 (2) 32

(3) 40 (4) 20

Ans. (1)

Sol. $R_1 = \frac{R_2}{2}$

$R_0 (A_1)^{1/3} = \frac{R_0}{2}(A_2)^{1/3}$

$A_1 = \frac{1}{8} A_2$

$A_1 = \frac{192}{8} = 24$

46. The mass of the moon is 1/144 times the mass of a planet and its diameter 1/16 times the diameter of a planet. If the escape velocity on the planet is v, the escape velocity on the moon will be:

(1) $\frac{v}{3}$ (2) $\frac{v}{4}$

(3) $\frac{v}{12}$ (4) $\frac{v}{6}$

Ans. (1)

Sol. $V_{escape} = \sqrt{\frac{2GM}{R}}$

$V_{planet} = \sqrt{\frac{2GM}{R}} = V$

$V_{Moon} = \sqrt{\frac{2GM \times 16}{144\, R}} = \frac{1}{3}\sqrt{\frac{2GM}{R}}$

$V_{Moon} = \frac{V_{Planet}}{3} = \frac{V}{3}$

47. A small spherical ball of radius r, falling through a viscous medium of negligible density has terminal velocity 'v'. Another ball of the same mass but of radius 2r, falling through the same viscous medium will have terminal velocity:

(1) $\frac{v}{2}$ (2) $\frac{v}{4}$

(3) 4v (4) 2v

Ans. (1)

Sol. Since density is negligible hence Buoyancy force will be negligible

At terminal velocity.

$Mg = 6\pi\eta r v$

$V \propto \frac{1}{r}$ (as mass is constant)

Now, $\frac{v}{v'} = \frac{r'}{r}$

$r' = 2r$

So, $v' = \frac{v}{2}$

48. A body of mass 2 kg begins to move under the action of a time dependent force given by $\vec{F} = \left(6t\,\hat{i} + 6t^2\,\hat{j}\right)N$. The power developed by the force at the time t is given by:

(1) $\left(6t^4 + 9t^5\right)W$

(2) $\left(3t^3 + 6t^5\right)W$

(3) $\left(9t^5 + 6t^3\right)W$

(4) $\left(9t^3 + 6t^5\right)W$

Ans. **(4)**

Sol. $\vec{F} = \left(6t\,\hat{i} + 6t^2\,\hat{j}\right)N$

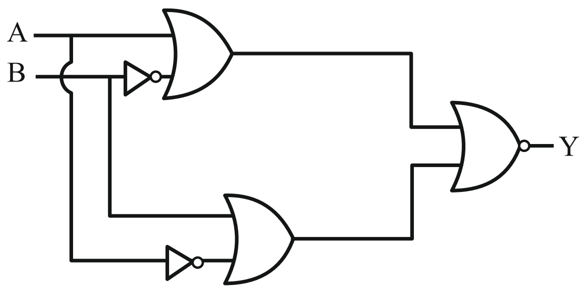
$$\vec{F} = m\vec{a} = \left(6t\hat{i} + 6t^2\hat{j}\right)$$

$$\vec{a} = \frac{\vec{F}}{m} = \left(3t\hat{i} + 3t^2\hat{j}\right)$$

$$\vec{v} = \int_0^t \vec{a}dt = \frac{3t^2}{2}\hat{i} + t^3\hat{j}$$

$$P = \vec{F}.\vec{v} = \left(9t^3 + 6t^5\right)W$$

49.



The output of the given circuit diagram is

(1)

A	B	Y
0	0	0
1	0	0
0	1	0
1	1	1

(2)

A	B	Y
0	0	0
1	0	1
0	1	1
1	1	0

(3)

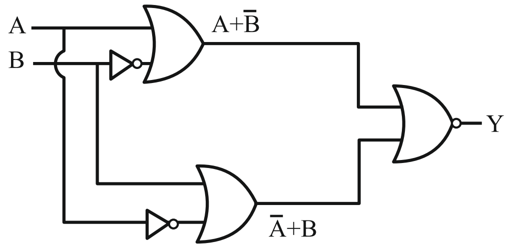
A	B	Y
0	0	0
1	0	0
0	1	0
1	1	0

(4)

A	B	Y
0	0	0
1	0	0
0	1	1
1	1	0

Ans. **(3)**

Sol.



If $A = 0;\ \bar{A} = 1$

$A = 1;\ \bar{A} = 0$

$B = 0;\ \bar{B} = 1$

$B = 1;\ \bar{B} = 0$

$$Y = \overline{\left(A + \bar{B}\right) + \left(\bar{A} + B\right)} = \overline{(1+1)} = 0$$

50. Consider two physical quantities A and B related to each other as $E = \frac{B - x^2}{At}$ where E, x and t have dimensions of energy, length and time respectively. The dimension of AB is

(1) $L^{-2}M^1T^0$

(2) $L^2M^{-1}T^1$

(3) $L^{-2}M^{-1}T^1$

(4) $L^0M^{-1}T^1$

Ans. **(2)**

Sol. $[B] = L^2$

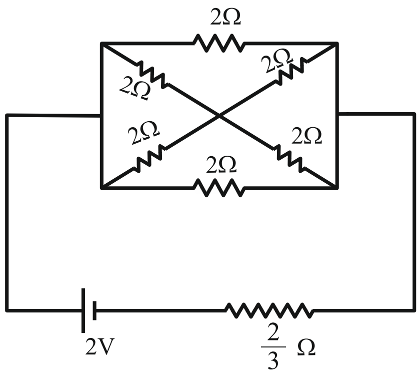
$$A = \frac{x^2}{tE} = \frac{L^2}{TML^2T^{-2}} = \frac{1}{MT^{-1}}$$

$$[A] = M^{-1}T$$

$$[AB] = [L^2M^{-1}T^1]$$

SECTION-B

51. In the following circuit, the battery has an emf of 2 V and an internal resistance of $\frac{2}{3}\Omega$. The power consumption in the entire circuit is ______ W.



Ans. **(3)**

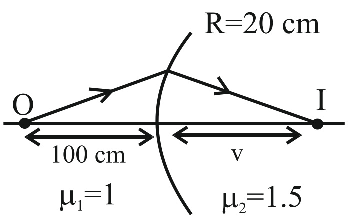
Sol. $R_{eq} = \frac{4}{3}\Omega$

$$\therefore P = \frac{V^2}{R_{eq}} = \frac{4}{4/3} = 3\text{ W}$$

52. Light from a point source in air falls on a convex curved surface of radius 20 cm and refractive index 1.5. If the source is located at 100 cm from the convex surface, the image will be formed at____ cm from the object.

Ans. **(200)**

Sol.



$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.5}{v} - \frac{1}{-100} = \frac{1.5-1}{20}$$

$$v = 100\text{ cm}$$

Distance from object

= 100+100

= 200 cm

53. The magnetic flux ϕ (in weber) linked with a closed circuit of resistance 8 Ω varies with time (in seconds) as $\phi = 5t^2 - 36t + 1$. The induced current in the circuit at t = 2s is _______ A.

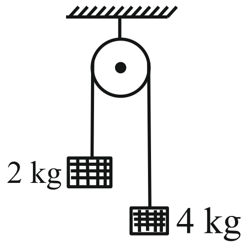
Ans. **(2)**

Sol. $\varepsilon = -\left(\frac{d\phi}{dt}\right) = 10t - 36$

at $t = 2, \varepsilon = 16$ V

$$i = \frac{\varepsilon}{R} = \frac{16}{8} = 2\text{ A}$$

54. Two blocks of mass 2 kg and 4 kg are connected by a metal wire going over a smooth pulley as shown in figure. The radius of wire is 4.0×10^{-5} m and Young's modulus of the metal is $2.0 \times 10^{11}\text{ N/m}^2$. The longitudinal strain developed in the wire is $\frac{1}{\alpha\pi}$. The value of α is____. [Use $g = 10\text{ m/s}^2$)



Ans. **(12)**

Sol. $T = \left(\frac{2m_1m_2}{m_1 + m_2}\right)g = \frac{80}{3}$ N

$A = \pi r^2 = 16\pi \times 10^{-10}\text{ m}^2$

$$\text{Strain} = \frac{\Delta \ell}{\ell} = \frac{F}{AY} = \frac{T}{AY}$$

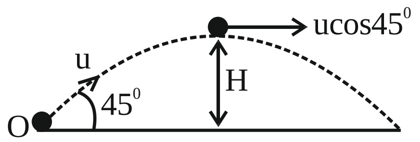
$$= \frac{80/3}{16\pi \times 10^{-10} \times 2 \times 10^{11}} = \frac{1}{12\pi}$$

$\alpha = 12$

55. A body of mass 'm' is projected with a speed 'u' making an angle of 45° with the ground. The angular momentum of the body about the point of projection, at the highest point is expressed as $\frac{\sqrt{2}\,mu^3}{Xg}$. The value of 'X' is_______.

Ans. **(8)**

Sol.



$$L = mu\cos\theta \frac{u^2 \sin^2\theta}{2g}$$

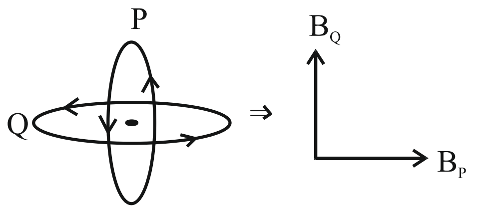
$$= mu^3 \frac{1}{4\sqrt{2}\,g} \Rightarrow x = 8$$

56. Two circular coils P and Q of 100 turns each have same radius of π cm. The currents in P and R are 1 A and 2 A respectively. P and Q are placed with their planes mutually perpendicular with their centers coincide. The resultant magnetic field induction at the center of the coils is $\sqrt{x}$ mT, where x =__________.

[Use $\mu_0 = 4\pi \times 10^{-7}\ TmA^{-1}$]

Ans. (20)

Sol.



$$B_P = \frac{\mu_0 N i_1}{2r} = \frac{\mu_0 \times 1 \times 100}{2\pi} = 2\times10^{-3}\ T$$

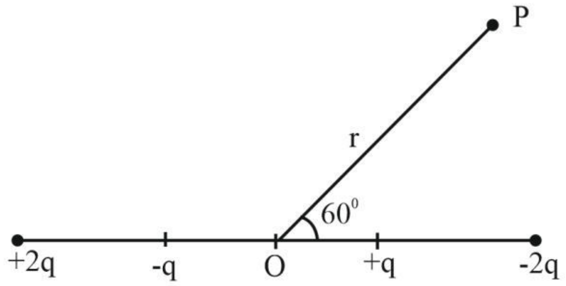
$$B_Q = \frac{\mu_0 N i_2}{2r} = \frac{\mu_0 \times 2 \times 100}{2\pi} = 4\times10^{-3}\ T$$

$$B_{net} = \sqrt{B_P^2 + B_Q^2}$$

$$= \sqrt{20}\ mT$$

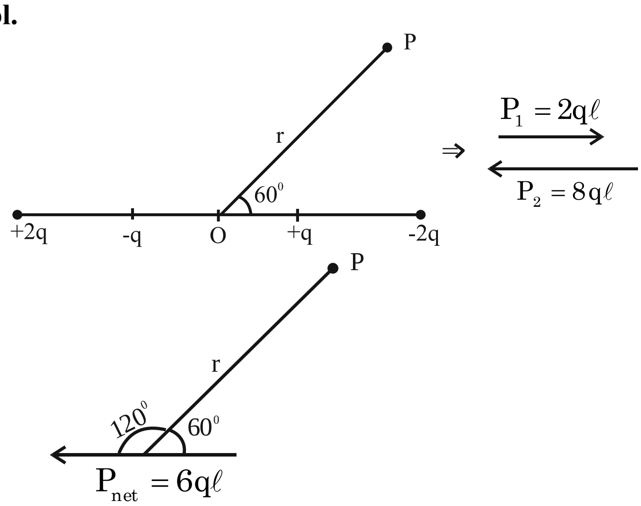
x = 20

57. The distance between charges +q and –q is $2l$ and between +2 q and -2 q is $4l$. The electrostatic potential at point P at a distance r from centre O is $-\alpha\left[\frac{ql}{r^2}\right]\times10^9 V$, where the value of α is ______. (Use $\frac{1}{4\pi\varepsilon_0} = 9\times10^9\ Nm^2C^{-2}$)



Ans. (27)

Sol.



$$V = \frac{K\,\vec{p}.\vec{r}}{r^3} = \frac{9\times10^9(6q\ell)}{r^2}\cos(120^0)$$

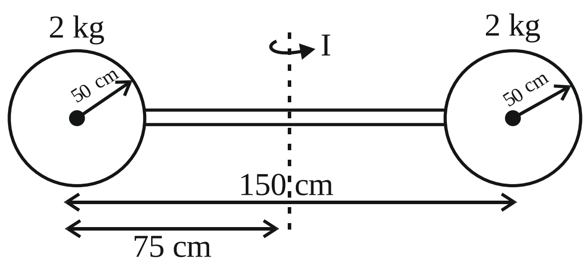
$$= -(27)\left(\frac{q\ell}{r^2}\right)\times10^9\ Nm^2c^{-2}$$

$$\Rightarrow \alpha = 27$$

58. Two identical spheres each of mass 2 kg and radius 50 cm are fixed at the ends of a light rod so that the separation between the centers is 150 cm. Then, moment of inertia of the system about an axis perpendicular to the rod and passing through its middle point is $\frac{x}{20} kg\ m^2$, where the value of x is ____.

Ans. (53)

Sol.

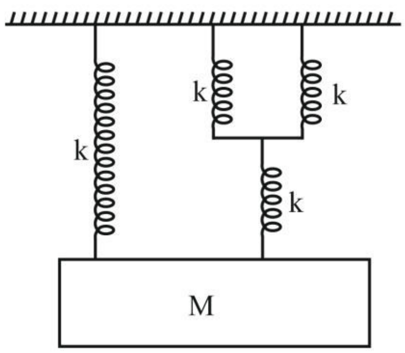


$$I=\left(\frac{2}{5}mR^2+md^2\right)\times 2$$

$$I=2\left(\frac{2}{5}\times 2\times\left(\frac{1}{2}\right)^2+2\times\left(\frac{3}{4}\right)^2\right)=\frac{53}{20}\ kg-m^2$$

$X = 53$

59. The time period of simple harmonic motion of mass M in the given figure is $\pi\sqrt{\frac{\alpha M}{5K}}$, where the value of α is _______.



Ans. **(12)**

Sol. $k_{eq}=\frac{2k.k}{3k}+k=\frac{5k}{3}$

Angular frequency of oscillation $(\omega)=\sqrt{\frac{k_{eq}}{m}}$

$(\omega)=\sqrt{\frac{5k}{3m}}$

Period of oscillation $(\tau)=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{3m}{5k}}$

$=\pi\sqrt{\frac{12m}{5k}}$

60. A nucleus has mass number A_1 and volume V_1. Another nucleus has mass number A_2 and volume V_2. If relation between mass number is $A_2=4A_1$, then $\frac{V_2}{V_1}=$ ________.

Ans. **(4)**

Sol. **For a nucleus**

Volume: $V=\frac{4}{3}\pi R^3$

$R=R_0(A)^{1/3}$

$V=\frac{4}{3}\pi R_0^3 A$

$\Rightarrow \frac{V_2}{V_1}=\frac{A_2}{A_1}=4$

CHEMISTRY	TEST PAPER WITH SOLUTION

SECTION-A

61. Match List I with List II

	LIST – I (Complex ion)		LIST – II (Electronic Configuration
A.	$\left[Cr(H_2O)_6\right]^{3+}$	I.	$t_{2g}^{2}\ e_g^{0}$
B.	$\left[Fe(H_2O)_6\right]^{3+}$	II.	$t_{2g}^{3}\ e_g^{0}$
C.	$\left[Ni(H_2O)_6\right]^{2+}$	III.	$t_{2g}^{3}\ e_g^{2}$
D.	$\left[V(H_2O)_6\right]^{3+}$	IV.	$t_{2g}^{6}\ e_g^{2}$

Choose the correct answer from the options given below :

(1) A-III, B-II, C-IV, D-I

(2) A-IV, B-I, C-II, D-III

(3) A-IV, B-III, C-I, D-II

(4) A-II, B-III, C-IV, D-I

Ans. **(4)**

Sol:- $\left[Cr(H_2O)_6\right]^{3+}$ Contains $Cr^{3+}:[Ar]3d^3:t_{2g}^3e_g^0$

$\left[Fe(H_2O)_6\right]^{3+}$ Contains $Fe^{3+}:[Ar]3d^5:t_{2g}^3\ e_g^2$

$\left[Ni(H_2O)_6\right]^{2+}$ Contains $Ni^{2+}:[Ar]3d^8:t_{2g}^6\ e_g^2$

$\left[V(H_2O)_6\right]^{3+}$ Contains $V^{3+}:[Ar]3d^2:t_{2g}^2\ e_g^0$

62. A sample of $CaCO_3$ and $MgCO_3$ weighed 2.21 g is ignited to constant weight of 1.152 g. The composition of mixture is :

(Given molar mass in g mol^{-1} $CaCO_3:100,\ MgCO_3:84$)

(1) $1.187\ g\ CaCO_3 + 1.023\ g\ MgCO_3$

(2) $1.023\ g\ CaCO_3 + 1.023\ g\ MgCO_3$

(3) $1.187\ g\ CaCO_3 + 1.187\ g\ MgCO_3$

(4) $1.023\ g\ CaCO_3 + 1.187\ g\ MgCO_3$

Ans. **(1)**

Sol:- $CaCO_3(s)\xrightarrow{\Delta}CaO(s)+CO_2(g)$

$MgCO_3(s)\xrightarrow{\Delta}MgO(s)+CO_2(g)$

Let the weight of $CaCO_3$ be x gm

$\therefore$ weight of $MgCO_3=(2.21-x)gm$

Moles of $CaCO_3$ decomposed = moles of CaO formed

$\frac{x}{100}$ = moles of CaO formed

$\therefore$ weight of CaO formed $=\frac{x}{100}\times 56$

Moles of $MgCO_3$ decomposed = moles of MgO formed

$\frac{(2.21-x)}{84}$ = moles of MgO formed

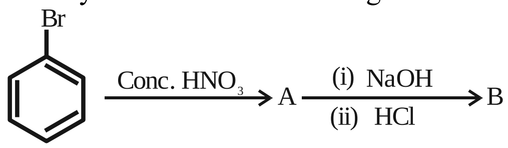
$\therefore$ weight of MgO formed $=\frac{2.21-x}{84}\times 40$

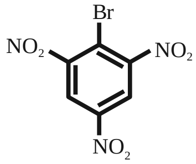
$\Rightarrow \frac{2.21-x}{84}\times 40+\frac{x}{100}\times 56=1.152$

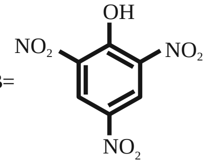
$\therefore x=1.1886\ g =$ weight of $CaCO_3$

& weight of $MgCO_3=1.0214\ g$

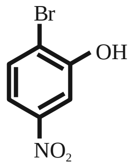
63. Identify A and B in the following reaction sequence.

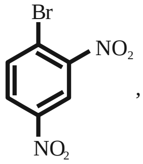


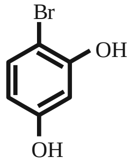
(1) A= 

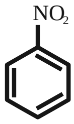
, B= 

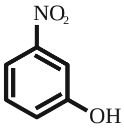
(2) A= 

, B= 

(3) A= 

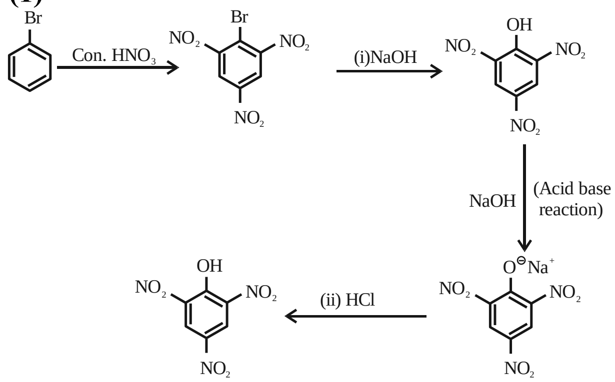
, B= 

(4) A= 

, B= 

Ans. (1)

Sol:-



64. Given below are two statements :

Statement I: S_8 solid undergoes disproportionation reaction under alkaline conditions to form S^{2-} and $S_2O_3^{2-}$

Statement II: ClO_4^- can undergo disproportionation reaction under acidic condition.

In the light of the above statements, choose the ***most appropriate answer*** from the options given below :

(1) Statement I is correct but statement II is incorrect.

(2) Statement I is incorrect but statement II is correct

(3) Both statement I and statement II are incorrect

(4) Both statement I and statement II are correct

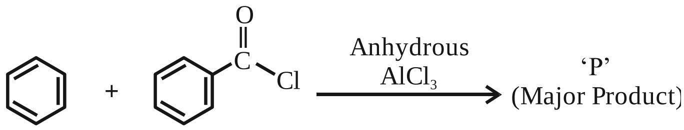
Ans. (1)

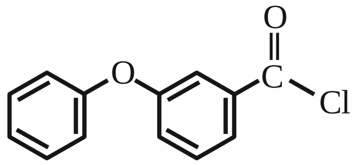
Sol:-

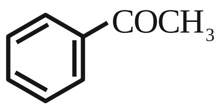
$S_1 : S_8 + 12\,OH^{\ominus} \rightarrow 4S^{2-} + 2S_2O_3^{2-} + 6H_2O$

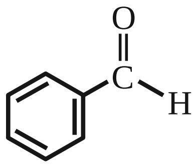
$S_2 : ClO_4^{\ominus}$ cannot undergo disproportionation reaction as chlorine is present in it's highest oxidation state.

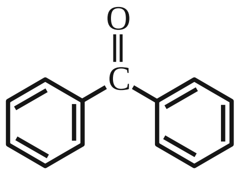
65. Identify major product 'P' formed in the following reaction.



(1) 

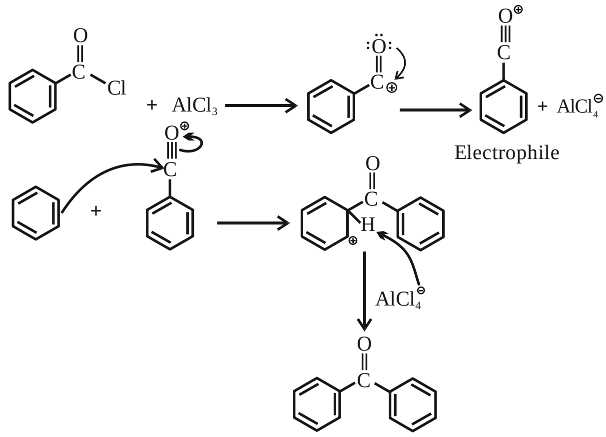
(2) 

(3) 

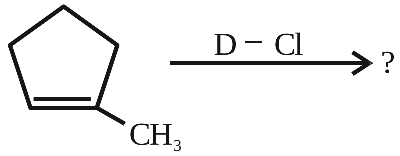
(4) 

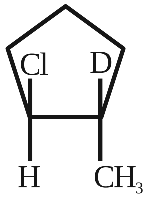
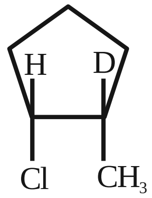
Ans. (4)

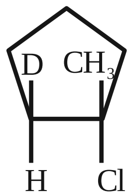
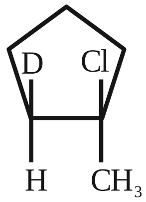
Sol:-



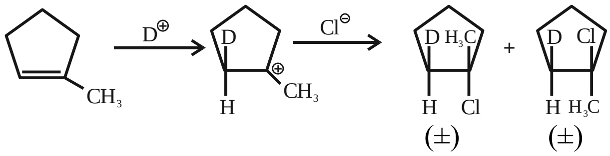
66. Major product of the following reaction is –



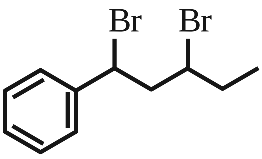
(1)  (2) 

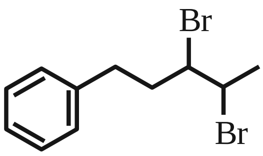
(3)  (4) 

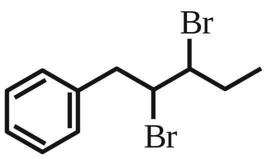
Ans. (3 or 4)

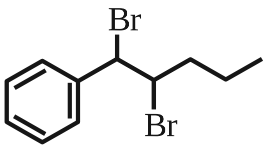
Sol:- 

67. Identify structure of 2,3-dibromo-1-phenylpentane.

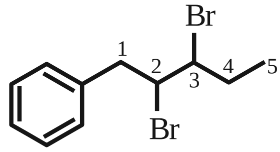
1. 

2. 

3. 

4. 

Ans. (3)

Sol:- 

2, 3-dibromo -1-phenylpentane

68. Select the option with correct property -

(1) $\left[Ni(CO)_4\right]$ and $\left[NiCl_4\right]^{2-}$ both diamagnetic

(2) $\left[Ni(CO)_4\right]$ and $\left[NiCl_4\right]^{2-}$ both paramagnetic

(3) $\left[NiCl_4\right]^{2-}$ diamagnetic, $\left[Ni(CO)_4\right]$ paramagnetic

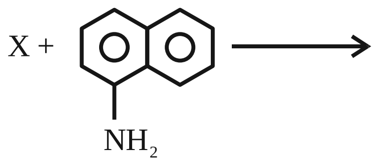
(4) $\left[Ni(CO)_4\right]$ diamagnetic, $\left[NiCl_4\right]^{2-}$ paramagnetic

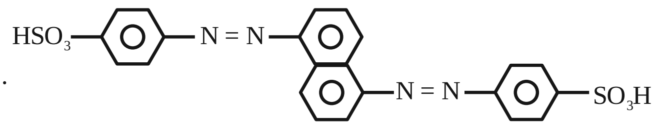
Ans. (4)

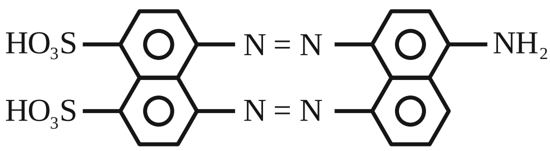
Sol:- $\left[Ni(CO)_4\right] \rightarrow$ diamagnetic, sp^3 hybridisation, number of unpaired electrons = 0

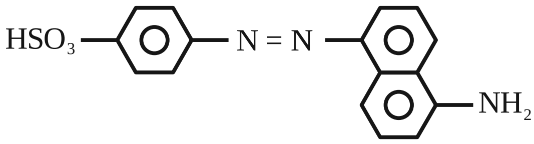
$\left[NiCl_4\right]^{2-}$, $\rightarrow$ paramagnetic, sp^3 hybridisation, number of unpaired electrons = 2

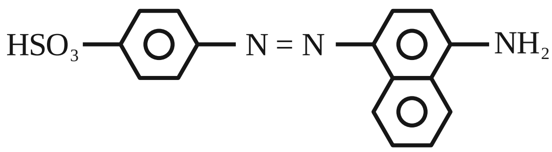
69. The azo-dye (Y) formed in the following reactions is Sulphanilic acid $+ NaNO_2 + CH_3COOH \rightarrow X$



1. 

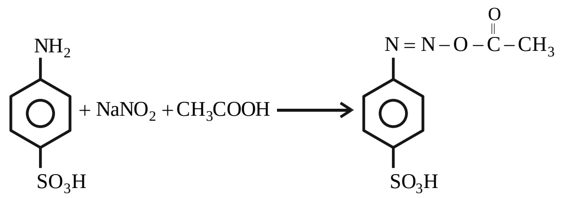
2. 

3. 

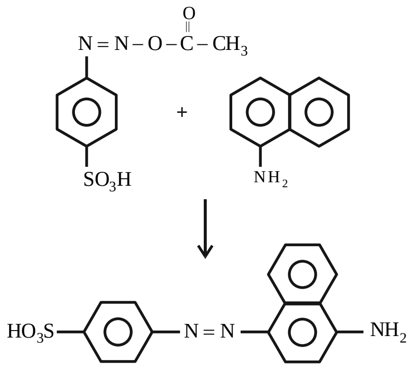
4. 

Ans. (4)

Sol:-



(X)



(Y)

Red azo-dye

This is known as Griess-Ilosvay test.

70. Given below are two statements :

Statement I: Aniline reacts with con. H_2SO_4 followed by heating at 453-473 K gives p-aminobenzene sulphonic acid, which gives blood red colour in the 'Lassaigne's test'.

Statement II: In Friedel - Craft's alkylation and acylation reactions, aniline forms salt with the $AlCl_3$ catalyst. Due to this, nitrogen of aniline aquires a positive charge and acts as deactivating group.

In the light of the above statements, choose the ***correct answer*** from the options given below :

1. Statement I is false but statement II is true

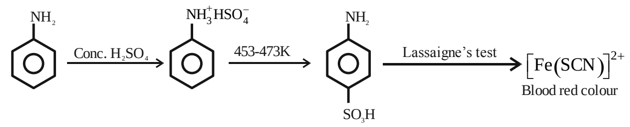
2. Both statement I and statement II are false

3. Statement I is true but statement II is false

4. Both statement I and statement II are true

Ans. (4)

Sol:-



71. $A_{(g)} \rightleftharpoons B_{(g)} + \frac{C}{2}_{(g)}$ The correct relationship between K_P, α and equilibrium pressure P is

(1) $K_P = \dfrac{\alpha^{1/2} P^{1/2}}{(2+\alpha)^{1/2}}$

(2) $K_P = \dfrac{\alpha^{3/2} P^{1/2}}{(2+\alpha)^{1/2}(1-\alpha)}$

(3) $K_P = \dfrac{\alpha^{1/2} P^{3/2}}{(2+\alpha)^{3/2}}$

(4) $K_P = \dfrac{\alpha^{1/2} P^{1/2}}{(2+\alpha)^{3/2}}$

Ans. (2)

Sol:- $A_{(g)} \rightleftharpoons B_{(g)} + \frac{C}{2}_{(g)}$

$t = t_{eq}$ $(1-\alpha)$ α $\frac{\alpha}{2}$

$$P_B = \frac{\alpha}{\left(1+\frac{\alpha}{2}\right)}.P,\quad P_A = \frac{(1-\alpha)}{\left(1+\frac{\alpha}{2}\right)}.P,\quad P_C = \frac{\frac{\alpha}{2}}{\left(1+\frac{\alpha}{2}\right)}.P$$

$$K_P = \frac{P_B \cdot P_C^{\frac{1}{2}}}{P_A}$$

$$= \frac{(\alpha)^{\frac{3}{2}}\,(P)^{\frac{1}{2}}}{(1-\alpha)(2+\alpha)^{\frac{1}{2}}}$$

72. Choose the correct statements from the following

A. All group 16 elements form oxides of general formula EO_2 and EO_3 where E = S, Se, Te and Po. Both the types of oxides are acidic in nature.

B. TeO_2 is an oxidising agent while SO_2 is reducing in nature.

C. The reducing property decreases from H_2S to H_2Te down the group.

D. The ozone molecule contains five lone pairs of electrons.

Choose the correct answer from the options given below:

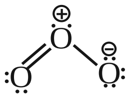
1. A and D only
2. B and C only
3. C and D only
4. A and B only

Ans. (4)

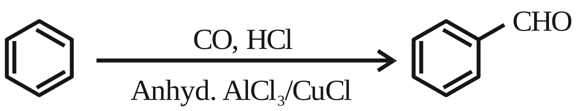
Sol:- **(A)** All group 16 elements form oxides of the EO_2 and EO_3 type where $E = S, Se, Te$ or Po.

(B) SO_2 is reducing while TeO_2 is an oxidising agent.

(C) The reducing property increases from H_2S to H_2Te down the group.

(D)  have six lone pairs

73. Identify the name reaction.



(1) Stephen reaction

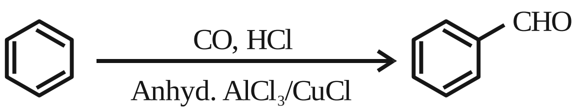
(2) Etard reaction

(3) Gatterman-koch reaction

(4) Rosenmund reduction

Ans. **(3)**

Sol:-



Gatterman-Koch reaction

74. Which of the following is least ionic ?

(1) $BaCl_2$ (2) $AgCl$

(3) KCl (4) $CoCl_2$

Ans. **(2)**

Sol:- $AgCl < CoCl_2 < BaCl_2 < KCl$ (ionic character)

Reason : Ag^+ has pseudo inert gas configuration.

75. The fragrance of flowers is due to the presence of some steam volatile organic compounds called essential oils. These are generally insoluble in water at room temperature but are miscible with water vapour in vapour phase. A suitable method for the extraction of these oils from the flowers is -

1. crystallisation
2. distillation under reduced pressure
3. distillation
4. steam distillation

Ans. **(4)**

Sol:- Steam distillation technique is applied to separate substances which are steam volatile and are immiscible with water.

76. Given below are two statements :

Statement I: Group 13 trivalent halides get easily hydrolyzed by water due to their covalent nature.

Statement II: $AlCl_3$ upon hydrolysis in acidified aqueous solution forms octahedral $\left[Al(H_2O)_6\right]^{3+}$ ion.

In the light of the above statements, choose the ***correct answer*** from the options given below :

1. Statement I is true but statement II is false
2. Statement I is false but statement II is true
3. Both statement I and statement II are false
4. Both statement I and statement II are true

Ans. **(4)**

Sol:- In trivalent state most of the compounds being covalent are hydrolysed in water. Trichlorides on hydrolysis in water form tetrahedral $\left[M(OH)_4\right]^-$ species, the hybridisation state of element M is sp^3.

In case of aluminium, acidified aqueous solution forms octahedral $\left[Al(H_2O)_6\right]^{3+}$ ion.

77. The four quantum numbers for the electron in the outer most orbital of potassium (atomic no. 19) are

(1) $n = 4,\ l = 2,\ m = -1,\ s = +\frac{1}{2}$

(2) $n = 4,\ l = 0,\ m = 0,\ s = +\frac{1}{2}$

(3) $n = 3,\ l = 0,\ m = 1,\ s = +\frac{1}{2}$

(4) $n = 2,\ l = 0,\ m = 0,\ s = +\frac{1}{2}$

Ans. **(2)**

Sol:- $_{19}K\ \ 1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 4s^1$.

Outermost orbital of potassium is 4s orbital

$n = 4, l = 0, m_l = 0, s = \pm\frac{1}{2}$.

78. Choose the correct statements from the following

A. Mn_2O_7 is an oil at room temperature

B. V_2O_4 reacts with acid to give VO_2^{2+}

C. CrO is a basic oxide

D. V_2O_5 does not react with acid

Choose the correct answer from the options given below :

1. A, B and D only

2. A and C only

3. A, B and C only

4. B and C only

Ans. **(2)**

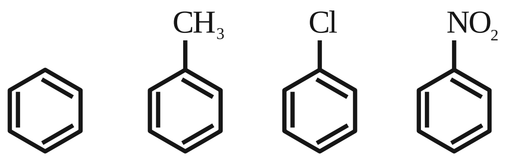
Sol:- **(A)** Mn_2O_7 is green oil at room temperature.

(B) V_2O_4 dissolve in acids to give VO^{2+} salts.

(C) CrO is basic oxide

(D) V_2O_5 is amphoteric it reacts with acid as well as base.

79. The correct order of reactivity in electrophilic substitution reaction of the following compounds is :



A B C D

1. $B > C > A > D$

2. $D > C > B > A$

3. $A > B > C > D$

4. $B > A > C > D$

Ans. **(4)**

Sol:- $-CH_3$ shows $+M$ and $+I$.

$-Cl$ shows $+M$ and $-I$ but inductive effect dominates.

$-NO_2$ shows $-M$ and $-I$.

Electrophilic substitution $\alpha \dfrac{1}{-M \text{ and } -I}$

$\alpha + M \text{ and } + I$

Hence, order is $B > A > C > D$.

80. Consider the following elements.

Group ↓ $A'B' \longrightarrow$ Period
$C'D'$

Which of the following is/are true about A', B', C' and D' ?

A. Order of atomic radii: $B' < A' < D' < C'$

B. Order of metallic character : $B' < A' < D' < C'$

C. Size of the element : $D' < C' < B' < A'$

D. Order of ionic radii : $B'^{+} < A'^{+} < D'^{+} < C'^{+}$

Choose the correct answer from the options given below :

1. A only
2. A, B and D only
3. A and B only
4. B, C and D only

Ans. **(2)**

Sol:- In general along the period from left to right, size decreases and metallic character decrease.

In general down the group, size increases and metallic character increases.

$B' < A'(\text{size})$ $C' > A'(\text{size})$

$D' < C'(\text{size})$ $D' > B'(\text{size})$

$B' < A'(\text{metallic character})$

$D' < C'(\text{metallic character})$

$B'^{+} < A'^{+}$ (size)

$D'^{+} < C'^{+}$ (size)

$\therefore$ C statement is incorrect.

SECTION-B

81. A diatomic molecule has a dipole moment of 1.2 D. If the bond distance is 1Å, then fractional charge on each atom is ______ $\times 10^{-1}$ esu.

(Given $1\,D = 10^{-18}$ esu cm)

Ans. (0)

Sol:- $\mu = 1.2\,D = q \times d$

$\Rightarrow 1.2 \times 10^{-10}\,\text{esu Å} = q \times 1\text{Å}$

$\therefore q = 1.2 \times 10^{-10}$ esu

82. $r = k[A]$ for a reaction, 50% of A is decomposed in 120 minutes. The time taken for 90% decomposition of A is ______ minutes.

Ans. (399)

Sol:- $r = k[A]$

So, order of reaction = 1

$t_{1/2} = 120$ min

For 90% completion of reaction

$$\Rightarrow k = \frac{2.303}{t}\log\left(\frac{a}{a-x}\right)$$

$$\Rightarrow \frac{0.693}{t_{1/2}} = \frac{2.303}{t}\log\frac{100}{10}$$

$\therefore t = 399$ min.

83. A compound (x) with molar mass $108\,g\,mol^{-1}$ undergoes acetylation to give product with molar mass $192\,g\,mol^{-1}$. The number of amino groups in the compound (x) is ______.

Ans. (2)

Sol:- $$R-NH_2 + CH_3-\overset{\overset{\large O}{\|}}{C}-Cl \longrightarrow R-NH-\overset{\overset{\large O}{\|}}{C}-CH_3$$

Gain in molecular weight after acylation with one $-NH_2$ group is 42.

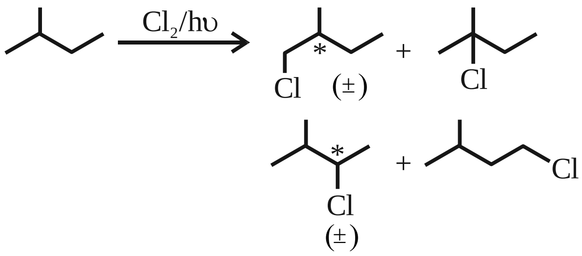
Total increase in molecular weight = 84

$\therefore$ Number of amino group in $x = \frac{84}{42} = 2$

84. Number of isomeric products formed by mono-chlorination of 2-methylbutane in presence of sunlight is _______.

Ans. (6)

Sol:-



$\therefore$ Number of isomeric products = 6

85. Number of moles of H^+ ions required by 1 mole of MnO_4^- to oxidise oxalate ion to CO_2 is ____.

Ans. (8)

Sol:-

$$2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \longrightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$$

$\therefore$ Number of moles of H^+ ions required by 1 mole of MnO_4^- to oxidise oxalate ion to CO_2 is 8

86. In the reaction of potassium dichromate, potassium chloride and sulfuric acid (conc.), the oxidation state of the chromium in the product is (+)_______.

Ans. (6)

Sol:- $K_2Cr_2O_7(s) + 4KCl(s) + 6H_2SO_4(conc.)$

$\rightarrow 2CrO_2Cl_2(g) + 6KHSO_4 + 3H_2O$

This reaction is called chromyl chloride test.

Here oxidation state of Cr is +6.

87. The molarity of 1L orthophosphoric acid (H_3PO_4) having 70% purity by weight (specific gravity $1.54\,g\,cm^{-3}$) is ______M.

(Molar mass of $H_3PO_4 = 98\,g\,mol^{-1}$)

Ans. (11)

Sol:- Specific gravity (density) = 1.54 g/cc.
Volume $=1L=1000\,ml$
Mass of solution $=1.54\times1000$
$=1540\,g$
% purity of H_2SO_4 is 70%
So weight of $H_3PO_4=0.7\times1540=1078\,g$
Mole of $H_3PO_4=\frac{1078}{98}=11$
Molarity $=\frac{11}{1L}=11$

88. The values of conductivity of some materials at 298.15 K in Sm^{-1} are 2.1×10^3, $1.0\times10^{-16}, 1.2\times10,\ 3.91, 1.5\times10^{-2}$, $1\times10^{-7}, 1.0\times10^3$. The number of conductors among the materials is ______.

Ans. **(4)**

Sol:-

Conductivity $(S\,m^{-1})$

$\left.\begin{matrix}2.1\times10^3\\1.2\times10\\3.91\\1\times10^3\end{matrix}\right\}$ conductors at 298.15 K

1×10^{-16} Insulator at 298.15 K

$\left.\begin{matrix}1.5\times10^{-2}\\1\times10^{-7}\end{matrix}\right\}$ Semiconductor at 298.15 K

Therefore number of conductors is 4.

89. From the vitamins A, B_1, B_6, B_{12}, C, D, E and K, the number vitamins that can be stored in our body is ______.

Ans. **(5)**

Sol:- Vitamins A, D, E, K and B_{12} are stored in liver and adipose tissue.

90. If 5 moles of an ideal gas expands from 10 L to a volume of 100 L at 300 K under isothermal and reversible condition then work, w, is $-x$ J. The value of x is ______.

(Given $R=8.314\,J\,K^{-1}mol^{-1}$)

Ans. **(28721)**

Sol:- It is isothermal reversible expansion, so work done negative

$$W=-2.303\,nRT\log\left(\frac{V_2}{V_1}\right)$$

$$=-2.303\times5\times8.314\times300\log\left(\frac{100}{10}\right)$$

$$=-28720.713\,J$$

$$\equiv-28721\,J$$

FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

(Held On Thursday 01st February, 2024) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS **TEST PAPER WITH SOLUTION**

SECTION-A

1. A bag contains 8 balls, whose colours are either white or black. 4 balls are drawn at random without replacement and it was found that 2 balls are white and other 2 balls are black. The probability that the bag contains equal number of white and black balls is:

(1) $\frac{2}{5}$ (2) $\frac{2}{7}$

(3) $\frac{1}{7}$ (4) $\frac{1}{5}$

Ans. (2)

Sol.

P(4W4B/2W2B) =

$$\frac{P(4W4B)\times P(2W2B/4W4B)}{P(2W6B)\times P(2W2B/2W6B)+P(3W5B)\times P(2W2B/3W5B)+.............+P(6W2B)\times P(2W2B/6W2B)}$$

$$=\frac{\frac{1}{5}\times\frac{{}^4C_2\times{}^4C_2}{{}^8C_4}}{\frac{1}{5}\times\frac{{}^2C_2\times{}^6C_2}{{}^8C_4}+\frac{1}{5}\times\frac{{}^3C_2\times{}^5C_2}{{}^8C_4}+...+\frac{1}{5}\times\frac{{}^6C_2\times{}^2C_2}{{}^8C_4}}$$

$$=\frac{2}{7}$$

2. The value of the integral

$$\int_0^{\frac{\pi}{4}}\frac{xdx}{\sin^4(2x)+\cos^4(2x)}\ equals:$$

(1) $\frac{\sqrt{2}\pi^2}{8}$ (2) $\frac{\sqrt{2}\pi^2}{16}$

(3) $\frac{\sqrt{2}\pi^2}{32}$ (4) $\frac{\sqrt{2}\pi^2}{64}$

Ans. (3)

Sol. $\int_0^{\frac{\pi}{4}}\frac{xdx}{\sin^4(2x)+\cos^4(2x)}$

Let $2x=t$ then $dx=\frac{1}{2}dt$

$$I=\frac{1}{4}\int_0^{\frac{\pi}{2}}\frac{tdt}{\sin^4 t+\cos^4 t}$$

$$I=\frac{1}{4}\int_0^{\frac{\pi}{2}}\frac{\left(\frac{\pi}{2}-t\right)dt}{\sin^4\left(\frac{\pi}{2}-t\right)+\cos^4\left(\frac{\pi}{2}-t\right)}$$

$$I=\frac{1}{4}\int_0^{\frac{\pi}{2}}\frac{\frac{\pi}{2}dt}{\sin^4 t+\cos^4 t}-I$$

$$2I=\frac{\pi}{8}\int_0^{\frac{\pi}{2}}\frac{dt}{\sin^4 t+\cos^4 t}$$

$$2I=\frac{\pi}{8}\int_0^{\frac{\pi}{2}}\frac{\sec^4 tdt}{\tan^4 t+1}$$

Let tant = y then $\sec^2$t dt = dy

$$2I=\frac{\pi}{8}\int_0^{\infty}\frac{(1+y^2)dy}{1+y^4}$$

$$=\frac{\pi}{16}\int_0^{\infty}\frac{1+\frac{1}{y^2}}{y^2+\frac{1}{y^2}}dy$$

Put $y-\frac{1}{y}=p$

$$I=\frac{\pi}{16}\int_{-\infty}^{\infty}\frac{dp}{p^2+\left(\sqrt{2}\right)^2}$$

$$=\frac{\pi}{16\sqrt{2}}\left[\tan^{-1}\left(\frac{p}{\sqrt{2}}\right)\right]_{-\infty}^{\infty}$$

$$I=\frac{\pi^2}{16\sqrt{2}}$$

3. If A = $\begin{bmatrix}\sqrt{2} & 1\\ -1 & \sqrt{2}\end{bmatrix}$, B = $\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$, C = ABA^T and X = A^TC^2A, then det X is equal to :

(1) 243

(2) 729

(3) 27

(4) 891

Ans. (2)

Sol.

$$A=\begin{bmatrix}\sqrt{2} & 1\\ -1 & \sqrt{2}\end{bmatrix}\Rightarrow \det(A)=3$$

$$B=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}\Rightarrow \det(B)=1$$

Now C = ABA^T $\Rightarrow$det(C) = (dct (A))^2 x det(B)

$|C|=9$

Now |X| = $|A^TC^2A|$

= $|A^T|\,|C|^2\,|A|$

= $|A|^2\,|C|^2$

= 9 x 81

= 729

4. If tanA = $\dfrac{1}{\sqrt{x(x^2+x+1)}}$, $\tan B=\dfrac{\sqrt{x}}{\sqrt{x^2+x+1}}$

and

$\tan C=\left(x^{-3}+x^{-2}+x^{-1}\right)^{\frac{1}{2}}, 0<A,B,C<\dfrac{\pi}{2}$, *then*

A + B is equal to :

(1) C

(2) $\pi-C$

(3) $2\pi-C$

(4) $\dfrac{\pi}{2}-C$

Ans. (1)

Sol.

Finding tan (A + B) we get

$\Rightarrow$ tan (A + B) =

$$\frac{\tan A+\tan B}{1-\tan A\tan B}=\frac{\dfrac{1}{\sqrt{x(x^2+x+1)}}+\dfrac{\sqrt{x}}{\sqrt{x^2+x+1}}}{1-\dfrac{1}{x^2+x+1}}$$

$$\Rightarrow \tan(A+B)=\frac{(1+x)\left(\sqrt{x^2+x+1}\right)}{(x^2+x)\left(\sqrt{x}\right)}$$

$$\frac{(1+x)\left(\sqrt{x^2+x+1}\right)}{(x^2+x)\left(\sqrt{x}\right)}$$

$$\tan(A+B)=\frac{\sqrt{x^2+x+1}}{x\sqrt{x}}=\tan C$$

$$A+B=C$$

5. If n is the number of ways five different employees can sit into four indistinguishable offices where any office may have any number of persons including zero, then n is equal to:

(1) 47

(2) 53

(3) 51

(4) 43

Ans. (3)

Sol.

Total ways to partition 5 into 4 parts are :

5, 0, 0, 0 $\Rightarrow$1 way

4, 1, 0, 0 $\Rightarrow \dfrac{5!}{4!}=5$ ways

3, 2, 0, 0, $\Rightarrow \dfrac{5!}{3!2!}=10$ ways

$2,2,0,1\Rightarrow \dfrac{5!}{2!2!2!}=15$ ways

$2,1,1,1\Rightarrow \dfrac{5!}{2!(1!)^3 3!}=10$ ways

$3,1,1,0\Rightarrow \dfrac{5!}{3!2!}=10$ ways

Total $\Rightarrow$ 1+5+10+15+10+10 = 51 ways

6. Let $S=\{z\in C:|z-1|=1$ and $(\sqrt{2}-1)(z+\bar{z})-i(z-\bar{z})=2\sqrt{2}\}$. Let z_1, z_2 $\in S$ be such that $|z_1|=\max\limits_{z\in s}|z|$ and $|z_2|=\min\limits_{z\in s}|z|$.

Then $\left|\sqrt{2}z_1-z_2\right|^2$ equals :

(1) 1 (2) 4

(3) 3 (4) 2

Ans. (4)

Sol. Let Z = x + iy

Then $(x-1)^2+y^2=1 \quad \to (1)$

& $(\sqrt{2}-1)(2x)-i(2iy)=2\sqrt{2}$

$\Rightarrow (\sqrt{2}-1)x+y=\sqrt{2} \to (2)$

Solving (1) & (2) we get

Either x = 1 or $x=\dfrac{1}{2-\sqrt{2}} \to (3)$

On solving (3) with (2) we get

For x = 1 $\Rightarrow$ y = 1 $\Rightarrow Z_2 = 1+i$

& for

$x=\dfrac{1}{2-\sqrt{2}} \Rightarrow y=\sqrt{2}-\dfrac{1}{\sqrt{2}} \Rightarrow Z_1=\left(1+\dfrac{1}{\sqrt{2}}\right)+\dfrac{i}{\sqrt{2}}$

Now

$\left|\sqrt{2}z_1-z_2\right|^2$

$=\left|\left(\dfrac{1}{\sqrt{2}}+1\right)\sqrt{2}+i-(1+i)\right|^2$

$=\left(\sqrt{2}\right)^2$

$=2$

7. Let the median and the mean deviation about the median of 7 observation 170, 125, 230, 190, 210, a, b be 170 and $\dfrac{205}{7}$ respectively. Then the mean deviation about the mean of these 7 observations is :

(1) 31

(2) 28

(3) 30

(4) 32

Ans. (3)

Sol. Median = 170 $\Rightarrow$ 125, a, b, 170, 190, 210, 230

Mean deviation about

Median =

$\dfrac{0+45+60+20+40+170-a+170-b}{7}=\dfrac{205}{7}$

$\Rightarrow$ a + b = 300

Mean = $\dfrac{170+125+230+190+210+a+b}{7}=175$

Mean deviation

About mean =

$\dfrac{50+175-a+175-b+5+15+35+55}{7}=30$

8. Let $\vec{a}=-5\hat{i}+\hat{j}-3\hat{k},\vec{b}=\hat{i}+2\hat{j}-4\hat{k}$ and $\vec{c}=\left(\left(\left(\vec{a}\times\vec{b}\right)\times\hat{i}\right)\times\hat{i}\right)\times\hat{i}$. Then $\vec{c}\cdot\left(-\hat{i}+\hat{j}+\hat{k}\right)$ is equal to

(1) –12 (2) –10

(3) –13 (4) –15

Ans. (1)

Sol. $\vec{a}=-5\hat{i}+j-3\hat{k}$

$\vec{b}=\hat{i}+2\hat{j}-4\hat{k}$

$(\vec{a}\times\vec{b})\times\hat{i}=\left(\vec{a}\cdot\hat{i}\right)\vec{b}-\left(\vec{b}\cdot\hat{i}\right)\vec{a}$

$=-5\vec{b}-\vec{a}$

$=\left(\left(\left(-5\vec{b}-\vec{a}\right)\times\hat{i}\right)\times\hat{i}\right)$

$=\left(\left(-11\hat{j}+23\hat{k}\right)\times\hat{i}\right)\times\hat{i}$

$\Rightarrow\left(11\hat{k}+23\hat{j}\right)\times\hat{i}$

$\Rightarrow\left(11\hat{j}-23\hat{k}\right)$

$\vec{c}.\left(-\hat{i}+\hat{j}+\hat{k}\right)=11-23=-12$

9. Let S = $\{x \in R : (\sqrt{3}+\sqrt{2})^x + (\sqrt{3}-\sqrt{2})^x = 10\}$.

Then the number of elements in S is :

(1) 4 (2) 0

(3) 2 (4) 1

Ans. (3)

Sol. $\left(\sqrt{3}+\sqrt{2}\right)^x + \left(\sqrt{3}-\sqrt{2}\right)^x = 10$

Let $\left(\sqrt{3}+\sqrt{2}\right)^x = t$

$t + \frac{1}{t} = 10$

$t^2 - 10t + 1 = 0$

$t = \frac{10 \pm \sqrt{100-4}}{2} = 5 \pm 2\sqrt{6}$

$\left(\sqrt{3}+\sqrt{2}\right)^x = \left(\sqrt{3}\pm\sqrt{2}\right)^2$

x = 2 or x = –2

Number of solutions = 2

10. The area enclosed by the curves xy + 4y = 16 and x + y = 6 is equal to :

(1) $28 - 30 \log_e 2$ (2) $30 - 28 \log_e 2$

(3) $30 - 32 \log_e 2$ (4) $32 - 30 \log_e 2$

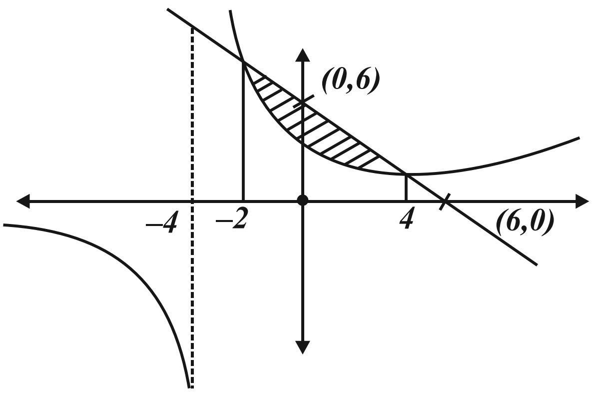
Ans. (3)

Sol. xy + 4y = 16 , x + y = 6

y(x + 4) = 16 ____(1) , x + y = 6___(2)

on solving, (1) & (2)

we get x = 4, x = –2



$$\text{Area} = \int_{-2}^{4} \left((6-x) - \left(\frac{16}{x+4} \right) \right) dx$$

$= 30 - 32 \ln 2$

11. Let f : **R** → **R** and g : **R** → **R** be defined as

$f(x) = \begin{cases} \log_e x & , \quad x > 0 \\ e^{-x} & , \quad x \le 0 \end{cases}$ and

$g(x) = \begin{cases} x & , \quad x \ge 0 \\ e^x & , \quad x < 0 \end{cases}$. Then, gof : **R** → **R** is :

(1) one-one but not onto

(2) neither one-one nor onto

(3) onto but not one-one

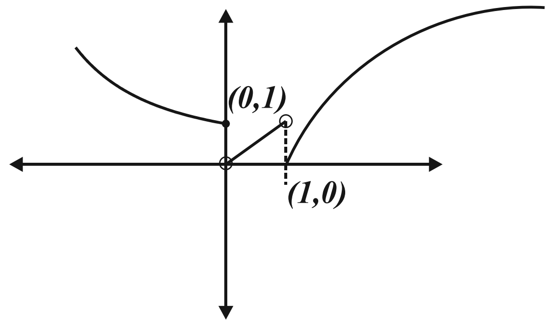
(4) both one-one and onto

Ans. (2)

Sol.

$g(f(x)) = \begin{cases} f(x), f(x) \ge 0 \\ e^{f(x)}, f(x) < 0 \end{cases}$

$g(f(x)) = \begin{cases} e^{-x}, (-\infty, 0] \\ e^{\ln x}, (0,1) \\ \ln x, [1, \infty) \end{cases}$



Graph of g(f(x))

g(f(x)) ⇒ Many one into

12. If the system of equations

2x + 3y – z = 5

x + αy + 3z = –4

3x – y + βz = 7

has infinitely many solutions, then 13 αβ is equal to

(1) 1110 (2) 1120

(3) 1210 (4) 1220

Ans. (2)

Sol. Using family of planes

$2x + 3y - z - 5 = k_1 (x + \alpha y + 3z + 4) + k_2 (3x - y + \beta z - 7)$

$2 = k_1 + 3k_2$, $3 = k_1\alpha - k_2$, $-1 = 3k_1 + \beta k_2$, $-5 = 4k_1 - 7k_2$

On solving we get

$$k_2 = \frac{13}{19}, k_1 = \frac{-1}{19}, \alpha = -70, \beta = \frac{-16}{13}$$

$$13\alpha\beta = 13(-70)\left(\frac{-16}{13}\right)$$

$$= 1120$$

13. For $0 < \theta < \pi/2$, if the eccentricity of the hyperbola $x^2 - y^2\text{cosec}^2\theta = 5$ is $\sqrt{7}$ times eccentricity of the ellipse $x^2\text{cosec}^2\theta + y^2 = 5$, then the value of θ is :

(1) $\frac{\pi}{6}$ (2) $\frac{5\pi}{12}$

(3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

Ans. (3)

Sol.

$$e_h = \sqrt{1+\sin^2\theta}$$

$$e_c = \sqrt{1-\sin^2\theta}$$

$$e_h = \sqrt{7}e_c$$

$$1+\sin^2\theta = 7(1-\sin^2\theta)$$

$$\sin^2\theta = \frac{6}{8} = \frac{3}{4}$$

$$\sin\theta = \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{3}$$

14. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = 2x(x+y)^3 - x(x+y) - 1$, $y(0) = 1$. Then, $\left(\frac{1}{\sqrt{2}} + y\left(\frac{1}{\sqrt{2}}\right)\right)^2$ equals :

(1) $\frac{4}{4+\sqrt{e}}$ (2) $\frac{3}{3-\sqrt{e}}$

(3) $\frac{2}{1+\sqrt{e}}$ (4) $\frac{1}{2-\sqrt{e}}$

Ans. (4)

Sol. $\frac{dy}{dx} = 2x(x+y)^3 - x(x+y) - 1$

$$x + y = t$$

$$\frac{dt}{dx} - 1 = 2xt^3 - xt - 1$$

$$\frac{dt}{2t^3 - t} = xdx$$

$$\frac{tdt}{2t^4 - t^2} = xdx$$

Let $t^2 = z$

$$\int \frac{dz}{2(2z^2 - z)} = \int xdx$$

$$\int \frac{dz}{4z\left(z - \frac{1}{2}\right)} = \int xdx$$

$$\ln\left|\frac{z - \frac{1}{2}}{z}\right| = x^2 + k$$

$$z = \frac{1}{2-\sqrt{e}}$$

15. Let $f : R \to R$ be defined as

$$f(x) = \begin{cases} \frac{a - b\cos 2x}{x^2} & ; \quad x < 0 \\ x^2 + cx + 2 & ; \quad 0 \le x \le 1 \\ 2x + 1 & ; \quad x > 1 \end{cases}$$

If f is continuous everywhere in **R** and m is the number of points where f is **NOT** differential then m + a + b + c equals :

(1) 1 (2) 4

(3) 3 (4) 2

Ans. (4)

Sol. At x = 1, f(x) is continuous therefore,

$f(1^-) = f(1) = f(1^+)$

$f(1) = 3 + c$ ....(1)

$f(1^+) = \lim_{h\to 0} 2(1 + h) + 1$

$f(1^+) = \lim_{h\to 0} 3 + 2h = 3$ ....(2)

from (1) & (2)

c = 0

at x = 0, f(x) is continuous therefore,

$f(0^-) = f(0) = f(0^+)$ ....(3)

$f(0) = f(0^+) = 2$ ....(4)

$f(0^-)$ has to be equal to 2

$$\lim_{h\to 0}\frac{a-b\cos(2h)}{h^2}$$

$$\lim_{h\to 0}\frac{a-b\left\{1-\frac{4h^2}{2!}+\frac{16h^4}{4!}+...\right\}}{h^2}$$

$$\lim_{h\to 0}\frac{a-b+b\left\{2h^2-\frac{2}{3}h^4...\right\}}{h^2}$$

for limit to exist a – b = 0 and limit is 2b ....(5)

from (3), (4) & (5)

a = b = 1

checking differentiability at x = 0

$$\text{LHD}: \lim_{h\to 0}\frac{\frac{1-\cos 2h}{h^2}-2}{-h}$$

$$\lim_{h\to 0}\frac{1-\left(1-\frac{4h^2}{2!}+\frac{16h^4}{4!}...\right)-2h^2}{-h^3}=0$$

$$\text{RHD}: \lim_{h\to 0}\frac{(0+h)^2+2-2}{h}=0$$

Function is differentiable at every point in its domain

$\therefore$ m = 0

m + a + b + c = 0 + 1 + 1 + 0 = 2

16. Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, a > b be an ellipse, whose eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latus rectum is $\sqrt{14}$. Then the square of the eccentricity of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is :

(1) 3 (2) 7/2

(3) 3/2 (4) 5/2

Ans. (3)

Sol.

$$e=\frac{1}{\sqrt{2}}=\sqrt{1-\frac{b^2}{a^2}}\Rightarrow\frac{1}{2}=1-\frac{b^2}{a^2}$$

$$\frac{2b^2}{a}=14$$

$$e_H=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{1+\frac{1}{2}}=\sqrt{\frac{3}{2}}$$

$$(e_H)^2=\frac{3}{2}$$

17. Let 3, a, b, c be in A.P. and 3, a – 1, b + 1, c + 9 be in G.P. Then, the arithmetic mean of a, b and c is :

(1) –4 (2) –1

(3) 13 (4) 11

Ans. (4)

Sol.

3, a, b, c $\rightarrow$ A.P $\Rightarrow$ 3, 3+d, 3+2d, 3+3d

3, a–1, b+1, c + 9 $\rightarrow$ G.P $\Rightarrow$ 3, 2+d, 4+2d, 12+3d

a = 3 + d $(2+d)^2=3(4+2d)$

b = 3 + 2d d = 4, -2

c = 3 + 3d

If d = 4 G.P $\Rightarrow$ 3, 6, 12, 24

a = 7

b = 11

c = 15

$$\frac{a+b+c}{3}=11$$

18. Let C : $x^2 + y^2 = 4$ and C' : $x^2 + y^2 - 4\lambda x + 9 = 0$ be two circles. If the set of all values of λ so that the circles C and C' intersect at two distinct points, is **R**– [a, b], then the point (8a + 12, 16b – 20) lies on the curve :

(1) $x^2 + 2y^2 - 5x + 6y = 3$

(2) $5x^2 - y = -11$

(3) $x^2 - 4y^2 = 7$

(4) $6x^2 + y^2 = 42$

Ans. (4)

Sol. $x^2 + y^2 = 4$

C (0, 0) $\qquad r_1 = 2$

C' $(2\lambda, 0)$ $\qquad r_2 = \sqrt{4\lambda^2 - 9}$

$|r_1 - r_2| < CC' < |r_1 + r_2|$

$$\left|2 - \sqrt{4\lambda^2 - 9}\right| < |2\lambda| < 2 + \sqrt{4\lambda^2 - 9}$$

$$4 + 4\lambda^2 - 9 - 4\sqrt{4\lambda^2 - 9} < 4\lambda^2$$

True $\lambda \in$ R…. (1)

$$4\lambda^2 < 4 + 4\lambda^2 - 9 + 4\sqrt{4\lambda^2 - 9}$$

$$5 < 4\sqrt{4\lambda^2 - 9} \text{ and } \qquad \lambda^2 \ge \frac{9}{4}$$

$$\frac{25}{16} < 4\lambda^2 - 9 \qquad \lambda \in \left(-\infty, -\frac{3}{2}\right] \cup \left[\frac{3}{2}, \infty\right)$$

$$\frac{169}{64} < \lambda^2$$

$$\lambda \in \left(-\infty, -\frac{13}{8}\right) \cup \left(\frac{13}{8}, \infty\right) \qquad \ldots(2)$$

from (1) and (2) $\lambda \in$

$$\lambda \in \left(-\infty, -\frac{13}{8}\right) \cup \left(\frac{13}{8}, \infty\right) \Rightarrow R - \left[-\frac{13}{8}, \frac{13}{8}\right]$$

as per question $a = -\frac{13}{8}$ and $b = \frac{13}{8}$

$\therefore$ required point is (–1, 6) with satisfies option (4)

19. If $5f(x) + 4f\left(\frac{1}{x}\right) = x^2 - 2, \forall x \ne 0$ and $y = 9x^2 f(x)$, then y is strictly increasing in :

(1) $\left(0, \frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$

(2) $\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$

(3) $\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$

(4) $\left(-\infty, \frac{1}{\sqrt{5}}\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$

Ans. (2)

Sol. $5f(x) + 4f\left(\frac{1}{x}\right) = x^2 - 2, \ \forall x \ne 0 \ldots(1)$

Substitute $x \to \frac{1}{x}$

$$5f\left(\frac{1}{x}\right) + 4f(x) = \frac{1}{x^2} - 2 \quad \ldots(2)$$

On solving (1) and (2)

$$f(x) = \frac{5x^4 - 2x^2 - 4}{9x^2}$$

$y = 9x^2 f(x)$

$y = 5x^4 - 2x^2 - 4 \quad \ldots(3)$

$$\frac{dy}{dx} = 20x^3 - 4x$$

for strictly increasing

$$\frac{dy}{dx} > 0$$

$4x(5x^2 - 1) > 0$

$$x \in \left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$$

20. If the shortest distance between the lines $\frac{x - \lambda}{-2} = \frac{y - 2}{1} = \frac{z - 1}{1}$ and $\frac{x - \sqrt{3}}{1} = \frac{y - 1}{-2} = \frac{z - 2}{1}$ is 1, then the sum of all possible values of λ is :

(1) 0 $\qquad$ (2) $2\sqrt{3}$

(3) $3\sqrt{3}$ $\qquad$ (4) $-2\sqrt{3}$

Ans. (2)

Sol. Passing points of lines L_1 & L_2 are

$(\lambda, 2, 1)$ & $(\sqrt{3}, 1, 2)$

$$S.D = \frac{\begin{vmatrix} \sqrt{3}-\lambda & -1 & 1 \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix}}$$

$$1 = \left|\frac{\sqrt{3}-\lambda}{\sqrt{3}}\right|$$

$\lambda = 0, \lambda = 2\sqrt{3}$

SECTION-B

21. If x = x(t) is the solution of the differential equation $(t + 1)dx = (2x + (t + 1)^4)\,dt$, $x(0) = 2$, then, x(1) equals ___________.

Ans. (14)

Sol. $(t + 1)dx = (2x + (t + 1)^4)dt$

$$\frac{dx}{dt} = \frac{2x + (t+1)^4}{t+1}$$

$$\frac{dx}{dt} - \frac{2x}{t+1} = (t + 1)^3$$

$$I \cdot F = e^{-\int \frac{2}{t+1}dt} = e^{-2\ln(t+1)} = \frac{1}{(t+1)^2}$$

$$\frac{x}{(t+1)^2} = \int \frac{1}{(t+1)^2}(t+1)^3 dt + c$$

$$\frac{x}{(t+1)^2} = \frac{(t+1)^2}{2} + c$$

$$\Rightarrow c = \frac{3}{2}$$

$$x = \frac{(t+1)^4}{2} + \frac{3}{2}(t+1)^2$$

put, t = 1

$x = 2^3 + 6 = 14$

22. The number of elements in the set

$S = \{(x, y, z) : x, y, z \in \mathbf{Z}, x + 2y + 3z = 42, x, y, z \geq 0\}$ equals __________.

Ans. (169)

Sol. $x + 2y + 3z = 42, \quad x, y, z \geq 0$

$z = 0 \quad x + 2y = 42 \Rightarrow 22$

$z = 1 \quad x + 2y = 39 \Rightarrow 20$

$z = 2 \quad x + 2y = 36 \Rightarrow 19$

$z = 3 \quad x + 2y = 33 \Rightarrow 17$

$z = 4 \quad x + 2y = 30 \Rightarrow 16$

$z = 5 \quad x + 2y = 27 \Rightarrow 14$

$z = 6 \quad x + 2y = 24 \Rightarrow 13$

$z = 7 \quad x + 2y = 21 \Rightarrow 11$

$z = 8 \quad x + 2y = 18 \Rightarrow 10$

$z = 9 \quad x + 2y = 15 \Rightarrow 8$

$z = 10 \quad x + 2y = 12 \Rightarrow 7$

$z = 11 \quad x + 2y = 9 \Rightarrow 5$

$z = 12 \quad x + 2y = 6 \Rightarrow 4$

$z = 13 \quad x + 2y = 3 \Rightarrow 2$

$z = 14 \quad x + 2y = 0 \Rightarrow 1$

Total : 169

23. If the Coefficient of x^{30} in the expansion of $\left(1+\frac{1}{x}\right)^6 (1 + x^2)^7 (1 - x^3)^8$; $x \neq 0$ is α, then |α| equals ___________.

Ans. (678)

Sol. coeff of x^{30} in $\dfrac{(x+1)^6(1+x^2)^7(1-x^3)^8}{x^6}$

coeff. of x^{36} in $(1+x)^6(1+x^2)^7(1-x^3)^8$

General term

$^6C_{r_1}\,^7C_{r_2}\,^8C_{r_3}(-1)^{r_3}x^{r_1+2r_2+3r_3}$

$r_1+2r_2+3r_3=36$

Case-I : $r_1+2r_2=12$ (Taking r_3 = 8)

r_1	r_2	r_3
0	6	8
2	5	8
4	4	8
6	3	8

Case-II : $r_1+2r_2=15$ (Taking r_3 = 7)

r_1	r_2	r_3
1	7	7
3	6	7
5	5	7

Case-III : $r_1+2r_2=18$ (Taking r_3 = 6)

r_1	r_2	r_3
4	7	6
6	6	6

Coeff. = 7 + (15 × 21) + (15 × 35) + (35)

–(6 × 8) – (20 × 7 × 8) – (6 × 21 × 8) + (15 × 28)

+ (7 × 28) = –678 = α

|α| = 678

24. Let 3, 7, 11, 15, ..., 403 and 2, 5, 8, 11, . . ., 404 be two arithmetic progressions. Then the sum, of the common terms in them, is equal to _________.

Ans. (6699)

Sol. 3, 7, 11, 15, ….., 403

2, 5, 8, 11, …., 404

LCM (4, 3) = 12

11, 23, 35,….. let (403)

403 = 11 + (n - 1) x 12

$\dfrac{392}{12}=n-1$

$33\cdot 66=n$

$n=33$

Sum $\dfrac{33}{2}(22+32\times 12)$

=6699

25. Let {x} denote the fractional part of x and $f(x)=\dfrac{\cos^{-1}(1-\{x\}^2)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}$, $x\neq 0$. If L and R respectively denotes the left hand limit and the right hand limit of f(x) at x = 0, then $\dfrac{32}{\pi^2}(L^2+R^2)$ is equal to _____________.

Ans. (18)

Sol. Finding right hand limit

$\lim\limits_{x\to 0^+} f(x)=\lim\limits_{h\to 0} f(0+h)$

$=\lim\limits_{h\to 0} f(h)$

$=\lim\limits_{h\to 0}\dfrac{\cos^{-1}(1-h^2)\sin^{-1}(1-h)}{h(1-h^2)}$

$=\lim\limits_{h\to 0}\dfrac{\cos^{-1}(1-h^2)}{h}\left(\dfrac{\sin^{-1}1}{1}\right)$

Let $\cos^{-1}(1-h^2)=\theta\Rightarrow\cos\theta=1-h^2$

$=\dfrac{\pi}{2}\lim\limits_{\theta\to 0}\dfrac{\theta}{\sqrt{1-\cos\theta}}$

$=\dfrac{\pi}{2}\lim\limits_{\theta\to 0}\dfrac{1}{\sqrt{\dfrac{1-\cos\theta}{\theta^2}}}$

$=\dfrac{\pi}{2}\dfrac{1}{\sqrt{1/2}}$

$R=\dfrac{\pi}{\sqrt{2}}$

Now finding left hand limit

$$L=\lim_{x\to0^-} f(x)$$

$$=\lim_{h\to0} f(-h)$$

$$=\lim_{h\to0}\frac{\cos^{-1}\left(1-\{-h\}^2\right)\sin^{-1}\left(1-\{-h\}\right)}{\{-h\}-\{-h\}^3}$$

$$=\lim_{h\to0}\frac{\cos^{-1}\left(1-(-h+1)^2\right)\sin^{-1}\left(1-(-h+1)\right)}{(-h+1)-(-h+1)^3}$$

$$=\lim_{h\to0}\frac{\cos^{-1}\left(-h^2+2h\right)\sin^{-1}h}{(1-h)\left(1-(1-h)^2\right)}$$

$$=\lim_{h\to0}\left(\frac{\pi}{2}\right)\frac{\sin^{-1}h}{\left(1-(1-h)^2\right)}$$

$$=\frac{\pi}{2}\lim_{h\to0}\left(\frac{\sin^{-1}h}{-h^2+2h}\right)$$

$$=\frac{\pi}{2}\lim_{h\to0}\left(\frac{\sin^{-1}h}{h}\right)\left(\frac{1}{-h+2}\right)$$

$$L=\frac{\pi}{4}$$

$$\frac{32}{\pi^2}\left(L^2+R^2\right)=\frac{32}{\pi^2}\left(\frac{\pi^2}{2}+\frac{\pi^2}{16}\right)$$

$= 16 + 2$

$= 18$

26. Let the line L : $\sqrt{2}x + y = \alpha$ pass through the point of the intersection P (in the first quadrant) of the circle $x^2 + y^2 = 3$ and the parabola $x^2 = 2y$. Let the line L touch two circles C_1 and C_2 of equal radius $2\sqrt{3}$. If the centres Q_1 and Q_2 of the circles C_1 and C_2 lie on the y-axis, then the square of the area of the triangle PQ_1Q_2 is equal to ___________.

Ans. (72)

Sol. $x^2 + y^2 = 3$ and $x^2 = 2y$

$y^2 + 2y - 3 = 0 \Rightarrow (y + 3)(y - 1) = 0$

$y = -3$ or $y = 1$

$y = 1$ $x = \sqrt{2} \Rightarrow P(\sqrt{2},1)$

p lies on the line

$$\sqrt{2}x+y=\alpha$$
$$\sqrt{2}(\sqrt{2})+1=\alpha$$
$$\alpha=3$$

For circle C_1

Q_1 lies on y axis

Let Q_1 $(0,\alpha)$ coordinates

$R_1 = 2\sqrt{3}$ (Given

Line L act as tangent

Apply P = r (condition of tangency)

$$\Rightarrow\left|\frac{\alpha-3}{\sqrt{3}}\right|=2\sqrt{3}$$
$$\Rightarrow|\alpha-3|=6$$

$\alpha-3=6$ or $\alpha-3=-6$

$\Rightarrow\alpha=9$ $\quad\alpha=-3$

$$\triangle PQ_1Q_2=\frac{1}{2}\begin{vmatrix}\sqrt{2}&1&1\\0&9&1\\0&-3&1\end{vmatrix}$$

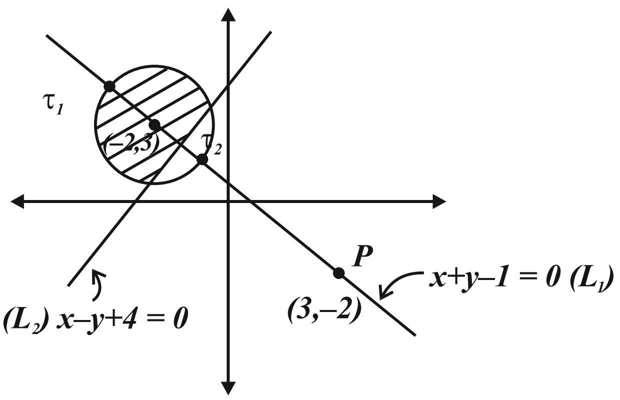
$$=\frac{1}{2}\left(\sqrt{2}(12)\right)=6\sqrt{2}$$

$$\left(\triangle PQ_1Q_2\right)^2=72$$

27. Let P = $\{z \in \ : |z + 2 - 3i| \le 1\}$ and Q = $\{z \in \mathbb{C} : z(1 + i) + \bar{z}(1 - i) \le -8\}$. Let in P ⋂ Q, $|z - 3 + 2i|$ be maximum and minimum at z_1 and z_2 respectively. If $|z_1|^2 + 2|z|^2 = \alpha + \beta\sqrt{2}$, where α, β are integers, then α + β equals ________.

Ans. (36)

Sol.



Clearly for the shaded region z_1 is the intersection of the circle and the line passing through P (L_1) and z_2 is intersection of line L_1 & L_2

Circle : $(x + 2)^2 + (y - 3)^2 = 1$

$L_1 : x + y - 1 = 0$

$L_2 : x - y + 4 = 0$

On solving circle & L_1 we get

$$z_1 : \left(-2 - \frac{1}{\sqrt{2}}, 3 + \frac{1}{\sqrt{2}}\right)$$

On solving L_1 and z_2 is intersection of line L_1 & L_2

we get $z_2 : \left(\frac{-3}{2}, \frac{5}{2}\right)$

$$|z_1|^2 + 2|z_2|^2 = 14 + 5\sqrt{2} + 17$$

$$= 31 + 5\sqrt{2}$$

So $\alpha = 31$

$\beta = 5$

$\alpha + \beta = 36$

28. If $\int_{-\pi/2}^{\pi/2} \frac{8\sqrt{2}\cos x dx}{(1 + e^{\sin x})(1 + \sin^4 x)} = \alpha\pi + \beta \log_e(3 + 2\sqrt{2})$, where α, β are integers, then $\alpha^2 + \beta^2$ equals ________.

Ans. (8)

Sol. $$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x}{\left(1 + e^{\sin x}\right)\left(1 + \sin^4 x\right)} dx$$

Apply king

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x\left(e^{\sin x}\right)}{\left(1 + e^{\sin x}\right)\left(1 + \sin^4 x\right)} dx \quad ....(2)$$

adding (1) & (2)

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x}{1 + \sin^4 x} dx$$

$$I = \int_{0}^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x}{1 + \sin^4 x} dx,$$

$\sin x = t$

$$I = \int_0^1 \frac{8\sqrt{2}}{1 + t^4} dx$$

$$I = 4\sqrt{2}\int_0^1 \left(\frac{1 + \frac{1}{t^2}}{t^2 + \frac{1}{t^2}} - \frac{1 - \frac{1}{t^2}}{t^2 + \frac{1}{t^2}}\right) dt$$

$$I = 4\sqrt{2}\int_0^1 \frac{\left(1 + \frac{1}{t^2}\right)}{\left(t - \frac{1}{t}\right)^2 + 2} - \frac{\left(1 - \frac{1}{t^2}\right)}{\left(t + \frac{1}{t}\right)^2 - 2} dt$$

$$Let\ t - \frac{1}{t} = z \ \&\ t + \frac{1}{t} = k$$

$$= 4\sqrt{2}\left[\int_{-\infty}^{0}\frac{dz}{z^2+2} - \int_{\infty}^{2}\frac{dk}{k^2-2}\right]$$

$$= 4\sqrt{2}\left[\left[\frac{1}{\sqrt{2}}\tan^{-1}\frac{z}{\sqrt{2}}\right]_{-\infty}^{0} - \left[\frac{1}{2\sqrt{2}}\ln\left(\frac{k-\sqrt{2}}{k+\sqrt{2}}\right)\right]_{\infty}^{2}\right]$$

$$= 4\sqrt{2}\left[\frac{\pi}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}\left[\ln\frac{2-\sqrt{2}}{2+\sqrt{2}}\right]\right]$$

$$= 2\pi + 2\ln(3+2\sqrt{2})$$

$\alpha = 2$

$\beta = 2$

29. Let the line of the shortest distance between the lines

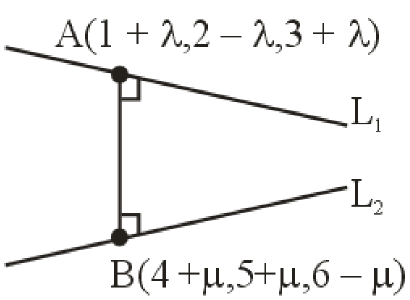
$L_1 : \vec{r} = \left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}+\hat{k}\right)$ and

$L_2 : \vec{r} = \left(4\hat{i}+5\hat{j}+6\hat{k}\right)+\mu\left(\hat{i}+\hat{j}-\hat{k}\right)$

intersect L_1 and L_2 at P and Q respectively. If (α, β, γ) is the midpoint of the line segment PQ, then $2(\alpha + \beta + \gamma)$ is equal to ____________.

Ans. (21)

Sol.



$\vec{b} = \hat{i}-\hat{j}+\hat{k}$ (DR's of L_1)

$\vec{d} = \hat{i}+\hat{j}-\hat{k}$ (DR's of L_2)

$$\vec{b}\times\vec{d} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 1\\ 1 & 1 & -1\end{vmatrix}$$

$= 0\hat{i}+2\hat{j}+2\hat{k}$ (DR's of Line perpendicular to L_1 and L_2)

DR of AB line

$= (0,2,2) = (3+\mu-\lambda, 3+\mu+\lambda, 3-\mu-\lambda)$

$$\frac{3+\mu-\lambda}{0} = \frac{3+\mu+\lambda}{2} = \frac{3-\mu-\lambda}{2}$$

Solving above equation we get $\mu = -\frac{3}{2}$ and $\lambda = \frac{3}{2}$

point $A = \left(\frac{5}{2}, \frac{1}{2}, \frac{9}{2}\right)$

$B = \left(\frac{5}{2}, \frac{7}{2}, \frac{15}{2}\right)$

Point of AB $= \left(\frac{5}{2}, 2, 6\right) = (\alpha, \beta, \gamma)$

$2(\alpha + \beta + \gamma) = 5 + 4 + 12 = 21$

30. Let A= {1, 2, 3, . . 20}. Let R_1 and R_2 two relation on A such that

R_1 = {(a, b) : b is divisible by a}

R_2 = {(a, b) : a is an integral multiple of b}.

Then, number of elements in $R_1 - R_2$ is equal to _________.

Ans. (46)

Sol. $n(R_1) = 20 + 10 + 6 + 5 + 4 + 3 + 2 + 2 + 2$

$+2+\underbrace{1+\ldots+1}_{10\text{ times}}$

$n(R_1) = 66$

$R_1 \cap R_2 = \{(1,1),(2,2),...(20,20)\}$

$n(R_1 \cap R_2) = 20$

$n(R_1 - R_2) = n(R_1) - n(R_1 \cap R_2)$

$= n(R_1) - 20$

$= 66 - 20$

$R_1 - R_2 = 46$ Pair

SECTION-A

31. With rise in temperature, the Young's modulus of elasticity

(1) changes erratically

(2) decreases

(3) increases

(4) remains unchanged

Ans. (2)

Sol. Conceptual questions

32. If R is the radius of the earth and the acceleration due to gravity on the surface of earth is $g = \pi^2$ m/s^2, then the length of the second's pendulum at a height h = 2R from the surface of earth will be,:

(1) $\frac{2}{9}$m

(2) $\frac{1}{9}$m

(3) $\frac{4}{9}$m

(4) $\frac{8}{9}$m

Ans. (2)

Sol. $g' = \frac{GMe}{(3R)^2} = \frac{1}{9}g$

$T = 2\pi\sqrt{\frac{\ell}{g'}}$

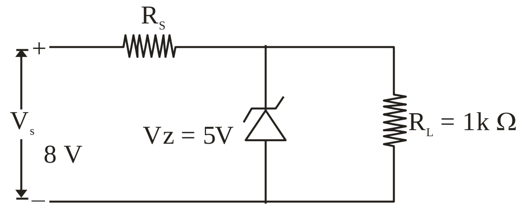
Since the time period of second pendulum is 2 sec.

T = 2 sec

$2 = 2\pi\sqrt{\frac{\ell}{g}9}$

$\boxed{\ell = \frac{1}{9}\text{m}}$

33. In the given circuit if the power rating of Zener diode is 10 mW, the value of series resistance R_s to regulate the input unregulated supply is :

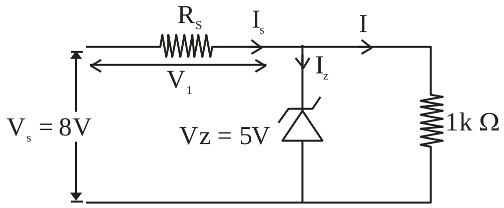


(1) 5kΩ (2) 10Ω

(3) 1kΩ (4) 10kΩ

Ans. (BONUS)

Sol.



Pd across R_s

$V_1 = 8 - 5 = 3V$

Current through the load resistor

$I = \frac{5}{1\times10^3} = 5mA$

Maximum current through Zener diode

$I_{z\ max.} = \frac{10}{5} = 2mA$

And minimum current through Zener diode

$I_{z\ min.} = 0$

$\therefore I_{s\ max.} = 5 + 2 = 7mA$

And $R_{s\ min} = \frac{V_1}{I_{s\ max.}} = \frac{3}{7}k\Omega$

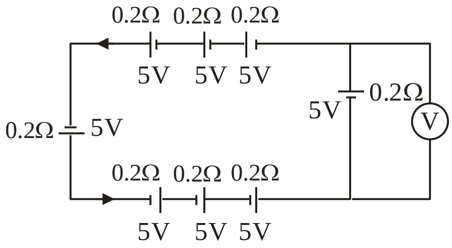
Similarly

$I_{s\ min.} = 5mA$

And $R_{s\ max.} = \frac{V_1}{I_{s\ min.}} = \frac{3}{5}k\Omega$

$\therefore \frac{3}{7}k\Omega < R_s < \frac{3}{5}k\Omega$

34. The reading in the ideal voltmeter (V) shown in the given circuit diagram is :



(1) 5V (2) 10V

(3) 0 V (4) 3V

Ans. **(3)**

Sol. $i = \frac{E_{eq}}{r_{eq}} = \frac{8 \times 5}{8 \times 0.2}$

$I = 25A$

$V = E - ir$

$= 5 - 0.2 \times 25$

$= 0$

35. Two identical capacitors have same capacitance C. One of them is charged to the potential V and other to the potential 2V. The negative ends of both are connected together. When the positive ends are also joined together, the decrease in energy of the combined system is :

(1) $\frac{1}{4}CV^2$

(2) $2\ CV^2$

(3) $\frac{1}{2}CV^2$

(4) $\frac{3}{4}CV^2$

Ans. **(1)**

Sol. $V_C = \frac{q_{net}}{C_{net}} = \frac{CV + 2CV}{2C}$

$V_C = \frac{3V}{2}$

Loss of energy

$= \frac{1}{2}CV^2 + \frac{1}{2}C(2V)^2 - \frac{1}{2}2C\left(\frac{3V}{2}\right)^2$

$= \left(\frac{CV^2}{4}\right)$

36. Two moles a monoatomic gas is mixed with six moles of a diatomic gas. The molar specific heat of the mixture at constant volume is :

(1) $\frac{9}{4}R$ (2) $\frac{7}{4}R$

(3) $\frac{3}{2}R$ (4) $\frac{5}{2}R$

Ans. **(1)**

Sol. $C_V = \frac{n_1C_{V_1} + n_2C_{V_2}}{n_1 + n_2}$

$= \frac{2 \times \frac{3}{2}R + 6 \times \frac{5}{2}R}{2+6}$

$= \frac{9}{4}R$

37. A ball of mass 0.5 kg is attached to a string of length 50 cm. The ball is rotated on a horizontal circular path about its vertical axis. The maximum tension that the string can bear is 400 N. The maximum possible value of angular velocity of the ball in rad/s is,:

(1) 1600 (2) 40

(3) 1000 (4) 20

Ans. **(2)**

Sol. $T = m\omega^2\ell$

$400 = 0.5\omega^2 \times 0.5$

$\omega = 40$ rad/s.

38. A parallel plate capacitor has a capacitance C = 200 pF. It is connected to 230 V ac supply with an angular frequency 300 rad/s. The rms value of conduction current in the circuit and displacement current in the capacitor respectively are :

(1) 1.38 μA and 1.38 μA

(2) 14.3 μA and 143 μA

(3) 13.8 μA and 138 μA

(4) 13.8 μA and 13.8 μA

Ans. **(4)**

Sol. $I = \frac{V}{X_C} = 230 \times 300 \times 200 \times 10^{-12} = 13.8\ \mu A$

39. The pressure and volume of an ideal gas are related as $PV^{3/2} = K$ (Constant). The work done when the gas is taken from state A (P_1, V_1, T_1) to state B (P_2, V_2, T_2) is :

(1) $2(P_1V_1 - P_2V_2)$

(2) $2(P_2V_2 - P_1V_1)$

(3) $2(\sqrt{P_1}V_1 - \sqrt{P_2}V_2)$

(4) $2(P_2\sqrt{V_2} - P_1\sqrt{V_1})$

Ans. **(1 or 2)**

Sol. For PV^x = constant

If work done by gas is asked then

$$W = \frac{nR\Delta T}{1-x}$$

$$\text{Here } x = \frac{3}{2}$$

$$\therefore W = \frac{P_2V_2 - P_1V_1}{-\frac{1}{2}}$$

$= 2(P_1V_1 - P_2V_2)$ ….. Option (1) is correct

If work done by external is asked then

$W = -2(P_1V_1 - P_2V_2)$ ….. Option (2) is correct

40. A galvanometer has a resistance of 50 Ω and it allows maximum current of 5 mA. It can be converted into voltmeter to measure upto 100 V by connecting in series a resistor of resistance

(1) 5975 Ω

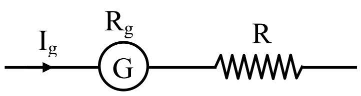
(2) 20050 Ω

(3) 19950 Ω

(4) 19500 Ω

Ans. **(3)**

Sol.



$$R = \frac{V}{I_g} - R_g = \frac{100}{5\times10^{-3}} - 50$$

$= 20000 - 50$

$= 19950\Omega$

41. The de Broglie wavelengths of a proton and an α particle are λ and 2 λ respectively. The ratio of the velocities of proton and α particle will be :

(1) 1 : 8

(2) 1 : 2

(3) 4 : 1

(4) 8 : 1

Ans. **(4)**

Sol. $\lambda = \frac{h}{p} = \frac{h}{mv} \Rightarrow v = \frac{h}{m\lambda}$

$$\frac{v_p}{v_\alpha} = \frac{m_\alpha}{m_p} \times \frac{\lambda_\alpha}{\lambda_p}$$

$= 4 \times 2 = 8$

42. 10 divisions on the main scale of a Vernier calliper coincide with 11 divisions on the Vernier scale. If each division on the main scale is of 5 units, the least count of the instrument is :

(1) $\frac{1}{2}$

(2) $\frac{10}{11}$

(3) $\frac{50}{11}$

(4) $\frac{5}{11}$

Ans. **(4)**

Sol. 10 MSD = 11 VSD

$$1 \text{ VSD} = \frac{10}{11}\text{MSD}$$

$$LC = 1MSD - 1VSD$$

$$= 1 \text{ MSD} - \frac{10}{11}\text{MSD}$$

$$= \frac{1MSD}{11}$$

$$= \frac{5}{11} \text{ units}$$

43. In series LCR circuit, the capacitance is changed from C to 4C. To keep the resonance frequency unchanged, the new inductance should be :

(1) reduced by $\frac{1}{4}L$

(2) increased by 2L

(3) reduced by $\frac{3}{4}L$

(4) increased to 4L

Ans. **(3)**

Sol. $\omega' = \omega$

$$\frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{LC}}$$

$\therefore$ $L'C' = LC$

$L'(4C) = LC$

$L' = \frac{L}{4}$

$\because$ Inductance must be decreased by $\frac{3L}{4}$

44. The radius (r), length (l) and resistance (R) of a metal wire was measured in the laboratory as

$r = (0.35 \pm 0.05)$ cm

$R = (100 \pm 10)$ ohm

$l = (15 \pm 0.2)$ cm

The percentage error in resistivity of the material of the wire is :

(1) 25.6% (2) 39.9%

(3) 37.3% (4) 35.6%

Ans. **(2)**

Sol. $\rho = R\frac{\rho}{\ell}$

$$\frac{\Delta\rho}{\rho} = \frac{\Delta R}{R} + 2\frac{\Delta r}{r} + \frac{\Delta\ell}{\ell}$$

$$= \frac{10}{100} + 2 \times \frac{0.05}{0.35} + \frac{0.2}{15}$$

$$= \frac{1}{10} + \frac{2}{7} + \frac{1}{75}$$

$$\frac{\Delta\rho}{\rho} = 39.9\%$$

45. The dimensional formula of angular impulse is :

(1) $[M L^{-2} T^{-1}]$ (2) $[M L^{2} T^{-2}]$

(3) $[M L\ T^{-1}]$ (4) $[M L^{2} T^{-1}]$

Ans. **(4)**

Sol. Angular impulse = change in angular momentum.

[Angular impulse] = [Angular momentum] = [mvr]

$= [M L^2 T^{-1}]$

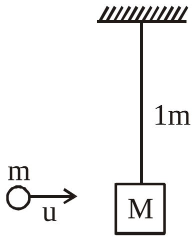
46. A simple pendulum of length 1 m has a wooden bob of mass 1 kg. It is struck by a bullet of mass 10^{-2} kg moving with a speed of 2×10^2 ms^{-1}. The bullet gets embedded into the bob. The height to which the bob rises before swinging back is. (use $g = 10$ m/s^2)

(1) 0.30 m (2) 0.20 m

(3) 0.35 m (4) 0.40 m

Ans. **(2)**

Sol.



$mu = (M + m)V$

$10^{-2} \times 2 \times 10^2 \cong 1 \times V$

$V \cong 2$m/s

$h = \frac{V^2}{2g} = 0.2$ m

47. A particle moving in a circle of radius R with uniform speed takes time T to complete one revolution. If this particle is projected with the same speed at an angle θ to the horizontal, the maximum height attained by it is equal to 4R. The angle of projection θ is then given by :

(1) $\sin^{-1}\left[\frac{2gT^2}{\pi^2 R}\right]^{\frac{1}{2}}$ (2) $\sin^{-1}\left[\frac{\pi^2 R}{2gT^2}\right]^{\frac{1}{2}}$

(3) $\cos^{-1}\left[\frac{2gT^2}{\pi^2 R}\right]^{\frac{1}{2}}$ (4) $\cos^{-1}\left[\frac{\pi R}{2gT^2}\right]^{\frac{1}{2}}$

Ans. **(1)**

Sol. $\frac{2\pi R}{T} = V$

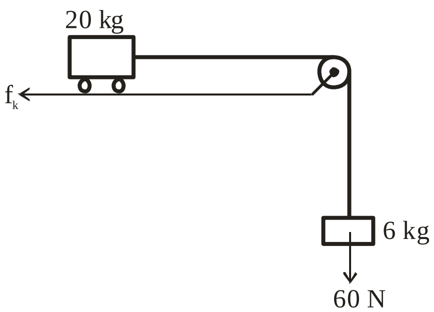
Maximum height $H = \frac{v^2 \sin^2\theta}{2g}$

$4R = \frac{4\pi^2 R^2}{T^2 2g} \sin^2\theta$

$\sin\theta = \sqrt{\frac{2gT^2}{\pi^2 R}}$

$\theta = \sin^{-1}\left(\frac{2gT^2}{\pi^2 R}\right)^{\frac{1}{2}}$

48. Consider a block and trolley system as shown in figure. If the coefficient of kinetic friction between the trolley and the surface is 0.04, the acceleration of the system in ms^{-2} is :
(Consider that the string is massless and unstretchable and the pulley is also massless and frictionless) :



(1) 3 (2) 4
(3) 2 (4) 1.2

Ans. **(3)**

Sol. $f_k = \mu N = 0.04 \times 20g = 8$ Newton

$a = \frac{60-8}{26} = 2 m/s^2$

49. The minimum energy required by a hydrogen atom in ground state to emit radiation in Balmer series is nearly :

(1) 1.5 eV (2) 13.6 eV
(3) 1.9 eV (4) 12.1 eV

Ans. **(4)**

Sol. Transition from n = 1 to n = 3

$\Delta E = 12.1 eV$

50. A monochromatic light of wavelength 6000Å is incident on the single slit of width 0.01 mm. If the diffraction pattern is formed at the focus of the convex lens of focal length 20 cm, the linear width of the central maximum is :

(1) 60 mm
(2) 24 mm
(3) 120 mm
(4) 12 mm

Ans. **(2)**

Sol. Linear width

$W = \frac{2\lambda d}{a} = \frac{2 \times 6 \times 10^{-7} \times 0.2}{1 \times 10^{-5}}$

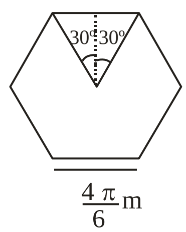
$= 2.4 \times 10^{-2} = 24$ mm

SECTION-B

51. A regular polygon of 6 sides is formed by bending a wire of length 4 π meter. If an electric current of $4\pi\sqrt{3}$ A is flowing through the sides of the polygon, the magnetic field at the centre of the polygon would be $x \times 10^{-7}$ T. The value of x is ____.

Ans. **(72)**

Sol.



$B = 6\left(\frac{\mu_0 I}{4\pi r}\right)(\sin 30° + \sin 30°)$

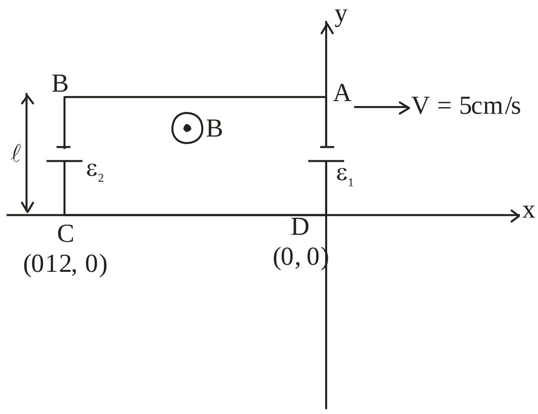
$= 6 \frac{10^{-7} \times 4\pi\sqrt{3}}{\left(\frac{\sqrt{3} \times 4\pi}{2 \times 6}\right)}$

$= 72 \times 10^{-7}$T

52. A rectangular loop of sides 12 cm and 5 cm, with its sides parallel to the x-axis and y-axis respectively moves with a velocity of 5 cm/s in the positive x axis direction, in a space containing a variable magnetic field in the positive z direction. The field has a gradient of 10^{-3}T/cm along the negative x direction and it is decreasing with time at the rate of 10^{-3} T/s. If the resistance of the loop is 6 mΩ, the power dissipated by the loop as heat is _______ $\times 10^{-9}$ W.

Ans. **(216)**

Sol.



B_0 is the magnetic field at origin

$$\frac{dB}{dx} = -\frac{10^{-3}}{10^{-2}}$$

$$\int_{B_0}^{B} dB = -\int_0^x 10^{-1}dx$$

$$B - B_0 = -10^{-1}x$$

$$B = \left(B_0 - \frac{x}{10}\right)$$

Motional emf in AB = 0

Motional emf in CD = 0

Motional emf in AD = $\varepsilon_1 = B_0\ell v$

Magnetic field on rod BC B

$$= \left(B_0 - \frac{(-12\times10^{-2})}{10}\right)$$

Motional emf in BC = $\varepsilon_2 = \left(B_0 + \frac{12\times10^{-2}}{10}\right)\ell\times v$

$\varepsilon_{eq} = \varepsilon_2 - \varepsilon_1 = 300\times 10^{-7}$ V

For time variation

$(\varepsilon_{eq})' = A\,\frac{dB}{dt} = 60\times10^{-7}$ V

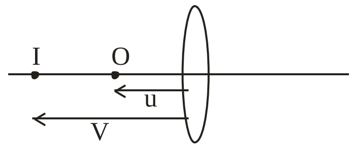
$(\varepsilon_{eq})_{net} = \varepsilon_{eq} + (\varepsilon_{eq})' = 360\times10^{-7}$ V

Power = $\frac{(\varepsilon_{eq})_{net}^2}{R} = 216\times10^{-9}$ W

53. The distance between object and its 3 times magnified virtual image as produced by a convex lens is 20 cm. The focal length of the lens used is __________ cm.

Ans. **(15)**

Sol.



$v = 3u$

$v - u = 20$ cm

$2u = 20$ cm

$u = 10$ cm

$$\frac{1}{(-30)} - \frac{1}{(-10)} = \frac{1}{f}$$

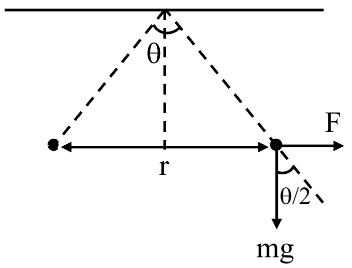
$f = 15$ cm

54. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle θ with each other. When suspended in water the angle remains the same. If density of the material of the sphere is 1.5 g/cc, the dielectric constant of water will be ________

(Take density of water = 1 g/cc)

Ans. **(3)**

Sol.



In air $\tan\frac{\theta}{2} = \frac{F}{mg} = \frac{q^2}{4\pi\varepsilon_0 r^2 mg}$

In water $\tan\frac{\theta}{2} = \frac{F'}{mg'} = \frac{q^2}{4\pi\varepsilon_0\varepsilon_r r^2 mg_{eff}}$

Equate both equations

$\varepsilon_0 g = \varepsilon_0\ \varepsilon_r\ g\left[1-\frac{1}{1.5}\right]$

$\varepsilon_r = 3$

55. The radius of a nucleus of mass number 64 is 4.8 fermi. Then the mass number of another nucleus having radius of 4 fermi is $\frac{1000}{x}$, where x is ______.

Ans. (27)

Sol. $R = R_0 A^{1/3}$

$R^3 \propto A$

$\left(\frac{4.8}{4}\right)^3 = \frac{64}{A}$

$= \frac{64}{A} = (1.2)^3$

$\frac{64}{A} = 1.44\times 1.2$

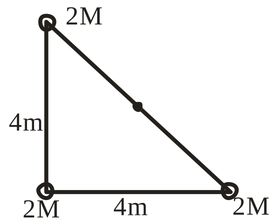
$A = \frac{64}{1.44\times 1.2} = \frac{1000}{x}$

$x = \frac{144\times 12}{64} = 27$

56. The identical spheres each of mass 2M are placed at the corners of a right angled triangle with mutually perpendicular sides equal to 4 m each. Taking point of intersection of these two sides as origin, the magnitude of position vector of the centre of mass of the system is $\frac{4\sqrt{2}}{x}$, where the value of x is ____________

Ans. (3)

Sol.



Position vector $\vec{r}_{COM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1 + m_2 + m_3}$

$\vec{r}_{COM} = \frac{2M\times 0 + 2M\times 4\hat{i} + 2M\times 4\hat{j}}{6M}$

$\vec{r} = \frac{4}{3}\hat{i} + \frac{4}{3}\hat{j}$

$|\vec{r}| = \frac{4\sqrt{2}}{3}$

$x = 3$

57. A tuning fork resonates with a sonometer wire of length 1 m stretched with a tension of 6 N. When the tension in the wire is changed to 54 N, the same tuning fork produces 12 beats per second with it. The frequency of the tuning fork is ________ Hz.

Ans. (6)

Sol. $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

$f_1 = \frac{1}{2}\sqrt{\frac{6}{\mu}}$ $f_2 = \frac{1}{2}\sqrt{\frac{54}{\mu}}$

$\frac{f_1}{f_2} = \frac{1}{3}$ $f_2 - f_1 = 12$

$f_1 = 6HZ$

58. A plane is in level flight at constant speed and each of its two wings has an area of 40 m^2. If the speed of the air is 180 km/h over the lower wing surface and 252 km/h over the upper wing surface, the mass of the plane is ________kg. (Take air density to be 1 kg m^{-3} and g = 10 ms^{-2})

Ans. **(9600)**

Sol. A = 80 m^2

Using Bernonlli equation

$$A(P_2 - P_1) = \frac{1}{2}\rho\left(V_1^2 - V_2^2\right)A$$

$$mg = \frac{1}{2} \times 1\ (70^2 - 50^2) \times 80$$

mg = 40 × 2400

m = 9600 kg

59. The current in a conductor is expressed as $I = 3t^2 + 4t^3$, where I is in Ampere and t is in second. The amount of electric charge that flows through a section of the conductor during t = 1s to t = 2s is ____________ C.

Ans. **(22)**

Sol. $q = \int_1^2 i\,dt = \int_1^2 (3t^2 + 4t^3)dt$

$$q = \left(t^3 + t^4\right)\Big|_1^2$$

q = 22C

60. A particle is moving in one dimension (along x axis) under the action of a variable force. It's initial position was 16 m right of origin. The variation of its position (x) with time (t) is given as $x = -3t^3 + 18t^2 + 16t$, where x is in m and t is in s. The velocity of the particle when its acceleration becomes zero is _________ m/s.

Ans. **(52)**

Sol. $x = 3t^3 + 18t^2 + 16t$

$\mathbf{v} = -9t^2 + 36 + 16$

$a = -18t + 36$

a = 0 at t = 2s

$v = -9(2)^2 + 36 \times 2 + 16$

v = 52 m/s

SECTION-A

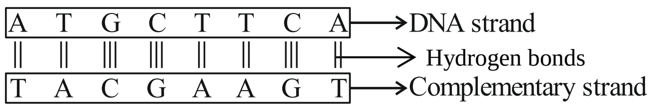
61. If one strand of a DNA has the sequence ATGCTTCA, sequence of the bases in complementary strand is:

(1) CATTAGCT (2) TACGAAGT

(3) GTACTTAC (4) ATGCGACT

Ans. (2)

Sol. Adenine base pairs with thymine with 2 hydrogen bonds and cytosine base pairs with guanine with 3 hydrogen bonds.



62. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Haloalkanes react with KCN to form alkyl cyanides as a main product while with AgCN form isocyanide as the main product.

Reason (R) : KCN and AgCN both are highly ionic compounds.

In the light of the above statement, choose the most appropriate answer from the options given below:

(1) (A) is correct but (R) is not correct

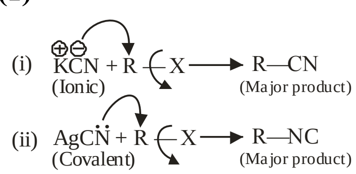
(2) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(3) (A) is not correct but (R) is correct

(4) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Ans. (1)

Sol.



AgCN is mainly covalent in nature and nitrogen is available for attack, so alkyl isocyanide is formed as main product.

63. In acidic medium, $K_2Cr_2O_7$ shows oxidising action as represented in the half reaction

$$Cr_2O_7^{2-} + XH^+ + Ye^- \rightarrow 2A + ZH_2O$$

X, Y, Z and A are respectively are:

(1) 8, 6, 4 and Cr_2O_3 (2) 14, 7, 6 and Cr^{3+}

(3) 8, 4, 6 and Cr_2O_3 (4) 14, 6, 7 and Cr^{3+}

Ans. (4)

Sol. The balanced reaction is,

$$Cr_2O_7^{2-} + 14H^+ + 6e^- \rightarrow 2Cr^{3+} + 7H_2O$$

X = 14

Y = 6

A = 7

64. Which of the following reactions are disproportionation reactions?

(A) $Cu^+ \rightarrow Cu^{2+} + Cu$

(B) $3MnO_4^{2-} + 4H^+ \rightarrow 2MnO_4^- + MnO_2 + 2H_2O$

(C) $2KMnO_4 \rightarrow K_2MnO_4 + MnO_2 + O_2$

(D) $2MnO_4^- + 3Mn^{2+} + 2H_2O \rightarrow 5MnO_2 + 4H^+$

Choose the correct answer from the options given below:

(1) (A), (B) (2) (B), (C), (D)

(3) (A), (B), (C) (4) (A), (D)

Ans. (1)

Sol. When a particular oxidation state becomes less stable relative to other oxidation state, one lower, one higher, it is said to undergo disproportionation.

$Cu^+ \rightarrow Cu^{2+} + Cu$

$3MnO_4^{2-} + 4H^+ \rightarrow 2MnO_4^- + MnO_2 + 2H_2O$

65. In case of isoelectronic species the size of F^-, Ne and Na^+ is affected by:

(1) Principal quantum number (n)

(2) None of the factors because their size is the same

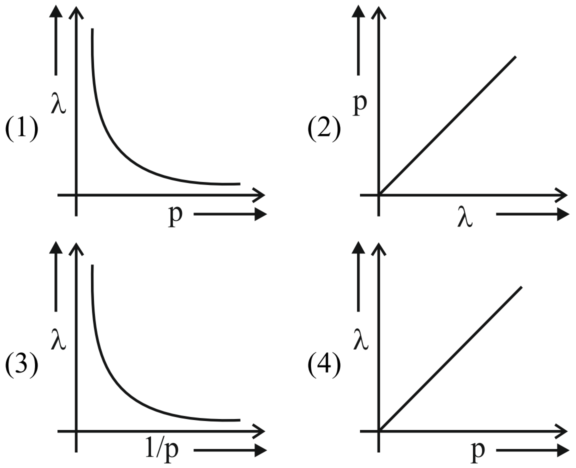
(3) Electron-electron interaction in the outer orbitals

(4) Nuclear charge (z)

Ans. (4)

Sol. In F^-, Ne, Na^+ all have $1s^2, 2s^2, 2p^6$ configuration. They have different size due to the difference in nuclear charge.

66. According to the wave-particle duality of matter by de-Broglie, which of the following graph plot presents most appropriate relationship between wavelength of electron (λ) and momentum of electron (p)?

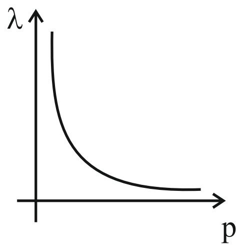


Ans. **(1)**

Sol. $\lambda = \frac{h}{p}\left[\lambda \propto \frac{1}{p}\right]$

$\Rightarrow \lambda p = h$ (constant)

So, the plot is a rectangular hyperbola.



67. Given below are two statements:

Statement (I): A solution of $[Ni(H_2O)_6]^{2+}$ is green in colour.

Statement (II): A solution of $[Ni(CN)_4]^{2-}$ is colourless.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both Statement I and Statement II are incorrect

(2) Both Statement I and Statement II are correct

(3) Statement I is incorrect but Statement II is correct

(4) Statement I is correct but Statement II is incorrect

Ans. **(2)**

Sol. $[Ni(H_2O)_6]^{+2} \rightarrow$ Green colour solution due to d-d transition.

$[Ni(CN)_4]^{-2} \rightarrow$ is diamagnetic and it is colourless.

68. Given below are two statements: one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : PH_3 has lower boiling point than NH_3.

Reason (R) : In liquid state NH_3 molecules are associated through vander waal's forces, but PH_3 molecules are associated through hydrogen bonding.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) Both (A) and (R) are correct and (R) is not the correct explanation of (A)

(2) (A) is not correct but (R) is correct

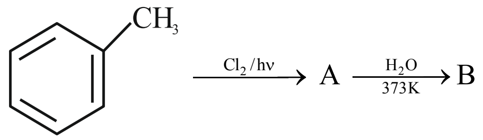
(3) Both (A) and (R) are correct but (R) is the correct explanation of (A)

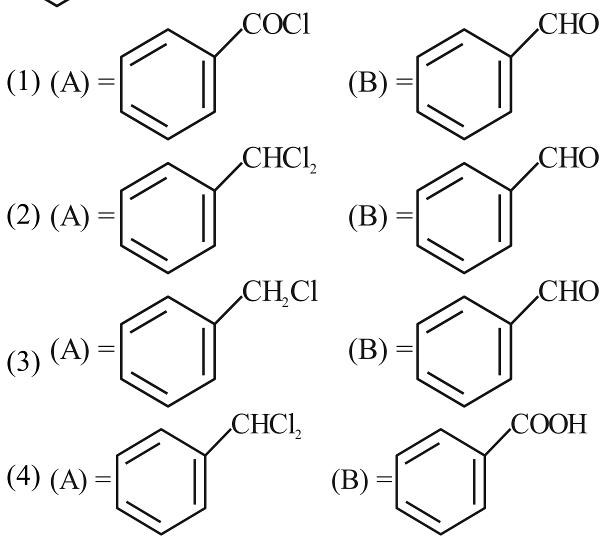
(4) (A) is correct but (R) is not correct

Ans. **(4)**

Sol. Unlike NH_3, PH_3 molecules are not associated through hydrogen bonding in liquid state. That is why the boiling point of PH_3 is lower than NH_3.

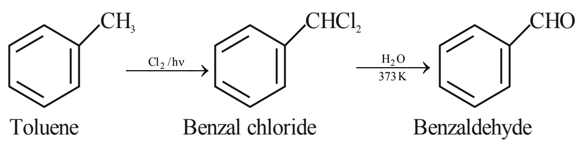
69. Identify A and B in the following sequence of reaction





Ans. **(2)**

Sol.



70. Given below are two statements:

Statement (I) : Aminobenzene and aniline are same organic compounds.

Statement (II) : Aminobenzene and aniline are different organic compounds.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) Both Statement I and Statement II are correct

(2) Statement I is correct but Statement II is incorrect

(3) Statement I is incorrect but Statement II is correct

(4) Both Statement I and Statement II are incorrect

Ans. (2)

Sol. Aniline is also known as amino benzene.

71. Which of the following complex is homoleptic?

(1) $[Ni(CN)_4]^{2-}$

(2) $[Ni(NH_3)_2Cl_2]$

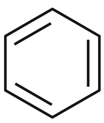
(3) $[Fe(NH_3)_4Cl_2]^+$

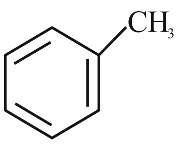
(4) $[Co(NH_3)_4Cl_2]^+$

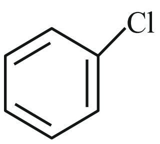
Ans. (1)

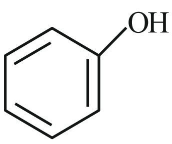
Sol. In Homoleptic complex all the ligand attached with the central atom should be the same. Hence $[Ni(CN)_4]^{2-}$ is a homoleptic complex.

72. Which of the following compound will most easily be attacked by an electrophile?

(1) 

(2) 

(3) 

(4) 

Ans. (4)

Sol. Higher the electron density in the benzene ring more easily it will be attacked by an electrophile. Phenol has the highest electron density amongst all the given compound.

73. Ionic reactions with organic compounds proceed through:

(A) Homolytic bond cleavage

(B) Heterolytic bond cleavage

(C) Free radical formation

(D) Primary free radical

(E) Secondary free radical

Choose the correct answer from the options given below:

(1) (A) only

(2) (C) only

(3) (B) only

(4) (D) and (E) only

Ans. (3)

Sol. Heterolytic cleavage of Bond lead to formation of ions.

74. Arrange the bonds in order of increasing ionic character in the molecules. LiF, K_2O, N_2, SO_2 and CIF_3.

(1) $CIF_3 < N_2 < SO_2 < K_2O < LiF$

(2) $LiF < K_2O < CIF_3 < SO_2 < N_2$

(3) $N_2 < SO_2 < CIF_3 < K_2O < LiF$

(4) $N_2 < CIF_3 < SO_2 < K_2O < LiF$

Ans. (3)

Sol. Increasing order of ionic character

$N_2 < SO_2 < ClF_3 < K_2O < LiF$

Ionic character depends upon difference of electronegativity (bond polarity).

75. We have three aqueous solutions of NaCl labelled as 'A', 'B' and 'C' with concentration 0.1 M, 0.01M & 0.001 M, respectively. The value of van t' Haft factor (i) for these solutions will be in the order.

(1) $i_A < i_B < i_C$

(2) $i_A < i_C < i_B$

(3) $i_A = i_B = i_C$

(4) $i_A > i_B > i_C$

Ans. (1)

Sol.

Salt	Values of i (for different conc. of a Salt)		
	0.1 M	0.01 M	0.001 M
NaCl	1.87	1.94	1.94

i approach 2 as the solution become very dilute.

76. In Kjeldahl's method for estimation of nitrogen, $CuSO_4$ acts as :

(1) Reducing agent (2) Catalytic agent

(3) Hydrolysis agent (4) Oxidising agent

Ans. (2)

Sol. Kjeldahl's method is used for estimation of Nitrogen where $CuSO_4$ acts as a catalyst.

77. Given below are two statements :

Statement (I) : Potassium hydrogen phthalate is a primary standard for standardisation of sodium hydroxide solution.

Statement (II) : In this titration phenolphthalein can be used as indicator.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) Both Statement I and Statement II are correct

(2) Statement I is correct but Statement II is incorrect

(3) Statement I is incorrect but Statement II is correct

(4) Both Statement I and Statement II are incorrect

Ans. (1)

Sol. **Statement (I) :** Potassium hydrogen phthalate is a primary standard for standardisation of sodium hydroxide solution as it is economical and its concentration does not changes with time.

Phenophthalin can acts as indicator in acid base titration as it shows colour in pH range 8.3 to 10.1

78. Match List – I with List –II.

	List – I (Reactions)		List – II (Reagents)
(A)	$CH_3(CH_2)_5-\underset{\underset{O}{\parallel}}{C}-OC_2H_5 \rightarrow CH_3(CH_2)_5CHO$	(I)	CH_3MgBr, H_2O
(B)	$C_6H_5COC_6H_5 \rightarrow C_6H_5CH_2C_6H_5$	(II)	Zn(Hg) and conc. HCl
(C)	$C_6H_5CHO \rightarrow C_6H_5CH(OH)CH_3$	(III)	$NaBH_4$, H^+
(D)	$CH_3COCH_2COOC_2H_5 \rightarrow CH_3\underset{\underset{H}{\mid}}{C}(OH)CH_2COOC_2H_5$	(IV)	DIBAL-H, H_2O

Choose the correct answer from options given below:

(1) A-(III), (B)-(IV), (C)-(I), (D)-(II)

(2) A-(IV), (B)-(II), (C)-(I), (D)-(III)

(3) A-(IV), (B)-(II), (C)-(III), (D)-(I)

(4) A-(III), (B)-(IV), (C)-(II), (D)-(I)

Ans. (2)

Sol. $CH_3(CH_2)_5COOC_2H_5 \xrightarrow{\text{DIBAL-H, } H_2O} CH_3(CH_2)_5CHO$

$C_6H_5COC_6H_5 \xrightarrow{\text{Zn(Hg) \& conc. HCl}} C_6H_5CH_2C_6H_5$

$C_6H_5CHO \xrightarrow[H_2O]{CH_3MgBr} C_6H_5CH(OH)CH_3$

$CH_3COCH_2COOC_2H_5 \xrightarrow{NaBH_4, H^+} CH_3CH(OH)CH_2COOC_2H_5$

79. Choose the correct option for free expansion of an ideal gas under adiabatic condition from the following :

(1) $q = 0, \Delta T \neq 0, w = 0$

(2) $q = 0, \Delta T < 0, w \neq 0$

(3) $q \neq 0, \Delta T = 0, w = 0$

(4) $q = 0, \Delta T = 0, w = 0$

Ans. (4)

Sol. During free expansion of an ideal gas under adiabatic condition $q = 0, \Delta T = 0, w = 0$.

80. Given below are two statements:

Statement (I) : The NH_2 group in Aniline is ortho and para directing and a powerful activating group.

Statement (II) : Aniline does not undergo Friedel-Craft's reaction (alkylation and acylation).

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both Statement I and Statement II are correct

(2) Both Statement I and Statement II are incorrect

(3) Statement I is incorrect but Statement II is correct

(4) Statement I is correct but Statement II is incorrect

Ans. (1)

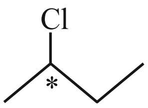
Sol. The NH_2 group in Aniline is ortho and para directing and a powerful activating group as NH_2 has strong +M effect.

Aniline does not undergo Friedel-Craft's reaction (alkylation and acylation) as Aniline will form complex with $AlCl_3$ which will deactivate the benzene ring.

SECTION-B

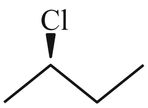
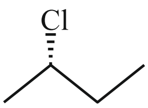
81. Number of optical isomers possible for 2 – chlorobutane ....

Ans. (2)

Sol. 

There is one chiral centre present in given compound.

So, Total optical isomers = 2

 and 

82. The potential for the given half cell at 298K is (-)........... × 10^{-2} V.

$2H^+_{(aq)} + 2e^- \rightarrow H_2\ (g)$

$[H^+] = 1M,\ P_{H_2} = 2$ atm

(Given: 2.303 RT/F = 0.06 V, log2 = 0.3)

Ans. (1)

Sol. $$E = E^o_{H^+/H_2} - \frac{0.06}{2}\log\frac{P_{H_2}}{[H^+]^2}$$

$$E = 0.00 - \frac{0.06}{2}\log\frac{2}{[1]^2}$$

$$E = -0.03 \times 0.3 = -0.9 \times 10^{-2}\ V$$

83. The number of white coloured salts among the following is ....

(A) $SrSO_4$ (B) $Mg(NH_4)PO_4$ (c) $BaCrO_4$

(D) $Mn(OH)_2$ (E) $PbSO_4$ (F) $PbCrO_4$

(G) AgBr (H) PbI_2 (I) CaC_2O_4

(J) $[Fe(OH)_2(CH_3COO)]$

Ans. (5)

Sol. $SrSO_4$ – white

$Mg(NH_4)PO_4$ – white

$BaCrO_4$ – yellow

$Mn(OH)_2$ – white

$PbSO_4$ – white

$PbCrO_4$ – yellow

AgBr – pale yellow

PbI_2 – yellow

CaC_2O_4 – white

$[Fe(OH)_2(CH_3COO)]$ – Brown Red

84. The ratio of $\frac{^{14}C}{^{12}C}$ in a piece of wood is $\frac{1}{8}$ part that of atmosphere. If half life of ^{14}C is 5730 years, the age of wood sample is ..... years.

Ans. (17190)

Sol. $$\lambda t = \ln\frac{(^{14}C/^{12}C)_{atmosphere}}{(^{14}C/^{12}C)_{wood\ sample}}$$

As per the question,

$$\frac{(^{14}C/^{12}C)_{wood}}{(^{14}C/^{12}C)_{atmosphere}} = \frac{1}{8}$$

So, $\lambda t = \ln 8$

$$\frac{\ln 2}{t_{1/2}} t = \ln 8$$

$t = 3 \times t_{1/2} = 17190$ years

85. The number of molecules/ion/s having trigonal bipyramidal shape is ....

PF_5, BrF_5, PCl_5, $[PtCl_4]^{2-}$, BF_3, $Fe(CO)_5$

Ans. (3)

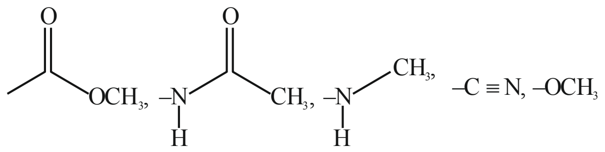
Sol. PF_5, PCl_5, $Fe(CO)_5$; Trigonal bipyramidal

BrF_5 ; square pyramidal

$[PtCl_4]^{-2}$; square planar

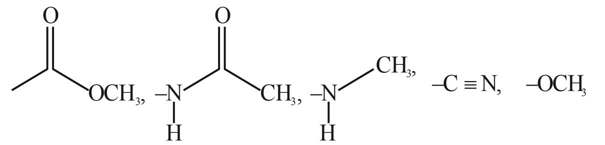
BF_3 ; Trigonal planar

86. Total number of deactivating groups in aromatic electrophilic substitution reaction among the following is



Ans. (2)

Sol.



(–M group) (+M group) (+M group) (–M group) (+M group)

87. Lowest Oxidation number of an atom in a compound A_2B is -2. The number of an electron in its valence shell is

Ans. (6)

Sol. $A_2B \rightarrow 2A^{+} + B^{-2}$, B^{-2} has complete octet in its di-anionic form, thus in its atomic state it has 6 electrons in its valence shell. As it has negative charge, it has acquired two electrons to complete its octet.

88. Among the following oxide of p - block elements, number of oxides having amphoteric nature is

Cl_2O_7, CO, PbO_2, N_2O, NO, Al_2O_3, SiO_2, N_2O_5, SnO_2

Ans. (3)

Sol. Acidic oxide: Cl_2O_7, SiO_2, N_2O_5

Neutral oxide: CO, NO, N_2O

Amphoteric oxide: Al_2O_3 , SnO_2 , PbO_2

89. Consider the following reaction:

$3PbCl_2 + 2(NH_4)_3PO_4 \rightarrow Pb_3(PO_4)_2 + 6NH_4Cl$

If 72 mmol of $PbCl_2$ is mixed with 50 mmol of $(NH_4)_3PO_4$, then amount of $Pb_3(PO_4)_2$ formed is …… mmol. (nearest integer)

Ans. (24)

Sol. Limiting Reagent is $PbCl_2$

mmol of $Pb_3(PO_4)_2$ formed

$$= \frac{\text{mmol of } PbCl_2 \text{ reacted}}{3}$$

= 24 mmol

90. K_a for CH_3COOH is 1.8×10^{-5} and K_b for NH_4OH is 1.8×10^{-5}. The pH of ammonium acetate solution will be

Ans. (7)

Sol. $pH = \dfrac{pK_w + pK_a - pK_b}{2}$

$pK_a = pK_b$

$\Rightarrow pH = \dfrac{pK_w}{2} = 7$

FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

(Held On Thursday 01st February, 2024) **TIME : 3 : 00 PM to 06 : 00 PM**

MATHEMATICS	TEST PAPER WITH SOLUTION

SECTION-A

1. Let $f(x) = |2x^2+5|x|-3|, x\in R$. If m and n denote the number of points where f is not continuous and not differentiable respectively, then m + n is equal to :

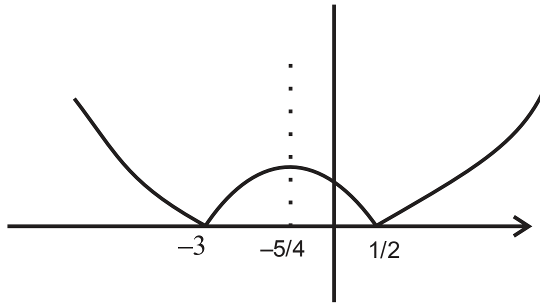
(1) 5 (2) 2

(3) 0 (4) 3

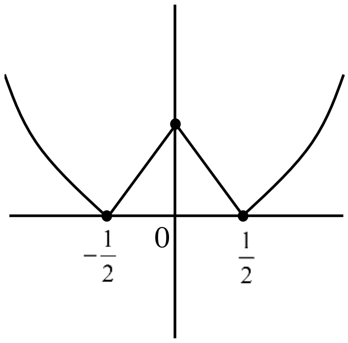
Ans. (4)

Sol. $f(x) = |2x^2+5|x|-3|$

Graph of $y = |2x^2 + 5x - 3|$



Graph of f(x)



Number of points of discontinuity = 0 = m

Number of points of non-differentiability = 3 = n

2. Let α and β be the roots of the equation $px^2 + qx - r = 0$, where $p \neq 0$. If p, q and r be the consecutive terms of a non-constant G.P and $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{3}{4}$, then the value of $(\alpha-\beta)^2$ is :

(1) $\frac{80}{9}$ (2) 9

(3) $\frac{20}{3}$ (4) 8

Ans. (1)

Sol. $px^2 + qx - r = 0 \quad <^{\alpha}_{\beta}$

$p = A,\ q = AR,\ r = AR^2$

$Ax^2 + ARx - AR^2 = 0$

$x^2 + Rx - R^2 = 0 \quad <^{\alpha}_{\beta}$

$\because \frac{1}{\alpha}+\frac{1}{\beta}=\frac{3}{4}$

$\therefore \frac{\alpha+\beta}{\alpha\beta}=\frac{3}{4} \Rightarrow \frac{-R}{-R^2}=\frac{3}{4} \Rightarrow R=\frac{4}{3}$

$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta = R^2 - 4(-R^2) = 5\left(\frac{16}{9}\right)$

$= 80/9$

3. The number of solutions of the equation $4\sin^2x - 4\cos^3x + 9 - 4\cos x = 0$; $x \in [-2\pi, 2\pi]$ is :

(1) 1

(2) 3

(3) 2

(4) 0

Ans. (4)

Sol. $4\sin^2x - 4\cos^3x + 9 - 4\cos x = 0$; $x \in [-2\pi, 2\pi]$

$4 - 4\cos^2x - 4\cos^3x + 9 - 4\cos x = 0$

$4\cos^3x + 4\cos^2x + 4\cos x - 13 = 0$

$4\cos^3x + 4\cos^2x + 4\cos x = 13$

L.H.S. ≤ 12 can't be equal to 13.

4. The value of $\int_0^1 (2x^3 - 3x^2 - x + 1)^{\frac{1}{3}}\,dx$ is equal to:

(1) 0

(2) 1

(3) 2

(4) –1

Ans. (1)

Sol. $I = \int_0^1 \left(2x^3 - 3x^2 - x + 1\right)^{\frac{1}{3}} dx$

Using $\int_0^{2a} f(x)dx$ where $f(2a-x) = -f(x)$

Here $f(1-x) = f(x)$

$\therefore\ I = 0$

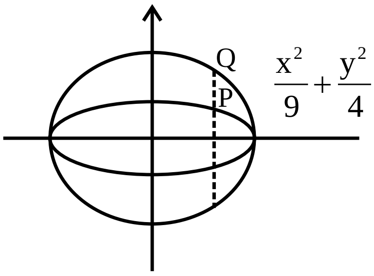
5. Let P be a point on the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$. Let the line passing through P and parallel to y-axis meet the circle $x^2 + y^2 = 9$ at point Q such that P and Q are on the same side of the x-axis. Then, the eccentricity of the locus of the point R on PQ such that PR : RQ = 4 : 3 as P moves on the ellipse, is :

(1) $\frac{11}{19}$ (2) $\frac{13}{21}$

(3) $\frac{\sqrt{139}}{23}$ (4) $\frac{\sqrt{13}}{7}$

Ans. (4)

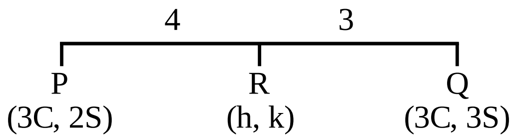
Sol.



$\frac{x^2}{9}+\frac{y^2}{4}=1$

P(3cosθ, 2sinθ)

Q(3cosθ, 3sinθ)



$h = 3\cos\theta$;

$k=\frac{18}{7}\sin\theta$

$\therefore$ locus $=\frac{x^2}{9}+\frac{49y^2}{324}=1$

$e=\sqrt{1-\frac{324}{49\times 9}}=\frac{\sqrt{117}}{21}=\frac{\sqrt{13}}{7}$

6. Let m and n be the coefficients of seventh and thirteenth terms respectively in the expansion of $\left(\frac{1}{3}x^{\frac{1}{3}}+\frac{1}{2x^{\frac{2}{3}}}\right)^{18}$. Then $\left(\frac{n}{m}\right)^{\frac{1}{3}}$ is :

(1) $\frac{4}{9}$ (2) $\frac{1}{9}$

(3) $\frac{1}{4}$ (4) $\frac{9}{4}$

Ans. (4)

Sol. $\left(\frac{x^{\frac{1}{3}}}{3}+\frac{x^{\frac{-2}{3}}}{\ }\right)^{18}$

$t_7={}^{18}C_6\left(\frac{x^{\frac{1}{3}}}{3}\right)^{12}\left(\frac{x^{\frac{-2}{3}}}{2}\right)^{6}={}^{18}C_6\frac{1}{(3)^{12}}\cdot\frac{1}{2^6}$

$t_{13}={}^{18}C_{12}\left(\frac{x^{\frac{1}{3}}}{3}\right)^{6}\left(\frac{x^{\frac{-2}{3}}}{2}\right)^{12}={}^{18}C_{12}\frac{1}{(3)^{6}}\cdot\frac{1}{2^{12}}.x^{-6}$

$m={}^{18}C_6.3^{-12}.2^{-6}$: $n={}^{18}C_{12}.2^{-12}.3^{-6}$

$\left(\frac{n}{m}\right)^{\frac{1}{3}}=\left(\frac{2^{-12}.3^{-6}}{3^{-12}.2^{-6}}\right)^{\frac{1}{3}}=\left(\frac{3}{2}\right)^{2}=\frac{9}{4}$

7. Let α be a non-zero real number. Suppose f : R → R is a differentiable function such that $f(0) = 2$ and $\lim\limits_{x\to-\infty}f(x)=1$. If $f'(x) = \alpha f(x) + 3$, for all $x \in$ R, then $f(-\log_e 2)$ is equal to____.

(1) 3 (2) 5

(3) 9 (4) 7

Ans. (3 OR BONUS)

Sol. $f(0) = 2$, $\lim\limits_{x\to-\infty}f(x)=1$

$f'(x) - x.f(x) = 3$

I.F $= e^{-\alpha x}$

$y(e^{-\alpha x}) = \int 3.e^{-\alpha x}dx$

$f(x).(e^{-\alpha x})=\frac{3e^{-\alpha x}}{-\alpha}+c$

$x=0 \Rightarrow 2=\frac{-3}{\alpha}+c \Rightarrow \frac{3}{\alpha}=c-2$ (1)

$f(x)=\frac{-3}{\alpha}+c.e^{\alpha x}$

$x\to-\infty \Rightarrow 1=\frac{-3}{\alpha}+c(0)$

$\alpha = -3 \therefore c = 1$

$f(-\ln 2)=\frac{-3}{\alpha}+c.e^{\alpha x}$

$= 1 + e^{3\ln 2} = 9$

(But α should be greater than 0 for finite value of c)

8. Let P and Q be the points on the line $\frac{x+3}{8}=\frac{y-4}{2}=\frac{z+1}{2}$ which are at a distance of 6 units from the point R (1,2,3). If the centroid of the triangle PQR is (α, β, γ), then $\alpha^2+\beta^2+\gamma^2$ is:

(1) 26

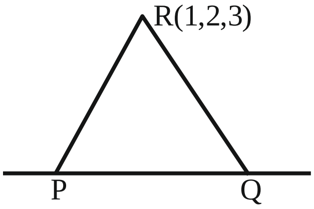
(2) 36

(3) 18

(4) 24

Ans. **(3)**

Sol.



$P(8\lambda - 3, 2\lambda + 4, 2\lambda - 1)$

$PR = 6$

$(8\lambda - 4)^2 + (2\lambda + 2)^2 + (2\lambda - 4)^2 = 36$

$\lambda = 0, 1$

Hence $P(-3, 4, -1)$ & $Q(5, 6, 1)$

Centroid of $\Delta PQR = (1, 4, 1) \equiv (\alpha, \beta, \gamma)$

$\alpha^2 + \beta^2 + \gamma^2 = 18$

9. Consider a ΔABC where A(1,2,3,), B(–2,8,0) and C(3,6,7). If the angle bisector of $\angle BAC$ meets the line BC at D, then the length of the projection of the vector $\vec{AD}$ on the vector $\vec{AC}$ is:

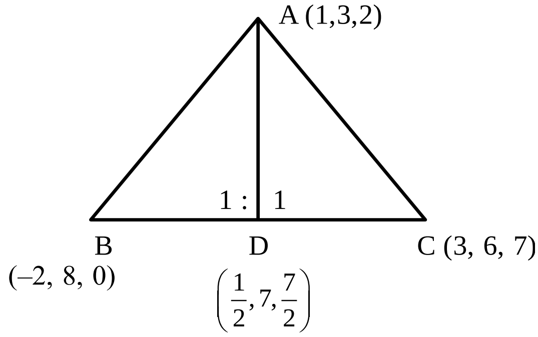
(1) $\frac{37}{2\sqrt{38}}$

(2) $\frac{\sqrt{38}}{2}$

(3) $\frac{39}{2\sqrt{38}}$

(4) $\sqrt{19}$

Ans. **(1)**



Sol.

A(1, 3, 2); B(–2, 8, 0); C(3, 6, 7);

$\overrightarrow{AC} = 2\hat{i} + 3\hat{j} + 5\hat{k}$

$AB = \sqrt{9+25+4} = \sqrt{38}$

$AC = \sqrt{4+9-25} = \sqrt{38}$

$\overrightarrow{AD} = \frac{1}{2}\hat{i} - 4\hat{j} - \frac{3}{2}\hat{k} = \frac{1}{2}(\hat{i} - 8\hat{j} - 3\hat{k})$

Length of projection of $\overrightarrow{AD}$ on $\overrightarrow{AC}$

$= \left|\frac{\overrightarrow{AD}.\overrightarrow{AC}}{|\overrightarrow{AC}|}\right| = \frac{37}{2\sqrt{38}}$

10. Let S_n denote the sum of the first n terms of an arithmetic progression. If $S_{10} = 390$ and the ratio of the tenth and the fifth terms is 15 : 7, then $S_{15} - S_5$ is equal to:

(1) 800

(2) 890

(3) 790

(4) 690

Ans. **(3)**

Sol. $S_{10} = 390$

$\frac{10}{2}\left[2a + (10-1)d\right] = 390$

$\Rightarrow 2a + 9d = 78$ (1)

$\frac{t_{10}}{t_5} = \frac{15}{7} \Rightarrow \frac{a+9d}{a+4d} = \frac{15}{7} \Rightarrow 8a = 3d$ (2)

From (1) & (2) $a = 3$ & $d = 8$

$S_{15} - S_5 = \frac{15}{2}(6 + 14 \times 8) - \frac{5}{2}(6 + 4 \times 8)$

$= \frac{15 \times 118 - 5 \times 38}{2} = 790$

11. If $\int_0^{\frac{\pi}{3}} \cos^4 x\, dx = a\pi + b\sqrt{3}$, where a and b are rational numbers, then 9a + 8b is equal to :

(1) 2 (2) 1

(3) 3 (4) $\frac{3}{2}$

Ans. (1)

Sol. $\int_0^{\pi/3} \cos^4 x dx$

$$= \int_0^{\pi/3} \left(\frac{1+\cos 2x}{2}\right)^2 dx$$

$$= \frac{1}{4}\int_0^{\pi/3} (1 + 2\cos 2x + \cos^2 2x) dx$$

$$= \frac{1}{4}\left[\int_0^{\pi/3} dx + 2\int_0^{\pi/3} \cos 2x\, dx + \int_0^{\pi/3} \frac{1+\cos 4x}{2} dx\right]$$

$$= \frac{1}{4}\left[\frac{\pi}{3} + (\sin 2x)_0^{\pi/3} + \frac{1}{2}\left(\frac{\pi}{3}\right) + \frac{1}{8}(\sin 4x)_0^{\pi/3}\right]$$

$$= \frac{1}{4}\left[\frac{\pi}{3} + (\sin 2x)_0^{\pi/3} + \frac{1}{2}\left(\frac{\pi}{3}\right) + \frac{1}{8}(\sin 4x)_0^{\pi/3}\right]$$

$$= \frac{1}{4}\left[\frac{\pi}{2} + \frac{\sqrt{3}}{2} + \frac{1}{8} \times \left(-\frac{\sqrt{3}}{2}\right)\right]$$

$$= \frac{\pi}{2} + \frac{7\sqrt{3}}{64}$$

$$\therefore a = \frac{1}{8};\ b = \frac{7}{64}$$

$$\therefore 9a + 8b = \frac{9}{8} + \frac{7}{8} = 2$$

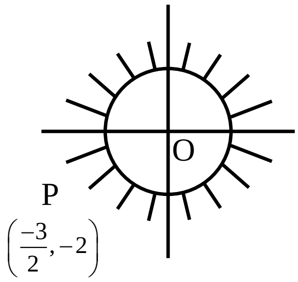
12. If z is a complex number such that $|z| \ge 1$, then the minimum value of $\left|z + \frac{1}{2}(3 + 4i)\right|$ is:

(1) $\frac{5}{2}$ (2) 2

(3) 3 (4) $\frac{3}{2}$

Ans. (Bonus)

Sol. $|z| \ge 1$



Min. value of $\left|z + \frac{3}{2} + 2i\right|$ is actually zero.

13. If the domain of the function $f(x) = \frac{\sqrt{x^2 - 25}}{(4 - x^2)}$ $+ \log_{10}(x^2 + 2x - 15)$ is $(-\infty, \alpha) \cup [\beta, \infty)$, then $\alpha^2 + \beta^3$ is equal to :

(1) 140 (2) 175

(3) 150 (4) 125

Ans. (3)

Sol. $f(x) = \frac{\sqrt{x^2 - 25}}{4 - x^2} + \log_{10}(x^2 + 2x - 15)$

Domain : $x^2 - 25 \ge 0 \Rightarrow x \in (-\infty, -5] \cup [5, \infty)$

$4 - x^2 \ne 0 \Rightarrow x \ne \{-2, 2\}$

$x^2 + 2x - 15 > 0 \Rightarrow (x + 5)(x - 3) > 0$

$\Rightarrow x \in (-\infty, -5) \cup (3, \infty)$

$\therefore x \in (-\infty, -5) \cup [5, \infty)$

$\alpha = -5;\ \beta = 5$

$\therefore \alpha^2 + \beta^3 = 150$

14. Consider the relations R_1 and R_2 defined as $aR_1b \Leftrightarrow a^2 + b^2 = 1$ for all a , b, $\in$ R and (a, b) R_2(c, d) $\Leftrightarrow a + d = b + c$ for all (a,b), (c,d) $\in N \times N$. Then

(1) Only R_1 is an equivalence relation

(2) Only R_2 is an equivalence relation

(3) R_1 and R_2 both are equivalence relations

(4) Neither R_1 nor R_2 is an equivalence relation

Ans. (2)

Sol. $aR_1 b \Leftrightarrow a^2 + b^2 = 1;\ a, b \in R$

(a, b) R_2 (c, d) $\Leftrightarrow a + d = b + c$; (a, b), (c, d) $\in$ N

for R_1 : Not reflexive symmetric not transitive

for R_2 : R_2 is reflexive, symmetric and transitive

Hence only R_2 is equivalence relation.

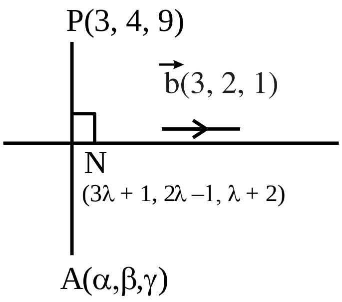
15. If the mirror image of the point P(3,4,9) in the line $\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-2}{1}$ is (α, β,γ), then 14 (α + β + γ) is :

(1) 102 (2) 138

(3) 108 (4) 132

Ans. (3)

Sol.



$\overrightarrow{PN}.\vec{b}=0\,?$

3(3 λ – 2) + 2 (2 λ – 5) + (λ – 7) = 0

$14\lambda = 23 \Rightarrow \lambda = \frac{23}{14}$

$N\left(\frac{83}{14},\frac{32}{14},\frac{51}{14}\right)$

$\therefore\ \frac{\alpha+3}{2}=\frac{83}{14}\Rightarrow \alpha=\frac{62}{7}$

$\frac{\beta+4}{2}=\frac{32}{14}\Rightarrow \beta=\frac{4}{7}$

$\frac{\gamma+9}{2}=\frac{51}{14}\Rightarrow \gamma=\frac{-12}{7}$

Ans. 14 (α + β + r) = 108

16. Let $f(x)=\begin{cases} x-1, & x \text{ is even,} \\ 2x, & x \text{ is odd,}\end{cases}$ $x \in N$. If for some $a \in N$, $f(f(f(a)))=21$, then $\lim\limits_{x\to a^-}\left\{\frac{|x|^3}{a}-\left[\frac{x}{a}\right]\right\}$, where [t] denotes the greatest integer less than or equal to t, is equal to :

(1) 121

(2) 144

(3) 169

(4) 225

Ans. (2)

Sol. $f(x)=\begin{cases} x-1; & x=\text{even} \\ 2x; & x=\text{odd}\end{cases}$

$f(f(f(a)))=21$

C–1: If a = even

$f(a) = a - 1 = \text{odd}$

$f(f(a)) = 2(a - 1) = \text{even}$

$f(f(f(a))) = 2a - 3 = 21 \Rightarrow a = 12$

C–2: If a = odd

$f(a) = 2a = \text{even}$

$f(f(a)) = 2a - 1 = \text{odd}$

$f(f(f(a))) = 4a - 2 = 21$ (Not possible)

Hence a = 12

Now

$$\lim_{x\to 12^-}\left(\frac{|x|^3}{2}-\left[\frac{x}{12}\right]\right)$$

$$=\lim_{x\to 12^-}\frac{|x|^3}{12}-\lim_{x\to 12^-}\left[\frac{x}{12}\right]$$

= 144 – 0 = 144.

17. Let the system of equations x + 2y +3z = 5, 2x + 3y + z = 9, 4x + 3y + λz = μ have infinite number of solutions. Then λ + 2μ is equal to :

(1) 28 (2) 17

(3) 22 (4) 15

Ans. (2)

Sol. x + 2y + 3z = 5

2x + 3y + z = 9

4x + 3y + λz = μ

for infinite following $\Delta=\Delta_1=\Delta_2=\Delta_3=0$

$$\Delta=\begin{vmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 4 & 3 & \lambda\end{vmatrix}=0\Rightarrow \lambda=-13$$

$$\Delta_1=\begin{vmatrix}5 & 2 & 3\\ 9 & 3 & 1\\ \mu & 3 & -13\end{vmatrix}=0\Rightarrow \mu=15$$

$$\Delta_2=\begin{vmatrix}1 & 5 & 3\\ 2 & 9 & 1\\ 4 & 15 & -13\end{vmatrix}=0$$

$$\Delta_3 = \begin{vmatrix} 1 & 2 & 5 \\ 2 & 3 & 9 \\ 4 & 3 & 15 \end{vmatrix} = 0$$

for $\lambda = -13$, $\mu = 15$ system of equation has infinite solution hence $\lambda + 2\mu = 17$

18. Consider 10 observation x_1, $x_{2,\ldots,}$ x_{10}. such that $\sum_{i=1}^{10} (x_i - \alpha) = 2$ and $\sum_{i=1}^{10} (x_i - \beta)^2 = 40$, where α, β are positive integers. Let the mean and the variance of the observations be $\frac{6}{5}$ and $\frac{84}{25}$ respectively. The $\frac{\beta}{\alpha}$ is equal to :

(1) 2 (2) $\frac{3}{2}$

(3) $\frac{5}{2}$ (4) 1

Ans. **(1)**

Sol. $x_1, x_2 \ldots\ldots x_{10}$

$$\sum_{i=1}^{10}(x_i - \alpha) = 2 \Rightarrow \sum_{i=1}^{10} x_i - 10\alpha = 2$$

$$\text{Mean } \mu = \frac{6}{5} = \frac{\sum x_i}{10}$$

$\therefore$ $\Sigma x_i = 12$

$10\alpha + 2 = 12 \ \therefore \ \alpha = 1$

Now $\sum_{i=1}^{10}(x_i - \beta)^2 = 40$ Let $y_i = x_i - \beta$

$$\therefore \sigma_y^2 = \frac{1}{10}\sum y_i^2 - (\bar{y})^2$$

$$\sigma_x^2 = \frac{1}{10}\sum (x_i - \beta)^2 - \left(\frac{\sum_{i=1}^{10}(x_i - \beta)}{10}\right)^2$$

$$\frac{84}{25} = 4 - \left(\frac{12 - 10\beta}{10}\right)^2$$

$$\therefore \left(\frac{6 - 5\beta}{5}\right)^2 = 4 - \frac{84}{25} = \frac{16}{25}$$

$6 - 5\beta = \pm 4 \Rightarrow \beta = \frac{2}{5}$ (not possible) or $\beta = 2$

Hence $\frac{\beta}{\alpha} = 2$

19. Let Ajay will not appear in JEE exam with probability $p = \frac{2}{7}$, while both Ajay and Vijay will appear in the exam with probability $q = \frac{1}{5}$. Then the probability, that Ajay will appear in the exam and Vijay will not appear is :

(1) $\frac{9}{35}$

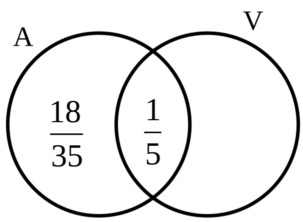
(2) $\frac{18}{35}$

(3) $\frac{24}{35}$

(4) $\frac{3}{35}$

Ans. (2)

Sol.



$$P(\bar{A}) = \frac{2}{7} = p$$

$$P(A \cap V) = \frac{1}{5} = q$$

$$P(A) = \frac{5}{7}$$

Ans. $P(A \cap \bar{V}) = \frac{18}{35}$

20. Let the locus of the mid points of the chords of circle $x^2+(y-1)^2 = 1$ drawn from the origin intersect the line $x+y = 1$ at P and Q. Then, the length of PQ is :

(1) $\frac{1}{\sqrt{2}}$

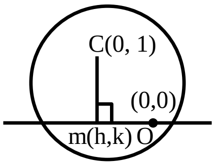
(2) $\sqrt{2}$

(3) $\frac{1}{2}$

(4) 1

Ans. **(1)**

Sol.

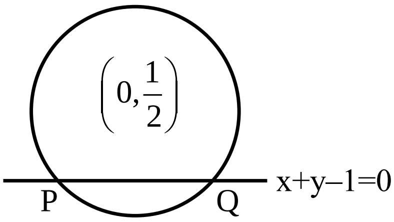


$m_{OM} \cdot m_{CM} = -1$

$\frac{k}{h} \cdot \frac{k-1}{h} = -1$

$\therefore$ locus is $x^2 + y(y-1) = 0$

$x^2 + y^2 - y = 0$



$p = \left|\frac{1/2}{\sqrt{2}}\right| \qquad p = \frac{1}{2\sqrt{2}}$

$PQ = 2\sqrt{r^2 - p^2}$

$= 2\sqrt{\frac{1}{4} - \frac{1}{8}} = \frac{1}{\sqrt{2}}$

SECTION-B

21. If three successive terms of a G.P. with common ratio r(r > 1) are the lengths of the sides of a triangle and [r] denotes the greatest integer less than or equal to r, then 3[r] + [–r] is equal to :

Ans. (1)

Sol. a, ar, $ar^2 \rightarrow$ G.P.

Sum of any two sides > third side

$a + ar > ar^2, \; a + ar^2 > ar, \; ar + ar^2 > a$

$r^2 - r - 1 < 0$

$r \in \left(\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right)$ (1)

$r^2 - r + 1 > 0$

always true

$r^2 + r - 1 > 0$

$r \in \left(-\infty, \frac{-1-\sqrt{5}}{2}\right) \cup \left(\frac{-1+\sqrt{5}}{2}, \infty\right)$ (2)

Taking intersection of (1), (2)

$r \in \left(\frac{-1+\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right)$

As $r > 1$

$r \in \left(1, \frac{1+\sqrt{5}}{2}\right)$

$[r] = 1 \quad [-r] = -2$

$3[r] + [-r] = 1$

22. Let $A = I_2 - MM^T$, where M is real matrix of order 2×1 such that the relation $M^T M = I_1$ holds. If λ is a real number such that the relation $AX = \lambda X$ holds for some non-zero real matrix X of order 2×1, then the sum of squares of all possible values of λ is equal to :

Ans. (2)

Sol. $A = I_2 - 2MM^T$

$A^2 = (I_2 - 2MM^T)(I_2 - 2MM^T)$

$= I_2 - 2MM^T - 2MM^T + 4MM^TMM^T$

$= I_2 - 4MM^T + 4MM^T$

$= I_2$

$AX = \lambda X$

$A^2X = \lambda AX$

$X = \lambda(\lambda X)$

$X = \lambda^2 X$

$X(\lambda^2 - 1) = 0$

$\lambda^2 = 1$

$\lambda = \pm 1$

Sum of square of all possible values = 2

23. Let f : (0, ∞) → R and $F(x)=\int_0^x tf(t)dt$. If $F(x^2)$ = $x^4 + x^5$, then $\sum_{r=1}^{12} f(r^2)$ is equal to :

Ans. **(219)**

Sol. $F(x)=\int_0^x t\cdot f(t)dt$

$F^1(x) = xf(x)$

Given $F(x^2) = x^4 + x^5$, let $x^2 = t$

$F(t) = t^2 + t^{5/2}$

$F'(t) = 2t + 5/2\ t^{3/2}$

$t\cdot f(t) = 2t + 5/2\ t^{3/2}$

$f(t) = 2 + 5/2\ r^{1/2}$

$\sum_{r=1}^{12} f(r^2)=\sum_{r=1}^{12} 2+\frac{5}{2}r$

$= 24 + 5/2\left[\frac{12(13)}{2}\right]$

$=219$

24. If $y=\frac{\left(\sqrt{x}+1\right)\left(x^2-\sqrt{x}\right)}{x\sqrt{x}+x+\sqrt{x}}+\frac{1}{15}(3\cos^2 x-5)\cos^3 x$, then $96y'\left(\frac{\pi}{6}\right)$ is equal to :

Ans. (105)

Sol. $y=\frac{\left(\sqrt{x}+1\right)\left(x^2-\sqrt{x}\right)}{x\sqrt{x}+x+\sqrt{x}}+\frac{1}{15}(3\cos^2x-5)\cos^3x$

$y=\frac{\left(\sqrt{x}+1\right)\left(\sqrt{x}\right)\left(\left(\sqrt{x}\right)^3-1\right)}{\left(\sqrt{x}\right)\left(\left(\sqrt{x}\right)^2+\left(\sqrt{x}\right)+1\right)}+\frac{1}{5}\cos^5x-\frac{1}{3}\cos^3x$

$y=\left(\sqrt{x}+1\right)\ \left(\sqrt{x}-1\right)+\frac{1}{5}\ \cos^5x-\frac{1}{3}\cos^3x$

$y' = 1 - \cos^4x\cdot (\sin x) + \cos^2x\ (\sin x)$

$y'\left(\frac{\pi}{6}\right)=1-\frac{9}{16}\times\frac{1}{2}+\frac{3}{4}\times\frac{1}{2}$

$=\frac{32-9+12}{32}\ =\frac{35}{32}$

$= 96\ y'\left(\frac{\pi}{6}\right)\ = 105$

25. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=-\hat{i}-8\hat{j}+2\hat{k}$ and $\vec{c}=4\hat{i}+c_2\hat{j}+c_3\hat{k}$ be three vectors such that $\vec{b}\times\vec{a}=\vec{c}\times\vec{a}$. If the angle between the vector $\vec{c}$ and the vector $3\hat{i}+4\hat{j}+\hat{k}$ is θ, then the greatest integer less than or equal to $\tan^2\theta$ is :

Ans. **(38)**

Sol. $\vec{a}=\hat{i}+\hat{j}+k$

$\vec{b}=\hat{i}+8\hat{j}+2k$

$\vec{c}=4\hat{i}+c_2\hat{j}+c_3k$

$\vec{b}\times\vec{a}=\vec{c}\times\vec{a}$

$\left(\vec{b}-\vec{c}\right)\times\vec{a}=0$

$\vec{b}-\vec{c}=\lambda\vec{\alpha}$

$\vec{b}=\vec{c}+\lambda\vec{\alpha}$

$-\hat{i}-8\hat{j}+2k=\left(4\hat{i}+c_2\hat{j}+c_3k\right)+\lambda\left(\hat{i}+\hat{j}+k\right)$

$\lambda + 4 = -1 \Rightarrow \lambda = -5$

$\lambda + c_2 = -8 \Rightarrow c_2 = -3$

$\lambda + c_3 = 2 \Rightarrow c_3 = 7$

$\vec{c}=4\hat{i}-3\hat{j}+7k$

$\cos\theta=\frac{12-12+7}{\sqrt{26}\cdot\sqrt{74}}=\frac{7}{\sqrt{26}\cdot\sqrt{74}}=\frac{7}{2\sqrt{481}}$

$\tan^2\theta=\frac{625\times 3}{49}$

$[\tan^2\theta] = 38$

26. The lines $L_1, L_2, \ldots, l_{20}$ are distinct. For $n = 1, 2, 3, \ldots, 10$ all the lines L_{2n-1} are parallel to each other and all the lines L_{2n} pass through a given point P. The maximum number of points of intersection of pairs of lines from the set $\{L_1, L_2, \ldots, L_{20}\}$ is equal to :

Ans. (101)

Sol. $L_1, L_3, L_5, - - L_{19}$ are Parallel

$L_2, L_4, L_6, - - L_{20}$ are Concurrent

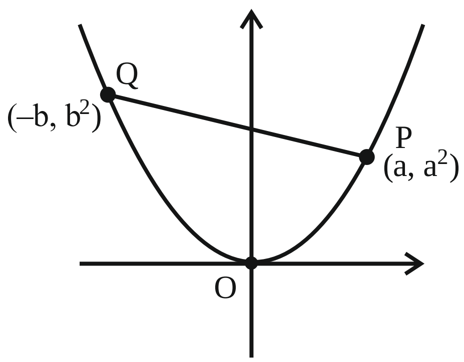
Total points of intersection $= {}^{20}C_2 - {}^{10}C_2 - {}^{10}C_2 + 1$

$= 101$

27. Three points O(0,0), P(a, a^2), Q(–b, b^2), a > 0, b > 0, are on the parabola $y = x^2$. Let S_1 be the area of the region bounded by the line PQ and the parabola, and S_2 be the area of the triangle OPQ. If the minimum value of $\frac{S_1}{S_2}$ is $\frac{m}{n}$, gcd(m, n) = 1, then m + n is equal to :

Ans. (7)

Sol.



$$S_2 = 1/2 \left| \begin{vmatrix} 0 & 0 & 1 \\ a & a^2 & 1 \\ -b & b^2 & 1 \end{vmatrix} \right| = 1/2\left(ab^2 + a^2b\right)$$

PQ:- $y - a^2 = \frac{a^2 - b^2}{a+b}(x-a)$

$y - a^2 = (a - b)x - (a - b)a$

$y = (a - b)x + ab$

$$S_1 = \int_{-b}^{a} \left((a-b)x + ab - x^2\right)dx$$

$$= (a-b)\frac{x^2}{2} + (ab)x - \frac{x^3}{3}\Big|_{-b}^{a}$$

$$= \frac{(a-b)^2(a+b)}{2} + ab(a+b) - \frac{(a^3+b^3)}{3}$$

$$\frac{S_1}{S_2} = \frac{\frac{(a-b)^2}{2} + ab - \frac{(a^2+b^2-ab)}{3}}{\frac{ab}{2}}$$

$$= \frac{3(a-b)^2 + 6ab - 2(a^2+b^2-ab)}{3ab}$$

$$= \frac{1}{3}\left[\underbrace{\frac{a}{b} + \frac{b}{a}}_{\min=2} + 2\right]$$

$$= \frac{4}{3} = \frac{m}{n} \qquad m + n = 7$$

28. The sum of squares of all possible values of k, for which area of the region bounded by the parabolas $2y^2 = kx$ and $ky^2 = 2(y - x)$ is maximum, is equal to :

Ans. (8)

Sol. $ky^2 = 2(y - x)$ $\quad\quad 2y^2 = kx$

Point of intersection →

$$ky^2 = \left(y - \frac{2y^2}{k}\right)$$

$y = 0$ $\quad\quad ky = 2\left(1 - \frac{2y}{k}\right)$

$$ky + \frac{4y}{k} = 2$$

$$y = \frac{2}{k + \frac{4}{k}} = \frac{2k}{k^2 + 4}$$

$$A = \int_0^{\frac{2k}{k^2+4}} \left(\left(y - \frac{ky^2}{2}\right) - \left(\frac{2y^2}{k}\right)\right).dy$$

$$= \frac{y^2}{2} - \left(\frac{k}{2} + \frac{2}{k}\right)\cdot\frac{y^3}{3}\Bigg|_0^{\frac{2k}{k^2+4}}$$

$$= \left(\frac{2k}{k^2+4}\right)^2\left[\frac{1}{2} - \frac{k^2+4}{2k}\times\frac{1}{3}\times\frac{2k}{k^2+4}\right]$$

$$= \frac{1}{6}\times 4 \times \left(\frac{1}{k + \frac{4}{k}}\right)^2$$

$$A\cdot M \geq G\cdot M \quad \frac{\left(k + \frac{4}{k}\right)}{2} \geq 2$$

$$k + \frac{4}{k} \geq 4$$

Area is maximum when $k = \frac{4}{k}$

$k = 2, -2$

29. If $\frac{dx}{dy} = \frac{1 + x - y^2}{y}$, $x(1) = 1$, then $5x(2)$ is equal to :

Ans. **(5)**

Sol. $\frac{dx}{dy} - \frac{x}{y} = \frac{1 - y^2}{y}$

Integrating factor $= e^{\int -\frac{1}{y}dy} = \frac{1}{y}$

$$x \cdot \frac{1}{y} = \int \frac{1 - y^2}{y^2} dy$$

$$\frac{x}{y} = \frac{-1}{y} - y + c$$

$x = -1 - y^2 + cy$

$x(1) = 1$

$1 = -1 - 1 + c \Rightarrow c = 3$

$x = -1 - y^2 + 3y$

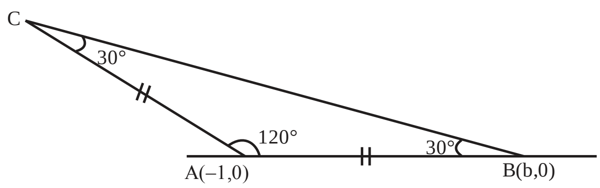
$5x(2) = 5(-1 - 4 + 6)$

$= 5$

30. Let ABC be an isosceles triangle in which A is at $(-1, 0)$, $\angle A = \frac{2\pi}{3}$, AB = AC and B is on the positive x-axis. If $BC = 4\sqrt{3}$ and the line BC intersects the line $y = x + 3$ at (α, β), then $\frac{\beta^4}{\alpha^2}$ is :

Ans. **(36)**

Sol.



$$\frac{c}{\sin 30°} = \frac{4\sqrt{3}}{\sin 120°} \text{ [By sine rule]}$$

$2c = 8 \Rightarrow c = 4$

$AB = |(b+1)| = 4$

$b = 3,\ m_{AB} = 0$

$m_{BC} = \frac{-1}{\sqrt{3}}$

BC:- $y = \frac{-1}{\sqrt{3}}(x-3)$

$\sqrt{3}y + x = 3$

Point of intersection : $y = x + 3,\ \sqrt{3}y + x = 3$

$$\left(\sqrt{3}+1\right)y = 6$$

$$y = \frac{6}{\sqrt{3}+1}$$

$$x = \frac{6}{\sqrt{3}+1} - 3$$

$$= \frac{6 - 3\sqrt{3} - 3}{\sqrt{3}+1}$$

$$= 3\frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)} = \frac{-6}{\left(1+\sqrt{3}\right)^2}$$

$$\frac{\beta^4}{\alpha^2} = 36$$

SECTION-A

31. In an ammeter, 5% of the main current passes through the galvanometer. If resistance of the galvanometer is G, the resistance of ammeter will be :

(1) $\frac{G}{200}$

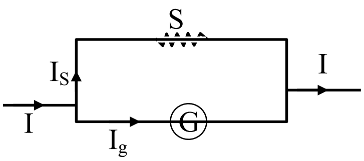
(2) $\frac{G}{199}$

(3) 199 G

(4) 200 G

Ans. (Bonus)

Sol.



$I_S S = I_g G$

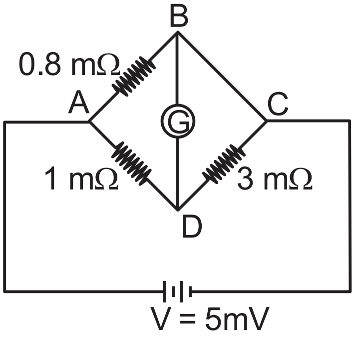
$$\frac{95}{100} IS = \frac{5I}{100} G$$

$$S = \frac{G}{19}$$

$$R_A = \frac{SG}{S+G} = \frac{\frac{G^2}{19}}{\frac{20G}{19}}$$

$$R_A = \frac{G}{20}$$

32. To measure the temperature coefficient of resistivity α of a semiconductor, an electrical arrangement shown in the figure is prepared. The arm BC is made up of the semiconductor. The experiment is being conducted at 25°C and resistance of the semiconductor arm is 3 mΩ. Arm BC is cooled at a constant rate of 2°C/s. If the galvanometer G shows no deflection after 10s, then α is :



(1) $-2 \times 10^{-2}\,°C^{-1}$

(2) $-1.5 \times 10^{-2}\,°C^{-1}$

(3) $-1 \times 10^{-2}\,°C^{-1}$

(4) $-2.5 \times 10^{-2}\,°C^{-1}$

Ans. (3)

Sol. For no deflection $\frac{0.8}{1} = \frac{R}{3}$

$\Rightarrow R = 2.4 m\Omega$

Temperature fall in 10s = 20°C

$\Delta R = R\,\alpha\,\Delta t$

$$\alpha = \frac{\Delta R}{R\Delta t} = \frac{-0.6}{3 \times 20}$$

$= -10^{-2} C^{-1}$

33. From the statements given below :

(A) The angular momentum of an electron in n^{th} orbit is an integral multiple of h.

(B) Nuclear forces do not obey inverse square law.

(C) Nuclear forces are spin dependent.

(D) Nuclear forces are central and charge independent.

(E) Stability of nucleus is inversely proportional to the value of packing fraction.

Choose the correct answer from the options given below :

(1) (A), (B), (C), (D) only

(2) (A), (C), (D), (E) only

(3) (A), (B), (C), (E) only

(4) (B), (C), (D), (E) only

Ans. **(3)**

Sol. Part of theory

34. A diatomic gas (γ = 1.4) does 200 J of work when it is expanded isobarically. The heat given to the gas in the process is :

(1) 850 J (2) 800 J

(3) 600 J (4) 700 J

Ans. **(4)**

Sol. $\gamma = 1 + \frac{2}{f} = 1.4 \Rightarrow \frac{2}{f} = 0.4$

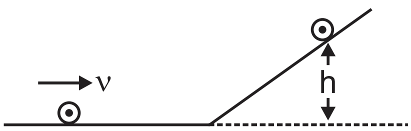
$\Rightarrow f = 5$

$W = n R \Delta T = 200J$

$Q = \left(\frac{f+2}{2}\right) nR\Delta T$

$= \frac{7}{2} \times 200 \quad = 700 \text{ J}$

35. A disc of radius R and mass M is rolling horizontally without slipping with speed v. It then moves up an inclined smooth surface as shown in figure. The maximum height that the disc can go up the incline is :



(1) $\frac{v^2}{g}$ (2) $\frac{3}{4}\frac{v^2}{g}$

(3) $\frac{1}{2}\frac{v^2}{g}$ (4) $\frac{2}{3}\frac{v^2}{g}$

Ans. **(3)**

Sol. Only the translational kinetic energy of disc changes into gravitational potential energy. And rotational KE remains unchanged as there is no friction.

$\frac{1}{2}mv^2 = mgh$

$h = \frac{v^2}{2g}$

36. Conductivity of a photodiode starts changing only if the wavelength of incident light is less than 660 nm. The band gap of photodiode is found to be $\left(\frac{X}{8}\right)$eV . The value of X is :

(Given, $h = 6.6 \times 10^{-34}$ Js, $e = 1.6 \times 10^{-19}$C)

(1) 15 (2) 11

(3) 13 (4) 21

Ans. **(1)**

Sol. $E_g = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{660 \times 10^{-9}} J$

$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{660 \times 10^{-9} \times 1.6 \times 10^{-19}} eV$

$= \frac{15}{8} eV$

So x = 15

37. A big drop is formed by coalescing 1000 small droplets of water. The surface energy will become :

(1) 100 times (2) 10 times

(3) $\frac{1}{100}$th (4) $\frac{1}{10}$th

Ans. **(4)**

Sol. Lets say radius of small droplets is r and that of big drop is R

$\frac{4}{3}\pi R^3 = 1000\frac{4}{3}\pi r^3$

R = 10r

$U_i = 1000\,(4\pi r^2 S)$

$U_f = 4\pi R^2 S$

$= 100\,(4\pi r^2 S)$

$U_f = \frac{1}{10} U_i$

38. If frequency of electromagnetic wave is 60 MHz and it travels in air along z direction then the corresponding electric and magnetic field vectors will be mutually perpendicular to each other and the wavelength of the wave (in m) is :

(1) 2.5 (2) 10

(3) 5 (4) 2

Ans. **(3)**

Sol. $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{60 \times 10^6} = 5\text{m}$

39. A cricket player catches a ball of mass 120 g moving with 25 m/s speed. If the catching process is completed in 0.1 s then the magnitude of force exerted by the ball on the hand of player will be (in SI unit):

(1) 24 (2) 12

(3) 25 (4) 30

Ans. **(4)**

Sol. $F_{av} = \frac{\Delta p}{\Delta t}$

$= \frac{0.12 \times 25}{0.1} = 30\text{N}$

40. Monochromatic light of frequency 6×10^{14} Hz is produced by a laser. The power emitted is 2×10^{-3} W. How many photons per second on an average, are emitted by the source ?

(Given $h = 6.63 \times 10^{-34}$ Js)

(1) 9×10^{18} (2) 6×10^{15}

(3) 5×10^{15} (4) 7×10^{16}

Ans. **(3)**

Sol. $P = nh\nu$

$n = \frac{P}{h\nu} = \frac{2 \times 10^{-3}}{6.63 \times 10^{-34} \times 6 \times 10^{14}}$

$= 5 \times 10^{15}$

41. A microwave of wavelength 2.0 cm falls normally on a slit of width 4.0 cm. The angular spread of the central maxima of the diffraction pattern obtained on a screen 1.5 m away from the slit, will be:

(1) 30° (2) 15°

(3) 60° (4) 45°

Ans. **(3)**

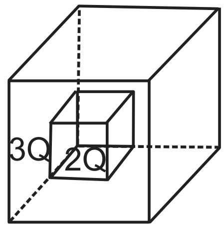
Sol. For first minima $a \sin\theta = \lambda$

$\sin\theta = \frac{\lambda}{a} = \frac{1}{2}$

$\theta = 30°$

Angular spread = 60°

42. C_1 and C_2 are two hollow concentric cubes enclosing charges 2Q and 3Q respectively as shown in figure. The ratio of electric flux passing through C_1 and C_2 is :



(1) 2 : 5 (2) 5 : 2

(3) 2 : 3 (4) 3 : 2

Ans. **(1)**

Sol. $\phi_{\text{smaller cube}} = \frac{2Q}{\epsilon_0}$

$\phi_{\text{bigger cube}} = \frac{5Q}{\epsilon_0}$

$\frac{\phi_{\text{smaller cube}}}{\phi_{\text{bigger cube}}} = \frac{2}{5}$

43. If the root mean square velocity of hydrogen molecule at a given temperature and pressure is 2 km/s, the root mean square velocity of oxygen at the same condition in km/s is :

(1) 2.0 (2) 0.5

(3) 1.5 (4) 1.0

Ans. **(2)**

Sol. $V_{rms} = \sqrt{\frac{3RT}{M}}$

$\frac{V_1}{V_2} = \sqrt{\frac{M_2}{M_1}} \Rightarrow \frac{2}{V_2} = \sqrt{\frac{32}{2}}$

$V_2 = 0.5$ km/s

44. Train A is moving along two parallel rail tracks towards north with speed 72 km/h and train B is moving towards south with speed 108 km/h. Velocity of train B with respect to A and velocity of ground with respect to B are (in ms^{-1}) :

(1) –30 and 50

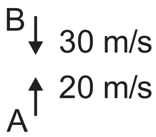
(2) –50 and –30

(3) –50 and 30

(4) 50 and –30

Ans. **(3)**

Sol.



$V_A = 20$ m/s

$V_B = -30$ m/s

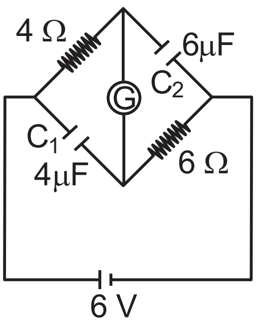
Velocity of B w.r.t. A

$V_{B/A} = -50$ m/s

Velocity of ground w.r.t. B

$V_{G/B} = 30$ m/s

45. A galvanometer (G) of 2Ω resistance is connected in the given circuit. The ratio of charge stored in C_1 and C_2 is :



(1) $\frac{2}{3}$

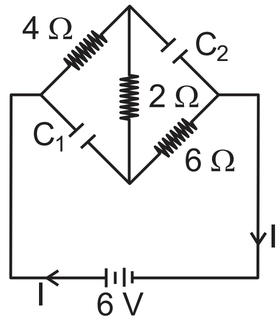
(2) $\frac{3}{2}$

(3) 1

(4) $\frac{1}{2}$

Ans. **(4)**

Sol.



In steady state

Req = 12Ω

$I = \frac{6}{12} = 0.5A$

P.D across $C_1 = 3V$

P.D acoross $C_2 = 4V$

$q_1 = C_1V_1 = 12\ \mu C$

$q_2 = C_2V_2 = 24\ \mu C$

$\frac{q_1}{q_2} = \frac{1}{2}$

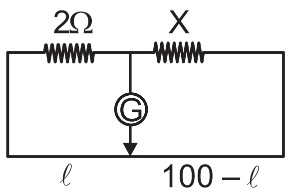
46. In a metre-bridge when a resistance in the left gap is 2Ω and unknown resistance in the right gap, the balance length is found to be 40 cm. On shunting the unknown resistance with 2Ω, the balance length changes by :

(1) 22.5 cm (2) 20 cm

(3) 62.5 cm (4) 65 cm

Ans. **(1)**

Sol.



First case $\frac{2}{40} = \frac{X}{60} \Rightarrow X = 3\Omega$

In second case $X' = \frac{2 \times 3}{2+3} = 1.2\Omega$

$\frac{2}{\ell} = \frac{1.2}{100-\ell}$

$200 - 2\ell = 1.2\ell$

$\ell = \frac{200}{3.2} = 62.5\text{cm}$

Balance length changes by 22.5 cm

47. Match List - I with List - II.

List - I (Number)	**List - II (Significant figure)**
(A) 1001	(I) 3
(B) 010.1	(II) 4
(C) 100.100	(III) 5
(D) 0.0010010	(IV) 6

Choose the correct answer from the options given below :

(1) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)
(2) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)
(3) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)
(4) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

Ans. **(3)**

Sol. Theoretical

48. A transformer has an efficiency of 80% and works at 10 V and 4 kW. If the secondary voltage is 240 V, then the current in the secondary coil is :

(1) 1.59 A (2) 13.33 A
(3) 1.33 A (4) 15.1 A

Ans. **(2)**

Sol. Efficiency $= \frac{E_S I_S}{E_P I_P}$

$$0.8 = \frac{240 I_S}{4000}$$

$$I_S = \frac{3200}{240} = 13.33A$$

49. A light planet is revolving around a massive star in a circular orbit of radius R with a period of revolution T. If the force of attraction between planet and star is proportional to $R^{-3/2}$ then choose the correct option :

(1) $T^2 \propto R^{5/2}$ (2) $T^2 \propto R^{7/2}$
(3) $T^2 \propto R^{3/2}$ (4) $T^2 \propto R^3$

Ans. **(1)**

Sol. $F = \frac{GMm}{R^{3/2}} = m\omega^2 R$

$\omega^2 \propto \frac{1}{R^{5/2}}$ $\because T = \frac{2\pi}{\omega}$ so

$T^2 \propto R^{5/2}$

50. A body of mass 4 kg experiences two forces $\vec{F}_1 = 5\hat{i} + 8\hat{j} + 7\hat{k}$ and $\vec{F}_2 = 3\hat{i} - 4\hat{j} - 3\hat{k}$. The acceleration acting on the body is :

(1) $-2\hat{i} - \hat{j} - \hat{k}$

(2) $4\hat{i} + 2\hat{j} + 2\hat{k}$

(3) $2\hat{i} + \hat{j} + \hat{k}$

(4) $2\hat{i} + 3\hat{j} + 3\hat{k}$

Ans. **(3)**

Sol. Net force $= 8\hat{i} + 4\hat{j} + 4\hat{k}$

$$\vec{a} = \frac{\vec{F}}{m} = 2\hat{i} + \hat{j} + \hat{k}$$

SECTION-B

51. A mass m is suspended from a spring of negligible mass and the system oscillates with a frequency f_1. The frequency of oscillations if a mass 9 m is suspended from the same spring is f_2. The value of $\frac{f_1}{f_2}$ is ____.

Ans. **(3)**

Sol. $f_1 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$f_2 = \frac{1}{2\pi}\sqrt{\frac{k}{9m}}$

$\frac{f_1}{f_2} = \sqrt{\frac{9}{1}} = \frac{3}{1}$

52. A particle initially at rest starts moving from reference point. x = 0 along x-axis, with velocity v that varies as $v = 4\sqrt{x}\,m/s$. The acceleration of the particle is ____ms^{-2}.

Ans. **(8)**

Sol. $V = 4\sqrt{x}$

$a = V\frac{dv}{dx}$

$= 4\sqrt{x} \times 4 \times \frac{1}{2} x^{-1/2} = 8\ m/s^2$

53. A moving coil galvanometer has 100 turns and each turn has an area of 2.0 cm^2. The magnetic field produced by the magnet is 0.01 T and the deflection in the coil is 0.05 radian when a current of 10 mA is passed through it. The torsional constant of the suspension wire is $x \times 10^{-5}$ N-m/rad. The value of x is___.

Ans. **(4)**

Sol. $\tau = BINA\sin\phi$

$C\theta = BINA\sin 90°$

$$C = \frac{BINA}{\theta} = \frac{0.01 \times 10 \times 10^{-3} \times 100 \times 2 \times 10^{-4}}{0.05}$$

$= 4 \times 10^{-5}$ N-m/rad.

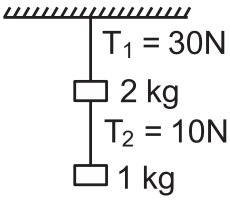
$x = 4$

54. One end of a metal wire is fixed to a ceiling and a load of 2 kg hangs from the other end. A similar wire is attached to the bottom of the load and another load of 1 kg hangs from this lower wire. Then the ratio of longitudinal strain of upper wire to that of the lower wire will be___.

[Area of cross section of wire = 0.005 cm^2, $Y = 2 \times 10^{11}$ Nm^{-2} and $g = 10$ ms^{-2}]

Ans. **(3)**

Sol.



$$\Delta L = \frac{FL}{AY}$$

$$\frac{\Delta L}{L} = \frac{F}{AY}$$

$$\frac{\frac{\Delta L_1}{L_1}}{\frac{\Delta L_2}{L_2}} = \frac{F_1}{F_2} = \frac{30}{10} = 3$$

55. A particular hydrogen - like ion emits the radiation of frequency 3×10^{15} Hz when it makes transition from n = 2 to n = 1. The frequency of radiation emitted in transition from n = 3 to n = 1 is $\frac{x}{9} \times 10^{15}$ Hz, when x = ______.

Ans. **(32)**

Sol. $E = -13.6z^2\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$

$$E = C\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$h\nu = C\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

$$\frac{\nu_1}{\nu_2} = \frac{\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]_{2-1}}{\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]_{3-1}}$$

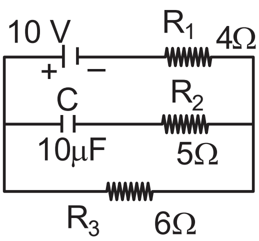
$$= \frac{\left[\frac{1}{1} - \frac{1}{4}\right]}{\left[\frac{1}{1} - \frac{1}{9}\right]} = \frac{3/4}{8/9}$$

$$= \frac{3}{4} \times \frac{9}{8}$$

$$\frac{\nu_1}{\nu_2} = \frac{27}{32}$$

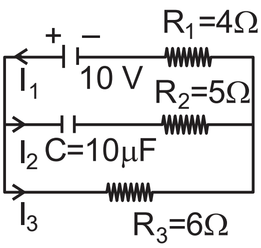
$$\nu_2 = \frac{32}{27}\nu_1 = \frac{32}{27} \times 3 \times 10^{15}\,Hz = \frac{32}{9} \times 10^{15}\,Hz$$

56. In an electrical circuit drawn below the amount of charge stored in the capacitor is ___μC.



Ans. **(60)**

Sol.



In steady state there will be no current in branch of capacitor, so no voltage drop across $R_2 = 5\Omega$

$I_2 = 0$

$$I_1 = I_3 = \frac{10}{4+6} = 1A$$

$$V_{R_3} = V_c + V_{R_2} \qquad V_{R_2} = 0$$

$I_3R_3 = V_c$

$V_c = 1 \times 6 = 6$ volt

$q_c = CV_c = 10 \times 6 = 60\ \mu C$

57. A coil of 200 turns and area 0.20 m^2 is rotated at half a revolution per second and is placed in uniform magnetic field of 0.01 T perpendicular to axis of rotation of the coil. The maximum voltage generated in the coil is $\frac{2\pi}{\beta}$ volt. The value of β is__.

Ans. **(5)**

Sol. $\phi = NAB\cos(\omega t)$

$$\varepsilon = -\frac{d\phi}{dt} = NAB\omega\sin(\omega t)$$

$\varepsilon_{max} = NAB\omega$

$= 200 \times 0.2 \times 0.01 \times \pi$

$$= \frac{4\pi}{10} = \frac{2\pi}{5}\text{volt}$$

58. In Young's double slit experiment, monochromatic light of wavelength 5000 Å is used. The slits are 1.0 mm apart and screen is placed at 1.0 m away from slits. The distance from the centre of the screen where intensity becomes half of the maximum intensity for the first time is___$\times 10^{-6}$ m.

Ans. **(125)**

Sol. Let intensity of light on screen due to each slit is I_0

So internity at centre of screen is $4I_0$

Intensity at distance y from centre-

$$I = I_0 + I_0 + 2\sqrt{I_0 I_0}\cos\phi$$

$I_{max} = 4I_0$

$$\frac{I_{max}}{2} = 2I_0 = 2I_0 + 2I_0\cos\phi$$

$\cos\phi = 0$

$$\phi = \frac{\pi}{2}$$

$$K\Delta x = \frac{\pi}{2}$$

$$\frac{2\pi}{\lambda} d\sin\theta = \frac{\pi}{2}$$

$$\frac{2}{\lambda} d \times \frac{y}{D} = \frac{1}{2}$$

$$y = \frac{\lambda D}{4d} = \frac{5\times10^{-7}\times1}{4\times10^{-3}}$$

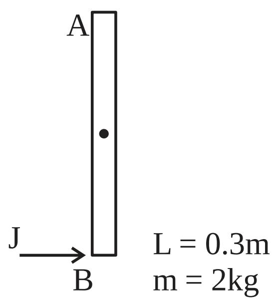
$= 125 \times 10^{-6}$

$= 125$

59. A uniform rod AB of mass 2 kg and Length 30 cm at rest on a smooth horizontal surface. An impulse of force 0.2 Ns is applied to end B. The time taken by the rod to turn through at right angles will be $\frac{\pi}{x}$s, where x = ____.

Ans. **(4)**

Sol.



Impulse J = 0.2 N-S

$$J = \int F dt = 0.2 N - s$$

Angular impuls $(\vec{M})$

$$\vec{M}_c = \int \tau dt$$

$$= \int F \frac{L}{2} dt$$

$$= \frac{L}{2} \int F dt = \frac{L}{2} \times J$$

$$= \frac{0.3}{2} \times 0.2$$

= 0.03

$$I_{cm} = \frac{ML^2}{12} = \frac{2 \times (0.3)^2}{12} = \frac{0.09}{6}$$

$$M = I_{cm}(\omega_f - \omega_i)$$

$$0.03 = \frac{0.09}{6}(\omega_f)$$

$\omega_f = 2$ rad/s

$\theta = \omega t$

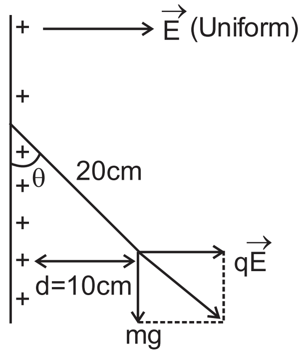
$$t = \frac{\theta}{\omega} = \frac{\pi}{2 \times 2} = \frac{\pi}{4} \text{sec}.$$

X = 4

60. Suppose a uniformly charged wall provides a uniform electric field of 2×10^4 N/C normally. A charged particle of mass 2 g being suspended through a silk thread of length 20 cm and remain stayed at a distance of 10 cm from the wall. Then the charge on the particle will be $\frac{1}{\sqrt{x}} \mu C$ where x =______. [use g = 10 m/s^2]

Ans. **(3)**

Sol.



$$\sin\theta = \frac{10}{20} = \frac{1}{2}$$

$\theta = 30°$

$$\tan\theta = \frac{qE}{mg}$$

$$\tan 30° = \frac{q \times 2 \times 10^4}{1 \times 10^{-3} \times 10}$$

$$\frac{1}{\sqrt{3}} = q \times 10^6$$

$$q = \frac{1}{\sqrt{3}} \times 10^{-6} C$$

x = 3

SECTION-A

61. The transition metal having highest 3rd ionisation enthalpy is :

(1) Cr (2) Mn

(3) V (4) Fe

Ans. **(2)**

Sol. 3rd Ionisation energy : [NCERT Data]

V : 2833 KJ/mol

Cr : 2990 KJ/mol

Mn : 3260 KJ/mol

Fe : 2962 KJ/mol

alternative

Mn : $3d^5\ 4s^2$

Fe : $3d^6\ 4s^2$

Cr : $3d^5\ 4s^1$

V : $3d^3\ 4s^2$

So Mn has highest 3rd IE among all the given elements due to d^5 configuration.

62. Given below are two statements :

Statement (I) : A π bonding MO has lower electron density above and below the inter-nuclear asix.

Statement (II) : The π* antibonding MO has a node between the nuclei.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both Statement I and Statement II are false

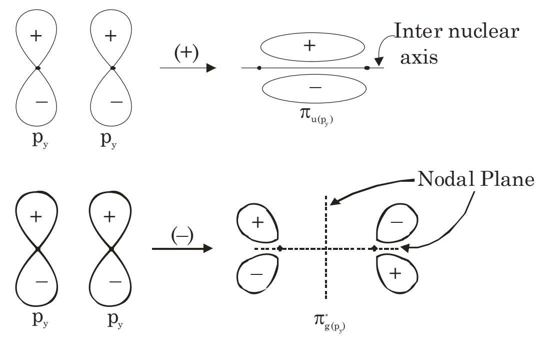
(2) Both Statement I and Statement II are true

(3) Statement I is false but Statement II is true

(4) Statement I is true but Statement II is false

Ans. **(3)**

Sol. A π bonding molecular orbital has higher electron density above and below inter nuclear axis



63. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : In aqueous solutions Cr^{2+} is reducing while Mn^{3+} is oxidising in nature.

Reason (R) : Extra stability to half filled electronic configuration is observed than incompletely filled electronic configuration.

In the light of the above statement, choose the most appropriate answer from the options given below:

(1) Both (A) and (R) are true and (R) is the correct explanation of (A)

(2) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(3) (A) is false but (R) is true

(4) (A) is true but (R) is false

Ans. **(1)**

Sol. Cr^{2+} is reducing as it configuration changes from d^4 to d^3 due to formation of Cr^{3+}, which has half filled t_{2g} level, on other hand, the change Mn^{3+} to Mn^{2+} result half filled d^5 configuration which has extra stability.

64. Match List - I with List - II.

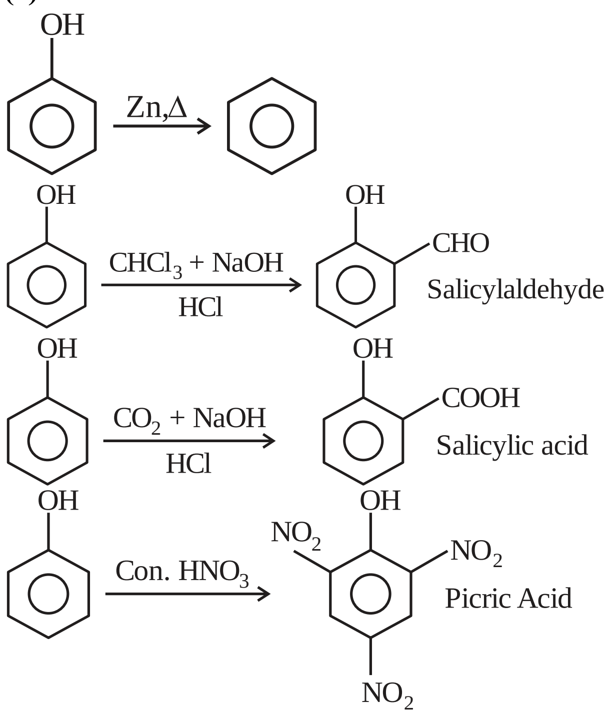
List-I (Reactants)	List-II Products
(A) Phenol, Zn/Δ	(I) Salicylaldehyde
(B) Phenol, $CHCl_3$, NaOH, HCl	(II) Salicylic acid
(C) Phenol, CO_2, NaOH, HCl	(III) Benzene
(D) Phenol, Conc. HNO_3	(IV) Picric acid

Choose the correct answer from the options given below.

(1) (A)-(IV), (B), (II), (C)-(I), (D)-(III)
(2) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)
(3) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)
(4) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

Ans. (3)

Sol.



65. Given below are two statements :

Statement (I) : Both metal and non-metal exist in p and d-block elements.

Statement (II) : Non-metals have higher ionisation enthalpy and higher electronegativity than the metals.

In the light of the above statements, choose the most appropriate answer from the option given below:

(1) Both Statement I and Statement II are false
(2) Statement I is false but Statement II is true
(3) Statement I is true but Statement II is false
(4) Both Statement I and Statement II are true

Ans. (2)

Sol. **I.** In p-Block both metals and non metals are present but in d-Block only metals are present.

II. EN and IE of non metals are greater than that of metals

I - False, **II-**True

66. The strongest reducing agent amont the following is:

(1) NH_3 (2) SbH_3
(3) BiH_3 (4) PH_3

Ans. (3)

Sol. Strongest reducing agent : BiH_3 explained by its low bond dissociation energy.

67. Which of the following compounds show colour due to d-d transition?

(1) $CuSO_4.5H_2O$ (2) $K_2Cr_2O_7$
(3) K_2CrO_4 (4) $KMnO_4$

Ans. (1)

Sol. $CuSO_4.5H_2O$

Cu^{2+} : $3d^9\ 4s^0$

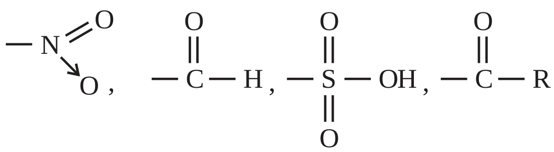
unpaired electron present so it show colour due to d-d transition.

68. The set of meta directing functional groups from the following sets is:

(1) –CN, $–NH_2$, –NHR, $–OCH_3$
(2) $–NO_2$, $–NH_2$, –COOH, –COOR
(3) $–NO_2$, –CHO, $–SO_3H$, –COR
(4) –CN, –CHO, $–NHCOCH_3$, –COOR

Ans. (3)

Sol.



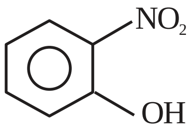
All are –M, Hence meta directing groups.

69. Select the compound from the following that will show intramolecular hydrogen bonding.

(1) H_2O

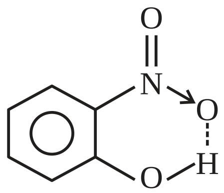
(2) NH_3

(3) C_2H_5OH

(4) 

Ans. **(4)**

Sol. H_2O, NH_3, $C_2H_5OH \Rightarrow$ Intermolecular H-Bonding



$\Rightarrow$ Intramolecular H-Bonding

70. Lassaigne's test is used for detection of :

(1) Nitrogen and Sulphur only

(2) Nitrogen, Sulphur and Phosphorous Only

(3) Phosphorous and halogens only

(4) Nitrogen, Sulphur, phosphorous and halogens

Ans. **(4)**

Sol. Lassaigne's test is used for detection of all element N, S, P, X.

71. Which among the following has highest boiling point?

(1) $CH_3CH_2CH_2CH_3$

(2) $CH_3CH_2CH_2CH_2–OH$

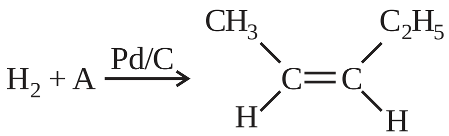
(3) $CH_3CH_2CH_2CHO$

(4) $H_5C_2 – O – C_2H_5$

Ans. **(2)**

Sol. Due to H-bonding boiling point of alcohol is High.

72. In the given reactions identify A and B.



$$CH_3 - C \equiv C - CH_3 + H_2 \xrightarrow{Na/Liquid\,NH_3} "B"$$

(1) A : 2–Pentyne B : trans – 2 – butene

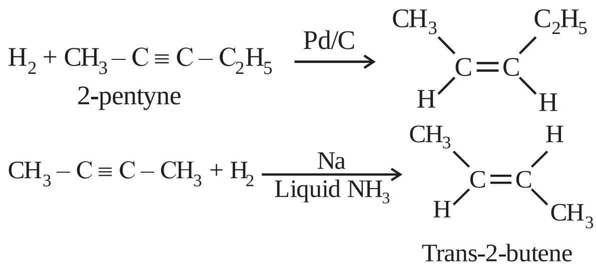
(2) A : n – Pentane B : trans – 2 – butene

(3) A : 2 – Pentyne B : Cis – 2 – butene

(4) A : n – Pentane B : Cis – 2 – butene

Ans. **(1)**

Sol.



73. The number of radial node/s for 3p orbital is:

(1) 1 (2) 4

(3) 2 (4) 3

Ans. **(1)**

Sol. For 3p : n = 3, $\ell = 1$

Number of radial node = $n - \ell - 1$

= 3 – 1 – 1 = 1

74. Match List - I with List - II.

List - I	**List - II**
Compound	**Use**
(A) Carbon tetrachloride	(I) Paint remover
(B) Methylene chloride	(II) Refrigerators and air conditioners
(C) DDT	(III) Fire extinguisher
(D) Freons	(IV) Non Biodegradable insecticide

Choose the correct answer from the options given below :

(1) (A)-(I), (B), (II), (C)-(III), (D)-(IV)

(2) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(3) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(4) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

Ans. **(2)**

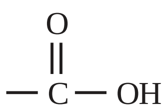
Sol. CCl_4 used in fire extinguisher. CH_2Cl_2 used as paint remover. Freons used in refrigerator and AC. DDT used as non Biodegradable insecticide.

75. The functional group that shows negative resonance effect is:

(1) $–NH_2$ (2) $–OH$

(3) $–COOH$ (4) $–OR$

Ans. **(3)**

Sol.  shows –R effect, while rest 3 groups shows +R effect via lone pair.

76. $[Co(NH_3)_6]^{3+}$ and $[CoF_6]^{3-}$ are respectively known as:

(1) Spin free Complex, Spin paired Complex

(2) Spin paired Complex, Spin free Complex

(3) Outer orbital Complex, Inner orbital Complex

(4) Inner orbital Complex, Spin paired Complex

Ans. **(2)**

Sol. $[Co(NH_3)_6]^{3+}$

Co^{3+} (strong field ligand) $\Rightarrow 3d^6\left(t_{2g}^6, e_g^0\right)$,

Hybridisation : d^2sp^3

Inner obital complex(spin paired complex)

Pairing will take place.

$[CoF_6]^{3-}$

Co^{3+} (weak field ligand) $\Rightarrow 3d^6\left(t_{2g}^4, e_g^2\right)$

Hybridisation : sp^3d^2

Outer orbital complex (spin free complex)

no pairing will take place

77. Given below are two statements :

Statement (I) : SiO_2 and GeO_2 are acidic while SnO and PbO are amphoteric in nature.

Statement (II) : Allotropic forms of carbon are due to property of catenation and $p\pi$-$d\pi$ bond formation.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both Statement I and Statement II are false

(2) Both Statement I and Statement II are true

(3) Statement I is true but Statement II is false

(4) Statement I is false but Statement II is true

Ans. **(3)**

Sol. SiO_2 and GeO_2 are acidic and SnO, PbO are amphoteric.

Carbon does not have d-orbitals so can not form $p\pi$-$d\pi$ Bond with itself. Due to properties of catenation and $p\pi$-$p\pi$ bond formation. carbon is able to show allotropic forms.

78. $C_2H_5Br \xrightarrow{\text{alc. KOH}} A \xrightarrow[CCl_4]{Br_2} B \xrightarrow[\text{Excess}]{KCN} C \xrightarrow[\text{Excess}]{H_3O^+} D$

Acid D formed in above reaction is :

(1) Gluconic acid

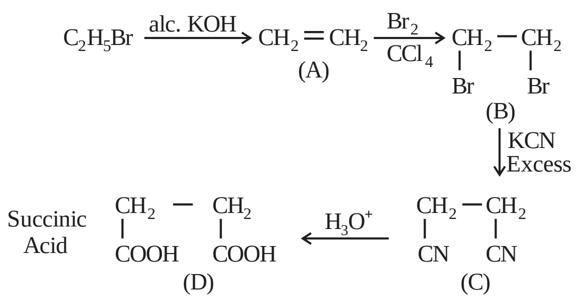
(2) Succinic acid

(3) Oxalic acid

(4) Malonic acid

Ans. **(2)**

Sol.



79. Solubility of calcium phosphate (molecular mass, M) in water is W_g per 100 mL at 25° C. Its solubility product at 25°C will be approximately.

(1) $10^7\left(\frac{W}{M}\right)^3$ (2) $10^7\left(\frac{W}{M}\right)^5$

(3) $10^3\left(\frac{W}{M}\right)^5$ (4) $10^5\left(\frac{W}{M}\right)^5$

Ans. **(2)**

Sol. $S = \frac{W \times 10}{M}$

$$Ca_3(PO_4)_2(s) \rightleftharpoons \underset{3s}{3Ca^{2+}}(aq.) + \underset{2s}{2PO_4^{3-}}(aq.)$$

$$S = \frac{W \times 1000}{M \times 100} = \frac{W \times 10}{M}$$

$K_{sp} = (3s)^3\,(2s)^2$

$= 108\, s^5$

$$= 108 \times 10^5 \times \left(\frac{W}{M}\right)^5$$

$$= 1.08 \times 10^7\left(\frac{W}{M}\right)^5$$

80. Given below are two statements :

Statement (I) : Dimethyl glyoxime forms a six-membered covalent chelate when treated with $NiCl_2$ solution in presence of NH_4OH.

Statement (II) : Prussian blue precipitate contains iron both in (+2) and (+3) oxidation states. In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Statement I is false but Statement II is true

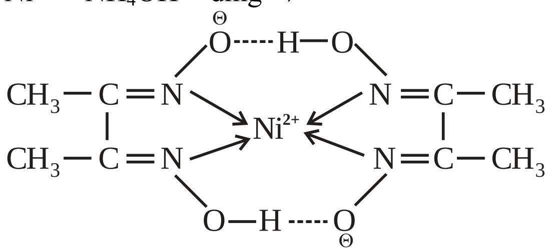
(2) Both Statement I and Statement II are true

(3) Both Statement I and Statement II are false

(4) Statement I is true but Statement II is false

Ans. (1)

Sol. $Ni^{2+} + NH_4OH + dmg \rightarrow$



2 Five member ring

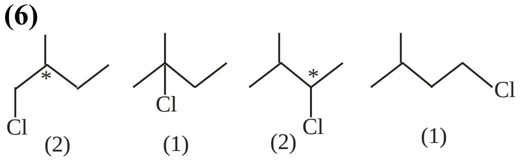
III II
$Fe_4[Fe(CN)_6]_3$
Prussian Blue

SECTION-B

81. Total number of isomeric compounds (including stereoisomers) formed by monochlorination of 2-methylbutane is________.

Ans. (6)

Sol.



82. The following data were obtained during the first order thermal decomposition of a gas A at constant volume:

$A(g) \rightarrow 2B(g) + C(g)$

S.No	Time/s	Total pressure/(atm)
1.	0	0.1
2.	115	0.28

The rate constant of the rection is______ $\times 10^{-2} s^{-1}$ (nearest integer)

Ans. (2)

Sol.

	A(g)	→	2B(g)	+	C(g)
t = 0	0.1				
t = 115 sec.	0.1 – x		2x		x

$0.1 + 2x = 0.28$

$2x = 0.18$

$x = 0.09$

$$K = \frac{1}{115} \ell n \frac{0.1}{0.1 - 0.09}$$

$= 0.0200 \text{ sec}^{-1}$

$= 2 \times 10^{-2} \text{ sec}^{-1}$

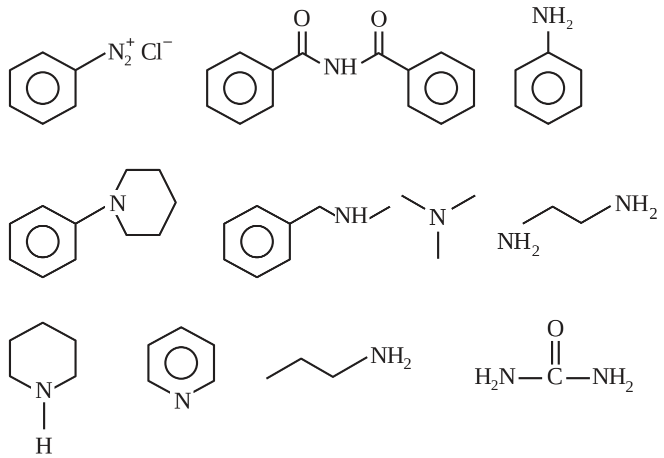
83. The number of tripeptides formed by three different amino acids using each amino acid once is ______.

Ans. (6)

Sol. Let 3 different amino acid are A, B, C then following combination of tripeptides can be formed-

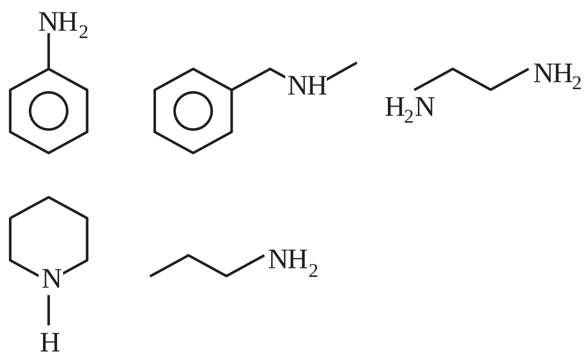
ABC, ACB, BAC, BCA, CAB, CBA

84. Number of compounds which give reaction with Hinsberg's reagent is_______.



Ans. (5)

Sol.



85. Mass of ethylene glycol (antifreeze) to be added to 18.6 kg of water to protect the freezing point at –24°C is________ kg (Molar mass in g mol^{-1} for ethylene glycol 62, K_f of water = 1.86 K kg mol^{-1})

Ans. **(15)**

Sol. $\Delta T_f = iK_f \times$ molality

$$24 = (1) \times 1.86 \times \frac{W}{62 \times 18.6}$$

W = 14880 gm

= 14.880 kg

86. Following Kjeldahl's method, 1g of organic compound released ammonia, that neutralised 10 mL of 2M H_2SO_4. The percentage of nitrogen in the compound is _______%.

Ans. **(56)**

Sol. $H_2SO_4 + 2NH_3 \rightarrow (NH_4)_2SO_4$

Millimole of $H_2SO_4 \rightarrow 10 \times 2$

So Millimole of $NH_3 = 20 \times 2 = 40$

Organic Compound $\rightarrow$ NH_3 40 Millimole

$$\therefore \text{Mole of N} = \frac{40}{1000}$$

$$\text{wt. of N} = \frac{40}{1000} \times 14$$

% composition of N in organic compound

$$= \frac{40 \times 14}{1000 \times 1} \times 100$$

= 56%

87. The amount of electricity in Coulomb required for the oxidation of 1 mol of H_2O to O_2 is ____$\times 10^5$C.

Ans. **(2)**

Sol. $2H_2O \rightarrow O_2 + 4H^+ + 4e^-$

$$\frac{W}{E} = \frac{Q}{96500}$$

$$\text{mole} \times \text{n-factor} = \frac{Q}{96500}$$

$$1 \times 2 = \frac{Q}{96500}$$

Q = 2 × 96500 C

= 1.93×10^5 C

88. For a certain reaction at 300K, K = 10, then $\Delta G°$ for the same reaction is –_______$\times 10^{-1}$ kJ mol^{-1}. (Given R = 8.314 JK^{-1} mol^{-1})

Ans. **(57)**

Sol. $\Delta G° = -RT \ln K$

$= -8.314 \times 300 \ln (10)$

= 5744.14 J/mole

= 57.44×10^{-1} kJ/mole

89. Consider the following redox reaction :

$$MnO_4^- + H^+ + H_2C_2O_4 \rightleftharpoons Mn^{2+} + H_2O + CO_2$$

The standard reduction potentials are given as below $\left(E^\circ_{red}\right)$

$$E^\circ_{MnO_4^-/Mn^{2+}} = +1.51V$$

$$E^\circ_{CO_2/H_2C_2O_4} = -0.49V$$

If the equilibrium constant of the above reaction is given as $K_{eq} = 10^x$, then the value of x = _______ (nearest integer)

Ans. **(338 OR 339)**

Sol. Cell Rx^n ; $MnO_4^- + H_2C_2O_4 \rightarrow Mn^{2+} + CO_2$

$E^\circ_{cell} = E^\circ_{op}$ of anode + E°_{RP} of cathode

= 0.49 + 1.51 = 2.00V

At equilibrium

$E_{cell} = 0,$

$$E^\circ_{cell} = \frac{0.059}{n}\log K$$

(As per NCERT $\frac{RT}{F} = 0.059$ But $\frac{RT}{F} = 0.0591$

can also be taken.)

$$2 = \frac{0.059}{10}\log K$$

logK = 338.98

90. 10 mL of gaseous hydrocarbon on combustion gives 40 mL of $CO_2(g)$ and 50 mL of water vapour. Total number of carbon and hydrogen atoms in the hydrocarbon is _____.

Ans. **(14)**

Sol. $\boxed{\begin{array}{c}CxHy\\10ml\end{array}} + O_2 \rightarrow CO_2 + H_2O$

$$CxHy + \left(x + \frac{y}{4}\right)O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$$

10x 5y

10x = 40

x = 4

5y = 50

y = 10

C_4H_{10}