MET 2013 Question Paper with Solution

Manipal Entrance Test (MET)

Manipal

Engineering Entrance Exam

Solved Paper 2013

Physics

1. The angular velocities of three bodies in simple harmonic motion are $\omega_1, \omega_2, \omega_3$ with their respective amplitudes as A_1, A_2, A_3. If all the three bodies have same mass and velocity, then

(a) $A_1^2\omega_1^2 = A_2^2\omega_2^2 = A_3^2\omega^2$ (b) $A_1^2\omega_1 = A_2^2\omega_2 = A_3^2\omega_3$
(c) $A_1\omega_1^2 = A_2\omega_2^2, A_3\omega_3^2$ (d) $A_1\omega_1 = A_2\omega_2 = A_3\omega_3$

2. If value of surface tension of a liquid is 70 dyne/cm, then its value in N/m will be

(a) 7×10^3 N/m (b) 7×10^2 N/m
(c) 7×10^{-2} N/m (d) 70 N/m

3. A ballon contains 1500 m^3 of a helium at 27°C and 4 atmospheric pressure. The volume of helium at -3°C temperature and 2 atmo spheric pressure will be

(a) 2700 m^3 (b) 1900 m^3
(c) 1700 m^3 (d) 1500 m^3

4. A car is moving along a straight horizontal road with a speed v_0. If the coefficient of friction between the tyres and the road is μ, the shortest distance in which the car be stopped is

(a) $\frac{v_0^2}{\mu}$ (b) $\left(\frac{v_0}{\mu_g}\right)^2$
(c) $\frac{v_0^2}{\mu_g}$ (d) $\frac{v_0^2}{2\mu_g}$

5. A car travelling on a straight track moves with uniform velocity of v_1 for some time and with unifrom velocity v_2 for the next equal time the average velocity on the car is

(a) $\frac{v_1 v_2}{2}$ (b) $\frac{v_1 v_2}{4}$
(c) $\frac{v_1 + v_2}{2}$ (d) $\frac{v_1 - v_2}{2}$

6. Threshold wavelength of a metal is 4000A°. If light of wavelength 3000Å irradiates the surface, the maximum kinetic energy of photoelectron is

(a) 1.7 eV (b) 1.6 eV (c) 1.5 eV (d) 1.0 eV

7. Simple pendulum of length l has a maximum angular displacement θ. The maximum kinetic energy of the bob is

(a) $mgl(1 - \cos\theta)$ (b) 0.5 mgl
(c) mgl (d) 2 mgl

8. Radius of orbit of satellite of earth is R. Its kinetic energy is proportional to

(a) $\frac{1}{R}$ (b) $\frac{1}{\sqrt{R}}$
(c) R (d) $\frac{1}{R^{3/2}}$

9. The radius R of the soap bubble is doubled under isothermal condition. If T be the surface tension of soap bubble, the work done in doing so is given by

(a) $32\ \pi R^2T$ (b) $24\pi R^2T$
(c) $8\pi R^2T$ (d) $4\pi R^2T$

10. A body of specific heat 0.2 kcal/kg°C is heated through 100°C. The percentage increase in its mass is

(a) 9% (b) 9.3×10^{-11}%
(c) 10% (d) None of these

11. Two similar coils are kept mutually perpendicular such that their centres coincide. At the centre, find the ratio of the magnetic field due to one coil and the resultant magnetic field through both coils, if the same current is flown

(a) $1:\sqrt{2}$ (b) $1:2$
(c) $1:3$ (d) $\sqrt{3}:1$

12. A prism of refractive index $\sqrt{2}$ has refracting angle of 60°. At what angle a ray must be incident one it so that it suffers a minimum deviation?

(a) 45° (b) 60° (c) 90° (d) 180°

13. A elevator car whose floor to distance is 2.7 m starts ascending with a constant acceleration of 1.2 m/s^2, 2s after a bolt is begin to fall from the ceiling of the car. The free fall time of the bolt is $(g = 9.8\ \mathrm{m}/s^e)$

(a) $\sqrt{\frac{2.7}{9.8}}$ s (b) $\sqrt{\frac{5.4}{9.8}}$ s
(c) $\sqrt{\frac{5.4}{8.6}}$ s (d) $\sqrt{\frac{5.4}{11}}$ s

14. A wet open umbrella is held vertical and it whirld about the handle at a uniform rate of 21 revolutions in 44s. If the rim of the umbrella is circle of 1 m in diameter and the height of the rim above the flour is 4.9 m, the locus of the drop is a circle of radius

(a) $\sqrt{2.5}$ m (b) 1 m
(c) 3 m (d) 1.5 m

15. The moment of inertia of a body about a given axis is 1.2 kg-m^2. To produce a rotational kinetic energy of 1500 J an angular acceleration of 25 rad/s^2 must be applied for

(a) 8.5 s (b) 5 s
(c) 2 s (d) 1 s

16. A running man has half the kinetic energy of that of a boy of half of his mass. The man speed up 1 m/s, so as the have same kinetic energy as that of a boy. The original speed of the man is

(a) $(\sqrt{2}-1)$ m/s
(b) $\sqrt{2}$ m/s
(c) $\frac{1}{(\sqrt{2}-1)\ \mathrm{m/s}}$
(d) $\frac{1}{\sqrt{2}}$ m/s

17. An earth satelites S has orbit radius which is 4 times that of communication satellite C. The period of revolution of S will be

(a) 32 days (b) 18 days
(c) 8 days (d) 9 days

18. n small balls, each of mass m impinge elastically each second on a surface with a velocity u, then the force experienced by the surface in one second, will be

(a) 4 mnu (b) 2 mnu
(c) 1.5 mnu (d) 0.8 mnu

19. A circular disc is rotating with angular velocity ω. If a man standing at the edge of the disc walks towards its centre then the angular velocity of the disc will

(a) decraese (b) increase
(c) be halved (d) not change

20. The moment of inertia of a disc of mass m and radius R about an axis, which is tangential to the circumference of the disc and parallel to its diameter is

(a) $\frac{3}{2}mR^2$ (b) $\frac{2}{3}mR^2$
(c) $\frac{5}{4}mR^2$ (d) $\frac{4}{5}mR^2$

21. A body weights 500 N on the surface of the earth. How much would it weigh half way below the surface of the earth ?

(a) 1000 N (b) 500 N
(c) 250 N (d) 125 N

22. Escape velocity at surface of earth is 11.2 Km/s. Escape velocity from a planet whose mass is the same as that of earth and radius 1/4 that of earth, is

(a) 2.8 km/s (b) 15.6 km/s
(c) 22.4 km/s (d) 44.8 km/s

23. The Bulk Modulus for an incompressible liquid is

(a) zero (b) unity
(c) infinity (d) between 0 and 1

24. In a capillary tube, water rises to 3 mm the height of water that will rise in another capillary tube having one-third radius of the first is

(a) 1 mm (b) 3 mm (c) 6 mm (d) 9 mm

25. An object is placed at a distance 20 cm from the pole of a convex mirror of focal length 20 cm. The image is produced

(a) 13.3 cm (b) 20 cm (c) 25 cm (d) 10 cm

26. Angular momentum is conserved

(a) always
(b) never
(c) when external force is absent
(d) when external torque is absent

27. The plano-convex lens of focal length 20 cm and 30 cm are placed together to form a double convex lens, the final focal length will be

(a) 12 cm (b) 60 cm (c) 20 cm (d) 30 cm

28. Due to a force of $(6\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$ N the displacement of a body is $(3\hat{\mathbf{i}} - \hat{\mathbf{j}})\ m$, then the work done is

(a) 16 j (b) 12 j (c) 8 j (d) zero

29. For a body moving with relativistic speed if the velocity is doubled, then

(a) its linear momentum is doubled
(b) its linear momentum will be less than double
(c) its linear momentum will be more than double
(d) its linear momentum remains unchanged

30. Position of a body with acceleration a is given by $x = ka^m t^n$, here t is time, find the dimension of m and n

(a) $m = 1, n = 1$ (b) $m = 1, n = 2$
(c) $m = 2, n = 1$ (d) $m = 2, n = 2$

31. The correct relation between α and β in a transistor is

(a) $\beta = \dfrac{\alpha}{1-\alpha}$ (b) $\beta = \dfrac{\alpha}{1+\alpha}$

(c) $\beta = \dfrac{1+\alpha}{\alpha}$ (d) $\beta = 1-\alpha$

32. A man crosses a 320 m wide river perpendicular to the current in 4 min. If in still water he can swim with a speed 5/3 times that of the current, then the speed of the current, in mm^{-1} in is

(a) 30 (b) 40 (c) 50 (d) 60

33. Two spheres of equal masses, one of which is a thin spherical shell and the other a solid, have the same moment of inertia about their respective diameters. The ratio of their radii will be

(a) 5 : 7 (b) 3 : 5
(c) $\sqrt{3} : \sqrt{5}$ (d) $\sqrt{3} : \sqrt{7}$

34. The speed with which the earth have to rotate on its axis so that a person on the equator would weight $(3/5)^{th}$ as much as present. [Radius of earth = 6400 km]

(a) 4.83×10^{-3} rads^{-1}
(b) 5.41×10^{-3} rads^{-1}
(c) 7.82×10^{-4} rads^{-1}
(d) 8.88×10^{-14} rads^{-1}

35. The rate of flow of glycerine of density 1.25×10^3 kgm^{-3} through the conical section of a pipe if the radii of its ends are 0.1 m and 0.04 m and the pressure drop across its length 10 Nm^{-2} is

(a) 6.93×10^{-4} m^{3}s^{-1}
(b) 7.8×10^{-4} m^{3}s^{-1}
(c) 10.4×10^{-5} m^{3}s^{-1}
(d) 14.5×10^{-5} m^{3}s^{-1}

36. The temperature of the black body increases from T to $2T$. The factor by which the rate of emission will increases is

(a) 4 (b) 2 (c) 16 (d) 8

37. A police jeep is chasing with velocity of 45 km/h a thief in another jeep moving with velocity 153 km/h. Police fires a bullet with muzzle velocity of 180 m/s. The velocity it will strike the car of the thief is

(a) 150 m/s (b) 27 m/s
(c) 450 m/s (d) 250 m/s

38. If the energy of a hydrogen atom in n^{th} orbit is E_n then energy in the n^{th} orbit of a singly ionized helium atom will be

(a) $4\ E_n$ (b) $E_n/4$
(c) $2\ E_n$ (d) $E_n/2$

39. If the work function of a potential is 6.875 eV, its threshold wavelength will be (Take $= c = 3 \times 10^8$ m/s)

(a) 3600 Å (b) 2400 Å
(c) 1800 Å (d) 1200 Å

40. Which of the following is unipoler transistor ?

(a) *p-n-p* transistor
(b) *n-p-n* transistor
(c) field effect transistor
(d) point confact transistor

41. If the length of a penduleum is made 9 times and mass of bob is made 4 times, then the value of time period becomes

(a) 3 T (b) 3/2 T
(c) 4 T (d) 2 T

42. Two weights w_1 and w_2 are suspended to be the two strings on a frictionless pulley. When the pulled up with an acceleration g then the tension in the string is

(a) $\frac{4\,w_1\,w_2}{w_1 + w_2}$ (b) $\frac{w_1\,w_2}{w_1 + w_2}$
(c) $\frac{2\,w_1\,w_2}{w_1 + w_2}$ (d) $\frac{w_1 + w_2}{2}$

43. An observer loop at a tree of height 15 m with a telescope of magnifying power 10. To him the tree appears

(a) 10 times taller (b) 15 times taller
(c) 10 times nearer (d) 15 times nearer

44. An inductor is connected to a battary through a switch induced emf is a when the switch is pressed and e_2 when the switch is opened. Then

(a) $e_1 = e_2$ (b) $e_1 > e_2$
(c) $e_1 < e_2$ (d) $e_1 >/< e_2$

45. The speed of a wave on a string is 150 m/s when the tension is 120 N. The percentage increase in the tension in order to raise the wave speed by 20% is

(a) 44% (b) 40% (c) 20% (d) 10%

46. The energy supplied to Kolkata by the state electricity board during an average November week day was 40 GWh. If this energy could be obtained by the conservation of matter, how much mass would have to be annihilated?

(a) 1.6 g (b) 2.2 g (c) 4.0 g (d) 1.6 kg

47. A simple pendulum has time period T_1. The point of suspension is now moved upward according to the relation

$y = kt^2\ (k = 1\text{ms}^2)$ where y is the vertical displacement. The time period now become T_2.

The ratio of $\frac{T_1^2}{T_2^2}$ is $g = 10\ \text{m}/\text{s}^2$

(a) 6/5 (b) 5/6
(c) 1 (d) 4/5

48. An electron is moving round the nucleus of a hydrogen atom in a circular orbit of radius r. The coulomb force $\mathbf{F}$ between the two is

(a) $k\,\frac{e^2}{r^3}\,r$ (b) $-\,k\,\frac{e^2}{r^3}\,r$ (c) $k\,\frac{e^2}{r^2}\,r$ (d) $-\,k\,\frac{e^2}{r^2}\,r$

49. A charge q is located at the centre of a cube. The electric flux through any face is

(a) $\frac{\pi q}{6\,(4\pi\,\varepsilon_0)}$ (b) $\frac{q}{6\,(4\pi\,\varepsilon_0)}$
(c) $\frac{2\,\pi q}{6\,(4\pi\,\varepsilon_0)}$ (d) $\frac{4\,\pi q}{6\,(4\pi\,\varepsilon_0)}$

50. In the Boolean Algebra $\mathbf{A} \cdot \mathbf{B}$ is same

(a) A + B (b) A · B (c) A – B (d) A + B

51. When a force F_1 acts on a particle, frequency is 6Hz and when a force F_2 acts, frequency is 8 Hz. what is the frequency when both the forces act simultaneously in same direction?

(a) 12 Hz (b) 25 Hz (c) 10 Hz (d) 5 Hz

52. For a particle executing simple harmonic motion, the kinetic energy k is given by, $k = k_0 \cos^2 \omega t$. The maximum value of potential energy is

(a) k_0 (b) zero
(c) $k_0/2$ (d) not obtainable

53. A train of 150 m length is going towards north direction at a speed of 10 m/s. A parrot flies at a speed of 5 m/s towards south direction parallel to the railway track, the time taken by the parrot to cross the train is equal to

(a) 12 s (b) 8 s (c) 15 s (d) 10 s

54. Ice starts freezing in a lake with water at 0°C when the atmospheric temperature is – 10 °C. If the time taken for 1 cm of ice to be formed is 12 min, the time taken for the thickness of the ice to change from 1 to 2cm will be

(a) 12 min
(b) less than 12 min
(c) more than 12 min but less than 24 min
(d) more than 24 min

55. The wavelength of the k_a line for an element of atomic number 43 is λ. Then the wavelength of the K_α line for an element of atomic number 29 is

(a) $\left(\frac{43}{29}\right)\lambda$ (b) $\left(\frac{42}{28}\right)\lambda$ (c) $\left(\frac{9}{4}\right)\lambda$ (d) $\left(\frac{4}{9}\right)\lambda$

56. The ratio of the speed of an object to the speed of its real image of magnification m of a convex mirror is

(a) $-\frac{1}{m^2}$ (b) m^2

(c) $-m$ (d) $\frac{1}{m}$

57. The maximum current that flow in the fuse wire before it blows out, varies with the radius r as

(a) $r^{3/2}$ (b) r

(c) $r^{2/3}$ (d) $r^{1/2}$

58. A diatomic gas is heated at certain pressure. What fraction of the heat energy is used to increase the internal energy?

(a) 3/5 (b) 3/7 (c) 5/7 (d) 5/9

59. In interference pattern, the energy is

(a) created at the minimum
(b) destroyed at the minimum
(c) conserved but redistributed
(d) All of the above

60. A red flower kept in green light will appear

(a) red (b) yellow
(c) black (d) white

Chemistry

1. 0.01 M solution of KCl and $CaCl_2$ are prepared in water. The freezing point of KCl is found to be $-2°C$. What is the freezing point of $CaCl_2$ to be completely ionised?

(a) $-3°C$ (b) $+3°C$
(c) $-2°C$ (d) $-4°C$

2. 1 mol He and 3 mol N_2 exert a pressure of 16 atm. Due to a hole in the vessel in which mixture in placed, mixture leaks out. What is the composition of mixture effusing out initially?

(a) 0.22 (b) 044 (c) 0.66 (d) 0.88

3. What are the four quantum numbers of 19th electron in Sc ($Z = 21$)?

(a) $n = 4, l = 0, m = 0, m_s = +\frac{1}{2}$

(b) $n = 3, l = 1, m = -1, m_s = +\frac{1}{2}$

(c) $n = 4, l = 1, m = +1, m_s = -\frac{1}{2}$

(d) $n = 3, l = 2, m = +2, m_s = -\frac{1}{2}$

4. (A) +tap water $\rightarrow$ white turbidity soluble in *aq.* NH_3

$$(A) \xrightarrow{\Delta} \text{residue } (B) + NO_2 + O_2$$

Aqueous (A) gives brown ring on adding $FeSO_4$ and Conc. H_2SO_4. Identify (A).

(a) $NaNO_3$ (b) $AgNO_3$
(c) $NaNO_2$ (d) $AgNO_2$

5. Each of the following compounds has been dissolved in water to make its 0.001 M solution. Rank them in order of their increasing conductivity in solution (assume 100% ionisation in each case)

(a) $c < d < a < b$
(b) $c < d < b < a$
(c) $a < b < c < d$
(d) $d < c < a < b$

6. Metal carbonyls can have formula $M(CO)_x$, where $x =$ number of CO units coordinated to metal M. What is the formula of the carbonyl of Fe (26) if EAN of Fe in metal carbonyl is 36?

(a) 2 (b) 3
(c) 4 (d) 5

7. Following method of extracting Zn is based on thermodynamics

$A: 2ZnS + 3O_2 \longrightarrow 2ZnO + 2SO_2$

$B: ZnO + C \longrightarrow Zn + CO$

If ΔG_f° (standard free energies of formation, in $kJmol^{-1}$) of ZnS = – 205.4,

$ZnO = -318.0, SO_2 = -300.4$ and of $CO = -137.3$

Free energy changes of the above reaction A and B (respectively) will be

(a) -826.4 kJ, $+180.9$ kJ
(b) $+826.4$ kJ, -180.9 kJ
(c) -826.4 kJ, -180.9 kJ
(d) $+826.4$ kJ, $+180.9$ kJ

8. The final step for the extraction of copper form copper pyrites in Bessemer convertor involves the reaction.

(a) $Cu_2S + 2FeO \longrightarrow 2Cu + 2Fe + SO_2$
(b) $Cu_2S + 2CuO \longrightarrow 6Cu + SO_2$
(c) $4Cu_2O + FeS \longrightarrow 8Cu + 2FeSO_4$
(d) $2Cu_2O + FeS \longrightarrow 4Cu + Fe + SO_2$

9. If $CO_2(g)$ under pressure is passed into Na_2CrO_4 (aq), yellow colour solution changes to

(a) blue
(b) green
(c) red
(d) orange-red

10. Select incorrect statements.

(a) Ionisation energies of 5*d*- elements are greater than those of 3*d* and 4*d* elements.
(b) Cu (I) is diamagnetic while Cu (II) is paramagnetic.
(c) $[Ti(H_2O)_6]^{3+}$ is colored while $[Sc(H_2O)_6]^{+}$ is colourless.
(d) Transition elements cannot form complexes.

11. ClO_2 is an/a

(a) anhydride of $HClO_2$
(b) anhydride of $HClO_3$
(c) mixed anhydride of $HClO_2$ and $HClO_3$
(d) mixed anhydride of $HClO_3$ and $HClO_4$

12. Based on the following reaction identify alkene A

$$A \xrightarrow{O_3/(H_2O)/Zn} 2HCHO + \underset{\displaystyle CHO}{\underset{|}{C}HO}$$

(a) $CH_3-\overset{\overset{\displaystyle CH_2}{\|}}{C}-CH_3$
(b) $CH_2=CH-CH=CH_2$
(c) $CH_3-CH=CH-CH_3$
(d) $CH_3-CH_2-CH=CH_2$

13. Which one of the following statments about NO is wrong?

(a) NO is an odd electron molecule
(b) It is a free radical and highly reactive
(c) It readily forms complexes with transition metal ions
(d) It can be prepared by heating NH_4NO_3 at 250°C

14. KO_2 is used in life supports in space crafts, submarines and emergency breathing apparatus since it

(a) absorbs CO_2
(b) release O_2
(c) releases CO_2
(d) absorbs CO_2 and releases O_2

15. Calculate second electron affinity of oxygen for the process,

$$O^-(g) + e^-(g) \rightarrow O^{2-}(g)$$

by using the following data

(i) Heat of sublimation of
$$Mg\ (s) = +147.7\ kJ\ mol^{-1}$$
(ii) Ionisation energy of $Mg(g)$ to form
$$Mg^{2+}(g) = +2189.0\ kJ\ mol^{-1}$$
(iii) Bond dissociation energy for
$$O_2 = +498.4\ kJ\ mol^{-1}$$
(iv) First electron affinity of
$$O(g) = -141.0\ kJ\ mol^{-1}$$
(v) Heat formation of
$$MgO = -601.7\ kJ\ mol - 1$$
(vi) Lattice energy of
$$MgO = -3791.0\ kJ\ mol^{-1}$$

(a) $235.6\ kJ\ mol^{-1}$
(b) $468.7\ kJ\ mol^{-1}$
(c) $544.4\ kJ\ mol^{-1}$
(d) $744.4\ kJ\ mol^{-1}$

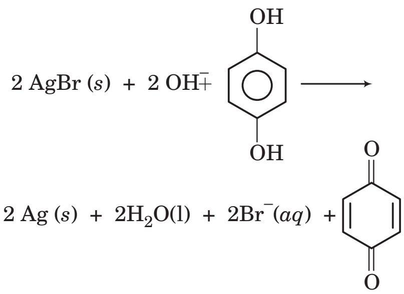
16. 1 g of fuming H_2SO_4 (oleum : It is a mixture of concentrated H_2SO_4 saturated with SO_3 and having formula $H_2S_2O_7$) is diluted with H_2O. This solution is completely neutralised by 26.7mL of 0.8 N NaOH. Find the percentage of free SO_3 in the oleum.

(a) 20.73% (b) 43.80%
(c) 79.27% (d) 60.74%

17. ${}^{14}_{7}N + {}^{1}_{0}n \rightarrow {}^{14}_{6}C + {}^{1}_{1}H$ is written as

(a) ${}^{14}_{7}N(n, e){}^{1}_{1}H$
(b) ${}^{14}_{7}N\ (p, n)\ {}^{14}_{6}C$
(c) ${}^{14}_{7}N\ (n, p)\ {}^{14}_{6}C$
(d) ${}^{14}_{6}C\ (p, n)\ {}^{14}_{7}N$

18. Photographic paper is developed with alkaline hydroquinone



Select correct statement.

(a) Hydroquinone is the oxidant
(b) Ag^+ is the oxidant
(c) Br^- is the oxidant
(d) Ag^+ is the reductant

19. Which has maximum ionisation potential?

(a) N (b) O
(c) O^+ (d) Na

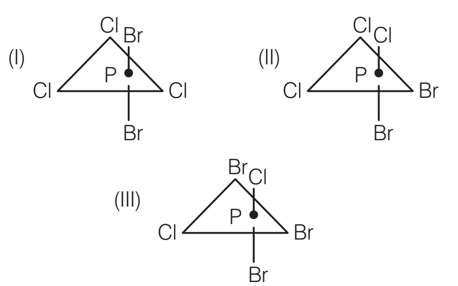
20. Electronic configuration of Gd (64) is written as

(a) [Xe] $4f^7\ 5d^1 6s^2$ (b) [Xe] $4f^8\ 6s^2$
(c) [Xe] $4f^9\ 6s^1$ (d) [Xe] $4f^{10}$

21. Screening effect is not observed in

(a) He^+ (b) Li^{2+}
(c) Be^{3+} (d) all cases

22. Which one of the following isomers of $PBrCl_3$ have no dipole moment?



(a) Only I (b) I and II
(c) II and III (d) Only III

23. The type of hybridisation of P atom in PCl_5, PCl_4^+ and PCl_6^- is (respectively)

(a) sp^3, sp^3d sp^3d^2 (b) sp^3d, sp^3, sp^2d^2
(c) sp^3, sp^3d^2, sp^3 (d) sp^3d^2, sp^3, sp^3d

24. Which has a maximum repulsive interaction?

(a) $bp - bp$ (b) $lp - lp$ (c) $lp - bp$ (d) Equal

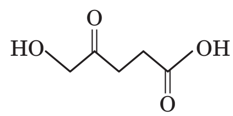
25. Calculate the electronegativity of chlorine from bond energy of Cl—F bond (61 kcal mol^{-1}) F—F (38 kcal mol^{-1}) and Cl—Cl bond (58 kcal mol^{-1}) and electronegativity of fluorine 4.0 eV

(a) 1.42 eV (b) 1.89 eV (c) 2.67 eV (d) 3.22 eV

26. A sample of ammonium phosphate $(NH_4)_3PO_4$ contains 3.18 moles of H-atom. The number of moles of O-atom in the sample is

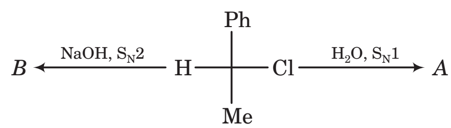
(a) 0.265 (b) 0.795 (c) 1.06 (d) 3.18

27. In the following compound keto group is at position



(a) 1 (b) 2 (c) 3 (d) 4

28.



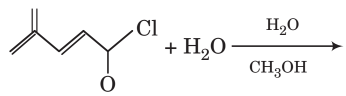
I. Formation of A has proceeded with racemisation.

II. Formation of B has proceeded with inversion.

Select the correct statement.

(a) I and II (b) Only I
(c) Only III (d) None of these

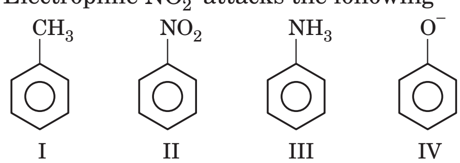
29.



In this reaction, we get types of substituted alcohols (stereoisomers not considered)

(a) one (b) two (c) three (d) four

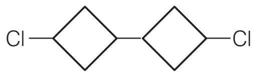
30. Electrophile $NO_2^{\oplus}$ attacks the following

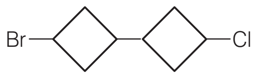


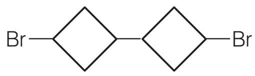
In which cases, $\overset{\oplus}{N}O_2$ will be at *meta* position?

(a) II and IV (b) I, II and III
(c) II and III (d) Only I

31. The reaction of 1- bromo-3- chlorocyclobutane with metallic sodium in dioxane gives

(a) 

(b) 

(c) 

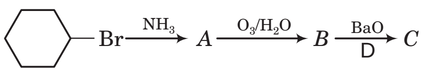
(d) 

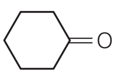
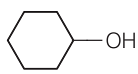
32. $CH_3CH{=}CH_2 \xrightarrow[H_2O_2/OH^-]{NaBD_4}$ Product X, X is

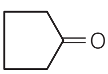
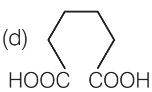
(a) $CH_3\underset{\underset{OH}{|}}{C}HCH_2D$ (b) $CH_3\underset{\underset{D}{|}}{C}HCH_2OH$

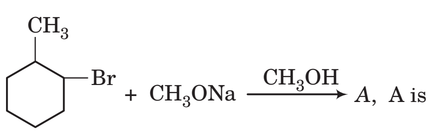
(c) $CH_3\underset{\underset{CO}{|}}{C}HCH_3$ (d) None of these

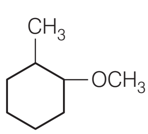
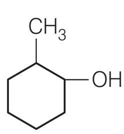
33. End product of following sequence of reaction is

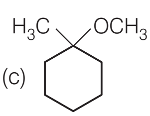
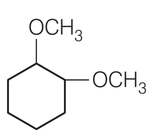


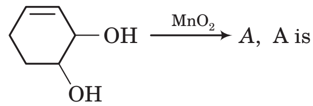
(a)  (b) 

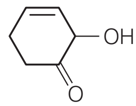
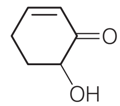
(c)  (d) 

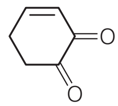
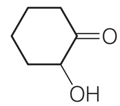
34. 

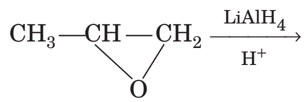
(a)  (b) 

(c)  (d) 

35. 

(a)  (b) 

(c)  (d) 

36.  A, A can be

(a) $CH_3\underset{\underset{OH}{|}}{C}HCH_3$ (b) $CH_3CH_2CH_2OH$

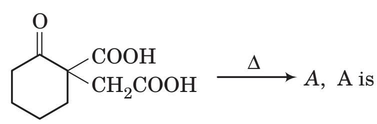
(c) Both (a) and (b) (d) None of these

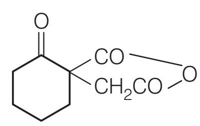
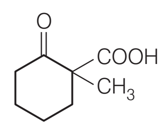
37. $2CH_3\overset{\overset{O}{||}}{C}OC_2H_5 \xrightarrow{C_2H_5ONa} A$, A is formed by Claisen condensation. Which is/are true about A?

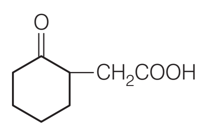
(a) A forms oxime
(b) A shows tautomerism
(c) A shows iodoform test
(d) All of the above are true

38. When a nucelophile encounters a ketone the site of attack is

(a) the carbon atom of the carbonyl
(b) the oxygen atom of the carbonyl
(c) Both the carbon and oxygen atoms with equal probability
(d) no attack occurs- ketones do not react with nucleophiles

39. 

(a)  (b) 

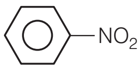
(c)  (d) None of these

40. The structure of N- propyl acetamide is

(a) $CH_3CH_2CONHCH_3$
(b) $CH_3CON(CH_2CH_2CH_3)_2$
(c) $CH_3CONHCH_2CH_2CH_3$
(d) $CH_3CH{=}NCH_2CH_2CH_3$

41. Which A gives red colour in the reaction

$$A \xrightarrow[\text{(ii) NaOH}]{\text{(i) HNO}_2} \text{red colour}$$

(a) $CH_3CH_2NO_2$ (b) $(CH_3)_2—CHNO_2$
(c) $(CH_3)_3CNO_2$ (d) 

42. Alanine forms Zwitter ion which exists as

(a) $CH_3\underset{}{C}HCOO^-$ (with $\oplus NH_3$ on C) in acidic medium
(b) $CH_3CHCOOH$ (with $\oplus NH_3$ on C) in a medium of pH = 4
(c) CH_3CHCOO^- (with NH_2 on C) in a medium of pH = 1
(d) CH_3CHCOO^- (with NH_2 on C) in a medium of pH = 2

43. Which gives only glucose by hydrolysis?

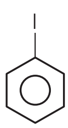
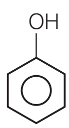
(a) Sucrose (b) Raffinose
(c) Maltose (d) Galactose

44. Which polysaccharide has α-glycoside linkage ?

(a) Amylose (b) Amylopection
(c) Cellulose (d) All of these

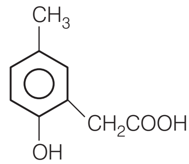
45.  $—O—XH_3$ + HI $\longrightarrow$ Product (mixture)

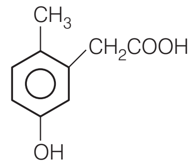
Mixture can be of

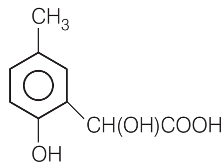
(a)  , CH_3OH (b)  , CH_3I

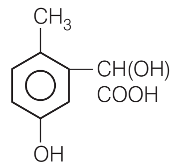
(c) Both (a) and (b) (d) None of these

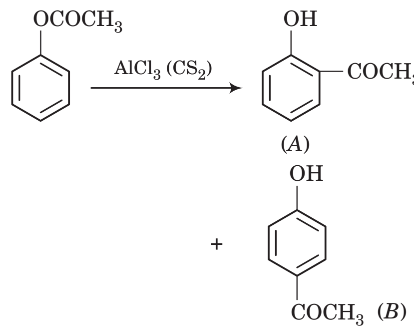
46. p-cresol reacts with chloroform in alkaline medium to give the compound A which adds hydrogen cyanide to form the compound B. The latter on acidic hydrolysis gives chiral carboxylic acid 'C' which is

(a) 

(b) 

(c) 

(d) 

47. 

Choose the correct statement.

(a) A is more volatile than B
(b) B is more volatile than A
(c) A is formed more rapidly at a higher temperature
(d) A is formed in higher yield at a low temperature

48. Sodium extract of thiourea will be ...... colour in Lassaigne's test.

(a) blue (b) red (c) yellow (d) green

49. Consider the following redox reaction occurring in acidic medium

$$BrO_3^- \xrightarrow{E^\circ = 1.5\text{V}} BrO^- \xrightarrow{E^\circ = 1.6\text{V}} Br_2$$

$BrO_3^- \longrightarrow Br_2$: E°=? V

The unknown standard reduction potential is

(a) –1.6 V (b) 1.6 V (c) –1.52 V (d) 1.52 V

50. The spontaneous reaction that takes place in this cell is

(a) $Zn + Ni \longrightarrow Zn^{2+} + Ni^{2+}$
(b) $Zn + Ni^{2+} \longrightarrow Zn^{2+} + Ni$
(c) $Ni + Zn^{2+} \longrightarrow Ni^{2+} + Zn$
(d) $Ni^{2+} + 2K \longrightarrow 2K^+ + Ni$

51. A 0.15 molal solution of $K_4[Fe(CN)_6]$ in water freezes at –0.65°C. What is the apparent percentage of dissociation of this compound in this solution? (K_f for water = 1.86°C mol^{-1})

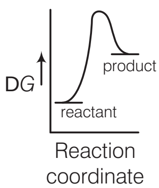
(a) 0.33 (b) 0.52
(c) 0.63 (d) 0.79

52. 10g of non-volatile solute is dissolved in 180 g of H_2O resulting in lowering of vapour pressure by 0.5%. Determine the boiling point of solution if K_b of water is 0.52 K kg mol^{-1}.

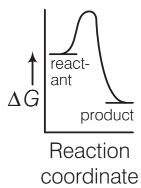
(a) 100.01° C (b) 100.15° C
(c) 100.23° C (d) 100.32°C

53. Which one of the following reaction energy diagrams best represents a reaction in the reverse direction; that it is the most endothermic?

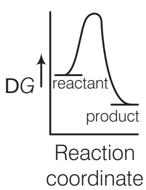
(a)



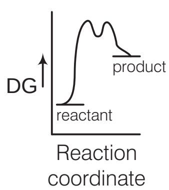
(b)



(c)



(b)



54. A certain reaction rate increases 1000 folds in the presence of a catalyst at 27°C. The activation energy of the original pathway is 98 kJ/mol. What is the activation energy of the new pathway?

(a) 80.77 kJ (b) 56.38 kJ
(c) 24.67 kJ (d) 90.43 kJ

55. Consider the following reaction in an aqueous solution

$$I^-(aq) + Cl^- \rightarrow IO^-(aq) + Cl^-(aq)$$

and the following initial concentration and initial rate data for this reaction.

Exp. No	$[I^-]$ M	$[OCl^-]$ M	Initial rate Ms^{-1}
1.	0.1000	0.0500	3.05×10^{-4}
2.	0.2000	0.0500	6.10×10^{-4}
3.	0.3000	0.0100	1.83×10^{-4}
4.	0.3000	0.0200	3.66×10^{-4}

Which of the following is the correct rate law for this reaction?

(a) Rate = $k\ [I^-]^2[OCl^-]$
(b) Rate = $k[OCl^-]$
(c) Rat = $k\ [I^-]^2$
(d) Rate = $k[I^-][OCl^-]$

56. Determine the pH of 0.024 M hydroxylamine hydrochloride solution. (K_b of hydroxyl amine is 10^{-8}).

(a) 8.4 (b) 6.2
(c) 3.8 (d) 5.6

57. Determine solubility of AgBr in a 0.1 M KCN solution. [K_f of $[Ag(CN)_2]^- = 5.6 \times 10^8$; K_{sp} of AgBr = 7.7×10^{-13}]

(a) 2×10^3 M (b) 4×10^{-6} M
(c) 2×10^{-6} M (d) 4×10^{-3} M

58. Which of the following forms cationic micelles above a certain minimum concentration?

(a) Sodium dodecyl sulphate
(b) Sodium acetate
(c) Urea
(d) Cetyl trimethyl ammonium bromide

59. Determine the relative size of Cs atom compared to a Li atom, if their densities are 1.87 g/cc and 0.53 g/cc respectively

(a) $r(Cs) = 1.753\, r(Li)$
(b) $r(Cs) = 1.936\, r(Li)$
(c) $r(Cs) = 2.753\, r(Li)$
(d) $r(Cs) = 2.936\, r(Li)$

60. Nylon threads are made of

(a) polyvinyl polymer
(b) Polyester polymer
(c) polyamide polymer
(d) polyethylene polymer

Mathematics

1. The least positive integer n, for which

$$n! < \left(\frac{n+1}{2}\right)^n \text{ holds, is}$$

(a) 1 (b) 2
(c) 3 (d) 4

2. The sum to n terms of the series:

$$\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + \dots \text{ is}$$

(a) $\frac{n^2+1}{2(n^2+n+1)}$ (b) $\frac{n^2+n}{(n^2+n+1)}$
(c) $\frac{n^2+n}{2(n^2+n+1)}$ (d) None of these

3. The sum of n terms of the series $+1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+\dots$ is

(a) $\frac{5}{4}+\frac{15}{16}\left(1-\frac{1}{5^{n-1}}\right)-\frac{(3n-2)}{4\cdot 5^{n-1}}$
(b) $\frac{5}{4}+\frac{1}{16}\left(1-\frac{1}{5^{n-1}}\right)-\frac{3n}{4\cdot 5^{n-1}}$
(c) $\left(1-\frac{1}{5^{n-1}}\right)-\frac{(3n+2)}{4\cdot 5^{n-1}}$
(d) None of the above

4. Sum of the series to n terms $5+7+13+31+85+\dots$ is

(a) 3^n+8n+1
(b) $\frac{1}{2}[3^n+8n-1]$
(c) $\frac{1}{2}(3^n+8n+1)$
(d) None of the above

5. If $b<0$, then the roots x_1 and x_2 of the equation $2x^2+6x+b=0$, satisfy the condition $\left(\frac{x_1}{x_2}\right)+\left(\frac{x_2}{x_1}\right)<K$, where K is equal to

(a) 2 (b) – 2
(c) 0 (d) 4

6. If the roots of the equation $(a-1)(x^2+x+1)^2 = (a+1)(x^4+x^2+1)$ are real and distinct then the value of $a \in$

(a) $(-\infty, 3]$ (b) $(-\infty, -2) \cup (2, \infty)$
(c) $[-2, 2]$ (d) $[-3, \infty)$

7. If the sum of two of the roots of $x^3+px^2+qx+r=0$ is zero, then $pq =$

(a) $-r$ (b) $2r$ (c) $-2r$ (d) r

8. If m parallel lines in a plane are intersected by a family of n parallel lines, then the number of parallelograms that can be formed is

(a) $\frac{1}{4}mn(m-1)(n-1)$ (b) $\frac{1}{2}mn(m-1)(n-1)$
(c) $\frac{1}{4}m^2n^2$ (d) None of these

9. A person is permitted to select at least one and at most n coins from a collection of $(2n+1)$ distinct coins. If the total number of ways in which he can select coins is 255, then n equals

(a) 4 (b) 8 (c) 16 (d) 32

10. The value of x in the expression $(x+x^{\log_{10} x})^5$, if the third term in the expansion is 1,000,000, is

(a) $10, 10^{-3/2}$ (b) 100 or $10^{-3/2}$
(c) 10 or $10^{-5/2}$ (d) None of these

11. $\sum_{k=0}^{10} {}^{20}C_k$ is equal to

(a) $2^{19}+\frac{1}{2}\,{}^{20}C_{10}$ (b) 2^{19}
(c) ${}^{20}C_{10}$ (d) None of these

12. If the point (a, a) are placed in between the lines $|x+y|=4$ then

(a) $|a|=2$ (b) $|a|=3$
(c) $|a|<2$ (d) $|a|<3$

13. The number of rational values of m for which the y-coordinate of the point of intersection of the lines $3x+2y=10$ and $x=my+2$ is an integer is

(a) 2 (b) 4
(c) 6 (d) 8

14. A straight line cuts intercepts from the axis of coordinates the sum of the reciprocals of which is a constant K. Then it always passes through a fixed point

(a) (K, K) (b) $\left(\frac{1}{K}, \frac{1}{K}\right)$
(c) $(-K, -K)$ (d) $(K-1, K-1)$

15. If the line $\frac{x}{a}+\frac{y}{b}=1$ moves in such a way that $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$ where, c is a constant, then the locus of the foot of perpendicular from the origin on the straight line is

(a) Straight line (b) Parabola
(c) Ellipse (d) Circle

16. The pair of lines $\sqrt{3}x^2-4xy+\sqrt{3}y^2=0$ are rotated about the origin by $\frac{\pi}{6}$ in the anticlockwise sense. The equation of the pair in the new position is

(a) $x^2-\sqrt{3}xy=0$
(b) $xy-\sqrt{3}y^2=0$
(c) $\sqrt{3}x^2-xy=0$
(d) None of the above

17. The value of $\lim_{x\to 1}\frac{\left[\sum_{K=1}^{100}-x^K\right]-100}{x-1}$ is

(a) – 5050 (b) 0
(c) 5050 (d) None of these

18. The value of $\lim_{x\to\infty}\left\{\frac{x}{x+\frac{\sqrt[3]{x}}{x+\frac{\sqrt[3]{x}}{x+\sqrt[3]{x}}}\dots}\right\}$ is

(a) 0 (b) 1 (c) 2 (d) 1/2

19. If $f(x)$ defined by $f(x)=\begin{cases}\frac{|x^2-x|}{x^2-x}, & x\neq 0,1\\ 1, & x=0\\ -1, & x=1\end{cases}$

then $f(x)$ is continuous for all

(a) x
(b) x except at $x=0$
(c) x except at $x=1$
(d) x excexpt at $x=0$ and $x=1$

20. Function $f(x)$ is defined as follows

$$f(x)=\begin{cases}ax-b, & x\le 1\\ 3x, & 1<x<2\\ bx^2-a, & x\ge 2\end{cases}$$

If $f(x)$ is continuous at $x=1$, but discontinuous at $x=2$ then the locus of the point (a,b) is a straight line excluding the point where it cuts the line

(a) $y=3$ (b) $y=2$
(c) $y=0$ (d) $y=1$

21. The maximum value of $(\cos\alpha_1)\cdot(\cos\alpha_2)\dots(\cos\alpha_n)$. Under the restrictions $0\le\alpha_1,\alpha_2,\dots\alpha_n\le\frac{\pi}{2}$ and $(\cot\alpha_1)\cdot(\cot\alpha_2)\dots(\cot\alpha_n)=1$ is

(a) $\frac{1}{2^{n/2}}$ (b) $\frac{1}{2^n}$ (c) $\frac{1}{2n}$ (d) 1

22. The minimum value of the expression $\sin\alpha+\sin\beta+\sin\gamma$, where α,β,γ are real numbers satisfying $\alpha+\beta+\gamma=\pi$ is

(a) positive (b) zero
(c) negative (d) – 3

23. If θ be the angle between the unit vectors **a** and **b**, then $\cos\frac{\theta}{2}$ is equal to

(a) $\frac{1}{2}|\mathrm{a}-\mathrm{b}|$ (b) $\frac{1}{2}|\mathrm{a}+\mathrm{b}|$
(c) $\frac{|\mathrm{a}-\mathrm{b}|}{|\mathrm{a}+\mathrm{b}|}$ (d) $\frac{|\mathrm{a}+\mathrm{b}|}{|\mathrm{a}-\mathrm{b}|}$

24. If $\mathbf{a}\cdot\mathbf{i}=\mathbf{a}\cdot(\mathbf{j}+\mathbf{i})=\mathbf{a}\cdot(\mathbf{i}+\mathbf{j}+\mathbf{k})$, then **a** is equal to

(a) i (b) k
(c) j (d) (i + j + k)

25. If the scalar projection of the vector $x\,\mathbf{i}-\mathbf{j}+\mathbf{k}$ on the vector $2\mathbf{i}-\mathbf{j}+5\mathbf{k}$ is $\frac{1}{\sqrt{30}}$ then value of x is equal to

(a) $\frac{-5}{2}$ (b) 6 (c) – 6 (d) 3

26. If $\mathbf{a}\times\mathbf{b}=\mathbf{c}$, $\mathbf{b}\times\mathbf{c}=\mathbf{a}$ and a, b, c be the moduli of the vectors **a**, **b**, **c** respectively, then

(a) $a=1, b=1$ (b) $c=1, a=1$
(c) $\mathrm{a}\cdot(\mathrm{b}\times\mathrm{c})=1$ (d) $b=1, c=a$

27. A unit vectors coplanar with $\mathbf{i}+\mathbf{j}+2\mathbf{k}$ and $\mathbf{i}+2\mathbf{j}+\mathbf{k}$ and perpendicular to $\mathbf{i}+\mathbf{j}+\mathbf{k}$ is

(a) $\frac{\mathrm{j}-\mathrm{k}}{\sqrt{2}}$ (b) $\frac{\mathrm{i}+\mathrm{j}+\mathrm{k}}{\sqrt{3}}$
(c) $\frac{\mathrm{i}+\mathrm{j}+2\mathrm{k}}{\sqrt{6}}$ (d) $\frac{-\mathrm{j}+2\mathrm{k}}{\sqrt{5}}$

28. Let $A = \begin{bmatrix} 0 & \alpha \\ 0 & 0 \end{bmatrix}$ and $(A + I)^{50} - 50A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then the value of $a + b + c + d$ is

(a) 2 (b) 1
(c) 4 (d) None of these

29. For two unimodular complex numbers z_1 and z_2, $\begin{bmatrix} \bar{z}_1 & z_2 \\ \bar{z}_2 & z_1 \end{bmatrix}^{-1} \begin{bmatrix} z_1 & z_2 \\ -\bar{z}_2 & \bar{z}_1 \end{bmatrix}^{-1}$ is equal to

(a) $\begin{bmatrix} z_1 & z_2 \\ \bar{z}_1 & \bar{z}_2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
(c) $\begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$ (d) None of these

30. If A is a square matrix of order n such that $|\text{adj}(\text{adj } A)| = |A|^9$, then the value of n can be

(a) 4 (b) 2
(c) either 4 or 2 (d) None of these

31. Coefficient of x in $f(x) = \begin{vmatrix} x & (1 + \sin x)^3 & \cos x \\ 1 & \log(1 + x) & 2 \\ x^2 & (1 + x)^2 & 0 \end{vmatrix}$ is

(a) 0
(b) 1
(c) – 2
(d) Cannot be determined

32. Let α_1, α_2 and β_1, β_2 be the roots of $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$ respectively. If the system of equations $\alpha_1 y + \alpha_2 z = 0$ and $\beta_1 y + \beta_2 z = 0$ has a non-trivial solution, then

(a) $b^2 pr = q^2 ac$ (b) $bpr^2 = qac^2$
(c) $bp^2 r = qa^2 c$ (d) None of these

33. The values of α for which the point $(\alpha - 1, \alpha + 1)$ lies in the larger segment of the circle $x^2 + y^2 - x - y - 6 = 0$ made by the chord whose equation is $x + y - 2 = 0$ is

(a) $-1 < \alpha < 1$ (b) $1 < \alpha < \infty$
(c) $-\infty < \alpha < -1$ (d) $\alpha \le 0$

34. The circles whose equations are $x^2 + y^2 + c^2 = 2ax$ and $x^2 + y^2 + c^2 - 2by = 0$ will touch each other externally if

(a) $\frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{a^2}$ (b) $\frac{1}{c^2} + \frac{1}{a^2} = \frac{1}{b^2}$
(c) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$ (d) None of these

35. The tangents to $x^2 + y^2 = a^2$ having inclinations α and β intersect at P. If $\cot\alpha + \cot\beta = 0$, then the locus of P is

(a) $x + y = 0$ (b) $x - y = 0$
(c) $xy = 0$ (d) None of these

36. An equlateral triangle SAB is inscribed in the parabola $y^2 = 4ax$ having its focus at S. If chord AB lies towards the lefit of S, then side length of this triangle is

(a) $2a(2 - \sqrt{3})$ (b) $4a(2 - \sqrt{3})$
(c) $a(2 - \sqrt{3})$ (d) $8a(2 - \sqrt{3})$

37. Minimum distance between the curves $y^2 = 4x$ and $x^2 + y^2 - 12x + 31 = 0$ is

(a) $\sqrt{5}$ (b) $\sqrt{21}$
(c) $\sqrt{28} - \sqrt{5}$ (d) $\sqrt{21} - \sqrt{5}$

38. If the line $lx + my + n = 0$ cuts the ellipse $\frac{x^2}{a^2} + \frac{y^2}{25} = 1$ in points whose eccentoric angles differ by $\frac{\pi}{2}$, then $\frac{a^2 l^2 + b^2 m^2}{n^2}$ is equal to

(a) 1 (b) 2 (c) 4 (d) $\frac{3}{2}$

39. If the tangent to ellipse $x^2 + 2y = 1$ at point $P\left(\frac{1}{\sqrt{2}}, \frac{1}{2}\right)$ meets the auxiliary circle at the points R and Q, then tangents to circle at Q and R intersect at

(a) $\left(\frac{1}{\sqrt{2}}, 1\right)$ (b) $\left(1, \frac{1}{\sqrt{2}}\right)$ (c) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (d) $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$

40. Which one of the following is independent of α in the hyperbola $(0 < \alpha < \pi/2)$ $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$

(a) Eccentricity (b) Abscissa of foci
(c) Directrix (d) Vertex

41. If PQ is a double ordinate of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ such that OPQ is an equilateral triangle, O bing the centre of the hyperbola, then the eccentricity e of the hyperbola satisfies.

(a) $1 < e < \frac{2}{\sqrt{3}}$ (b) $e = \frac{2}{\sqrt{3}}$
(c) $e = \frac{\sqrt{3}}{2}$ (d) $e > \frac{2}{\sqrt{3}}$

42. If $iz^4 + 1 = 0$, then z can take the value

(a) $\frac{1+i}{\sqrt{2}}$ (b) $\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}$

(c) $\frac{1}{4i}$ (d) i

43. If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$, then the value of $\cos 3\alpha + \cos 3\beta + \cos 3\gamma$ is

(a) 0 (b) $\cos(\alpha + \beta + \gamma)$

(c) $3\cos(\alpha + \beta + \gamma)$ (d) $3\sin(\alpha + \beta + \gamma)$

44. If Q is real and z_1, z_2 are connected by $z_1^2 + z_2^2 + 2z_1z_2\cos\theta = 0$, then triangle with vertices $0, z_1$ and z_2 is

(a) equilateral (b) right-angled

(c) isosceles (d) None of these

45. Let $f(xy) = f(x)\cdot f(y)$ for all $x, y \in R$. If $f'(1) = 2$ and $f(4) = 4$, then $f'(4)$ equal to

(a) 4 (b) 1

(c) $\frac{1}{2}$ (d) 8

46. If $y = \sqrt{(a-x)(x-b)} - (a-b)\tan^{-1}\sqrt{\frac{a-x}{x-b}}$, then $\frac{dy}{dx}$ is equal to

(a) 1 (b) $\sqrt{\frac{a-x}{x-b}}$

(c) $\sqrt{(a-x)(x-b)}$ (d) $\frac{1}{\sqrt{(a-x)(b-x)}}$

47. If $\sin^{-1}\left(\frac{x^2 - y^2}{x^2 + y^2}\right) = \log a$, then $\frac{d^2y}{dx^2}$ equals

(a) $\frac{x}{y}$ (b) $\frac{y}{x^2}$

(c) $\frac{y}{x}$ (d) 0

48. A man 1.6 m high walks at the rate of 30m/min away from a lamp which is 4 m above ground. How fast is the man's shadow lengthening?

(a) 22m/min (b) 20m/min

(c) 15m/min (d) 25m/min

49. The value P such that the length of subtangent and subnormal is equal for the curve $y = e^{Px} + Px$ at the point $(0, 1)$ is

(a) $P = \pm 1$ (b) $P = \pm 2$

(c) $P = \pm\frac{1}{2}$ (d) None of these

50. AB is a diameter of a circle and C is any point on the circumference of the circle. then

(a) The area of ΔABC is maximum when it is isosceles

(b) The area of ΔABC is minimum when it is isosceles

(c) The perimeter of ΔABC is minimum when it is isosceles

(d) None of these

51. The value of $\tan\left\{\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right\}$ is

(a) $\frac{3+\sqrt{5}}{2}$ (b) $3+\sqrt{5}$

(c) $\frac{1}{2}(3-\sqrt{5})$ (d) None of these

52. The interval for which $2\tan^{-1}x + \sin^{-1}\frac{2x}{1+x^2}$ is independent of x is

(a) $|x| < 1$ (b) $|x| > 1$

(c) $|x| = 1$ (d) ϕ

53. The number of solutions of the equation $1 + \sin x \cdot \sin^2\frac{x}{2} = 0$ in $[-\pi, \pi]$ is

(a) zero (b) 1

(c) 2 (d) 3

54. If $x, y \in [0, 2\pi]$ then the total number of ordered pairs (x, y) satisfying $\sin x \cdot \cos y = 1$ is equal to

(a) 1 (b) 3

(c) 5 (d) 7

55. $\int x^x \log(ex)\, dx$ is equal to

(a) $x^x + c$ (b) $x \cdot \log x + c$

(c) $(\log x)^x + c$ (d) $x^{\log x} + c$

56. $\int \sqrt{1 + \text{cosec}\, x}\, dx$ is equal to

(a) $\pm\sin^{-1}(\tan x - \sec x) + c$

(b) $2\sin^{-1}(\cos x) + c$

(c) $\sin^{-1}\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right) + c$

(d) $\pm 2\sin^{-1}\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right) + c$

57. If $\int \frac{dx}{x^2(x^n + 1)^{\frac{(n-1)}{n}}} = -[f(x)]^{1/n} + c$ then $f(x)$ is

(a) $1 + x^n$ (b) $1 + x^{-n}$

(c) $x^n + x^{-n}$ (d) None of these

58. The value of

$$\lim_{n \to \infty} \left\{ \frac{1}{na} + \frac{1}{na+1} + \frac{1}{na+2} + \ldots + \frac{1}{n_b} \right\} \text{ is}$$

(a) $\log(ab)$ (b) $\log(a/b)$
(c) $\log(b/a)$ (d) $-\log(a/b)$

59. The value of $\int_a^b \frac{|x|}{x} dx$ is

(a) $|b| - |a|$ (b) $|a| - |b|$
(c) $|b| + |a|$ (d) $-|b| - |a|$

60. $y = \int_{1/8}^{\sin^2 x} \sin^{-1} \sqrt{t} dt + \int_{1/8}^{\cos^2 x} \cos^{-1} \sqrt{t}\ dt$, $0 \le x \le x/2$.

(a) Is the equation of a straight line parallel to the x-axis
(b) Is the euqation of a straight line which is the bisector of first quadrant
(c) Is the equation of a straight line which is the bisector of second quadrant
(d) None of the above

61. The area of the region

$$[(x, y): x^2 + y^2 \le 1 \le x + y] \text{ is;}$$

(a) $\frac{\pi}{5}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi^2}{3}$ (d) $\frac{\pi}{4} - \frac{1}{2}$

62. The area of the figure bounded by the parabola $(y-2)^2 = x - 1$, the tangent to it at the point with the ordinate 3 and the x-axis is

(a) 3 (b) 6
(c) 9 (d) None of these

63. The degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x \sin\left(\frac{d^2y}{dx^2}\right) \text{ is}$$

(a) 1 (b) 2
(c) 3 (d) None of these

64. The slope of the tangent at (x, y) to a curve passing through $\left(1, \frac{\pi}{4}\right)$ is given by $\frac{y}{x} - \cos^2\left(\frac{y}{x}\right)$ then the equation of the curve is

(a) $y = \tan^{-1}\left(\log\frac{c}{x}\right)$ (b) $y = x\tan^{-1}\left(\log\frac{x}{c}\right)$
(c) $y = x\tan^{-1}\left(\log\frac{c}{x}\right)$ (d) None of these

65. The solution of the equation

$$\frac{dy}{dx} = \frac{x(2\log x + 1)}{\sin y + y\cos y} \text{ is}$$

(a) $y\sin y = x^2 \log x + \frac{x^2}{y} + c$
(b) $y\cos y = x^2(\log x + 1) + c$
(c) $y\cos y = x^2 \log x + \frac{x^2}{2} + c$
(d) $y\sin y = x^2 \log x + c$

66. The curve for which the length of the normal is equal to the length of the radius vector, are

(a) only circles
(b) only rectangular hyperbolas
(c) either circles or rectangular hyperbolas
(d) None of the above

67. If $\frac{1}{x(x+1)(x+2)\ldots(x+n)} = \frac{A_0}{x} + \frac{A_1}{x+1} + \frac{A_2}{x+2} + \ldots + \frac{A_n}{x+n}$ then A_r is equal to

(a) $\frac{r!\,(1)^r}{(n-r)!}$ (b) $\frac{(-1)^r}{r!\,(n-r)!}$
(c) $\frac{1}{r!\,(n-r)!}$ (d) None of these

68. $\log_2(9 - 2^x) = 10^{\log(3-x)}$, solve for x.

(a) 0 (b) 3
(c) both (a) and (b) (d) 0 and 6

69. 3 numbers are in GP therefore, their logarithms are in

(a) GP (b) HP
(c) AP (d) None of these

70. In an equilateral triangle, the in-radius, circum-radius and one of the ex-radii are in the ratio

(a) 2 : 3 : 5 (b) 1 : 2 : 3
(c) 1 : 3 : 7 (d) 3 : 7 : 9

71. In a triangle, the length of the two larger sides are 24 and 22, respectively. If the angles are in AP, then the third side is

(a) $12 + 2\sqrt{3}$
(b) $12 - 2\sqrt{3}$
(c) $2\sqrt{3} + 2$
(d) $2\sqrt{3} - 2$

72. G is a set of all rational numbers except -1 and $*$ is defined by $a * b = a + b + ab$ for all $a, b \in G$, in the group $(G, *)$, the solution of $2^{-1} * x * 3^{-1} = 5$ is

(a) 71 (b) 68 (c) 63/5 (d) 72/5

73. If G is an abelian group then for all $a, b \in G$ $b^{-1} * a^{-1} * b * a$ is equal to

(a) $a * b$
(b) $a^{-1} * b^{-1}$
(c) e
(d) None of these

74. If $(G, *)$ is a group such that $(a * b)^2 = (a * a) * (b * b)$ for all $a, b * G$, then G is

(a) abelian
(b) finite
(c) infinite
(d) None of these

75. A graph which has no edges or node is known as

(a) Digraph
(b) Mixed graph
(c) Nullgraph
(d) Multigraph

76. If all edges are directed in a graph then it is called as

(a) Null graph
(b) Digraph
(c) Mixed graph
(d) None of these

77. Find the greatest value of xyz for positive values of x, y, z subject to the condition $xy + yz + zx = 12$.

(a) 64 (b) 8 (c) 16 (d) 32

78. a, b, c are prime numbers, x is an even number, y is an odd number. Which of the following is/are never true?

I. $a + x = b$ II. $b + y = c$
III. $ab = c$ IV. $a + b = c$

(a) I and II
(b) II and III
(c) Only III
(d) III and IV

79. $(p \wedge \sim q) \wedge (\sim p \wedge q)$

(a) A tautology
(b) A contradiction
(c) Both a tautology and a contradiction
(d) Neither a tautology nor a contradiction

80. $(\sim p \wedge q)$ is logically equivalent to

(a) $p \to q$
(b) $q \to p$
(c) $\sim(p \to q)$
(d) $\sim(q \to p)$

English

Directions (Q. Nos. 1-5) *In the following questions (a) part of the sentence is printed in bold. Below are given alternatives to the bold part at (a), (b) and (c), which may improve the sentence. Choose the correct alternative. In case no improvement is needed your answer is (d).*

1. No sooner **he reached** home than all the villagers gathered at his home to listen to his story.

(a) would he reach
(b) did he reach
(c) had he reached
(d) No improvement

2. Because of his mastery in this field, his suggestions **are wide accepted.**

(a) are widely accepted
(b) are wide acceptance
(c) have widely accepted
(d) No improvement

3. You are warned **against committing** the same mistake again.

(a) for committing
(b) against to commit
(c) to commit
(d) No improvement

4. The teacher gave the students some **advice**.

(a) advise
(b) advises
(c) advices
(d) No improvement

5. I shall **ring him** tomorrow in the afternoon.

(a) ring to him
(b) ring up to him
(c) ring him up
(d) No improvement

Directions (Q. Nos. 6-10) *In the following questions, out of the four alternatives, choose the one, which best expresses the meaning of the given word.*

6. RADIANT

(a) Bright
(b) Beautiful
(c) Plight
(d) Influential

7. PRUNE

(a) Lend (b) Reduce
(c) Expand (d) Prolong

8. DILETTANTE

(a) Opponent (b) Specialist
(c) Amateur (d) Expert

9. FOSTER

(a) Encourage (b) Fabricate
(c) Forment (d) Nurture

10. ENIGMA

(a) Elusive (b) Clear (c) Puzzle (d) Praise

Directions (Q. Nos. 11-15) *In the following questions, choose the word opposite in meaning to the given word.*

11. CONFORM

(a) Disappoint (b) Reform
(c) Deform (d) Dissent

12. ABORIGINAL

(a) Modern (b) Popular
(c) Current (d) Contemporary

13. AMENABLE

(a) Stubborn (b) Docile
(c) Obedient (d) Offensive

14. ACQUIT

(a) Confirm (b) Blame
(c) Punish (d) Indict

15. FORBIDDEN

(a) Allowed
(b) Prohibited
(c) Agreed
(d) Foresaken

Directions (Q. Nos. 16-20) *In the following questions, four alternatives are given for the idiom/phrase printed in bold in the sentence. Choose the alternative, which best expresses the meaning of the idiom/Phrase.*

16. Rahul **fought tooth and nail** to save his company.

(a) with weapons
(b) as best as he could
(c) using unfair means
(d) with strength and fury

17. Mr Roy is known as a **shop-lifter** in the city commercial centre.

(a) daily visitor
(b) buyer of all new things
(c) smuggler
(d) a thief in guise of customer

18. We should guard against our **green-eyed** friends.

(a) rich (b) jealous
(c) handsome (d) enthusiastic

19. This place affords **a bird's eye view** of the green valley below.

(a) beautiful view (b) general view
(c) narrow view (d) ugly view

20. I won't mind even if he **goes to dogs**.

(a) goes mad (b) is insulted
(c) is ruined (d) becomes brutal

Directions (Q. Nos. 21-25) *In the following questions out of the four alternatives, choose the one, which can be substituted for the given words/sentence.*

21. A person who lives alone and avoids other people.

(a) Ascetic (b) Recluse
(c) Unsocial (d) Agnostic

22. Explicit undertaking to do something.

(a) Agreement (b) Decision
(c) Settlement (d) Promise

23. Murder of a king.

(a) Matricide
(b) Genocide
(c) Regicide
(d) Homicide

24. A person interested in reading books and nothing else.

(a) Student (b) Book worn
(c) Scholar (d) Book-keeper

25. A book or picture produced merely to bring in money.

(a) Money-spinner
(b) Pot-hook
(c) Pot-boiler
(d) Blue-bird

Directions (Q. Nos. 26-30) *In the following questions, you have to rearrange the parts P, Q, R and S to produce a proper sentence.*

26. (P) took place
(Q) when militants opened fire on BSF men
(R) the encounter lasting over two hours
(S) in the town around 8.30 am
(a) QPRS (b) QSRP
(c) RPQS (d) RPSQ

27. They are plant eaters
(P) and various kinds of vegetation
(Q) browsing on grass
(R) and consume
(S) vast quantities of pasture
(a) QPRS (b) QSRP
(c) RSPQ (d) RSQP

28. (P) is becomes difficult
(Q) satisfying our desire
(R) but once we set about
(S) if not impossible to restrain them
(a) PQRS (b) QRPS (c) RQPS (d) RQSP

29. The captain
(P) when engaged against the enemy
(Q) who was himself a brave man
(R) never to lose heart
(S) advised the soliders
(a) PQSR (b) QPSR
(c) QSRP (d) SQRP

30. Some remarks
(P) put the police on the right scent
(Q) by a woman
(R) and they discoverd
(S) casually dropped
(6) the whole gang of brigands
(a) QSPR
(b) RPSQ
(c) RQSP
(d) SQPR

Directions (Q. Nos. 31-35) *In the following questions, some of the sentences have errors and some have none. Find out which part of a sentence has an error. The letter of that part is your answer. If there is no error, your answer is (d) i.e., 'No error'.*

31. Unless you do not listen to his advice (a)/I am not going (b)/ to help you. (c)/No error (d)

32. The teacher called Ravi (a)/and asked him (b)/to describe about the incident. (c)/No error (d)

33. Be smart (a)/not only in dress (b)/and also inaction. (c)/No error (d)

34. The reason for (a)/his failure is because (b)/he did not work hard. (c)/No error (d)

35. If I was you (a)/I would have (b)/terminated his services them and there. (c)/No error (d)

Directions (Q. Nos. 36-40) *In the following questions, groups of four words are given. In each group, one word is wrong spelt. Find the missplet word.*

36. (a) Laudable (b) Honourable
(c) Lovable (d) Honourary

37. (a) Behaviour (b) Commend
(c) Mentenance (d) Appraise

38. (a) Focal
(b) Vocal
(c) Vehical
(d) Mystical

39. (a) Dairy
(b) Dafodil
(c) Dainty
(d) Damage

40. (a) Cureable
(b) Currency
(c) Campaign
(d) Chronicle

Answers

Physics

1. (d)	**2.** (c)	**3.** (a)	**4.** (d)	**5.** (c)	**6.** (d)	**7.** (a)	**8.** (a)	**9.** (a)	**10.** (b)
11. (a)	**12.** (a)	**13.** (d)	**14.** (a)	**15.** (c)	**16.** (c)	**17.** (c)	**18.** (b)	**19.** (b)	**20.** (c)
21. (c)	**22.** (c)	**23.** (b)	**24.** (d)	**25.** (d)	**26.** (a)	**27.** (a)	**28.** (a)	**29.** (c)	**30.** (b)
31. (b)	**32.** (d)	**33.** (c)	**34.** (c)	**35.** (a)	**36.** (c)	**37.** (a)	**38.** (a)	**39.** (c)	**40.** (c)
41. (a)	**42.** (a)	**43.** (c)	**44.** (a)	**45.** (a)	**46.** (a)	**47.** (a)	**48.** (b)	**49.** (d)	**50.** (a)
51. (c)	**52.** (b)	**53.** (d)	**54.** (d)	**55.** (c)	**56.** (a)	**57.** (a)	**58.** (c)	**59.** (c)	**60.** (c)

Chemistry

1. (a)	**2.** (d)	**3.** (a)	**4.** (b)	**5.** (b)	**6.** (d)	**7.** (a)	**8.** (b)	**9.** (d)	**10.** (d)
11. (c)	**12.** (b)	**13.** (d)	**14.** (d)	**15.** (d)	**16.** (a)	**17.** (c)	**18.** (b)	**19.** (c)	**20.** (a)
21. (d)	**22.** (a)	**23.** (b)	**24.** (b)	**25.** (d)	**26.** (c)	**27.** (d)	**28.** (a)	**29.** (c)	**30.** (b)
31. (a)	**32.** (b)	**33.** (c)	**34.** (c)	**35.** (b)	**36.** (a)	**37.** (d)	**38.** (a)	**39.** (c)	**40.** (c)
41. (a)	**42.** (b)	**43.** (c)	**44.** (a)	**45.** (b)	**46.** (c)	**47.** (a)	**48.** (b)	**49.** (d)	**50.** (b)
51. (a)	**52.** (b)	**53.** (b)	**54.** (a)	**55.** (d)	**56.** (c)	**57.** (a)	**58.** (d)	**59.** (a)	**60.** (c)

Mathematics

1. (b)	**2.** (c)	**3.** (a)	**4.** (b)	**5.** (b)	**6.** (b)	**7.** (d)	**8.** (a)	**9.** (a)	**10.** (c)
11. (a)	**12.** (c)	**13.** (c)	**14.** (b)	**15.** (d)	**16.** (c)	**17.** (c)	**18.** (b)	**19.** (c)	**20.** (a)
21. (a)	**22.** (c)	**23.** (b)	**24.** (a)	**25.** (a)	**26.** (d)	**27.** (a)	**28.** (a)	**29.** (c)	**30.** (a)
31. (c)	**32.** (a)	**33.** (a)	**34.** (c)	**35.** (c)	**36.** (b)	**37.** (a)	**38.** (b)	**39.** (a)	**40.** (b)
41. (d)	**42.** (b)	**43.** (c)	**44.** (c)	**45.** (d)	**46.** (b)	**47.** (d)	**48.** (b)	**49.** (c)	**50.** (a)
51. (c)	**52.** (b)	**53.** (a)	**54.** (b)	**55.** (a)	**56.** (d)	**57.** (b)	**58.** (c)	**59.** (a)	**60.** (a)
61. (d)	**62.** (c)	**63.** (d)	**64.** (c)	**65.** (d)	**66.** (c)	**67.** (b)	**68.** (a)	**69.** (c)	**70.** (b)
71. (a)	**72.** (a)	**73.** (c)	**74.** (a)	**75.** (d)	**76.** (b)	**77.** (b)	**78.** (c)	**79.** (b)	**80.** (d)

English

1. (b)	**2.** (a)	**3.** (d)	**4.** (d)	**5.** (c)	**6.** (a)	**7.** (b)	**8.** (c)	**9.** (d)	**10.** (c)
11. (d)	**12.** (a)	**13.** (a)	**14.** (d)	**15.** (a)	**16.** (d)	**17.** (d)	**18.** (b)	**19.** (b)	**20.** (c)
21. (b)	**22.** (d)	**23.** (c)	**24.** (b)	**25.** (c)	**26.** (c)	**27.** (a)	**28.** (c)	**29.** (c)	**30.** (d)
31. (a)	**32.** (c)	**33.** (c)	**34.** (b)	**35.** (a)	**36.** (d)	**37.** (c)	**38.** (c)	**39.** (b)	**40.** (a)

Hints & Solutions

Physics

1. Rate of change of displacement is known as velocity. The equation for displacment (y) of a body in SHM with angular velocity ω is given by

$$y = a\sin \omega t$$

where a is amplitude

velocity v = rate of change of displacement $= \dfrac{dy}{dt}$

$$v = \frac{dy}{dt} = \frac{d}{dt}(a\sin \omega t) = a\omega \cos \omega t$$

Given, $v_1 = v_2 = v_3$ and $a_1 = A_1$, $a_2 = A_2$.

$$a_3 = A_3$$

$$A_1\omega_1 = A_2\omega_2 = A_3\omega_3$$

2. The unit dyne/cm is CGS system of unit.

The CGS unit of force is dyne and SI unit is Newton. Also there are 10^5 dynes in one newton and 100 cm in 1 cm.

$$\therefore \quad \frac{70\text{ dynes}}{\text{cm}} = \frac{70 \times 10^{-5}}{10^{-2}} = 7 \times 10^{-2}\text{ N/m}$$

3. The general gas equation is $\dfrac{p_1 V_1}{T_1} = \dfrac{p_2 V_2}{T_2}$ = gas constant

where p_1, V_1, p_2, V_2 are pressure and volume at temperatures T_1 and T_2 respectively.

Given $V_1 = 1500\text{ m}^3$, $T_1 = 27 + 273 = 300\text{ K}$,

$p_1 = 4$ atm.

$T_2 = -3°\text{C} = 273 - 3 = 270\text{ K}$, $p_2 = 2$ atm.

$$\therefore \quad V_2 = \frac{p_1 V_1}{T_1} \times \frac{T_2}{p_2}$$

$$V_2 = \frac{4 \times 1500}{300} \times \frac{270}{2}$$

$$V_2 = 2700\text{ m}^3$$

4. Work done against frictional force equals the kinetic energy of the body.

when a body of mass m, moves with velocity v, it has kinetic energy $k = \dfrac{1}{2}mv^2$, this energy is utilized in doing work against the frictional force between the tyres of the car and road.

$\therefore$ kinetic energy = work done against friction force

$$\frac{1}{2}m^2 = \mu\ mgs$$

where s is the distance in which the car is stopped and μ is coefficient of kinetic friction.

Given $\quad v = v_0$

$$s = \frac{v_0^2}{2\mu\ g}.$$

5. Average velocity is equal to ratio of total displacement to total time. Velocity is a vector quantity and average velocity is defined as the total displacement undergone by the body per unit time.

Displacement in first instance $= v_1 t$

Displacement in second instance $= v_2\ t$

$$\therefore \quad \text{Average velocity} = \frac{v_1\ t + v_2\ t}{t + t} = \frac{v_1 + v_2}{2}$$

6. If the maximum kinetic energy of photoelectron emitted from the surface of a metal is E_K and W is the work function of the metal, the from Einstein's photoelectric equation we have

$$E_K = h\nu - W$$

where, $h\nu$ is the energy of the photon absorbed by the electron in the metal.

Also $\quad \nu = \dfrac{c}{\lambda}$

$$= \frac{\text{velocity}}{\text{wavelength}}$$

$$= E_K = \frac{hc}{\lambda} - \frac{hc}{\lambda_0} = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

Given, $\lambda = 3 \times 10^{-7}$ m,

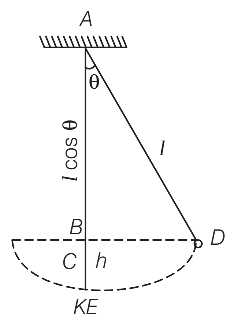
$\lambda_0 = 4 \times 10^{-7}$ m.

$$\therefore \quad E_K = 6.6 \times 10^{-34} \times 3 \times 10^8 \times \left(\frac{1}{3 \times 10^{-7}} - \frac{1}{4 \times 10^{-7}}\right)\text{ J}$$

$$= \frac{19.8 \times 10^{-19}}{12 \times 1.6 \times 10^{-19}}\text{ eV}$$

$$= 1.03 \approx 1\text{eV}$$

7. In the equilibrium position energy is in the form of kinetic energy.



Height of bob at maximum angular displacement is

$$h = l - l\cos\theta$$

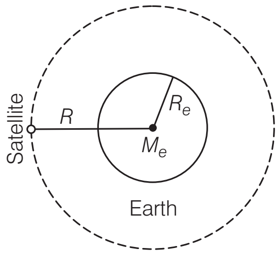
$$= l(1 - \cos\theta)$$

Also PE = KE

$$mgh = mgl(1 - \cos\theta)$$

8. Gravitational force provides the required centripetal force.

The gravitational force provides the required centripetal force in orbit of earth.



$$\therefore \quad \frac{GM_e M}{R^2} = \frac{mv_0^2}{R}$$

$$v_0 = \sqrt{\frac{GM_e}{R}}$$

$$\text{kinetic energy} = \frac{1}{2} mv_0^2$$

$$\therefore \quad \text{KE} = \frac{1}{2} m\left(\frac{GM_e}{R}\right)^{2/2}$$

$$= \frac{1}{2} \frac{mGM_e}{R}$$

$$\text{KE} \propto \frac{1}{R}$$

9. The surface tension (T) of a liquid is equal to the work (W) required to increase the surface area of the liquid film by unity at constant temperature.

As per key idea, tension

$$= \frac{\text{work done}}{\text{surface area}}$$

or $$T = \frac{W}{\Delta A}$$

Since soap bubble has two surface and surface area of soap bubble is $4\pi R^2$

where R is radius of bubble.

then $W = T \times 2 \times 4\pi R^2$

Given $R = 2R$

therefore $W = T \times 2 \times 4\pi \times (2R)^2$

$W = 32\ \pi R^2 T$

10. Form Einstein's mass energy relation

$$E = \Delta mc^2 \quad \ldots\text{(i)}$$

where Δm is mass lost, c is speed of light. Also heat given by body is

$$E = mC\Delta\theta \quad \ldots\text{(ii)}$$

where, C is specific heat.

Equating Eqs. (i) and (ii) we get

$$\Delta m = \frac{MC\Delta\theta}{C^2} = \frac{M \times 0.2 \times 100 \times 4.2 \times 10^3}{(3 \times 10^8)^2}\ \text{J}$$

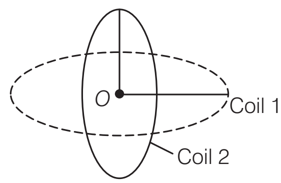
$$\frac{\Delta m}{M} = \frac{20 \times 4.2 \times 10^3}{(3 \times 10^8)^2}$$

% increase in mass

$$= \frac{\Delta m}{M} \times 100 = \frac{20 \times 4.2 \times 10^3}{(3 \times 10^8)^2} \times 100$$

$$= 9.3 \times 10^{-11}\%$$

11. Where the two coils are kept perpendicular to each other then resultant field in one coil is directed along the axis of coil and the other downwards, hence resultant field is

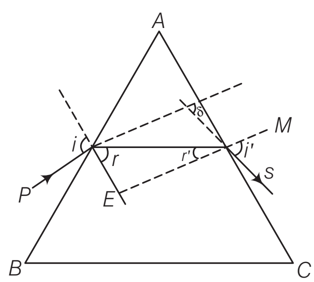


$$B' = \sqrt{B^2 + B^2} = \sqrt{2}\ B$$

Ratio of magnetic field due to one coil and resultant is $\dfrac{B}{B\sqrt{2}} = \dfrac{1}{\sqrt{2}}$

12. In the position of minimum deviation angle of incidence is equal to angle of emergence.

Let a ray of monochromatic light PQ be incident on face AB. $PQRS$ is path of light ray,



where i is angle of incidence r angle of refraction, r angle of incidence and i angle of emergence.

In position of minimum deviation

$$i' = i,\ r' = r,\ \delta = \delta m$$

$$2r = A \text{ or } r = \frac{A}{2}$$

Given $A = 60°$,

$$r = \frac{60°}{2} = 30°$$

Also from Snell's law

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin i}{\sin 30°}$$

$$\sqrt{2} = \frac{\sin i}{\sin 30°}$$

$$\Rightarrow \quad \sin i = \sqrt{2} \times \frac{1}{2} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad i = 45°$$

13. Net acceleration on the elevator car is more than acceleration due to gravity.

Since elevator car is ascending upwards, from Newton's second law, the net force is acting upwards hence resultant acceleration is

$$a_r = (g + a)$$
$$= (9.8 + 1.2) = 11 \text{ m/s}^2$$

Relative velocity of observer to elevator is

$$u_r = 0$$

from the equation

$$s = u_r t + \frac{1}{2} a_r t^2$$

$$2.7 = 0 + \frac{1}{2} \times 11 \times t^2$$

$$t^2 = \frac{5.4}{11}$$

$$t = \sqrt{\frac{5.4}{11}} \text{ s}$$

14. From equation of motion

$$h = ut + \frac{1}{2} gt^2$$

where u is initial velocity and t is time.

Since, $u = 0$

$$\therefore \quad t = \sqrt{\frac{2h}{g}} = \frac{\sqrt{2 \times 4.9}}{9.8} = 1 \text{ s}$$

The horizontal range of the drop = x, then

$$x = \left(\frac{v_t}{o}\right) t$$

Also, $\omega = \frac{\Delta\theta}{\Delta t} = \frac{21 \times 2\pi}{44} = 3 \text{ rad/s}$

Tangential speed $v_t = r\omega = 0.5 \times 3 \times 1.5$ m/s

$$x = 1.5 \times 1 = 1.5 \text{ m}$$

Locus of drop = $\sqrt{x^2 + r^2} = \sqrt{(1.5)^2 + (0.5)^2}$

$$= \sqrt{2.5} \text{ m}$$

15. Kinetic energy of rotation is half the product of the moment of inertia (I) of the body and the square of the angular velocity (ω) of the body.

Kinetic energy of rotation = $\frac{1}{2} \times$ moment of inertia $\times$ angular velocity

i.e., $K = \frac{1}{2} I \omega^2$

$$\omega^2 = \frac{2K}{I}$$

Given $I = 1.2$ kg m^2, $K = 1500$ J

$$\omega^2 = \frac{2 \times 1500}{1.2}$$

$$\Rightarrow \quad \omega = 50 \text{ rad/s}$$

From the equation of angular motion, we have

$$\omega = \omega_0 + \alpha t$$

where ω_0 is initial angular velocity, α is angular acceleration and t is time

given $\omega_0 = 0$,

$\omega = 50$ rad/s,

$\alpha = 25$ rad/s^2

$$t = \frac{\omega}{\alpha} = \frac{50}{25} = 2s$$

16. The kinetic energy of moving body is equal to half the product of the mass (m) of the body and the square of its speed (v^2).

kinetic energy = $\frac{1}{2} \times$ mass $\times$ (speed)^2

i.e., $= \frac{1}{2} mv^2$

Let mass of man is M kg and speed is v and speed of body is v_1, then

$$\frac{1}{2} Mv^2 = \frac{1}{2}\left\{\frac{1}{2}\frac{M}{2} v_1^2\right\} \quad \ldots\text{(i)}$$

when man speed up by 1 m/s, than

$$v = v + 1$$

$$\therefore \quad \frac{1}{2} M (v+1)^2 = \frac{1}{2}\left\{\frac{M}{2}\right\} \cdot v_1^2 \quad \ldots\text{(ii)}$$

Dividing Eqs. (i) by (ii), we get

$$\frac{v^2}{(v+1)^2} = \frac{1}{2}$$

$$\sqrt{2}v = v + 1$$

$$v = \frac{1}{\sqrt{2} - 1} \text{ m/s}$$

17. Larger the distance of planet from the sun, larger will be its period of revolution around the sun.

Kepler's third low planetary motion the square of the period of revolution of any planet around the sun is directly proportional to the cube of the semi-major axis of its elliptical orbit

$$\therefore \quad T^2 \propto R^3$$

$$\frac{T_s}{T_c} = \left(\frac{R_s}{R_c}\right)^{3/2}$$

given $R_s = 4R_c$

$$\frac{T_s}{T_c} = \left(\frac{4R_c}{R_c}\right)^{3/2} = 8$$

for $T_c = 1$ day

$T_s = 8$ days.

18. Force experienced by the surface is equal to change in momentum in one second.

From Newton's second law of motion, the rate of change of momentum of a body is equal to the external force applied on the body and the change in momentum always takes place in the direction of the force.

Momentum of ball before impact = mu

Momentum after impact = $-mu$

Change in momentum = $mu - (-mu) = 2mu$

Change in momentum due to n balls in one second is = $2mnu$.

Change of momentum = force experienced per second = $2mnu$.

19. When no external force acts on a body its angular moment remains conserved.

From law of conservation of angular momentum, we have

$$J = I\omega = \text{constant}$$

where I is moment of inertia of the body and ω is angular velocity.

Also $I = \frac{1}{2}MR^2$ (for disc) where M is mass and R is radius of disc.

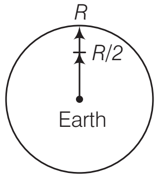
$$J = \frac{1}{2}MR^2\,\omega = \text{constant}$$

Since, man moves towards the centre of the disc, distance of mass distribution R decreases hence, ω increases.

20. Use parallel axis theorem, from the theorem of parallel axis, the moment of inertia (I) of a body about given axis is equal to its moment of inertia I about its diameter, plus the product of the mass M of the body and square of perpendicular distance between the two axes. That is

$$I = I_d + MR^2 = \frac{1}{4}MR^2 + MR^2 = \frac{5}{4}MR^2$$

21. Value of acceleration due to gravity decreases on going below the surface of the earth.



weight (w) of a body is defined as product of mass (m) and acceleration due to gravity (g)

$w = mg$

The value of g below the surface of earth at a distance h is given by

$$g' = g\left(1 - \frac{h}{R}\right)$$

$$\Rightarrow \quad W' = W\left(1 - \frac{h}{R}\right)$$

Given, $h = \frac{R}{2}$, $W = 500$ N

$$W' = 500\left(1 - \frac{1}{2}\right)$$

$$= \frac{500}{2} = 250\text{ N}$$

22. At a crtain velocity of projection the body will go out of the gravitational field of the earth and will never return to the earth, this velocity is known as escape velocity.

$$v_e = \sqrt{\frac{2GM_e}{R_e}}$$

Given, $M_e = M_p,\ R_p = \frac{R_e}{4}$

$$\therefore \quad \frac{v_p}{v_e} = \sqrt{\frac{M_e}{M_e} \times \frac{R_e}{R_e/4}} = \sqrt{4} = 2$$

$$\Rightarrow \quad v_p = 2v_e = 2 \times 11.2$$

$$= 22.4\text{ km/s}$$

23. Since liquid is incompressible there is no change in volume.

When strain is small the ratio of the normal stress to the volume strain is called the bulk modulus of material of the body.

$$B = \frac{\text{normal stress}}{\text{volume strain}}$$

$$= \text{normal stress}$$

change in volume/original volume

since, the liquid is incompressible change in volume is zero.

$$B = \frac{\text{normal stress}}{0} = \infty$$

Hence, bulk modulus is infinite.

24. The narrower the tube, the higher is the rise in water.

when a glass capillary

$$h = \frac{2T\cos\theta}{R\rho g}$$

$$h \propto \frac{1}{R}$$

$$\frac{h_1}{h_2} = \frac{R_2}{R_1}$$

$$\frac{3}{h_2} = \frac{1}{3}$$

$$h_2 = 9\text{ mm}$$

25. $u = -20\,\text{cm}, f = 20\,\text{cm}$

From mirror formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{20} = \frac{1}{v} + \frac{1}{-20}$$

$$\frac{1}{v} = \frac{1}{20} + \frac{1}{20}$$

$$\frac{1}{v} = \frac{2}{20} = v = 10\,\text{cm}$$

26. According to law of conservation of angular momentum if there is no torque on the system, then the angular momentum remains constant.

27. Equivalent focal length

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$= \frac{1}{20} + \frac{1}{30}$$

$$F = \frac{20 \times 30}{20 + 30} = \frac{600}{50} = 12\text{ cm}$$

$$F = 12\text{ cm}$$

28. Work done $W = \text{F} \times \text{d}$

$$= (6\,\text{i} + 2\,\text{j}) \times (3\,\text{i} - \text{j})$$

$$= 6 \times 3 + 2 \times -1$$

$$= 18 - 2 = 16\,\text{J}$$

$$W = 16\,\text{J}$$

29. From Einstein's theorem

Relativistic momentum $= \dfrac{m_0 v}{\sqrt{1 - v^2/c^2}}$

Putting $v = 2v$

we get $p = \dfrac{m_0\,(2v)}{\sqrt{1 - (2v)^2/c^2}}$

$$= 2\left(\frac{m_0 v}{\sqrt{1 - (2v)^2/c^2}}\right)$$

So the momentum becomes more than the double.

30. Position $x = Ka^m\, t^n$

Writing the dimension on the both sides.

$$[\text{M}^0\ \text{L}\ \text{T}^0] = [\text{L}\ \text{T}^{-2}]^m\ [\text{T}]^n$$

$$[\text{M}^0\ \text{L}^m\ \text{T}^{-2m+n}]$$

On comparing both sides.

$$m = 1$$

$$-2m + n = 0$$

$$n = 2m = 2 \times 1 = 2$$

31. We know that

$$\alpha = \left(\frac{\Delta i_C}{\Delta i_E}\right) \qquad \ldots\text{(i)}$$

and $$\beta = \left(\frac{\Delta i_C}{\Delta i_B}\right) = \frac{\Delta i_C}{\Delta i_E} \times \frac{\Delta i_E}{\Delta i_B}$$

$$= \alpha\,\frac{\Delta i_E}{\Delta i_B} \qquad \ldots\text{(ii)}$$

But $\Delta i_B = \Delta i_E - \Delta i_C$

from (ii)

$$\beta = \alpha\,\frac{\Delta i_E}{\Delta i_E - \Delta i_C} = \frac{\alpha}{1 - \dfrac{\Delta i_C}{\Delta i_E}}$$

$\therefore$ $$\beta = \frac{\alpha}{1 - \alpha}$$

32. $v_r^2 = v_m^2 - v^2$

$v = \dfrac{320}{4}$ m/min = 80 m/min

$$v_m = \frac{5}{3}\,v_r$$

$$v_r^2 = t\,\frac{5}{3}\,(v_r)^2 - (80)^2$$

$$\frac{16}{9}\,v_r^2 = (80)^2$$

$$v_r = 60\,\text{m/min}$$

u^2, u_0 form canours linear

33. Let the radii of the thin spherical and the solid sphere are R_1 and R_2 respectively.

Then the moment of inertia of the spherical shell about their diameter

$$I = \frac{2}{3}\,MR_1^2 \qquad \ldots\text{(i)}$$

and the moment of inertia of the solid sphere is given by

$$I = \frac{2}{5}\,MR_2^2 \qquad \ldots\text{(ii)}$$

Given that the masses and moment of inertia for both the bodies are equal, then from Eqs. (i)and (ii)

$$\frac{2}{3}\,MR_1^2 = \frac{2}{5}\,MR_2^2 \Rightarrow \frac{R_1^2}{R_2^2} = \frac{3}{5}$$

$\Rightarrow$ $$\frac{R}{R_2} = \sqrt{\frac{3}{5}} \Rightarrow R_1 : R_2 = \sqrt{3} : \sqrt{5}$$

34. The apparent weight of person on the equator (latitude $\lambda = 0$) is given by

$$\omega' = \text{w} - mR\omega^2$$

Here, $\therefore$ $\omega' = (3/5)\,\text{w} = (3/5)\,\text{mg}$ $\quad [\because\ \omega = mg]$

or $mR\omega^2 = mg - (3/5)\,mg = \left(\frac{2}{5}\right) mg$

or $\omega^2 = \frac{2g}{5R}$

$$\omega = \sqrt{\frac{2g}{5R}}$$

Here, $g = 9.8\,\text{ms}^{-2}$

and $R = 6400\,\text{km} = 6400 \times 10^3$ m

$\therefore$ $$\omega = \sqrt{\left(\frac{2}{5} \times \frac{9.8}{6400 \times 10^3}\right)}\ \text{rads}^{-1}$$

$$= 7.82 \times 10^{-4}\ \text{rads}^{-1}$$

35. From Bernoulli's theorem

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{\alpha}\rho v_2^2$$

$\therefore$ $$p_1 - p_2 = \frac{1}{2}\rho\,(v_2^2 - v_1^2)$$

$\therefore$ $$10 = \frac{1}{2} \times 1.25 \times 10^3\,(v_2^2 - v_1^2)$$

$\therefore$ $$v_2^2 - v_1^2 = \frac{10 \times 2}{1.25 \times 10^3} = 16 \times 10^{-3} \qquad \ldots\text{(i)}$$

Also from equation of continuity

$$= A_1 v_1 = A_2 v_2$$

$$\pi r_1^2 v_1 = \pi r_2^2\, v_2$$

$\therefore$ $$\frac{v_1}{v_2} = \left[\frac{r_2}{r_1}\right]^2 = \frac{0.04}{0.1} = 0.4$$

$$v_1 = 0.4\, v_2 \qquad \ldots\text{(ii)}$$

Substituting this value in Eq. (i)

$$v_2^2 - (0.4\, v_2)^2 = 16 \times 10^{-3}$$

$$v_2 = 1.38 \times 10^{-1} = 0.138\,\text{ms}^{-1}$$

Rate of flow of glycrine $v = A_2 v_2$

$$= \pi r_2^2 v_2$$

$$= 6.93 \times 10^{-4}\ \text{m}^3\text{s}^{-1}$$

36. $E = \sigma T^4$

$T_1 = T, \qquad T_2 = 2T$

$$\frac{E_1}{E_2} = \frac{T_1^{\,4}}{T_2^{\,4}} = \left(\frac{T}{2T}\right)^4 = \frac{1}{16}$$

$$E_2 = 16\,E$$

37. Since, police jeep and thief are moving in the same direction relative velocity is

$$v_\sigma = v_1 - v_2 = 153 - 45$$

$$= 108\ \text{km/h}$$

$$= 108 \times \frac{5}{18} = 30\ \text{min}$$

38. $E = \frac{-mz^2e^4}{8\,\varepsilon_0\,h^2} \cdot \frac{1}{n^2}$

$Z_H = 1,$

$Z_{He} = 2$

$$\frac{E_H}{E_{He}} = \frac{Z_H^2}{Z_{He}^2} = \frac{1}{4}$$

$$E_H = \frac{Z_H^2}{Z_{He}^2} = \frac{2}{4}$$

$E_{He} = 4E_H$

$E_H = E_n$

$E_{He} = 4E_n$

39. $W = \frac{hc}{\lambda}$

where h is planck's constant, c is speed of light, λ is wavelength

$$\lambda = \frac{hc}{W} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{6.825 \times 1.6 \times 10^{-19}}$$

$$\lambda = 1800\,\text{Å}$$

40. A field effert transistor is a unipolar device since the conduction of current is due to majority charge carriers only.

41. The periodic-time of a simple pendulum is given by

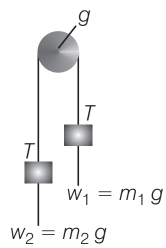
$$T = 2\pi\sqrt{\frac{l}{g}}$$

$T_1 = T\ ,\ l_1 = l,\ l_2 = 9l$

$$\frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}} = \sqrt{\frac{1}{9}} = \frac{1}{3}$$

$T_2 = 3T_1 = 3T$

42. The free body diagram of the given situation is shown.



Taking the upward and downward motions respectively, we get

$$T_1 - mg = m_1\,(g + a)$$

$$T - 2mg = m_1 a$$

for second weight

$$m_2 g - T = m_2\,(g - a)$$

$$2m_2 g - T = m_2 a$$

$$T = \frac{4m_1 m_2 g}{(m_1 + m_2)}$$

$w_1 = m_1 g,\ m_2 = m_2 g$ hence switch

$$T = \frac{4\,w_1\,w_2}{(w_1 + w_2)}$$

43. The angle subtended at the eye becomes 10 times larger, this happens only when the tree appear 10 times nearer.

44. When the switch is closed, the current grows in the circuit but due to self-inductance of the switch inductor the growth is slow. On disclosing the switch the current suddenly decrease to zero on account of the infinite resistance of the circuit. Thus, rate cf growth of the current is lower as compared to the rate of decays

45. We know that $v = \sqrt{\frac{T}{m}}$

$v \propto \sqrt{T}$

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$$

$$\frac{T_1}{T_2} = \left(\frac{v_1}{v_2}\right)^2$$

$$\frac{T_2 - T_1}{T_1} = \frac{v_1^2 - v_1^2}{v_1^2}$$

$$v_2 = v_1 + \frac{20}{100}\, v_1 = \frac{120}{100} v_1 = \frac{6}{5} v_1$$

$$= \frac{6}{5} \times 150$$

$$\frac{T_2 - T_1}{T_1} = \frac{(180)^2 - (150)^2}{(150)^2}$$

$$= \frac{30 \times 330}{150 \times 180} = 0.47$$

$$= \frac{T_2 - T_1}{T_1} \times 100 = 0.44 \times 100 = 44\%$$

46. $E - 40\,\text{GWh} = 40 \times 10^9\,\text{J/s}$

$$(60 \times 60) = 1.44 \times 10^{14}\ \text{J}$$

$$E = mc^2$$

$$m = \frac{E}{c^2} = \frac{1.44 \times 10^{14}}{(3 \times 10^8)^2} = 1 \times 10^3\ \text{kg}$$

$$m = 1.6\ \text{g}$$

47. $y = k\,t^2$

$$\frac{d^2y}{dt^2} = 2k$$

$$a = 2\ \text{m/s}^2$$

$$T_1 = 2\pi\sqrt{\frac{l}{g}}$$

$$T_2 = 2\pi\sqrt{\frac{l}{g + a_y}}$$

$$\frac{T_1^2}{T_2^2} = \frac{g + a_y}{g}$$

$$= \frac{10 + 2}{10} = \frac{6}{5}$$

48. $\text{F} = -\frac{1}{4\,\pi\,\varepsilon_0}\,\frac{q_1\,q_2}{r^2} = \text{r}$

$$\text{F} = -k\,\frac{q_1\,q_2}{r^2} = \text{r}$$

$$\text{F} = -k\,\frac{e.e}{r^2}\,\hat{\text{r}} = -k\,\frac{e^2}{r^2}\,\hat{\text{r}}$$

$$\hat{\text{r}} = \frac{\text{r}}{|\text{r}|} = \frac{\text{r}}{r}$$

$$F = -k\,\frac{e^2}{r^2} \cdot \frac{\text{r}}{r} = -k\frac{e^2}{r^3}\,\text{r}$$

49. Gauss's law

$$\phi = \frac{q}{\varepsilon_0}$$

$$\phi' = \frac{\phi}{6} = \frac{q}{6\varepsilon_0} = \frac{4\pi\,q}{6\,(4\pi\varepsilon_0)}$$

50. $\overline{A} \cdot \overline{B} = \overline{A + B}$

51. $F = mr\,\omega^2 = mr \times (2\,\pi mrn)^2 = 4\pi^2 mrn^2$

$F \propto 2n^2$

$$F = F_1 + F_2$$

$$n^2 = n_1^2 + n_2^2$$

$$n = \sqrt{n_1^2 + n_2^2}$$

$$n = \sqrt{(6)^2 + (8)^2}$$

$$n = \sqrt{100} = 10\,\text{Hz}$$

52. $K = K_0 =$ total energy

As total energy remains conserved itself hence when U is maximum in SHM, $K = 0$

i.e., E is also equal to $U_{\max}$ *i.e.*,

$$U_{\max} = E = K_0$$

53. Relative velocity of the parrot w.r.t. the train

$$[10 - (-5)] = (10 - (-5))\ \text{m/s} = 15\,\text{m/s}$$

Time taken by the parrot to cross the train

$$= \frac{150}{15} = 10\,\text{s}$$

54. Time taken by ice to grow a thickness of

$$t = \frac{eL}{2K\theta}\,y^2$$

Hence time interval to change thickness from 0 to y, from y to $2y$ and so on will be in the ratio

$$\Delta t_1 : \Delta t_2 : \Delta t_3$$

$$\therefore \quad = (1 - 0^2) : (2^2 - 1^2) : (3^2 - 2^2)$$

$$\Delta t_1 : \Delta t_2 : \Delta t_3 = 1 : 3 : 5$$

According to question $= \Delta t_1 <$ terims

$$\Delta t_2 = 3\,\Delta H$$

$$= 3 \times 12 = 36\,\text{min}$$

55. As, $\sqrt{V} = a(Z-b)$

$$\sqrt{V} = a(Z-1)$$
$$V = a^2 (Z-1)^2$$
$$\frac{C}{\lambda} = a^2(Z-1)^2$$
$$\lambda = \frac{c}{a^2(Z-1)^2}$$

$z = 43$, wavelength $= \lambda$

$$\lambda = \frac{c}{a^2(43-1)^2}$$
$$\lambda = \frac{c}{a^2 \times 42}$$

for $z = 29$, wavelength $= \lambda'$

$$\lambda' = \frac{C}{a^2(29-1)^2}$$
$$\lambda' = \frac{c}{28a^2}$$
$$\frac{\lambda'}{\lambda} = -\left(\frac{42}{28}\right)^2 = \left(\frac{3}{2}\right)^2$$
$$\lambda' = \left(\frac{9}{4}\right)\lambda$$

56. $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

$$-\frac{1}{u^2}\frac{du}{dt} - \frac{1}{v^2}\frac{dv}{dt} = 0$$
$$= -\frac{1}{u^2}u_0 - \frac{1}{v^2}v_0 = 0$$
$$\frac{u_0}{v_0} = -\frac{u^2}{v^2} = -\frac{1}{m^2}$$

57. As, $I^2R = hA$

$$I^2\left(\frac{\rho L}{\pi r^2}\right) = h \times 2\pi r l, \quad I \propto r^{3/2}$$

58. $f = \frac{\Delta V}{(\Delta Q_P)} = \frac{\Delta Q_V}{(\Delta Q_P)_p} = \frac{\mu\, Cv\, \Delta T}{\mu\, Cp\, \Delta T} = \frac{1}{T}$

$$f = \frac{7}{5}$$
$$f = \frac{5}{7}$$

59. In interference of light energy is neither created nor destroyed.

60. From colour triangle for pigment and dyes we observe that the given combination give black colour.

Chemistry

1. i for KCl = 2, i for for $CaCl_2$= 3

$$\Delta T_f \propto i$$
$$\frac{\Delta T_f(KCl)}{\Delta T_f(CaCl_2)} = \frac{2}{3}$$
$$\Delta T_f(CaCl_2) = \frac{3}{2} \times 2 = 3°C$$

Freezing point of $CaCl_2 = -3°C$

2. $\frac{r_1}{r_2} = \frac{p_1}{p_2}\sqrt{\frac{M_2}{M_1}}$

$P_{He} = x_{He} \cdot p_{total}$

$$= \frac{1}{4} \times 16 = 4 \text{ atom}$$

$P_{N_2} = x_{N_2} \cdot p_{total}$

$$= \frac{3}{4} \times 16 = 12 \text{ atom} = (p_{total}^{-} p_{He})$$
$$\frac{r_{He}}{r_{N_2}} = \frac{p_{He}}{p_{N_2}}\sqrt{\frac{M(N_2)}{M(He)}}$$
$$= \frac{4}{12}\sqrt{\frac{28}{4}} = 0.88$$

Hence, moles of He and N_2 effusing out initially are in the ratio 0.88 : 1.

3. Sc (21) = $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^1\ s^2$

19th electron goes into $4s$

Hence, $n = 4$, $l = 0$,

$m = 0$, $m_s = +\frac{1}{2}$

4. (A) + tap water $\rightarrow$ white turbidity soluble in aq. NH_3.

Tap water has Cl^- and turbidity is soluble in $aq\,NH_3$.

Turbidity is of AgCl and thus (A) has Ag^+

(A) also gives ring test of NO_3^-

(A) has nitrate and thus (A) is $AgNO_3$

5. Greater the number of ions, greater the conduction hence, order of increasing conductivity is

$c<d<b<a$

(a) $[Pt(NH_3)_6]Cl_4 \rightleftharpoons [Pt(NH_3)_6]^{4+} + 4Cl^- = 5$ ions

(b) $[Cr(NH_3)_6]Cl_3 \rightleftharpoons [Cr(NH_3)_6] + 3Cl^- = 4$ ions

(c) $[Co(NH_3)_4Cl_2]Cl \rightleftharpoons [Co(NH_3)_4Cl_2] + Cl^- = 2$ ions

(d) $K_2[PtCl_6] \rightleftharpoons 2K + [PtCl_6]^{2-} = 3$ ions

6. EAN $= Z - (O.N.) + 2x$ or $36 = 26 - 0 + 2x$ or $x = 5$

7. **For reaction *A***

$\Delta G° = 2\Delta G°_f\,(ZnO) + 2\Delta\, G°_f\,(SO_2) \;\; - 2G°_f\,(ZnS)$

$= 2\,[-318.2 - 300.4 + 205.4]$

$= -826.4$ kJ $\qquad\qquad (\Delta G°_f$ (element) $= 0)$

For reaction *B*

$\Delta G° = \Delta G°_f\,(CO) - \Delta G°_f\,(ZnO)$

$= -137.3 + 328.2$

$= +180.9$ kJ

8. Ore- copper pyrites , $CuFeS_2$

Concentrated pyrites ore (froth- floatation process) is roasted below 800°C. FeS is converted to FeO while copper remains as sulphide.

Roasting $2FeS + 3O_2 \xrightarrow{\Delta} 2FeO + 2\,SO_2\uparrow$

Smelting $FeO + SiO_2 \longrightarrow \underset{\text{slag}}{FeSIO_3}$

(SiO_2 either is present ore or is added)

Bessemerisation

$2Cu_2S + 3O_2 \longrightarrow 2Cu_2O + 2SO_2$

$2Cu_2O + Cu_2S \longrightarrow 6Cu + SO_2$ (final step)

9. $CO_2 + H_2O \rightleftharpoons H_2CO_3 \rightleftharpoons H^+ + HCO_3^-$

$\underset{\text{yellow}}{2CrO_4^{2-}} + 2H^+ \longrightarrow \underset{\text{orange red}}{Cr_2O_7^{2-}} + H_2O$

Aqueous CO_2 is acidic and changes yellow CrO_4^{2-} into orange $Cr_2O_7^{2-}$.

10. Transition elements form complexers due to their smaller size, higher nuclear charge and presence of low energy vacant orbitals to accept lone pair of electrons donated by ligands.

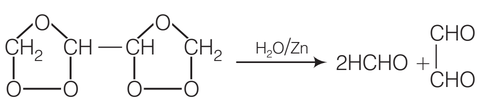
11. ClO_2 reacts with KOH forming KCl_3 and $KClO_2$

$2KOH + 2ClO_2 \longrightarrow KClO_2 + KClO_3 + H_2O$

This reaction indicates that ClO_2 is a mixed anhydride of $HClO_2$ and $HClO_3$.

12. Alkene *A* is $CH_2{=}CH{-}CH{=}CH_2$

$CH_2{=}CH{-}CH{=}CH_2 + O_3 \longrightarrow$



13. It can be prepared in the laboratory by reacting sodium, nitrite ($NaNO_2$) with a reducing agent such as Fe^{2+} in an acidic medium. NH_4NO_3 on heating at 250°C gives N_2O.

14. It absorbs CO_2 and releases O_2.

$4KO_2 + 2CO_2 \longrightarrow 2K_2CO_3 + 3O_2$

15. Stepwise formation of MgO involves

Step	Reaction	Δ *H* in kJ mol^{-1}
(i)	$Mg(s) \to Mg(g)$	147.7
(ii)	$Mg(g) \to Mg^{2+} + 2e^-$	2189.0
(iii)	$\frac{1}{2}O_2(g) \to O(g)$	498.4/2
(iv)	$O(g) + 1e^- \to O^-(g)$	−141.0
(v)	$O^-(g)te^- \to O^{2-}(g)$	*q*
(vi)	$Mg^{2+}(g) + O^{2-}(g) \to MgO(s)$	−3791.0

$Mg(s) + \frac{1}{2}O_2(g) \longrightarrow MgO(s)\;\Delta\,H = -1346.1 + q$

$Mg(s) + \frac{1}{2}O_2(g) \longrightarrow MgO(s)\;\Delta H_2 = -\,601.7$

By Born Herber cycle (based on Hes's law)

$\Delta H_1 = \Delta H_2$

$-\,1346.1 + q = -\,601.7$

$q = 744.4\,\text{Jmol}^{-1}$

16. $H_2S_2O_7 + H_2O \longrightarrow 2H_2SO_4$

SO_3 of $H_2S_2O_7$ is converted into H_2SO_4, hence SO_3 acts also as a dibasic acid.

Eq. wt. $\qquad (SO_3) = \frac{M}{2} = 40$

Let H_2SO_4 in fuming $H_2SO_4 = x$ g

$SO_3 = (1 - x)$ g

Equivalent of $H_2SO_4 = \frac{x}{49}$

$SO_3 = \frac{1 - x}{40}$

Equivalent of NaOH used $= \frac{26.7 \times 0.8}{1000}$

$= 0.02136$

$\frac{x}{49} + \frac{1 - x}{40} = 0.02136$

$x = 0.7927$ g H_2SO_4 in 1 g oleum.

Percentage of $H_2SO_4 = 79.27\%$

$SO_3 = 20.73\%$

17. ${}^{14}_{7}N + {}^{1}_{0}n \longrightarrow {}^{14}_{6}C + {}^{1}_{1}H$

This reaction can be represented as

${}^{14}_{7}N(n, p){}^{14}_{6}C$

18. Ag^+ is reduced hence, it is oxidant Hydroquinone is oxidised hence, it is reductant.

19. N(7) and $O^+ (8-1=7) = 1s^2\ 2s^2\ \underbrace{2p_x^1 2p_y^1 2p_z^1}_{\text{Maximum multiplicity most stable}}$

$$O = 1s^2, 2s^2\ 2p_x^2, 2p_y^1, 2p_z^1$$

Ionisation potential is dependent on the value of Z.

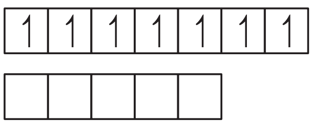
$$Z \text{ of } O^+(8) > Z \text{ of } N(7)$$

Thus, ionisation potential of $O^+ > N$

Na with one valence electron has least ionisation potential.

20. Gd. (64) = $[Xe]4f^7 5d^1$

$4f^4$



$5d^1$

All electrons are unpaired, hence greater stability

21. He^+, Li^{2+} and Be^{3+} each has one electron, hence no screening.

22. Symmetrical molecules will have net dipole zero while unsymmetrical will have dipoles. I is symmetrical and have no dipole while II and III have dipoles.

23. In PCl_5 No. of e^- pairs around central atom P = 5

No of lone pairs = 0 and hybridisation-sp^3d Shape = trigonal bipyramidal. In PCl_4^+– number. of e^- pairs = 4, lone pairs = 0 , hybridisation = sp^3 and shape – tetrahedral

In PCl_6^-– number of e^- pairs = 6, lone pairs = 0, hybridisation = sp^3d^2 and shape is octahedral

24. Repulsive interactions in decreasing order

$$= lp - lp > lp - bp > bp - bp$$

25. Based on Pauling's scale

$$(EN)_F - (EN)_{Cl} = k\sqrt{\Delta} = 0.208\sqrt{\Delta}$$

where, Δ = resonance energy

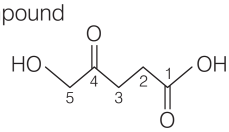
and k = conversion factor which is 0.208 for converting k cal into eV.

$$\Delta = (BE)_{Cl-F} - \sqrt{(BE)_{Cl-Cl}(BE)_{F-F}}$$
$$= 61 - \sqrt{58 \times 38}$$
$$= 61 - 46.95 = 14.05 \text{ kcal}$$
$$(EN)_{Cl} = (EN)_F - 0.208\sqrt{\Delta}$$
$$= 4.0 - 0.208\sqrt{14.05}$$
$$= 4.0 - 0.78 = 3.22 \text{ eV.}$$

26. Number of moles of O atoms = $\frac{1}{3}$ moles of H atoms

$$= \frac{1}{3} \times 3.18 = 1.06 \text{ mol}$$

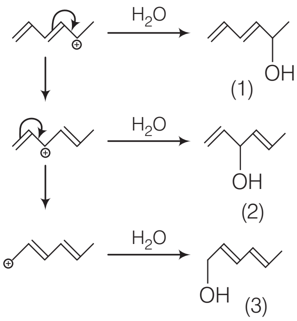
27. In the compound



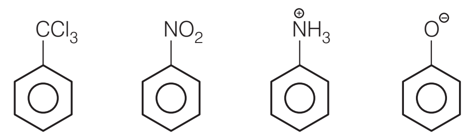
keto group is at 4C – atom

28. S_N1 reaction proceed through formation of carbocation which can form a pair of enantiomers. Thus, recemisation takes place (statement I). S_N2 reaction takes place forming pantavalent intermediate which loses Cl^- giving inverted alcohol. (Statement II) Hence, statements I and II both are correct.

29.

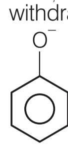


30. Electrophile $\overset{\oplus}{N}O_2$ attacks the following



I II III IV

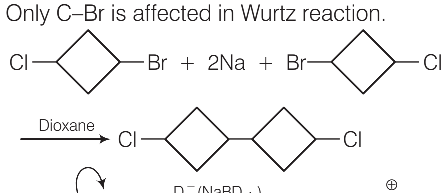
CCl_3, NO_2 and NH_3 groups are electron withdrawing groups. All deactivating and m-directing.



In negative charge on O is delocalised increasing electron density at o-and p-positions.

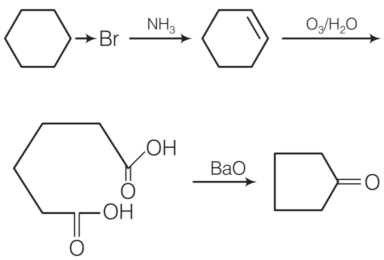
31. BE of C– Br bond < BE of C–Cl

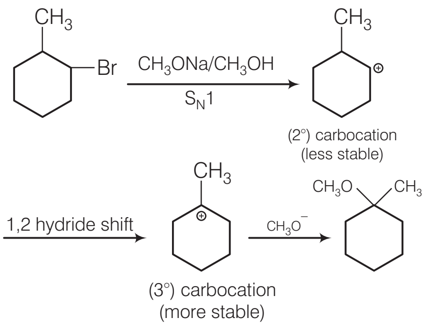
Only C–Br is affected in Wurtz reaction.



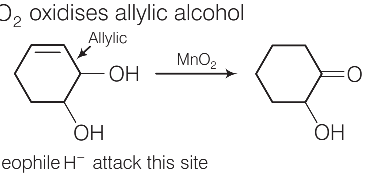
32. $CH_3CH{=}CH_2 \xrightarrow{D^-(NaBD_4)} CH_3\underset{\displaystyle D}{\underset{|}{C}}H\overset{\oplus}{C}H_2$

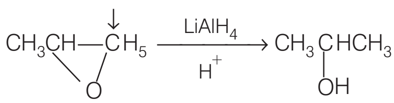
$\xrightarrow{H_2O_2/OH^-} CH_3\underset{\displaystyle D}{\underset{|}{C}}HCH_2OH$

33. 

34. 

35. MnO_2 oxidises allylic alcohol



36. 

37. $CH_3\overset{O}{\overset{\|}{C}}OC_2H_5 + HCH_2\overset{O}{\overset{\|}{C}}OC_2H_5$

$$\xrightarrow{C_2H_5ONa} CH_3\overset{O}{\overset{\|}{C}}CH_2\overset{O}{\overset{\|}{C}}OC_2H_5$$

(*A*) Keto ester

(b) *A* forms oxime due to keto group

(c) *A* shows tautomerism

(d) *A* shows iodoform test due to $CH_3\overset{O}{\overset{\|}{C}}—$

38. When a nucleophile encounters a ketone a site of attack is the carbon atom of the carbonyl group.

39. —COOH attached to ring is eliminated due to presence of keto group at β – position.

40. $CH_3—\overset{O}{\overset{\|}{C}}—NHCH_2CH_2CH_3$

N- propylacetamide.

41. (a) $CH_3CH_2NO_2 \xrightarrow{HNO_2} CH_3\underset{\underset{N—OH}{\|}}{C}NO_2$

blue (nutroic acid)

$$\xrightarrow{NaOH} CH_3—\underset{\underset{NONa}{\|}}{C}—NO_2$$

red

(b) $CH_3\overset{CH_3}{\overset{|}{C}}HNO_2 \xrightarrow{HNO_2} CH_3—\overset{CH_3}{\overset{|}{\underset{\underset{N=O}{|}}{C}}}—NO_2$

preudonitrol (blue)

$\xrightarrow{NaOH}$ No reaction blue

(c) $(CH_3)_3CNO_2 \xrightarrow{HNO_2}$ No reaction

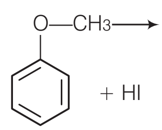
42. $CH_3\underset{\underset{NH_2}{|}}{C}HCOOH \quad CH_3\underset{\underset{\oplus NH_3}{|}}{C}HCOO^-$

$$\underset{\text{acidic}}{\xrightarrow{H^+}} CH_3\underset{\underset{\oplus NH_3}{|}}{C}HCOOH$$

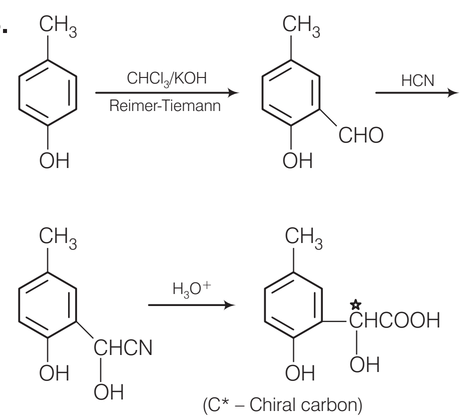
(pH = 4, 1 or 2)

43. Sucrose has glucose and fructose, maltose has two units of glucose, raffinose has glucose, fructose and galactose. Galactose is a monosaccharide.

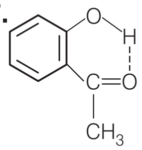
44. Amylose is a straight chain polymer have D- glucose units joined to gather by α – glycosidic linkage.

45. 

This bond is cleaved due to greater stability of phenxide ion. Thus, C_6H_5OH and CH_3I is formed.

46. 

(C* – Chiral carbon)

47. 

Boiling point of of A is lower than that of B. (In A, there is interamolecular H-bonding and in B, there is intermolecular H- bonding.) Thus, A is more volatile than B.

48. Sodium extract of thiourea will give red colour in Lassaigne's test due to the presence of both the elements nitrogen and sulphur.

49. The given half reactions are

	$E°$(inV)	$\Delta G° = -nFE°$
$BrO_3^- + 4e^- + 4H^+ \longrightarrow BrO^- + 2H_2O$	1.50	-6 F
$BrO^- + e^- + 2H^+ \longrightarrow \frac{1}{2}Br_2 + H_2O$	1.60	-1.6 F

Adding $BrO_3^- + 5e^- + 6H^+ \longrightarrow \frac{1}{2} Br_2 + 3H_2O$

$$\Delta G° = -7.6\,F = -5\,E°F$$

$$E° = \frac{7.6}{5} = 1.52 \text{ V}$$

50. Zn is at lower potential, it will be oxidised and the cell reaction will be

$$Zn(s) + Ni^{2+}(aq) \longrightarrow Zn^{2+}(aq) + Ni(s)$$

51. $0.65 = i \times 1.86 \times 0.15$

$$i = 2.33 = 1 + 4\alpha$$

$$\alpha = -0.33$$

52. $p = p° x_1$

$$99.50 = 100\frac{n_1}{n_1 + n_2}$$

$$\frac{n_2}{n_1} = \frac{1}{191}$$

$$\Delta T_b = k_b m = K_b \frac{n_2}{n_1} \times \frac{1000}{m_1}$$

$$= 0.52 \times \frac{1}{191} \times \frac{1000}{18} = 0.15$$

$$T_b = 100.15°\text{C}$$

53. In reverse direction, product becomes reactant and *vice versa*. Hence, (b) is most endothermic in reverse direction

54. $\log \frac{k_2}{k_1} = \log 1000$

$$= \frac{E_1 - E_2}{RT}$$

$$= \frac{98000 - E_2}{8.314 \times 300}$$

$$E_2 \text{ (catalyst)} = 80.77 \text{ kJ}$$

55. From experiment no –3 and 4, doubling $[OCl^-]$ doubles the rate; *i.e.*, order with respect to OCl^- is one. Also, from experiment no 1 and 2, doubes the concentration of I^- double the rate, *i.e.* order with respect to I^- is also one.

rate $= k[I^-][OCl^-]$

56. $HON\overset{+}{H_3} + H_2O \rightleftharpoons HONH_3OH + H^+$

$$K_h = \frac{K_w}{k_b} = \frac{10^{-14}}{10^{-8}} = 10^{-6}$$

$$[H^+] = \sqrt{K_h C}$$

$$= \sqrt{10^{-6} \times 0.024}$$

$$= 1.5 \times 10^{-4}$$

and pH = 3.8

57. $\underset{0.1-2s}{AgBr} + 2CN^- \rightleftharpoons \underset{s}{Ag(CN)_2^-} + \underset{s}{Br^-}$

$$K_{sp} = k_{sp} \cdot k = 4.3 \times 10^{-4}$$

$$4.3 \times 10^{-4} = \left(\frac{s}{0.1 - 2s}\right)$$

$$s = 2 \times 10^{-3}$$

58. Cetyl trimethyl ammonium bromide forms cationic micelles above a certain minimum concentration.

59. $\frac{d(Cs)}{d(Li)} = \frac{1.87}{0.53} = \frac{133}{7}\left(\frac{r(Li)}{r(Cs)}\right)^3$

$$\frac{r(Cs)}{r(Li)} = 1.753$$

or $\quad r_{Cs} = 1.753\, r(Li)$

60. Nylon threads are made up of polyamide. Some common examples are nylon 6 and nylon 66.

Nylon 6 $\underset{\text{Monomer}}{H_2N-(CH_2)_5-COOH}$

$$\underset{\text{Polymer}}{-NH-(CH_2)_5-\overset{\overset{\displaystyle O}{\|}}{C}-NH-(CH_2)_5-\overset{\overset{\displaystyle O}{\|}}{C}-NH-}$$

Nylon 66 $HO-\overset{\overset{\displaystyle O}{\|}}{C}-(CH_2)_4COOH$

and $H_2N-(CH_2)_6-NH_2$ are monomers

$$\underset{\text{nylon-66 (polymer)}}{HN-\overset{\overset{\displaystyle O}{\|}}{C}-(CH_2)_4-\overset{\overset{\displaystyle O}{\|}}{C}-NH-(CH_2)_6-NH}$$

$$-\overset{\overset{\displaystyle O}{\|}}{C}-(CH_2)_4-\overset{\overset{\displaystyle O}{\|}}{C}-$$

Mathematics

1. For $n = 1$, $1! < (1)^1$

that is, $1 < 1$ is not true.

For $n = 2$, $2! < \left(\frac{3}{2}\right)^1$

that is, $2 < \frac{9}{4}$ is true,

By induction, $n! < \left(\frac{n+1}{2}\right)^n \ \forall\, n \in N$ for $n > 2$.

$\therefore$ least $n = 2$.

2. Let T_r be the r^{th} term of the given series.

Then, $T_r = \frac{r}{1 + r^2 + r^4}, \qquad r = 1, 2, 3, \ldots, n$

$$= \frac{r}{(r^2 + r + 1)(r^2 - r + 1)}$$

$$= \frac{1}{2}\left[\frac{1}{r^2 - r + 1} - \frac{1}{r^2 + r + 1}\right]$$

$\therefore$ Sum of the series $= \sum_{r=1}^{n} T_r$

$$= \frac{1}{2}\left\{\sum_{r=1}^{n}\left(\frac{1}{r^2 - r + 1} - \frac{1}{r^2 + r + 1}\right)\right\}$$

$$= \frac{1}{2}\left\{\left(1 - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{7}\right) + \left(\frac{1}{7} - \frac{1}{13}\right) + \ldots + \left(\frac{1}{n^2 - n + 1} - \frac{1}{n^2 + n + 1}\right)\right\}$$

$$= \frac{1}{2}\left\{1 - \frac{1}{n^2 + n + 1}\right\} = \frac{n^2 + n}{2(n^2 + n + 1)}$$

3. Clearly, the given series is an arithmetico-geometric series whose corresponding AP and GP are respectively, $1, 4, 7, 10, \ldots$ and $1, \frac{1}{5}, \frac{1}{5^2}, \frac{1}{5^3}, \ldots$

The n^{th} term of AP $= [1 + (n-1) \times 3] = 3n - 2$

The n^{th} term of GP $= \left[1 \times \left(\frac{1}{5}\right)^{n-1}\right] = \left(\frac{1}{5}\right)^{n-1}$

So, the n^{th} term of the given series is

$$(3n - 2) \times \frac{1}{5^{n-1}} = \frac{3n - 2}{5^{n-1}}$$

Let, $S_n = 1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \ldots + \frac{3n - 5}{5^{n-2}} + \frac{3n - 2}{5^{n-1}} \qquad \ldots\text{(i)}$

$$\frac{1}{5}S_n = \frac{1}{5} + \frac{4}{5^2} + \frac{7}{5^3} + \ldots + \frac{(3n - 5)}{5^{n-1}} + \frac{3n - 2}{5^n} \ldots\text{(ii)}$$

Subtracting (ii) from (i), we get

$$S_n - \frac{1}{5}S_n = 1 + \left\{\frac{3}{5} + \frac{3}{5^2} + \frac{3}{5^3} + \ldots + \frac{3}{5^{n-1}}\right\} - \frac{(3n - 2)}{5^n}$$

$$\Rightarrow \frac{4}{5}S_n = 1 + \frac{3}{5}\,\frac{\left\{1 - \left(\frac{1}{5}\right)^{n-1}\right\}}{\left(1 - \frac{1}{5}\right)} - \frac{(3n - 2)}{5^n}$$

$$\Rightarrow \frac{4}{5}S_n = 1 + \frac{3}{5}\,\frac{\left\{1 - \frac{1}{5^{n-1}}\right\}}{\left(\frac{4}{5}\right)} - \frac{(3n - 2)}{5^n}$$

$$\Rightarrow \frac{4}{5}S_n = 1 + \frac{3}{4}\left(1 - \frac{1}{5^{n-1}}\right) - \frac{(3n - 2)}{5^n}$$

$$\Rightarrow S_n = \frac{5}{4} + \frac{15}{16}\left(1 - \frac{1}{5^{n-1}}\right) - \frac{(3n - 2)}{4 \cdot 5^{n-1}}$$

4. The sequence of differences between successive terms is $2, 6, 18, 54, \ldots$.

Clearly, it is a GP. Let T_n be the n^{th} term of the given series and S_n be the sum of its n terms.

Then,

$$S_n = 5 + 7 + 13 + 31 + 85 + \ldots + T_{n-1} + T_n \qquad \ldots\text{(i)}$$

$$S_n = 5 + 7 + 13 + 31 + \ldots + T_{n-1} + T_n \qquad \ldots\text{(ii)}$$

Subtracting (ii) from (i), we get

$$O = 5 + [2 + 6 + 18 + 54 + \ldots + (T_n - T_{n-1})] - T_n$$

$$\Rightarrow O = 5 + 2 \cdot \frac{3^{n-1} - 1}{3 - 1} - T_n$$

$$\Rightarrow T_n = 5 + (3^{n-1} - 1) = 4 + 3^{n-1}$$

$$\therefore S_n = \sum_{k=1}^{n} T_k = \sum_{k=1}^{n} (4 + 3^{K-1})$$

$$= \sum_{k=1}^{n} 4 + \sum_{k=1}^{n} 3^{k-1}$$

$$= 4n + (1 + 3 + 3^2 + \ldots + 3^{n-1})$$

$$= 4n + 1 \times \left(\frac{3^n - 1}{3 - 1}\right)$$

$$= 4n + \left(\frac{3^n - 1}{2}\right)$$

$$= \frac{1}{2}(3^n + 8n - 1)$$

5. The discriminant of the quadratic equation $2x^2 + 6x + b = 0$ is given by $D = 36 - 8b > 0$.
Therefore, the given equation has real roots.
we have,

$$\frac{x_1}{x_2} + \frac{x_2}{x_1} = \frac{x_1^2 + x_2^2}{x_1 \cdot x_2} = \frac{(x_1 + x_2)^2 - 2x_1x_2}{x_1 \cdot x_2}$$

$$= \frac{(-3)^2 - 2(b/2)}{(b/2)} = \frac{18}{b} - 2 < -2 \qquad [\because\ b < 0]$$

6. $x^4 + x^2 + 1 = (x^2 + 1)^2 - x^2$

$$= (x^2 + x + 1)(x^2 - x + 1)$$

$$x^2 + x + 1 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4} \neq 0 \qquad \forall\ x.$$

Now, $(a - 1)(x^2 + x + 1)^2 = (a + 1)(x^4 + x^2 + 1)$

$$(a - 1)(x^2 + x + 1) = (a + 1)(x^2 - x + 1).$$

$\Rightarrow$ $x^2 - ax + 1 = 0$ which has real and distinct roots

$\therefore$ $\Delta = a^2 - 4 > 0$

$\Rightarrow$ $a^2 > 4$

$\Rightarrow$ $a \in (-\infty, -2) \cup (2, \infty)$

7. Given that, $\alpha + \beta = 0$

$$\alpha + \beta + \gamma = -p$$

$\Rightarrow$ $\gamma = -p$

Substituting $\gamma = -p$ in the given equation,

$\Rightarrow$ $-p^3 + p^3 - pq + r = 0$

$\Rightarrow$ $pq = r.$

8. Selection of 2 parallel lines from $m = {}^mC_2$

Selection of 2 parallel lines from $n = {}^nC_2$

Hence, number of parallelograms $= {}^mC_2 \cdot {}^nC_2$

$$= \frac{1}{4}(m - 1)(n - 1)\,mn$$

9. We have

$${}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots + {}^{2n+1}C_n = 255 \quad \ldots \text{(i)}$$

Also, the sum of binomial coefficients

$${}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots + {}^{2n+1}C_n + {}^{2n+1}C_{n+1} + + {}^{2n+1}C_{n+2} + \ldots + {}^{2n+1}C_{2n+1}$$

$$= (1 + 1)^{2n+1} = 2^{2n+1}$$

$\Rightarrow$ ${}^{2n+1}C_0 + 2\{{}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots + {}^{2n+1}C_n\} + {}^{2n+1}C_{2n+1} = 2^{2n+1}$

$\Rightarrow$ $1 + 2(255) + 1 = 2^{2n+1}$

$\Rightarrow$ $1 + 255 = 2^{2n}$

$\Rightarrow$ $2^{2n} = 2^8$

$\Rightarrow$ $n = 4$

10. $\log x$ is defined only when $x > 0$

Now, the 3rd term in the expansion

$T_{2+1} = {}^5C_2 \cdot x^{5-2} \cdot (x^{\log_{10} x})^2 = 1{,}000{,}000$ (given)

$\Rightarrow$ $x^{3 + 2\log_{10} x} = 10^5.$

Taking logarithm of both sides, we get

$$(3 + 2\log_{10} x) \cdot \log_{10} x = 5$$

$\Rightarrow$ $2y^2 + 3y - 5 = 0,$

where $\log_{10} x = y$

$\Rightarrow$ $(y - 1)(2y + 5) = 0$

$\Rightarrow$ $y = 1$ or $-5/2$

$\Rightarrow$ $\log_{10} x = 1$ or $-5/2$

$\Rightarrow$ $x = 10^1 = 10$ or $10^{-5/2}$

11. $\sum_{k=0}^{10} {}^{20}C_k = {}^{20}C_0 + {}^{20}C_1 + \ldots + {}^{20}C_{10}$

$$= \frac{1}{2}[2 \cdot {}^{20}C_0 + 2 \cdot {}^{20}C_1 + \ldots + 2 \cdot {}^{20}C_{10}]$$

$$= \frac{1}{2}[({}^{20}C_0 + {}^{20}C_{20}) + ({}^{20}C_1 + {}^{20}C_{19}) + \ldots + 2\,{}^{20}C_{10}]$$

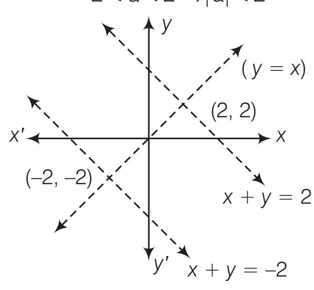
$$= \frac{1}{2}[({}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \ldots + {}^{20}C_{19} + {}^{20}C_{20}) + {}^{20}C_{10}]$$

$$= \frac{1}{2}[2^{29} + {}^{20}C_{10}]$$

$$= 2^{19} + \frac{1}{2}\,{}^{20}C_{10}.$$

12. Lines $x + y = 4$ and $x + y = -4$ are parallel and points $(2, 2)$ and $(-2, -2)$ lie on these lines.
If point (a, a) lies between the lines then $a > -2$ and $a < 2$ *i.e.*,

$$-2 < a < 2 \Rightarrow |a| < 2$$



13. From the given equation,

$$3(my + 2) + 2y = 0$$

$\Rightarrow$ $y(3m + 2) = 4$

$\Rightarrow$ $y = \frac{4}{3m + 2},$

since, $y \in I \Rightarrow 3m + 2$

$\Rightarrow \qquad \pm 1, \pm 2, \pm 4$

$\Rightarrow \quad m = -1, -\frac{1}{3}, -\frac{4}{3}, 0, \frac{2}{3}, -2$

14. Let the equation of the line be $\frac{x}{a} + \frac{y}{b} = 1 \qquad \ldots(i)$

Its intercepts on y and x axes are b and a respectively.

According to the question,

$$\frac{1}{a} + \frac{1}{b} = \text{constant} = K \qquad \text{(say)}$$

$$\therefore \quad \frac{1}{aK} + \frac{1}{bK} = 1$$

$$\Rightarrow \quad \frac{1/K}{a} + \frac{1/K}{b} = 1 \qquad \ldots(ii)$$

From (ii) it follows that line (i) passes through the fixed point $\left(\frac{1}{K}, \frac{1}{K}\right)$.

15. Variable line is $\frac{x}{a} + \frac{y}{b} = 1 \qquad \ldots(i)$

Any line perpendicular to (i) and passing through the origin will be

$$\frac{x}{b} - \frac{y}{a} = 0 \qquad \ldots(ii)$$

Now foot of the perpendicular from the origin to line (i) is the point of intersection (i) and (ii)

Let it be $P(\alpha, \beta)$, then $\quad \frac{\alpha}{a} + \frac{\beta}{b} = 1 \qquad \ldots(iii)$

and $\quad \frac{\alpha}{b} - \frac{\beta}{a} = 1 \qquad \ldots(iv)$

Squarring and adding Eqs. (iii) and (iv), we get

$$\alpha^2\left(\frac{1}{a^2} + \frac{1}{b^2}\right) + \beta^2\left(\frac{1}{b^2} + \frac{1}{a^2}\right) = 1$$

$$\therefore \quad (\alpha^2 + \beta^2)\frac{1}{c^2} = 1$$

Hence, the locus of $P(\alpha, \beta)$ is $x^2 + y^2 = c^2$

which is a circle.

16. The given equation of pair of straight lines can be rewritten as $(\sqrt{3}x - y)(x - \sqrt{3}y) = 0$. Their separate equations are,

$$y = \sqrt{3}\,x \text{ and } y = \frac{1}{\sqrt{3}}x$$

$$\Rightarrow \quad y = \tan 60° x \quad \text{and} \quad y = \tan 30° x$$

After rotation, the separate equations are

$$y = \tan 90° x$$

and $\quad y = \tan 60° x$

$\Rightarrow \quad x = 0$ and $y = \sqrt{3} \cdot x$

$\therefore$ Combined equation in the new position is

$x(\sqrt{3}x - y) = 0$ or $\sqrt{3}x^2 - xy = 0$

17. $\lim_{x \to 1} \frac{\left[\sum_{K=1}^{100} x^K\right] - 100}{(x-1)}$

$$= \lim_{x \to 1} \frac{(x + x^2 + x^3 + \ldots + x^{100}) - 100}{(x-1)}$$

$$= \lim_{x \to 1} \frac{(x-1) + (x^2 - 1) + (x^3 - 1) + \ldots + (x^{100} - 1)}{(x-1)}$$

$$= \lim_{x \to 1}\left\{\left(\frac{x-1}{x-1}\right) + \left(\frac{x^2-1}{x-1}\right) + \left(\frac{x^3-1}{x-1}\right) + \ldots + \left(\frac{x^{100}-1}{x-1}\right)\right\}$$

$$= \lim_{x \to 1}\left(\frac{x-1}{x-1}\right) + \lim_{x \to 1}\left(\frac{x^2-1}{x-1}\right) + \lim_{x \to 1}\left(\frac{x^3-1}{x-1}\right) + \ldots + \lim_{x \to 1}\left(\frac{x^{100}-1}{x-1}\right)$$

$$= 1 + 2 + 3 + \ldots + 100$$

$$= \Sigma 100 = \frac{100 \times (100 + 1)}{2}$$

$$= 50 \times 101 = 5050 \qquad \left\{\because \; \Sigma n = \frac{n(n+1)}{2}\right\}.$$

18. Let $y = \cfrac{x}{x + \cfrac{\sqrt[3]{x}}{x + \sqrt[3]{x} \ldots \text{infinity}}}$

$$= \frac{x}{x + \frac{1}{x^{2/3}} \cdot \frac{x}{x + \sqrt[3]{x} \ldots \infty}} = \frac{x}{x + \frac{y}{x^{2/3}}}$$

$$\Rightarrow \quad y = \frac{x^{5/3}}{x^{5/3} + y}$$

$$\Rightarrow \quad y^2 + (x^{5/3}) \cdot y - x^{5/3} = 0$$

$$\therefore \quad y = \frac{-x^{5/3} \pm \sqrt{x^{10/3} + 4x^{5/3}}}{2}$$

$$= \frac{-x^{5/3} + \sqrt{x^{10/3} + 4x^{5/3}}}{2} \qquad (\because \; y > 0)$$

$$= \frac{4x^{5/3}}{2\left(\sqrt{x^{10/3} + 4x^{5/3}} + x^{5/3}\right)} = \frac{2}{\sqrt{\left(1 + \frac{4}{x^{5/3}}\right)} + 1}$$

$$\therefore \quad \lim_{x \to \infty} y = \frac{2}{\sqrt{1 + 0} + 1}$$

$$= \frac{2}{2} = 1.$$

19. We have; $f(x)=\begin{cases}\dfrac{x^2-x}{x^2-x}=1, & \text{if } x<0 \text{ or } x>1\\ -\dfrac{(x^2-x)}{(x^2-x)}=-1, & \text{if } 0<x<1\\ 1, & \text{if } x=0\\ -1, & \text{if } x=1\end{cases}$

$=\begin{cases}1, & \text{if } x\le 0 \text{ or } x>1\\ -1, & \text{if } 0<x\le 1\end{cases}$.

Now, $\lim_{x\to 0^-} 1=1$ and $\lim_{x\to 0^+} f(x)=\lim_{x\to 0} -1=-1$

Clearly, $\lim_{x\to 0^-} f(x)\ne \lim_{x\to 0^+} f(x)$.

So, $f(x)$ is not continuous at $x=0$. It can be easily seen that it is not continuous at $x=1$ also.

20. Given; $f(x)$ is continuous at $x=1$

$\therefore\ f(1)=\text{RHL}$

$\Rightarrow\ f(1)=\lim_{x\to 1}+f(x)\Rightarrow f(x)=\lim_{h\to 0} f(1+h)$

$\Rightarrow\ a-b=\lim_{h\to 0} 3(1+h)\Rightarrow a-b=3$ ...(i)

Again, given $f(x)$ is discontinuous at $x=2$.

$\therefore$ LHL $\ne f(x)$

$\Rightarrow\ \lim_{x\to 2^-} f(x)\ne f(2)\Rightarrow \lim_{h\to 0} f(2-h)\ne f(2)$

$\Rightarrow\ \lim_{h\to 0} 3(2-h)\ne 4b-a\Rightarrow 6\ne 4b-a$ ...(ii)

Assume, $6=4b-a$

then from (i) and (ii), we get $b=3$.

$\therefore$ locus $y=3$

Which is impossible $(\because\ 6\ne 4b-a)$

Hence, locus of (a, b) is $x-y=3$ excluding the point when it cuts the line $y=3$.

21. We are given that,

$(\cot\alpha_1)\cdot(\cot\alpha_2)\dots(\cot\alpha_n)=1$

$\Rightarrow\ (\cos\alpha_1\cdot(\cos\alpha_2)\dots(\cos\alpha_n)=(\sin\alpha_1)$
$\cdot(\sin\alpha_2)\dots(\sin\alpha_n)$ ...(i)

Let $y=(\cos\alpha_1)\cdot(\cos\alpha_2)\dots(\cos\alpha_n)$ (to be max.)

Squarring both sides, we get

$y^2=(\cos^2\alpha_1)\cdot(\cos^2\alpha_2)\dots(\cos^2\alpha_n)$

$=\cos\alpha_1\cdot\sin\alpha_1\cdot\cos\alpha_2\cdot\sin\alpha_2\dots\cos\alpha_n\cdot\sin\alpha_n$ [using (i)]

$=\dfrac{1}{2^n}[\sin 2\alpha_1\cdot\sin 2\alpha_2\dots\sin 2\alpha_n]$

As $0\le\alpha_1,\alpha_2,\dots,\alpha_n\le\dfrac{\pi}{2}$

$\therefore\ 0\le 2\alpha_1, 2\alpha_2,\dots,2\alpha_n\le\pi$

$\Rightarrow\ 0\le\sin 2\alpha_1,\sin 2\alpha_2,\dots,\sin 2\alpha_n\le 1$

$\therefore\ y^2\le\dfrac{1}{2^n}\cdot 1 \quad \Rightarrow y\le\dfrac{1}{2^{n/2}}$

$\therefore$ Maximum value of y is $\dfrac{1}{2^{n/2}}$.

22. For $\alpha=-\dfrac{\pi}{2}, \beta=-\dfrac{\pi}{2}$ and $r=2\pi$

$\sin\alpha+\sin\beta+\sin\gamma=-2$

$\Rightarrow$ minimum value of the expression is negative

23. $(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}+\mathbf{b})=|\mathbf{a}^2|+|\mathbf{b}^2|+2\mathbf{a}\cdot\mathbf{b}$

$=1+1+2(1)(1)\cos\theta=2+2\cos\theta$

$\Rightarrow\ |\mathbf{a}+\mathbf{b}|^2=2\cdot 2\cos^2\dfrac{\theta}{2}$

$\Rightarrow\ \cos\dfrac{\theta}{2}=\dfrac{1}{2}|\mathbf{a}+\mathbf{b}|$

24. Let $\mathbf{a}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$

then, $\mathbf{a}\cdot\mathbf{i}=(x\mathbf{i}+y\mathbf{j}+z\mathbf{k})\cdot\mathbf{i}=x$

and $\mathbf{a}\cdot(\mathbf{i}+\mathbf{j})=(x\mathbf{i}+y\mathbf{j}+z\mathbf{k})\cdot(\mathbf{i}+\mathbf{j})=x+y$

and $\mathbf{a}\cdot(\mathbf{i}+\mathbf{j}+\mathbf{k})=(x\mathbf{i}+y\mathbf{j}+z\mathbf{k})\cdot(\mathbf{i}+\mathbf{j}+\mathbf{k})$
$=x+y+z$

Given that $\mathbf{a}\cdot\mathbf{i}=\mathbf{a}\cdot(\mathbf{j}+\mathbf{i})=\mathbf{a}\cdot(\mathbf{i}+\mathbf{j}+\mathbf{k})$

$\because\ x=x+y=x+y+z$

Now, $x=x+y \quad\Rightarrow y=0$

and $x+y=x+y+z \quad\Rightarrow z=0$

Hence; $x=1$

$\therefore\ \mathbf{a}=\mathbf{i}$

25. Projection of $x\mathbf{i}-\mathbf{j}+\mathbf{k}$ on $2\mathbf{i}-\mathbf{j}+5\mathbf{k}$

$=\dfrac{(x\mathbf{i}-\mathbf{j}+\mathbf{k})\cdot(2\mathbf{i}-\mathbf{j}+5\mathbf{k})}{\sqrt{4+1+25}}=\dfrac{2x+1+5}{\sqrt{30}}$

But, given, $\dfrac{2x+6}{\sqrt{30}}=\dfrac{1}{\sqrt{30}}$

$\Rightarrow\ 2x+6=1$

$\Rightarrow\ x=\dfrac{-5}{2}$

26. Since, $\mathbf{a}\times\mathbf{b}=\mathbf{c}$

$\therefore\ (\mathbf{b}\times\mathbf{c})\times\mathbf{b}=\mathbf{c}$

$\Rightarrow\ (\mathbf{b}\cdot\mathbf{b})\mathbf{c}-(\mathbf{b}\cdot\mathbf{c})\mathbf{b}=\mathbf{c}$

$\Rightarrow\ (\mathbf{b}\cdot\mathbf{b})=1, \quad \mathbf{c}\cdot\mathbf{b}=0$

$\Rightarrow\ b^2=1$ *i.e.*, $b=1$

$\therefore$ $\mathbf{b}$ is a unit vector

$\therefore |\mathbf{c}|=|\mathbf{a}| \quad\Rightarrow\ c=a.$

27. Let $\mathbf{a}=t(\mathbf{i}+\mathbf{j}+2\mathbf{k})+s(\mathbf{i}+2\mathbf{j}+\mathbf{k})$

$=(t+3s)\mathbf{i}+(t+2s)\mathbf{j}+(2t+s)\mathbf{k}$

given, $\mathbf{a}\cdot(\mathbf{i}+\mathbf{j}+\mathbf{k})=0$

$\therefore\ t+s+t+2s+2t+s=0$

$\Rightarrow\ 4(t+s)=0$

$\Rightarrow\ t+s=0$

$\therefore\ \mathbf{a}=t(\mathbf{i}+\mathbf{j}+2\mathbf{k})-t(\mathbf{i}+2\mathbf{j}+\mathbf{k})$
$=t(-\mathbf{j}+\mathbf{k})$

But $|\mathbf{a}|=1$

$\therefore\ 1=2t^2$

$$\Rightarrow \qquad t = \pm \frac{1}{\sqrt{2}}$$

$$\therefore \qquad a = \pm \left(\frac{k - j}{\sqrt{2}}\right).$$

28. As, $A^2 = O,\ A^K = O \quad \forall\ K \geq 2$

Thus, $\qquad (A + I)^{50} = I + 50\,A$

$$\Rightarrow \quad (A + I)^{50} - 50\,A = I$$

$$\therefore \qquad I = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\Rightarrow \ a = 1, b = 0, c = 0, d = 1$$

$$\therefore \ a + b + c + d = 1 + 0 + 0 + 1 = 2.$$

29. $\begin{bmatrix} \bar{z}_1 & -z_2 \\ \bar{z}_2 & z_1 \end{bmatrix}^{-1} \begin{bmatrix} z_1 & z_2 \\ -\bar{z}_2 & \bar{z}_1 \end{bmatrix}^{-1}$

$$= \left\{\begin{bmatrix} z_1 & z_2 \\ -\bar{z}_2 & \bar{z}_1 \end{bmatrix} \begin{bmatrix} \bar{z}_1 & -z_2 \\ \bar{z}_2 & z_1 \end{bmatrix}\right\}^{-1}$$

$$= \begin{bmatrix} z_1\bar{z}_1 + z_2\bar{z}_2 & 0 \\ 0 & z_2\bar{z}_2 + z_1\bar{z}_1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} |z_1|^2 + |z_2|^2 & 0 \\ 0 & |z_1|^2 + |z_2|^2 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$$

30. We know that, in a square matrix of order n

$$|\text{adj } A| = |A|^{n-1}$$

$$\Rightarrow \ |\text{adj (adj } A)| = |\text{adj } A|^{n-1} = |A|^{(n-1)^2}$$

$$\Rightarrow \ n^2 - 2n - 8 = 0$$

$$\Rightarrow \quad n = 4 \text{ as } n = -2 \text{ is not possible.}$$

31. Coefficient x in $f(x)$ is equal to coefficient of x in

$$\begin{vmatrix} x & (1 + x - \frac{x^3}{3!} \dots)^3 & 1 - \frac{x^2}{2!} \\ 1 & x - \frac{x^2}{2} & 2 \\ x^2 & 1 + x^2 & 0 \end{vmatrix}$$

$$= \text{coefficient of } x \text{ in } \begin{vmatrix} x & 1 & 1 \\ 1 & x & 2 \\ x^2 & 1 & 0 \end{vmatrix}$$

$$= \text{Coefficient of } x \text{ in } [x(0-2) - 1(0 - 2x^2) + 1(1 - x^3)]$$

$$= -2$$

32. Given system has non-trivial solution then

$$\begin{vmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{vmatrix} = 0$$

$$\Rightarrow \qquad \alpha_1\beta_2 = \alpha_2\beta_1$$

$$\Rightarrow \qquad \frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = \frac{\alpha_1 + \alpha_2}{\beta_1 + \beta_2} = \sqrt{\frac{\alpha_1\,\alpha_2}{\beta_1\,\beta_2}}$$

$$\Rightarrow \qquad \frac{-b/a}{-q/p} = \sqrt{\frac{c/a}{r/p}}$$

$$\Rightarrow \qquad \frac{bp}{aq} = \sqrt{\frac{cp}{ar}}$$

$$\Rightarrow \qquad b^2p^2ar = a^2q^2cp$$

$$\Rightarrow \qquad b^2pr = q^2ac$$

33. The given circle

$$S(x, y) \equiv x^2 + y^2 - x - y - 6 = 0 \qquad \dots\text{(i)}$$

has centre at $C \equiv \left(\frac{1}{2}, \frac{1}{2}\right)$.

According to the given conditions, the given point $P(\alpha - 1, \alpha + 1)$ must lie inside the given circle.

i.e., $\qquad S(\alpha - 1, \alpha + 1) < 0$

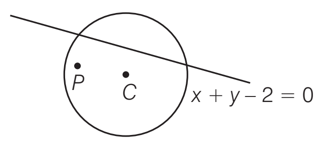
$$\Rightarrow \ (\alpha - 1)^2 + (\alpha + 1)^2 - (\alpha - 1) - (\alpha + 1) - 6 < 0$$

$$\Rightarrow \ \alpha^2 - \alpha - 2 < 0 \ \ i.e., \ \ (\alpha - 2)(\alpha + 1) < 0$$

$$\Rightarrow \ -1 < \alpha < 2$$

[using sign – scheme from algebra] $\qquad \dots$(ii)

and also P and C must lie on the same side of the line.



$$L(x, y) = x + y - 2 = 0$$

i.e., $L\left(\frac{1}{2}, \frac{1}{2}\right)$ and $L(\alpha - 1, \alpha + 1)$ must have the same sign.

Now, Isince $L\left(\frac{1}{2}, \frac{1}{2}\right) = \frac{1}{2} + \frac{1}{2} - 2 < 0$

therefore, we have

$$L(\alpha - 1, \alpha + 1) = (\alpha - 1) + (\alpha + 1) - 2 < 0$$

$$\Rightarrow \qquad \alpha < 1 \qquad \dots\text{(iv)}$$

Ineqalities (ii) and (iv) together give the permissible values of α as $-1 < \alpha < 1$.

34. The two circles are

$$x^2 + y^2 - 2ax + c^2 = 0$$

and $\qquad x^2 + y^2 - 2by + c^2 = 0$

centres : $C_1\ (a, 0) \qquad C_2\ (0, b)$

radii : $\ r_1 = \sqrt{a^2 - c^2}, \quad r_2 = \sqrt{b^2 - c^2}$

Since, the two circles touch each other externally, therefore

$$C_1C_2 = r_1 + r_2$$

$$\Rightarrow \quad \sqrt{a^2+b^2}=\sqrt{a^2-c^2}+\sqrt{b^2-c^2}$$

$$\Rightarrow \quad a^2+b^2=a^2-c^2+b^2-c^2 + 2\sqrt{a^2-c^2}\sqrt{b^2-c^2}$$

$$\Rightarrow \quad c^4=a^2b^2-c^2(a^2+b^2)+c^4$$

$$\Rightarrow \quad a^2b^2=c^2(a^2+b^2)$$

$$\Rightarrow \quad \frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$$

35. Let the coordinates of P be (h, k). Let the equation of a tangent from $P(h, K)$ to the circle $x^2+y^2=a^2$ be $y=mx+a\sqrt{1+m^2}$

Since, $P(h, K)$ lies on $y=mx+a\sqrt{1+m^2}$ therefore, $k=mh+a\sqrt{1+m^2}=(k-mh)^2=a^2(1+m^2)$ this is a quadratic equation in m. Let the two roots be m_1 and m_2, then

$$m_1+m_2=\frac{2hK}{K^2-a^2}$$

But $\tan\alpha=m_1$, $\tan\beta=m_2$ and it is given that

$$\cot\alpha+\cot\beta=0$$

$$\therefore \quad \frac{1}{m_1}+\frac{1}{m_2}=0$$

$$\Rightarrow \quad m_1+m_2=0$$

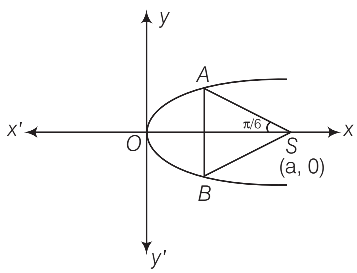
$$\Rightarrow \quad \frac{2hK}{K^2-a^2}=0$$

$$\Rightarrow \quad hK=0$$

Hence, the locus of (h, K) is $xy=0$.

36. Let $A(at_1^2, 2at_2)$, $B(at_2^2, -2at_2)$.

We have,



$$m_{AS}=\tan\left(\frac{5\pi}{6}\right)$$

$$\Rightarrow \quad \frac{2at_1}{at_1^2-a}=-\frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad t_1^2+2\sqrt{3}\,t_1-1=0 \quad \Rightarrow t_1=-\sqrt{3}\pm 2$$

Clearly, $t_1=-\sqrt{3}-2$ is rejected.

thus, $t_1=(2-\sqrt{3})$,

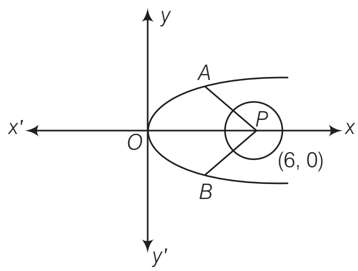
Hence, $AB=4at_1=4a(2-\sqrt{3})$.

37. Centre and radius of the given circle is $P(6, 0)$ and $\sqrt{5}$, respectively.

Now minimum distance between two curves always occurs along a line which normal to both the curves.

Equation of normal to $y^2=4x$ at $(t^2, 2t)$ is

$$y=-tx+2t+t^3.$$



If it is normal to circle also, then it must pass though (6, 0).

$$\therefore \quad 0=t^3-4t \quad \Rightarrow \quad t=0 \quad \text{or} \quad t=\pm 2.$$

$\Rightarrow$ $A(4, 4)$ and $C(4, -4)$.

$\Rightarrow$ $PA=PC=\sqrt{20}=2\sqrt{5}$.

$\Rightarrow$ Required minimum distance $=2\sqrt{5}-\sqrt{5}=\sqrt{5}$.

38. Let the points of intersection of the line and the ellipse be $(a\cos\theta, b\sin\theta)$ and $\left\{a\cos\left(\frac{\pi}{2}+\theta\right), b\sin\left(\frac{\pi}{2}+\theta\right)\right\}$, since they lie on the given line $lx+my+n=0$.

$$la\cos\theta+mb\sin\theta+n=0$$

$$\Rightarrow \quad la\cos\theta+mb\sin\theta=-n$$

and $\quad -la\sin\theta+mb\cos\theta+n=0$

$$la\sin\theta-mb\cos\theta=n$$

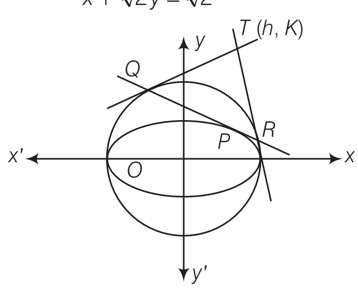
squarring and adding, we get,

$$a^2l^2+b^2m^2=2n^2 \quad \Rightarrow \frac{a^2l^2+b^2m^2}{n^2}=2.$$

39. Equation of tangent to ellipse at given point is

$$x\left(\frac{1}{\sqrt{2}}\right)+2y\left(\frac{1}{2}\right)=1.$$

$$\Rightarrow \quad x+\sqrt{2}y=\sqrt{2} \qquad \ldots\text{(i)}$$



Now, QR is chord of contact of circle $x^2 + y^2 = 1$ with respect to point $T(h, K)$.

then, $QR \equiv hx + Ky = 1$ ...(ii)

Equations (i) and (ii) represent same straight line, then comparing ratio of coefficients we have

$$\frac{h}{1} = \frac{K}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

Hence; $(h, K) \equiv \left(\frac{1}{\sqrt{2}}, 1\right)$

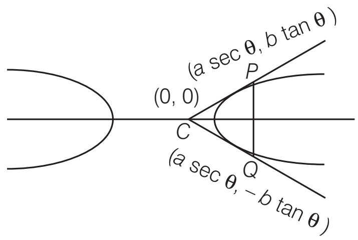
40. $e^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{\sin^2\alpha}{\cos^2\alpha} = \frac{1}{\cos^2\alpha}$

and $a^2 = \cos^2\alpha$

$\therefore$ $a^2e^2 = 1$

$\therefore$ *foci* $(\pm ae, 0) = (\pm 1, 0)$, which is independent of α.

41. Let the hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and any double ordinate PQ be $(a\sec\theta, b\tan\theta)$, $(a\sec\theta, -b\tan\theta)$ and O is centre $(0, 0)$.



ΔOPQ being equilateral.

$\therefore$ $\tan 30° = \frac{b\tan\theta}{a\sec\theta}$

$\Rightarrow$ $3 \cdot \frac{b^2}{a^2} = \text{cosce}^2\,\theta$

$\Rightarrow$ $3(e^2 - 1) = \text{cosce}^2\theta$

Now, $\text{cosec}^2\theta \geq 1$

$\therefore$ $3(e^2 - 1) \geq 1 \Rightarrow e^2 \geq \frac{4}{3}$

$\Rightarrow$ $e > 2/\sqrt{3}$

42. $iz^4 = -1$

$\Rightarrow$ $z^4 = -\frac{1}{i}$

$\Rightarrow$ $z^4 = i$

$\Rightarrow$ $z = (i)^{1/4} = (0 + i)^{1/4}$

$\Rightarrow$ $z = \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^{1/4}$

$\Rightarrow$ $z = \cos\frac{\pi}{8} + i\sin\frac{\pi}{8}$

(using De Moirrels theorem)

43. Let $a = \cos\alpha + i\sin\alpha$,

$b = \cos\beta + i\sin\beta$,

$c = \cos\gamma + i\sin\gamma$

then; $a + b + c = (\cos\alpha + \cos\beta + \cos\gamma) + i(\sin\alpha + \sin\beta + \sin\gamma)$

$= 0 + i0 = 0$

$\Rightarrow$ $a^3 + b^3 + c^3 = 3abc$

$\Rightarrow$ $(\cos 3\alpha + i\sin 3\alpha) + (\cos 3\beta + i\sin 3\beta) + (\cos 3\gamma + i\sin 3\gamma)$

$= 3[\cos(\alpha + \beta + \gamma) + i\sin(\alpha + \beta + \gamma)]$

$\Rightarrow$ $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$

44. $z_1^2 + z_2^2 + 2z_1z_2\cos\theta = 0$

$\Rightarrow$ $\left(\frac{z_1}{z_2}\right)^2 + 2\left(\frac{z_1}{z_2}\right)\cos\theta + 1 = 0$

$\Rightarrow$ $\left(\frac{z_1}{z_2} + \cos\theta\right)^2 = -(1 - \cos^2\theta) = -\sin^2\theta$

$\Rightarrow$ $\frac{z_1}{z_2} = -\cos\theta \pm i\sin\theta$

$\Rightarrow$ $\left|\frac{z_1}{z_2}\right| = \sqrt{(-\cos\theta)^2 + (\sin\theta)^2} = 1$

$\Rightarrow$ $|z_1| = |z_2|$

$\Rightarrow$ $|z_1 - 0| = |z_2 - 0|$

Thus, triangle with vertices 0, z_1, z_2 is iscosceles.

45. We have $f(xy) = f(x)f(y)$ for all $x, y \in R$.

Putting $x = y = 1$, we get

$f(1) = f(1)f(1)$

$\Rightarrow$ $f(1)[1 - f(1)] = 0$

$\Rightarrow$ $f(1) = 1$ $[\because f(1) \neq 0]$ ...(i)

Now, $f'(1) = 2$

$\Rightarrow$ $\lim_{h \to 0} \frac{f(1 + h) - f(1)}{h} = 2$

$\Rightarrow$ $f(1)\lim_{h \to 0} \frac{f(h) - 1}{h} = 2$

$\Rightarrow$ $\lim_{h \to 0} \frac{f(1)f(h) - f(1)}{h} = 2$ [using $f(1) = 1$] ...(ii)

$\Rightarrow$ $\lim_{h \to 0} \frac{f(h) - 1}{h} = 2$

Now $f'(4) = \lim_{h \to 0} \frac{f(4 + h) - f(4)}{h}$

$= \lim_{h \to 0} \frac{f(4) \cdot f(h) - f(4)}{h}$

$= \left\{\lim_{h \to 0} \frac{f(h) - 1}{h}\right\} \cdot f(4) = 2f(4)$ {from (i)}

$= 2 \times 4 = 8$

46. Let $x = a\cos^2\theta + b\sin^2\theta$

$\therefore \quad a - x = a - a\cos^2\theta - b\sin^2\theta = (a-b)\sin^2\theta$

and $x - b = a\cos^2\theta - b\sin^2\theta - b = (a-b)\cos^2\theta$

$$\therefore \quad y = (a-b)\sin\theta\cdot\cos\theta - (a-b)\tan^{-1}\tan\theta$$

$$= \frac{a-b}{2}\sin 2\theta - (a-b)\theta$$

$$\therefore \quad \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{(a-b)\cos 2\theta - (a-b)}{(b-a)\sin 2\theta}$$

$$= \frac{1-\cos 2\theta}{\sin\theta} = \tan\theta = \sqrt{\frac{a-x}{x-b}}$$

47. We have $\sin^{-1}\left(\dfrac{x^2-y^2}{x^2+y^2}\right) = \log a$

$$\Rightarrow \quad \frac{x^2-y^2}{x^2+y^2} = \sin(\log a)$$

$$\Rightarrow \quad \frac{1-\tan^2\theta}{1+\tan^2\theta} = \sin(\log a)$$

(on putting $y = x\tan\theta$.)

$$\Rightarrow \quad \cos 2\theta = \sin(\log a)$$

$$\Rightarrow \quad 2\theta = \cos^{-1}(\sin\log a)$$

$$\Rightarrow \quad \theta = \frac{1}{2}\cos^{-1}\{\sin(\log a)\}$$

$$\tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\cos^{-1}\{\sin\log a\}$$

$$\Rightarrow \quad \frac{1}{1+\dfrac{y^2}{x^2}}\cdot\frac{x\dfrac{dy}{dx} - y}{x^2} = 0$$

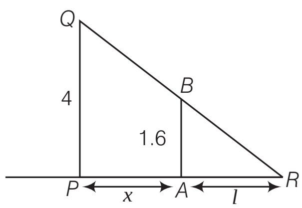
$$\Rightarrow \quad x\frac{dy}{dx} - y = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x} \qquad \ldots\text{(i)}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{-y}{x^2} + \frac{1}{x}\cdot\frac{dy}{dx} = \frac{-y}{x^2} + \frac{1}{x}\left(\frac{y}{x}\right); \qquad \{\text{from (i)}\}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{-y}{x^2} + \frac{y}{x^2} = 0.$$

48. Let $PQ = 4m$ be the height of pole and, $AB = 1.6m$ be height of man. Let the end of shadow is R and it is at a distance of l from A when the man is at a distance x from PQ at some instant.



Since, ΔPQR and ΔABR are similar,

we have, $\dfrac{PQ}{AB} = \dfrac{PR}{AR}$

$$\Rightarrow \quad \frac{4}{1.6} = \frac{x+l}{l}$$

$$\Rightarrow \quad 2x = 3l$$

$$\Rightarrow \quad \frac{2\,dx}{dt} = 3\cdot\frac{dl}{dt} \qquad \left\{\text{given } \frac{dx}{dt} = 30\text{ m/min}\right\}$$

$$\Rightarrow \quad \frac{dl}{dt} = \frac{2}{3}\times 30\text{ m/min} = 20\text{ m/min.}$$

49. $\dfrac{dy}{dx} = Pe^{px} + P$

$$\left(\frac{dy}{dx}\right)_{(0,\,1)} = 2P$$

Subtangent $= \left|y\dfrac{dx}{dy}\right|$, subnormal $= \left|y\dfrac{dy}{dx}\right|$

Given; Subtangent = Subnormal

$$\Rightarrow \quad \frac{dy}{dx} = \pm 1 \Rightarrow 2p = \pm 1 \quad \Rightarrow \quad p = \pm\frac{1}{2}.$$

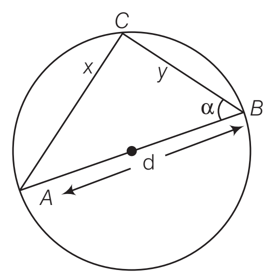
50. Area of ΔABC, $A = \dfrac{1}{2}\,x\sqrt{d^2 - x^2}$

for max/min $\dfrac{dA}{dx} = 0$

$$\Rightarrow \quad \frac{1}{2}\sqrt{d^2-x^2} + \frac{1}{2}\times\left\{\frac{-2x}{2\sqrt{d^2-x^2}}\right\} = 0$$

$$\Rightarrow \quad \frac{d^2 - x^2 - x^2}{2\sqrt{d^2-x^2}} = 0$$

$$\Rightarrow \quad x = \frac{d}{\sqrt{2}}$$



Also for $\quad x < \dfrac{d}{\sqrt{2}}, \dfrac{dA}{dx} > 0$

and for $\quad x > \dfrac{d}{\sqrt{2}}, \dfrac{dA}{dx} < 0.$

Hence; $x = \dfrac{d}{\sqrt{2}}$ is the point of maxima.

$\therefore$ A is max at $x = \dfrac{d}{\sqrt{2}}$, the area is maximum $y = \dfrac{d}{\sqrt{2}}$

$\therefore$ when Δ is isosceles,

51. Let $\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)=\alpha$. then $\cos\alpha=\frac{\sqrt{5}}{3}$, where $0<\alpha<\frac{\pi}{2}$.

Now, $\tan\frac{\alpha}{2}=\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}=\sqrt{\frac{1-\sqrt{5}/3}{1+\sqrt{5}/3}}$

$=\sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}}=\sqrt{\frac{(3-\sqrt{5})^2}{9-5}}=\frac{1}{2}(3-\sqrt{5})$

52. We know that;

if $|x|\le 1$, then $2\tan^{-1}x=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

and if $|x|>1$, then $\pi-2\tan^{-1}x=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

thus, if $|x|>1$.

$2\tan^{-1}x+\sin^{-1}\frac{2x}{1+x^2}=2\tan^{-1}x+\pi$

$-2\tan^{-1}x=\pi$

which is independent of x.

53. $1+\sin x\cdot\sin^2\frac{x}{2}=0$

$\Rightarrow\quad 2+2\sin x\cdot\sin^2\frac{x}{2}=0$

$\Rightarrow\quad 2+\sin x(1-\cos x)=0$

$\Rightarrow\quad 4+2\sin x(1-\cos x)=0$

$\Rightarrow\quad 4+2\sin x-\sin 2x=0$

$\Rightarrow\quad \sin 2x=2\sin x+4$

Above is not possible for any value of x as LHS has maximum value 1 and RHS has minimum value 2.

Hence, there is no solution.

54. $\sin x\cdot\cos y=1$

$\Rightarrow\quad \sin x=1, \cos y=1$

or $\quad \sin x=-1, \cos y=-1$.

if $\quad \sin x=1, \cos y=1 \quad\Rightarrow\quad x=\frac{\pi}{2}, y=0, 2\pi$

if $\quad \sin x=-1, \cos y=-1 \quad\Rightarrow\quad x=\frac{3\pi}{2}, y=\pi$

Thus possible ordered pairs are

$\left(\frac{\pi}{2},0\right),\left(\frac{\pi}{2},2\pi\right),\left(\frac{3\pi}{2},\pi\right)$.

55. Let $I=\int x^x(\log ex)\,dx$

$=\int x^x(1+\log x)\,dx$

let $t=x^x=e^{x\log x}$

$\Rightarrow\quad \frac{dt}{dx}=x^x\left\{x\cdot\frac{1}{x}+\log x\right\}$

$\Rightarrow\quad dt=x^x(1+\log x)\,dx$

$\therefore\quad I=\int t+c=x^x+c.$

56. Let $I=\int\sqrt{1+\operatorname{cosec}x}\,dx=\int\frac{\sqrt{1+\sin x}}{\sqrt{\sin x}}\,dx$

$=\int\pm\frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\sqrt{2\sin\frac{x}{2}\cdot\cos\frac{x}{2}}}\cdot dx$

$=\pm\int\frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\sqrt{1-\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)^2}}\cdot dx$

Put, $\quad \sin\frac{x}{2}-\cos\frac{x}{2}=t$

$\Rightarrow\quad \left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)dx=2dt$

$\therefore\quad I=\pm\int\frac{2\,dt}{\sqrt{1-t^2}}=\pm 2\sin^{-1}t+c$

$=\pm 2\sin^{-1}\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)+c$

57. We have;

$\int\frac{dx}{x^2(x^n+1)^{\frac{(n-1)}{n}}}=\int\frac{dx}{x^2\cdot x^{n-1}\left(1+\frac{1}{x^n}\right)^{\frac{(n-1)}{n}}}$

$=\int\frac{dx}{x^{n+1}(1+x^{-n})^{\frac{(n-1)}{n}}}$

Put, $\quad 1+x^{-n}=t$

$\therefore\quad -nx^{-n-1}dx=dt$

$\Rightarrow\quad \frac{dx}{x^{n+1}}=\frac{-dt}{n}$

$\Rightarrow\quad \int\frac{dx}{x^2(x^n+1)^{\frac{(n-1)}{n}}}=-\frac{1}{n}\int\frac{dt}{t^{\frac{(n-1)}{n}}}$

$=-\frac{1}{n}\int t^{(1/n)-1}\,dt=-\frac{1}{n}\cdot\frac{t^{\left(\frac{1}{n}-1+1\right)}}{\left(\frac{1}{n}-1+1\right)}+c$

$=-t^{1/n}+c=-(1+x^{-n})^{\frac{1}{n}}+c$

58. The given limit

$L=\lim_{n\to\infty}\left\{\frac{1}{na}+\frac{1}{na+1}+\frac{1}{na+2}+\ldots+\frac{1}{na+n(b-a)}\right\}$

$$= \lim_{n \to \infty} \sum_{r=0}^{n(b-a)} \frac{1}{na + r}$$

$$= \lim_{n \to \infty} \frac{1}{n} \sum_{r=0}^{(b-a)n} \frac{1}{a + r/n}$$

$$= \int_0^{(b-a)} \frac{dx}{a + x} = [\log(a + x)]_0^{b-a} \qquad \left(\frac{r}{n} = x\right)$$

$$= \log b - \log a$$

$$= \log\left(\frac{b}{a}\right).$$

59. Case I. If $0 \le a < b$, then $\frac{|x|}{x} = 1$

$\therefore \qquad I = \int_a^b 1 \cdot dx = b - a = |b| - |a|$

Case II : If $a < b \le 0$, then $|x| = -x$

$\therefore \qquad I = \int_a^b \frac{-x}{x} dx = \int_a^b (-1)\, dx = [-x]_a^b$

$$= -b - (-a) = |b| - |a|$$

Case III If $a < 0 < b$.

then $|x| = -x$ when $a < x < 0$

and $|x| = x$ when $0 < x < b$

$$I = \int_a^b \frac{|x|}{x} dx = \int_a^0 \frac{|x|}{x} dx + \int_0^b \frac{|x|}{x} dx$$

$$= \int_a^0 \frac{-x}{x} dx + \int_0^b \frac{x}{x} dx$$

$$= \int_a^0 (-1)\, dx + \int_0^b (1)\, dx$$

$$= [-x]_a^0 + [x]_0^b$$

$$= a + b = b - (-a)$$

$$= |b| - |a|$$

Hence, in all cases,

$$I = \int_a^b \frac{|x|}{x} dx = |b| - |a|$$

60. Here, we have to prove that,

y = constant or derivative of y with respect to x is zero.

$$y = \int_{1/8}^{\sin^2 x} \sin^{-1}\sqrt{t}\, dt + \int_{1/8}^{\cos^2 x} \cos^{-1}\sqrt{t}\, dt \quad \ldots\text{(i)}$$

$$\frac{dy}{dx} = \sin^{-1}\sqrt{\sin^2 x} \cdot 2\sin x \cdot \cos x + \cos^{-1}\sqrt{\cos^2 x}$$

$(-2\sin x \cdot \cos x) = 2x \sin x \cdot \cos x - 2x \cdot \sin x \cdot \cos x$

$= 0$ for all x.

61. $x^2 + y^2 = 1$, $x + y = 1$ meet when

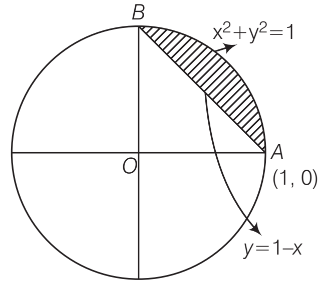
$$x^2 + (1 - x)^2 = 1$$

$\Rightarrow \qquad x^2 + 1 + x^2 - 2x = 1$

$\Rightarrow \qquad 2x^2 - 2x = 0$

$\Rightarrow \qquad 2x(x - 1) = 0$

$\Rightarrow \quad x = 0, x = 1$



$\Rightarrow$ Meet at points $A(1, 0)$, $B(0, 1)$

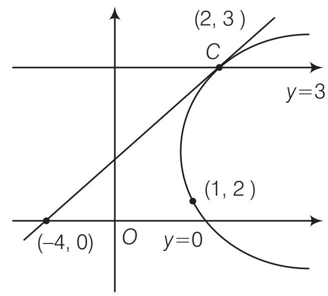
$\therefore$ Required area $= \frac{\pi}{4} - \frac{1}{2}(1) \cdot (1)$

$= \frac{\pi}{4} - \frac{1}{2}$ *i.e.,* [quad. $OAB - \Delta OAB$].

62. Given parabola, is

$$(y - 2)^2 = x - 1$$

$$\Rightarrow \qquad \frac{dy}{dx} = \frac{1}{2(y - 2)}$$



when, $y = 3$, $x = 2$

$$\therefore \qquad \frac{dy}{dx} = \frac{1}{2(3 - 2)} = \frac{1}{2}$$

Tangent at $(x, 3)$ is

$$y - 3 = \frac{1}{2}(x - 2)$$

$\Rightarrow \qquad x - 2y + 4 = 0.$

$\therefore$ Required area

$$\int_0^3 \{(y - 2)^2 + 1\}\, dy - \int_0^3 (2y - 4)\, dy$$

$$= \left[\frac{(y - 2)^3}{3} + y\right]_0^3 - [y^2 - 4y]_0^3$$

$$= \frac{1}{3} + 3 + \frac{8}{3} - (9 - 12) = 9.$$

63. Clearly, the given differential equation is not a polynomial indifferential coefficients. So, its degree is not defined.

64. We have, $\frac{dy}{dx} = \frac{y}{x} - \cos^2\left(\frac{y}{x}\right)$

Putting $y = vx$, so that

$$\frac{dy}{dx} = v + x\frac{dv}{dx},$$

$\therefore \quad v + x\frac{dv}{dx} = v - \cos^2 v$

$\Rightarrow \quad \frac{dv}{\cos^2 v} = -\frac{dx}{x}$

$\Rightarrow \quad \sec^2 v\, dv = -\frac{1}{x}dx$

on integration, we get

$$\tan v = -\log x + \log c$$

$\Rightarrow \quad \tan\left(\frac{y}{x}\right) = -\log x + \log c$

this passes through $(1, \pi/4)$. therefore,

$$1 = \log c$$

So; $\quad \tan\left(\frac{y}{x}\right) = -\log x + 1$

$\Rightarrow \quad \tan\left(\frac{y}{x}\right) = -\log x + \log c$

$\Rightarrow \quad y = x\tan^{-1}\left(\log\frac{c}{x}\right).$

65. $(y\cos y + \sin y)\,dy = (2x\log x + x)\,dx$

$y\sin y - \int \sin y\, dy + \int \sin y\, dx$

$$= x^2\log x - \int x^2 \cdot \frac{1}{x}dx + \int x\,dx + c$$

$\therefore \quad y\sin y = x^2\log x + c$

66. We have length of the

normal = radius vector

$\Rightarrow \quad y\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{x^2+y^2}$

$\Rightarrow \quad y^2\left\{1+\left(\frac{dy}{dx}\right)^2\right\} = x^2 + y^2$

$\Rightarrow \quad y^2\left(\frac{dy}{dx}\right)^2 = x^2$

$\Rightarrow \quad x = \pm\, y\frac{dy}{dx}$

$\Rightarrow \quad x = y\frac{dy}{dx} \quad$ or $\quad x = -y\frac{dy}{dx}$

$\Rightarrow \quad x\,dx - y\,dy = 0$

or $\quad x dx + y dy = 0$

$\Rightarrow \quad x^2 - y^2 = c_1 \quad$ or $\quad x^2 + y^2 = c_2$

Clearly, $x^2 - y^2 = c_1$ represents a rectangular hyperbola and $x^2 + y^2 = c_2$ represents circle.

67. We have,

$$\frac{1}{x(x+1)(x+2)\ldots(x+n)} = \frac{A_0}{x} + \frac{A_1}{x+1} + \ldots + \frac{A_r}{x+r} + \ldots + \frac{A_n}{x+n}$$

$\Rightarrow \quad 1 = A_0\,(x+1)(x+2)\ldots(x+n)$

$+ \ldots + A_r\; x(x+1)(x+2)\ldots(x+r-1)(x+r+1)$

$\ldots(x+n) + \ldots + A_n x(x+1)\ldots(x+n-1)$

Putting $x = -r$, we get;

$1 = A_r\,(-r)(-r+1)(-r+2)\ldots(-3)(-2)(-1)(1)(2)\ldots(n-r)$

$\Rightarrow \quad 1 = A_r\,(-1)^r\,\{r(r-1)(r-2)\ldots(3\cdot2\cdot1\}(1\cdot2\cdot3\ldots(n-r))$

$\Rightarrow \quad A_r = (-1)^r / r!\,(n-r)!$

68. $\log_2(9-2^x) = 10^{\log_{10}(3-x)}$

$\Rightarrow \quad \log_2(9-2^x) = (3-x) \qquad [\because a^{\log_a b} = b]$

$\Rightarrow \quad 2^{3-x} = 9 - 2^x$

$\Rightarrow \quad \frac{2^3}{2^x} = 9 - 2^x$

$\Rightarrow \quad 8 = 2^x \times (9 - 2^x)$

$\Rightarrow \quad 2^{2x} - 2^x \times 9 + 8 = 0$

Let $2^x = y$, then;

$$y^2 - 9y + 8 = 0$$

$\Rightarrow \quad (y-8)(y-1) = 0$

$\Rightarrow \quad y = 8 \quad$ or $\quad y = 1.$

$\Rightarrow \quad 2^x = 2^3 \quad$ or $\quad 2^x = 2^0$

$\Rightarrow \quad x = 3 \quad$ or $\quad x = 0.$

But $x = 3$ does not satisfy the given equation, since $\log 0$ is not defined.

69. a, b, c are in GP $\Rightarrow b^2 = ac$

taking log on both the sides, we get

$$2\log b = \log a + \log c$$

$\Rightarrow \quad \log b = \frac{\log a + \log c}{2}$

$\Rightarrow \quad \log a, \log b, \log c$ are in AP.

70. We have; $\Delta = \frac{\sqrt{3}}{4}a^2,\ s = \frac{3a}{2}$

$\therefore \quad r = \frac{\Delta}{S} = \frac{a}{2\sqrt{3}},\ R = \frac{abc}{4\Delta}$

$$= \frac{a^3}{\sqrt{3}\cdot a^2} = \frac{a}{\sqrt{3}}$$

and $\quad r_1 = \frac{\Delta}{S-a} = \frac{\sqrt{3}/4\cdot a^2}{a/2} = \frac{\sqrt{3}}{2}\cdot a$

Hence; $\; r : R : r_1 = \frac{a}{2\sqrt{3}} : \frac{a}{\sqrt{3}} : \frac{\sqrt{3}}{2}\cdot a = 1 : 2 : 3$

71. Since the angles are in AP, therefore

$$2B = A + C \quad \Rightarrow \quad 3B = A + B + C$$

$$\Rightarrow \quad 3B = 180° \quad \Rightarrow \quad B = 60°$$

using the formula,

$$\frac{\sin A}{a} = \frac{\sin B}{b}, \text{ we get}$$

$$\sin A = \frac{a}{b} \sin B = \frac{24}{22} \cdot \sin 60° = \frac{6\sqrt{3}}{11}$$

Now; $\sin C = \sin(180° - (A + B)) = \sin(A + B)$

$$= \sin A \cdot \cos B + \cos A \cdot \sin B$$

$$= \left(\frac{6\sqrt{3}}{11}\right)\left(\frac{1}{2}\right) + \sqrt{\left\{1 - \left(\frac{6\sqrt{3}}{11}\right)^2\right\}} \cdot \left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{1}{22}\left(\frac{\sqrt{3}}{2}\right)(12 + 2\sqrt{3})$$

Now, $c = \dfrac{b}{\sin B} \sin C$

$$\Rightarrow \quad c = 12 + 2\sqrt{3}.$$

72. $2^{-1} * x * 3^{-1} = 5$

$$\Rightarrow \quad x = 2 * (5 * 3) = 2 * (5 + 3 + 15)$$

$$= 2 * (23) = 2 + 23 + 46 = 71$$

73. $b^{-1} * a^{-1} * b * a = (b^{-1} * a^{-1}) * (b * a) = (a * b)^{-1}$

$$* (b * a) = (a * b)^{-1} * (a * b) = e.$$

74. $(a * b)^2 = (a * a) * (b * b)$ for all $a, b \in G$

$\Rightarrow \quad (a * b) * (a * b) = (a * a) * (b * b)$ for all $a, b \in G$

$a * (b * a) * b = a * ((a * b) * b$

for all $a, b \in G$

$\Rightarrow \quad b * a = a * b$ for all $a, b \in G$

{by cancellation laws}

$\Rightarrow \quad G$ is abelian.

77. By hit and trial method taking $x = y = z = 2$,

we get $xy + yz + zx = 12$

and $xyz = 8.$

78. Since, c is a prime number so it can have only two factors 1 and c. $ab = c$ gives two more factors of c as a and b, which is not possible.

79. $(p \wedge \sim q)(\sim p \wedge q) = (p \wedge \sim p) \wedge (\sim q \wedge q)$

$$= f \wedge f = f$$

(By using associative laws and commutative laws.)

$\therefore \quad (p \wedge \sim q) \wedge (\sim p \wedge q)$ is a contradiction.

80. $\sim p \wedge q = \sim (q \rightarrow p)$

English

6. **Radiant** means giving a warm bright light.

7. **Prune** means to make something smaller by removing parts.

8. **Dilettante** means a person, who does something, but is not serious about it.

9. **Foster** means to encourage something to develop.

10. **Enigma** means a person or thing that is mysterious or difficult to understand.

11. **Conform** means to agree with or match something.

12. **Aboriginal** means relating to the original people, animals etc.

13. **Amenable** means easy to control.

14. **Acquit** means to state officially that somebody is not guilty of a crime.

15. **Forbidden** means not allowed.

31. Remove 'do not'.

32. Remove 'about'.

33. Use 'but' in place of 'and'.

34. Use 'that' in place of 'because'.

35. Use 'were' in place of 'was'.

36. The correct spelling is 'Honorary'.

37. The correct spelling is 'Maintenance'.

38. The correct spelling is 'Vehicle'.

39. The correct spelling is 'Daffodil'.

40. The correct spelling is 'Curable'.