VITEEE 2021 Question Paper

Vellore Institute of Technology Engineering Entrance Examination

SOLVED PAPER (memory based) VITEEE 2021

GENERAL INSTRUCTIONS

- This question paper contains total 80 questions divided into four parts :
  Part I : Physics Q. No - 1 to 25
  Part II : Chemistry Q. No - 26 to 50
  Part III : Mathematics Q. No - 51 to 75
  Part IV : English & Logical reasoning Q. No - 76 to 80
- All questions are multiple choice questions with four options, only one of them is correct.
- For each correct response, the candidate will get 1 mark.
- There is no negative marking for the wrong answer.
- The test is of 1½ hours duration.

PART - I (PHYSICS)

1. The distance of the centres of moon and earth is D. The mass of earth is 81 times the mass of the moon. At what distance from the centre of the earth, the gravitational force will be zero?

(a) $\frac{D}{2}$ (b) $\frac{2D}{3}$

(c) $\frac{4D}{3}$ (d) $\frac{9D}{10}$

2. Two wires A and B are of the same material. Their lengths are in the ratio of 1 : 2 and the diameter are in the ratio 2 : 1. If they are pulled by the same force, then increase in length will be in the ratio of

(a) 2 : 1 (b) 1 : 4

(c) 1 : 8 (d) 8 : 1

3. If $x = at + bt^2$, where x is the distance travelled by the body in kilometers while t is the time in seconds, then the unit of b is

(a) km/s (b) kms

(c) km/s^2 (d) kms^2

4. A soap bubble of radius r_1 is placed on another soap bubble of radius r_2 ($r_1 < r_2$). The radius R of the soapy film separating the two bubbles is

(a) $r_2 + r_2$ (b) $\frac{r_2 - r_1}{r_1 r_2}$

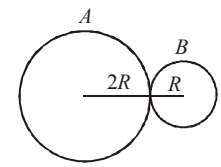
(c) $\frac{r_1 r_2}{r_2 - r_1}$ (d) $\sqrt{r_1^2 + r_2^2}$

5. A charge q is moving with a velocity v parallel to a magnetic field B. Force on the charge due to magnetic field is

(a) q v B (b) q B/v

(c) zero (d) B v/q

6. Two spheres *A* and *B* of masses *m* and 2*m* and radii 2*R* and *R* respectively are placed in contact as shown. The COM of the system lies



(a) inside *A*

(b) inside *B*

(c) at the point of contact

(d) None of these

7. Identify the correct statement.
(a) Static friction depends on the area of contact
(b) Kinetic friction depends on the area of contact
(c) Coefficient of kinetic friction is more than the coefficient of static friction
(d) Coefficient of kinetic friction is less than the coefficient of static friction

8. The distance travelled by a particle starting from rest and moving with an acceleration $\frac{4}{3}\text{ms}^{-2}$, in the third second is:
(a) 6m (b) 4m
(c) $\frac{10}{3}$m (d) $\frac{19}{3}$m

9. Photoelectric work function of a metal is 1eV. Light of wavelength $\lambda = 3000$ Å falls on it. The photo electrons come out with a maximum velocity of :
(a) 10 metres/sec (b) 10^2 metres/sec
(c) 10^4 metres/sec (d) 10^6 metres/sec

10. The coefficient of apparent expansion of mercury in a glass vessel is 153×10^{-6}/°C and in a steel vessel is 144×10^{-6}/°C. If α for steel is 12×10^{-6}/°C , then that of glass is
(a) 9×10^{-6}/°C (b) 6×10^{-6}/°C
(c) 36×10^{-6}/°C (d) 27×10^{-6}/°C

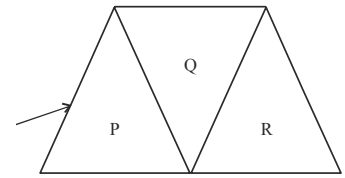
11. A step-up transformer operates on a 230 *V* line and supplies a load of 2 ampere. The ratio of the primary and secondary windings is 1 : 25. The current in the primary is
(a) 15 *A* (b) 50 *A*
(c) 25 *A* (d) 12.5 *A*

12. Two bodies of same mass are projected with the same velocity at an angle 30° and 60° respectively. The ratio of their horizontal ranges will be
(a) 1 : 1 (b) 1 : 2
(c) 1 : 3 (d) $2:\sqrt{2}$

13. Two point charges $+3\mu C$ and $+8\mu C$ repel each other with a force of 40*N*. If a charge of $-5\mu C$ is added to each of them, then the force between them will become
(a) –10*N* (b) +10*N*
(c) +20*N* (d) –20*N*

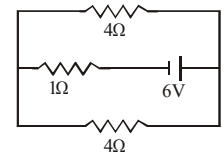
14. A sphere rolls down on an inclined plane of inclination θ. What is the acceleration as the sphere reaches the bottom?
(a) $\frac{5}{7}g\sin\theta$ (b) $\frac{3}{5}g\sin\theta$
(c) $\frac{2}{7}g\sin\theta$ (d) $\frac{2}{5}g\sin\theta$

15. A given ray of light suffers minimum deviation in an equilateral prism P. Additional prisms Q and R of identical shape and of same material as that of P are now combined as shown in figure. The ray will now suffer



(a) greater deviation
(b) no deviation
(c) same deviation as before
(d) total internal reflection

16. The current in the 1Ω resistor shown in the circuit is



(a) $\frac{2}{3}$ A (b) 3 A
(c) 6 A (d) 2 A

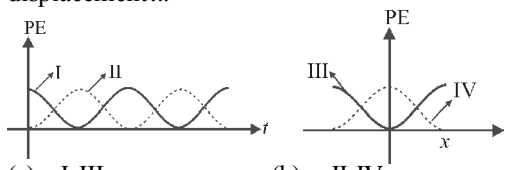
17. The root mean square velocity of hydrogen molecules at 300 K is 1930 metre/sec. Then the r.m.s velocity of oxygen molecules at 1200 K will be
(a) 482.5 metre/sec (b) 965 metre/sec
(c) 1930 metre/sec (d) 3860 metre/sec

18. Lenz's law gives
(a) the magnitude of the induced e.m.f.
(b) the direction of the induced current
(c) both the magnitude and direction of the induced current
(d) the magnitude of the induced current

19. A parallel plate capacitor with air between the plates has a capacitance of 8 pF. Calculate the capacitance if the distance between the plates is reduced by half and the space between them is filled with a substance of dielectric constant. (k = 6)

(a) 72 pF (b) 81 pF
(c) 84 pF (d) 96 pF

20. For a particle executing S.H.M. the displacement x is given by $x = A \cos \omega t$. Identify the graph which represents the variation of potential energy (P.E.) as a function of time t and displacement x.



(a) I, III (b) II, IV
(c) II, III (d) I, IV

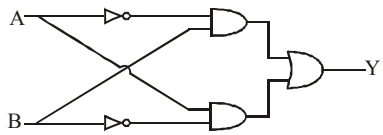
21. A radioactive sample contains 10^{-3} kg each of two nuclear species A and B with half-life 4 days and 8 days respectively. The ratio of the amounts of A and B after a period of 16 days is

(a) 1 : 2 (b) 4 : 1
(c) 1 : 4 (d) 2 : 1

22. A string of 7 m length has a mass of 0.035 kg. If tension in the string is 60.5 N, then speed of a wave on the string is

(a) 77 m/s (b) 102 m/s
(c) 110 m/s (d) 165 m/s

23. The following circut represents



(a) OR gate (b) AND gate
(c) NAND gate (d) None of these

24. A straight section PQ of a circuit lies along the X-axis from $x = -\frac{a}{2}$ to $x = \frac{a}{2}$ and carries a steady current i. The magnetic field due to the section PQ at a point $X = +a$ will be

(a) proportional to a
(b) proportional to a^2
(c) proportional to $1/a$
(d) zero

25. A source producing sound of frequency 170 Hz is approaching a stationary observer with a velocity 17 ms^{-1}. The apparent change in the wavelength of sound heard by the observer is (speed of sound in air = 340 ms^{-1})

(a) 0.1 m (b) 0.2 m
(c) 0.4 m (d) 0.5 m

PART - II (CHEMISTRY)

26. Consider the following reactions:

$NaCl + K_2Cr_2O_7 + H_2SO_4$ (Conc.) $\rightarrow$ (A) + Side products

(A) + NaOH $\rightarrow$ (B) + Side products

(B) + H_2SO_4 (dilute) + H_2O_2 $\rightarrow$ (C) + Side products

The sum of the total number of atoms in one molecule each of (A), (B) and (C) is _____.

(a) 18 (b) 15
(c) 21 (d) 20

27. Xenon hexafluoride on partial hydrolysis produces compounds 'X' and 'Y'. Compounds 'X', 'Y' and the oxidation state of Xe are respectively :

(a) $XeOF_4$(+6) and XeO_3(+6)
(b) XeO_2(+4) and XeO_3(+6)
(c) $XeOF_4$(+6) and XeO_2F_2(+6)
(d) XeO_2F_2(+6) and XeO_2(+4)

28. The edge length of unit cell of a metal having molecular weight 75 g/mol is 5Å which crystallizes in cubic lattice. If the density is 2g/cc then find the radius of metal atom. ($N_A = 6 \times 10^{23}$). Give the answer in pm.

(a) 217 pm (b) 210 pm
(c) 220 pm (d) 205 pm

29. Consider the following statements:

I. Increase in concentration of reactant increases the rate of a zero order reaction.
II. Rate constant k is equal to collision frequency A if $E_a = 0$.
III. Rate constant k is equal to collision frequency A if $E_a = \infty$.
IV. In k vs T is a straight line.
V. In k vs 1/T is a straight line.

Correct statements are

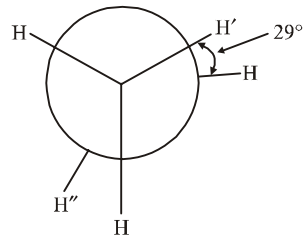
(a) I and IV (b) II and V
(c) III and IV (d) II and III

30. To deposit 0.634 g of copper by electrolysis of aqueous cupric sulphate solution, the amount of electricity required (in coulombs) is

(a) 1930 (b) 3960

(c) 4825 (d) 9650

31. In the following skew conformation of ethane, H′ – C – C – H″ dihedral angle is :

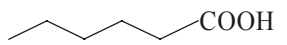


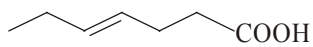
(a) 58° (b) 149°

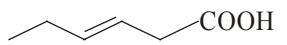
(c) 151° (d) 120°

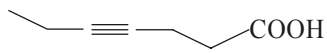
32. What is the product of following reaction?

$$\text{Hex-3-ynal} \xrightarrow[\text{(ii) } PBr_3,\ \text{(iii) Mg/ether, (iv) } CO_2/H_3O^+]{\text{(i) } NaBH_4} ?$$

(a) 

(b) 

(c) 

(d) 

33. In the following sequence of reactions,

$$CH_3CH_2OH \xrightarrow{P+I_2} A \xrightarrow[\text{ether}]{Mg} B \xrightarrow{HCHO} C \xrightarrow{H_2O} D$$

the compound D is

(a) propanal (b) butanal

(c) *n*-butyl alcohol (d) *n*-propyl alcohol.

34. Which of the following reactions can produce aniline as main product?

(a) $C_6H_5NO_2 + Zn/KOH$

(b) $C_6H_5NO_2 + Zn/NH_4Cl$

(c) $C_6H_5NO_2 + LiAlH_4$

(d) $C_6H_5NO_2 + Zn/HCl$

35. Secondary structure of protein refers to

(a) mainly denatured proteins and structure of prosthetic groups

(b) three -dimensional structure, especially the bond between amino acid residues that are distinct from each other in the polypeptide chain

(c) linear sequence of amino acid residues in the polypetide chain

(d) regular folding patterns of continous portions of the polypeptide chain

36. The increasing order for the values of e/m (charge/mass) is

(a) e, p, n, α (b) n, p, e, α

(c) n, p, α, e (d) n, α, p, e

37. In which of the following pairs both the ions are coloured in aqueous solutions ?

(a) Sc^{3+}, Ti^{3+} (b) Sc^{3+}, Co^{2+}

(c) Ni^{2+}, Cu^{+} (d) Ni^{2+}, Ti^{3+}

38. The total number of possible isomers for square-planar $[Pt(Cl)(NO_2)(NO_3)(SCN)]^{2-}$ is:

(a) 16 (b) 12

(c) 8 (d) 24

39. For the reaction,

$2SO_2(g) + O_2(g) \rightleftharpoons 2\,SO_3(g)$,

$\Delta H = -57.2\ kJ\ mol^{-1}$ and $K_c = 1.7\times10^{16}$

Which of the following statement is INCORRECT ?

(a) The equilibrium constant is large suggestive of reaction going to completion and so no catalyst is required.

(b) The equilibrium will shift in forward direction as the pressure increases.

(c) The equilibrium constant decreases as the temperature increases.

(d) The addition of inert gas at constant volume will not affect the equilibrium constant.

40. The half-life of a reaction is inversely proportional to the square of the initial concentration of the reactant. Then the order of the reaction is

(a) 0 (b) 1

(c) 2 (d) 3

41. A galvanic cell is set up from electrodes A and B

Electrode A : $Cr_2O_7^{2-}/Cr^{3+}$, $E^\circ_{red} = +1.33\ V$

Electrode B : Fe^{3+}/Fe^{2+}, $E^\circ_{red} = 0.77\ V$

Which of the following statements is false ?

(a) Standard e.m.f of the cell is 0.56 V

(b) Current will flow from electrode A to B in the external circuit

(c) A will act as cathode and have positive polarity

(d) None of these

42. Keto-enol tautomerism is observed in :

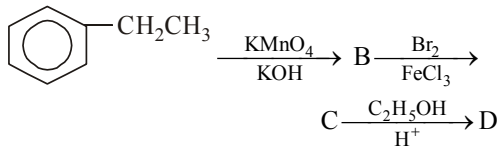
(a) $H_5C_6-\overset{\overset{O}{\|}}{C}-CH_2-\overset{\overset{O}{\|}}{C}-C_6H_5$

(b) $H_5C_6-\overset{\overset{O}{\|}}{C}-CH_3$

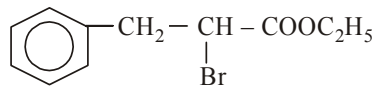
(c) $H_5C_6-\overset{\overset{O}{\|}}{C}-H$

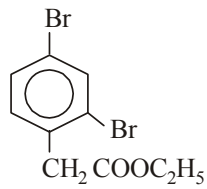
(d) Both (a) and (b)

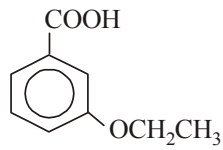
43. In a set of reactions, ethylbenzene yield a product D.

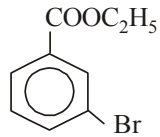


Identify D :

(a) 

(b) 

(c) 

(d) 

44. What will be the final product in the following reaction sequence –

$$CH_3CH_2CN \xrightarrow{H^+/H_2O} A \xrightarrow[\Delta]{NH_3} B \xrightarrow{NaOBr} C$$

(a) $CH_3CH_2CONH_2$

(b) CH_3CH_2COBr

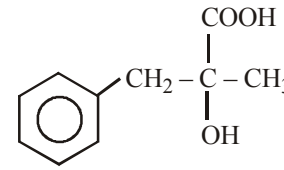
(c) $CH_3CH_2NH_2$

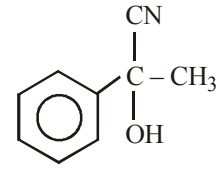
(d) $CH_3CH_2CH_2NH_2$

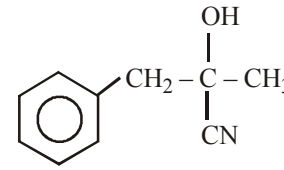
45. In a set of reactions acetic acid yielded a product D.

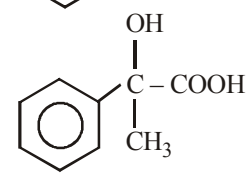
$$CH_3COOH \xrightarrow{SOCl_2} (A) \xrightarrow[AlCl_3]{Benzene} (B) \xrightarrow{HCN} (C) \xrightarrow{HOH} (D)$$

The structure of (D) would be –

(a) 

(b) 

(c) 

(d) 

46. In fructose, the possible optical isomers are

(a) 12 (b) 8

(c) 16 (d) 4

47. The position of both, an electron and a helium atom is known within 1.0 nm. Further the momentum of the electron is known within 5.0×10^{-26} kg ms^{-1}. The minimum uncertainty in the measurement of the momentum of the helium atom is

(a) 50 kg ms^{-1} (b) 80 kg ms^{-1}

(c) 8.0×10^{-26} kg ms^{-1} (d) 5.0×10^{-26} kgms^{-1}

48. The value of $\log_{10} K$ for a reaction $A \rightleftharpoons B$ is

(Given : $\Delta_r H^\circ_{298K} = -54.07$ kJ mol^{-1}, $\Delta_r S^\circ_{298K} = 10$ JK^{-1} mol^{-1} and $R = 8.314$ JK^{-1} mol^{-1}; $2.303 \times 8.314 \times 298 = 5705$)

(a) 5 (b) 10
(c) 95 (d) 100

49. If $C(s) + O_2(g) \longrightarrow CO_2(g)$; $\Delta H = R$ and

$CO(g) + \frac{1}{2}O_2(g) \longrightarrow CO_2(g)$; $\Delta H = S$,

then heat of formation of CO is:

(a) R + S (b) R – S
(c) R × S (d) S – R

50. Which of the following compounds does not follow Markownikoff's law ?

(a) $CH_3CH=CH_2$ (b) CH_2CHCl
(c) $CH_3CH=CHCH_3$ (d) None

PART - III (MATHEMATICS)

51. The value of c in Rolle's Theorem for the function $f(x) = e^x \sin x$, $x \in [0, \pi]$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{2}$ (d) $\frac{3\pi}{4}$

52. The equations $2x + 3y + 4 = 0$; $3x + 4y + 6 = 0$ and $4x + 5y + 8 = 0$ are

(a) consistent with unique solution
(b) inconsistent
(c) consistent with infinitely many solutions
(d) None of the above

53. The shortest distance between the lines $x = y + 2 = 6z - 6$ and $x + 1 = 2y = -12z$ is

(a) $\frac{1}{2}$ (b) 2
(c) 1 (d) $\frac{3}{2}$

54. If the tangent at P(1, 1) on $y^2 = x(2 - x)^2$ meets the curve again at Q, then Q is

(a) (2, 2) (b) (–1, –2)
(c) $\left(\frac{9}{4}, \frac{3}{8}\right)$ (d) None of these

55. If $f(x) = x + \frac{x}{1+x} + \frac{x}{(1+x)^2} + \ldots$ to ∞, then at $x = 0$, f(x)

(a) has no limit
(b) is discontinuous
(c) is continuous but not differentiable
(d) is differentiable

56. Radius of the circle $(x + 5)^2 + (y - 3)^2 = 36$ is

(a) 2 (b) 3
(c) 6 (d) 5

57. If $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{c} = -\hat{i} + 2\hat{k}$ then $|\vec{c}|$. $\vec{a}$ is equal to :

(a) $2\sqrt{5}\,\hat{i} + 2\sqrt{5}\,\hat{j} + \sqrt{5}\,\hat{k}$
(b) $2\sqrt{5}\,\hat{i} - 2\sqrt{5}\,\hat{j} + \sqrt{5}\,\hat{k}$
(c) $\sqrt{5}\,\hat{i} + \sqrt{5}\,\hat{j} + \sqrt{5}\,\hat{k}$
(d) $\sqrt{5}\,\hat{i} + 2\sqrt{5}\,\hat{j} + \sqrt{5}\,\hat{k}$

58. If $(-4, 5)$ is one vertex and $7x - y + 8 = 0$ is one diagonal of a square, then the equation of second diagonal is

(a) $x + 3y = 21$ (b) $2x - 3y = 7$
(c) $x + 7y = 31$ (d) $2x + 3y = 21$

59. $p \Rightarrow q$ can also be written as

(a) $p \Rightarrow \sim q$ (b) $\sim p \vee q$
(c) $\sim q \Rightarrow \sim p$ (d) None of these

60. Let $\int \frac{x^{1/2}}{\sqrt{1-x^3}}\,dx = \frac{2}{3}\,gof(x) + C$, then

(a) $f(x) = \sqrt{x}$
(b) $f(x) = x^{3/2}$ and $g(x) = \sin^{-1}x$
(c) $f(x) = x^{2/3}$
(d) None of these

61. Which one of the following is an infinite set ?

(a) The set of human beings on the earth
(b) The set of water drops in a glass of water
(c) The set of trees in a forest
(d) The set of all primes

62. The domain of the function $\sqrt{x^2 - 5x + 6} + \sqrt{2x + 8 - x^2}$ is

(a) [2, 3] (b) [–2, 4]
(c) $[-2, 2] \cup [3, 4]$ (d) $[-2, 1] \cup [2, 4]$

63. Area bounded by the curve y = log x and the coordinate axes is

(a) 2 (b) 1

(c) 5 (d) $2\sqrt{2}$

64. The angle of intersection to the curve $y = x^2$, $6y = 7 - x^3$ at (1, 1) is :

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) π

65. Angle formed by the positive Y-axis and the tangent to $y = x^2 + 4x - 17$ at $\left(\frac{5}{2}, \frac{-3}{4}\right)$ is

(a) $\tan^{-1} 9$ (b) $\frac{\pi}{2} - \tan^{-1} 9$

(c) $\frac{\pi}{2} + \tan^{-1} 9$ (d) $\frac{\pi}{2}$

66. The value of $(1 + i)^4 \left(1 + \frac{1}{i}\right)^4$ is

(a) 12 (b) 2

(c) 8 (d) 16

67. The relation R defined on the set A = {1, 2, 3, 4, 5} by R = {(x, y) : $|x^2 - y^2| < 16$} is given by

(a) {(1, 1), (2, 1), (3, 1), (4, 1), (2, 3)}

(b) {(2, 2), (3, 2), (4, 2), (2, 4)}

(c) {(3, 3), (4, 3), (5, 4), (3, 4)}

(d) None of these

68. $\int \frac{2dx}{(e^x + e^{-x})^2} =$

(a) $\frac{-e^{-x}}{(e^x + e^{-x})} + C$ (b) $\frac{-1}{(e^x + e^{-x})} + C$

(c) $\frac{1}{(e^x + 1)^2} + C$ (d) $\frac{1}{(e^x + e^{-x})} + C$

69. The value of $\tan^{-1}(1) + \tan^{-1}(0) + \tan^{-1}(2) + \tan^{-1}(3)$ is equal to

(a) π (b) $\frac{5\pi}{4}$

(c) $\frac{\pi}{2}$ (d) None of these

70. In a culture the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2,00,000 if the rate of growth of bacteria is proportional to the number present.

(a) $\frac{2}{\log \frac{11}{10}}$ (b) $\frac{2 \log 2}{\log\left(\frac{11}{10}\right)}$

(c) $\frac{\log 2}{\log 11}$ (d) $\frac{\log 2}{\log\left(\frac{11}{10}\right)}$

71. What is the angle between the two straight lines $y = (2 - \sqrt{3})x + 5$ and $y = (2 + \sqrt{3})x - 7$?

(a) 60° (b) 45°

(c) 30° (d) 15°

72. If the angle θ between the line $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ and the plane $2x - y + \sqrt{\lambda}\, z + 4 = 0$ is such that $\sin \theta = \frac{1}{3}$ then the value of λ is

(a) $\frac{5}{3}$ (b) $\frac{-3}{5}$

(c) $\frac{3}{4}$ (d) $\frac{-4}{3}$

73. The distance of the point (–5, –5, –10) from the point of intersection of the line $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$ is

(a) 13 (b) 12

(c) $4\sqrt{15}$ (d) $10\sqrt{2}$

74. $\int_{\log \sqrt{\pi/2}}^{\log \sqrt{\pi}} e^{2x} \sec^2\left(\frac{1}{3}e^{2x}\right) dx$ is equal to :

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\frac{3\sqrt{3}}{2}$ (d) $\frac{1}{2\sqrt{3}}$

75. If $\frac{|x + 3| + x}{x + 2} > 1$, then $x \in$

(a) (–5, –2) (b) (–1, ∞)

(c) (–5, –2) ∪ (–1, ∞) (d) None of these

PART - IV (ENGLISH & LOGICAL REASIONING)

Directions (76-78): Study the paragraph and answer the questions that follow.

A training calendar and schedule for Fire Agency Specialties Team (F.A.S.T.) membership is available in this office to all applicants for F.A.S.T. membership. Training will take place the third week of each month. Classes will be taught on Monday afternoons, Wednesday evenings, and Saturday afternoons.

So that the F.A.S.T. can maintain a high level of efficiency and preparedness for emergency response situations, its members must meet certain requirements.

First, in order for you to be considered for membership on F.A.S.T., your department must be a member of the F.A.S.T. organization, and you must have written permission from your fire chief or your department's highest ranking administrator.

Once active, you must meet further requirements to maintain active status. These include completion of technician-level training and certification in hazardous material (hazmat) operations. In addition, after becoming a member, you must also attend a minimum of 50% of all drills conducted by F.A.S.T. and go to at least one F.A.S.T. conference. You may qualify for alternative credit for drills by proving previous experience in actual hazmat emergency response.

If you fail to meet minimum requirements, you will be considered inactive, and the director of your team will be notified. You will be placed back on active status only after you complete the training necessary to meet the minimum requirements.

76. Potential F.A.S.T. members can attend less than half of F.A.S.T. drills if they
- (a) complete technician-level training requirements.
- (b) indicate prior real emergency experience.
- (c) receive permission from their fire chief.
- (d) enroll in three weekly training sessions.

77. Which of the following is the main subject of the passage?
- (a) preparing for hazmat certification
- (b) the main goal of F.A.S.T.
- (c) completing F.A.S.T. membership requirements
- (d) learning about your department's F.A.S.T. membership

78. Applicants must be available for training
- (a) three days each month.
- (b) three days each week.
- (c) every third month.
- (d) for 50% of classes.

79. Jatin starting from a fixed point, goes 15 m towards North and then after turning to his right, he goes 15 m. Then, he goes 10 m, 15 m and 15 m after turning to his left each time. How far is he from his starting point ?
- (a) 15 m (b) 5m
- (c) 10 m (d) 20 m

80. Examine the following statements:
1. All members of Mohan's family are honest.
2. Some members of Mohan's family are not employed.
3. Some employed persons are not honest.
4. Some honest persons are not employed.

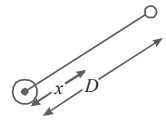
Which one of the following inferences can be drawn from the above statements?
- (a) All members of Mohan's family are employed
- (b) The employed members of Mohan's family are honest
- (c) The honest members of Mohan's family are not employed
- (d) The employed member of Mohan's family are not honest

SOLUTIONS

PART - I (PHYSICS)

1. (d) $\frac{Gm_e}{x^2} = \frac{Gm_m}{(D-x)^2}$

or $\frac{G(81m)}{x^2} = \frac{m}{(D-x)^2}$



$\therefore \quad x = \frac{9D}{10}$.

2. (c) We know that Young's modulus

$$Y = \frac{F}{\pi r^2} \times \frac{L}{\ell}$$

Since Y, F are same for both the wires, we have,

$$\frac{1}{r_1^2}\frac{L_1}{\ell_1} = \frac{1}{r_2^2}\frac{L_2}{\ell_2} \quad \text{or,} \quad \frac{\ell_1}{\ell_2} = \frac{r_2^2 \times L_1}{r_1^2 \times L_2} =$$

$$\frac{(D_2/2)^2 \times L_1}{(D_1/2)^2 \times L_2}$$

$$\text{or,} \quad \frac{\ell_1}{\ell_2} = \frac{D_2^2 \times L_1}{D_1^2 \times L_2} = \frac{D_2^2}{(2D_2)^2} \times \frac{L_2}{2L_2} = \frac{1}{8}$$

So, $\ell_1 : \ell_2 = 1:8$

3. (c) $[x] = [bt^2]$. Hence $[b] = [x/t^2] = km/s^2$.

4. (c)

5. (c)

6. (c) As mass of B is twice that of A.

$$\text{And,} \frac{r_1}{r_2} = \frac{m_2}{m_1}$$

7. (d) $\mu_k < \mu_s$ coefficient of static friction is always greater than kinetic friction.

8. (c) Distance travelled in the nth second is given by $d_n = u + \frac{a}{2}(2n-1)$

put $u = 0$, $a = \frac{4}{3} ms^{-2}$, $n = 3$

$$\therefore d = 0 + \frac{4}{3 \times 2}(2 \times 3 - 1) = \frac{4}{6} \times 5 = \frac{10}{3} m$$

9. (d) $h\nu = W + \frac{1}{2}mv^2$ or $\frac{hc}{\lambda} = W + \frac{1}{2}mv^2$

Here $\lambda = 3000$ Å $= 3000 \times 10^{-10}$ m

and $W = 1$ eV $= 1.6 \times 10^{-19}$ joule

$$\therefore \frac{(6.6\times10^{-34})(3\times10^{8})}{3000\times10^{-10}}$$

$$= (1.6\times10^{-19}) + \frac{1}{2}\times(9.1\times10^{-31})v^2$$

Solving we get, $v \cong 10^6$ m/s

10. (a) $\gamma_{real} = \gamma_{app.} + \gamma_{vessel}$

So $(\gamma_{app.} + \gamma_{vessel})_{glass} = (\gamma_{app} + \gamma_{vessel})_{steel}$

$$\Rightarrow 153\times10^{-6} + (\gamma_{vessel})_{glass}$$

$$= 144\times10^{-6} + (\gamma_{vessel})_{steel}$$

Further,

$$(\gamma_{vessel})_{steel} = 3\alpha = 3\times(12\times10^{-6})$$

$$= 36\times10^{-6}/°C$$

$$\Rightarrow 153\times10^{-6} + (\gamma_{vessel})_{glass}$$

$$= 144\times10^{-6} + 36\times10^{-6}$$

$$\Rightarrow (\gamma_{vessel})_{glass} = 3\alpha = 27\times10^{-6}/°C$$

$$\Rightarrow \alpha = 9\times10^{-6}/°C$$

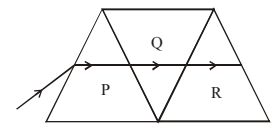
11. (b)

12. (a) Horizontal range is same when angle of projection is θ or $(90° - \theta)$.

13. (a)

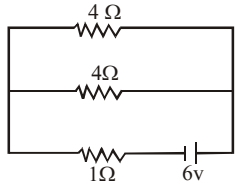
14. (a) $a = \frac{g\sin\theta}{1 + \frac{K^2}{R^2}} = \frac{g\sin\theta}{1 + \frac{2}{5}} = \frac{5}{7}g\sin\theta$

15. (c) When the ray suffers minimum deviation, it becomes parallel to the base of prism P. As prisms Q and R are of same material and have identical shape, therefore, the ray continues to be parallel to base of Q and R. Hence final deviation of the ray remains the same as before.



16. (d) Two 4Ω resistors are in parallel combination. Their equivalent resistance

$$= \frac{4\times4}{4+4} = \frac{16}{8} = 2\Omega$$



$\therefore$ Total resistance of the network = 2 + 1 = 3Ω

$\therefore$ Current through 1Ω resistor $= \frac{6}{3} = 2A$

17. (b) Root-mean square-velocity is given by

$$v_{rms} = \sqrt{\frac{3RT}{M}} \text{ i.e., } v_{rms} \propto \sqrt{\left(\frac{T}{M}\right)}$$

$$\frac{(v_{rms})O_2}{(v_{rms})H_2} = \sqrt{\left[\frac{T_{O_2}}{T_{H_2}} \times \frac{M_{H_2}}{M_{O_2}}\right]}$$

$$= \sqrt{\left[\left(\frac{1200}{300}\right) \times \left(\frac{2}{32}\right)\right]} = \frac{1}{2}$$

$$\therefore \quad (v_{rms})O_2 = (v_{rms})H_2 \times \frac{1}{2} = \frac{1930}{2} =$$

965 m/s

18. (b) Lenz's law helps to identify the direction of induced current.

19. (d) Capacity of parallel plate capacitor

$$C = \frac{k\varepsilon_0 A}{d} \qquad (\text{For air } k_r = 1)$$

$$\text{So, } \frac{\varepsilon_0 A}{d} = 8 \times 10^{-12}$$

$$\text{If } d \to \frac{d}{2} \text{ and } k_r \to 6 \text{ then new capacitance}$$

$$C' = 6 \times \frac{\varepsilon_0 A}{d/2} = 12\frac{\varepsilon_0 A}{d} = 12 \times 8\,pF = 96\,pF$$

20. (a) In $x = A\cos\omega t$, the particle starts oscillating from extreme position. So at t = 0, its potential energy is maximum.

21. (c) Ratio of number of half life taken is given as:

After 16 days

$$n_{A1/2} = \frac{16}{4} = 4; \quad n_{B1/2} = \frac{16}{8} = 2$$

$$N = N_0\left(\frac{1}{2}\right)^n \Rightarrow \frac{N_A}{N_B}$$

$$= \frac{1}{2^4} : \frac{1}{2^2} = 2^2 : 2^4$$

$$= 4 : 16, = 1 : 4$$

22. (c) Given : Length (l) = 7 m

Mass (M) = 0.035 kg and tension (T) = 60.5 N. We know that mass of string per unit length (m)

$$= \frac{0.035}{7} = 0.005 \text{ kg/m}$$

and speed of

$$\text{wave} = \sqrt{\frac{T}{m}} = \sqrt{\frac{60.5}{0.005}} = 110 \text{ m/s}.$$

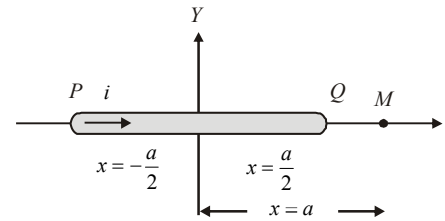
23. (d) Output of upper AND gate = $\overline{A}B$

Output of lower AND gate = $A\overline{B}$

$\therefore$ Output of OR gate, $Y = A\overline{B} + B\overline{A}$

This is boolean expression for XOR gate.

24. (d) Magnetic field at a point on the axis of a current carrying wire is always zero.



25. (a) $\lambda = \frac{v}{n} = \frac{340}{170} = 2m, \quad n' = \frac{340}{340-17} \times 170$

$n' = 178.9\,Hz$

$$\text{Now} \quad \lambda' = \frac{v}{n'} = \frac{340}{178.9} = 1.9$$

$\Rightarrow \lambda - \lambda' = 2 - 1.9 = 0.1$m

PART - II (CHEMISTRY)

26. (a) $4NaCl + K_2Cr_2O_4 + 3H_2SO_4 \longrightarrow \underset{(A)}{2CrO_2Cl_2} + K_2SO_4 + \underset{(Conc.)}{2NaSO_4} + 3H_2O$

$$\underset{(A)}{CrO_2Cl_2} + NaOH \longrightarrow \underset{(B)}{Na_2CrO_4} + H_2O + NaCl$$

$$\underset{(B)}{Na_2CrO_4} + \underset{(dilute)}{H_2SO_4} + H_2O_2 \longrightarrow \underset{(C)}{CrO_5} + Na_2SO_4 + H_2O$$

The sum of total no. of atoms in one molecules each of A, B & C = 5 + 7 + 6 = 18

27. (c) $XeF_6 + H_2O \xrightarrow[\text{hydrolysis}]{\text{partial}} \underset{\text{'X'}}{\overset{+6}{Xe}}OF_4 + 2HF$

$$\underset{\text{'X'}}{XeOF_4} + H_2O \longrightarrow \underset{\text{'Y'}}{\overset{+6}{Xe}O_2F_2} + 2HF$$

28. **(a)** $\rho = \dfrac{ZM}{N_A V}$

$$Z = \frac{\rho N_A V}{M} = \frac{2 \times 6 \times 10^{23} \times (5 \times 10^{-8})^3}{75}$$

$Z = 2$, which represents *bcc* structure

$\therefore \quad r = \dfrac{\sqrt{3}}{4}a = \dfrac{\sqrt{3}}{4} \times 5 = 2.165\text{Å} = 216.5 \text{ pm} \approx 217 \text{ pm}$

29. **(b)** According to Arrhenius equation,

$k = Ae^{-E_a/RT}$

$\therefore$ when $E_a = 0$, $k = A$

Also ln k vs 1/T is a straight line with slope $= -E_a/R$.

$\therefore$ Statements (ii) and (v) are correct.

30. **(a)** In the electrolysis of cupric sulphate, the reaction that occurs at cathode is

$$Cu^{2+} + 2e^- \longrightarrow Cu$$

Thus 2F or 2 × 96500 C of electricity is required to deposit

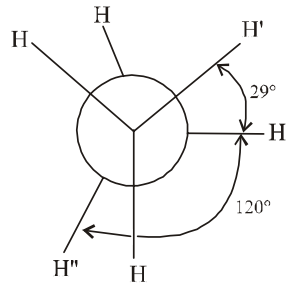
= 1 mol of Cu = 63.5 g of Cu

It means that to deposit 63.5 g of Cu, the amount of electricity required = 2 × 96500 C

So, to deposit 0.634 g of Cu, the amount of electricity required $= \dfrac{2 \times 96500}{63.5} \times 0.634$

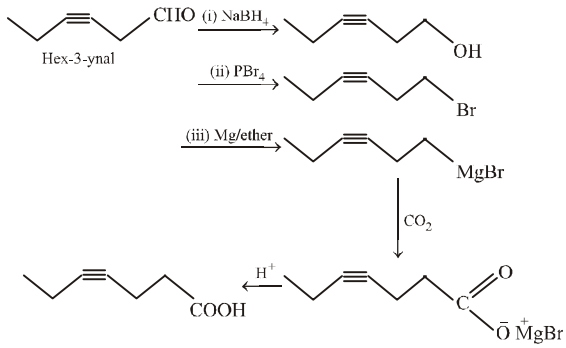
$\approx 1930 \text{ C}$

31. **(b)**



$\therefore$ Angle between H' and H" = 120° + 29° = 149°

32. **(d)**



33. **(d)** $CH_3CH_2OH \xrightarrow{P+I_2} \underset{A}{CH_3CH_2I}$

$\xrightarrow[\text{Ether}]{Mg} \underset{(B)}{CH_3CH_2MgI}$

$\xrightarrow{HCHO}$ $H-\overset{H}{\overset{|}{C}}-OMgI$ (with CH_2CH_3)

(C)

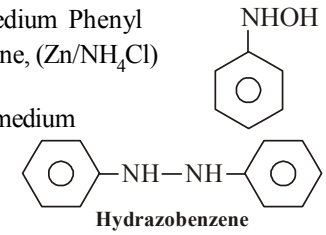
$\xrightarrow{H_2O}$ $H-\overset{H}{\overset{|}{C}}-OH$ (with CH_2CH_3)

(D)

n–Propylalcohol

34. **(d)** Various products are formed when nitroarenes are reduced. These are given below for $C_6H_5NO_2$.

	Medium	Main product
(d)	In acidic medium (metal/HCl)	Aniline ($C_6H_5NH_2$)
(b)	In neutral medium	Phenyl hydroxylamine, (Zn/NH_4Cl)
(a)	In alkaline medium	Hydrazobenzene



Thus, aniline will be main product in case of (d).

35. **(d)**

36. **(d)** $\dfrac{e}{m}$ for (i) neutron $= \dfrac{0}{1} = 0$

(ii) α-particle $= \dfrac{2}{4} = 0.5$

(iii) proton $= \dfrac{1}{1} = 1$

(iv) electron $= \dfrac{1}{1/1837} = 1837$

37. **(d)** Sc^{3+} (Z = 18) : $1s^2, 2s^2p^6, 3s^2p^6d^0, 4s^0$; no unpaired electron.

Cu^+ (Z = 28) : $1s^2, 2s^2p^6, 3s^2p^6d^{10}, 4s^0$; no unpaired electron.

$Ni^{2+}(Z=26): 1s^2, 2s^2p^6, 3s^2p^6d^8, 4s^0$;
unpaired electrons are present.
$Ti^{3+}(Z=19): 1s^2, 2s^2p^6, 3s^2p^6d^1, 4s^0$;
unpaired electron is present
$Co^{2+}(Z=25): 1s^2, 2s^2p^6, 3s^2p^6d^7, 4s^0$;
unpaired electrons are present
So from the given options the only correct combination is Ni^{2+} and Ti^{3+}.

38. **(b)** The square planar complex of the type $[Mabcd]^{n\pm}$, where all four ligands are different, has 3 geometrical isomers. But if one of the ligands is ambidentate, then $2 \times 3 = 6$ geometrical isomers are possible. But if two ligands are ambidentate, then $4 \times 3 = 12$ geometrical isomers are possible. In the given example, NO_2^- and SCN^- are ambidentate ligands.

39. **(a)** Equilibrium constant has no relation with catalyst. Catalyst only affects the rate of the reaction.
Catalyst, V_2O_5 in the given reaction, is used to speed up the reaction.

40. **(d)** $t_{1/2} \propto \frac{1}{a^2}$

We know that $t_{1/2} \propto \frac{1}{a^{n-1}}$

i.e. $n = 3$
Thus reaction is of 3rd order.

41. **(d)**

42. **(d)** Keto-enol tautomerism is shown by carbonyl compounds having α-hydrogen atom.

44. **(c)** $CH_3CH_2CN \xrightarrow{H^+/H_2O} \underset{(A)}{CH_3CH_2COOH} \xrightarrow[\Delta]{NH_3}$

$\underset{(B)}{CH_3CH_2CONH_2} \xrightarrow[NaOBr]{\text{Hoffman bromide reaction}} \underset{(C)}{CH_3CH_2NH_2}$

45. **(d)** $CH_3COOH \xrightarrow{SOCl_2} \underset{(A)}{CH_3-\overset{O}{\overset{\|}{C}}-Cl}$

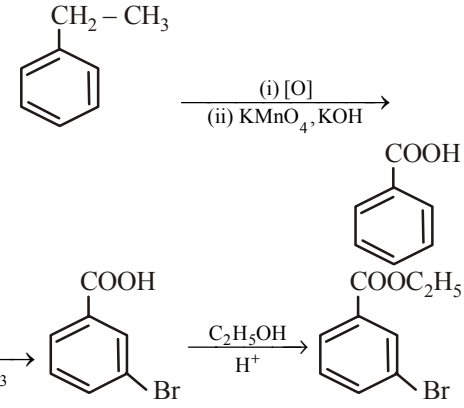
$\xrightarrow[AlCl_3]{Benzene}$ $\underset{(B)}{C_6H_5-\overset{O}{\overset{\|}{C}}-CH_3} \xrightarrow{HCN} \underset{(C)}{C_6H_5-\overset{OH}{\overset{|}{C}}(CN)-CH_3}$

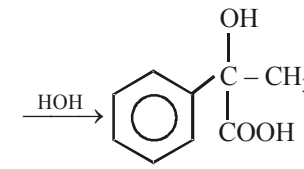
(a) $H_5C_6-\overset{O}{\overset{\|}{C}}-\overset{\alpha}{C}H_2-\overset{O}{\overset{\|}{C}}-C_6H_5 \rightleftharpoons$
keto form

$H_5C_6-\overset{O}{\overset{\|}{C}}-\underset{H}{\underset{|}{C}}=\overset{OH}{\overset{|}{C}}-C_6H_5$
enol form

(b) $H_5C_6-\overset{O}{\overset{\|}{C}}-\overset{\alpha}{C}H_3 \rightleftharpoons C_6H_5-\overset{OH}{\overset{|}{C}}=CH_2$
keto form enol form

(c) $C_6H_5-\overset{O}{\overset{\|}{C}}-H$
no α–H atom

43. **(d)** 



(D)

46. **(b)** Fructose has 3 chiral centres and hence number of optical isomers are $2^3 = 8$

47. **(d)** By Heisenberg uncertainty Principle

$\Delta x \times \Delta p = \frac{h}{4\pi}$ (which is constant)

As Δx for electron and helium atom is same thus momentum of electron and helium will also be same therefore the momentum of helium atom is equal to 5×10^{-26} kg. m.s^{-1}.

48. **(b)** $A \rightleftharpoons B$

$\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$

$\Delta G^\circ = -2.303\, RT \log_{10} K$

$-2.303\, RT \log_{10} K = \Delta H^\circ - T\Delta S^\circ$

$2.303\, RT \log_{10} K = T\Delta S^\circ - \Delta H^\circ$

$$\log_{10} K = \frac{T\Delta S^\circ - \Delta H^\circ}{2.303RT}$$

$$= \frac{298\times10 + 54.07\times1000}{2.303\times8.314\times298}$$

$$= 9.998 \approx 10$$

49. **(b)** Let, $C(s) + O_2(g) \longrightarrow CO_2(g);\ \Delta H = R$ ..... (i)

and $CO(g) + \frac{1}{2}O_2(g) \longrightarrow CO_2(g);\ \Delta H = S$ ..... (ii)

Subtracting equ. (i) from equ (ii), we get

$C(s) + \frac{1}{2}O_2(g) \longrightarrow CO(g);\ \Delta H = R - S$

Hence, heat of formation of CO = R – S

50. **(c)** Both the doubly bonded carbon atoms are identical.

PART - III (MATHEMATICS)

51. **(d)** Since Rolle's theorem is satisfied

$\therefore f'(c) = 0 \Rightarrow e^c \sin c + e^c \cos c = 0$

$\Rightarrow e^c \{\sin c + \cos c\} = 0$

$\therefore \sin c + \cos c = 0 \quad (\because e^c \neq 0)$

$\Rightarrow \tan c = -1 \Rightarrow c = \tan^{-1}(-1) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

52. **(a)** Consider first two equations :

$2x + 3y = -4$ and $3x + 4y = -6$

We have $\Delta = \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} = -1 \neq 0$

$\Delta_x = \begin{vmatrix} -4 & 3 \\ -6 & 4 \end{vmatrix} = 2$ and $\Delta_y = \begin{vmatrix} 2 & -4 \\ 3 & -6 \end{vmatrix} = 0$

$\therefore\ x = -2$ and $y = 0$

Now this solution satisfies all the equations, so the equations are consistent with unique solution.

53. **(b)** The lines are $\frac{x}{6} = \frac{y+2}{6} = \frac{z-1}{1}$

and $\frac{x+1}{12} = \frac{y}{6} = \frac{z}{-1}$

Here, $\vec{a}_1 = -2\hat{j} + \hat{k}$, $\vec{b}_1 = 6\hat{i} + 6\hat{j} + \hat{k}$, $\vec{a}_2 = -\hat{i}$,

$\vec{b}_2 = 12\hat{i} + 6\hat{j} - \hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 6 & 6 & 1 \\ 12 & 6 & -1 \end{vmatrix} = -12\hat{i} + 18\hat{j} - 36\hat{k}$$

Shortest distance $= \frac{|(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 - \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$

$$= \frac{\left|\left(-\hat{i} + 2\hat{j} - \hat{k}\right).\left(-12i + 18\hat{j} - 36\hat{k}\right)\right|}{\sqrt{(-12)^2 + (18)^2 + (-36)^2}}$$

$$= \frac{|+12 + 36 + 36|}{\sqrt{1764}} = \frac{84}{42} = 2$$

54. **(c)** $y^2 = x(2-x)^2 \Rightarrow y^2 = x^3 - 4x^2 + 4x$ ...(i)

$\Rightarrow 2y\frac{dy}{dx} = 3x^2 - 8x + 4 \Rightarrow \frac{dy}{dx} = \frac{3x^2 - 8x + 4}{2y}$

$\Rightarrow \left[\frac{dy}{dx}\right]_P = \frac{3 - 8 + 4}{2} = -\frac{1}{2}$

$\therefore$ Equation of tangent at P is: $y - 1 = -\frac{1}{2}(x - 1)$

$\Rightarrow x + 2y - 3 = 0$

Using $y = \frac{3-x}{2}$ in (i),

we get: $\left(\frac{3-x}{2}\right)^2 = x^3 - 4x^2 + 4x$

$\Rightarrow 4x^3 - 17x^2 + 22x - 9 = 0$ ...(ii)

which has two roots 1, 1

(Because of (ii) being tangent at (1, 1)).

Sum of 3 roots $= \frac{17}{4}$ $\therefore$ 3rd root $= \frac{17}{4} - 2 = \frac{9}{4}$

Then, $y = \frac{3 - \frac{9}{4}}{2} = \frac{3}{8}$ $\therefore$ Q is $\left(\frac{9}{4}, \frac{3}{8}\right)$

55. **(b)** For $x \neq 0$, we have,

$$f(x) = x + \frac{x/1+x}{1-\frac{1}{1+x}} = x + \frac{x/1+x}{x/1+x} = x+1$$

For $x = 0$, $f(x) = 0$. Thus, $f(x) = \begin{cases} x+1, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Clearly, $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^+} f(x) = 1 \neq f(0)$.

So, f(x) is discontinuous and hence not differentiable at $x = 0$.

56. **(c)** Comparing the equation of the circle

$(x+5)^2 + (y-3)^2 = 36$

with $(x-h)^2 + (y-k)^2 = r^2$

$\therefore$ $-h = 5$ or $h = -5, k = 3, r^2 = 36 \Rightarrow r = 6$

$\therefore$ Centre of the circle is $(-5, 3)$ and radius $= 6$

57. **(b)** If $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{c} = -\hat{i} + 2\hat{k}$

$|\vec{c}| = \sqrt{(-1)^2 + 2^2} = \sqrt{1+4} = \sqrt{5}$

$|\vec{c}|.\vec{a} = \sqrt{5}.(2\hat{i} - 2\hat{j} + \hat{k})$

$\therefore$ $|\vec{c}|.\vec{a} = 2\sqrt{5}\hat{i} - 2\sqrt{5}\hat{j} + \sqrt{5}\hat{k}$

58. **(c)** One vertex of square is $(-4, 5)$ and equation of one diagonal is $7x - y + 8 = 0$

Diagonal of a square are perpendicular and bisect each other

Let the equation of the other diagonal be $y = mx + c$ where m is the slope of the line and c is the y-intercept.

Since this line passes through $(-4, 5)$

$\therefore$ $5 = -4m + c$ ... (i)

Since this line is at right angle to the line $7x - y + 8 = 0$ or $y = 7x + 8$, having slope $= 7$,

$\therefore$ $7 \times m = -1$ or $m = \frac{-1}{7}$

Putting this value of m in equation (i) we get

$$5 = -4 \times \left(\frac{-1}{7}\right) + c$$

or $5 = \frac{4}{7} + c$ or $c = 5 - \frac{4}{7} = \frac{31}{7}$

Hence equation of the other diagonal is

$y = -\frac{1}{7}x + \frac{31}{7}$ or $7y = -x + 31$

or $x + 7y - 31 = 0$ or $x + 7y = 31$.

59. **(b)** $p \Rightarrow q \equiv \sim p \vee q$

60. **(b)** Put $x^{3/2} = t \Rightarrow \frac{3}{2}x^{1/2}dx = dt$

$\therefore$ integral is

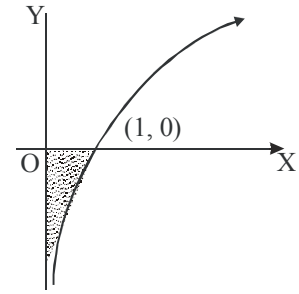
$$\int \frac{\frac{2}{3}dt}{\sqrt{1-t^2}} = \frac{2}{3}\sin^{-1} t + C = \frac{2}{3}\sin^{-1}(x^{3/2}) + C$$

61. **(d)** In the given sets, the set of all primes is an infinite set.

62. **(c)** $f(x) = \sqrt{(x-2)(x-3)} + \sqrt{-(x-4)(x+2)}$

The first part is real outside (2, 3) and the second is real in $[-2, 4]$ so that the domain is $[-2, 2] \cup [3, 4]$.

63. **(b)** Observing the graph of log x, we find that the required area lies below x-axis between $x = 0$ and $x = 1$.



So required area $= \left|\int_0^1 \log x\, dx\right| = |(x\log x - x)|_0^1$

$= |-1| = 1$

64. **(a)** Let m_1 and m_2 be slope of curve $y = x^2$ and $6y = 7 - x^3$ respectively.

Now, $y = x^2 \Rightarrow \frac{dy}{dx} = 2x$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)} = 2$ i.e. $m_1 = 2$

and $6y = 7 - x^3 \Rightarrow 6\frac{dy}{dx} = -3x^2$

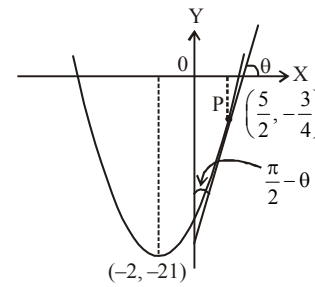
$\Rightarrow \frac{dy}{dx} = -\frac{3}{6}x^2 = -\frac{1}{2}x^2$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)} = -\frac{1}{2}(1)^2 = -\frac{1}{2}$

$\therefore$ $m_2 = -\frac{1}{2}$ $\therefore$ $m_1 m_2 = 2 . -\frac{1}{2} = -1$

$\therefore$ Angle of intersection is 90° i.e. $\frac{\pi}{2}$

65. **(b)**



$y = x^2 + 4x + 4 - 4 - 17$

$y = (x+2)^2 - 21 \Rightarrow$ Vertex is $(-2, -21)$

Also $y = x^2 + 4x - 17 \Rightarrow \frac{dy}{dx} = 2x + 4$

$\Rightarrow$ Slope of tangent at $\left(\frac{5}{2}, -\frac{3}{4}\right)$

$m = \frac{dy}{dx} = 2 \times \frac{5}{2} + 4 = 9$; $\theta = \tan^{-1} 9$

$\therefore$ angle by y-axis $= \frac{\pi}{2} - \tan^{-1} 9 = \cot^{-1} 9$

66. **(d)** $(1+i)^4 \times \left(1 + \frac{1}{i}\right)^4 = (1+i)^4 \times (1-i)^4 = 2^4$

67. **(d)** We have R = $\{(x, y) : |x^2 - y^2| < 16\}$
Let x = 1, $|x^2 - y^2| < 16 \Rightarrow |1 - y^2| < 16$
$\Rightarrow$ $|y^2 - 1| < 16 \Rightarrow y = 1, 2, 3, 4$
Let x = 2, $|x^2 - y^2| < 16 \Rightarrow |4 - y^2| < 16$
$\Rightarrow$ $|y^2 - 4| < 16 \Rightarrow y = 1, 2, 3, 4$
Let x = 3, $|x^2 - y^2| < 16 \Rightarrow |9 - y^2| < 16$
$\Rightarrow$ $|y^2 - 9| < 16 \Rightarrow y = 1, 2, 3, 4$
Let x = 4, $|x^2 - y^2| < 16 \Rightarrow |16 - y^2| < 16$
$\Rightarrow$ $|y^2 - 16| < 16 \Rightarrow y = 1, 2, 3, 4, 5$
Let x = 5, $|x^2 - y^2| < 16 \Rightarrow |25 - y^2| < 16$
$\Rightarrow$ $|y^2 - 25| < 16 \Rightarrow y = 4, 5$
$\therefore$ R = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (5, 4), (5, 5)}.

68. **(a)** $\int \frac{2dx}{(e^x + e^{-x})^2} = \int \frac{2e^{2x}}{(e^{2x}+1)^2} dx$

$= -\frac{1}{(e^{2x}+1)} + c = -\frac{e^{-x}}{e^x + e^{-x}} + c$

69. **(a)** $\tan^{-1}(1) + \tan^{-1}(0) + \tan^{-1}(2) + \tan^{-1}(3)$

$= \frac{\pi}{4} + \pi + \tan^{-1}\left(\frac{2+3}{1-2.3}\right)$ (as $2.3 > 1$)

$= \frac{5\pi}{4} + \tan^{-1}(-1) = \frac{5\pi}{4} - \frac{\pi}{4} = \pi$

70. **(b)** Let y denote the number of bacteria at any instant t · then according to the question

$\frac{dy}{dt} \alpha y \Rightarrow \frac{dy}{y} = k\, dt$ ... (i)

k is the constant of proportionality, taken to be + ve on integrating (i), we get

$\log y = kt + c$ ... (ii)

c is a parameter. let y_0 be the initial number of bacteria
i.e., at t = 0 using this in (ii), $c = \log y_0$

$\Rightarrow \log y = kt + \log y_0 \Rightarrow \log \frac{y}{y_0} = kt$ ...(iii)

$y = \left(y_0 + \frac{10}{100} y_0\right) = \frac{11y_0}{10}$, when t = 2

So, from (iii), we get $\log \frac{\frac{11y_0}{10}}{y_0} = k$ (2)

$\Rightarrow k = \frac{1}{2} \log \frac{11}{10}$ ... (iv)

Using (iv) in (iii) $\log \frac{y}{y_0} = \frac{1}{2}\left(\log \frac{11}{10}\right) t$ ... (v)

let the number of bacteria become 1, 00, 000 to 2,00,000 in t_1 hours. i.e., $y = 2y_0$
when $t = t_1$ hours. from (v)

$\log \frac{2y_0}{y_0} = \frac{1}{2}\left(\log \frac{11}{10}\right) t_1 \Rightarrow t_1 = \frac{2 \log 2}{\log \frac{11}{10}}$

Hence, the reqd. no. of hours $= \frac{2 \log 2}{\log \frac{11}{10}}$

71. **(a)** The given lines are:
$y = (2 - \sqrt{3})\, x + 5$ and $y = (2 + \sqrt{3})\, x - 7$
Therefore, slope of first line $= m_1 = 2 - \sqrt{3}$ and slope of second line $= m_2 = 2 + \sqrt{3}$
$\therefore$

$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{2+\sqrt{3} - 2 + \sqrt{3}}{1 + (4-3)}\right|$

$= \left|\frac{2\sqrt{3}}{2}\right| = \sqrt{3} = \tan\frac{\pi}{3} \Rightarrow \theta = \frac{\pi}{3} = 60°$

72. **(a)** If θ is the angle between line and plane then $\left(\frac{\pi}{2} - \theta\right)$ is the angle between line and normal to plane given by

$\cos\left(\frac{\pi}{2} - \theta\right) = \frac{\left(\hat{i} + 2\hat{j} + 2\hat{k}\right).\left(2\hat{i} - \hat{j} + \sqrt{\lambda}\hat{k}\right)}{3\sqrt{4 + 1 + \lambda}}$

$\cos\left(\frac{\pi}{2} - \theta\right) = \frac{2 - 2 + 2\sqrt{\lambda}}{3 \times \sqrt{5 + \lambda}}$

$\Rightarrow \sin\theta = \frac{2\sqrt{\lambda}}{3\sqrt{5+\lambda}} = \frac{1}{3} \Rightarrow 4\lambda = 5 + \lambda \Rightarrow \lambda = \frac{5}{3}$.

73. (a) Given equation of line is

$$\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$$

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right) = (2+3\lambda)\hat{i} + (-1+4\lambda)\hat{j} + (2+2\lambda)\hat{k}$$

Any point on the line is
$(2+3\lambda, -1+4\lambda, 2+2\lambda)$

Since it also lie on the plane $\vec{r}\cdot\left(\hat{i}-\hat{j}+\hat{k}\right)$

So, $\left[(2+3\lambda)\hat{i}+(-1+4\lambda)\hat{j}+(2+2\lambda)\hat{k}\right].\left(\hat{i}-\hat{j}+\hat{k}\right)=5$

$\Rightarrow \quad 2+3\lambda+1-4\lambda+2+2\lambda=5 \Rightarrow \lambda=0$

Therefore, coordinate of the point of intersection of line and plane is (2,–1, 2).

$\therefore$ Distance

$$d = \sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2} = 13$$

74. (a) $$I = \int_{\log\sqrt{\pi/2}}^{\log\sqrt{\pi}} e^{2x}\sec^2\left(\frac{1}{3}e^{2x}\right)dx$$

Put $e^{2x} = t \Rightarrow 2e^{2x}dx = dt$

When $x = \log\sqrt{\pi/2}$, $t = e^{2\log\sqrt{\pi/2}}$

$$= e^{\log\pi/2} = \frac{\pi}{2}$$

When $x = \log\sqrt{\pi}$, $t = e^{2\log\sqrt{\pi}} = \pi$

$$\therefore I = \int_{\frac{\pi}{2}}^{\pi}\frac{1}{2}\sec^2\left(\frac{1}{3}t\right)dt = \frac{1}{2}\cdot\frac{1}{\frac{1}{3}}\left[\tan\frac{t}{3}.\right]_{\pi/2}^{\pi}$$

$$= \frac{3}{2}\left[\tan\frac{\pi}{3} - \tan\frac{\pi}{6}\right] = \frac{3}{2}\left[\sqrt{3} - \frac{1}{\sqrt{3}}\right] = \sqrt{3}$$

75. (c) We have $\dfrac{|x+3|+x}{x+2} > 1$

$$\Rightarrow \frac{|x+3|+x}{x+2} - 1 > 0 \Rightarrow \frac{|x+3|-2}{x+2} > 0$$

Now, two cases arise :

Case I : When $x + 3 \geq 0$, i.e. $x \geq -3$. Then,

$$\frac{|x+3|-2}{x+2} > 0 \Rightarrow \frac{x+3-2}{x+2} > 0$$

$$\Rightarrow \frac{x+1}{x+2} > 0$$

$\Rightarrow \{(x + 1) > 0$ and $x + 2 > 0\}$
or $\{x + 1 < 0$ and $x + 2 < 0\}$
$\Rightarrow \{x > -1$ and $x > -2\}$
or $\{x < -1$ and $x < -2\}$
$\Rightarrow x > -1$ or $x < -2$
$\Rightarrow x \in (-1, \infty)$ or $x \in (-\infty, -2)$
$\Rightarrow x \in (-3, -2) \cup (-1, \infty)$ [Since $x \geq -3$] ... (i)

Case II : When $x + 3 < 0$, i.e. $x < -3$

$$\frac{|x+3|-2}{x+2} > 0 \Rightarrow \frac{-x-3-2}{x+2} > 0$$

$$\Rightarrow \frac{-(x+5)}{x+2} > 0 \Rightarrow \frac{x+5}{x+2} < 0$$

$\Rightarrow (x + 5 < 0$ and $x + 2 > 0)$ or $(x + 5 > 0)$ and $x + 2 < 0)$
$\Rightarrow (x < -5$ and $x > -2)$ or $(x > -5$ and $x < -2)$ it is not possible.
$\Rightarrow x \in (-5, -2)$ ... (ii)
Combining (i) and (ii), the required solution is $x \in (-5, -2) \cup (-1, \infty)$.

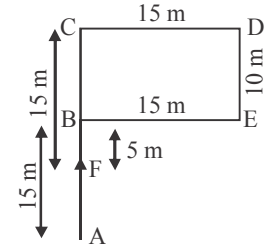
PART - IV (ENGLISH & LOGICAL REASONING)

76. (b) See the last sentence of the fourth paragraph.

77. (c) Virtually, the whole passage deals with F.A.S.T. membership requirements. The other choices are too narrow to be main ideas.

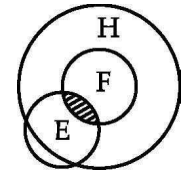
78. (a) See the first paragraph.

79. (c) Let the fixed point from where Jatin starts his journey be A. Also, his walking directions are as follows.



$\therefore AF = AB - FB = 15 - 5 = 10$ meters
So, Jatin is 10 meters away from the starting point.

80. (b) F — Mohan's family members
E — Employed members
H — Honest members



Here, shaded area denotes the employed members of Mohan's family members, who are honest.