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VITEEE 2017 Question Paper

Vellore Institute of Technology Engineering Entrance Examination

SOLVED PAPER (memory based)

VITEEE 2017

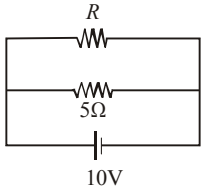
GENERAL INSTRUCTIONS

- This question paper contains total 125 questions divided into four parts :
  Part I : Physics Q. No - 1 to 40
  Part II : Chemistry Q. No - 41 to 80
  Part III : Mathematics Q. No - 81 to 120
  Part IV : English Q. No - 121 to 125
- All questions are multiple choice questions with four options, only one of them is correct.
- For each correct response, the candidate will get 1 mark.
- There is no negative marking for the wrong answer.
- The test is of 2½ hours duration.

PART - I (PHYSICS)

1. A 5000 kg rocket is set for vertical firing. The exhaust speed is 800 m/s. To give an initial upward acceleration of 20 m/s^2, the amount of gas ejected per second to supply the needed thrust will be (Take g = 10 m/s^2)
(a) 127.5 kg/s (b) 137.5 kg/s
(c) 155.5 kg/s (d) 187.5 kg/s

2. The power dissipated in the circuit shown in the figure is 30 Watts. The value of *R* is



(a) 20 Ω (b) 15 Ω
(c) 10 Ω (d) 30 Ω

3. If the kinetic energy of a moving particle is E, then the de-Broglie wavelength is

(a) $\lambda = h\sqrt{2mE}$ (b) $\lambda = \sqrt{\frac{2mE}{h}}$

(c) $\lambda = \frac{h}{\sqrt{2mE}}$ (d) $\lambda = \frac{hE}{\sqrt{2mE}}$

4. Two bodies A and B having masses in the ratio of 3 : 1 possess the same kinetic energy. The ratio of linear momentum of B to A is
(a) 1 : 3 (b) 3 : 1
(c) $1:\sqrt{3}$ (d) $\sqrt{3}:1$

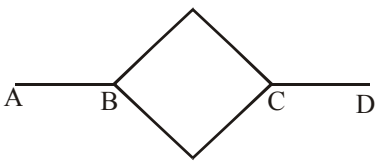
5. In which sequence the radioactive radiations are emitted in the following nuclear reaction?

$${}_ZX^A \longrightarrow {}_{Z+1}Y^A \longrightarrow {}_{Z-1}K^{A-4} \longrightarrow {}_{Z-1}K^{A-4}$$

(a) γ, α, β (b) α, β, γ
(c) β, γ, α (d) β, α, γ

6. Which of the following does not support the wave nature of light?
(a) Interference (b) Diffraction
(c) Polarisation
(d) Photoelectric effect

7. Six identical conducting rods are joined as shown in figure. Points A and D are maintained at 200°C and 20°C respectively. The temperature of junction B will be



(a) 120°C (b) 100°C
(c) 140°C (d) 80°C

8. A hydrogen atom is in ground state. Then to get six lines in emission spectrum, wavelength of incident radiation should be
(a) 800 Å (b) 825 Å
(c) 975 Å (d) 1025 Å

9. A conducting circular loop of radius *r* carries a constant current *i*. It is placed in a uniform magnetic field $\vec{B}_0$ such that $\vec{B}_0$ is perpendicular to the plane of the loop. The magnetic force acting on the loop is
(a) $ir B_0$ (b) $2\pi ir B_0$
(c) zero (d) $\pi ir B_0$

10. A vessel of depth 2d cm is half filled with a liquid of refractive index μ_1 and the upper half with a liquid of refractive index μ_2. The apparent depth of the vessel seen perpendicularly is

(a) $\left(\frac{\mu_1\mu_2}{\mu_1+\mu_2}\right)d$ (b) $\left(\frac{1}{\mu_1}+\frac{1}{\mu_2}\right)d$

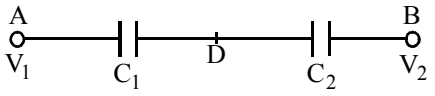
(c) $\left(\frac{1}{\mu_1}+\frac{1}{\mu_2}\right)2d$ (d) $\left(\frac{1}{\mu_1\mu_2}\right)2d$

11. A smooth sphere of mass M moving with velocity u directly collides elastically with another sphere of mass m at rest. After collision, their final velocities are V and v respectively. The value of v is

(a) $\frac{2uM}{m}$ (b) $\frac{2um}{M}$

(c) $\frac{2u}{1+\frac{m}{M}}$ (d) $\frac{2u}{1+\frac{M}{m}}$

12. Two capacitors C_1 and C_2 in a circuit are joined as shown in figure. The potentials of points A and B are V_1 and V_2 respectively. Then the potential of point D will be



(a) $\frac{(V_1+V_2)}{2}$ (b) $\frac{C_2V_1+C_1V_2}{C_1+C_2}$

(c) $\frac{C_1V_1+C_2V_2}{C_1+C_2}$ (d) $\frac{C_2V_1+C_1V_2}{C_1+C_2}$

13. Light of wavelength 500 nm is incident on a metal with work function 2.28 eV. The de Broglie wavelength of the emitted electron is:
(a) $< 2.8 \times 10^{-9}\, m$ (b) $\geq 2.8 \times 10^{-9}\, m$
(c) $\leq 2.8 \times 10^{-12}\, m$ (d) $< 2.8 \times 10^{-10}\, m$

14. Kerosene oil rises up in a wick of a lantern because of
(a) diffusion of the oil through the wick
(b) capillary action
(c) buoyant force of air
(d) the gravitational pull of the wick

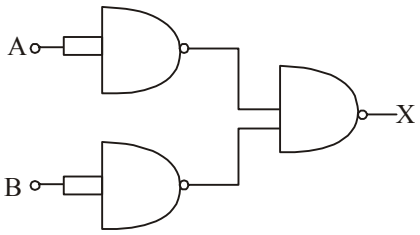
15. The current in a coil of L = 40 mH is to be increased uniformly from 1A to 11A in 4 milli sec. The induced e.m.f. will be
(a) 100 V (b) 0.4 V
(c) 440 V (d) 40 V

16. An alternating voltage of 220 V, 50 Hz frequency is applied across a capacitor of capacitance 2 μF. The impedence of the circuit is

(a) $\frac{\pi}{5000}$ (b) $\frac{1000}{\pi}$

(c) 500π (d) $\frac{5000}{\pi}$

17. The combination of gates shown below yields



(a) OR gate (b) NOT gate
(c) XOR gate (d) NAND gate

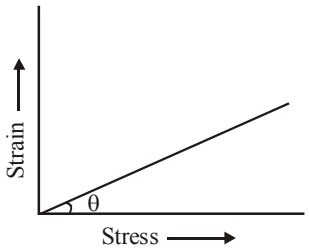
18. A hollow insulated conduction sphere is given a positive charge of 10 μC. What will be the electric field at the centre of the sphere if its radius is 2 metres?
(a) Zero (b) $5\ \mu Cm^{-2}$
(c) $20\ \mu Cm^{-2}$ (d) $8\ \mu Cm^{-2}$

19. Two mercury drops (each of radius r) merge to form a bigger drop. The surface energy of the bigger drop, if T is the surface tension, is
(a) $2^{5/3}\pi r^2T$ (b) $4\pi r^2T$
(c) $2\pi r^2T$ (d) $2^{8/3}\pi r^2T$

20. Resistances 1 Ω, 2 Ω and 3 Ω are connected to form a triangle. If a 1.5 V cell of negligible internal resistance is connected across the 3 Ω resistor, the current flowing through this resistor will be
(a) 0.25 A (b) 0.5 A
(c) 1.0 A (d) 1.5 A

21. A current carrying coil is subjected to a uniform magnetic field. The coil will orient so that its plane becomes
(a) inclined at 45° to the magnetic field
(b) inclined at any arbitrary angle to the magnetic field
(c) parallel to the magnetic field
(d) perpendicular to the magnetic field

22. The value of tan (90° – θ) in the graph gives



(a) Young's modulus of elasticity
(b) compressibility
(c) shear strain
(d) tensile strength

23. An electron makes a transition from an excited state to the ground state of a hydrogen - like atom. Then
(a) kinetic energy decreases, potential energy increases but total energy remains same
(b) kinetic energy and total energy decrease but potential energy increases
(c) its kinetic energy increases but potential energy and total energy decrease
(d) kinetic energy, potential energy and total energy decrease

24. An A.C. source is connected to a resistive circuit. Which of the following is true?
(a) Current leads ahead of voltage in phase
(b) Current lags behind voltage in phase
(c) Current and voltage are in same phase
(d) Any of the above may be true depending upon the value of resistance.

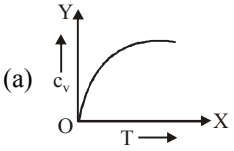
25. A milli voltmeter of 25 milli volt range is to be converted into an ammeter of 25 ampere range. The value (in ohm) of necessary shunt will be
(a) 0.001 (b) 0.01
(c) 1 (d) 0.05

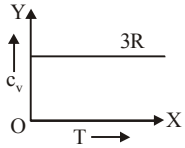
26. In young's double-slit experiment, the intensity of light at a point on the screen where the path difference is λ is I, λ being the wavelength of light used. The intensity at a point where the path difference is $\frac{\lambda}{4}$ will be
(a) $\frac{I}{4}$ (b) $\frac{I}{2}$
(c) I (d) zero

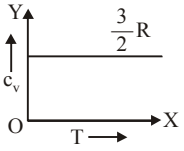
27. Which of the following is a self adjusting force?
(a) Static friction (b) Limiting friction
(c) Dynamic friction (d) Sliding friction

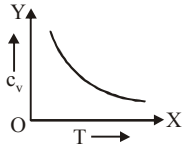
28. Which of the following are not electromagnetic waves?
(a) Cosmic rays (b) Gamma rays
(c) β-rays (d) X-rays

29. Graph of specific heat at constant volume for a monatomic gas is

(a) 

(b) 

(c) 

(d) 

30. A charge +q is at a distance L/2 above a square of side L. Then what is the flux linked with the surface?
(a) $\frac{q}{4\varepsilon_0}$ (b) $\frac{2q}{3\varepsilon_0}$
(c) $\frac{q}{6\varepsilon_0}$ (d) $\frac{6q}{\varepsilon_0}$

31. The potential energy of a system increases if work is done
(a) upon the system by a non conservative force
(b) by the system against a conservative force
(c) by the system against a non conservative force
(d) upon the system by a conservative force

32. Two capacitors when connected in series have a capacitance of 3 μF, and when connected in parallel have a capacitance of 16 μF. Their individual capacities are

(a) 1 μF, 2 μF (b) 6 μF, 2 μF
(c) 12 μF, 4 μF (d) 3 μF, 16 μF

33. Resonance frequency of LCR series a.c. circuit is f_0. Now the capacitance is made 4 times, then the new resonance frequency will become

(a) $f_0/4$ (b) $2f_0$
(c) f_0 (d) $f_0/2$.

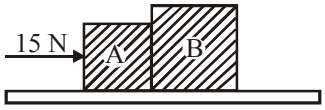
34. If the light is polarised by reflection, then the angle between reflected and refracted light is

(a) 180° (b) 90°
(c) 45° (d) 36°

35. The velocity of efflux of a liquid through an orifice in the bottom of the tank does not depend upon

(a) size of orifice
(b) height of liquid
(c) acceleration due to gravity
(d) density of liquid

36. On a smooth plane surface (figure) two block A and B are accelerated up by applying a force 15 N on A. If mass of B is twice that of A, the force on B is



(a) 30 N (b) 15 N
(c) 10 N (d) 5 N

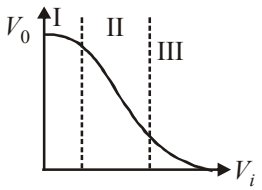
37. A potentiometer wire, 10 m long, has a resistance of 40Ω. It is connected in series with a resistance box and a 2 V storage cell. If the potential gradient along the wire 0.1 m is V/cm, the resistance unplugged in the box is

(a) 260 Ω (b) 760 Ω
(c) 960 Ω (d) 1060 Ω

38. A prism has a refracting angle of 60°. When placed in the position of minimum deviation, it produces a deviation of 30°. The angle of incidence is

(a) 30° (b) 45°
(c) 15° (d) 60°

39. Transfer characteristics [output voltage (V_0) vs input voltage (V_i)] for a base biased transistor in CE configuration is as shown in the figure. For using transistor as a switch, it is used



(a) in region (III)
(b) both in region (I) and (III)
(c) in region (II)
(d) in region (I)

40. A bar magnet of magnetic moment M, is placed in a magnetic field of induction B. The torque exerted on it is

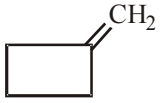
(a) $\vec{M}.\vec{B}$ (b) $-\vec{M}.\vec{B}$
(c) $\vec{M}\times\vec{B}$ (d) $-\vec{B}.\vec{M}$

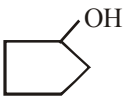
PART - II (CHEMISTRY)

41. Schottky defect in crystals is observed when

(a) unequal number of cations and anions are missing from the lattice
(b) equal number of cations and anions are missing from the lattice
(c) an ion leaves its normal site and occupies an interstitial site
(d) density of the crystal is increased

42. The cyclobutyl methylamine with nitrous acid gives

(a) 

(b) 

(c) 
(d) All of these

43. The exothermic formation of ClF_3 is represented by the equation :

$$Cl_{2(g)} + 3F_{2(g)} \rightleftharpoons 2ClF_{3(g)} ; \Delta H = -329\ kJ$$

Which of the following will increase the quantity of ClF_3 in an equilibrium mixture of Cl_2, F_2 and ClF_3 ?

(a) Adding F_2
(b) Increasing the volume of the container
(c) Removing Cl_2
(d) Increasing the temperature

44. For the reaction

$$2NO_{2(g)} \rightleftharpoons 2NO_{(g)} + O_{2(g)},$$

$(K_c = 1.8 \times 10^{-6}\ at\ 184°C)$ (R = 0.0831 kJ/ (mol. K)) When K_p and K_c are compared at 184°C, it is found that

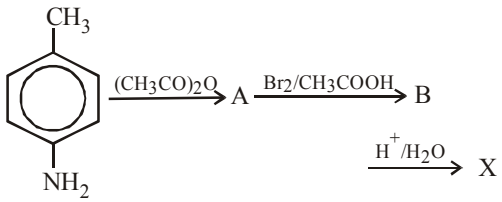
(a) Whether K_p is greater than, less than or equal to K_c depends upon the total gas pressure

(b) $K_p = K_c$

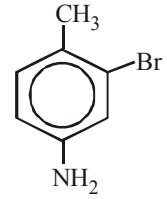
(c) K_p is less than K_c

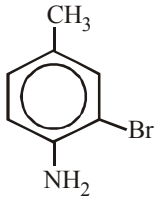
(d) K_p is greater than K_c

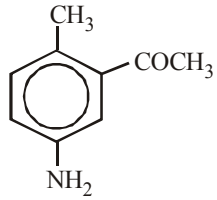
45.

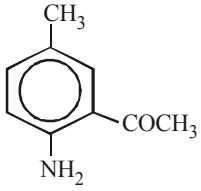


What is X?

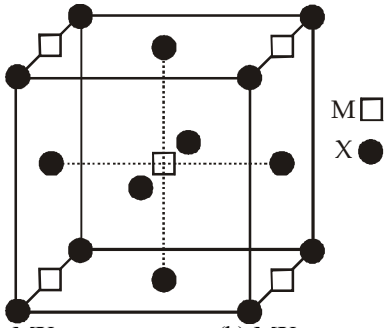
(a) 

(b) 

(c) 

(d) 

46. A compound M_pX_q has cubic close packing (ccp) arrangement of X. Its unit cell structure is shown below. The empirical formula of the compound is



(a) MX (b) MX_2

(c) M_2X (d) M_5X_{14}

47. What is Z in the following sequence of reactions?

$$\text{Phenol} \xrightarrow[\text{dust}]{\text{Zn}} X \xrightarrow[\text{Anhyd. } AlCl_3]{CH_3Cl} Y \xrightarrow[KMnO_4]{\text{Alkaline}} Z$$

(a) Benzene (b) Toluene

(c) Benzaldehyde (d) Benzoic acid

48. Which of the following oxy-acids has the maximum number of hydrogens directly attached to phosphorus?

(a) $H_4P_2O_7$ (b) H_3PO_2

(c) H_3PO_3 (d) H_3PO_4

49. The number of geometrical isomers of $CH_3CH{=}CH{-}CH{=}CH{-}CH{=}CHCl$ is

(a) 2 (b) 4

(c) 6 (d) 8

50. If '*a*' stands for the edge length of the cubic systems : simple cubic, body centred cubic and face centred cubic, then the ratio of radii of the spheres in these systems will be respectively,

(a) $\frac{1}{2}a : \frac{\sqrt{3}}{4}a : \frac{1}{2\sqrt{2}}a$

(b) $\frac{1}{2}a : \sqrt{3}a : \frac{1}{\sqrt{2}}a$

(c) $\frac{1}{2}a : \frac{\sqrt{3}}{2}a : \frac{\sqrt{3}}{2}a$

(d) $1a : \sqrt{3}a : \sqrt{2}a$

51. For a first order reaction A→P, the temperature (T) dependent rate constant (*k*) was found to follow the equation $\log k = -(2000)\frac{1}{T} + 6.0$. The pre-exponential factor A and the activation energy E_a, respectively, are

(a) $1.0 \times 10^6\ s^{-1}$ and 9.2 kJ mol^{-1}

(b) $6.0\ s^{-1}$ and 16.6 kJ mol^{-1}

(c) $1.0 \times 10^6\ s^{-1}$ and 16.6 kJ mol^{-1}

(d) $1.0 \times 10^6\ s^{-1}$ and 38.3 kJ mol^{-1}

52. 1-Propanol and 2-propanol can be distinguished by

(a) oxidation with alkaline $KMnO_4$ followed by reaction with Fehling solution

(b) oxidation with acidic dichromate followed by reaction with Fehling solution

(c) oxidation by heating with copper followed by reaction with Fehling solution

(d) oxidation with concentrated H_2SO_4 followed by reaction with Fehling solution

53. Which group contains coloured ions out of

1. Cu^{2+} 2. Ti^{4+}

3. Co^{2+} 4. Fe^{2+}

(a) 1, 2, 3, 4 (b) 1, 3, 4

(c) 2, 3 (d) 1, 2

54. The half life period of a first order chemical reaction is 6.93 minutes. The time required for the completion of 99% of the chemical reaction will be (log 2 = 0.301)

(a) 23.03 minutes (b) 46.06 minutes

(c) 460.6 minutes (d) 230.03 minutes

55. A mixture of benzaldehyde and formaldehyde on heating with aqueous NaOH solution gives

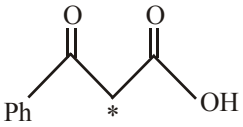
(a) benzyl alcohol and sodium formate

(b) sodium benzoate and methyl alcohol

(c) sodium benzoate and sodium formate

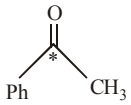
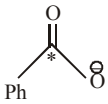
(d) benzyl alcohol and methyl alcohol

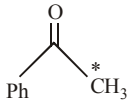
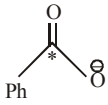
56. In the following reaction sequence, the correct structures of E, F and G are

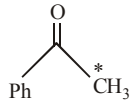
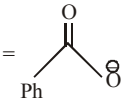


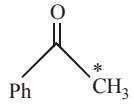
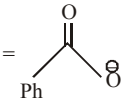
$\xrightarrow{\text{Heat}}[E]\xrightarrow[\text{NaOH}]{I_2}[F]+[G]$

[* implies ^{13}C labelled carbon)

(a) E=  F =  G = CHI_3

(b) E=  F =  G = CHI_3

(c) E=  F =  G = $\overset{*}{C}HI_3$

(d) E=  F =  G = $\overset{*}{C}H_3I$

57. Standard entropies of X_2, Y_2 and XY_3 are 60, 30 and 50 $JK^{-1}mol^{-1}$ respectively. For the reaction $\frac{1}{2}X_2+\frac{3}{2}Y_2 \rightleftharpoons XY_3$, $\Delta H = -30$ kJ to be at equilibrium, the temperature should be:

(a) 750 K (b) 1000 K

(c) 1250 K (d) 500 K

58. An organic compound (A) on reduction gives compound (B). (B) on treatment with $CHCl_3$ and alcoholic KOH gives (C). (C) on catalytic reduction gives N-methylaniline. The compound A is

(a) Methylamine (b) Nitromethane

(c) Aniline (d) Nitrobenzene

59. The standard reduction potential for Cu^{2+}/Cu is + 0.34. Calculate the reduction potential at pH = 14 for the above couple. (K_{sp} $Cu(OH)_2 = 1\times10^{-19}$)

(a) −0.22 V (b) +0.22 V

(c) −0.44 V (d) +0.44 V

60. A substance $C_4H_{10}O$ yields on oxidation a compound, C_4H_8O which gives an oxime and a positive iodoform test. The original substance on treatment with conc. H_2SO_4 gives C_4H_8. The structure of the compound is

(a) $CH_3CH_2CH_2CH_2OH$

(b) $CH_3CHOHCH_2CH_3$

(c) $(CH_3)_3COH$

(d) CH_3CH_2–O–CH_2CH_3

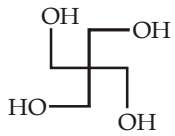
61. The emf of a particular voltaic cell with the cell reaction $Hg_2^{2+}+H_2 \rightleftharpoons 2Hg+2H^+$ is 0.65 V. The maximum electrical work of this cell when 0.5 g of H_2 is consumed.

(a) -3.12×10^4 J (b) -1.25×10^5 J

(c) 25.0×10^6 J (d) None

62. The number of aldol reaction(s) that occurs in the given transformation is :

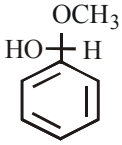
$CH_3CHO+4HCHO$

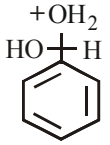
$\xrightarrow{\text{conc. aq. NaOH}}$ 

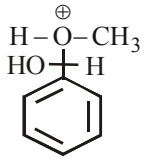
(a) 1 (b) 2

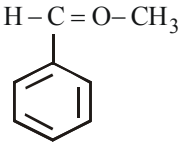
(c) 3 (d) 4

63. Which of the following is not intermediate in the acid catalysed reaction of benzaldehyde with 2 equivalent of methanol to give acetal ?

(a) 

(b) 

(c) 

(d) 

64. Iron crystallizes in several modifications. At about 911°C, the bcc 'α' form undergoes a trasition to fcc 'γ' form. If the distance between the two nearest neighbours is the same in the two forms at the transition temperature, the ratio of the density of iron in fcc form (ρ_2) to that of iron of bcc form (ρ_1) at the transition temperature

(a) $\frac{\rho_1}{\rho_2} = 0.918$ (b) $\frac{\rho_1}{\rho_2} = 0.718$

(c) $\frac{\rho_1}{\rho_2} = 0.518$ (d) $\frac{\rho_1}{\rho_2} = 0.318$

65. The half life of the first order reaction

$$CH_3.CHO\ (g) \longrightarrow CH_4\ (g) + CO\ (g)$$

If initial pressure of CH_3CHO (g) is 80 mm Hg and the total pressure at the end of 20 minutes is 120 mm Hg

(a) 80 min (b) 120 min
(c) 20 min (d) 40 min

66. A compound is soluble in conc. H_2SO_4. It does not decolourise bromine in carbon tetrachloride but is oxidised by chromic anhydride in aqueous sulphuric acid within two seconds, turning orange solution to blue, green and then opaque. The original compound is

(a) a primary alcohol (b) a tertiary alcohol
(c) an alkane (d) an ether

67. The values of Planck's constant is 6.63×10^{-34} Js. The velocity of light is 3.0×10^8 m s^{-1}. Which value is closest to the wavelength in nanometres of a quantum of light with frequency of 8×10^{15} s^{-1}?

(a) 5×10^{-18} (b) 4×10^1
(c) 3×10^7 (d) 2×10^{-25}

68. The number of stereoisomers possible for a compound of the molecular formula $CH_3 - CH = CH - CH(OH) - Me$ is:

(b) 2 (c) 4
(d) 6 (d) 3

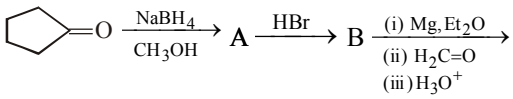
69. The optically active tartaric acid is named as D - (+) - tartaric acid because it has a positive

(a) optical rotation and is drived from D - glucose
(b) pH in organic solvent
(c) optical rotation and is derived from D - (+) - glyceraldehyde
(d) optical rotation when substituted by deuterium

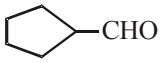
70. Consider the reaction : $N_2 + 3H_2 \rightarrow 2NH_3$ carried out at constant temperature and pressure. If ΔH and ΔU are the enthalpy and internal energy changes for the reaction, which of the following expressions is true ?

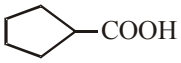
(a) $\Delta H > \Delta U$ (b) $\Delta H < \Delta U$
(c) $\Delta H = \Delta U$ (d) $\Delta H = 0$

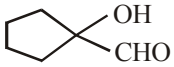
71. What is D in the following sequence of reactions ?



$$C \xrightarrow[CH_2Cl_2]{PCC} D$$

(a) 

(b) 

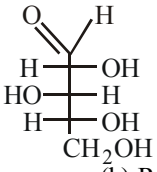
(c) 

(d) 

72. Knowing that the chemistry of lanthanoids(Ln) is dominated by its + 3 oxidation state, which of the following statements is incorrect?

(a) The ionic size of Ln (III) decrease in general with increasing atomic number
(b) Ln (III) compounds are generally colourless.
(c) Ln (III) hydroxide are mainly basic in character.
(d) Because of the large size of the Ln (III) ions the bonding in its compounds is predominantly ionic in character.

73. What is the R and S configuration for each stereogenic centre in this sugar from top to bottom ?

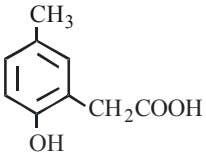


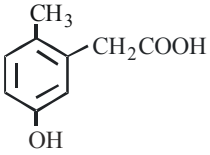
(a) R, R, S (b) R, S, S
(c) R, S, R (d) S, S, R

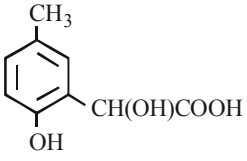
74. Saponification of coconut oil yields glycerol and
(a) palmitic acid (b) sodium palmitate
(c) oleic acid (d) stearic acid

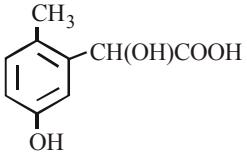
75. A certain reaction is non spontaneous at 298K. The entropy change during the reaction is 121 JK^{-1}. Is the reaction is endothermic or exothermic ? The minimum value of ΔH for the reaction is
(a) endothermic, $\Delta H = 36.06$ kJ
(b) exothermic, $\Delta H = -36.06$ kJ
(c) endothermic, $\Delta H = 60.12$ kJ
(d) exothermic, $\Delta H = -60.12$ kJ

76. p -cresol reacts with chloroform in alkaline medium to give the compound A which adds hydrogen cyanide to form, the compound B. The latter on acidic hydrolysis gives chiral carboxylic acid. The structure of the carboxylic acid is

(a) 

(b) 

(c) 

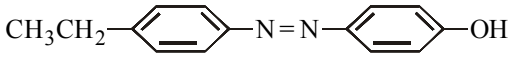
(d) 

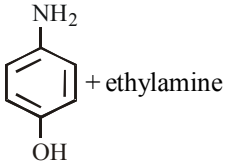
77. Which of the following has maximum number of lone pairs associated with Xe ?
(a) XeF_4 (b) XeF_6
(c) XeF_2 (d) XeO_3

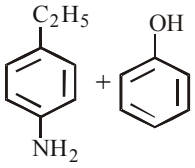
78. Which one of the following statements is not true regarding (+) Lactose ?
(a) On hydrolysis (+) Lactose gives equal amount of D(+) glucose and D(+) galactose.
(b) (+) Lactose is a β-glycoside formed by the union of a molecule of D(+) glucose and a molecule of D(+) galactose.
(c) (+) Lactose is a reducing sugar and does not exhibit mutarotation.
(d) (+) Lactose, $C_{12}H_{22}O_{11}$ contains 8-OH groups.

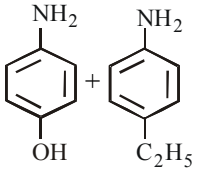
79. If one strand of DNA has the sequence ATGCTTGA, the sequence in the complimentary strand would be
(a) TACGAACT (b) TCCGAACT
(c) TACGTACT (d) TACGTAGT

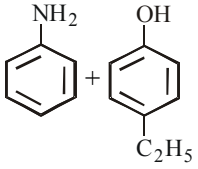
80. The starting reagents needed to make the azo compound shown below



(a) 

(b) 

(c) 

(d) 

PART - III (MATHEMATICS)

81. $\sin^{-1}(\sin 5) > x^2 - 4x$ holds if

(a) $x = 2 - \sqrt{9 - 2\pi}$

(b) $x = 2 + \sqrt{9 - 2\pi}$

(c) $x > 2 + \sqrt{9 - 2\pi}$

(d) $x \in (2 - \sqrt{9 - 2\pi}, 2 + \sqrt{9 - 2\pi})$

82. A value of c for which conclusion of Mean Value Theorem holds for the function $f(x) = \log_e x$ on the interval [1, 3] is

(a) $\log_3 e$ (b) $\log_e 3$

(c) $2 \log_3 e$ (d) $\frac{1}{2}\log_3 e$

83. Negation of the proposition : If we control population growth, we prosper

(a) If we do not control population growth, we prosper

(b) If we control population growth, we do not prosper

(c) We control population but we do not prosper

(d) We do not control population, but we prosper

84. The equation $z\bar{z} + (2 - 3i)z + (2 + 3i)\bar{z} + 4 = 0$ represents a circle of radius

(a) 2 (b) 3

(c) 4 (d) 6

85. The function $f(x) = \sin x - kx - c$, where k and c are constants, decreases always when

(a) $k > 1$ (b) $k \geq 1$

(c) $k < 1$ (d) $k \leq 1$

86. Equation $\frac{1}{r} = \frac{1}{8} + \frac{3}{8}\cos\theta$ represents

(a) A rectangular hyperbola

(b) A hyperbola

(c) An ellipse

(d) A parabola

87. The acceleration of a sphere falling through a liquid is $(30 - 3v)$ cm/s^2 where v is its speed in cm/s. The maximum possible velocity of the sphere and the time when it is achieved are

(a) 10 cm/s after 10 second

(b) 10 cm/s instantly

(c) 10 cm/s, will never be achieved

(d) 30 cm/s, after 30 second

88. A straight line parallel to the line $2x - y + 5 = 0$ is also a tangent to the curve $y^2 = 4x + 5$. Then the point of contact is

(a) (2, 1) (b) (–1, 1)

(c) (1, 3) (d) (3, 4)

89. Value of $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ is

(a) $\frac{\pi}{2}$ (b) $\frac{-\pi}{2}$

(c) $\frac{\pi}{4}$ (d) None of these

90. The range of the function $f(x) = \frac{1}{2 - \cos 3x}$ is

(a) $(-2, \infty)$ (b) $[-2, 3]$

(c) $\left[\frac{1}{3}, 1\right]$ (d) $\left(\frac{1}{2}, 1\right)$

91. The area bounded by $y - 1 = |x|$, $y = 0$ and $|x| = \frac{1}{2}$ will be :

(a) $\frac{3}{4}$ (b) $\frac{3}{2}$

(c) $\frac{5}{4}$ (d) None of these

92. The value of x obtained from the equation

$$\begin{vmatrix} x+\alpha & \beta & \gamma \\ \gamma & x+\beta & \alpha \\ \alpha & \beta & x+\gamma \end{vmatrix} = 0$$ will be

(a) 0 and $-(\alpha+\beta+\gamma)$

(b) 0 and $\alpha+\beta+\gamma$

(c) 1 and $(\alpha-\beta-\gamma)$

(d) 0 and $\alpha^2+\beta^2+\gamma^2$

93. The solution of the differential equation

$\log x \frac{dy}{dx} + \frac{y}{x} = \sin 2x$ is

(a) $y \log|x| = C - \frac{1}{2}\cos x$

(b) $y \log|x| = C + \frac{1}{2}\cos 2x$

(c) $y \log|x| = C - \frac{1}{2}\cos 2x$

(d) $xy \log|x| = C - \frac{1}{2}\cos 2x$

94. $\lim_{x\to\infty}\left(\frac{x^2}{3x-2} - \frac{x}{3}\right) =$

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{-2}{3}$ (d) $\frac{2}{9}$

95. If $((\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}))\cdot(\vec{a}\times\vec{d}) = 0$, then which of the following is always true ?

(a) $\vec{a},\vec{b},\vec{c},\vec{d}$ are necessarily coplanar

(b) either $\vec{a}$ or $\vec{d}$ must lie in the plane of $\vec{b}$ and $\vec{c}$

(c) either $\vec{b}$ or $\vec{c}$ must lie in the plane of $\vec{a}$ and $\vec{d}$

(d) either $\vec{a}$ or $\vec{b}$ must lie in the plane of $\vec{c}$ and $\vec{d}$

96. Let A be the centre of the circle $x^2+y^2-2x-4y-20=0$, and $B(1,7)$ and $D(4,-2)$ are points on the circle then, if tangents be drawn at B and D, which meet at C, then area of quadrilateral $ABCD$ is -

(a) 150 (b) 75

(c) 75/2 (d) None of these

97. $\int_0^1 [f(x)g''(x) - f''(x)g(x)]dx$ is equal to :

[Given $f(0)=g(0)=0$]

(a) $f(1)g(1) - f(1)g'(1)$

(b) $f(1)g'(1) + f'(1)g(1)$

(c) $f(1)g'(1) - f'(1)g(1)$

(d) none of these

98. If $z = \frac{7-i}{3-4i}$ then $z^{14} =$

(a) 2^7 (b) $2^7 i$

(c) $2^{14} i$ (d) $-2^7 i$

99. The difference between greatest and least value of $f(x) = 2\sin x + \sin 2x$, $x \in \left[0, \frac{3\pi}{2}\right]$ is –

(a) $\frac{3\sqrt{3}}{2}$ (b) $\frac{3\sqrt{3}}{2} - 2$

(c) $\frac{3\sqrt{3}}{2} + 2$ (d) None of these

100. A and B are two independent witnesses (i.e. there is no collision between them) in a case. The probability that A will speak the truth is x and the probability that B will speak the truth is y. A and B agree in a certain statement. The probability that the statement is true is

(a) $\frac{x-y}{x+y}$ (b) $\frac{xy}{1+x+y+xy}$

(c) $\frac{x-y}{1-x-y+2xy}$ (d) $\frac{xy}{1-x-y+2xy}$

101. A and B are events such that $P(A \cup B)=3/4$, $P(A \cap B)=1/4$, $P(\overline{A})=2/3$ then $P(\overline{A} \cap B)$ is

(a) 5/12 (b) 3/8

(c) 5/8 (d) 1/4

102. The line which passes through the origin and intersect the two lines

$$\frac{x-1}{2}=\frac{y+3}{4}=\frac{z-5}{3}, \frac{x-4}{2}=\frac{y+3}{3}=\frac{z-14}{4}, \text{ is}$$

(a) $\frac{x}{1}=\frac{y}{-3}=\frac{z}{5}$ (b) $\frac{x}{-1}=\frac{y}{3}=\frac{z}{5}$

(c) $\frac{x}{1}=\frac{y}{3}=\frac{z}{-5}$ (d) $\frac{x}{1}=\frac{y}{4}=\frac{z}{-5}$

103. If $u_n=\int_0^{\pi/4}\tan^n\theta\,d\theta$ then u_n+u_{n-2} is :

(a) $\frac{1}{n-1}$ (b) $\frac{1}{n+1}$

(c) $\frac{1}{2n-1}$ (d) $\frac{1}{2n+1}$

104. Ten different letters of an alphabet are given, words with five letters are formed from these given letters. Then the number of words which have at least one letter repeated is

(a) 69760 (b) 30240

(c) 99784 (d) None of these

105. The area bounded by $f(x)=x^2,\ 0\le x\le 1$, $g(x)=-x+2, 1\le x\le 2$ and x - axis is

(a) $\frac{3}{2}$ (b) $\frac{4}{3}$

(c) $\frac{8}{3}$ (d) None of these

106. The condition that the line $\frac{x}{p}+\frac{y}{q}=1$ be a normal to the parabola $y^2=4ax$ is]

(a) $p^3=2ap^2+aq^2$ (b) $p^3=2aq^2+ap^2$

(c) $q^3=2ap^2+aq^2$ (d) None of these

107. A random variable X has the probability distribution

X	1	2	3	4	5	6	7	8
p(X)	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the events E = {X is a prime number} and $F=\{X<4\}$, then $P(E\cup F)$ is

(a) 0.50 (b) 0.77

(c) 0.35 (d) 0.87

108. The value of $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{7}{8}$ is

(a) $\tan^{-1}\frac{7}{8}$ (b) $\cot^{-1}15$

(c) $\tan^{-1}15$ (d) $\tan^{-1}\frac{15}{24}$

109. The parabola having its focus at (3, 2) and directrix along the y-axis has its vertex at

(a) $(2,2)$ (b) $\left(\frac{3}{2},2\right)$

(c) $\left(\frac{1}{2},2\right)$ (d) $\left(\frac{2}{3},2\right)$

110. The rank of the matrix $\begin{bmatrix}-1 & 2 & 5\\ 2 & -4 & a-4\\ 1 & -2 & a+1\end{bmatrix}$ is

(a) 1 if $a=6$ (b) 2 if $a=1$

(c) 3 if $a=2$ (d) 1 if $a=4$

111. If $f(x)=\begin{vmatrix}\cos x & 1 & 0\\ 1 & 2\cos x & 1\\ 0 & 1 & 2\cos x\end{vmatrix}$, then

$\int_0^{\pi/2} f(x)dx$ is equal to

(a) $\frac{1}{4}$ (b) $-\frac{1}{3}$

(c) $\frac{1}{2}$ (d) 1

112. The distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z-1}{-6}$ is

(a) 1 (b) 2

(c) 4 (d) $2\sqrt{3}$

113. The tangent lines to the curve $y^2 = 4ax$ at points where $x = a$, are

(a) parallel (b) perpendicular

(c) inclined at 60° (d) inclined at 30°

114. If the eccentricity of the hyperbola $x^2 - y^2 \operatorname{cosec}^2\alpha = 25$ is $\sqrt{5}$ times the eccentricity of the ellipse $x^2 \operatorname{cosec}^2\alpha + y^2 = 5$, then α is equal to :

(a) $\tan^{-1}\sqrt{2}$ (b) $\sin^{-1}\sqrt{\frac{3}{4}}$

(c) $\tan^{-1}\sqrt{\frac{2}{5}}$ (d) $\sin^{-1}\sqrt{\frac{2}{5}}$

115. The conditional $(p \wedge q) \Rightarrow p$ is

(a) A tautology

(b) A fallacy *i.e.*, contradiction

(c) Neither tautology nor fallacy

(d) None of these

116. The set of points of discontinuity of the function $f(x) = \lim_{n\to\infty} \frac{(2\sin x)^{2n}}{3^n - (2\cos x)^{2n}}$ is given by

(a) R (b) $\left\{n\pi \pm \frac{\pi}{3}, n \in I\right\}$

(c) $\left\{n\pi \pm \frac{\pi}{6}, n \in I\right\}$ (d) None of these

117. The volume V and depth x of water in a vessel are connected by the relation $V = 5x - \frac{x^2}{6}$ and the volume of water is increasing , at the rate of 5 cm^3/sec, when x = 2 cm. The rate at which the depth of water is increasing, is

(a) $\frac{5}{18}$ cm/sec (b) $\frac{1}{4}$ cm/sec

(c) $\frac{5}{16}$ cm/sec (d) None of these

118. If vectors $a\hat{i} + \hat{j} + \hat{k}, \hat{i} + b\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + c\hat{k}$ $(a \neq b \neq c \neq 1)$ are coplanar, then find $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$.

(a) 0 (b) 1

(c) –1 (d) 2

119. If matrix $A = \begin{bmatrix} 3 & -2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \frac{1}{k}\text{adj}(A)$, then k is

(a) 7 (b) -7

(c) 15 (d) -11

120. The angle between a pair of tangents drawn from a point T to the circle $x^2 + y^2 + 4x - 6y + 9\sin^2\alpha + 13\cos^2\alpha = 0$ is 2α. The equation of the locus of the point T is

(a) $x^2 + y^2 + 4x - 6y + 4 = 0$

(b) $x^2 + y^2 + 4x - 6y - 9 = 0$

(c) $x^2 + y^2 + 4x - 6y - 4 = 0$

(d) $x^2 + y^2 + 4x - 6y + 9 = 0$

PART - IV (ENGLISH)

Directions (Qs. 121-123): Study the paragraph and answer the questions that follow:

At this stage of civilisation, when many nations are brought into close and vital contact for good and evil, it is essential, as never before, that their gross ignorance of one another should be diminished, that they should begin to understand a little of one another's historical experience and resulting mentality. It is the fault of the English to expect the people of other countries to react as they do, to political and international situations. Our genuine goodwill and good intentions are often brought to nothing, because we expect other people to be like us. This would be corrected if we knew the history, not necessarily in detail but in broad outlines, of the social and political conditions which have given to each nation its present character.

121. The character of a nation is the result of its
- (a) gross ignorance
- (b) cultural heritage
- (c) socio-political conditions
- (d) mentality

122. According to the author Mentality' of a nation is mainly product of its
- (a) present character
- (b) international position
- (c) politics
- (d) history

123. The need for a greater understanding between nations
- (a) is more today than ever before
- (b) was always there
- (c) is no longer there
- (d) will always be there

Directions (Q. 124) : In the question below a sentence is given, a part of which is printed in bold and underline. This part may contain a grammatical error. Each sentence is followed by phrases a, b, c and d. Find out which phrase should replace the phrase given in bold/underline to correct the error, if there is any to make the sentence grammatically meaningful and correct.

124. There are any number of skilled writers who can develop content and create marketing materials **with a keen eye to using proven methods, but also to developing new and innovative techniques.**
- (a) with a keen eye to using proven methods, but also to developing new and innovative techniques.
- (b) with a keen eye for using proven methods, and also to developing new and innovative techniques.
- (c) with a keen eye not only to using proven methods, but also to developing new and innovative techniques.
- (d) with a keen eye to using proven methods, but to developing new and innovative techniques.

125. Choose the best pronunciation of the word, Sorbet from the following options.
- (a) Sore-bet
- (b) Sore-bay
- (c) Sorb rhymes with orb
- (d) Shore-bay

SOLUTIONS

PART - I (PHYSICS)

1. (d) **Given :** Mass of rocket (m) = 5000 Kg
Exhaust speed (v) = 800 m/s
Acceleration of rocket (a) = 20 m/s^2
Gravitational acceleration (g) = 10 m/s^2
We know that upward force
F = m (g + a) = 5000 (10 +20)
= 5000 × 30 = 150000 N.
We also know that amount of gas ejected

$$\left(\frac{dm}{dt}\right)=\frac{F}{v}=\frac{150000}{800}=187.5\ kg/s$$

2. (c) The power dissipated in the circuit.

$$P=\frac{V^2}{R_{eq}} \qquad ...(i)$$

$v = 10$ volt

$$\frac{1}{R_{eq}}=\frac{1}{R}+\frac{1}{5}=\frac{5+R}{5R}$$

$$R_{eq}=\left(\frac{5R}{5+R}\right)$$

$P = 30$ W
Substituting the values in equation (i)

$$30=\frac{(10)^2}{\left(\frac{5R}{5+R}\right)}$$

$$\frac{15R}{5+R}=10 \Rightarrow 15R=50+10R$$

$$5R=50 \Rightarrow R=10\,\Omega$$

3. (c) $E=\frac{1}{2}mv^2$ or $mv=\sqrt{2mE}$

$$\text{so } \lambda=\frac{h}{mv}=\frac{h}{\sqrt{2mE}}$$

4. (c) As $\frac{1}{2}m_A v_A^2=\frac{1}{2}m_B v_B^2$

$$\frac{v_A}{v_B}=\sqrt{\frac{m_B}{m_A}};$$

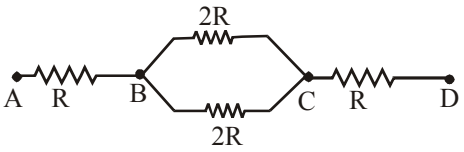
$$\frac{P_B}{P_A}=\frac{m_B v_B}{m_A v_A}$$

$$=\frac{m_B}{m_A}\sqrt{\frac{m_A}{m_B}}=\sqrt{\frac{m_B}{m_A}}=\frac{1}{\sqrt{3}}$$

5. (d)

6. (d) Photoelectric effect does not support the wave nature of light.

7. (c) The equivalent electrical circuit of the arrangement is shown in figure.



Temperature difference between the end points A and D = 200 – 20 = 180°C
As the resistances for the three parts are equal, the temperature difference must be distribuited equally in the three parts (= 180/3 = 60°C)
∴ Temperature of B = 200°C – 60° = 140°C.

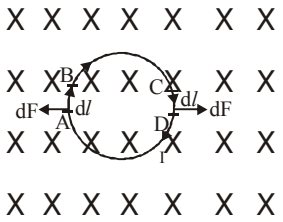
8. (c) Number of possible spectral lines emitted when an electron jumps back to ground state from n^{th} orbit $= \frac{n(n-1)}{2}$

Here, $\frac{n(n-1)}{2}=6 \Rightarrow n=4$

Wavelength λ from transition from n = 1 to n = 4 is given by,

$$\frac{1}{\lambda}=R\left(\frac{1}{1}-\frac{1}{4^2}\right) \Rightarrow \lambda=\frac{16}{15R}=975\ \text{Å}$$

9. (c) The magnetic field is perpendicular to the plane of the paper. Let us consider two diametrically opposite elements. By Fleming's Left hand rule on element AB the direction of force will be Leftwards and the magnitude will be
$dF = Id\ell B \sin 90° = Id\ell B$



On element CD, the direction of force will be towards right on the plane of the papper and the magnitude will be dF = Id*l*B.

10. (b) Apparent depth = $d/\mu_1 + d/\mu_2$

11. (c) By law of conservation of momentum,

$Mu = MV + mv$ ....(i)

$$\text{Also } e = \frac{|v_1 - v_2|}{|u_1 - u_2|} \Rightarrow Mu = Mv - MV$$ ....(ii)

From (i) and (ii), $2Mu = (M+m)v$

$$\Rightarrow v = \frac{2uM}{M+m} \Rightarrow v = \frac{2u}{1+\frac{m}{M}}$$

12. (c) Consider the potential at D be 'V'.

Potential drop across C_1 is $(V - V_1)$ and C_2 is $(V_2 - V)$

$\therefore q_1 = C_1(V - V_1),\ q_2 = C_2(V_2 - V)$

As $q_1 = q_2$ [capacitors are in series]

$\therefore C_1(V - V_1) = C_2(V_2 - V)$

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

13. (b) Given : work function ϕ of metal = 2.28 eV

Wavelength of light $\lambda = 500\text{ nm} = 500 \times 10^{-9}\text{m}$

$$KE_{max} = \frac{hc}{\lambda} - \phi$$

$$KE_{max} = \frac{6.6\times10^{-34}\times3\times10^{8}}{5\times10^{-7}} - 2.82$$

$KE_{max} = 2.48 - 2.28 = 0.2\text{ eV}$

$$\lambda_{min} = \frac{h}{p} = \frac{h}{\sqrt{2m(KE)_{max}}}$$

$$= \frac{\frac{20}{3}\times10^{-34}}{\sqrt{2\times9\times10^{-31}\times0.2\times1.6\times10^{-19}}}$$

$$\lambda_{min} = \frac{25}{9}\times10^{-9}$$

$= 2.80\times10^{-9}\text{ nm} \quad \therefore \lambda \geq 2.8\times10^{-9}\text{ m}$

14. (b) Kerosene oil rises up in wick of a lantern because of capillary action. If the surface tension of oil is zero, then it will not rise, so oil rises up in a wick of a lantern due to surface tension.

15. (a) $$e = \frac{LdI}{dt} = \frac{40\times10^{-3}(11-1)}{4\times10^{-3}} = 100\text{V}$$

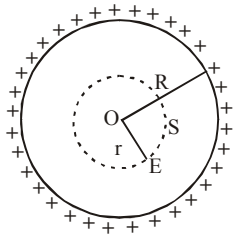
16. (d) Impedence of a capacitor is $X_C = 1/\omega C$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi\times50\times2\times10^{-6}} = \frac{5000}{\pi}.$$

17. (a) The final boolean expression is,

$$X = \overline{(\overline{A}.\overline{B})} = \overline{\overline{A}} + \overline{\overline{B}} = A + B \Rightarrow \text{OR gate}$$

18. (a) Charge resides on the outer surface of a conducting hollow sphere of radius R. We consider a spherical surface of radius r < R. By Gauss theorem



$$\int_s \vec{E}.\vec{ds} = \frac{1}{\varepsilon_0}\times \text{charge enclosed or}$$

$$E\times4\pi r^2 = \frac{1}{\varepsilon_0}\times0 \Rightarrow E = 0$$

i.e electric field inside a hollow sphere is zero.

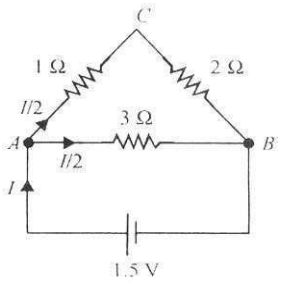
19. (d) Let R be the radius of the bigger drop, then Volume of bigger drop = 2 × volume of small drop

$$\frac{4}{3}\pi R^3 = 2\times\frac{4}{3}\pi r^3 \Rightarrow R = 2^{1/3}r$$

Surface energy of bigger drop,

$$E = 4\pi R^2T = 4\times2^{2/3}\pi r^2T = 2^{8/3}\pi r^2T$$

20. **(b)** Equivalent resistance between A and B = series combination of 1 Ω and 2 Ω in parallel with 3 Ω resistor..



$\frac{1}{R}=\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$ or $R=1.5\,\Omega$.

$\therefore$ Current in the circuit is $I=V/R=1.5/1.5=1A$.

Since the resistance in arm ACB = resistance in arm AB = 3 Ω, the current divides equally in the two arms. Hence the current through the 3 Ω resistor = I/2 = 0.5 A.

21. **(d)**

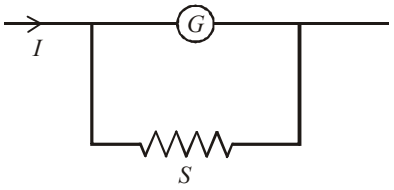
22. **(a)** $\tan(90°-\theta)=\frac{stress}{strain}$

23. **(c)** $U=-K\frac{ze^2}{r}$; $T.E=-\frac{k}{2}\frac{ze^2}{r}$

$K.E=\frac{k}{2}\frac{ze^2}{r}$. Here r decreases

24. **(c)** When resistance is connected to A.C source, then current & voltage are in same phase.

25. **(a)** Galvanometer is converted into ammeter, by connected a shunt, in parallel with it.



$\frac{GS}{G+S}=\frac{V_G}{I}=\frac{25\times10^{-3}}{25}$

$\frac{GS}{G+S}=0.001\Omega$

Here $S<<G$ so $S=0.001\,\Omega$

26. **(b)** For path difference λ, phase

$\text{difference}=2\pi\left(Q=\frac{2\pi}{\lambda}x=\frac{2\pi}{\lambda}.\lambda=2\pi\right)$

$\Rightarrow I=I_0+I_0+2I_0\cos 2\pi$

$\Rightarrow I=4I_0\ (\because \cos 2\pi=1)$

For $x=\frac{\lambda}{4}$, phase difference $=\frac{\pi}{2}$

$\therefore I'=I_1+I_2+2\sqrt{I_1}\sqrt{I_2}\cos\frac{\pi}{2}$

If $I_1=I_2=I_0$ then $I'=2I_0=2.\frac{I}{4}=\frac{I}{2}$

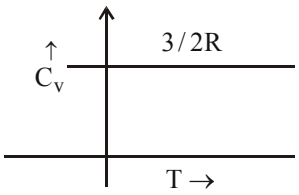
27. **(a)** Static friction is a self adjusting force in magnitude and direction.

28. **(a)** Cosmic rays are coming from outer space, having high energy charged particles, like a-particle, proton etc. β-rays are stream of high energy electrons, coming from the nucleus of radioactive atoms.

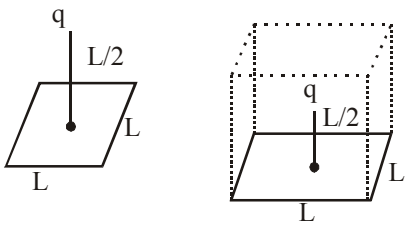
29. **(c)** For a monatomic gas

$C_v=\frac{3}{2}R$

So correct graph is



30. **(c)**



The given square of side L may be considered as one of the faces of a cube with edge L. Then given charge q will be considered to be placed at the centre of the cube. Then according to Gauss's theorem, the magnitude of the electric flux through the faces (six) of the cube is given by

$$\phi = q/\varepsilon_0$$

Hence, electric flux through one face of the cube for the given square will be

$$\phi' = \frac{1}{6}\phi = \frac{q}{6\varepsilon_0}$$

31. (d) When work is done upon a system by a conservative force then its potential energy increases.

32. **(c)** $C_s = \frac{C_1 C_2}{C_1 + C_2} = 3$

$C_p = C_1 + C_2 = 16 \quad \therefore \quad C_1 C_2 = 48$

$C_1 - C_2 = \sqrt{(C_1 + C_2)^2 - 4C_1 C_2}$

$= \sqrt{16^2 - 4 \times 48} = \sqrt{64} = 8$

$C_1 + C_2 = 16\,\mu F$

$C_1 - C_2 = 8\,\mu F$

$\Rightarrow 2C_1 = 24\mu F \Rightarrow C_1 = 12\mu F$

$\therefore \; C_2 = \frac{48}{12} = 4\mu F$

33. **(d)** In LCR series circuit, resonance frequency f_0 is given by

$$L\omega = \frac{1}{C\omega} \Rightarrow \omega^2 = \frac{1}{LC} \quad \therefore \quad \omega = \sqrt{\frac{1}{LC}} = 2\pi f_0$$

$$\therefore \quad f_0 = \frac{1}{2\pi\sqrt{LC}} \qquad \text{or} \qquad f_0 \alpha \frac{1}{\sqrt{C}}$$

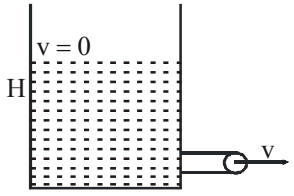
When the capacitance of the circuit is made 4 times, its resonant frequency become f_0'

$$\therefore \quad \frac{f_0'}{f_0} = \frac{\sqrt{C}}{\sqrt{4C}} \qquad \text{or} \qquad f_0' = \frac{f_0}{2}$$

34. **(b)** On polarisation by reflection, the reflected and refracted waves are at 90° to each other.

35. **(a)** v = velocity of efflux through an orifice

$= \sqrt{2gH}$



It is independent of the size of orifice.

36. **(c)** The acceleration of both the blocks = $\frac{15}{3x} = \frac{5}{x}$

$\therefore$ Force on $B = \frac{5}{x} \times 2x = 10$ N

37. **(b)** Potential gradient along wire

$$= \frac{\text{potential difference along wire}}{\text{length of wire}}$$

or, $0.1 \times 10^{-3} = \frac{I \times 40}{1000} V/cm$

or, Current in wire, $I = \frac{1}{400} A$

or, $\frac{2}{40 + R} = \frac{1}{400}$ or $R = 800 - 40 = 760\,\Omega$

38. **(b)** $i = \frac{A + \delta_m}{2} = \frac{60 + 30}{2} = 45°$

39. **(b)** $I \rightarrow ON$

$II \rightarrow OFF$

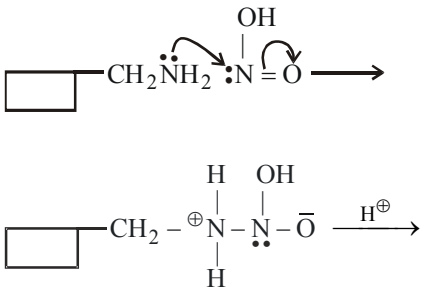
In II[nd] state it is used as a amplifier it is active region.

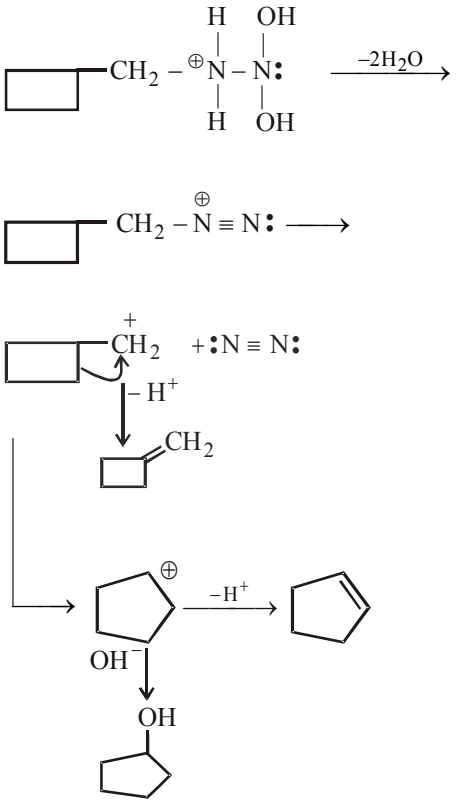
40. **(c)** $\tau = MB \sin\theta = \vec{M} \times \vec{B}$

PART - II (CHEMISTRY)

41. **(b)** It is stoichiometric defect and it is observed when equal number of cations and anions are missing from the lattice site.

42. **(d)**





43. **(a)** The reaction given is an exothermic reaction thus according to Le chatalier's principle lowering of temperature, addition of F_2 and Cl_2 favour the forward direction and hence the production of ClF_3.

44. **(d)** For the reaction:-

$$2NO_2(g) \rightleftharpoons 2NO(g) + O_2(g)$$

Given $K_c = 1.8 \times 10^{-6}$ at 184 °C

$R = 0.00831$ kJ/mol. K

$$K_p = K_c (RT)^{\Delta n_g}$$

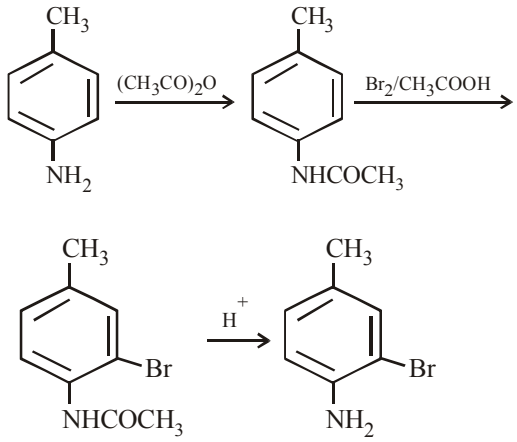
$$\Delta n_g = 3 - 2 = 1$$

$K_p = 1.8 \times 10^{-6} \times 0.00831 \times 457$

$= 6.836 \times 10^{-6}$

Hence it is clear that $K_p > K_c$

45. **(b)**



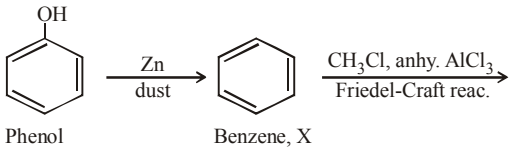
($-NHCOCH_3$ is more electron-releasing than $-CH_3$)

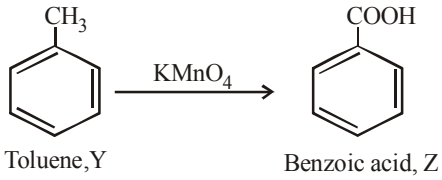
46. **(b)** No. of M atoms $= \frac{1}{4} \times 4 + 1 = 1 + 1 = 2$

No. of X atoms $= \frac{1}{2} \times 6 + \frac{1}{8} \times 8 = 3 + 1 = 4$

So, formula $= M_2X_4 = MX_2$

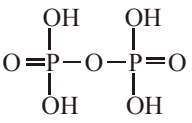
47. **(c)**



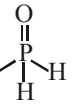


Toluene, Y — Benzoic acid, Z

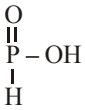
48. **(b)**

(a) $H_4P_2O_7$ $\Rightarrow$ 

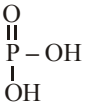
Pyrophosphoric acid

(b) H_3PO_2 $\Rightarrow$ 

Hypophosphorous acid

(c) H_3PO_3 $\Rightarrow$ 

Phosphorous acid

(d) H_3PO_4 $\Rightarrow$ 

orthophosphoric acid

49. (d) The given structure has three double bonds whose each carbon atom is differently substituted hence number of geometrical isomers will be $2^n = 2^{3'} = 8$, where n is the number of double bonds whose each carbon atom is differently substituted.

50. (a) Following generalization can be easily derived for various types of lattice arrangements in cubic cells between the edge length (a) of the cell and r the radius of the sphere.

For simple cubic : $a = 2r$ or $r = \frac{a}{2}$

For body centred cubic :

$$a = \frac{4}{\sqrt{3}}r \text{ or } r = \frac{\sqrt{3}}{4}a$$

For face centred cubic :

$$a = 2\sqrt{2}r \text{ or } r = \frac{1}{2\sqrt{2}}\text{a}$$

Thus the ratio of radii of spheres for these will be

simple : bcc : fcc

$$= \frac{a}{2} : \frac{\sqrt{3}}{4}a : \frac{1}{2\sqrt{2}}\ a \quad \text{i.e.}$$

option (a) is correct answer.

51. (d) $\log k = \log A - \frac{E_a}{2.303RT}$ ...(1)

Also given $\log k = 6.0 - (2000)\frac{1}{T}$ ...(2)

On comparing equations, (1) and (2)

$\log A = 6.0 \Rightarrow A = 10^6\ s^{-1}$

and $\frac{E_a}{2.303\ R} = 2000$;

$\Rightarrow E_a = 2000 \times 2.303 \times 8.314$

$= 38.29\ kJ\ mol^{-1}$

52. (c)

(a)

$$CH_3CH_2CH_2OH \xrightarrow[\text{or } K_2Cr_2O_7/H+]{\text{alk. } KMnO_4} CH_3CH_2CHO$$

$$\longrightarrow CH_3CH_2COOH \xrightarrow{\text{Fehling sol.}} \text{No reaction}$$

(b)

$$CH_3CHOHCH_3 \xrightarrow[\text{or } K_2Cr_2O_7/H+]{\text{alk. } KMnO_4} CH_3COCH_3$$

$$\longrightarrow CH_3COOH \xrightarrow{\text{Feh sol.}} \text{No reaction}$$

(c) $CH_3CH_2CH_2OH \xrightarrow{Cu} CH_3CH_2CHO$

$$\xrightarrow[\text{solution}]{\text{Fehling}} Cu_2O\downarrow \text{ (red)}$$

$$CH_3CHOHCH_3 \xrightarrow{Cu} CH_3COCH_3$$

$$\xrightarrow[\text{solution}]{\text{Fehling}} \text{No reaction}$$

(d) $CH_3CH_2CH_2OH$ or

$$CH_3CHOHCH_3 \xrightarrow{\text{Conc. } H_2SO_4} CH_3CH = CH_2$$

53. (b) $Cu^{2+}[Ar]3d^9$, $Ti^{4+}[Ar]3d^0$

$Co^{2+}[Ar]3d^7$, $Fe^{2+}[Ar]3d^6$

1,3,4 are coloured ions hence the answer is b.

54. (b) For first order reaction,

$$k = \frac{0.693}{t_{1/2}} = \frac{0.693}{6.93}$$

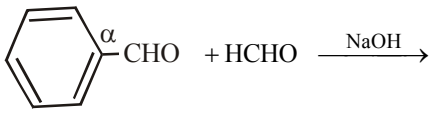
$$k = \frac{2.303}{t}\log\frac{100}{100-99}$$

$$\frac{0.693}{6.93} = \frac{2.303}{t}\log\frac{100}{1}$$

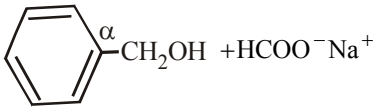
$$\frac{0.693}{6.93} = \frac{2.303\times 2}{t}$$

$t = 46.06$ min

55. (a) Benzaldyde and formaldehyde, both do not have α - hydrogen atom, so both will undergo Cannizzaro reaction; here formaldehyde will always be oxidised to formate while the other aldehyde (C_6H_5CHO or any other aldehyde not having α-H, viz- Me_3CCHO) will always be reduced to corresponding alcohol *(crossed Cannizzaro reaction)*

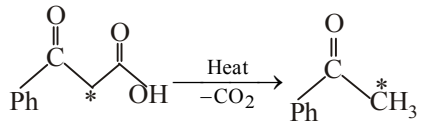


Benzaldehyde Formaldehyde

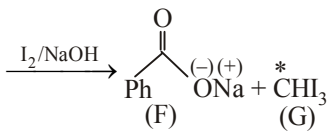


Benzyl alcohol sod. formate

56. (c)



(β - keto acid) (E)



(F) (G)

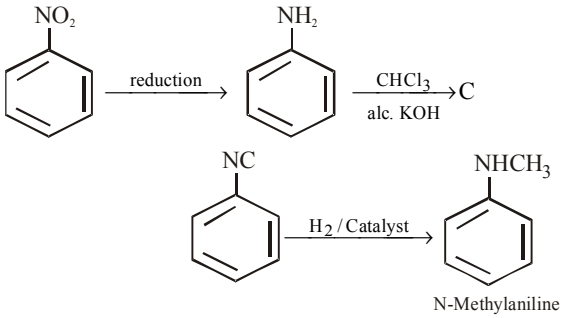
57. (a) ΔS for the reaction $\frac{1}{2}X_2 + \frac{3}{2}Y_2 \rightleftharpoons XY_3$

$\Delta S = 50 - (30 + 60) = -40$ J

For equilibrium $\Delta G = 0 = \Delta H - T\Delta S$

$$T = \frac{\Delta H}{\Delta S} = \frac{-30000}{-40} = 750\text{ K}$$

58. (d)



N-Methylaniline

59. (a) When pH = 14 $[H^+] = 10^{-14}$

and $[OH^-] = 1$ M

$$K_{sp} = [Cu^{2+}][OH^-]^2 = 10^{-19}$$

$$\therefore \quad [Cu^{2+}] = \frac{10^{-19}}{[OH^-]^2} = 10^{-19}$$

The half cell reaction

$$Cu^{2+} + 2e^- \longrightarrow Cu$$

$$E = E° - \frac{0.059}{2}\log\frac{1}{[Cu^{2+}]}$$

$$= 0.34 - \frac{0.059}{2}\log\frac{1}{10^{-19}} = -0.22\text{ V}$$

60. (b) $C_4H_8 \xleftarrow[(-H_2O)]{\text{Conc.}H_2SO_4} C_4H_{10}O \xrightarrow{\text{Oxi}}$

C_4H_8O (R-$COCH_3$)

Thus C_4H_8O should be $CH_3CH_2COCH_3$, hence $C_4H_{10}O$ should be $CH_3CH_2CHOHCH_3$

61. (a) $W_{max} = -n.FE;$

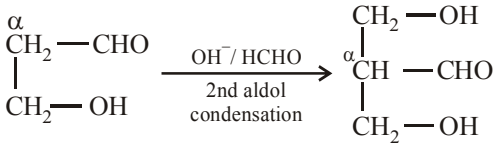
$W_{max} = -2 \times 96500 \times 0.65 = -1.25 \times 10^5 \text{ J}$

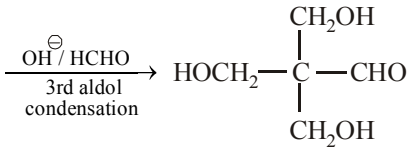
$0.5\text{g } H_2 = 0.25$ mole.

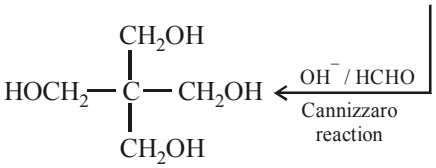
Hence,

$W_{max} = 1.25 \times 10^5 \times 0.25 = -3.12 \times 10^4 \text{ J}$

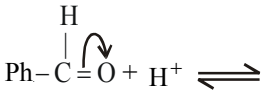
62. (c) $\overset{\alpha}{CH_3} - CHO + HCHO \xrightarrow[\text{1st aldol condensation}]{OH^{\ominus}}$

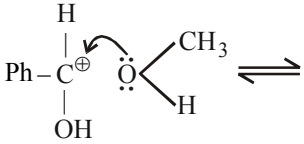


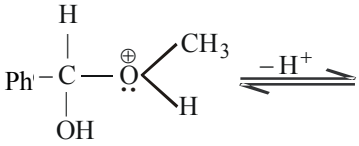


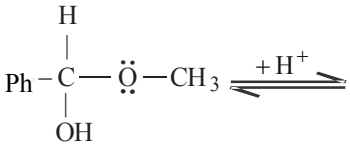


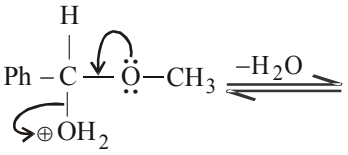
63. (b)

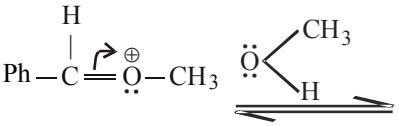


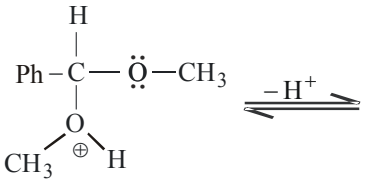


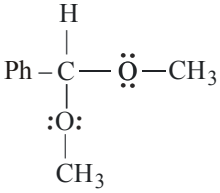












64. (a) In $\alpha-$ form distance between nearest neighbour atom is $\frac{\sqrt{3}a_1}{2}$.

In γ form distance between nearest neighbour atom is $\frac{a_2}{\sqrt{2}}$.

$\therefore \quad \frac{\sqrt{3}\,a_1}{2} = \frac{a_2}{\sqrt{2}}$ (given)

or $\frac{a_2}{a_1} = \sqrt{\frac{3}{2}}$

$$\frac{\rho_1}{\rho_2} = \frac{z_1}{z_2}\left(\frac{a_2}{a_1}\right)^3 = \frac{1}{2}\left(\sqrt{\frac{3}{2}}\right)^3 = 0.918$$

65. (c) $CH_3CHO(g) \longrightarrow CH_4(g) + CO(g)$

When t = 0 p^0 0 0

When t = t $p^0 - p$ p p

$\therefore\ p^0 - p + p + p = 120$ mm Hg

or, $p^0 + p = 120$ mm Hg;

$p = 120 - 80 = 40$ mm Hg

$$k = \frac{1}{t}\ln\frac{p^0}{p^0 - p} = \frac{1}{20}\ln\frac{80}{80-40} = \frac{1}{20}\ln 2$$

Again, $t_{1/2} = \frac{\ell n 2}{k}$

$$\therefore\ t_{1/2} = \frac{\ln 2}{\ln 2} \times 20 = 20 \text{ min.}$$

66. (a) Solubility of the compound in conc. H_2SO_4 indicates that it can be an alkene, alcohol or an ether. The inability to discharge bromine colour indicates absence of an alkene. Hence the compound is an alcohol which should be primary because it is readily (within 2 seconds) oxidised by CrO_3 in sulphuric acid.

67. (b) $E = h\upsilon = \frac{ch}{\lambda}$; and $\upsilon = \frac{c}{\lambda}$

$$8\times10^{15} = \frac{3.0\times10^8}{\lambda}$$

$$\therefore\ \lambda = \frac{3.0\times10^8}{8\times10^{15}} = 0.37\times10^{-7}$$

$$= 37.5\times10^{-9} \text{ m} = 4\times10^1 \text{ nm}$$

68. (b) $CH_3 - CH = CH - \overset{*}{C}HCH_3$ (OH)

exhibits both geometrical as well as optical isomerism.

cis - R *cis* - S

trans - R *trans* - S

69. (c) Positive sign is for optical rotation (dextro rotatory) and D - is for configuration. It is derived from

D (+) glyceraldehyde $\left(\begin{array}{l}\text{CHO}\\ |\\ \text{H}-\text{C}-\text{OH}\\ |\\ \text{CH}_2\text{OH}\end{array}\right)$

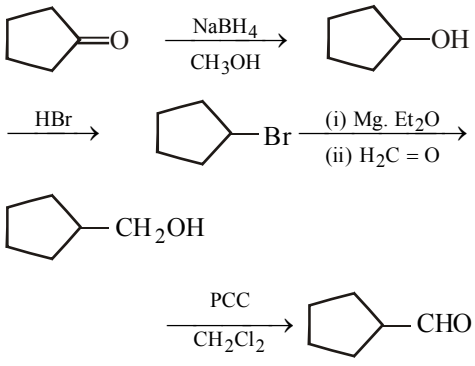
L (–)glyceraldehyde is $\left(\begin{array}{l}\text{CHO}\\ |\\ \text{HO}-\text{C}-\text{H}\\ |\\ \text{CH}_2\text{OH}\end{array}\right)$

70. (b) $\Delta H = \Delta U + \Delta nRT$ for

$N_2 + 3H_2 \longrightarrow 2NH_3$

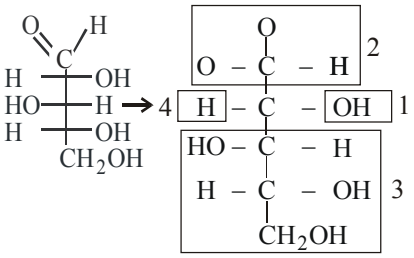
$\Delta n_g = 2 - 4 = -2$

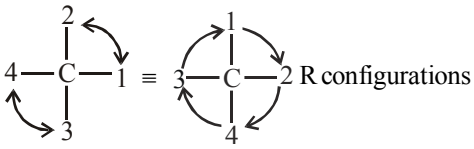
$\therefore \Delta H = \Delta U - 2RT$ or $\Delta U = \Delta H + 2RT$ $\therefore \Delta U > \Delta H$

71. (a) 

72. (b) Most of the Ln^{3+} compounds except La^{3+} and Lu^{3+} are coloured due to the presence of *f*-electrons.

73. (c)





Arrange the groups in order of priority by following the text.

74. (b) Saponification (alkaline hydrolysis) of oils and fats gives glycerol and sodium salt of fatty acids, which is sodium palmitate in the present question

75. (a) For non spontaneous reaction

$\Delta G = +ve$

$\Delta G = \Delta H - T\Delta S$ and

$\Delta S = 121\ JK^{-1}$

For $\Delta G = +ve$

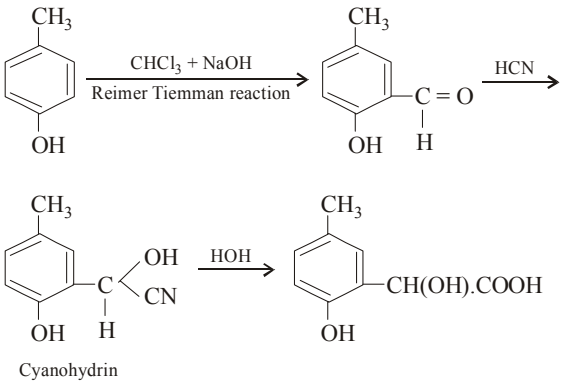
ΔH has to be positive. Hence the reaction is endothermic.

The minimum value of ΔH can be obtained by putting $\Delta G = 0$

$\Delta H = T\Delta S = 298 \times 121\ J$

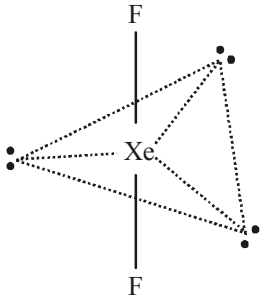
$= 36.06\ kJ$

76. (c)



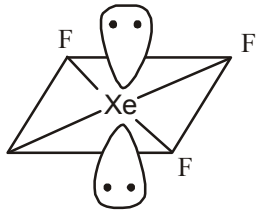
Cyanohydrin

77. (c) XeF_2 :



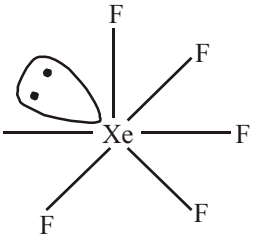
$3\ell p$

XeF_4 :

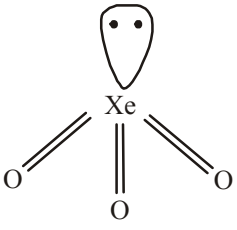


$2\ell p$

XeF_6 :



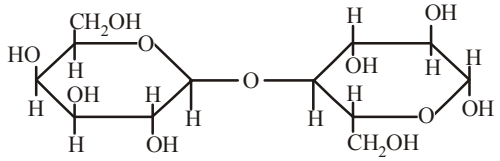
XeO_3 :



$1\ell p$

Hence XeF_2 has maximum no. of lone pairs of electrons.

78. (c)



(Lactose)

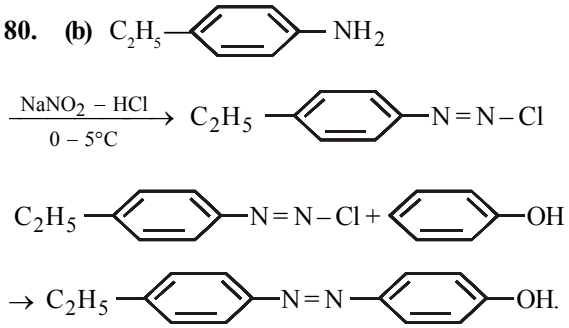
All reducing sugar shows mutarotation.

79. (a) On the basis of structure of guanine and complementary bases present in them, we can say that if the sequence of bases in one strand of DNA is I, then the sequence in the second strand should be II

A : T : G : C : T : T : G : A I

T : A : C : G : A : A : C : T II

80. (b)



PART - III (MATHEMATICS)

81. (d) $\frac{3\pi}{2} < 5 < \frac{5\pi}{2}$

$\Rightarrow \sin^{-1}(\sin 5) = 5 - 2\pi$

Given $\sin^{-1}(\sin 5) > x^2 - 4x$

$\Rightarrow x^2 - 4x + 4 < 9 - 2\pi$

$\Rightarrow (x-2)^2 < 9 - 2\pi$

$\Rightarrow -\sqrt{9-2\pi} < x - 2 < \sqrt{9-2\pi}$

$\Rightarrow 2 - \sqrt{9-2\pi} < x < 2 + \sqrt{9-2\pi}$

82. (c) Using Lagrange's Mean Value Theorem

Let f(x) be a function defined on [a, b]

then, $f'(c) = \frac{f(b) - f(a)}{b - a}$ ....(i)

$c \in [a, b]$

$\therefore$ Given $f(x) = \log_e x$ $\therefore f'(x) = \frac{1}{x}$

$\therefore$ equation (i) become

$\frac{1}{c} = \frac{f(3) - f(1)}{3 - 1}$

$\Rightarrow \frac{1}{c} = \frac{\log_e 3 - \log_e 1}{2} = \frac{\log_e 3}{2} \Rightarrow c = \frac{2}{\log_e 3}$

$\Rightarrow c = 2\log_3 e$

83. (c) p : we control population, q : we prosper

$\therefore$ we have $p \Rightarrow q$

Its negation is $\sim(p \Rightarrow q)$ i.e. $p \wedge \sim q$

i.e., we control population but we do not prosper.

84. (b) Consider the equation

$z\bar{z} + (2 - 3i)z + (2 + 3i)\bar{z} + 4 = 0$ ...(1)

Let $z = x + iy$ and $\bar{z} = x - iy$, $z\bar{z} = x^2 + y^2$

Put value of z, $\bar{z}$ and $z\bar{z}$ in equation (1), we get

$(x^2 + y^2) + (2 - 3i)(x + iy) + (2 + 3i)(x - iy) + 4 = 0$

$\Rightarrow 4x + 6y + 4 + x^2 + y^2 = 0$

Now, we make it perfect square

$\Rightarrow x^2 + y^2 + 4x + 6y + 4 + 4 + 9 = 4 + 9$

$\Rightarrow (x+2)^2 + (y+3)^2 = 9$

This represents a circle of radius 3.

85. (b) Let $f(x) = \sin x - kx - c$ where k and c are constants

$f'(x) = \cos x - k$

$\therefore f$ decreases *if* $\cos x \le k$

Thus, $f(x) = \sin x - kx - c$ decrease always when

$k \ge 1$.

86. **(b)** Given, equation is

$$\frac{1}{r}=\frac{1}{8}+\frac{3}{8}\cos\theta \text{ or } \frac{8}{r}=1+3\cos\theta$$

which is the form of $\frac{l}{r}=1+e\cos\theta$

$\because \quad e=3>0,$

$\therefore$ Given equation is a hyperbola.

87. **(c)** The differential equation of the motion is

$$\frac{dv}{dt}=30-3v \qquad \text{....(i)}$$

$$\Rightarrow \frac{dv}{30-3v}=dt\,.$$

Integrating we get

$$-\frac{1}{3}\log(30-3v)=t+C$$

$$\Rightarrow \log(30-3v)=-3(t+c)$$

$$\Rightarrow 30-3v=e^{-3t-3c}=Ae^{-3t},\ A=e^{-3c}$$

$$\Rightarrow 3v=30-Ae^{-3t} \qquad \text{....(ii)}$$

For maximum velocity $\frac{dv}{dt}=0$

$\Rightarrow 30-3v=0$ from (i)

$$\therefore v=\frac{30}{3}=10\,cm/s$$

which is the maximum velocity

However from (ii) $\frac{dv}{dt}=3Ae^{-3t}$

Clearly $\frac{dv}{dt}=0$ if $t\to\infty$

$\therefore$ The maximum velocity will be achieved after infinite time in other words, the maximum velocity will hever be reached.

88. **(b)** Given curve is $y^2=4x+5$

on differentiating, we get

$$2y\frac{dy}{dx}=4\Rightarrow\frac{dy}{dx}=\frac{2}{y}$$

Given line is $2x-y+5=0$

$\Rightarrow y=2x+5$

slope of line is 2. Therefore,

$$\frac{2}{y}=2\Rightarrow y=1$$

put y = 1 in the equation of curve, we get

$1=4x+5$

$x=-1$

Hence, point of contact is (– 1, 1)

89. **(c)** Let $I=\int_0^{\pi/2}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ ...(i)

Then, I =

$$\int_0^{\pi/2}\frac{\sqrt{\sin(\pi/2-x)}}{\sqrt{\sin(\pi/2-x)}+\sqrt{\cos(\pi/2-x)}}dx$$

$$\Rightarrow \quad I=\int_0^{\pi/2}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx \qquad \text{...(ii)}$$

Adding (i) and (ii), we get

2I

$$\int_0^{\pi/2}\frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx+\int_0^{\pi/2}\frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

$$\int_0^{\pi/2}\frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx=\int_0^{\pi/2}1.dx=[x]_0^{\pi/2}=\frac{\pi}{2}-0$$

$$\Rightarrow \quad I=\frac{\pi}{4}\Rightarrow\int_0^{\pi/2}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx=\frac{\pi}{4}$$

90. **(c)** We have

$-1\le\cos 3x\le 1 \ \Rightarrow -1\le-\cos 3x\le 1$

Add '2' on both side

$1\le 2-\cos 3x\le 3$

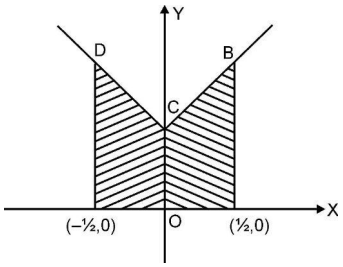
$$\Rightarrow \quad 1\ge\frac{1}{2-\cos 3x}\ge\frac{1}{3}\,.$$

91. (c) The given lines are,

$y-1=x,\ x\geq 0;\quad y-1=-x,\ x<0$

$y=0;\quad x=-\frac{1}{2},\ x<0;\quad x=\frac{1}{2},\ x\geq 0$

so that the area bounded is as shown in the figure.



$$\text{Required area} = 2\int_0^{1/2}(1+x)\,dx$$

$$=2\left(x+\frac{x^2}{2}\right)_0^{1/2}=2\left(\frac{1}{2}+\frac{1}{8}\right)=\frac{5}{4}$$

92. (a) Given $\begin{vmatrix} x+\alpha & \beta & \gamma \\ \gamma & x+\beta & \alpha \\ \alpha & \beta & x+\gamma \end{vmatrix}=0$

Operate $C_1 \to C_1+C_2+C_3$

$$\begin{vmatrix} x+\alpha+\beta+\gamma & \beta & \gamma \\ x+\alpha+\beta+\gamma & x+\beta & \alpha \\ x+\alpha+\beta+\gamma & \beta & x+\gamma \end{vmatrix}=0$$

$$=(x+\alpha+\beta+\gamma)\begin{vmatrix} 1 & \beta & \gamma \\ 1 & x+\beta & \alpha \\ 1 & \beta & x+\gamma \end{vmatrix}=0$$

$\Rightarrow x+\alpha+\beta+\gamma=0 \Rightarrow x=-(\alpha+\beta+\gamma)$

Again if

$$\begin{vmatrix} 1 & \beta & \gamma \\ 1 & x+\beta & \alpha \\ 1 & \beta & \gamma \end{vmatrix}=0 \Rightarrow \begin{vmatrix} 1 & \beta & \gamma \\ 0 & x & \alpha-\gamma \\ 0 & 0 & x \end{vmatrix}=0$$

$\Rightarrow x^2=0 \Rightarrow x=0$

$\therefore$ Solutions of the equation are $x = 0$, $-(\alpha+\beta+\gamma)$

93. (c) $\frac{dy}{dx}+\frac{y}{x\log x}=\frac{\sin 2x}{\log x}$

$\text{I.F.}=e^{\int \frac{dx}{x\log x}}$

$\therefore\ \text{I.F.}=e^{\int \frac{1}{t}dt}=e^{\log t}=t=\log|x|$

solution is given by

$y(\text{I.F.})=\int Q.(\text{I.F.})\,dx+C$

$$y\log|x|=\int \frac{\sin 2x}{\log|x|}(\log|x|)\,dx+C$$

$$=-\frac{\cos 2x}{2}+C$$

94. (d) Consider $\lim_{x\to\infty}\left[\frac{x^2}{3x-2}-\frac{x}{3}\right]$

$$=\lim_{x\to\infty}\left[\frac{3x^2-x(3x-2)}{3(3x-2)}\right]$$

$$=\lim_{x\to\infty}\frac{2x}{3(3x-2)}=\lim_{x\to\infty}\frac{2x}{3x\left[3-\frac{2}{x}\right]}$$

$$=\lim_{x\to\infty}\frac{2}{3}\frac{1}{\left(3-\frac{2}{x}\right)}=\frac{2}{3}\times\frac{1}{3-0}=\frac{2}{9}$$

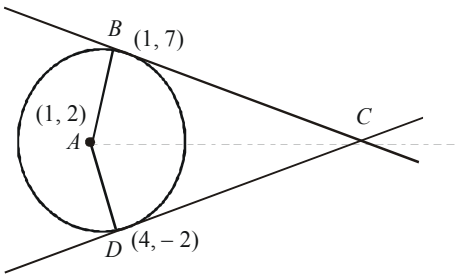
95. **(c)** $((\vec{a}\times b)\times(\vec{c}\times d)).(\vec{a}\times d)=0$

$([\vec{a}\,\vec{c}\,\vec{d}]\vec{b}-[\vec{b}\,\vec{c}\,\vec{d}]\vec{a}).(\vec{a}\times\vec{d})=0$

$[\vec{a}\,\vec{c}\,\vec{d}][\vec{b}\,\vec{a}\,\vec{d}]=0$

Either $\vec{c}$ or $\vec{b}$ must lie in the plane of $\vec{a}$ and $\vec{d}$.

96. **(b)**



Here, centre is A (1,2), and Tangent at B (1,7) is

$x.1+y.7-1\,(x+1)-2\,(y+7)-20=0$

or $y=7$ ...(1)

Tangent at D (4,–2) is

$3x-4y-20=0$ ...(2)

Solving (1) and (2), we get C is (16, 7)

Area $ABCD$ = 2 (Area of Δ ABC)

$=2\times\frac{1}{2}AB\times BC$

$AB\times BC=5\times 15=75$ units

97. **(c)** Integrating by parts.

$\int f(x)g''(x)\,dx-\int f''(x)g(x)\,dx$

$=f(x)g'(x)-\int f'(x)g'(x)\,dx$

$-f'(x)g(x)+\int f'(x)g'(x)dx$

$=f(x)g'(x)-f'(x)g(x)$

Hence, $\int_0^1 f(x)g''(x)\,dx-\int_0^1 f''(x)g(x)\,dx$

$=f(1)g'(1)-f'(1)g(1)-f(0)g'(0)+f'(0)g(0)$

$=f(1)g'(1)-f'(1)g(1)$

98. **(d)** $z=\frac{7-i}{3-4i}\times\frac{3+4i}{3+4i}$

$=\frac{21+25i+4}{16+9}=\frac{25(1+i)}{25}=(1+i)$

$z^{14}=(1+i)^{14}=[(1+i)^2]^7=(2i)^7$

$=2^7i^7=-2^7i$

99. **(c)** $f(x)=2\sin x+\sin 2x$

$f'(x)=2\cos x+2\cos 2x=2(\cos x+\cos 2x)$

$\therefore\ f'(x)=0\Rightarrow 2\cos^2 x+\cos x-1=0$

$\cos x=\frac{-1\pm 3}{4}=-1,\frac{1}{2}\quad\therefore\ x=\pi,\frac{\pi}{3}$

Now, $f(0)=0,\ f\left(\frac{3\pi}{2}\right)=-2$

$f(\pi)=0,\ f\left(\frac{\pi}{3}\right)=2\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}=\frac{3\sqrt{3}}{2}$

$\therefore$ difference between greatest value and least value

$=\frac{3\sqrt{3}}{2}+2$

100. **(d)** A and B will agree in a certain statement if both speak truth or both tell a lie. We define following events

E_1 = A and B both speak truth $\Rightarrow P(E_1)=xy$

E_2 = A and B both tell a lie

$\Rightarrow\ P(E_2)=(1-x)(1-y)$

E = A and B agree in a certain statement

Clearly, $P(E/E_1)=1$ and $P(E/E_2)=1$

The required probability is $P(E_1/E)$.

Using Baye's theorem

$P(E_1/E)$

$=\frac{P(E_1)P(E/E_1)}{P(E_1)P(E/E_1)+P(E_2)P(E/E_2)}$

$=\frac{xy.1}{xy.1+(1-x)(1-y).1}=\frac{xy}{1-x-y+2xy}$

101. (a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$;

$\Rightarrow \frac{3}{4} = 1 - P(\overline{A}) + P(B) - \frac{1}{4}$

$\Rightarrow 1 = 1 - \frac{2}{3} + P(B) \Rightarrow P(B) = \frac{2}{3}$;

Now, $P(\overline{A} \cap B) = P(B) - P(A \cap B) = \frac{2}{3} -$

$\frac{1}{4} = \frac{5}{12}$.

102. (a) Let the line be $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$ …(i)

If line (i) intersects with the line $\frac{x-1}{2} = \frac{y+3}{4} = \frac{z-5}{3}$, then

$$\begin{vmatrix} a & b & c \\ 2 & 4 & 3 \\ 4 & -3 & 14 \end{vmatrix} = 0$$

$\Rightarrow \quad 9a - 7b - 10c = 0$ …(ii)

from (i) and (ii), we have

$$\frac{a}{1} = \frac{b}{-3} = \frac{c}{5}$$

$\therefore$ The line is $\frac{x}{1} = \frac{y}{-3} = \frac{z}{5}$

103. (a) Given: $u_n = \int_0^{\pi/4} \tan^n \theta \, d\theta$

$$= \int_0^{\pi/4} \tan^2 \theta \tan^{n-2} \theta \, d\theta$$

$$= \int_0^{\pi/4} (\sec^2 \theta - 1) \tan^{n-2} \theta \, d\theta$$

$$= \int_0^{\pi/4} \sec^2 \theta \tan^{n-2} \theta \, d\theta$$

$$- \int_0^{\pi/4} \tan^{n-2} \theta \, d\theta$$

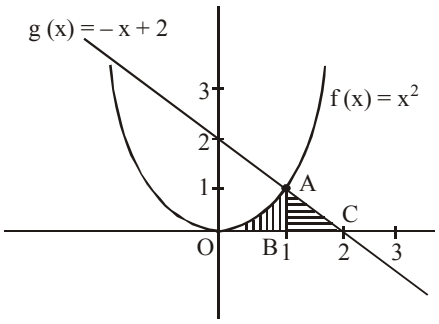
$$= \int_0^{\pi/4} \sec^2 \theta \tan^{n-2} \theta \, d\theta - u_{n-2}$$

$$\Rightarrow u_n + u_{n-2} = \int_0^{\pi/4} \sec^2 \theta \tan^{n-2} \theta \, d\theta$$

$$= \frac{\tan^{n-1} \theta}{n-1}\Big|_0^{\pi/4} = \frac{1}{n-1}$$

104. (a) Total number of words that can be formed $= 10^5$. Number of words in which no letter is repeated $= {}^{10}P_5$. So, number of words in which at least one letter is repeated $= 10^5 - {}^{10}P_5 = 69760$.

105. (d) Required area = Area of OAB + Area of ABC



Now, Area of $OAB = \int_0^1 f(x)\,dx + \int_1^2 g(x)\,dx$

$$= \int_0^1 x^2 dx + \int_1^2 (-x + 2)\,dx$$

$$= \frac{x^3}{3}\Big|_0^1 + \left[\frac{-x^2}{2} + 2x\right]_1^2$$

$$= \frac{1}{3} + \left[\left(\frac{-4}{2} + 4\right) - \left(\frac{-1}{2} + 2\right)\right]$$

$$= \frac{1}{3} + \left[(-2 + 4) - \left(\frac{3}{2}\right)\right]$$

$$= \frac{1}{3} + \frac{1}{2} = \frac{5}{6} \text{ sq unit}$$

106. (a) The line $\frac{x}{p} + \frac{y}{q} = 1$ will be a normal to the parabola $y^2 = 4ax$ if, for some value of m, it is identical with

$y = mx - 2am - am^3$ i.e. $mx - y = (2am + am^3)$

Comparing coefficients, we get

$$\frac{m}{1/p} = \frac{-1}{1/q} = \frac{2am + am^3}{1} \Rightarrow \quad mp = -q, \quad \therefore$$

$$m = -\frac{q}{p} \text{ and } mp = m(2a + am^2)$$

or $$P = 2a + am^2 = 2a + a\left(\frac{-q}{p}\right)^2$$

or $$p = 2a + \frac{aq^2}{p^2}$$

or $p^3 = 2ap^2 + aq^2$,

Which is the required condition.

107. (b) $P(E) = P(2 \text{ or } 3 \text{ or } 5 \text{ or } 7)$

$$= 0.23 + 0.12 + 0.20 + 0.07 = 0.62$$

$P(F) = P(1 \text{ or } 2 \text{ or } 3)$

$$= 0.15 + 0.23 + 0.12 = 0.50$$

$P(E \cap F) = P(2 \text{ or } 3)$

$$= 0.23 + 0.12 = 0.35$$

$\therefore P(EUF) = P(E) + P(F) - P(E \cap F)$

$$= 0.62 + 0.50 - 0.35 = 0.77$$

108. (c) $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{7}{8}$

$$= \left[\frac{\frac{1}{2} + \frac{1}{3} + \frac{7}{8} - \frac{1}{2}\times\frac{1}{3}\times\frac{7}{8}}{1 - \frac{1}{2}\times\frac{1}{3} - \frac{1}{3}\times\frac{7}{8} - \frac{7}{8}\times\frac{1}{2}}\right]$$

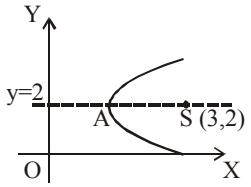
$$\left[\because \quad \tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left(\frac{x + y + z - xyz}{1 - xy - yz - zx}\right)\right]$$

$$= \tan^{-1}\left[\frac{\frac{41}{24} - \frac{7}{48}}{1 - \frac{1}{6} - \frac{7}{24} - \frac{7}{16}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{75}{48}}{1 - \frac{43}{48}}\right] = \tan^{-1}\left(\frac{75}{48 - 43}\right)$$

$$= \tan^{-1}\left[\frac{75}{5}\right] = \tan^{-1}15$$

109. (b) Vertex of the parabola is a point which lies on the axis of the parabola, which is a line $\perp$ to the directrix through the focus, i.e., $y = 2$ and equidistant from the focus and directrix $x = 0$, so that the vertex is $\left(\frac{3}{2}, 2\right)$.



110. (b) Let

$$A=\begin{bmatrix}-1 & 2 & 5\\ 2 & -4 & a-4\\ 1 & -2 & a+1\end{bmatrix}\sim\begin{bmatrix}-1 & 2 & 5\\ 0 & 0 & a+6\\ 0 & 0 & a+6\end{bmatrix}$$

$$[R_2\to R_2+2R_1,\ R_3\to R_3+R_1]$$

Clearly rank of A is 1 if $a=-6$

Also, for $a=1$, $|A|=\begin{vmatrix}-1 & 2 & 5\\ 2 & -4 & -3\\ 1 & -2 & 2\end{vmatrix}=0$

and $\begin{vmatrix}2 & 5\\ -4 & -3\end{vmatrix}=-6+20=14\neq 0$

$\therefore$ rank of A is 2 if $a=1$

111. (b) $f(x)=4\cos^3 x-\cos x-2\cos x=\cos 3x$

[Expansion of determinant]

$$\therefore\quad \int_0^{\pi/2} f(x)dx=\frac{\sin 3x}{3}\Big]_0^{\pi/2}=-\frac{1}{3}$$

112. (a) Equation of the line through (1, –2, 3) parallel to the line $\frac{x}{2}=\frac{y}{3}=\frac{z-1}{-6}$ is

$$\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{-6}=r\ (\text{say})\qquad ...(1)$$

Then any point on (1) is $(2r+1, 3r-2, -6r+3)$

If this point lies on the plane $x-y+z=5$ then

$$(2r+1)-(3r-2)+(-6r+3)=5\Rightarrow r=\frac{1}{7}$$

Hence the point is $\left(\frac{9}{7},-\frac{11}{7},\frac{15}{7}\right)$

Distance between (1, –2, 3) and $\left(\frac{9}{7},-\frac{11}{7},\frac{15}{7}\right)$

$$=\sqrt{\left(\frac{4}{49}+\frac{9}{49}+\frac{36}{49}\right)}=\sqrt{\left(\frac{49}{49}\right)}=1$$

113. (b) The given equation of the curve is $y^2=4ax$ ....(1)

Differentiating both sides of (1) with respect to x, we get

$$2y\frac{dy}{dx}=4a;\ \Rightarrow\frac{dy}{dx}=\frac{4a}{2y}=\frac{2a}{y}\qquad ...(2)$$

If ψ be the angle which the tangent to the curve at (x, y) makes with the positive direction of x-axis then $\tan\psi=\frac{dy}{dx}$ or

$$\tan\psi=\frac{2a}{y}\quad ....(3),\qquad [\text{using (2)}]$$

At x = a, then from (1), $y^2=4a.a=4a^2\Rightarrow y=\pm 2a$.

Hence, we get two points (a, 2a) and (a, –2a) on the curve.

At (a, 2a) x = a, y = 2a and let $\psi=\psi_1$.

$$\therefore\ from\ (3),\ \tan\psi_1=\frac{2a}{2a}=1=\tan 45°;$$

$$\Rightarrow\psi_1=45°.$$

At (a, –2a), x = a, y = –2a and let $\psi=\psi_2$.

$$\therefore\ from\ (3),\ \tan\psi_2=\frac{2a}{-2a}=-1=\tan 135°;$$

$$or\ \ \psi_2=135°.$$

Hence the required angle between tangents to (1) at (a, 2a) and (a, –2a) $=\psi_2-\psi_1=135°-45°=90°$.

This shows that the tangent lines to (1) at (a, 2a) and (a, –2a) are perpendicular to each other.

114. (a) Eccentrcity of $\frac{x^2}{25}-\frac{y^2}{25\sin^2\alpha}=1$ is $\sqrt{1+\sin^2\alpha}$.

Eccentricity of $\frac{x^2}{5\sin^2\alpha}+\frac{y^2}{5}=1$ is $\sqrt{1-\sin^2\alpha}$

Given, $\sqrt{1+\sin^2\alpha}=\sqrt{5}\sqrt{1-\sin^2\alpha}$

$\Rightarrow \sin^2\alpha=\frac{2}{3}$

$\Rightarrow \alpha=\sin^{-1}\sqrt{\frac{2}{3}}=\tan^{-1}\sqrt{2}$

115. (a)

p	q	$p\wedge q$	$(p\wedge q)\Rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

$\therefore (p\wedge q)\Rightarrow p$ is a tautology.

116. (c) We have, $f(x)=\lim_{n\to\infty}\frac{(2\sin x)^{2n}}{3^n-(2\cos x)^{2n}}$

$=\lim_{n\to\infty}\frac{(2\sin x)^{2n}}{(\sqrt{3})^{2n}-(2\cos x)^{2n}}$

f(x) is discontinuous when

$(\sqrt{3})^{2n}-(2\cos x)^{2n}=0$

i.e. $\cos x=\pm\frac{\sqrt{3}}{2}\Rightarrow x=n\pi\pm\frac{\pi}{6}$

117. (d) $V=5x-\frac{x^2}{6}\Rightarrow\frac{dV}{dt}=5\frac{dx}{dt}\frac{x}{3}\cdot\frac{dx}{dt}$

$\Rightarrow\frac{dx}{dt}=\frac{\frac{dV}{dt}}{\left(5-\frac{x}{3}\right)}$

$\Rightarrow\left(\frac{dx}{dt}\right)_{x=2}=\frac{5}{5-\frac{2}{3}}=\frac{15}{13}\text{cm/sec}$.

118. (b) Since vectors are coplanar

$\therefore \begin{vmatrix} a & 1 & 1\\ 1 & b & 1\\ 1 & 1 & c\end{vmatrix}=0$

$\Rightarrow \begin{vmatrix} a & 1 & 1\\ 1-a & b-1 & 0\\ 0 & 1-b & c-1\end{vmatrix}=0$ [Using R_2-R_1, R_3-R_2]

$\Rightarrow a(b-1)(c-1)-(1-a)\{(c-1)-(1-b)\}=0$

$\Rightarrow a(1-b)(1-c)+(1-a)(1-c)+(1-a)(1-b)=0$

$\Rightarrow (a-1+1)(1-b)(1-c)+(1-a)(1-c)+(1-a)(1-b)=0$

$\Rightarrow (1-b)(1-c)+(1-a)(1-c)+(1-a)(1-b)=(1-a)(1-b)(1-c)$

$\Rightarrow \frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1$

119. (c) If $A=\begin{bmatrix}3 & -2 & 4\\ 1 & 2 & -1\\ 0 & 1 & 1\end{bmatrix}$

and $A^{-1}=\frac{1}{k}\text{adj}(A)$ .....(i)

Also, we know $A^{-1}=\frac{\text{adj}(A)}{|A|}$ .....(ii)

$\therefore$ By comparing (i) and (ii)

$|A|=k$

$$\Rightarrow |A| = \begin{vmatrix} 3 & -2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{vmatrix}$$

$$= 3(2+1) + 2(1+0) + 4(1-0)$$

$$= 9 + 2 + 4 = 15$$

120. (d) Radius of circle

$$= \sqrt{4 + 9 - 9\sin^2\alpha - 13\cos^2\alpha} = 2|\sin\alpha|$$

If T be (h, k) then as in Q. 44

$$\tan\alpha = \frac{2|\sin\alpha|}{\sqrt{h^2 + k^2 + 4h - 6k + 9\sin^2\alpha + 13\cos^2\alpha}}$$

$$\Rightarrow h^2 + k^2 + 4h - 6k + 9\sin^2\alpha + 13\cos^2\alpha = 4\cos^2\alpha$$

$$\Rightarrow h^2 + k^2 + 4h - 6k + 9 = 0$$

$\therefore$ Locus of T is $x^2 + y^2 + 4x - 6y + 9 = 0$

PART - IV (ENGLISH)

121. (c)

122. (d)

123. (a)

124. (c)

125. (b) The best pronunciation of the word 'sorbet' is sore-bay.