WBJEE 2017 Question Paper

West Bengal Joint Entrance Examinations

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WB JEE

Engineering Entrance Exam

Solved Paper 2017

PHYSICS

Category-I (Q.1 to Q. 30)

Direction *Only one answer is correct. Correct answer will fetch full marks 1. Incorrect answer or any combination of more than one answer will fetch* $-\frac{1}{4}$ *marks. No answer will fetch 0 marks.*

Unless otherwise specified in the question, the following values should be used.

Mechanical equivalent of heat, $J = 4.2\text{ J cal}^{-1}$

Acceleration due to gravity, $g = 9.8\text{ ms}^{-2}$

Absolute zero temperature $= -273\,°\text{C}$

The following symbols usually carry meaning as given below

ε_0 : electric permittivity of free space

μ_0 : magnetic permeability of free space

R : universal gas constant

1. The velocity of a particle executing a simple harmonic motion is 13 ms^{-1}, when its distance from the equilibrium position (Q) is 3 m and its velocity is 12 ms^{-1}, when it is 5 m away from Q. The frequency of the simple harmonic motion is

(a) $\frac{5\pi}{8}$ (b) $\frac{5}{8\pi}$

(c) $\frac{8\pi}{5}$ (d) $\frac{8}{5\pi}$

2. A uniform string of length L and mass M is fixed at both ends while it is subject to a tension T. It can vibrate at frequencies (v) given by the formula (where n = 1, 2, 3, ...)

(a) $v = \frac{n}{2}\sqrt{\frac{T}{ML}}$ (b) $v = \frac{n}{2L}\sqrt{\frac{T}{M}}$

(c) $v = \frac{1}{2n}\sqrt{\frac{T}{ML}}$ (d) $v = \frac{n}{2}\sqrt{\frac{TL}{M}}$

3. A uniform capillary tube of length l and inner radius r with its upper end sealed is submerged vertically into water. The outside pressure is p_0 and surface tension of water is γ. When a length x of the capillary is submerged into water, it is found that water levels inside and outside the capillary coincide. The value of x is

(a) $\frac{l}{\left(1 + \frac{p_0 r}{4\gamma}\right)}$ (b) $l\left(1 - \frac{p_0 r}{4\gamma}\right)$

(c) $l\left(1 - \frac{p_0 r}{2\gamma}\right)$ (d) $\frac{l}{\left(1 + \frac{p_0 r}{2\gamma}\right)}$

4. A liquid of bulk modulus k is compressed by applying an external pressure such that its density increases by 0.01%. The pressure applied on the liquid is

(a) $\frac{k}{10000}$ (b) $\frac{k}{1000}$

(c) 1000 k (d) 0.01 k

5. Temperature of an ideal gas, initially at 27° C, is raised by 6° C. The rms velocity of the gas molecules will
(a) increase by nearly 2%
(b) decrease by nearly 2%
(c) increase by nearly 1%
(d) decrease by nearly 1%

6. 2 moles of an ideal monoatomic gas is carried from a state (p_0, V_0) to state $(2p_0, 2V_0)$ along a straight line path in a p-V diagram. The amount of heat absorbed by the gas in the process is given by
(a) $3\,p_0V_0$ (b) $\frac{9}{2}\,p_0V_0$ (c) $6\,p_0V_0$ (d) $\frac{3}{2}\,p_0V_0$

7. A solid rectangular sheet has two different coefficients of linear expansion α_1 and α_2 along its length and breadth respectively. The coefficient of surface expansion is
(for $\alpha_1 t << 1, \alpha_2 t << 1$)
(a) $\frac{\alpha_1 + \alpha_2}{2}$ (b) $2(\alpha_1 + \alpha_2)$
(c) $\frac{4\alpha_1\alpha_2}{\alpha_1 + \alpha_2}$ (d) $\alpha_1 + \alpha_2$

8. A positive charge Q is situated at the centre of a cube. The electric flux through any face of the cube is (in SI units)
(a) $\frac{Q}{6\varepsilon_0}$ (b) $4\pi Q$ (c) $\frac{Q}{4\pi\varepsilon_0}$ (d) $\frac{Q}{6\pi\varepsilon_0}$

9. Three capacitors of capacitance 1.0, 2.0 and 5.0 μF are connected in series to a 10 V source. The potential difference across the 2.0 μF capacitor is
(a) $\frac{100}{17}$ V (b) $\frac{20}{17}$ V (c) $\frac{50}{17}$ V (d) 10 V

10. A charge of 0.8 coulomb is divided into two charges Q_1 and Q_2. These are kept at a separation of 30 cm. The force on Q_1 is maximum when
(a) $Q_1 = Q_2 = 0.4$ C
(b) $Q_1 \approx 0.8$ C, Q_2 negligible
(c) Q_1 negligible, $Q_2 \approx 0.8$ C
(d) $Q_1 = 0.2$ C, $Q_2 = 0.6$ C

11. The magnetic field due to a current in a straight wire segment of length L at a point on its perpendicular bisector at a distance r (r >> L)
(a) decreases as $\frac{1}{r}$
(b) decreases as $\frac{1}{r^2}$
(c) decreases as $\frac{1}{r^3}$
(d) approaches a finite limit as $r \to \infty$

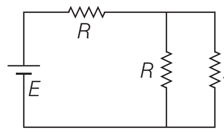
12. The magnets of two suspended coil galvanometers are of the same strength so that they produce identical uniform magnetic fields in the region of the coils. The coil of the first one is in the shape of a square of side a and that of the second one is circular of radius $\frac{a}{\sqrt{\pi}}$. When the same current is passed through the coils, the ratio of the torque experienced by the first coil to that experienced by the second one is
(a) $1 : \frac{1}{\sqrt{\pi}}$ (b) 1 : 1 (c) π : 1 (d) 1 : π

13. A proton is moving with a uniform velocity of 10^6 ms^{-1} along the Y-axis, under the joint action of a magnetic field along Z-axis and an electric field of magnitude 2×10^4 Vm^{-1} along the negative *X*-axis. If the electric field is switched off, the proton starts moving in a circle. The radius of the circle is nearly $\left(\text{given : } \frac{e}{m} \text{ ratio for proton} \approx 10^8 \text{ Ckg}^{-1}\right)$
(a) 0.5 m (b) 0.2 m (c) 0.1 m (d) 0.05 m

14. When the frequency of the AC voltage applied to a series *LCR* circuit is gradually increased from a low value, the impedance of the circuit
(a) monotonically increases
(b) first increases and then decreases
(c) first decreases and then increases
(d) monotonically decreases

15. Six wires, each of resistance r, are connected so as to form a tetrahedron. The equivalent resistance of the combination when current enters through one corner and leaves through some other corner is
(a) r (b) 2r (c) $\frac{r}{3}$ (d) $\frac{r}{2}$

16. Consider the circuit shown in the figure.

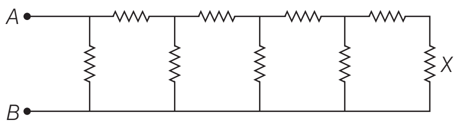


The value of the resistance X for which the thermal power generated in it is practically independent of small variation of its resistance is

(a) $X = R$ (b) $X = \frac{R}{3}$

(c) $X = \frac{R}{2}$ (d) $X = 2R$

17.



Consider the circuit shown in the figure where all the resistances are of magnitude 1 kΩ. If the current in the extreme right resistance X is 1 mA, the potential difference between A and B is

(a) 34 V (b) 21 V

(c) 68 V (d) 55 V

18. The ratio of the diameter of the sun to the distance between the earth and the sun is approximately 0.009. The approximate diameter of the image of the sun formed by a concave spherical mirror of radius of curvature 0.4 m is

(a) 4.5×10^{-6} m (b) 4.0×10^{-6} m

(c) 3.6×10^{-3} m (d) 1.8×10^{-3} m

19. Two monochromatic coherent light beams A and B have intensities L and $\frac{L}{4}$, respectively. If these beams are superposed, the maximum and minimum intensities will be

(a) $\frac{9L}{4}, \frac{L}{4}$ (b) $\frac{5L}{4}, 0$

(c) $\frac{5L}{2}, 0$ (d) $2L, \frac{L}{2}$

20. A point object is held above a thin equiconvex lens at its focus. The focal length is 0.1 m and the lens rests on a horizontal thin plane mirror. The final image will be formed at

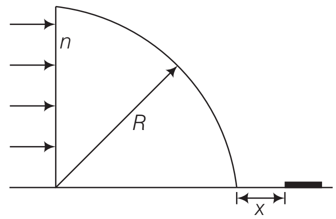
(a) infinite distance above the lens

(b) 0.1 m above the center of the lens

(c) infinite distance below the lens

(d) 0.1 m below the center of the lens

21.



A parallel beam of light is incident on a glass prism in the shape of a quarter cylinder of radius $R = 0.05$ m and refractive index $n = 1.5$, placed on a horizontal table as shown in the figure. Beyond the cylinder, a patch of light is found whose the nearest distance x from the cylinder is

(a) $(3\sqrt{3} - 4) \times 10^{-2}$ m

(b) $(2\sqrt{3} - 2) \times 10^{-2}$ m

(c) $(3\sqrt{5} - 5) \times 10^{-2}$ m

(d) $(3\sqrt{2} - 3) \times 10^{-2}$ m

22. The de-Broglie wavelength of an electron is 0.4×10^{-10} m when its kinetic energy is 1.0 keV. Its wavelength will be 1.0×10^{-10} m, when its kinetic energy is

(a) 0.2 keV (b) 0.8 keV

(c) 0.63 keV (d) 0.16 keV

23. When light of frequency v_1 is incident on a metal with work function W (where $hv_1 > W$), then photocurrent falls to zero at a stopping potential of V_1. If the frequency of light is increased to v_2, the stopping potential changes to V_2. Therefore, the charge of an electron is given by

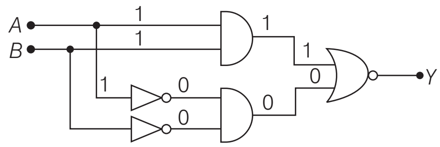
(a) $\frac{W(v_2 + v_1)}{v_1V_2 + v_2V_1}$ (b) $\frac{W(v_2 + v_1)}{v_1V_1 + v_2V_2}$

(c) $\frac{W(v_2 - v_1)}{v_1V_2 - v_2V_1}$ (d) $\frac{W(v_2 - v_1)}{v_2V_2 - v_1V_1}$

24. Radon-222 has a half-life of 3.8 days. If one starts with 0.064 kg of radon-222, the quantity of radon-222 left after 19 days will be

(a) 0.002 kg (b) 0.062 kg

(c) 0.032 kg (d) 0.024 kg

25. 

In the given circuit, the binary inputs at A and B are both 1 in one case and both 0 in the next case. The respective outputs at Y in these two cases will be

(a) 1, 1 (b) 0, 0
(c) 0, 1 (d) 1, 0

26. When a semiconducting device is connected in series with a battery and a resistance, a current is found to flow in the circuit. If however, the polarity of the battery is reversed, practically no current flows in the circuit. The device may be

(a) a p-type semiconductor
(b) a n-type semiconductor
(c) an intrinsic semiconductor
(d) a p-n junction

27. The dimension of the universal constant of gravitation, G is

(a) $[ML^2T^{-1}]$ (b) $[M^{-1}L^3T^{-2}]$
(c) $[M^{-1}L^2T^{-2}]$ (d) $[ML^3T^{-2}]$

28. Two particles A and B (both initially at rest) start moving towards each other under a mutual force of attraction. At the instant, when the speed of A is v and the speed of B is 2v, the speed of the centre of mass is

(a) zero (b) v
(c) $\frac{3v}{2}$ (d) $-\frac{3v}{2}$

29. Three vectors $A = ai + j + k$; $B = i + bj + k$ and $C = i + j + ck$ are mutually perpendicular (i, j and k are unit vectors along X, Y and Z-axes respectively). The respective values of a, b and c are

(a) 0, 0, 0 (b) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}$
(c) 1, – 1, 1 (d) $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$

30. A block of mass 1 kg starts from rest at $x = 0$ and moves along the X-axis under the action of a force $F = kt$, where t is time and $k = 1\ Ns^{-1}$. The distance the block will travel in 6 seconds is

(a) 36 m (b) 72 m
(c) 108 m (d) 18 m

Category-II (Q. 31 to Q. 35)

Direction *Only one answer is correct. Correct answer will fetch full marks 2. Incorrect answer or any combination of more than one answer will fetch $-\frac{1}{2}$ marks. No answer will fetch 0 marks.*

31. A particle with charge Q coulomb, tied at the end of an inextensible string of length R metre, revolves in a vertical plane. At the centre of the circular trajectory, there is a fixed charge of magnitude Q coulomb. The mass of the moving charge M is such that $Mg = \frac{Q^2}{4\pi\varepsilon_0 R^2}$. If at the highest position of the particle, the tension of the string just vanishes, the horizontal velocity at the lowest point has to be

(a) 0 (b) $2\sqrt{gR}$ (c) $\sqrt{2gR}$ (d) $\sqrt{5gR}$

32. A bullet of mass 4.2×10^{-2} kg, moving at a speed of 300 ms^{-1}, gets stuck into a block with a mass 9 times that of the bullet. If the block is free to move without any kind of friction, the heat generated in the process will be

(a) 45 cal (b) 405 cal
(c) 450 cal (d) 1701 cal

33. A particle with charge e and mass m, moving along the *X*-axis with a uniform speed u, enters a region where a uniform electric field E is acting along the *Y*-axis. The particle starts to move in a parabola. Its focal length (neglecting any effect of gravity) is

(a) $\frac{2mu^2}{eE}$ (b) $\frac{eE}{2mu^2}$ (c) $\frac{mu}{2eE}$ (d) $\frac{mu^2}{2eE}$

34. A unit negative charge with mass M resides at the mid-point of the straight line of length 2a adjoining two fixed charges of magnitude +Q each. If it is given a very small displacement $x (x << a)$ in a direction perpendicular to the straight line, it will

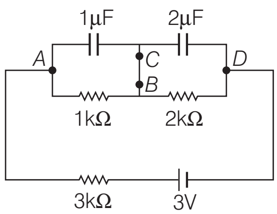
(a) come back to its original position and stay there

(b) execute oscillations with frequency $\dfrac{1}{2\pi}\sqrt{\dfrac{Q}{4\pi\varepsilon_0 Ma^3}}$

(c) fly to infinity

(d) execute oscillations with frequency $\dfrac{1}{2\pi}\sqrt{\dfrac{Q}{4\pi\varepsilon_0 Ma^2}}$

35. Consider the circuit given here. The potential difference V_{BC} between the points B and C is



(a) 1 V (b) 0.5 V (c) 0 V (d) – 1 V

Category-III (Q. 36 to Q. 40)

Direction *One or more answer(s) is (are) correct. Correct answer(s) will fetch full marks 2. Any combination containing one or more incorrect answer will fetch 0 marks. Also, no answer will fetch 0 marks. If all correct answers are not marked and also no incorrect answer is marked then score = 2 × number of correct answers marked ÷ actual number of correct answers.*

36. If the pressure, temperature and density of an ideal gas are denoted by p, T and ρ, respectively, the velocity of sound in the gas is

(a) proportional to $\sqrt{p}$, when T is constant.

(b) proportional to $\sqrt{T}$.

(c) proportional to $\sqrt{p}$, when ρ is constant.

(d) proportional to T.

37. Two long parallel wires separated by 0.1 m carry currents of 1 A and 2 A, respectively in opposite directions. A third current-carrying wire parallel to both of them is placed in the same plane such that it feels no net magnetic force. It is placed at a distance of

(a) 0.5 m from the 1st wire, towards the 2nd wire

(b) 0.2 m from the 1st wire, towards the 2nd wire

(c) 0.1 m from the 1st wire, away from the 2nd wire

(d) 0.2 m from the 1st wire, away from the 2nd wire

38. If χ stands for the magnetic susceptibility of a substance, μ for its magnetic permeability and μ_0 for the permeability of free space, then

(a) for a paramagnetic substance: $\chi < 0$, $\mu > 0$

(b) for a paramagnetic substance: $\chi > 0$, $\mu > \mu_0$

(c) for a diamagnetic substance: $\chi > 0$, $\mu < 0$

(d) for a ferromagnetic substance: $\chi > 1$, $\mu > \mu_0$

39. Let v_n and E_n be the respective speed and energy of an electron in the *n*th orbit of radius r_n, in a hydrogen atom, as predicted by Bohr's model. Then.

(a) plot of $\dfrac{E_n r_n}{E_1 r_1}$ as a function of n is a straight line of slope 0

(b) plot of $\dfrac{r_n v_n}{r_1 v_1}$ as a function of n is a straight line of slope 1

(c) plot of $\ln\left(\dfrac{r_n}{r_1}\right)$ as a function of ln (n) is a straight line of slope 2

(d) plot of $\ln\left(\dfrac{r_n E_1}{E_n r_1}\right)$ as a function of ln (n) is a straight line of slope 4

40. A small steel ball bounces on a steel plate held horizontally. On each bounce the speed of the ball arriving at the plate is reduced by a factor e (coefficient of restitution) in the rebound, so that $V_{upward} = eV_{downward}$

If the ball is initially dropped from a height of 0.4 m above the plate and if 10 seconds later the bouncing ceases, the value of e is

(a) $\sqrt{\dfrac{2}{7}}$ (b) $\dfrac{3}{4}$

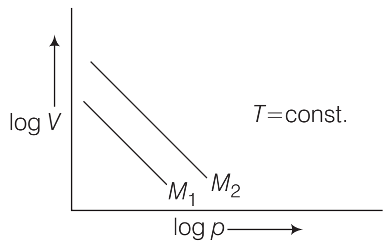
(c) $\dfrac{13}{18}$ (d) $\dfrac{17}{18}$

CHEMISTRY

Category-I (Q.41 to 70)

Direction *Only one answer is correct. Correct will fetch full marks 1. Incorrect answer or any combination of more than one answer will detch – 1/4 marks. No answer will fetch 0 marks.*

41.



For same mass of two different ideal gases of molecular weights M_1 and M_2, Plots of log V *vs* log p at a given constant temperature are shown. Identify the correct option.

(a) $M_1 > M_2$
(b) $M_1 = M_2$
(c) $M_1 < M_2$
(d) Can be predicted only if temperature is known

42. Which of the following has the dimension if $[ML^0T^{-2}]$?

(a) Coefficient of viscosity
(b) Surface tension
(c) Vapour pressure
(d) Kinetic energy

43. If the given four electronic configurations.

(i) n = 4, l = 1 (ii) n = 4, l = 0
(iii) n = 3, l = 2 (iv) n = 3, l = 1

are arranged in order of increasing energy, then the order will be

(a) (iv) < (ii) < (iii) < (i)
(b) (ii) < (iv) < (i) < (iii)
(c) (i) < (iii) < (ii) < (iv)
(d) (iii) < (i) < (iv) < (ii)

44. Which of the following sets of quantum numbers represents the 19^{th} electron of Cr (Z = 24)?

(a) $\left(4, 1, -1, +\frac{1}{2}\right)$ (b) $\left(4, 0, 0, +\frac{1}{2}\right)$
(c) $\left(3, 2, 0, -\frac{1}{2}\right)$ (d) $\left(3, 2, -2, +\frac{1}{2}\right)$

45. 0.126 g of an acid is needed to completely neutralise 20 mL 0.1 (N) NaOH solution. The equivalent weight of the acid is

(a) 53 (b) 40
(c) 45 (d) 63

46. In a flask, the weight ratio of $CH_4(g)$ and $SO_2(g)$ at 298 K and 1 bar is 1 : 2. The ratio of the number of molecules of $SO_2(g)$ and $CH_4(g)$ is

(a) 1 : 4 (b) 4 : 1 (c) 1 : 2 (d) 2 : 1

47. $C_6H_5F^{18}$ is a F^{18} radio-isotope labelled organic compound. F^{18} decays by positron emission. The product resulting on decay is

(a) $C_6H_5O^{18}$ (b) $C_6H_5Ar^{19}$
(c) $B^{12}C_5H_5F$ (d) $C_6H_5O^{16}$

48. Dissolving NaCN in de-ionised water will result in a solution having

(a) pH < 7 (b) pH = 7
(c) pOH = 7 (d) pH > 7

49. Among Me_3N, C_5H_5N and MeCN (Me = methyl group) the electronegativity of N is in the order

(a) MeCN > C_5H_5N > Me_3N
(b) C_5H_5N > Me_3N > MeCN
(c) Me_3N > MeCN > C_5H_5N
(d) Electronegativity same in all

50. The shape of XeF_5^- will be

(a) square pyramid
(b) trigonal bipyramidal
(c) planar
(d) pentagonal bipyramid

51. The ground state magnetic property of B_2 and C_2 molecules will be

(a) B_2 paramagnetic and C_2 diamagnetic
(b) B_2 diamagnetic and C_2 paramagnetic
(c) Both are diamagnetic
(d) Both are paramagnetic

52. The number of unpaired electrons in $[NiCl_4]^{2-}$, $Ni(CO)_4$ and $[Cu(NH_3)_4]^{2+}$ respectively are

(a) 2, 2, 1 (b) 2, 0, 1
(c) 0, 2, 1 (d) 2, 2, 0

53. Which of the following atoms should have the highest Ist electron affinity?

(a) F (b) O (c) N (d) C

54. $PbCl_2$ is insoluble in cold water. Addition of HCl increases its solubility due to

(a) formation of soluble complex anions like $[PbCl_3]^-$
(b) oxidation of Pb(II) to PB(IV)
(c) formation of $[Pb(H_2O)_6]^{2+}$
(d) formation of polymeric lead complexes

55. Of the following compounds, which one is the strongest Bronsted acid in a aqueous solution?

(a) $HClO_3$ (b) $HClO_2$ (c) HOCl (d) HOBr

56. The correct basicity order of the following lanthanide ions is

(a) $La^{3+} > Lu^{3+} > Ce^{3+} > Eu^{3+}$
(b) $Ce^{3+} > Lu^{3+} > La^{3+} > Eu^{3+}$
(c) $Lu^{3+} > Ce^{3+} > Eu^{3+} > La^{3+}$
(d) $La^{3+} > Ce^{3+} > Eu^{3+} > Lu^{3+}$

57. When $BaCl_2$ is added to an aqueous salt solution, a white precipitate is obtained. The anion among CO_3^{2-}, SO_3^{2-} and SO_4^{2-} that was present in the solution can be

(a) CO_3^{2-} but not any of the other two
(b) SO_3^{2-} but not any of the other two
(c) SO_4^{2-} but not any of the other two
(d) Any of them

58. In the IUPAC system, $PhCH_2CH_2CO_2H$ is named as

(a) 3-phenylpropanoic acid
(b) benzylacetic acid
(c) carboxyethylbenzene
(d) 2-phenylpropanoic acid

59. The isomerisation of 1-butyne to 2-butyne can be achieved by treatment with

(a) hydrochloric acid
(b) ammoniacal silver nitrate
(c) ammoniacal cuprous chloride
(d) ethanolic potassium hydroxide

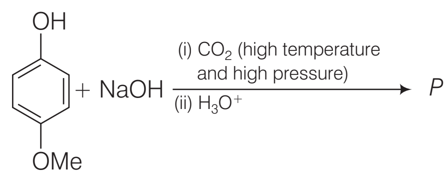
60. The correct order of acid strengths of benzoic acid (X), peroxybenzoic acid (Y) and *p*-nitrobenzoic acid (*Z*) is

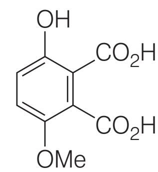
(a) $Y > Z > X$
(b) $Z > Y > X$
(c) $Z > X > Y$
(d) $Y > X > Z$

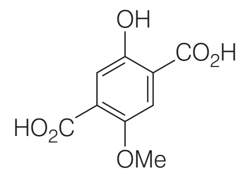
61. The yield of acetanilide in the reaction (100% conversion) of 2 moles of aniline with 1 mole of acetic anhydride is

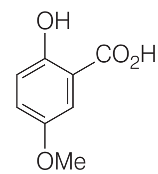
(a) 270 g (b) 135 g
(c) 67.5 g (d) 177 g

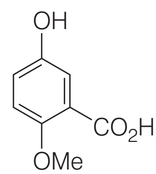
62. The structure of the product *P* of the following reaction is



(a) 

(b) 

(c) 

(d) 

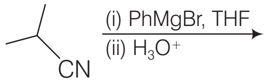
63. ADP and ATP differ in the number of

(a) phosphate units
(b) ribose units
(c) adenine base
(d) nitrogen atom

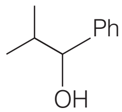
64. The compound that would produce a nauseating smell/odour with a hot mixture of chloroform and ethanolic potassium hydroxide is

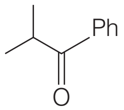
(a) $PhCONH_2$
(b) $PhNHCH_3$
(c) $PhNH_2$
(d) PhOH

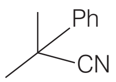
65. For the reaction below

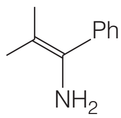


the structure of the product Q is

(a) 

(b) 

(c) 

(d) 

66. You are supplied with 500 mL each of 2N HCl and 5N HCl. What is the maximum volume of 3M HCl that you can prepare using only these two solutions?

(a) 250 mL (b) 500 nL

(c) 750 mL (d) 1000 mL

67. Which one of the following corresponds to a photon of highest energy?

(a) $\lambda = 300$ mm

(b) $v = 3 \times 10^{8}$ s^{-1}

(c) $\bar{v} = 30$ cm^{-1}

(d) $\varepsilon = 6.626 \times 10^{-27}$ J

68. Assuming the compounds to be completely dissociated in aqueous solution, identify the pair of the solutions that can be expected to be isotonic at the same temperature.

(a) 0.01 M urea and 0.01 M NaCl

(b) 0.02 M NaCl and 0.01 M Na_2SO_4

(c) 0.03 M NaCl and 0.02 M $MgCl_2$

(d) 0.01 M sucrose and 0.02 M glucose

69. How many faradays are required to reduce 1 mol of $Cr_2O_7^{2-}$ to Cr^{3+} in acid medium?

(a) 2 (b) 3

(c) 5 (d) 6

70. Equilibrium constants for the following reactions at 1200 K are given

$$2H_2O(g) \rightleftharpoons 2H_2(g) + O_2(g), \quad K_1 = 6.8 \times 10^{-8}$$

$$2CO_2(g) \rightleftharpoons 2CO(g) + O_2(g), \quad K_2 = 1.6 \times 10^{-6}$$

The equilibrium constant for the reaction?

$$H_2(g) + CO_2(g) \rightleftharpoons CO(g) + H_2O(g)$$

at 1200 K will be

(a) 0.05 (b) 20

(c) 0.2 (d) 5.0

Category-II (Q.71 to Q.75)

Direction *Only one answer is correct. Correct answer will fetch full marks 2. Incorrect answer or any combination of more than one answer will fetch* – 1/2 *marks. No answer will fetch* 0 *marks.*

71. In a close-packed body-centred cubic lattice of potassium, the correct relation between the atomic radius (r) of potassium and the edge-length (a) of the cube is

(a) $r = \frac{a}{\sqrt{2}}$ (b) $r = \frac{a}{\sqrt{3}}$

(c) $r = \frac{\sqrt{3}}{2} a$ (d) $r = \frac{\sqrt{3}}{4} a$

72. Which of the following solutions will turn violet when a drop of lime juice is added to it?

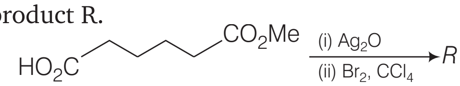
(a) A solution of NaI

(b) A solution mixture of KI and $NaIO_3$

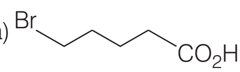
(c) A solution mixture of NaI and KI

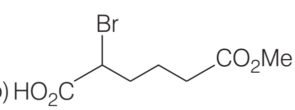
(d) A solution mixture of KIO_3 and $NaIO_3$

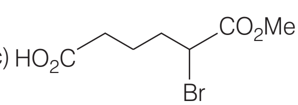
73. The reaction sequence given below given product R.

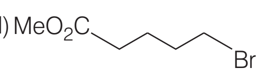


The structure of the product R is

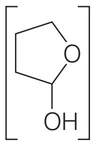
(a) 

(b) 

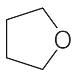
(c) 

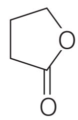
(d) 

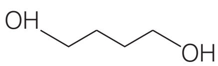
74. Reduction of the lactol S

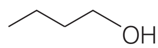


OH with sodium borohydride gives

(a) 

(b) 

(c) 

(d) 

75. What will be the normality of the salt solution obtained by neutralising x mL y (N) HCl with y mL x(N) NaOH, and finally adding $(x + y)$ mL distilled water

(a) $\frac{2(x+y)}{xy}$ N (b) $\frac{xy}{2(x+y)}$ N

(c) $\left(\frac{2xy}{x+y}\right)$ N (d) $\left(\frac{x+y}{xy}\right)$ N

Category-III (Q. 76 to Q.80)

Direction *One or more answer(s) is (are) correct. Correct answer(s) will fetch full marks 2. Any combination containing one or more incorrect answer will fetch 0 marks. Also no answer will fetch 0 marks. If all correct answers are not marked and also no incorrect answer is marked then score $= 2 \times$ number of correct answers marked $\div$ actual number of correct answers.*

76. During electrolysis of molten NaCl, some water was added. What will happen?

(a) Electrolysis will stop

(b) Hydrogen will be evolved

(c) Some amount of caustic soda will be formed

(d) A fire is likely

77. The role of fluorspar, which is added in small quantities in the electrolytic reduction of alumina dissolved in fused cryolite is

(a) as a catalyst

(b) to make fused mixture conducting

(c) to lower the melting temperature of the mixture

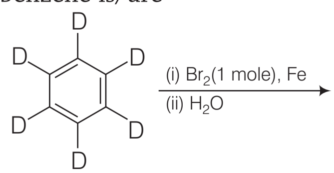
(d) to decrease the rate of oxidation of carbon at anode

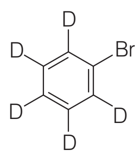
78. The reduction of benzenediazonium chloride to phenyl hydrazine can be accomplished by

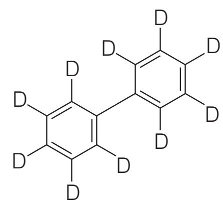
(a) $SnCl_2$, HCl (b) Na_2SO_3

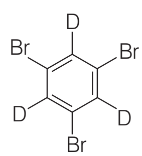
(c) CH_3CH_2OH (d) H_3PO_2

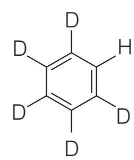
79. The major product(s) obtained form the following reaction of 1 mole of hexadeuteriobenzene is/are



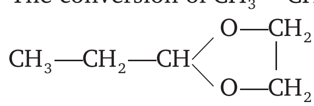
(a) 

(b) 

(c) 

(d) 

80. The conversion of $CH_3—CH_2—COOH$ to



can be accomplished by

(a) $SOCl_2$, $LiAlH_4$, ethylene glycol.

(b) $SOCl_2$, $KMnO_4$, NH_2NH_2

(c) $SnCl_2$, HCl, Na_2SO_3

(d) HCl, $SnCl_2$ ethylene glycol

MATHEMATICS

Category-I (Q.1 to Q.50)

Direction *Only one answer is correct. Correct answer will fetch full marks 1. Incorrect answer or any combination of more than one answer will fetch-1/4 marks. No answer will fetch 0 marks.*

1. The number of all numbers having 5 digits, with distinct digits is

(a) 99999 (b) $9 \times {}^9P_4$

(c) ${}^{10}P_5$ (d) ${}^9P_4$

2. The greatest integer which divides $(p+1)(p+2)(p+3)\ldots(p+q)$ for all $p \in N$ and fixed $q \in N$ is

(a) $p!$ (b) $q!$ (c) p (d) q

3. Let $(1+x+x^2)^9 = a_0 + a_1x + a_2x^2 + \ldots + a_{18}x^{18}$. Then,

(a) $a_0 + a_2 + \ldots + a_{18} = a_1 + a_3 + \ldots + a_{17}$

(b) $a_0 + a_2 + \ldots + a_{18}$ is even

(c) $a_0 + a_2 + \ldots + a_{18}$ is divisible by 9

(d) $a_0 + a_2 + \ldots + a_{18}$ is divisible by 3 but not by 9

4. The linear system of equations

$$\left.\begin{aligned} 8x - 3y - 5z &= 0 \\ 5x - 8y + 3z &= 0 \\ 3x + 5y - 8z &= 0 \end{aligned}\right\} \text{has}$$

(a) only zero solution

(b) only finite number of non-zero solutions

(c) no non-zero solution

(d) infinitely many non-zero solutions

5. Let P be the set of all non-singular matrices of order 3 over R and Q be the set of all orthogonal matrices of order 3 over R. Then,

(a) P is proper subset of Q

(b) Q is proper subset of P

(c) Neither P is proper subset of Q nor Q is proper subset of P

(d) $P \cap Q = \phi$, the void set

6. Let $A = \begin{pmatrix} x+2 & 3x \\ 3 & x+2 \end{pmatrix}$, $B = \begin{pmatrix} x & 0 \\ 5 & x+2 \end{pmatrix}$. Then all solutions of the equation $\det(AB) = 0$ is

(a) 1, – 1, 0, 2 (b) 1, 4, 0, – 2

(c) 1, – 1, 4, 3 (d) – 1, 4, 0, 3

7. The value of det A, where

$$A = \begin{pmatrix} 1 & \cos\theta & 0 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{pmatrix}, \text{ lies}$$

(a) in the closed interval [1, 2]

(b) in the closed interval [0, 1]

(c) in the open interval (0, 1)

(d) in the open interval (1, 2)

8. Let $f: R \to R$ be such that f is injective and $f(x)f(y) = f(x+y)$ for $\forall\ x, y \in R$. If $f(x)$, $f(y)$, $f(z)$ are in G.P., then x, y, z are in

(a) AP always

(b) GP always

(c) AP depending on the value of x, y, z

(d) GP depending on the value of x, y, z

9. On the set R of real numbers we define xPy if and only if $xy \geq 0$. Then, the relation P is

(a) reflexive but not symmetric

(b) symmetric but not reflexive

(c) transitive but not reflexive

(d) reflexive and symmetric but not transitive

10. On R, the relation ρ be defined by '$x \rho y$ holds if and only if $x - y$ is zero or irrational'. Then,

(a) ρ is reflexive and transitive but not symmetric

(b) ρ is reflexive and symmetric but not transitive

(c) ρ is symmetric and transitive but not reflexive

(d) ρ is equivalence relation

11. Mean of n observations $x_1, x_2, \ldots, x_n$ is $\bar{x}$. If an observation x_q is replaced by x'_q then the new mean is

(a) $\bar{x} - x_q + x'_q$ (b) $\dfrac{(n-1)\bar{x} + x'_q}{n}$

(c) $\dfrac{(n-1)\bar{x} - x'_q}{n}$ (d) $\dfrac{n\bar{x} - x_q + x'_q}{n}$

12. The probability that a non-leap year selected at random will have 53 Sunday is

(a) 0 (b) 1/7

(c) 2/7 (d) 3/7

13. The equation $\sin x(\sin x + \cos x) = k$ has real solutions, where k is a real number. Then,

(a) $0 \le k \le \frac{1+\sqrt{2}}{2}$

(b) $2-\sqrt{3} \le k \le 2+\sqrt{3}$

(c) $0 \le k \le 2-\sqrt{3}$

(d) $\frac{1-\sqrt{2}}{2} \le k \le \frac{1+\sqrt{2}}{2}$

14. The possible values of x, which satisfy the trigonometric equation

$\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$ are

(a) $\pm \frac{1}{\sqrt{2}}$ (b) $\pm \sqrt{2}$

(c) $\pm \frac{1}{2}$ (d) ± 2

15. Transforming to parallel axes through a point (p, q), the equation $2x^2 + 3xy + 4y^2 + x + 18y + 25 = 0$ becomes $2x^2 + 3xy + 4y^2 = 1$. Then,

(a) $p = -2, q = 3$ (b) $p = 2, q = -3$

(c) $p = 3, q = -4$ (d) $p = -4, q = 3$

16. Let $A(2, -3)$ and $B(-2, 1)$ be two angular points of ΔABC. If the centroid of the triangle moves on the line $2x + 3y = 1$, then the locus of the angular point C is given by

(a) $2x + 3y = 9$

(b) $2x - 3y = 9$

(c) $3x + 2y = 5$

(d) $3x - 2y = 3$

17. The point $P(3, 6)$ is first reflected on the line $y = x$ and then the image point Q is again reflected on the line $y = -x$ to get the image point Q'. Then, the circumcentre of the $\Delta PQQ'$ is

(a) $(6, 3)$ (b) $(6, -3)$

(c) $(3, -6)$ (d) $(0, 0)$

18. Let d_1 and d_2 be the lengths of the perpendiculars drawn from any point of the line $7x - 9y + 10 = 0$ upon the lines $3x + 4y = 5$ and $12x + 5y = 7$, respectively. Then,

(a) $d_1 > d_2$ (b) $d_1 = d_2$

(c) $d_1 < d_2$ (d) $d_1 = 2d_2$

19. The common chord of the circles $x^2 + y^2 - 4x - 4y = 0$ and $2x^2 + 2y^2 = 32$ subtends at the origin an angle equal to

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

20. The locus of the mid-points of the chords of the circle $x^2 + y^2 + 2x - 2y - 2 = 0$, which make an angle of $90°$ at the centre is

(a) $x^2 + y^2 - 2x - 2y = 0$

(b) $x^2 + y^2 - 2x + 2y = 0$

(c) $x^2 + y^2 + 2x - 2y = 0$

(d) $x^2 + y^2 + 2x - 2y - 1 = 0$

21. Let P be the foot of the perpendicular from focus S of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ on the line $bx - ay = 0$ and let C be the centre of the hyperbola. Then, the area of the rectangle whose sides are equal to that of SP and CP is

(a) $2ab$ (b) ab

(c) $\frac{(a^2+b^2)}{2}$ (d) $\frac{a}{b}$

22. B is an extremity of the minor axis of an ellipse whose foci are S and S'. If $\angle SBS'$ is a right angle, then the eccentricity of the ellipse is

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

23. The axis of the parabola $x^2 + 2xy + y^2 - 5x + 5y - 5 = 0$ is

(a) $x + y = 0$ (b) $x + y - 1 = 0$

(c) $x - y + 1 = 0$ (d) $x - y = \frac{1}{\sqrt{2}}$

24. The line segment joining the foci of the hyperbola $x^2 - y^2 + 1 = 0$ is one of the diameters of a circle. The equation of the circle is

(a) $x^2 + y^2 = 4$ (b) $x^2 + y^2 = \sqrt{2}$

(c) $x^2 + y^2 = 2$ (d) $x^2 + y^2 = 2\sqrt{2}$

25. The equation of the plane through $(1, 2, -3)$ and $(2, -2, 1)$ and parallel to X-axis is

(a) $y - z + 1 = 0$ (b) $y - z - 1 = 0$

(c) $y + z - 1 = 0$ (d) $y + z + 1 = 0$

26. Three lines are drawn from the origin O with direction cosines proportional to $(1, -1, 1)$, $(2, -3, 0)$ and $(1, 0, 3)$. The three lines are

(a) not coplanar
(b) coplanar
(c) perpendicular to each other
(d) coincident

27. Consider the non-constant differentiable function f of one variable which obeys the relation $\frac{f(x)}{f(y)} = f(x-y)$. If $f'(0) = p$ and $f'(5) = q$, then $f'(-5)$ is

(a) $\frac{p^2}{q}$ (b) $\frac{q}{p}$
(c) $\frac{p}{q}$ (d) q

28. If $f(x) = \log_5 \log_3 x$, then $f'(e)$ is equal to

(a) $e \log_e 5$ (b) $e \log_e 3$
(c) $\frac{1}{e \log_e 5}$ (d) $\frac{1}{e \log_e 3}$

29. Let $F(x) = e^x$, $G(x) = e^{-x}$ and $H(x) = G(F(x))$, where x is a real variable. Then, $\frac{dH}{dx}$ at $x = 0$ is

(a) 1 (b) -1 (c) $-\frac{1}{e}$ (d) $-e$

30. If $f''(0) = k$, $k \neq 0$, then the value of

$$\lim_{x \to 0} \frac{2f(x) - 3f(2x) + f(4x)}{x^2} \text{ is}$$

(a) k (b) $2k$ (c) $3k$ (d) $4k$

31. If $y = e^{m \sin^{-1} x}$ then

$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - ky = 0$, where k is equal to

(a) m^2 (b) 2 (c) -1 (d) $-m^2$

32. The chord of the curve $y = x^2 + 2ax + b$, joining the points where $x = \alpha$ and $x = \beta$, is parallel to the tangent to the curve at abscissa x is equal to

(a) $\frac{a+b}{2}$ (b) $\frac{2a+b}{3}$
(c) $\frac{2\alpha+\beta}{3}$ (d) $\frac{\alpha+\beta}{2}$

33. Let $f(x) = x^{13} + x^{11} + x^9 + x^7 + x^5 + x^3 + x + 19$. Then, $f(x) = 0$ has

(a) 13 real roots
(b) only one positive and only two negative real roots
(c) not more than one real root
(d) has two positive and one negative real root

34. Let $f(x) = \begin{cases} \frac{x^p}{(\sin x)^q}, & \text{if } 0 < x \leq \frac{\pi}{2}, \\ 0, & \text{if } x = 0 \end{cases}$

$(p, q \in R)$. Then, Lagrange's mean value theorem is applicable to $f(x)$ in closed interval $[0, x]$

(a) for all p, q
(b) only when $p > q$
(c) only when $p < q$
(d) for no value of p, q

35. $\lim_{x \to 0} (\sin x)^{2\tan x}$ is equal to

(a) 2 (b) 1
(c) 0 (d) does not exist

36. $\int \cos(\log x)\, dx = F(x) + C$, where C is an arbitrary constant. Here, $F(x)$ is equal to

(a) $x[\cos(\log x) + \sin(\log x)]$
(b) $x[\cos(\log x) - \sin(\log x)]$
(c) $\frac{x}{2}[\cos(\log x) + \sin(\log x)]$
(d) $\frac{x}{2}[\cos(\log x) - \sin(\log x)]$

37. $\int \frac{x^2 - 1}{x^4 + 3x^2 + 1} dx \, (x > 0)$ is

(a) $\tan^{-1}\left(x + \frac{1}{x}\right) + C$
(b) $\tan^{-1}\left(x - \frac{1}{x}\right) + C$
(c) $\log_e \left| \frac{x + \frac{1}{x} - 1}{x + \frac{1}{x} + 1} \right| + C$
(d) $\log_e \left| \frac{x - \frac{1}{x} - 1}{x - \frac{1}{x} + 1} \right| + C$

38. Let $I = \int_{10}^{19} \frac{\sin x}{1 + x^8} dx$. Then,

(a) $|I| < 10^{-9}$ (b) $|I| < 10^{-7}$
(c) $|I| < 10^{-5}$ (d) $|I| > 10^{-7}$

39. Let $I_1 = \int_0^n [x]\, dx$ and $I_2 = \int_0^n \{x\}\, dx$, where $[x]$ and $\{x\}$ are integral and fractional parts of x and $n \in N - \{1\}$. Then, I_1 / I_2 is equal to

(a) $\frac{1}{n-1}$ (b) $\frac{1}{n}$ (c) n (d) $n - 1$

40. The value of

$\lim_{n \to \infty}\left[\frac{n}{n^2 + 1^2} + \frac{n}{n^2 + 2^2} + \ldots + \frac{1}{2n}\right]$ is

(a) $\frac{n\pi}{4}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{4n}$ (d) $\frac{\pi}{2n}$

41. The value of the integral $\int_0^1 e^{x^2} dx$

(a) is less than 1
(b) is greater than 1
(c) is less than or equal to 1
(d) lies in the closed interval $[1, e]$

42. $\int_0^{100} e^{x - [x]} dx$ is equal to

(a) $\frac{e^{100} - 1}{100}$
(b) $\frac{e^{100} - 1}{e - 1}$
(c) $100(e - 1)$
(d) $\frac{e - 1}{100}$

43. Solution of $(x + y)^2 \frac{dy}{dx} = a^2$ ('a' being a constant) is

(a) $\frac{(x + y)}{a} = \tan \frac{y + C}{a}$, C is an arbitrary constant
(b) $xy = a \tan Cx$, C is an arbitrary constant
(c) $\frac{x}{a} = \tan \frac{y}{C}$, C is an arbitrary constant
(d) $xy = \tan(x + C)$, C is an arbitrary constant

44. The integrating factor of the first order differential equation

$x^2(x^2 - 1)\frac{dy}{dx} + x(x^2 + 1)\, y = x^2 - 1$ is

(a) e^x (b) $x - \frac{1}{x}$
(c) $x + \frac{1}{x}$ (d) $\frac{1}{x^2}$

45. In a GP series consisting of positive terms, each term is equal to the sum of next two terms. Then, the common ratio of this GP series is

(a) $\sqrt{5}$ (b) $\frac{\sqrt{5} - 1}{2}$
(c) $\frac{\sqrt{5}}{2}$ (d) $\frac{\sqrt{5} + 1}{2}$

46. If $(\log_5 x)(\log_x 3x)(\log_{3x} y) = \log_x x^3$, then y equals

(a) 125 (b) 25
(c) 5/3 (d) 243

47. The expression $\frac{(1 + i)^n}{(1 - i)^{n-2}}$ equals

(a) $-i^{n+1}$ (b) i^{n+1}
(c) $-2i^{n+1}$ (d) 1

48. Let $z = x + iy$, where x and y are real. The points (x, y) in the X-Y plane for which $\frac{z + i}{z - i}$ is purely imaginary, lie on

(a) a straight line
(b) an ellipse
(c) a hyperbola
(d) a circle

49. If p, q are odd integers, then the roots of the equation $2px^2 + (2p + q)\, x + q = 0$ are

(a) rational (b) irrational
(c) non-real (d) equal

50. Out of 7 consonants and 4 vowels, words are formed each having 3 consonants and 2 vowels. The number of such words that can be formed is

(a) 210 (b) 25200
(c) 2520 (d) 302400

Category-II (Q.51 to Q.65)

Direction *Only one answer is correct. Correct answer will fetch full marks 2. Incorrect answer or any combination of more than one answer will fetch -1/2 marks. No answer will fetch 0 marks.*

51. Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$. Then, for positive integer n, A^n is

(a) $\begin{pmatrix} 1 & n & n^2 \\ 0 & n^2 & n \\ 0 & 0 & n \end{pmatrix}$

(b) $\begin{pmatrix} 1 & n & n\left(\frac{n+1}{2}\right) \\ 0 & 1 & n \\ 0 & 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & n^2 & n \\ 0 & n & n^2 \\ 0 & 0 & n^2 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & n & 2n-1 \\ 0 & \frac{n+1}{2} & n^2 \\ 0 & 0 & \frac{n+1}{2} \end{pmatrix}$

52. Let a, b, c be such that $b(a+c) \neq 0$.

If $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0,$

then the value of n is

(a) any integer (b) zero
(c) any even integer (d) any odd integer

53. On set $A = \{1, 2, 3\}$, relations R and S are given by

$R = \{(1,1), (2,2), (3,3), (1,2), (2,1)\}$,
$S = \{(1,1), (2,2), (3,3), (1,3), (3,1)\}$.

Then,

(a) $R \cup S$ is an equivalence relation
(b) $R \cup S$ is reflexive and transitive but not symmetric
(c) $R \cup S$ is reflexive and symmetric but not transitive
(d) $R \cup S$ is symmetric and transitive but not reflexive

54. If one of the diameters of the curve $x^2 + y^2 - 4x - 6y + 9 = 0$ is a chord of a circle with centre (1, 1), the radius of this circle is

(a) 3 (b) 2
(c) $\sqrt{2}$ (d) 1

55. Let $A(-1, 0)$ and $B(2, 0)$ be two points. A point M moves in the plane in such a way that $\angle MBA = 2\angle MAB$. Then, the point M moves along

(a) a straight line (b) a parabola
(c) an ellipse (d) a hyperbola

56. If $f(x) = \int_{-1}^{x} |t|\, dt$, then for any $x \geq 0$, $f(x)$ is equal to

(a) $\frac{1}{2}(1-x^2)$ (b) $1-x^2$
(c) $\frac{1}{2}(1+x^2)$ (d) $1+x^2$

57. Let for all $x > 0$, $f(x) = \lim_{n \to \infty} n\left(x^{1/n} - 1\right)$, then

(a) $f(x) + f\left(\frac{1}{x}\right) = 1$
(b) $f(xy) = f(x) + f(y)$
(c) $f(xy) = xf(y) + yf(x)$
(d) $f(xy) = xf(x) + yf(y)$

58. Let $I = \int_0^{100\pi} \sqrt{(1-\cos 2x)}\, dx$, then

(a) $I = 0$ (b) $I = 200\sqrt{2}$
(c) $I = \pi\sqrt{2}$ (d) $I = 100$

59. The area of the figure bounded by the parabolas $x = -2y^2$ and $x = 1 - 3y^2$ is

(a) $\frac{4}{3}$ sq units (b) $\frac{2}{3}$ sq units
(c) $\frac{3}{7}$ sq units (d) $\frac{6}{7}$ sq units

60. Tangents are drawn to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ at the ends of both latusrectum. The area of the quadrilateral, so formed is

(a) 27 sq units (b) $\frac{13}{2}$ sq units

(c) $\frac{15}{4}$ sq units (d) 45 sq units

61. The value of K in order that $f(x)=\sin x-\cos x-Kx+5$ decreases for all positive real values of x is given by

(a) $K<1$ (b) $K\ge 1$

(c) $K>\sqrt{2}$ (d) $K<\sqrt{2}$

62. For any vector x, where $\hat{i}, \hat{j}, \hat{k}$ have their usual meanings the value of $(x\times\hat{i})^2+(x\times\hat{j})^2+(x\times\hat{k})^2$ where $\hat{i}, \hat{j}, \hat{k}$ have their usual meanings, is equal to

(a) $|x|^2$ (b) $2|x|^2$ (c) $3|x|^2$ (d) $4|x|^2$

63. If the sum of two unit vectors is a unit vector, then the magnitude of their difference is

(a) $\sqrt{2}$ units (b) 2 units

(c) $\sqrt{3}$ units (d) $\sqrt{5}$ units

64. Let α and β be the roots of $x^2+x+1=0$. If n be a positive integer, then $\alpha^n+\beta^n$ is

(a) $2\cos\frac{2n\pi}{3}$ (b) $2\sin\frac{2n\pi}{3}$

(c) $2\cos\frac{n\pi}{3}$ (d) $2\sin\frac{n\pi}{3}$

65. For real x, the greatest value of $\frac{x^2+2x+4}{2x^2+4x+9}$ is

(a) 1 (b) -1 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

Category-III (Q. 66 to Q.75)

Direciton *One or more answer(s) is/are correct. Correct answer(s) will fetch full marks 2. Any combination containing one or more incorrect answer will fetch 0 marks. Also no answer will fetch 0 marks. If all correct answers are not marked and also no incorrect answer is marked, then score $=2\times$ number of correct answers marked $\div$ actual number of correct answers.*

66. If $a, b\in\{1, 2, 3\}$ and the equation $ax^2+bx+1=0$ has real roots, then

(a) $a>b$

(b) $a\le b$

(c) number of possible ordered pairs (a, b) is 3

(d) $a<b$

67. If the tangent to $y^2=4ax$ at the point $(at^2, 2at)$ where $|t|>1$ is a normal to $x^2-y^2=a^2$ at the point $(a\sec\theta, a\tan\theta)$, then

(a) $t=-\operatorname{cosec}\theta$

(b) $t=-\sec\theta$

(c) $t=2\tan\theta$

(d) $t=2\cot\theta$

68. The focus of the conic $x^2-6x+4y+1=0$ is

(a) (2, 3) (b) (3, 2)

(c) (3, 1) (d) (1, 4)

69. Let $f:R\to R$ be twice continuously differentiable. Let $f(0)=f(1)=f'(0)=0$. Then,

(a) $f''(x)\ne 0$ for all x

(b) $f''(c)=0$ for some $c\in R$

(c) $f''(x)\ne 0$ if $x\ne 0$

(d) $f'(x)>0$ for all x

70. If $f(x)=x^n$, n being a non-negative integer, then the values of n for which $f'(\alpha+\beta)=f'(\alpha)+f'(\beta)$ for all $\alpha, \beta>0$ is

(a) 1 (b) 2

(c) 0 (d) 5

71. Let f be a non-constant continuous function for all $x\ge 0$. Let f satisfy the relation $f(x)f(a-x)=1$ for some $a\in R^+$. Then, $I=\int_0^a \frac{dx}{1+f(x)}$ is equal to

(a) a (b) $\frac{a}{4}$

(c) $\frac{a}{2}$ (d) $f(a)$

72. If the line $ax+by+c=0$, $ab\ne 0$, is a tangent to the curve $xy=1-2x$, then

(a) $a>0, b<0$

(b) $a>0, b>0$

(c) $a<0, b>0$

(d) $a<0, b<0$

73. Two particles move in the same straight line starting at the same moment from the same point in the same direction. The first moves with constant velocity u and the second starts from rest with constant acceleration f. Then,

(a) they will be at the greatest distance at the end of time $\frac{u}{2f}$ from the start

(b) they will be at the greatest distance at the end of time $\frac{u}{f}$ from the start

(c) their greatest distance is $\frac{u^2}{2f}$

(d) their greatest distance is $\frac{u^2}{f}$

74. The complex number z satisfying the equation $|z - i| = |z + 1| = 1$ is

(a) 0

(b) $1 + i$

(c) $-1 + i$

(d) $1 - i$

75. On R, the set of real numbers, a relation ρ is defined as 'aρb if and only if $1 + ab > 0$'. Then,

(a) ρ is an equivalence relation

(b) ρ is reflexive and transitive but not symmetric

(c) ρ is reflexive and symmetric but not transitive

(d) ρ is only symmetric

ANSWERS

Physics

1. (b)	**2.** (a)	**3.** (d)	**4.** (a)	**5.** (c)	**6.** (c)	**7.** (d)	**8.** (a)	**9.** (c)	**10.** (a)
11. (b)	**12.** (b)	**13.** (a)	**14.** (b)	**15.** (d)	**16.** (c)	**17.** (a)	**18.** (d)	**19.** (a)	**20.** (b)
21. (c)	**22.** (d)	**23.** (c)	**24.** (a)	**25.** (b)	**26.** (d)	**27.** (b)	**28.** (a)	**29.** (b)	**30.** (a)
31. (b)	**32.** (b)	**33.** (d)	**34.** (*)	**35.** (b)	**36.** (b,c)	**37.** (c)	**38.** (b,d)	**39.** (a,b,c,d)	**40.** (d)

Chemistry

41. (a)	**42.** (b)	**43.** (a)	**44.** (b)	**45.** (d)	**46.** (c)	**47.** (a)	**48.** (d)	**49.** (a)	**50.** (c)
51. (a)	**52.** (b)	**53.** (a)	**54.** (a)	**55.** (a)	**56.** (d)	**57.** (d)	**58.** (a)	**59.** (d)	**60.** (c)
61. (b)	**62.** (c)	**63.** (a)	**64.** (c)	**65.** (b)	**66.** (c)	**67.** (a)	**68.** (c)	**69.** (d)	**70.** (d)
71. (d)	**72.** (b)	**73.** (d)	**74.** (c)	**75.** (b)	**76.** (b,c,d)	**77.** (b, c)	**78.** (a,b)	**79.** (a)	**80.** (a,d)

Mathematics

1. (b)	**2.** (b)	**3.** (b)	**4.** (d)	**5.** (b)	**6.** (b)	**7.** (a)	**8.** (a)	**9.** (d)	**10.** (b)
11. (d)	**12.** (b)	**13.** (d)	**14.** (a)	**15.** (b)	**16.** (a)	**17.** (d)	**18.** (b)	**19.** (d)	**20.** (c)
21. (b)	**22.** (b)	**23.** (a)	**24.** (c)	**25.** (d)	**26.** (b)	**27.** (a)	**28.** (c)	**29.** (c)	**30.** (c)
31. (a)	**32.** (d)	**33.** (c)	**34.** (b)	**35.** (b)	**36.** (c)	**37.** (a)	**38.** (b)	**39.** (d)	**40.** (b)
41. (d)	**42.** (c)	**43.** (a)	**44.** (b)	**45.** (b)	**46.** (a)	**47.** (c)	**48.** (d)	**49.** (a)	**50.** (b)
51. (b)	**52.** (d)	**53.** (c)	**54.** (a)	**55.** (d)	**56.** (c)	**57.** (b)	**58.** (b)	**59.** (a)	**60.** (a)
61. (c)	**62.** (b)	**63.** (c)	**64.** (a)	**65.** (c)	**66.** (c)	**67.** (a, c)	**68.** (c)	**69.** (b)	**70.** (b)
71. (c)	**72.** (b, d)	**73.** (b,c)	**74.** (a,c)	**75.** (c)					

HINTS & SOLUTIONS

Physics

1. (b) The speed of a particle executing simple harmonic motion is $v = \omega\sqrt{a^2 - x^2}$

where, a = Amplitude

ω = Angular frequency

x = Displacement

or $v^2 = \omega^2(a^2 - x^2)$

According to the question,

$$v_1^2 = \omega^2(a^2 - x_1^2)$$
$$v_2^2 = \omega^2(a^2 - x_2^2)$$
$$v_1^2 - v_2^2 = \omega^2(x_2^2 - x_1^2)$$
$$\omega = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}}$$

Here $v_1 = 13$ m/s

$v_2 = 12$ m/s

$x_1 = 3$ m

$x_2 = 5$ m

$$\omega = \sqrt{\frac{(13)^2 - (12)^2}{(5)^2 - (3)^2}}$$
$$= \sqrt{\frac{169 - 144}{25 - 9}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$$
$$\omega = 2\pi r$$
$$v = \frac{\omega}{2\pi} = \frac{5/4}{2\pi}$$
$$v = \frac{5}{8\pi}$$

2. (a) The formula of frequency of 3 vibrations produced in tensed wire is

$$v = \frac{n}{2}\sqrt{\frac{T}{m}}$$

m = Mass per unit length of the wire $= \dfrac{m}{L}$

M = Mass of wire

L = Length of wire

n = Number of loops produced in wire

$$v = \frac{n}{2L}\sqrt{\frac{T}{M/L}} = \frac{n}{2}\sqrt{\frac{T}{ML}}$$

3. (d) When the sealed capillary tube is submerged vertically into water, the pressure inside the tube changes

For the inside capillary,

$$p_1V_1 = p_2V_2$$
$$\therefore \quad p_0(lA) = p'(l - x)A$$

where p' is pressure in capillary after being submerged

$$\therefore \quad p' = \frac{p_0 l}{l - x}$$

According to question, since level of water inside capillary coincides with outside.

$$\therefore \quad p' - p_0 = \frac{2\gamma}{r}$$
$$\therefore \quad \frac{p_0 l}{l - x} - p_0 = \frac{2\gamma}{r} = \frac{l}{\left(1 + \frac{p_0 r}{2\gamma}\right)}$$

4. (a) The Bulk modulus k

$$k = -\frac{p}{\Delta V / V} \quad \ldots \text{(i)}$$

Where, p = Pressure

ΔV = Change in volume

V = Volume of liquid

From (i)

$$\frac{p}{k} = -\frac{\Delta V}{V} = \frac{\Delta p}{p}$$
$$\Rightarrow \quad p = \frac{k\Delta p}{p}$$
$$\Delta p = 0 \cdot 01\% = 0 \cdot 01/100$$
$$p = \frac{k}{10000}$$

5. (c) Initial temperature of ideal gas,

$$T_1 = 273 + 27 = 300 \text{ K}$$

when temperature of gas is raised by 6°C, the final temperature of gas

$$T_2 = 273 + 6 + 27$$
$$= 306 \text{ K}$$

Let initial velocities are v_{rms_1}, and v_{rms_2}

$$v_{rms} \propto \sqrt{T}$$

$$\frac{v_{rms_1}}{v_{rms_2}} = \sqrt{\frac{T_1}{T_2}}$$

$$v_{rms_2} = \sqrt{\frac{T_2}{T_1}} \times v_{rms_1}$$

$$= \sqrt{\frac{306}{300}} \times v_{rms_1}$$

$$= 1 \cdot 00 \times v_{rms_1}$$

So, it will increase by 1%

6. (c) The internal energy

$\Delta U = nC_v \Delta T$

C_v = Specific heat of gas at constant volume

$$\Rightarrow \quad \Delta U = n \cdot \frac{3R}{2}\left(\frac{4p_0V_0}{nR} - \frac{p_0V_0}{nR}\right)$$

$$= n \cdot \frac{3R}{2}\left(\frac{4p_0V_0 - p_0V_0}{nR}\right)$$

$$= n \cdot \frac{3R}{2} \cdot \frac{3p_0V_0}{nR}$$

$$= \frac{9}{2} p_0V_0 \qquad \ldots(i)$$

Work done by the gas

$$W = (2p_0 + p_0)\frac{V_0}{2} = \frac{3p_0V_0}{2} \qquad \ldots(ii)$$

From first law of thermodynamics,

$$\Delta Q = dW + dU$$

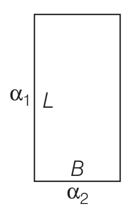
$$= \frac{3p_0V_0}{2} + \frac{9}{2}p_0V_0$$

[from Eqs. (i) and (ii)]

$$= \frac{3p_0V_0}{2} + \frac{9}{2}p_0V_0$$

$$= \frac{12p_0V_0}{2} = 6p_0V_0$$

7. (d) The coefficient of linear expansion along its length $= \alpha_1$



The coefficient of linear expansion along its breadth $= \alpha_2$

Increase in length,

$$L_t = l_0(1 + \alpha_1 \Delta t)$$

Increase in breadth,

$$B_t = b_0(1 + \alpha_2 \Delta t_2)$$

Let coefficient of surface expansion is β

$$\text{Area} = \text{length} \times \text{breadth}$$

$$= l_0(1 + \alpha_1 \Delta t) \times b_0(1 + \alpha_2 \Delta t)$$

$$= l_0 b_0 (1 + \alpha_1 \Delta t)(1 + \alpha_2 \Delta t)$$

$$= S_0(1 + \alpha_1 \Delta t + \alpha_2 \Delta t + \ldots)$$

where, $S_0 = l_0 \cdot b_0$

= Initial area of surface

In state of expansion,

$$S_t = L_t \times B_t$$

$$= l_0 b_0 (1 + \alpha_1 \Delta t)\,(1 + \alpha_2 \Delta t)$$

$$= S_0(1 + \alpha_1 \Delta t + \alpha_2 \Delta t + \ldots)$$

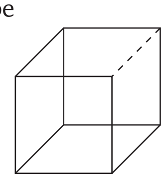
$$S_t = S_0(1 + \beta \Delta t)$$

$$\therefore\ S_0(1 + \beta \Delta t) = S_0\,(1 + \alpha_1 \Delta t + \alpha_2 \Delta t + \ldots)$$

$$\beta \cdot \Delta t = \alpha_1 \Delta t + \alpha_2 \Delta t$$

$$\beta = \alpha_1 + \alpha_2$$

8. (a) The positive charge Q is situated at the centre of a cube

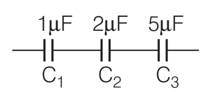


According to Gauss theorem, the flux from any closed surface

$$\phi_E = \frac{Q}{\varepsilon_0}$$

Cube has 6 faces, so electric flux from any face $\phi_E = \dfrac{Q}{6\varepsilon_0}$

9. (c)



When the capacitors are connected in series, the resultant capacitance of combination

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$= \frac{1}{1} + \frac{1}{2} + \frac{1}{5} = \frac{17}{10}\,\mu F$$

$$C = \frac{10}{17}\,\mu F$$

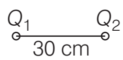
The charge will be same on all the capacitors in series

$$Q = CV = \frac{10}{17} \times 10 = \frac{100}{17}$$

The potential difference across $2.0\mu F$ capacitor

$$V' = \frac{Q}{C} = \frac{100/17}{2} = \frac{50}{17}\text{ volt}$$

10. (a)



Checking the given options one by one

(i) $Q_1 = Q_2 = 0 \cdot 4C$

The force on Q_1 due to Q_2

$$F = k_p \frac{Q_1 \times Q_2}{30 \times 10^{-2}}$$

$$= k \frac{Q_1 Q_2 \times 100}{30}$$

$$= k \frac{0 \cdot 4 \times 0 \cdot 4 \times 100}{30}$$

$$= \frac{k \times 0 \cdot 16 \times 100}{30}$$

(ii) When $Q_1 = 0 \cdot 8C\ Q_2 \approx 0$

$$F = \frac{k \times Q_1 Q_2}{30 \times 10^{-2}} \approx 0$$

(iii) when $Q_1 \approx 0, Q_2 = 0 \cdot 8C$

$$F = 0$$

(iv) $Q_1 = 0 \cdot 2C\ Q_2 = 0 \cdot 6C$

$$F = \frac{k \cdot 0 \cdot 2 \times 0 \cdot 6}{30 \times 10^{-2}}$$

$$= \frac{k \times 0 \cdot 12 \times 10^2}{30}$$

Thus we find that, in option(a), the force on Q_1 will be maximum.

$\therefore$ correct answer is $Q_1 = Q_2 = 0 \cdot 4C$

11. (b) By Biot-Savart's law, the magnetic field due to a current carrying wire is

$$B = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin\theta}{r^2}$$

So, $\quad B \propto \frac{1}{r^2}$

So, the magnetic field due to current decreases as inverse square of distance of the point of observation.

12. (b) In galvanometer $I \propto \theta = G\theta$

Where, $\quad G = \frac{k}{NAB}$

When coil is in square of shape,

$$G = \frac{k}{NAB} = \frac{k}{Na^2B} \quad \ldots \text{(i)}$$

When coil is in circular shape of radius $a/\sqrt{\pi}$

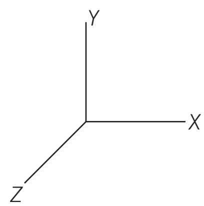
$$G = \frac{k}{NAB} = \frac{k}{N\pi r^2 \cdot B}$$

$$= \frac{k}{N\pi\left(\frac{a}{\sqrt{\pi}}\right)^2 \cdot B} = \frac{k}{Na^2B} \quad \ldots \text{(ii)}$$

So, value of G is same for both type of shape of coil. So, the torque $e = NBIA \sin\theta$ will be same in both the case

$$e : e = 1 : 1$$

13. (a) Velocity of proton $= 10^6$ m/s along y-direction



Electric field $= 2 \times 10^4$ V/m

$\frac{e}{m}$ for proton $= 10^{-8}C/kg$

The magnetic field,

$$B = \frac{E}{v} \quad (\because q_p vB = q_p E)$$

$$= \frac{2 \times 10^4}{10^6} = 2 \times 10^{-2}\,T$$

The radius of circular path

$$\gamma = \frac{mv}{q_p \cdot B} = \frac{10^{-8} \times 10^6}{2 \times 10^{-2}}$$

$$= \frac{1}{2} = 0 \cdot 5\text{ m}$$

14. (b) The impedance of *LCR* circuit

$$Z = \sqrt{R^2 + \left(\omega_C - \frac{1}{\omega_C}\right)^2}$$

Where, R = Resistance

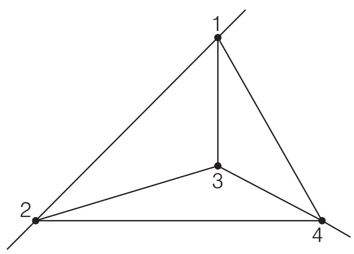
L = Inductance

C = Capacitance

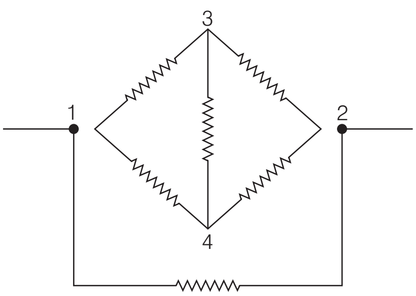
When frequency of AC voltage applied to a series LCR circuit, it will first increase and resonance state $\omega_L = \frac{1}{\omega_C}$, it will decrease. In that state (Z = R), it will be minimum.

So, it will first increases and then decreases.

15. (d) Six wires each of resistance r form a tetrahedron as shown in the following figure



The equivalent circuit of this tetrahedron



It is circuit of Wheatstone the equivalent resistance of upper circuit

$$\frac{1}{R} = \frac{1}{2r} + \frac{1}{2r} = \frac{2}{2r} = \frac{1}{r}$$

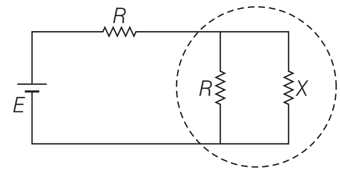
$$R = r$$

It will in parallel with outer resistance

$$\frac{1}{R_{eq}} = \frac{1}{r} + \frac{1}{r} = \frac{2}{r}$$

$$R_{eq} = \frac{r}{2}$$

16. (c)



For above circuit

$$\frac{1}{R'} = \frac{1}{R} + \frac{1}{X} = \frac{R+X}{RX}$$

$$R' = \frac{RX}{R+X}$$

The current in the circuit

$$i = \frac{E}{R+R'} = \frac{E}{\left(R + \frac{RX}{R+X}\right)}$$

$$V_{RX} = \frac{E \cdot \frac{RX}{R+X}}{\left(R + \frac{RX}{R+X}\right)} = \frac{EX}{R+2X}$$

Power dissipated in the circuit

$$P_X = \frac{V_{RX}^2}{X} = \frac{E^2 \cdot X^2}{X(R+2X)^2}$$

$$= \frac{E^2 X}{(R+2X)^2}$$

$$\frac{dP_X}{dX} = E^2 \frac{(R-2X)}{(R+2X)^3}$$

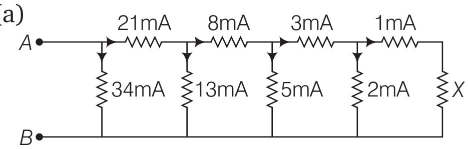
$$\Rightarrow \qquad dP_X = \frac{E^2(R-2X)}{(R+2X)^3} \cdot dX$$

(dP_X) will be zero for all (dX) if

$$X = \frac{R}{2} \qquad \left[dP_X = \frac{E^2\left(R - 2 \times \frac{R}{2}\right)}{\left(R + 2 \times \frac{R}{2}\right)^3} \right.$$

$$\left. = 0 \right]$$

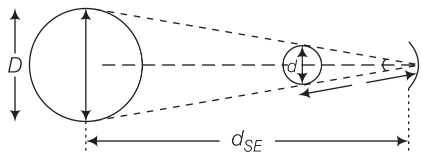
17. (a)



The potential between points A and B

$V_{AB} = 34$ volt

18. (d)



The angle subtended by sun on mirror

$$Q = \frac{D}{d_{SE}} = \frac{d}{f}$$

$$\therefore \quad d = \frac{D}{d_{SE}} \times f$$

According to the question,

$$\frac{D}{d_{SE}} = 0 \cdot 009$$

$$f = \frac{r}{2} = \frac{0.4}{2} = 0 \cdot 2 \text{ m}$$

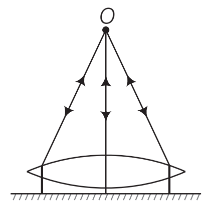
$$\therefore \quad d = 0 \cdot 009 \times 0 \cdot 2 \text{ m}$$
$$= 9 \times 2 \times 10^{-4} \text{ m}$$
$$= 18 \times 10^{-4} \text{m}$$
$$= 1 \cdot 8 \times 10^{-3} \text{m}$$

19. (a) For maximum intensity,

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\delta$$
$$\delta = 0$$
$$I_{max} = L + \frac{L}{4} + 2\sqrt{L \times \frac{L}{4}} \cdot \cos 0$$
$$= \frac{5L}{4} + \frac{2L}{2}$$
$$= \frac{5L + 4L}{4}$$
$$= \frac{9L}{4}$$
$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2} \cos\theta$$
$$\theta = 0°$$
$$= L + \frac{L}{4} - 2\sqrt{L \times \frac{L}{4}} \times 1$$
$$= \frac{5L}{4} - L = \frac{L}{4}$$

Thus, maximum and minimum intensities will be $\left(\frac{9L}{4}, \frac{L}{4}\right)$.

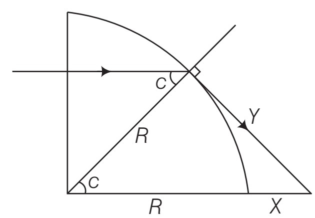
20. (b) As the lens is placed on a horizontal thin plane mirror.



So due to role of plane mirror the image will be formed at the same point O above 0.1 m distance above the center of the lens.

21. (c) Given, radius $R = 0 \cdot 05$ m

Refractive index $n = 1 \cdot 5$



The critical angle, $\sin c = \frac{1}{n} = \frac{1}{1 \cdot 5}$

$$= \frac{10}{15} = \frac{2}{3}$$

From above figure,

$$\cos c = \frac{R}{R + X}$$
$$\sqrt{1 - \sin^2 c} = \frac{R}{R + X}$$
$$\sqrt{1 - \frac{4}{9}} = \frac{R}{R + X}$$
$$\frac{\sqrt{5}}{3} = \frac{R}{R + X}$$
$$\sqrt{5}(R + X) = 3R$$
$$\sqrt{5}R + \sqrt{5}X = 3R$$
$$\therefore \quad X = (3\sqrt{5} - 5) \times 10^{-2} \text{m}$$

22. (d) de-Broglie wavelength,

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{E_2}{E_1}}$$

λ_1 = Wave length of electron

$= 0 \cdot 4 \times 10^{-10}$ m

$E_1 = 1 \cdot 0$keV

$\lambda_2 = 1 \cdot 0 \times 10^{-10}$m

$E_2 = ?$

$$\frac{0 \cdot 4 \times 10^{-10}}{1 \times 10^{-10}} = \sqrt{\frac{E_2}{1 \cdot 0 \text{ keV}}}$$

$$\frac{4}{10} = \sqrt{\frac{E_2}{1}}$$

$$\frac{16}{100} = \frac{E_2}{1}$$

$$E_2 = 0 \cdot 16\text{keV}$$

23. (c) From Einstein's photoelectric equation,

$$hv = W + eV$$

W = Work function

v = Frequency of incident photon

V = Stopping potential

For a photon of frequency v_1,

$$hv_1 = W + eV_1 \quad \ldots(i)$$

For photon of frequency v_2,

$$hv_2 = W + eV_2 \quad \ldots(ii)$$

Divided Eq. (i) by Eq. (ii),

$$\frac{hv_1}{hv_2} = \frac{W + eV_1}{W + eV_2}$$

$$\frac{v_1}{v_2} = \frac{W + eV_1}{W + eV_2}$$

$\Rightarrow \quad Wv_1 + eV_2v_1 = Wv_2 + eV_1v_2$

Solving above equation,

$$\Rightarrow \quad e = \frac{W(v_2 - v_1)}{V_2v_1 - V_1v_2}$$

24. (a) Half life of radon $= 3 \cdot 8$ days

Total half-lives in 19 days

$$= \frac{19}{3 \cdot 8}$$

$$= \frac{190}{38}$$

$= 5$ half-lives

Initially mass of radon $= 0 \cdot 064$ kg

$$0 \cdot 064 \xrightarrow{T_1} 0 \cdot 032 \xrightarrow{T_2}$$

$$0 \cdot 016 \xrightarrow{T_3} 0 \cdot 008$$

$$\xrightarrow{T_4} 0 \cdot 004 \xrightarrow{T_5} 0 \cdot 002$$

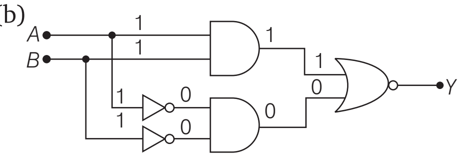
So in five half-life periods, the Radon sample will reduce to 0.002 kg

$$\text{Or } \frac{N}{N_0} = \left(\frac{1}{2}\right)^5$$

$$\Rightarrow \quad N = 0 \cdot 064 \times \frac{1}{32}$$

$$= 0 \cdot 002 \text{ kg}$$

25. (b)



The output of logic gate circuit

$$Y = \overline{A \cdot B + \overline{A}\ \overline{B}}$$

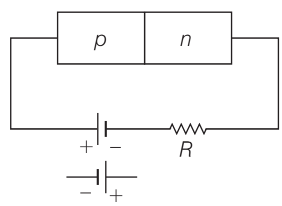
for $\quad A = 1$

$B = 1 \Rightarrow Y = 0$

and for $\quad A = 0, B = 0$

$\Rightarrow \quad Y = 0$

26. (d) When the battery and resistance is connected with pn diode,



In case of forward bias that p is connected to + ve terminal of battery, current flows.

If one reverse the polarity i.e., p-end is made negative, no current flows in semiconductor diode.

27. (b) The dimensions of universal constant G, is obtained by Newton's law of gravitation

$$F = \frac{Gm_1m_2}{r^2}$$

$$G = \frac{F \times r^2}{m_1 m_2}$$

F = Force

r = Distance

m_1 and m_2 are masses

$$G = \frac{[M^1L^1T^{-2}][L^2]}{[M][M]}$$

$$= \frac{[M^1L^3T^{-2}]}{[M^2]} = [M^{-1}L^3T^{-2}]$$

28. (a) In the absence of external force the speed of centre of mass is always zero i.e.

$$u_{cm} = 0$$

$\because$ $F_{ext} = 0$ then,

$v_{cm.} = 0$ always

29. (b) Three vectors are mutually perpendicular, so the scalar or dot product of two vectors will zero. So,

$A \cdot B = 0, B \cdot C = 0, A \cdot C = 0$

So, $A \cdot B = (a\hat{i} + \hat{j} + \hat{k}) \cdot (\hat{i} + b\hat{j} + \hat{k}) = 0$

$\Rightarrow a + b + 1 = 0$ ...(i)

$B \cdot C = (\hat{i} + b\hat{j} + \hat{k}) \cdot (\hat{i} + \hat{j} + c\hat{k}) = 0$

$\Rightarrow 1 + b + c = 0$ ... (ii)

$A \cdot C = (a\hat{i} + \hat{j} + \hat{k}) \cdot (\hat{i} + \hat{j} + c\hat{k}) = 0$

$\Rightarrow$ $a + 1 + c = 0$ ... (iii)

Addition of Eqs. (i), (ii) and (iii),

$$2(a + b + c) + 3 = 0$$

$$a + b + c = -\frac{3}{2}$$

$$\Rightarrow \quad -1 + c = -\frac{3}{2}$$

$$c = -\frac{3}{2} + 1$$

$$= -\frac{1}{2} = 0 \cdot 5$$

$$\therefore \quad a = -\frac{1}{2}$$

$$b = -\frac{1}{2}$$

$$c = -\frac{1}{2}$$

30. (a) From Newton's second law

$F = ma$

$m = 1$ kg

$$F = 1 \times a = \frac{dv}{dt}$$

According to the question,

$$F = kt = \frac{dv}{dt}$$

$$dv = kt \cdot dt$$

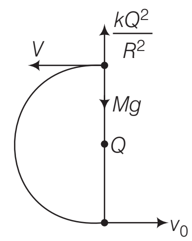
Integration gives

$$v = \frac{kt^2}{2} = \frac{dx}{dt}$$

$$x = \frac{kt^3}{6} = \frac{1 \times 6 \times 6 \times 6}{6}$$

$= 36$ m

31. (b) According to the question, when tension in the string is zero.



As $T = 0$

$$Mg - \frac{KQ^2}{R^2} = \frac{mv^2}{R} \qquad \left(k = \frac{1}{4\pi\varepsilon_0}\right)$$

$\frac{mv^2}{R}$ = centripetal force required to keep the body in a circular path

$$\Rightarrow \quad v = 0 \qquad \left[\because Mg = \frac{kQ^2}{R^2}\right]$$

So the required work done to keep charge in vertical motion

$\therefore$ $W_g = \Delta KE$ = change in K.E.

$$\Rightarrow \quad mg(2R) = \frac{1}{2} mv_0^2$$

$$v_0^2 = 2g \cdot 2R = 4gR$$

$$v_0 = \sqrt{4gR} = 2\sqrt{gR}$$

32. (b) Let mass of bullet = m

Mass of block = M

Velocity of bullet = v = 300 m/s

Velocity of combined system M + m = V

Here, from momentum conservation

$$\frac{M+m}{m} V = v$$

$$\Rightarrow \qquad V = \frac{300 \times 4.2 \times 10^{-2}}{4.2 \times 10^{-2} + 9\,(4.2 \times 10^{-2})}$$

$$= 30 \text{ m/s}$$

Now, heat produced = Loss in kinetic energy of bullet

$$= \frac{1}{2} mv^2 - \frac{1}{2}(M+m)V^2$$

$$= \frac{1}{2} \times 4.2 \times 10^{-2}\,(300)^2 - \frac{1}{2}(4.2 \times 10^{-2} + 9 \times 4.2 \times 1)\,(30)^2$$

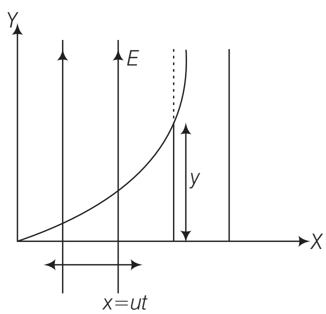
$$= 6.3 \times 270$$

$$= 1701 \text{ J}$$

$$= \frac{1701}{4.2} \text{ Cal}$$

$$= 405 \text{ Cal}$$

33. (d) From parabola



$$y = \frac{1}{2} \times \left[\frac{Ee}{m}\right] \times \frac{X^2}{u^2}$$

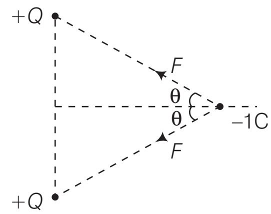
$$= \frac{E \cdot e}{2mu^2} \cdot X^2$$

As $\qquad X = 4ay \qquad$ (for parabola)

$$\therefore \qquad X^2 = \frac{2mu^2}{E \cdot e} \cdot y$$

$$a = \frac{2mu^2}{4E \cdot e} = \frac{mu^2}{2E \cdot e}$$

34. (*)



From figure the net force

$$F_{net} = -F\cos\theta + (-F\cos\theta)$$

$$= -2F\cos\theta$$

$$= -2 \times \frac{kQ \times 1}{(x^2 + a^2)} \times \frac{x}{\sqrt{x^2 + a^2}}$$

$$= -\frac{2kQ}{(x^2 + a^2)^{3/2}} \cdot x$$

$$\therefore \qquad F_{net} = -\left(\frac{2kQ}{a^3}\right) \cdot x$$

Frequency of oscillation

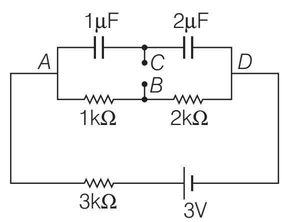
$$= \frac{1}{2\pi}\sqrt{\frac{2kQ}{Ma^3}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{2 \times \frac{1}{4\pi\varepsilon_0} \cdot Q}{Ma^3}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{Q}{2\pi\varepsilon_0 Ma^3}}$$

$\Rightarrow$ (* None of the option matches)

35. (b)



From circuit the current,

$$I = \frac{E}{R}$$

The equivalent resistance of above circuit

$$= 3 + 2 + 1$$

$$= 6\text{k } \Omega$$

$$= 6 \times 10^3 \Omega$$

$$E = 3 \text{ volt}$$

$$I = \frac{3}{6 \times 10^3}$$

$$= 0.5 \times 10^{-3}\ A$$

Potential,

$$V_{AD} = iR = 0 \cdot 5 \times 10^{-3} \times 3 \times 10^3$$

$$= 1 \cdot 5V$$

Charge,

$$\frac{1}{C} = \frac{1}{1} + \frac{1}{2} = \frac{3}{2}$$

$$C = \frac{2}{3}$$

$$Q = C \cdot V_{AD}$$

$$= \frac{2}{3} \times 1 \cdot 5$$

$$= 1\ \mu C$$

Applying KVL (Kirchhoff's Voltage Law) from B to C

$$V_B - 0 \cdot 5 \times 10^{-3} \times 2 \times 10^3 + \frac{1}{2} = V_C$$

$$V_B - V_C = 1 - \frac{1}{2} = 0 \cdot 5V$$

36. (b,c) $v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{\gamma P}{\rho}}$

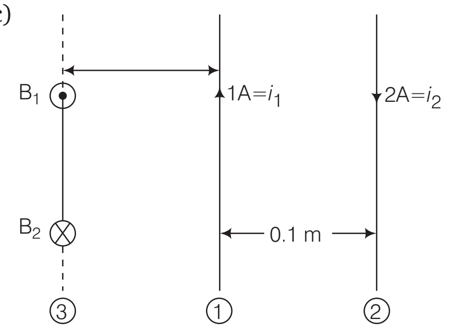
(i) Velocity of sound is proportional to $\sqrt{T}$

$\therefore$ $\quad v \propto \sqrt{T}$

(ii) When ρ is constant, the velocity of sound is proportional to root of pressure P

$\therefore$ $\quad v \propto \sqrt{P}$

37. (c)



According to question, the third wire should not feel magnetic force due to wire (1) and (2). It can be only possible in the condition that fields due to wire (1) and (2) must act opposite to each other and must be equal.

The magnetic field due to long straight wire,

$$B = \frac{\mu_0 \cdot I}{2\pi x}$$

$\therefore$ $\quad B_1 = B_2$

$$\frac{\mu_0 i_1}{2\pi x} = \frac{\mu_0 i_2}{2\pi(0 \cdot 1 + x)}$$

Here $\quad i_1 = 1A$

$\quad i_2 = 2A$

$$\frac{\mu_0 \times 1}{2\pi x} = \frac{\mu_0 \times 2}{2\pi(0 \cdot 1 + x)}$$

$$2x = (0 \cdot 1 + x)$$

$$x = 0 \cdot 1\ m$$

38. (b, d) We know that magnetic susceptibility

$$\chi = \mu_{r-1}$$

and $\quad \mu_r = \frac{\mu}{\mu_0}$

= Relative permeability

For paramagnetic substance,

$$\chi > 0$$

$\therefore$ $\quad \mu_r > 1$

$\therefore$ $\quad u > \mu_0$

For diamagnetic substance,

$$\chi < 0$$

$\therefore$ $\quad \mu_r < 1$

$\therefore$ $\quad \mu < \mu_0$

For ferromagnetic substance,

$$\chi >> 1$$

$$\mu >> M_0$$

If $\quad 0 < \mu < \mu_0$

Then substance will not be paramagnetic. Hence option (a) is incorrect and option (b) and (d) are correct.

39. (a, b, c, d) We know that, $v \propto \frac{1}{n}$ (speed)

$$E_n \propto \frac{1}{n^2} \quad \text{(energy)}$$

$$r_n \propto n^2$$

$\therefore$ $\quad E_n r_n \propto n^0$

$\therefore$ $\quad E_n r_n \propto E_1 r_1$

$$\frac{E_n r_n}{E_1 r_1} = \text{constant} \qquad (\because \text{slope} = 0)$$

$$r_n v_n \propto n^2 \times \frac{1}{n} \propto n$$

$$\therefore \quad \frac{r_n v_n}{r_1 v_1} = n \qquad (\because \text{slope} = 1)$$

$$r_n \propto n^2$$

$$\therefore \quad \frac{r_n}{r_1} = n^2$$

$$\log\left(\frac{r_n}{r_1}\right) = 2 \log n \qquad (\because \text{slope} = 2)$$

$$\frac{r_n}{E_n} \propto n^4$$

$$\therefore \quad \frac{r_n}{E_n} \times \frac{E_1}{r_1} = n^4$$

$$\log\left(\frac{r_n E_1}{E_n r_1}\right) = 4 \log(n) (\because \text{slope} = 4)$$

40. (d) Given, steel ball bounces on a steel plate held horizontally, so

$$V_{upward} = eV_{downward}$$

According to the relation,

$$h_n = e^{2n} h$$

$$\therefore \quad t = \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2he^2}{g}} + 2\sqrt{\frac{2he^4}{g}} + \dots$$

$$= \sqrt{\frac{2h}{g}} [1 + 2e + 2e^2 + \dots]$$

$$= \sqrt{\frac{2h}{g}} \left[\frac{1+e}{1-e}\right]$$

$$\therefore \quad t = 10 \text{ sec}$$

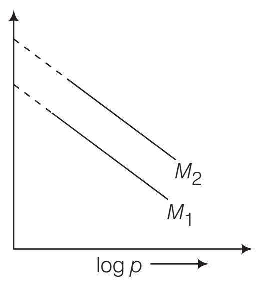
$$\therefore \quad 10 = \sqrt{\frac{2(0 \cdot 4)}{10}} \left(\frac{1+e}{1-e}\right)$$

Solving above relation

$$\therefore \quad e = \frac{25\sqrt{2} - 1}{25\sqrt{2} + 1} \approx \frac{17}{18}$$

Chemistry

41. (a)For ideal gas, $\log \frac{k}{M_2}$, $\log \frac{k}{M_1}$



$$\because \quad pV = \frac{W}{M} RT \qquad \dots(i)$$

Let, $\quad WRT = K$

Taking log of both the sides of Eq. (i)

$$\log p + \log V = \log \frac{K}{M}$$

$$\text{or} \quad \log V = -\log p + \log \frac{K}{M} \qquad \dots(ii)$$

[On compairing Eq. (ii) with $y = mx + C$]

$$\log \frac{K}{M_2} > \log \frac{K}{M_1} \quad \text{or} \quad M_1 > M_2$$

42. (b) $\because$ Surface tension (T) = Work done per unit area

$$\therefore \quad T = \frac{dW}{dA}$$

$$= \frac{F \cdot dx}{dA} \qquad (FL^{-1} = MT^{-2})$$

$\therefore$ Dimension of surface tension = ML^0T^{-2}

43. (a) According to $(n + l)$ rule;

(i) More be the sum of $(n + l)$ value, more be the energy.

(ii) For same value of sum, more be the value of n, more be the energy.

For

(i) $n + l \rightarrow 4 + 1 = 5$

(ii) $n + l \rightarrow 4 + 0 = 4$

(iii) $n + l \rightarrow 3 + 2 = 5$

(iv) $n + l \rightarrow 3 + 1 = 4$

Hence, order of energy will be

(iv) < (ii) < (iii) < (i)

44. (b) Electronic configuration for Cr(24)
$= [Ar]_{18} 4s^1 3d^5$
$\therefore$ 19^{th} electron enters in 4*s*-orbital and its set of quantum numbers is
$n = 4, l = 0, m = 0, m_s = +\frac{1}{2}$

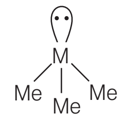
45. (d) $\because$ Number of milli equivalents of NaOH $(n') = NV = 0.1 \times 20 = 2$
$\therefore$ Number of equivalent = 0.002
For Neutralisation
Number of equivalent of base (NaOH) = Number of equivalent of acid
$$0.002 = \frac{W}{E} = \frac{0.126}{E}$$
$$\therefore \quad E = \frac{0.126}{0.002} = 63$$

46. (c) Given, weight ratio : $W_{CH_4} : W_{SO_2} = 1 : 2$
$$\because \quad n = \frac{W}{m} \quad \left[\begin{array}{l} n = \text{no. of moles} \\ w = \text{mass} \\ M = \text{molar mass} \end{array}\right.$$
$$\therefore \quad \frac{n_1}{n_2} = \frac{W_1}{M_1} \times \frac{M_2}{W_2} \left\{\begin{array}{l} n_1 = n_{SO_2} \\ n_2 = n_{CH_4} \end{array}\right.$$
$$\frac{n_1}{n_2} = \frac{W_{SO_2}}{M_{SO_2}} \times \frac{M_{CH_4}}{W_{CH_4}}$$
$$= \frac{2}{64} \times \frac{16}{1}$$
$$\frac{n_1}{n_2} = \frac{1}{2}, \text{ i.e. } 1 : 2$$
Also $n \propto N$
$\therefore$ Ratio of number of molecules is 1 : 2.

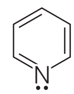
47. (a) ${}_9F^{18} \longrightarrow {}_xE^y + {}_{+1}e^0$
$$x = 8, y = 18,$$
$$\therefore \quad {}_xE^y = {}_8O^{18}$$
$\because$ Position has one unit of the charge and zero mass.
Thus, F^{18} changes to O^{18} therefore resulting decay product is $C_6H_5O^{18}$.

48. (d) $\because$ On dissolving NaCN in de-ionised water, following reaction takes place.
$$CN^- + H_2O \rightleftharpoons HCN + OH^-$$
$\because$ CN^- ions come from the salt of strong base and weak acid.
$\therefore$ The solution becomes basic and pH > 7.

49. (a) $\because$ In $Me_3N \rightarrow$ N is sp^3-hybrid

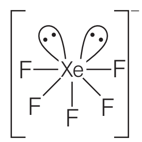


In $C_5H_5N \rightarrow$ N is sp^2-hybrid



and in MeCN $\rightarrow$ N is sp-hybrid
Also electronegativity of any element α% *s*-character in hybrid orbitals.
Thus, order of electronegativity for nitrogen is
$$MeCN > C_5H_5N < Me_3N$$

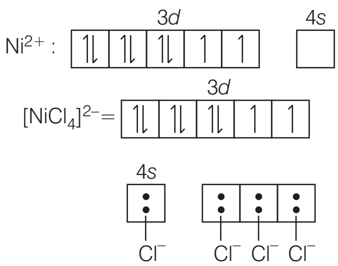
50. (c) **Shape of** $XeF_5^- \rightarrow$



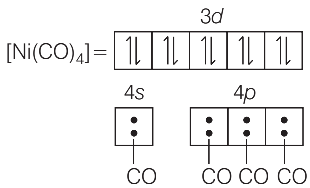
The Xe show sp^3d^3-hybridisation in XeF_5^-,
Thus, its geometry is
Pentagonal-bipyramidal.
But, number of electron pair $= \frac{8 + 5 + 1}{2} = 7$
Number of bond pair $= 5^2$, number of lone pair = 2
A lot of electrons are present at axil position and all bonds are in same plane. Hence, the shape is planar.

51. (a) According to molecular orbital theory, outer configuration for
$$[C_2] = C_{12} \rightarrow \pi p_x{}^2 = \pi p_y{}^2$$
Thus, is diamagnetic in nature.
$$[B_2] = B_{10} -> \pi p_x^1 = \pi p_y^1$$
Thus, is paramagnetic in nature.

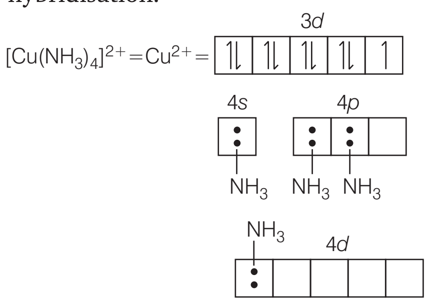
52. (b) Configuration for metal (M) in various complex.



Ni^{2+} has 2 unpaired electrons



Ni^0 has zero unpaired electrons with sp^3-hybridisation.



Cu^{2+} has one unpaired electron with dsp^2-hybridisation.

$\therefore$ (b) is the correct answer, i.e.

Number of unpaired electron (2, 0, 1)

53. (a) $\because$ F-atom has 2nd highest electron affinity (Cl has highest) in the periodic table

$\therefore$ Among the given options

F-has highest electron gain affinity

54. (a) $PbCl_2$ react with HCl as follows

$$PbCl_2(s) + Cl^- \xrightarrow{\text{Cold}} [PbCl_3]^-(aq)$$

$$PbCl_2(s) + 2Cl^- \xrightarrow[\text{of HCl}]{\text{Excess}} [PbCl_4]^{2-}(aq)$$

Thus, addition of excess of Cl^- ions change the $PbCl_2$ as soluble complex of $[PbCl_4]^{-2}$. Hence, becomes soluble.

55. (a) $\because$ Higher be the oxidation number of central atom in oxo-acids, more strongly it behave as Bronsted-acid.

Oxidation number for Cl in $HClO_3 \to +5$

Oxidation number for Cl in $HClO_2 \to +3$

Oxidation number for Cl in $HClO \to +1$

Oxidation number for Cl in $HBrO \to +1$

Hence, $HClO_3$ is the strongest Bronsted acid in aqueous solution.

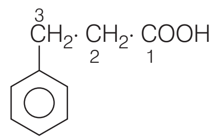
56. (d) As size of cation decreases, ionic character decreases, thus basicity also decreases

Hence, correct order is

$La^{3+} > Ce^{3+} > Eu^{3+} > Lu^{3+}$

57. (d) $\because$ If CO_3^{2-}, SO_3^{2-} and SO_4^{2-} are present alongwith $BaCl_2$, these can also show white precipitate (as precipitate of all these are also white).

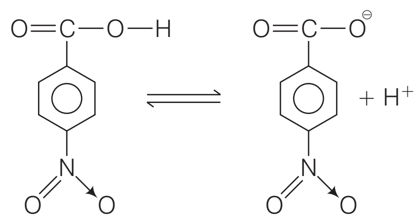
58. (a) IUPAC name of $Ph \cdot CH_2CH_2CO_2H$ is, 3-phenylpropanoic acid



59. (d) Isomerisation of 1-butyne to 2-butyne can be achived by treatment with ethanolic KOH

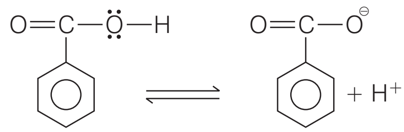
$$\underset{\text{1-butyne}}{H_3C-CH_2-C\equiv CH} \xrightarrow[\text{KOH (ethanolic)}]{\text{Isomerisation}} \underset{\text{2-butyne}}{CH_3-C\equiv C-CH_3}$$

60. (c) $\because$ In p-nitrobenzoic acid (Z) — NO_2 present as electron withdrawing group, thus is most acidic



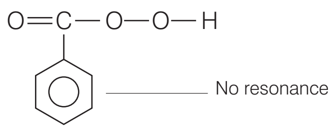
(p-introbenzoic acid)

(X) $\because$ Benzoic acid is stabilised *via* resonance, thus is more acidic than per oxybenzoic acid.



Benzoic acid

(Y) peroxy benzoic acid does not show any such resonance.

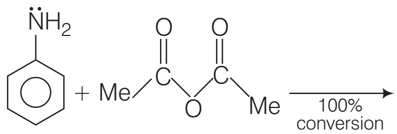


(Peroxy benzoic acid)

Thus is least acidic.

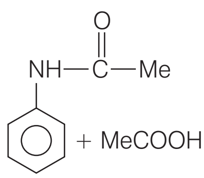
Hence, correct order is $Z > X > Y$.

61. (b) The reaction between aniline and acetic anhydride is as follows :



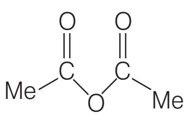
Initial moles 2 1

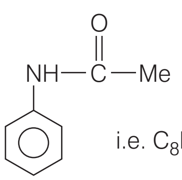
Final moles 1 0

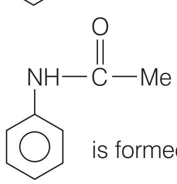


0 0

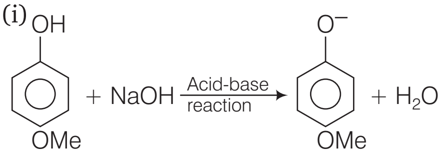
1 1

As  is limiting reagent

M. nt of  i.e. C_8H_9ON is 135 g/mol

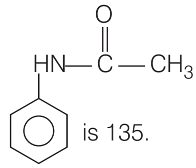
As 1 mole of  is formed

62. (c) (i)

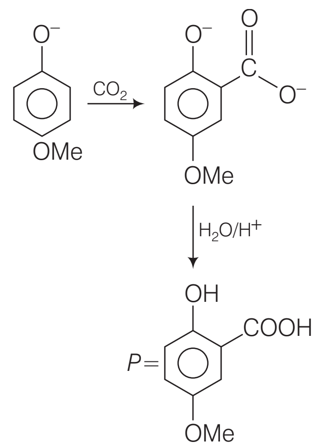


($\because$ Acetic anhydride is limiting reagent)

Thus, we get one mole of acetanilide and molar mass of

 is 135.

(ii)



(Product)

$\because O^-$ is more reactive than —OMe and p-position is occupied by —OMe, the substitution occurs at *ortho*-position w.r.t —O^- group.

63. (a) ADP is adenosine diphosphate (2-phosphate groups) and ATP is adenosine triphosphate (3 phosphate groups)

Thus, they differ in number of phosphate units.

64. (c) When primary amine react with chloroform and ethanolic potassium hydroxide, they produce –isocyamides as main product.

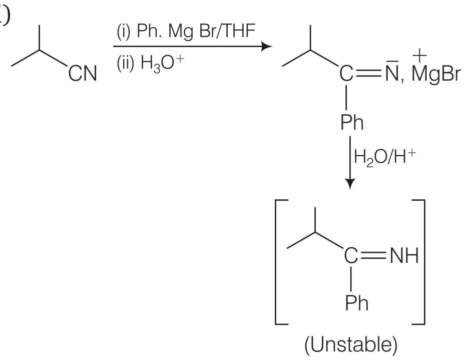
The reaction is also known as carbyl amine reaction.

$$Ph-NH_2 \xrightarrow[\text{KOH (ethanolic)}]{CHCl_3} Ph\cdot N \longrightarrow C$$

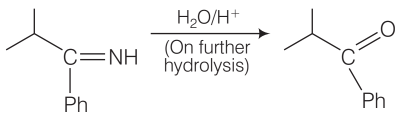
(Nauseating smell)

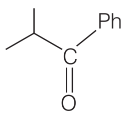
65. (b) The given reaction occurs as follows :

(i)



(ii)





66. (c) As per given relation,

$N_1V_1 + N_2V_2 = N(V_1 + V_2)$

$N_1 = 2 \quad N_2 = 5$

$V_1 = 500\, V_2 = \text{say-x}$

Thus,

$2 \times 500 + 5x = 3(x + 500)$ or $2x = 500$

i.e. $x = 250$

Hence, maximum volume of

3M HCl $= 500 + 250 = 750$ mL

67. (a) $\because E = hv = \dfrac{hc}{\lambda} = hc \cdot \bar{v}$ $\left(\because \dfrac{1}{\lambda} = \bar{v}\right)$

where E = energy of photon

c = velocity of photon (= light)

λ = wavelength of photon

h = plank's constant.

$\therefore$ **For (a)**

$E = 6.63 \times 10^{-34} \times 3 \times 10^8 / 300 \times 10^{-9}$

$E = 1.98 \times 10^{-25} \cdot J / 300 \times 10^{-9}$

$\therefore \quad E = \dfrac{1.98 \times 10^{-25} J}{300 \times 10^{-9} m}$

(a) $\rightarrow E = 6.6 \times 10^{-19}$ J

For (b) $E = hv = 6.63 \times 10^{-34} \times 3 \times 10^8$

(b) $\rightarrow E = 1.98 \times 10^{-25}$ J

For (c) $E = hc \times \bar{v}$ $\left(\because \dfrac{1}{\lambda} = \bar{v}\right)$

$E = 6.63 \times 10^{-34} \times 3 \times 10^8 \times 30 \times 10^{-2}$

(c) $\Rightarrow E = 5.96 \times 10^{-26}$

For (d) d $\rightarrow E = 6.62 \times 10^{-27}$ J

Hence, highest energy for photon is in (a).

68. (c) $\because$ For Isotonic solution

$\pi_1 = \pi_2$ i.e. $i_1C_1RT = i_2C_2RT$

For option (c)

$i_1 = 2, i_2 = 3$

$C_1 = 0.03$

$C_2 = 0.02$

Thus, $\quad i_1 \times C_1 = i_2 \times C_2$

Hence, are isotonic, i.e.

$0.06 = 0.06$

In all other options, $\quad i_1 \times C_1 \neq i_2 \times C_2$

Thus, are not isotonic.

69. (d) $Cr_2O_7^{2-} \longrightarrow Cr^{3+}$ Change in + 3

Oxi no. = + 6

Total change in number of electrons

$= 2 \times 3 = 6$ mole $= 6$ F

70. (d) For the given reactions,

$2H_2O \rightleftharpoons 2H_2 + O_2,\ K_1 = 6.4 \times 10^{-8}$

or, $H_2O \rightleftharpoons H_2 + \dfrac{1}{2}O_2,\ K'_1 = \sqrt{K_1}$

or $H_2 + \dfrac{1}{2}O_2 \rightleftharpoons H_2O,\ K''_1 = 1/\sqrt{K_1}$ ... (i)

$2CO_2 \rightleftharpoons 2CO + O_2,\ K_2 = 1.6 \times 10^{-6}$

or $CO_2 \rightleftharpoons CO + \dfrac{1}{2}O_2,\ K'_2 = \sqrt{K_2}$ ... (ii)

Form (i) and (ii) [on adding]

$$K = \frac{\sqrt{K_2}}{\sqrt{K_1}} = \frac{\sqrt{1.6 \times 10^{-6}}}{\sqrt{6.4 \times 10^{-8}}}$$

$$K = \sqrt{\frac{10^2}{4}} = \sqrt{25} = 5$$

71. (d) $\because$ For be packing,

$$\sqrt{3}a = 4r \ \therefore\ r = \frac{\sqrt{3}}{4} \cdot a$$

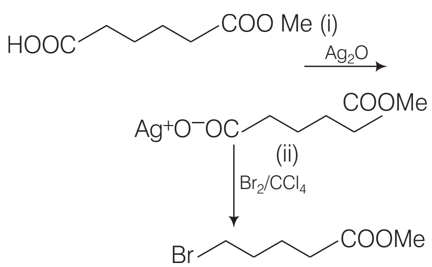
where a = edge length

r = radius of lattice sphere

72. (b) When drop of line juice is added to mixture of $I^- + IO_3^-$. The following reaction take place and violet colour appears due to formation of I_2

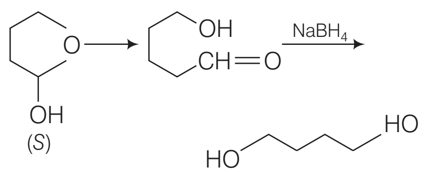
$$I^- + IO_3^- + H^+ \longrightarrow I_2 + H_2O$$

73. (d) The reaction occurs as follows :



The above reaction is an example of Borodine Hunsdiecker reaction.

74. (c) The reaction occurs as follows :



75. (b) The reaction for given reaction is

	HCl +	NaOH ⟶	NaCl +	H_2O
Number of moles	xy	xy	0	0
Added	NV	NV		
Number of moles left	0	0	xy	xy

$$\therefore N\text{ (solution)} = \frac{\text{Number of milliequivalent}}{\text{vol. of solution in mililetre}}$$

76. (b,c,d) During electroysis of molten NaCl, when water is added, the following statements are true

(a) false

(b) $\because Na + H_2O \longrightarrow NaOH + \frac{1}{2}H_2$

and discharge potential of hydrogen is less than of sodium. So, H_2 will evolve.

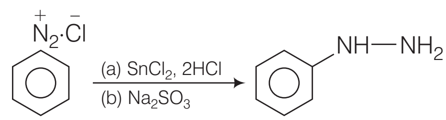
(c) Some amount of caustic soda (NaOH) will be formed [As shown in (b)]

(d) Fire can likely take place because relations given given in (b) is highly exothermic.

$\therefore$ (b), (c) and (d) are correct.

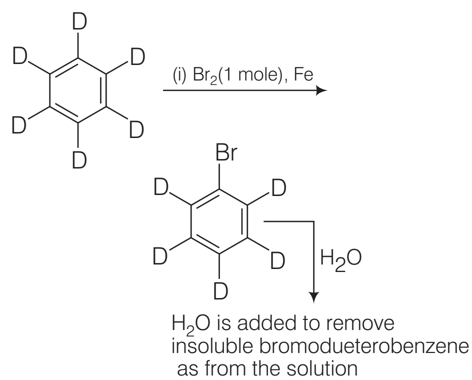
77. (b), (c) Among the given statements (b) and (c) are correct these are the facts.

78. (a, b) Reduction of benzene diazoniums chloride occurs as follows:



Hence, (a) and (b) are the correct answers.

79. (a) The reaction for 1 mole of hexadeuterisobenzene occurs as follows :

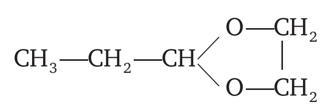


80. (a,d) $CH_3—CH_2—COOH$

$$\xrightarrow[\text{(d) HCl}]{\text{(a) SoCl}_2} CH_3—CH_2—\underset{\underset{O}{\|}}{C}—Cl$$

$$\xrightarrow[\text{(d)SnCl}_2]{\text{(a)LiAlH}_4} CH_3—CH_2—CHO$$

$$\xrightarrow{\text{(a)/(d)}\ \ CH_2—OH,\ CH_2—OH}$$



Hence, (a) and (d) are correct answers.

Mathematics

1. (b) Total number of 5-digit numbers having all the digits distinct $= {}^{10}P_5 - {}^9P_4$

$$= \frac{10!}{5!} - \frac{9!}{5!} = \frac{10 \times 9!}{5!} - \frac{9!}{5!}$$
$$= \frac{9!}{5!}(10-1) = \frac{9!}{(9-4)!}(9)$$
$$= 9 \times {}^9P_4$$

2. (b) $(p+1)(p+2)(p+3)\ldots(p+q)$ is the product of q consecutive natural number $(p, q \in N)$. The product of q consecutive natural number is always divisible by $q!$.

3. (b) $(1+x+x^2)^9 = a_0 + a_1x + a_2x^2 + \ldots + a_{18}x^{18}$

Put $x = -1$, we get

$$(1-1+1)^9 = a_0 - a_1 + a_2 + \ldots + a_{18}$$
$$\Rightarrow \quad 1 = a_0 - a_1 + a_2 + \ldots + a_{18} \quad \ldots\text{(i)}$$

Put $x = 1$, we get

$$(1+1+1)^9 = a_0 + a_1 + a_2 + \ldots + a_{18}$$
$$\Rightarrow \quad 3^9 = a_0 + a_1 + a_2 + \ldots + a_{18} \quad \ldots\text{(ii)}$$

On adding Eqs. (i) and (ii), we get

$$3^9 + 1 = 2(a_0 + a_2 + \ldots + a_{18})$$
$$\Rightarrow \quad a_0 + a_2 + a_4 + \ldots + a_{18} = \frac{3^9+1}{2}$$
$$= \frac{19683+1}{2}$$
$$= \frac{19684}{2}$$

$= 9842$, which is even number.

4. (d) We have, $8x - 3y - 5z = 0$,
$5x - 8y + 3z = 0$,
$3x + 5y - 8z = 0$

$$\therefore \; D = \begin{vmatrix} 8 & -3 & -5 \\ 5 & -8 & 3 \\ 3 & 5 & -8 \end{vmatrix}$$
$$= 8(64-15) + 3(-40-9) - 5(25+24)$$
$$= 8 \times 49 + 3 \times (-49) - 5 \times 49$$
$$= 0$$

$\therefore$ The system has infinitely many non-zero solutions.

5. (b) All the orthogonal matrix are non-singular matrix.

$\therefore$ Q is proper subset of P.

6. (b) We know that,

$$\det(AB) = \det(A)\det(B)$$
$$\therefore \quad \det(AB) = 0$$
$$\Rightarrow \quad \det(A)\cdot\det(B) = 0$$
$$\Rightarrow \begin{vmatrix} x+2 & 3x \\ 3 & x+2 \end{vmatrix}\begin{vmatrix} x & 0 \\ 5 & x+2 \end{vmatrix} = 0$$
$$\Rightarrow \{(x+2)^2 - 9x\}\{x(x+2) - 0\} = 0$$
$$\Rightarrow (x^2 + 4x + 4 - 9x)\, x(x+2) = 0$$
$$\Rightarrow x(x+2)(x^2 - 5x + 4) = 0$$
$$\Rightarrow x(x+2)(x-1)(x-4) = 0$$
$$\Rightarrow x = 0, -2, 1, 4$$

7. (a) We have,

$$|A| = \begin{vmatrix} 1 & \cos\theta & 0 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{vmatrix}$$
$$= 1[1 - (-\cos\theta)(\cos\theta)]$$
$$-\cos\theta[-\cos\theta + \cos\theta] \;\; +0(\cos^2\theta + 1)$$
$$= 1 + \cos^2\theta$$

Now, we know that

$$-1 \le \cos\theta \le 1$$
$$\Rightarrow \quad 0 \le \cos^2\theta \le 1$$
$$\Rightarrow \quad 1 \le 1 + \cos^2\theta \le 2$$
$$\Rightarrow \quad 1 \le |A| \le 2$$
$$\therefore \quad |A| \in [1, 2]$$

8. (a) Since, $f: R \to R$ is injective and $f(x)f(y) = f(x+y), \forall\, x, y \in R$.

$\therefore \quad f(x) = a^x$

Again, $f(x), f(y), f(z)$ are in GP.

$$\Rightarrow \quad (f(y))^2 = f(x)\cdot f(y)$$
$$\Rightarrow \quad a^{2y} = a^x \cdot a^z$$
$$\Rightarrow \quad a^{2y} = a^{x+z}$$
$$\Rightarrow \quad 2y = x + z$$

$\therefore$ x, y, z are in AP.

9. (d) For every real number x, $x^2 \ge 0$

$\therefore \quad (x, x) \in P$

Hence, P is reflexive.

Now, let $(x, y) \in P$

$\Rightarrow \quad xy \geq 0$

$\Rightarrow \quad yx \geq 0$

$\Rightarrow \quad (y, x) \in P$

Hence, P is symmetric.

Again, $(-1, 0) \in P$ and $(0, 2) \in P$.

But $(-1, 2) \notin P$

as $(-1)(2) = -2 < 0$

$\therefore$ P is not transitive.

10. (b) We have,

$x \rho y \Rightarrow x - y$ is zero or irrational.

Now, $x - x = 0$

$\Rightarrow \quad (x, x) \in \rho$

$\therefore$ ρ is reflexive.

Again, if $x - y$ is either zero or irrational, then $y - x$ will also be either zero or irrational.

$\Rightarrow \quad (x, y) \in \rho$

$\Rightarrow \quad (y, x) \in \rho$

$\therefore$ ρ is symmetric.

Again, $(2, \sqrt{3}) \in \rho$ and $(\sqrt{3}, 4) \in \rho$

But $(2, 4) \notin \rho$

$\therefore \rho$ is not transitive.

11. (d) We have,

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_q + \ldots + x_n}{n}$$

$$\Rightarrow \quad \Sigma x = n\bar{x} \quad \ldots(i)$$

If x_q is replaced by x'_q, then new total will be

$\Sigma x' = \Sigma x - x_q - x'_q$

$\therefore$ New mean will be

$$\bar{x}' = \frac{\Sigma x'}{n}$$

$$\bar{x}' = \frac{\Sigma x - x_q + x'_q}{n}$$

$$\bar{x}' = \frac{n\bar{x} - x_q + x'_q}{n} \quad \text{[from Eq. (i)]}$$

12. (b) In a non-leap year, total number of days is 365. Out of them, there are 52 weeks and 1 day extra. Thus, a non-leap year always has 52 Sunday. The remaining 1 day can be Sunday, Monday, Tuesday, Wednesday, Thursday, Friday and Saturday.

Out of these 7 cases, we have Sunday in one case.

$\therefore$ Total number of outcomes = 7

Number of favourable outcomes = 1

Hence, required probability $= \frac{1}{7}$

13. (d) Let, $f(x) = \sin x (\sin x + \cos x)$

$$= \sin^2 x + \sin x \cos x$$

$$= \frac{1 - \cos 2x}{2} + \frac{2 \sin x \cos x}{2}$$

$$= \frac{1}{2} - \frac{1}{2}\cos 2x + \frac{1}{2}\sin 2x$$

$$= \frac{1}{2} + \frac{1}{2}(\sin 2x - \cos 2x)$$

Now, $-\sqrt{(1)^2 + (1)^2} \leq \sin 2x - \cos 2x$

$$\leq \sqrt{(1)^2 + (1)^2}$$

$$\left[\because \sqrt{a^2 + b^2} \leq a \sin x + b \cos x \leq \sqrt{a^2 + b^2}\right]$$

$$\Rightarrow \quad -\sqrt{2} \leq \sin 2x - \cos 2x \leq \sqrt{2}$$

$$\Rightarrow \quad -\frac{\sqrt{2}}{2} \leq \frac{\sin 2x - \cos 2x}{2} \leq \frac{\sqrt{2}}{2}$$

$$\Rightarrow \frac{1}{2} - \frac{\sqrt{2}}{2} \leq \frac{1}{2} + \frac{\sin 2x - \cos 2x}{2} \leq \frac{1}{2} + \frac{\sqrt{2}}{2}$$

$$\Rightarrow \frac{1 - \sqrt{2}}{2} \leq f(x) \leq \frac{1 + \sqrt{2}}{2}$$

$$\therefore \frac{1 - \sqrt{2}}{2} \leq k \leq \frac{1 + \sqrt{2}}{2}$$

14. (a) We have,

$$\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$$

$$\Rightarrow \quad \tan^{-1}\left[\frac{\frac{x-1}{x-2} + \frac{x+1}{x+2}}{1 - \frac{x-1}{x-2} \cdot \frac{x+1}{x+2}}\right] = \frac{\pi}{4}$$

$$\Rightarrow \frac{(x-1)(x+2) + (x-2)(x+1)}{(x-2)(x+2) - (x-1)(x+1)} = \tan\frac{\pi}{4}$$

$$\Rightarrow \quad \frac{x^2 + x - 2 + x^2 - x - 2}{x^2 - 4 - x^2 + 1} = 1$$

$$\Rightarrow \quad \frac{2x^2-4}{-3}=1$$

$$\Rightarrow \quad 2x^2-4=-3$$

$$\Rightarrow \quad 2x^2=1$$

$$\Rightarrow \quad x^2=\frac{1}{2}$$

$$\Rightarrow \quad x=\pm\frac{1}{\sqrt{2}}$$

15. (b) Given equations are

$$2x^2+3xy+4y^2+x+18y+25=0 \quad \text{...(i)}$$

$$2x^2+3xy+4y^2+1=0 \quad \text{...(ii)}$$

Let the origin be transferred to (p, q) axes being parallel to the previous axes; then the equation (i) becomes.

$$2(x'+p)^2+3(x'+p)(y'+q)$$
$$+4(y'+q)^2+(x'+p)+18(y'+q)+25=0$$

$$\Rightarrow 2x'^2+2p^2+4x'p+3x'y'+3x'q+3py'$$
$$+3pq+4y'^2+4q^2+8y'q+x'+p$$
$$+18y'+18q+25=0$$

$$\Rightarrow 2x'^2+4y'^2+3x'y'+(4p+3q+1)x'$$
$$1+(3p+8q+18)y'+2p^2+3pq$$
$$+4q^2+p+25=0$$

From Eq. (ii) coefficient of x' and y' must be zero.

$$\therefore \quad 4p+3q+1=0 \quad \text{...(iii)}$$

$$3p+8q+18=0 \quad \text{...(iv)}$$

By solving Eqs. (iii) and (iv), we get

$$p=2,\ q=-3$$

16. (a) Let the coordinates of C be (α, β).

$\therefore$ Coordinates of centroid

$$=\left(\frac{2-2+\alpha}{3}, \frac{-3+1+\beta}{3}\right)$$

$$=\left(\frac{\alpha}{3}, \frac{\beta-2}{3}\right)$$

Since, centroid lie on $2x+3y=1$.

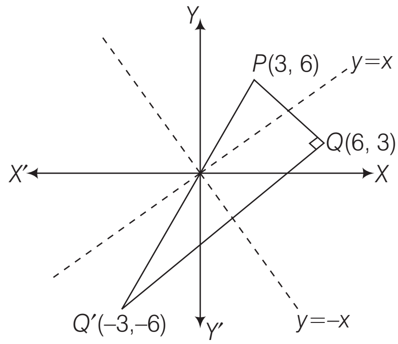
$$\therefore \quad \frac{2\alpha}{3}+3\left(\frac{\beta-2}{3}\right)=1$$

$$\Rightarrow \quad \frac{2\alpha}{3}+\frac{3\beta-6}{3}=1$$

$$\Rightarrow \quad 2\alpha+3\beta-6=3 \Rightarrow 2\alpha+3\beta=9$$

$\therefore$ Locus of point C will be $2x+3y=9$.

17. (d)



The coordinates of Q and Q′ will be (6, 3) and (– 3, – 6), respectively.

Now, Slope of $PQ=\frac{6-3}{3-6}=-1$

and Slope of $QQ'=\frac{-6-3}{-3-6}=1$

$\therefore$ Slope of PQ $\times$ Slope of $QQ'=-1\times 1=-1$

$\therefore \Delta PQQ'$ is right angled triangle at Q.

$\therefore$ Circumcentre will be the mid-point of

hypotenuse $PQ'=\left(\frac{-3+3}{2}, \frac{-6+6}{2}\right)$

$$=(0, 0)$$

18. (b) Let (h, k) be any point on the line $7x-9y+10=0$, then $7h-9k+10=0$

$$\Rightarrow \quad 7h=9k-10$$

$$\Rightarrow \quad h=\frac{9k-10}{7} \quad \text{...(i)}$$

Now, perpendicular distance from point (h, k) to the line $3x+4y=5$ is d_1

$$d_1=\frac{3h+4k-5}{\sqrt{3^2+4^2}}$$

$$\Rightarrow \quad d_1=\frac{3h+4k-5}{5} \quad \text{...(ii)}$$

and perpendicular distance from (h, k) to the line $12x+5y=7$ is d_2

$$\therefore \quad d_2=\frac{12h+5k-7}{\sqrt{12^2+5^2}}$$

$$\Rightarrow \quad d_2=\frac{12h+5k-7}{13} \quad \text{...(iii)}$$

Now, $d_1-d_2=\frac{3h+4k-5}{5}-\frac{12h+5k-7}{13}$

$\Rightarrow d_1 - d_2$

$$= \frac{13(3h + 4k - 5) - 5(12h + 5k - 7)}{65}$$

$$= \frac{39h + 52k - 65 - 60h - 25k + 35}{65}$$

$$= \frac{-21h + 27k - 30}{65}$$

$$= \frac{-21\left(\frac{9k - 10}{7}\right) + 27k - 30}{65} \quad \text{[from Eq.(i)]}$$

$$= \frac{-27k + 30 + 27k - 30}{65} = 0$$

$\Rightarrow d_1 - d_2 = 0 \Rightarrow d_1 = d_2$

19. (d) Given, equation of circles are

$x^2 + y^2 - 4x - 4y = 0$ and $2x^2 + 2y^2 = 32$

or $x^2 + y^2 - 4x - 4y = 0$

and $x^2 + y^2 = 16$

$\therefore$ Equation of common chord is

$(x^2 + y^2 - 4x - 4y) - (x^2 + y^2 - 16) = 0$

$\Rightarrow \quad -4x - 4y + 16 = 0$

$\Rightarrow \quad x + y = 4$

This common chord passes through (2, 2), i.e. centre of first circle.

Also, (0, 0) is at the circumference of the first circle.

$\therefore$ Common chord will subtent $\frac{\pi}{2}$ angle at (0, 0).

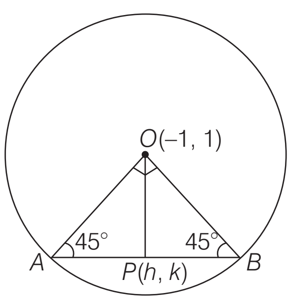
20. (c) Given, equation of circle is

$$x^2 + y^2 + 2x - 2y - 2 = 0$$

$\Rightarrow \quad (x+1)^2 + (y-1)^2 = 4$

$\therefore$ Centre $(-1, 1)$ and radius $= 2$

Let (h, k) be the mid-point of chord.



From figure,

$$OP = \sqrt{(h+1)^2 + (k-1)^2}$$

In ΔOAP,

$$\sin 45° = \frac{OP}{OA}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{\sqrt{(h+1)^2 + (k-1)^2}}{2}$$

On squaring both sides, we get

$(h+1)^2 + (k-1)^2 = 2$

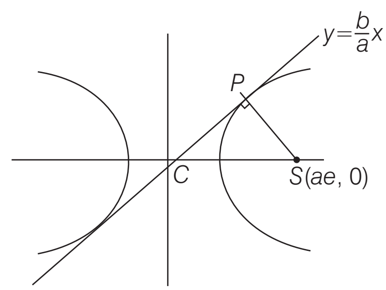
$\Rightarrow \quad h^2 + k^2 + 2h - 2k = 0$

$\therefore$ Locus of P will be

$$x^2 + y^2 + 2x - 2y = 0$$

21. (b) Given, equation of hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



From figure,

$$SP = \left|\frac{abe}{\sqrt{b^2 + a^2}}\right|$$

$$= \left|\frac{abe}{ae}\right| = b$$

and $CS = ae$

Again, ΔSPC is right angled triangle at P.

$\therefore \quad CP = \sqrt{CS^2 - SP^2}$

$$= \sqrt{a^2e^2 - b^2}$$

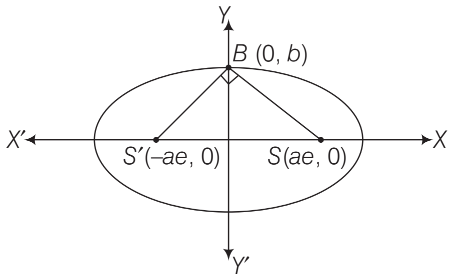
$$= \sqrt{a^2\left(1 + \frac{b^2}{a^2}\right) - b^2}$$

$$= \sqrt{a^2 + b^2 - b^2} = a$$

$\therefore$ Area of rectangle $= CP \times SP$

$= ab$

22. (b)



Slope of SB, $m_1 = \frac{b-0}{0-ae} = -\frac{b}{ae}$

and slope of S′ B, $m_2 = \frac{b-0}{0-(-ae)} = \frac{b}{ae}$

Since, $\angle SBS'$ is a right angle.

$\therefore \quad m_1 m_2 = -1$

$\Rightarrow \quad \frac{-b}{ae} \times \frac{b}{ae} = -1$

$\Rightarrow \quad b^2 = a^2e^2$

$\Rightarrow \quad \frac{b^2}{a^2} = e^2$

$\Rightarrow \quad 1 - e^2 = e^2$

$\Rightarrow \quad 2e^2 = 1$

$\Rightarrow \quad e^2 = \frac{1}{2}$

$\Rightarrow \quad e = \pm \frac{1}{\sqrt{2}}$

$\Rightarrow \quad e = \frac{1}{\sqrt{2}}$ or $e = -\frac{1}{\sqrt{2}}$

23. (a) Given, equation

$$x^2 + 2xy + y^2 - 5x + 5y - 5 = 0$$

$\Rightarrow \quad (x+y)^2 = 5x - 5y + 5$

$\Rightarrow \quad (x+y)^2 = 5(x - y + 1)$

$\therefore$ Axis of the parabola is $x + y = 0$

24. (c) Given, equation of hyperbola is

$$x^2 - y^2 + 1 = 0$$

$\Rightarrow \quad y^2 - x^2 = 1$

On comparing it with $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$, we get

$$a = b = 1$$

Now, $e = \sqrt{1 + \frac{a^2}{b^2}} = \sqrt{1 + \frac{1}{1}} = \sqrt{2}$

$\therefore$ Foci $= (0, \pm be) = (0, \pm\sqrt{2})$

Since, line joining foci of hyperbola is diameter of circle.

$\therefore$ Centre of circle $= \left(\frac{0+0}{2}, \frac{\sqrt{2}-\sqrt{2}}{2}\right) = (0, 0)$

and radius $= \frac{1}{2}\sqrt{(0-0)^2 + (\sqrt{2} - (-\sqrt{2}))^2}$

$= \frac{1}{2}\sqrt{(2\sqrt{2})^2} = \sqrt{2}$

$\therefore$ Equation of circle will be

$$(x-0)^2 + (y-0)^2 = (\sqrt{2})^2$$

$\Rightarrow \quad x^2 + y^2 = 2$

25. (d) Equation of the plane will be

$$\begin{vmatrix} x-1 & y-2 & z+3 \\ 2-1 & -2-2 & 1-(-3) \\ 1 & 0 & 0 \end{vmatrix} = 0$$

$\Rightarrow \quad \begin{vmatrix} x-1 & y-2 & z+3 \\ 1 & -4 & 4 \\ 1 & 0 & 0 \end{vmatrix} = 0$

$\Rightarrow \quad 1[4(y-2) + 4(z+3)] = 0$

[Expanding along R_3]

$\Rightarrow \quad 4y - 8 + 4z + 12 = 0$

$\Rightarrow \quad 4y + 4z + 4 = 0$

$\Rightarrow \quad y + z + 1 = 0$

26. (b) $\Delta = \begin{vmatrix} 1 & -1 & 1 \\ 2 & -3 & 0 \\ 1 & 0 & 3 \end{vmatrix}$

$= 1(-9-0) - (-1)\cdot(6-0) + 1(0-(-3))$

$= -9 + 6 + 3 = 0$

Since, $\Delta = 0$

$\therefore$ Lines are coplanar.

27. (a) We have,

$$\frac{f(x)}{f(y)} = f(x-y)$$

$\Rightarrow \quad f(x) = a^{kx}$

$\therefore \quad f'(x) = ka^{kx} \log a$

Again, $\quad f'(0) = P$

$\Rightarrow \quad ka^0 \log a = P$

$\Rightarrow \quad k \log a = P$

Also, $\quad f'(5) = q$

$\Rightarrow \quad ka^{5k} \log a = q$

$$\Rightarrow \quad a^{5k}P = q$$

$$\Rightarrow \quad a^{5k} = \frac{q}{p}$$

Now, $\quad f'(-5) = ka^{-5k}\log a$

$$= \frac{k\log a}{a^{5k}}$$

$$= \frac{p}{\left(\frac{q}{p}\right)} = \frac{p^2}{q}$$

28. (c) We have,

$$f(x) = \log_5 \log_3 x$$

$$= \frac{\log \log_3 x}{\log 5}$$

$$= \frac{\log\left(\frac{\log x}{\log 3}\right)}{\log 5}$$

$$= \frac{\log\log x - \log\log 3}{\log 5}$$

$$\therefore \quad f'(x) = \frac{1}{\log 5}\cdot\frac{1}{\log x}\cdot\frac{1}{x}$$

$$\therefore \quad f'(e) = \frac{1}{e\log 5\cdot\log e}$$

$$= \frac{1}{e\log 5} \qquad [\because \log e = 1]$$

$$= \frac{1}{e\log_e 5}$$

29. (c) We have, $F(x) = e^x, G(x) = e^{-x}$

$$\therefore \quad H(x) = G(F(x))$$

$$= G(e^x)$$

$$= e^{-e^x}$$

$$\therefore \quad \frac{dH}{dx} = e^{-e^x}\cdot(-e^x)$$

$$= -e^x e^{-e^x}$$

$$\therefore \quad \left.\frac{dH}{dx}\right|_{x=0} = -e^0\cdot e^{-e^0}$$

$$= -1.\, e^{-1}$$

$$= -e^{-1}$$

$$= \frac{-1}{e}$$

30. (c) $\lim_{x\to 0}\frac{2f(x) - 3f(2x) + f(4x)}{x^2}$

$$= \lim_{x\to 0}\frac{2f'(x) - 3f'(2x)\cdot 2 + f'(4x)\cdot 4}{2x}$$

$$= \lim_{x\to 0}\frac{f'(x) - 3f'(2x) + 2f'(4x)}{x}$$

$$= \lim_{x\to 0}\frac{f''(x) - 3f''(2x)\cdot 2 + 2f''(4x)\cdot 4}{1}$$

$$= \lim_{x\to 0} f''(x) - 6f''(2x) + 8f''(4x)$$

$$= f''(0) - 6f''(0) + 8f''(0)$$

$$= k - 6k + 8k \qquad [\because f''(0) = k]$$

$$= 3k$$

31. (a) $y = e^{m\sin^{-1}x}$

$$\Rightarrow \quad \frac{dy}{dx} = e^{m\sin^{-1}x}\cdot\frac{m}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad \sqrt{1-x^2}\,\frac{dy}{dx} = me^{m\sin^{-1}x}$$

$$\Rightarrow \quad \sqrt{1-x^2}\,\frac{dy}{dx} = my \qquad \ldots(i)$$

$$\Rightarrow \sqrt{1-x^2}\,\frac{d^2y}{dx^2} + \frac{1}{2\sqrt{1-x^2}}(-2x)\frac{dy}{dx} = m\frac{dy}{dx}$$

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m\sqrt{1-x^2}\,\frac{dy}{dx}$$

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m\cdot(my)$$

[From Eq. (i)]

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0$$

$$\therefore \quad k = m^2$$

32. (d) Given curve,

$$y = x^2 + 2ax + b$$

At $\quad x = \alpha,$

$$y = \alpha^2 + 2a\alpha + b$$

and at $\quad x = \beta,$

$$y = \beta^2 + 2a\beta + b$$

$\therefore$ Slope of line joining $(\alpha, \alpha^2 + 2a\alpha + b)$ and $(\beta, \beta^2 + 2a\beta + b)$ is

$$= \frac{(\alpha^2 + 2a\alpha + b) - (\beta^2 + 2a\beta + b)}{\alpha - \beta}$$

$$= \frac{\alpha^2 + 2a\alpha + b - \beta^2 - 2a\beta - b}{\alpha - \beta}$$

$$= \frac{(\alpha^2 - \beta^2) + 2a(\alpha - \beta)}{\alpha - \beta}$$

$$= \frac{(\alpha - \beta)(\alpha + \beta) + 2a(\alpha - \beta)}{\alpha - \beta}$$

$$= \alpha + \beta + 2a$$

Slope of given curve $= \frac{dy}{dx}$

$$= 2x + 2a$$

Now, according to question, tangent is parallel to the chord. Therefore,

$$2x + 2a = \alpha + \beta + 2a$$

$$\Rightarrow \quad 2x = \alpha + \beta \Rightarrow x = \frac{\alpha + \beta}{2}$$

33. (c) We have,

$$f(x) = x^{13} + x^{11} + x^9 + x^7 + x^5 + x^3 + x + 19$$

$$\Rightarrow \quad f'(x) = 13x^{12} + 11x^{10} + 9x^8 + 7x^6 + 5x^4 + 3x^2 + 1$$

$\therefore f'(x)$ has no real root.

$\therefore f(x) = 0$ has not more than one real root.

34. (b) Since, Lagrange's mean value theorem is applicable on $f(x)$.

$$\therefore \quad \lim_{x \to 0} \frac{x^P}{(\sin x)^q} = f(0)$$

$$\Rightarrow \quad \lim_{x \to 0} \frac{x^P}{(\sin x)^q} = 0$$

Above equation holds only when $p > q$.

35. (b) Let $y = \lim_{x \to 0} (\sin x)^{2\tan x}$

$$\Rightarrow \quad \log y = \lim_{x \to 0} \log (\sin x)^{2\tan x}$$

$$= 2 \lim_{x \to 0} \tan x \log \sin x$$

$$= 2 \lim_{x \to 0} \frac{\log \sin x}{\cot x}$$

$$= 2 \lim_{x \to 0} \frac{\frac{1}{\sin x} \cdot \cos x}{-\operatorname{cosec}^2 x}$$

$$= 2 \lim_{x \to 0} (-\sin x \cos x)$$

$$= 2 \times 0 = 0$$

$$\therefore \quad \log y = 0 \Rightarrow y = e^0 = 1$$

36. (c) Let $I = \int \cos(\log x)\, dx$

Put $\log x = t$

$$\Rightarrow \quad x = e^t$$

$$\therefore \quad dx = e^t dt$$

$$\therefore \quad I = \int e^t \cos t \, dt$$

$$= \frac{e^t}{1^2 + 1^2} [\cos t + \sin t] + C$$

$$\left[\because \int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2 + b^2} [a \cos bx + b \sin bx] + C\right]$$

$$\Rightarrow \quad I = \frac{e^t}{2} [\cos t + \sin t] + C$$

$$= \frac{x}{2} [\cos(\log x) + \sin(\log x] + C$$

$$\therefore f(x) = \frac{x}{2} [\cos(\log x) + \sin(\log x)]$$

37. (a) Let $I = \int \frac{x^2 - 1}{x^4 + 3x^2 + 1} dx$

$$= \int \frac{1 - 1/x^2}{x^2 + 3 + 1/x^2} dx$$

$$= \int \frac{1 - 1/x^2}{\left(x^2 + \frac{1}{x^2}\right) + 3} dx$$

$$= \int \frac{1 - 1/x^2}{\left(x + \frac{1}{x}\right)^2 - 2 + 3} dx$$

$$= \int \frac{1 - 1/x^2}{\left(x + \frac{1}{x}\right)^2 + 1} dx$$

Let $x + \frac{1}{x} = t$

$$\Rightarrow \quad \left(1 - \frac{1}{x^2}\right) dx = dt$$

$$\therefore \quad I = \int \frac{dt}{t^2 + 1}$$

$$= \tan^{-1} t + C$$

$$= \tan^{-1}\left(x + \frac{1}{x}\right) + C \quad \left[\because t = x + \frac{1}{x}\right]$$

38. (b) For $x > 10$, we have

$|\sin x| < 1$ and $1 + x^8 > 10^8$

$$\Rightarrow \quad \frac{1}{1+x^8} \le 10^{-8}$$

$$\therefore \quad \left| \int_{10}^{19} \frac{\sin x}{1+x^8}\, dx \right| \le \int_{10}^{19} \frac{|\sin x|}{1+x^8}\, dx$$

$$\le \int_{10}^{19} 10^{-8} = 9 \times 10^{-8} < 10^{-7}$$

39. (d) We have,

$$I_1 = \int_0^n [x]\, dx$$

$$= \int_0^1 [x]\, dx + \int_1^2 [x]\, dx + \int_2^3 [x]\, dx + \dots + \int_{n-1}^n [x]\, dx$$

$$= \int_0^1 0\, dx + \int_2^3 1\, dx + \int_2^3 2\, dx + \dots + \int_{n-1}^n (n-1)\, dx$$

$$= 0 + [x]_1^2 + 2[x]_2^3 + \dots + (n-1)[x]_{n-1}^n$$

$$= (2-1) + 2(3-2) + \dots + (n-1)(n-(n-1))$$

$$= 1 + 2 + 3 + \dots + (n-1)$$

$$= \frac{(n-1)(n-1+1)}{2} \qquad \left[\because \Sigma n = \frac{n(n+1)}{2}\right]$$

$$= \frac{n(n-1)}{2}$$

Now, $I_2 = \int_0^n \{x\}\, dx = \int_0^n x - [x]\, dx$

$$= \int_0^n x\, dx - \int_0^n [x]\, dx$$

$$= \left[\frac{x^2}{2}\right]_0^n - I_1$$

$$= \frac{n^2}{2} - \frac{n(n-1)}{2}$$

$$= \frac{n^2 - n^2 + n}{2}$$

$$= \frac{n}{2}$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{\frac{n(n-1)}{2}}{\frac{n}{2}}$$

$$= (n-1)$$

40. (b) $\lim_{n \to \infty} \left[\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \dots + \frac{1}{2n} \right]$

$$= \lim_{n \to \infty} \left[\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \dots + \frac{n}{n^2+n^2} \right]$$

$$= \lim_{n \to \infty} \sum_{r=1}^{\infty} \frac{n}{n^2+r^2}$$

$$= \lim_{n \to \infty} \frac{1}{n} \sum_{r=1}^{\infty} \frac{1}{1+\left(\frac{r}{n}\right)^2}$$

$$= \int_0^1 \frac{1}{1+x^2}\, dx$$

$$= [\tan^{-1} x]_0^1$$

$$= \tan^{-1} 1 - \tan^{-1} 0$$

$$= \frac{\pi}{4} - 0 = \frac{\pi}{4}$$

41. (d) We know that, $0 \le x \le 1$

$$\Rightarrow \quad x^2 \le 1$$

$$\Rightarrow \quad e^{x^2} \le e$$

$$\Rightarrow \quad \int_0^1 e^{x^2} dx \le \int_0^1 e\, dx$$

$$\Rightarrow \quad \int_0^1 e^{x^2}\, dx \le e$$

$$\therefore \quad \int_0^1 e^{x^2} \in [1, e]$$

42. (c) Let $I = \int_0^{100} e^{x-[x]} dx$

$$= 100 \int_0^1 e^{x-[x]}\, dx$$

[$\because x - [x]$ is a periodic function of period 1 and $\int_0^{mT} f(x)\, dx = m\int_0^T f(x)\, dx$, where T is period of $f(x)$]

$$= 100 \int_0^1 e^x\, dx \quad [\because x - [x] = x \text{ for } 0 < x < 1]$$

$$= 100\,[e^x]_0^1$$

$$= 100\,[e^1 - e^0]$$

$$= 100\,(e-1)$$

43. (a) We have,

$$(x+y)^2 \frac{dy}{dx} = a^2$$

Let $\quad x + y = v$

$$\Rightarrow \quad 1+\frac{dy}{dx}=\frac{dv}{dx}$$
$$\Rightarrow \quad \frac{dy}{dx}=\frac{dv}{dx}-1$$
$$\therefore \quad v^2\left(\frac{dv}{dx}-1\right)=a^2$$
$$\Rightarrow \quad v^2\frac{dv}{dx}=v^2+a^2$$
$$\Rightarrow \quad \frac{dv}{dx}=\frac{v^2+a^2}{v^2}$$
$$\Rightarrow \quad \frac{v^2}{v^2+a^2}dv=dx$$

On integrating both sides, we get

$$\int\frac{v^2}{v^2+a^2}dv=\int dx$$
$$\Rightarrow \quad \int\left(1-\frac{a^2}{v^2+a^2}\right)dv=x+C'$$
$$\Rightarrow \quad v-\frac{a^2}{a}\tan^{-1}\frac{v}{a}=x+C'$$
$$\Rightarrow \quad x+y-a\tan^{-1}\frac{x+y}{a}=x+C'$$
$$\Rightarrow \quad y=a\tan^{-1}\frac{x+y}{a}+C'$$
$$=\frac{y-C'}{a}=\tan^{-1}\frac{x+y}{a}$$
$$\Rightarrow \quad \left(\frac{x+y}{a}\right)=\tan\left(\frac{y+C}{a}\right),$$

where $-C'=C$.

44. (b) We have,

$$x^2(x^2-1)\frac{dy}{dx}+x(x^2+1)y=x^2-1$$
$$\Rightarrow \quad \frac{dy}{dx}+\frac{x^2+1}{x(x^2-1)}y=\frac{1}{x^2}$$
$$\therefore \quad IF=e^{\int\frac{x^2+1}{x(x^2-1)}dx}$$
$$=e^{\int\frac{x^2+1}{x(x-1)(x+1)}dx}$$

Let $\dfrac{x^2+1}{x(x-1)(x+1)}=\dfrac{A}{x}+\dfrac{B}{x-1}+\dfrac{C}{x+1}$

$$\Rightarrow x^2+1=A(x-1)(x+1)+Bx(x+1)+Cx(x-1)$$

Put $x=0$,

$\therefore \quad 1=-A$

$\Rightarrow \quad A=-1$

Put $\quad x=1$,

$\therefore \quad 2=2B$

$\Rightarrow \quad B=1$

Put $\quad x=-1$,

$\therefore \quad 2=2C$

$\Rightarrow \quad C=1$

$$\therefore \quad \frac{x^2+1}{x(x-1)(x+1)}=\frac{-1}{x}+\frac{1}{x-1}+\frac{1}{x+1}$$
$$\therefore \quad IF=e^{\int\left(\frac{-1}{x}+\frac{1}{x-1}+\frac{1}{x+1}\right)dx}$$
$$=e^{[-\log x+\log(x-1)+\log(x+1)]}$$
$$=e^{\log\left(\frac{x^2-1}{x}\right)}=\frac{x^2-1}{x}$$
$$=x-\frac{1}{x}$$

45. (b) Let a_n be the general term of a GP whose first term is a and common ratio is r.

Now according to the question,

$$a_n=a_{n+1}+a_{n+2}$$
$$\Rightarrow \quad ar^{n-1}=ar^n+ar^{n+1}$$
$$\Rightarrow \quad r^{n-1}=r^n+r^{n+1}$$
$$\Rightarrow \quad 1=\frac{r^n}{r^{n-1}}+\frac{r^{n+1}}{r^{n-1}}$$
$$\Rightarrow \quad 1=r+r^2$$
$$\Rightarrow \quad r^2+r-1=0$$
$$\Rightarrow \quad r=\frac{-1\pm\sqrt{(1)^2-4(1)(-1)}}{2(1)}$$
$$=\frac{-1\pm\sqrt{1+4}}{2}=\frac{-1\pm\sqrt{5}}{2}$$

Since, GP consists only positive terms

$$\therefore \quad r=\frac{\sqrt{5}-1}{2}$$

46. (a) We have,

$$\log_5 x\cdot\log_x 3x\cdot\log_{3x} y=\log_x x^3$$
$$\Rightarrow \quad \frac{\log x}{\log 5}\times\frac{\log 3x}{\log x}\times\frac{\log y}{\log 3x}=3\log_x x$$

$$\left[\because \log_b a = \frac{\log a}{\log b} \text{ and } \log a^m = m \log a\right]$$

$$\Rightarrow \quad \frac{\log y}{\log 5} = 3 \quad [\because \log_a a = 1]$$

$$\Rightarrow \quad \log y = 3 \log 5$$

$$\Rightarrow \quad \log y = \log 5^3$$

$$\Rightarrow \quad y = 5^3 = 125$$

47. (c) $\dfrac{(1+i)^n}{(1-i)^{n-2}} = \dfrac{(1+i)^n}{(1-i)^n\,(1-i)^{-2}}$

$$= \left(\frac{1+i}{1-i}\right)^n (1-i)^2$$

$$= \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^n (1 + i^2 - 2i)$$

$$= \left(\frac{1+2i+i^2}{1-i^2}\right)^n (1 + i^2 - 2i)$$

$$= \left(\frac{1+2i-1}{1-(-1)}\right)^n (1 - 1 - 2i)$$

$$[\because i^2 = -1]$$

$$= \left(\frac{2i}{2}\right)^n (-2i)$$

$$= i^n(-2i) = -2i^{n+1}$$

48. (d) $\dfrac{z+i}{z-i} = \dfrac{(x+iy)+i}{(x+iy)-i}$

$$= \frac{x+(1+y)\,i}{x+(y-1)\,i}$$

$$= \frac{x+(y+1)\,i}{x+(y-1)\,i} \times \frac{x-(y-1)\,i}{x-(y-1)\,i}$$

$$= \frac{x^2 - x(y-1)\,i + x(y+1)\,i - (y+1)(y-1)\,i^2}{x^2-(y-1)^2 i^2}$$

$$= \frac{x^2 + i[-xy + x + xy + x] + (y^2-1)}{x^2+(y-1)^2}$$

$$[\because i^2 = -1]$$

$$= \frac{x^2+y^2-1}{x^2+(y-1)^2} + \frac{2x}{x^2+(y-1)^2}\, i$$

Now, $\dfrac{z+i}{z-i}$ is purely imaginary.

$$\therefore \quad \text{Re}\left(\frac{z+i}{z-i}\right) = 0$$

$$\Rightarrow \quad \frac{x^2+y^2-1}{x^2+(y-1)^2} = 0$$

$$\Rightarrow \quad x^2+y^2 = 1$$

$\therefore$ (x, y) lies on a circle.

49. (a) Given equation,

$2px^2 + (2p+q)\,x + q = 0$

$\therefore D = (2p+q)^2 - 4(2p)(q)$

$= 4p^2 + q^2 + 4pq - 8pq$

$= 4p^2 + q^2 - 4pq$

$= (2p-q)^2$

= a perfect square

$\therefore$ Given equation has rational roots

50. (b) Out of 7 consonants, the number of ways of selecting 3 consonants = ${}^7C_3$

Similarly, number of ways of selecting 2 vowels out of 4 vowels = ${}^4C_2$

$\therefore$ Total number of words formed

$$= {}^7C_3 \times {}^4C_2 \times {}^5P_5$$

$$= \frac{7\times 6\times 5}{3\times 2\times 1} \times \frac{4\times 3}{2\times 1} \times 5!$$

$$= 7\times 5\times 2\times 3\times 120 = 25200$$

51. (b) We have, $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$$\therefore \quad A^2 = A \cdot A$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+0 & 1+1+0 & 1+1+1 \\ 0+0+0 & 0+1+0 & 0+1+1 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & \frac{2(2+1)}{2} \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Again, $A^3 = A^2 \cdot A$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$=\begin{bmatrix}1+0+0 & 1+2+0 & 1+2+3\\ 0+0+0 & 0+1+0 & 0+1+2\\ 0+0+0 & 0+0+0 & 0+0+1\end{bmatrix}$$

$$=\begin{bmatrix}1 & 3 & 6\\ 0 & 1 & 3\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 3 & \frac{3(3+1)}{2}\\ 0 & 1 & 3\\ 0 & 0 & 1\end{bmatrix}$$

$$\therefore \quad A^n=\begin{bmatrix}1 & n & \frac{n(n+1)}{2}\\ 0 & 1 & n\\ 0 & 0 & 1\end{bmatrix}$$

52. (d) We have,

$$\begin{vmatrix}a & a+1 & a-1\\ -b & b+1 & b-1\\ c & c-1 & c+1\end{vmatrix}+\begin{vmatrix}a+1 & b+1 & c-1\\ a-1 & b-1 & c+1\\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c\end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix}a & a+1 & a-1\\ -b & b+1 & b-1\\ c & c-1 & c+1\end{vmatrix}+\begin{vmatrix}a+1 & a-1 & (-1)^{n+2}a\\ b+1 & b-1 & (-1)^{n+1}b\\ c-1 & c+1 & (-1)^n c\end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix}a & a+1 & a-1\\ -b & b+1 & b-1\\ c & c-1 & c+1\end{vmatrix}+\begin{vmatrix}(-1)^{n+2}a & a+1 & a-1\\ (-1)^{n+1}b & b+1 & b-1\\ (-1)^n c & c-1 & c+1\end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix}a(1+(-1)^{n+2}) & a+1 & a-1\\ b(-1+(-1)^{n+1}) & b+1 & b-1\\ c(1+(-1)^n) & c-1 & c+1\end{vmatrix}=0$$

$\therefore$ n is any odd integer.

53. (c) We have,

$R=\{(1,1),(2,2),(3,3),(1,2),(2,1)\}$,

$S=\{(1,1),(2,2),(3,3),(1,3),(3,1)\}$.

$\therefore R\cup S=\{(1,1),(2,2),(3,3),(1,2),(2,1),(1,3),(3,1)\}$.

Since, $(2,1)\in R\cup S$, $(1,3)\in R\cup S$ but $(2,3)\notin R\cup S$

$\therefore R\cup S$ is reflexive and symmetric but not transitive.

54. (a) Given circle

$x^2+y^2-4x-6y+9=0$

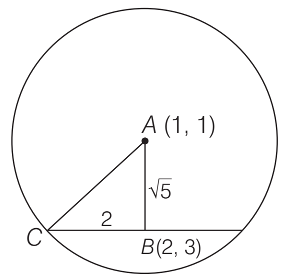
$\therefore$ Centre $=\left(-\frac{1}{2}\times(-4),-\frac{1}{2}\times(-6)\right)$

$=(2,3)$

Radius $=\sqrt{(-2)^2+(-3)^2-9}$

$=\sqrt{4+9-9}$

$=2$



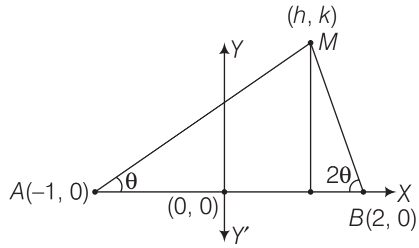
$\therefore \quad AB=\sqrt{(2-1)^2+(3-1)^2}=\sqrt{1+4}=\sqrt{5}$

$\therefore \quad AC=\sqrt{AB^2+BC^2}=\sqrt{(\sqrt{5})^2+(2)^2}$

$=\sqrt{5+4}=\sqrt{9}=3$

$\therefore$ Radius of required circle = 3

55. (d)



Let $\angle MAB = \theta$, then $\angle MBA = 2\theta$

$\tan\theta = \dfrac{k}{1+h}$ and $\tan 2\theta = \dfrac{k}{2-h}$,

then $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta} = \dfrac{k}{2-h}$

$$\Rightarrow \quad \frac{2(k/1+h)}{1-(k/1+h)^2} = \frac{k}{2-h}$$

$$\Rightarrow \quad \frac{2/(1+h)}{\dfrac{(1+h)^2-k^2}{(1+h)^2}} = \frac{1}{2-h}$$

$$\Rightarrow \quad \frac{2}{1+h} \times \frac{(1+h)^2}{(1+h)^2-k^2} = \frac{1}{2-h}$$

$$\Rightarrow \quad \frac{2(1+h)}{(1+h)^2-k^2} = \frac{1}{2-h}$$

$$\Rightarrow \quad 1+h^2+2h-k^2 = 2(1+h)(2-h)$$

$$\Rightarrow \quad 1+h^2+2h-k^2$$
$$= 2(2-h+2h-h^2)$$

$$\Rightarrow \quad 1+h^2+2h-k^2 = 2(2+h-h^2)$$

$$\Rightarrow \quad 1+h^2+2h-k^2 = 4+2h-2h^2$$

$$\Rightarrow \quad 1+3h^2-k^2 = 4$$

$$\Rightarrow \quad 3h^2-k^2 = 3$$

which represents hyperbola (d).

56. (c) We have,

$$f(x) = \int_{-1}^{x} t|dt = \int_{-1}^{0} |t|dt + \int_{0}^{x} |t|dt$$

$$= -\int_{-1}^{0} t\,dt + \int_{0}^{x} t\,dt$$

$$= \left[-\frac{t^2}{2}\right]_{-1}^{0} + \left[\frac{t^2}{2}\right]_{0}^{x}$$

$$= -\frac{1}{2}(0^2-(-1)^2) + \frac{1}{2}(x^2-0^2)$$

$$= -\frac{1}{2}(-1) + \frac{1}{2}(x^2) = \frac{1}{2}(1+x^2)$$

57. (b) We have, $f(x) = \lim\limits_{n\to\infty} n(x^{1/n}-1)$

$$= \lim_{n\to\infty} \frac{x^{1/n}-1}{1/n}$$

Let $\quad \dfrac{1}{n} = y$

$$\therefore \quad f(x) = \lim_{y\to 0} \frac{x^y-1}{y}$$

$$= \log x$$

$$\therefore \quad f(xy) = \log(xy)$$
$$= \log x + \log y$$
$$= f(x) + f(y)$$

58. (b) $I = \int_0^{100\pi} \sqrt{1-\cos 2x}\,dx$

$$= \int_0^{100\pi} \sqrt{2\sin^2 x}\,dx$$

$$= \sqrt{2}\int_0^{100\pi} \sin x|\,dx$$

$$= \sqrt{2} \times 100 \int_0^{\pi} \sin x|\,dx|$$

[| sin x | has period of π]

$$= 100\sqrt{2}\int_0^{\pi} \sin x\,dx$$

$$= 100\sqrt{2}\,[-\cos x]_0^{\pi}$$

$$= 100\sqrt{2}[(-\cos\pi)-(-\cos 0)]$$

$$= 100\sqrt{2}\,[-(-1)-(-1)]$$

$$= 100\sqrt{2} \times 2$$

$$= 200\sqrt{2}$$

59. (a) We have,

$$x = -2y^2$$

$$\Rightarrow \quad y^2 = -\frac{1}{2}x \qquad \ldots(i)$$

and $\quad x = 1-3y^2$

$$\Rightarrow \quad y^2 = -\frac{1}{3}(x-1) \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$-\frac{1}{2}x = -\frac{1}{3}(x-1)$$

$$\Rightarrow \quad 3x = 2(x-1)$$

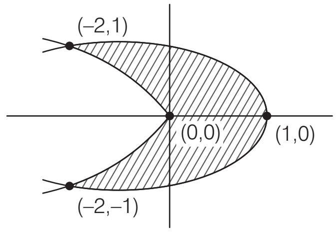
$$\Rightarrow \quad 3x = 2x-2$$

$$\Rightarrow \quad x = -2$$

$$\therefore \quad y^2 = -\frac{1}{2}(-1) = 1$$

$$\Rightarrow \quad y = \pm 1$$

$\therefore$ Point of intersection of two curves is $(-2, \pm 1)$



$\therefore$ Required Area $= 2\int_0^1 \sin x \, dx$

$$= 2\left[y - \frac{y^3}{3}\right]_0^1$$

$$= 2\left[1 - \frac{1}{3}\right]$$

$$= \frac{4}{3} \text{ sq units}$$

60. (a) We have,

$$\frac{x^2}{9} + \frac{y^2}{5} = 1$$

$\Rightarrow$ $a = 3$ and $b = \sqrt{5}$

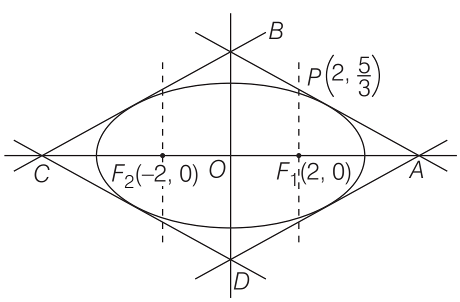
$\therefore$ $e = \sqrt{1 - \frac{b^2}{a^2}}$

$$= \sqrt{1 - \frac{5}{9}} = \frac{2}{3}$$

$\therefore$ Foci $= (\pm ae, 0)$

$$= \left(\pm 3 \times \frac{2}{3}, 0\right) = (\pm 2, 0)$$

$\therefore$ Ends of latusrectum $= \left(\pm 2, \pm \frac{5}{3}\right)$



Equation of tangent at the end of latusrectum P is

$$\frac{x \times 2}{9} + \frac{y \times 5/3}{5} = 1$$

$\Rightarrow$ $\frac{2}{9}x + \frac{1}{3}y = 1$

$\Rightarrow$ $2x + 3y = 9$

$\therefore$ $OA = \frac{9}{2}$ and $OB = 3$

$\therefore$ Area of quadrilateral ABCD

$$= 4 \times \text{Area of } \Delta OAB$$

$$= 4 \times \frac{1}{2} \times OA \times OB$$

$$= 4 \times \frac{1}{2} \times \frac{9}{2} \times 3$$

$$= 27 \text{ sq units.}$$

61. (c) We have,

$$f(x) = \sin x - \cos x - Kx + 5$$

$\Rightarrow$ $f'(x) = \cos x + \sin x - K$

For decreasing, $f'(x) < 0$

$\Rightarrow \cos x + \sin x - K < 0$

$\Rightarrow$ $K > \cos x + \sin x$

$\Rightarrow$ $K > \sqrt{2}\left[\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x\right]$

$\Rightarrow$ $K > \sqrt{2}\sin\left(\frac{\pi}{4} + x\right)$

$\therefore$ $K > \sqrt{2}$

62. (b) Let $x = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$

Then, $x \times \hat{i} = -\beta\hat{k} + \gamma\hat{j}$

$$x \times \hat{j} = \hat{k} - \gamma\hat{i}$$

$$x \times \hat{k} = -\alpha\hat{j} + \beta\hat{i}$$

Now, $(x \times \hat{i})^2 = (x \times \hat{i}) \cdot (x \times \hat{i})$

$$= (-\beta\hat{k} + \gamma\hat{j}) \cdot (-\beta\hat{k} + \gamma\hat{j})$$

$$= \beta^2 + \gamma^2$$

Similarly, $(x \times \hat{j})^2 = \alpha^2 + \gamma^2$

and $(x \times \hat{k})^2 = \alpha^2 + \beta^2$

$\therefore (x \times \hat{i})^2 + (x \times \hat{j})^2 + (x \times \hat{k})^2$

$$= \beta^2 + \gamma^2 + \alpha^2 + \gamma^2 + \alpha^2 + \beta^2$$

$$= 2(\alpha^2 + \beta^2 + \gamma^2) = 2|x|^2$$

63. (c) Let $\hat{a}$ and $\hat{b}$ are two unit vectors.

Then, $|\hat{a}+\hat{b}|=1$

$\Rightarrow \quad |\hat{a}+\hat{b}|^2=1$

$\Rightarrow \quad (\hat{a}+\hat{b})\cdot(\hat{a}+\hat{b})=1$

$\Rightarrow |\hat{a}|^2+|\hat{b}|^2+2\hat{a}\cdot\hat{b}=1$

$\Rightarrow \quad 1+1+2\hat{a}\cdot\hat{b}=1$

$\Rightarrow \quad 2\hat{a}\cdot\hat{b}=-1$

Again, $|\hat{a}-\hat{b}|^2=(\hat{a}-\hat{b})\cdot(\hat{a}-\hat{b})$

$=|\hat{a}|^2+|\hat{b}|^2-2\hat{a}\cdot\hat{b}$

$=1+1-(-1)$

$=1+1+1=3$

$\therefore \quad |\hat{a}-\hat{b}|=\sqrt{3}$

64. (a) We have, $x^2+x+1=0$

$$\Rightarrow \quad x=\frac{-1\pm\sqrt{3}i}{2}$$

$$\Rightarrow \quad \alpha=\frac{-1+\sqrt{3}i}{2} \text{ and } \beta=\frac{-1-\sqrt{3}i}{2}$$

or $\quad \alpha=e^{\frac{i\,2\pi}{3}}$ and $\beta=e^{\frac{-2\pi i}{3}}$

$$\therefore \quad \alpha^n+\beta^n=e^{\frac{2n\pi i}{3}}+e^{\frac{-2n\pi i}{3}}$$

$$=2\left(\frac{e^{\frac{2n\pi i}{3}}+e^{\frac{-2n\pi i}{3}}}{2}\right)$$

$$=2\cos\left(\frac{2n\pi}{3}\right)$$

65. (c) Let $y=\dfrac{x^2+2x+4}{2x^2+4x+9}$

$\Rightarrow 2x^2y+4xy+9y=x^2+2x+4$

$\Rightarrow (2y-1)x^2+(4y-2)x+9y-4=0$

$$\Rightarrow x=\frac{-(4y-2)\pm\sqrt{(4y-2)^2-4(2y-1)(9y-4)}}{2(2y-1)}$$

Since, x is real number.

$\therefore \quad (4y-2)^2-4(2y-1)(9y-4)\geq 0$

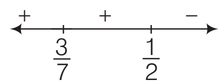
$\Rightarrow \quad 4(2y-1)^2-4(2y-1)(9y-4)\geq 0$

$\Rightarrow \quad 4(2y-1)(2y-1-9y+4)\geq 0$

$\Rightarrow \quad 4(2y-1)(3-7y)\geq 0$

$\Rightarrow \quad (2y-1)(7y-3)\leq 0$

$$\Rightarrow \quad \frac{3}{7}\leq y\leq\frac{1}{2}$$



$\therefore$ Maximum value of $y=\dfrac{1}{2}$

66. (c, d) We have,

$$ax^2+bx+1=0$$

For real roots, $D\geq 0$

$\therefore \quad b^2-4a\geq 0$

$\Rightarrow \quad b^2\geq 4a$

$\therefore \quad (a, b)=(1, 2), (1, 3), (2, 3)$

$\therefore$ Number of ordered pairs $(a, b)=3$

and a is always less than b.

67. (a, c) Equation of tangent to $y^2=4ax$ at $(at^2, 2at)$ will be

$$x-yt=-at^2 \qquad \ldots\text{(i)}$$

Also, equation of normal to $x^2-y^2=a^2$ at $(a\sec\theta, a\tan\theta)$ will be

$$\frac{x}{a\sec\theta}+\frac{y}{a\tan\theta}=2$$

$\Rightarrow x+y\,\text{cosec}\,\theta=2a\sec\theta \qquad \ldots\text{(ii)}$

Since, Eqs. (i) and (ii) are identical.

$\therefore \quad t=-\text{cosec}\,\theta$ or $t=2\tan\theta$

68. (c) We have,

$$x^2-6x+4y+1=0$$

$\Rightarrow \quad (x-3)^2-9+4y+1=0$

$\Rightarrow \quad (x-3)^2+4y-8=0$

$\Rightarrow \quad (x-3)^2=-4(y-2)$

It represents parabola whose vertex is (3, 2)

$\therefore$ Focus $=(3, -1+2)=(3, 1)$

69. (b) Let function $f(x)=x^2(x-1)$

$\Rightarrow \quad f'(x)=3x^2-2x$

and $\quad f''(x)=6x-2$

Now, $\quad f(0)=f(1)=f'(0)=0$

Then, according to question,

at $x=\dfrac{1}{3}, \quad f''(x)=0$

(i.e. $c=1/3$ for some $C\in R$)

70. (b, c) We have,

$$f(x) = x^n$$

$$\Rightarrow \quad f'(x) = nx^{n-1}$$

Now, $f'(\alpha+\beta) = f'(\alpha) + f'(\beta)$

$$\Rightarrow \quad n(\alpha+\beta)^{n-1} = n\alpha^{n-1} + n\beta^{n-1}$$

$$\Rightarrow \quad (\alpha+\beta)^{n-1} = \alpha^{n-1} + \beta^{n-1}$$

From options we see that n = 2, satisfy the above equation.

$$\therefore \quad n = 2$$

71. (c) $I = \int_0^a \frac{dx}{1+f(x)}$ ...(i)

$$= \int_0^a \frac{1}{1+f(a-x)}\,dx$$

$$\left[\because \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\right]$$

$$= \int_0^a \frac{dx}{1+\frac{1}{f(x)}} \qquad [\because f(x)\,f(a-x) = 1]$$

$$I = \int_0^a \frac{f(x)}{f(x)+1}\,dx \qquad \text{...(ii)}$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^a \frac{f(x)+1}{f(x)+1}\,dx$$

$$\Rightarrow \quad 2I = \int_0^a 1\,dx$$

$$\Rightarrow \quad 2I = [x]_0^a$$

$$\Rightarrow \quad 2I = a$$

$$\Rightarrow \quad I = \frac{a}{2}$$

72. (b, d) We have,

$$xy = 1 - 2x$$

$$\Rightarrow y = \frac{1-2x}{x}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-2x-(1-2x)\cdot 1}{x^2}$$

$$= \frac{-2x-1+2x}{x^2} = \frac{-1}{x^2} < 0$$

Since, $ax + by + c = 0$ is tangent to the curve $xy = 1 - 2x$.

$$\therefore \quad \frac{-a}{b} < 0 \Rightarrow \frac{a}{b} > 0$$

$\Rightarrow$ Either $a > 0, b > 0$ or $a < 0, b < 0$.

73. (b, c) We know, $S = ut + \frac{1}{2}at^2$ ...(i)

Condition for first,

As, velocity is constant u = u, v = u (constant)

$\therefore \quad a = 0, \; S = ut$ ... (ii) [from Eq. (i)]

Condition for second u = 0, a = f (constant)

$$\therefore \quad S = \frac{1}{2}at^2$$

$$S = \frac{1}{2}f\,t^2 \text{ ... (iii)} \quad \text{[from Eq. (i)]}$$

On differentiating Eqs. (ii) and (iii), we get

$$\frac{dS}{dt} = u \qquad \text{... (iv)}$$

and $$\frac{dS}{dt} = \frac{1}{2}f(2t)$$

$$\frac{dS}{dt} = ft$$

$$u = ft \qquad \text{[from Eq. (iv)]}$$

$$t = \frac{u}{f} \qquad \text{... (v)}$$

Thus, they will be at the greatest distance at the end of the $\frac{u}{f}$ from the start.

For greatest distance

$$S = \frac{1}{2}ft^2$$

Put $$t = \frac{u}{f},$$

$$S = \frac{1}{2}f\left(\frac{u^2}{f^2}\right)$$

$$S = \frac{u^2}{2f}$$

Hence, option (b) and (c) are correct.

74. (a, c) We have, $|z - i| = |z + 1| = 1$

Let, $z = x + iy$

$$\therefore \quad |z - i| = 1$$

$$\Rightarrow \quad |x + iy - i| = 1$$

$$\Rightarrow \quad |x + (y-1)i| = 1$$

$$\Rightarrow \quad x^2 + (y-1)^2 = 1 \qquad \text{... (i)}$$

Also, $|z + 1| = 1$

$$\Rightarrow \quad |x + iy + 1| = 1$$

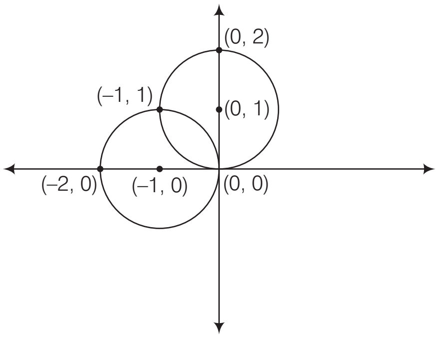
$$\Rightarrow \quad |(x+1) + iy| = 1$$

$\Rightarrow \quad (x+1)^2 + y^2 = 1 \quad \dots \text{(ii)}$

From Eqs. (i) and (ii), we get

$x = y = 0$ and $x = -1, y = 1$

$\therefore \quad z = 0, -1 + i$



75. (c) We know that,

$$1 + a^2 > 0, a \in R$$

$\Rightarrow \quad (a, a) \in \rho$

$\therefore$ ρ is reflexive

Again, $\quad$ let $(a, b) \in \rho$

$\Rightarrow \quad 1 + ab > 0$

$\Rightarrow \quad 1 + ba > 0$

$\Rightarrow \quad (b, a) \in \rho$

$\therefore$ ρ is symmetric

Now, $\quad (1, -0 \cdot 1) \in \rho$

and $\quad (-0 \cdot 1, -9) \in \rho$

but $\quad (1, -9) \notin \rho$

$\therefore$ ρ is not transitive.