VITEEE 2008 Question Paper

Vellore Institute of Technology Engineering Entrance Examination

SOLVED PAPER VITEEE 2008

PART - I (PHYSICS)

1. Two beams of light will not give rise to an interference patern, if
   (a) they are coherent
   (b) they have the same wavelength
   (c) they are linearly polarized perpendicular to each other
   (d) they are not monochromatic

2. A slit of width 'a' is illuminated with a monochromatic light of wavelength λ from a distant source and the diffraction pattern is observed on a screen placed at a distance 'D' from the slit. To increase the width of the central maximum one should
   (a) decrease D
   (b) decrease a
   (c) decrease λ
   (d) the width cannot be changed

3. A thin film of soap solution (n = 1.4) lies on the top of a glass plate (n = 1.5). When visible light is incident almost normal to the plate, two adjacent reflection maxima are observed at two wavelengths 420 and 630 nm. The minimum thickness of the soap solution is
   (a) 420 nm (b) 450 nm
   (c) 630 nm (d) 1260 nm

4. If the speed of a wave doubles as it passes from shallow water into deeper water, its wavelength will be
   (a) unchanged (b) halved
   (c) doubled (d) quadrupled

5. A light whose frequency is equal to 6×10^{14} Hz is incident on a metal whose work function is

   $2\text{eV}.\left[h = 6.63\times10^{-34}\text{ Js}\right]\ \left[1\text{eV} = 1.6\times10^{-19}\text{ J}\right]$

   The maximum energy of the electrons emitted will be
   (a) 2.49 eV (b) 4.49 eV
   (c) 0.49 eV (d) 5.49 eV

6. An electron microscope is used to probe the atomic arrangements to a resolution of 5Å. What should be the electric potential to which the electrons need to be accelerated ?
   (a) 2.5 V (b) 5 V
   (c) 2.5 kV (d) 5 kV

7. Which phenomenon best supports the theory that matter has a wave nature ?
   (a) Electron momentum
   (b) Electron diffraction
   (c) Photon momentum
   (d) Photon diffraction

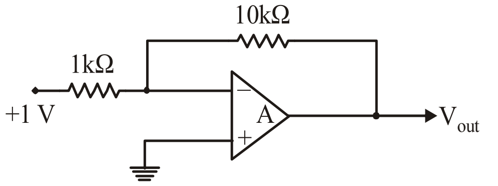
8. The radioactivity of a certain material drops to $\frac{1}{16}$ of the initial value in 2 hours. The half life of this radionuclide is
   (a) 10 min (b) 20 min
   (c) 30 min (d) 40 min

9. An observer 'A' sees an asteroid with a radioactive element moving by at a speed = 0.3 c and measures the radioactivity decay time to be T_A. Another observer 'B' is moving with the asteroid and measures its decay time as T_B. Then T_A and T_B are related as below
   (a) $T_B < T_A$
   (b) $T_A = T_B$
   (c) $T_B > T_A$
   (d) Either (A) or (C) depending on whether the asteroid is approaching or moving away from A

10. ^{234}U has 92 protons and 234 nucleons total in its nucleus. It decays by emitting an alpha particle. After the decay it becomes
    (a) ^{232}U (b) ^{232}Pa
    (c) ^{230}Th (d) ^{230}Ra

11. K_α and K_β x-rays are emitted when there is a transition of electron between the levels
    (a) n = 2 to n = 1 and n = 3 to n = 1 respectively
    (b) n = 2 to n = 1 and n = 3 to n = 2 respectively
    (c) n = 3 to n = 2 and n = 4 to n = 2 respectively
    (d) n = 3 to n = 2 and n = 4 to n = 3 respectively

12. A certain radioactive material ${}_{Z}X^{A}$ starts emitting α and β particles successively such that the end product is ${}_{Z-3}Y^{A-8}$. The number of α and β particles emitted are
(a) 4 and 3 respectively
(b) 2 and 1 respectively
(c) 3 and 4 respectively
(d) 3 and 8 respectively

13. 

In the circuit shown above, an input of 1V is fed into the inverting input of an ideal Op-amp A. The output signal V_{out} will be
(a) +10 V (b) –10 V
(c) 0 V (d) infinity

14. When a solid with a band gap has a donor level just below its empty energy band, the solid is
(a) an insulator
(b) a conductor
(c) a p-type semiconductor
(d) an n-type semiconductor

15. A p-n junction has acceptor impurity concentration of 10^{17} cm^{-3} in the p-side and donor impurity concentration of 10^{16} cm^{-3} in the n-side. What is the contact potential at the junction (kT = thermal energy, intrinsic carrier concentration $n_i = 1.4\times10^{10}$ cm^{-3}) ?
(a) $(kT/e)\ln(4\times10^{12})$
(b) $(kT/e)\ln(2.5\times10^{23})$
(c) $(kT/e)\ln(10^{23})$
(d) $(kT/e)\ln(10^{9})$

16. A Zener diode has a contact potential of 1V in the absence of biasing. It undergoes Zener breakdown for an electric field of 10^6 V/m at the depletion region of p-n junction. If the width of the depletion region is 2.5 μm, what should be the reverse biased potential for the Zener breakdown to occur ?
(a) 3.5 V (b) 2.5 V
(c) 1.5 V (d) 0.5 V

17. In Colpitt oscillator the feedback network consists of
(a) two inductors and a capacitor
(b) two capacitors and an inductor
(c) three pairs of RC circuit
(d) three pairs of RL circuit

18. The reverse saturation of p-n diode
(a) depends on doping concentrations
(b) depends on diffusion lengths of carriers
(c) depends on the doping concentrations and diffusion lengths
(d) depends on the doping concentrations, diffusion length and device temperature

19. A radio station has two channels. One is AM at 1020 kHz and the other FM at 89.5 MHz. For good results you will use
(a) longer antenna for the AM channel and shorter for the FM
(b) shorter antenna for the AM channel and longer for the FM
(c) same length antenna will work for both
(d) information given is not enough to say which one to use for which

20. The communication using optical fibers is based on the principle of
(a) total internal reflection
(b) Brewster angle
(c) polarization
(d) resonance

21. In nature, the electric charge of any system is always equal to
(a) half integral multiple of the least amount of charge
(b) zero
(c) square of the least amount of charge
(d) integral multiple of the least amount of charge

22. The energy stored in the capacitor as shown in Fig. (a) is 4.5×10^{-6} J. If the battery is replaced by another capacitor of 900 pF as shown in Fig. (b), then the total energy of system is

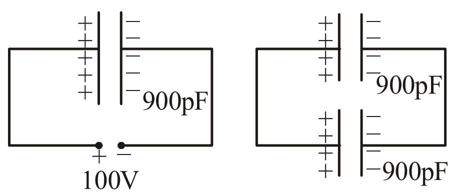


Fig. (a) Fig. (b)

(a) 4.5×10^{-6} J (b) 2.25×10^{-6} J
(c) zero (d) 9×10^{-6} J

23. Equal amounts of a metal are converted into cylindrical wires of different lengths (L) and cross-sectional area (A). The wire with the maximum resistance is the one, which has
(a) length = L and area = A
(b) length = $\frac{L}{2}$ and area = 2A
(c) length = 2L and area = $\frac{A}{2}$
(d) all have the same resistance, as the amount of the metal is the same

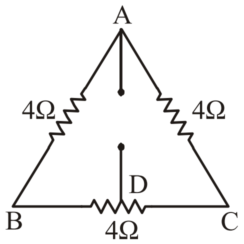
24. If the force exerted by an electric dipole on a charge q at a distance of 1m is F, the force at a point 2m away in the same direction will be
(a) $\frac{F}{2}$ (b) $\frac{F}{4}$
(c) $\frac{F}{6}$ (d) $\frac{F}{8}$

25. A solid sphere of radius R_1 and volume charge density $\rho = \frac{\rho_0}{r}$ is enclosed by a hollow sphere of radius R_2 with negative surface charge density σ, such that the total charge in the system is zero. ρ_0 is a positive constant and r is the distance from the centre of the sphere. The ratio $\frac{R_2}{R_1}$ is
(a) $\frac{\sigma}{\rho_0}$ (b) $\sqrt{2\sigma/\rho_0}$
(c) $\sqrt{\rho_0/(2\sigma)}$ (d) $\frac{\rho_0}{\sigma}$

26. A solid spherical conductor of radius R has a spherical cavity of radius a (a < R) at its centre. A charge + Q is kept at the centre. The charge at the inner surface, outer surface and at a position r (a < r < R) are respectively
(a) + Q, – Q, 0 (b) – Q, + Q, 0
(c) 0, – Q, 0 (d) + Q, 0, 0

27. A cylindrical capacitor has charge Q and length L. If both the charge and length of the capacitor are doubled, by keeping other parameters fixed, the energy stored in the capacitor
(a) remains same
(b) increases two times
(c) decreases two times
(d) increases four times

28. Three resistances of $4\,\Omega$ each are connected as shown in figure. If the point D divides the resistance into two equal halves, the resistance between point A and D will be



(a) $12\,\Omega$ (b) $6\,\Omega$
(c) $3\,\Omega$ (d) $\frac{1}{3}\,\Omega$

29. The resistance of a metal increases with increasing temperature because
(a) the collisions of the conducting electrons with the electrons increase
(b) the collisions of the conducting electrons with the lattice consisting of the ions of the metal increase
(c) the number of conduction electrons decreases
(d) the number of conduction electrons increases

30. In the absence of applied potential, the electric current flowing through a metallic wire is zero because
(a) the electrons remain stationary
(b) the electrons are drifted in random direction with a speed of the order of 10^{-2} cm/s
(c) the electrons move in random direction with a speed of the order close to that of velocity of light
(d) electrons and ions move in opposite direction

31. A meter bridge is used to determine the resistance of an unknown wire by measuring the balance point length *l*. If the wire is replaced by another wire of same material but with double the length and half the thickness, the balancing point is expected to be
(a) $\frac{1}{8\ell}$ (b) $\frac{1}{4\ell}$
(c) 8ℓ (d) $16\,\ell$

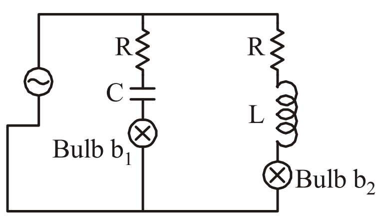
32. Identify the INCORRECT statement regarding a superconducting wire
   (a) transport current flows through its surface
   (b) transport current flows through the entire area of cross-section of the wire
   (c) it exhibits zero electrical resistivity and expels applied magnetic field
   (d) it is used to produce large magnetic field

33. A sample of HCl gas is placed in an electric field $3\times10^4\ NC^{-1}$. The dipole moment of each HCl molecule is 6×10^{-30}cm. The maximum torque that can act on a molecule is
   (a) $2\times10^{-34}\ C^2Nm^{-1}$ (b) 2×10^{-34} Nm
   (c) 18×10^{-26} Nm (d) $0.5\times10^{34}\ C^{-2}\ Nm^{-1}$

34. When a metallic plate swings between the poles of a magnet
   (a) no effect on the plate
   (b) eddy currents are set up inside the plate and the direction of the current is along the motion of the plate
   (c) eddy currents are set up inside the plate and the direction of the current oppose the motion of the plate
   (d) eddy currents are set up inside the plate

35. When an electrical appliance is switched on, it responds almost immediately, because
   (a) the electrons in the connecting wires move with the speed of light
   (b) the electrical signal is carried by electromagnetic waves moving with the speed of light
   (c) the electrons move with the speed which is close to but less than speed of light
   (d) the electrons are stagnant.

36. Two identical incandescent light bulbs are connected as shown in the Figure. When the circuit is an AC voltage source of frequency f, which of the following observations will be correct ?



   (a) both bulbs will glow alternatively
   (b) both bulbs will glow with same brightness provided frequency $f = \dfrac{1}{2\pi\sqrt{(1/LC)}}$
   (c) bulb b_1 will light up initially and goes off, bulb b_2 will be ON constantly
   (d) bulb b_1 will blink and bulb b_2 will be ON constantly

37. A transformer rated at 10 kW is used to connect a 5kV transmission line to a 240V circuit. The ratio of turns in the windings of the transformer is
   (a) 5 (b) 20.8
   (c) 104 (d) 40

38. Three solenoid coils of same dimension, same number of turns and same number of layers of winding are taken. Coil 1 with inductance L_1 was wound using a Mn wire of resistance $11\,\Omega$/m; Coil 2 with inductance L_2 was wound using the similar wire but the direction of winding was reversed in each layer; Coil 3 with inductance L_3 was wound using a superconducting wire. The self inductance of the coils L_1, L_2, L_3 are
   (a) $L_1 = L_2 = L_3$ (b) $L_1 = L_2;\ L_3 = 0$
   (c) $L_1 = L_3;\ L_2 = 0$ (d) $L_1 > L_2 > L_3$

39. Light travels with a speed of 2×10^8 m/s in crown glass of refractive index 1.5. What is the speed of light in dense flint glass of refractive index 1.8 ?
   (a) 1.33×10^8 m/s (b) 1.67×10^8 m/s
   (c) 2.0×10^8 m/s (d) 3.0×10^8 m/s

40. A parallel beam of fast moving electrons is incident normally on a narrow slit. A screen is placed at a large distance from the slit. If the speed of the electrons is increased, which of the following statement is correct ?
   (a) diffraction pattern is not observed on the screen in the case of electrons
   (b) the angular width of the central maximum of the diffraction pattern will increase
   (c) the angular width of the central maximum will decrease
   (d) the angular width of the central maximum will remain the same

PART - II (CHEMISTRY)

41. $CH_3CH_3 + HNO_3 \xrightarrow{675\,K}$
(a) $CH_3CH_2NO_2$
(b) $CH_3CH_2NO_2 + CH_3NO_2$
(c) $2CH_3NO_2$
(d) $CH_2 = CH_2$

42. When acetamide is hydrolysed by boiling with acid, the product obtained is :
(a) acetic acid (b) ethyl amine
(c) ethanol (d) acetamide

43. Which will not go for diazotization ?
(a) $C_6H_5NH_2$ (b) $C_6H_5CH_2NH_2$
(c) $\begin{matrix} H_2N \\ H_3C \end{matrix} > C_6H_4$ (d) $\begin{matrix} H_2N \\ O_2N \end{matrix} > C_6H_4$

44. Secondary nitroalkanes can be converted into ketones by using Y. Identify the Y from the following

$$\begin{matrix} R \\ R \end{matrix} > CHNO_2 + Y \longrightarrow \begin{matrix} R \\ R \end{matrix} > C{=}O$$

(a) Aqueous HCl (b) Aqueous NaOH
(c) $KMnO_4$ (d) CO

45. Alkyl cyanides undergo Stephen reduction to produce
(a) aldehyde (b) secondary amine
(c) primary amine (d) amide

46. The continuous phase contains the dispersed phase throughout, Example is
(a) Water in milk
(b) Fat in milk
(c) Water droplets in mist
(d) Oil in water

47. The number of hydrogen atoms present in 25.6 g of sucrose ($C_{12}H_{22}O_{11}$) which has a molar mass of 342.3 g is
(a) 22×10^{23} (b) 9.91×10^{23}
(c) 11×10^{23} (d) 44×10^{23}

48. Milk changes after digestion into :
(a) cellulose (b) fructose
(c) glucose (d) lactose

49. Which of the following sets consists only of essential amino acids ?
(a) Alanine, tyrosine, cystine
(b) Leucine, lysine, tryptophane
(c) Alanine, glutamine, lycine
(d) Leucine, proline, glycine

50. Which of the following is ketohexose ?
(a) Glucose (b) Sucrose
(c) Fructose (d) Ribose

51. The oxidation number of oxygen in KO_3, Na_2O_2 is
(a) 3, 2 (b) 1, 0
(c) 0, 1 (d) –0.33, –1

52. Reaction of PCl_3 and PhMgBr would give
(a) bromobenzene
(b) chlorobenzene
(c) triphenylphosphine
(d) dichlorobenzene

53. Which of the following is not a characteristic of transition elements ?
(a) Variable oxidation states
(b) Formation of colored compounds
(c) Formation of interstitial compounds
(d) Natural radioactivity

54. Cl - P - Cl bond angles in PCl_5 molecule are
(a) 120° and 90° (b) 60° and 90°
(c) 60° and 120° (d) 120° and 30°

55. The magnetic moment of a salt containing Zn^{2+} ion is
(a) 0 (b) 1.87
(c) 5.92 (d) 2

56. The number of formula units of calcium flouride CaF_2 present in 146.4 g of CaF_2 are (molar mass of CaF_2 is 78.08 g/mol)
(a) $1.129 \times 10^{24}\ CaF_2$ (b) $1.146 \times 10^{24}\ CaF_2$
(c) $7.808 \times 10^{24}\ CaF_2$ (d) $1.877 \times 10^{24}\ CaF_2$

57. The IUPAC name of the given compound $[Co(NH_3)_5Cl]Cl_2$ is
(a) pentaamino cobalt chloride chlorate
(b) cobalt pentaamine chloro chloride
(c) pentaamine chloro cobalt (III) chloride
(d) pentaamino cobalt (III) chlorate

58. When SCN^- is added to an aqueous solution containing $Fe(NO_3)_3$, the complex ion produced is
(a) $\left[Fe(OH_2)_2(SCN^-)\right]^{2+}$
(b) $\left[Fe(OH_2)_5(SCN^-)\right]^{2+}$
(c) $\left[Fe(OH_2)_8(SCN^-)\right]^{2+}$
(d) $\left[Fe(OH_2)(SCN^-)\right]^{6+}$

59. Hair dyes contain
(a) copper nitrate (b) gold chloride
(c) silver nitrate (d) copper sulphate

60. Schottky defects occurs mainly in electrovalent compounds where
(a) positive ions and negative ions are of different size
(b) positive ions and negative ions are of same size
(c) positive ions are small and negative ions are big
(d) positive ions are big and negative ions are small

61. The number of unpaired electrons calculated in $\{Co(NH_3)_6\}^{3+}$ and $\{Co(F_6)\}^{3-}$ are
(a) 4 and 4 (b) 0 and 2
(c) 2 and 4 (d) 0 and 4

62. The standard free energy change of a reaction is $\Delta G^\circ = -115 kJ$ at 298 K. Calculate the equilibrium constant k_p in log k_p $(R = 8.314\ Jk^{-1}\ mol^{-1})$.
(a) 20.16 (b) 2.303
(c) 2.016 (d) 13.83

63. If an endothermic reaction occurs spontaneously at constant temperature T and P, then which of the following is true ?
(a) $\Delta G > 0$ (b) $\Delta H < 0$
(c) $\Delta S > 0$ (d) $\Delta S < 0$

64. If a plot of $\log_{10}C$ versus t gives a straight line for a given reaction, then the reaction is
(a) zero order (b) first order
(c) second order (d) third order

65. A spontaneous process is one in which the system suffers :
(a) no energy change
(b) a lowering of free energy
(c) a lowering of entropy
(d) an increase in internal energy

66. The half life period of a first order reaction is 1 min 40 secs. Calculate its rate constant.
(a) 6.93×10^{-3} min^{-1} (b) 6.93×10^{-3} sec^{-1}
(c) 6.93×10^{-3} sec (d) 6.93×10^{3} sec

67. The molar conductivities of KCl, NaCl and KNO_3 are 152, 128 and 111 S cm^2 mol^{-1} respectively. What is the molar conductivity of $NaNO_3$?
(a) 101 S cm^2 mol^{-1} (b) 87 S cm^2 mol^{-1}
(c) –101 S cm^2 mol^{-1} (d) –391 S cm^2 mol^{-1}

68. The electrochemical cell stops working after sometime because :
(a) electrode potential of both the electrodes becomes zero
(b) electrode potential of both the electrodes becomes equal
(c) one of the electrodes is eaten away
(d) the cell reaction gets reversed

69. The amount of electricity required to produce one mole of copper from copper sulphate solution will be
(a) 1 Faraday (b) 2.33 Faraday
(c) 2 Faraday (d) 1.33 Faraday

70. Dipping iron article into a strongly alkaline solution of sodium phosphate
(a) does not affect the article
(b) forms $Fe_2O_3 . xH_2O$ on the surface
(c) forms iron phosphate film
(d) forms ferric hydroxide

71. Hydroboration oxidation of 4-methyl-octene would give
(a) 4-methyl octanol
(b) 2-methyl decane
(c) 4-methyl heptanol
(d) 4-methyl-2-octanone

72. When ethyl alcohol is heated with conc. H_2SO_4, the product obtained is :
(a) $CH_3COOC_2H_5$ (b) C_2H_2
(c) C_2H_6 (d) C_2H_4

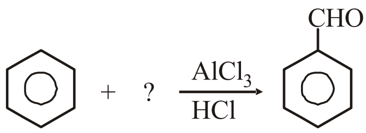
73. Anisole is the product obtained from phenol by the reaction known as
(a) coupling (b) etherification
(c) oxidation (d) esterification

74. Ethylene glycol gives oxalic acid on oxidation with
(a) acidified $K_2Cr_2O_7$
(b) acidified $KMnO_4$
(c) alkaline $KMnO_4$
(d) periodic acid

75. Diamond is hard because
(a) all the four valence electrons are bonded to each carbon atoms by covalent bonds
(b) it is a giant molecule
(c) it is made up of carbon atoms
(d) it cannot be burnt

76. A wittig reaction with an aldehyde gives
(a) ketone compound
(b) a long chain fatty acid
(c) olefin compound
(d) epoxide

77. Cannizaro reaction is given by
(a) HCHO (b) CH_3COCH_3
(c) CH_3CHO (d) CH_3CH_2OH

78. 

Identify the reactant
(a) H_2O (b) HCHO
(c) CO (d) CH_3CHO

79. Maleic acid and Fumaric acids are
(a) Position Isomers (b) Geometric Isomers
(c) Enentiomers (d) Functional Isomers

80. The gas evolved on heating alkali formate with soda-lime is
(a) CO (b) CO_2
(c) Hydrogen (d) water vapor

PART - III (MATHEMATICS)

81. If $\vec{a}, \vec{b}, \vec{c}$ be three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{2}\vec{b}, \vec{b}$ and $\vec{c}$ being non-parallel. If θ_1 is the angle between $\vec{a}$ and $\vec{b}$ and θ_2 is the angle between $\vec{a}$ and $\vec{c}$, then
(a) $\theta_1 = \frac{\pi}{6}, \theta_2 = \frac{\pi}{3}$ (b) $\theta_1 = \frac{\pi}{3}, \theta_2 = \frac{\pi}{6}$
(c) $\theta_1 = \frac{\pi}{2}, \theta_2 = \frac{\pi}{3}$ (d) $\theta_1 = \frac{\pi}{3}, \theta_2 = \frac{\pi}{2}$

82. The equation $\vec{r}^2 - 2\vec{r}.\vec{c} + h = 0, |\vec{c}| > \sqrt{h}$, represents
(a) circle (b) ellipse
(c) cone (d) sphere

83. The simplified expression of $\sin(\tan^{-1} x)$, for any real number x is given by
(a) $\frac{1}{\sqrt{1+x^2}}$ (b) $\frac{x}{\sqrt{1+x^2}}$
(c) $-\frac{1}{\sqrt{1+x^2}}$ (d) $-\frac{x}{\sqrt{1+x^2}}$

84. If $\left|\frac{z-25}{z-1}\right| = 5$, the value of $|z|$
(a) 3 (b) 4
(c) 5 (d) 6

85. Argument of the complex number $\left(\frac{-1-3i}{2+i}\right)$ is
(a) 45° (b) 135°
(c) 225° (d) 240°

86. In a triangle ABC, the sides b and c are the roots of the equation $x^2 - 61x + 820 = 0$ and $A = \tan^{-1}\left(\frac{4}{3}\right)$, then a^2 is equal to
(a) 1098 (b) 1096
(c) 1097 (d) 1095

87. The shortest distance between the straight lines through the points $A_1 = (6, 2, 2)$ and $A_2 = (-4, 0, -1)$, in the directions of $(1, -2, 2)$ and $(3, -2, -2)$ is
(a) 6 (b) 8
(c) 12 (d) 9

88. The center and radius of the sphere $x^2 + y^2 + z^2 + 3x - 4z + 1 = 0$ are
(a) $\left(-\frac{3}{2}, 0, -2\right); \frac{\sqrt{21}}{2}$
(b) $\left(\frac{3}{2}, 0, 2\right); \sqrt{21}$
(c) $\left(-\frac{3}{2}, 0, 2\right); \frac{\sqrt{21}}{2}$
(d) $\left(-\frac{3}{2}, 2, 0\right); \frac{21}{2}$

89. Let A and B are two fixed points in a plane then locus of another point C on the same plane such that CA + CB = constant, (> AB) is
(a) circle (b) ellipse
(c) parabola (d) hyperbola

90. The directrix of the parabola $y^2 + 4x + 3 = 0$ is
(a) $x - \frac{4}{3} = 0$ (b) $x + \frac{1}{4} = 0$
(c) $x - \frac{3}{4} = 0$ (d) $x - \frac{1}{4} = 0$

91. If g (x) is a polynomial satisfying $g(x)\,g(y) = g(x) + g(y) + g(xy) - 2$ for all real x and y and g (2) = 5 then $\underset{x \to 3}{\text{Lt}}\, g(x)$ is
(a) 9 (b) 10
(c) 25 (d) 20

92. The value of f(0) so that $\frac{(-e^x + 2^x)}{x}$ may be continuous at x = 0 is

(a) $\log\left(\frac{1}{2}\right)$ (b) 0

(c) 4 (d) $-1 + \log 2$

93. Let [ ] denote the greatest integer function and $f(x) = [\tan^2 x]$. Then

(a) $\lim_{x \to 0} f(x)$ does not exist

(b) f(x) is continuous at x = 0

(c) f(x) is not differentiable at x = 0

(d) f(x) = 1

94. A spherical balloon is expanding. If the radius is increasing at the rate of 2 centimeters per minute, the rate at which the volume increases (in cubic centimeters per minute) when the radius is 5 centimetres is

(a) 10π (b) 100π

(c) 200π (d) 50π

95. The length of the parabola $y^2 = 12x$ cut off by the latus-rectum is

(a) $6\left(\sqrt{2} + \log(1+\sqrt{2})\right)$

(b) $3\left(\sqrt{2} + \log(1+\sqrt{2})\right)$

(c) $6\left(\sqrt{2} - \log(1+\sqrt{2})\right)$

(d) $3\left(\sqrt{2} - \log(1+\sqrt{2})\right)$

96. If $I = \int \frac{x^5}{\sqrt{1+x^3}}dx$, then I is equal to

(a) $\frac{2}{9}(1+x^3)^{\frac{5}{2}} + \frac{2}{3}(1+x^3)^{\frac{3}{2}} + C$

(b) $\log\left|\sqrt{x} + \sqrt{1+x^3}\right| + C$

(c) $\log\left|\sqrt{x} - \sqrt{1+x^3}\right| + C$

(d) $\frac{2}{9}(1+x^3)^{\frac{3}{2}} - \frac{2}{3}(1+x^3)^{\frac{1}{2}} + C$

97. Area enclosed by the curve $\pi\left[4\left(x-\sqrt{2}\right)^2 + y^2\right] = 8$ is

(a) π (b) 2

(c) 3π (d) 4

98. The value of $\int_0^a \sqrt{\frac{a-x}{x}}dx$ is

(a) $\frac{a}{2}$ (b) $\frac{a}{4}$

(c) $\frac{\pi a}{2}$ (d) $\frac{\pi a}{4}$

99. Let y be the number of people in a village at time t. Assume that the rate of change of the population is proportional to the number of people in the village at any time and further assume that the population never increases in time. Then the population of the village at any fixed time t is given by

(a) $y = e^{kt} + c$, for some constants $c \le 0$ and $k \ge 0$

(b) $y = ce^{kt}$, for some constants $c \ge 0$ and $k \le 0$

(c) $y = e^{ct} + k$, for some constants $c \le 0$ and $k \ge 0$

(d) $y = ke^{ct}$, for some constants $c \ge 0$ and $k \le 0$

100. The differential equation of all straight lines touching the circle $x^2 + y^2 = a^2$ is

(a) $\left(y - \frac{dy}{dx}\right)^2 = a^2\left[1 + \left(\frac{dy}{dx}\right)^2\right]$

(b) $\left(y - x\frac{dy}{dx}\right)^2 = a^2\left[1 + \left(\frac{dy}{dx}\right)^2\right]$

(c) $\left(y - x\frac{dy}{dx}\right) = a^2\left[1 + \left(\frac{dy}{dx}\right)\right]$

(d) $\left(y - \frac{dy}{dx}\right) = a^2\left[1 - \frac{dy}{dx}\right]$

101. The differential equation $\left|\frac{dy}{dx}\right| + |y| + 3 = 0$ admits

(a) infinite number of solutions

(b) no solution

(c) a unique solution

(d) many solutions

102. Solution of the differential equation

$xdy - ydx - \sqrt{x^2 + y^2}dx = 0$ is

(a) $y - \sqrt{x^2 + y^2} = Cx^2$

(b) $y + \sqrt{x^2 + y^2} = Cx^2$

(c) $x + \sqrt{x^2 + y^2} = Cy^2$

(d) $x - \sqrt{x^2 + y^2} = Cy^2$

103. Let P, Q, R and S be statements and suppose that $P \to Q \to R \to P$. if $\sim S \to R$, then

(a) $S \to \sim Q$ (b) $\sim Q \to S$

(c) $\sim S \to \sim Q$ (d) $Q \to \sim S$

104. In how many number of ways can 10 students be divided into three teams, one containing four students and the other three?

(a) 400 (b) 700

(c) 1050 (d) 2100

105. If R be a relation defined as a R b iff $|a - b| > 0$, then the relation is

(a) reflexive (b) symmetric

(c) transitive

(d) symmetric and transitive

106. Let S be a finite set containing n elements. Then the total number of commutative binary operation on S is

(a) $n^{\left[\frac{n(n+1)}{2}\right]}$ (b) $n^{\left[\frac{n(n-1)}{2}\right]}$

(c) $n^{(n^2)}$ (d) $2^{(n^2)}$

107. A manufacturer of cotter pins knows that 5% of his product is defective. He sells pins in boxes of 100 and guarantees that not more than one pin will be defective in a box. In order to find the probability that a box will fail to meet the guaranteed quality, the probability distribution one has to employ is

(a) Binomial (b) Poisson

(c) Normal (d) Exponential

108. The probability that a certain kind of component will survive a given shock test is $\frac{3}{4}$. The probability that exactly 2 of the next 4 components tested survive is

(a) $\frac{9}{41}$ (b) $\frac{25}{128}$

(c) $\frac{1}{5}$ (d) $\frac{27}{128}$

109. Mean and standard deviation of marks obtained in some particular subject by four classes are given below. Report the class with best performance

(a) 80, 18 (b) 75, 5

(c) 80, 21 (d) 76, 7

110. A random variable X follows binomial distribution with mean α and variance β. Then

(a) $0 < \alpha < \beta$ (b) $0 < \beta < \alpha$

(c) $\alpha < 0 < \beta$ (d) $\beta < 0 < \alpha$

111. The system of equations

$x + y + z = 0$

$2x + 3y + z = 0$

$x + 2y = 0$

has

(a) a unique solution; $x = 0, y = 0, z = 0$

(b) infinite solutions

(c) no solution

(d) finite number of non-zero solutions

112. $\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}^4 = I$, then

(a) $a = 1 = 2b$ (b) $a = b$

(c) $a = b^2$ (d) $ab = 1$

113. If $D = \text{diag}(d_1, d_2, ..., d_n)$ where $d_1 \neq 0$, for $i = 1, 2, ..., n$, then D^{-1} is equal to

(a) D^T

(b) D

(c) Adj (D)

(d) $\text{diag}(d_1^{-1}, d_2^{-1}, ...d_n^{-1})$

114. If x, y, z are different from zero and

$\Delta = \begin{vmatrix} a & b-y & c-z \\ a-x & b & c-z \\ a-x & b-y & c \end{vmatrix} = 0$ then the value of the expression $\frac{a}{x} + \frac{b}{y} + \frac{c}{z}$ is

(a) 0 (b) –1

(c) 1 (d) 2

115. Probability of getting positive integral roots of the equation $x^2 - n = 0$ for the integer n, $1 \le n \le 40$ is

(a) $\frac{1}{5}$ (b) $\frac{1}{10}$

(c) $\frac{3}{20}$ (d) $\frac{1}{20}$

116. The number of real roots of the equation $x^4 + \sqrt{x^4 + 20} = 22$ is

(a) 4 (b) 2

(c) 0 (d) 1

117. Let α, β be the roots of the equation $x^2 - ax + b = 0$ and $A_n = \alpha^n + \beta^n$.

Then $A_{n+1} - aA_n + bA_{n-1}$ is equal to

(a) $-a$ (b) b

(c) 0 (d) $a - b$

118. If the sides of a right-angle triangle form an A.P., the 'Sin' of the acute angles are

(a) $\left(\frac{3}{5}, \frac{4}{5}\right)$

(b) $\left(\sqrt{3}, \frac{1}{\sqrt{3}}\right)$

(c) $\left(\sqrt{\frac{\sqrt{5}-1}{2}}, \sqrt{\frac{\sqrt{5}-1}{2}}\right)$

(d) $\left(\sqrt{\frac{\sqrt{3}-1}{2}}, \sqrt{\frac{\sqrt{3}-1}{2}}\right)$

119. The plane through the point (–1, –1, –1) and containing the line of intersection of the planes $\vec{r}.(\hat{i} + 3\hat{j} - \hat{k}) = 0$ and $\vec{r}.(\hat{j} + 2\hat{k}) = 0$ is

(a) $\vec{r}.(\hat{i} + 2\hat{j} - 3\hat{k}) = 0$

(b) $\vec{r}.(\hat{i} + 4\hat{j} + \hat{k}) = 0$

(c) $\vec{r}.(\hat{i} + 5\hat{j} - 5\hat{k}) = 0$

(d) $\vec{r}.(\hat{i} + \hat{j} + 3\hat{k}) = 0$

120. $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + 4\hat{j} + 3\hat{k}$ are one of the sides and medians respectively, of a triangle through the same vertex, then area of the triangle is

(a) $\frac{1}{2}\sqrt{83}$ (b) $\sqrt{83}$

(c) $\frac{1}{2}\sqrt{85}$ (d) $\sqrt{86}$

SOLUTIONS

PART - I (PHYSICS)

1. (d) Two beams of light give rise to an interference pattern if they are coherent, they have same wavelength/frequency in same phase or having a constant phase difference and same state of polarisation.
Interference pattern can not be obtained if the two beams are not mono chromatic.

2. (b) By the theory of diffraction at a single slit, the width of the central maximum is given by

$$W = \frac{2D\lambda}{a} \Rightarrow W \propto D,$$

$$W \propto \lambda \text{ and } W \propto \frac{1}{a}$$

Therefore, to increase the width of the central maximum a should be decreased.

3. (b) R_1 and R_2 are the two rays considered for interference. R_1 is the result of reflection at denser medium; hence it suffers an additional path difference of $\frac{\lambda}{2}$. Ray R_2 originates after reflection at narer medium.

Net path difference $= 2\mu t + \frac{\lambda}{2}$,

where t is the thickness of the soap solution.
For constructive interference,

$$2\mu t + \frac{\lambda}{2} = m\lambda, \text{ where } m = 0, 1, 2.....$$

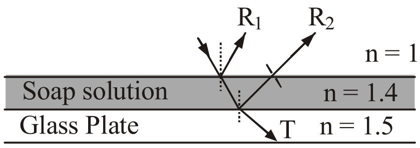
Let the two adjacent reflection maxima be observed at m and m–1. Then

$$2\mu t + \frac{\lambda_1}{2} = m\lambda_1$$

$$\text{or } 2\mu t = \left(m - \frac{1}{2}\right)\lambda_1 \quad ...(1)$$

$$\text{Similarly, } 2\mu t = \left(m - \frac{3}{2}\right)\lambda_2 \quad ...(2)$$

Solving (1) & (2), we get



$$2\mu t = \left(\frac{\lambda_2}{\frac{\lambda_2}{\lambda_1} - 1}\right)$$

Putting, $\lambda_1 = 420$ nm, $\lambda_2 = 630$ nm, $\mu = 1.4$
We get, $t = 450$ nm

4. (c) We have,
speed = frequency × wavelength
Also, when a wave passes from one medium to the other, its frequency remains constant.
$\therefore$ speed $\propto$ wavelength
Therefore, when the speed of a wave doubles, its wavelength also doubles.

5. (c) Here,
Frequency, $\nu = 6 \times 10^{14}$ Hz
Work-function, $\phi = 2eV = 2 \times 1.6 \times 10^{-19}$ J
$= 3.2 \times 10^{-19}$ J
Maximum energy, $T_{max} = ?$
By Einstein's photo electric equation, we have

$$h\nu = \phi + T_{max}$$
$$\Rightarrow T_{max} = h\nu - \phi$$
$$= (6.63 \times 10^{-34} \times 6 \times 10^{14}) - (3.2 \times 10^{-19})$$
$$= (3.97 \times 10^{-19}) - (3.2 \times 10^{-19})$$
$$= 0.77 \times 10^{-19} \text{ J}$$
$$= \frac{0.77 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} = 0.49 \text{ eV}$$

6. (b) We have, $d \sin \varphi = n\lambda$

For $\phi = 90°$ and $n = 1$, we get $d = \lambda$

$$\text{But } \lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}} = \sqrt{\frac{h^2}{2meV}}$$

$$= \sqrt{\left(\frac{(6.63 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times V}\right)}$$

$$= \sqrt{\frac{1.5}{V}} \times 10^{-9} \text{ m} \quad \therefore d = \sqrt{\frac{1.5}{V}} \times 10^{-9}$$

$$\text{or } 5 \times 10^{-10} = \sqrt{\frac{1.5}{V}} \times 10^{-9} \text{ or } 0.5 = \sqrt{\frac{1.5}{V}}$$

$$\text{or } 0.5 \times 0.5 = \frac{1.5}{V} \Rightarrow V = \frac{1.5}{0.5 \times 0.5} = 6V$$

No option is matching with the exact answer. But 5V is approximately equal to the exact potential. Therefore, option (b) should be the correct option.

7. (b) Davison and Germer performed an experiment to prove that matter has a wave nature. The experiment was based on electron diffraction.

8 (c) We have, $N_t = N_0\left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$,

When
N_t = number of atoms present after time t
N_0 = initial number of atoms
$T_{1/2}$ = half life of the nuclide

$$\therefore \frac{N_t}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}} \text{ or } \frac{1}{16} = \left(\frac{1}{2}\right)^{\frac{2}{T_{1/2}}}$$

$$\text{or } \left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^{\frac{2}{T_{1/2}}} \Rightarrow 4 = \frac{2}{T_{1/2}}$$

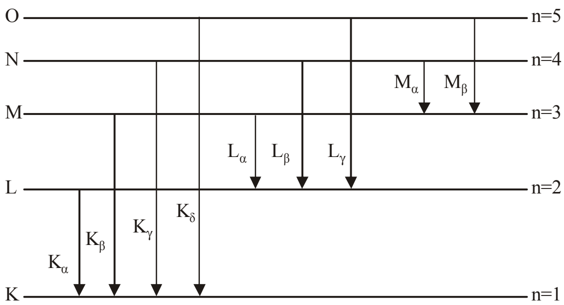
$$\Rightarrow T_{1/2} = \frac{2}{4}\text{Hr} = \frac{1}{2}\text{Hr} \quad \therefore T_{1/2} = 30\text{ min.}$$

9. (c) Due to time dilation the interval between two events at the same point in a moving frame appears to be longer by a factor $\gamma\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right)$ to an observer in a stationary frame.
Time dilation is independent of the direction of velocity and depends only on its magnitude.

10. (c) The α- decay in the case of ^{234}U takes place as follows :

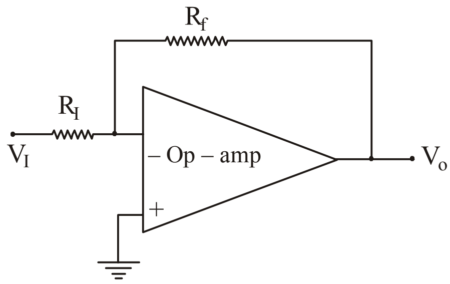
$$^{234}_{92}U \longrightarrow {}^{230}_{90}Th + {}^{4}_{2}He$$

11. (a) When electron jumps from the level n = 2 to the level n = 1, K_α x - rays are emitted. Similarly, K_β x-rays are emitted when there is a transition of electron from the level n = 3 to the level n = 1. X-ray spectra has been shown below.



12. (b) Let the number of α and β particles emitted be m and n respectively. Then
$A-4m = A-8 \Rightarrow m = 2$
[∵ the mass number of a radioactive nuclide decreases by 4 due to emission of one α - particle]
Again,
$(Z-2m)+n = Z-3$
[∵ the atomic number decreases by 2 due to emission of 1 α-particle but increases by 1 due to emission of 1 β - particle]
or $-2m+n = -3$
or $2m-n = 3$
or $(2\times 2)-n = 3$ (∵ m=2)
⇒ $n = 1$

13. (b)



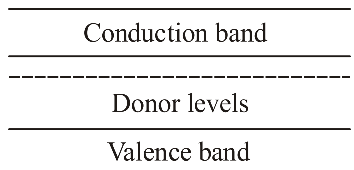
For the Op-amp shown above, we have

$$\frac{V_o}{V_I} = -\frac{R_f}{R_I}$$

Comparing this circuit with the given one,
We get $V_I = 1V$, $R_f = 10k\Omega = 10\times10^3\Omega$
$R_I = 1k\Omega = 1\times10^3\Omega$

$$\therefore \frac{V_o}{1} = -\frac{10\times10^3}{1\times10^3} \Rightarrow V_o = -10\text{ V}$$

14. (d) The solid is an n-type semiconductor. In an n-type semiconductor, the impurity is pentavalent which is also called the Donar impurity because one impurity atom generate one electron. The Donor energy level lies just below the conduction band as shown in the figure.



15. (a) We have, $I_D = I_S(e^{V_D/V_T}-1)$

where, $V_T = \frac{kT}{e}$

$$\therefore \left(\frac{I_D}{I_S}+1\right) = e^{\left(\frac{V_0}{V_T}\right)} \quad \text{or} \quad \ln\left(1+\frac{I_D}{I_S}\right) = \frac{V_D}{nV_T}$$

$$\therefore V_D = nV_T \ln\left(1+\frac{I_D}{I_S}\right) = V_T \ln\left(1+\frac{I_D}{I_S}\right)$$

$$= V_T \ln\left(\frac{I_{majority}}{I_S}\right)$$

Here, $n_e = (10^{17} - 10^{16})\text{cm}^{-3}$

$$= 10^{16}(10-1)\text{cm}^{-3} = 9\times10^{16}\,\text{cm}^{-3}$$

We know that, $n_e \times n_h = n_i^2$

$$\therefore n_h = \frac{n_i^2}{n_e} = \frac{(1.4\times10^{10})^2}{9\times10^{16}}\text{cm}^3$$

Also, $\frac{I_{majority}}{I_S} \propto \frac{n_e}{n_h}$

$$\therefore V_D = \frac{kT}{e}\ln\left[\frac{9\times10^{16}}{\frac{(1.4\times10^{10})^2}{9\times10^{16}}}\right]$$

$$= \frac{kT}{e}\ln(4\times10^{12})$$

16. (c) Here, electric field, $E = 10^6$ V/m
width of depletion region,
$d = 2.5\ \mu\text{m} = 2.5\times10^{-6}$m
$\therefore$ Potential required for breakdown $(V) = Ed = (10^6\times2.5\times10^{-6})\ V = 2.5\ V$
Contact Potential = 1.0 V
$\therefore$ Reverse biased potential for zener breakdown $= (2.5 - 1.0)\ V = 1.5\ V$

17. (b) In a colpitt oscillator, the feed-back network consists of two capacitors and one inductor.

18. (d) The reverse saturation of p-n diode depends on the doping concentrations, diffusion length and device temperature.

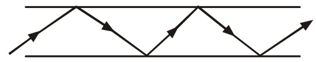
19. (a) Antenna length $= \frac{\lambda}{4} \propto \lambda$

$\because \lambda \propto \frac{1}{\nu}$ for cosntant velocity,

$\therefore$ Antenna length $\propto \frac{1}{\nu}$

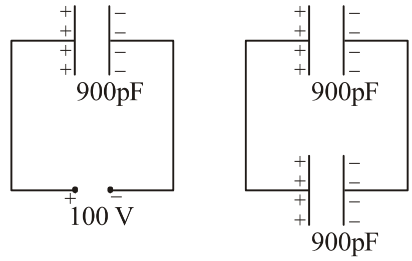
Here, $\nu_{AM} < \nu_{FM}$
$\therefore$ Antenna length for AM should be longer than that of FM.

20. (a) The principle of communication using optical fibers is based on the principle of total internal reflection.



21. (d) According to quantization of charge, the charge of any system is an integral multiple of the charge of electron which is the least amount of charge on any system.

22. (b) The charge on the capacitor when connected to the battery is given by



$Q = CV = (900\times10^{-12}\,F)\times100\,V = 9\times10^{-8}\,C$
When the battery is replaced by another capacitor of 900 pF capacitance, the charge of 9×10^{-8} C is distributed on both. Let Q_1 and Q_2 be the charge on each of them.
$\therefore Q = Q_1 + Q_2$
$= C_1V + C_2V$, where V is the common potential.

or $V = \frac{Q}{C_1+C_2}$

As the two capacitors are in parallel, the equivalent capacitance is given by $C = C_1 + C_2$

$\therefore$ Total energy of the capacitors $= \frac{1}{2}CV^2$

$$= \frac{1}{2}\times(C_1+C_2)\times\frac{Q^2}{(C_1+C_2)^2} = \frac{Q^2}{2(C_1+C_2)}$$

$$= \frac{(9\times10^{-8})^2}{2(900\times10^{-12}+900\times10^{-12})}$$

$$= \frac{(9)^2\times10^{-16}}{2\times2\times9\times10^{-10}} = 2.25\times10^{-6}\,J$$

23. (c) We know that $R = \rho\frac{l}{A}$

$\therefore$ For $l = L$ and $A = A$; $R_1 = \rho\frac{L}{A}$

For $l = \frac{L}{2}$ and $A = 2A$;

$$R_2 = \rho\frac{\left(\frac{L}{2}\right)}{2A} = \frac{1}{4}\left(\rho\frac{L}{A}\right) = \frac{1}{4}R_1$$

For $l = 2L$ and $A = \frac{A}{2}$;

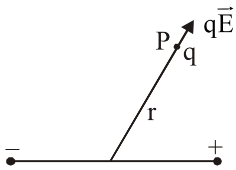
$$R_3 = \rho\frac{2L}{\left(\frac{A}{2}\right)} = 4\left(\rho\frac{L}{A}\right) = 4R_1$$

Thus $R_3 > R_1 > R_2$

Therefore, the wire having length 2L and area $\frac{A}{2}$ has the maximum resistance.

24. (d) We know that the electric field at any point due to an electric dipole varies inversely with the cube of the distance of the point from the centre of the dipole, that is,

$$E \propto \frac{1}{r^3}$$



Also, force on a charge (q) is given by

$$F = qE \Rightarrow F \propto \frac{1}{r^3}$$

When the distance of the charge becomes 2m, i.e. double of its initial value, then new force (F′) will become

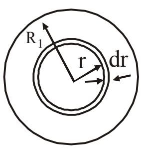
$$F' = \frac{1}{(2)^3}.F = \frac{F}{8}$$

25. (c) Let us first calculate the total charge on the soild sphere.

Let us consider a concentric sphere of radius r and thickness dr.

Then volume of the sphere, $dV = 4\pi r^2 dr$

Given, the volume charge density of the sphere $= \rho = \frac{\rho_0}{r}$



∴ Charge on this sphere,

$$dQ = \rho.dV = \frac{\rho_0}{r}.4\pi r^2 dr = 4\pi\rho.rdr.$$

∴ Total charge on the whole solid sphere,

$$Q_s = \int_0^{R_1} dQ = 4\pi\rho_0\int_0^{R_1} rdr$$

$$= 4\pi\rho_0\frac{R_1^2}{2} = 2\pi\rho_0 R_1^2$$

$$\therefore Q_S = 2\pi\rho_0 R_1^2 \quad ...(1)$$

Now, the total charge on the hollow sphere,

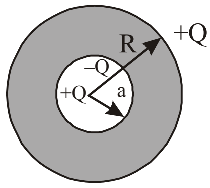
$$Q_h = -(4\pi R_2^2)\sigma \quad ...(2)$$

By question, $Q_s + Q_h = 0$

$\Rightarrow 2\pi\rho_0 R_1^2 = 4\pi R_2^2\sigma$

$$\Rightarrow \left(\frac{R_2}{R_1}\right)^2 = \frac{\rho_0}{2\sigma} \Rightarrow \frac{R_2}{R_1} = \sqrt{\frac{\rho_0}{2\sigma}}$$

26. (b) A charge Q will be induced on the inner surface of the solid spherical conductor. An equal but opposite charge will be induced on the outer surface of the conductor. There will be no charge at a position between the inner and outer surface.



27. (b) The energy stored in a capacitor is given by

$$E = \frac{Q^2}{2C}$$

Here, $C = \frac{2\pi\epsilon_0 L}{\ln\left(\frac{R_2}{R_1}\right)}$, where

L = length of the cylindrical conductor

R_1 = inner radius

R_2 = outer radius

$$\therefore C \propto L \Rightarrow E \propto \frac{Q^2}{L}$$

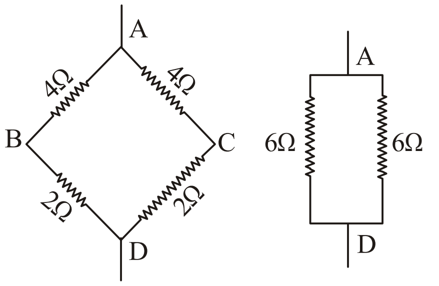
When both Q and L are doubled, by keeping other parameters fixed, the energy stored (E′) becomes

$$E' \propto \frac{(2Q)^2}{2L}$$

$$\propto \frac{2Q^2}{L}$$

$\therefore E' = 2E$

28. (c) Redrawing the figure, we get



(a) (b)

The 4Ω and 2Ω resistors are in series and the same is the case with another 4Ω and 2Ω resistors. So, the four resistors are equivalent to following two resistors.

Now, in fig (b) these two 6Ω resistors are in parallel.

$\therefore$ Equivalent resistance, $R = \frac{6 \times 6}{6+6} = \frac{36}{12}$
$= 3\Omega.$

29. (b) In a conductor, the charge carriers are electrons. As the temperature is increased, the collisions of these conduction electrons with the fixed ions of the lattice of the metal increases and hence the resistance of the conductor also increases.

30. (b) If there is no potential difference through a metallic wire, the current is zero because the electrons drift in a random direction with a speed of the order of 10^{-2} cm/s so that the net charge crossing a particular cross-section in a given time is zero.

31. If the wire is replaced by another wire of same material but with double the length and half the thickness, the resistance of the wire as a whole will change. Let us calculate this change.

Initial resistance, $R = \rho\frac{L}{A} = \rho\frac{L}{\pi r^2}$

Fubak resistance, $R' = \rho\frac{2L}{A'} = \rho\frac{2L}{\pi\left(\frac{r}{2}\right)^2}$

$= \rho\frac{8L}{\pi r^2} = 8\left(\rho\frac{L}{\pi r^2}\right) = 8R$

Therefore, the resistance will increase by eight times.

In a meter-bridge, we have

$\frac{R}{S} = \frac{l}{100-l}$

where R = unknown resistor
S = known resistor
l = balancing length

$\therefore l = \frac{100R}{S+R}$

When the resistance of the new wire is 8R then the new balancing length will be

$l = \frac{100 \times 8R}{S+8R} = \frac{100 \times 8R}{(S+R)+7R}$

$= \frac{100 \times 8R}{\left(\frac{100R}{l}\right)+7R} \quad \therefore l' = \frac{800l}{100+7l}.$

No option is correct.

32. (b) The transport current flows through the surface of the superconducting wire and not through the entire area of cross-section of the wire.

33. (c) Here, $E = 3 \times 10^4\,NC^{-1}$
$p = 6 \times 10^{-30}$ Cm
$\Gamma_{max} = ?$

We have,

torque acting on a dipole $\Gamma = pE\sin\theta$

The maximum value of Γ, i.e. $\Gamma_{max} = pE$

$\therefore \Gamma_{max} = (3 \times 10^4 \times 6 \times 10^{-30})$ Nm
$= 18 \times 10^{-26}$ Nm

34. (c) When a metallic plate swings between the poles of a magnet, eddy currents are set up inside the plate. These currents set up their own magnetic field which opposes the magnetic field of the poles. Thus, the direction of the current opposes the motion of the plate.

35. (b) When an electrical appliance is switched on, the electrons in the conducting wires move with their drift speed which is very less than the speed of light. But as soon as the switch is on, an electromagnetic wave is set up inside the conductor and the electrical signal is carried by them. The speed of electromagnetic wave is, of course, equal to the speed of light and hence the appliance responds almost immediately after the switch is made on.

36. (b) The circuit shown is a parallel resonant circuit. The frequency is $f = \frac{1}{2\pi\sqrt{LC}}$ at

resonance. Also, at resonance, the capacitative reactance is equal to the inductive reactance. Therefore, equal current will flow through both the bulbs b_1 and b_2. So, both will glow with same brightness.

37. (b) In a transformer, the ratio of turns in the windings is given by

$$\frac{N_P}{N_S} = \frac{V_P}{V_S}$$

$$\therefore \frac{N_P}{N_S} = \frac{5kV}{240V} = \frac{5000V}{240V} = 20.8$$

38. (a) Self-inductance (L) of a solenoid $= \frac{\mu_0 N^2 A}{l}$

where μ_o = absolute permittivily of space/ air

N = number of turns in the coil

A = area of cross-section of the solenoid

l = length of solenoid.

This expression shows that for all three solenoids, the self-inductances will be equal.

39. (b) We know that the refractive index of a medium is given by

$$\mu = \frac{c_o}{c}$$

Where c = velocity of light in the medium

c_o = velocity of light in vacuum

For crown glass,

$$1.5 = \frac{c_o}{2 \times 10^8} \quad ...(1)$$

For flint glass, $1.8 = \frac{c_o}{c}$ ...(2)

Dividing (2) by (1), we get $\frac{2 \times 10^8}{c} = \frac{1.8}{1.5}$

or $c = \frac{1.5 \times 2 \times 10^8}{1.8} m/s = 1.67 \times 10^8$ m/s

40. (c) Fast moving electrons create electromagnetic waves. So, the diffraction pattern will be observed. Also, the angular width of the central maximum is given by

$W = \frac{2\lambda}{a}$, where a = width of the slit

λ = wavelength of the light.

When the speed of electrons will increase, the frequency will increase which will result in decreasing the wavelength as the speed of light is a constant. Therefore, the angular width will decrease.

PART - II (CHEMISTRY)

41. (b) Aliphatic nitro compounds are prepared by vapour phase nitration of alkanes at 693-793K, under pressure. Alkanes though less reactive do undergo nitration to give a mixture of nitro alkanes resulting through cleavage of carbon-carbon bond alongwith oxidation product like CO_2, NO_2, H_2O etc.

$$\underset{}{CH_3CH_3} + \underset{\text{(fuming)}}{HNO_3} \xrightarrow{675K} \underset{(80\%)}{CH_3CH_2NO_2} + \underset{(20\%)}{CH_3 - NO_2}$$

42. (a) Amides are hydrolysed rapidly by acids to produce carboxylic acid and ammonium salt;

$$RCONH_2 \xrightarrow{H_3O^+} RCOOH + NH_4^+$$

Hence acetamide will give acetic acid on hydrolysis.

43. (b) Diazotisation reactions are shown by primary aromatic amine only as the arene diazonium salt formed is stable at 273-278 K. Compound $C_6H_5CH_2NH_2$ is not an aromatic amine, hence will not give the test/reaction.

44. (a) Secondary nitro alkanes upon hydrolysis with boiling HCl gives a ketone & nitrous oxide.

$$2R_2CHNO_2 \xrightarrow[\text{boiling}]{HCl} 2R_2CO + N_2O + H_2O$$

45. (a) Stephen reduction is partial reduction of alkyl or aryl cyanides to give corresponding aldehydes with a suspension of anhydrous stannous chloride in ether.

$$SnCl_2 + 2HCl \longrightarrow SnCl_4 + 2[H]$$

$$CH_3C \equiv N + 2[H] + HCl$$

$$\xrightarrow[A]{\text{Ether}} \underset{\text{diamine hydrochloride}}{CH_3CH = NH.HCl}$$

$$\xrightarrow[\text{boiling } H_2O]{\text{hydrolysis}} CH_3CHO + NH_4Cl$$

46. (a) In a colloidal system, the substance present in large amount in the mixture is called the dispersed medium & the solute is called dispersed phase. In case of milk and water solution the dispersed phase is milk protein & fat and water is dispersed medium.

47. (b) No. of moles of a compound

$$= \frac{\text{given mass (gm)}}{\text{Molar mass (gm)}}$$

i.e. $\frac{25.6}{342.3} = 0.0748$ moles.

1 mole of sucrose ($C_{12}H_{22}O_{11}$) contains 6.022 × 10^{23} molecules of it.

Hence 0.0748 moles contains

$= 6.022 \times 10^{23} \times 0.0748$

$= 0.4504 \times 10^{23}$ molecules.

1 molecule of sucrose by formula is having 22 atoms of hydrogen.

∴ 0.4504 × 10^{23} × 10^{23} × 22 = 9.91 × 10^{23} atoms of hydrogen.

48. (c) Milk contains lactose as milk sugar. After digestion of milk lactose is broken down by enzymes lactase to form glucose and galactose before it enter the blood stream.

49. (b) Out of 20 amino acids, the 10 amino acids which human body cannot synthesize are called essential amino acids. The ten essential amino acids are :

1) Valine 2) Leucine 3) Isolucine 4) Histidine 5) Phenylanaline 6) Methionine 7) Tryptophan 8) Lysine 9) Arginine 10) Threonine.

50. (c) Among the given examples, glucose is an alcohexose, sucrose is a disaccharide, fructose is a ketohexose while ribose is a aldopentose.

51. (d) To find the oxidation number of a given compound we have to equate the charge on the overall compound with the charge on individual atom of which the compound is made of.

In KO_3 . K is an alkali metal.

hence its oxidation number is +1.

$$(+1) + 3 \times (x) = 0 \text{ or } x = \frac{-1}{3} \cong -0.33$$

hence oxidation number of oxygen

i.e. x = –0.33.

In Na_2O_2, again Na is an alkali metal.

Hence $2 \times (+1) + 2 \times x = 0$

$$x = \frac{-2}{2} = -1$$

52. (c) In a reaction the alkyl part of grignard reagent acts as a nucleophile as carbon is more electronegative than magnesium. Hence the alkyl part will get attached to the electron deficient species.

In PCl_3, chlorine is more electronegative than phosphorous.

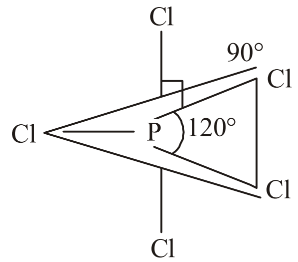
Hence the Ph^- will attack the phosphorous in PCl_3 to form organic phosphine with formula Ph_3P.

$$3PhMgBr + PCl_3 \longrightarrow \underset{\text{Triphenylphosphine}}{Ph_3P} + 3MgBrCl$$

53. (d) Transition elements or d-block elements have variable oxidation states, they form coloured compounds because of partially filled d-orbitals and also because of small size they form interstitial compounds. They are stable elements and does not show radio activity.

54. (a) In PCl_5, phosphorous undergoes sp^3d hybridization and has trigonal bipyramidal geometry. It has two axial chlorine atoms & three equatorial chlorine atoms bonded to the central P.

Hence bond angles for axial are 90°, Cl–P–Cl & for equatorial Cl–P–Cl it is 120°.



55. (a) Magnetic moment of a salt depends upon the number of unpaired d-electrons. In Zn^{2+} salt configuration of cation is $4s^0 3d^{10}$. Hence total no. of unpaired electron, n is zero. So magnetic moment i.e.

B.M. $= \sqrt{n(n+2)} = 0.$

56. (a) Formula unit = no. of molecules of CaF_2.

$$\text{Moles} = \frac{\text{mass in gm}}{\text{molar mass}} = \frac{146.4\,\text{gm}}{78.08\,\text{gm}} = 1.875$$

Molecules = Mole × 6.022 × 10^{23}

= 1.875 × 6.022 × 10^{23}

= 1.129 × 10^{24} units of CaF_2

57. (c) For writing IUPAC name of a co-ordination compound we first write the name of (+) ive complex here. $[Co(NH_3)_5Cl]^{2+}$

The names of ligands will come first in alphabatical order, followed by metal ion with its oxidation state written in bracket or parentheis in Roman number i.e. Co (III) here.

IUPAC name for cationic complex

→ Pentaamine chloro cobalt (III).

This will follow the name of anion with a gap.

i.e. Pentaamine chloro cobalt (III) chloride.

58. (b) Fe (III) ion from ferric nitrate will react with thiocyanate, SCN^- ion to form a blood red complex i.e. $FeSCN^{2+}$. But in presence of water it forms a complex containing five water molecules. i.e. $[Fe(H_2O)_5(SCN^-)]^{2+}$.

59. (c) Silver nitrate has been used since the begining of nineteenth century to dye hair. Silver salts darken when exposed to light and silver combines with protein yeilding a dark coloured proteinate.

60. (b) Schottky defect is generally shown by compounds which have ionic nature and small difference in the size of cations & anions.
In this defect equal no. of cations & anions is found missing from their lattice sites.

61. (d) In $[Co(NH_3)_6]^{3+}$ and $\{CoF_6\}^{3-}$ both the oxidation state of cobalt ion is +3. In first case NH_3 is the neutral ligand which is a strong field ligand.
hence the electrons in Co (+III) i.e. $4s^0 3d^6$ get paired to form inner orbital complex. Hence no unpaired electron.
On the other hand F^- is a weak field ligand hence it forms an outer orbital complex with 4 unpaired electrons.

62. (a) $\Delta G° = -115$ kJ at 298K.
Now, $\Delta G° = -2.303\ RT \log k_P$
$R = 8.314\ JK^{-1}\ mol^{-1}$ & $T = 298K$.
$\Delta G° = -2.303 \times 8.314 \times 298 \times \log k_P$

$$\log k_P = \frac{-115 \times 10^3}{-2.303 \times 8.314 \times 298} = 20.155$$
$= 20.16$

63. (c) For a reaction to take place spontaneously the value of ΔG must be negative i.e. $\Delta G < 0$.
Now, $\Delta G = \Delta H - T\Delta S$.
As the reaction is endothermic, so value of ΔH must be positive, i.e. $\Delta H > 0$.
Hence to have a negative ΔG.
$\Delta H < T\Delta S$. As T & P are constant.
$T\Delta S$ must be positive to give the total value a negative sign.
Hence $\Delta S > 0$.

64. (b) Any first order reaction follows the equation

$$\log [A] = \frac{-k}{2.303} t + \log [A]_o$$

$\because$ it resembles equation of straight line
$y = mx + C$
$y = \log [A]$ i.e. $\log_{10} C$

$$m = \frac{-k}{2.303} \quad \text{if } x = t\ \&\ C = \log [A]_0$$

hence the plot is for a 1^{st} order reaction.

65. (b) The driving forces which are responsible for a process to be spontaneous are :
i) Tendency for minimizing energy
ii) Tendency for maximum randomness. i.e. maximum entropy

66. (b) For 1^{st} order reaction.

$$t_{1/2}\ (\text{half life time}) = \frac{0.693}{K}$$

$$\text{Hence } K = \frac{0.693}{t_{1/2}} = \frac{0.693}{(60+40)\ \text{sec}} = 0.693 \times$$
$10^{-2}\ sec^{-1} = 6.93 \times 10^{-3}\ sec^{-1}$

67. (b) Since $NaNO_3$ is formed by the reaction

$$NaCl + KNO_3 \rightarrow NaNO_3 + KCl$$

hence, using Kohlrausch's law

$$\overset{o}{\Lambda}_{m\,NaNO_3} = \overset{o}{\Lambda}_{NaCl} + \overset{o}{\Lambda}_{KNO_3} - \overset{o}{\Lambda}_{KCl}$$
$$= 128 + 111 - 152 = 87\ S\ cm^2\ mol^{-1}$$

68. (c) As we know that
$\Delta E_{cell} = E_{cathode} - E_{anode}$
when $E_{cathode} = E_{anode}$
$\Delta E_{cell} = 0$
If $\Delta E_{cell} = 0$ no net reaction occurs. The reactants and products are at equilibrium and no current will flow.

Note that it is only possible to obtain electrical work from a system that is not at equilibrium. In order for current to flow, there must be a net reaction occurring. As the oxidation- reduction reaction proceeds toward equilibrium, and the concentrations of the reacting species approach their equilibrium values, the EMF of the cell decreases to zero. When the system is at equilibrium, the cell potential is zero and we have a dead battery.

69. (c) In $CuSO_4$ solution, oxidation state of Cu is +2. Hence one mole of copper sulphate will require charge equal to two moles of electrons to form metallic Cu. Mole charge = IF. Hence 2 Faraday is required.

70. (c) Rusting of iron is generally promoted in an acidic aqueous medium. Alkaline medium prevents availability of H^+ ions. Sodium phosphate will cause formation of a protective film of iron phosphate on the iron preventing rusting. These solutions are used in car radiators to prevent rusting of iron parts.

71. (a) Hydroboration-oxidation of alkenes give alcohols containing same number of carbon atoms. The addition follow anti-Markowi Koff's rule. Boron atom act as an electrophile. Main two steps re involved. Reagent used BH_3 & $NaOH/H_2O_2$.

$$3H_3C-CH_2-CH_2-CH_2-CH(Me)-CH_2-CH=CH_2+BH_3$$

4 - methyl octene

$$\longrightarrow R_3B$$

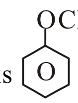
where R=4-methyl octyl

$$\xrightarrow[NaOH]{H_2O_2} 3CH_3(CH_2)_3-\underset{\underset{Me}{|}}{CH}-(CH_2)_3-OH$$

4−Methyloctanol

72. (d) Ethyl alcohol on treating with conc. H_2SO_4 undergoes dehydration to form alkene i.e. ethylene

$$C_2H_5OH \xrightarrow[H_2SO_4]{Conc} C_2H_4 + H_2O$$

73. (b) Anisole is  , Phenyl methyl ether. It can be prepared by treating phenol first with a base like NaOH to form phenoxide ion. The phenoxide ion will then substitute the halide of an R-X molecule, to form methyl phenyl ether.

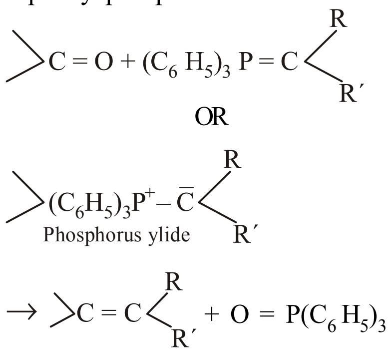
$$C_6H_5ONa + CH_3-Cl \rightarrow \underset{Anisole}{CH_3-O-C_6H_5}$$

74. (c) Alkaline Potassium permanganate is a strong oxidising agent. It oxidises ethylene glycol to oxalic acid.

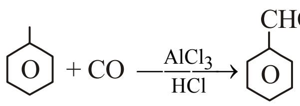
$$\underset{\text{Ethylene glycol}}{\begin{array}{c}CH_2OH\\|\\CH_2OH\end{array}} \xrightarrow[KMnO_4]{alk.} \underset{\text{Oxalic acid}}{\begin{array}{c}COOH\\|\\COOH\end{array}}$$

75. (a) In the structure of diamond each carbon atom in sp^3 hybridised & is covalently bonded with four other carbon atom held at the corners of a regular tetrahedron by covalent bonds. This results in a very big three dimensional polymeric structure in which C – C distance is 154 pm and bond angle is 109.5°. Owing to very strong covalent bonds by which atoms are held to gether diamond is the hardest substance known.

76. (c) The witting reaction is a chemical reaction of an aldehyde or ketone with triphenyl phosphonium ylide to give an alkene and triphenyl phosphine oxide.



77. (a) Cannizaro's reaction is for those aldehydes which does not contain α- hydrogen atom. This is also called self oxidation - reduction reaction. Among the given carbonyl compounds only HCHO does not have α-hydrogen.

78. (c) 

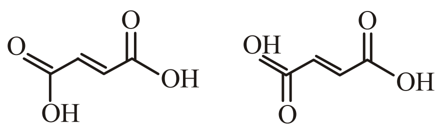
When a mixture of CO and HCl gas is passed through benzene in presence of catalyst consisting anhydrous $AlCl_3$, benzaldehyde is formed.

$$CO + HCl \rightarrow HOCl$$

formyl chloride

$$C_6H_6 + HOCl \xrightarrow{AlCl_3} C_6H_5CHO + HCl$$

79. (b) Maleic acid & fumaric acid are both the isomers of butene dioic acid. Maleic acid is the cis isomer & fumaric is the trans-isomer.



Maleic acid Fumaric acid

80. (c) Alkali formate i.e. HCOONa with soda-lime i.e. NaOH + CaO will react to give Na_2CO_3 and hydrogen gas is liberated.

$$HCOONa + NaOH \xrightarrow{CaO} Na_2CO_3 + H_2 \uparrow$$

PART - III (MATHS)

81. (c) $\vec{a}\times(\vec{b}\times\vec{c})=\frac{1}{2}\vec{b}$

$(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}=\frac{1}{2}\vec{b}$ [Vector triple product]

$(|\vec{a}|.|\vec{c}|\cos\theta_2)\vec{b}-(|\vec{a}|.|\vec{b}|\cos\theta_1)\vec{c}=\frac{1}{2}\vec{b}$

$[\because |\vec{a}||\vec{b}|=|\vec{c}|=1]$

Equating the coefficients of $\vec{b}$ and $\vec{c}$ on both sides, we get

$\cos\theta_2=\frac{1}{2}$ and $-\cos\theta_1=0$

$\Rightarrow \cos\theta_2=\cos\frac{\pi}{3}$ and $\cos\theta_1=\frac{\pi}{2}$

$\Rightarrow \theta_2=\frac{\pi}{3}$ and $\theta_1=\frac{\pi}{2}$

82. (d) Vector equation of a sphere in central form with centre having position vector $\vec{c}$ and radius R is

$|\vec{r}-\vec{c}|=R \Rightarrow |\vec{r}-\vec{c}|^2=R^2$

$\Rightarrow (\vec{r}-\vec{c}).(\vec{r}-\vec{c})=R^2$

$\Rightarrow \vec{r}.\vec{r}-\vec{r}.\vec{c}-\vec{c}.\vec{r}+\vec{c}.\vec{c}=R^2$

$\Rightarrow |\vec{r}|^2-2(\vec{r}.\vec{c})+|\vec{c}|^2=R^2$

$\Rightarrow |\vec{r}|^2-2(\vec{r}.\vec{c})+|\vec{c}|^2-R^2=0$

If $h=|\vec{c}|^2-R^2$ i.e. $|\vec{c}|>\sqrt{h}$, then

$|\vec{r}|^2-2(\vec{r}.\vec{c})+h=0$

Hence, the given equation represents a sphere.

83. (b) Let $\tan^{-1}x=\theta$

$\Rightarrow x=\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$

$\Rightarrow x=\sqrt{1-\sin^2\theta}=\sin\theta$

$\Rightarrow x^2(1-\sin^2\theta)=\sin^2\theta$

$\Rightarrow x^2=\sin^2\theta(1+x^2)$

$\Rightarrow \sin^2\theta=\frac{x^2}{1+x^2} \Rightarrow \sin\theta=\frac{x}{\sqrt{1+x^2}}$

$\Rightarrow \theta=\sin^{-1}\frac{x}{\sqrt{1+x^2}}$

$\Rightarrow \tan^{-1}x=\sin^{-1}\frac{x}{\sqrt{1+x^2}}$

Now, $\sin(\tan^{-1}x)=\sin\left(\sin^{-1}\frac{x}{\sqrt{1+x^2}}\right)$

$=\frac{x}{\sqrt{1+x^2}}$

84. (c) Given that

$\left|\frac{z-25}{z-1}\right|=5 \Rightarrow |Z-25|=5|Z-1|$

Let $Z=x+iy$, then

$|x+iy-25|=5|x+iy-1|$

$\Rightarrow |(x-25)+iy|=5|(x-1+iy|$

Squaring both sides, we get

$(x-25)^2+y^2=25\{(x-1)^2+y^2\}$

$\Rightarrow x^2-50x+625+y^2$

$=25x^2-50x+25+25y^2$

$\Rightarrow 24x^2+24y^2-600=0$

$\Rightarrow x^2+y^2-25=0$

$\Rightarrow |x+iy|^2=25 \Rightarrow |Z|^2=5^2$

$\Rightarrow |Z|=5$

85. (c) We have,

$\frac{-1-3i}{2+i}=\frac{-1-3i}{2+i}\times\frac{2-i}{2-i}=\frac{-2-5i+3i^2}{4-i^2}$

$=-1-i$

Now, let us put $-1=r\cos\theta$, $-1=r\sin\theta$

Squaring and adding, $r^2=2$ i.e., $r=\sqrt{2}$

So that $\cos\theta=\frac{-1}{\sqrt{2}}$, $\sin\theta=\frac{-1}{\sqrt{2}}$

Therefore, $\theta=225°$

Thus, argument is 225°.

86. (c) Since b and C are the roots of $x^2-61x+820=0$, so

$b+c=61$, $bc=820$

$A=\tan^{-1}\left(\frac{4}{3}\right) \Rightarrow \tan A=\frac{4}{3} \Rightarrow \cos A=\frac{3}{5}$

Now, using the formula,

$\cos A=\frac{b^2+c^2-a^2}{2bc}$

$\Rightarrow a^2 = b^2 + c^2 - 2bc\cos A$
$= (b + c)^2 - 2bc - 2bc\cos A$
$= (b + c)^2 - 2bc(1 + \cos A)$

$$= (61)^2 - 2 \times 820\left(1+\frac{3}{5}\right)$$

$= 3721 - 2624 = 1097$

87. (d) Equation of first line,

$$\frac{x-6}{1} = \frac{y-2}{-2} = \frac{z-2}{2} = k(\text{say})$$

$\therefore\ x = k + 6, y = -2k + 2, z = 2k + 2$
Hence, general point on the first line,
$P \equiv (k + 6, -2k + 2, 2k + 2)$
Equation of second line,

$$\frac{x+4}{3} = \frac{y}{-2} = \frac{Z+1}{-2} = l \quad (\text{say})$$

$\therefore\ x = 3l - 4, y = -2l, z = -2l - 1$
Hence, general point on the second line,
$Q \equiv (3l - 4, -2l, -2l - 1)$
Direction ratios of PQ are
$3l - 4 - k - 6, -2l + 2k - 2, -2l - 1 - 2k - 2$
i.e. $3l - k - 10, -2l + 2k - 2, -2l - 2k - 3$
Now |PQ| will be the shortest distance between the two lines if PQ is perpendicular to both the lines. Hence,

$$1(3l - k - 10) + (-2)(-2l + 2k - 2) + 2(-2l - 2k - 3) = 0$$

and $3(3l - k - 10) + (-2)(-2l + 2k - 2) + (-2)(-2l - 2k - 3) = 0$
i.e. $3l - 9k = 12$ or $l - 3k = 4$ ...(i)
and $17l - 3k = 20$ ... (ii)
Subtracting equation (i) from (ii), we get
$16l = 16 \quad \therefore l = 1$
Putting this value of l in equation (i), we get
$-3k = 3, \therefore k = -1$
$\therefore P \equiv (-1 + 6, -2(-1) + 2, 2(-1) + 2)$
$\equiv (5, 4, 0)$
Similarly, $Q = (-1, -2, -3)$
Hence, shortest distance, PQ,

$$= \sqrt{(-1-5)^2 + (-2-4)^2 + (-3-0)^2}$$

$$= \sqrt{(-6)^2 + (-6)^2 + (-3)^2} = \sqrt{36+36+9}$$

$= 9$ units

88. (c) Since the centre and radius of the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ are $(-u, -v, -w)$ and $\sqrt{u^2 + v^2 + w^2 - d}$ respectively. So, for the sphere

$x^2 + y^2 + z^2 + 3x - 4z + 1 = 0,$

$$\text{Centre} \equiv \left(-\frac{3}{2}, 0, 2\right), \text{ and}$$

$$\text{Radius} = \sqrt{\left(-\frac{3}{2}\right)^2 + 0^2 + (2)^2 - 1}$$

$$= \sqrt{\frac{9}{4} + 4 - 1} = \frac{\sqrt{21}}{2}$$

89. (b) Let two fixed points be A (ae, 0) and B (–ae, 0). Let C (x, y) be a moving point such that AC + CB = constant = 2a (say)

$$\text{i.e., } \sqrt{(x-ae)^2 + (y-0)^2} + \sqrt{(x+ae)^2 + (y-0)^2} = 2a$$

$$\text{Or } \sqrt{x^2 + y^2 + a^2e^2 - 2aex} + \sqrt{x^2 + y^2 + a^2e^2 + 2aex} = 2a \quad ...(1)$$

Or $l + m = 2a$ ...(2)
Where, $l^2 = x^2 + y^2 + a^2e^2 - 2aex$ ...(3)
and $m^2 = x^2 + y^2 + a^2e^2 + 2aex$ ...(4)
From, (3) and (4)
$m^2 - l^2 = 4aex$
or $(m - l)(l + m) = 4aex$
$2a(m - l) = 4aex$ [From (2)]
$m - l = 2ex$ ...(5)
Adding (2) and (5), we get
$m = a + ex$ ...(6)
From (4) and (6),
$a^2 + e^2x^2 + 2aex = x^2 + y^2 + a^2e^2 + 2aex$
$\Rightarrow x^2(1 - e^2) + y^2 = a^2(1 - e^2)$
Dividing both sides by $a^2(1 - e^2)$, we get

$$\frac{x^2}{a^2} + \frac{y^2}{a^2(1-e^2)} = 1$$

Or $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $b^2 = a^2(1 - e^2)$
This is the equation of ellipse.

90. (d) The equation of the parabola is

$$y^2 + 4x + 3 = 0$$

$$\text{or} \quad y^2 = -4\left(x + \frac{3}{4}\right) \quad ...(1)$$

The directrix of the parabola
$Y^2 = -4aX$ ... (2)
is $X = a$.

On comparing the equation (1) and (2), we get $4a = 4$ and $X = x + \frac{3}{4}$

or $a = 1$ and $X = x + \frac{3}{4}$

Hence the directrix of the parabola (1) is

$x + \frac{3}{4} = 1$ or $x - \frac{1}{4} = 0.$

91. (b) $g(x).\,g(y) = g(x) + g(y) + g(xy) - 2$ ...(1)

Put $x = 1, y = 2$, then

$g(1).\,g(2) = g(1) + g(2) + g(2) - 2$

$5g(1) = g(1) + 5 + 5 - 2$

$4g(1) = 8$ $\quad\therefore g(1) = 2$

Put $y = \frac{1}{x}$ in equation (1), we get

$$g(x).g\left(\frac{1}{x}\right) = g(x) + g\left(\frac{1}{x}\right) + g(1) - 2$$

$$g(x).g\left(\frac{1}{x}\right) = g(x) + g\left(\frac{1}{x}\right) + 2 - 2$$

$[\because g(1) = 2]$

This is valid only for the polynomial

$\therefore \quad g(x) = 1 \pm x^n$ ... (2)

Now $g(2) = 5$ (Given)

$\therefore 1 \pm 2^n = 5$ [Using equation (2)]

$\pm 2^n = 4, \Rightarrow 2^n = 4, -4$

Since the value of 2^n cannot be –Ve.

So, $2^n = 4, \Rightarrow n = 2$

Now, put $n = 2$ in equation (2), we get

$g(x) = 1 \pm x^2$

$$\therefore \underset{x\to 3}{\text{Lt}}\, g(x) = \underset{x\to 3}{\text{Lt}}\,(1 \pm x^2) = 1 \pm (3)^2$$

$$= 1 \pm 9 = 10, -8$$

92. (d) $f(x) = \frac{-e^x + 2^x}{x}$

$$= \frac{1}{x}\left[-(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + ..) + 1 + \frac{\log 2}{1!}x + \frac{(bg\,2)^2}{2!}x^2 + \frac{(\log 2)^3}{3!}x^3 + ..\right]$$

$$f(x) = \log 2 - 1 + \frac{x}{2!}\{(\log 2)^2 - 1\} + \frac{x^2}{3!}\{(\log^2)^3 - 1\} + ...$$

Putting $x = 0$, we get

$f(0) = \log 2 - 1 + 0 + 0 + ... = -1 + \log 2.$

93. (b) Check the continuity of the function $f(x) = [\tan^2 x]$ at $x = 0$.

L.H.L. (at $x = 0$)

$$= \lim_{x\to 0^-}[\tan^2 x] = \lim_{h\to 0}[\tan^2(0-h)]$$

$$= \lim_{h\to 0}[\tan^2 h] = [\tan^2 0] = [0] = 0$$

R.H.L. (at $x = 0$)

$$= \lim_{x\to 0^+}[\tan^2 x] = \lim_{h\to 0}[\tan^2(0+h)]$$

$$= \lim_{h\to 0}[\tan^2 h] = [\tan^2 0] = [0] = 0$$

Now, determine the value of f(x) at $x = 0$.

$f(0) = [\tan^2 0] = [0] = 0$

Hence, $f(x)$ is continuous at $x = 0$.

94. (c) Let r and V be the respectively radius and volume of the balloon. Let t represents the time. The rate of increament in radius is $\frac{dr}{dt} = 2$ cm/minute. The volume of the balloon is given by

$$V = \frac{4}{3}\pi r^3$$

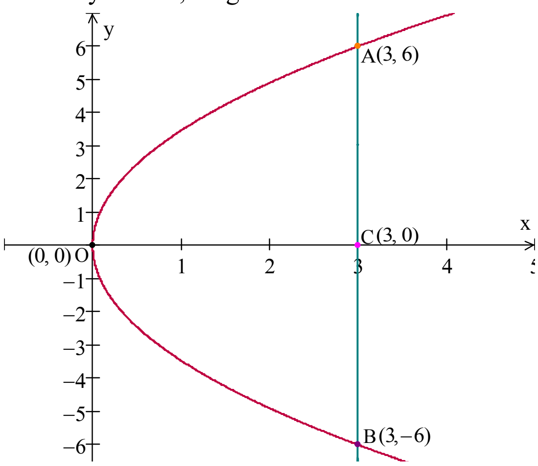
Differentiating w.r. to t, we get

$$\frac{dV}{dt} = \frac{4}{3}\pi(3r^2\frac{dr}{dt})$$

Substituting the values of and $\frac{dr}{dt}$, we get

$$\frac{dV}{dt} = \frac{4}{3}\pi(3 \times 5^2 \times 2) = 200\pi\, cm^3 / \text{minute}$$

95. (a) On comparing the equation of the parabola $y^2 = 12x$ with the standard equation, $y^2 = 4ax$, we get $4a = 12$ or $a = 3$.



Hence, the focus, point C will be at (3, 0) and the extremities of the latus-rectum AB will be at (a, 2a) and (a, –2a). So the coordinates of A and B are (3, 6) and (3, –6) respectively. Now we need to find the length (curve AOB) of the parabola. As it is not a straight line so we cannot directly find the length of this curve as we cannot directly apply Pythagorous theorem. Let us consider a small length ds on the parabola. Using pythagorous theorem for this length,

$$ds = \sqrt{(dx)^2+(dy)^2} = \sqrt{\left(\frac{dx}{dy}\right)^2+1}\,dy$$

$$\Rightarrow s = \int_{-6}^{6}\left[\sqrt{\left(\frac{dx}{dy}\right)^2+1}\right].dy \qquad ...(1)$$

From $y^2 = 12x \Rightarrow x = \frac{y^2}{12}$

$\frac{dx}{dy} = \frac{2y}{12} = \frac{y}{6}$ Putting in (1),

$$s = \int_{-6}^{6}\left[\sqrt{\left(\frac{y}{6}\right)^2+1}\right]dy = 2\int_{0}^{6}\sqrt{\frac{y^2+36}{36}}\,dy$$

$$= \frac{2}{6}\int_{0}^{6}\sqrt{y^2+6^2}\;dy$$

Using $\int\sqrt{x^2+a^2} = \frac{x}{2}\sqrt{a^2+x^2}$

$+\frac{a^2}{2}\log\left(x+\sqrt{a^2+x^2}\right)+C$

We get $s = \frac{1}{3}\left[\frac{y}{2}\sqrt{6^2+y^2}+\frac{6^2}{2}\right.$

$\left.\log\left(y+\sqrt{6^2+y^2}\right)+C\right]_0^6$

$$= \frac{1}{3}\left[\frac{6}{2}\sqrt{6^2+6^2}+18\log\left(6+\sqrt{6^2+6^2}\right)\right.$$

$$\left.+C-0-18\log\left(0+\sqrt{6^2+0}\right)-C\right]$$

$$= \frac{1}{3}\left[3.6\sqrt{2}+18\log\left(6+6\sqrt{2}\right)-18\log 6\right]$$

$$= 6\sqrt{2}+6\log\frac{6(1+\sqrt{2})}{6}$$

$$= 6\left[(\sqrt{2}+\log(1+\sqrt{2})\right]$$

96. (d) $I = \int\frac{x^5}{\sqrt{1+x^3}}dx = \int\frac{x^3.x^2}{\sqrt{1+x^3}}dx$

Let $1 + x^3 = t^2$, so that $3x^2\,dx = 2t\,dt$

$\Rightarrow x^2 dx = \frac{2}{3}t\,dt$

$$\therefore I = \int\frac{(t^2-1)\frac{2}{3}t\,dt}{t} = \frac{2}{3}\int t^2-1)dt$$

$$= \frac{2}{3}\left(\frac{t^3}{3}-t\right)+C = \frac{2}{3}\left[\frac{(1+x^3)^{3/2}}{3}-(1+x^3)^{\frac{1}{2}}\right]+C$$

$$= \frac{2}{9}(1+x^3)^{3/2}-\frac{2}{3}(1+x^3)^{1/2}+C$$

97. (d) The given curve is

$$\pi[4(x-\sqrt{2})^2+y^2] = 8$$

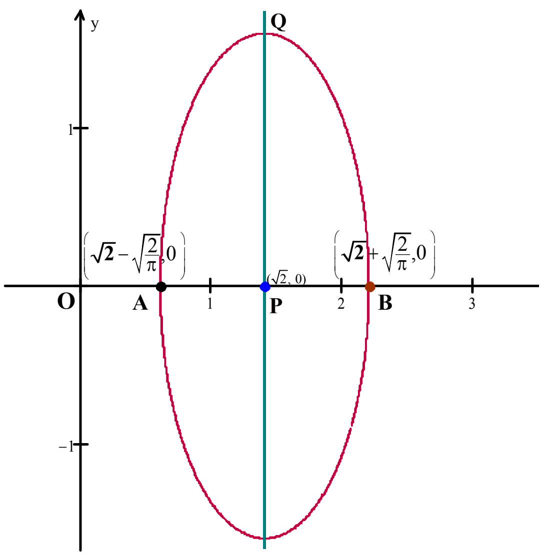
$$4(x-\sqrt{2})^2+y^2 = \frac{8}{\pi}$$

$$(x-\sqrt{2})^2+\left(\frac{y}{2}\right)^2 = \frac{2}{\pi}$$

$$\frac{(x-\sqrt{2})^2}{\left(\sqrt{\frac{2}{\pi}}\right)^2}+\frac{y^2}{\left(2\sqrt{\frac{2}{\pi}}\right)^2} = 1 \qquad ...(1)$$

This is the equation of the ellipse having centre ($\sqrt{2}, 0$).

Observe the figure of ellipse (1). The centre P is ($\sqrt{2}, 0$). A and B are $\left(\sqrt{2}, -\sqrt{\frac{2}{\pi}}, 0\right)$ and $\left(\sqrt{2}+\sqrt{\frac{2}{\pi}}, 0\right)$ respectively.



The required area = 4 × area of figure PQB

$$= 4\times\int_{\sqrt{2}}^{\sqrt{2}+\sqrt{\frac{2}{\pi}}} y\,dx$$

$$= 4\times\int_{\sqrt{2}}^{\sqrt{2}+\sqrt{\frac{2}{\pi}}} \sqrt{\frac{8}{\pi}-4(x-\sqrt{2})^2}\,dx$$

$$= 4\times 2\int_{\sqrt{2}}^{\sqrt{2}+\sqrt{\frac{2}{\pi}}} \sqrt{\left(\sqrt{\frac{2}{\pi}}\right)^2-(x-\sqrt{2})^2}\,dx$$

$$= 8\left[\frac{x-\sqrt{2}}{2}\sqrt{\frac{2}{\pi}-(x-\sqrt{2})^2}+\frac{(2/\pi)}{2}\sin^{-1}\left(\frac{x-\sqrt{2}}{\sqrt{2/\pi}}\right)\right]_{\sqrt{2}}^{\sqrt{2}+\sqrt{\frac{2}{\pi}}}$$

$$\left[\because \int\sqrt{a^2-x^2}\,dx=\frac{1}{2}x\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C\right]$$

$$= 8\left[\left\{\frac{\sqrt{2}+\sqrt{\frac{2}{\pi}}-\sqrt{2}}{2}\sqrt{\frac{2}{\pi}-\left(\sqrt{2}+\sqrt{\frac{2}{x}}-\sqrt{2}\right)^2}+\frac{1}{\pi}\sin^{-1}\left(\frac{\sqrt{2}+\sqrt{\frac{2}{\pi}}-\sqrt{2}}{\sqrt{\frac{2}{\pi}}}\right)\right\}-\left\{\frac{\sqrt{2}-\sqrt{2}}{2}\sqrt{\frac{2}{\pi}-(\sqrt{2}-\sqrt{2})^2}+\frac{1}{\pi}\sin^{-1}\left(\frac{\sqrt{2}-\sqrt{2}}{\sqrt{2/\pi}}\right)\right\}\right]$$

$$=8\left[\frac{1}{\sqrt{2\pi}}(0)+\frac{1}{\pi}\sin^{-1}(1)-0-0\right]$$

$$=8\left(\frac{1}{\pi}\times\frac{\pi}{2}\right)=4 \text{ square units.}$$

98. (c) Let $x = \cos^2\theta$, so that $dx = -2a\sin\theta\cos\theta\,d\theta$

Now $\int_0^a\sqrt{\frac{a-x}{x}}dx=\int_{\frac{\pi}{2}}^{0}\sqrt{\frac{a-a\cos^2\theta}{a\cos^2\theta}}(-a\sin\theta\cos\theta)d\theta$

$$\left[\because \theta=\frac{\pi}{2}\text{ at } x=0; \theta=0 \text{ at } x=a\right]$$

$$= a\int_0^{\pi/2}\sqrt{\frac{1-\cos^2\theta}{\cos^2\theta}}\,2\sin\theta\cos\theta\,d\theta$$

$$\left[\because \int_t^0 f(x)\,dx=-\int_0^t f(x)\,dx\right]$$

$$= a\int_0^{\pi/2}2\frac{\sin\theta}{\cos\theta}.\sin\theta\cos\theta\,d\theta$$

$$= a\int_0^{\pi/2}2\sin^2\theta\,d\theta = a\int_0^{\pi/2}(1-\cos 2\theta)\,d\theta$$

$$[\because \cos 2\theta = 1-2\sin^2\theta]$$

$$= a\left[\theta-\frac{\sin 2\theta}{2}\right]_0^{\pi/2}$$

$$= a\left[\left(\frac{\pi}{2}-\frac{\sin\pi}{2}\right)-(0-0)\right]$$

$$= a\left(\frac{\pi}{2}-0\right)=\frac{a\pi}{2}$$

99. (b) According to the question,

$$\frac{dy}{dt}\propto y \Rightarrow \frac{dy}{dt}=ky$$

Separating the variables, we get $\frac{dy}{dt}=kdt$

Integrating both sides, we get $\int\frac{dy}{y}=\int k\,dt$

$\log y = k\,t + M$ (as y cannot be –ve)

$\Rightarrow y = e^{kt+M} \Rightarrow y = e^M . e^{kt}$

$y = C e^{kt}$, where $C = e^M$

Constant k cannot be positive because the population never increases in time. And

another constant C cannot be negative because of $e^M > 0$ always.
Hence $y = Ce^{kt}$, for some constants $C \geq 0$ and $k \leq 0$.

100. (b) The given circle is, $x^2 + y^2 = a^2$
Differentiating with respect to x, we get

$$2x + 2y\frac{dy}{dx} = 0 \Rightarrow \quad x + y\frac{dy}{dx} = 0$$

$$\Rightarrow \left(x + y\frac{dy}{dx}\right)^2 = 0 \quad \text{(Squaring both sides)}$$

$$\Rightarrow \quad x^2 + 2xy\frac{dy}{dx} + y^2\left(\frac{dy}{dx}\right)^2 = 0$$

$$\Rightarrow \quad -2xy\frac{dy}{dx} = x^2 + y^2\left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow \quad y^2 + x^2\left(\frac{dy}{dx}\right)^2 - 2xy\frac{dy}{dx}$$

$$= \quad x^2 + y^2 + x^2\left(\frac{dy}{dx}\right)^2 + y^2\left(\frac{dy}{dx}\right)^2$$

(Adding $y^2 + x^2\left(\frac{dy}{dx}\right)^2$ both sides)

$$\Rightarrow \quad \left(y - x\frac{dy}{dx}\right)^2 = a^2\left[1 + \left(\frac{dy}{dx}\right)^2\right]$$

$$\left(\because x^2 + y^2 = a^2\right)$$

101. (b) $\left|\frac{dy}{dx}\right| + |y| + 3 = 0$ Since $\left|\frac{dy}{dx}\right| \geq 0, \ |y| \geq 0$

$$\therefore \left|\frac{dy}{dx}\right| + |y| + 3 \geq 3$$

Hence $\left|\frac{dy}{dx}\right| + |y| + 3 = 0$ is not possible.

Therefore, the given differential equation has no solution.

102. (b) The given differential equation is

$$xdy - ydx - \sqrt{x^2 + y^2}\,dx = 0$$

$$\Rightarrow \quad xdy = \left(y + \sqrt{x^2 + y^2}\right)dx$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{(y + \sqrt{x^2 + y^2})}{x}$$

This is the linear differential equation.

Put $y = vx$, so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$. Then

$$v + x\frac{dv}{dx} = \frac{vx + \sqrt{x^2 + v^2x^2}}{x}$$

$$\Rightarrow \quad v + x\frac{dv}{dx} = v + \sqrt{1 + v^2}$$

$$\frac{dv}{\sqrt{1 + v^2}} = \frac{dx}{x}$$

Integrating both sides, we get

$$\int \frac{dv}{\sqrt{1 + v^2}} = \int \frac{dx}{x}$$

$$\Rightarrow \quad \log(v + \sqrt{1 + v^2}) = \log x + \log C$$

$$\Rightarrow \quad v + \sqrt{1 + v^2} = Cx$$

$$\Rightarrow \quad \frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}} = Cx \qquad [\because y = vx]$$

$$y + \sqrt{x^2 + y^2} = Cx^2$$

103. (b) $\sim Q \to S \qquad \therefore \sim S \to Q$
But $\sim S \to R$
$\therefore Q \to R$, True [As $P \to Q \to R \to P$]
Hence $\sim Q \to S$ is true

104. (d) We know that the number of ways of dividing (m+n+p) things into three groups containing m, n and p things respectively

$$= \frac{(m + n + p)!}{m!\,n!\,p!}$$

Further if any two groups out of the three have same number of things then number of ways

$$= \frac{(m + n + p)!}{m!\,n!\,p! \times 2}$$

Hence number of ways to divide 10 students into three teams one containing four students and each remaining two teams contain three

$$= \frac{10!}{4!\,3!\,3! \times 2} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5}{3 \times 2 \times 3 \times 2 \times 2} = 2100$$

105. (d) We observe the following properties :
Reflexivity - Let a be an arbitrary element. Then,

$$|a - a| = 0 \not> 0 \Rightarrow a \not R\ a$$

This, R is not reflexive on R.

Symmetry – Let a and b be two distinct elements, then $(a,b) \in R$

$\Rightarrow |a-b|>0 \Rightarrow \quad |b-a|>0$

$(\because |a-b| = |b-a|)$

$\Rightarrow (b,a) \in R$

Thus, (a, b) $\in R \Rightarrow$ (b, a) $\in$ R. So, R is symmetric.

Transitivity – Let (a, b) $\in$ R and (b, c) $\in$ R.

Then $\quad |a-b|>0$ and $|b-c|>0$

$\Rightarrow \quad |a-c|>0 \Rightarrow$ (a, c) $\in$ R

So, R is transitive.

106. (a) Let $S = \{a_i\}$ where i = 1.2.....n

From commutative operations,

$a_i * a_j = a_j * a_i \quad \ldots (i) \ \forall i, j = 1, 2, 3....n$

where * represents a binary operation

$\therefore$ Number of distinct elements in $S \times S$

i.e., $\underset{i=1,2...n}{\{a_i\}} \times \underset{j=1,2...n}{\{a_j\}}$ subject to the condition (i)

$= n\{(a_1,a_1),(a_1,a_2)......(a_1,a_n),$

$(a_2, a_2), (a_2,a_3),....(a_2,a_n),$

$...(a_{n-1},a_{n-1}),(a_{n-1},a_n),(a_n,a_n)$

$= n+(n-1)+(n-2)+....+2+1 = \frac{n(n+1)}{2}$

$\therefore$ No. of commutative binary operations

= No. of functions $f: S \times S \to S$ subject to (i)

$= n.n.n....\frac{n(n+1)}{2}\text{times} = n^{\frac{n(n+1)}{2}}$

107. (b) Poisson distribution is a probability distribution which is obtained when the probability (p) of the happening of an event is same in all the trials and there are only two events in each trials generally says successes and failures probability (p) of the happening of the event in trial is very less but number of trials (n) is very large.

Here, p = $5\% = \frac{5}{100} = \frac{1}{20}$ is very less and

n = 100, is very large. Hence, one has to employ the Poisson distribution in the given question.

108. (d) The probability that a component survives is $p = \frac{3}{4}$. Then $q = 1-p = 1-\frac{3}{4} = \frac{1}{4}$

$[\because p+q=1]$

n takes the value 4 and r = 2. Hence the required probability is

$${}^nC_r p^r q^{n-r} = {}^4C_2 \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right)^2$$

$$= 6 \times \frac{3 \times 3 \times 1}{4 \times 4 \times 4 \times 4} = \frac{27}{128}$$

109. (b) Performance of the class will be best if mean of the marks obtained is maximum but standard deviation of the marks obtained is minimum.

Hence the class which has mean and standard deviation of the marks obtained as 75 and 5 respectively performs best.

110. (b) Mean, $np = \alpha$; and variance, $npq = \beta$ where n = number of trials and p + q = 1.

So, $\frac{npq}{np} = \frac{\beta}{\alpha} \Rightarrow q = \frac{\beta}{\alpha} \Rightarrow (1-p) = \frac{\beta}{\alpha}$

$\because 0 < p < 1$

$\therefore -1 < -p < 0 \Rightarrow 0 < 1-p < 1$

$\Rightarrow 0 < \frac{\beta}{\alpha} < 1 \Rightarrow 0 < \beta < \alpha$

111. (b) $x+y+z=0 \quad ...(1)$

$2x+3y+z=0 \quad ...(2)$

$x+2y=0 \quad ...(3)$

From equation (3), we have

$x = -2y$

Putting this value of x in equations (1), we get $\quad -2y+y+z=0 \Rightarrow y = z$

Hence $x = -2z$

Thus, the solution of the given system of equations is (–2z, z, z), where z is a parameter $(Z \in R)$. Hence the system has infinite number of solutions including zero solution.

112. (d) Here, $\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}^2$

$= \begin{bmatrix} 0+ab & 0+0 \\ 0+0 & ab+0 \end{bmatrix} = \begin{bmatrix} ab & 0 \\ 0 & ab \end{bmatrix}$

Similarly, $\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}^4 = \begin{bmatrix} ab & 0 \\ 0 & ab \end{bmatrix}\begin{bmatrix} ab & 0 \\ 0 & ab \end{bmatrix}$

$= \begin{bmatrix} a^2b^2+0 & 0+0 \\ 0+0 & 0+a^2b^2 \end{bmatrix} = \begin{bmatrix} a^2b^2 & 0 \\ 0 & a^2b^2 \end{bmatrix}$

Now,

$\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}^4 = I \Rightarrow \begin{bmatrix} a^2b^2 & 0 \\ 0 & a^2b^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow a^2b^2 = 1 \Rightarrow ab = 1$

113. (d) $D = \text{diag}\,(d_1, d_2, ..., d_n)$

$$D = \begin{vmatrix} d_1 & 0 & 0 & - & - & 0 \\ 0 & d_2 & 0 & - & - & 0 \\ 0 & 0 & d_3 & - & - & 0 \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ 0 & 0 & 0 & 0 & - & 0 \end{vmatrix}$$

$$|D| = \begin{vmatrix} d_1 & 0 & 0 & - & - & 0 \\ 0 & d_2 & 0 & - & - & 0 \\ 0 & 0 & d_3 & - & - & 0 \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ 0 & 0 & 0 & 0 & - & 0 \end{vmatrix}$$

$$= d_1\, d_2\, d_3 \,.......\, d_n$$

$$\text{adj}(D) = \begin{bmatrix} d_2d_3d_4...dn & 0 & 0 & - & - & 0 \\ 0 & d_1d_3d_4...dn & 0 & - & - & 0 \\ 0 & 0 & d_1d_2d_4...dn & - & - & 0 \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ 0 & 0 & 0 & 0 & - & d_1d_2d_3...d_{n-1} \end{bmatrix}$$

$$d^{-1} = \frac{1}{|D|}\,\text{adj}\,(D)$$

$$\text{adj}(D) = \frac{1}{d_1d_2d_3....d_n}$$

$$\begin{bmatrix} d_2d_3d_4...dn & 0 & 0 & - & - & 0 \\ 0 & d_1d_3d_4...dn & 0 & - & - & 0 \\ 0 & 0 & d_1d_2d_4...dn & - & - & 0 \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ 0 & 0 & 0 & 0 & - & d_1d_2d_3...d_{n-1} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{d_1} & 0 & 0 & - & - & 0 \\ 0 & \frac{1}{d_2} & 0 & - & - & 0 \\ 0 & 0 & \frac{1}{d_3} & - & - & 0 \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ 0 & 0 & 0 & 0 & - & \frac{1}{d_n} \end{bmatrix}$$

$$= \begin{bmatrix} d_1^{-1} & 0 & 0 & - & - & 0 \\ 0 & d_2^{-1} & 0 & - & - & 0 \\ 0 & 0 & d_3^{-1} & - & - & 0 \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ 0 & 0 & 0 & 0 & - & d_n^{-1} \end{bmatrix}$$

$$= \text{diag}\,(\,d_1^{-1}, d_2^{-1}, ..., d_n^{-1}\,)$$

114. (d) $\begin{bmatrix} a & b-y & c-z \\ a-x & b & c-z \\ a-x & b-y & c \end{bmatrix} = 0$

$\Rightarrow a(bc - bc + bz + cy - yz) - (b - y)$

$(ac - cx - ac + az + cx - zx) + (c - z)$

$(ab - ay - bx + xy - ab + bx) = 0$

$\Rightarrow abz + acy - ayz - abz + bzx + ayz -$

$xyz - acy + cxy + ayz - xyz = 0$

$\Rightarrow ayz + bzx + cxy - 2xyz = 0$

$\Rightarrow \frac{a}{x} + \frac{b}{y} + \frac{c}{z} - 2 = 0$ (Dividing by xyz)

$\Rightarrow \frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$

115. (c) $x^2 - n = 0 \;\Rightarrow\; x = \sqrt{n}$

For each integral value of $x \in [1, 40]$, There is a positive root. Hence for 40 integral values of x from 1 to 40, there are 40 positive roots, out of which only six roots 1, 2, 3, 4, 5 and 6 are positive integral roots. Hence, probability

of getting positive integral roots $= \frac{6}{40} = \frac{3}{20}$

116. (a) $x^4 + \sqrt{x^4 + 20} = 22$

or $x^4 - 22 = -\sqrt{x^4 + 20}$

Put $x^4 = y$ and square both the sides.

$(y - 22)^2 = y + 20$

$y^2 + 484 - 44y = y + 20$

$y^2 - 45y + 464 = 0$

$y^2 - 29y - 16y + 464 = 0$

$(y - 29)\,(y - 16) = 0$

$y = 16, 29$

$\therefore\; x^4 = 16, 29$ or $x = \pm 2, \pm 2.31$

117. (c) Since α, β be the roots of the equation $x^2 - ax + b = 0$, so each of them must satisfy the equation. Therefore

$\alpha^2 - a\,\alpha + b = 0$ ...(1) and $\beta^2 - a\beta + b = 0$ ...(2)

Now, $A_{n+1} - a\,A_n + b\,A_{n-1} = \alpha^{n+1} + \beta^{n+1} - a\,(\alpha^n + \beta^n) + b\,(\alpha^{n-1} + \beta^{n-1})$

$= \alpha^{n-1}(\alpha^2 - a\,\alpha + b) + \beta^{n-1}(\beta^2 - a\,\beta + b)$

$= \alpha^{n-1}(0) + \beta^{n-1}(0) = 0$ [From (1) & (2)]

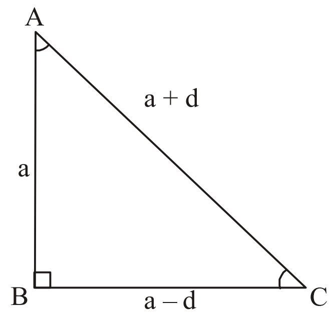
118. (a) Let the sides of the triangle be a – d, a, a+d, where d is greater than zero. From the figure, it is clear that the angles A and C are acute angles. Now, by the theorem of Pythagorus,

$AC^2 = AB^2 + BC^2$

$(a + d)^2 = a^2 + (a - d)^2$

$a^2 + d^2 + 2ad = a^2 + a^2 + d^2 - 2ad$

$4ad = a^2 \Rightarrow d = a/4$



$$\sin A = \frac{a-d}{a+d} = \frac{a - \frac{a}{4}}{a + \frac{a}{4}} = \frac{3}{5}$$

$$\sin C = \frac{a}{a+d} = \frac{a}{a + \frac{a}{4}} = \frac{4}{5}$$

119. (a) The plane containing the line of intersection of the planes $\vec{r}.(\hat{i} + 3\hat{j} - \hat{k}) = 0$ and $\vec{r}.(\hat{j} + 2\hat{k}) = 0$ is

$\vec{r}.(\hat{i} + 3\hat{j} - \hat{k}) + \lambda[\vec{r}.(\hat{j} + 2\hat{k})] = 0$

$\Rightarrow \vec{r}.[\hat{i} + (3+\lambda)\hat{j} + (2\lambda - 1)\hat{k}] = 0$ ...(1)

Since the plane (i) passes through the point $(-1, -1, -1)$ or $(-\hat{i} - \hat{j} - \hat{k})$, so this point must satisfy (i). Hence,

$\left(-\hat{i} - \hat{j} - \hat{k}\right).[\hat{i} + (3 \times \lambda)\hat{j} + (2\lambda - 1)\hat{k}] = 0$

$\Rightarrow -1 - (3 + \lambda) - (2\lambda - 1) = 0$

$\Rightarrow -3\lambda - 3 = 0 \ \Rightarrow \lambda = -1$

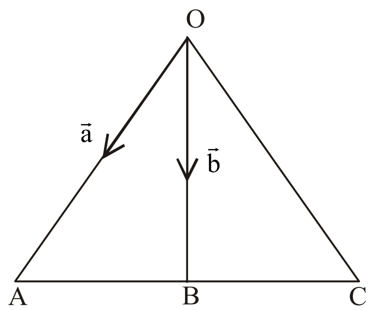
Substituting this value for λ in (i), we get the required plane $\vec{r}.(\hat{i} + 2\hat{j} - 3\hat{k}) = 0$

120. (d) In the figure, OAC is a triangle and OB is a median such that

$\overrightarrow{OA} = \vec{a} = \hat{i} - \hat{j} + \hat{k}$

$\overrightarrow{OB} = \vec{b} = 2\hat{i} + 4\hat{j} + 3\hat{k}$

$\overrightarrow{OC} = \vec{c}$ (say)



$\overrightarrow{OA} + \overrightarrow{OC} = 2\overrightarrow{OB}$

$\vec{a} + \vec{c} = 2\vec{b}$

$\Rightarrow \vec{c} = 2\vec{b} - \vec{a} = 2(2\hat{i} + 4\hat{j} + 3\hat{k}) - (\hat{i} - \hat{j} + \hat{k})$

$= (3\hat{i} + 9\hat{j} + 5\hat{k})$

Now, the area of the triangle,

$$\Delta = \frac{1}{2}\left|\overrightarrow{OA} \times \overrightarrow{OC}\right| = \frac{1}{2}\left|\vec{a} \times \vec{c}\right|$$

Here,

$$\vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 3 & 9 & 5 \end{vmatrix}$$

$= \hat{i}(-5 - 9) - \hat{j}(5 - 3) + \hat{k}(9 + 3)$

$= -14\hat{i} - 2\hat{j} + 12\hat{k} = 2(-7\hat{i} - \hat{j} + 6\hat{k})$

$\therefore \left|\vec{a} \times \vec{c}\right| = 2\sqrt{(-7)^2 + (-1)^2 + (6)^2}$

$= 2\sqrt{49 + 1 + 36} = 2\sqrt{86}$

$\therefore \Delta = \frac{1}{2} \times 2\sqrt{86} = \sqrt{86}$.