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MHT CET 2021 Question Paper with Solution

Paper - 1st and Paper - 2nd

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Question 1

With usual notations, in any $\triangle ABC$, if $a \cos B = b \cos A$, then the triangle is

Options:

- A. an isosceles triangle
- B. an equilateral triangle
- C. a right angled triangle
- D. a scalene triangle

Answer: A

Solution:

Solution:

Using sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = K$$

$$\text{We have, } \frac{\cos A}{a} = \frac{\cos B}{b}$$

$$\Rightarrow \frac{\cos A}{K \sin A} = \frac{\cos B}{K \sin B}$$

$$\Rightarrow K \sin B \cos A = K \sin A \cos B$$

$$\Rightarrow \cos A \sin B - \cos B \sin A = 0$$

$$\Rightarrow \sin(A - B) = 0$$

$$A = B$$

\therefore Triangle is an isosceles triangle.

Question 2

If θ lies in the first quadrant and $5 \tan \theta = 4$, then $\frac{5 \sin \theta - 3 \cos \theta}{\sin \theta + 2 \cos \theta}$ is equal to

Options:

A. $\frac{5}{14}$

B. $\frac{3}{14}$

C. $\frac{1}{14}$

D. 0

Answer: A

Solution:

Solution:

Given, $\tan \theta = \frac{4}{5}$

$\therefore \sin \theta = \frac{4}{\sqrt{41}}$ and $\cos \theta = \frac{5}{\sqrt{41}}$

Now,
$$\frac{5 \sin \theta - 3 \cos \theta}{\sin \theta + 2 \cos \theta} = \frac{5 \times \frac{4}{\sqrt{41}} - 3 \times \frac{5}{\sqrt{41}}}{\frac{4}{\sqrt{41}} + 2 \times \frac{5}{\sqrt{41}}} = \frac{5}{14}$$

Question 3

If $\cos^{-1}x > \sin^{-1}x$, then

Options:

A. $\frac{1}{\sqrt{2}} < x \leq 1$

B. $0 \leq x \leq \frac{1}{\sqrt{2}}$

C. $-1 \leq x < \frac{1}{\sqrt{2}}$

D. $x > 0$

Answer: C

Solution:

Solution:

$\cos^{-1}x > \sin^{-1}x$ [where $x \in [-1, 1]$]

$\therefore \cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x$

$\therefore \frac{\pi}{2} - \sin^{-1}x > \sin^{-1}x \Rightarrow 2\sin^{-1}x < \frac{\pi}{2}$

$\Rightarrow \sin^{-1}x < \frac{\pi}{4} \Rightarrow x < \frac{1}{\sqrt{2}}$

$\therefore -1 \leq x \leq 1$

$-1 \leq x < \frac{1}{\sqrt{2}}$

Question 4

The value of $\tan 3A - \tan 2A - \tan A$ is

Options:

- A. $\tan 3A \tan 2A \tan A$
- B. $-\tan 3A \tan 2A \tan A$
- C. $\tan A \tan 2A - \tan 2A \tan 3A$
- D. $\tan 3A \tan A - \tan 2A \tan 3A$

Answer: A**Solution:****Solution:**Let $3A = A + 2A$ $\tan 3A = \tan(A + 2A)$

$$\tan 3A = \frac{\tan A + \tan 2A}{1 - \tan A \cdot \tan 2A}$$

$$\Rightarrow \tan A + \tan 2A = \tan 3A - \tan 3A \tan 2A \cdot \tan A$$

$$\Rightarrow \tan 3A - \tan 2A - \tan A = \tan 3A \cdot \tan 2A \cdot \tan A$$

Question 5

The three straight lines $ax + by = c$, $bx + cy = a$ and $cx + ay = b$ are collinear, if

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Options:

- A. $b + c = a$
- B. $c + a = b$
- C. $a + b + c = 0$
- D. $a + b = c$

Answer: C**Solution:****Solution:**Given lines are $ax + by = c$, $bx + cy = a$ and $cx + ay = b$

On adding the given three equations, we get

$$ax + by + bx + cy + cx + ay = a + b + c$$

$$\Rightarrow (a + b + c)x + (a + b + c)y = (a + b + c)$$

On comparing with $0x + 0y = 0$ for collinearity, we get

$$a + b + c = 0$$

Question 6

A line through the point $A(2, 0)$ which makes an angle of 30° with the positive direction of X-axis is rotated about A in clockwise direction

through an angle of 15° . Then, the equation of the straight line in the new position is

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Options:

A. $(2 - \sqrt{3})x + y - 4 + 2\sqrt{3} = 0$

B. $(2 - \sqrt{3})x - y - 4 + 2\sqrt{3} = 0$

C. $(2 - \sqrt{3})x - y + 4 + 2\sqrt{3} = 0$

D. $(2 - \sqrt{3})x + y + 4 + 2\sqrt{3} = 0$

Answer: B

Solution:

Solution:

The equation of line in new position is

$$y - 0 = \tan 15^\circ (x - 2)$$

$$\Rightarrow y = (2 - \sqrt{3})(x - 2)$$

$$\Rightarrow (2 - \sqrt{3})x - y - 4 + 2\sqrt{3} = 0$$

Question 7

Find the value of $\cos\left(\frac{x}{2}\right)$, if $\tan x = \frac{5}{12}$ and x lies in third quadrant.

Options:

A. $\frac{5}{\sqrt{13}}$

B. $\frac{5}{\sqrt{26}}$

C. $\frac{5}{13}$

D. $\sqrt{\frac{1}{26}}$

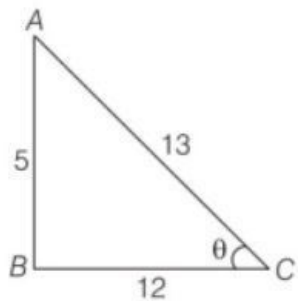
Answer: D

Solution:

Solution:

Given, $\tan x = \frac{5}{12}$ and x lies in III quadrant.

$$\therefore \sin x = \frac{-5}{13} \text{ and } \cos x = \frac{-12}{13}$$



Now, $\cos x = 2\cos^2 \frac{x}{2} - 1$

$$\Rightarrow \cos^2 \frac{x}{2} = \frac{1}{2}(\cos x + 1)$$

$$= \frac{1}{2} \left(\frac{-12}{13} + 1 \right) = \frac{1}{2} \left(\frac{1}{13} \right) = \frac{1}{26}$$

$$\therefore \cos \frac{x}{2} = \sqrt{\frac{1}{26}}$$

Question 8

The locus of a point which moves, so that the ratio of the length of the tangents to the circles $x^2 + y^2 + 4x + 3 = 0$ and $x^2 + y^2 - 6x + 5 = 0$ is $2 : 3$, is

Options:

A. $5x^2 + 5y^2 - 60x + 7 = 0$

B. $5x^2 + 5y^2 + 60x - 7 = 0$

C. $5x^2 + 5y^2 - 60x - 7 = 0$

D. $5x^2 + 5y^2 + 60x + 7 = 0$

Answer: D

Solution:

Solution:

Since, $\frac{\sqrt{S_1}}{\sqrt{S_2}} = \frac{2}{3}$

$$\therefore \frac{\sqrt{x_1^2 + y_1^2 + 4x_1 + 3}}{\sqrt{x_1^2 + y_1^2 - 6x_1 + 5}} = \frac{2}{3}$$

$$\Rightarrow 9x_1^2 + 9y_1^2 + 36x_1 + 27 - 4x_1^2 - 4y_1^2 + 24x_1 - 20 = 0$$

$$\Rightarrow 5x_1^2 + 5y_1^2 + 60x_1 + 7 = 0$$

\therefore Locus of point (x, y) is

$$5x^2 + 5y^2 + 60x + 7 = 0$$

Question 9

The circles $x^2 + y^2 + 6x + 6y = 0$ and $x^2 + y^2 - 12x - 12y = 0$

Options:

- A. cut orthogonally
- B. touch each other internally
- C. intersect two points
- D. touch each other externally

Answer: D**Solution:****Solution:**

The centres of given circles are $C_1(-3, -3)$, $C_2(6, 6)$ and radii are

$r_1 = \sqrt{9 + 9 + 0} = 3\sqrt{2}$, $r_2 = \sqrt{36 + 36 + 0} = 6\sqrt{2}$, respectively.

Now, $C_1C_2 = \sqrt{(6 + 3)^2 + (6 + 3)^2} = 9\sqrt{2}$

and $r_1 + r_2 = 3\sqrt{2} + 6\sqrt{2} = 9\sqrt{2}$

Here, $C_1C_2 = r_1 + r_2$

So, both circles touch each other externally.

Question 10

The mean and variance of n observations $x_1, x_2, x_3, \dots, x_n$ are 5 and 0, respectively. If $\sum_{i=1}^n x_i^2 = 400$, then the value of n is equal to

Options:

- A. 80
- B. 25
- C. 20
- D. 16

Answer: D**Solution:****Solution:**

$\bar{x} = 5$, variance = $\frac{1}{n} \sum x_i^2 - (\bar{x})^2$

$$\Rightarrow 0 = \frac{1}{n} \cdot 400 - 25$$

$$\Rightarrow n = \frac{400}{25} = 16$$

Question 11

The variance of first n natural numbers is

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Options:

A. $\frac{n(n+1)}{2}$

B. $\frac{(n+1)(n+5)}{12}$

C. $\frac{(n+1)(n-5)}{12}$

D. $\frac{(n^2-1)}{12}$

Answer: D

Solution:

Solution:

Variance of first n natural numbers

$$\begin{aligned} &= \frac{\sum n^2}{n} - \left(\frac{\sum n}{n} \right)^2 \\ &= \frac{n(n+1)(2n+1)}{6n} - \left(\frac{n(n+1)}{2n} \right)^2 \\ &= (n+1) \left[\frac{2n+1}{6} - \frac{(n+1)}{4} \right] \\ &= \frac{(n+1)}{12} [4n+2-3n-3] \\ &= \frac{(n+1)}{12} \times (n-1) = \frac{n^2-1}{12} \end{aligned}$$

Question 12

Out of 50 tickets numbered 00, 01, 02, ..., 49, one ticket is drawn randomly, the probability of the ticket having the product of its digits 7, given that the sum of the digits is 8, is

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Options:

A. $\frac{1}{14}$

B. $\frac{3}{14}$

C. $\frac{1}{5}$

D. None of these

Answer: C

Solution:

Solution:

Total number of cases = ${}^{50}C_1 = 50$

Let A be the event of selecting ticket with sum of digits ' 8 '.

Favourable cases to A are {08, 17, 26, 35, 44}.

Let B be the event of selecting ticket with product of its digits ' 7 '.

Favourable cases to B is only {17}.

$$\text{Now, } P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{1/50}{5/50} = \frac{1}{5}$$

Question 13

The probability of India winning a test match against South Africa is $\frac{1}{2}$ assuming independence form match to match played. The probability that in a match series India's second win occurs at the third day is

Options:

A. $\frac{1}{8}$

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. $\frac{2}{3}$

Answer: C

Solution:**Solution:**

Given, probability of winning a test match, $P(W) = \frac{1}{2}$

Probability of lossing a match, $P(L) = 1 - \frac{1}{2} = \frac{1}{2}$

Probability that India's second win occurs at the third day

$$= P(L) \cdot P(W) \cdot P(W) + P(W) \cdot P(L) \cdot P(W)$$

$$= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$$

Question 14

Consider the following statements

r : If a number is a multiple of 9 , then it is multiple of 3.

Let p and q denote the statements

p: A number is a multiple of 9.

q : A number is a multiple of 3 .

Then, if p then q is the same as

Options:

- A. p only if q
- B. q is a necessary condition for p
- C. $\sim q$ implies $\sim p$
- D. All of the above

Answer: D**Solution:****Solution:**

I. p only if q. This says that number is a multiple of 9 only, if it is a multiple of 3.

II. q is a necessary condition for p.

This says that when a number is a multiple of 9, it is necessarily a multiple of 3.

III. $\sim q$ implies $\sim p$. This says that if a number is not a multiple of 3, then it is not a multiple of 9.

Question 15

The negation of the statement 7 is greater than 4 or 6 is less than 7 .

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Options:

- A. 7 is not greater than 4 and 6 is not less than 7
- B. 7 is not greater than 4 or 6 is not less than 7
- C. 7 is greater than 4 and 6 is less than 7
- D. None of the above

Answer: A**Solution:****Solution:**

Let p : 7 is greater than 4

and q : 6 is less than 7 .

Then, the given statement is disjunction $p \vee q$.

Here, $\sim p$: 7 is not greater than 4 .

and $\sim q$: 6 is not less than 7 .

$\therefore \sim(p \vee q)$: 7 is not greater than 4 and 6 is not less than 7 .

Question 16

For an invertible matrix A, if $A(\text{adj} A) = \begin{vmatrix} 20 & 0 \\ 0 & 20 \end{vmatrix}$, then $|A| =$

Options:

- A. 20
- B. -200
- C. 200
- D. -20

Answer: A**Solution:****Solution:**

$$\text{Given, } A(\text{adj}A) = \begin{vmatrix} 20 & 0 \\ 0 & 20 \end{vmatrix}$$

$$20 \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 20I$$

We know that

$$A^{-1} = \frac{1}{|A|} \text{adj}A$$

$$\Rightarrow A(\text{adj}A) = |A| I$$

$$\Rightarrow 20I = |A| I$$

$$\Rightarrow |A| = 20$$

Question 17

If matrix $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$, then $A^2 - 4A + 5I$ is where I is a unit matrix.

Options:

- A. Null matrix
- B. Skew symmetric matrix
- C. symmetric Matrix
- D. None of the above

Answer: A**Solution:****Solution:**

$$\text{Given, } A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

$$\therefore A^2 = A \cdot A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \times 1 + (-1) \times 2 & 1 \times (-1) + (-1) \times 3 \\ 2 \times 1 + 3 \times 2 & 2 \times (-1) + 3 \times 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2 & -1-3 \\ 2+6 & -2+9 \end{bmatrix} = \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix}$$

Now, $A^2 - 4A + 5I$

$$= \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix} - 4 \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} + 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix} - \begin{bmatrix} 4 & -4 \\ 8 & 12 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -1-4+5 & -4+4+0 \\ 8-8+0 & 7-12+5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Question 18

If $f(x) = \begin{cases} \frac{3 \sin \pi x}{5x} & x \neq 0 \\ 2k & x = 0. \end{cases}$ is continuous at $x = 0$, then the value of k is

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Options:

A. $\frac{\pi}{10}$

B. $\frac{3\pi}{10}$

C. $\frac{3\pi}{2}$

D. $\frac{3\pi}{5}$

Answer: D

Solution:

Solution:

Given, $f(x) = \begin{cases} \frac{3 \sin \pi x}{5x} & x \neq 0 \\ 2k & x = 0. \end{cases}$

Now, $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \left(\frac{3 \sin \pi x}{5x} \right)$
 $= \frac{3}{5} \lim_{x \rightarrow 0} \left(\sin \frac{\pi x}{\pi x} \right) \times \pi = \frac{3}{5} \times 1 \times \pi = \frac{3}{5}\pi$

Also, $f(0) = 2k$

Since, $f(x)$ is continuous at $x = 0$.

$\therefore f(0) = \lim_{x \rightarrow 0} f(x)$

$\Rightarrow 2k = \frac{3}{5}\pi \Rightarrow k = \frac{3\pi}{10}$

Question 19

If $f(x) = \begin{cases} \frac{\sin^3(\sqrt{3}) \cdot \log(1+3x)}{(\tan^{-1}\sqrt{x})^2(e^{5\sqrt{3}}-1)x} & x \neq 0 \\ a & x = 0. \end{cases}$ is continuous in $[0, 1]$ then a equals

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Options:

A. 0

B. $\frac{3}{5}$

C. 2

D. $\frac{5}{3}$

Answer: B

Solution:

Solution:

$$\text{We have, } f(x) = \begin{cases} \frac{\sin^3(\sqrt{3}) \cdot \log(1+3x)}{(\tan^{-1}\sqrt{x})^2(e^{5\sqrt{3}}-1)x} & x \neq 0 \\ a & x = 0. \end{cases}$$

For continuity in $[0, 1]$, $f(0) = \lim_{x \rightarrow 0} f(x)$ otherwise it is discontinuous.

$$\begin{aligned} \therefore a &= \lim_{x \rightarrow 0} \frac{\sin^3(\sqrt{x}) \cdot \log(1+3x)}{x(\tan^{-1}\sqrt{x})^2 \cdot (e^{5\sqrt{x}}-1)} \\ &= \lim_{x \rightarrow 0} \left[\frac{3}{5} \cdot \frac{\sin^3\sqrt{x}}{(\sqrt{x})^3} \cdot \frac{(\sqrt{x})^3}{(\tan^{-1}\sqrt{x})^3} \right. \\ &\quad \left. \& \times \frac{\log(1+3x)}{3x} \cdot \frac{5\sqrt{x}}{e^{5\sqrt{x}}-1} \right] \\ &= \frac{3}{5} \lim_{x \rightarrow 0} \frac{\sin^3}{(\sqrt{x})^3} \cdot \frac{(\sqrt{x})^3}{\tan^{-1}\sqrt{x}} \\ \therefore a &= \frac{3}{5} \end{aligned}$$

Question 20

The differential equation of all parabolas having vertex at the origin and axis along positive Y-axis is

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Options:

A. $x^2 \frac{dy}{dx} - y = 0$

B. $x \frac{dy}{dx} + 2y = 0$

C. $x \frac{dy}{dx} = 2y$

D. $x^3 \frac{dy}{dx} = 3y$

Answer: C

Solution:

Solution:

The general equation of an parabola having vertex at the origin and axis along positive Y-axis is

$$x^2 = 4ay. \dots (i)$$

On differentiating Eq. (i), we get

$$2x = 4a \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{2a} \Rightarrow 2a = \frac{x}{\frac{dy}{dx}}$$

Putting value of 2a in Eq. (i), we get

$$x^2 = 2 \left(\frac{x}{\frac{dy}{dx}} \right) y \Rightarrow x \frac{dy}{dx} = 2y$$

Question 21

The order and degree of the differential equation $\sqrt{\frac{dy}{dx}} - 4 \frac{dy}{dx} - 7x = 0$ are

Options:

A. 1 and 1/2

B. 2 and 1

C. 1 and 1

D. 1 and 2

Answer: D

Solution:

Solution:

Given equation is $\sqrt{\frac{dy}{dx}} - 4 \frac{dy}{dx} - 7x = 0$

$$\Rightarrow \frac{dy}{dx} = 16 \left(\frac{dy}{dx} \right)^2 + 49x^2 + 56x \frac{dy}{dx}$$

Obviously, it is of first order and second degree differential equation.

Question 22

The solution of the differential equation

$e^{-x}(y+1)dy + (\cos^2 x - \sin 2x)ydx = 0$ subjected to the condition that $y = 1$, when $x = 0$ is

Options:

- A. $y + \log y + e^x \cos^2 x = 2$
- B. $\log(y + 1) + e^x \cos^2 x = 1$
- C. $y + \log y = e^x \cos^2 x$
- D. $(y + 1) + e^x \cos^2 x = 2$

Answer: A**Solution:****Solution:**

Given equation can be rewritten as

$$\left(1 + \frac{1}{y}\right) dy = -e^x(\cos^2 x - \sin 2x) dx$$

On integrating both sides, we get

$$y + \log y = -e^x \cos^2 x + \int e^x \sin 2x dx$$

$$- \int e^x \sin 2x dx + C$$

$$\Rightarrow y + \log y = -e^x \cos^2 x + C$$

At $x = 0$ and $y = 1$,

$$1 + 0 = -e^0 \cos 0 + C$$

$$C = 2 \text{ [given]}$$

\therefore Required solutions is

$$y + \log y = -e^x \cos^2 x + 2$$

$$\Rightarrow y + \log y + e^x \cos^2 x = 2$$

Question 23

$$\int_{-1}^1 \frac{17x^5 - x^4 + 29x^3 - 31x + 1}{x^2 + 1} dx \text{ is equal to}$$

Options:

- A. $\frac{4}{5}$
- B. $\frac{5}{4}$
- C. $\frac{4}{3}$
- D. $\frac{3}{4}$

Answer: C**Solution:****Solution:**

$$\int_{-1}^1 \frac{17x^5 - x^4 + 29x^3 - 31x + 1}{x^2 + 1} dx$$

$$= \int_{-1}^1 \frac{17x^5 + 29x^3 - 31x}{x^2 + 1} dx - \int_{-1}^1 \frac{x^4 - 1}{x^2 + 1} dx$$

Odd function Even function

$$= 0 - 2 \int_0^1 \frac{(x^2 - 1)(x^2 + 1)}{(x^2 + 1)} dx = -2 \left[\left(\frac{x^3}{3} - x \right) \right]_0^1 = \frac{4}{3}$$

Question 24

The number of ways of arranging letters of the 'HAVANA', so that V and N do not appear together, is

Options:

- A. 60
- B. 80
- C. 100
- D. 120

Answer: B

Solution:

Solution:
 Given word is 'HAVANA' (3A, 1H, 1N, 1V)
 Total number of ways of arranging the given word
 $= \frac{6!}{3!} = 120$
 Total number of words in which N, V together
 $= \frac{5!}{3!} \times 2! = 40$
 \therefore Required number of ways = $120 - 40 = 80$

Question 25

Out of 7 consonants and 4 vowels, the number of words (not necessarily meaningful) that can be made, each consisting of 3 consonants and 2 vowels, is

Options:

- A. 24800
- B. 25100
- C. 25200
- D. 25400

Answer: C

Solution:

Solution:

3 consonants can be selected from 7 consonants = 7C_3 ways

2 vowels can be selected from 4 vowels = 4C_2 ways

∴ Required number of words = ${}^7C_3 \times {}^4C_2 \times 5!$

[selected 5 letters can be arranged in $5!$ ways, to get a different word]
= $35 \times 6 \times 120 = 25200$

Question 26

If $f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cos\left(x + \frac{\pi}{3}\right)$ and $g\left(\frac{5}{4}\right) = 1$, then $g(f(x))$ is equal to

Options:

A. 1

B. -1

C. 2

D. -2

Answer: A

Solution:

Solution:

Given $f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x$

$\cos\left(x + \frac{\pi}{3}\right)$

$$= \sin^2 x + \left[\sin x \cos \frac{\pi}{3} + \cos x \sin \frac{\pi}{3} \right]^2$$

$$+ \cos x \left[\cos x \cos \frac{\pi}{3} - \sin x \sin \frac{\pi}{3} \right]$$

$$= \sin^2 x + \left[\frac{\sin x}{2} + \cos x \cdot \frac{\sqrt{3}}{2} \right]^2$$

$$\therefore = \cos x \left[\frac{\cos x}{2} - \sin x \cdot \frac{\sqrt{3}}{2} \right]$$

$$= \sin^2 x + \frac{\sin^2 x}{4} + \frac{3 \cos^2 x}{4} + \sin x \cos x \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{5 \sin^2 x}{4} + \frac{5 \cos^2 x}{4} = \frac{5}{4}$$

$$\Rightarrow f(x) = g[f(x)] = g\left(\frac{5}{4}\right) = 1$$

Question 27

Range of the function $f(x) = \frac{x}{1+x^2}$ is

Options:

A. $(-\infty, \infty)$

B. $[-1, 1]$

C. $\left[-\frac{1}{2}, \frac{1}{2}\right]$

D. $[-\sqrt{2}, \sqrt{2}]$

Answer: C

Solution:

Solution:

$$\text{Let } y = \frac{x}{1+x^2} \Rightarrow x^2y - x + y = 0$$

$$\text{For } x \text{ to be real, } 1 - 4y^2 \geq 0 \{ \text{Discriminant} = 1 - 4y^2 \}$$

$$\Rightarrow 4y^2 - 1 \leq 0$$

$$\Rightarrow (2y - 1)(2y + 1) \leq 0$$

$$\Rightarrow -\frac{1}{2} \leq y \leq \frac{1}{2}$$

$$\therefore y = f(x) \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

Question 28

The domain of $f(x) = \sin^{-1} \left[\log_2 \left(\frac{x}{2} \right) \right]$ is

Options:

A. $0 \leq x \leq 1$

B. $0 \leq x \leq 4$

C. $1 \leq x \leq 4$

D. $4 \leq x \leq 6$

Answer: C

Solution:

Solution:

$$\text{Given, } f(x) = \sin^{-1} \left[\log_2 \left(\frac{x}{2} \right) \right]$$

$$\Rightarrow -1 \leq \log_2 \left(\frac{x}{2} \right) \leq 1 \Rightarrow 2^{-1} \leq \frac{x}{2} \leq 2^1$$

$$\Rightarrow \frac{1}{2} \leq \frac{x}{2} \leq 2 \Rightarrow 1 \leq x \leq 4$$

Question 29

If $f(x) = \frac{k}{2^x}$ is a probability distribution of a random variable X that can take on the values $x = 0, 1, 2, 3, 4$. Then, k is equal to

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Options:

- A. $\frac{16}{15}$
- B. $\frac{15}{16}$
- C. $\frac{31}{16}$
- D. None of these

Answer: D

Solution:

Solution:

We have, $f(x) = \frac{k}{2^x}$, $x = 0, 1, 2, 3, 4$

Since, $f(x)$ is a probability distribution of a random variable X , therefore we have

$$\sum_{x=0}^4 f(x) = 1 \Rightarrow \sum_{x=0}^4 \left(\frac{k}{2^x} \right) = 1$$

$$\Rightarrow k \sum_{x=0}^4 \frac{1}{2^x} = 1$$

$$\Rightarrow k \left(1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} \right) = 1$$

$$\Rightarrow k \left(\frac{16 + 8 + 4 + 2 + 1}{2^4} \right) = 1$$

$$\Rightarrow k \times \left(\frac{31}{16} \right) = 1$$

$$\therefore k = \frac{16}{31}$$

Question 30

$\lim_{x \rightarrow 0} \frac{e^{x^2} - \cos x}{x^2}$ is equal to

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Options:

- A. 0
- B. $\frac{1}{2}$
- C. 1

D. $\frac{3}{2}$

Answer: D

Solution:

Solution:

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{e^{x^2} - \cos x}{x^2} & \left[\frac{0}{0} \text{ form} \right] \text{ [using L'Hospital's rule]} \\ &= \lim_{x \rightarrow 0} \frac{2xe^{x^2} + \sin x}{2x} \left[\frac{0}{0} \text{ form} \right] \\ &= \lim_{x \rightarrow 0} \frac{2e^{x^2} + 4x^2e^{x^2} + \cos x}{2} \\ &= \frac{2 + 0 + 1}{2} = \frac{3}{2} \text{ [using L'Hospital's rule]} \end{aligned}$$

Question 31

Evaluate $\lim_{x \rightarrow \sqrt{2}^-} \frac{x^4 - 4}{x^2 + 3\sqrt{2}x - 8}$.

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Options:

A. $\frac{8}{5}$

B. $\frac{8}{3}$

C. $\frac{5}{8}$

D. $\frac{3}{8}$

Answer: A

Solution:

Solution:

$$\begin{aligned} \lim_{x \rightarrow \sqrt{2}} \frac{x^4 - 4}{x^2 + 3\sqrt{2}x - 8} \\ &= \lim_{x \rightarrow \sqrt{2}} \frac{(x + \sqrt{2})(x - \sqrt{2})(x^2 + 2)}{(x - \sqrt{2})(x + 4\sqrt{2})} \\ &= \lim_{x \rightarrow \sqrt{2}} \frac{(x + \sqrt{2})(x^2 + 2)}{(x + 4\sqrt{2})} \\ &= \frac{(\sqrt{2} + \sqrt{2})(2 + 2)}{(\sqrt{2} + 4\sqrt{2})} = \frac{(2\sqrt{2})(4)}{5\sqrt{2}} = \frac{8\sqrt{2}}{5\sqrt{2}} = \frac{8}{5} \end{aligned}$$

Question 32

What will be projection of the vector $4\hat{i} - 3\hat{j} + \hat{k}$ on the line joining the

points $(2, 3, -1)$ and $(-2, -4, 3)$?

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Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: A

Solution:

Solution:

Let point A = $(2, 3, -1)$ and point B = $(-2, -4, 3)$.

Now the position vector of line joining A and B,

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$= -2\hat{i} - 4\hat{j} + 3\hat{k} - 2\hat{i} - 3\hat{j} + \hat{k}$$

$$= -4\hat{i} - 7\hat{j} + 4\hat{k}$$

Again let $\vec{a} = \vec{AB} = -4\hat{i} - 7\hat{j} + 4\hat{k}$

and $\vec{b} = 4\hat{i} - 3\hat{j} + \hat{k}$

Then, $\vec{a} \cdot \vec{b} = (-4\hat{i} - 7\hat{j} + 4\hat{k}) \cdot (4\hat{i} - 3\hat{j} + \hat{k})$
 $= -16 + 21 + 4 = 9$

$$|\vec{a}| = \sqrt{(-4)^2 + (-7)^2 + (4)^2}$$
$$= \sqrt{16 + 49 + 16} = 9$$

Now projection of \vec{b} on \vec{a}

$$= \frac{(\vec{a} \cdot \vec{b})}{|\vec{a}|} = \frac{9}{9} = 1$$

Question 33

If $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 4\hat{j} - 3\hat{k}$, then $|\vec{a} - 2\vec{b}|$ will be

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Options:

- A. 9
- B. $\sqrt{86}$
- C. $\sqrt{94}$
- D. 10

Answer: B

Solution:

Solution:

$$a = 3\hat{i} - 2\hat{j} + \hat{k} \dots (i)$$

$$b = 2\hat{i} - 4\hat{j} - 3\hat{k} \dots (ii)$$

Multiplying by 2 both sides, we get

$$2b = 4\hat{i} - 8\hat{j} - 6\hat{k} \dots (iii)$$

Subtracting Eq. (iii) from Eq. (i), we get

$$\begin{aligned} a - 2b &= (3\hat{i} - 2\hat{j} + \hat{k}) - (4\hat{i} - 8\hat{j} - 6\hat{k}) \\ &= -\hat{i} + 6\hat{j} + 7\hat{k} \\ &= \sqrt{(-1)^2 + (6)^2 + (7)^2} = \sqrt{86} \end{aligned}$$

Question 34

What will be the equation of plane passing through a point $(1, 4, -2)$ and parallel to the given plane $-2x + y - 3z = 9$?

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Options:

A. $2x - y + 3z + 8 = 0$

B. $2x = y + 3z + 8 = 0$

C. $2x - 2y + 2z + 10 = 0$

D. $2x + 2y - 3z + 8 = 0$

Answer: A

Solution:

Solution:

$$r \cdot (2\hat{i} - 3\hat{j} + 5\hat{k}) + 2 = 0$$

Equation of plane parallel to the given plane is

$$-2x + y - 3z = \lambda \dots (i)$$

Since, plane (i) passes through the point $(1, 4, -2)$.

Therefore this point satisfy the given plane. Put $x = 1$, $y = 4$ and $z = -2$ in Eq. (i)

$$-2(1) + 4 - 3(-2) = \lambda$$

$$-2 + 4 + 6 = \lambda$$

$$\lambda = 8$$

Put $\lambda = 8$ in Eq. (i), we get

$$-2x + y - 3z = 8$$

$$2x - y + 3z + 8 = 0$$

Question 35

If the line joining two points $A(2, 0)$ and $B(3, 1)$ is rotated about A in anticlockwise direction through an angle of 15° , then the equation of the line in new position is

Options:

A. $y = -\sqrt{3}x + 2\sqrt{3}$

B. $y = 3x - 6$

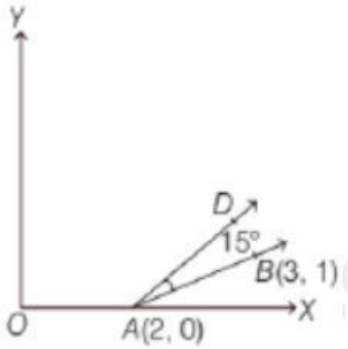
C. $y = \frac{1}{\sqrt{3}}x - \frac{2}{\sqrt{3}}$

D. $y = \sqrt{3}x - 2\sqrt{3}$

Answer: D

Solution:

Solution:



The slope of the line is $\frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 0}{3 - 2} = 1$ or $\tan 45^\circ$ rotated anti-clockwise direction the line through 15° the slope of line AD in new position. New position will be $\tan 60^\circ = \sqrt{3}$.
Therefore, the equation of the new line AD is
 $y - 0 = \sqrt{3}(x - 2)$
 $y = \sqrt{3}x - 2\sqrt{3}$

Question 36

$$\int \frac{(\log x)^2}{x} dx$$

Options:

A. $\left(\frac{\log x}{x} \right)^3 + C$

B. $\left(\frac{\log x}{3} \right)^3 + C$

C. $\frac{(\log x)^3}{2} + C$

D. $\frac{(\log x)^3}{3} + C$

Answer: D

Solution:

Solution:

Let $I = \int \frac{(\log x)^2}{x} dx$

Put $\log x = t \Rightarrow \frac{1}{x} dx = dt$

$$\begin{aligned}\therefore I &= \int \frac{(\log x)^2}{x} dx = \int t^2 dt \\ &= \frac{t^3}{3} + C = \frac{(\log x)^3}{3} + C \quad [\text{put } t = \log x]\end{aligned}$$

Question 37

$$\int \frac{\tan^4 \sqrt{x} \cdot \sec^2 \sqrt{x}}{\sqrt{x}} dx =$$

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Options:

A. $\frac{-5}{2}[\tan \sqrt{x}]^5 + C$

B. $[\tan \sqrt{x}]^5 + C$

C. $\frac{2}{5}[\tan \sqrt{x}]^5 + C$

D. $\frac{5}{2}[\tan \sqrt{x}]^5 + C$

Answer: C

Solution:

Solution:

$$\text{Let } I = \int \frac{\tan^4 \sqrt{x} \cdot \sec^2 \sqrt{x}}{\sqrt{x}} dx$$

$$\text{Put } \tan \sqrt{x} = t$$

$$\Rightarrow \frac{\sec^2 \sqrt{x}}{2\sqrt{x}} dx = dt$$

$$\therefore I = \int \frac{\tan^4 \sqrt{x} \cdot \sec^2 \sqrt{x}}{\sqrt{x}} dx = \int 2t^4 dt$$

$$= \frac{2t^5}{5} + C = \frac{2(\tan \sqrt{x})^5}{5} + C$$

Question 38

The slant height of a right circular cone is 3cm. The height of the cone for maximum volume is

Options:

A. 5cm

B. $\sqrt{3}$ cm

C. 3cm

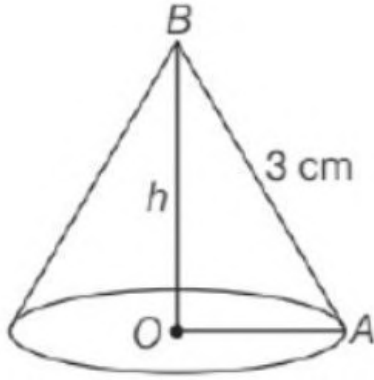
D. $\sqrt{5}$ cm

Answer: B

Solution:

Solution:

Let height of a right circular cone = h cm and $OA = r$ cm



Given, slant height of a right circular cone = 3 cm In $\triangle OAB$,

$$\angle BOA = 90^\circ$$

$$(OB)^2 + (OA)^2 = (AB)^2$$

[apply pythagoras theorem]

$$(h)^2 + (r)^2 = (3)^2$$

$$r = \sqrt{9 - h^2} \dots (i)$$

We know that, Volume of cone

$$= \frac{\pi}{3} r^2 h$$

$$V = \frac{\pi}{3} (9 - h^2) \times h$$

$$[\text{from Eq. (i), } r = \sqrt{9 - h^2}]$$

$$V = \frac{\pi}{3} (9h - h^3)$$

$$\frac{dV}{dh} = \frac{\pi}{3} (9 - 3h^2)$$

$$\therefore \frac{dV}{dh} = 0 \Rightarrow \frac{\pi}{3} (9 - 3h^2) = 0$$

$$\Rightarrow \frac{h^2}{3} = \frac{9}{3} = 3 \Rightarrow h = \sqrt{3}$$

$$\frac{d^2V}{dh^2} = \frac{-6 \times h}{3} < 0$$

So, $h = \sqrt{3}$ of the cone for maximum volume

Question 39

If $y = p[x(x - 2)]^2$ is an increasing function then the value of x is

Options:

A. 1, 2, 3

B. 0, 1, 2

C. 2, 3

D. None of these

Answer: B

Solution:

Solution:

Given, function is $y = [x(x - 2)]^2 = [x^2 - 2x]^2$

On differentiating both sides w.r.t. x , we get

$$\frac{dy}{dx} = 2(x^2 - 2x) \frac{d}{dx}(x^2 - 2x)$$

$$= 2(x^2 - 2x)(2x - 2) = 4x(x - 2)(x - 1)$$

On putting $\frac{dy}{dx} = 0$, we get

$$4x(x - 2)(x - 1) = 0 \Rightarrow x = 0, 1 \text{ and } 2$$

Now, we find interval in which $f(x)$ is strictly increasing or strictly decreasing.

Interval	$\frac{dy}{dx} = 4x(x - 2)(x - 1)$	Sign of $f'(x)$
$(-\infty, 0)$	$(-)(-)(-)$	$-ve$
$(0, 1)$	$(+)(-)(-)$	$+ve$
$(1, 2)$	$(+)(-)(+)$	$-ve$
$(2, \infty)$	$(+)(+)(+)$	$+ve$

Hence, y is strictly increasing in $(0, 1)$ and $(2, \infty)$. Also, y is a polynomial function, so it continuous at $X = 0, 1$ and 2 .

Hence, y is increasing in $[0, 1] \cup [2, \infty]$.

Question 40

If $y = \tan^{-1} \sqrt{\frac{1 + \cos x}{1 - \cos x}}$, then $\frac{dy}{dx} =$

Options:

A. $\frac{3}{2}$

B. 0

C. 1

D. $-\frac{1}{2}$

Answer: D

Solution:

Solution:

$$y = \tan^{-1} \sqrt{\frac{1 + \cos x}{1 - \cos x}}$$

$$y = \tan^{-1} \sqrt{\frac{2\cos^2 \frac{x}{2}}{2\sin^2 \frac{x}{2}}}$$

$$y = \tan^{-1} \left(\cot \frac{x}{2} \right)$$

$$y = \tan^{-1} \left[\tan \left(\frac{\pi}{2} - \frac{x}{2} \right) \right]$$

$$y = \frac{\pi}{2} - \frac{x}{2} \Rightarrow \frac{dy}{dx} = -\frac{1}{2}$$

Question 41

If $\left(\frac{\sqrt{1+x^2}-1}{x} \right)$ w.r.t. $\sin^{-1} \left(\frac{2x}{1+x^2} \right)$ is

Options:

A. $\frac{1}{2}$

B. $\frac{1}{4}$

C. $\frac{3}{2}$

D. $\frac{3}{4}$

Answer: B

Solution:

Solution:

$$\text{Let } u = \tan^{-1} \left(\frac{\sqrt{1+x^2}-1}{x} \right)$$

Put $x = \tan \theta \Rightarrow \theta = \tan^{-1} x$, then

$$u = \tan^{-1} \left[\frac{\sqrt{1+\tan^2 \theta}-1}{\tan \theta} \right] = \tan^{-1} \left[\frac{\sqrt{\sec^2 \theta}-1}{\tan \theta} \right]$$

$$= \tan^{-1} \left[\frac{\sec \theta - 1}{\tan \theta} \right] = \tan^{-1} \left[\frac{1 - \cos \theta}{\sin \theta} \right]$$

$$= \tan^{-1} \left[\frac{2 \sin^2 \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}} \right] = \tan^{-1} \left[\tan \frac{\theta}{2} \right]$$

$$\Rightarrow u = \frac{\theta}{2} = \frac{1}{2} \tan^{-1} x$$

$$r \cdot \tan^{-1}(\tan \theta) = \theta$$

On differentiating both sides w.r.t. x , we get

$$\frac{du}{dx} = \frac{1}{2(1+x^2)} \left[\because \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2} \right] \dots (i)$$

$$\text{Also, let } v = \sin^{-1} \left(\frac{2x}{1+x^2} \right)$$

Put $x = \tan \theta \Rightarrow \theta = \tan^{-1} x$, then we get

$$v = \sin^{-1} \left[\frac{2 \tan \theta}{1 + \tan^2 \theta} \right]$$

$$\Rightarrow v = \sin^{-1}[\sin 2\theta]$$

$$\Rightarrow v = 2\theta \Rightarrow v = 2 \tan^{-1} x$$

On differentiating both sides w.r.t. x , we get

$$\frac{dv}{dx} = \frac{2}{1+x^2}$$

$$\text{Now, } \frac{du}{dv} = \frac{du}{dx} \times \frac{dx}{dv} = \frac{1}{2(1+x^2)} \times \frac{(1+x^2)}{2}$$

[from Eqs. (i) and (ii)]

$$\therefore \frac{du}{dv} = \frac{1}{4}$$

Question 42

The region represented by the inequation system $x, y \geq 0, y \leq 6, x + y \leq 3$ is

Options:

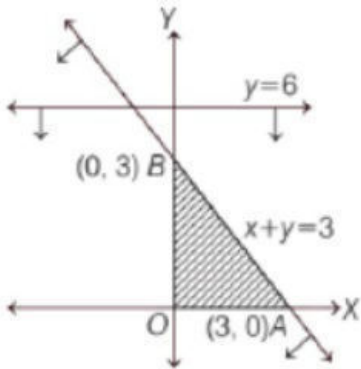
- A. unbounded in first quadrant
- B. unbounded in first and second quadrants
- C. bounded in first quadrant
- D. None of the above

Answer: C

Solution:

Solution:

The given region is bounded in first quadrant.



Question 43

The constraints $-x_1 + x_2 \leq 1, -x_1 + 3x_2 \leq 9$ and $x_1, x_2 \geq 0$ defines on

Options:

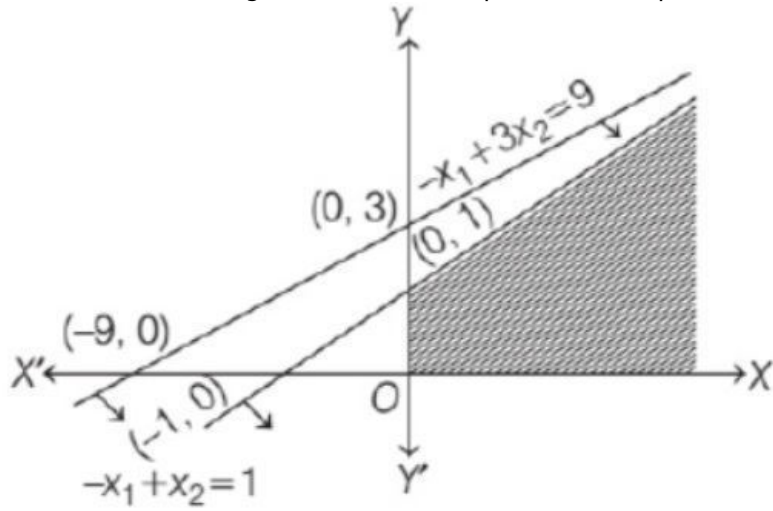
- A. bounded feasible space
- B. unbounded feasible space
- C. both bounded and unbounded feasible space
- D. None of the above

Answer: B

Solution:

Solution:

It is clear from the figure that feasible space (shaded portion) is unbounded.



Question 44

If $z = \frac{(\sqrt{3} + i)^3(3i + 4)^2}{(8 + 6i)^2}$, then $|z|$ is equal to

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Options:

- A. 8
- B. 2
- C. 5
- D. 4

Answer: B

Solution:**Solution:**

$$\text{Given, } z = \frac{(\sqrt{3} + i)^2(3i + 4)^2}{(8 + 6i)^2}$$

$$\begin{aligned} \text{Now, } |z| &= \left| \frac{(\sqrt{3} + i)^3(3i + 4)^2}{(8 + 6i)^2} \right| \\ &= \frac{|(\sqrt{3} + i)^3| |3i + 4|^2}{|(8 + 6i)^2|} \left[\because \left| \frac{z_1}{z_2} \right| = \left| \frac{z_1}{z_2} \right| \right] \\ &= \frac{|\sqrt{3} + i|^3 |3i + 4|^2}{|8 + 6i|^2} \left[\because |z^n| = |z|^n \right] \\ &= \frac{(\sqrt{3} + 1)^3 (\sqrt{9 + 16})^2}{(\sqrt{64 + 36})^2} \\ &= \frac{(2)^3 (5)^2}{(10)^2} = \frac{10^2 \cdot 2}{(10)^2} = 2 \end{aligned}$$

Question 45

If $\frac{3}{2 + \cos \theta + i \sin \theta} = a + ib$, then $[(a - 2)^2 + b^2]$ is equal to

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Options:

- A. 0
- B. 1
- C. -1
- D. 2

Answer: B

Solution:

Solution:

$$\text{Given, } \frac{3}{2 + \cos \theta + i \sin \theta} = a + ib$$

$$\Rightarrow \frac{3[(2 + \cos \theta) - i \sin \theta]}{(2 + \cos \theta)^2 + \sin^2 \theta} = a + ib$$

$$\Rightarrow \frac{3[2 + \cos \theta - i \sin \theta]}{5 + 4 \cos \theta} = a + ib$$

$$\Rightarrow a = \frac{3(2 + \cos \theta)}{5 + 4 \cos \theta} \text{ and } b = -\frac{3 \sin \theta}{5 + 4 \cos \theta}$$

$$\therefore (a - 2)^2 + b^2 = \left(\frac{6 + 3 \cos \theta}{5 + 4 \cos \theta} - 2 \right)^2 + \frac{9 \sin^2 \theta}{(5 + 4 \cos \theta)^2}$$

$$= \frac{(-4 - 5 \cos \theta)^2 + 9 \sin^2 \theta}{(5 + 4 \cos \theta)^2}$$

$$= \frac{16 + 25 \cos^2 \theta + 40 \cos \theta + 9 \sin^2 \theta}{(5 + 4 \cos \theta)^2}$$

$$= \frac{16 + 16 \cos^2 \theta + 40 \cos \theta + 9}{(5 + 4 \cos \theta)^2}$$

$$= \frac{(5 + 4 \cos \theta)^2}{(5 + 4 \cos \theta)^2} = 1$$

Question 46

A box contains 100 bulbs, out of which 10 are defective. A sample of 5 bulbs is drawn. The probability that none is defective, is

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Options:

- A. $\frac{9}{10}$
- B. $\left(\frac{1}{10} \right)^5$
- C. $\left(\frac{9}{10} \right)^5$
- D. $\left(\frac{1}{2} \right)^5$

Answer: C

Solution:

Solution:

Let probability of defective bulb,

$$p = \frac{10}{100} = \frac{1}{10} = 0.1$$

and probability of non-defective bulb,

$$q = 1 - 0.1 = 0.9$$

Here, $n = 5$

$$\therefore P(\text{none is defective}) = P(X = 0)$$

$$= {}^5C_0(0.1)^0(0.9)^5$$

$$= 1 \times (0.9)^5 = \left(\frac{9}{10}\right)^5$$

Question 47

A random variable X has the probability distribution given below.

X	1	2	3	4	5
$P(X=x)$	K	$2K$	$3K$	$2K$	K

Its variance is

Options:

A. $\frac{16}{3}$

B. $\frac{4}{3}$

(C) $\frac{5}{3}$

C. $\frac{10}{3}$

Answer: B

Solution:

Solution:

X	1	2	3	4	5
$P(X=x)$	K	$2K$	$3K$	$2K$	K

$$\therefore \text{Variance} = \sum x_i^2 p - (\sum x_i p)^2$$

$$= (1k + 8k + 27k + 32k + 25k)$$

$$- (k + 4k + 9k + 8k + 5k)^2$$

$$= (93k) - (27k)^2 = \left(93 \times \frac{1}{9}\right) - \left(27 \times \frac{1}{9}\right)^2$$

$$= \frac{93}{9} - 9 = \frac{93 - 81}{9} = \frac{12}{9} = \frac{4}{3}$$

Question 48

The area (in sq. units) of the region bounded by the curves $y^2 = 4ax$ and $x^2 = 4ay$, $a > 0$ is

Options:

A. $\frac{16a^2}{3}$

B. $\frac{14a^2}{3}$

C. $\frac{13a^2}{3}$

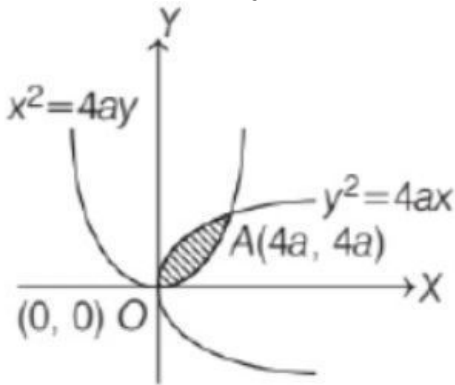
D. $16a^2$

Answer: A

Solution:

Solution:

The equations of given curves are
 $y^2 = 4ax$ and $x^2 = 4ay$



On solving these equations, we get the intersection points, i.e. $(0, 0)$ and $(4a, 4a)$.

$$\begin{aligned}\therefore \text{Required area} &= \int_0^{4a} \left(2\sqrt{a}\sqrt{x} - \frac{x^2}{4a} \right) dx \\ &= 2\sqrt{a} \left[\frac{x^{3/2}}{3/2} \right]_0^{4a} - \left[\frac{x^3}{12a} \right]_0^{4a} \\ &= \frac{32a^2}{3} - \frac{16a^2}{3} \\ &= \frac{16a^2}{3}\end{aligned}$$

Question 49

The area in the positive quadrant enclosed by the circles $x^2 + y^2 = 4$, the line $x = y\sqrt{3}$ and X-axis is

Options:

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

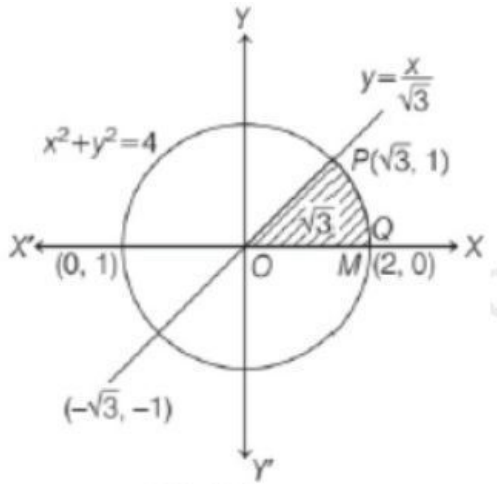
D. π

Answer: C

Solution:

Solution:

The intersection points of curves $x^2 + y^2 = 4$ and $y = \frac{x}{\sqrt{3}}$ are $(0, 0)$ and $P(\sqrt{3}, 1)$.



$$\therefore \text{Area of } \triangle OPM = \frac{1}{2} \times \sqrt{3} \times 1 = \frac{\sqrt{3}}{2}$$

$$\text{and area of curve MPQ} = \int_{\sqrt{3}}^2 \sqrt{4 - x^2} dx$$

$$= \left[\frac{x}{2} \sqrt{4 - x^2} + \frac{4}{2} \sin^{-1} \left(\frac{x}{2} \right) \right]_{\sqrt{3}}^2$$

$$= \left[0 + 2 \left(\frac{\pi}{2} \right) - \left(\frac{\sqrt{3}}{2} + 2 \times \frac{\pi}{3} \right) \right]$$

$$= \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2} \right)$$

Question 50

$$\int_{\pi/6}^{\pi/2} \frac{\operatorname{cosec} x \cdot \cot x}{1 + \operatorname{cosec}^2 x} dx =$$

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Options:

A. $\tan^{-1} 2$

B. $\tan^{-1} \left(\frac{1}{3} \right)$

C. $\tan^{-1} 1$

D. $\frac{\pi}{4} - \tan^{-1} 2$

Answer: B

Solution:

Solution:

$$\text{Let } I = \int_{\pi/6}^{\pi/2} \frac{\operatorname{cosec} x \cdot \cot x}{1 + \operatorname{cosec}^2 x} dx$$

$$\text{Let } \operatorname{cosec} x = t$$

$$\Rightarrow -\operatorname{cosec} x \cot x dx = dt$$

$$\text{When } x = \frac{\pi}{6}, \text{ then } t = \operatorname{cosec} \frac{\pi}{6} = 2$$

$$\text{and when } x = \frac{\pi}{2}, \text{ then } t = \operatorname{cosec} \frac{\pi}{2} = 1$$

$$\therefore I = \int_2^1 \frac{dt}{1+t^2} = -\int_1^2 \frac{dt}{1+t^2}$$

$$= \int_1^2 \frac{dt}{1+t^2} = [\tan^{-1}(t)]_1^2$$

$$= \tan^{-1}(2) - \tan^{-1}(1)$$

$$= \tan^{-1}\left(\frac{2-1}{1+2 \times 1}\right) = \tan^{-1}\left(\frac{1}{1+2}\right)$$

$$= \tan^{-1}\left(\frac{1}{3}\right)$$

Question 51

If $\sin A + \cos A = \sqrt{2}$, then the value of $\cos^2 A$ is

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Options:

A. $\sqrt{2}$

B. $\frac{1}{2}$

C. 4

D. -4

Answer: B

Solution:**Solution:**

$$\text{Given, } \sin A + \cos A = \sqrt{2}$$

$$\therefore \frac{1}{\sqrt{2}} \sin A + \frac{1}{\sqrt{2}} \cos A = 1$$

$$\Rightarrow \sin \frac{\pi}{4} \sin A + \cos \frac{\pi}{4} \cos A = 1$$

$$\Rightarrow \cos\left(A - \frac{\pi}{4}\right) = 1$$

$$\Rightarrow A - \frac{\pi}{4} = 0$$

$$\Rightarrow A = \frac{\pi}{4}$$

$$\text{Now, } \cos^2 A = \cos^2 \frac{\pi}{4} = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$$

Question 52

The number of solutions of equation $\tan x + \sec x = 2 \cos x$ lying in the interval $[0, 2\pi]$ is

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Options:

- A. 0
- B. 1
- C. 2
- D. 3

Answer: C

Solution:

Solution:

$$\tan x + \sec x = 2 \cos x$$

$$\frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2 \cos x$$

$$\Rightarrow 1 + \sin x = 2 \cos^2 x$$

$$\Rightarrow 1 + \sin x = 2(1 - \sin^2 x)$$

$$\Rightarrow 1 + \sin x = 2 - 2\sin^2 x$$

$$\Rightarrow 2\sin^2 x + \sin x - 1 = 0$$

$$\Rightarrow 2\sin^2 x + 2\sin x - \sin x - 1 = 0$$

$$\Rightarrow 2\sin x(\sin x + 1) - 1(\sin x + 1) = 0$$

$$\Rightarrow (\sin x + 1)(2\sin x - 1) = 0$$

$$\Rightarrow \sin x + 1 = 0 \Rightarrow 2\sin x - 1 = 0$$

$$\Rightarrow \sin x = -1, \sin x = \frac{1}{2}$$

$$\Rightarrow x = \frac{3\pi}{2}, x = \frac{\pi}{6}$$

Question 53

The value of $\tan 75^\circ - \cot 75^\circ$ is

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Options:

- A. $2\sqrt{3}$
- B. $2 - \sqrt{3}$
- C. $2 + \sqrt{3}$
- D. 0

Answer: A

Solution:

Solution:

$$\tan 75^\circ - \cot 75^\circ$$

$$\begin{aligned}
&= \frac{\sin 75^\circ}{\cos 75^\circ} - \frac{\cos 75^\circ}{\sin 75^\circ} \\
&= \frac{\sin^2 75^\circ - \cos^2 75^\circ}{\sin 75^\circ \cdot \cos 75^\circ} = \frac{-2 \cos 150^\circ}{\sin 150^\circ} \\
&= \frac{-2 \cos(90^\circ + 60^\circ)}{\sin(90^\circ + 60^\circ)} \\
&= \frac{2 \cdot \frac{\sqrt{3}}{2}}{\frac{1}{2}} = 2\sqrt{3}
\end{aligned}$$

Question 54

$\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$, then $\alpha(\beta + \gamma) + \beta(\gamma + \alpha) + \gamma(\alpha + \beta)$ equals

Options:

- A. 8
- B. 1
- C. 6
- D. 12

Answer: C

Solution:

Solution:

$$\begin{aligned}
&\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi \\
&0 \leq \cos^{-1}x \leq \pi \\
&\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi \\
&\cos^{-1}\alpha = \cos^{-1}\beta = \cos^{-1}\gamma = \pi \\
&\cos \pi = \alpha = \beta = \gamma \\
&-1 = \alpha = \beta = \gamma \\
&\alpha = \beta = \gamma = -1 \\
&\alpha(\beta + \gamma) + \beta(\gamma + \alpha) + \gamma(\alpha + \beta) \\
&= -1(-1 - 1) - 1(-1 - 1) - 1(-1 - 1) \\
&= 2 + 2 + 2 = 6
\end{aligned}$$

Question 55

A line has slope m and y -intercept 4. The distance between the origin and the line is equal to

Options:

- A. $\frac{4}{\sqrt{1 - m^2}}$

B. $\frac{4}{\sqrt{m^2 - 1}}$

C. $\frac{4}{\sqrt{m^2 + 1}}$

D. $\frac{4m}{\sqrt{1 + m^2}}$

Answer: C

Solution:

Solution:

Equation of line is $y = mx + 4$

$$\therefore \text{Required distance} = \frac{4}{\sqrt{1 + m^2}}$$

Question 56

If a line with y-intercept 2 , is perpendicular to the line $3x - 2y = 6$, then its x-intercept is

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Options:

A. 1

B. 2

C. -4

D. 3

Answer: D

Solution:

Solution:

Let the equation of perpendicular line to the line

$$3x - 2y = 6 \text{ be } 3y + 2x = c$$

Since, it passes through (0, 2).

$$\therefore c = 6$$

On putting the value of c in Eq. (i), we get

$$\Rightarrow 3y + 2x = 6$$

$$\frac{x}{3} + \frac{y}{2} = 1$$

Hence, x-intercept is 3 .

Question 57

A line passes through the point of intersection of the lines $3x + y + 1 = 0$ and $2x - y + 3 = 0$ and makes equal intercepts with axes. Then, equation

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of the line is

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Options:

A. $5x + 5y - 3 = 0$

B. $x + 5y - 3 = 0$

C. $5x - y - 3 = 0$

D. $5x + 5y + 3 = 0$

Answer: A

Solution:

Solution:

The point of intersection of the lines $3x + y + 1 = 0$ and $2x - y + 3 = 0$ is $\left(-\frac{4}{5}, \frac{7}{5}\right)$

The equation of line, which makes equal intercepts with axes, is $x + y = a$.

$$\therefore -\frac{4}{5} + \frac{7}{5} = a \Rightarrow a = \frac{3}{5}$$

Now, equation of line is $x + y - \frac{3}{5} = 0$

$$\Rightarrow 5x + 5y - 3 = 0$$

Question 58

If the circles $x^2 + y^2 = 9$ and $x^2 + y^2 + 2\alpha x + 2y + 1 = 0$ touch each other internally, then α is equal to

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Options:

A. $\pm \frac{4}{3}$

B. 1

C. $\frac{4}{3}$

D. $-\frac{4}{3}$

Answer: A

Solution:

Solution:

Centres and radii of the given circles are $C_1(0, 0)$, $r_1 = 3$ and $C_2(-\alpha, -1)$

$$r_2 = \sqrt{\alpha^2 + 1 - 1} = |\alpha|$$

Since, two circles touch internally.

$$\therefore C_1C_2 = r_1 - r_2$$

$$\Rightarrow \sqrt{\alpha^2 + 1} = 3 - |\alpha|$$

$$\Rightarrow \alpha^2 + 1 = 9 + \alpha^2 - 6|\alpha|$$

$$\Rightarrow 6|\alpha| = 8 \Rightarrow |\alpha| = \frac{4}{3}$$

$$\therefore \alpha = \pm \frac{4}{3}$$

Question 59

The length of the common chord of the two circles $x^2 + y^2 - 4y = 0$ and $x^2 + y^2 - 8x - 4y + 11 = 0$ is

Options:

A. $\frac{\sqrt{145}}{4}$ cm

B. $\frac{\sqrt{11}}{2}$ cm

C. $\sqrt{135}$ cm

D. $\frac{\sqrt{135}}{4}$ cm

Answer: D

Solution:

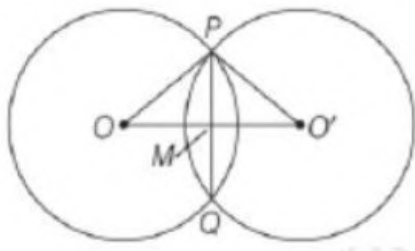
Solution:

Given, equation of circles are $x^2 + y^2 - 4y = 0$ and $x^2 + y^2 - 8x - 4y + 11 = 0$

\therefore Equation of chord is

$$x^2 + y^2 - 4y - (x^2 + y^2 - 8x - 4y + 11) = 0$$

$$\Rightarrow 8x - 11 = 0$$



So, centre and radius of first circle are $O(0, 2)$ and $OP = r = 2$.

Now, perpendicular distance from $O(0, 2)$ to the line $8x - 11 = 0$ is

$$d = OM = \frac{|8 \times 0 - 11|}{\sqrt{8^2}} = \frac{11}{8}$$

$$\text{In } \triangle OMP, PM = \sqrt{OP^2 - OM^2}$$

$$= \sqrt{2^2 - \left(\frac{11}{8}\right)^2} = \sqrt{4 - \frac{121}{64}}$$

$$= \sqrt{\frac{256 - 121}{64}} = \frac{\sqrt{135}}{8}$$

\therefore Length of chord $PQ = 2PM$

$$= 2 \times \frac{\sqrt{135}}{8} = \frac{\sqrt{135}}{4} \text{ cm}$$

Question 60

If x_1, x_2, \dots, x_{18} are observations such that $\sum_{j=1}^{18} (x_j - 8) = 9$ and $\sum_{j=1}^{18} (x_j - 8)^2 = 45$, then standard deviation of these observations is

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Options:

A. $\sqrt{\frac{81}{34}}$

B. 5

C. $\sqrt{5}$

D. $\frac{3}{2}$

Answer: D

Solution:

Solution:

Standard deviation

$$\begin{aligned} & \sqrt{\frac{\sum_{j=1}^{18} (x_j - 8)^2}{n} - \left(\frac{\sum_{j=1}^{18} (x_j - 8)}{n} \right)^2} \\ &= \sqrt{\frac{45}{18} - \left(\frac{9}{18} \right)^2} = \sqrt{\frac{45}{18} - \frac{1}{4}} \\ &= \sqrt{\frac{81}{36}} = \frac{9}{6} = \frac{3}{2} \end{aligned}$$

Question 61

The sum of the deviations of the variates from the arithmetic mean is always

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Options:

A. +1

B. 0

C. -1

D. real number

Answer: B

Solution:

Solution:

Let x_1, x_2, \dots, x_n be n variates. Then, their arithmetic mean will be

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n} \dots (i)$$

Now, the sum of deviation of the variates from the AM, i.e. $\sum (x_i - \bar{x})$

$$\begin{aligned} &= \sum_{i=1}^n (x_i - \bar{x}) = \sum_{i=1}^n x_i - \sum_{i=1}^n \bar{x} \\ &= \sum_{i=1}^n x_i - n\bar{x} = \sum_{i=1}^n x_i - \sum_{i=1}^n x_i \quad [\text{from Eq. (i)}] \\ &= 0 \end{aligned}$$

Question 62

A student answers a multiple choice question with 5 alternatives, of which exactly one is correct. The probability that he knows the correct answer is p , $0 < p < 1$. If he does not know the correct answer, he randomly ticks one answer. Given that he has answered the question correctly, the probability that he did not tick the answer randomly, is

Options:

A. $\frac{3p}{4p+3}$

B. $\frac{5p}{3p+2}$

C. $\frac{5p}{4p+1}$

D. $\frac{4p}{3p+1}$

Answer: C

Solution:

Solution:

Let E_1 = Student does not know the answer E_2 = Student knows the answer

and E = student answer correctly

$$\therefore P(E_1) = 1 - p \Rightarrow P(E_2) = p$$

$$\Rightarrow P\left(\frac{E}{E_2}\right) = 1 \text{ and } P\left(\frac{E}{E_1}\right) = \frac{1}{5}$$

Note that, the probability that student did not know the answer randomly = The probability that student know the answer.

$$\begin{aligned} \therefore P\left(\frac{E_2}{E}\right) &= \frac{P(E_2)P\left(\frac{E}{E_2}\right)}{P(E_1)P\left(\frac{E}{E_1}\right) + P(E_2)P\left(\frac{E}{E_2}\right)} \\ &= \frac{p(1)}{(1-p)\frac{1}{5} + p(1)} = \frac{p}{\frac{1-p+5p}{5}} \\ &= \frac{5p}{1+4p} \end{aligned}$$

Question 63

Box A contains 2 black and 3 red balls, while box B contains 3 black and 4 red balls. Out of these two boxes one is selected at random and the probability of choosing box A is double that of box B. If a red ball is drawn from the selected box, then the probability that it has come from box B, is

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Options:

A. $\frac{21}{41}$

B. $\frac{10}{31}$

C. $\frac{12}{31}$

D. $\frac{13}{41}$

Answer: B

Solution:

Solution:

Let probability of choosing box B, $P(B) = p$

According to the given condition,

$$P(A) = 2P(B) = 2p$$

$$\text{Now, } P\left(\frac{R}{A}\right) = \frac{{}^3C_1}{{}^5C_1} = \frac{3}{5}$$

$$\text{and } P\left(\frac{R}{B}\right) = \frac{{}^4C_1}{{}^7C_1} = \frac{4}{7}$$

$$\begin{aligned} \therefore P\left(\frac{B}{R}\right) &= \frac{P(B) \cdot P\left(\frac{R}{B}\right)}{P(A) \cdot P\left(\frac{R}{A}\right) + P(B) \cdot P\left(\frac{R}{B}\right)} \\ &= \frac{p \cdot \frac{4}{7}}{2p \cdot \frac{3}{5} + p \cdot \frac{4}{7}} = \frac{10}{31} \end{aligned}$$

Question 64

The contrapositive of the statement. 'If $2^2 = 5$, then I get first class' is

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Options:

A. If I do not get a first class, then $2^2 = 5$

B. If I do not get a first class, then $2^2 \neq 5$

C. If I get a first class, then $2^2 = 5$

D. None of the above

Answer: B

Solution:

Solution:

Let p and q be two propositions given by $p : 2^2 = 5$, $q : I \text{ get first class}$.

Then, given statement is $p \rightarrow q$.

The contrapositive of this statement is

$\sim q \rightarrow \sim p$, i.e. if I do not get first class, then $2^2 \neq 5$.

Question 65

The truth value of the statement 'Patna is in Bihar or $5 + 6 = 11$ ' is

Options:

A. true

B. false

C. Cannot say anything

D. None of these

Answer: A

Solution:

Solution:

Let $p : \text{Patna is in Bihar}$ and $q : 5 + 6 = 11$

Then, the given statement is disjunction $p \vee q$.

Since, p is true and q is false.

\therefore The disjunction $p \vee q$ is true.

Hence, truth value of given statement is true.

Question 66

If $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, then A^3 is

Options:

A. $3A$

B. $2A$

C. $4A$

D. A

Answer: C

Solution:

Solution:

$$\text{We have, } A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\text{Now, } A^2 = A \cdot A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & -1-1 \\ -1-1 & 1+1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$$\text{and } A^3 = A^2 \cdot A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2 & -2-2 \\ -2-2 & 2+2 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix}$$

$$\therefore A^3 = 4A$$

Question 67

If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A \operatorname{adj} A = AA^T$, then $5a + b$ is equal to

Options:

A. 5

B. 4

C. 13

D. -1

Answer: A

Solution:

Solution:

$$A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix} \text{ and } A \operatorname{adj} A = AA^T$$

$$\therefore \operatorname{adj} A = \begin{bmatrix} 2 & b \\ -3 & 5a \end{bmatrix}$$

$$\text{Now, } AA^T = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 13 \end{bmatrix}$$

$$\text{and } A \cdot \operatorname{adj} A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & b \\ -3 & 5a \end{bmatrix}$$

$$= \begin{bmatrix} 10a + 3b & 0 \\ 0 & 10a + 3b \end{bmatrix}$$

$\therefore A \cdot (\text{adj} A) = AA^T$ is given, so equating the two expression, we get

$$\begin{bmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 13 \end{bmatrix} = \begin{bmatrix} 10a + 3b & 0 \\ 0 & 10a + 3b \end{bmatrix}$$

We have, $10a + 3b = 13$ and $15a - 2b = 0$

On solving, we get

$$a = \frac{2}{5} \text{ and } b = 3$$

$$\Rightarrow 5a + b = 5 \times \frac{2}{5} + 3$$

$$\Rightarrow 5a + b = 2 + 3$$

$$\Rightarrow 5a + b = 5$$

Question 68

The value of f at $x = 0$, so that function $f(x) = \frac{2^x - 2^{-x}}{x}$, $x \neq 0$ is continuous at $x = 0$, is

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Options:

A. 0

B. $\log 4$

C. 4

D. e^4

Answer: B

Solution:

Solution:

$$\lim_{x \rightarrow 0} \frac{2^x - 2^{-x}}{x} = \lim_{x \rightarrow 0} 2^x \log 2 + 2^{-x} \log 2$$

[using L'Hospital's rule]

$$= \log 2 + \log 2 = \log 4$$

Since, function is continuous at $x = 0$.

$$\therefore f(0) = \lim_{x \rightarrow 0} \frac{2^x - 2^{-x}}{x} = \log 4$$

Question 69

If $f(x) = \begin{cases} ax + 3 & x \leq 2 \\ a^2x - 1 & x > 2. \end{cases}$, then the values of a for which f is continuous for all x are

Options:

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- A. 1 and -2
- B. 1 and 2
- C. -1 and 2
- D. -1 and -2

Answer: C

Solution:

Solution:

Given, $f(x) = \begin{cases} ax + 3, & x \leq 2 \\ a^2x - 1, & x > 2 \end{cases}$ end cases
Continuity at $x = 2$,

$$\text{LHL} = \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2} (ax + 3) = 2a + 3$$

$$\text{RHL} = \lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2} (a^2x - 1) = 2a^2 - 1$$

Since, $f(x)$ is continuous for all values of x .

$$\begin{aligned} \therefore \text{LHL} &= \text{RHL} \\ \Rightarrow 2a + 3 &= 2a^2 - 1 \\ \Rightarrow 2a^2 - 2a - 4 &= 0 \\ \Rightarrow a^2 - a - 2 &= 0 \\ \Rightarrow a^2 - 2a + a - 2 &= 0 \\ \Rightarrow a(a - 2) + 1(a - 2) &= 0 \\ \Rightarrow (a + 1)(a - 2) &= 0 \\ \therefore a &= -1, 2 \end{aligned}$$

Question 70

$\lim_{x \rightarrow 0} \frac{1 - \cos^3 x}{x \sin x \cos x}$ is equal to

Options:

- A. $\frac{2}{5}$
- B. $\frac{3}{5}$
- C. $\frac{3}{2}$
- D. $\frac{3}{4}$

Answer: C

Solution:

Solution:

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{1 - \cos^3 x}{x \sin x \cos x} \\ = \lim_{x \rightarrow 0} \frac{(1 - \cos x)(1 + \cos^2 x + \cos x)}{x^2 \cos x \cdot \frac{\sin x}{x}} \end{aligned}$$

$$= 3 \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = 3 \times \frac{1}{2} = \frac{3}{2}$$

Question 71

The value of $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x}{3} \right)^{2/x}$, ($a, b, c > 0$) is

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Options:

- A. $(abc)^3$
- B. abc
- C. $(abc)^{1/3}$
- D. None of these

Answer: D

Solution:

Solution:

$$\text{Let } y = \lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x}{3} \right)^{2/x}$$

$$\Rightarrow \log y = \lim_{x \rightarrow 0} \frac{2}{x} \log \left(\frac{a^x + b^x + c^x}{3} \right)$$

$$\Rightarrow \log y = \lim_{x \rightarrow 0} \frac{2 \log(a^x + b^x + c^x) - \log 3}{x}$$

$$\Rightarrow \log y = \log(abc)^{2/3}$$

\therefore [using L'Hospital's rule]

$$\therefore y = (abc)^{2/3}$$

Question 72

If the vectors $2\hat{i} - \hat{j} - \hat{k}$, $\hat{i} + 2\hat{j} - 3\hat{k}$ and $3\hat{i} + \lambda\hat{j} + 5\hat{k}$ are coplanar, then the value of λ is

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Options:

- A. -8
- B. -4
- C. -2
- D. -1

Answer: B

Solution:

Solution:

$$\text{Let } \vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \quad \vec{b} = \hat{i} + 2\hat{j} - 3\hat{k},$$

$$\vec{c} = 3\hat{i} + \lambda\hat{j} + 5\hat{k}$$

Now given vectors \vec{a} , \vec{b} , \vec{c} will be coplanar if

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = 0, \text{ i.e. } [\vec{a} \ \vec{b} \ \vec{c}] = 0$$

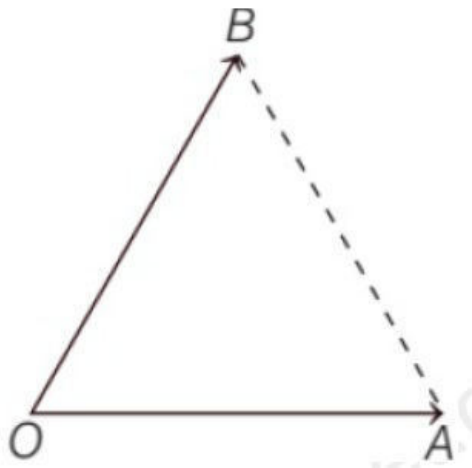
$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & \lambda & 5 \end{vmatrix} = 0$$

$$\Rightarrow 2(10 + 3\lambda) + 1(5 + 9) + 1(\lambda - 6) = 0$$

$$\Rightarrow 7\lambda = -28 \Rightarrow \lambda = -4$$

Question 73

A $\triangle OAB$ is determined by the vector \vec{a} and \vec{b} as show in the figure what will be area of triangle?



Options:

A. $\frac{1}{2} \sqrt{|\vec{a}|^2 + |\vec{b}|^2 - (\vec{a} \cdot \vec{b})}$

B. $\frac{1}{2} \sqrt{|\vec{a}| |\vec{b}| - (\vec{a} \cdot \vec{b})^2}$

C. $\frac{1}{4} \sqrt{|\vec{a}|^2 + |\vec{b}|^2 - (\vec{a} \cdot \vec{b})}$

D. $\frac{1}{4} \sqrt{|\vec{a}| |\vec{b}| - (\vec{a} \cdot \vec{b})^2}$

Answer: B

Solution:

Solution:

$$\text{We know that area of triangle} = \frac{1}{2} |\vec{OA} \times \vec{OB}|$$

$$\Delta^2 = \frac{1}{4} |\vec{a} \times \vec{b}|^2 \dots (i)$$

$$\begin{aligned} \text{Now } |\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 \\ = |\vec{a}|^2 |\vec{b}|^2 \sin^2 \theta + |\vec{a}|^2 |\vec{b}|^2 \cos^2 \theta \end{aligned}$$

$$\begin{aligned}
 &= |a|^2 |b|^2 (\sin^2 \theta + \cos^2 \theta) \\
 &= |a|^2 |b|^2 \times 1 = |a|^2 |b|^2 \\
 &\Rightarrow |a \times b|^2 = |a|^2 |b|^2 - (a \cdot b)^2 \\
 &\text{From Eq. (i), we get} \\
 &\Delta^2 = \frac{1}{4} [|a|^2 |b|^2 - (a \cdot b)^2] \\
 &\Delta = \frac{1}{2} \sqrt{|a|^2 |b|^2 - (a \cdot b)^2}
 \end{aligned}$$

Question 74

The line $\frac{x-2}{3} - \frac{y-1}{-5} = \frac{z+2}{2}$ lies in the plane $x + 3y - \alpha z + \beta = 0$, then value of $\alpha\beta$ is

Options:

- A. -42
- B. 1
- C. -2
- D. 42

Answer: A

Solution:

Solution:

Given equation of line

$$\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2} \dots (i)$$

The direction ratios of the normal are $(1, 3, -\alpha)$.

The direction ratios of the line are $(3, -5, 2)$ and equation of given plane

$$x + 3y - \alpha z + \beta = 0 \dots (ii)$$

Four lines are perpendicular

$$\Rightarrow a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$

$$\Rightarrow 3 - 15 + 2\alpha = 0$$

$$\Rightarrow 2\alpha = -12 \Rightarrow \alpha = -6$$

$(2, 1, -2)$ lies on the plane, so

$$2 + 3 + 6(-2) + \beta = 0 \Rightarrow \beta = 7$$

$$\alpha \cdot \beta = -6 \times 7 = -42$$

Question 75

Find the angle between the lines whose direction cosines are given by the equations $3l + m + 5n = 0$, $6mn - 2nl + 5lm = 0$

Options:

- A. $\cos^{-1} \frac{1}{6}$

B. $\cos^{-1} \frac{2}{6}$

C. $\sin^{-1} \frac{1}{6}$

D. $\sin^{-1} \frac{2}{6}$

Answer: A

Solution:

Solution:

The given equations are $3l + m + 5n = 0 \dots (i)$

and $6mn - 2nl + 5l/m = -0 \dots (ii)$

Now, from Eq. (i), we get

$$m = -3l - 5n$$

On substituting $m = -3l - 5n$ in Eq. (ii), we get

$$6(-3l - 5n)n - 2nl + 5l(-3l - 5n) = 0$$

$$\Rightarrow 30n^2 + 45ln + 15l^2 = 0$$

$$\Rightarrow 2n^2 + 3ln + l^2 = 0$$

$$\Rightarrow 2n^2 + 2nl + nl + l^2 = 0$$

$$\Rightarrow 2n(n + l) + l(n + l) = 0$$

$$\Rightarrow (n + l)(2n + l) = 0$$

$$\Rightarrow \text{Either } l = -n \text{ or } l = -2n$$

If $l = -n$, then $m = -2n$ and if $l = -2n$, then $m = n$

Thus, the direction ratios of two lines are proportional $(-n, -2n, n)$ and $(-2n, n, n)$ i.e. $(-1, -2, 1)$ and $(-2, 1, 1)$, respectively.

Now, let θ be the acute angle between the lines,

$$\text{Then, } \cos \theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{|2 - 2 + 1|}{\sqrt{1 + 4 + 1} \sqrt{4 + 1 + 1}} = \frac{1}{6}$$

$$\Rightarrow \theta = \cos^{-1} \left(\frac{1}{6} \right)$$

Question 76

Find the shortest distance between the lines $r = \hat{i} + \hat{j} + \lambda (2\hat{i} - \hat{j} + \hat{k})$ and $r = 2\hat{i} + \hat{j} - \hat{k} + \mu (3\hat{i} - 5\hat{j} + 2\hat{k})$.

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Options:

A. $\frac{10}{\sqrt{59}}$

B. $\frac{8}{\sqrt{57}}$

C. $\frac{9}{\sqrt{89}}$

D. $\frac{10}{\sqrt{39}}$

Answer: A

Solution:

Solution:

$$r = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k}) \dots (i)$$

$$r = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} - 5\hat{j} + 2\hat{k}) \dots (ii)$$

Compare with vector equation $r = a + \lambda b$

$$a_1 = \hat{i} + \hat{j} = 2\hat{i} - \hat{j} + \hat{k}$$

$$a_2 = 2\hat{i} + \hat{j} - \hat{k}, b_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$$

$$d = \left| \frac{(b_1 \times b_2) \cdot (a_2 - a_1)}{|b_1 \times b_2|} \right| \dots (iii)$$

$$b_1 \times b_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$$

$$b_1 \times b_2 = \hat{i}(-2 + 5) - \hat{j}(4 - 3) + \hat{k}(-10 + 3)$$

$$= 3\hat{i} - \hat{j} - 7\hat{k}$$

$$|b_1 \times b_2| = \sqrt{(3)^2 + (-1)^2 + (-7)^2}$$

$$\text{Also } a_2 - a_1 = (2\hat{i} + \hat{j} - \hat{k}) + (\hat{i} + \hat{j}) = \hat{i} - \hat{k} \dots (iv)$$

$$d = \left| \left(3\hat{i} - \hat{j} - 7\hat{k} \right) \left(\hat{i} - \frac{\hat{k}}{\sqrt{59}} \right) \right| = \left| \frac{3 - 0 + 7}{\sqrt{59}} \right| = \frac{10}{\sqrt{59}}$$

Question 77

$\int \frac{dx}{\sin^2 x \cos^2 x}$ is equal to

Options:

A. $\tan x + \cot x + C$

B. $\tan x - \cot x + C$

C. $\tan x - \cot^2 x + C$

D. None of these

Answer: B

Solution:**Solution:**

$$\begin{aligned} I &= \int \frac{dx}{\sin^2 x \cos^2 x} \\ &= \int \frac{(\sin^2 x + \cos^2 x)}{\sin^2 x \cdot \cos^2 x} [\because \sin^2 \theta + \cos^2 \theta = 1] \\ &= \int (\sec^2 x + \operatorname{cosec}^2 x) dx \\ &= \int (\sec^2 x dx + \operatorname{cosec}^2 x dx) \\ &= \tan x - \cot x + C \end{aligned}$$

Question 78

$\int \frac{e^{\tan^{-1}x}}{1+x^2} dx$ is equal to

©

Options:

A. $\frac{\tan^{-1}x}{x} + C$

B. $\frac{\tan^{-1}x}{x} + C$

C. $e^{\tan^{-1}x} + C$

D. $\frac{e^{\tan^{-1}x}}{x^2} + C$

Answer: C

Solution:

Solution:

$$\text{Let } I = \int \frac{e^{\tan^{-1}x}}{1+x^2} dx$$

$$\text{Put } \tan^{-1}x = t$$

$$\Rightarrow \frac{1}{1+x^2} dx = dt$$

$$\therefore I = \int \frac{e^{\tan^{-1}x}}{1+x^2} dx = \int e^t dt$$

$$= e^t + C \quad [\because \int e^x dx = e^x]$$

$$= e^{\tan^{-1}x} + C \quad [\text{put } t = \tan^{-1}x]$$

Question 79

A spherical raindrop evaporates at a rate proportional to its surface originally is 3mm and 1h later has been reduced to 2mm, then radius r of the raindrop at any time t is (where $0 \leq t < 3$)

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Options:

A. $r = t + 3$

B. $r = t + 5$

C. $r = t - 5$

D. $r = 3 - t$

Answer: D

Solution:

Solution:

Let r be the radius, V be the volume and S be the surface area of the spherical raindrop at time t .

Then, $V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$

The rate at which the raindrop evaporates is $\frac{dV}{dt}$

which is proportional to the surface area.

$$\therefore \frac{dV}{dt} \propto S \Rightarrow \frac{dV}{dt} = -kS, \text{ where } k > 0. \dots (i)$$

Now, $V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$

$$\therefore \frac{dV}{dt} = \frac{4\pi}{3} \times 3r^2 \frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$4\pi r^2 \frac{dr}{dt} = -k(4\pi r^2) \quad [\text{from Eq. (i)}]$$

$$\frac{dr}{dt} = -k \Rightarrow dr = -k dt$$

On integrating, we get

$$\int dr = -k \int dt + C$$

$$\therefore r = -kt + C$$

Initially, i.e. when $t = 0, r = 3$

$$\therefore 3 = -k \times 0 + C$$

$$\therefore C = 3$$

$$\therefore r = -kt + 3$$

When $t = 1, r = 2$

$$\therefore 2 = -k \times 1 + 3$$

$$\therefore k = 1$$

$$\therefore r = -t + 3$$

$$\therefore r = 3 - t, \text{ where } 0 \leq t \leq 3$$

This is the required expression for the radius of the raindrop at any time t .

Question 80

If $y = \sin^{-1}(6x\sqrt{1-9x^2})$, $-\frac{1}{3\sqrt{2}} < x < \frac{1}{3\sqrt{2}}$ then $\frac{dy}{dx}$ is

Options:

A. $\frac{6}{\sqrt{1-9x^2}}$

B. $\frac{5}{\sqrt{1-3x^2}}$

C. $\frac{6}{\sqrt{1-3x^2}}$

D. $\frac{5}{\sqrt{1-4x^2}}$

Answer: A

Solution:

Solution:

$$\text{Given, } y = \sin^{-1}(6x\sqrt{1-9x^2})$$

$$\Rightarrow y = \sin^{-1}(2 \cdot 3x\sqrt{1-(3x)^2})$$

$$\text{Put } 3x = \sin \theta \Rightarrow y = \sin^{-1}(2 \sin \theta \cdot \cos \theta)$$

$$\Rightarrow y = \sin^{-1}(\sin 2\theta)$$

$$\Rightarrow y = 2\theta \Rightarrow y = 2\sin^{-1}(3x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{1-9x^2}}(3)$$

$$\Rightarrow \frac{dy}{dx} = \frac{6}{\sqrt{1-9x^2}}$$

Question 81

If $y = (\sin x)^x + \sin^{-1}\sqrt{x}$, then $\frac{dy}{dx}$

Options:

A. $(\sin x)^x [\cot x + \log \sin x] + \frac{1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}}$

B. $(\sin x)[x \cot x + \log \sin x] + \frac{1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}}$

C. $(\sin x)^x [x \cot x + \log \sin x] + \frac{1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}}$

D. None of the above

Answer: C

Solution:

Solution:

Given, $y = (\sin x)^x + \sin^{-1}\sqrt{x} \dots (i)$

Let $u = (\sin x)^x \dots (ii)$

Then, Eq. (i) becomes,

$y = u + \sin^{-1}\sqrt{x} \dots (iii)$

On taking log both sides of Eq. (ii), we get

$\log u = x \log \sin x$

On differentiating both sides w.r.t., x , we get

$$\frac{1}{u} \frac{du}{dx} = x \frac{d}{dx}(\log \sin x) + \log \sin x \frac{d}{dx}(x)$$

$$\Rightarrow \frac{du}{dx} = u \left[x \times \frac{1}{\sin x} \frac{d}{dx}(\sin x) + \log \sin x(1) \right]$$

$$\Rightarrow \frac{du}{dx} = (\sin x)^x \left[\frac{x}{\sin x} \times \cos x + \log \sin x \right]$$

$$\Rightarrow \frac{du}{dx} = (\sin x)^x [x \cot x + \log \sin x] \dots (iv)$$

On differentiating both sides of Eq. (iii) w.r.t. x , we get

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{1}{\sqrt{1-(\sqrt{x})^2}} \frac{d}{dx}(\sqrt{x})$$

$$\therefore \frac{dy}{dx} = (\sin x)^x [x \cot x + \log \sin x] + \frac{1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}}$$

[from Eq. (iv)]

Question 82

The side of an equilateral triangle is increasing at the rate of 2cm / s. If the side of the triangle is 20cm² the rate of area increasing is

Options:

- A. $20\sqrt{3}\text{cm}^2$
 B. 20cm^2
 C. 60cm^2
 D. $\frac{20\sqrt{3}}{3}\text{cm}^2$

Answer: A**Solution:****Solution:**

Let a be the side of an equilateral triangle and A be the area of an equilateral triangle. Then, $\frac{da}{dt} = 2\text{cm/s}$

We know that, area of an equilateral triangle

$$A = \frac{\sqrt{3}}{4}a^2$$

On differentiating both sides w.r.t. t , we get

$$\frac{dA}{dt} = \frac{\sqrt{3}}{4} \times 2a \times \frac{da}{dt}$$

$$\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{4} \times 2 \times 20 \times 2 \quad [\text{given } a = 20]$$

$$\therefore \frac{dA}{dt} = 20\sqrt{3}\text{cm}^2/\text{s}$$

Thus, the rate of area increasing is $20\sqrt{3}\text{cm}^2/\text{s}$.

Question 83

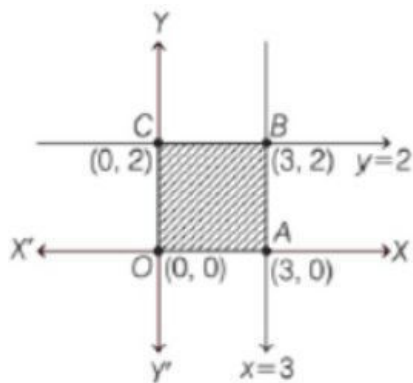
The maximum value $Z = 11x + 7y$, subject to $x \leq 3$, $y \leq 2$, $x \geq 0$ and $y \geq 0$ is

Options:

- A. 44
 B. 46
 C. 54
 D. 47

Answer: D**Solution:****Solution:**

Maximise $Z = 11x + 7y$, subject to the constraints $x \leq 3$, $y \leq 2$, $x \geq 0$, $y \geq 0$



The shaded region as shown in the figure as OABC is bounded and the coordinates of corner points are $(0, 0)$, $(3, 0)$, $(3, 2)$ and $(0, 2)$ respectively.

Corner points	Corresponding value of Z
$(0, 0)$	0
$(3, 0)$	33
$(3, 2)$	47 ← Maximum
$(0, 2)$	14

Hence, Z is maximum at $(3, 2)$ and its maximum value is 47 .

Question 84

The minimum value $Z = 13x - 15y$ subject to $x + y \leq 7$, $2x - 3y + 6 \geq 0$, $x \geq 0$ and $y \geq 0$ is

Options:

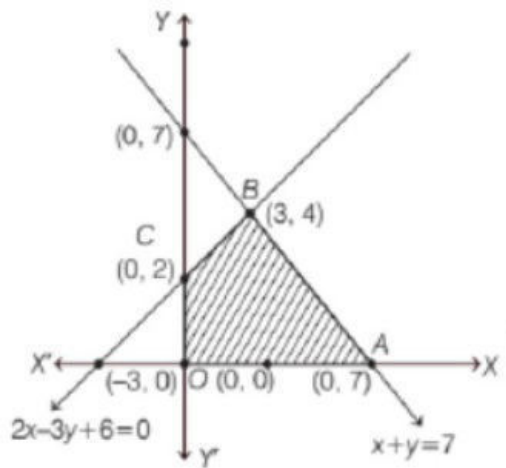
- A. 80
- B. -21
- C. -30
- D. 91

Answer: C

Solution:

Solution:

Minimize $Z = 13x - 15y$ subject to the constraints $x + y \leq 7$, $2x - 3y + 6 \geq 0$, $x \geq 0$, $y \geq 0$



Shaded region shown as OABC is bounded and coordinates of its corner points are (0, 0), (7, 0), (3, 4) and (0, 2) respectively,

Corner points	Corresponding value of Z
(0, 0)	0
(7, 0)	91
(3, 4)	-21
(0, 2)	-30 ← Minimum

Hence, the minimum value of Z is (-30) at (0, 2).

Question 85

If z_1, z_2, \dots, z_n are complex numbers such that $|z_1| = |z_2| = \dots = |z_n| = 1$, then $|z_1 + z_2 + \dots + z_n|$ is equal to

Options:

- A. $|z_1 z_2 z_3 \dots z_n|$
- B. $|z_1| + |z_2| + \dots + |z_n|$
- C. $\left| \frac{1}{z_1} + \frac{1}{z_2} + \dots + \frac{1}{z_n} \right|$
- D. n

Answer: C

Solution:

Solution:

Given, $|z_1| = |z_2| = \dots = |z_n| = 1$
 $\Rightarrow |z_1|^2 = |z_2|^2 = \dots = |z_n|^2 = 1$
 $\Rightarrow z_1 \bar{z}_1 = z_2 \bar{z}_2 = \dots = z_n \bar{z}_n = 1$

$$\Rightarrow \bar{z}_1 = \frac{1}{z_1}, \bar{z}_2 = \frac{1}{z_2}, \dots, \bar{z}_n = \frac{1}{z_n} \dots (i)$$

$$\text{Now, } |z_1 + z_2 + \dots + z_n|$$

$$= |z_1 + z_2 + \dots + z_n| = |\bar{z}_1 + \bar{z}_2 + \dots + \bar{z}_n|$$

$$= \left| \frac{1}{z_1} + \frac{1}{z_2} + \dots + \frac{1}{z_n} \right| \quad [\text{using Eq. (i)}]$$

Question 86

If $2\alpha = -1 - i\sqrt{3}$ and $2\beta = -1 + i\sqrt{3}$, then $5\alpha^4 + 5\beta^4 + 7\alpha^{-1}\beta^{-1}$ is equal to

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Options:

A. -1

B. 2

C. 0

D. 1

Answer: B

Solution:

Solution:

$$\text{Given, } 2\alpha = -1 - i\sqrt{3} \text{ and } 2\beta = -1 + i\sqrt{3}$$

$$\therefore \alpha + \beta = -1 \text{ and } \alpha\beta = 1$$

$$\text{Now, } 5\alpha^4 + 5\beta^4 + \frac{7}{\alpha\beta} = 5\{(\alpha + \beta)^2 - 2\alpha\beta\}^2 - 2(\alpha\beta)^2]$$

$$= 5\{(-1)^2 - 2 \times 1\}^2 - 2(1)^2] + \frac{7}{1}$$

$$= 5[(1 - 2)^2 - 2] + 7 = 2$$

Question 87

If a fair coin is tossed 20 times and we get head n times, then probability that n is odd, is

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Options:

A. $\frac{1}{2}$

B. $\frac{1}{6}$

C. $\frac{5}{8}$

D. $\frac{7}{8}$

Answer: A

Solution:

Solution:

Probability of getting head in one trial, $p = \frac{1}{2}$ and probability of not getting head,

$$q = \frac{1}{2}$$

Probability of getting head odd times

$$\begin{aligned} &= {}^{20}C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{19} + {}^{20}C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{17} \\ &+ \dots + {}^{20}C_{19} \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{19} \\ &= \frac{1}{2^{20}} [{}^{20}C_1 + {}^{20}C_3 + \dots + {}^{20}C_{19}] \\ &= \frac{1}{2^{20}} \times 2^{20-1} = \frac{2^{19}}{2^{20}} = \frac{1}{2} \end{aligned}$$

Question 88

If the records of a hospital show that 10% of the cases of a certain disease are fatal. If 6 patients are suffering from the disease, then the probability that only three will die is

Options:

A. 8748×10^{-5}

B. 1458×10^{-5}

C. 1468×10^{-6}

D. 41×10^{-6}

Answer: B

Solution:

Solution:

Since, the probability of person die, due to suffering from a disease is 10%.

$$\therefore p = \frac{10}{100} = \frac{1}{10} \text{ and } q = \frac{9}{10}$$

Total number of patients, $n = 6$

$$\begin{aligned} \therefore \text{Required probability} &= {}^6C_3 \left(\frac{1}{10}\right)^3 \left(\frac{9}{10}\right)^3 \\ &= \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} \times \frac{1}{1000} \times \frac{9 \times 9 \times 9}{1000} \\ &= \frac{2}{10^5} \times 729 = 1458 \times 10^{-5} \end{aligned}$$

Question 89

Area bounded by the curve $x = 0$ and $x + 2|y| = 1$ is

Options:

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. 1

D. 2

Answer: B

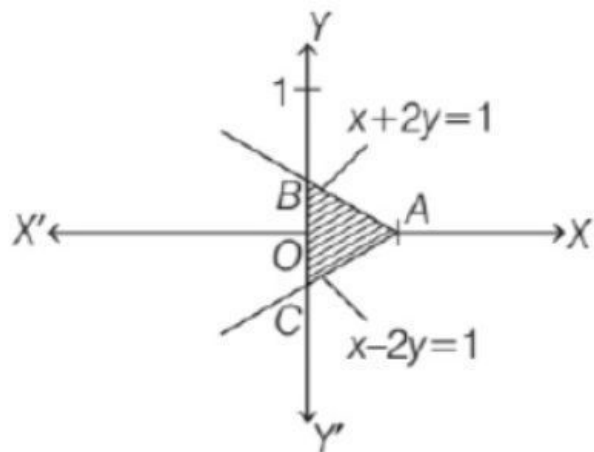
Solution:

Solution:

Given curves are $x = 0$ and $x + 2|y| = 1$

Now, $x + 2|y| = 0$

When $y > 0$, $x + 2y = 1$; when $y < 0$, $x - 2y = 1$



\therefore Area of bounded region ABC

$$= 2AOB = 2 \int_0^1 \left(\frac{1-x}{2} \right) dx$$

$$= \left[x - \frac{x^2}{2} \right]_0^1 = \left[1 - \frac{1}{2} - (0 - 0) \right] = \frac{1}{2}$$

Question 90

The area bounded by the curves $y = \sqrt{5 - x^2}$ and $y = |x - 1|$ is

Options:

A. $\left(\frac{5\pi}{4} - 2 \right)$ sq units

B. $\frac{(5\pi - 2)}{4}$ sq units

C. $\frac{(5\pi - 2)}{2}$ sq units

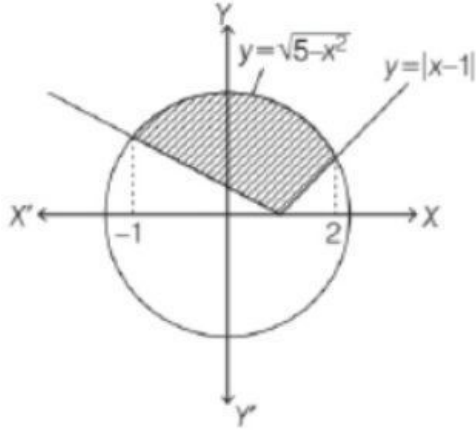
D. $\left(\frac{\pi}{2} - 5\right)$ sq units

Answer: B

Solution:

Solution:

Given, $y = \sqrt{5 - x^2} \Rightarrow y^2 + x^2 = 5$
and $y = |x - 1|$



\therefore Required area

$$\begin{aligned} &= \int_{-1}^2 \sqrt{5 - x^2} dx - \int_{-1}^1 (1 - x) dx - \int_1^2 (x - 1) dx \\ &= \left[\frac{x}{2} \sqrt{5 - x^2} + \frac{5}{2} \sin^{-1} \frac{x}{\sqrt{5}} \right]_{-1}^2 - \left[x - \frac{x^2}{2} \right]_{-1}^1 - \left[\frac{x^2}{2} - x \right]_1^2 \\ &= \left[1 + \frac{5}{2} \sin^{-1} \frac{2}{\sqrt{5}} + 1 + \frac{5}{2} \sin^{-1} \frac{1}{\sqrt{5}} \right] \\ &\quad - \left[1 - \frac{1}{2} - \left(-1 - \frac{1}{2} \right) \right] - \left[2 - 2 - \left(\frac{1}{2} - 1 \right) \right] \\ &= 2 + \frac{5}{2} \sin^{-1} \left(\frac{2}{\sqrt{5}} \sqrt{1 - \frac{1}{5}} + \frac{1}{\sqrt{5}} \sqrt{1 - \frac{4}{5}} \right) - \frac{5}{2} \\ &= \frac{5}{2} \sin^{-1}(1) - \frac{1}{2} = \frac{5\pi}{4} - \frac{1}{2} = \left(\frac{5\pi - 2}{4} \right) \text{ sq units} \end{aligned}$$

Question 91

The value of $\int_3^5 \frac{x^2}{x^2 - 4} dx$ is

Options:

A. $2 - \log_e \left(\frac{15}{7} \right)$

B. $2 + \log_e \left(\frac{15}{7} \right)$

C. $2 + 4\log_e 3 - 4\log_e 7 + 4\log_e 5$

D. $2 - \tan^{-1} \left(\frac{15}{7} \right)$

Answer: B

Solution:

Solution:

$$\begin{aligned}\int_3^5 \frac{x^2}{x^2-4} dx &= \int_3^5 \left(\frac{x^2-4}{x^2-4} + \frac{4}{x^2-4} \right) dx \\&= \int_3^5 \left(1 + \frac{4}{x^2-4} \right) dx = \left[x + \frac{4}{2 \times 2} \log_e \left(\frac{x-2}{x+2} \right) \right]_3^5 \\&= \left[5 + \log_e \left(\frac{5-2}{5+2} \right) - 3 - \log_e \left(\frac{3-2}{3+2} \right) \right] \\&= 2 + \log_e \left(\frac{3}{7} \right) - \log_e \left(\frac{1}{5} \right) \\&= 2 + \log_e \left(\frac{3}{7} \times \frac{5}{1} \right) = 2 + \log_e \left(\frac{15}{7} \right)\end{aligned}$$

Question 92

The particular solution of the differential equation $\frac{dy}{dx} = \frac{y+1}{x^2-x}$, when $x = 2$ and $y = 1$ is

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Options:

- A. $xy = 3x - 4$
- B. $xy = 2x - 2$
- C. $xy = 4x - 6$
- D. $xy = -x + 4$

Answer: A

Solution:

Solution:

Given, differential equation

$$\frac{dy}{dx} = \frac{y+1}{x^2-x} \Rightarrow \frac{dy}{y+1} = \frac{dx}{x^2-x}$$

On integrating both sides, we get

$$\int \frac{dy}{y+1} = \int \frac{dx}{x^2-x}$$

$$\text{Now, } \frac{1}{x^2-x} = \frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$$

$$\therefore \frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1} \dots (i)$$

$$\Rightarrow 1 = A(x-1) + B(x)$$

Putting $x = 0$, then

$$1 = A(0-1) \Rightarrow A = -1$$

Putting $x-1 = 0$, then $x = 1$

$$\therefore 1 = A(0) + B(1)$$

$$\Rightarrow B = 1$$

From Eq. (i), we get

$$\int \frac{dy}{y+1} = \int -\frac{1}{x} + \int \frac{1}{x-1} dx$$

$$\Rightarrow \log(y+1) = -\log x + \log(x-1) + \log C$$

$$\Rightarrow \log(y+1) + \log x - \log(x-1) = \log C$$

$$\Rightarrow \log \left\{ \frac{x(y+1)}{x-1} \right\} = \log C$$

$$\Rightarrow \frac{x(y+1)}{x-1} = C \dots (ii)$$

On putting $x = 2$ and $y = 1$ in Eq. (ii), we get

$$\frac{2(1+1)}{2-1} = C \Rightarrow C = (2)(2) = 4$$

Putting value of $C = 4$ in Eq. (ii), we get

$$\frac{x(y+1)}{x-1} = 4$$

$$\Rightarrow xy + x = 4x - 4 \Rightarrow xy = 3x - 4$$

Question 93

The differential equation of all circles which passes through the origin and whose centre lies on Y-axis, is

Options:

A. $(x^2 - y^2) \frac{dy}{dx} - 2xy = 0$

B. $(x^2 - y^2) \frac{dy}{dx} + 2xy = 0$

C. $(x^2 - y^2) \frac{dy}{dx} - xy = 0$

D. $(x^2 - y^2) \frac{dy}{dx} + xy = 0$

Answer: A

Solution:

Solution:

$$\text{Let } x^2 + y^2 - 2ky = 0$$

On differentiating w.r.t x , we get

$$2x + 2y \frac{dy}{dx} - 2k \frac{dy}{dx} = 0$$

$$\Rightarrow k = \frac{x}{\left(\frac{dy}{dx}\right)} + y$$

From Eq. (i),

$$x^2 + y^2 - 2 \left(\frac{x}{(dy/dx)} + y \right) y = 0$$

$$\Rightarrow (x^2 - y^2) \frac{dy}{dx} - 2xy = 0$$

Question 94

The solution of the equation $(x^2 + xy)dy = (x^2 + y^2)dx$ is

Options:

A. $\log x = \log(x - y) + \frac{y}{x} + C$

B. $\log x = 2 \log(x - y) + \frac{y}{x} + C$

C. $\log x = \log(x - y) + \frac{y}{x} + C$

D. None of the above

Answer: B

Solution:

Solution:

Given, $(x^2 + xy)dy = (x^2 + y^2)dx$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2 + y^2}{x^2 + xy}$$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$

$$\Rightarrow v + x \frac{dv}{dx} = \frac{x^2 + v^2 x^2}{x^2 + x^2 v}$$

$$\Rightarrow v + x \frac{dv}{dx} = \frac{1 + v^2}{1 + v}$$

$$\Rightarrow \frac{xdv}{dx} = \frac{1 + v^2}{1 + v} - v$$

$$\Rightarrow = \frac{1 + v^2 - v - v^2}{1 + v} = \frac{1 - v}{1 + v}$$

$$\Rightarrow dv \left(\frac{1 + v}{1 - v} \right) = \frac{dx}{x} \Rightarrow dv \left(-1 + \frac{2}{1 - v} \right) = \frac{dx}{x}$$

On integrating both sides, we get

$$-v - 2 \log(1 - v) = \log x + C$$

$$\Rightarrow \frac{y}{x} - 2 \log \left(1 - \frac{y}{x} \right) = \log x + C$$

$$\Rightarrow \frac{y}{x} - 2 \log(x - y) + 2 \log(x - y) + C = \log x + C$$

Question 95

The value of $\int_{\pi/6}^{\pi/2} \left(\frac{1 + \sin 2x + \cos 2x}{\sin x \cos x} \right) dx$ is equal to

Options:

A. 16

B. 8

C. 4

D. 1

Answer: D

Solution:

Solution:

$$\int_{\pi/6}^{\pi/2} \left(\frac{1 + \sin 2x + \cos 2x}{\sin x \cos x} \right) dx$$

$$\begin{aligned}
&= \int_{\pi/6}^{\pi/2} \left(\frac{1 + \sin x \cos x + 2\cos^2 x - 1}{(\sin x + \cos x)} \right) dx \\
&= \int_{\pi/6}^{\pi/2} \left(\frac{2\cos x(\sin x + \cos x)}{(\sin x + \cos x)} \right) dx \\
&= \int_{\pi/6}^{\pi/2} 2\cos x dx = 2[\sin x]_{\pi/6}^{\pi/2} \\
&= 2 \left(\sin \frac{\pi}{2} - \sin \frac{\pi}{6} \right) = 2 \left(1 - \frac{1}{2} \right) = 2 \times \frac{1}{2} = 1
\end{aligned}$$

Question 96

How many 5-digit telephone number can be constructed using the digits 0 to 9 , if each number starts with 67 and no digits appears more than once?

Options:

- A. 335
- B. 336
- C. 338
- D. 337

Answer: B

Solution:

Solution:

Since, telephone number start with 67 , so two digits is already fixed. Now, we have to do arrangement of three digits from remaining eight digits.

\therefore Possible number of ways = 8P_3

$$= \frac{8!}{(8-3)!} = \frac{8!}{5!} = 8 \times 7 \times 6 = 336 \text{ ways}$$

Question 97

The number of selecting atleast 4 candidates from 8 candidates is

Options:

- A. 270
- B. 70
- C. 163
- D. None of these

Answer: C

Solution:

Solution:

$$\begin{aligned}\text{Required number of selections} &= {}^8C_4 + {}^8C_5 + {}^8C_6 + {}^8C_7 + {}^8C_8 \\ &= 70 + 56 + 28 + 8 + 1 = 163\end{aligned}$$

Question 98

If $f(x) = \frac{2x-1}{x+5}$, $x \neq -5$, then $f^{-1}(x)$ is equal to

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Options:

A. $\frac{x+5}{2x-1}$, $x \neq \frac{1}{2}$

B. $\frac{5x+1}{2-x}$, $x \neq 2$

C. $\frac{x-3}{2x+1}$, $x \neq \frac{1}{2}$

D. $\frac{5x-1}{2-x}$, $x \neq 2$

Answer: D

Solution:

Solution:

$$\begin{aligned}\text{Let } y &= \frac{2x-1}{x+5} \\ \Rightarrow x &= \frac{5y+1}{2-y} \\ \therefore f^{-1}(x) &= \frac{5x-1}{2-x}, x \neq 2\end{aligned}$$

Question 99

If $f(x) = \frac{\alpha x}{x+1}$, $x \neq -1$, for what values of α is $f(f(x)) = x$?

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Options:

A. $\sqrt{2}$

B. $-\sqrt{2}$

C. -1

D. 2

Answer: C

Solution:

Solution:

$$f(x) = \frac{\alpha x}{x+1}, x \neq -1$$

$$\therefore f(f(x)) = f\left(\frac{\alpha x}{x+1}\right) = \frac{\alpha\left(\frac{\alpha x}{x+1}\right)}{\left(\frac{\alpha x}{x+1}\right) + 1} = \frac{\alpha^2 x}{\alpha x + x + 1}$$

$$\Rightarrow \frac{\alpha^2 x}{\alpha x + x + 1} = x \quad [\text{given}]$$

$$\Rightarrow x[\alpha^2 - \alpha x - x - 1] = 0$$

$$\Rightarrow x(\alpha + 1)(\alpha - 1 - x) = 0$$

$$\Rightarrow x = 0 \text{ or } \alpha + 1 = 0$$

$$\text{or } \alpha = 1 + x \quad [\because \alpha - 1 - x \neq 0]$$

$$\Rightarrow \alpha = -1 \text{ or } \alpha = 1 + x$$

$$\therefore \alpha = -1$$

$$[\because \alpha = 1 + x \text{ gives value for particular } x, \text{ not for all } x]$$

Question 100

If A and B are two events with $P(A^c) = 0.3$, $P(B) = 0.4$ and $P(A \cap B^c) = 0.5$. Then, $P[(B) / (A \cap B^c)]$ is equal to

Options:

A. $\frac{1}{4}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{2}{3}$

Answer: A

Solution:

Solution:

$$\text{Given, } P(A^c) = 0.3, P(B) = 0.4$$

$$\text{and } P(A \cap B^c) = 0.5$$

$$\therefore P(A^c) = 0.3$$

$$\Rightarrow P(A) = 1 - P(A^c) = 0.7$$

$$\text{and } P(B) = 0.4 \Rightarrow P(B^c) = 1 - P(B) = 0.6$$

$$\text{Consider, } P(A \cap B^c) = P(A) - P(A \cap B)$$

$$0.5 = 0.7 - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = 0.2$$

$$\text{Now, } P\left[\frac{B}{(A \cup B^c)}\right] = \frac{P[B \cap (A \cup B^c)]}{P(A \cup B^c)}$$

$$= \frac{P[(B \cap A) \cup (A \cap B^c)]}{P(A) + P(B^c) - P(A \cap B^c)}$$

$$= \frac{P(B \cap A)}{0.7 + 0.6 - 0.5} = \frac{0.2}{0.8} = \frac{1}{4}$$

Question 1

Two cars A and B are travelling in the same direction with velocities v_A and v_B ($v_A > v_B$). When the car is at a distance s behind car B, then the driver of the car A applies the brakes producing a uniform retardation a , there will be no collision when

Options:

A. $s \leq \frac{v_A - v_B}{2}$

B. $s \leq \frac{(v_A - v_B)^2}{2a}$

C. $s \leq (v_A - v_B)^2$

D. $s = \frac{(v_A - v_B)^2}{a}$

Answer: B

Solution:

Solution:

(b) For no collision, the speed of car A should be reduced to v_B before the cars meet, i.e. final relative velocity of car A with respect to car B is zero, i.e.

$$v_{\text{relative}} = 0$$

Here, initial relative velocity, $u_r = v_A - v_B$

Relative acceleration, $a_r = -a - 0 = -a$

Let relative displacement be s_r .

Thus, from third equation of motion, we get

$$v_{\text{relative}}^2 = u_r^2 + 2a_r s_r = (v_A - v_B)^2 - 2as_r$$

$$\Rightarrow s_r = \frac{(v_A - v_B)^2}{2a}$$

For no collision, $s \leq s_r$

$$\text{i.e., } s \leq \frac{(v_A - v_B)^2}{2a}$$

Question 2

A point moves along X-axis initially at rest. Its acceleration is $a = (6t + 5) \text{ m/s}^2$. The distance covered in $2s$, if it starts from origin is given by

Options:

- A. 6m
- B. 12m
- C. 16m
- D. 18m

Answer: D**Solution:****Solution:**

(d) Given , acceleration, $a = (6t + 5)\text{m} / \text{s}^2$

$$a = \frac{dv}{dt} = 6t + 5$$

$$\Rightarrow dv = (6t + 5)dt$$

$$\Rightarrow \int dv = \int (6t + 5)dt$$

$$\Rightarrow v = 3t^2 + 5t + c$$

where, c is constant of integration.

When $t = 0$, $v = 0$, so $c = 0$

Therefore, $v = 3t^2 + 5t$

$$\Rightarrow ds = (3t^2 + 5t)dt \quad \left[\because v = \frac{ds}{dt} \right]$$

From 0 to 2s, we have

$$\int_0^s ds = \int_0^2 (3t^2 + 5t)dt$$

$$s = \left(t^3 + \frac{5}{2}t^2 \right)_0^2 = 8 + 10 = 18\text{m}$$

Question 3

A ball released from the top of a tower falls $\frac{11}{36}$ of the height of the tower in the last second of its fall. The height of the tower is

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Options:

- A. 180m
- B. 120m
- C. 140m
- D. 110m

Answer: A**Solution:****Solution:**

(a) The distance travelled in nth second is given by

$$s = u + \frac{1}{2}a(2n - 1)$$

$$s = \frac{g}{2}(2n - 1) \quad [\text{here } u = 0, a = g]$$

According to the question,

$$\frac{11}{36}h = \frac{9.8}{2}(2n - 1) \quad \dots\dots (i)$$

From second equation of motion,

$$h = \frac{1}{2}gn^2$$

[$\because u = 0$]... (ii)

From Eqs. (i) and (ii), we have

$$\frac{11}{36} \times \frac{9.8}{2} \times n^2 = \frac{9.8}{2}(2n - 1) \text{ [here, } g = 9.8 \text{ m / s}^2 \text{]}$$

$$\Rightarrow 2n - 1 = \frac{11}{36}n^2$$

$$\Rightarrow 11n^2 - 72n + 36 = 0$$

$$\Rightarrow 11n^2 - 66n - 6n + 36 = 0$$

$$\Rightarrow 11n(n - 6) - 6(n - 6) = 0$$

$$\Rightarrow n = 6$$

(rejecting fractional values).

$$\text{Therefore, } h = \frac{1}{2} \times 10 \times 6 \times 6 = 180 \text{ m}$$

Question 4

A person throws a ball with speed 10m / s at an angle of 30° with horizontal from the top of 10m high tower. The distance of ball from the foot of the tower after falling on the ground will be

Options:

A. 7m

B. 8.6m

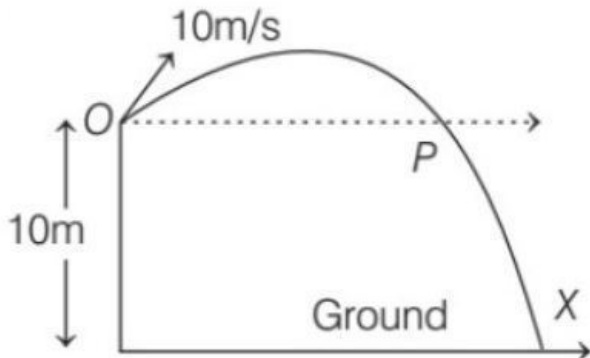
C. 9.6m

D. 10m

Answer: B

Solution:

Solution:



The ball will be at a point P when it is at a height of 10m from the ground. So, we have to find the distance OP, which can be calculated directly by considering it as a projectile on a levelled plane OX.

$$\text{Therefore, maximum range, } OP = R = \frac{u^2 \sin 2\theta}{g}$$

$$= \frac{10^2 \times \sin(2 \times 30^\circ)}{10} = \frac{10\sqrt{3}}{2} = 5\sqrt{3} = 8.6 \text{ m}$$

Question 5

A 1kg mass is tied to a light inextensible string of length $l = \frac{10}{3}\text{m}$, is wheeled in circular path of radius 1m in a vertical plane. The ratio of maximum to the minimum tension in the string is 4. The speed of stone at the highest point of circle is

Options:

- A. 5m / s
- B. 10m / s
- C. 15m / s
- D. 20m / s

Answer: B

Solution:

Solution:

(b) The maximum tension in the string will be at lowest point i.e.,

$$T_{\max} = \frac{mv_1^2}{l} + mg$$

and minimum tension in the string will be the highest point

$$\text{i.e., } T_{\min} = \frac{mv_2^2}{l} - mg$$

$$\text{Therefore, } \frac{T_{\max}}{T_{\min}} = \frac{\frac{mv_1^2}{l} + mg}{\frac{mv_2^2}{l} - mg} = 4$$

$$\Rightarrow \frac{v_1^2 + gl}{v_2^2 - gl} = 4 \quad \dots\dots\dots (i)$$

$$\text{As we know, } v_1^2 = v_2^2 + 4gl \quad \dots\dots\dots (ii)$$

So from Eqs. (i) and (ii), we get

$$v_2^2 + 4gl + gl = 4v_2^2 - 4gl$$

$$\Rightarrow 3v_2^2 = 9gl$$

$$v_2^2 = 3gl = 3 \times 10 \times \frac{10}{3}$$

$$\Rightarrow v_2^2 = 100 \Rightarrow v_2 = 10\text{m / s}$$

Question 6

The spheres of masses 2kg and 4kg are situated at the opposite end wooden bar of length 9m. Where does centre of mass of the system will

Options:

- A. 6m from 2kg sphere

- B. 2m from 4kg sphere
- C. 3m from 2kg sphere
- D. 6m from 4kg sphere

Answer: A

Solution:

Solution:

(a) Let 2kg mass be placed at $x = 0$, therefore 4kg mass will be situated at $x = 9$.

Therefore,

$$x_{\text{COM}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$= \frac{0 + 4 \times 9}{2 + 4} = \frac{36}{6} = 6\text{m}$$

Thus, centre of mass will be situated at 6m from 2 kg mass.

Question 7

Assuming the expression for the pressure exerted by the gas, it can be shown that pressure is

Options:

- A. $\left(\frac{3}{2}\right)$ nd times kinetic energy per unit volume of gas
- B. $\left(\frac{1}{3}\right)$ rd of kinetic energy per unit volume of a gas
- C. $\left(\frac{3}{4}\right)$ th of kinetic energy per unit volume of a gas
- D. $\left(\frac{2}{3}\right)$ rd of kinetic energy per unit volume of a gas

Answer: D

Solution:

Solution:

(d) The rms velocity of gas molecule,

$$v_{\text{rms}} = \sqrt{\frac{3RT}{m}}$$

$$\text{As, } K = \frac{1}{2} m v_{\text{rms}}^2$$

$$= \frac{1}{2} m \left(\frac{3RT}{m} \right) = \frac{3}{2} RT$$

$$\Rightarrow K = \frac{3}{2} RT \quad \dots\dots (i)$$

From ideal gasequation for one mole,

$$pV = RT \quad \dots\dots (ii)$$

From Eqs. (i) and (ii), we get,

$$p = \frac{2}{3} \frac{K}{V}$$

Hence, pressure is $\left(\frac{2}{3}\right)$ rd of kinetic energy per unit volume of gas.

Question 8

Out of the following which statement is NOT true about black body radiation?

Options:

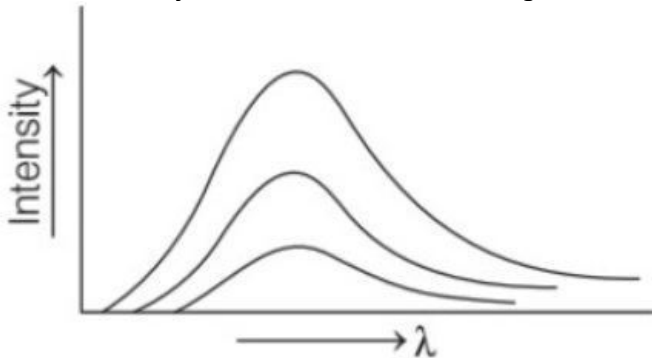
- A. A black body emits all wavelengths.
- B. Intensity is less for longer wavelengths.
- C. Intensity is same for all wavelengths.
- D. Intensity is more for shorter wavelengths.

Answer: C

Solution:

Solution:

(c) The intensity curve at different wavelength for different temperature is given below



A black body emits all wavelengths.

According to the graph, intensity is less for longer wavelength and more for shorter wavelength.

Question 9

An ideal gas ($\gamma = 1.5$) is expanded adiabatically. How many times has the gas to be expanded to reduce root mean square velocity of molecules two times?

Options:

- A. 8 times
- B. 20 times
- C. 16 times
- D. 12 times

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Answer: C

Solution:

Solution:

(c) Given, $\gamma = 1.5$

rms velocity of gas molecule,

$$v_{\text{rms}} = \sqrt{\frac{3RT}{m}} \Rightarrow T \propto v^2$$

$$\Rightarrow \frac{T_2}{T_1} = \frac{v_2^2}{v_1^2}$$

$$\text{Here, } v_2 = \frac{v_1}{2}$$

$$\therefore \frac{T_2}{T_1} = \frac{(v_1/2)^2}{v_1^2} = \frac{1}{4} \quad \dots\dots\dots (i)$$

For adiabatic process,

$$TV^{\gamma-1} = \text{constant}$$

$$\Rightarrow \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{T_1}{T_2}\right)$$

$$\Rightarrow \frac{V_2}{V_1} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}} = (4)^{\frac{1}{1.5-1}} \quad [\text{using Eq. (i)}]$$
$$= (4)^2 = 16$$

Hence, gas has to be expanded to 16 times.

Question 10

An ideal gas having pressure p , volume V and temperature T undergoes a thermodynamic process in which $dW = 0$ and $dQ < 0$. Then, for the gas

Options:

- A. p may increase or decrease
- B. T will decrease
- C. V will increase
- D. T will increase

Answer: B

Solution:

Solution:

(b) According to first law of thermodynamics,

$$\Delta U = \Delta Q - dW$$

It is given that, $dW = 0$ and $\Delta Q < 0$

Thus, $\Delta U = C_V dT = \Delta Q$ is negative.

Since, C_V is specific heat, which remains constant, the temperature will decrease.

Question 11

A particle executing SHM has velocities v_1 and v_2 at distances x_1 and x_2 respectively, from the mean position. Its time-period is

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Options:

A. $2\pi \sqrt{\frac{x_2^2 - x_1^2}{v_2^2 - v_1^2}}$

B. $2\pi \sqrt{\frac{x_1^2 - x_2^2}{v_2^2 - v_1^2}}$

C. $2\pi \sqrt{\frac{x_1 x_2}{v_2^2 - v_1^2}}$

D. $2\pi \sqrt{\frac{x_1 + x_2}{v_1 v_2}}$

Answer: B

Solution:

Solution:

As, we know in SHM,

$$v^2 = \omega^2(a^2 - x^2)$$

$$\Rightarrow v_1^2 = \omega^2(a^2 - x_1^2) \dots\dots (i)$$

$$\text{and } v_2^2 = \omega^2(a^2 - x_2^2) \dots\dots\dots (ii)$$

Subtracting Eq. (i) from Eq. (ii), we get

$$v_2^2 - v_1^2 = \omega^2(x_1^2 - x_2^2)$$

$$= \frac{4\pi^2}{T^2}(x_1^2 - x_2^2) \left(\because \omega = \frac{2\pi}{T} \right)$$

$$\Rightarrow T = 2\pi \sqrt{\frac{x_1^2 - x_2^2}{v_2^2 - v_1^2}}$$

Question 12

A spring of force constant 400N / m is loaded with a mass 0.25kg. The amplitude of oscillations is 4cm. When mass comes to equilibrium position. Its velocity is

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Options:

A. 16m / s

B. 1.6m / s

C. 0.16m / s

D. 0.016m / s

Answer: B

Solution:

Solution:

(b) Given, mass, $m = 0.25\text{kg}$

Force constant, $k = 400\text{N / m}$

Amplitude of oscillations, $A = 4\text{cm} = 0.04\text{m}$

$$\text{Angular frequency, } \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{400}{0.25}}$$

$$= \sqrt{1600} = 40\text{rad / s}$$

Velocity at equilibrium position $= \omega A$

$$= 40 \times 0.04 = 1.6\text{m / s}$$

Question 13

A progressive wave of frequency 500H z is travelling with a velocity of 360m / s. How far apart are two points 60° out of phase?

Options:

A. 0.05m

B. 0.10m

C. 0.12m

D. 0.72m

Answer: C

Solution:

Solution:

(c) Given,

$$v = 360\text{ms}^{-1}$$

$$v = 500\text{H z}$$

$$\Delta\phi = 60^\circ = \frac{\pi}{3}$$

Since, velocity of a wave, $v = v\lambda$

$$\Rightarrow \lambda = \frac{v}{v} = \frac{360}{500} = 0.72\text{m}$$

As, phase difference, $\Delta\phi = \frac{2\pi}{\lambda} \times \text{path difference } (\Delta x)$

$$\Rightarrow \Delta x = \frac{\lambda}{2\pi} \times \Delta\phi$$

$$= \frac{0.72}{2\pi} \times \frac{\pi}{3} = 0.12\text{m}$$

Question 14

A body sends waves 100mm long through medium P and 0.25m long in medium Q. If the velocity of wave in medium P is 80cms^{-1} . The velocity of wave in medium Q is

Options:

- A. 1ms^{-1}
- B. 2ms^{-1}
- C. 5ms^{-1}
- D. 7ms^{-1}

Answer: B

Solution:

Solution:

(b) Given, $\lambda_P = 100\text{mm} = 0.10\text{m}$

$\lambda_Q = 0.25\text{m}$

$v_P = 80\text{cms}^{-1} = 0.80\text{ms}^{-1}$

Since, frequency of wave remains same in the two media,

$$\frac{v_P}{\lambda_P} = \frac{v_Q}{\lambda_Q} \left(\because v = \frac{v}{\lambda} \right)$$

$$\Rightarrow v_Q = \frac{\lambda_Q}{\lambda_P} \times v_P = \frac{0.25}{0.10} \times 0.80 = 2\text{ms}^{-1}$$

Question 15

Separation between the plates of parallel plate capacitor is d and the area of each plate is A . When a slab of material of dielectric constant k and thickness t ($t < d$) is introduced between the plates, its capacitance becomes ($\epsilon_0 =$ permittivity of free space)

Options:

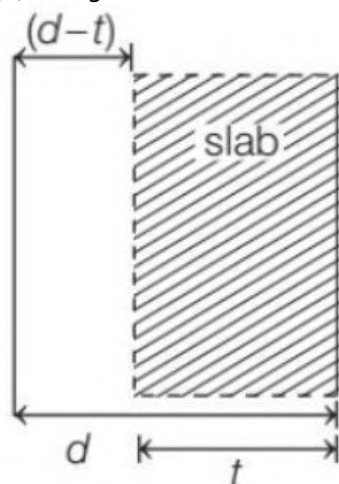
- A. $\frac{A\epsilon_0}{2d - t \left[1 - \frac{1}{k} \right]}$
- B. $\frac{A\epsilon_0}{d + t \left[1 + \frac{1}{k} \right]}$
- C. $\frac{A\epsilon_0}{d - t \left[1 - \frac{1}{k} \right]}$
- D. $\frac{A\epsilon_0}{d - t \left[1 + \frac{1}{k} \right]}$

Answer: C

Solution:

Solution:

(c) The given situation can be shown as



The capacitance of a parallel plate capacitor is given by

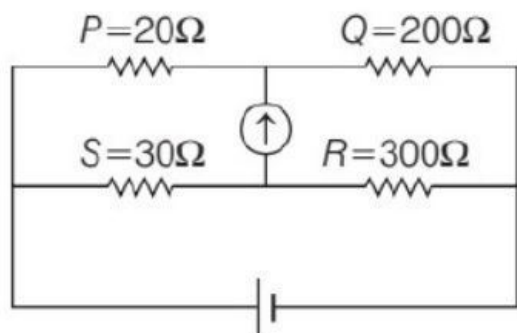
$$C = \frac{k\epsilon_0 A}{d}$$

The above circuit can be considered to be combination of two series capacitors as

$$\begin{aligned}\therefore \frac{1}{C} &= \frac{1}{C_1} + \frac{1}{C_2} = \frac{(d-t)}{\epsilon_0 A} + \frac{t}{k\epsilon_0 A} \\ &= \frac{k(d-t) + t}{k\epsilon_0 A} = \frac{d-t + \frac{t}{k}}{\epsilon_0 A} = \frac{d-t\left(1 - \frac{1}{k}\right)}{\epsilon_0 A} \\ \Rightarrow C &= \frac{\epsilon_0 A}{d-t\left(1 - \frac{1}{k}\right)}\end{aligned}$$

Question 16

Figure below shows a balanced Wheatstone's network. If it is disturbed by changing P to 22 Ω , then which of the following steps will bring the bridge again in balanced state?



Options:

- A. By increasing S by 3 Ω
- B. By increasing Q by 20 Ω
- C. By increasing R by 30 Ω
- D. Both (a) and (b)

Answer: D

Solution:

Solution:

(d) At balanced condition,

$$\frac{P}{Q} = \frac{S}{R} \Rightarrow \frac{22}{200} = \frac{30}{300}$$

So, it can be balanced again by increasing the resistance S by 3Ω or by increasing Q by 20Ω .

Question 17

Two circular coils A and B are made from the same wire but the radius of coil A is twice that of coil B. If the magnetic fields at their centres are the same, then the ratio of potential differences applied across A to that of B is

Options:

A. 1 : 4

B. 4 : 1

C. 2 : 1

D. 1 : 2

Answer: B

Solution:

Solution:

(b) According to the question,

Magnetic field at centre of coil A = Magnetic field at centre of coil B

$$\frac{\mu_0 I_1}{2(2r)} = \frac{\mu_0 I_2}{2r}$$

$$\Rightarrow \frac{I_1}{I_2} = 2 \dots\dots (i)$$

We know, $R = \rho \left(\frac{l}{A} \right)$, where ρ is resistivity, l is length

and A is area of cross-section.

$$\Rightarrow I_1 = \frac{V_1}{R_1} = \frac{V_1}{\rho \left(\frac{l_1}{A} \right)}$$

$$\Rightarrow \frac{V_1}{l_1} = \rho \cdot \frac{l_1}{A} \dots\dots (ii)$$

$$\text{and } I_2 = \frac{V_2}{R_2} = \frac{V_2}{\rho \cdot \left(\frac{l_2}{A} \right)}$$

$$\Rightarrow \frac{V_2}{l_2} = \rho \cdot \frac{l_2}{A} \dots\dots (iii)$$

From Eqs. (ii) and (iii), we get

$$\frac{V_1}{l_1} \times \frac{l_2}{V_2} = \frac{l_1}{l_2} = \frac{2\pi r_1}{2\pi r_2}$$

$$\Rightarrow \frac{V_1}{V_2} \cdot \frac{l_2}{l_1} = \frac{r_1}{r_2}$$

$$\Rightarrow \frac{V_1}{V_2} \cdot \frac{l_2}{l_1} = \frac{2r}{r}$$

$$\Rightarrow \frac{V_1}{V_2} = 2 \cdot \frac{2}{l_2} = 2 \times 2 \quad [\text{from Eq. (i)}]$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{4}{1} = 4 : 1$$

Question 18

A bar magnet of magnetic moment M_1 is cut into two pieces along its axis. The pieces are kept perpendicular to each other with their unlike poles in contact. The magnetic moment of the arrangement is M_2 . The ratio of M_1 / M_2 is

Options:

- A. $1 / 2\sqrt{2}$
- B. 1
- C. $\sqrt{2}$
- D. $1 / \sqrt{2}$

Answer: C

Solution:

Solution:

(c) In given case, the magnetic moment of the pieces get halved.

$$\text{i.e., } M_1' = M_2' = \frac{M_1}{2}$$

The magnetic moment of given arrangement,

$$M_2 = \sqrt{(M_1')^2 + (M_2')^2}$$

$$= \sqrt{\left(\frac{M_1}{2}\right)^2 + \left(\frac{M_1}{2}\right)^2} = \frac{M_1}{\sqrt{2}}$$

$$\Rightarrow \frac{M_1}{M_2} = \sqrt{2}$$

Question 19

Frequency of the series limit of Balmer series of hydrogen atom of Rydberg's constant R and velocity of light c is

Options:

A. $4Rc$

B. $\frac{4}{Rc}$

C. Rc

D. $\frac{Rc}{4}$

Answer: D

Solution:

Solution:

(d) We know that, wavelength of spectrum in hydrogen atom is given as

$$\frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

For the series limit of Balmer series, $n_1 = 2$ and $n_2 = \infty$

$$\therefore \frac{1}{\lambda} = R \left[\frac{1}{2^2} - \frac{1}{\infty^2} \right]$$

$$\Rightarrow \frac{1}{\lambda} = \frac{R}{4} \Rightarrow \lambda = \frac{4}{R} \Rightarrow \frac{c}{f} = \frac{4}{R} [\because c = f \lambda]$$

$$\Rightarrow f = \frac{Rc}{4}$$

Question 20

A spherical solid ball of volume V is made up of a material of density ρ_1 . It is falling through a liquid of density ρ_2 ($\rho_2 < \rho_1$). Assume that, the liquid applies a viscous force on the ball that is proportional to the square of the speed v_t , i.e., $F_{\text{viscous}} = K V_t^2$, then the terminal speed of ball is
(g = acceleration due to gravity)

Options:

A. $\sqrt{\frac{K(\rho_1 - \rho_2)}{Vg}}$

B. $\sqrt{\frac{Vg(\rho_1 - \rho_2)}{K}}$

C. $\sqrt{\frac{Vg(\rho_1 - \rho_2)}{K}}$

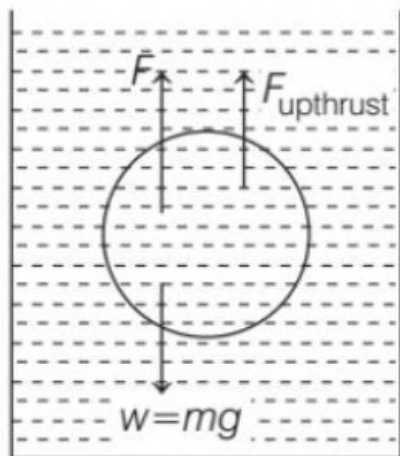
D. $\sqrt{\frac{Vg(\rho_1 - \rho_2)}{2K}}$

Answer: B

Solution:

Solution:

(b) The given situation is shown below



When ball moves with terminal velocity (v_t), then

$$F_{\text{viscous}} + F_{\text{upthrust}} = W$$

$$K v_t^2 + V \rho_2 g = mg$$

$$\Rightarrow K v_t^2 + V \rho_2 g = V \rho_1 g$$

$$\Rightarrow K v_t^2 = V (\rho_1 - \rho_2) g$$

$$v_t = \sqrt{\frac{V (\rho_1 - \rho_2) g}{K}}$$

Question 21

A charge of magnitude $3e$ and mass $2m$ is moving in an electric field E . The acceleration imparted to the charge is

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Options:

A. $\frac{3E e}{2m}$

B. $\frac{3m}{2E e}$

C. $\frac{2E e}{3m}$

D. $\frac{2m}{3E e}$

Answer: A

Solution:

Solution:

(a) Given, charge on particle, $q = 3e$

Mass, $m = 2m$

Force on charged particle in electric field,

$$F = qE = 3eE$$

\therefore Acceleration imparted to charged particle,

$$a = \frac{F}{m} = \frac{3eE}{2m}$$

Question 22

In a two input logic gate, when one input is 1 and the other is 0 the output is 1 . But even if both inputs are 0, the outputs is 1 . The logic gate is

Options:

- A. a X-OR gate
- B. an AND gate
- C. a NOR gate
- D. a NAND gate

Answer: D

Solution:

Solution:

(d) According to given situation, truth table can be given as

Input			Output
A	B	AB	$(Y) = \overline{(AB)}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

Hence the given logic gate is NAND gate

Question 23

The angular separation of the central maximum in the Fraunhofer diffraction pattern is measured. The slit is illuminated by the light of wavelength 6000\AA . If the slit is illuminated by light of another wavelength, the angular separation decrease by 30%. The wavelength of light used is

Options:

- A. 6000\AA
- B. 3500\AA
- C. 4200\AA

D. 4700\AA

Answer: C

Solution:

Solution:

(c) Given, wavelength of used light, $\lambda_1 = 6000\text{\AA}$

Initial angular separation, $\beta_1 = \beta$

Final angular separation, $\beta_2 = \beta - 30\% \text{ of } \beta$

$$\beta_2 = \beta - 0.3\beta = 0.7\beta$$

We know that, $\beta \propto \lambda$

$$\Rightarrow \frac{\beta_1}{\beta_2} = \frac{\lambda_1}{\lambda_2}$$

$$\Rightarrow \lambda_2 = \frac{\beta_2 \times \lambda_1}{\beta_1} = \frac{0.7\beta \times 6000}{\beta} = 4200\text{\AA}$$

Question 24

A series L – C – R circuit is connected to a source of alternating emf of 50V and the potential difference across inductor and capacitor is 90V and 60V , respectively. The potential difference across the resistor is

Options:

A. 70V

B. 60V

C. 40V

D. 80V

Answer: C

Solution:

Solution:

(c) Given, $V = 50\text{V}$, $V_L = 90\text{V}$, $V_C = 60\text{V}$

In L – C – R circuit,

$$\therefore V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\Rightarrow V^2 = V_R^2 + (V_L - V_C)^2$$

$$50^2 = V_R^2 + (90 - 60)^2$$

$$\Rightarrow 2500 = V_R^2 + 900$$

$$\Rightarrow V_R^2 = 1600$$

$$\Rightarrow V_R = \sqrt{1600} = 40\text{V}$$

Question 25

A particle of mass 5kg moves in a circle of radius 20cm. Its linear speed is

at a time t is given by $v = 4t$, t is in second and v is in ms^{-1} . Find the net force acting on the particle at $t = 0.5\text{s}$.

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Options:

- A. 20N
- B. $20\sqrt{23}\text{N}$
- C. $20\sqrt{26}\text{N}$
- D. 10N

Answer: C

Solution:

Solution:

(c) Given, $r = 20\text{cm} = 0.2\text{m}$, $t = 0.5\text{s}$, $v = 4t$ and $m = 5\text{kg}$

$$\begin{aligned}\text{Radial acceleration, } a_r &= \frac{v^2}{r} = \frac{(4t)^2}{0.2} = \frac{16t^2}{0.2} = 80t^2 \\ &= 80 \times (0.5)^2 \\ \text{At, } &= 20\text{ms}^{-2}\end{aligned}$$

Tangential acceleration of particle,

$$a_t = \frac{dv}{dt} = \frac{d}{dt}(4t) = 4\text{ms}^{-2}$$

\therefore Net acceleration,

$$a_n = \sqrt{a_r^2 + a_t^2} = \sqrt{(20)^2 + (4)^2} = 4\sqrt{26}\text{ms}^{-2}$$

$$\begin{aligned}\text{So, net force, } F_n &= ma_n = 5 \times 4\sqrt{26} \\ &= 20\sqrt{26}\text{N}\end{aligned}$$

Question 26

A 1kg stone attached to the end of a 60cm chain is revolving at the rate of 3rev / s. If after 30s, it is making only 1rev / s. Find the mean torque acting on it.

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Options:

- A. 0.15N – m
- B. 0.32N – m
- C. 0.25N – m
- D. 0.20N – m

Answer: A

Solution:

Solution:

(a) Given, $r = 60\text{cm} = 0.6\text{m}$
 $m = 1\text{kg}$
 $\omega_1 = 3\text{rev/s} = 2\pi \times 3\text{rad/s}$
 $\omega_2 = 1\text{rev/s} = 2\pi \times 1\text{rad/s}$
 $t = 30\text{s}$

$$\begin{aligned}\text{Torque, } \tau &= l \alpha = l \frac{d\omega}{dt} = mr^2 \frac{d\omega}{dt} \\ &= 1 \times (0.6)^2 \times \frac{2\pi(3) - 2\pi(1)}{30} \\ &= 0.15\text{N} \cdot \text{m}\end{aligned}$$

Question 27

A toroid is a long coil of wire wound over a circular core. Major radius and cross-sectional radius of toroid is R and r , respectively. The coefficient of mutual induction of the toroid is (The magnetic field in it is uniform, N = number of turns, $R \gg r$, μ_0 = permeability of free space)

Options:

- A. $\frac{\mu_0 N R}{2r}$
- B. $\frac{\mu_0 N^2 R^2}{2r}$
- C. $\frac{\mu_0 N r}{2R}$
- D. $\frac{\mu_0 N^2 r^2}{2R}$

Answer: B

Solution:

Solution:

(b) The coefficient of mutual induction is given by

$$M = \frac{\mu_0 N_1 N_2 A}{l} \dots\dots (i)$$

where, μ_0 is the permeability of free space, N_1 is the number of turns in primary coil, N_2 is the number of turns in secondary coil, A is the common area of cross-section and l is the length of coils.

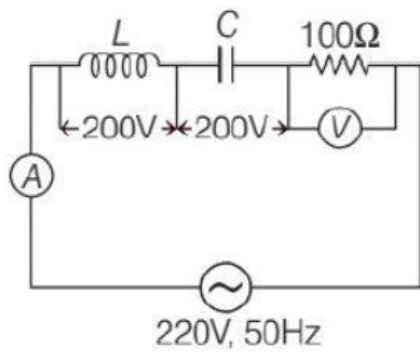
Thus, for toroid the Eq. (i) is given as

$$M = \frac{\mu_0 N \cdot N \cdot \pi R^2}{2\pi r} = \frac{\mu_0 N^2 R^2}{2r} \left[\begin{array}{l} \because N_1 = N_2 = N \\ A = \pi R^2 \quad l = 2\pi r \end{array} \right]$$

where, R is the major radius and r is the minor radius.

Question 28

The readings of ammeter and voltmeter in the given circuit are respectively



Options:

- A. 2.2A, 220V
- B. 2A, 200V
- C. 2.5A, 200V
- D. 2A, 220V

Answer: A

Solution:

Solution:

(a) Since, the voltage across inductor and capacitor is same, so they are in resonance i.e.,

$$X_L = X_C$$

The impedance of circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = R$$

\therefore Voltage across, $R = 220V$

By Ohm's law, $V = IR$

$$\Rightarrow I = \frac{V}{R} = \frac{220}{100} = 2.2A$$

Hence, ammeter reading is 2.2A and voltmeter reading is 220V .

Question 29

Let E_e and E_p represent kinetic energy of electron and photon, respectively. If de-Broglie wavelength λ_p of a photon is twice the de-Broglie wavelength λ_e of an electron, then $\frac{E_p}{E_e}$ is (speed of electron $= \frac{c}{100}$, $c =$ velocity of light)

Options:

- A. 10^{-1}
- B. 10^2
- C. 10^{-2}
- D. 10^4

Answer: B

Solution:

Solution:

(b) For electron, $\lambda_e = \frac{h}{\sqrt{2mE_e}} \Rightarrow E_e = \frac{h^2}{(2m)\lambda_e^2}$

For photon, $\lambda_p = \frac{hc}{E_p}$

$$\Rightarrow E_p = \frac{hc}{\lambda_p} = \frac{hc}{2\lambda_e} \quad (\because \lambda_p = 2\lambda_e)$$

$$\Rightarrow \frac{E_p}{E_e} = \frac{hc}{2\lambda_e} \times \frac{2m\lambda_e^2}{h^2} = mc \times \frac{\lambda_e}{h}$$

Also, $\lambda_e = \frac{h}{mv_e} \Rightarrow \frac{\lambda_e}{h} = \frac{1}{mv_e}$

$$\Rightarrow \frac{E_p}{E_e} = mc \times \frac{1}{mv_e} = \frac{c}{v_e}$$

$$= \frac{c}{c/100} \quad \left(\because v_e = \frac{c}{100} \right)$$
$$= 100 = 10^2$$

Question 30

A nucleus of rest splits into two nuclear parts having radii in the ratio 1 : 2. Their velocities are in the ratio

Options:

A. 1 : 8

B. 1 : 4

C. 4 : 1

D. 8 : 1

Answer: D

Solution:

Solution:

(d) If A_1 and A_2 are the mass number of two parts.

The radius of nucleus is given by

$$R = R_0(A)^{1/3}$$

So, $R_1 = R_0(A_1)^{1/3}$ and $R_2 = R_0(A_2)^{1/3}$

$$\Rightarrow \frac{R_1}{R_2} = \left(\frac{A_1}{A_2} \right)^{1/3}$$

$$\Rightarrow \frac{A_1}{A_2} = \left(\frac{R_1}{R_2} \right)^3 = \left(\frac{1}{2} \right)^3 = \frac{1}{8}$$

\therefore Ratio of masses, $\frac{m_1}{m_2} = \frac{A_1}{A_2} = \frac{1}{8}$

From conservation of momentum,

$$\Rightarrow m_1 v_1 = m_2 v_2$$

$$\Rightarrow \frac{v_1}{v_2} = \frac{m_2}{m_1} = \frac{8}{1} \text{ or } 8 : 1$$

Question 31

For a common-emitter amplifier, the voltage gain is 40 . Its input and output impedances are 100Ω and 400Ω , respectively. The power gain of the CE amplifier will be

Options:

- A. 450
- B. 400
- C. 300
- D. 500

Answer: B

Solution:

Solution:

(b) Given, voltage gain = 40, $R_{in} = 100\Omega$,

$R_{out} = 400\Omega$

Since, voltage gain = $\beta \left(\frac{R_{out}}{R_{in}} \right)$

$$\Rightarrow \beta = \text{Voltage gain} \times \frac{R_{in}}{R_{out}}$$

$$= 40 \times \frac{100}{400} = 10$$

Power gain = $\beta \times \text{voltage gain}$

$$= 10 \times 40 = 400$$

Question 32

Two particles A and B having same mass have charge $+q$ and $+4q$, respectively. When they are allowed to fall from rest through same electric potential difference the ratio of their speeds v_A to v_B will become

Options:

- A. 2 : 1
- B. 1 : 4
- C. 4 : 1

D. 1 : 2

Answer: D

Solution:

Solution:

(d) Given, mass of both particle = m

Charge of particle, A = +q

Charge of particle, B = +4q

Potential difference = V

Kinetic energy is given by

$$K = \frac{1}{2}mv^2 \dots\dots (i)$$

This energy is equal to electrostatic potential energy is

$$K = V \times Q \dots\dots (ii)$$

From Eqs. (i) and (ii), we have

$$\frac{1}{2}mv^2 = V \times Q$$

For particle A,

$$qV = \frac{1}{2}mv_A^2 \dots\dots (iii)$$

For particle B,

$$4qV = \frac{1}{2}mv_B^2 \dots\dots (iv)$$

Dividing Eq. (iii) by Eq. (iv), we get

$$\frac{1}{4} = \frac{v_A^2}{v_B^2}$$

$$\Rightarrow \frac{v_A}{v_B} = \frac{1}{2}$$

Hence, the ratio of their speed $\frac{v_A}{v_B} = \frac{1}{2}$

Question 33

The potential at a point x (measured in μm) due to some charges situated on the X-axis is given by $v(x) = \frac{20}{(x^2 - 4)}\text{V}$

The electric field E at $x = 4\mu\text{m}$ is given by

Options:

A. $\frac{5}{2}\text{V} / \mu\text{m}$ and in the - ve x-direction

B. $\frac{5}{2}\text{V} / \mu\text{m}$ and in the + ve x-direction

C. $\frac{10}{9}\text{V} / \mu\text{m}$ and in the - ve x-direction

D. $\frac{10}{9}\text{V} / \mu\text{m}$ and in the + ve x-direction

Answer: D

Solution:

Solution:

$$(d) \mathbf{E} = -\frac{\partial V}{\partial x} \hat{i} - \frac{\partial V}{\partial y} \hat{j} - \frac{\partial V}{\partial z} \hat{k}$$

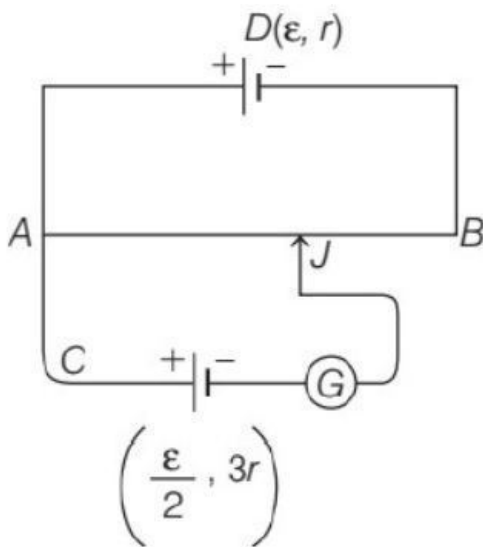
$$\Rightarrow E_x = -\frac{\partial V}{\partial x} = -\frac{d}{dx} \left[\frac{20}{x^2 - 4} \right] = \frac{40x}{(x^2 - 4)^2}$$

$$\Rightarrow E_x \text{ at } x = 4\mu\text{m} = \frac{10}{9} \text{V} / \mu\text{m}$$

and is along +ve x-direction.

Question 34

A potentiometer wire AB having length L and resistance 12r is joined to a cell D of emf ε and internal resistance r. A cell C having emf $\frac{\varepsilon}{2}$ and internal resistance 3r is connected. The length AJ at which galvanometer as shown in figure shows no deflection is



Options:

A. $\frac{5}{12}L$

B. $\frac{11}{12}L$

C. $\frac{13}{24}L$

D. $\frac{11}{24}L$

Answer: C

Solution:

Solution:

(c) Given, length of wire, AB = L

Resistance of wire, AB = 12r

emf of cell D = ε , internal resistance of D = r

emf of cell C = $\frac{\varepsilon}{2}$, internal resistance of C = 3r

$$\text{Current in potentiometer wire (i)} = \frac{\text{Total emf}}{\text{Total resistance}}$$

$$i = \frac{\varepsilon}{r + 12r} = \frac{\varepsilon}{13r}$$

Potential drop across the balance length AJ of potentiometer wire is $V_{AJ} = i \times R_{AJ}$

$$\Rightarrow V_{AJ} = i \text{ (resistance per unit length} \times \text{length AJ)}$$

$$V_{AJ} = i \left(\frac{12r}{L} \times x \right)$$

where, x is the balance length AJ.

As null point occurs at J, so potential drop across balance length,

AJ = emf of cell C

$$V_{AJ} = \frac{\varepsilon}{2} \Rightarrow i \left(\frac{12r}{L} \times x \right) = \frac{\varepsilon}{2}$$

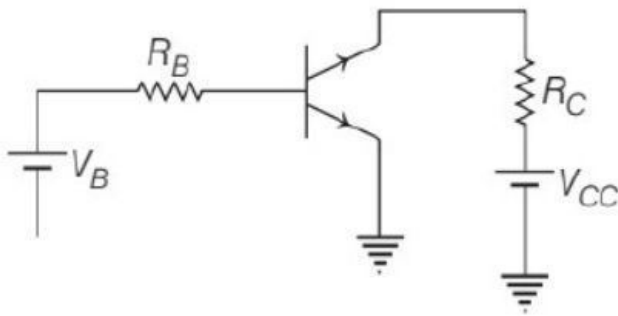
$$\Rightarrow \frac{\varepsilon}{13r} \times \frac{12r}{L} \times x = \frac{\varepsilon}{2}$$

$$\Rightarrow x = \frac{13}{24}L$$

Question 35

A common emitter amplifier circuit built using an n – p – n transistor is shown in figure. Its DC current gain is 300, $R_C = 4k\Omega$ and $V_{CC} = 20V$.

What is the minimum base current for V_{CE} to reach saturation?



Options:

- A. $40\mu A$
- B. $26.66\mu A$
- C. $16.66\mu A$
- D. $10\mu A$

Answer: C

Solution:

Solution:

(c) For CE n – p – n transistor, DC current gain,

$$\beta_{DC} = \frac{I_C}{I_B}$$

At saturation state, V_{CE} becomes zero,

$$\therefore V_{CC} - I_C R_C = 0$$

$$I_C \approx \frac{V_{CC}}{R_C} = \frac{20}{4000} = \frac{1}{200} A$$

Hence, saturation base current,

$$I_B = \frac{I_C}{\beta_{DC}} = \frac{1}{200 \times 300} = \frac{1}{60000} A = 16.66\mu A$$

Question 36

A satellite is revolving round a planet in a circular orbit close to its surface and ρ is mean density and R is the radius of planet, then the period of (G = universal constant of gravitation)

Options:

A. $\sqrt{\frac{3\pi}{\rho G}}$

B. $\sqrt{\frac{\pi}{\rho G}}$

C. $\sqrt{\frac{2\pi}{\rho G}}$

D. $\sqrt{\frac{4\pi}{\rho G}}$

Answer: A

Solution:

Solution:

(a) According to deduction of Kepler's third law and with the help of Newton's law, the law of period is given by

$$\text{or } T^2 = \frac{4\pi^2}{GM} r^3$$
$$\Rightarrow T = 2\pi \sqrt{\frac{r^3}{GM}}$$

where, r is the radius of orbit and M is mass of the planet or star.

As satellite is very close of the planet, thus $r = R$

Also mass (M) = density (ρ) \times volume (V)

$$= \rho \times \frac{4}{3}\pi R^3$$

$$\therefore T = 2\pi \sqrt{\frac{R^3}{G \times \frac{4}{3}\pi R^3}} = \sqrt{\frac{3\pi}{\rho G}}$$

Question 37

Halley's Comet revolves around the sun with time period of 76 years. The aphelion distance if perihelion is given by 8.9×10^{10} m, will be (Take, mass of sun = 2×10^{30} kg and $G = 6.67 \times 10^{-11}$ N m³ / kg²)

Options:

A. 4.7×10^{11} m

B. 3.23×10^{12} m

C. $5.3 \times 10^{12}\text{m}$

D. $7.63 \times 10^{11}\text{m}$

Answer: C

Solution:

Solution:

(c) According to Kepler's third law, time period

$$T^2 = \frac{4\pi^2}{GM}a^3$$

where, a is the semi-major axis,

$$\Rightarrow a = \left[\frac{76 \times 86400 \times 365 \times 6.67 \times 2 \times 10^{30}}{4 \times 3.14 \times 3.14} \right]^{1/3}$$

$$= 2.7 \times 10^{12}\text{m}$$

Also in case of ellipse,

$$2a = \text{perihelion} + \text{aphelion}$$

$$\Rightarrow \text{Aphelion} = 2a - \text{perihelion}$$

$$= 2 \times 2.7 \times 10^{12} - 8.9 \times 10^{10}$$

$$\approx 5.3 \times 10^{12}\text{m}$$

Question 38

540g of ice at 0°C is mixed with 540g of water at 80°C . The final temperature of the mixture will be

Options:

A. 53.3°C

B. 56.6°C

C. 60.6°C

D. 62.3°C

Answer: A

Solution:

Solution:

(a) From the principle of calorimetry,

$$m_1s_1\Delta T_1 = m_2s_2\Delta T_2$$

$$540 \times S_w(80 - T) = 540 \times \frac{S_w}{2} \times (T - 0) \text{ where, } s_w \text{ is specific heat of water.}$$

$$\Rightarrow T = \frac{160}{3}^\circ\text{C} = 53.3^\circ\text{C}$$

Question 39

The temperature of a liquid drops from 365K to 361K in 2min. Find the

**time during which temperature of the liquid drops from 344K to 342K .
(Take, room temperature = 293K)**

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Options:

- A. 72s
- B. 66s
- C. 60s
- D. 84s

Answer: D

Solution:

Solution:

(d) According to Newton's law of cooling,

$$\frac{T_1 - T_2}{2} = K \left(\frac{T_1 + T_2}{2} - T_0 \right)$$
$$\frac{365 - 361}{2} = K \left[\frac{365 + 361}{2} - 293 \right]$$
$$K = \frac{1}{35}$$
$$\Rightarrow t = \frac{14}{10} \text{min} = 84\text{s}$$

Question 40

The speed of sound in a mixture of 1 mole of He and 2 mole of oxygen at 27°C is

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Options:

- A. 400m / s
- B. 401m / s
- C. 402m / s
- D. 403m / s

Answer: B

Solution:

Solution:

(b) Molecular weight of the mixture,

$$M_{\text{mix}} = \frac{n_1 M_1 + n_2 M_2}{n_1 + n_2}$$
$$= \frac{1 \times 4 + 2 \times 32}{1 + 2} = \frac{68}{3} \times 10^{-3} \text{kgmol}^{-1}$$

For helium, $C_{V_1} = \frac{3}{2}R$

For oxygen, $C_{V_2} = \frac{5}{2}R$

$$(C_V)_{\text{mix}} = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$$
$$= \frac{1 + \frac{3R}{2} + 2 \times \frac{5R}{2}}{1 + 2} = \frac{13R}{6}$$

Now, $(C_p)_{\text{mix}} = (C_V)_{\text{mix}} + R$

$$= \frac{13R}{6} + R = \frac{19R}{6}$$

$$r_{\text{mix}} = \frac{(C_p)_{\text{mix}}}{(C_V)_{\text{mix}}} = \frac{19}{13}$$

Speed of sound in the mixture, $v = \sqrt{\frac{r_{\text{mix}} \times RT}{M_{\text{mix}}}}$

$$= \sqrt{\frac{19}{13} \times \frac{8.31 \times 300}{\frac{68}{3} \times 10^{-3}}}$$

$$\approx 401 \text{ms}^{-1}$$

Question 41

The lower half of a concave mirror is covered with opaque material. Then,

Options:

- A. the magnification will be halved
- B. the intensity of the image will be reduced
- C. the distance of the image will increase
- D. All of the above

Answer: B

Solution:

Solution:

(b) The law of reflection is true for all points of the remaining part of the mirror, so the image will be that of the whole object. However, as the area of the reflecting surface has been reduced, hence the intensity of the image will be low.

Question 42

An LED is placed at a depth h below the water surface. An opaque disc is floating on the surface of water such that the bulb is not visible from the surface. The minimum radius of the disc will be

Options:

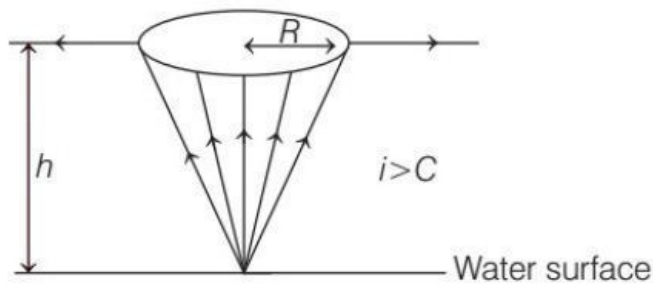
- A. $\frac{2h}{\mu}$
- B. $\frac{h}{\sqrt{\mu} - 1}$
- C. $\frac{h}{2\mu - 1}$
- D. $\frac{\mu}{2h}$

Answer: B

Solution:

Solution:

(b) The given situation is shown below



From the figure we can see the light from the LED will not emerge out of the water, if at the edge of the disc, the incidence angle is greater than critical angle.

i.e., $i > C$
or $\sin i > \sin C$

Now, if R is the radius of disc and h is the depth of the LED, then

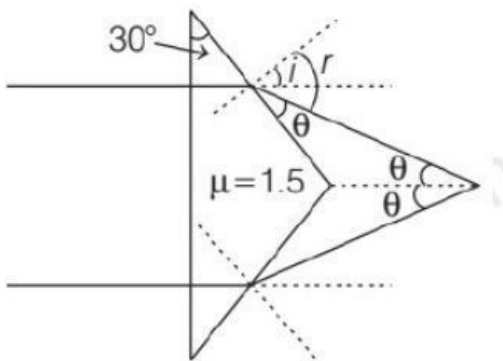
$$\sin i = \frac{R}{\sqrt{R^2 + h^2}} \text{ and } \sin C = \frac{1}{\mu}$$

From Eq (i), we have

$$\frac{R}{\sqrt{R^2 + h^2}} > \frac{1}{\mu} \Rightarrow R > \frac{h}{\sqrt{\mu} - 1}$$

Question 43

For the prism given below. Two light rays are incident normally on the surface of the prism. The angle between emergent rays will be



Options:

- A. 27°
- B. 32°

C. 37°

D. 42°

Answer: C

Solution:

Solution:

(c) The angle between any two lines is equal to the angle between their perpendiculars.

$\therefore i = 30^\circ$

From Snell's law, we have

$$\frac{1}{1.5} = \frac{\sin 30^\circ}{\sin r}$$

$$\Rightarrow \sin r = 0.75 \text{ or } r = 48.6^\circ$$

Therefore, $\theta = r - i$

$$= 48.6^\circ - 30^\circ = 18.6^\circ$$

\therefore Required angle between two emergent rays

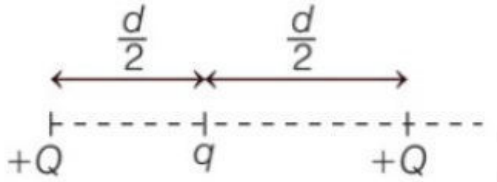
$$= 2 \times 18.6^\circ$$

$$= 37.2^\circ \approx 37^\circ$$

Question 44

Three charges are arranged as the configuration given below.

If net force experienced by $+Q$ placed at $x = 0$ is zero, then the value of q is



Options:

A. $\frac{-Q}{4}$

B. $\frac{Q}{2}$

C. $\frac{-Q}{2}$

D. $\frac{Q}{4}$

Answer: A

Solution:

Solution:

(a) From the principle of superposition, we have

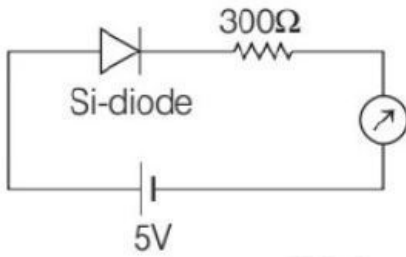
$$F_{\text{net}} = \frac{1}{4\pi\epsilon_0} \frac{Q \times q}{\left(\frac{d}{2}\right)^2} + \frac{1}{4\pi\epsilon_0} \frac{Q \times Q}{(d)^2}$$

$$\text{Since, } F_{\text{net}} = 0$$

$$\Rightarrow \frac{1}{4\pi\epsilon_0} \frac{Q \times q}{\left(\frac{d}{2}\right)^2} = -\frac{1}{4\pi\epsilon_0} \frac{Q \times Q}{d^2} \Rightarrow q = -\frac{Q}{4}$$

Question 45

What will be the reading in the ammeter for the circuit given below



Options:

- A. 12.3mA
- B. 12.3mA
- C. 11.3mA
- D. 14.3mA

Answer: D

Solution:

Solution:

(d) Potential drop across silicon diode in forward bias is around 0.7V .
In the given circuit, potential drop across 300Ω resistor is

$$\Delta V = IR$$

$$\Rightarrow I = \frac{\Delta V}{R} = \frac{5 - 0.7}{300} = 0.01433A$$

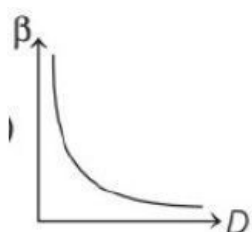
$$\text{or } I = 14.33mA$$

Question 46

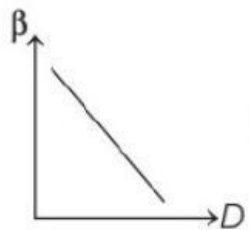
In Young's double slit experiment, which of the following graph represents correct variation of fringe width β versus distance D between sources and screen?

Options:

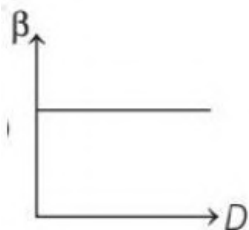
A.



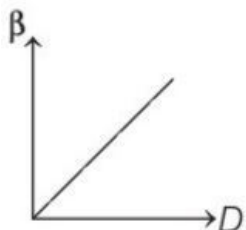
B.



C.



D.



Answer: D

Solution:

Solution:

(d) We know that, fringe width,

$$\beta = \frac{D\lambda}{d}$$

$$\Rightarrow \beta \propto D$$

Hence, graph (d) is the correct.

Question 47

An electron in hydrogen atom have energy of -3.4eV . The difference of its kinetic and potential energy is

Options:

A. 10.2eV

B. -10.2eV

C. -13.6eV

D. Both (a) and (b)

Answer: D

Solution:

Solution:

(d) According to Bohr's model, the kinetic energy of moving electron in n th orbit is given as

$$K = \frac{Rhc}{n^2} \dots\dots (i)$$

where, R = Rydberg constant, h = Planck's constant and c = speed of light.

Similarly, potential energy of electron moving in n th orbit,

$$P = -2 \frac{Rhc}{n^2} \dots\dots (ii)$$

From Eqs. (i) and (ii), we have

$$P = -2K$$

Total energy of electron moving in n th orbit,

$$E = K + P = K - 2K$$

$$E = -K$$

$$\Rightarrow K = -E = -(-3.4\text{eV}) \text{ (given } E = -3.4\text{eV) }$$

$$K = 3.4 \text{ eV}$$

From Eq. (iii), we have

$$P = -2K = -2(3.4) = -6.8\text{eV}$$

$$\therefore P - K = -6.8 - 3.4 = -10.2\text{eV}$$

$$\text{and } K - P = 3.4 - (-6.8) = 10.2\text{eV}$$

Question 48

In most liquids, with rise in temperature, surface tension of a liquid

Options:

- A. remains unchanged
- B. decreases
- C. increases
- D. first decreases and then increases

Answer: B

Solution:

Solution:

(b) In most of liquids, the cause of dynamic viscosity is the intermolecular force of attraction or cohesive forces and on increasing the temperature, molecules of liquids try to move away from each other which in turn reduces the cohesive force and hence dynamic viscosity decreases with increase in temperature in case of liquid.

Question 49

Three identical thin rods each of length l and mass M are joined together to form a letter H. What is the moment of inertia of the system about one of the sides of H ?

Options:

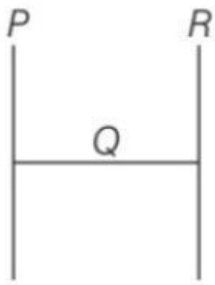
- A. $\frac{MI^2}{4}$
- B. $\frac{2MI^2}{3}$
- C. $\frac{4MI^2}{3}$
- D. $\frac{MI^2}{3}$

Answer: C

Solution:

Solution:

(c) The given situation can be shown as



So, moment of inertia of the system about P,

$$I = I_P + I_Q + I_R$$

$$= 0 + \left(\frac{MI^2}{12} + \frac{MI^2}{4} \right) + MI^2 = \frac{4}{3}MI^2$$

Question 50

One thousand small water drops of equal radii combine to form a big drop. The ratio of final surface energy to the total initial surface energy is

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Options:

- A. 1000: 1
- B. 1 : 10
- C. 10 : 1
- D. 1 : 1000

Answer: B

Solution:

Solution:

(b) Let initial radius be r_1 .

$$\text{Surface area of 1000 small water drops} = 1000 \times 4\pi r_1^2$$

$$\text{Volume of 1000 small drops} = 1000 \times \frac{4}{3}\pi r_1^3$$

Let final radius be r_2 .

Since, initial volume = final volume

$$1000 \times \frac{4}{3}\pi r_1^3 = \frac{4}{3}\pi r_2^3$$

$$\Rightarrow 1000r_1^3 = r_2^3 \Rightarrow \frac{r_2}{r_1} = 10$$

Let us assume surface tension remain constant, i.e. S .

$$\text{Therefore, } \frac{(\text{Surface energy})_{\text{final}}}{(\text{Surface energy})_{\text{initial}}}$$

$$= \frac{S \times 4\pi r_2^2}{S \times 4\pi r_1^2 \times 1000}$$

$$= \left(\frac{r_2}{r_1}\right)^2 \times \frac{1}{1000} = \frac{1}{10}$$

Question 51

Water rises to height 3.2cm in glass capillary tube. Find the height to which same water rises in another capillary having half the area of cross-section.

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Options:

A. 4.5cm

B. 5.0cm

C. 5.4cm

D. 6.2cm

Answer: A

Solution:

Solution:

(a) Given, $h_1 = 3.2\text{cm}$

As per question,

$$\frac{1}{2} (\text{area of first capillary}) = \text{Area of second capillary}$$

$$\Rightarrow \frac{1}{2} \times \pi r_1^2 = \pi r_2^2 \Rightarrow r_1 = \sqrt{2}r_2$$

For same fluid,

$$h_1 r_1 = h_2 r_2$$

$$\Rightarrow h_2 = \frac{h_1 r_1}{r_2} = \frac{3.2 \times \sqrt{2}r_2}{r_2} = 4.5\text{cm}$$

Question 52

If the pressure of an ideal gas is decreased by 10% isothermally, then its volume will

Options:

- A. decrease by 8%
- B. increase by 8%
- C. decrease by 9%
- D. increase by 11.1%

Answer: D**Solution:****Solution:**(d) From ideal gas equation, $pV = nRT$

For isothermal process, temperature remains constant,

So, for one mole of gas,

$$pV = \text{constant}$$

So, if pressure decreases, then volume increases.

$$\Rightarrow p_1 V_1 = p_2 V_2$$

$$\text{Here, } p_2 = p_1 \left(1 - \frac{1}{10} \right) = \frac{9}{10} p_1$$

$$\Rightarrow V_2 = \frac{10}{9} V_1$$

 \therefore Percentage increase in volume

$$= \left(\frac{V_2 - V_1}{V_1} \right) \times 100$$

$$= \left(\frac{10}{9} - 1 \right) \times 100$$

$$= \frac{1}{9} \times 100 = 11.1\%$$

Question 53

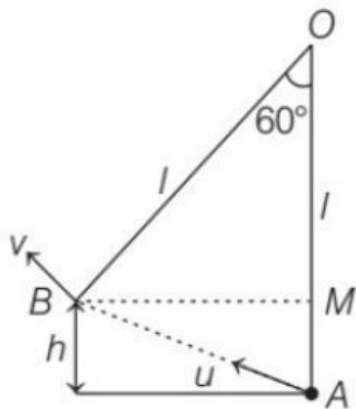
The pendulum bob has a speed of 3ms^{-1} at its lowest position and the length of pendulum is 0.5m. The speed of the bob when the length of the pendulum makes an angle of 60° with the vertical will be

Options:

- A. 1.5ms^{-1}
- B. 2ms^{-2}
- C. 2.7ms^{-1}
- D. 3.2ms^{-2}

Answer: B**Solution:****Solution:**(b) Given, $l = 0.5\text{m}$, $u = 3\text{ms}^{-1}$

The situation is shown below



Applying energy conservation at points A and B

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgh$$

$$\Rightarrow u^2 = v^2 + 2gh$$

$$v^2 = u^2 - 2gh = u^2 - 2g(I - I \cos 60^\circ)$$

$$[\text{since, } h = MA = OA - OM = I - OB \cos 60^\circ \\ = I - I \cos 60^\circ]$$

$$= (3)^2 - 2(9.8) \left(0.5 - 0.5 \times \frac{1}{2} \right)$$

$$\Rightarrow v^2 \approx 4$$

$$\therefore v = 2\text{ms}^{-1}$$

Question 54

In Young's double slit experiment using monochromatic light of wavelength λ , the maximum intensity of light at a point on the screen is K units. The intensity of light at point where the path difference is $\frac{\lambda}{3}$ is

$$\left[\cos 60^\circ = \sin 30^\circ \frac{1}{2} \right]$$

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Options:

A. $\frac{K}{4}$

B. $\frac{K}{2}$

C. K

D. $\frac{3K}{4}$

Answer: A

Solution:

Solution:

(a) For monochromatic light the resultant intensity is

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \theta$$

$$= 2I + 2I \cos \theta$$

$$(\because I_1 = I_2 = I) \dots (i)$$

For maximum intensity, $\theta = 0^\circ$

$$I_R = 2I + 2I \cos 0^\circ = 4I \quad (\because \cos 0^\circ = 1)$$

$$\text{or } K = 4I$$

$$\text{or } I = \frac{K}{4}$$

For path difference $\lambda / 3$, phase difference,

$$\phi = 2\pi \times \frac{\text{Path difference}}{\lambda} = 2\pi \times \frac{\lambda / 3}{\lambda} = \frac{2\pi}{3}$$

$$\therefore I_R = 2I + 2I \cos \frac{2\pi}{3}$$

[From Eq. (i)]

$$= 2I + 2I \left(-\frac{1}{2} \right) = 2I - I = I$$

$$= \frac{K}{4}$$

[using Eq. (ii)]

Question 55

In Young's double slit experiment, minimum intensity is obtained when phase difference of the superimposing waves is ($n = 1, 2, 3 \dots$)

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Options:

A. $(2n - 1)\pi$

B. $(n + 1)\pi$

C. zero

D. $n\pi$

Answer: A

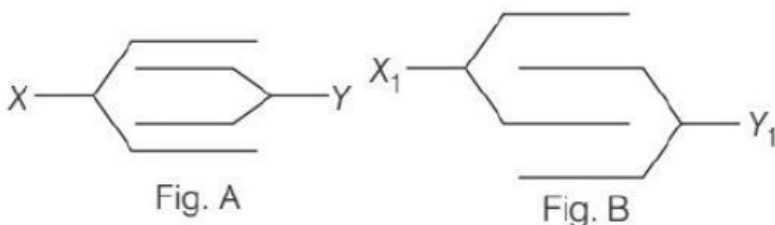
Solution:

Solution:

(a) For minima in YDSE, the phase difference as superimposing waves should be odd integral multiple of π .
i.e., $\phi = (2n - 1)\pi$ (where, $n = 1, 2, 3, \dots$)

Question 56

Four plates of equal area A are separated by equal distance d and are arranged as shown in figure A. Their equivalent capacity is C_1 . Then, the same four plates are arranged as shown in figure B, their equivalent capacity is C_2 , then $C_1 : C_2$ is



Options:

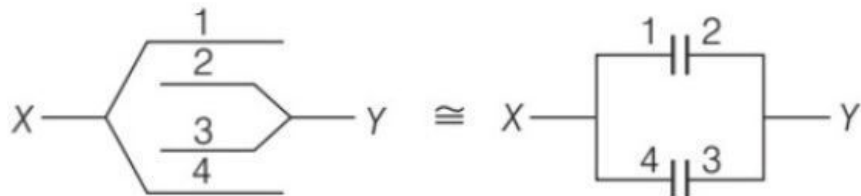
- A. 3 : 4
- B. 2 : 3
- C. 3 : 2
- D. 4 : 3

Answer: B

Solution:

Solution:

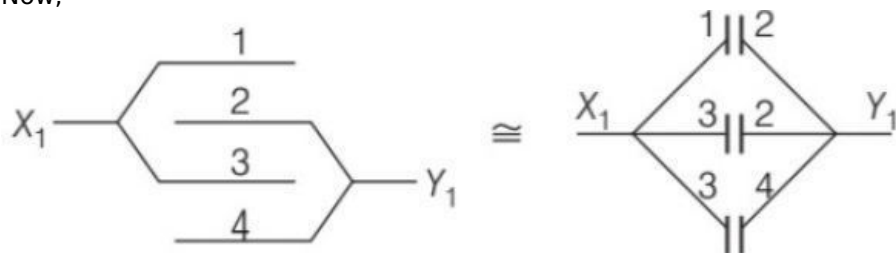
(b) Let X (1, 4) be positive plate and Y (2, 3) be negative plates.



The equivalent capacitance is just a parallel combination of two capacitors, therefore,

$$C_1 = \frac{\epsilon_0 A}{d} + \frac{\epsilon_0 A}{d} = \frac{2\epsilon_0 A}{d}$$

Now,



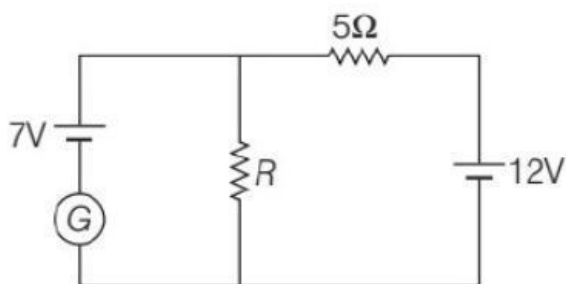
the equivalent capacitance is just a parallel combination of three capacitors. Therefore,

$$C_2 = \frac{\epsilon_0 A}{d} + \frac{\epsilon_0 A}{d} + \frac{\epsilon_0 A}{d} = \frac{3\epsilon_0 A}{d}$$

Therefore, $\frac{C_1}{C_2} = \frac{2}{3}$

Question 57

For what value of R in the circuit shown below will the current in galvanometer be zero?



Options:

- A. 2Ω
- B. 4Ω
- C. 5Ω

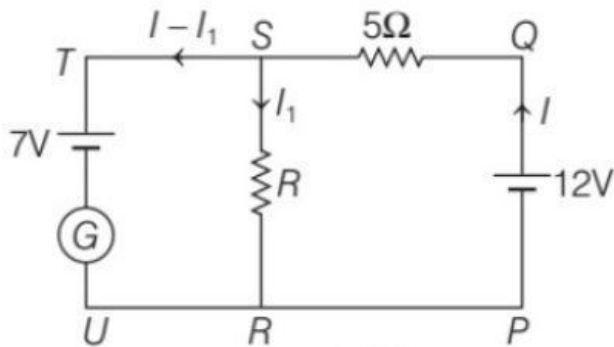
D. 7Ω

Answer: D

Solution:

Solution:

(d) The current distribution in the circuit is shown below



Applying KVL in loop PQSRP, we get
 $-5I - RI_1 + 12 = 0 \dots\dots\dots (i)$

Similarly, for loop SRUTS, we get
 $r - I_1R + 7 = 0$

$$\Rightarrow I_1R = 7 \dots\dots\dots (ii)$$

As per question, $I - I_1 = 0 \Rightarrow I = I_1 \dots\dots\dots (iii)$

Solving Eqs. (i), (ii) and (iii), we get

$$-5I - 7 + 12 = 0 \Rightarrow I = 1A$$

From Eq. (ii),

$$R = 7\Omega$$

Question 58

A long wire carrying a steady current is bent into a circle of single turn. The magnetic field at the centre of the coil is B . If it is bent into a circular loop of radius r having n turns, the magnetic field at the centre of the coil for same current is

Options:

A. nB

B. n^2B

C. $\frac{B}{n}$

D. $\frac{B}{n^2}$

Answer: B

Solution:

Solution:

(b) The magnetic field at the centre of coils is

$$B = \frac{\mu_0 I}{2r} \dots\dots\dots (i)$$

where, r = radius of the coil.

Let L be the length of wire, then

$$L = 2\pi r \Rightarrow 2r = \frac{L}{\pi}$$

From Eq. (i), we get

$$B = \frac{\mu_0 I \pi}{L} \dots\dots\dots (ii)$$

For a loop of n turns, $L = 2\pi nr$

$$\Rightarrow 2r = \frac{L}{n\pi}$$

And current, $I' = nI$

$$\therefore B = \frac{\mu_0 I'}{2r} = \frac{\mu_0 nI}{L / n\pi} = \frac{\mu_0 n^2 I / \pi}{L} = n^2 B \text{ [from Eq. (ii)]}$$

Question 59

The mutual inductance between a primary and secondary circuit is 0.5H . The resistances of the primary and the secondary circuits are 20Ω and 5Ω , respectively. To generate a current of 0.4A in the secondary, current in the primary must be changed at the rate of

Options:

A. $16\text{A} / \text{s}$

B. $1.9\text{A} / \text{s}$

C. $4\text{A} / \text{s}$

D. $8\text{A} / \text{s}$

Answer: C

Solution:

Solution:

(c) Given, coefficient of mutual inductance, $m = 0.5\text{H}$

Resistance of primary, $R_1 = 20\Omega$

Resistance of secondary, $R_2 = 5\Omega$

Let, current in primary be I_1 , such that current in secondary is $I_2 = 0.4\text{A}$.

Now, the emf induced in secondary due to change primary current is given by

$$\Rightarrow \varepsilon = M \frac{dI_1}{dt}$$

$$\Rightarrow (0.4) \times 5 = (0.5) \frac{dI_1}{dt} \quad (\because \varepsilon = I_2 R_2)$$

$$\Rightarrow \frac{dI_1}{dt} = 4\text{A} / \text{s}$$

Question 60

A condenser of capacitance $2.4\mu\text{F}$ is used in a transmitter to transmit at

wavelength λ . If the inductor of 10^{-8}H is used for resonant circuit, then the value of λ is

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Options:

- A. 200m
- B. 254m
- C. 292m
- D. 320m

Answer: C

Solution:

Solution:

(c) Given, $C = 2.4\mu\text{F} = 2.4 \times 10^{-6}\text{F}$
 $L = 10^{-8}\text{H}$

At resonant frequency, $v = \frac{1}{2\pi\sqrt{LC}}$

$$\Rightarrow LC = \frac{1}{4\pi^2 v^2} = \frac{\lambda^2}{4\pi^2 c^2} \quad \left(\because v = \frac{c}{\lambda} \right)$$

$$\begin{aligned}\Rightarrow \lambda &= \sqrt{4\pi^2 c^2 LC} \\ &= \sqrt{4 \times \pi^2 \times (3 \times 10^8)^2 \times 10^{-8} \times 2.4 \times 10^{-6}} \\ &= 292\text{m}\end{aligned}$$

Question 61

The work function for a metal is 3.6V and threshold wavelength is 3000Å. If work function for another metal is 1.8eV, then threshold wavelength must be

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Options:

- A. 6000Å
- B. 5600Å
- C. 5200Å
- D. 4800Å

Answer: A

Solution:

Solution:

(a) Given, work function of metal, $\phi_1 = 3.6\text{eV}$
Threshold wavelength, $\lambda_1 = 3000\text{Å}$
Work function for another metal, $\phi_2 = 1.8\text{eV}$

Threshold wavelength, $\lambda_2 = ?$

According to Einstein's photoelectric equation,

$$\frac{hc}{\lambda_0} = \phi_0$$

$$\text{Therefore, } \frac{\frac{hc}{\lambda_1}}{\frac{hc}{\lambda_2}} = \frac{\phi_1}{\phi_2} = \frac{3.6}{1.8}$$

$$\Rightarrow \lambda_2 = 2 \times \lambda_1$$

$$\Rightarrow \lambda_2 = 6000\text{\AA}$$

Question 62

If the ionisation energy for the hydrogen atom is 13.6eV, then the energy required to excite it from the ground state to the next higher state is nearly

Options:

A. 13.6eV

B. -3.4eV

C. 10.2eV

D. -10.2eV

Answer: C

Solution:

Solution:

(c) Given, $E_1 = 13.6\text{eV}$

Energy of H-atom in nth excited state is

$$E_n = -\frac{13.6}{n^2}\text{eV}$$

For

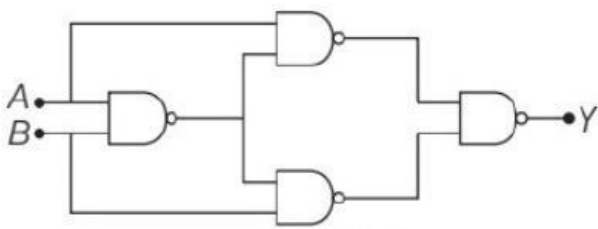
$$n = 2, E_2 = -\frac{13.6}{2^2} = -3.4\text{eV}$$

So, energy required to excite H-atom from ground state to next higher state is

$$E = E_2 - E_1 = -3.4 - (-13.6) = 10.2\text{eV}$$

Question 63

The output Y of the logic circuit shown is

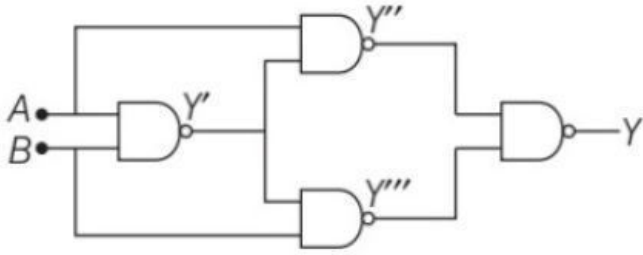


Options:

- A. $A \cdot \bar{B} + A \cdot B$
 B. $\bar{A} \cdot B + A \cdot \bar{B}$
 C. $\bar{A} \cdot \bar{B} + A \cdot B$
 D. $A \cdot \bar{B} + \bar{A} \cdot \bar{B}$

Answer: C**Solution:****Solution:**

(c) The respective output of each logic gate is shown below



$$\begin{aligned}
 \text{Here, } Y' &= \overline{A \cdot B} = \bar{A} + \bar{B} \\
 Y'' &= A \cdot (A + B) = \bar{A} + (\bar{A} + B) \\
 [\text{By de Morgan's theorem}] \\
 &= \bar{A} + (\bar{A} \cdot B) = \bar{A} + (A \cdot B) \\
 Y''' &= B \cdot (A + B) = \bar{B} + (\bar{A} + B) = \bar{B} + (\bar{A} \cdot B) \\
 &= \bar{B} + (A \cdot B) \\
 Y &= Y'' \cdot Y''' = \bar{A} + (A \cdot B) \cdot [\bar{B} + (A \cdot B)] \\
 &= [\bar{A} + A \cdot B] + [\bar{B} + (A \cdot B)] \\
 &= \bar{A} \cdot (\bar{A} \cdot B) + \bar{B} \cdot (A \cdot B) \\
 &= A(\bar{A}B) + B(\bar{A}B) \\
 &= (A + B)(\bar{A} \cdot B) = (A + B)(\bar{A} + \bar{B}) \\
 &= A\bar{A} + A\bar{B} + B\bar{A} + B\bar{B} \\
 &= 0 + A\bar{B} + B\bar{A} + 0 \\
 &= \bar{A} \cdot B + A \cdot \bar{B}
 \end{aligned}$$

Question 64

In a single slit diffraction pattern, width of slit is a . The first minimum is observed at angle of 45° . When light of wavelength 6000\AA is incident on the slit. The first secondary maximum is observed at an angle of

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Options:

- A. $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$
 B. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$
 C. $\sin^{-1}\left(\frac{3}{2\sqrt{2}}\right)$
 D. Zero

Answer: C

Solution:

Solution:

(c) For the first minima, $\sin \theta = \frac{\lambda}{a}$

$$\Rightarrow \frac{\lambda}{a} = \sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{\lambda}{a} = \frac{1}{\sqrt{2}} \dots\dots (i)$$

$$\text{For first secondary maximum, } \sin \theta = \frac{3\lambda}{2a} = \frac{3}{2} \times \frac{1}{\sqrt{2}}$$

[From Eq. (i)]

$$\Rightarrow \theta = \sin^{-1} \left(\frac{3}{2\sqrt{2}} \right)$$

Question 65

A beam of light of wavelength 600nm from a distant source falls on a single-slit 1mm wider and the resulting diffraction pattern is observed on a screen 2m away. The distance between first dark on either side of the central bright fringes is

Options:

A. 1.2cm

B. 1.2mm

C. 2.4cm

D. 2.4mm

Answer: D

Solution:

Solution:

(d) Distance between the first dark fringes on either side of the central bright fringe
= width of central maxima

$$\Rightarrow 2y = \frac{2\lambda D}{d}$$

$$\text{Here, } \lambda = 600\text{nm} = 600 \times 10^{-9}\text{m}$$

$$d = 1\text{mm} = 1 \times 10^{-3}\text{m}$$

$$D = 2\text{m}$$

$$\Rightarrow 2y = \frac{2 \times 600 \times 10^{-9} \times 2}{1 \times 10^{-3}} = 24 \times 10^{-4}\text{m} = 2.4 \text{ mm}$$

Question 66

Eight dipoles of charges of magnitude e each one placed inside a cube.

The total flux coming out of the cube will be

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Options:

A. $\frac{8e}{\epsilon_0}$

B. $\frac{16e}{\epsilon_0}$

C. $\frac{e}{\epsilon_0}$

D. Zero

Answer: D

Solution:

Solution:

(d) Net charge of one dipole = $-e + e = 0$

Net charge of 8 dipoles = $8 \times 0 = 0$

\therefore Net charge inside cube, $q = 0$

By Gauss's law

$$\text{Total flux emerging out from surface} = \frac{q}{\epsilon_0} = 0$$

Question 67

A parallel plate air capacitor has a capacitance $18\mu\text{F}$. If the distance between the plates is tripled and a dielectric medium is introduced, the capacitance becomes $72\mu\text{F}$. The dielectric constant of the medium is

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Options:

A. 4

B. 9

C. 12

D. 2

Answer: C

Solution:

Solution:

(c) For air capacitor, $C_0 = \frac{\epsilon_0 A}{d} = 18\mu\text{F}$

When dielectric slab is introduced between plates and distance is tripled, then

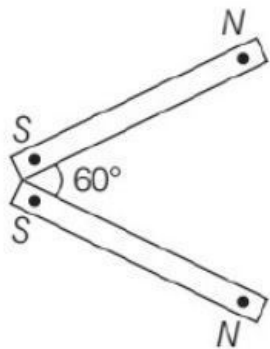
$$C = \frac{K \epsilon_0 A}{3d} = 72\mu\text{F}$$

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{K}{3} = \frac{72}{18} \Rightarrow K = 12$$

Question 68

Two bar magnet having magnetic dipole moments $4A - m^2$ and $5A - m^2$ are kept as shown below. The resultant dipole moment will be



Options:

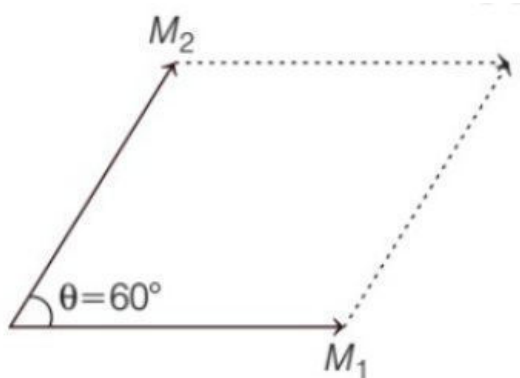
- A. $\sqrt{21}Am^2$
- B. $\sqrt{41}Am^2$
- C. $\sqrt{61}A - m^2$
- D. $\sqrt{31}A - m^2$

Answer: C

Solution:

Solution:

(c) Here,
 $M_1 = 4A - m^2$
 $M_2 = 5A - m^2$

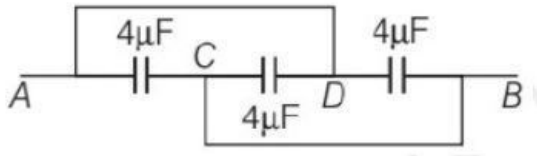


According to parallelogram law of vector addition, resultant dipole moment M

$$\begin{aligned} M &= \sqrt{M_1^2 + M_2^2 + 2M_1M_2 \cos 60^\circ} \\ &= \sqrt{4^2 + 5^2 + 2 \times 4 \times 5 \times \cos 60^\circ} = \sqrt{61}A - m^2 \end{aligned}$$

Question 69

The equivalent capacitance between points A and B is



Options:

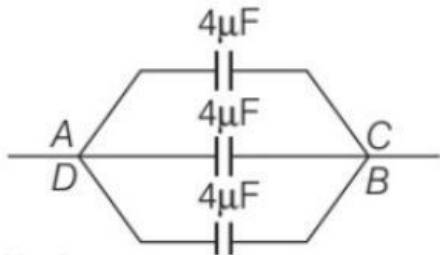
- A. $\frac{4}{3}\mu\text{F}$
- B. $3\mu\text{F}$
- C. $4\mu\text{F}$
- D. $12\mu\text{F}$

Answer: D

Solution:

Solution:

The given circuit diagram is drawn as



Hence, given capacitors are in parallel combination.

$$\therefore C_{AB} = 4 + 4 + 4 = 12\mu\text{F}$$

Question 70

A point charge Q is placed at the centre of a cube. The flux linked with the each face of the cube is

Options:

- A. $\frac{Q}{6\epsilon_0}$
- B. $\frac{Q}{\epsilon_0}$
- C. $\frac{Q}{12\epsilon_0}$
- D. $\frac{Q}{8\epsilon_0}$

Answer: A

Solution:

Solution:

(a) The cube is a symmetrical body with 6 faces and the point charge is at its centre, so electric flux linked with each face will be

$$\phi = \frac{\phi_{\text{total}}}{6} = \frac{Q}{6\epsilon_0}$$

Question 71

Maximum velocity of photoelectrons emitted from a metal surface is v_1 when frequency of incident radiation is 2ν . When frequency of incident radiation is increased to 5ν , then maximum velocity of emitted photoelectrons becomes v_2 . The ratio $v_1 : v_2$ will be

Options:

- A. 1 : 4
- B. 4 : 1
- C. 1 : 2
- D. 2 : 1

Answer: C

Solution:

Solution:

(c) According to Einstein's photoelectric equation, maximum energy of emitted electrons is given by

$$K_{\text{max}} = \frac{1}{2}mv_{\text{max}}^2 = h\nu - h\nu_0$$

$$\Rightarrow \frac{1}{2}mv_{\text{max}}^2 = h(\nu - \nu_0) \quad \dots\dots (i)$$

When, frequency of incident radiation, becomes 2ν , then velocity of emitted electron is v_1 , hence from Eq. (i), we get

$$\frac{1}{2}mv_1^2 = h(2\nu - \nu_0)$$

$$\frac{1}{2}mv_1^2 = h\nu_0 \quad \dots\dots (ii)$$

Again, when frequency becomes $5\nu_0$, the velocity of emitted electron is v_2 .

\therefore From Eq. (i), we have

$$\frac{1}{2}mv_2^2 = h(5\nu - \nu)$$

$$\Rightarrow \frac{1}{2}mv_2^2 = 4h\nu$$

Dividing Eq. (ii) by Eq. (iii), we get

$$\frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2} = \frac{h\nu}{4h\nu}$$

$$\Rightarrow \left(\frac{v_1}{v_2}\right)^2 = \left(\frac{1}{4}\right)^2$$

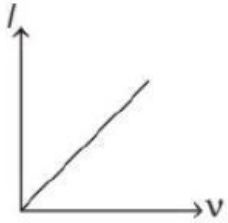
$$\Rightarrow v_1 : v_2 = 1 : 2$$

Question 72

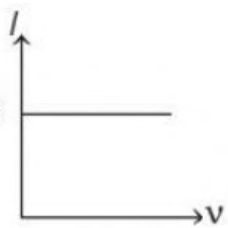
Which of the following graphs shows correct variation of photoelectric current / with frequency of incident radiation?

Options:

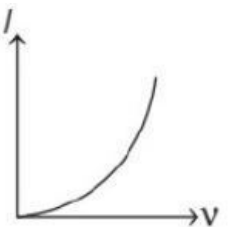
A.



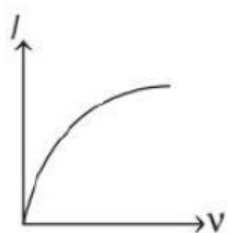
B.



C.



D.



Answer: B

Solution:

Solution:

Since, photoelectric current (I) does not depend on the frequency (ν) of incident radiation. Hence, graph showing in option (b) is correct.

Question 73

The frequency of a stretched uniform wire of length L under tension is in resonance with the fundamental frequency of air column in a closed pipe of the same length. If the tension in the wire is increased by 8N, it is in resonance with the first overtone of the same closed pipe. The initial tension in the wire is

Options:

- A. 3N
- B. 4N
- C. 1N
- D. 2N

Answer: C

Solution:

Solution:

(c) As we know the, frequency of sound wave produced in a stretched wire is directly proportional to square root of tension in the wire

i.e., $f \propto \sqrt{T}$

Let T be the initial tension.

Therefore, for air column, $f_c \propto \sqrt{T}$ (i)

According to question, first overtone

i.e., $3f_c \propto \sqrt{T + 8}$ (ii)

From Eq. (i) and Eq. (ii), we get

$$\frac{f_c}{3f_c} = \frac{\sqrt{T}}{\sqrt{T + 8}}$$

$$\Rightarrow T + 8 = 9T$$

$$\Rightarrow T = 1N$$

Question 74

A vertical column 50cm long at 50°C balances another column of same liquid 60cm long at 100°C. The coefficient of absolute expansion of the liquid is

Options:

- A. 0.002 / °C
- B. 0.003 / °C
- C. 0.004 / °C
- D. 0.005 / °C

Answer: D

Solution:

Solution:

(d) Given, $h_1 = 50\text{cm}$, $T_1 = 50^\circ\text{C}$

$h_2 = 60\text{cm}$, $T_2 = 100^\circ\text{C}$

Let the density of the given liquid at STP be ρ_0 , if both vertical columns balance each other, then their pressure should be equal.

i.e., $p = \rho gh$

$$\Rightarrow \rho_1 gh_1 = \rho_2 gh_2$$

$$\Rightarrow \frac{\rho_1}{\rho_2} = \frac{h_2}{h_1}$$

If r be the coefficient of absolute expansion of liquid, then, $\rho_1 = \frac{\rho_0}{1 + rT_1}$ and $\rho_2 = \frac{\rho_0}{1 + rT_2}$

\therefore From Eq. (i) we have

$$\frac{\frac{\rho_0}{1 + rT_1}}{\frac{\rho_0}{1 + rT_2}} = \frac{h_2}{h_1} = \frac{60}{50}$$

$$\Rightarrow \frac{1 + rT_2}{1 + rT_1} = \frac{6}{5} \Rightarrow 5rT_2 - 6rT_1 = 1$$

$$\Rightarrow r = \frac{1}{200} = 0.005 / ^\circ\text{C}$$

Question 75

A source of sound emitting a note of frequency n is approaching a stationary listener. If the frequency of the note heard by the listener is $2n$, the velocity of the source versus is equal to [v = velocity of sound in air]

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Options:

A. $2v$

B. $\frac{2v}{3}$

C. $\frac{v}{2}$

D. $\frac{v}{4}$

Answer: C

Solution:**Solution:**

(c) The Doppler's effect in sound is given by

$$f_o = \frac{v + v_o}{v + v_s} f_s$$

where, f_o = observer frequency of sound,

v = speed of sound waves,

v_o = observer velocity,

v_s = source velocity

and f_s = actual frequency of sound wave.

Thus, according to question,

$$2n = \left(\frac{v+0}{v+v_s} \right) n$$

$$\Rightarrow 2(v+v_s) = v$$

$$\Rightarrow 2v + 2v_s = v$$

$$-2v_s = v$$

$$v_s = -\frac{v}{2}$$

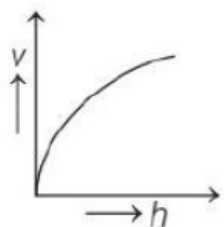
where -ve sign shows that source is approaching to the listener.

Question 76

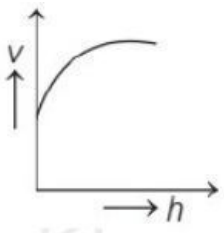
A uniform rope having a mass m is suspended with a rigid support. It is struck as its lower end. The speed of transverse wave v produced in this way will vary with height h as

Options:

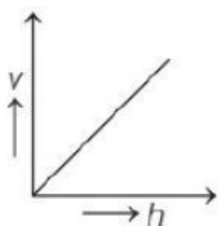
A.



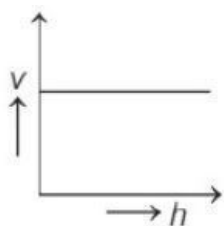
B.



C.



D.



Answer: A

Solution:

Solution:

(a) Let m be the total mass of the rope of length L . Tension in the rope at a height h from lower end = weight of rope length h ,

i.e.,

As

$$T = \frac{mg}{L}h$$

$$v = \sqrt{\frac{T}{\left(\frac{m}{L}\right)}}$$

$$v = \sqrt{\frac{mg(h)}{L\left(\frac{m}{L}\right)}} = \sqrt{gh}$$

$$\Rightarrow v^2 = gh$$

Which represents a parabola symmetric along h -axis.

Thus, option (a) represents the correct graph.

Question 77

A thin prism P_1 with angle 4° and made from glass of refractive index 1.54 is combined with another thin prism P_2 made from glass of refractive index 1.72 to produce dispersion without deviation. The angle of prism P_2 is

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Options:

A. 2.6°

B. 5.33°

C. 3°

D. 4°

Answer: C

Solution:**Solution:**

(c) For dispersion without deviation,

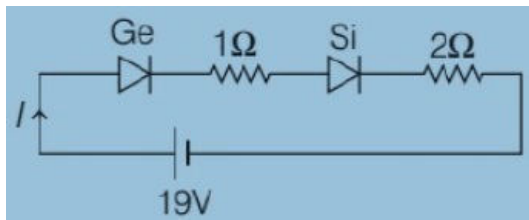
$$\frac{A}{A_1} = \frac{\mu - 1}{\mu' - 1}$$

$$\Rightarrow \frac{4}{A_1} = \frac{1.72 - 1}{1.54 - 1} = \frac{0.72}{0.54}$$

$$\Rightarrow A_1 = \frac{4 \times 0.54}{0.72} = 3^\circ$$

Question 78

A silicon and germanium diode is connected in series as shown below



The current flowing through the circuit will be

Options:

- A. 6A
- B. 3A
- C. 2A
- D. 1A

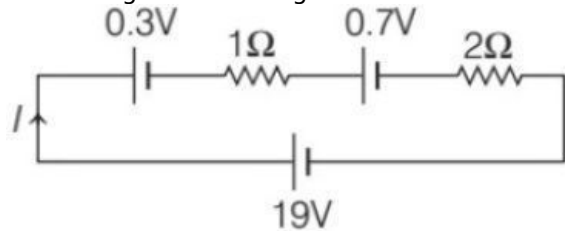
Answer: A

Solution:

Solution:

(a) We know that, barrier potential of Ge, $V_{B_1} = 0.3 \text{ V}$ and barrier potential of Si, $V_{B_2} = 0.7 \text{ V}$

Redrawing the above figure as



Applying KVL, we get

$$-19 + 0.3 + 1 \cdot I + 0.7 + 2I = 0$$

$$\Rightarrow 3I = 18 \Rightarrow I = 6\text{A}$$

Question 79

In hydrogen atom, an electron jumps from third excited state to ground state, then frequency of emitted light will be

[Take, $h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$]

Options:

- A. $1.5 \times 10^{14} \text{ Hz}$
- B. $3 \times 10^{15} \text{ Hz}$
- C. $4 \times 10^{13} \text{ Hz}$
- D. $6.5 \times 10^{12} \text{ Hz}$

Answer: B

Solution:

Solution:(b) Energy of electron in n th orbit in H-atom,

$$E_n = \frac{-13.6}{n^2} \text{eV}$$

For third excited state, $n = 4$

$$\therefore E_4 = \frac{-13.6}{4^2} = -0.85 \text{eV}$$

For ground state, $n = 1$

$$\therefore E_1 = \frac{-13.6^2}{1^2} = -13.6 \text{eV}$$

If ν be the frequency of emitted photon, then

$$\begin{aligned} h\nu &= E_4 - E_1 \\ &= -0.85 - (-13.6) \\ &= -0.85 + 13.6 = 12.75 \text{eV} \end{aligned}$$

$$\begin{aligned} \nu &= \frac{12.75 \times 1.6 \times 10^{-19}}{h} \\ &= \frac{12.75 \times 16 \times 10^{-19}}{6.63 \times 10^{-34}} = 3 \times 10^{15} \text{Hz} \end{aligned}$$

Question 80

An asteroid is falling towards the centre of earth under the influence of earth's gravitational field. At $10R_e$ (R_e = radius of earth) it has speed of 12km / s . What will be the speed of the asteroids when it hits the earth's surface?

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Options:A. 12.4km / s B. 14.6km / s C. 16km / s D. 18km / s **Answer: C****Solution:****Solution:**

(c) According to conservation of energy,

Total energy of asteroid at $10R_e$ = Total energy of asteroid at surface of earth

$$\Rightarrow U_1 + K_1 = U_2 + K_2$$

$$\Rightarrow \frac{-GM_em}{10R_e} + \frac{1}{2}mv_0^2 = \frac{-Gm_em}{R_e} + \frac{1}{2}mv^2$$

$$\Rightarrow \frac{9}{10} \frac{GM_em}{R_e} + \frac{1}{2}mv_0^2 = \frac{1}{2}mv^2$$

$$\Rightarrow \frac{9}{10} \times \frac{2GM}{R_e} + v_0^2 = v^2$$

$$\Rightarrow v^2 = \frac{9}{10}(v_e)^2 + v_0^2$$

where, v_e = escape velocity = 11.2km / s

$$\Rightarrow v^2 = \frac{9}{10} \times (11.2)^2 + (12)^2$$

$$= \frac{9}{10} \times (11.2)^2 + 144 = 16 \text{kms}^{-1}$$

Question 81

A geostationary satellite at 36000km height has 24h time period. The time-period of a spy satellite at $R = 6400\text{km}$ will be

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Options:

- A. 1.2h
- B. 1.3h
- C. 1.53h
- D. 2h

Answer: C

Solution:

Solution:

(c) For time period of a satellite, we can write

$$T = \frac{2\pi r}{v} = \frac{2 \times r}{\sqrt{\frac{GM}{r}}} = 2\pi \left(\frac{r^3}{GM} \right)^{1/2}$$

According to question,

$$24 = 2\pi \left[\frac{(6400 + 36000)^3}{GM} \right]^{1/2}$$

$$\text{and for spy satellite, } T' = 2\pi \left[\frac{(6400)^3}{GM} \right]^{1/2}$$

$$\text{II } \therefore \frac{T'}{24} = \left[\frac{(6400)^3}{(6400 + 36000)^3} \right]^{\frac{1}{2}}$$

$$\Rightarrow T' = (24) \times (0.4)^3$$

$$\Rightarrow T' = 1.53\text{h}$$

Question 82

A lead bullet at 27°C melts when hit on a target. Assuming only 75% of heat is used to melt the bullet, the velocity of bullet at time of striking is

(Take, melting point of lead = 327°C , specific heat of lead = $0.03\text{cal} / \text{g}^\circ\text{C}$, latent heat of fusion of lead = $6\text{cal} / \text{g}$)

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Options:

- A. $330\text{m} / \text{s}$
- B. $410\text{m} / \text{s}$
- C. $470\text{m} / \text{s}$

D. 510m / s

Answer: B

Solution:

Solution:

(b) Let mass of the bullet be m gram, then total heat required for bullet to just melt down

$$Q_1 = mc \Delta T + mL$$

$$= m \times (0.03)(327 - 27) + m \times 6$$

$$= 15m - \text{cal}$$

$$= (16m \times 4.2) \text{J}$$

Now, when bullet is struck by obstacles, the loss in its mechanical energy

$$= \frac{1}{2}(m \times 10^{-3})v^2$$

The energy absorbed by bullet,

$$Q_2 = \frac{72}{100} \times \frac{1}{2}mv^2 \times 10^{-3}$$

$$= \frac{3}{8}mv^2 \times 10^{-3} \text{J}$$

Now, the bullet will melt if $Q_2 \geq Q_1$

$$\text{i.e., } \frac{3}{8}mv^2 \times 10^{-3} \geq 15m \times 4.2$$

$$\Rightarrow v_{\min} = 410 \text{ m / s}$$

Question 83

A cell can be balanced against 120cm and 100cm of potentiometer wire, respectively.

When in open circuit and when short-circuited through a resistance of 10Ω , then the internal resistance of the cell is

Options:

A. 1Ω

B. 1.5Ω

C. 2Ω

D. 0.5Ω

Answer: C

Solution:

Solution:

(c) Given, $l_1 = 120\text{cm}$, $l_2 = 100\text{cm}$

$R = 10\Omega$, $r = ?$

$$\therefore r = \left(\frac{l_1}{l_2} - 1 \right) R$$

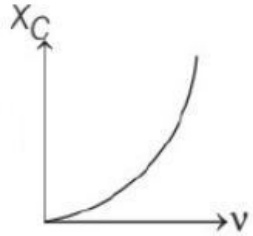
$$= \left(\frac{120}{100} - 1 \right) \times 10 = \frac{20}{100} \times 10 = 2\Omega$$

Question 84

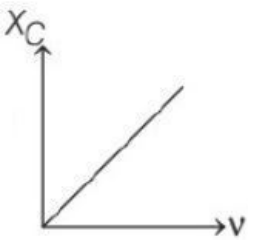
Which of the following curves shows correct variation of capacitive reactance X_C with frequency ν ?

Options:

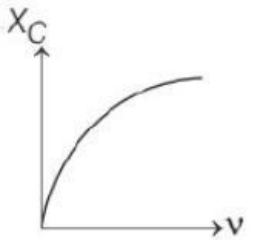
A.



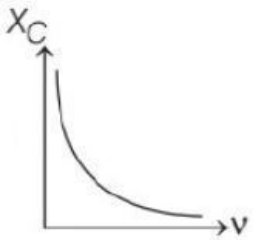
B.



C.



D.



Answer: D

Solution:

Solution:

(d) We know that, capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$$

$$\Rightarrow X_C \propto \frac{1}{\nu}$$

Hence, graph shown in option (d) represents correct variation of X_C with ν .

Question 85

An α -particle and a proton, accelerated through same potential difference, enter into a region of uniform magnetic field with their velocities perpendicular to the field. If radius of circular path traversed by proton is $\sqrt{2}\text{cm}$, then radius of circular path traversed by α -particle is

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Options:

- A. 1cm
- B. 2cm
- C. 3cm
- D. 4cm

Answer: B

Solution:

Solution:

(b) We know that, radius of circular path in magnetic field B,

$$r = \frac{mv}{Bq}$$

When a charge q is accelerated by V volts, it acquires a kinetic energy, $E_k = qV$

$$\therefore \text{Momentum } p = \sqrt{2mE_k} = \sqrt{2mqV}$$

$$\Rightarrow mv = \sqrt{2mqV}$$

From Eq. (i) and (ii), we get

$$cr = \sqrt{\frac{2mqV}{Bq}} = \sqrt{\frac{2mV}{qB^2}}$$

$$\text{Thus, } \frac{r_\alpha}{r_p} = \sqrt{\frac{m_\alpha}{m_p}} \sqrt{\frac{q_p}{q_\alpha}} = \sqrt{\frac{4m_p}{m_\alpha}} \cdot \sqrt{\frac{q_p}{2q_\alpha}} = \sqrt{2}$$

$$\Rightarrow r_\alpha = \sqrt{2}r_p = \sqrt{2} \times \sqrt{2} = 2\text{cm}$$

Question 86

If two waves of the same frequency and amplitude respectively on superposition produce a resultant disturbance of the same amplitude. The waves differ in phase by

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Options:

- A. $\frac{\pi}{2}$
- B. $\frac{\pi}{6}$
- C. $\frac{2\pi}{3}$

D. $\frac{\pi}{4}$

Answer: C

Solution:

Solution:

(c) According to given situation,

$$a_1 = a_2 = a = a_R \text{ (resultant)}$$

$$\therefore a_R = \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos \varphi}$$

$$a = \sqrt{a^2 + a^2 + 2a^2 \cos \varphi}$$

$$a^2 = 2a^2(1 + \cos \varphi)$$

$$\Rightarrow 1 + \cos \varphi = 1 / 2$$

$$\Rightarrow \cos \varphi = -\frac{1}{2} \Rightarrow \varphi = \frac{2\pi}{3}$$

Question 87

The radii of two columns in a U-tube are r_1 and r_2 . When a liquid of density ρ (angle of contact = 0°) is filled in it, the level difference of liquid in two arms is h . The surface tension of liquid is

Options:

A. $\frac{(r_1 - r_2)\rho gh}{2r_1r_2}$

B. $\frac{r_1r_2\rho gh}{(r_1 - r_2)}$

C. $\frac{r_1r_2\rho gh}{2(r_2 - r_1)}$

D. $\frac{r_1r_2(\rho gh)}{2(r_1 - r_2)}$

Answer: C

Solution:

Solution:

(c) The height of liquid in tube of radius r_1 is

$$h_1 = \frac{2T \cos \theta}{r_1 \rho g} = \frac{2T}{r_1 \rho g}$$

$$(\because \theta = 0^\circ)$$

Similarly, for tube of radius r_2 ,

$$h_2 = \frac{2T}{r_2 \rho g}$$

$$\text{Given, } h = h_1 - h_2$$

$$= \frac{2T}{\rho g} \left(\frac{1}{r_1} - \frac{1}{r_2} \right) = \frac{2T(r_2 - r_1)}{\rho g r_1 r_2}$$

$$\Rightarrow T = \frac{r_1 r_2 \rho g h}{2(r_2 - r_1)}$$

Question 88

The ratio of the wavelength of the last line of the Balmer series to last line of Lyman series is

Options:

- A. 1 : 2
- B. 2 : 1
- C. 4 : 1
- D. 1 : 4

Answer: C

Solution:

Solution:

(c) Wavelength of spectral line in hydrogen atom ($Z = 1$) given as

$$\frac{1}{\lambda} = R \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] \dots (i) \quad [\because Z = 1]$$

For last line of Lyman series,

$$n_1 = 1 \text{ and } n_2 = \infty$$

\therefore From Eq. (i), we have

$$\frac{1}{\lambda_L} = R \left(\frac{1}{1^2} - \frac{1}{\infty} \right) = R$$

$$\Rightarrow \lambda_L = \frac{1}{R} \dots \dots \dots (ii)$$

Similarly, for last line of Balmer series, $n_1 = 2$ and $n_2 = \infty$

\therefore From Eq. (i), we get

$$\frac{1}{\lambda_B} = R \left(\frac{1}{2^2} - \frac{1}{\infty} \right) = \frac{R}{4}$$

$$\Rightarrow \lambda_B = \frac{4}{R}$$

Hence, from Eq. (i) and Eq. (ii), we get

$$\frac{\lambda_L}{\lambda_B} = \frac{1/R}{4/R} = \frac{1}{4}$$

$$\text{or } \lambda_L : \lambda_B = 1 : 4$$

$$\Rightarrow \lambda_B : \lambda_L = 4 : 1$$

Question 89

A body of mass m is tied to one end of a spring and whirled round in a horizontal circle with a constant angular velocity. The elongation in the spring is 1cm. If the angular velocity is doubled, the elongation in the spring is 5cm. The original length of the spring is

Options:

- A. 13cm
- B. 15cm
- C. 14cm
- D. 16cm

Answer: B**Solution:****Solution:**

(b) Let original length be xcm.

Initial angular velocity be ω .Elongation, $\Delta x = 1\text{cm}$ According to Newton's law $F = -k\Delta x = \frac{mv^2}{r} \Rightarrow -k\Delta x = m\omega^2 r$ Since, $r = (x + 1)$ and $\Delta x = 1\text{cm}$ Therefore, $-k(1) = m\omega^2(x + 1)$

...(i)

Again angular velocity is doubled and elongation produced is 5cm.

Therefore, $-k(5) = m(2\omega)^2(x + 5)$ $-5k = 4m\omega^2(x + 5)$

From Eq. (i) and Eq. (ii), we get

$$\frac{5k}{k} = \frac{4m\omega^2(x + 5)}{m\omega^2(x + 1)}$$

$$\Rightarrow (x + 1)5 = 4(x + 5)$$

$$\Rightarrow 5x + 5 = 4x + 20$$

$$\Rightarrow 5x - 4x = 20 - 5$$

$$\Rightarrow x = 15\text{cm}$$

Question 90

A particle is moving in a circle of radius R with constant speed v. The magnitude of average acceleration after half revolution is

Options:

A. $\frac{v^2}{2\pi R}$

B. $\frac{2v^2}{\pi R}$

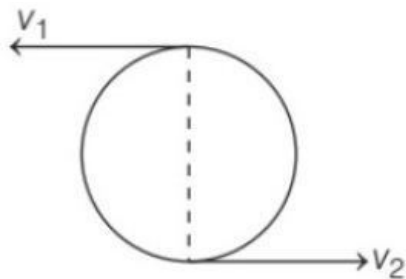
C. $\frac{2v^2}{\pi R^2}$

D. $\frac{v^2}{\pi R}$

Answer: B**Solution:****Solution:**

(b) For half revolution, the position of the particle is given below,

Let velocity be v.



Change in velocity $= v_2 - v_1 \Rightarrow v(-v) = 2v$

Time taken by the particle to cover half revolution

$$\therefore = \frac{\pi R}{v}$$

$$\therefore \text{Average acceleration} = \frac{\text{Velocity}}{\text{Time}} = \frac{2v}{\frac{\pi R}{v}} = \frac{2v^2}{\pi R}$$

Question 91

For a given gas at 1atm pressure, rms speed of the molecules is 100m / s at 27°C. At 2atm pressure and at 927°C temperature, the rms speed of the molecules will be

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Options:

- A. 50m / s
- B. 100m / s
- C. 200m / s
- D. 400m / s

Answer: C

Solution:

Solution:

(c) Given, $T_1 = 27 + 273 = 300\text{K}$ $T_2 = 927 + 273 = 1200\text{K}$

$(v_{\text{rms}})_1 = 100\text{m / s}$

$(v_{\text{rms}})_2 = ?$

We know that, $v_{\text{rms}} \propto \sqrt{T}$

$$\frac{(v_{\text{rms}})_2}{(v_{\text{rms}})_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{1200}{300}} = 2$$

$$(v_{\text{rms}})_2 = 2 \times (v_{\text{rms}})_1 = 2 \times 100 = 200\text{m / s}$$

Question 92

The magnitude of electric field intensity that balances the weight of an α -particle is

[Take, mass of proton, $m_p = 1.67 \times 10^{-27}\text{kg}$, $g = 10\text{m / s}^2$ and

$e = 1.6 \times 10^{-19}\text{C}$]

Options:

A. $3.3 \times 10^{-6} \text{ N / C}$

B. $4.2 \times 10^{-8} \text{ N / C}$

C. $2.5 \times 10^{-9} \text{ N / C}$

D. $2.1 \times 10^{-7} \text{ N / C}$

Answer: D

Solution:

Solution:

. (d) For equilibrium of α -particle,

$$qE = mg$$

$$\Rightarrow E = \frac{mg}{q}$$

$$= \frac{4m_p \cdot g}{2e} = \frac{4 \times 167 \times 10^{-27} \times 10}{2 \times 1.6 \times 10^{-19}}$$

$$= 2.1 \times 10^{-7} \text{ N / C}$$

Question 93

A particle executes simple harmonic motion with an amplitude of 4cm. When the particle is at 2cm from the mean position, then the magnitude of its velocity is equal to that of its acceleration. Then, its time period is

Options:

A. $\sqrt{3}\pi\text{s}$

B. $\frac{\pi}{\sqrt{3}}\text{s}$

C. πs

D. $\frac{2\pi}{\sqrt{3}}\text{s}$

Answer: B

Solution:

Solution:

(b) Magnitude of velocity of particle when it is at displacement x from mean position.

$$v = \omega \sqrt{A^2 - x^2}$$

when $x = 2\text{cm}$, then

$$v = a$$

$$\Rightarrow \omega \sqrt{A^2 - x^2} = \omega^2 x$$

$$\Rightarrow \omega = \frac{\sqrt{A^2 - x^2}}{x} = \frac{\sqrt{4^2 - 2^2}}{2} = \frac{\sqrt{12}}{2} = \frac{4\sqrt{3}}{2}$$

$$\Rightarrow \omega = 2\sqrt{3} \text{ rad / s}$$

$$\therefore \text{Time period, } T = \frac{2\pi}{\omega} = \frac{2\pi}{2\sqrt{3}} = \frac{\pi}{\sqrt{3}} \text{ s}$$

Question 94

Molar specific heat capacity of an ideal gas at constant volume is $\frac{3}{4}R$, the molar specific heat capacity of the ideal gas at constant pressure will be

Options:

A. 1.75R

B. 1.45R

C. 1.33R

D. 1.20R

Answer: A

Solution:

Solution:

(a) We know that, according to Mayer's formula,

$$C_p = C_v + R$$

$$= \frac{3}{4}R + R = \frac{7}{4}R = 1.75R$$

Question 95

Which of the following is correct for adiabatic process?

Options:

A. Work done is independent of path.

B. Work done is equal to negative of change in internal energy.

C. Both (a) and (b)

D. Neither (a) nor (b)

Answer: C

Solution:

Solution:

(c) According to first law of dynamics.

$$\Delta Q = \Delta U + \Delta W$$

$$0 = \Delta U + \Delta W \Rightarrow \Delta W = -\Delta U$$

In adiabatic process, work done does not depend on the path, i.e., it is independent with path.

Question 96

A force of $(3\hat{i} + 2\hat{j} - \hat{k})$ N acts on a particle with position vector $(\hat{i} + \hat{j} - \hat{k})$ m. The magnitude of torque of given force is

Options:

A. $\sqrt{10}\text{N} - \text{m}$

B. $\sqrt{6}\text{N} - \text{m}$

C. $\sqrt{8}\text{N} - \text{m}$

D. $\sqrt{5}\text{N} - \text{m}$

Answer: B

Solution:**Solution:**

(b) Given, $F = (3\hat{i} + 2\hat{j} - \hat{k})$ N

$r = (\hat{i} + \hat{j} - \hat{k})$ m

\therefore Torque, $\tau = r \times F = (\hat{i} + \hat{j} - \hat{k}) \times (3\hat{i} + 2\hat{j} - \hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 3 & 2 & -1 \end{vmatrix}$$

$$= \hat{i}(-1 + 2) - \hat{j}(-1 + 3) + \hat{k}(2 - 3)$$

$$\tau = \hat{i} - 2\hat{j} - \hat{k}$$

$$\therefore \tau = |\tau| = \sqrt{1^2 + (-2)^2 + (-1)^2}$$

$$= \sqrt{1 + 4 + 1} = \sqrt{6}\text{N} - \text{m}$$

Question 97

In the phenomena of refraction of light, which of the following characteristics remains unchanged?

Options:

A. Intensity

B. Amplitude

C. Frequency

D. Velocity

Answer: C

Solution:

Solution:

(c) In the phenomenon of refraction, frequency of light remains unchanged.

Question 98

**A 2kW motor is used to pump water from a well 20m deep. The quantity of water pumped out per second is nearly
[Take, $g = 10 \text{ m/s}^2$]**

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Options:

A. 10kg

B. 20kg

C. 5kg

D. 25kg

Answer: A

Solution:

Solution:

(a) Given, $P = 2 \text{ kW} = 2 \times 10^3 \text{ W}$

$h = 20 \text{ m}$

$$\therefore P = \frac{W}{t}$$

$$P = \frac{mgh}{t}$$

$$\Rightarrow m = \frac{Pt}{gh} = \frac{2 \times 10^3 \times 1}{10 \times 20} = 10 \text{ kg}$$

Question 99

The angle of projection of a projectile for which horizontal range and height attain same magnitude is

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Options:

A. $\tan^{-1}(2)$

- B. $\tan^{-1}(4)$
 C. $\tan^{-1}(3)$
 D. $\tan^{-1}(0.5)$

Answer: B

Solution:

Solution:

(b) Here, $H = R$

$$\frac{u^2 \sin^2 \theta}{2g} = \frac{u^2 \sin 2\theta}{g}$$

$$\Rightarrow \frac{\sin^2 \theta}{2} = \sin 2\theta$$

$$\Rightarrow \frac{\sin^2 \theta}{2} = 2 \sin \theta \cos \theta$$

$$\Rightarrow \frac{\sin \theta}{\cos \theta} = 4 \Rightarrow \tan \theta = 4$$

$$\Rightarrow \theta = \tan^{-1}(4)$$

Question 100

A thin rod of length $2l$ is rotating about an axis passing through its centre and perpendicular to its length. The radius of gyration of for the rod is

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Options:

A. $\frac{l}{\sqrt{3}}$

B. $\frac{l}{\sqrt{4}}$

C. $\frac{1}{\sqrt{14}}$

D. $\frac{l}{\sqrt{12}}$

Answer: A

Solution:

Solution:

(a) The moment of inertia of rod.

$$I = \frac{ML^2}{12} = \frac{M(2l)^2}{12} \text{ (Here, } L=2l\text{)}$$

$$I = \frac{MI^2}{3}$$

$$Mk^2 = \frac{MI^2}{3} \text{ [} k = \text{ radius of gyration]}$$

$$k = \frac{l}{\sqrt{3}}$$

Question 1

Identify the last element in lanthanoid series.

Options:

- A. Nd
- B. Tm
- C. Lu
- D. Tb

Answer: C

Solution:

Solution:

Last element of lanthanoid series is Lu i.e. leutetium with atomic number 71 and mass number 175.5. It is a silver white metal, which resists corrosion in dry air, but not in moist air.

Question 2

0.05F electricity is passed through CuSO_4 solution. Calculate the mass of Cu produced at cathode? (molar mass of Cu = 63.5gmol^{-1})

Options:

- A. 3.17g
- B. 2.54g
- C. 0.795g
- D. 1.59g

Answer: D

Solution:

Solution:

Equivalent mass of copper

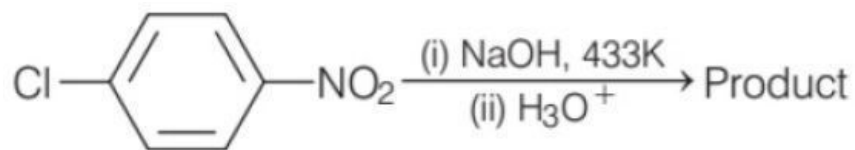
$$= \frac{\text{Atomic mass}}{\text{Valency}} = \frac{63.5}{2} = 31.75$$

Amount of copper deposited by 0.05F

$$= 0.05 \times 31.75 = 1.5875 \sim 1.59\text{g}$$

Question 3

Identify the product obtained in the following reaction.



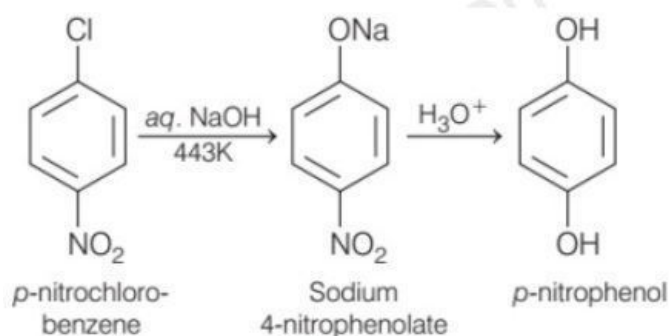
Options:

- A. o-nitrophenol
- B. m-nitrophenol
- C. p-nitrophenol
- D. None of these

Answer: C

Solution:

Solution:



Question 4

Formation of gluconic acid from glucose by oxidation using Br₂ water

Options:

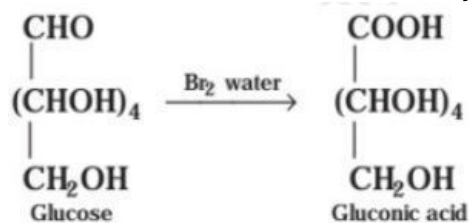
- A. glucose contains five hydroxyl groups.
- B. the presence of aldehyde group in glucose.
- C. glucose contains one primary alcoholic group.
- D. the six carbon atoms of glucose are in a straight chain.

Answer: B

Solution:

Solution:

Glucose gets oxidised to six carbon. Carboxylic acid (gluconic acid) on reaction with a mild oxidising agent like bromine water. This indicates that the carbonyl group is present as an aldehyde group.



Question 5

What is the density of potassium, if it has bcc structure with edge length 4\AA ?

(Atomic mass of K = 39)

Options:

- A. 3gcm^{-3}
- B. 2gcm^{-3}
- C. 5gcm^{-3}
- D. 4gcm^{-3}

Answer: B

Solution:

Solution:

For density, $\rho = \frac{Z \times M}{a^3 \times N_A}$

Edge length, $a = 4\text{\AA} = 4 \times 10^{-8}\text{cm}$

For bcc structure, $Z = 2$

$$\begin{aligned}
 \rho &= \frac{2 \times 39}{(4 \times 10^{-8})^3 \times 6.022 \times 10^{23}} \\
 &= 0.202 \times 10^{24} \times 10^{-23} \text{gcm}^{-3} \\
 &= 0.202 \times 10 \text{gcm}^{-3} = 2.02 \text{gcm}^{-3}
 \end{aligned}$$

Question 6

Which formula from following is used to determine molar mass of solute from depression in freezing point?

Options:

A. $M_2 = \frac{1000 \times K_f \times W_1}{\Delta T_f \times W_2}$

$$\text{B. } M_2 = \frac{1000 \times K_f \times W_2}{\Delta T_f \times W_1}$$

$$\text{C. } M_2 = \frac{1000 \times \Delta T_f \times W_2}{K_f \times W_1}$$

$$\text{D. } M_2 = \frac{1000 \times \Delta T_f \times W_1}{K_f \times W_2}$$

Answer: B

Solution:

Solution:

The formula used to determine the molar mass of solute from depression in freezing point is

$$M_2 = \frac{1000 \times K_f \times W_2}{\Delta T_f \times W_1}$$

where, W_2 = weight of solute

W_1 = weight of solvent

K_f = molal depression constant

Question 7

A gas is heated from 273K to 373K at 1atm pressure. If the initial volume of the gas is 10 L, its final volume would be

Options:

A. 20 d m^3

B. 13.66 d m^3

C. 10 d m^3

D. 7.32 d m^3

Answer: B

Solution:

Solution:

$$T_1 = 273\text{K}$$

$$T_2 = 373\text{K}$$

$$V_1 = 10\text{L}$$

$$V_2 = ?$$

From Charles law,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\&V_2 = 373 \times \frac{10}{273} = 13.66\text{L} = 13.66 \text{ d m}^3$$

Question 8

The ratio between the root mean square velocity of H_2 at 50K and that of O_2 at 800K is

Options:

- A. 4
- B. 2
- C. 1
- D. 0.25

Answer: B

Solution:

Solution:

$$T_1 = 50K \text{ for } H_2$$

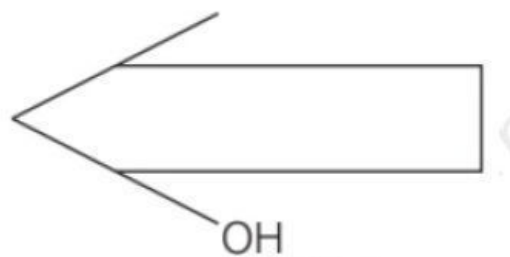
$$T_2 = 800K \text{ for } O_2$$

$$u_{ms} = \sqrt{\frac{3RT}{M}}$$

$$\Rightarrow \frac{u_{H_2}}{u_{O_2}} = \sqrt{\frac{T_1}{M_1} \times \frac{M_2}{T_2}} = \sqrt{\frac{50}{2} \times \frac{32}{800}} = 1$$

Question 9

What is the IUPAC name of following compound?



Options:

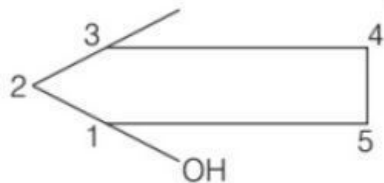
- A. 1-methylcyclopentan-3-ol
- B. 3 -hydroxy-1-methyl cyclopentane
- C. 3-methyl cyclopentanol
- D. 3-methylpentan-1-ol

Answer: C

Solution:

Solution:

The correct IUPAC name of the given compound is 3 -methylcyclopentanol.



Question 10

An organic compound on analysis gave C = 42.8%, H = 7.2% and N = 50%. Volume of 1g of the compound was found to be 50mL at STP. Molecular formula of the compound is

Options:

- A. $C_2H_4N_2$
- B. $C_{12}H_{24}N_{12}$
- C. $C_{16}H_{32}N_{16}$
- D. $C_4H_8N_4$

Answer: C

Solution:

Solution:

Given, C = 42.8%, H = 7.2% and N = 50%

Element	%	Moles	Simplest ratio	Whole number
C	42.8	$\frac{42.8}{12} = 3.56$	1	1
H	7.2	$\frac{7.2}{1} = 7.2$	2	2
N	50	$\frac{50}{14} = 3.57$	1	1

\therefore Empirical formula = CH_2N

Empirical formula mass = 28

Now, number of moles = $\frac{\text{volume given at STP}}{22400\text{mL}}$

$$= \frac{50}{22400} = 2.23 \times 10^{-3}$$

Also, number of moles = $\frac{\text{weight}}{\text{molecular weight}}$

$$\Rightarrow \text{Molecular weight} = \frac{1}{2.23 \times 10^{-3}} = 448$$

$$\therefore n = \frac{\text{molecular weight}}{\text{empirical weight}} = \frac{448}{28} = 16$$

Hence, molecular formula = (empirical formula)_n

$$= (CH_2N)_{16} = C_{16}H_{32}N_{16}$$

Question 11

How much $\text{BaCl}_2 \cdot 2\text{H}_2\text{O}$ and pure water are to be mixed to prepare 50g of 12.0% (by mass) BaCl_2 solution?

Options:

- A. 40.4g
- B. 42.9g
- C. 52.7g
- D. 50.0g

Answer: B

Solution:

Solution:

$w = 12\text{g}$ of BaCl_2 , $W = 100\text{g}$ of solution.

For 50g of solution, $w = 6\text{g}$ of BaCl_2

$W = 50\text{g}$ of solution

$$\therefore w_{\text{BaCl}_2 \cdot 2\text{H}_2\text{O}} = \left(\frac{6 \times 244}{208} \right) \text{g} = 7.038\text{g}$$

$$w_{\text{H}_2\text{O}} = (50 - 7.038)\text{g} = 42.96\text{g} \sim 42.9\text{g}$$

Question 12

What is enthalpy of formation of NH_3 , if bond enthalpies as $(\text{N} \equiv \text{N}) = -941\text{kJ/mol}$, $(\text{H} - \text{H}) = 436\text{kJ/mol}$ and $(\text{N} - \text{H}) = 389\text{kJ/mol}$?

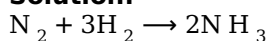
Options:

- A. -84.5kJ/mol
- B. -63.45kJ/mol
- C. -21.25kJ/mol
- D. -42.5kJ/mol

Answer: D

Solution:

Solution:



$$\Delta_f H (\text{NH}_3) = \text{Bond energy of reactant} - \text{Bond energy of product}$$

$$\begin{aligned}
 &= \left(\frac{1}{2} \times \text{BE of N} \equiv \text{N bond} + \frac{3}{2} \text{BE of H} - \text{H bond} \right) - (3 \text{ BE of N} - \text{H bond}) \\
 &= \left(\frac{1}{2} \times 941 + \frac{3}{2} \times 436 \right) - (3 \times 389) \\
 &= 470.5 + 654 - 1167 = -42.5 \text{ kJ / mol}
 \end{aligned}$$

Question 13

The enthalpy change that accompanies a reaction in which 1 mole of its standard state is formed from its elements in their standard states

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Options:

- A. bond enthalpy of reaction
- B. standard enthalpy of formation
- C. standard enthalpy of reaction.
- D. standard enthalpy of combustion

Answer: B

Solution:

Solution:

Standard enthalpy of formation is the change in enthalpy that accompanies the formation of one mole of a compound from its elements, with all substances in their standard states, it is also called as 'standard heat of formation'.

Question 14

What is the SI unit of conductivity?

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Options:

- A. Ω^{-1}
- B. $\Omega^{-1} \text{cm}^2 \text{mol}^{-1}$
- C. $\text{Sm}^2 \text{mol}^{-1}$
- D. Sm^{-1}

Answer: D

Solution:

Solution:

$$\text{Conductivity } (\sigma) = \frac{1}{\text{Resistivity } (\rho)}$$

Question 15

What is rate constant of a first order reaction, if 0.08 mole of reactant reduces to 0.02 mole in 23.03 minute?

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Options:

- A. 1.6021min^{-1}
- B. 0.2303min^{-1}
- C. 0.4031min^{-1}
- D. 0.06020min^{-1}

Answer: D

Solution:

Solution:

For first order reaction,

$$t = \frac{2.303}{k} \log \frac{a}{a-x}$$

where, t = time, k = rate constant

a = initial concentration

a - x = remaining concentration

$$\Rightarrow k = \frac{2.303}{23.03} \log \frac{0.08}{0.02} = \frac{1}{10} \log 4$$

$$= \frac{0.6020}{10} = 0.06020\text{min}^{-1}$$

Question 16

Which of the following statement is not true for a reaction having rate law $r = k[\text{H}_2][\text{I}_2]$?

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Options:

- A. Overall order of reaction is 2 .
- B. The reaction is first order in H_2 .
- C. The reaction is first order in I_2
- D. Overall order of reaction is 1

Answer: D

Solution:

Solution:

Among the given statements, (d) is not correct regarding the rate $= k[H_2][I_2]$ because overall order of the reaction is 2 not 1.

Question 17

At room temperature, polonium crystallises in a primitive cubic unit cell. If $a = 3.36\text{\AA}$. Calculate the theoretical density of polonium. [It's atomic weight is 209g/mol .]

Options:

- A. 9.15g/cm^3
- B. 10.33g/cm^3
- C. 6.22g/cm^3
- D. 9.74g/cm^3

Answer: A

Solution:

Solution:

Given number of atoms per unit cell, $Z = 1$

Molecular weight, $M = 209\text{g/mol}$

Avogadro number, $N_A = 6.022 \times 10^{23}$

$V = a^3 = (3.36 \times 10^{-8})^3\text{cm}^3$ [$\because Z = 1$]

We know that,

$$d = \frac{ZM}{N_A V} = \frac{1 \times 209}{6.022 \times 10^{23} \times (3.36 \times 10^{-8})^3}$$
$$= 9.15\text{g/cm}^3$$

Question 18

The vapour pressure of 100g of water reduces from 17.53mm to 17.22mm when 17.10g of substance X is dissolved in it. Substance X can be

Options:

- A. methanol
- B. glucose

C. carbon dioxide

D. cannot predict

Answer: B

Solution:

Solution:

Given $p^\circ = 17.53$, $p_s = 17.22$ and $W = 17.10$

$$\frac{p^\circ - p_s}{p^\circ} = \frac{n}{n + N} = \frac{\frac{w}{m}}{\frac{w}{m} + \frac{W}{M}}$$

$$\therefore \frac{w}{m} < \frac{W}{M}$$

$$\therefore \frac{p^\circ - p_s}{p^\circ} = \frac{w / m}{W / M} = \frac{w}{m} \times \frac{M}{W}$$

$$\frac{17.53 - 17.22}{17.53} = \frac{17.10}{m} \times \frac{18}{100}$$

$$\Rightarrow m = \frac{17.10 \times 18 \times 17.53}{0.31 \times 100} = 174.05$$

174 is nearest to the molecular weight of glucose ($C_6H_{12}O_6$), thus the substance X can be glucose.

Question 19

The process by which sites of adsorbent are made free so, that more adsorbent can occupy them is called

Options:

A. sorption

B. desorption

C. unbalanced distribution

D. dissociation

Answer: B

Solution:

Solution:

Desorption is the process of removing an adsorbed substance from the surface of adsorbent, so that more adsorbate can occupy surface of adsorbent.

Question 20

Starch is an example of which of the following type of colloid?

Options:

- A. Macromolecular colloid
- B. Associated colloid
- C. Molecular colloid
- D. Electrolytic solution

Answer: A**Solution:****Solution:**

Starch is an example of macromolecular colloid. In this type of colloid, the particles are themselves large molecules which on dissolution form size in the colloidal range.

Question 21

Hybridisation of $[\text{Ni}(\text{CN})_4]^{2-}$ is

Options:

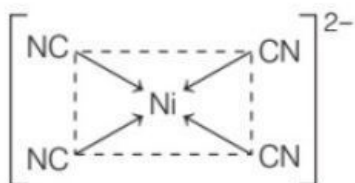
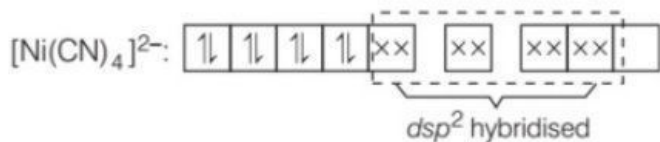
- A. $d sp^2$
- B. $sp^3 d^2$
- C. $d^2 sp^3$
- D. sp^3

Answer: A**Solution:****Solution:**

Hybridisation of $[\text{Ni}(\text{CN})_4]^{2-}$ is $d sp^2$.

$\text{Ni} = 1s^2, 2s^2, 2p^6, 3s^2 3p^6 3d^8 4s^2$

$\text{Ni}^{2+} = 1s^2, 2s^2, 2p^6, 3s^2 3p^6 3d^8, 4s^0 4p^0$



Question 22

The IUPAC name of the coordination compound $[\text{Co}(\text{H}_2\text{O})_2(\text{NH}_3)_4]\text{Cl}_3$ is

Options:

- A. tetraamminediaquacobalt (III) chloride
- B. cobalt (III) tetraamminediaquachloride
- C. diaquatetraammine cobalt (III) chloride
- D. tetraammine diaquacobalt (II) chloride

Answer: A

Solution:

Solution:

The IUPAC name of the coordination compound $[\text{Co}(\text{H}_2\text{O})_2(\text{NH}_3)_4]\text{Cl}_3$ is tetraamminediaquacobalt (III) chloride.

Question 23

The 3d -block elements that exhibits maximum number of oxidation states is

Options:

- A. Sc
- B. Ti
- C. Mn
- D. Zn

Answer: C

Solution:

Solution:

Mn exhibits the maximum number of oxidation states. $\text{Mn}(Z = 25) = [\text{Ar}]3d^5 4s^2$. It shows +2, +3, +4, +5, +6 and +7 oxidation states.

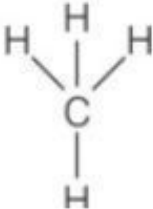
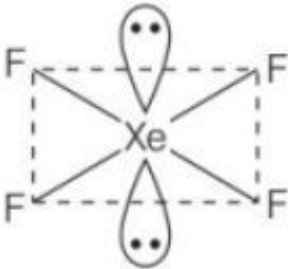
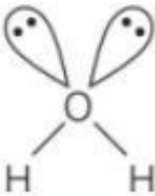
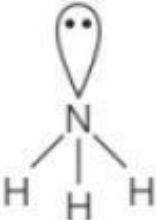
Question 24

Which one of the following does not have sp^3 hybridisation?

Options:

- A. CH_4
- B. XeF_4
- C. H_2O
- D. NH_3

Answer: B**Solution:****Solution:**

Molecular formula	Geometry	Bond and hybridisation
CH_4		4 σ bond sp^3 hybridisation
XeF_4		4 σ bond + 2 lone pairs sp^3d^2 hybridisation
H_2O		2 σ bond + 2 lone pairs sp^3 hybridisation
NH_3		3 σ bond + 1 lone pair sp^3 hybridisation

Question 25

Identify the molecule having dipole moment.

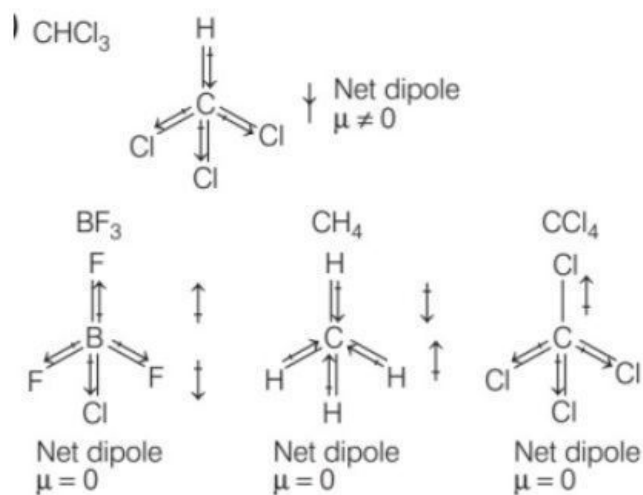
Options:

- A. BF_3
- B. CH_4
- C. CHCl_3
- D. CCl_4

Answer: C

Solution:

Solution:



Among the given compound CHCl_3 is having dipole moment.

Question 26

The major product obtained in the following reaction is
Chlorobenzene + Chlorine $\xrightarrow[\text{FeCl}_3]{\text{Anhydrous}}$ **product (Major)**

Options:

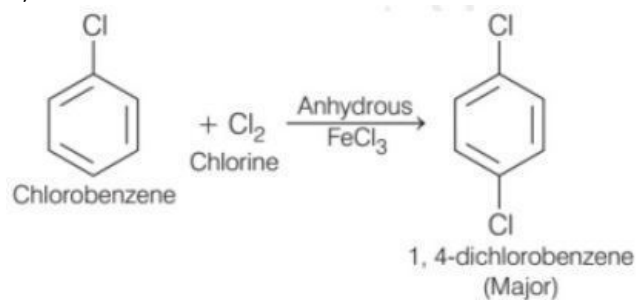
- A. 1,2 -dichlorobenzene
- B. 1, 3, 5-trichlorobenzene
- C. 1 , 4-dichlorobenzene
- D. 1 , 3-dichlorobenzene

Answer: C

Solution:

Solution:

1,4 -dichlorobenzene



Question 27

Which among the following compounds is not a colourless gas?

Options:

- A. ClF_3
- B. ClF
- C. IF_7
- D. IF_3

Answer: D

Solution:

Solution:

IF_3 is a yellow powder whereas ClF_3 , ClF and IF_7 is a colourless gas.

Question 28

${}_6^{14}\text{C}$ and ${}_8^{16}\text{O}$ are the examples of

Options:

- A. isotopes
- B. isobars
- C. isoelectronic species
- D. isotones

Answer: D

Solution:

Solution:

${}^6_{14}\text{C}$ and ${}^8_{16}\text{O}$ have same number of neutrons, i.e. 8. Therefore, they are the examples of isotones.

Question 29

Which of the following series of transition of hydrogen spectrum falls in visible region?

Options:

- A. Lyman series
- B. Balmer series
- C. Paschen series
- D. Brackett series

Answer: B

Solution:

Solution:

Balmer series of transitions in the spectrum of hydrogen atom fall in visible region. Lyman series falls in ultraviolet region, while Paschen and Brackett series fall in infrared region.

Question 30

Among the following order, which one is correct order of reactivity of group 16 elements?

Options:

- A. $\text{O} > \text{S} > \text{Se} > \text{Te} > \text{Po}$
- B. $\text{O} > \text{Se} > \text{Te} > \text{S} > \text{Po}$
- C. $\text{Po} > \text{Te} > \text{Se} > \text{S} > \text{O}$
- D. $\text{S} > \text{O} > \text{Se} > \text{Te} > \text{Po}$

Answer: A

Solution:

Solution:

Oxygen is the most reactive among the other elements of group 16. As we move down the group, the reactivity decreases because there is an increase in the atomic radius down the group and hence, the effective nuclear charge decreases which leads to decrease in chemical reactivity. So, the reactivity order will be $O > S > Se > Te > Po$.

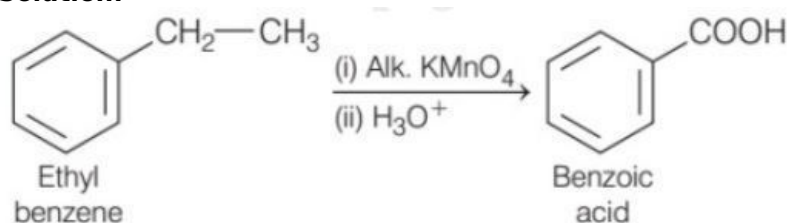
Question 31

Identify the product formed in following reaction.

**Options:**

- A. $\text{C}_6\text{H}_5\text{OH}$
- B. $\text{C}_6\text{H}_5\text{COOH}$
- C. $\text{C}_6\text{H}_5\text{CH}_2\text{COOH}$
- D. $\text{C}_6\text{H}_5\text{CH}_2\text{CH}_2\text{COOH}$

Answer: B

Solution:**Solution:**

Question 32

Identify electron withdrawing group from following when attached with π -bond.

Options:

- A. - COOH
- B. - OR
- C. - Cl
- D. - OH

Answer: A

Solution:

Solution:

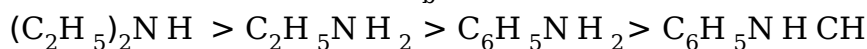
An electron withdrawing group (EWG) is a group that reduces electron density in a molecule through the carbon atom it is bonded to. Here -COOH is one such example of electron withdrawing group.

Question 33

Which of the following is incorrectly matched.

Options:

A. Decreasing order of pK_b values :



B. Decreasing order of basic strength :



C. Increasing order of basic strength : p-nitroaniline < aniline < p-toluidine

D. Increasing order of solubility in water : $C_6H_5NH_2 < (C_2H_5)_2NH < C_2H_5NH_2$

Answer: A

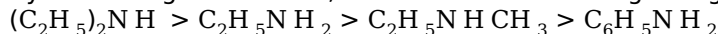
Solution:

Solution:

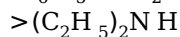
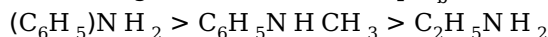
$C_6H_5NH_2$ and $C_6H_5NHCH_3$ are less basic than $C_2H_5NH_2$ and $(C_2H_5)_2NH$ it is due to the delocalisation of lone pair of electrons of N-atom over the benzene ring.

Next, $C_6H_5NHCH_3$ is little more basic than $C_6H_5NH_2$ it is due to the +effect of the CH_3 group. Among $C_2H_5NH_2$ and $(C_2H_5)_2NH$, $(C_2H_5)_2NH$ is more basic than $C_2H_5NH_2$ due to greater +I-effect of two C_2H_5 group.

By combining above facts, the relative basic strength of given amines decreases as :

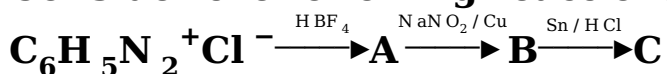


As a stronger base has a lower pK_b value, therefore, pK_b values decreases in the reverse order.



Question 34

Consider the following reactions.



Identify the compound formed C in the above reaction.

Options:

A. Nitrobenzene

B. Aniline

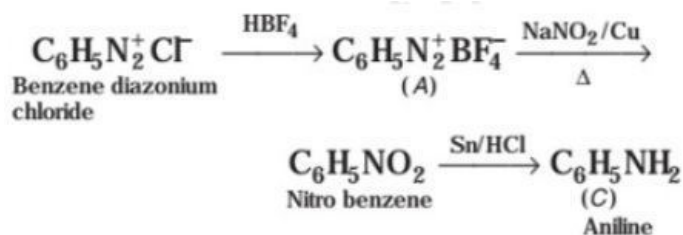
C. Chloro benzene

D. Phenol

Answer: B

Solution:

Solution:

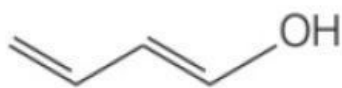


Question 35

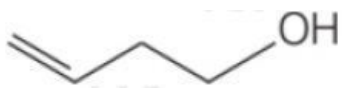
The bond line structure of crotonyl alcohol is

Options:

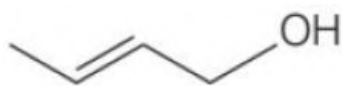
A.



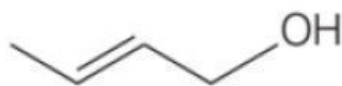
B.



C.



D.

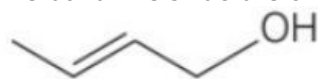


Answer: C

Solution:

Solution:

The bond line structure of crotonyl alcohol is



Question 36

Which among the following compounds has highest melting point?

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Options:

- A. Phenol
- B. p-cresol
- C. p-nitrophenol
- D. O-nitrophenol

Answer: C

Solution:

Solution:

p-nitrophenol has highest melting point due to the presence of intermolecular hydrogen bonding, which is not present in all other given compounds.

Question 37

Which of the following alkenes on oxidation by KMnO_4 in dil. H_2SO_4 forms adipic acid?

©

Options:

- A. Hex-2-ene
- B. Hex-1-ene
- C. Cyclohexene
- D. Hex-3-ene

Answer: C

Solution:

Solution:

Cyclohexene on oxidation with KMnO_4 in presence of dil. H_2SO_4 will form adipic acid, in which oxidative cleavage takes place. The double bond is broken to which oxygen atoms are going to be added forming a carboxylic acid group at each. Thus, the cyclic structure is broken forming adipic acid.

Question 38

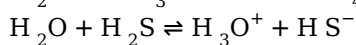
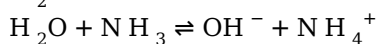
Which of the following species is amphoteric in nature?

Options:

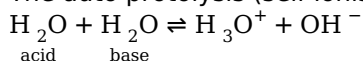
- A. HCl
- B. H₂O
- C. CH₃COOH
- D. NaOH

Answer: B**Solution:****Solution:**

Water is amphoteric in nature i.e. it can act as an acid as well as a base. It acts as an acid with NH₃ and a base with H₂S.



The auto protolysis (self ionisation) of water takes place as follows



Question 39

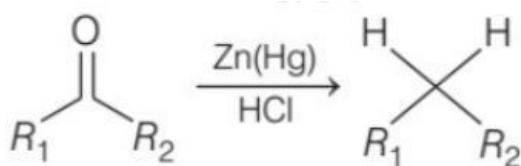
Identify the reaction in which carbonyl group of aldehydes and ketones is reduced to methylene group on treatment with zinc-amalgam and conc. HCl.

Options:

- A. Wolf-Kishner reduction
- B. Clemmensen reduction
- C. Stephen reduction
- D. Etard reaction

Answer: B**Solution:****Solution:**

The reduction of carbonyl groups (in aldehydes and ketones) to methylene groups with zinc amalgam and hydrochloric acid is known as the Clemmensen reduction reaction.



Question 40

Which of the following is an example of green chemistry?

Options:

- A. Recycled carpet
- B. A product made on Earth's day
- C. A sublimation reaction
- D. Bio-plastics or bio-diesel

Answer: D

Solution:

Solution:

Bio-plastics or bio-diesel is a product of green chemistry so, other options (a, b, c) are incorrect.

Question 41

Which of the following alkyl halide is treated with sodium metal to obtain 2, 2, 3, 3-tetramethyl butane?

Options:

- A. sec-butyl bromide
- B. n-propyl bromide
- C. n-butyl bromide
- D. tert-butyl bromide

Answer: D

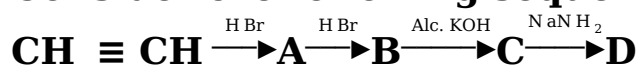
Solution:

Solution:

In the Wurtz reaction, alkyl halides react with sodium in dry ether to give hydrocarbon contain double the number of carbon atoms present in the halide.

Question 42

Consider the following sequence of reactions

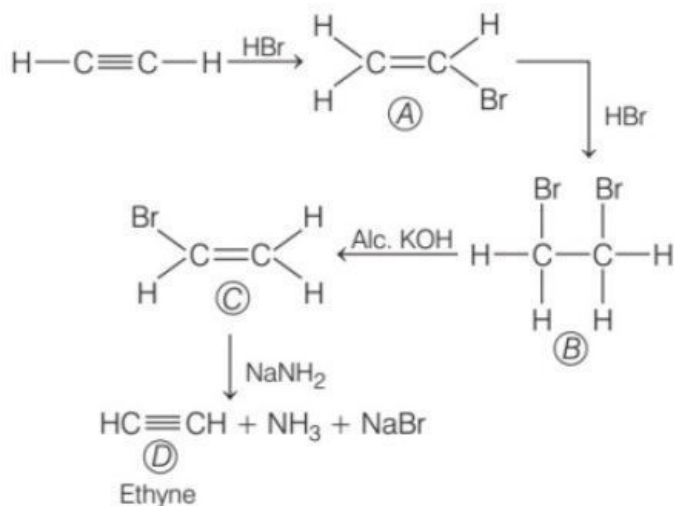


Identify D.

Options:

- A. Ethanol
- B. Ethane
- C. Ethyne
- D. Ethanal

Answer: C

Solution:**Solution:**

Question 43

Identify the polymer used in making floor tiles.

Options:

- A. PVC
- B. LDPE
- C. HDPE
- D. PETE

Answer: A

Solution:**Solution:**

Polyvinyl chloride or PVC is used in making floor tiles. Flexible PVC is made flexible and strong to prepare floor tiles. So, the correct option is (a).

Question 44

Which of the following is a thermosetting polymer?

Options:

- A. Polythene
- B. Bakelite
- C. Polystyrene
- D. Polyvinyls

Answer: B

Solution:

Solution:

Bakelite is thermosetting polymer as it contains cross links, or heavily branched polymer chains. It gets hardened on heating and cannot be softened again.

Question 45

A sample of air turns lime water milky and also turns acidified potassium dichromate green in aqueous solution has low pH . This is due to the presence of pollutants.

Options:

- A. CO_2
- B. SO_2
- C. Both (a) and (b)
- D. None of these

Answer: B

Solution:

Solution:

SO_2 turns lime water milky.

SO_2 in aqueous solution gives H_2SO_3 .

$\text{H}_2\text{SO}_3 \rightleftharpoons \text{H}^+ + \text{HSO}_3^-$

CO_2 also turns lime water milky.

CO_2 in aqueous solution gives H_2CO_3 .

$\text{H}_2\text{CO}_3 \rightleftharpoons \text{H}^+ + \text{HCO}_3^-$

H_2SO_3 is stronger acid than H_2CO_3 . Thus, ionisation of H_2SO_3 is larger than H_2CO_3 . Thus, pH of H_2SO_3 is lower than that of H_2CO_3 .

Question 46

Solubility product of AgBr is 4.9×10^{-13} . What is its solubility?

Options:

A. $2.4 \times 10^{-7} \text{ mol d m}^{-3}$

B. $7.0 \times 10^{-7} \text{ mol d m}^{-3}$

C. $4.9 \times 10^{-7} \text{ mol d m}^{-3}$

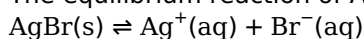
D. $3.2 \times 10^{-7} \text{ mol d m}^{-3}$

Answer: C

Solution:

Solution:

The equilibrium reaction of AgBr is



$$\begin{aligned}\text{Molar solubility (S) of AgBr} &= \sqrt{4.9 \times 10^{-13}} \\ &= 7.0 \times 10^{-7} \text{ mol d m}^{-3}\end{aligned}$$

Question 47

Consider the following reaction



Here, 'A' and 'B' are respectively

Options:

A. glucose oxime and saccharic acid

B. gluconic acid and saccharic acid

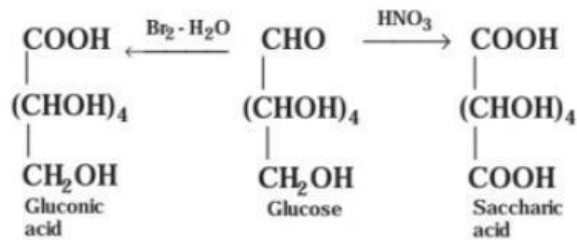
C. saccharic acid and sorbitol

D. gluconic acid and sorbitol

Answer: B

Solution:

Solution:



Question 48

Which of the following has maximum solubility?

Options:

- A. BeF_2
- B. MgF_2
- C. CaF_2
- D. SrF_2

Answer: A

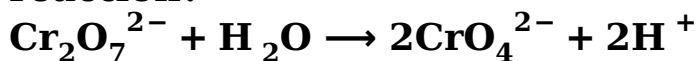
Solution:

Solution:

The solubility of fluorides of alkaline earth metals decreases down the group due to increase in hydration enthalpy. Thus, the order of solubility of the given compounds is $\text{BeF}_2 > \text{MgF}_2 > \text{SrF}_2$. Hence, BeF_2 has maximum solubility.

Question 49

Which of the following explanation is correct about the given below reaction?



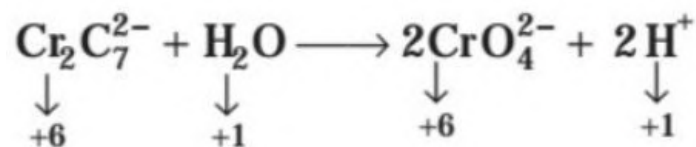
Options:

- A. Cr is reduced
- B. Cr is oxidised
- C. Oxidation number of Cr has neither decreased nor increased
- D. Hydrogen is reduced

Answer: C

Solution:

Solution:



The oxidation number of Cr in $\text{Cr}_2\text{O}_7^{2-}$ and CrO_4^{2-} is same, i.e. 6. Thus, oxidation of Cr has neither decreased nor increased.

Question 50

Aldehydes are readily oxidised to yield carboxylic acids but ketones are inert to oxidation. Which is the most likely explanation regarding this difference in reactivity?

Options:

- A. Aldehydes have a proton attached to the carbonyl that is abstracted during oxidation. Ketones lack this proton and so, cannot be oxidised.
- B. Reducing agents like HNO_3 are sterically hindered by ketone's carbonyl carbon.
- C. Aldehydes and ketones are of similar hybridisation.
- D. The rate of the forward oxidation reaction is equal to the rate of the reverse reduction reaction in ketones.

Answer: A

Solution:

Solution:

Abstraction of H-atom, attached to carbonyl group in an aldehyde, is much easier than the abstraction from bulkier alkyl group attached to carbonyl group in a ketone.

Question 51

If an alkane contains n number of carbon atoms, the number of oxygen molecules required for combustion of alkane are

Options:

- A. $\frac{3n+1}{2}$
- B. $\frac{2n+1}{2}$

C. $2n + 1$

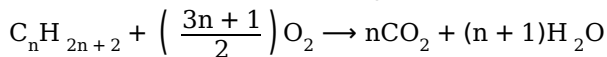
D. n

Answer: A

Solution:

Solution:

The chemical reaction of complete combustion of alkane is as follows :



where n = number of carbon atoms.

Question 52

Which among the following is branched polymer?

Options:

A. Bakelite

B. HDP

C. PVC

D. LDP

Answer: D

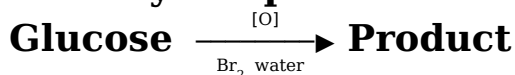
Solution:

Solution:

LDP i.e. low density polythene has highly branched structure. It is mainly used for manufacturing containers, wash bottles, plastic bags etc.

Question 53

Identify the product obtained in the following conversion.



Options:

A. n-hexane

B. Gluconic acid

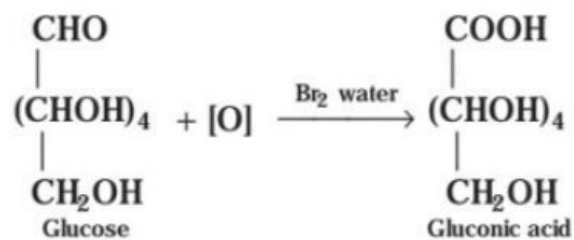
C. Saccharic acid

D. Glucose cyanohydrine

Answer: B

Solution:

Solution:



Question 54

Arrange the following free radicals in order of decreasing stability. Methyl (I), Vinyl (II), Allyl (III), Benzyl (IV) Codes

Options:

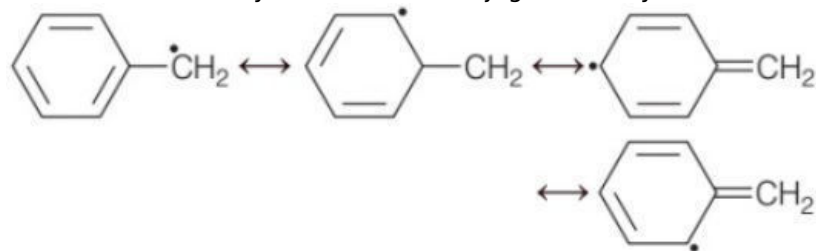
- A. I > II > III > IV
- B. III > II > I > IV
- C. II > I > IV > III
- D. IV > III > I > II

Answer: D

Solution:

Solution:

Among the alkyl free radicals, tertiary alkyl free radicals are most stable and methyl free radical is least stable. Benzyl and allyl free radicals are resonance stabilised and are more stable than alkyl free radicals. Benzyl free radical is more stable than allyl due to more conjugation. Vinyl radical is least stable as the odd electron is present on sp^3C -atom.



\therefore Benzyl > Allyl > Methyl > Vinyl

Question 55

The decreasing order of the rate of nitration of benzene (I), C_6D_6 (II), nitrobenzene (III), chlorobenzene (IV) is

Options:

- A. I > II > III > IV
 B. I > I > IV > II
 C. I = I > IV > III
 D. I = II > III > IV

Answer: C

Solution:

Solution:

Rate of nitration is faster when substituent activates the ring (+I -effect or +R-effect, ortho/para directing) and rate is slower when substituent deactivates the ring (-I -effect or -R-effect, meta directing group).

- Halogen deactivates ring but orientation is ortho para directing group.

- Rate of $C_6H_6 = C_6D_6$, as no effect is observed when H is replaced by D. Hence, order is

I = II > IV > III

Question 56

The chemical test that distinguish between benzoic acid and phenol is

Options:

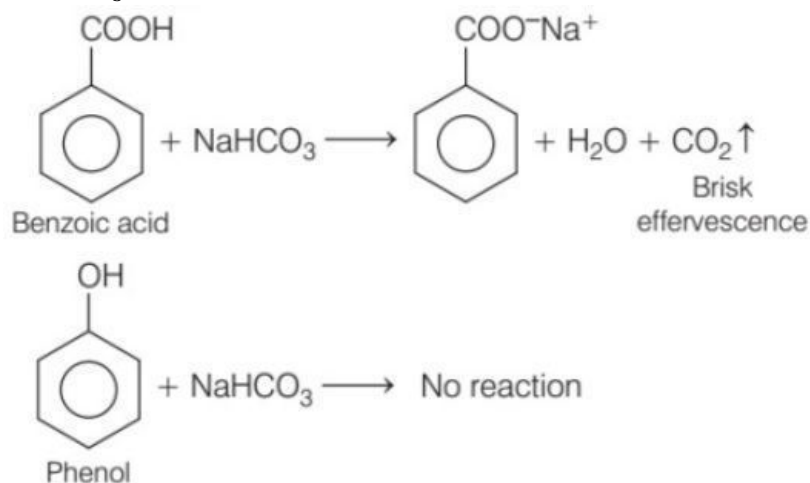
- A. NaHCO₃ test
 B. Tollen's reagent
 C. Biuret test
 D. None of these

Answer: A

Solution:

Solution:

NaHCO₃ test is used to distinguish benzoic acid and phenol.



Question 57

The order of reactivity of hydrogen halides with ether is as follows

©

Options:

- A. $\text{H Br} > \text{H I} > \text{H Cl}$
- B. $\text{H Cl} > \text{H Br} > \text{H I}$
- C. $\text{H I} > \text{H Br} > \text{H Cl}$
- D. $\text{H Cl} > \text{H I} > \text{H Br}$

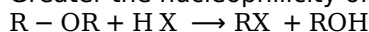
Answer: C

Solution:

Solution:

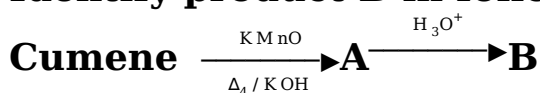
The correct order of hydrogen halides is $\text{HI} > \text{H Br} > \text{H Cl}$. The cleavage of ethers usually takes place with conc. HI or HBr at high temperature.

Greater the nucleophilicity of halide ion, more is the reactivity of hydrogen halide.



Question 58

Identify product B in following reaction.



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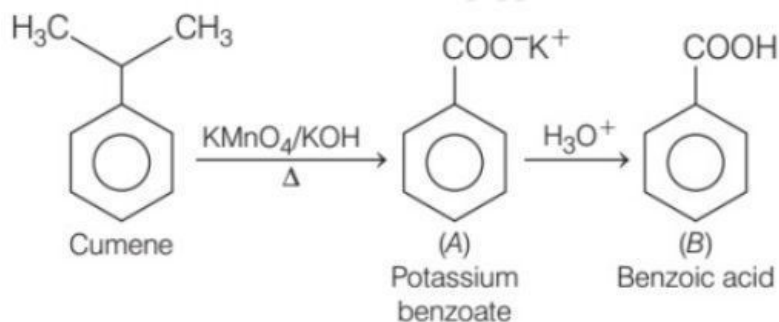
Options:

- A. Phenol
- B. Benzophenone
- C. Benzaldehyde
- D. Benzoic acid

Answer: D

Solution:

Solution:



Question 59

The most effective pair of reagents for the preparation of tert-butyl ether is

Options:

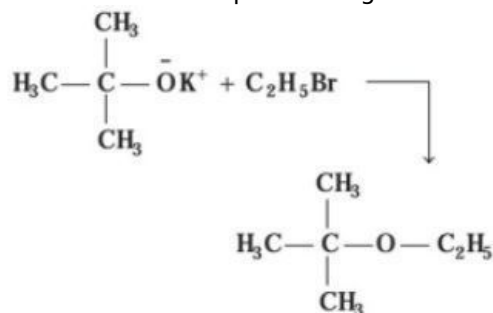
- A. tert-butyl alcohol and ethyl bromide
- B. sodium ethoxide and tert-butyl bromide
- C. potassium tert-butoxide and ethanol
- D. potassium tert-butoxide and ethyl bromide

Answer: D

Solution:

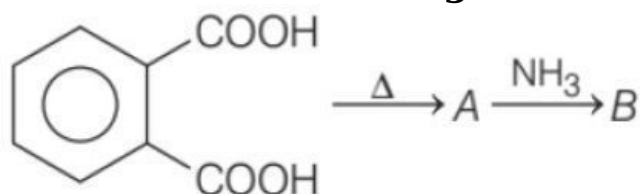
Solution:

The most effective pair of reagents for the preparation of tert-butyl ether is potassium tert-butoxide and ethyl bromide.

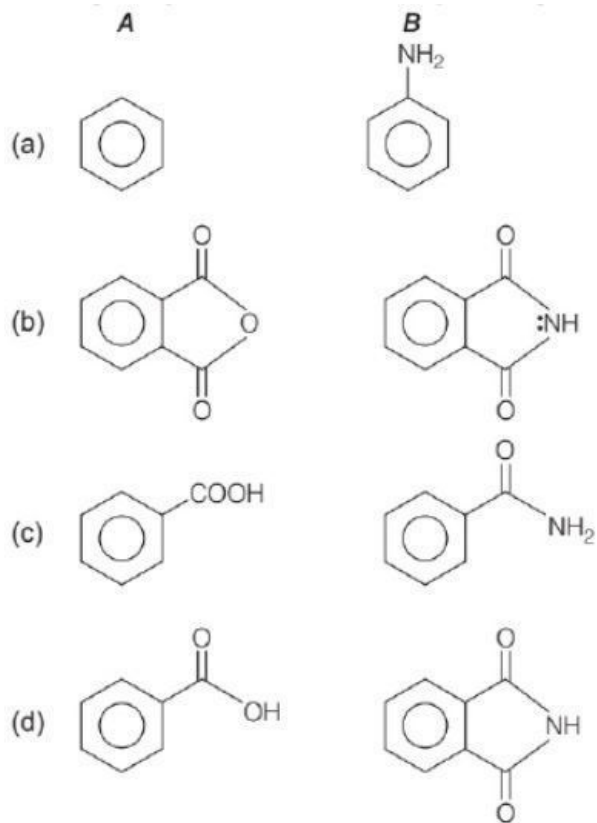


Question 60

Consider the following reaction



Identify the products A and B respectively.



Options:

A. (a)

B. (b)

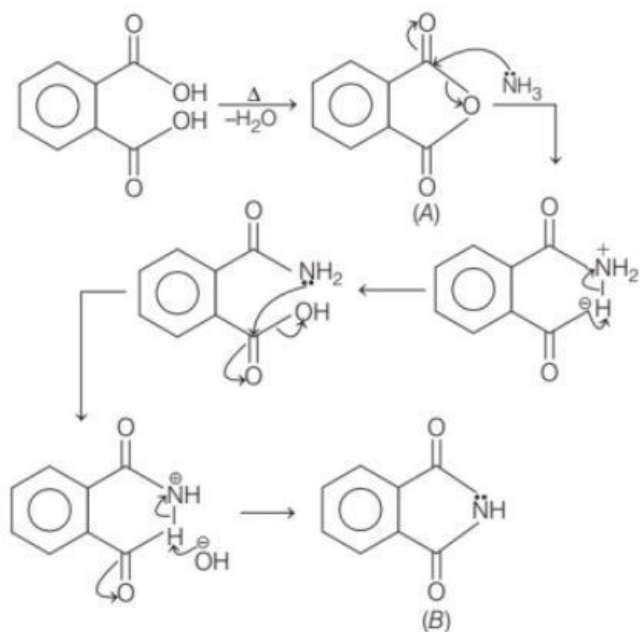
C. (c)

D. (d)

Answer: B

Solution:

Solution:



Question 61

The major product obtained in the following reaction is



Options:

- A. 4-iodo-3-methylhexane
- B. 4-iodo-4-methylhexane
- C. 3-iodo-3-methylhexane
- D. 3-iodo-4-methylhexane

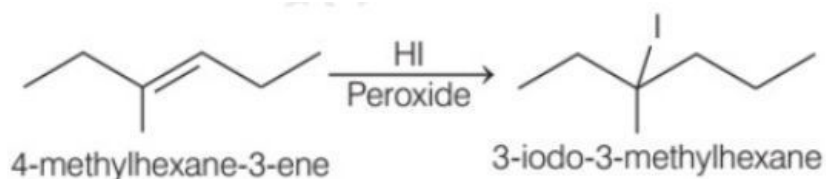
Answer: C

Solution:

Solution:

3-iodo-3-methylhexane

Here, a molecule HI is added to C = C double bond. The addition follows anti-Markownikoff's rule.



Question 62

Which of the following is likely to undergo racemisation during alkaline hydrolysis?

Options:

- A. $(\text{CH}_3)_3\text{C} - \text{CH}_2 - \text{Cl}$
- B. $\text{H}_3\text{C} - \underset{\text{Cl}}{\text{CH}} - \text{CH}_3$
- C. $\text{H}_3\text{C} - \text{CH}_2 - \underset{\text{Cl}}{\text{CH}} - \text{CH}_3$
- D. $\text{H}_3\text{C} - \text{CH}_2 - \underset{\text{Cl}}{\text{CH}} - \text{CH}_2 - \text{CH}_3$

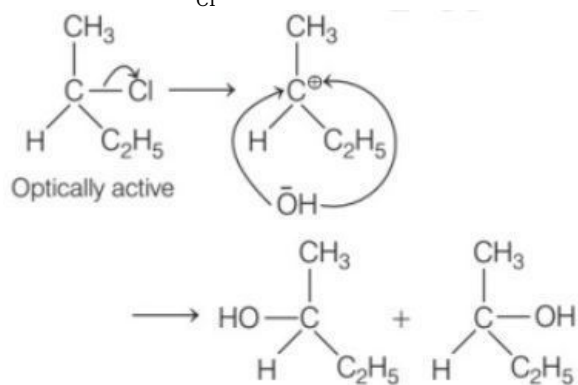
Answer: C

Solution:

Solution:

Among the given compounds,

$\text{H}_3\text{C}-\text{CH}_2-\underset{\text{Cl}}{\text{CH}}-\text{CH}_3$ will undergo racemisation during alkaline hydrolysis. The reaction involved is as follows:



Question 63

The 'green' chemical used in household cleaners to remove stains and also a favourite dressing on salads is

Options:

- A. vinegar
- B. citric acid
- C. hydrochloric acid
- D. water

Answer: A

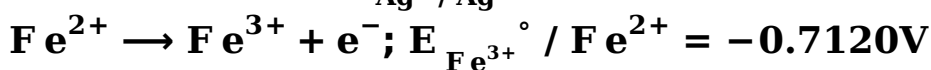
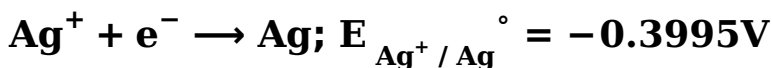
Solution:

Solution:

Vinegar (acetic acid) is used as household cleaner to remove stains and is also a favourite dressing on salads.

Question 64

The two half-cell reactions of an electrochemical cell is given as



The value of EMF will be

Options:

- A. -0.3125V
- B. 0.3125V

C. 1.114V

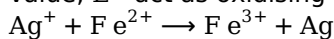
D. -1.114V

Answer: B

Solution:

Solution:

Species with more negative E° (standard reduction potential) generally acts as reducing agent, while with less negative value, E° act as oxidising agent. Thus, the overall reaction is



The value of EMF will be,

$$\begin{aligned}\Delta E^\circ &= E_{\text{oxidation}}^\circ - E_{\text{reduction}}^\circ \\ &= -0.3995 - (-0.7120) \\ &= +0.3125\text{V}\end{aligned}$$

Question 65

Ethers when dissolved in cold concentrated sulphuric acid forms

Options:

A. oxonium salts

B. alkanoic acids

C. alkanols

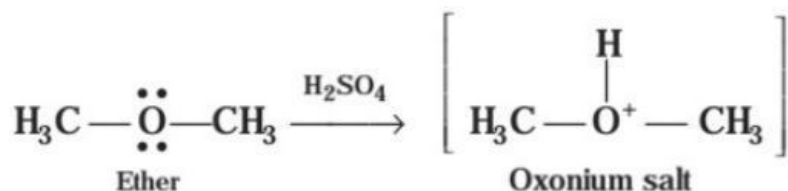
D. alkyl hydrogen sulphate

Answer: A

Solution:

Solution:

Ethers react with cold concentrated sulphuric acid to form oxonium salts. In ethers due to the presence of lone pairs of electrons on oxygen atoms, it behaves as a Lewis base and reacts with a strong acid to form protonated oxonium salt.



Question 66

The value of Rydberg constant in joule is

Options:

A. 0.082

B. 2.18×10^{-18}

C. 2.0×10^{-19}

D. 8.314

Answer: B

Solution:

Solution:

The value of Rydberg constant is $2.18 \times 10^{-18} \text{ J}$.

Question 67

What is the number of lone pair of electrons in IF_7 ?

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Options:

A. 1

B. 2

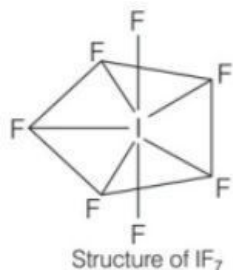
C. 3

D. 0

Answer: D

Solution:

Solution:



Number of lone pair of electron in IF_7 is 0 .

Question 68

The $\text{H} - \text{N} - \text{H}$ bond angle in ammonia molecule is

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Options:

A. 107°

- B. 111°
- C. 104°
- D. 109°

Answer: A

Solution:

Solution:

The $\text{H} - \text{N} - \text{H}$ bond angle in ammonia molecule is 107° . Since, the repulsion between lone pair- lone pair and lone pair-bond pair is more than that between bond pair-bond pair. The repulsion in H_2O is much greater than that in NH_3 which results in contraction of bond angle from $109^\circ 28'$ to 104.5° in water while in NH_3 contraction i.e. less is from $109^\circ 28'$ to 107° .

Question 69

Identify compound having square pyramidal shape from following

Options:

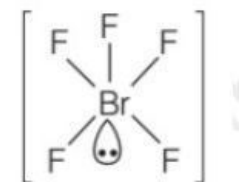
- A. ClF_5
- B. BrF_5
- C. ICl
- D. BrF_3

Answer: B

Solution:

Solution:

According to VSEPR theory, the shape of BrF_5 is square pyramidal and its electron geometry is octahedral because bromine being the central atom has five bonds connected with surrounding fluorine atoms. Each $\text{F} - \text{Br} - \text{F}$ bond making an angle of 90° in the same plane.



Square pyramidal structure of BrF_5

Question 70

The IUPAC name of complex ion $[\text{Cr}(\text{NH}_3)_4\text{Cl}_2]^+$ is

Options:

- A. tetraaminedichloridochromium (III) ion
- B. dichloridotetraaminechromium (III) ion
- C. dichloridotetraaminechromate (II) ion
- D. tetraaminedichloridochromate (III) ion

Answer: A**Solution:****Solution:**

The IUPAC name of the complex ion $[\text{Cr}(\text{NH}_3)_4\text{Cl}_2]^+$ is tetraaminedichloridochromium (III) ion.

Question 71

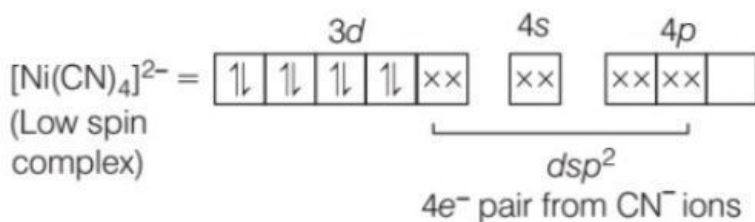
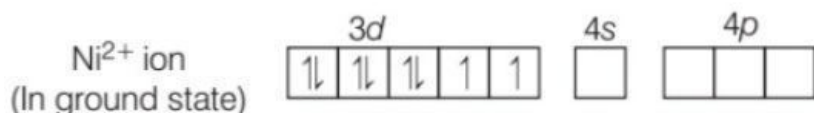
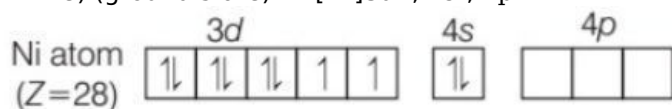
Which of the following statement is incorrect?**Options:**

- A. $[\text{NiCl}_4]^{2-}$ is paramagnetic.
- B. $[\text{Ni}(\text{CO})_4]$ is diamagnetic.
- C. $[\text{Cr}(\text{NH}_3)_6]^{3+}$ is paramagnetic.
- D. $[\text{Ni}(\text{CN})_4]^{2-}$ is paramagnetic.

Answer: D**Solution:****Solution:**

$[\text{Ni}(\text{CN})_4]^{2-}$ is diamagnetic not paramagnetic.

Ni atom ($Z = 28$) (ground state) = $[\text{Ar}]3d^8, 4s^2, 4p^0$



CN^- is a strong field ligand, due to which pairing of electron takes place.

Question 72

Which of the following statement is incorrect?

Options:

- A. $[\text{NiCl}_4]^{2-}$ is paramagnetic.
- B. $[\text{Ni}(\text{CO})_4]$ is diamagnetic.
- C. $[\text{Cr}(\text{NH}_3)_6]^{3+}$ is paramagnetic.
- D. $[\text{Ni}(\text{CN})_4]^{2-}$ is paramagnetic.

Answer: D

Solution:

Solution:

In lanthanide series as atomic number increases, atomic radius decreases. Therefore, Pr having lowest atomic number among the given element is having largest atomic radius.

Question 73

Which element from following has largest atomic radius?

Options:

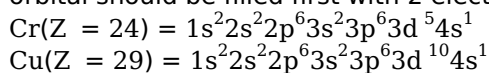
- A. Pm
- B. Eu
- C. Gd
- D. Pr

Answer: A

Solution:

Solution:

The 3d-orbitals of Cr and Cu have half-filled and completely filled electronic configuration respectively which are the most stable electronic configurations. Thus, these both elements do not follow Aufbau principle according to which 4s orbital should be filled first with 2 electrons as it have lower energy than 3d-orbital. So, their electronic configuration are



Question 74

Identify positively charged sol from following.

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Options:

- A. Sols of starch
- B. Congo red sol
- C. Methylene blue sol
- D. Gelatin sol

Answer: C

Solution:

Solution:

Sols of starch, congo red sol and gelatin sol are the examples of negatively charged sol, while methylene blue sol is positively charged sol.

Question 75

Which one of the following laws will represent the pairing of electrons in a subshell after each orbital is filled with one electron?

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Options:

- A. Pauli's exclusion principle
- B. Hund's rule
- C. Heisenberg's uncertainty principle
- D. Hess's law

Answer: B

Solution:

Solution:

Hund's rule represent the pairing of electrons in a subshell after each orbital is filled with one electron. Thus, this rule states that "during filling of electrons in a subshell, pairing of electrons cannot take place until there is no empty orbital available". This rule is also called Hund's rule of maximum multiplicity.

Question 76

What is the energy of an electron in stationary state corresponding to $n = 2$?

Options:

- A. $-3.45 \times 10^{-18}\text{J}$
 B. $-0.545 \times 10^{-18}\text{J}$
 C. $0.545 \times 10^{-18}\text{J}$
 D. $3.45 \times 10^{-18}\text{J}$

Answer: B**Solution:****Solution:**

Energy of an electron is given by

$$E_n = \frac{-13.6}{n^2} \text{eV}$$

For $n = 2$

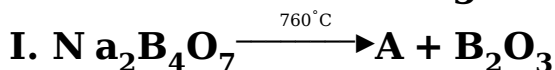
$$E = \frac{-13.6}{2 \times 2} \text{eV} = \frac{-13.6}{4} \text{eV}$$

$$[1 \text{eV} = 1.602 \times 10^{-19} \text{J}]$$

$$= -3.4 \times 1.602 \times 10^{-19} \text{J}$$

$$= -5.45 \times 10^{-19} \text{J}$$

$$\text{or } = -0.545 \times 10^{-18} \text{J}.$$

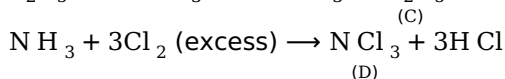
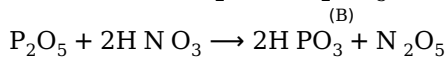
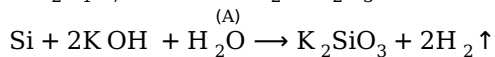
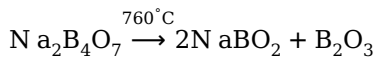
Question 77**Consider the following reactions.****Identify the missing products (A, B, C and D) of the given reactions.**

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Options:

- A. (A) $\rightarrow \text{N a}_2\text{O}$ (B) $\rightarrow \text{K SiO}_3$ (C) $\rightarrow \text{N}_2\text{O}_4$ (D) $\rightarrow \text{N Cl}_3$
 B. (A) $\rightarrow \text{N aBO}_2$ (B) $\rightarrow \text{K}_2\text{SiO}_3$ (C) $\rightarrow \text{N}_2\text{O}_5$ (D) $\rightarrow \text{N Cl}_3$
 C. (A) $\rightarrow \text{N aBO}_2$ (B) $\rightarrow \text{K}_2\text{SiO}_3$ (C) $\rightarrow \text{N}_2\text{O}_4$ (D) $\rightarrow \text{N}^\oplus\text{Cl}_4$
 D. (A) $\rightarrow \text{N a}_2\text{O}$ (B) $\rightarrow \text{K}_2\text{SiO}_3$ (C) $\rightarrow \text{N}_2\text{O}_5$ (D) $\rightarrow \text{N Cl}_3$

Answer: B**Solution:****Solution:**



Question 78

A buffer solution is prepared in which the concentration of NH_3 is **0.30M** and the concentration of NH_4^+ is **0.20M**. If the equilibrium constant, K_b for NH_3 equals 1.8×10^{-5} , what is the pH of the solution?

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Options:

- A. 8.73
- B. 9.08
- C. 9.44
- D. 11.72

Answer: C

Solution:

Solution:

Given, $K_b = 1.8 \times 10^{-5}$

$\text{p}K_b = -\log K_b$

$= -\log 1.8 \times 10^{-5} = 4.74$

$\text{pOH} = \text{p}K_b + \log \frac{[\text{salt}]}{[\text{base}]}$

$= 4.74 + \log \frac{0.20}{0.30}$

$= 4.74 - 0.176 = 4.56$

$\text{pH} + \text{pOH} = 14$

$\text{pH} = 14 - 4.56 = 9.44$

Question 79

A synthetic polymer which is an ester is

Options:

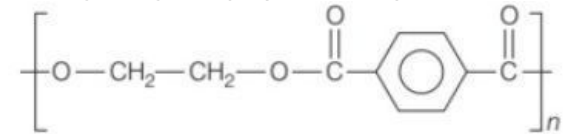
- A. dacron
- B. teflon
- C. PMMA
- D. PVC

Answer: A

Solution:

Solution:

Among the given polymers, only dacron is the ester. It's structure is as follows :



Question 80

Identify the correct decreasing order of precipitation power of flocculating ion added, from following.

Options:

- A. $\text{Na}^+ > \text{Ba}^{2+} > \text{Al}^{3+}$
- B. $\text{Al}^{3+} > \text{Ba}^{2+} > \text{Na}^+$
- C. $\text{Al}^{3+} > \text{Na}^+ > \text{Ba}^{2+}$
- D. $\text{Ba}^{2+} > \text{Al}^{3+} > \text{Na}^+$

Answer: B

Solution:

Solution:

The flocculation power of cation decreases with decrease in the valence of cation. So, the correct order is $\text{Al}^{3+} > \text{Ba}^{2+} > \text{Na}^+$

Question 81

When electrons are trapped into the crystal in anion vacancy, the defect is known as

Options:

- A. Schottky defect
- B. Frenkel defect
- C. stoichiometric defect
- D. F-centres

Answer: D

Solution:

Solution:

Due to missing of a negative ion from its lattice site, leaving a hole which is occupied by an electron. The electron, thus trapped in anion vacancy is called F-centres.

Question 82

Gas equation, $pV = nRT$ is obeyed by a gas in

Options:

- A. adiabatic process
- B. isothermal process
- C. Both (a) and (b)
- D. None of these

Answer: C

Solution:

Solution:

Ideal gas equation,

$$pV = nRT$$

is obeyed by an ideal gas in both adiabatic and isothermal processes.

Question 83

What will be the formula of an oxide of iodine (atomic mass = 127) which contains 25.4g of iodine and 8g of oxygen?

Options:

- A. I_2O
- B. I_2O_3
- C. I_2O_5
- D. I_2O_7

Answer: C

Solution:

Solution:

$\therefore 25.4 \text{ g of } \text{I}_2 \text{ combines with } 8 \text{ g of oxygen.}$

$\therefore 254 \text{ g of } \text{I}_2 \text{ will combine with } 80 \text{ g of oxygen.}$

Element	Mass%	Atomic mass	Moles of element	Molar ratio	Simplest whole number ratio
I	25.4	127	$\frac{25.4}{127} = 0.2$	$\frac{0.2}{0.2} = 1$	2
O	8	16	$\frac{8}{16} = 0.5$	$\frac{0.5}{0.2} = 2.5$	5

\therefore Formula of oxide of iodine will be I_2O_5 .

Question 84

When 0.5g of sulphur is burnt to form SO_2 and 4.6kJ of heat liberated. Calculate enthalpy of formation of $\text{SO}_2(\text{g})$.

(Atomic mass: S = 32, O = 16)

Options:

A. $-294.4 \text{ kJ mol}^{-1}$

B. $-172.4 \text{ kJ mol}^{-1}$

C. $-81.2 \text{ kJ mol}^{-1}$

D. $-258.6 \text{ kJ mol}^{-1}$

Answer: A

Solution:**Solution:**

$\text{S} + \text{O}_2 \rightarrow \text{SO}_2; \Delta H_f = -4.6 \text{ kJ}$

0.5g of sulphur on burning produce 1g of SO_2 .

$\therefore 32 \text{ g of sulphur on burning will produce } 64 \text{ g of } \text{SO}_2$.

$\therefore \Delta H_f = (-4.6 \text{ kJ}) \times 64$

$= -294.4 \text{ kJ mol}^{-1}$

Question 85

What is the coordination number of atoms in bcc crystal lattice?

Options:

A. 2

B. 8

C. 6

D. 4

Answer: B

Solution:

Solution:

The body centered cubic lattice has a coordination number of 8 and contains 2 atoms per unit cell.

Question 86

Which of the following scientific notation of some figures is not correct?

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Options:

A. $2080 = 2.08 \times 10^3$

B. $0.0034 = 3.4 \times 10^3$

C. $5000 = 5.0 \times 10^3$

D. $8008 = 8.008 \times 10^3$

Answer: B

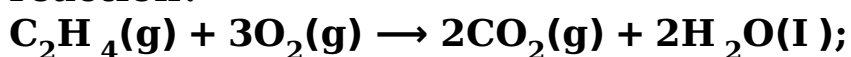
Solution:

Solution:

The correct scientific notation of 0.0034 will be 3.4×10^{-3} . Thus, $0.0034 = 3.4 \times 10^3$ is incorrect.

Question 87

What is the difference between ΔH and ΔU at 298K for the following reaction?



$$\Delta H = -1410.0\text{kJ}$$

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Options:

A. -14.8kJ

B. -2.45kJ

C. -4.95kJ

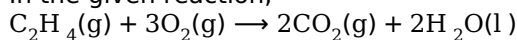
D. -7.30kJ

Answer: C

Solution:

Solution:

In the given reaction,



$$\Delta H = -1410.0\text{kJ}$$

For ideal gas, $\Delta H = \Delta U + RT(\Delta n_g)$

where, $R = 8.314\text{J / mol K}$

$$T = 298\text{K}$$

$\Delta n_g = \text{number of gaseous products} - \text{number of gaseous reactants}$

$$= 2 - 4 = -2$$

$$\therefore -1410 = \Delta U + [8.314 \times 298(-2)]$$

$$\Delta U = -1405.045\text{kJ}$$

$$\sim \text{eq} - 1405.05\text{kJ}$$

$$\text{Now, } \Delta H - \Delta U = -1410 - (-1405.05) = -4.95\text{kJ}$$

Question 88

What is the relation between $\frac{d[\text{H}_2]}{dt}$ and $\frac{d[\text{NH}_3]}{dt}$?

Options:

A. $\frac{1}{2} \frac{d[\text{NH}_3]}{dt} = \frac{d[\text{H}_2]}{dt}$

B. $\frac{d[\text{NH}_3]}{dt} = -\frac{2}{3} \frac{d[\text{H}_2]}{dt}$

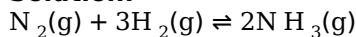
C. $\frac{2}{3} \frac{d[\text{NH}_3]}{dt} = \frac{d[\text{H}_2]}{dt}$

D. $\frac{d[\text{NH}_3]}{dt} = \frac{d[\text{H}_2]}{dt}$

Answer: B

Solution:

Solution:



Rate expression can be written as,

$$\frac{-d[\text{N}_2]}{dt} = \frac{-1}{3} \frac{d[\text{H}_2]}{dt} = \frac{1}{2} \frac{d[\text{NH}_3]}{dt}$$

$$-\frac{1}{3} \frac{d[\text{H}_2]}{dt} = \frac{1}{2} \frac{d[\text{NH}_3]}{dt}$$

$$\therefore \frac{d[\text{NH}_3]}{dt} = -\frac{2}{3} \frac{d[\text{H}_2]}{dt}$$

Question 89

The half-life of first order reaction is 1.7hr. What is its rate constant?

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Options:

- A. 0.245hr^{-1}
- B. 1.07hr^{-1}
- C. 0.407hr^{-1}
- D. 3.45hr^{-1}

Answer: C

Solution:

Solution:

For first order reaction.

$$k = \frac{0.693}{t_{1/2}} = \frac{0.693}{1.7}\text{hr}^{-1}$$

$$k = 0.407\text{hr}^{-1}$$

Question 90

Galvanic cell is a device in which

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Options:

- A. chemical energy is converted into electrical energy
- B. electrical energy is converted into chemical energy
- C. chemical energy is seen in the form of heat
- D. thermal energy from an outside source is used to derive the cell reaction

Answer: A

Solution:

Solution:

Galvanic cell is an electrochemical cell that converts the chemical energy of a spontaneous reaction into electrical energy.

Question 91

The solubility product (K_{sp}) of solid barium sulphate at 298K is 1.1×10^{-10} . The molar solubility, S of $[\text{Ba}^{2+}]$ and $[\text{SO}_4^{2-}]$ is

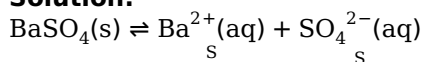
Options:

- A. $1.05 \times 10^{-7} \text{ mol L}^{-1}$
- B. $1.05 \times 10^{-10} \text{ mol L}^{-1}$
- C. $1.05 \times 10^{-6} \text{ mol L}^{-1}$
- D. $1.05 \times 10^{-5} \text{ mol L}^{-1}$

Answer: D

Solution:

Solution:



$$K_{\text{sp}} = S \times S = S^2$$

$$1.1 \times 10^{-10} = S^2$$

$$S = 1.05 \times 10^{-5} \text{ mol L}^{-1}$$

Question 92

If two molecules of A and B have mass 100kg and 64kg and rate of diffusion of A is 12×10^{-3} then, what will be rate of diffusion of B ?

Options:

- A. 15×10^{-3}
- B. 64×10^{-3}
- C. 5×10^{-3}
- D. 46×10^{-3}

Answer: A

Solution:

Solution:

According to Graham's law, $\frac{r_A}{r_B} = \sqrt{\frac{M_B}{M_A}}$

Here, $M_A = \left(\frac{100}{2} \right) \text{ kg/molecule}$,

$M_B = \left(\frac{64}{2} \right) \text{ kg/molecule}$

$r_A = 12 \times 10^{-3}$ and $r_B = ?$

$$\frac{12 \times 10^{-3}}{r_B} = \sqrt{\frac{64/2}{100/2}} = \sqrt{\frac{64}{100}} = \frac{8}{10}$$

$$r_B = \frac{12 \times 10^{-3} \times 10}{8} = 15 \times 10^{-3}$$

Question 93

Buna-N synthetic rubber is obtained by the copolymerisation of

Options:

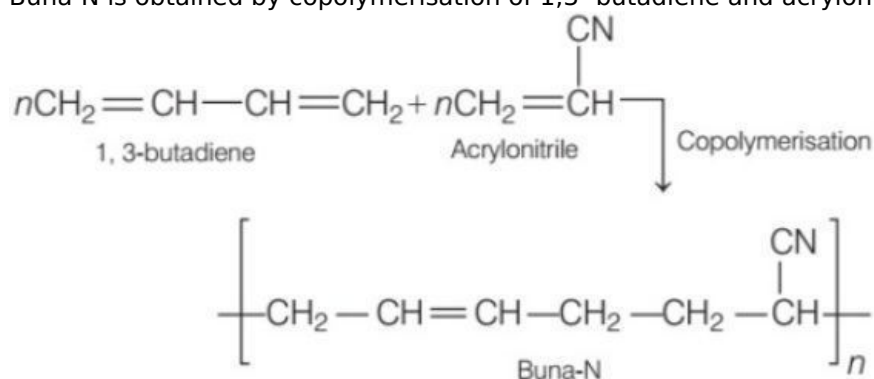
- A. $\text{CH}_2 = \text{CH} - \text{CH} = \text{CH}_2$ and $\text{H}_5\text{C}_6 - \text{CH} = \text{CH}_2$
- B. $\text{CH}_2 = \text{CH} - \text{CN}$ and $\text{H}_2\text{C} = \text{CH} - \text{CH} = \text{CH}_2$
- C. $\text{H}_2\text{C} = \text{CH} - \text{CN}$ and $\text{CH}_2 = \text{CH} - \text{C}(\text{CH}_3) = \text{CH}$
- D. $\text{H}_2\text{C} = \text{CH} - (\text{CCl}) = \text{CH}_2$ and $\text{H}_2\text{C} = \text{CH} - \text{CH} = \text{CH}_2$

Answer: B

Solution:

Solution:

Buna-N is obtained by copolymerisation of 1,3-butadiene and acrylonitrile in the presence of a peroxide catalyst.



Question 94

The oxidation number of oxygen in oxygen difluoride (OF_2) and dioxygen difluoride (O_2F_2) respectively is

Options:

- A. +1 and +2
- B. +2 and +1
- C. +1 and +1
- D. +2 and +2

Answer: B

Solution:

Solution:

In oxygen difluoride (OF_2) and dioxygen difluoride (O_2F_2), fluorine gains electron with negative charge as F is more electronegative than O. Thus, O will have positive charge.

\therefore Oxidation number of O in $\text{OF}_2 = +2$

\therefore Oxidation number of O in $\text{O}_2\text{F}_2 = +1$

Question 95

The correct order of radii of F^- , F^- , O and O^{2-} is

Options:

A. $\text{O}^{2-} > \text{F}^- > \text{F} > \text{O}$

B. $\text{F}^- > \text{O}^{2-} > \text{F} > \text{O}$

C. $\text{O}^{2-} > \text{O} > \text{F}^- > \text{F}$

D. $\text{O}^{2-} > \text{F}^- > \text{O} > \text{F}$

Answer: D

Solution:

Solution:

Radii of the given species vary in the order.

$\text{O}^{2-} > \text{F}^- > \text{O} > \text{F}$

The size of the anion is larger than their parent atom. Also, the more the nuclear charge, the lesser is the size.

Question 96

Which of the following can reduce? $\text{RCOOH} \rightarrow \text{RCH}_2\text{OH}$

Options:

A. NaBH_4

B. $\text{Na} / \text{C}_2\text{H}_5\text{OH}$

C. $\text{BH}_3 / \text{THF} / \text{H}_3\text{O}^+$

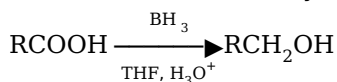
D. $\text{H}_2 / \text{catalyst}$

Answer: C

Solution:

Solution:

The reduction of carboxylic acids to alcohols is carried out by LiAlH_4 and boranes (BH_3 or B_2H_6) in THF.



Question 97

Which of the following trend is correct with respect to the reactivity of alkalimetal oxides with water?

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Options:

- A. $\text{Li}_2\text{O} > \text{Na}_2\text{O} > \text{K}_2\text{O} > \text{Rb}_2\text{O} > \text{Cs}_2\text{O}$
- B. $\text{Cs}_2\text{O} > \text{Rb}_2\text{O} > \text{K}_2\text{O} > \text{Na}_2\text{O} > \text{Li}_2\text{O}$
- C. $\text{Li}_2\text{O} > \text{Na}_2\text{O} > \text{K}_2\text{O} < \text{Rb}_2\text{O} < \text{Cs}_2\text{O}$
- D. $\text{Na}_2\text{O} > \text{K}_2\text{O} < \text{Li}_2\text{O} < \text{Rb}_2\text{O} > \text{Cs}_2\text{O}$

Answer: B

Solution:**Solution:**

Down the group, reactivity of alkali metal oxides with water increases and thus, the reaction of Rb_2O with water and Cs_2O with water are explosive. Hence, the correct reactivity order is $\text{Cs}_2\text{O} > \text{Rb}_2\text{O} > \text{K}_2\text{O} > \text{Na}_2\text{O} > \text{Li}_2\text{O}$

Question 98

Which of the following is an extensive property?

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Options:

- A. Viscosity
- B. Heat capacity
- C. Density
- D. Surface tension

Answer: B

Solution:**Solution:**

Heat capacity is an extensive property. It is a property that changes when the size of the system changes, whereas viscosity, density and surface tension are intensive properties.

Question 99

Which of the following polymer is used in the manufacture of insulators?

Options:

- A. Polyacrylonitrile
- B. Teflon
- C. Bakelite
- D. Nomex

Answer: B

Solution:

Solution:

Teflon is used in the manufacture of insulators, gaskets etc. It is also known as polytetrafluoro ethylene having monomer unit of tetrafluoro ethylene.

Question 100

According to Faraday's first law,

Options:

- A. $W = \frac{96500 \times E}{I \times t}$
- B. $W = \frac{I \times t \times E}{96500}$
- C. $E = \frac{I \times W}{t \times 96500}$
- D. $E = \frac{I \times t \times 96500}{W}$

Answer: B

Solution:

Solution:

According to Faraday's first law,
 $W = ZIt$ and

$$Z = \frac{E}{96500}$$

$$W = \frac{I \times t \times E}{96500}$$

where, I = current, t = time

E = equivalent weight

W = weight deposited.
