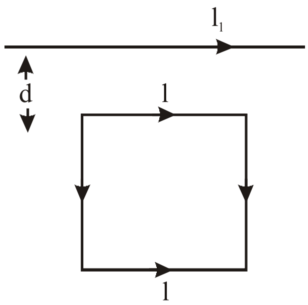
VITEEE 2012 Question Paper

Vellore Institute of Technology Engineering Entrance Examination

SOLVED PAPER VITEEE 2012

PART - I (PHYSICS)

1. A square loop, carrying a steady current I, is placed in horizontal plane near a long straight conductor carrying a steady current I_1 at a distance of d from the conductor as shown in figure. The loop will experience

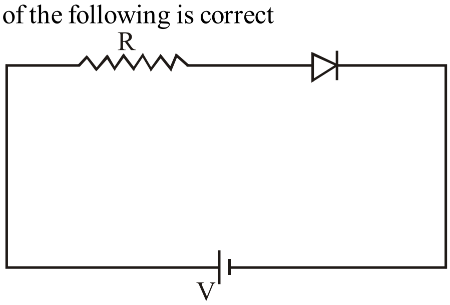


(a) a net repulsive force away from the conductor
(b) a net torque acting upward perpencicular to the horizontal plane
(c) a net torque acting downward normal to the horizontal plane
(d) a net attractive force towards the conductor

2. The threshold frequency for a photo-sensitine metal is 3.3×10^{14} Hz. If light of frequency 8.2×10^{14} Hz is incident on this metal, the cut-off voltage for the photo-electric emission is nearly

(a) 2V (b) 3V
(c) 5V (d) 1V

3. For the given circuit of p-n junction diode which of the following is correct

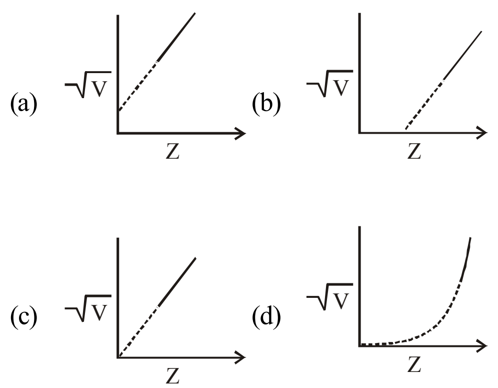


(a) In forward biasing the voltage across R is V
(b) In forward biasing the voltage across R is 2V
(c) In reverse biasing the voltage across R is V
(d) In reverse biasing the voltage across R is 2V

4. If the binding energy per nuclear in Li^7 and He^4 nuclei are respectively 5.60 MeV and 7.06 MeV, then energy of reactor
$Li^7 + P \rightarrow 2\ _2He^4$ is

(a) 19.6 MeV (b) 2.4 MeV
(c) 8.4 MeV (d) 17.3 MeV

5. The graph between the square root of the frequency of a specific line of characterstic spectrum of X-ray and the atomic number of the ttarget will be



6. A resistor R, an inductor L and capacitor C are connected in series to an oscillator of frequency n. If the resonant frequency is n_r, then the current lags behind voltage, when

(a) $n = 0$ (b) $n < n_r$
(c) $n = n_r$ (d) $n > n_r$

7. A parallel plate capacitor has capacitance C. If it is equally filled the parallel layers of materials of dielectric constant K_1 and K_2 its capacity becomes C_1. The ratio of C_1 and C is

(a) $K_1 + K_2$ (b) $\frac{K_1K_2}{K_1 + K_2}$

(c) $\frac{K_1 + K_2}{K_1K_2}$ (d) $\frac{2K_1K_2}{K_1 + K_2}$

8 The potential of the electric field produced by point charge at any point (x, y, z) is given by $V = 3x^2 + 5$, where x, y are in metres and V is in volts. The intensity of the electric field at (–2, 1, 0) is

(a) $+17\ Vm^{-1}$ (b) $-17\ Vm^{-1}$

(c) $+12\ Vm^{-1}$ (d) $-12\ Vm^{-1}$

9. The potential of a large liquid drop when eight liquid drops are combined is 20 V. Then the potential of each single drop was

(a) 10 V (b) 7.5 V

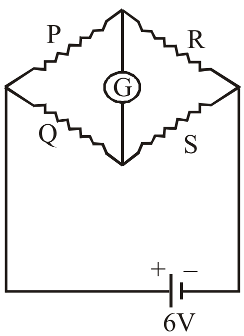
(c) 5 V (d) 2.5 V

10 A and B are two metals with threshold frequencies 1.8×10^{14} Hz and 2.2×10^{4} Hz. Two identical photons of energy 0.825 eV each are incident on them. Then photoelectrons are emitted by (Take $h = 6.6 \times 10^{-34}$ J-s)

(a) B alone (b) A alone

(c) Neither A nor B (d) Both A and B

11 In the Wheatstone's network given, $P = 10\Omega$, $Q = 20\Omega$, $R = 15\Omega$, $S = 30\Omega$, the current passing through the battery (of negligible internal resistance) is



(a) 0.36 A (b) Zero

(c) 0.18 A (d) 0.72 A

12 Three resistors 1Ω, 2Ω and 3Ω are connected to form a triangle. Across 3Ω resistor a 3V battery is connected. The current through 3Ω resistor is

(a) 0.75 A (b) 1 A

(c) 2 A (d) 1.5 A

13. In a common emitter amplifier the input signal is applied across

(a) anywhere (b) emitter-collector

(c) collector-base (d) base-emitter

14 The kinetic energy of an electron get tripled then the de-Broglie wavelength associated with it changes by a factor

(a) $\frac{1}{3}$ (b) $\sqrt{3}$

(c) $\frac{1}{\sqrt{3}}$ (d) 3

15 A radioactive substance contains 10000 nuclei and its half-life period is 20 days. The number of nuclei present at the end of 10 days is

(a) 7070 (b) 9000

(c) 8000 (d) 7500

16. A direct X-ray photograph of the intenstines is not generally taken by radiologists because

(a) intenstines would burst an exposure to X-rays

(b) the X-rays would be not pass through the intenstines

(c) the X-rays will pass through the intenstines without causing a good shadow for any useful diagnosis

(d) a very small exposure of X-rays causes cancer in the intenstines

17. Charge passing through a conductor of crosssection area $A = 0.3\ m^2$ is given by $q = 3t^2 + 5t + 2$ in coulomb, where t is in second. What is the value of drift velocity at t = 2s? (Given, $m = 2 \times 10^{25}/m^3$)

(a) 0.77×10^{-5} m/s (b) 1.77×10^{-5} m/s

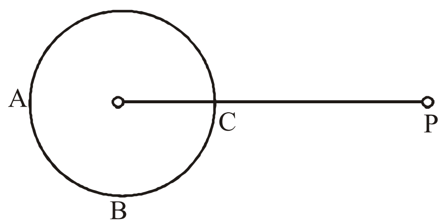
(c) 2.08×10^{-5} m/s (d) 0.57×10^{-5} m/s

18. Two capacitors of capacities 1 μF and C μF are connected in series and the combination is charged to a potential difference of 120 V. If the charge on the combination is 80 μC, the energy stored in the capacitor of capacity C in μJ is

(a) 1800 (b) 1600

(c) 14400 (d) 7200

19. A hollow conducting sphere is placed in an electric field produced by a point charg placed at P as shown in figure. Let V_A, V_B, V_C be the potentials at points A, B and C respectively. Then



(a) $V_C > V_B$ (b) $V_B > V_C$
(c) $V_A > V_B$ (d) $V_A = V_C$

20. In a hydrogen discharged tube it is observed that through a given cross-section 3.13×10^{15} electrons are moving from right to left and 3.12×10^{15} protons are moving from left to right. What is the electric current in the discharge tube and what is its direction?
(a) 1 mA towards right
(b) 1 mA towards left
(c) 2 mA towards left
(d) 2 mA towards right

21. In $CuSO_4$ solution when electric current equal to 2.5 faraday is passed, the gm equivalent deposited on the cathode is
(a) 1 (b) 1.5
(c) 2 (d) 2.5

22. In hydrogen a atom, an electron is revolving in the orbit of radius 0.53 Å with 6.6×10^{15} radiations/s. Magnetic field produced at the centre of the orbit is
(a) 0.125 Wb/m^2 (b) 1.25 Wb/m^2
(c) 12.5 Wb/m^2 (d) 125 Wb/m^2

23. The dipole moment of the short bar magnet is 12.5 $A\text{-}m^2$. The magnetic field on its axis at a distance of 0.5 m from the centre of the magnet is
(a) 1.0×10^{-4} N/A-m (b) 4×10^{-2} N/A-m
(c) 2×10^{-6} N/A-m (d) 6.64×10^{-8} N/A-m

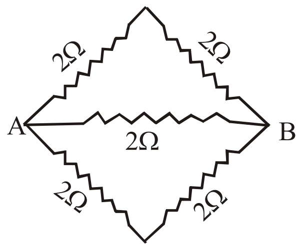
24. The turn ratio of transformers is given as 2:3. If the current through the primary coil is 3 A, thus calculate the current through load resistance
(a) 1A (b) 4.5 A
(c) 2 A (d) 1.5 A

25. In an AC circuit, the potential across an inductance and resistance joined in series are respectively 16 V and 20 V. The total potential difference across the circuit is
(a) 20.0 V (b) 25.6 V
(c) 31.9 V (d) 33.6 V

26. If hydrogen atom is its ground state absorbs 10.2 eV of energy. The orbital angular momentum is increase by
(a) 1.05×10^{-34} J/s (b) 3.16×10^{-34} J/s
(c) 2.11×10^{-34} J/s (d) 4.22×10^{-34} J/s

27. Highly energetic electrons are bombarded on a target of an element containing 30 neutrons. The ratio of radii of nucleus to that of Helium nucleus is $(14)^{1/3}$. The atomic number of nucleus will be
(a) 25 (b) 26
(c) 56 (d) 30

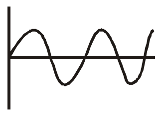
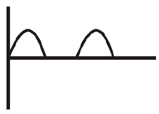
28. Each resistance shown in figure is 2 Ω. The eqivalent resistance between A and B is

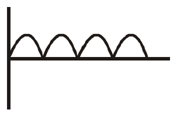
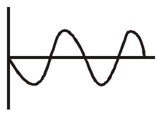


(a) 2 Ω (b) 4 Ω
(c) 8 Ω (d) 1 Ω

29. If in a triode value amplification factor is 20 and plate resistance is 10 kΩ, then its mutual conductance is
(a) 2 milli mho (b) 20 milli mho
(c) (1/2) milli mho (d) 200 milli mho

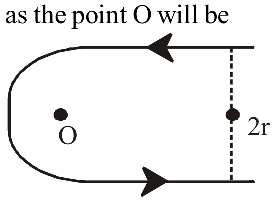
30. The output wave form of full-wave rectifier is

(a)  (b) 

(c)  (d) 

31. Calculate the energy released when three α-particles combined to form a ^{12}C nucleus, the mass defect is
(Atomic mass of $_2He^4$ is 4.002603 u)
(a) 0.007809 u (b) 0.002603 u
(c) 4.002603 u (d) 0.5 u

32. In the figure shown, the magnetic field induction as the point O will be



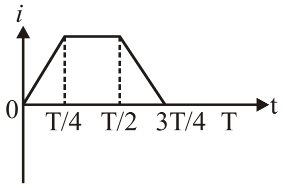
(a) $\frac{\mu_0 i}{2\pi r}$ (b) $\left(\frac{\mu_0}{4\pi}\right)\left(\frac{i}{r}\right)(\pi + 2)$

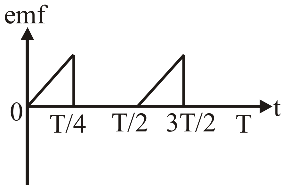
(c) $\left(\frac{\mu_0}{4\pi}\right)\left(\frac{i}{r}\right)(\pi + 1)$ (d) $\frac{\mu_0}{4\pi}\frac{i}{r}(\pi - 2)$

33. In photoelectric emission process from a metal of work function 1.8 eV, the kinetic energy of most energetic electrons is 0.5 eV. The corresponding stopping potential is
(a) 1.3 V (b) 0.5 V
(c) 2.3 V (d) 1.8 V

34. A current of 2 A flows through a 2 Ω resistor when connected across a battery. The same battery supplies a current of 0.5 A when connected across a 9 Ω resistor. The internal resistance of the battery is
(a) 1/3 Ω (b) 1/4 Ω
(c) 1 Ω (d) 0.5 Ω

35. The current i in a coil varies with time as shown in the figure. The variation of induced emf with time would be



(a)


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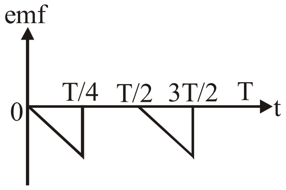
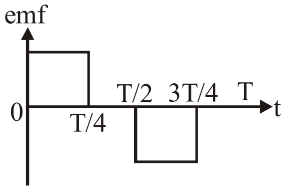
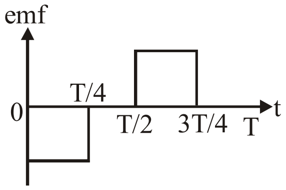


(c)


(d)


36. A transistor is operated in common emitter configuration at $V_C = 2$ V such that a change in the base current from 100 μA to 300 μA produces a change in the collector current from 10 mA to 20 mA. The current gain is
(a) 75 (b) 100
(c) 25 (d) 50

37. A uniform electric field and a uniform magnetic field are acting along the same direction in a certain region. If an electron is projected in the region such that its velocity is pointed along the direction of fields, then the electron
(a) speed will decrease
(b) speed will increase
(c) will turn towards left of direction of motion
(d) will turn towards right of direction a motion

38. Charge q is uniformly spread on a thin ring of radius R. The ring rotates about its axis with a uniform frequency f Hz. The magnitude of magnetic induction at the centre of the ring is

(a) $\frac{\mu_0 qf}{2R}$ (b) $\frac{\mu_0 q}{2fR}$

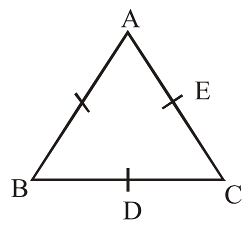
(c) $\frac{\mu_0 q}{2\pi fR}$ (d) $\frac{\mu_0 qf}{2\pi R}$

39. A galvanometer of resistance, G is shunted by a resistance S ohm. To keep the main current in the circuit unchanged, the resistance to be put in series with the galvanometer is

(a) $\frac{S^2}{(S+G)}$ (b) $\frac{SG}{(S+G)}$

(c) $\frac{G^2}{(S+G)}$ (d) $\frac{G}{(S+G)}$

40. Three charges, each + q, are placed at the corners of an isosceles triagle ABC of sides BC and AC, 2a. D and E are the mid-points of BC and CA. The work done in taking a charge Q from D to E is



(a) $\frac{eqQ}{8\pi\varepsilon_0 a}$ (b) $\frac{qQ}{4\pi\varepsilon_0 a}$

(c) Zero (d) $\frac{3qQ}{4\pi\varepsilon_0 a}$

PART - II (CHEMISTRY)

41. A bubble of air is underwater at temperature 15°C and the pressure 1.5 bar. If the bubble rises to the surface where the temperature is 25°C and the pressure is 1.0 bar, what will happen to the volume of the bubble ?
(a) Volume will become greater by a factor of 1.6
(b) Volume will become greater by a factor of 1.1
(c) Volume will become smaller by a factor of 0.70
(d) Volume will become greater by a factor of 2.9

42. Match List-I with List-II for the compositions of substances and select the correct answer using the codes given below the lists.

List-I (Substances)		List-II (Composition)	
A.	Plaster of Paris	1.	$CaSO_4.2H_2O$
B.	Epsomite	2.	$CaSO_4.\frac{1}{2}H_2O$
C.	Kieserite	3.	$MgSO_4.7H_2O$
D.	Gypsum	4.	$MgSO_4.H_2O$
		5.	$CaSO_4$

Codes :

	A	B	C	D
(a)	3	4	1	2
(b)	2	3	4	1
(c)	1	2	3	5
(d)	4	3	2	1

43. The pairs of species of oxygen and their magnetic behaviours are noted below. Which of the following presents the correct description?
(a) O_2^-, O_2^{2-} – Both diamagnetic
(b) O^+, O_2^{2-} – Both paramagnetic
(c) O_2^+, O_2 – Both paramagnetic
(d) O, O_2^{2-} – Both paramagnetic

44. Consider the reactions
(i) $(CH_3)_2CH-CH_2Br \xrightarrow{C_2H_5OH} (CH_3)_2CH-CH_2OC_2H_5 + HBr$
(ii) $(CH_3)_2CH-CH_2Br \xrightarrow{C_2H_5O^-} (CH_3)_2CH-CH_2OC_2H_5 + Br^-$
The mechanisms of reactions (i) and (ii) are respectively
(a) S_N1 and S_N2 (b) S_N1 and S_N1
(c) S_N2 and S_N2 (d) S_N2 and S_N1

45. Which of the following complex compounds will exhibit highest paramagnetic behaviour? (At. no. Ti = 22, Cr = 24, Co = 27, Zn = 30)
(a) $[Ti(NH_3)_6]^{3+}$ (b) $[Cr(NH_3)_6]^{3+}$
(c) $[Co(NH_3)_6]^{3+}$ (d) $[Zn(NH_3)_6]^{2+}$

46. Which of the following oxide is amphoteric?
(a) SnO_2 (b) CaO
(c) SiO_2 (d) CO_2

47. The following reactions take place in the blast furnace in the preparation of impure iron. Identify the reaction pertaining to the formation of the slag.
(a) $Fe_2O_3(s) + 3CO(g) \longrightarrow 2Fe(l) + 3CO_2(g)$
(b) $CaCO_3(s) \longrightarrow CaO(s) + CO_2(g)$
(c) $CaO(s) + SiO_2(s) \longrightarrow CaSiO_3(s)$
(d) $2C(s) + O_2(g) \longrightarrow 2CO(g)$

48. Among the elements Ca, Mg, P and Cl, the order of increasing atomic radii is
(a) Mg < Ca < Cl < P (b) Cl < P < Mg < Ca
(c) P < Cl < Ca < Mg (d) Ca < Mg < P < Cl

49. The reaction,
$$2A(g) + B(g) \rightleftharpoons 3C(g) + D(g)$$
is begun with the concentrations of A and B both at an initial value of 1.00 M. When equilibrium is reached, the concentration of D is measured and found to be 0.25 M. The value for the equilibrium constant for this reaction is given by the expression
(a) $[(0.75)^3(0.25)] \div [(1.00)^2(1.00)]$
(b) $[(0.75)^3(0.25)] \div [(0.50)^2(0.75)]$
(c) $[(0.75)^3(0.25)] \div [(0.50)^2(0.25)]$
(d) $[(0.75)^3(0.25)] \div [(0.75)^2(0.25)]$

50. Which of the following expressions correctly represents the equivalent conductance at infinite dilution of $Al_2(SO_4)_3$? Given that $\Lambda^\circ_{Al^{3+}}$ and $\Lambda^\circ_{SO_4^{2-}}$ are the equivalent conductances at infinite dilution of the respective ions?

(a) $2\Lambda^{\circ}_{Al^{3+}} + 3\Lambda^{\circ}_{SO_4^{2-}}$

(b) $\Lambda^{\circ}_{Al^{3+}} + \Lambda^{\circ}_{SO_4^{2-}}$

(c) $\left(\Lambda^{\circ}_{Al^{3+}} + 3\Lambda^{\circ}_{SO_4^{2-}}\right) \times 6$

(d) $\frac{1}{3}\Lambda^{\circ}_{Al^{3+}} + \frac{1}{2}\Lambda^{\circ}_{SO_4^{2-}}$

51. The pressure exerted by 6.0g of methane gas in a 0.03 m^3 vessel at 129°C is
(Atomic masses : C =12.01, H = 1.01 and R = 8.314 JK^{-1} mol^{-1})
(a) 215216 Pa (b) 13409 Pa
(c) 41648 Pa (d) 31684 Pa

52. Match List I (Equations) with List II (Types of process) and select the correct option.

List-I (Equations)		List-II (Types of process)	
A.	$K_P > Q$	1.	Non-spontaneous
B.	$\Delta G^{\circ} < RT \ln Q$	2.	Equilibrium
C.	$K_P = Q$	3.	Spontaneous and endothermic
D.	$T > \frac{\Delta H}{\Delta S}$	4.	Spontaneous

Codes :

	A	B	C	D
(a)	1	2	3	4
(b)	3	4	2	1
(c)	4	1	2	3
(d)	2	1	4	3

53. Among the following which one has the highest cation of anion size ratio?
(a) CsI (b) CsF
(c) LiF (d) NaF

54. Which of the following species is not electrophilic in nature?
(a) $\overset{\oplus}{Cl}$ (b) BH_3
(c) $H_3\overset{\oplus}{O}$ (d) $\overset{\oplus}{NO_2}$

55. Match List I (Substances) with List II (Processes employed in the manufacture of the substances) and select the correct option.

List-I (Substances)		List-II (Processes)	
A.	Sulphuric acid	1.	Haber's process
B.	Steel	2.	Bessemer's process
C.	Sodium hydroride	3.	Leblanc process
D.	Ammonia	4.	Contact process

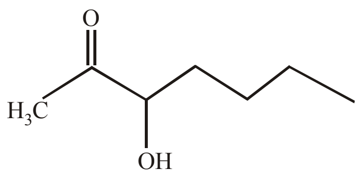
Codes :

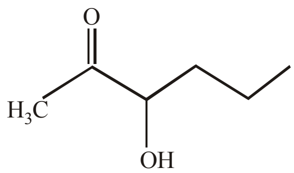
	A	B	C	D
(a)	1	4	2	3
(b)	1	2	3	4
(c)	4	3	2	1
(d)	4	2	3	1

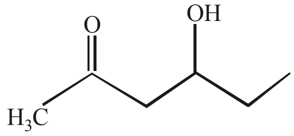
56. When glycerol is treated with excess of HI, it produces
(a) 2-iodopropane (b) allyl iodide
(c) propene (d) glycerol triiodide

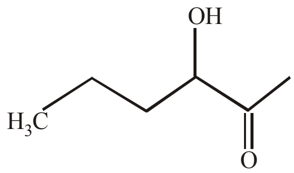
57. Some statements about heavy water are given below.
(i) Heavy water is used as moderator in nuclear reactors
(ii) Heavy water is more associated than ordinary water
(iii) Heavy water is more effective solvent than ordinary water
Which of the above statements are correct?
(a) (i) and (ii) (b) (i), (ii) and (iii)
(c) (ii) and (iii) (d) (i) and (iii)

58. Which one of the following compounds will be most readily dehydrated ?

(a) 

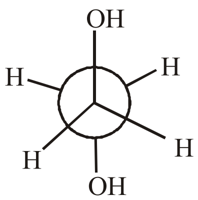
(b) 

(c) 

(d) 

59. Which one of the following complexes is not expected to exhibit isomerism?
(a) $[Ni(NH_3)_4(H_2O)_2]^{2+}$
(b) $[Pt(NH_3)_2Cl_2]$
(c) $[Ni(NH_3)_2Cl_2]$
(d) $[Ni(en)_3]^{2+}$

60. Which of the following conformers for ethylene glycol is most stable ?

(a)


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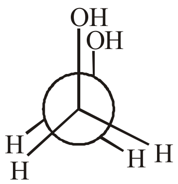

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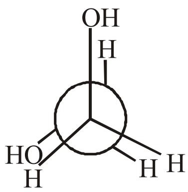
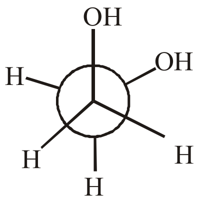


(d)


61. The IUPAC name of the compound $CH_3CH=CHC\equiv CH$ is
(a) pent-4-yn-2-ene (b) pent-3-en-1-yne
(c) pent-2-en-4-yne (d) pent-1-yn-3-ene

62. Which of the following oxidation states is the most common among the lanthanoids?
(a) 4 (b) 2
(c) 5 (d) 3

63. Some of the properties of the two species, NO_3^- and H_3O^+ are described below. Which one of them is correct?
(a) Dissimilar in hybridisation for the central atom with different structures
(b) Isostructural with same hybridisation for the central atom
(c) Isostructural with different hybridisation for the central atom
(d) Similar in hybridisation for the central atom with different structures

64. Following compounds are given
(i) CH_3CH_2OH (ii) CH_3COCH_3
(iii) $CH_3-CH(OH)-CH_3$ (iv) CH_3OH

Which of the above compound(s) on being warmed with iodine solution and NaOH, will give iodoform?
(a) (i), (iii) and (iv) (b) Only (ii)
(c) (i), (ii) and (iii) (d) (i) and (ii)

65. Fructose reduces Tollen's reagent due to
(a) asymmetric carbons
(b) primary alcoholic group
(c) secondary alcoholic group
(d) enolisation of fructose followed by conversion to aldehyde by base

66. In the following reaction,

$$C_6H_5CH_2Br \xrightarrow[\text{(ii) } H_3O^+]{\text{(i) Mg, Ether}} X,$$

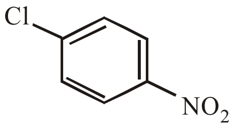
the product 'X' is
(a) $C_6H_5CH_2OCH_2C_6H_5$
(b) $C_6H_5CH_2OH$
(c) $C_6H_5CH_3$
(d) $C_6H_5CH_2CH_2C_6H_5$

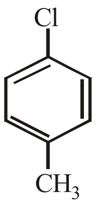
67. Which of the following is not a fat soluble vitamin?
(a) Vitamin-B complex
(b) Vitamin-D
(c) Vitamin-E
(d) Vitamin-A

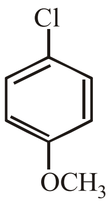
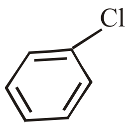
68. Which of the statements about 'Denaturation' given below are correct?
Statements :
(i) denaturation of proteins causes loss of secondary and tertiary structures of the protein.
(ii) Denaturation leads to the conversion of double strand of DNA into single strand'.
(iii) Denaturation affects primary structure which gets destroyed.
(a) (ii) and (iii) (b) (i) and (iii)
(c) (i) and (ii) (d) (i), (ii) and (iii)

69. Which has the maximum number of molecules among the following ?
(a) 44 g CO_2 (b) 48 g O_3
(c) 8 g H_2 (d) 64 g SO_2

70. Which of the following compounds undergoes nucleophilic substitution reaction most easily ?

(a) 

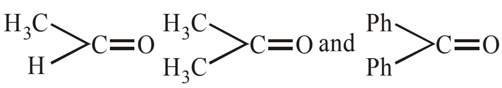
(b) 

(c)  (d) 

71. A 0.1 molal aqueous solution of a weak acid is 30% ionised. If K_f for water is 1.86° C/m, the freezing point of the solution will be
(a) –0.18°C (b) –0.54°C
(c) –0.36°C (d) –0.24°C

72. Which of the following carbonyls will have the strongest C – O bond?
(a) $Mn(CO)_6^+$ (b) $Cr(CO)_6$
(c) $V(CO)_6^-$ (d) $Fe(CO)_5$

73. The order of reactivity of phenyl magnesium bromide (PhMgBr) with the following compounds



(I) (II) (III)

(a) III > II > I (b) II > I > III
(c) I > III > II (d) I > II > III

74. A solid compound XY has NaCl structure. If the radius of the cation is 100 pm, the radius of the anion (Y^-) will be
(a) 275.1 pm (b) 322.5 pm
(c) 241.5 pm (d) 165.7 pm

75. Consider the following processes
ΔH (kJ/mol)

$$\frac{1}{2}A \longrightarrow B + 150$$

$$3B \longrightarrow 2C + D - 125$$

$$E + A \longrightarrow 2D + 350$$

For $B + D \longrightarrow E + 2C$, ΔH will be
(a) 525 kJ/mol (b) –175 kJ/mol
(c) –325 kJ/mol (d) 325 kJ/mol

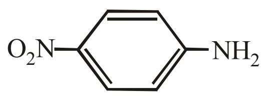
76. Match the compounds given in List-I with List-II and select the suitable option using the codes given below

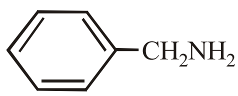
List-I		List-II	
A.	Benzaldehyde	1.	Phenolphthalein
B.	Phthalic anhydride	2.	Benzoin condensation
C.	Phenyl benzoate	3.	Oil of wintergreen
D.	Methyl salicylate	4.	Fries rearrangement

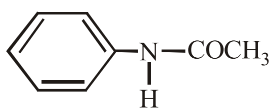
Codes :

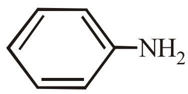
	A	B	C	D
(a)	4	1	3	2
(b)	4	2	3	1
(c)	2	3	4	1
(d)	2	1	4	3

77. Which of the following compound is the most basic ?

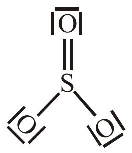
(a) 

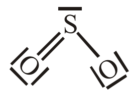
(b) 

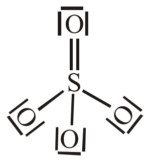
(c) 

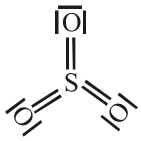
(d) 

78. Which of the following structures is the most preferred and hence of lowest energy for SO_3?

(a) 

(b) 

(c) 

(d) 

79. What is the value of electron gain enthalpy of Na^+ if IE_1 of Na = 5.1 eV ?

(a) –5.1 eV (b) –10.2 eV
(c) +2.55 eV (d) +10.2 eV

80. The unit of rate constant for a zero order reaction is

(a) $mol\,L^{-1}s^{-1}$ (b) $L\,mol^{-1}s^{-1}$
(c) $L^2 mol^{-2}s^{-1}$ (d) s^{-1}

PART - III (MATHEMATICS)

81. The solution of the differential equation

$$\frac{dy}{dx}+\frac{2yx}{1+x^2}=\frac{1}{(1+x^2)^2}$$ is

(a) $y(1+x^2)=C+\tan^{-1}x$
(b) $\frac{y}{1+x^2}=C+\tan^{-1}x$
(c) $y\log(1+x^2)=C+\tan^{-1}x$
(d) $y(1+x^2)=C+\sin^{-1}x$

82. If x, y and z are all distinct and

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix}=0,$$ then the value of xyz is

(a) –2 (b) –1
(c) –3 (d) None of these

83. The probability that atleast one of the events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.2, then $P(\bar{A})+P(\bar{B})$ is

(a) 0.4 (b) 0.8
(c) 1.2 (d) 1.4

84. If 3p and 4p are resultant of a force 5p, then the angle between 3p and 5p is

(a) $\sin^{-1}\left(\frac{3}{5}\right)$ (b) $\sin^{-1}\left(\frac{4}{5}\right)$
(c) 90° (d) None of these

85. If $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$, then the value of x is

(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$ (d) None of these

86. Let *a* be any element in a boolean algebra *B*. If $a+x=1$ and $ax=0$, then

(a) $x=1$ (b) $x=0$
(c) $x=a$ (d) $x=a'$

87. Dual of $(x+y).(x+1)=x+x.y+y$ is

(a) $(x.y)+(x.0)=x.(x+y).y$
(b) $(x+y)+(x.1)=x.(x+y).y$
(c) $(x.y)(x.0)=x.(x+y).y$
(d) None of the above

88. The function $f:R\to R$ defined by $f(x)=(x-1)(x-2)(x-3)$ is

(a) one-one but not onto
(b) onto but not one-one
(c) both one-one and onto
(d) neither one-one nor onto

89. If the complex numbers z_1, z_2 and z_3 are in AP, then they lie on a

(a) a circle (b) a parabola
(c) line (d) ellipse

90. Let a, b and c be in AP and $|a|<1$, $|b|<1$, $|c|<1$. If $x=1+a+a^2+\ldots$ to ∞,
$y=1+b+b^2+\ldots$ to ∞,
$z=1+c+c^2+\ldots$ to ∞, then x, y and z are in

(a) AP (b) GP
(c) HP (d) None of these

91. The number of real solutions of the equation

$$\left(\frac{9}{10}\right)=-3+x-x^2$$ is

(a) 0 (b) 1
(c) 2 (d) None of these

92. The lines 2x – 3y – 5 = 0 and 3x – 4y = 7 are diameters of a circle of area 154 sq units, then the equation of the circle is

(a) $x^2+y^2+2x-2y-62=0$
(b) $x^2+y^2+2x-2y-47=0$
(c) $x^2+y^2-2x+2y-47=0$
(d) $x^2+y^2-2x+2y-62=0$

93. The angle of depressions of the top and the foot of a chimney as seen from the top of a second chimney, which is 150 m high and standing on the same level as the first are θ and ϕ respectively, then the distance between their tops when $\tan\theta = \frac{4}{3}$ and $\tan\phi = \frac{5}{2}$ is

(a) $\frac{150}{\sqrt{3}}$ m (b) $100\sqrt{3}$ m
(c) 150 m (d) 100 m

94. If one root is square of the other root of the equation $x^2 + px + q = 0$, then the relations between p and q is
(a) $p^3 - (3p - 1)q + q^2 = 0$
(b) $p^3 - q(3p + 1) + q^2 = 0$
(c) $p^3 + q(3p - 1) + q^2 = 0$
(d) $p^3 + q(3p + 1) + q^2 = 0$

95. The coefficient of x^{53} in the following expansions $\sum_{m=0}^{100} {}^{100}C_m (x-3)^{100-m}.2^m$ is
(a) ${}^{100}C_{47}$ (b) ${}^{100}C_{53}$
(c) $-{}^{100}C_{53}$ (d) $-{}^{100}C_{100}$

96. If (–3, 2) lies on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, which is concentric with the circle $x^2 + y^2 + 6x + 8y - 5 = 0$, then c is equal to
(a) 11 (b) –11
(c) 24 (d) 100

97. If $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$ and $\mathbf{c} = 7\mathbf{i} + 9\mathbf{j} + 11\mathbf{k}$, then the area of Parallelogram having diagonals $\mathbf{a} + \mathbf{b}$ and $\mathbf{b} + \mathbf{c}$ is
(a) $4\sqrt{6}$ sq. units (b) $\frac{1}{2}\sqrt{21}$ sq. units
(c) $\frac{\sqrt{6}}{2}$ sq. units (d) $\sqrt{6}$ sq. units

98. If $A = \begin{bmatrix} 1 & -5 & 7 \\ 0 & 7 & 9 \\ 11 & 8 & 9 \end{bmatrix}$, then trace of matrix A is
(a) 17 (b) 25
(c) 3 (d) 12

99. The value of the determinant
$$\begin{vmatrix} \cos\alpha & -\sin\alpha & 1 \\ \sin\alpha & \cos\alpha & 1 \\ \cos(\alpha+\beta) & -\sin(\alpha+\beta) & 1 \end{vmatrix}$$ is
(a) independent of α
(b) independent of β
(c) independent of α and β
(d) None of the above

100. The maximum value of $4\sin^2 x - 12\sin x + 7$ is
(a) 25 (b) 4
(c) does not exist (d) None of these

101. A straight line through the point A(3, 4) is such that its intercept between the axes is bisected at A, its equation is
(a) $3x - 4y + 7 = 0$ (b) $4x + 3y = 24$
(c) $3x + 4y = 25$ (d) $x + y = 7$

102. The tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ at
(a) (6, 7) (b) (–6, 7)
(c) (6, –7) (d) (–6, –7)

103. The equation of straight line through the intersection of the lines $x - 2y = 1$ and $x + 3y = 2$ and parallel $3x + 4y = 0$ is
(a) $3x + 4y + 5 = 0$ (b) $3x + 4y - 10 = 0$
(c) $3x + 4y - 5 = 0$ (d) $3x + 4y + 6 = 0$

104. $\int \frac{dx}{\sin x - \cos x + \sqrt{2}}$ equals to
(a) $-\frac{1}{\sqrt{2}}\tan\left(\frac{x}{2} + \frac{\pi}{8}\right) + C$
(b) $\frac{1}{2}\tan\left(\frac{x}{2} + \frac{\pi}{8}\right) + C$
(c) $\frac{1}{\sqrt{2}}\cot\left(\frac{x}{2} + \frac{\pi}{8}\right) + C$
(d) $-\frac{1}{\sqrt{2}}\cot\left(\frac{x}{2} + \frac{\pi}{8}\right) + C$

105. The value of integral $\int_0^1 \sqrt{\frac{1-x}{1+x}}\,dx$ is
(a) $\frac{\pi}{2} + 1$ (b) $\frac{\pi}{2} - 1$
(c) –1 (d) 1

106. The value of $I = \int_0^1 x\left|x - \frac{1}{2}\right| dx$ is
(a) $\frac{1}{3}$ (b) $\frac{1}{4}$
(c) $\frac{1}{8}$ (d) None of these

107. The eccentricity of the ellipse, which meets the straight line $\frac{x}{7}+\frac{y}{2}=1$ on the axis of x and the straight line $\frac{x}{3}-\frac{y}{5}=1$ on the axis of y and whose axes lie along the axes of coordinates, is

(a) $\frac{3\sqrt{2}}{7}$ (b) $\frac{2\sqrt{6}}{7}$

(c) $\frac{\sqrt{3}}{7}$ (d) None of there

108. If $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\,(a>b)$ and $x^2-y^2=c^2$ cut at right angles, then

(a) $a^2+b^2=2c^2$ (b) $b^2-a^2=2c^2$

(c) $a^2-b^2=2c^2$ (d) $a^2b^2=2c^2$

109. The equation of the conic with focus at (1, –1) directrix along $x-y+1=0$ and with eccentricity $\sqrt{2}$ is

(a) $x^2-y^2=1$

(b) $xy=1$

(c) $2xy-4x+4y+1=0$

(d) $2xy+4x-4y-1=0$

110. There are 5 letters and 5 different envelopes. The number of ways in which all the letters can be put in wrong envelope, is

(a) 119 (b) 44

(c) 59 (d) 40

111. The sum of the series

$$1+\frac{1^2+2^2}{2!}+\frac{1^2+2^2+3^2}{3!}+\frac{1^2+2^2+3^2+4^2}{4!}+...$$

is

(a) 3e (b) $\frac{17}{6}e$ (c) $\frac{13}{6}e$ (d) $\frac{19}{6}e$

112. The coefficient of x^n in the expansion of $\log_a(1+x)$ is

(a) $\frac{(-1)^{n-1}}{n}$ (b) $\frac{(-1)^{n-1}}{n}\log_a e$

(c) $\frac{(-1)^{n-1}}{n}\log_e a$ (d) $\frac{(-1)^{n}}{n}\log_a e$

113. If a plane meets the coordinate axes at A, B and C in such a way that the centroid of ΔABC is at the point (1, 2, 3), then equation of the plane is

(a) $\frac{x}{1}+\frac{y}{2}+\frac{z}{3}=1$ (b) $\frac{x}{3}+\frac{y}{6}+\frac{z}{9}=1$

(c) $\frac{x}{1}+\frac{y}{2}+\frac{z}{3}=\frac{1}{3}$ (d) None of these

114. Area lying in the first quadrant and bounded by the circle $x^2+y^2=4$, the line $x=\sqrt{3}y$ and x-axis is

(a) π sq units (b) $\frac{\pi}{2}$ sq units

(c) $\frac{\pi}{3}$ sq units (d) None of these

115. The value of $\lim_{x\to\infty}\left(\frac{\pi}{2}-\tan^{-1}x\right)^{1/x}$ is

(a) 0 (b) 1

(c) –1 (d) e

116. If $f(x)=\begin{cases} mx+1, & x\le\frac{\pi}{2} \\ \sin x+n, & x>\frac{\pi}{2}\end{cases}$ is continuous at $x=\frac{\pi}{2}$, then

(a) $m=1, n=0$ (b) $m=\frac{n\pi}{2}+1$

(c) $n=m\frac{\pi}{2}$ (d) $m=n=\frac{\pi}{2}$

117. The domain of the function $f(x)=\frac{\sqrt{4-x^2}}{\sin^{-1}(2-x)}$ is

(a) [0, 2] (b) [0, 2)

(c) [1, 2) (d) [1, 2]

118. The general solution of the differential equation $(1+y^2)\,dx+(1+x^2)\,dy=0$ is

(a) $x-y=C(1-xy)$ (b) $x-y=C(1+xy)$

(c) $x+y=C(1-xy)$ (d) $x+y=C(1+xy)$

119. The order and degree of the differential equation

$$\rho=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

are, respectively

(a) 2, 2 (b) 2, 3

(c) 2, 1 (d) None of these

120. The relation *R* defined on the set of natural numbers as {(*a*, *b*) : *a* differs from *b* by 3} is given

(a) {(1, 4), (2, 5), (3, 6),... }

(b) {(4, 1), (5, 2), (6, 3),... }

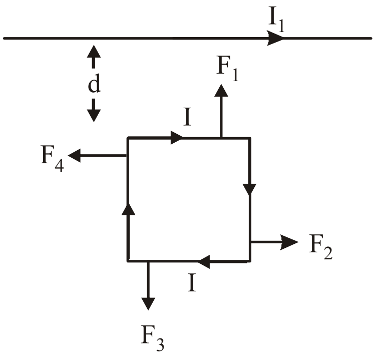
(c) {(1, 3), (2, 6), (3, 9),... }

(d) None of the above

SOLUTIONS

PART - I (PHYSICS)

1. (d)



$F_2 = -F_4$

$F_1 = \dfrac{\mu_0 I_1 I l}{2\pi d l}$

$F_2 = \dfrac{\mu_0 I_1 I}{2\pi(d+1)}$

$F_1 > F_3$

$F_{net} = F_1 - F_3$

So, wire attract loop.

2. (a) Here, $V_0 = \dfrac{E-V}{e} = \dfrac{h(v-v_0)}{e}$

$= \dfrac{6.62\times10^{-34}(8.2\times10^{14} - 3.3\times10^{14})}{1.6\times10^{-19}}$

$= \dfrac{6.62\times10^{-34}}{1.6}\times 4.9\times10^{33}$

$= \dfrac{6.62\times4.9\times10^{-1}}{1.6}$

$V_0 = 2$ volt

3. (a) In forward biasing, resistance of p–n junctin diode is zero, so whole voltage appears across the resistance.

4. (d) BE of $Li^7 = 39.20$ MeV

and $He^4 = 28.24$ MeV

Hence binding energy of ${}_2He^4 = 56.84$ MeV

Energy of reaction $= 56.84 - 39.20$

$= 17.28$ MeV

5. (b) $\sqrt{v} \propto (Z-b)$

6. (d) When reactance of inductance is mroe than the reactance of cndenser, the current will lag behind the voltage.

Thus $\omega L < \dfrac{1}{\omega c}$ or $\omega > \dfrac{1}{\sqrt{LC}}$

or $n > \dfrac{1}{2\pi\sqrt{LC}}$ or $n > n_r$

n_r = resonant frequency

7. (c) Capacitance, $C_A = \dfrac{K_1\varepsilon_0 A}{\frac{d}{2}}, C_B = \dfrac{K_2\varepsilon_0 A}{\frac{d}{2}}$

$C_{eq} = \dfrac{C_1}{C_2} = \dfrac{2K_1K_2}{K_1+K_2}$

$= \dfrac{C_A C_B}{C_A + C_B} = \left(\dfrac{2K_1K_2}{K_1+K_2}\right)\dfrac{\varepsilon_0 A}{d}\left(\because e = \dfrac{\varepsilon_0 A}{d}\right)$

8. (d) Intensity of the electric field, $E = \dfrac{dV}{dx} = 6x$

Potential $(v) = 3x^2 + 5$

$\therefore$ E at $x = -2$

$= 6(-2) = -12$ V/m

9. (c) Volume of 8 small drops = Volume of big drop

$\therefore \left(\dfrac{4}{3}\pi r^3\right)\times 8 = \dfrac{4}{3}\pi R^3$

$\Rightarrow 2r = R$

...(i)

According to charge conservation

$8q = Q$

...(ii)

Potential of one small drop $(V') = \dfrac{q}{4\pi\varepsilon_0 r}$

Similarly, potential of big drop $(V) = \dfrac{Q}{4\pi\varepsilon_0 R}$

Now, $\dfrac{V'}{V} = \dfrac{q}{Q}\times\dfrac{R}{r} \Rightarrow \dfrac{V'}{20} = \dfrac{9}{8q}\times\dfrac{2r}{r}$

$\therefore V' = 5V$

10. (b) Threshold energy of A $E_A = h\nu_A$
$= 6.6 \times 10^{-34} \times 1.8 \times 10^{14}$
$= 11.88 \times 10^{-20}$ J

$$= \frac{11.88 \times 10^{-20}}{1.6 \times 10^{-19}} \text{eV} = 0.74 \text{ eV}$$

Similarly, $E_B = 0.91$ eV
As the incident photons have energy greater than E_A but less than E_B
So, photoelectrons will be emitted from metal A only.

11. (a) Balanced wheatstone bridge condition

$$\frac{P}{Q} = \frac{R}{S}$$

No, current flows through galvanometer
Now, P and R are in series, so
Resistance $R_1 = P + R$

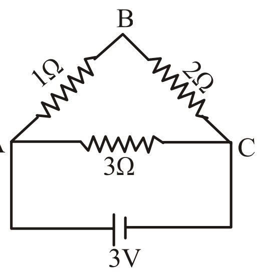
$= 10 + 15 = 25\Omega$
Similarly, Q and S are in series, so
Resistance $R_2 = R + S$
$= 20 + 30 = 50\Omega$
Net resistance of the network as R_1 and R_2 are in parallel

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\therefore R = \frac{25 \times 50}{25 + 50} = \frac{50}{3}\Omega$$

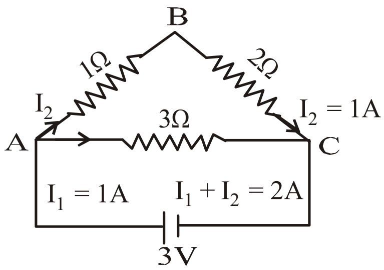
Hence, current, $I = \frac{V}{R} = \frac{6}{50/3} = 0.36$A

12. (b) The arrangement is shown in figure.



Here, two reisistance of 1Ω and 2Ω are in series, which form 3Ω which is in parallel with 3Ω resistance.
Therefore, the effective resistance

$$\frac{(1+2) \times 3}{(1+2) + 3} = \frac{3}{2}\Omega$$

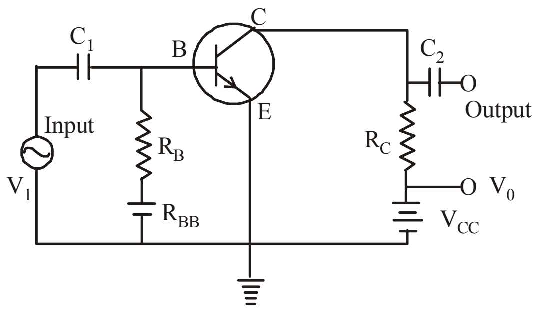


$\therefore$ Current in the circuit,

$$I = \frac{3}{(3/2)} = 2A$$

$\therefore$ Current in 3Ω resistor $= \frac{I}{2} = $ IA

13. (b) In CE amplifier, the input signal is applied across base–emitter junction.



14. (c) de–Broglie wavelength of an electron

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mK}} \text{ or } \lambda \propto \frac{1}{\sqrt{K}}$$

$$\therefore \frac{\lambda'}{\lambda} = \frac{1}{\sqrt{3K}} = \frac{\sqrt{K}}{1} = \frac{1}{\sqrt{3}}$$

or $\lambda' = \frac{\lambda}{\sqrt{3}}$

i.e. de–Broglie wavelength will change by factor $\frac{1}{\sqrt{3}}$.

15. (a) We know,

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T} \Rightarrow \frac{N}{10000} = \left(\frac{1}{2}\right)^{\frac{10}{20}}$$

$$\Rightarrow N = \frac{10000}{\sqrt{2}} = \frac{10000}{1.414} = 7070$$

16. (c) As X-rays pass through the intestine without casting a clear shadow.

17. (b) Given : $A = 0.3\ m^2$ $n = 2 \times 10^{25}/m^3$

$q = 3t^2 + 5t + 2$

$$i = \frac{dq}{dt} = 6t + 5 = 17$$

Drift velocity, $v_d = \frac{i}{neA}$

$$= \frac{17}{2\times10^{25}\times1.6\times10^{-19}\times0.3}$$

$$= \frac{17}{0.96\times10^{6}} = 1.77 \times 10^{-5}\ m/s$$

18. (b) Capacitance 1 μF and C μF are connected in series,

$$\therefore C_{eq} = \frac{C}{1+C}$$

Given, V = 120 V and q = 80 μC

$\because q = C_{eq}V$

$$80 = \frac{C}{C+1}\times 20$$

or C = 2 μF

Energy stored in the capacitor of capicity C

$$U = \frac{1}{2}\frac{q^2}{C}$$

$$= \frac{1}{2}\times\frac{(80\times10^{-6})^2}{2\times10^{-6}}$$

$$= \frac{1}{2}\times\frac{80\times10^{-6}\times80\times10^{-6}}{2\times10^{-6}}$$

U = 1600 μJ

19. (d) Conducting surface behaves as equipotential surface.

20. (a) $I = n_e q_e + n_p q_E = 1$ mA (towards right)

21. (a) 1 faraday deposited 1 g equivalent

22. (c) The magnetic field

$$B = \frac{\mu_0}{4\pi}.\frac{2\pi(qv)}{r}$$

$$= 10^{-7}\times\frac{2\times3.14\times(1.6\times10^{-19}\times1.6\times10^{15})}{0.53\times10^{-10}}$$

$= 12.5\ Wb/m^3$

23. (c) The magnetic field, $B = \frac{\mu_0}{4\pi}.\frac{2N}{d^3}$

$$= 10^{-7}\times\frac{2\times1.25}{(0.5)^3} = 2\times10^{-6}\ N/A\text{–}m$$

24. (c) Transformation ratio, $\frac{I_P}{I_S} = \frac{n_S}{n_S}$

i.e. $\frac{3}{I_S} = \frac{3}{2}$ or, $I_S = 2A$

25. (b) Voltage

$$V = \sqrt{V_R^2 + V_C^2} = \sqrt{(20)^2 + (16)^2}$$

=25.6V

26. (a) Electron goes to its first excited state (n = 2) from ground state (n = 1) after absorbing 10.2 eV energy

$\therefore$ Increase in momentum $= \frac{h}{2\pi}$

$$= \frac{6.6\times10^{-34}}{6.28}$$

$= 1.05 \times 10^{-34}$ J–s

27. (b) Using $R = R_0 A^{1/3}$

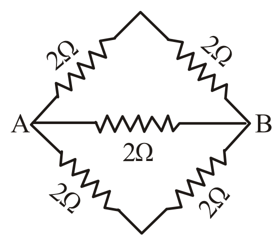
$$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3}$$

$$\frac{R}{R_{He}} = \left(\frac{A}{4}\right)^{\frac{1}{3}}$$

$$(14)^{1/3} = \left(\frac{A}{4}\right)^{\frac{1}{3}} \Rightarrow A = 56$$

So, $= 56 - 30 = 26$

28. (a) Given circuit is a balanced Wheatstone bridge.



Equivalent resistance of upper arms

$= 2 + 2 = 4\Omega$

Equivalent resistance of lowre arms

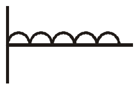
$= 2 + 2 = 4\Omega$

$\therefore R_{AB} = \frac{4 \times 4}{4 + 4} = 2\Omega$

29. (a) Mutual conductance $g_m = \frac{\mu}{R_P}$

$= \frac{20}{10 \times 10^3} = 2 \times 10^{-3} = 2$ milli mho

30. (c) Full-wave rectifier output wave form



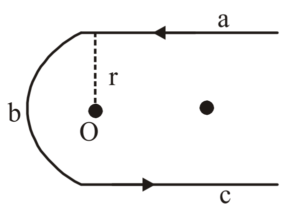
31. (c) Mass defect

$\Delta m =$ Total mass of α–particles – mass of ^{12}C nucleus

$= 3 \times 4.002603 - 12 = 12.007809 - 12$

$= 0.007809$ unit

32. (a) Field due to a straight wire of infinite length is $\frac{\mu_0 i}{4\pi r}$ if the point is on a line perpendicular to its length while at the centre of a semicurcular coil is $\frac{\mu_0 \pi i}{4\pi r}$



$\therefore B = B_a + B_b + B_c$

$= \frac{\mu_0}{4\pi}\frac{i}{r} + \frac{\mu_0}{4\pi}\frac{\pi i}{r} + \frac{\mu_0}{4\pi}\frac{i}{r}$

$= \frac{\mu_0}{4\pi}\frac{i}{r}(\pi + 2)$ out of the phase

33. (b) Stopping potential = Maximum KE

$eV = KE_{max}$

34. (d) Current $i = \frac{E}{R + r}$

$2 = \frac{E}{2 + r}$ ... (i)

$0.5 = \frac{E}{9 + r}$ ...(ii)

From Eqs. (i) and (ii), we have

$\frac{2}{0.5} = \frac{9 + r}{2 + r} \Rightarrow 4 = \frac{9 + r}{2 + r}$

$3r = 1 \therefore r = \frac{1}{3}\Omega$

35. (d) We know, induced emf

$e = -L\frac{di}{dt}$

During 0 to $\frac{T}{4}, \frac{di}{dt} =$ constant

So, e = –ve

For $\frac{T}{4}$ to $\frac{T}{2}, \frac{di}{dt} = 0$

i.e., e = 0

For $\frac{T}{4}$ to $\frac{3T}{4}, \frac{di}{dt} =$ constant

i.e., e = +ve

36. (d) Current gain, $\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{(20 - 100)\text{mA}}{(300 - 100)\text{mA}}$

$= \frac{10 \times 10^{-3}}{200 \times 10^{-6}} = 50$

37. (a) Field B not applied only force. Field E will apply a force opposite to velocity of the electron hence, speed will decreases.

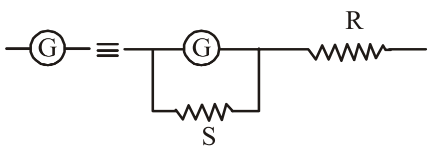
38. (a) We know magnetic field

$\beta = \frac{\mu_0 i}{2R}$

$q = it \Rightarrow i = \frac{q}{t} = qf$

$\therefore \beta = \frac{\mu_0 qf}{2R}$

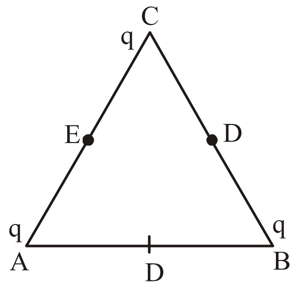
39. (c) If resistance remains same so current will be unchanged.



$G = \frac{GS}{G + S} + R \Rightarrow R = G - \frac{GS}{G + S}$

or, $R = \frac{G^2}{G + S}$

40. (c) Here, AC = BC



$V_D = V_E = V$

$W = Q[V_E - V_D]$

$W = Q[V - V]$

$W = 0$

PART - II (CHEMISTRY)

41. (a) $\because \frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$ (By ideal gas equation)

or $\frac{1.5 \times V_1}{288} = \frac{1 \times V_2}{298}$

$\therefore V_2 = 1.55\, V_1$

i.e, volume of bubble will be almost 1.6 times to initial volume of bubble.

42. (b) (A) Plaster or Paris = $CaSO_4 \cdot \frac{1}{2} H_2O$

(B) Epsomite = $MgSO_4 \cdot 7H_2O$

(C) Kieserite = $MgSO_4 \cdot H_2O$

(D) Gypsum= $CaSO_4 \cdot 2H_2O$

43. (c) The molecular orbital configurations of O_2^+, O_2^-, O_2^{2-} and O_2 are

$$O_2^+ = \sigma 1s^2, \overset{*}{\sigma} 1s^2, \sigma 2s^2, \overset{*}{\sigma} 2s^2, \sigma 2p_z^2, \pi 2p_x^2$$

$$\approx \pi 2p_y^2, \overset{*}{\pi} 2p_x^1 \approx \overset{*}{\pi} 2p_y^0$$

$$O_2^- = \sigma 1s^2, \overset{*}{\sigma} 1s^2, \sigma 2s^2, \overset{*}{\sigma} 2s^2, \sigma 2p_z^2, \pi 2p_x^2$$

$$\approx \pi 2p_y^2, \overset{*}{\pi} 2p_x^2 \approx \overset{*}{\pi} 2p_y^1$$

$$O_2^{2-} = \sigma 1s^2, \overset{*}{\sigma} 1s^2, \sigma 2s^2, \overset{*}{\sigma} 2s^2, \sigma 2p_z^2, \pi 2p_x^2$$

$$\approx \pi 2p_y^2, \overset{*}{\pi} 2p_x^2 \approx \overset{*}{\pi} 2p_y^2$$

$$O_2 = \sigma 1s^2, \overset{*}{\sigma} 1s^2, \sigma 2s^2, \overset{*}{\sigma} 2s^2, \sigma 2p_z^2, \pi 2p_x^2$$

$$\approx \pi 2p_y^2, \overset{*}{\pi} 2p_x^1 \approx \overset{*}{\pi} 2p_y^1$$

and the electronic configuration of O and O^+ are

$O = 1s^2, 2s^2, 2p_x^2, 2p_y^1, 2p_z^1$

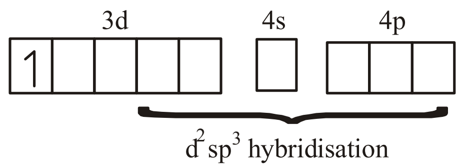
$O^+ = 1s^2, 2s^2, 2p_x^1, 2p_y^1, 2p_z^1$

As O_2^+, O_2, O_2^-, O and O^+ have unpaired electrons, hence are paramagnetic.

44. (a) C_2H_5OH being a weaker nucleopbile, when used as a solvent in case of hindered 1° halide, favours S_N1 mechanism while $C_2H_5O^-$ being a strong nucleophile in this reaction favours S_N2 mechanism.

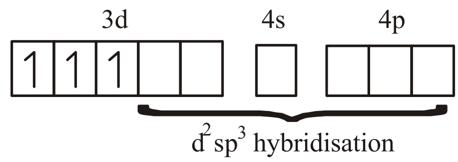
45. (b) (a) Electronic configuration of Ti^{3+} in $[Ti(NH_3)_6]^{3+}$

$Ti^{3+} = 3d^1$;



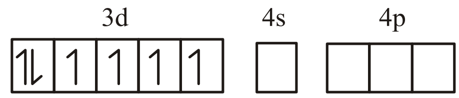
(b) Electronic configuration of Cr^{3+} in $[Cr(NH_3)_6]^{3+}$

$Cr^{3+} = 3d^3$;



(c) Electronic configuration of Co^{3+} in $[Co(NH_3)_6]^{3+}$;

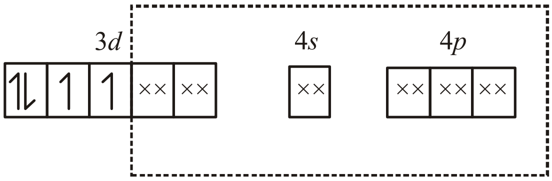
$Co^{3+} = 3d^6$.



In the presence of strong field ligand NH_3, pairing of electrons takes place and hence, octahedral complex, $[Co(NH_3)_6]^{3+}$ is diamagnetic.

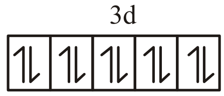
$[Co(NH_3)_6]^{3+}$ [inner orbital or low spin complex]

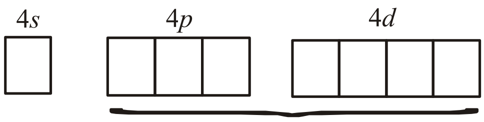
($6NH_3$ molecles)



d^2sp^3 hybridisation

(d) Electronic configuration of Zn^{2+} in $[Zn(NH_3)_6]^{2+}$

$Zn^{2+} = 3d^{10}$; 



sp^3d^2 hybridisation

$\therefore$ $[Zn(NH_3)_6]^{2+}$ is an outer orbital complex and is diamagnetic.

46. (a) SnO_2 reacts with acids as well as bases to form corresponding salts. So it is an amphoteric oxide.

$$SnO_2 + 4HCl \rightarrow SnCl_4 + 2H_2O$$

$$SnO_2 + 2NaOH \rightarrow \underset{\text{sod.stannate}}{Na_2SnO_3} + H_2O$$

47. (c) A slag is an easily fusible material which is formed when gangue still present in the roasted or the calcined ore combines with the flux. For example, in the metallurgy of iron, CaO (flux) combines with silica gangue to form easily fusible calcium silicate ($CaSiO_3$) slag.

$$CaO + SiO_2 \rightarrow CaSiO_3 \text{ (slag)}$$

48. (b) Atomic radii increases, as the number of shells increases. Thus, on moving down a group atomic radii increases.

The electronic configuration of the given element is

$Mg_{12} = [Ne]\ 3s^2$
$Ca_{20} = [Ar]\ 4s^2$
$P_{15} = [Ne]\ 3s^2 3p^3$
$Cl_{17} = [Ne] 3s^2 3p^5$

On the other hand, on increasing the number of electron in the same shell, the atomic radii decreases because effective nuclear charge is increases.

In Mg, P and C1, the number of electrons are increasing in the same shell, thus the order of their atomic radii is

C1 < P < Mg

In case of Ca, the electron is entering in higher shell. So, its atomic radii is highest. Thus, the order of radii is

Cl < P < Mg < Ca

49. (b) The reaction-

$$2A(g) + B(g) \rightleftharpoons 3C(g) + D(g)$$

	$2A$	B	$3C$	D
Initial	1	1	0	0
At equil	1 – 0.50	1 – 0.25	0.75	0.25

$$K = \frac{(0.75)^3(0.25)}{(0.50)^2(0.75)}$$

50. (b) $Al_2(SO_4)_3 \rightleftharpoons 2Al^{3+} + 3SO_4^{2-}$

We can calculate the equivalent conductance only for ions, so the equivalent conductance at infinite dilution,

$$\Lambda_{eq}^{\infty} = \Lambda^{\circ}_{Al^{3+}} + \Lambda^{\circ}_{SO_4^{2-}}$$

51. (c)

$$\left[\begin{array}{l} \text{w(given mass of methane)} = 6\text{g} \\ \text{temperature, } T = 129 + 273 = 402 \text{ K} \\ \text{mol mass of methane, } M = 12.01 + 4 \times 1.01 \\ = 16.05 \end{array}\right]$$

From, ideal gas equation,

$$pV = nRT \Rightarrow P = \frac{nRT}{v}$$

$$p = \frac{6}{16.05} \times \frac{8.314 \times 402}{0.03} = 41648 \text{ Pa}$$

52. (c) (A) If $k_p > Q$ and goes in forward direction than reaction is spontaneous

(B) Given, $\Delta G^{\circ} < RT \ln Q$,
thus, $\Delta G^{\circ} = +\text{ve}$
and hence, the reaction is non–spontaneous.

(C) At equilibrium, $K_p = Q$

(D) $T > \dfrac{\Delta H}{\Delta S}$

or $T\Delta S = \Delta H$

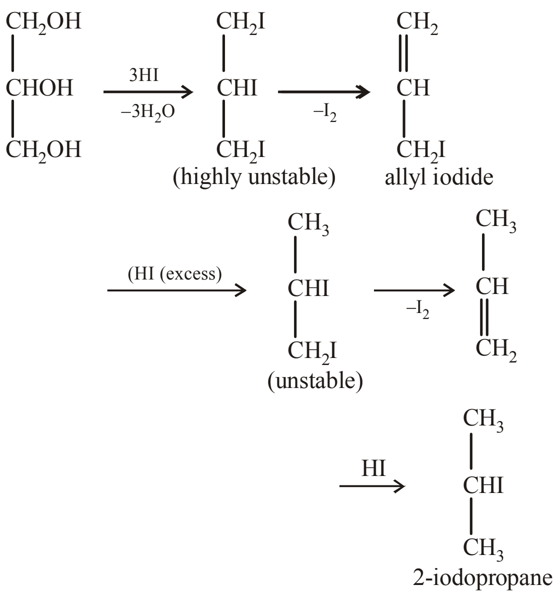
This is valid condition for spontaneous endothermic reactions (as $\Delta G \geq \Delta H - T\Delta S$)

53. (b) The size of cation is in order of-
$Li^+ < Na^+ < Cs^+$
and the size of anions in the order of-
$I^- > F^-$
Thus, when the cation is largest and anion is smallest, the ratio of their sizes is maximum.
Hence, cation to anion size ratio is maximum for CsF.

54. (c) Electron deficient species are known as electrophiles.
Among the given, $H_3O^{\oplus}$ has lone pair of electrons for donation, so it is not electron deficient and hence, not an electrophile.

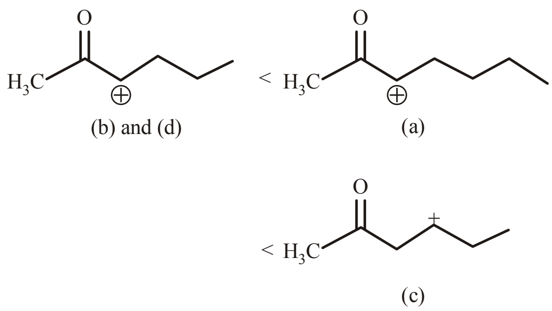
55. (d) Contact process is used for sulphuric acid, steel is manufactured by Bessemer's process, Leblanc process is used for the production of NaOH while Haber's process is used for NH_3 production.

56. (a)



57. (a) In nuclear reactors heavy water is used as a moderator. It has higher boiling point as compared to the ordinary water. Thus, it is more associated as compared to ordinary water. The dielectric constant is however higher for H_2O, thus, H_2O is a more effective solvent as compare to heavy water (D_2O).

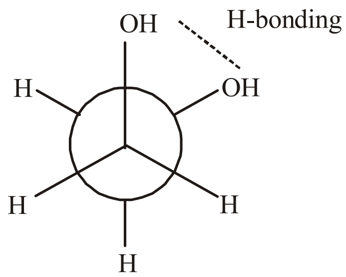
58. (c) Dehydration of alcohols involve formation of carbocation intermediate. Higher the stability of carbocation, higher is the ease of dehydration. The order of stability of carbocation, is



Hence, compound given in option (c) readily undergoes dehydration.

59. (c) Compounds having tetrahedral geometry does not exhibit isomerism due to presence of symmetry elements. Here, $[Ni(NH_3)_2Cl_2]$ has tetrated ral geometry.

60. (d)



This conformation is most stable due to intramolecular H-bonding.

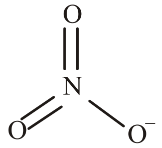
61. (b) $\overset{5}{C}H_3 - \overset{4}{C}H = \overset{3}{C}H - \overset{2}{C} \equiv \overset{1}{C}H$
pent-3-en-1-yne

62. (d) The most common oxidation state exhibited by lanthanoids is +3.

63. (a) In NO_3^-,

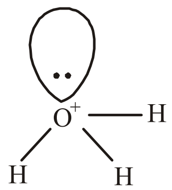
$H = \dfrac{1}{2}[5 + 0 - 0 + 1] = 3$. So, sp^3

hybridization.
Thus, it has trigonal planar geometry.



In H_3O^+,

$H = \frac{1}{2}[6 + 3 - 1 + 0] = 4$; So, sp^3 hybridization and it has pyramidal geometry due to the presence of one lone pair of electrons.



64. (c) Compounds having either $CH_3\overset{O}{\overset{||}{C}}—$ group or $CH_3CHOH—$ group, give iodoform when warmed with I_2 and NaOH.
Thus, compounds

$$CH_3\underset{H}{\underset{|}{C}}HOH, CH_3—\overset{O}{\overset{||}{C}}—CH_3, CH_3—\underset{CH_3}{\underset{|}{CHOH}}$$

give iodoform when heated with I_2 and NaOH. (Note : NaOI oxidises CH_3CH_2OH to CH_3CHO, and gives positive iodoform test.)

65. (d) In aqueous medium, fructose is enolised and converted into aldehyde in basic medium. Generally all aldehydes reduce Tollen's reagent, thus fructose can also reduces Tollen's reagent.

66. (c) $C_6H_5CH_2Br \xrightarrow{\text{Mg, Ether}} \underset{\text{Grignard reagent}}{C_6H_5CH_2MgBr}$

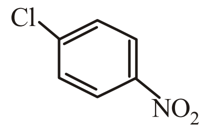
$$\xrightarrow{H_3O^+/H^+} \underset{\text{toluene}}{C_6H_5CH_3} + Mg\begin{matrix} Br \\ OH \end{matrix}$$

67. (a) Fat soluble vitamins are A, D and E. Whereas vitamin-B complex is soluble in water.

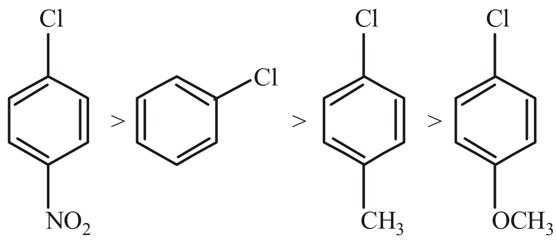
68. (c) In the process denaturation secondary and tertiary structures of protein destroyed but primary structure remains undisturbed.
Heat, acid and alkali denature DNA molecule and double strand of DNA converts into single strand.

69. (c) 44 g CO_2 = 1 mol CO_2 = N_A molecules of CO_2
48g O_3 = 1 mol O_3 = N_A molecules of O_3
8 g H_2 = 4 mol H_2 = $4 \times N_A$ molecules of H_2
64 g SO_2 = 1 mol SO_2 = N_A molecules of SO_2
$N_A = 6.023 \times 10^{23}$

70. (a)



It has electron withdrawing group $—NO_2$ which reduces the double bond character between carbon of benzene ring and chlorine. Hence, the correct order of nucleophilic substitution reactions are,



71. (d) Freezing point depression $(\Delta T_f) = iK_f m$

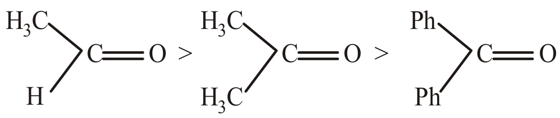
HA →	H^+ +	A^-
$1 - \alpha$	α	α
$1 - 0.3$	0.3	0.3

$i = 1 - 0.3 + 0.3 + 0.3$
$i = 1.3$
$\therefore \Delta T_f = 1.3 \times 1.86 \times 0.1 = 0.2418°C$
$T_f = 0 - 0.2418°C$
$= -0.2418°C$

72. (a) As positive charge on the central metal atom increases, the less readily the metal can donate electron density into the anti–bonding π–orbitals of C–O ligand to weaken the C—O bond. Thus, the C–O bond would be strongest in $Mn(CO)_6^+$

73. (d) Since alkyl group has +*I*–effect and aryl group has + *R*–effect, Hence greater the number of alkyl and aryl groups attached to the carbonyl group, its reactivity towards nucleophilic addition reaction. Secondly, as the steric crowding on carbonyl group increases, the reactivity decreases accordingly.

$\therefore$ Correct reactivity order for reaction with PhMgBr is



(I) (II) (III)

74. (c) Radius ratio of NaCl like crystal = $\frac{r^+}{r^-}$

$= 0.414$ or $r^- = \frac{100}{0.414} = 241.5$ pm

75. (b) $\frac{1}{2}A \rightarrow B$; $\Delta H = 150$ kJ/mol ...(i)

$3B \rightarrow 2C + D$; $\Delta H = -125$ kJ/mol ...(ii)

$E + A \rightarrow 2D$; $\Delta H = +350$ kJ/mol ...(iii)

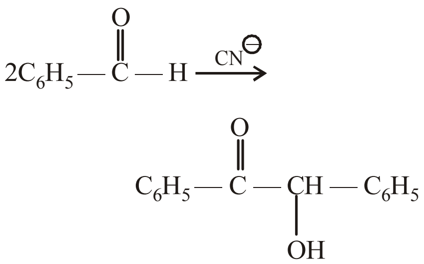
By [2 × (i) + (ii)] – (iii), we have

$B + D \rightarrow E + 2C$

$\therefore \Delta H = 150 \times 2 + (-125) - 350$

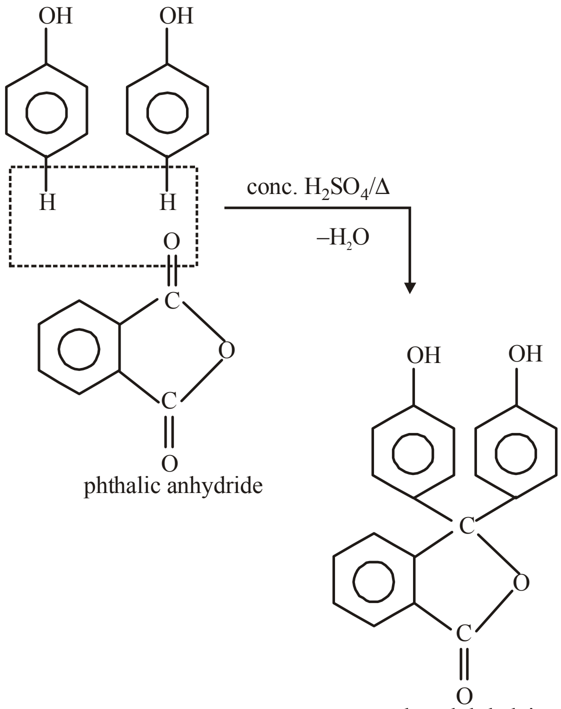
$= -175$ kJ/mol

76. (d) (a) **Benzoin condensation :** Heating ethanolic solution with strong alkali like KCN or NaCN, benzoin is obtained.



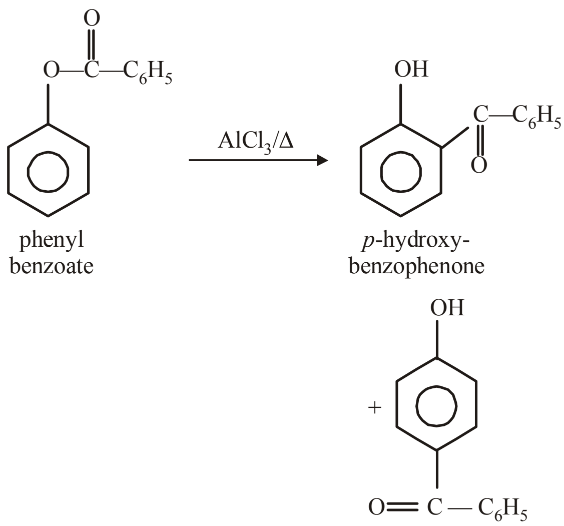
benzoin

(b) **Formation of phenolphthalein** phenol is treated with phthalic anhydride in the presence of conc. H_2SO_4, it gives phenolphthalein, an indicator.



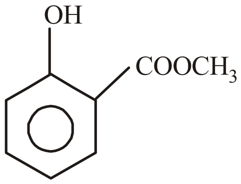
phenolphthalein

(c) **Fries rearrangement** Phenyl benzoate heated with anhydrous $AlCl_3$ in the presence of inert solvent gives *ortho*–and Para–hydroxybenzophenone. In this rearrangement, there is only a benzoyl group migration from the phenolic oxygen to an *ortho*–and *para*–position.

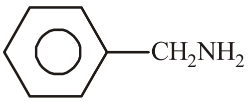


p-hydroxy-benzophenone

(d) Methylsalicylate



(A chief constituent of oil of wintergreen)

77. (b) 

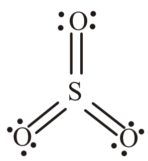
Compound is most basic due to localised lone pair of electrons on nitrogen atom While in other compounds, because of resonance, the lone pair of electrons on nitrogen atom gets delocalised over benzene ring and thus is less easily available for donation.

78. (d) Formal charges help in selection of the lowest energy structure from a number of possible Lewis structures for a given species. Generally the lowest energy structure is the one with the smallest formal charges on the atoms.

Formal charge on an atom

= total no. of valence electrons – non -bonding electrons – $\frac{1}{2}$ × bonding electrons.

For Lewis structure of SO_3



Formal charge on S atom

$= 6 - 0 - \frac{1}{2} \times 12 = 0$

Formal charge on three O atoms

$= 6 - 4 - \frac{1}{2} \times 4 = 0$

79. (a) IE_1 of Na = – Electron gain enthalpy of Na^+ ion = –5.1eV.

80. (a) For zero order reaction,

Rate = k [Reactants]°

∴ Rate = k

and unit of k = mol L^{-1} s^{-1}

PART - III (MATHEMATICS)

81. (a) $\frac{dy}{dx} + \frac{2yx}{1+x^2} = \frac{1}{(1+x^2)^2}$

which is a linear differential equation.

Here, $P = \frac{2x}{1+x^2}, Q = \frac{1}{(1+x^2)^2}$

Now, IF = $e^{\int P\,dx}$

$= e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = (1+x^2)$

∴ Solution of differential equation is

$y.(1+x^2) = \int \frac{1}{(1+x^2)^2}.(1+x^2)dx + C$

$\Rightarrow y(1+x^2) = \int \frac{1}{1+x^2}dx + C$

$\Rightarrow y(1+x^2) = \tan^{-1}x + C$

82. (b) $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + xyz\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$

$\Rightarrow (1+xyz)\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} = 0$

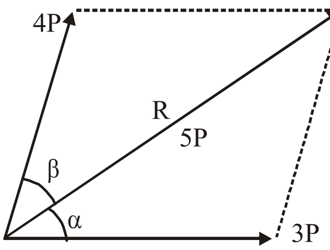
$\Rightarrow (1+xyz)[x(y^2-z^2) - y(x^2-z^2) + z(x^2-y^2)] = 0$

$\Rightarrow (1+xyz)(x-y)(y-z)(z-x) = 0$

$\Rightarrow 1+xyz = 0 \Rightarrow xyz = -1$

83. (c) $P(A \cup B) = 0.6$ and $P(A \cap B) = 0.2$
we know that
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$\Rightarrow 0.6 = P(A) + P(B) - 0.2$
$\Rightarrow P(A) + P(B) = 0.8$
$\Rightarrow 1 - P(\bar{A}) + 1 - P(\bar{B}) = 0.8$
$\Rightarrow -[P(\bar{A}) + P(\bar{B})] = 0.8 - 2$
$\Rightarrow P(\bar{A}) + P(\bar{B}) = 1.2$

84. (b) $Q = \dfrac{R\sin\alpha}{\sin(\alpha+\beta)}$
Also, $(5P)^2 = (4P)^2 + (3P)^2 + 2(4P)(3P)\cos(\alpha+\beta)$
$\Rightarrow 25P^2 = 16p^2 + 9P^2 + 24P^2\cos(\alpha+\beta)$
$\Rightarrow 24P^2\cos(\alpha+\beta) = 0$
$\Rightarrow \cos(\alpha+\beta) = 0 = \cos 90°$
$\Rightarrow \alpha + \beta = 90°$



Now, $4P = \dfrac{5P\sin\alpha}{\sin 90°}$
$\Rightarrow \sin\alpha = \dfrac{4}{5}$
$\Rightarrow \alpha = \sin^{-1}\left(\dfrac{4}{5}\right)$

85. (b) $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$
$\Rightarrow \tan^{-1}\left(\dfrac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}(2\operatorname{cosec} x)$
$\Rightarrow \dfrac{2\cos x}{1-\cos^2 x} = 2\operatorname{cosec} x$
$\Rightarrow \dfrac{2\cos x}{\sin^2 x} = 2\operatorname{cosec} x$
$\Rightarrow \sin x = \cos x \Rightarrow x = \dfrac{\pi}{4}$

86. (d) Given conditions are $a + x = 1$ and $ax = 0$.
These two conditions will be true, if $x = a'$.

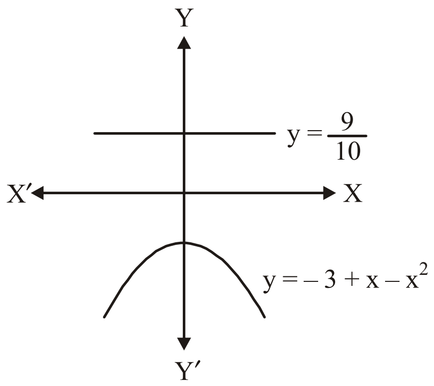
87. (a) $(x+y).(x+1) = x + x.y + y$
Replace '.' by '+', '+' by '.', '1' by '0', we get $(x.y) + (x.0) = x.(x+y).y$

88. (b) $f(x) = (x-1)(x-2)(x-3)$
$\Rightarrow f(1) = f(2) = f(3) = 0$
$\therefore$ f(x) is not one–one.
For each $y \in R$, there exists $x \in R$ such that $f(x) = y$.
$\therefore$ f is onto.
Note that if a continuous function has more than one roots, then the function is always many–one.

89. (c) Let z_1, z_2 and z_3 be affixes of points A, B and C, respectively. Since, z_1, z_2 and z_3 are in AP, therefore
$2z_2 = z_1 + z_3$
$\Rightarrow z_2 = \dfrac{z_1 + z_3}{2}$
So, B is the mid–point of the line AC.
$\Rightarrow$ A, B and C are collinear.
$\therefore$ z_1, z_2 and z_3 lie on a line.

90. (c) $x = 1 + a + a^2 + \ldots\infty = \dfrac{1}{1-a}$
$y = 1 + b + b^2 + \ldots\infty = \dfrac{1}{1-b}$
and $z = 1 + c + c^2 + \ldots\infty = \dfrac{1}{1-c}$
Since, a, b and c are in AP.
$\Rightarrow$ 1 – a, 1 – b and 1 – c are also in AP.
$\Rightarrow \dfrac{1}{1-a}, \dfrac{1}{1-b}$ and $\dfrac{1}{1-c}$ are in HP.
$\therefore$ x, y and z are in HP.
Note that if the common ratio of a GP is not less than 1, then we do not determined the sum of an infinite GP that series.

91. (a) Let $f(x) = -3 + x - x^2$
Then, $f(x) < 0$ for all x because coefficient of $x^2 < 0$ and disc < 0. Thus, LHS of the given equation is always positive whereas the RHS is always less than zero.
Hence, the given equation has no solution.
Alternate Solution :
Given, equation is
$\dfrac{9}{10} = -3 + x - x^2$



Let $y=\frac{9}{10}$, therefore

$y=-3+x-x^2$

$y=-\left[x^2-x+\frac{1}{4}\right]-3+\frac{1}{4}$

$\Rightarrow y+\frac{11}{4}=-\left(x-\frac{1}{2}\right)^2$

It is clear from the graph that two curves do not intersect. Hence, no solution exists.

92. (c) The centre of the required circle lies at the intersection of $2x-3y-5=0$ and $3x-4y-7=0$. Thus, the coordinates of the centre are $(1,-1)$.

Let r be the radius of the circle.

$\pi r^2=154$

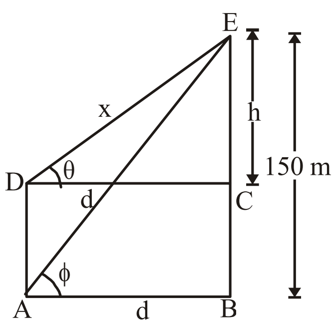
$\Rightarrow \frac{22}{7}r^2=154 \Rightarrow r=7$

Hence, the equation of required circle is

$(x-1)^2+(y+1)^2=7^2$

$\Rightarrow x^2+y^2-2x+2y-47=0$

93. (d) Given : $\tan\theta=\frac{4}{3}$ and $\tan\phi=\frac{5}{2}$



In ΔABE,

$\tan\phi=\frac{150}{d}$

$\Rightarrow d=150\cot\phi$

$=150\times\frac{2}{5}=60\,m$

In ΔDCE,

$\tan\theta=\frac{h}{d}$

$\Rightarrow \frac{4}{3}=\frac{h}{d} \Rightarrow h=\frac{4}{3}\times 60 \Rightarrow h=80\,m$

Now in ΔDCE,

$DE^2=DC^2+CE^2$

$\Rightarrow x^2=60^2+80^2=10000$

$\Rightarrow x=100\,m$

94. (a) Given equation $x^2+px+q=0$ has roots α and α^2.

$\Rightarrow$ Sum $=\alpha+\alpha^2=-p$ and Product $=\alpha^3=q$

$\Rightarrow \alpha(\alpha+1)=-p$

$\Rightarrow \alpha^3[\alpha^3+1+3\alpha(\alpha+1)]=-p^3$

$\Rightarrow q(q+1-3p)=-p^3$

$\Rightarrow p^3-(3p-1)q+q^2=0$

95. (c) $\sum_{m=0}^{100} {}^{100}C_m(x-3)^{100-m}.2^m$

Above expansion can be rewritten as

$[(x-3)+2]^{100}=(x-1)^{100}=(1-x)^{100}$

$\therefore x^{53}$ will occur in T_{54}.

$\Rightarrow T_{54}={}^{100}C_{53}(-x)^{53}$

$\therefore$ Required coefficient is $-{}^{100}C_{53}$.

96. (b) Equation of family of concentric circles to the circle $x^2+y^2+6x+8y-5=0$ is

$x^2+y^2+6x+8y+\lambda=0$

which is similar to

$x^2+y^2+2gx+2fy+c=0$

Thus, the point $(-3,2)$ lies on the cirlce

$x^2+y^2+6x+8y+c=0$

$\therefore (-3)^2+(2)^2+6(-3)+8(2)+c=0$

$\Rightarrow 9+4-18+16+c=0 \Rightarrow c=-11$

97. (a) $\mathbf{a}=\mathbf{i}+\mathbf{j}+\mathbf{k}$, $b=\mathbf{i}+3\mathbf{j}+5\mathbf{k}$

and $\mathbf{c}=7\mathbf{i}+9\mathbf{j}+11\mathbf{k}$

Let $\mathbf{A}=\mathbf{a}+\mathbf{b}$

$=(\mathbf{i}+\mathbf{j}+\mathbf{k})+(\mathbf{i}+3\mathbf{j}+5\mathbf{k})$

$=2\mathbf{i}+4\mathbf{j}+6\mathbf{k}$

and $\mathbf{B}=\mathbf{b}+\mathbf{c}$

$=(\mathbf{i}+3\mathbf{j}+5\mathbf{k})+(7\mathbf{i}+9\mathbf{j}+11\mathbf{k})$

$=8\mathbf{i}+12\mathbf{j}+16\mathbf{k}$

$\therefore$ Area of parallelogram

$= \frac{1}{2}|\mathbf{A} \times \mathbf{B}|$

($\because$ A and B are diagonals)

$$= \frac{1}{2}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 4 & 6 \\ 8 & 12 & 16 \end{vmatrix}$$

$$= \frac{1}{2}|\mathbf{i}(64-72) - \mathbf{j}(32-48) + \mathbf{k}(24-32)|$$

$$= \frac{1}{2}|-8\mathbf{i} + 16\mathbf{j} - 8\mathbf{k}|$$

$$= \sqrt{(-4)^2 + (8)^2 + (-4)^2}$$

$$= \sqrt{96} = 4\sqrt{6} \text{ sq units}$$

98. (a) We know that, $\text{tr}(A) = \sum_{i=1}^{n} a_{ii}$

$$\therefore \text{ If } A = \begin{bmatrix} 1 & -5 & 7 \\ 0 & 7 & 9 \\ 11 & 8 & 9 \end{bmatrix}, \text{ then}$$

$\text{tr}(A) = 1 + 7 + 9 = 17$

99. (a) Given, $\begin{vmatrix} \cos\alpha & -\sin\alpha & 1 \\ \sin\alpha & \cos\alpha & 1 \\ \cos(\alpha+\beta) & -\sin(\alpha+\beta) & 1 \end{vmatrix}$

[Applying $R_3 \to R_3 - R_1(\cos\beta) + R_2(\sin\beta)$]

$$= \begin{vmatrix} \cos\alpha & -\sin\alpha & 1 \\ \sin\alpha & \cos\alpha & 1 \\ 0 & 0 & 1+\sin\beta-\cos\beta \end{vmatrix}$$

$= (1 + \sin\beta - \cos\beta)(\cos^2\alpha + \sin^2\alpha)$

$= 1 + \sin\beta - \cos\beta$, which is independent of α.

100. (d) $4\sin^2 x - 12\sin x + 7$

$= 4(\sin^2 x - 3\sin x) + 7$

$$= 4\left[\left(\sin x - \frac{3}{2}\right)^2 - \frac{9}{4}\right] + 7$$

$$= 4\left(\sin x - \frac{3}{2}\right)^2 - 9 + 7$$

$$= 4\left(\sin x - \frac{3}{2}\right)^2 - 2$$

We know that, $-1 \le \sin x \le 1$

$$\Rightarrow -\frac{5}{2} \le \sin x - \frac{3}{2} \le -\frac{1}{2}$$

$$\Rightarrow \frac{1}{4} \le \left(\sin x - \frac{3}{2}\right)^2 \le \frac{25}{4}$$

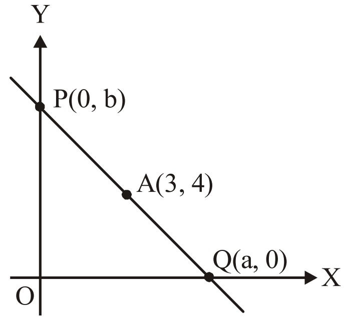
$$\Rightarrow 1 \le 4\left(\sin x - \frac{3}{2}\right)^2 \le 25$$

$$\Rightarrow -1 \le 4\left(\sin x - \frac{3}{2}\right)^2 - 2 \le 23$$

101. (b) A is mid point of line PQ.

$$\therefore 3 = \frac{a+0}{2} \Rightarrow a = 6$$

$$\text{and } 4 = \frac{0+b}{2} \Rightarrow b = 8$$



Thus, equation of line is

$$\frac{x}{6} + \frac{y}{8} = 1$$

$\Rightarrow 4x + 3y = 24$

102. (d) The tangent at (1, 7) to the curve $x^2 = y - 6$ is

$$x = \frac{1}{2}(y+7) - 6$$

$\Rightarrow 2x = y + 7 - 12$

$\Rightarrow y = 2x + 5$

which is also tangent to the circle

$x^2 + y^2 + 16x + 12y + c = 0$

i.e., $x^2 + (2x+5)^2 + 16x + 12(2x+5) + c = 0$

$\Rightarrow 5x^2 + 60x + 85 + c = 0$, which must have equal roots.

Let α and β are the roots of the equation.

Then $\alpha + \beta = -12 \Rightarrow \alpha = -6$

($\because \alpha = \beta$)

$\therefore x = -6, y = 2x + 5 = -7$

$\therefore$ Point of contact is (–6, –7).

103. (c) The intersection point of lines $x - 2y = 1$ and $x + 3y = 2$ is $\left(\frac{7}{5}, \frac{1}{5}\right)$

Since, required line is parallel to $3x + 4y = 0$.

Therefore, the slope of required line is $-\frac{3}{4}$.

$\therefore$ Equation of required line which passes through $\left(\frac{7}{5}, \frac{1}{5}\right)$ is given by

$$y - \frac{1}{5} = -\frac{3}{4}\left(x - \frac{7}{5}\right)$$

$$\Rightarrow \frac{3x}{4} + y = \frac{21}{20} + \frac{1}{5}$$

$$\Rightarrow 3x + 4y - 5 = 0$$

104. (c) Let $I = \int \frac{dx}{\sin x - \cos x + \sqrt{2}}$

$$= \int \frac{dx}{\sqrt{2}\left(\sin x \sin\frac{\pi}{4} - \cos x \cos\frac{\pi}{4} + 1\right)}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{1 - \cos\left(x + \frac{\pi}{4}\right)}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{2\sin^2\left(\frac{x}{2} + \frac{\pi}{8}\right)}$$

$$= \frac{-1}{\sqrt{2}} \int \operatorname{cosec}^2\left(\frac{x}{2} + \frac{\pi}{8}\right) dx$$

$$= \frac{-1}{2\sqrt{2}} \cdot \frac{-\cot\left(\frac{x}{2} + \frac{\pi}{8}\right)}{\frac{1}{2}} + C$$

$$= \frac{1}{\sqrt{2}} \cot\left(\frac{x}{2} + \frac{\pi}{8}\right) + C$$

105. (b) Let $I = \int_0^1 \sqrt{\frac{1-x}{1+x}}\,dx$

$$= \int_0^1 \frac{1-x}{\sqrt{1-x^2}}\,dx$$

$$= \int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx - \int_0^1 \frac{x}{\sqrt{1+x^2}}\,dx$$

$$= [\sin^{-1} x]_0^1 - \int_0^1 \frac{x}{\sqrt{1-x^2}}\,dx$$

Put $t^2 = 1 - x^2 \Rightarrow 2t\,dt = -2x\,dx$

$\Rightarrow t\,dt = -x\,dx$

$$\therefore I = (\sin^{-1} 1 - \sin^{-1} 0) + \int_1^0 \frac{t}{t}\,dt$$

$$= \frac{\pi}{2} + [t]_1^0 = \frac{\pi}{2} - 1$$

106. (c) Let $I = \int_0^1 x\left|x - \frac{1}{2}\right| dx$

$$= -\int_0^{1/2} x\left(x - \frac{1}{2}\right) dx + \int_{1/2}^1 x\left(x - \frac{1}{2}\right) dx$$

$$= \int_0^{1/2} \left(\frac{x}{2} - x^2\right) dx + \int_{1/2}^1 x\left(x^2 - \frac{x}{2}\right) dx$$

$$= \left[\frac{x^2}{4} - \frac{x^3}{3}\right]_0^{1/2} + \left[\frac{x^3}{3} - \frac{x^2}{4}\right]_{1/2}^1$$

$$= \left(\frac{1}{16} - \frac{1}{24}\right) + \left(\frac{1}{3} - \frac{1}{4} - \frac{1}{24} + \frac{1}{16}\right)$$

$$= \left(\frac{6-4}{96}\right) + \left(\frac{32-24-4+6}{96}\right)$$

$$= \frac{12}{96} = \frac{1}{8}$$

107. (b) Let the equation of the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

It is given that it passes through (7, 0) and (0, –5).

Therefore, $a^2 = 49$ and $b^2 = 25$

The eccentricity of the ellipse is given by

$$e=\sqrt{1-\frac{b^2}{a^2}}$$

$$=\sqrt{1-\frac{25}{49}}=\sqrt{\frac{24}{49}}=\frac{2\sqrt{6}}{7}$$

108. (c) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ...(i)

On differentiating w.r.t. x, we get

$$\frac{2x}{a^2}+\frac{2y}{b^2}\cdot\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}=-\frac{xb^2}{a^2y} \text{ and}$$

$x^2 - y^2 = c^2$

On differentiating w.r.t. x, we get

$$2x-2y\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}=\frac{x}{y}$$

The two curves will cut at right angles, if

$$\left(\frac{dy}{dx}\right)_{c_1}\times\left(\frac{dy}{dx}\right)_{c_2}=-1$$

$$\Rightarrow -\frac{b^2x}{a^2y}\cdot\frac{x}{y}=-1$$

$$\Rightarrow \frac{x^2}{a^2}=\frac{y^2}{b^2}$$

$$\Rightarrow \frac{x^2}{a^2}=\frac{y^2}{b^2}=\frac{1}{2}$$

[using eq. (i)]

On substituting these values in $x^2 - y^2 = c^2$, we get

$$\frac{a^2}{2}-\frac{b^2}{2}=c^2$$

$\Rightarrow a^2 - b^2 = 2c^2$

109. (c) Let P (x, y) be any point on the conic. Then,

$$\sqrt{(x-1)^2+(y+1)^2}=\sqrt{2}\left(\frac{x-y+1}{\sqrt{2}}\right)$$

$\Rightarrow (x - 1)^2 + (y + 1)^2 = (x - y + 1)^2$

$\Rightarrow 2xy - 4x + 4y + 1 = 0$

110. (b) Required numbers

$$=5!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right]=44$$

Note that if r $(0 \le r \le n)$ objects occupy the original places and none of the remaining (n – r) objects occupies its original places then the number of such arrangements $= {}^nC_r\cdot(n-r)!$

$$\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+....+(-1)^{n-2}\frac{1}{(n-r)!}\right]$$

111. (b) $T_n=\frac{1^2+2^2+3^2+...+n^2}{n!}$

$$=\frac{\sum n^2}{n!}=\frac{n(n+1)(2n+1)}{6n!}$$

$$=\frac{1}{6}\left(\frac{2n^3+3n^2+n}{n!}\right)$$

$$=\frac{1}{6}\left(2.\frac{n^3}{n!}+\frac{3n^2}{n!}+\frac{n}{n!}\right)$$

$\therefore$ Sum of the series

$$=\frac{1}{6}\left(2\sum_{n=1}^{\infty}\frac{n^3}{n!}+3\sum_{n=1}^{\infty}\frac{n^2}{n!}+\sum_{n=1}^{\infty}\frac{n}{n!}\right)$$

$$=\frac{1}{6}(2\times 5e+3\times 2e+e)$$

$$=\frac{1}{6}(10e+6e+e)=\frac{17}{6}e$$

112. (b) $\log_a(1+x) = \log_e(1+x)\log_a e$

$= \log_a e\left[\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}\right]$

So, the coefficient of x^n in $\log_a(1+x)$ is

$\frac{(-1)^{n-1}}{n}\log_a e.$

113. (b) Let the equation of the required plane be

$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1.$

This meets the coordinate axes at A, B and C, the coordinates of the centroid of ΔABC

are $\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)$

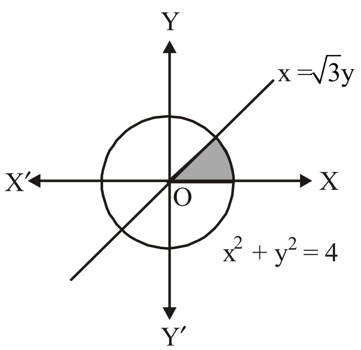
$\therefore \frac{a}{3}=1, \frac{b}{3}=2, \frac{c}{3}=3$

$\Rightarrow a=3, b=6, c=9$

Hence, the equation of the plane is

$\frac{x}{3}+\frac{y}{6}+\frac{z}{9}=1$

114. (c) Required area



$= \int_0^1 (x_2 - x_1)dy$

$= \int_0^1 (\sqrt{4-y^2}-\sqrt{3}y)dy$

$= \left[\frac{1}{2}y\sqrt{4-y^2}+\frac{1}{2}(4)\sin^{-1}\frac{y}{2}-\frac{\sqrt{3}y^2}{2}\right]_0^1$

$= \frac{\sqrt{3}}{2}+2\sin^{-1}\left(\frac{1}{2}\right)-\frac{\sqrt{3}}{2}-2\sin^{-1}0$

$= \frac{\sqrt{3}}{2}+2\left(\frac{\pi}{6}\right)-\frac{\sqrt{3}}{2}=\frac{\pi}{3}$ sq units

115. (b) Let $y = \lim_{x\to\infty}\left(\frac{\pi}{2}-\tan^{-1}x\right)$

Taking log on both sides, we get

$\log y = \lim_{x\to\infty}\frac{1}{x}\log\left(\frac{\pi}{2}-\tan^{-1}x\right)$ form

$\left(\text{form}\frac{\infty}{\infty}\right)$

$= \lim_{x\to\infty}\frac{\left(-\frac{1}{1+x^2}\right)}{\frac{\pi}{2}-\tan^{-1}x}$

(using L 'Hospitals' rule)

$= \lim_{x\to\infty}\frac{\frac{2x}{(1+x^2)^2}}{-\left(\frac{1}{1+x^2}\right)}$

(using L′ Hospital's rule)

$= \lim_{x\to\infty}\frac{-2x}{1+x^2}=0 \Rightarrow y=e^{\circ}=1$

116. (c) f(x) is continuous at $x=\frac{\pi}{2}$.

So, $\lim_{x\to\frac{\pi^-}{2}} f(x) = \lim_{x\to\frac{\pi^+}{2}} f(x)$

$\Rightarrow m\frac{\pi}{2}+1=\sin\frac{\pi}{2}+n$

$\Rightarrow m\frac{\pi}{2}+1=1+n \Rightarrow \frac{m\pi}{2}=n$

117. (c) $f(x)=\dfrac{\sqrt{4-x^2}}{\sin^{-1}(2-x)}$

$\sqrt{4-x^2}$ is defined for $4-x^2\ge 0$.

$\Rightarrow x^2\le 4$

$\Rightarrow -2\le x\le 2$

and $\sin^{-1}(2-x)$ is defined for $-1\le 2-x\le 1$

$\Rightarrow -3\le -x\le -1$

$\Rightarrow 1\le x\le 3$

Also, $\sin^{-1}(2-x)=0$ for $x=2$

$\therefore$ Domain of $f(x)=[-2,2]\cap[1,3]-\{2\}$

$=[1,2)$

118. (c) $(1+y^2)\,dx+(1+x^2)\,dy=0$

$\Rightarrow \dfrac{dx}{1+x^2}+\dfrac{dy}{1+y^2}=0$

On integrating, we get

$\tan^{-1}x+\tan^{-1}y=\tan^{-1}C$

$\Rightarrow \dfrac{x+y}{1-xy}=C$

$\Rightarrow x+y=C(1-xy)$

119. (a) $\rho=\dfrac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}}$

$\Rightarrow \rho\left(\dfrac{d^2y}{dx^2}\right)=\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2}$

On squaring both sides, we get

$\rho^2\left(\dfrac{d^2y}{dx^2}\right)^2=\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^3$

Clearly, it is a second order differential equation of degree 2.

Note that the higher order derivative is in the transcendental, then we do not determined the degree of that equation.

120. (b) Let $R=\{(a,b): a,b\in N, a-b=3\}$

$=[\{(n+3),n\}: n\in N]$

$=\{(4,1),(5,2),(6,3),\ldots\}$