MHT CET 2022 Question Paper with Solution

Paper - 1st and Paper - 2nd

Mathematics

Q.1

$\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equal to

Options:

A.

-2√2

B.

2

C.

-2-2√2

D.

-2

Answer: B

Solution:

$$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x} = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{2\cdot\cos^2\frac{x}{2}}$$

$$= \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \left(\frac{1}{2}\sec^2\frac{x}{2}\right)dx = \left[\tan\frac{x}{2}\right]_{\pi/4}^{3\pi/4}$$

$$= \tan\frac{3\pi}{8} - \tan\frac{\pi}{8} = \frac{\sin(3\pi/8)}{\sin(3\pi/8)} - \frac{\sin(\pi/8)}{\cos(\pi/8)}$$

$$= \frac{\sin(3\pi/8)\cdot\cos(\pi/8) - \cos(3\pi/8)\cdot\sin(\pi/8)}{\cos(3\pi/8)\cdot\cos(\pi/8)}$$

$$= \frac{\sin\left(\frac{3\pi}{8}-\frac{\pi}{8}\right)}{\frac{1}{2}\left[\cos\left(\frac{3\pi}{8}+\frac{\pi}{8}\right)+\cos\left(\frac{3\pi}{8}-\frac{\pi}{8}\right)\right]}$$

$$= \frac{2\times\sin\frac{\pi}{4}}{\cos\frac{\pi}{2}+\cos\frac{\pi}{4}} = \frac{2\times\frac{1}{\sqrt{2}}}{0+\frac{1}{\sqrt{2}}} = 2$$

Q.2

If $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x$, then x has the value

Options:

A.

3

B.

1

C.

1/√3

D.

√3

Answer: C

Solution:

$$\because \tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\cdot\tan^{-1}x$$
$$\therefore 2\left[\tan^{-1}\left(\frac{1-x}{1+1\cdot x}\right)\right] = \tan^{-1}x$$
$$\therefore 2\left[\tan^{-1}1 - \tan^{-1}x\right] = \tan^{-1}x$$
$$\therefore 2\left(\frac{\pi}{4} - \tan^{-1}x\right) = \tan^{-1}x$$
$$\therefore \frac{\pi}{2} = 3\cdot\tan^{-1}x$$
$$\therefore \frac{\pi}{6} = \tan^{-1}x$$
$$\therefore x = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

Q.3

If $p : \forall n \in IN, n^2 + n$ is an even number $q : \forall n \in IN, n^2 - n$ is an odd numer, then the truth values of $p \wedge q, p \vee q$ and $p \rightarrow q$ are respectively

Options:

A.

F,T,F

B.

F,F,T

C.

T,T,F

D.

F,T,T

Answer: A

Solution:

Let's analyze each of the propositions given, starting with p which states that for every natural number n, the expression $n^2 + n$ is even.

The expression $n^2 + n$ can be factored into $n(n+1)$. Since any integer n is either even or odd, if n is even, $n(n+1)$ will certainly be even because an even number times any other number is even. If n is odd, then $n+1$ is even, and again, an odd number times an even number is also even. Thus, in either case, the expression will be even, and p is true.

Now let's look at q, which posits that for every natural number n, the expression $n^2 - n$ is odd.

Similarly, we can factor the expression into $n(n-1)$, which is the product of two consecutive numbers. Just as in the first case, if n is even, $n-1$ is odd, resulting in an even number. If n is odd, then $n-1$ is even, which again results in an even number. Thus, q is false because the product of two consecutive integers is always even.

Therefore, the truth value of p is True (T), and the truth value of q is False (F).

Next, we'll find the truth values of the compound statements:

$p \wedge q$ is the conjunction of p and q, and it is true if and only if both p and q are true. Since q is false, $p \wedge q$ is false (F).

$p \vee q$ is the disjunction of p and q, and it is true if at least one of p or q is true. Since p is true, $p \vee q$ is true (T).

$p \rightarrow q$ is the conditional statement "if p then q" and is only false if p is true and q is false (since the only way for an implication to be false is if a true premise leads to a false conclusion). Since this is precisely the case here, $p \rightarrow q$ is false (F).

So the correct truth values for $p \wedge q$, $p \vee q$, and $p \rightarrow q$ are respectively False (F), True (T), and False (F).

The right option based on the above reasoning is:

Option A F, T, F.

Q.4

If the function $f(x) = x^3 - 3(a-2)x^2 + 3ax + 7$, for some $a \in IR$ is increasing in $(0, 1]$ and decreasing in $[1, 5)$, then a root of the equation $\frac{f(x)-14}{(x-1)^2} = 0 (x \neq 1)$ is

Options:

A.

14

B.

7

C.

-14

D.

-7

Answer: B

Solution:

$\because f$ is $\uparrow$ in $(0, 1]$ and $\downarrow$ in $[1, 5)$

$\therefore f$ is both $\uparrow$ and $\uparrow$ at $x = 1$

$\therefore f(1) =$ constant $\quad \therefore f'(1) = 0$

$\because f(x) = x^3 - 3(a-2)x^2 + 3ax + 7$
$\therefore f'(x) = 3x^2 - 6(a-2)x + 3a$
$\therefore f'(1) = 3 - 6(a-2) + 3a = 15 - 3a = 0$
$\therefore a = 5$
$\therefore f(x) = x^3 - 9x^2 + 15x + 7$
$\therefore f(x) - 14 = x^3 - 9x^2 + 15x - 7$
$= (x-7)(x-1)^2$ ... factorising

$\therefore \frac{f(x)-14}{(x-1)^2} = x - 7 = 0 \quad \therefore x = 7$

Q.5

If $\bar{a} = \hat{i} - \hat{k}$, $\bar{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}$ and
$\bar{c} = y\hat{i} + x\hat{j} + (1 + x - y)\hat{k}$, then $[\bar{a}\ \bar{b}\ \bar{c}]$
depends on

Options:

A.

only y

B.

neither x nor y

C.

both x and y

D.

only x

Answer: B

Solution:

$\bar{a} = \hat{i} - \hat{k} \equiv (1, 0, -1)$

$\bar{b} = x\hat{i} + \hat{j} + (1 - x)\hat{k} \equiv (x, 1, 1 - x)$

$\bar{c} = y\hat{i} + x\hat{j} + (1 + x - y)\hat{k} \equiv (y, x, 1 + x - y)$

$$\therefore [\bar{a}\ \bar{b}\ \bar{c}] = \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1 - x \\ y & x & 1 + x - y \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ x & 1 & 1 \\ y & x & 1 + x \end{vmatrix} \quad \text{... by } C_1 + C_3$$

$= 1\,[(1)(1 + x) - (x)(1)]$

$= 1 + x - x$

$= 1$

= contains neither x nor y

Q.6

If cot (A + B) = 0, then sin (A + 2B) is equal to

Options:

A.

sin A

B.

cos 2A

C.

sin 2A

D.

cos A

Answer: A

Solution:

The cotangent of an angle being equal to 0 implies that the angle in question is an odd multiple of $\frac{\pi}{2}$. This is because the cotangent function, defined as $\cot(x) = \frac{1}{\tan(x)}$, becomes undefined when $\tan(x)$ is 0, which occurs at multiples of π. And $\cot(x) = 0$ when $\tan(x)$ goes to infinity, which happens at odd multiples of $\frac{\pi}{2}$ since the tangent function has its asymptotes there.

Since $\cot(A+B) = 0$, the angle $A+B$ must be an odd multiple of $\frac{\pi}{2}$. For simplicity, let's assume $A+B = \frac{\pi}{2}$ as the result would be the same for any odd multiple:

$$A+B = \frac{\pi}{2} \Rightarrow B = \frac{\pi}{2} - A$$

We can use this to determine $\sin(A+2B)$:

$$\sin(A+2B) = \sin(A + 2(\frac{\pi}{2} - A))$$

$$\sin(A+2B) = \sin(A + \pi - 2A)$$

$$\sin(A+2B) = \sin(\pi - A)$$

Now, since sine is a function with symmetry in an odd fashion around the y-axis (odd function), we have:

$$\sin(\pi - A) = \sin A$$

Therefore, the answer is Option A, which is $\sin A$. This result holds true for any odd multiple of $\frac{\pi}{2}$ since the sine function is periodic with period 2π. Even if $A+B$ were, say, $\frac{3\pi}{2}$ (another odd multiple of $\frac{\pi}{2}$), the result would be the same due to the periodic nature of the sine function.

Q.7

The joint equation of pair of lines through the origin and making an equilateral triangle with the line y = 5 is

Options:

A.

$x^2 - 3y^2 = 0$

B.

$\sqrt{3}x^2 - y^2 = 0$

C.

$3x^2 - y^2 = 0$

D.

$5x^2 - y^2 = 0$

Answer: C

Solution:

Joint equation of a pair of lines, through the origin, making an equilateral triangle with the line

$y = b$, is

$\sqrt{3}x^2 - y^2 = 0$

Q.8

If $f(x) = \sqrt{\tan x}$ and $g(x) = \sin x \cdot \cos x$ then $\int \frac{f(x)}{g(x)} \mathrm{d}x$ is equal to (where C is a constant of integration)

Options:

A.

$2\sqrt{\tan x} + C$

B.

$\frac{1}{2}\sqrt{\tan x} + C$

C.

$\sqrt{\tan x} + C$

D.

$\frac{3}{2}\sqrt{\tan x} + C$

Answer: A

Solution:

$$\begin{aligned} I &= \int \frac{\sqrt{\tan x}}{\sin x \cdot \cos x} \mathrm{d}x \\ &= \int \frac{\sqrt{\tan x}}{\left(\frac{\sin x}{\cos x}\right) \cdot \cos^2 x} \mathrm{d}x \\ &= \int \frac{\sqrt{\tan x}}{\tan x} \cdot \frac{1}{\cos^2 x} \mathrm{d}x \\ &= \int \frac{1}{\sqrt{\tan x}} \cdot \sec^2 x \, \mathrm{d}x \\ &= \int \frac{1}{\sqrt{t}} \mathrm{d}t, \ldots t = \tan x \\ &= 2 \cdot \sqrt{t} + C \\ &= 2 \cdot \sqrt{\tan x} + C \end{aligned}$$

Q.9

The general solution of the differential equation $\frac{dy}{dx} = \frac{3x + y}{x - y}$ is (where C is a constant of integration)

Options:

A.

$$\tan^{-1}\left(\frac{y}{x}\right) + \log\left(\frac{y^2 + 3x^2}{x^2}\right) = \log(x) + C$$

B.

$$\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{y}{x\sqrt{3}}\right) + \log\left(\frac{y^2+3x^2}{x^2}\right)^{\frac{1}{2}} = \log(x) + C$$

C.

$$\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{y}{x\sqrt{3}}\right) - \log\left(\frac{y^2+3x^2}{x^2}\right)^{\frac{1}{2}} = \log(x) + C$$

D.

$$\tan^{-1}\left(\frac{x}{y}\right) + \log\left(\frac{y^2 + 3x^2}{x^2}\right) = \log(x) + C$$

Answer: C

Solution:

$$\because \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x+y}{x-y} \quad \therefore \text{ put } y = vx$$

$$\therefore v + x\cdot\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{3+v}{1-v} \quad \therefore x\cdot\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{3+v}{1-v} - v$$

$$\therefore x\cdot\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{3+v-v+v^2}{1-v} \quad \therefore x\cdot\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{3+v^2}{1-v}$$

$$\therefore \int \frac{1-v}{3+v^2}\,\mathrm{d}v = \int \frac{1}{x}\,\mathrm{d}v$$

$$\therefore \int \frac{1}{\sqrt{3}^2+v^2}\,\mathrm{d}v - \frac{1}{2}\int \frac{2v}{3+v^2}\,\mathrm{d}v = \log x$$

$$\therefore \frac{1}{\sqrt{3}}\cdot\tan^{-1}\left(\frac{v}{\sqrt{3}}\right) - \frac{1}{2}\cdot\log\left(3+v^2\right) = \log x + C$$

$$\therefore \frac{1}{\sqrt{3}}\cdot\tan^{-1}\left(\frac{y}{x\sqrt{3}}\right) - \log\left(\frac{y^2+3x^2}{x^2}\right)^{1/2} = \log x + C$$

Q.10

If $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 1 & 2 \\ 1 & 2 \end{bmatrix}$, then $(AB)^{-1} =$

Options:

A.

$$\begin{bmatrix} \frac{17}{5} & \frac{9}{5} \\ 2 & 1 \end{bmatrix}$$

B.

$$\begin{bmatrix} \frac{-17}{5} & \frac{9}{5} \\ 2 & -1 \end{bmatrix}$$

C.

$$\begin{bmatrix} \frac{17}{5} & 2 \\ \frac{9}{5} & 1 \end{bmatrix}$$

D.

$$\begin{bmatrix} \frac{-17}{5} & 2 \\ \frac{-9}{5} & -1 \end{bmatrix}$$

Answer: B

Solution:

Note : $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$A \cdot B = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 9 \\ 10 & 17 \end{bmatrix}$$

$$\therefore |AB| = 85 - 90 = -5$$

$$\therefore (AB)^{-1} = \frac{1}{-5}\begin{bmatrix} 17 & -9 \\ -10 & 5 \end{bmatrix} = \begin{bmatrix} -17/5 & 9/5 \\ 2 & -1 \end{bmatrix}$$

Q.11

The distance between parallel lines $\frac{x-1}{2} = \frac{y-2}{-2} = \frac{z-3}{1}$ and $\frac{x}{2} = \frac{y}{-2} = \frac{z}{1}$ is

Options:

A.

$\frac{2\sqrt{5}}{3}$ units

B.

$\frac{\sqrt{5}}{3}$ units

C.

$\frac{5\sqrt{5}}{3}$ units

D.

$\frac{4\sqrt{5}}{3}$ units

Answer: C

Solution:

$$L_1 : \frac{x-1}{2} = \frac{y-2}{-2} = \frac{z-3}{1}$$
$$L_2 = \frac{x-0}{2} = \frac{y-0}{-2} = \frac{z-0}{1}$$

$\therefore$ their vector equations are

$\bar{r} = \bar{a}_1 + m\bar{b}$ and $\bar{r} = \bar{a}_2 + n\bar{b}$, where

$\bar{a}_1 \equiv (1, 2, 3), \bar{a}_2 \equiv (0, 0, 0), \bar{b} \equiv (2, -2, 1)$

$$\therefore (\bar{a}_1 - \bar{a}_2) \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -2 & -3 \\ 2 & -2 & 1 \end{vmatrix} \equiv (-8, -5, 6)$$

$$\therefore |(\bar{a}_2 - \bar{a}_1) \times \bar{b}| = \sqrt{64 + 25 + 36} = 5\sqrt{5}$$
$$|\bar{b}| = \sqrt{4 + 4 + 1} = 3$$
$$\therefore d(L_1, L_2) = \left| \frac{(\bar{a}_2 - \bar{a}_1) \times \bar{b}}{b} \right| = \frac{5\sqrt{5}}{3}$$

Q.12

Maximum value of Z = 5x + 2y, subject to 2x – y ≥ 2, x + 2y ≤ 8 and x, y ≥ 0 is

Options:

A.

40

B.

17.6

C.

28

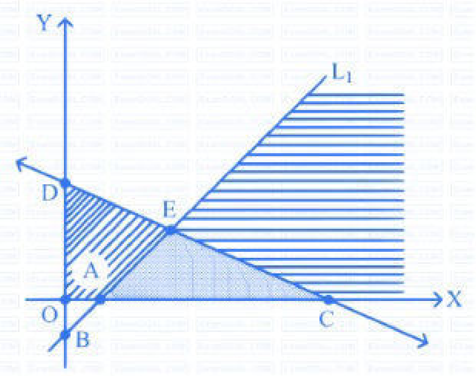
D.

25.6

Answer: A

Solution:

$$Z = 5x + 2y$$
$$2x - y \geq 2, x + 2y \leq 8, x, y \geq 0$$
$$L_1 : 2x - y = 2, L_2 : x + 2y = 8$$
$$\therefore A(1,0), B(0,-2) \text{ lie on } L_1$$
$$C(8,0), D(0,4) \text{ lie on } L_2$$
$$L_1 \cap L_2 \equiv \left(\frac{12}{5}, \frac{14}{5}\right) \equiv E$$



$\therefore$ Critical points are $A(1,0), C(8,0), E \equiv \left(\frac{12}{5}, \frac{14}{5}\right)$

$$\therefore Z_A = 5(1) + 2(0) = 5$$
$$Z_C = 5(8) + 2(0) = 40$$
$$Z_E = 5\left(\frac{12}{5}\right) + 2\left(\frac{14}{5}\right) = \frac{88}{5} = 17.6$$

$$\therefore Z_{\max} = 40$$

Q.13

The value of sin $(2\sin^{-1} 0.8)$ is equal to

Options:

A.

0.96

B.

0.16

C.

0.12

D.

0.48

Answer: A

Solution:

To find the value of $\sin\left(2\sin^{-1}0.8\right)$, we can use the double angle formula for sine, which states:

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

Firstly, we are given that $\sin^{-1}(0.8) = \theta$, hence $\sin(\theta) = 0.8$. We now need to find $\cos(\theta)$ to use in the double angle formula.

Since sin and cos are related by the Pythagorean identity:

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

We can solve for $\cos(\theta)$:

$$\cos^2(\theta) = 1 - \sin^2(\theta)$$

Substituting the value of $\sin(\theta)$:

$$\cos^2(\theta) = 1 - 0.8^2$$

$$\cos^2(\theta) = 1 - 0.64$$

$$\cos^2(\theta) = 0.36$$

The cosine has two possible values, +0.6 and -0.6, for angles in different quadrants, but since $\theta = \sin^{-1}(0.8)$ lies in the first quadrant (where sine and cosine are both positive), we select the positive value. Hence:

$$\cos(\theta) = 0.6$$

Now we can use the double angle formula for sine:

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta) = 2 \cdot 0.8 \cdot 0.6 = 2 \cdot 0.48 = 0.96$$

Therefore, the value of $\sin\left(2\sin^{-1}0.8\right)$ is 0.96, which corresponds to Option A.

Q.14

A line makes the same angle 'α' with each of the x and y axes. If the angle 'θ', which it makes with the z-axis, is such that $\sin^2\theta = 2\sin^2\alpha$, then the angle α is

Options:

A.

(π/4)

B.

(π/2)

C.

(π/3)

D.

(π/6)

Answer: A

Solution:

Let us consider the direction cosines of the line that makes the same angle α with the **x** and **y** axes. The direction cosines *l*,m, and n are related to the angles made by the line with the x,y, and z axes respectively.

The direction cosines are given by:

l = cos(α) (angle with x-axis)

m = cos(α) (angle with y-axis)

n = cos(θ) (angle with z-axis)

We know that the sum of the squares of the direction cosines equals 1, which is expressed as:

$$l^2 + m^2 + n^2 = 1$$

Substituting the values of l, m, and n, we get:

$$\cos^2(\alpha) + \cos^2(\alpha) + \cos^2(\theta) = 1$$

Given that $\sin^2(\theta) = 2\sin^2(\alpha)$, we can express $\cos^2(\theta)$ in terms of $\cos^2(\alpha)$ using the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$.

First, let's solve for $\cos^2(\theta)$:

$$\begin{aligned} \sin^2(\theta) &= 2\sin^2(\alpha) \\ 1 - \cos^2(\theta) &= 2(1 - \cos^2(\alpha)) \\ 1 - \cos^2(\theta) &= 2 - 2\cos^2(\alpha) \\ \cos^2(\theta) &= 2\cos^2(\alpha) - 1 \end{aligned}$$

We then substitute this expression into the sum of squares equation mentioned earlier:

$$2\cos^2(\alpha) + (2\cos^2(\alpha) - 1) = 1$$

Simplify and solve for $\cos^2(\alpha)$:

$$\begin{aligned} 4\cos^2(\alpha) - 1 &= 1 \\ 4\cos^2(\alpha) &= 2 \\ \cos^2(\alpha) &= \frac{1}{2} \\ \cos(\alpha) &= \frac{1}{\sqrt{2}} \text{ or } \cos(\alpha) = -\frac{1}{\sqrt{2}} \end{aligned}$$

Since angles with both axes are given to be the same and it is generally assumed that these are acute angles, we will consider only the positive value.

The positive angle α whose cosine is $\frac{1}{\sqrt{2}}$ corresponds to an angle of $\frac{\pi}{4}$ radians.

Therefore, the angle α is $\frac{\pi}{4}$, which corresponds to **Option A**.

Q.15

The negation of the statement pattern p∨(q→~r) is

Options:

A.

$\sim p \wedge (q \wedge \sim r)$

B.

$\sim p \wedge (q \wedge r)$

C.

$\sim p \wedge (\sim q \wedge r)$

D.

$\sim p \wedge (\sim q \wedge \sim r)$

Answer: B

Solution:

$$\begin{aligned} &\sim [p \vee (q \rightarrow \sim r)] \\ &= (\sim p) \wedge \sim (q \rightarrow \sim r) \\ &= \sim p \wedge (q \wedge \sim\sim r) \\ &= \sim p \wedge (q \wedge r) \end{aligned}$$

Q.16

Let $\bar{a} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\bar{b} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$ be two vectors. If $\bar{c}$ is a vector such that $\bar{b} \times \bar{c} = \bar{b} \times \bar{c}$ and $\bar{c} \cdot \bar{a} = 0$, then $\bar{c} \cdot \bar{b}$ is equal to

Options:

A.

- 1/2

B.

3/2

C.

1/2

D.

- 3/2

Answer: A

Q.17

The incidence of occupational disease in an industry is such that the workmen have a 10% chance of suffering from it. The probability that out of 5 workmen, 3 or more will contract the disease is

Options:

A.

0.000856

B.

0.856

C.

0.0000856

D.

0.00856

Answer: D

Solution:

To find the probability that out of 5 workmen, 3 or more will contract the disease, we can use the binomial probability formula. The binomial probability formula is used when there are exactly two mutually exclusive outcomes of a trial - 'success' and 'failure'. In this case, 'success' is contracting the disease, and 'failure' is not contracting the disease.

The binomial probability formula is given by:

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

Where:

$P(X = k)$ is the probability of having exactly k successes in n trials.
$\binom{n}{k}$ is the binomial coefficient, which equals $\frac{n!}{k!(n-k)!}$ and represents the number of ways to choose k successes from n trials.
p is the probability of success on a single trial.
$(1-p)$ is the probability of failure on a single trial.

In this problem, $p = 0.10$ and $n = 5$. We want to calculate the probability that 3 or more workmen will contract the disease, meaning we seek $P(X \geq 3)$. This is the sum of the probabilities of having exactly 3, exactly 4, and exactly 5 workmen contracting the disease, which is:

$$P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5)$$

Calculating the probabilities for each of these:

$$P(X = 3) = \binom{5}{3}(0.10)^3(0.90)^2$$
$$P(X = 4) = \binom{5}{4}(0.10)^4(0.90)^1$$
$$P(X = 5) = \binom{5}{5}(0.10)^5(0.90)^0$$

Now we calculate these probabilities:

$$P(X=3) = \binom{5}{3}(0.10)^3(0.90)^2$$
$$P(X=3) = \frac{5!}{3!2!} \times 0.001 \times 0.81$$
$$P(X=3) = 10 \times 0.001 \times 0.81$$
$$P(X=3) = 0.0081$$
$$P(X=4) = \binom{5}{4}(0.10)^4(0.90)^1$$
$$P(X=4) = \frac{5!}{4!1!} \times 0.0001 \times 0.9$$
$$P(X=4) = 5 \times 0.0001 \times 0.9$$
$$P(X=4) = 0.00045$$
$$P(X=5) = \binom{5}{5}(0.10)^5(0.90)^0$$
$$P(X=5) = \frac{5!}{5!0!} \times 0.00001 \times 1$$
$$P(X=5) = 1 \times 0.00001 \times 1$$
$$P(X=5) = 0.00001$$

Adding these probabilities gives us:

$$P(X \geq 3) = P(X=3) + P(X=4) + P(X=5) = 0.0081 + 0.00045 + 0.00001$$

$$P(X \geq 3) = 0.00856$$

The answer is Option D, 0.00856.

Q.18

If $y = \log \sqrt{\frac{1+\sin x}{1-\sin x}}$, then $\frac{dy}{dx}$ at $x = \frac{\pi}{3}$ is

Options:

A.

1/2

B.

- 1/2

C.

2

D.

1/4

Answer: C

Solution:

$$y=\log\left[\left(\frac{1+\sin x}{1-\sin x}\right)^{1/2}\right]$$

$$\therefore y=\frac{1}{2}[\log(1+\sin x)-\log(1-\sin x)]$$

$$\therefore \frac{dy}{dx}=\frac{1}{2}\left[\frac{1}{1+\sin x}(\cos x)-\frac{1}{1-\sin x}(-\cos x)\right]$$

$$=\frac{1}{2}(\cos x)\left[\frac{1}{1+\sin x}+\frac{1}{1-\sin x}\right]$$

$$=\frac{\cos x}{\cos^2 x}$$

$$=\frac{1}{\cos^2}$$

$$\therefore \left.\frac{dy}{dx}\right|_{x=\pi/3}=\frac{1}{\cos\frac{\pi}{3}}=\frac{1}{\left(\frac{1}{2}\right)}=2$$

Q.19

The variance and mean of 15 observations are respectively 6 and 10 . If each observation is increased by 8 then the new variance and new mean of resulting observations are respectively

Options:

A.

6, 18

B.

6, 10

C.

14, 10

D.

14, 18

Answer: A

Solution:

When each observation of a data set is increased by a constant, the mean of the data set increases by that constant, but the variance does not change. Variance is a measure of the dispersion of the data points around the mean and is not affected by the addition (or subtraction) of a constant to each data point.

Let the original observations be denoted as $x_1, x_2, \ldots, x_{15}$. The original mean (μ) is given by:

$\mu = 10$

The original variance (σ^2) is given by:

$\sigma^2 = 6$

Now, if each observation is increased by 8, the new observations will be:

$x_i' = x_i + 8$

for $i = 1, 2, \ldots, 15$. The new mean (μ') of these observations will be:

$\mu' = \mu + 8$

$\mu' = 10 + 8$

$\mu' = 18$

The variance, however, is not affected by the addition of a constant to each data point. Therefore, the new variance (σ'^2) is the same as the original variance:

$\sigma'^2 = \sigma^2$

$\sigma'^2 = 6$

Hence, the new mean is 18 and the new variance is 6. The correct option is:

Option A

6, 18

Q.20

If $y = \sin\left(2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right)$ then $\frac{\mathrm{d}y}{\mathrm{d}x}$ is equal to

Options:

A.

$\frac{-x}{\sqrt{1-x^2}}$

B.

$\frac{-2x}{\sqrt{1-x^2}}$

C.

$\frac{-1}{\sqrt{1-x^2}}$

D.

$\frac{1}{\sqrt{1-x^2}}$

Answer: A

Solution:

$$y = \sin\left\{2 \cdot \tan^{-1}\sqrt{\frac{1+x}{1-x}}\right\}$$

Putting $x = \cos 2\theta$,

$$\sqrt{\frac{1+x}{1-x}} = \ldots = \cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$$

$$\begin{aligned}\therefore y &= \left\{\sin 2 \cdot \tan^{-1}\left[\tan\left(\frac{\pi}{2} - \theta\right)\right]\right\} \\ &= \sin\left\{2\left(\frac{\pi}{2} - \theta\right)\right\} \\ &= \sin(\pi - 2\theta) \\ &= \sin 2\theta \\ &= \sqrt{1 - \cos^2 2\theta} \\ &= \sqrt{1 - x^2}\end{aligned}$$

$$\begin{aligned}\therefore \frac{dy}{dx} &= \frac{1}{2\sqrt{1-x^2}} \times \frac{d}{dx}(1 - x^2) \\ &= \frac{1}{2\sqrt{1-x^2}} \times (-2x) \\ &= \frac{-x}{\sqrt{1-x^2}}\end{aligned}$$

Q.21

The magnitude of the projection of the vector $2\hat{i} + 3\hat{j} + \hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$ is

Options:

A.

3√6 units

B.

√3/2 units

C.

1/√6 units

D.

$\sqrt{\frac{3}{2}}$ units

Answer: D

Solution:

To find the vector perpendicular to the plane containing the vectors $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$, we can use the cross product of these two vectors.

The cross product is defined as:

$$\mathbf{A} \times \mathbf{B} = (a_2b_3 - a_3b_2)\hat{\mathbf{i}} + (a_3b_1 - a_1b_3)\hat{\mathbf{j}} + (a_1b_2 - a_2b_1)\hat{\mathbf{k}}$$

For our vectors, this becomes:

$$(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) \times (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix}$$

$$= \hat{\mathbf{i}}(1 \cdot 3 - 1 \cdot 2) - \hat{\mathbf{j}}(1 \cdot 3 - 1 \cdot 1) + \hat{\mathbf{k}}(1 \cdot 2 - 1 \cdot 1)$$

$$= \hat{\mathbf{i}}(3 - 2) - \hat{\mathbf{j}}(3 - 1) + \hat{\mathbf{k}}(2 - 1)$$

$$= \hat{\mathbf{i}}(1) - \hat{\mathbf{j}}(2) + \hat{\mathbf{k}}(1)$$

$$= \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}.$$

Now that we have the perpendicular vector, let us compute the magnitude of the projection of the vector $2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$ onto $\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$.

The projection of vector a onto vector b is given by the formula:

$$\text{proj}_{\mathbf{b}}(\mathbf{a}) = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{b}\|}$$

But we are only interested in the magnitude of the projection, which simplifies to:

$$\|\text{proj}_{\mathbf{b}}(\mathbf{a})\| = \left| \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{b}\|} \right|$$

Let's find the dot product of vectors $2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$:

$$\mathbf{a} \cdot \mathbf{b} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}})$$

$$= 2(1) - 3(2) + 1(1)$$

$$= 2 - 6 + 1$$

$$= -3.$$

Next, let's find the magnitude of $\mathbf{b}$, which is $\|\mathbf{b}\|$:

$$\|\mathbf{b}\| = \sqrt{(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}})}$$

$$= \sqrt{1^2 + (-2)^2 + 1^2}$$

$$= \sqrt{1 + 4 + 1}$$

$$= \sqrt{6}.$$

Finally, the magnitude of the projection is:

$$\|\text{proj}_{\mathbf{b}}(\mathbf{a})\| = \left|\frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{b}\|}\right|$$

$$= \left|\frac{-3}{\sqrt{6}}\right|$$

$$= \left|-\frac{\sqrt{9}}{\sqrt{6}}\right|$$

$$= \left|-\sqrt{\frac{3}{2}}\right|$$

$$= \sqrt{\frac{3}{2}}$$

Q.22

$$f(x) = ax^2 + bx + 1, \quad if \quad |2x - 3| \geq 2$$
$$= 3x + 2, \quad if \quad \tfrac{1}{2} < x < \tfrac{5}{2}$$

is continuous on its domain, then $a + b$ has the value

Options:

A.

13/5

B.

31/5

C.

23/5

D.

1/5

Answer: C

Q.23

If $\bar{a} = \hat{i} + \hat{j} + \hat{k}$, $\bar{b} = \hat{i} - \hat{j} + \hat{k}$ and $\bar{c} = \hat{i} - \hat{j} - \hat{k}$ are three vectors then vector $\bar{r}$ in the plane of $\bar{a}$ and $\bar{b}$, whose projection on $\bar{c}$ is $\frac{1}{\sqrt{3}}$, is given by

Options:

A.

$(2t+1)\hat{i} - \hat{j} + (2t+1)\hat{k},\ \forall t \in IR$

B.

$(2t+1)\hat{i} - \hat{j} + (2t-1)\hat{k},\ \forall t \in IR$

C.

$(2t-1)\hat{i} - \hat{j} + (2t-1)\hat{k},\ \forall t \in IR$

D.

$(2t-1)\hat{i} - \hat{j} + (2t+1)\hat{k},\ \forall t \in IR$

Answer: A

Q.24

A tetrahedron has verticles $P(1,2,1), Q(2,1,3), R(-1,1,2)$ and $O(0,0,0)$. Then the angle between the faces OPQ and PQR is

Options:

A.

$\cos^{-1}\left(\frac{17}{35}\right)$

B.

$\cos^{-1}\left(\frac{17}{31}\right)$

C.

$\cos^{-1}\left(\frac{19}{35}\right)$

D.

$\cos^{-1}\left(\frac{19}{31}\right)$

Answer: A

Q.25

The principal value of $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$ is

Options:

A.

(π/3)

B.

(2π/3)

C.

- (2π/3)

D.

(5π/3)

Answer: A

Solution:

The principal value of the inverse sine function, denoted as $\sin^{-1}$ or arcsin, lies in the range $[-\frac{\pi}{2}, \frac{\pi}{2}]$. Given the expression $\sin^{-1}(\sin(\frac{2\pi}{3}))$, we need to find the angle within the principal range of the inverse sine function whose sine is equivalent to $\sin(\frac{2\pi}{3})$.

The sine function is periodic with a period of 2π, which means that $\sin(\theta) = \sin(\theta + 2k\pi)$ where k is an integer. However, because the inverse sine function has a limited range, we have to find an equivalent angle for $\frac{2\pi}{3}$ that lies within the range $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

Given that the angle $\frac{2\pi}{3}$ is in the second quadrant where sine is positive, and $\sin(\theta) = \sin(\pi - \theta)$ in the second quadrant, we can use the fact that:

$$\sin\left(\frac{2\pi}{3}\right) = \sin\left(\pi - \frac{2\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right)$$

Now, $\frac{\pi}{3}$ is within the range $[-\frac{\pi}{2}, \frac{\pi}{2}]$, which is the principal value range for $\sin^{-1}$.

Therefore, the principal value of $\sin^{-1}(\sin(\frac{2\pi}{3}))$ is indeed $\frac{\pi}{3}$.

Thus, the correct answer is:

Option A: $\frac{\pi}{3}$

Q.26

The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines a family of circles with

Options:

A.

fixed radius of 1 unit and variable centres along the X-axis

B.

fixed radius of 1 unit and variable centres along the Y-axis

C.

variable radii and a fixed centre at (0,1)

D.

variable radii and a fixed centre at (0,-1)

Answer: A

Solution:

D.E. : $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ $\quad \therefore \frac{y\,dy}{\sqrt{1-y^2}} = dx$

$$\therefore \int \frac{-2y}{\sqrt{1-y^2}}\,dy = -2\int dx$$

$$\therefore 2\sqrt{1-y^2} = -2x + 2c$$

$$\therefore \sqrt{1-y^2} = -x + c$$

$$\therefore \text{ Sq. } : 1 - y^2 = x^2 - 2cx + c^2$$

$$\therefore x^2 + y^2 - 2cx + \left(c^2 - 1\right) = 0$$

$$\therefore 2g = -2c, 2f = 0, k = c^2 - 1$$

$$\therefore C \equiv (-g, -f) \equiv (c, 0),$$

$$r = \sqrt{g^2 + f^2 - k} = \sqrt{c^2 + 0^2 - \left(c^2 - 1\right)} = 1$$

$\therefore r = 1$, fixed; $C \equiv (c, 0)$, moves on X-axis as c changes.

Q.27

The value of the integral $\int\limits_0^1 \sqrt{\frac{1-x}{1+x}}\,dx$ is

Options:

A.

$\left(\frac{\pi}{2}\right) + 1$

B.

$\left(\frac{\pi}{2}\right) - 1$

C.

1

D.

-1

Answer: B

Solution:

$$I=\int_0^1 \sqrt{\frac{1-x}{1+x}}\,dx=\int_0^1 \sqrt{\frac{1-x}{1+x}\cdot\frac{1-x}{1-x}}\,dx$$

$$=\int_0^1 \frac{1-x}{\sqrt{1-x^2}}\,dx$$

$$=\int_0^1 \left[\frac{1}{\sqrt{1-x^2}}-\frac{x}{\sqrt{1-x^2}}\right]dx$$

$$=\left[\sin^{-1}x+\sqrt{1-x^2}\right]_0^1$$

$$=\left(\sin^{-1}1+0\right)-\left(\sin^{-1}0+1\right)$$

$$=\frac{\pi}{2}-(0+1)$$

$$=\frac{\pi}{2}-1$$

Q.28

If a question paper consists of 11 questions divided into two sections I and II. Section I consists of 6 questions and section II consists of 5 questions, then the number of different ways can student select 6 questions, taking at least 2 questions from each section, is

Options:

A.

425

B.

275

C.

350

D.

225

Answer: A

Solution:

To solve this problem, we will consider the different ways in which the student can select questions from two distinct sections under the given constraint that at least 2 questions must be selected from each section. We can break it down into different cases and compute the number of possibilities for each case.

There are three cases to consider:

- Case 1: Selecting 2 questions from Section I and 4 questions from Section II.
- Case 2: Selecting 3 questions from Section I and 3 questions from Section II.
- Case 3: Selecting 4 questions from Section I and 2 questions from Section II.

Then we will add the number of ways from all the cases, as these are mutually exclusive events.

Let's calculate for each case:

Case 1: Choose 2 questions from Section I and 4 questions from Section II.

The number of ways to choose 2 questions from 6 in Section I is given by $C(6,2)$, and the number of ways to choose 4 questions from 5 in Section II is given by $C(5,4)$.

Therefore, the total number of ways for Case 1 = $C(6,2) \times C(5,4)$.

Case 2: Choose 3 questions from Section I and 3 questions from Section II.

The number of ways to choose 3 questions from 6 in Section I is given by $C(6,3)$, and the number of ways to choose 3 questions from 5 in Section II is given by $C(5,3)$.

Therefore, the total number of ways for Case 2 = $C(6,3) \times C(5,3)$.

Case 3: Choose 4 questions from Section I and 2 questions from Section II.

The number of ways to choose 4 questions from 6 in Section I is given by $C(6,4)$, and the number of ways to choose 2 questions from 5 in Section II is given by $C(5,2)$.

Therefore, the total number of ways for Case 3 = $C(6,4) \times C(5,2)$.

Let's calculate the number of combinations for each case:

For Case 1: $C(6,2) \times C(5,4) = \frac{6!}{2!(6-2)!} \times \frac{5!}{4!(5-4)!} = 15 \times 5 = 75$
For Case 2: $C(6,3) \times C(5,3) = \frac{6!}{3!(6-3)!} \times \frac{5!}{3!(5-3)!} = 20 \times 10 = 200$
For Case 3: $C(6,4) \times C(5,2) = \frac{6!}{4!(6-4)!} \times \frac{5!}{2!(5-2)!} = 15 \times 10 = 150$

Now to find the total number of different ways, we add the number from all three cases together:

Total number of ways = $75 + 200 + 150 = 425$

Thus, the total number of different ways a student can select 6 questions, taking at least 2 questions from each section, is 425, which corresponds to **Option A**.

Q.29

The area (in sq. units) of the region described by $A = \{(x,y)/x^2 + y^2 \leq 1$ and $y^2 \leq 1 - x\}$ is

Options:

A.

$\left(\frac{\pi}{2} + \frac{2}{3}\right)$

B.

$\left(\frac{\pi}{2} + \frac{4}{3}\right)$

C.

$\left(\frac{\pi}{2} - \frac{4}{3}\right)$

D.

$\left(\frac{\pi}{2} - \frac{2}{3}\right)$

Answer: B

Q.30

A firm is manufacturing 2000 items. It is estimated that the rate of change of production P with respect to additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is

Options:

A.

4500

B.

3000

C.

2500

D.

3500

Answer: D

Solution:

To find the new level of production when 25 more workers are employed, we need to integrate the given rate of change of production with respect to the number of workers over the interval from the current number of workers to the current number plus 25.

The rate of change of production is given by the differential equation:

$\frac{dP}{dx} = 100 - 12\sqrt{x}$

To find the production function $P(x)$, we integrate the differential equation with respect to x:

$P(x) = \int (100 - 12\sqrt{x})\, dx$

Integrating term by term, we get:

$$P(x) = \int 100\, dx - \int 12\sqrt{x}\, dx$$

$$= 100x - 12 \int x^{\frac{1}{2}}\, dx$$

$$= 100x - 12 \cdot \frac{2}{3} x^{\frac{3}{2}} + C$$

$$= 100x - 8x^{\frac{3}{2}} + C$$

Where C is the constant of integration.

We know that when the number of workers was 0, the firm was manufacturing 2000 items. We can use this information to find the constant C:

$2000 = P(0) = 100 \cdot 0 - 8 \cdot 0^{\frac{3}{2}} + C\, C = 2000$

Now, the production function with the constant C included is:

$P(x) = 100x - 8x^{\frac{3}{2}} + 2000$

The new level of production after employing 25 more workers is $P(25)$, provided that the initial number of workers was 0. If there was an initial number of workers x_0 where $x_0 \geq 0$, we need to evaluate the increase in production from x_0 to $x_0 + 25$.

However, without loss of generality and because no initial number of workers x_0 is provided, let's proceed by assuming the firm is starting from 0 workers. We add 25 workers to find:

$P(25) = 100 \cdot 25 - 8 \cdot 25^{\frac{3}{2}} + 2000$

$= 2500 - 8 \cdot (5^2)^{\frac{3}{2}} + 2000$

$= 2500 - 8 \cdot 5^3 + 2000$

$= 2500 - 8 \cdot 125 + 2000$

$= 2500 - 1000 + 2000$

$= 2500 + 1000$

$= 3500$

Thus, the correct answer is 3500, which corresponds to Option D.

Q.31

$\int \frac{3x-2}{(x+1)(x-2)^2}\, \mathrm{d}x =$

(where C is a constant of integration)

Options:

A.

$\frac{-5}{9}\log(x+1) + \frac{5}{9}\log(x-2) - \frac{4}{3} \times \frac{1}{(x-2)} + C$

B.

$\frac{-5}{9}\log(x+1) + \frac{5}{9}\log(x-2) - \frac{1}{x-2} + C$

C.

$\frac{1}{9}\log(x+1) + \frac{5}{9}\log(x-2) - \frac{4}{3} \times \frac{1}{(x-2)} + C$

D.

$\frac{-5}{9}\log(x+1) + \frac{1}{9}\log(x-2) - \frac{1}{x-2} + C$

Answer: A

Solution:

To solve the integral $\int \frac{3x-2}{(x+1)(x-2)^2}\,dx$, we can use partial fraction decomposition. We want to express the integrand as a sum of fractions of the form:

$$\frac{3x-2}{(x+1)(x-2)^2} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

For this, we need to find the constants A, B, and C. Multiplying both sides by the common denominator $(x+1)(x-2)^2$, we get:

$$3x-2 = A(x-2)^2 + B(x+1)(x-2) + C(x+1)$$

To find A, B, and C, we can equate coefficients or plug in convenient values for x that simplify the equation:

Let's plug in $x = 2$, the root of the denominator, to solve for C:

$$3(2)-2 = A(0) + B(2+1)(0) + C(2+1)$$

$$4 = 3C$$

$$C = \frac{4}{3}$$

Next, let's plug in $x = -1$, the other root:

$$3(-1)-2 = A(-1-2)^2 + B(0) + C(0)$$

$$-3-2 = A(9)$$

$$-5 = 9A$$

$$A = \frac{-5}{9}$$

To find B, we can either equate coefficients for the x term or choose another convenient value for x. Let's use $x = 0$ to have only B term remaining:

$$3(0)-2 = A(0-2)^2 + B(0+1)(0-2) + C(0+1)$$

$$-2 = A(4) - 2B + C$$

Inserting A and C we get:

$$-2 = \left(\frac{-5}{9}\right)(4) - 2B + \left(\frac{4}{3}\right)$$

Now, simplify and solve for B:

$$-2 = \frac{-20}{9} - 2B + \frac{4}{3}$$

Combining the fractions gives us:

$$-2 = -\frac{20}{9} + \frac{12}{9} - 2B$$

$$-2 = -\frac{8}{9} - 2B$$

Add $\frac{8}{9}$ to both sides:

$$-2 + \frac{8}{9} = -2B$$

$$-18/9 + 8/9 = -2B$$

$$-10/9 = -2B$$

$$B = \frac{5}{9}$$

Now, the integrand can be rewritten as:

$$\frac{3x-2}{(x+1)(x-2)^2} = \frac{-5/9}{x+1} + \frac{5/9}{x-2} + \frac{4/3}{(x-2)^2}$$

Integrating term by term:

$$\int \frac{-5/9}{x+1}\,dx + \int \frac{5/9}{x-2}\,dx + \int \frac{4/3}{(x-2)^2}\,dx$$

The integral of each fraction can be calculated as follows:

$$\frac{-5}{9}\ln|x+1| + \frac{5}{9}\ln|x-2| - \frac{4}{3} \cdot \frac{1}{x-2} + C$$

So the answer to the integral is Option A:

$$\frac{-5}{9}\ln(x+1) + \frac{5}{9}\ln(x-2) - \frac{4}{3} \times \frac{1}{(x-2)} + C$$

Q.32

If the normal to the curve $y = f(x)$ at the point $(3, 4)$ makes an angle $\left(\frac{3\pi}{4}\right)^c$ with positive X-axis, then $f'(3)$ is equal to

Options:

A.

-1

B.

4/3

C.

- 3/4

D.

1

Answer: D

Solution:

At the point $P(3,4), x=3$

$\therefore$ slope of normal to the curve $y=f(x)$ at P is

$$m_N=\frac{-1}{f'(3)}=\tan\frac{3\pi}{4}=-\cot\frac{\pi}{4}=-1$$
$$\therefore\frac{-1}{f'(3)}=-1$$
$$\therefore f'(3)=1$$

Q.33

If $P(A\cup B)=0.7, P(A\cap B)=0.2$, then $P(A')+P(B')$ is

Options:

A.

1.1

B.

1.6

C.

1.8

D.

0.6

Answer: A

Solution:

$$P(A\cup B)=0.7, P(A\cap B)=0.2$$
$$\because P(A\cup B)=P(A)+P(B)-P(A\cap B)$$
$$\therefore P(A)+P(B)=P(A\cup B)+P(A\cap B)$$
$$=0.7+0.2$$
$$=0.9$$

$$\begin{aligned}\therefore P(A')+P(B')&=[1-P(A)]+[1-P(B)]\\&=2-[P(A)+P(B)]\\&=2-0.9\\&=1.1\end{aligned}$$

Q.34

If $\lim_{x \to 1} \frac{x^2 - ax + b}{(x-1)} = 5$, then $(a+b)$ is equal to

Options:

A.

-4

B.

-7

C.

7

D.

-3

Answer: B

Solution:

The limit given is

$$\lim_{x \to 1} \frac{x^2 - ax + b}{(x-1)} = 5$$

For this limit to exist and be equal to 5, the term $x - 1$ in the denominator must cancel out with a similar term in the numerator, because otherwise the expression would be undefined at $x = 1$. Let's re-write the numerator by factoring it, assuming it has a factor of $x - 1$:

$$\lim_{x \to 1} \frac{x^2 - ax + b}{(x-1)} = \lim_{x \to 1} \frac{(x-1)(x-c)}{(x-1)}$$

Here, $x - c$ is the other factor of the numerator. Now, we can cancel the $x - 1$ term from the numerator and denominator:

$$\lim_{x \to 1} \frac{(x-1)(x-c)}{(x-1)} = \lim_{x \to 1} (x - c)$$

Since the limit as x approaches 1 of $(x - c)$ should be 5, this must mean that:

$1 - c = 5$

$-c = 5 - 1$

$-c = 4$

$c = -4$

This means that the original function can be expressed as:

$f(x) = x^2 - ax + b = (x - 1)(x + 4)$

Let's expand the right hand side of the equation:

$(x - 1)(x + 4) = x^2 + 4x - x - 4$

$f(x) = x^2 + 3x - 4$

This tells us that the coefficient of x (which is $-a$ in the equation) should be 3, and b, the constant term, should be -4. So from the expanded form, we can see that:

$-a = 3$

$\therefore a = -3$

And we already found:

$b = -4$

Finally, adding a and b we get:

$a + b = (-3) + (-4)$

$a + b = -7$

Therefore, $(a + b) = -7$ which corresponds to Option B.

Q.35

If $y = \cos\left(\sin x^2\right)$, then $\frac{dy}{dx}$ at $x = \sqrt{\frac{\pi}{2}}$ is

Options:

A.

0

B.

2

C.

-1

D.

-2

Answer: A

Solution:

$\because y = \cos\left(\sin x^2\right)$

$\therefore$ at $x = \sqrt{\frac{\pi}{2}}$,

$y = \cos\left(\sin\frac{\pi}{2}\right) = \cos 1$

$$(1) \Rightarrow \frac{dy}{dx} = 2x \cdot \cos x^2 \cdot \left[-\sin\left(\sin x^2\right)\right]$$
$$= -2x \cdot \cos x^2 \cdot \sin\left(\sin x^2\right)$$

$$\therefore \left.\frac{dy}{dx}\right|_{x=\sqrt{\pi/2}} = -2 \cdot \sqrt{\frac{\pi}{2}} \cdot \cos\frac{\pi}{2} \cdot \sin\left(\sin\frac{\pi}{2}\right)$$
$$= -2 \cdot \sqrt{\frac{\pi}{2}} \cdot (0) \cdot \sin 1$$
$$= 0$$

Q.36

Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Then mean of number of kings is

Options:

A.

4/169

B.

1/13

C.

1/169

D.

2/13

Answer: D

Solution:

The problem here speaks of drawing cards with replacement, which means that after drawing a card and noting whether it is a king or not, the card is placed back into the deck. As a result, the deck remains complete with 52 cards for each draw.

Let's define our random variable X as the number of kings drawn in two successive draws with replacement. There are four kings in a deck of 52 cards. The probability p of drawing a king in one draw is therefore:

$$p = \frac{\text{Number of kings}}{\text{Total number of cards}} = \frac{4}{52} = \frac{1}{13}$$

With replacement, the probability remains the same for each draw since the deck composition does not change.

For the mean (or expected value) of a binomial distribution, which is the appropriate distribution to model the number of successes (in this case, drawing a king) in n independent Bernoulli trials (draws from the deck) with the same probability p of success on each trial, we use the formula:

Mean $= E(X) = n \cdot p$

In our scenario, $n = 2$ because two cards are drawn, and $p = \frac{1}{13}$ as calculated before. Plugging these values into the formula, we get:

$$E(X) = n \cdot p = 2 \cdot \frac{1}{13} = \frac{2}{13}$$

So, the mean of the number of kings drawn in two successive draws with replacement is $\frac{2}{13}$, which corresponds to Option D.

Q.37

The polar co-ordinates of the point, whose Cartesian coordinates are (−2√3,2), are

Options:

A.

$(4, (\frac{3\pi}{4}))$

B.

$(4, (\frac{5\pi}{6}))$

C.

$(4, (\frac{2\pi}{3}))$

D.

$(4, (\frac{11\pi}{12}))$

Answer: B

Solution:

Cartesian $\equiv (-2\sqrt{3}, 2) \equiv (r \cdot \cos\theta, r \cdot \sin\theta)$

$\therefore r = \sqrt{x^2 + y^2} = \sqrt{12 + 4} = 4$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \tan^{-1}\left(-\tan\frac{\pi}{6}\right)$$

$$= \tan^{-1}\left[\tan\left(\pi - \frac{\pi}{6}\right)\right]$$

$$= \tan^{-1}\left(\tan\frac{5\pi}{6}\right)$$

$$= \frac{5\pi}{6}$$

$\therefore$ Polar $\equiv (4, \frac{5\pi}{6})$

Q.38

Let z be a complex number such that $|z| + z = 3 + i$, $i = \sqrt{-1}$, then $|z|$ is equal to

Options:

A.

5/4

B.

5/3

C.

$\frac{\sqrt{34}}{3}$

D.

$\frac{\sqrt{41}}{4}$

Answer: B

Solution:

First, let's write down the equation given:

|z| + z = 3 + i

To solve for |z|, we need to convert this equation into something that we can easily manipulate. Let's represent z as a complex number in the standard form:

z = x + iy

where x and y are the real and imaginary parts of z, respectively. Thus, the modulus of z, denoted by |z|, is given by:

$|z| = \sqrt{x^2 + y^2}$

Now, let's substitute z with x + iy in the original equation:

$\sqrt{x^2 + y^2}$ + x + iy = 3 + i

Separating the real and imaginary parts, we get:

\sqrt{x^2 + y^2} + x = 3

$y = 1$

Since we know that $y = 1$, we can substitute this into the first equation and then solve for x:

$\sqrt{x^2 + 1^2} + x = 3$

$\sqrt{x^2 + 1} + x = 3$

Let's isolate $\sqrt{x^2 + 1}$ on one side:

$\sqrt{x^2 + 1} = 3 - x$

Squaring both sides to eliminate the square root:

$x^2+1=(3-x)^2$

$x^2+1=9-6x+x^2$

Cancelling out x^2 from both sides and simplifying:

$1=9-6x$

$6x=9-1$

$6x=8$

$x=\frac{8}{6}$

$x=\frac{4}{3}$

Now we can find $|z|$ using the value of x and the fact that $y=1$:

$|z|=\sqrt{x^2+1}$

$|z|=\sqrt{\left(\frac{4}{3}\right)^2+1}$

$|z|=\sqrt{\frac{16}{9}+\frac{9}{9}}$

$|z|=\sqrt{\frac{25}{9}}$

$|z|=\frac{5}{3}$

Thus, the correct answer is **Option B:** $\frac{5}{3}$.

Q.39

Given $A=\begin{bmatrix} x & 3 & 2 \\ 1 & y & 4 \\ 2 & 2 & z \end{bmatrix}$, if $xyz=60$ and $8x+4y+3z=20$, then $A.$(adjA)

Options:

A.

$$\begin{bmatrix} 60 & 0 & 0 \\ 0 & 60 & 0 \\ 0 & 0 & 60 \end{bmatrix}$$

B.

$$\begin{bmatrix} 108 & 0 & 0 \\ 0 & 108 & 0 \\ 0 & 0 & 108 \end{bmatrix}$$

C.

$$\begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$

D.

$$\begin{bmatrix} 68 & 0 & 0 \\ 0 & 68 & 0 \\ 0 & 0 & 68 \end{bmatrix}$$

Answer: D

Solution:

To find the matrix product $A \cdot (\text{adj} A)$, we first need to understand the properties of a matrix and its adjugate (adjoint) matrix. The adjugate of a matrix A, denoted as adj$A$, is the transpose of the cofactor matrix of A. One important property relating a matrix and its adjugate involves the determinant of the matrix, given by the equation:

$$A \cdot (\text{adj} A) = (\det A) \cdot I$$

where I is the identity matrix of the same size as A, and $\det A$ is the determinant of the matrix A. This relation tells us that the product of a matrix and its adjugate is a diagonal matrix, where each diagonal element is equal to the determinant of the original matrix.

Given the matrix A and the equations $xyz = 60$ and $8x + 4y + 3z = 20$, we are asked to find $A \cdot (\text{adj} A)$. According to the property mentioned, this requires us to find the determinant of the given matrix A.

The determinant of A, $\det A$, for a 3x3 matrix $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ is given by:

$$\det A = a(ei - fh) - b(di - fg) + c(dh - eg)$$

Substituting the values from matrix A, we get:

$\det A = x(yz-8) - 3(z-8) + 2(2-2y)$

$\det A = xyz - 8x - 3z + 24 + 4 - 4y$

$\det A = xyz - (8x + 4y + 3z) + 24 + 4$

Since we know that $xyz = 60$ and have the equation $8x + 4y + 3z = 20$, we can substitute $xyz = 60$ and $8x + 4y + 3z = 20$ in ours determinant formula:

$\det A = 60 - 20 + 28 = 68$

But to proceed, we notice that directly applying the properties will be more efficient. By knowing that $\det A = 68$ (since it's given by the product xyz), all we need to use is the derived property of a matrix and its adjugate:

$A \cdot (\text{adj} A) = (\det A) \cdot I$

Given that $\det A = 68$, the right hand side of this equation becomes:

$A \cdot (\text{adj} A) = 68 \cdot I$

Where I is the identity matrix of the same dimensions as A, which is a 3x3 matrix. Therefore,

$$A \cdot (\text{adj} A) = \begin{bmatrix} 68 & 0 & 0 \\ 0 & 68 & 0 \\ 0 & 0 & 68 \end{bmatrix}$$

This matches with Option D. The correct answer is therefore Option D:

$$\begin{bmatrix} 68 & 0 & 0 \\ 0 & 68 & 0 \\ 0 & 0 & 68 \end{bmatrix}$$

Q.40

If $y^{\frac{1}{m}} + y^{\frac{-1}{m}} = 2x, x \neq 1$, then $(x^2 - 1)\left(\frac{dy}{dx}\right)^2$ is equal to

Options:

A.

my^2

B.

m^2y

C.

m^2y^2

D.

$\frac{my^2}{2}$

Answer: C

Solution:

$\because y^{1/m} + y^{-1/m} = 2x, \quad \ldots x \neq 1$ .... (1)

$\therefore$ Squaring both sides,

$y^{2/m} + y^{-2/m} + 2 = 4x^2$

$\therefore$ adding (-4) to both sides

$y^{2/m} + y^{-2/m} - 2 = 4x^2 - 4$

$\therefore \left(y^{1/m} - y^{-1/m}\right)^2 = 4\left(x^2 - 1\right)$

$\therefore y^{1/m} - y^{-1/m} = 2\sqrt{x^2 - 1}$ ..... (2)

$\therefore (1) + (2) \Rightarrow 2y^{1/m} = 2x + 2\sqrt{x^2 - 1}$

$\therefore \quad y^{1/m} = x + \sqrt{x^2 - 1}$

$\therefore \quad y = \left(x + \sqrt{x^2 - 1}\right)^m$

$\therefore \quad \log y = m \cdot \log\left(x + \sqrt{x^2 - 1}\right)$

$\therefore$ Differentiating b.s. w.r.t. x,

$$\frac{1}{y} \cdot \frac{\mathrm{d}y}{\mathrm{d}x} = m \cdot \frac{1}{x + \sqrt{x^2 - 1}} \cdot \left[1 + \frac{1}{2\sqrt{x^2 - 1}} \cdot 2x\right]$$

$$\therefore \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{my}{x + \sqrt{x^2 - 1}} \times \frac{\sqrt{x^2 - 1} + x}{\sqrt{x^2 - 1}}$$

$$\therefore \sqrt{x^2 - 1} \cdot \frac{\mathrm{d}y}{\mathrm{d}x} = my$$

$\therefore$ Squaring b.s. : $\left(x^2 - 1\right) \cdot \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = m^2 y^2$

Q.41

$$\int_0^2 [x]\mathrm{d}x + \int_0^2 |x - 1|\mathrm{d}x =$$

(where [x] denotes the greatest integer function.)

Options:

A.

4

B.

3

C.

1

D.

2

Answer: D

Solution:

To evaluate the given integrals, let's tackle them one by one.

First, let's evaluate the integral with the greatest integer function (also known as the floor function), $\int_0^2 [x] \mathrm{d}x$.

The greatest integer function [x] returns the greatest integer less than or equal to x. On the interval from 0 to 2, [x] takes on the values 0 and 1. Specifically:

For $0 \leq x < 1, [x] = 0$
For $1 \leq x < 2, [x] = 1$

Hence, the integral of the greatest integer function can be separated into two integrals:

$$\int_0^2 [x] \mathrm{d}x = \int_0^1 0 \, \mathrm{d}x + \int_1^2 1 \, \mathrm{d}x$$

Evaluating these integrals, we get:

$$= (0 \cdot x)\Big|_0^1 + (x)\Big|_1^2 = (0 \cdot 1 - 0 \cdot 0) + (2 - 1) = 0 + 1 = 1$$

Next, let's evaluate the integral of the absolute value function, $\int_0^2 |x - 1| \mathrm{d}x$.

The absolute function $|x - 1|$ equals $x - 1$ when $x \geq 1$ and equals $-x + 1$ when $x < 1$.
Consequently, the integral can be separated at the point where $x = 1$:

$$\int_0^2 |x - 1| \mathrm{d}x = \int_0^1 (-x + 1) \mathrm{d}x + \int_1^2 (x - 1) \mathrm{d}x$$

Evaluating the first part of the integral:

$$\int_0^1 (-x + 1) \mathrm{d}x = \left(-\frac{x^2}{2} + x\right)\Big|_0^1 = \left(-\frac{1^2}{2} + 1\right) - \left(-\frac{0^2}{2} + 0\right) = (-\frac{1}{2} + 1) - 0 = \frac{1}{2}$$

Now, evaluating the second part:

$$\int_1^2 (x - 1) \mathrm{d}x = \left(\frac{x^2}{2} - x\right)\Big|_1^2 = \left(\frac{2^2}{2} - 2\right) - \left(\frac{1^2}{2} - 1\right) = (2 - 2) - (\frac{1}{2} - 1) = 0 - (-\frac{1}{2})$$
$$= \frac{1}{2}$$

Adding the results of both parts of the absolute value integral:

$$\frac{1}{2} + \frac{1}{2} = 1$$

Finally, combining the results from both original integrals:

$$\int_0^2 [x] \mathrm{d}x + \int_0^2 |x - 1| \mathrm{d}x = 1 + 1 = 2$$

Therefore, the correct answer is Option D, which is 2.

Q.42

The equations of the lines passing through the point (3,2) and making an acute angle of 45° with the line x − 2y − 3 = 0 are

Options:

A.

3x + y – 11 = 0, x + 3y + 9 = 0

B.

3x – y – 7 = 0, x + 3y – 9 = 0

C.

3x + y – 11 = 0, x + 3y – 9 = 0

D.

x + 2y – 7 = 0, 2x – y – 4 = 0

Answer: B

Solution:

To find the equations of the lines passing through the point (3,2) and making an acute angle of 45° with the line $x-2y-3=0$, we first need to find the slope of the given line. The slope-intercept form of a line is $y=mx+b$, where m is the slope of the line.

The given line is:

$x-2y-3=0$

Let's rewrite it in slope-intercept form by isolating y:

$2y=x-3$

$y=\frac{1}{2}x-\frac{3}{2}$

Now we have the slope of the given line as $m=\frac{1}{2}$.

Lines that make an angle of 45° with a given line can be found using the angle between two lines formula. If m_1 is the slope of the first line and m_2 is the slope of the second line, the tangent of the angle θ between them is given by:

$\tan(\theta)=\left|\frac{m_2-m_1}{1+m_1m_2}\right|$

For our case, $\theta = 45^\circ$ and $m_1 = \frac{1}{2}$, so we can plug those values into the formula to find m_2, the slope of the lines we're looking for:

$$\tan(45^\circ) = \left| \frac{m_2 - \frac{1}{2}}{1 + m_2 \cdot \frac{1}{2}} \right|$$

$$\tan(45^\circ) = 1 = \left| \frac{m_2 - \frac{1}{2}}{1 + \frac{m_2}{2}} \right|$$

The absolute value gives us two equations to solve, one for each line:

$$\frac{m_2 - \frac{1}{2}}{1 + \frac{m_2}{2}} = 1$$

$$\frac{m_2 - \frac{1}{2}}{1 + \frac{m_2}{2}} = -1$$

Let's solve the first equation:

$$m_2 - \frac{1}{2} = 1 + \frac{m_2}{2}$$

$$2m_2 - 1 = 2 + m_2$$

$$m_2 = 3$$

Now the second equation:

$$m_2 - \frac{1}{2} = -\left(1 + \frac{m_2}{2}\right)$$

$$m_2 - \frac{1}{2} = -1 - \frac{m_2}{2}$$

$$2m_2 - 1 = -2 - m_2$$

$$3m_2 = -1$$

$$m_2 = -\frac{1}{3}$$

So we have two slopes: $m_2 = 3$ and $m_2 = -\frac{1}{3}$. Now we can use the point-slope form to find the equations of the lines passing through (3,2) with these slopes.

For $m_2 = 3$:

$$y - 2 = 3(x - 3)$$

$$y - 2 = 3x - 9$$

$$3x - y - 7 = 0$$

For $m_2 = -\frac{1}{3}$:

$$y - 2 = -\frac{1}{3}(x - 3)$$

$$y - 2 = -\frac{1}{3}x + 1$$

$$x + 3y - 9 = 0$$

Thus, the equations of the lines are $3x - y - 7 = 0$ and $x + 3y - 9 = 0$, which corresponds to Option B:

$$3x - y - 7 = 0$$

$$x + 3y - 9 = 0$$

Q.43

If [x] is greatest integer function and 2[2x − 5] − 1 = 7, then x lies in

Options:

A.

$\left(\frac{9}{2}, 5\right)$

B.

$\left(\frac{9}{2}, 5\right]$

C.

$\left[\frac{9}{2}, 5\right]$

D.

$\left[\frac{9}{2}, 5\right)$

Answer: D

Solution:

Let's start by solving the given equation step by step. We have: $2[2x-5]-1=7$

First, we add 1 to both sides: $2[2x-5]=8$

Then, we divide both sides by 2: $[2x-5]=4$

The greatest integer function $[y]$ outputs the greatest integer less than or equal to y. If $[y]=n$, where n is an integer, this implies that $n \leq y < n+1$. Therefore, $[2x-5]=4$ implies that: $4 \leq 2x-5 < 5$

Now let's solve for x by adding 5 to all parts of the inequality: $4+5 \leq 2x < 5+5$
$9 \leq 2x < 10$

Finally, we divide all parts of the inequality by 2 to solve for x: $\frac{9}{2} \leq x < \frac{10}{2}$ $\frac{9}{2} \leq x < 5$

This means that x can be equal to or greater than $\frac{9}{2}$, but must be less than 5. Hence, the correct option which portrays this interval is:

Option D: $\left[\frac{9}{2}, 5\right)$ where x is greater than or equal to $\frac{9}{2}$ and less than 5.

Q.44

The Cartesian equation of a line passing through (1, 2, 3) and parallel to x – y + 2z = 5 and 3x + y + z = 6 is

Options:

A.

$\frac{x-1}{-3} = \frac{y-2}{5} = \frac{z-3}{4}$

B.

$\frac{x-1}{3} = \frac{y-2}{1} = \frac{z-3}{1}$

C.

$\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-3}{1}$

D.

$\frac{x-1}{-3} = \frac{y-2}{-5} = \frac{z-3}{4}$

Answer: D

Solution:

Required line $L \rightarrow P(1, 2, 3)$

$L \parallel E_1 : 1x - 1y + 2z = 5$

$L \parallel E_2 : 3x + 1y + 1z = 6$

$\overline{n_1} \equiv (1, -1, 2), \overline{n_2} \equiv (3, 1, 1)$

$$\therefore \overline{n_1} \times \overline{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 3 & 1 & 1 \end{vmatrix} = -3\hat{i} - 5\hat{j} + 4\hat{k}$$

$\equiv (-3, -5, 4)$

$\therefore$ d.R.s. of line L are $(-3, -5, 4)$ and it passes through $(1, 2, 3)$

$\therefore$ its cartesian equations are

$$\frac{x-1}{-3} = \frac{y-2}{-5} = \frac{z-3}{4}$$

Q.45

If the lines 3x – 4y – 7 = 0 and 2x – 3y – 5 = 0 pass through diameters of a circle of area 49π square units, then the equation of the circle is

Options:

A.

$x^2 + y^2 - 2x + 2y - 47 = 0$

B.

$x^2 + y^2 - 2x + 2y + 51 = 0$

C.

$x^2 + y^2 + 2x - 2y - 51 = 0$

D.

$x^2 + y^2 + 2x + 2y + 47 = 0$

Answer: A

Solution:

The lines $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ are given to pass through the diameters of a circle. Since both lines are diameters of the circle, the point of intersection of the two lines will be the center of the circle.

To find the point of intersection, we can solve the two equations simultaneously. We can use the substitution or elimination method. Let's use the elimination method in this case by multiplying the first equation by 3 and the second by 4 so that the coefficients of y match and can thus be eliminated.

Multiplying the first equation by 3:

$$3 \cdot (3x - 4y - 7) = 3 \cdot 0$$

$$9x - 12y - 21 = 0$$

Multiplying the second equation by 4:

$$4 \cdot (2x - 3y - 5) = 4 \cdot 0$$

$$8x - 12y - 20 = 0$$

Subtracting the second equation from the first:

$$9x - 12y - 21 - (8x - 12y - 20) = 0$$

$$x - 1 = 0 \Rightarrow x = 1$$

Now we substitute $x = 1$ back into either original equation to solve for y. Using the first original equation:

$$3 \cdot 1 - 4y - 7 = 0$$

$$3 - 4y - 7 = 0$$

$$-4y = 4$$

$$y = -1$$

So the center of the circle is (1, -1).

The area of the circle is given by 49π, which means that the radius squared, r^2, is:

$$\pi r^2 = 49\pi \Rightarrow r^2 = 49$$

The standard form of the equation of a circle with center (h, k) and radius r is given by:

$(x-h)^2+(y-k)^2=r^2$

Substituting the center (1, -1) and radius $r=\sqrt{49}=7$ gives us:

$(x-1)^2+(y+1)^2=7^2$

$x^2-2x+1+y^2+2y+1=49$

Now we bring all terms to one side to set the equation equal to zero:

$x^2+y^2-2x+2y+2-49=0$

$x^2+y^2-2x+2y-47=0$

Therefore, the correct equation of the circle is:

Option A: $x^2+y^2-2x+2y-47=0$

Q.46

A spherical iron ball of 10 cm radius is coated with a layer of ice of uniform thickness that melts at the rate of 50 cm^3 /min. If the thickness of ice is 5 cm, then the rate at which the thickness of ice decreases is

Options:

A.

$\frac{-1}{18\pi}\mathrm{cm/min}$

B.

$\frac{2}{9\pi}\mathrm{cm/min}$

C.

$\frac{1}{18\pi}\mathrm{cm/min}$

D.

$\frac{1}{3\pi}\mathrm{cm/min}$

Answer: C

Q.47

$\int \frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}dx =$

(where C is a constant of integration.)

Options:

A.

x + sin x + sin 2x + C

B.

x + sin x + sin 2x – C

C.

x + 2 sin x + 2 sin 2x + C

D.

None of these

Answer: D

Solution:

$$I = \int \frac{\sin\left(\frac{5x}{2}\right)}{\sin\left(\frac{x}{2}\right)} dx$$

Putting $\frac{x}{2} = \theta$, i.e., $dx = 2d\theta$

$$I = \int \frac{\sin 5\theta}{\sin\theta} d\theta$$

$$= \int \frac{\sin\theta \cdot (2\cos 2\theta + 2\cos 4\theta + 1)}{\sin\theta} \cdot 2d\theta$$

$$= 2 \cdot \int (1 + 2\cos 2\theta + 2\cos 4\theta)\, d\theta$$

$$= 2\left[\theta + 2\left(\frac{\sin 2\theta}{2}\right) + 2\left(\frac{\sin 4\theta}{4}\right)\right] + c, \ldots x = 2\theta$$

$$= 2\theta + 2\sin 2\theta + \sin 4\theta + c$$

$$= x + 2\sin x + \sin 2x + c \qquad \textbf{... (Ans.)}$$

Note : None of the given option matches the answer.

Q.48

The equation of the plane passing through the points (2, 3, 1), (4, –5, 3) and parallel to X-axis is

Options:

A.

3y + 4z = 13

B.

y – 4z = –1

C.

2y + 4z = 19

D.

y + 4z = 7

Answer: D

Solution:

To find the equation of a plane passing through two points and parallel to the X-axis, we need to understand that a plane parallel to the X-axis would have a normal vector that is orthogonal (perpendicular) to the X-axis. Since the X-axis is represented by the vector (1,0,0), any normal vector to the plane will have the form (0,b,c), where b and c are real numbers. This means that the plane's normal vector does not have an X component.

The general equation of a plane in three dimensions is given by:

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

where (x_0, y_0, z_0) is a point on the plane, and (A, B, C) is a normal vector to the plane.

Since the plane is parallel to the X-axis, we know that $A = 0$. Therefore, the equation simplifies to:

$$B(y - y_0) + C(z - z_0) = 0$$

Now we need to find values for B and C that will allow the plane to pass through $(2, 3, 1)$ and $(4, -5, 3)$. These two points will provide us the direction vectors along the plane:

From point $(2, 3, 1)$ to $(4, -5, 3)$, we get the direction vector by subtracting the respective components:

$$(4 - 2, -5 - 3, 3 - 1) = (2, -8, 2).$$

However, we need a direction vector that lies on the plane and is perpendicular to the X-axis direction vector. Given that the plane is parallel to the X-axis, the direction vector of the plane can simply be taken from the YZ components of the direction vector we found:

$$(0, -8, 2).$$

Now we can normalize this vector to get the plane's normal vector. Multiplying by a constant does not change the direction, so we can use $(0, B, C) = (0, -8, 2)$ directly or choose to simplify it by dividing by a common factor, for instance, dividing by 2 to get $(0, -4, 1)$ so:

$$(B, C) = (-4, 1)$$

Now let's use one of the given points, $(2, 3, 1)$, to find the equation:

$$-4(y - 3) + 1(z - 1) = 0$$

$$-4y + 12 + z - 1 = 0$$

$$-4y + z + 11 = 0$$

This can be rearranged to:

$$4y - z = 11$$

So the correct equation of the plane should be:

$$4y - z = 11$$

This is not present in the given options. Let's check the options to see if any of them would be equivalent:

If we consider Option A, we get:

$3y + 4z = 13$

For this equation to represent the same plane as $4y - z = 11$, we would need the ratio between coefficients of y and z to be the same, and the independent term to scale accordingly.

Let's evaluate this by putting y and z from the point $(2, 3, 1)$ into Option A:

$3(3) + 4(1) = 9 + 4 = 13$

Option A satisfies the point $(2, 3, 1)$. Now we need to do the same for the second point $(4, -5, 3)$:

$3(-5) + 4(3) = -15 + 12 = -3$

But for this plane,

$4y - z = 11$

$4(-5) - 3 = -20 - 3$

$-23 \neq 11$

Hence Option A does not represent the same plane as $4y - z = 11$.

Option B:

$y - 4z = -1$

Using the given points we can test for Option B in a similar fashion:

$(2, 3, 1): \quad 3 - 4(1) = 3 - 4 = -1$

$(4, -5, 3): \quad -5 - 4(3) = -5 - 12 = -17 \neq -1$

So, Option B can be ruled out as well.

Performing a similar verification for Options C and D:

Option C:

$2y + 4z = 19$

Using the given points:

$(2, 3, 1): \quad 2(3) + 4(1) = 6 + 4 = 10 \neq 19$

$(4, -5, 3): \quad 2(-5) + 4(3) = -10 + 12 = 2 \neq 19$

Option C is not the correct option.

Option D:

$y + 4z = 7$

Using the given points:

$(2, 3, 1): \quad 3 + 4(1) = 3 + 4 = 7$

$(4, -5, 3): \quad -5 + 4(3) = -5 + 12 = 7$

Option D works with both points. Moreover, the ratio of coefficients between y and z in our correct equation $4y - z = 11$ is 4 to -1. We can invert this to get -1 to 4, which matches the ratio for the coefficients of y and z in Option D, after rearranging it to be $y + 4z - 7 = 0$. This indicates that Option D could represent the same plane as our equation but with the terms moved around.

Based upon this analysis, the correct answer should be:

Option D

$y + 4z = 7$

Q.49

If $\int e^{x^2} \cdot x^3 \, dx = e^{x^2} \cdot [f(x) + C]$ (where C is a constant of integration.) and $f(1) = 0$, then value of $f(2)$ will be

Options:

A.

$\frac{-3}{2}$

B.

$\frac{-1}{2}$

C.

3/2

D.

1/2

Answer: C

Solution:

$$I = \int e^{x^2} \cdot x^3 \, dx$$
$$= \frac{1}{2} \int x^2 \cdot e^{x^2} \cdot (2x) dx$$
$$= \frac{1}{2} \int (t \cdot e^t) dt, \quad \ldots t = x^2$$
$$= \frac{1}{2} \left[t \left(e^t\right) - \int \left(e^t\right)(1) dt \right]$$
$$= \frac{1}{2}(t-1) \cdot e^t$$
$$= \frac{1}{2}\left(x^2 - 1\right) \cdot e^{x^2}$$
$$= e^{x^2} \cdot [f(x) - c] \quad \ldots \text{(Given)}$$

$$\therefore f(x) = \frac{1}{2}\left(x^2 - 1\right) + c$$
$$\because f(1) = 0 \quad \therefore \frac{1}{2}(0) + c = 0 \quad \therefore c = 0$$
$$\therefore f(x) = \frac{1}{2}\left(x^2 - 1\right) + 0$$
$$\therefore f(2) = \frac{1}{2}(4-1) = \frac{3}{2}$$

Q.50

The negation of the statement, "The payment will be made if and only if the work is finished in time" is

Options:

A.

The work is finished in time and the payment is not made or the payment is made and the work is finished in time.

B.

The work is finished in time and the payment is not made.

C.

The payment is made and the work is not finished in time.

D.

Either the work is finished in time and the payment is not made or the payment is made and the work is not finished in time.

Answer: D

Solution:

The given statement is a biconditional statement, which has the form:

$P \Leftrightarrow Q$

where P is "the work is finished in time," and Q is "the payment will be made."

In propositional logic, the negation of a biconditional statement $P \Leftrightarrow Q$ is:

$\neg(P \Leftrightarrow Q)$

Which is logically equivalent to:

$(P \wedge \neg Q) \vee (\neg P \wedge Q)$

That is, for the negation of "if and only if," one part must be true while the other part must be false.

So, for the negation of the statement "The payment will be made if and only if the work is finished in time," we can interpret the negated statement as either:

1. The work is finished in time and the payment is not made. (This corresponds to $P \wedge \neg Q$).

2. The payment is made and the work is not finished in time. (This corresponds to $\neg P \wedge Q$).

Therefore, the negation of the statement would mean that one of these two possibilities must occur – either the work is finished on time and payment is not made, or the payment is made but the work is not finished on time.

Hence, the correct negation of the given statement is:

Option D

"Either the work is finished in time and the payment is not made or the payment is made and the work is not finished in time."

Physics

Q.1

The magnetic susceptibility of the material of a rod is 349 and permeability of vacuum μ_0 is $4\pi \times 10^{-7}$ SI units. Absolute permeability of the material of the rod in SI units is

Options:

A.

4400×10^{-7}

B.

4200×10^{-7}

C.

4800×10^{-7}

D.

4600×10^{-7}

Answer: A

Solution:

The relative permeability $\mu_R = 1 + \chi$

$\therefore$ Absolute permeability

$$\mu = \mu_R\mu_0 = \mu_0(1+\chi)$$
$$= 4\pi \times 10^{-7}[1+349]$$
$$= 350 \times 4 \times 3.142 \times 10^{-7}$$
$$\doteqdot 4400 \times 10^{-7} \text{ SI units}$$

Q.2

The magnetic flux through a coil of resistance 'R' changes by an amount '$\Delta\phi$' in time 'Δt'. The total quantity of induced electric charge 'Q' is

Options:

A.

$-\frac{\Delta\phi}{\Delta t} + R$

B.

$\frac{\Delta\phi}{\Delta t} \times R$

C.

$\frac{\Delta\phi}{R}$

D.

$\frac{\Delta\phi}{\Delta t}$

Answer: C

Solution:

By Faraday's law, $e = \frac{\Delta\phi}{\Delta t}$

$\therefore i = \frac{e}{R} = \frac{\Delta\phi}{R\Delta t}$

$\therefore (i\Delta t) = \frac{\Delta\phi}{R}$ ..... (1)

But $i = \frac{\Delta Q}{\Delta t}$ $\therefore \Delta Q = i\Delta t$

$\therefore$ From (1), $\Delta Q = \frac{\Delta\phi}{R}$

$\therefore Q = \frac{\Delta\phi}{R}$

Q.3

A body weighs 500 N on the surface of the earth. At what distance below the surface of the earth it weighs 250 N ? (Radius of earth, R = 6400 km)

Options:

A.

6400 km

B.

800 km

C.

1600 km

D.

3200 km

Answer: D

Solution:

The value of g at a depth h below the surface of the earth of radius R is

$$g' = g\left[1 - \frac{d}{R}\right]$$

$$\therefore \frac{g'}{g} = 1 - \frac{d}{R} \quad ..... (1)$$

It is given that $mg = 500\,N$ and $mg' = 250\,N$

$$\therefore \frac{g'}{g} = \frac{250}{500} = \frac{1}{2} \quad ..... (2)$$

$\therefore$ From (1) and (2), $\frac{1}{2} = 1 - \frac{d}{R}$

$$\therefore \frac{d}{R} = \frac{1}{2}$$

$$\therefore d = \frac{R}{2} = \frac{6400}{2} = 3200\,km$$

Q.4

Three discs x, y and z having radii 2 m, 3 m and 6 m respectively are coated on outer surfaces. The wavelength corresponding to maximum intensity are 300 nm, 400 nm and 500 nm respectively. If P_x , P_y and P_z are power radiated by them respectively then

Options:

A.

P_x is maximum

B.

P_z is maximum

C.

P_y is maximum

D.

$P_x = P_y = P_z$

Answer: B

Solution:

According to Wien's law, $\lambda_m T$ = constant (b)

$\therefore T = \frac{b}{\lambda_m}$ ..... (1)

and from Stefan's law, $Q = \sigma A T^4$ .... (2)

For the disc, area $(A) = \pi r^2$

$\therefore$ From (1) and (2),

$Q = \sigma \cdot \pi r^2 \cdot \frac{b^4}{\lambda_m^4} = \frac{Kr^2}{(\lambda m)^4}$

where $K = \pi \sigma b^4$ is a constant

Q is the quantity of heat radiated per second or power.

Hence P_x, P_y and P_z are the powers of x, y, z.

For x we have $r_1 = 2$ m and $\lambda_1 = 300$ nm

For y, we have $r_2 = 3$ m and $\lambda_2 = 400$ nm

and For z, we have $r_3 = 6$ m and $\lambda_3 = 500$ nm

$$\therefore P_x \propto \frac{r_1^2}{\lambda_1^4} \text{ or } \frac{2^2}{(3 \times 10^{-7})^4} \text{ or } \frac{4 \times 10^{+28}}{81}$$

$$P_y \propto \frac{r_2^2}{\lambda_2^4} \text{ or } \frac{9 \times 10^{+28}}{256}$$

$$P_z \propto \frac{r_3^2}{\lambda_3^4} \text{ or } \frac{36 \times 10^{28}}{625}$$

But $\frac{4}{81} = 0.049$, $\frac{9}{250} = 0.035$ and $\frac{36}{625} = 0.0576$

$\therefore$ $\mathbf{P_z}$ is maximum.

Q.5

A stationary wave is represented by $y = 10 \sin\left(\frac{\pi x}{4}\right) \cos(20\pi t)$ where x and y are in cm and t in second. The distance between two consecutive nodes is

Options:

A.

1 cm

B.

8 cm

C.

4 cm

D.

2 cm

Answer: C

Solution:

$y=10\sin\left(\frac{\pi x}{4}\right)\cos(20\pi t)$

Comparing with $y=2A\cos\left(\frac{2\pi x}{\lambda}\right)\sin\left(\frac{2\pi t}{T}\right)$, we get

$$\frac{2\pi x}{\lambda}=\frac{\pi x}{T} \quad \therefore \frac{2}{\lambda}=\frac{1}{4}$$
$$\therefore \lambda=8\text{ cm}$$

$\therefore$ The distance between two consecutive nodes

$=\frac{\lambda}{2}=\frac{8}{2}=4\text{ cm}$

Q.6

When the rms velocity of a gas is denoted by 'v', which one of the following relations is true ?

(T = Absolute temperature of the gas.)

Options:

A.

$\frac{v^2}{T}=$ constant

B.

$v^2T=$ constant

C.

$vT^2=$ constant

D.

$\frac{v}{T^2}=$ constant

Answer: A

Solution:

The r.m.s. speed (v) of a gas is $v=\sqrt{\frac{3RT}{M}}$ i.e. $v\propto\sqrt{T}$

$\therefore v^2\propto T \quad \therefore v^2=KT$ or $\frac{v^2}{T}=$ constant

----

Q.7

A parallel plate air capacitor has a uniform electric field 'E' in the space between the plates. Area of each plate is A and the distance between the plates is 'd'. The energy stored in the capacitor is [ε_0 = permittivity of free space)

Options:

A.

$2\varepsilon_0 EAd$

B.

$\frac{1}{2}\varepsilon_0 E^2 Ad$

C.

$\frac{\varepsilon_0 E^2}{2Ad}$

D.

$\frac{E^2 Ad}{2\varepsilon_0}$

Answer: B

Solution:

The intensity of the electric field (E) between two plane parallel sheets of equal and opposite charges is given by $E = \frac{\sigma}{\varepsilon_0}$

$\therefore \sigma = E\varepsilon_0$ where σ = surface density of charge $= \frac{Q}{A}$

$\therefore$ Charge on either plate of the capacitor is $Q = \sigma A = \varepsilon_0 EA$ and $C = \frac{\varepsilon_0 A}{d}$

$\therefore$ The energy stored in the capacitor is

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{(\varepsilon_0 EA)^2}{2 \cdot \frac{\varepsilon_0 A}{d}} = \frac{\varepsilon_0^2 A^2 E^2 \times d}{2\varepsilon_0 A}$$

$$\therefore \mathbf{U = \frac{1}{2}\varepsilon_0 E^2 Ad}$$

Q.8

Two massless springs of spring constant K_1 and K_2 are connected one after the other forming a single chain, suspended vertically and certain mass is attached to the free end. If 'e_1' and 'e_2' are their respective extensions and 'f' is their stretching force, the total extension produced is

Options:

A.

$f\left(\frac{1}{K_1}+\frac{1}{K_2}\right)$

B.

$f\left(\frac{1}{K_1}-\frac{1}{K_2}\right)$

C.

$f(K_1+K_2)$

D.

$f(K_1-K_2)$

Answer: A

Solution:

For a spring $F = Kx$

$\therefore x = \frac{F}{K}$

$\therefore x_1 = \frac{F}{K_1}$ and $x_2 = \frac{F}{K_2}$

or $e_1 = \frac{F}{K_1}$ and $e_2 = \frac{F}{K_2}$

$\therefore e_1 + e_2 = F\left(\frac{1}{K_1}+\frac{1}{K_2}\right)$

Note: The springs are connected in series.

$\therefore$ The effective spring constant $K = \frac{K_1 K_2}{K_1+K_2}$

$\therefore$ Total extension

$e = \frac{F}{K} = \frac{F(K_1+K_2)}{K_1 K_2} = \frac{F}{K_2} + \frac{F}{K_1}$

Q.9

The time taken by a particle executing simple harmonic motion of period 'T', to move from the mean position to half the maximum displacement is

Options:

A.

$\frac{T}{12}$ s

B.

$\frac{T}{2}$ s

C.

$\frac{T}{4}$ s

D.

$\frac{T}{6}$ s

Answer: A

Solution:

$$x = A\sin\omega t = A\sin\left(\frac{2\pi t}{T}\right) \text{ where } x = A/2$$
$$\therefore \frac{A}{2} = A\sin\left(\frac{2\pi t}{T}\right)$$
$$\therefore \frac{1}{2} = \sin\frac{\pi}{6} = \sin\left(\frac{2\pi t}{T}\right)$$
$$\therefore \frac{2\pi t}{T} = \frac{\pi}{6}$$
$$\therefore t = \frac{T}{12} \text{ s}$$

Q.10

Using Bohr's model, the orbital period of electron in hydrogen atom in the n^{th} orbit is (ε_0 = permittivity of vacuum, h = Planck's constant, m = mass of electron, e = electronic charge)

Options:

A.

$\frac{4\varepsilon_0 nh^3}{me^2}$

B.

$\frac{4\varepsilon_0 n^2 h^2}{me^2}$

C.

$\frac{4\varepsilon_0^2 n^3 h^3}{me^4}$

D.

$\frac{4\varepsilon_0^2 n^2 h^3}{me^3}$

Answer: C

Solution:

The period of an electron in the n^{th} orbit of a hydrogen atom is given by

$$T_n = \frac{2\pi r_n}{v_n}$$

and $r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2}$

and linear speed $v_n = \frac{e^2}{2\varepsilon_0 n h}$

$$\therefore T_n = \frac{2\pi \cdot \varepsilon_0 n^2 h^2}{\pi m e^2} \times \frac{2\varepsilon_0 n h}{e^2} = \frac{4\varepsilon_0^2 n^3 h^3}{m e^4}$$

Q.11

A parallel plate capacitor is charged and then disconnected from the charging battery. If the plates are now moved further apart by pulling them by means of insulating handles, then

Options:

A.

the capacitance of the capacitor increases

B.

the charge on the capacitor decreases

C.

the voltage across the capacitor increases

D.

the energy stored in the capacitor decreases

Answer: C

Solution:

For a parallel plate capacitor, $C = \frac{\varepsilon_0 A}{d}$

When the charging battery is disconnected and d is increased then

(a) Q remains constant

(b) $C \propto \frac{1}{d}$ hence if d is increased, C is decreased.

(c) $C = \frac{Q}{V}$ or $V = \frac{Q}{C}$ if C is decreased, V will increase

(d) $E = \frac{1}{2}\frac{Q^2}{C}$ if C is decreased, then E will increase

Thus (a), (b) and (d) are wrong. Only (c) is correct.

Q.12

If the kinetic energy of a free electron doubles, it's de Broglie wavelength (λ) changes by a factor

Options:

A.

$1/\sqrt{2}$

B.

1/2

C.

$\sqrt{2}$

D.

2

Answer: A

Solution:

$\lambda = \frac{h}{\sqrt{2mE}}$ where E is the K.E.

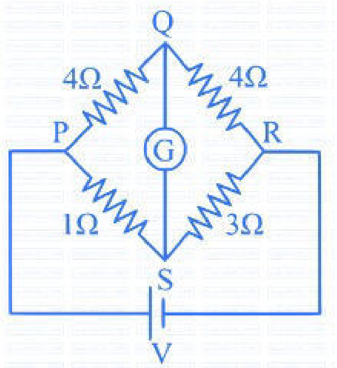
$$\therefore \lambda \propto \frac{1}{\sqrt{E}} \quad \therefore \frac{\lambda_2}{\lambda_1} = \sqrt{\frac{E_1}{E_2}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

$$\therefore \lambda_2 = \frac{\lambda_1}{\sqrt{2}}$$

Thus λ charges by a factor $\frac{1}{\sqrt{2}}$

Q.13

In the following network, the current through galvanometer will



Options:

A.

be zero

B.

flow from Q to S

C.

flow in a direction which depends on value of V

D.

flow from S to Q

Answer: D

Solution:

This is an unbalanced network.

Current through the branch PQR is $I_1 = \frac{V}{8}$ and current through the branch $PSR = I_2 = \frac{V}{4}$

$$\therefore V_P - V_Q = I_1 \times 4 = \frac{V}{8} \times 4 = \frac{V}{2}$$

and $V_P - V_S = I_2 \times 1 = \frac{V}{4} \times 1 = \frac{V}{4}$

$$\therefore (V_P - V_Q) - (V_P - V_S) = \frac{V}{2} - \frac{V}{4} = \frac{V}{4}$$

$$\therefore V_S - V_Q = \frac{V}{4}$$

$$\therefore V_S > V_Q$$

$\therefore$ The current will flow from $\mathbf{S}$ to $\mathbf{Q}$.

Q.14

In a medium, the phase difference between two particles separated by a distance 'x' is $\left(\frac{\pi}{5}\right)^c$. If the frequency of the oscillation of particles is 25 Hz and the velocity of propagation of the waves is 75 m/s, then the value of x is

Options:

A.

0.4 m

B.

0.1 m

C.

0.2 m

D.

0.3 m

Answer: D

Solution:

$\lambda = \frac{v}{n} = \frac{75}{25} = 3\,m$

and phase difference $= \frac{2\pi}{\lambda}$ (path difference)

$\therefore \frac{\pi}{5} = \frac{2\pi}{3}x$

$\therefore 10x = 3$

$\therefore x = \frac{3}{10} = 0.3\,m$

Q.15

The work done in blowing a soap bubble of radius R is 'W_1' at room temperature. Now the soap solution is heated. From the heated solution another soap bubble of radius 2R is blown and the work done is 'W_2'. Then

Options:

A.

$W_2 = W_1$

B.

$W_2 = 4W_1$

C.

$W_2 < 4W_1$

D.

$W_2 = 0$

Answer: C

Solution:

Understanding Surface Tension and Work Done

Surface tension (T): A property of liquids that causes their surface to behave like a stretched membrane, minimizing surface area. It's represented by the symbol T and has units of force per unit length.

Work done in forming a bubble: When you blow a bubble, you increase its surface area. This requires work to be done against the surface tension forces trying to minimize the area.

Relationship Between Work, Surface Area, and Surface Tension

The work done (W) in increasing the surface area of a liquid is:

$W = T\Delta A$

where:

T is the surface tension

ΔA is the change in surface area

Bubble 1:

Radius = R

Surface Area = $4\pi R^2$ (Since a soap bubble has two surfaces, inner and outer)

Work Done = W_1

Bubble 2:

Radius = 2R

Surface Area = $4\pi(2R)^2 = 16\pi R^2$

Work Done = W_2

Temperature's Effect:

Surface tension of liquids generally decreases with an increase in temperature.

Calculations

Since the surface area of bubble 2 is four times that of bubble 1, you might think $W_2 = 4W_1$. However, the surface tension is lower for bubble 2 due to the heated soap solution.

Conclusion

Because the surface tension decreases with temperature:

Work done (W_2) to form bubble 2 will be less than four times the work (W_1) needed to form bubble 1.

Therefore, the correct answer is C: $W_2 < 4W_1$

Q.16

A capacitor of capacitance 50μF is connected to a.c. source e = 220 sin 50t (e in volt, t in second). The value of peak current is

Options:

A.

$\frac{0.55}{\sqrt{2}}$ A

B.

$\frac{\sqrt{2}}{0.55}$ A

C.

0.55 A

D.

√2 A

Answer: C

Solution:

Given $e = 220\sin(50t)$

Comparing with $e = e_0 \sin \omega t$, we get

$e_0 = 220\ \mathrm{V}$, peak voltage

$C = 50\mu F = 50 \times 10^{-6}\ F$

$\therefore X_C = \frac{1}{\omega C} = \frac{1}{50 \times 50} \times 10^6 = 400\Omega$

$\therefore I_0 = \frac{e_0}{X_C} = \frac{220}{400} = 0.55\ A$

Peak current $= 0.55\ A$

Q.17

Two waves are superimposed whose ratio of intensities is 9:1. The ratio of maximum and minimum intensity is

Options:

A.

9 : 1

B.

4 : 1

C.

3 : 1

D.

5 : 3

Answer: B

Solution:

Given: $\frac{I_1}{I_2} = \frac{9}{1} = \frac{a_1^2}{a_2^2} \quad \therefore \frac{a_1}{a_2} = \frac{3}{1}$

$\therefore a_1 = 3a_2$

$\therefore \frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{(3a_2 + a_2)^2}{(3a_2 - a_2)^2}$

$= \frac{4^2}{2^2} = \frac{16}{4} = 4 : 1$

Q.18

The masses and radii of the moon and the earth are M_1,R_1 and M_2,R_2 respectively. Their centres are at a distance d apart. What should be the minimum speed with which a body of mass 'm' should be projected from a point midway between their

centres, so as to escape to infinity?

Options:

A.

$\frac{G(M_1+M_2)}{d}$

B.

$\sqrt[2]{\frac{G(M_1+M_2)}{d}}$

C.

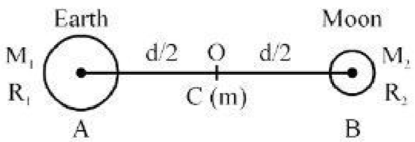
$\sqrt{\frac{Gd}{M_1+M_2}}$

D.

$\sqrt{\frac{M_1+M_2}{Gd}}$

Answer: B

Solution:



O is the midpoint of the line joining the centres of A and B. and a body (C) of mass 'm' is kept at O

The P.E. of C is

$$U = -\frac{GM_1m}{d/2} - \frac{GM_2m}{d/2} = -\frac{2Gm}{d}(M_1 + M_2)$$

Initially, C is at rest, its K.E. = 0

$\therefore$ Total energy of C $= -\frac{2Gm}{d}(M_1 + M_2)$

$\therefore$ Its binding energy $= \frac{2GM}{d}(M_1 + M_2)$ ... (1)

Let v_e be the velocity that should be given to the body to escape to infinity.

For this its K.E. = Binding energy

$$\therefore \frac{1}{2}mv_e^2 = \frac{2Gm}{d}(M_1 + M_2)$$

$$\therefore v_e^2 = \frac{4G(M_1 + M_2)}{d}$$

$$\therefore v_e = 2\sqrt{\frac{G(M_1 + M_2)}{d}}$$

Q.19

A monoatomic gas $\left(\gamma = \frac{5}{3}\right)$ initially at $27^{\circ}\mathrm{C}$ having volume 'V' is suddenly compressed to one-eighth of its original volume $\left(\frac{V}{8}\right)$. After the compression its temperature becomes

Options:

A.

580 K

B.

1200 K

C.

1160 K

D.

927 K

Answer: B

Solution:

For adiabatic charge, PV^{γ} = constant as well as $TV^{\gamma-1}$ = constant and for a monoatomic gas $\gamma = \frac{5}{3}$

$$\therefore T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\therefore \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{V}{V/8}\right)^{5/3-1}$$

$$\therefore \frac{T_2}{T_1} = (8)^{2/3} = \left(2^3\right)^{2/3} = 4$$

$\therefore T_2 = 4\,T_1$ but $T_1 = 273 + 27 = 300\,\mathrm{K}$

$\therefore T_2 = 4 \times 300 = 1200\,\mathrm{K}$

Q.20

Two parallel conducting wires of equal length are placed distance 'd' apart, carry currents 'I_1' and 'I_2' respectively in opposite directions. The resultant magnetic field at the midpoint of the distance between both the wires is

Options:

A.

$\frac{\mu_0(I_1-I_2)}{\pi d}$

B.

$\frac{\mu_0(I_1+I_2)}{2\pi d}$

C.

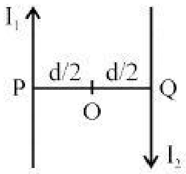
$\frac{\mu_0(I_1-I_2)}{2\pi d}$

D.

$\frac{\mu_0(I_1+I_2)}{\pi d}$

Answer: D

Solution:



The magnetic field at O, due to current in P is

$$B_1 = \frac{\mu_0}{4\pi}\left(\frac{2I_1}{d/2}\right) = \frac{\mu_0}{4\pi} \times \frac{4I_1}{d} = \frac{\mu_0 I_1}{\pi d}$$

and the magnetic field at O due to current I_2 in the wire Q is

$$B_2 = \frac{\mu_0}{4\pi}\left(\frac{2I_2}{d/2}\right) = \frac{\mu_0}{4\pi} \times \frac{4I_2}{d} = \frac{\mu_0 I_2}{\pi d}$$

The currents in the wires are in opposite directions.

Hence the magnetic fields will be added.

$\therefore$ Resultant field $B = B_1 + B_2 = \frac{\mu_0}{\pi d}(I_1 + I_2)$

Q.21

Self inductance of a solenoid cannot be increased by

Options:

A.

decreasing its length

B.

increasing its area of cross-section

C.

increasing the current through it

D.

increasing the number of turns in it

Answer: C

Solution:

The self inductance of a solenoid is given by

$$L = \frac{\mu_0 \mu_r \cdot N^2 A}{l}$$

L depends upon N,A and *l*. It does not depend upon the current flowing through it. Change in current does not affect L.

Q.22

For a NAND gate, the inputs and outputs are given below.

Input A	Input B	Output Y
0	1	C
0	0	D
1	0	E
1	1	F

The values taken by C, D, E, F are respectively

Options:

A.

0, 1, 0, 0

B.

1, 1, 1, 0

C.

1, 0, 1, 1

D.

0, 1, 0, 1

Answer: B

Solution:

For the NAND gate, the truth table is

Input A	Input B	Output $Y = \overline{AB}$	
0	1	1	(C)
0	0	1	(D)
1	0	1	(E)
1	1	0	(F)

Thus C = 1, D = 1, E = 1 and F = 0

For a NAND gate, there is a high output for a low input and a low output for high input.

Q.23

An electron and a proton having the same momenta enter perpendicularly into a magnetic field. What are their trajectories in the field?

Options:

A.

Path of the electron is more curved than that of proton

B.

They will travel undeflected

C.

Path of the proton is more curved than that of the electron

D.

Both the electron and the proton will move along the same curved path but they will move in opposite directions

Answer: D

Solution:

The force produced by the magnetic field on a moving charged particle is F = qvB and this gives a C.P. force

$$\frac{mv^2}{r}$$
$$\therefore \frac{mv^2}{r} = qvB$$
$$\therefore r = \frac{mv}{qB} = \frac{p}{qB} \text{ where } p = \text{ momentum}$$

It is given that both the particles (electron and proton) have the same momenta.

Similarly they have the same charge in magnitude (e and −e) and they move in the same field (B).

$$\therefore \frac{r_1}{r_2} = \frac{p_1}{q_1 B} \times \frac{q_2 B}{p_2} = 1$$
$$\because p_1 = p_2 \text{ and } q_1 = q_2$$
$$\therefore r_1 = r_2$$

∴ They will describe the same curved path.

[One will move clockwise and the other anticlockwise.]

Q.24

The resistance offered by an inductor ((X_L) in an a.c. circuit is

Options:

A.

inversely proportional to inductance and frequency of the alternating current

B.

inversely proportional to frequency of alternating current and directly proportional to inductance

C.

inversely proportional to inductance and directly proportional to the frequency of alternating current

D.

directly proportional to inductance and frequency of alternating current

Answer: D

Solution:

The resistance offered by an inductor (X_L) in an a.c. circuit is $X_L = \omega L = 2\pi f L$

$\therefore X_L \propto fL$ i.e. it is directly proportional to the inductance (L) and frequency (f).

Q.25

The force between the plates of a parallel plate capacitor of capacitance 'C' and distance of separation of the plates 'd' with a potential difference 'V' between the plates is

Options:

A.

$\frac{V^2 d}{C}$

B.

$\frac{C^2 V^2}{d^2}$

C.

$\frac{CV^2}{2d}$

D.

$\frac{C^2 V^2}{2d^2}$

Answer: C

Solution:

The force of attraction between the plates of a parallel plate capacitor is

$$F = \frac{\sigma^2 A}{2\varepsilon_0} = \frac{Q^2}{A^2} \times \frac{A}{2\varepsilon_0}\left(\because \sigma = \frac{Q}{A}\right)$$
$$= \frac{Q^2}{2\varepsilon_0 A} = \frac{C^2 V^2}{2\varepsilon_0 A}(\because Q = CV)$$
$$\therefore F = C\left[\frac{CV^2}{2\varepsilon_0 A}\right] = \frac{\varepsilon_0 A}{d} \times \frac{CV^2}{2\varepsilon_0 A}$$
$$\therefore F = \frac{1}{2}\frac{CV^2}{d}$$

Q.26

Consider the following statements about stationary waves.

A. The distance between two adjacent nodes or antinodes is equal to λ/2(λ = wavelength of the wave)

B. A node is always formed at the open end of the open organ pipe.

Choose the correct option from the following.

Options:

A.

Both statements A and B are wrong.

B.

Only the statement B is true.

C.

Only the statement A is true.

D.

Both statements A and B are true.

Answer: C

Solution:

The correct option is C: Only the statement A is true.

Statement A is indeed correct. In a stationary wave, or standing wave, nodes are points of zero amplitude, while antinodes are points of maximum amplitude. The distance between two adjacent nodes or two adjacent antinodes is half the wavelength of the wave, or λ/2. This is because one complete wavelength of the wave contains two node-to-node or antinode-to-antinode segments.

Statement B, however, is incorrect. A node represents a point of no displacement in a standing wave and is typically formed where there is a fixed end that cannot vibrate, like a clamped end of a string. In contrast, an open end of an organ pipe is free to move and thus supports an antinode, not a node. The pressure variation at an open end is minimal (corresponding to a displacement antinode), while pressure variations are maximal at a closed end (corresponding to a displacement node). Therefore, in an open organ pipe, there's actually an antinode at each open end if we're discussing a standing wave in terms of displacement rather than pressure.

Q.27

If the radius of the spherical gaussian surface is increased then the electric flux due to a point charge enclosed by the surface

Options:

A.

increases

B.

remains unchanged

C.

decreases

D.

zero

Answer: B

Solution:

If the radius of the spherical Gaussian surface is increased, then the electric flux due to a point charge enclosed by the surface remains constant.

Flux depends only on the enclosed charge. It does not depend upon the size or shape of the Gaussian surface.

Q.28

The wave number of the last line of the Balmer series in hydrogen spectrum will be

(Rydberg's constant $=10^7\ m^{-1}$)

Options:

A.

250 m^{-1}

B.

$2.5 \times 10^6\ m^{-1}$

C.

$0.25 \times 10^9\ m^{-1}$

D.

$2.5 \times 10^5\ m^{-1}$

Answer: B

Solution:

Wave number $= \frac{1}{\lambda} =$ Receiprocal of wavelength

For the last line of the Balmer series, $n = \infty$ and the transition is from $n = \infty$ to $n = 2$

$\therefore$ Wave number $\bar{v} = \frac{1}{\lambda} = \mathrm{R}\left[\frac{1}{2^2} - \frac{1}{\infty}\right] = \frac{\mathrm{R}}{4}$ and $\mathrm{R} = 10^7/\mathrm{m}$

$\therefore \bar{v} = \frac{R}{4} = \frac{10^7}{4} = \frac{10 \times 10^6}{4} = 2.5 \times 10^6\ \mathrm{m/s}$

Q.29

A bucket containing water is revolved in a vertical circle of radius r. To prevent the water from falling down, the minimum frequency of revolution required is

(g = acceleration due to gravity)

Options:

A.

$2\pi\sqrt{\frac{\mathrm{r}}{\mathrm{g}}}$

B.

$\frac{1}{2\pi}\sqrt{\frac{\mathrm{r}}{\mathrm{g}}}$

C.

$\frac{1}{2\pi}\sqrt{\frac{\mathrm{g}}{\mathrm{r}}}$

D.

$2\pi\sqrt{\frac{\mathrm{g}}{\mathrm{r}}}$

Answer: C

Solution:

To answer this question, we need to consider the forces in action when the bucket is at the topmost point in its circular path. At that point, the centripetal force required to keep the water moving in a circular path must be greater than or equal to the gravitational force acting on the water, to prevent it from falling out of the bucket.

The centripetal force (F_c) can be described by the following equation, where m is the mass of water, v is the linear velocity of the bucket, and r is the radius of the circle:

$F_c = \frac{mv^2}{r}$

At the minimum velocity needed to keep the water in the bucket, this centripetal force is provided entirely by the weight of the water, which is mg, where g is the acceleration due to gravity. Therefore, we can set $F_c = mg$ and solve for the velocity:

$mg = \frac{mv^2}{r}$

dividing both sides by m and then multiplying by r gives us:

$v^2 = rg$

Now, velocity can also be related to the frequency of revolution (f) and the circumference of the circle (C) using the relation:

$v = f \times C$

The circumference of the circle is given by:

$C = 2\pi r$

Let's substitute this into the velocity equation and solve for frequency:

$\sqrt{rg} = f \times 2\pi r$

dividing both sides by $2\pi r$ gives us:

$\frac{\sqrt{rg}}{2\pi r} = f$

To isolate f, since r is in a square root in the numerator and is not in a square root in the denominator, we can simplify:

$f = \frac{1}{2\pi}\sqrt{\frac{g}{r}}$

So the correct answer to the minimum frequency required to prevent the water from falling out of the bucket is Option C:

$\frac{1}{2\pi}\sqrt{\frac{g}{r}}$

Q.30

Two monatomic ideal gases A and B of molecular masses 'm_1' and 'm_2' respectively are enclosed in separate containers kept at the same temperature. The ratio of the speed of sound in gas A to that in gas B is given by

Options:

A.

$\sqrt{\frac{m_2}{m_1}}$

B.

$\frac{m_1}{m_2}$

C.

$\sqrt{\frac{m_1}{m_2}}$

D.

$\frac{m_2}{m_1}$

Answer: A

Solution:

$v = \sqrt{\frac{3RT}{M}}$ or $v \propto \sqrt{\frac{1}{M}}$ at constant T

$\therefore \frac{v_1}{v_2} = \sqrt{\frac{m_2}{m_1}}$

Q.31

A particle starts oscillating simple harmonically from its mean position with time period 'T'. At time $t = \frac{T}{12}$, the ratio of the potential energy to kinetic energy of the particle is $\left(\sin 30^\circ = \cos 60^\circ = 0.5, \cos 30^\circ = \sin 60^\circ = \sqrt{3}/2\right)$

Options:

A.

1 : 2

B.

3 : 1

C.

2 : 1

D.

1 : 3

Answer: D

Solution:

, $t = t/2$

The particle starts from the mean position.

$$x = A\sin\omega t = A\sin\left(\frac{2\pi}{T}\right)\times t$$

$$= A\sin\left(\frac{2\pi}{T}\times\frac{T}{12}\right)$$

$$= A\sin\left(\frac{\pi}{6}\right)$$

$$\therefore x = A\sin 30^{\circ} = \frac{A}{2} \quad \therefore x^2 = \frac{A^2}{4}$$

$\therefore$ The particle is at a distance A/2 from the mean position.

At this point its P.E. $= \frac{1}{2}Kx^2 = \frac{1}{2}m\omega^2x^2$ ... (1)

and its K.E. $= \frac{1}{2}mv^2$

$\therefore K = \frac{1}{2}m\omega^2(A^2 - x^2)$ ... (2)

$$\therefore \frac{\text{P.E.}(U)}{\text{K.E.}(K)} = \frac{x^2}{A^2 - x^2} = \frac{\frac{A^2}{4}}{A^2 - \frac{A^2}{4}} = \frac{\frac{A^2}{4}}{\frac{3A^2}{4}}$$

$$\therefore \frac{U}{K} = \frac{1}{3}$$

Q.32

A hollow pipe of length 0.8 m is closed at one end. At its open end, a 0.5 m long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of pipe. If the tension in the string is 50 N and speed of sound in air is 320 m/s, the mass of the string is

Options:

A.

20 g

B.

10 g

C.

40 g

D.

5 g

Answer: A

Solution:

The fundamental frequency of the closed pipe $(n) = \frac{V}{4L}$

$$n = \frac{320}{4 \times 0.8} = \frac{320}{3.2} = 100\ \mathrm{Hz}$$

For the vibrating wire, fundamental frequency (n) is

$$n' = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

$\therefore$ For the second harmonic, frequency $= 2n'$

$$= \frac{2}{2L}\sqrt{\frac{T}{m}} = \frac{1}{L}\sqrt{\frac{T}{m}}$$

It is given that $2n' = n = 100$ (Resonance)

$$\therefore 100 = \frac{1}{L}\sqrt{\frac{T}{m}} = \frac{1}{0.5}\sqrt{\frac{50}{m}}$$

$$\therefore 100 \times 0.5 = \sqrt{\frac{50}{m}}$$ on squaring

$$\therefore (50)^2 = \frac{50}{m}$$

$$\therefore 50\,m = 1$$

$$\therefore m = \frac{1}{50}\,kg = \frac{1}{50} \times 1000\,g = 20$$ gram

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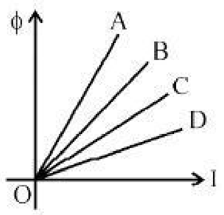
Q.33

A graph of magnetic flux (ϕ) versus current (I) is drawn for four inductors A, B, C, D. Larger value of self inductance is for inductor.



Options:

A.

D

B.

B

C.

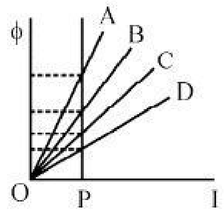
C

D.

A

Answer: D

Solution:



Draw a line parallel to ϕ axis

$\because \phi = \mathrm{LI} \quad \therefore \mathrm{L} = \frac{\phi}{\mathrm{I}}$ = slope of $\phi - \mathrm{I}$ curve

$\therefore$ We find that the slope is maximum for A

$\therefore$ A has the maximum self inductance as OP is same for all.

Q.34

A parallel beam of monochromatic light falls normally on a single narrow slit. The angular width of the central maximum in the resulting diffraction pattern

Options:

A.

decreases with increase of slitwidth

B.

may increase or decrease

C.

decreases with decrease of slitwidth

D.

increases with increase in slitwidth

Answer: A

Solution:

The angular width of the central maximum is

$2\theta = \frac{2\lambda}{a}$ where a is the slit width.

∴ If a is increased, the angular width is decreased.

Q.35

A body moving in a circular path with a constant speed has constant

Options:

A.

momentum

B.

velocity

C.

acceleration

D.

kinetic energy

Answer: D

Solution:

A body moving in a circular path at constant speed has constant kinetic energy. The directions of momentum, velocity and acceleration change from point to points. Hence they do not remain constant. K.E. is a scalar. Others are vectors.

Q.36

A steel coin of thickness 'd' and density '*p*' is floating on water of surface tension 'T'. The radius of the coin (R) is [g = acceleration due to gravity]

Options:

A.

$\frac{T}{\rho g d}$

B.

$\frac{4T}{3\rho g d}$

C.

$\frac{3T}{4\rho g d}$

D.

$\frac{2T}{\rho g d}$

Answer: D

Solution:

Upward force (F) for the steel coin due to S.T.

$= 2\pi r \times T \quad [\because T = \frac{F}{L} = \frac{F}{2\pi r}]$

and it is equal to downward force due to weight $= mg$

$=$ volume of coin $\times$ density $\times g$

$= \pi r^2 d \times \rho \times g$ [d $=$ thickness of the coin]

$\therefore 2\pi r T = \pi r^2 d\rho g$

$\therefore 2T = rd\rho g$

$\therefore r = \frac{2T}{\rho dg}$

Q.37

A door 1.2 m wide requires a force of 1 N to be applied perpendicular at the free end to open or close it. The perpendicular force required at a point 0.2 m distant from the hinges for opening or closing the door is

Options:

A.

3.6 N

B.

2.4 N

C.

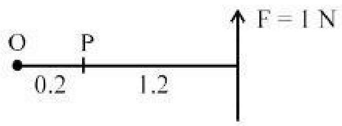
1.2 N

D.

6.0 N

Answer: D

Solution:



To open or close the door, a force of 1 N is applied at a distance of 1.2m from the hinges.

Moment of the force = F × d = 1 × 1.2 = 1.2 N-m

When the force is applied at P at a distance of 0.2 m from O, then the force required to have the same moment is given by

$1.2 = F \times 0.2 \quad \therefore F = \frac{1.2}{0.2} = 6$ N

Q.38

The thermodynamic process in which no work is done on or by the gas is

Options:

A.

isochoric process

B.

adiabatic process

C.

isothermal process

D.

isobaric process

Answer: A

Solution:

The thermodynamic process, in which no work is done on or by the system is isochoric process.

In an isochoric process, $\mathrm{V} = \text{constant} \quad \therefore \mathrm{dV} = 0$

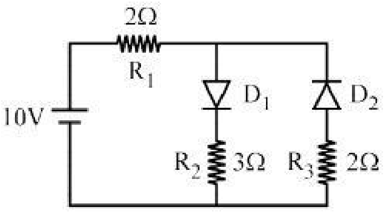
$\therefore$ Work done $(\mathrm{dW}) = \mathrm{PdV} = 0$

Note : $\mathrm{dQ} = 0$ in adiabatic, $\mathrm{dT} = 0$ in isothermal.

In isochoric, $\mathrm{dV} = 0$ and in isobaric, $\mathrm{dP} = 0$

Q.39

The given circuit has two ideal diodes D_1 and D_2 connected as shown in the figure. The current flowing through the resistance R_1 will be



Options:

A.

7 A

B.

3.3 A

C.

2 A

D.

2.5 A

Answer: C

Solution:

Diode D_1 is forward biased and diode D_2 is reverse biased. Hence no current till flow in the branch of D_2.

$\therefore$ The total effective resistance in the circuit is

$2 + 3 = 5\ \Omega$

$\therefore$ Current $I = \frac{10}{5} = 2\ A$

$\therefore$ Current through $R_1 = 2A$

Q.40

In a Fraunhofer diffraction at a single slit of width 'd' and incident light of wavelength 5500 Å, the first minimum is observed at an angle 30°. The first secondary maxima is observed at an angle θ, equal to

Options:

A.

$\sin^{-1}\left(\frac{1}{4}\right)$

B.

$\sin^{-1}\left(\frac{3}{4}\right)$

C.

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

D.

$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

Answer: B

Solution:

The first minimum in a Fraunhofer diffraction pattern occurs when the path difference between the light from the two edges of the slit is equal to the wavelength of the light, λ. The condition for the first minimum can be written as:

$d \sin\theta = m\lambda$

For the first minimum, $m = 1$ and we know that the angle is 30°. Using the given wavelength $\lambda = 5500\overset{o}{A} = 550 \times 10^{-9}$ meters (since $1\overset{o}{A} = 10^{-10}$ meters), we can write:

$d \sin 30^{\circ} = 1 \times 550 \times 10^{-9}$ m

$d \times \frac{1}{2} = 550 \times 10^{-9}$ m

$d = 2 \times 550 \times 10^{-9}$ m

$d = 1100 \times 10^{-9}$ m

$d = 1100\overset{o}{A}$

The secondary maxima occur in between the primary minima. The first secondary maxima (also known as the first 'bright' fringe other than the central maximum) occurs when the path difference is $3/2$ times the wavelength (this is the condition for the maximum that lies between the first and second minima, $m = 1$ and $m = 2$, respectively). This results in the following condition:

$d \sin\theta = \left(m + \frac{1}{2}\right)\lambda$

For the first secondary maxima $m = 1$:

$d \sin\theta = \left(1 + \frac{1}{2}\right) 550 \times 10^{-9}$ m

$d \sin\theta = \frac{3}{2} \times 550 \times 10^{-9}$ m

$1100 \sin\theta = 3 \times 550 \times 10^{-9}$ m

$\sin\theta = \frac{3}{2} \times \frac{550 \times 10^{-9}}{1100 \times 10^{-9}}$

$\sin\theta = \frac{3}{2} \times \frac{1}{2}$

$\sin\theta = \frac{3}{4}$

Therefore, the correct answer is:

Option B

$\sin^{-1}\left(\frac{3}{4}\right)$

Q.41

A galvanometer of resistance 200 Ω is to be converted into an ammeter. The value of shunt resistance which allows 3% of the mains current through the galvanometer is equal to (nearly)

Options:

A.

6 Ω

B.

7Ω

C.

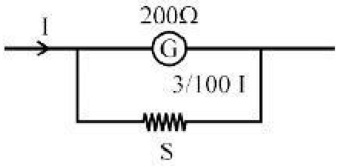
10Ω

D.

5Ω

Answer: A

Solution:



The value of the required shunt (S) is calculated by using

$$\frac{I_g}{I} = \frac{S}{S + G}$$

$$\therefore \frac{3}{100} = \frac{S}{S + 200}$$

$\therefore$ 100 S = 3S + 600 $\quad\therefore$ 97 S = 600

$$\therefore S = \frac{600}{97} \div 6\,\Omega$$

[Note for S = 6, 97 S = 582 and for S = 7, 97 S = 679]

Q.42

The speed of light in two media M_1 and M_2 are 1.5×10^8 m/s and 2×10^8 m/s respectively. If the light undergoes total internal reflection, the critical angle between the two media is

Options:

A.

$\sin^{-1}\left(\frac{3}{2}\right)$

B.

$\sin^{-1}\left(\frac{2}{3}\right)$

C.

$\sin^{-1}\left(\frac{4}{3}\right)$

D.

$\sin^{-1}\left(\frac{3}{4}\right)$

Answer: D

Solution:

$V_{M_1} = 1.5 \times 10^8 \, m/s, V_{M_2} = 2 \times 10^8 \, m/s$
$\because V_{M_2} > V_{M_1}$

$\therefore M_1$ is a denser medium and M_2 is a rarer medium.

$\therefore$ For critical angle, the ray must travel from M_1 to M_2

$\therefore {}_{M_1}\mu_{M_2} = \frac{V_{M_1}}{V_{M_2}} = \frac{1.5 \times 10^8}{2 \times 10^8} = \frac{3}{4}$

$\therefore M_1\mu_{M_2} = \sin C = \frac{3}{4}$

$\therefore C = \sin^{-1}\left(\frac{3}{4}\right)$

Q.43

The minimum distance between an object and its real image formed by a convex lens of focal length 'f' is

Options:

A.

2f

B.

4f

C.

1.5f

D.

2.5f

Answer: B

Solution:

To find the minimum distance between an object and its real image formed by a convex lens, we need to take into account the lens formula, which is given by:

$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$

where:

f is the focal length of the lens,
d_o is the distance of the object from the lens, and
d_i is the distance of the image from the lens.

To find the minimum distance between the object and its image, we should consider the case where the object is at a distance of $2f$ from the lens. In this case, according to the properties of a convex lens, the image will also be formed at a distance $2f$ on the other side of the lens. This situation corresponds to the object and the image being at the same distance from the lens and both being twice the focal length. The separation between the object and the image would then be the sum of these distances. Thus, the object distance $d_o = 2f$ and the image distance $d_i = 2f$.

To find the minimum total distance between the object and image (D), we simply add the object distance (d_o) and the image distance (d_i):

$$D = d_o + d_i = 2f + 2f = 4f$$

Therefore, the minimum distance between an object and its real image formed by a convex lens of focal length f is $4f$, which corresponds to Option B.

Q.44

Heat given to a body, which raises its temperature by 1ºC is known as

Options:

A.

specific heat

B.

thermal capacity

C.

water equivalent

D.

temperature gradient

Answer: B

Solution:

The heat given to a body, which raises its temperature by 1ºC (or 1 K), is known as the **thermal capacity** or sometimes referred to as the heat capacity of the body. The correct answer is Option B: thermal capacity.

The thermal capacity (C) of a body is defined as the amount of heat energy (Q) required to raise the temperature of the entire body by one degree Celsius (or one Kelvin). The formula for thermal capacity is given by:

$$C = \frac{Q}{\Delta T}$$

where **Q** is the heat energy supplied to the body and ΔT is the change in temperature.

To elaborate on the other options provided:

Option A: **Specific heat** (sometimes called specific heat capacity) is the amount of heat required to raise the temperature of one kilogram of the substance by one degree Celsius (or one Kelvin). It is an intrinsic property of the substance and is expressed in units such as joules per kilogram Kelvin (J/kg·K). The formula for specific heat (c) is given by:

$$c = \frac{Q}{m\Delta T}$$

where **m** is the mass of the substance.

Option C: **Water equivalent** is a somewhat outdated term used to describe a quantity of a substance that would absorb the same amount of heat as a given mass of water. It is based on the high specific heat capacity of water, which has historically been used as a benchmark. It's not precisely a term for the heat to raise the temperature but rather a comparative metric.

Option D: **Temperature gradient** refers to the rate of change of temperature with respect to distance in a particular direction. It is a vector quantity that illustrates how temperature changes from one point to another and is not related to the amount of heat energy that is supplied to a body. Therefore, it does not describe the amount of heat needed to increase the temperature of a body by 1ºC.

Q.45

A shell is fired at an angle of 30^o to the horizontal with velocity 196 m/s. The time of flight is

$[\sin 30^o = \frac{1}{2} = \cos 60^o]$

Options:

A.

6.5 s

B.

20 s

C.

16.5 s

D.

10 s

Answer: B

Solution:

$$\theta = 30^\circ, \mathrm{v} = 196\,\mathrm{m/s}$$

$$\text{Time of flight} = \frac{2\mathrm{v}\sin\theta}{\mathrm{g}} = \frac{2 \times 196 \times \sin 30^\circ}{9.8}$$

$$= 2 \times 20 \times \frac{1}{2} = 20\,\mathrm{s}$$

Q.46

Three equal charges 'q_1', ''q_2' and 'q_3' are placed on the three corners of a square of side 'a'. If the force between q_1 and q_2 is 'F_{12}' and that between q_1 and q_3 is 'F_{13}', then the ratio of magnitudes $\left(\frac{F_{12}}{F_{13}}\right)$ is

Options:

A.

1/2

B.

√2

C.

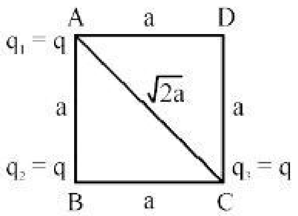
1/√2

D.

2

Answer: D

Solution:



Three equal charges are kept at the corners A, B, C of a square ABCD.

∴ The force between q_1 and q_2 is F_{12}

and $F_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q \times q}{a^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}$

and the force between q_1 and q_3 at A and C is

$F_{13} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2a^2}$

$\therefore \frac{F_{12}}{F_{13}} = \frac{q^2}{a^2} \times \frac{2a^2}{q^2} = 2$ (in magnitude)

Q.47

A coil having an inductance of $\frac{1}{\pi}$H is connected in series with a resistance of 300Ω. If A.C. Source $(20\text{ V} - 200\text{ Hz})$ is connected across the combination, the phase angle between voltage and current is

Options:

A.

$\tan^{-1}\left(\frac{3}{4}\right)$

B.

$\tan^{-1}\left(\frac{4}{3}\right)$

C.

$\tan^{-1}\left(\frac{5}{4}\right)$

D.

$\tan^{-1}\left(\frac{4}{5}\right)$

Answer: B

Solution:

The phase angle ϕ in an R-L circuit (a circuit with resistance and inductance) is determined by the ratio of the inductive reactance (X_L) to the resistance (R). The inductive reactance is given by the formula:

$$X_L = 2\pi f L$$

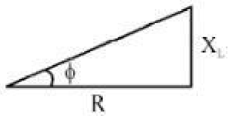
where:

f is the frequency of the alternating current (in hertz).
L is the inductance of the coil (in henrys).

Given that the inductance $L = \frac{1}{\pi}\,\mathrm{H}$ and the frequency $f = 200\,\mathrm{Hz}$, we can calculate X_L as follows:

$$X_L = 2\pi \cdot 200 \cdot \frac{1}{\pi} = 2 \cdot 200 = 400\,\Omega$$

The phase angle ϕ is the arctangent of the ratio of inductive reactance to resistance:



$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

Since the resistance $R = 300\,\Omega$, we can insert the values into the formula to find the phase angle:

$$\phi = \tan^{-1}\left(\frac{400\,\Omega}{300\,\Omega}\right) = \tan^{-1}\left(\frac{4}{3}\right)$$

Therefore, the correct answer is: Option B $\tan^{-1}\left(\frac{4}{3}\right)$.

Q.48

In a full wave rectifier circuit without filter, the output current is

Options:

A.

an eddy current

B.

a constant direct current

C.

a sinusoidal current

D.

unidirectional but not steady current

Answer: D

Solution:

The correct answer to this question is Option D: unidirectional but not steady current.

In a full-wave rectifier circuit, the objective is to convert alternating current (AC) into direct current (DC). The fundamental characteristic of a full-wave rectifier is that it inverts the negative half-cycles of the AC input voltage so that on the output side, the current flows in only one direction for both half-cycles. This makes the output current unidirectional. However, without a filter to smooth out the ripples, the output current is not a constant direct current but a pulsating direct current that still retains the frequency of the original AC signal.

The output current of a full-wave rectifier without a filter can be represented as an absolute value of a sinusoidal wave, meaning it maintains the sinusoidal shape but with all the values above the zero line since the negative values are inverted. This waveform is not constant or steady; instead, it rises and falls with the frequency of the AC input, typically at twice the frequency in the case of a full-wave rectifier because it rectifies both halves of the input sine wave.

So, to summarize:

Option A (an eddy current) is incorrect since eddy currents are localized currents induced in conductors when they are exposed to changing magnetic fields, which is not the output of a rectifier circuit.

Option B (a constant direct current) is incorrect because without a filter (such as a capacitor or an inductor), the output is not constant but pulsating with ripples.

Option C (a sinusoidal current) is incorrect because the resulting current is not sinusoidal; it's the absolute value of a sinusoidal wave, representing both halves of the wave above zero volts.

Option D (unidirectional but not steady current) is correct as the output is indeed unidirectional due to the inversion of the negative half-cycles, but not steady due to the absence of a filter to smooth out the ripples.

Q.49

The excess pressure inside a soap bubble of radius 2 cm is 50 dyne/cm^2 . The surface tension is

Options:

A.

50 dyne/cm

B.

60 dyne/cm

C.

75 dyne/cm

D.

25 dyne/cm

Answer: D

Solution:

The excess pressure (P) in a soap bubble of radius $2\ \mathrm{cm}$ is $50\ \mathrm{dyne/cm^2}$.

If T is the S.T., then $\mathrm{P}=\frac{4\mathrm{T}}{\mathrm{R}}$

$\therefore \mathrm{T}=\frac{\mathrm{PR}}{4}=\frac{50\times 2}{4}$

$\therefore \mathrm{T}=25\ \mathrm{dyne/cm}$

Q.50

Two bodies of masses 'm' and ' 3 m' are rotating in horizontal speed of the body of mass 'm' is n times that of the value of heavier body; while the centripetal force is same for both. The value of n is

Options:

A.

3

B.

1

C.

9

D.

6

Answer: A

Solution:

For body A, mass $= \mathrm{m}$, radius of the circle $= \mathrm{r}$

For body B, mass $= 3\ \mathrm{m}$ and radius of the circle $= \frac{r}{3}$ and $\mathbf{v}$ and v' are the tangential speeds

For A, C.P. force $= \frac{\mathrm{mv}^2}{\mathrm{r}}$ .... (1)

For B, C.P. force $= \frac{3\ \mathrm{m} \cdot \mathrm{v}'^2}{\mathrm{r}/3} = \frac{9\mathrm{mv}'^2}{\mathrm{r}}$ .... (2)

and it is given that $\mathrm{v} = \mathrm{nv}'$

Since the C.P. force is same for both

$$\therefore \frac{\mathrm{mv}^2}{\mathrm{r}} = \frac{9mv'^2}{\mathrm{r}}$$
$$\therefore \mathrm{v}^2 = 9\mathrm{v}'^2 \text{ but } \mathrm{v} = \mathrm{nv}'$$
$$\therefore \mathrm{n}^2\mathrm{v}'^2 = 9\mathrm{v}'^2$$
$$\therefore \mathrm{n}^2 = 9 \quad \therefore \mathrm{n} = 3$$

Chemistry

Q.1

For the reaction $N_{2(g)} + 3H_{2(g)} \rightarrow 2NH_{3(g)}$, rate of disappearance of $N_{2(g)}$ is $2.22 \times 10^{-3}\ \mathrm{mol\ dm}^{-3}$. What is the rate of appearance of $NH_{3(g)}$?

Options:

A.

2.22×10^{-3} mol dm^{-3}

B.

1.11×10^{-3} mol dm^{-3}

C.

4.44×10^{-3} mol dm^{-3}

D.

3.33×10^{-3} mol dm^{-3}

Answer: C

Solution:

$$\frac{\mathrm{d}\,[N_2]}{\mathrm{dt}} = \frac{1}{2}\frac{\mathrm{d}\,[NH_3]}{\mathrm{dt}};$$

$$\therefore \frac{\mathrm{d}\,[NH_3]}{\mathrm{dt}} = 2\frac{\mathrm{d}\,[N_2]}{\mathrm{dt}}$$

$$\frac{\mathrm{d}\,[NH_3]}{\mathrm{dt}} = 2 \times 2.22 \times 10^{-3} = 4.44 \times 10^{-3}$$

Q.2

Identify the products obtained when chlorine reacts with hot and conc. NaOH.

Options:

A.

$NaClO_3$, NaCl and H_2O

B.

NaCl and HOCl

C.

Na_2O and NaCl

D.

NaOCl and H_2O

Answer: A

Solution:

$$3Cl_2 + \underset{(conc.)}{6NaOH} \xrightarrow{\Delta} NaClO_3 + 5NaCl + 3H_2O$$

Q.3

Which from following elements does NOT react with water?

Options:

A.

Ca

B.

Sr

C.

Be

D.

Mg

Answer: C

Solution:

The element that does not react with water at room temperature among the options provided is Beryllium (Be), option C. Calcium (Ca), Strontium (Sr), and Magnesium (Mg) can all react with water, though Magnesium's reaction is very slow and often requires heat to be noticeable at room temperature.

Beryllium is an alkaline earth metal, and it is the least reactive among the group 2 elements. While other alkaline earth metals like calcium and strontium will react with water to form hydroxides and release hydrogen gas, beryllium does not react with water even when heated. It is protected by an oxide layer that forms on the surface, which prevents it from reacting with water. The reactions for Ca and Sr with water are as follows:

For Calcium (Ca):

$$Ca + 2H_2O \rightarrow Ca(OH)_2 + H_2$$

For Strontium (Sr):

$$Sr + 2H_2O \rightarrow Sr(OH)_2 + H_2$$

For Magnesium (Mg), the reaction with water is slower, and it usually occurs with hot water or steam:

$Mg + 2H_2O \rightarrow Mg(OH)_2 + H_2$ (with steam)

Consequently, the correct answer to the question is Option C, Beryllium (Be).

Q.4

Identify the type of hybridization involved in hexaaminecobalt (III) complex ion.

Options:

A.

sp^3

B.

dsp^2

C.

d^2sp^3

D.

sp^3d^2

Answer: C

Solution:

The hexaaminecobalt(III) complex ion has the formula $[Co(NH_3)_6]^{3+}$. The cobalt in this complex is in the +3 oxidation state, which means it has lost three electrons from its valence shell. Cobalt as a transition metal has the electronic configuration of $[Ar]3d^7 4s^2$ in its neutral state. When it loses three electrons to become Co^{3+}, it has an electronic configuration of $[Ar]3d^6$.

In this complex, cobalt is surrounded by six NH_3 (ammonia) ligands, which are all monodentate and donate a pair of electrons to the metal center for bonding. Due to this, cobalt needs to have six hybridized orbitals to accommodate the bond formation with these six ligands.

This coordination number of 6 typically leads to octahedral geometry, and the hybridization of the orbitals in cobalt required to form this geometry is d^2sp^3. This includes two d orbitals, one s orbital, and three p orbitals mixing to give six hybridized orbitals. Each of the hybrid orbitals will overlap with the s orbital of the nitrogen in ammonia to form a sigma bond.

Hence, the correct hybridization for the cobalt in the hexaaminecobalt(III) complex ion is d^2sp^3.

Therefore, the correct answer is:

Option C

d^2sp^3

Q.5

Calculate the solubility of a gas in water at 0.8 atm and 25^oC.

[Henry's law constant is 6.85 × 10^{-4} mol dm^{-3} atm^{-1}]

Options:

A.

2.74 × 10^{-4} mol dm^{-3}

B.

3.94 × 10^{-4} mol dm^{-3}

C.

6.85 × 10^{-4} mol dm^{-3}

D.

5.48 × 10^{-4} mol dm^{-3}

Answer: D

Solution:

The solubility of a gas in a liquid according to Henry's law can be determined using the formula:

$S = k_H \cdot P$

Where:

S is the solubility of the gas in the liquid (in $\mathrm{mol\,dm^{-3}}$).
k_H is Henry's law constant (in $\mathrm{mol\,dm^{-3}\,atm^{-1}}$).
P is the partial pressure of the gas (in atm).

Given the Henry's law constant (k_H) is $6.85 \times 10^{-4}\,\mathrm{mol\,dm^{-3}\,atm^{-1}}$ and the partial pressure (P) of the gas is $0.8\,atm$, we can calculate the solubility (S) of the gas in water at the given conditions:

$S = (6.85 \times 10^{-4}\,\mathrm{mol\,dm^{-3}\,atm^{-1}}) \cdot (0.8\,\mathrm{atm})$

$S = 5.48 \times 10^{-4}\,\mathrm{mol\,dm^{-3}}$

Therefore, the solubility of the gas in water at $0.8\,\mathrm{atm}$ and $25^\circ\mathrm{C}$ is $5.48 \times 10^{-4}\,\mathrm{mol\,dm^{-3}}$.
The correct option is:

Option D

$5.48 \times 10^{-4}\,\mathrm{mol\,dm^{-3}}$

Q.6

What is the value of temperature in degree Celsius at absolute zero ?

Options:

A.

273.15°C

B.

–373.15°C

C.

0°C

D.

–273.15°C

Answer: D

Solution:

The value of temperature in degree Celsius at absolute zero is Option D:

–273.15°C

Absolute zero is the lowest theoretical temperature where nothing could be colder and no heat energy remains in a substance. Absolute zero is the point at which the fundamental particles of nature have minimal vibrational motion, retaining only quantum mechanical, zero-point energy-induced particle motion. The Celsius scale is set up so that the freezing point of water is at 0°C, while the boiling point is at 100°C. By definition, absolute zero is 0 kelvin (K), but it can be converted into degrees Celsius by the equation:

$T(^\circ C) = T(K) - 273.15$

Thus, when the temperature is $0K$ (absolute zero in Kelvin), the equivalent temperature in degrees Celsius is:

$-273.15^\circ C$

Q.7

Which among the following reactions does NOT correctly match with its reagent?

Options:

A.

Stephen reaction : $SnCl_2$, HCl

B.

Etard reaction : CrO_2Cl_2

C.

Gatterman - Koch formulation : $CrO_3/(CH_3CO)_2O$

D.

Rosenmund reduction : $H_2/Pd - BaSO_4$

Answer: C

Solution:

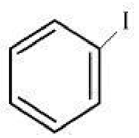
Gatterman Koch - CO, HCl and Anhydrous $AlCl_3$

Q.8

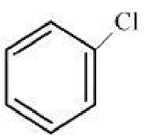
Which among the following compounds is NOT prepared by Sandmeyer's reaction ?

Options:

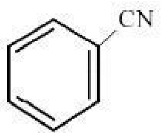
A.



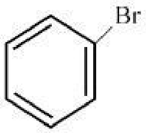
B.



C.



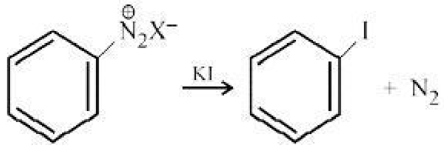
D.



Answer: A

Solution:

It requires only KI



Q.9

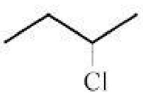
Which among the following compounds undergoes SN^2 reaction fastly ?

Options:

A.

B.

C.



D.

Answer: D

Solution:

Primary iodide will undergo SN^2 reaction fast.

Q.10

Which of the following molecules possesses highest dipole-dipole interactions ?

Options:

A.

HCl

B.

HI

C.

HBr

D.

HF

Answer: D

Solution:

Greater the dipole moment higher is dipole-dipole interaction.

Q.11

What is the total volume occupied by atoms in bcc unit cell ?

Options:

A.

52.36%

B.

68%

C.

80%

D.

74%

Answer: B

Solution:

The body-centered cubic (bcc) unit cell is one of the basic structures that atoms can be arranged in for a crystalline solid. In a bcc cell, there is one atom at each corner of the cube and one atom in the center of the cube. The volume occupied by the atoms in a bcc unit cell can be calculated if we know the atomic radius and the edge length of the cube.

The atoms in a bcc structure touch each other along the body diagonal of the cube. Therefore, we can use the relation between the body diagonal (d) of the cube, the cube's edge length (a), and the atomic radius (r) for a bcc lattice, which is given as:

$$d = \sqrt{3}a$$

The body diagonal is also equal to four times the atomic radius in a bcc cell because the body diagonal passes through two half atoms at the corners and one whole atom in the center:

$$d = 4r$$

Putting these equations together we get:

$$\sqrt{3}a = 4r$$

Now, we want to solve for the edge length a in terms of the atomic radius:

$$a = \frac{4r}{\sqrt{3}}$$

To find the total volume of the cube (the unit cell), we cube the edge length:

$$V_{\text{cell}} = a^3 = \left(\frac{4r}{\sqrt{3}}\right)^3 = \frac{64r^3}{3\sqrt{3}}$$

Each corner atom is shared by eight adjacent cubes and the center atom belongs entirely to one cube. Therefore, a single bcc unit cell contains

$$V_{\text{atoms}} = 1 \times \frac{4}{3}\pi r^3 + 8 \times \frac{1}{8} \times \frac{4}{3}\pi r^3 = 2 \times \frac{4}{3}\pi r^3$$

This means that the volume occupied by the actual atoms in a unit cell is equal to the volume of two atoms since each corner atom is shared among eight unit cells.

The packing efficiency or the fraction of the volume occupied by the atoms is given by

$$\text{Packing efficiency} = \frac{V_{\text{atoms}}}{V_{\text{cell}}} \times 100\%$$

$$\text{Packing efficiency} = \left(\frac{2 \times \frac{4}{3}\pi r^3}{\frac{64r^3}{3\sqrt{3}}}\right) \times 100\%$$

When we simplify this equation, we get:

$$\text{Packing efficiency} = \left(\frac{2 \times \frac{\pi}{\sqrt{3}}}{8}\right) \times 100\%$$

$$\text{Packing efficiency} = \left(\frac{\pi}{4\sqrt{3}}\right) \times 100\% \approx 68\%$$

Thus, the total volume occupied by atoms in a body-centered cubic (bcc) unit cell is approximately 68%. Therefore, the correct answer is:

Option B

68%

Q.12

Which among the following metals is involved in preparation of Grignard reagent ?

Options:

A.

Magnesium

B.

Sodium

C.

Silver

D.

Zinc

Answer: A

Solution:

Alkyl magnesium halide is Grignard reagent.

Q.13

Which among the following properties of lanthanoids is NOT true?

Options:

A.

Good conductors of heat and electricity

B.

All are non-radioactive elements

C.

Have greater co-ordination number than six

D.

Strongly paramagnetic

Answer: B

Solution:

The incorrect property among the listed options for lanthanoids is:

Option B: All are non-radioactive elements

This statement is not true because not all lanthanoids are non-radioactive. Most lanthanoids are indeed non-radioactive under normal conditions. However, among the lanthanides, promethium (Pm) is an exception as it does not have any stable isotopes and is radioactive. The most common isotope of promethium, promethium-145, has a half-life of 17.7 years and thus, it decays over time emitting radiation.

The other options given are typically true for the lanthanides (also known as lanthanoids):

Option A: Good conductors of heat and electricity – True. Lanthanoids are metals and like most metals, they are generally good conductors of heat and electricity.

Option C: Have greater coordination number than six – True. Lanthanoids have the ability to adopt coordination numbers greater than 6 because of their relatively large ionic radii and the availability of empty 4f, 5d, and 6s orbitals that can participate in bonding.

Option D: Strongly paramagnetic – True. Many lanthanides are strongly paramagnetic due to the presence of unpaired electrons in their 4f orbitals. Their magnetic properties are significant and are utilized in various applications such as in magnets, phosphors, and in magnetic resonance imaging (MRI) contrast agents.

Q.14

Which of the following is a Lewis acid but NOT a Bronsted acid?

Options:

A.

BCl_3

B.

HNO_3

C.

NH_3

D.

HSO_4^-

Answer: A

Solution:

To identify which of the given compounds is a Lewis acid but not a Bronsted acid, we must first understand the definitions of Lewis and Bronsted acids.

A Lewis acid is a compound that can accept an electron pair, whereas a Bronsted acid is a compound that can donate a proton (H^+). In other words, Lewis acids are electron pair acceptors, and Bronsted acids are proton donors.

Now let's evaluate the given options:

Option A: BCl_3 - Boron trichloride (BCl_3) is a Lewis acid because it has an incomplete octet; the boron atom has only six electrons in its valence shell and thus can accept an electron pair to complete its octet. However, it does not have a releasable proton, which means it cannot donate a proton in a reaction. Hence, BCl_3 is a Lewis acid but not a Bronsted acid.

Option B: HNO_3 - Nitric acid (HNO_3) is both a Lewis acid and a Bronsted acid. It can donate a proton to become NO_3^- (making it a Bronsted acid), and it can also accept an electron pair because the nitrogen atom can participate in coordinate covalent bonding. However, since it is a Bronsted acid, it does not fit the requirement for this question.

Option C: NH_3 - Ammonia (NH_3) is a Lewis base because it has a lone pair of electrons that can be donated to form a bond. It is not a Lewis acid. Ammonia is also not a Bronsted acid, as it doesn't donate protons but rather accepts them.

Option D: HSO_4^- - Hydrogen sulfate ion (HSO_4^-) is a Bronsted acid because it can donate a proton to become sulfate ion (SO_4^{2-}). It does not readily act as a Lewis acid in accepting an electron pair, so its primary function is as a Bronsted acid.

Therefore, the correct answer is:

Option A: BCl_3

Q.15

Which of the following aqueous solutions of salts will have highest pH value?

Options:

A.

CH_3COONH_4

B.

Na_2CO_3

C.

NH_4Cl

D.

NaCl

Answer: B

Solution:

Na_2CO_3 is the only alkaline solution having highest pH value. It is salt of weak acid (H_2CO_3) and strong base $(NaOH)$.

Q.16

Which among the following compounds represents a soap molecule?

Options:

A.

Ammonium salt of higher fatty acids

B.

Sodium salt of formic acid

C.

Potassium salt of higher fatty acids

D.

Ammonium salt of formic acid

Answer: C

Solution:

Soaps are best represented by the salts of higher fatty acids (long-chain carboxylic acids) with sodium or potassium. These salts form when a strong base reacts with a fatty acid in a process called saponification. Soaps have a hydrophobic tail that is typically composed of a long straight-chain hydrocarbon and a hydrophilic head that contains the ionic part of the molecule.

Let's analyze each option:

Option A: Ammonium salt of higher fatty acids – This features the right type of acid (higher fatty acids) but the wrong cation, ammonium (NH_4^+). Ammonium salts aren't typically used as soaps.

Option B: Sodium salt of formic acid – Formic acid is the simplest carboxylic acid and doesn't qualify as a higher fatty acid which is needed to form soap. Moreover, the resulting salt is not used as a soap due to its simple structure and lack of a long hydrophobic tail.

Option C: Potassium salt of higher fatty acids – This fits the definition of a soap, having a potassium ion (K^+) combined with a higher fatty acid anion, making it a potassium soap.

Option D: Ammonium salt of formic acid – Similar to option B, this uses formic acid and has the ammonium ion, which is also not typically used in soap.

Therefore, **Option C**, the Potassium salt of higher fatty acids, best represents a soap molecule.

Q.17

How long will it take to produce 5.4 g of Ag from molten AgCl by passing 5 amp current?

(Molar mass Ag = 108 g mol^{-1})

Options:

A.

1930 second

B.

193 second

C.

965 second

D.

9650 second

Answer: C

Solution:

$$t = \frac{m \times 96500}{\text{mol. ratio}} = \frac{5.4 \times 96500 \times 1}{108 \times 5} = 965 \text{ second}$$

Q.18

Which of the following is NOT an example of secondary voltaic cell?

Options:

A.

Lead storage battery

B.

Dry cell

C.

Nickel-cadmium cell

D.

Mercury cell

Answer: B

Solution:

Dry cell is an example of primary voltaic cell.

Q.19

What is the number of unpaired electrons in $[Co(NH_3)_6]^{3+}$ complex ?

Options:

A.

Four

B.

Two

C.

Zero

D.

Six

Answer: C

Solution:

Configuration of Co^{3+} is $1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 3d^6, 4s^0$

Q.20

Which among the following methods is used to prepare Grignard reagent?

Options:

A.

Action of magnesium powder on alkyl halide in aqueous medium

B.

Action of magnesium hydroxide on alkyl halide

C.

Action of magnesium metal on alkyl halide in presence of dry ether

D.

Action of $MgCl_2$ on alkyl halide in presence of dry ether

Answer: C

Solution:

Grignard reagent reacts in aqueous medium hence dry ether is used.

Q.21

Calculate the density of metal having volume of unit cell 64 × 10^{-24} cm^3 and molar mass of metal 192 g mol^{-1} containing 4 particles in unit cell.

Options:

A.

14.92 g cm^{-3}

B.

16.00 g cm^{-3}

C.

19.93 g cm^{-3}

D.

18.00 g cm^{-3}

Answer: C

Solution:

$$\begin{aligned} D = \frac{M \cdot N}{V \cdot N_A} &= \frac{192 \times 4}{64 \times 10^{-24} \times 6.022 \times 10^{23}} \\ &= \frac{192 \times 4}{6.4 \times 6.022} = 19.93 \text{ g cm}^{-3} \end{aligned}$$

Q.22

Calculate the work done when 2 moles of an ideal gas expand from a volume of 5 dm^3 to 7 × 10^{-3} m^3 against a constant external pressure of 2.02 × 10^5 Nm^{-2} ?

Options:

A.

20.2 J

B.

–404 J

C.

202 J

D.

–35.0 J

Answer: B

Solution:

$$\begin{aligned} W &= -P_{ext}(V_2 - V_1) \\ &= -2.02 \times 10^5 \left(7 \times 10^{-3} - 5 \times 10^{-3}\right) \\ &= -404\ J \end{aligned}$$

Q.23

Which among the following pair of monomers does not generate polyamide polymer?

Options:

A.

Urea and Formaldehyde

B.

Glycine and ε amino caproic acid

C.

Adipic acid and hexamethylene diamine

D.

3-Hydroxybutanoic acid and 3-Hydroxy pentanoic acid

Answer: D

Solution:

None of the monomers has $-NH_2$ group.

Q.24

What type of following phenomena is NOT exhibited by adsorption?

Options:

A.

Irreversible

B.

Bulk

C.

Exothermic

D.

Endothermic

Answer: B

Solution:

Among the listed options, Option B: "Bulk" is NOT a phenomenon that is typically exhibited by adsorption. To understand why let's examine all the options:

Option A: Irreversible - Adsorption can be either reversible or irreversible. Irreversible adsorption means that once the adsorbate molecules attach to the adsorbent surface, they are not easily detached; this is common with chemisorption, where strong chemical bonds are formed. Reversible adsorption is more characteristic of physisorption, where the forces involved are weaker (like van der Waals forces), allowing the adsorbate to be released from the adsorbent surface under certain conditions. So, adsorption can indeed be irreversible, but it's not exclusively so.

Option B: Bulk - Adsorption is a surface phenomenon. It involves the accumulation of substances at the interface between two phases, such as between a solid surface and a gas or liquid. "Bulk" refers to the volume of a material or a phenomenon that occurs throughout the volume, which is contrary to the localized nature of adsorption at surfaces or interfaces. Therefore, adsorption is not a bulk phenomenon.

Option C: Exothermic - Adsorption is typically an exothermic process. When adsorbate molecules attach to the adsorbent surface, they release energy in the form of heat. This is because the adsorbate molecules usually go to a lower energy state when they adhere to the adsorbent, leading to a release of energy. The enthalpy change (ΔH) of adsorption is negative, which is indicative of an exothermic reaction.

Option D: Endothermic - Adsorption is generally exothermic, but there are instances where adsorption can be endothermic. For example, when the adsorbate is bound with relatively stronger intermolecular forces in the bulk phase compared to the forces it experiences at the adsorbent surface, the process may end up absorbing energy from the surroundings to allow adsorption. However, these cases are the exception rather than the rule.

Therefore, the correct answer is Option B: "Bulk," as adsorption does not exhibit bulk characteristics; it is a surface-based phenomenon.

Q.25

Find the rate constant of first order reaction in second having half life of 2.5 hours.

Options:

A.

4.3×10^{-5} sec^{-1}

B.

7.7×10^{-5} sec^{-1}

C.

6.9×10^{-5} sec^{-1}

D.

8.4×10^{-5} sec^{-1}

Answer: B

Solution:

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{2.5 \times 60 \times 60} = 7.7 \times 10^{-5} \text{sec}^{-1}$$

Q.26

Which nitrogen atom of pyrimidine base numbered from 1 to 6 is bonded with furanose sugar ?

Options:

A.

4

B.

2

C.

1

D.

5

Answer: B

Solution:

2nd nitrogen atom of pyrimidine base is bonded to furanose sugar.

Q.27

Identify the element with smallest ionic radius in +3 oxidation state from following.

Options:

A.

Er

B.

Lu

C.

Eu

D.

Yb

Answer: B

Solution:

For elements in the same group of the periodic table, the ionic radii decrease with an increase in atomic number due to increasing effective nuclear charge, which pulls the electrons closer to the nucleus. However, when looking across the lanthanide series (or rare earth metals), there is an additional factor called the lanthanide contraction to consider.

Within lanthanides, as the atomic number increases, the filling up of the 4f sublevel takes place. Due to the poor shielding effect of 4f electrons, the effective nuclear charge experienced by the outer electrons increases. Thus, despite a constant charge state (+3 in this case), elements later in the series will have smaller radii due to the stronger attraction by the nucleus.

Between erbium (Er), lutetium (Lu), europium (Eu), and ytterbium (Yb), Er and Lu are actual lanthanides, while Eu and Yb have different electronic configurations that make them exceptions within the series. Europium tends to have a larger radius due to having it typically in a +2 state, and ytterbium has an anomalous configuration with a filled 4f level, imparting it a smaller radius than expected for its position.

Among the given options, lutetium (Lu) has the highest atomic number (71) and is the last element in the lanthanide series. Because of the continued lanthanide contraction, Lu^{+3} is expected to have the smallest ionic radius of the given options:

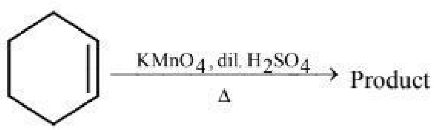
- Er (Erbium) +3 is before Lu in the lanthanide series.
- Eu (Europium) +3 would generally have a larger ionic radius due to being earlier in the series and also due to its typical +2 state.
- Yb (Ytterbium) +3, although having a filled 4f level resulting in a smaller radius than expected, it is still not as small as that of Lu^{+3} because Yb has a lower atomic number.

Therefore, the correct answer is:

Option B: Lu (Lutetium)

Q.28

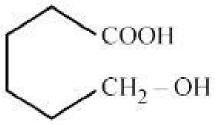
Identify the product in the following reaction.



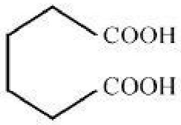
Options:

A.

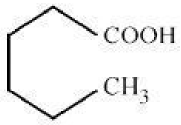
B.



C.



D.



Answer: C

Solution:

The carbon atoms of double bond will form –COOH on oxidation.

Q.29

Which among following compounds possesses highest number of N atoms in it ?

Options:

A.

Cytosine

B.

Uracil

C.

Guanine

D.

Thymine

Answer: C

Solution:

The number of nitrogen atoms present in the given compounds can be determined by looking at the molecular structures of each base. Cytosine, uracil, guanine, and thymine are all nitrogenous bases found in nucleic acids. Let's analyze each one:

Cytosine (Option A) has a single ring structure known as pyrimidine and it contains three nitrogen atoms within that ring.

Uracil (Option B) is also a pyrimidine base and it too contains two nitrogen atoms.

Guanine (Option C) is a purine base and has a two-ring structure. It contains five nitrogen atoms—three in the larger six-membered ring and two in the smaller five-membered ring.

Thymine (Option D) is another pyrimidine base and it contains two nitrogen atoms, similar to uracil.

Among these options, guanine contains the most nitrogen atoms. Therefore, the correct answer is:

Option C - Guanine

Q.30

What is the bond order of CO molecule?

Options:

A.

1

B.

2

C.

3

D.

0

Answer: C

Solution:

Bond order of $CO = \frac{6-0}{2} = 3$

Q.31

Which of the following is NOT hydrogen like species?

Options:

A.

He

B.

He^{+}

C.

Li^{2+}

D.

Be^{3+}

Answer: A

Solution:

Hydrogen-like species, also known as hydrogenic atoms or ions, are those that have only one electron surrounding the nucleus, irrespective of the charge on the nucleus. To evaluate which of the given species is not hydrogen-like, we need to look at the electron configuration of each.

Option A: He has two protons in its nucleus and two electrons. This is the normal helium atom and it has two electrons, hence it is not a hydrogen-like species since a hydrogen-like species can have only one electron.

Option B: He^+ is a helium ion with one electron removed, thus it has two protons in its nucleus but only one electron. This species is hydrogen-like because it has only one electron.

Option C: Li^{2+} has three protons in its nucleus and, in this ionic state, has had two electrons removed, leaving it with just one electron. Like He^+, this ion is hydrogen-like.

Option D: Be^{3+} has four protons in its nucleus and, as a trivalent cation, has had three electrons removed. Therefore, it has only one electron remaining, which makes it a hydrogen-like species.

Therefore, the correct answer is Option A: He, because it is not hydrogen-like due to its two electrons.

Q.32

What is the intermediate compound formed when chlorobenzene is treated with fused NaOH under pressure?

Options:

A.

Phenoxide ion

B.

Sodium phenoxide

C.

Benzene diazonium chloride

D.

Benzene

Answer: B

Solution:

$$C_6H_5Cl + 2NaOH \xrightarrow[-NaCl, -H_2O]{\text{fused}} C_6H_5ONa \xrightarrow{HCl} C_6H_5OH + NaCl$$

Q.33

If rate of reaction is given as

$$\frac{1}{3}\frac{d[x]}{dt} = -\frac{1}{2}\frac{d[y]}{dt} = -\frac{d[Z]}{dt},$$

the reaction can be represented as

Options:

A.

2y + Z → 3x

B.

2y → 3x + Z

C.

3x + 2y → Z

D.

3x → 2y + Z

Answer: A

Solution:

The given rate of reaction is:

$$\frac{1}{3}\frac{d[x]}{dt} = -\frac{1}{2}\frac{d[y]}{dt} = -\frac{d[Z]}{dt}$$

The negative sign in front of the derivatives of concentrations of Y and Z suggests these are reactants being consumed over time, while the positive sign for substance X indicates that it is a product being formed over time.

To relate the reaction rates of individual reactants and products to a balanced chemical equation, you have to equalize the rate of disappearance of the reactants with the rate of appearance of the products, taking into consideration their stoichiometric coefficients.

Using the stoichiometry of the balanced equation, we can understand that the coefficients of X, Y, and Z in the reaction equation would correspond to the ratios of the rates at which they appear or disappear.

For this, the coefficients of Y and Z can be taken as 2 and 1, respectively, which would mean that for every 1 mole of Z consumed, 2 moles of Y are consumed, as indicated by their rates being two times faster.

As per the given rate relationship, X is being formed thrice as fast as Z is being consumed and simultaneously 1.5 times (= 3/2) as fast as Y is being consumed. Adding these stoichiometric coefficients into the equation, you can balance them according to the reaction:

2Y + Z → 3X

Therefore, the correct representation of the reaction with stoichiometric coefficients is option A:

2y + Z → 3x

This is the only equation that correlates to the provided rate of reaction change for each compound, ensuring that the Law of Conservation of Mass is upheld, with reactants being converted into products at the rates described in the equation.

Q.34

Which among the following compounds contains highest number of

chlorine atoms in their single molecule ?

Options:

A.

Mustard gas

B.

Phosgene

C.

Tear gas

D.

Phosphine

Answer: C

Solution:

Tear gas $CCl_3(NO_2)$ contains three Cl-atoms.

Q.35

What is the heat of formation of $HCl_{(g)}$ from following equation?

$H_{2(g)} + Cl_{2(g)} \rightarrow 2HCl_{(g)} \Delta_f H = -194\,kJ$

Options:

A.

–388 kJ

B.

–194 kJ

C.

–97 kJ

D.

194 kJ

Answer: C

Solution:

$\Delta_f H \text{ of } HCl = \frac{1}{2}(-194) = -97\,kJ$

Q.36

Identify the concentration of the solution from following so that values of ,ΔT_f and K_f are same.

Options:

A.

1 m

B.

1 M

C.

1 N

D.

N/10

Answer: A

Solution:

To identify the correct option, we first need to understand the relationship between the freezing point depression (ΔT_f) and the molal freezing point depression constant (K_f) of the solvent. This relationship is given by the colligative property equation for freezing point depression:

$$\Delta T_f = K_f \times m$$

where:

ΔT_f is the freezing point depression,
K_f is the cryoscopic constant (also known as the molal freezing point depression constant), and
m is the molality of the solution.

The key to solving this problem is noting that the values for ΔT_f and K_f need to be the same, which means:

$$\Delta T_f = K_f \Rightarrow K_f = m$$

Thus, the molality of the solution needs to be 1 mol/kg since the only situation where the equality holds is if the molality m is equal to 1.

Now, let's analyze the options:

Option A: 1 m - This represents a concentration of 1 molal, which means 1 mole of solute per 1 kilogram of solvent.

Option B: 1 M - This represents a concentration of 1 molar, which is 1 mole of solute per 1 liter of solution. Molarity and molality are not the same and the density of the solution would matter to convert between the two.

Option C: 1 N - This represents a 1 normal solution, which is related to the equivalent concept of moles of reactive species. Normality can vary based on the equivalent factor of the solute, and is not purely a measure of the number of moles of solute per liter of solution.

Option D: N/10 - This represents one-tenth the normality of the solution, which would be equivalent to a 0.1 N solution.

From the analysis above, we can conclude that the correct option is A, as it provides a molality of 1 which is necessary for ΔT_f to equal K_f when they are numerically the same:

$$\Delta T_f = K_f = 1\,m$$

Q.37

What is the product formed when cumene is air oxidised in presence of Co-naphthenate and further treated with dilute acid?

Options:

A.

Cumene hydroperoxide

B.

Phenol and CO_2

C.

Acetone and Benzoic acid

D.

Phenol + Acetone

Answer: D

Solution:

The oxidation of cumene (isopropylbenzene) in the presence of a catalyst like cobalt naphthenate followed by treatment with a dilute acid such as sulfuric acid leads to a process known as the cumene process. This process is used in the industrial production of two important chemicals: phenol and acetone. Here’s how the process works:

Cumene is first oxidized by oxygen from the air to form cumene hydroperoxide (C6H5C(CH3)2OOH) in the presence of a catalyst like cobalt naphthenate.

The cumene hydroperoxide is then treated with dilute acid to initiate a cleavage reaction. This reaction is a form of acid-catalyzed decomposition called heterolytic cleavage.

The cumene hydroperoxide splits or cleaves to form phenol (C6H5OH) and acetone (CH3COCH3).

The reaction scheme is as follows:

$$C_6H_5C(CH_3)_2 + O_2 \xrightarrow[\text{Co-naphthenate}]{\text{Air}} C_6H_5C(CH_3)_2OOH$$

$$C_6H_5C(CH_3)_2OOH \xrightarrow[H^+]{\text{Dilute acid}} C_6H_5OH + (CH_3)_2CO$$

Thus, the correct product formed when cumene is air oxidized in the presence of cobalt naphthenate and further treated with dilute acid is Option D: Phenol + Acetone.

Q.38

Identify the use of polystyrene for household purposes.

Options:

A.

To prepare shopping bags

B.

To prepare microwavable food trays

C.

To manufacture disposable cups and plates

D.

To prepare bottles for storage of mouth wash

Answer: C

Solution:

Polystyrene is a versatile plastic that can be used for various purposes, including household items. Let's evaluate each option provided:

Option A: To prepare shopping bags – Polystyrene is not typically used for shopping bags. Shopping bags are commonly made from materials like polyethylene, which is more flexible and durable for this purpose. Therefore, Option A is not generally correct.

Option B: To prepare microwavable food trays – Polystyrene can be used to make food trays, but it is important to note that typical polystyrene, such as the one used in foam cups and takeout containers, is not microwave-safe. However, there are certain types of polystyrene that have been made to be microwavable, so this use is possible but not as common. Thus, Option B may be used in some cases with special types of polystyrene.

Option C: To manufacture disposable cups and plates – Polystyrene is widely used to produce disposable cups and plates. This material offers insulation properties useful for both hot and cold drinks and is lightweight, making it suitable for such disposable items. Therefore, Option C is correct.

Option D: To prepare bottles for storage of mouth wash – Polystyrene is generally not used for making bottles for mouthwash or other liquids that can be ingested. Such bottles are usually made from materials like polyethylene terephthalate (PET) or high-density polyethylene (HDPE) which are better suited to storing liquids safely. Consequently, Option D is not correct for typical household-purpose polystyrene.

The correct answer for the most common use of polystyrene for household purposes from the options provided is Option C: To manufacture disposable cups and plates.

Q.39

Identify compound A in following reaction

Benzene + Ozone (excess) → Benzenetriozonide $\xrightarrow{A}$ Glyoxal

Options:

A.

conc. HNO_3

B.

Ni

C.

$Zn + H_2O$

D.

Zn

Answer: C

Solution:

Zn + H_2O is used for ozonolysis.

Q.40

Which from following pairs of compounds is an example of metamerism?

Options:

A.

But-2-ene and But-1-ene

B.

m-Butane and 2-Methylpropane

C.

Ethoxyethane and methoxypropane

D.

Dimethyl ether and ethyl alcohol

Answer: C

Solution:

Divalent oxygen is bonded to two different alkyl group having same molecular formula.

Q.41

If Q is the heat liberated from the system and W is the work done on the system then first law of thermodynamics can be written as,

Options:

A.

$Q = W - \Delta U$

B.

$Q = \Delta U - W$

C.

$Q = \Delta U + W$

D.

Q = –W

Answer: B

Solution:

First law of thermodynamics is $\Delta U = Q + W$

Q.42

Calculate the number of atoms in 5 gram metal that crystallises to form simple cubic unit cell structure having edge length 336 pm. (Density of metal = 9.4 g cm^{-3})

Options:

A.

1.0×10^{22}

B.

2.1×10^{22}

C.

1.4×10^{22}

D.

1.8×10^{22}

Answer: C

Solution:

$$\text{No. of atoms} = \frac{m \times N}{D \times a^3} = \frac{5 \times 1}{9.4 \times \left(3.36 \times 10^{-8}\right)^3} = 1.4 \times 10^{22}$$

Q.43

Identify the molecule in which central atom undergoes sp^3 hybridisation?

Options:

A.

BF_3

B.

H_2O

C.

C_2H_4

D.

$BeCl_2$

Answer: B

Solution:

The central atom in a molecule undergoes sp^3 hybridization when it forms four sigma (σ) bonds and has no lone pairs of electrons. Let's examine each option to identify the correct one:

Option A: BF_3

The boron atom in boron trifluoride (BF_3) has three bonding pairs and no lone pairs of electrons. As a result, boron is surrounded by three regions of electron density. Boron undergoes sp^2 hybridization to form three sp^2 hybrid orbitals, which overlap with the p orbitals of fluorine to form three σ bonds. Hence, BF_3 does not have sp^3 hybridization.

Option B: H_2O

In water (H_2O), the central oxygen atom has two bonding pairs (with hydrogen atoms) and two lone pairs of electrons. The oxygen atom is thus surrounded by four regions of electron density, which necessitates the use of four orbitals (one s and three p orbitals) to hybridize into four equivalent sp^3 hybrid orbitals. Two of these sp^3 hybrid orbitals form σ bonds with hydrogen atoms, and two accommodate the lone pairs. Therefore, water exhibits sp^3 hybridization.

Option C: C_2H_4

In ethene (C_2H_4), each carbon atom forms three sigma (σ) bonds—two with hydrogen atoms and one with the other carbon atom. In addition, there is a pi (π) bond between the carbon atoms formed from unhybridized p orbitals. Each carbon atom in ethene undergoes sp^2 hybridization to form three sp^2 hybrid orbitals involved in sigma bonding, leaving one p orbital to form the pi bond. Thus, ethene does not exhibit sp^3 hybridization either.

Option D: $BeCl_2$

In beryllium chloride ($BeCl_2$), the central beryllium atom forms two sigma (σ) bonds with two chlorine atoms and has no lone pairs. It uses its two available orbitals (one s and one p orbital) to hybridize into two equivalent sp hybrid orbitals. Therefore, beryllium in $BeCl_2$ undergoes sp hybridization, not sp^3.

Only **Option B**, H_2O, has a central atom that undergoes sp^3 hybridization. The correct answer is **Option B: H_2O**.

Q.44

Which one of the following conversions does NOT involve either oxidation or reduction?

Options:

A.

$Na \rightarrow Na^+$

B.

$VO_2^+ \rightarrow V_2O_3$

C.

$Zn^{2+} \rightarrow Zn$

D.

$CrO_4^{2-} \rightarrow Cr_2O_7^{2-}$

Answer: D

Solution:

Oxidation and reduction are chemical processes often involving the transfer of electrons between species. Oxidation involves the loss of electrons or an increase in oxidation state, whereas reduction involves the gain of electrons or a decrease in oxidation state.

Option A: $Na \rightarrow Na^+$ represents the oxidation of sodium metal to sodium ions, with sodium losing an electron (oxidation).

Option B: $VO_2^+ \rightarrow V_2O_3$ involves a change in the oxidation state of vanadium. In VO_2^+, the vanadium is in a +4 oxidation state, whereas in V_2O_3, vanadium is in a +3 oxidation state, suggesting that vanadium has been reduced.

Option C: $Zn^{2+} \rightarrow Zn$ involves the reduction of zinc ions to zinc metal, with zinc ions gaining two electrons (reduction).

Option D: $CrO_4^{2-} \rightarrow Cr_2O_7^{2-}$ does not involve a change in the oxidation state of chromium. Both species contain chromium in a +6 oxidation state. The conversion between chromate, CrO_4^{2-}, and dichromate, $Cr_2O_7^{2-}$, involves a change in the arrangement of oxygen atoms around chromium atoms, but does not involve a change in the oxidation number of chromium. Therefore, this conversion does not involve oxidation or reduction.

Based on these considerations, the correct answer to the question of which conversion does NOT involve either oxidation or reduction is:

Option D: $CrO_4^{2-} \rightarrow Cr_2O_7^{2-}$

Q.45

Calculate $\wedge_0$ of $CH_2ClCOOH$ if $\wedge_0$ for HCl, KCl and $CH_2ClCOOK$ are $4.2, 1.5$ and $1.1 \Omega^{-1} cm^2 mol^{-1}$ respectively?

Options:

A.

$2.7 \Omega^{-1} cm^2 mol^{-1}$

B.

$3.8 \Omega^{-1} cm^2 mol^{-1}$

C.

$1.9 \Omega^{-1} cm^2 mol^{-1}$

D.

$4.2\Omega^{-1}\,\mathrm{cm}^2\,\mathrm{mol}^{-1}$

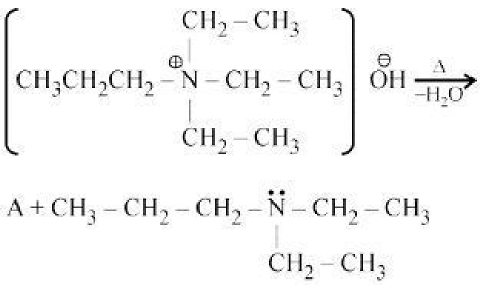
Answer: B

Solution:

$$\begin{aligned}\wedge_0 &= (\wedge_{CH_2ClCOOK} + \wedge_{HCl}) - \wedge_{KCl}\\ &= (1.1 + 4.2) - 1.5\\ &= 3.8\Omega^{-1}\,\mathrm{cm}^2\,\mathrm{mol}^{-1}\end{aligned}$$

Q.46

Identify the product A in the following reaction.



Options:

A.

$CH_3 - CH = CH_2$

B.

$H_2C = CH_2$

C.

$CH_3 - CH_2 - CH_3$

D.

$CH_3 - C \equiv CH$

Answer: B

Solution:

Hoffmann's β-elimination reaction.

Q.47

Calculate the amount of solute dissolved in 160 gram solvent that boils at $85^\circ C$, the molar mass of solute is $120\ \mathrm{g\,mol^{-1}}$. ($K_b$ for solvent $= 2.7^\circ C\ \mathrm{kg\,mol^{-1}}$ and boiling point of solvent $= 76^\circ C$)

Options:

A.

42 gram

B.

60 gram

C.

64 gram

D.

50 gram

Answer: C

Solution:

$$\Delta T_b = K_b \cdot m$$

$$9 = 2.7\left(\frac{m}{0.12} \times \frac{1}{0.16}\right)$$

$$m = \frac{9 \times 0.12 \times 0.16}{2.7} = 0.064\,kg = 64\,g$$

Q.48

Identify ether from the following compounds.

Options:

A.

Benzenol

B.

Benzene-1, 2-diol

C.

Methoxymethane

D.

Propan-2-ol

Answer: C

Solution:

Methoxymethane $CH_3 - O - CH_3$ is ether

Q.49

Which from following polymers is used to obtain bristles for brushes?

Options:

A.

Nylon 2 - nylon 6

B.

Nylon 6, 6

C.

Nylon 6

D.

Polyacrylamide

Answer: B

Solution:

The material used to obtain bristles for brushes is nylon, which is a type of synthetic polymer known for its strength, elasticity, and resistance to abrasion and chemicals. Let's examine the options you provided to determine the correct answer.

Option A: **Nylon 2 - nylon 6** doesn't typically refer to a commercial polymer used for bristles.

Option B: **Nylon 6,6** is a type of nylon made from hexamethylenediamine and adipic acid, which provides strong and durable fibers that are often used in bristles for brushes.

Option C: **Nylon 6** is another type of nylon, made from caprolactam, and it also has good properties to be used for bristles in brushes.

Option D: **Polyacrylamide** is not typically used for making bristles for brushes. It is more commonly used in water treatment processes, soil conditioner or as a flocculant.

Among the options given, both **Nylon 6,6** and **Nylon 6** are plausible materials for making brush bristles. However, traditionally, **Nylon 6,6** (Option B) is the most common polymer used for high-quality brush bristles due to its high tensile strength and stiffness. Therefore, Option B is likely the best answer.

Q.50

What is the pH of 2×10^{-3}M solution of monoacidic weak base if it ionises to the extent of 5% ?

Options:

A.

14

B.

6

C.

4

D.

2

Answer: C

Solution:

$$[H^+] = c\alpha = 2 \times 10^{-3} \times 5 \times 10^{-2} = 10^{-4} M$$
$$\therefore pH = 4$$
