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## **WBJEE 2018 Question Paper**

**West Bengal Joint Entrance Examinations**

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# WB JEE

## Engineering Entrance Exam

### Solved Paper 2018

### Physics

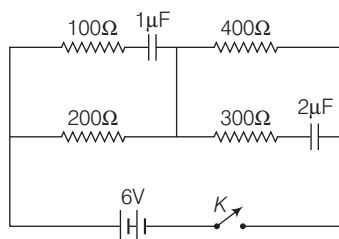
#### Category-I (Q. Nos. 1 to 30)

Only one answer is correct. Correct answer will fetch full marks 1. *Incorrect answer or any combination of more than one answer will fetch  $-\frac{1}{4}$  marks. No answer will fetch 0 marks.*

1. Four resistors,  $100\ \Omega$ ,  $200\ \Omega$ ,  $300\ \Omega$  and  $400\ \Omega$  are connected to form four sides of a square. The resistors can be connected in any order. What is the maximum possible equivalent resistance across the diagonal of the square?

(a)  $210\ \Omega$  (b)  $240\ \Omega$   
(c)  $300\ \Omega$  (d)  $250\ \Omega$

2. What will be current through the  $200\ \Omega$  resistor in the given circuit, a long time after the switch  $K$  is made on?



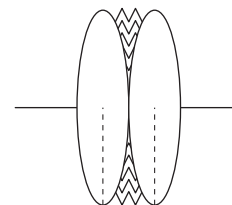
(a) Zero (b) 100 mA  
(c) 10 mA (d) 1 mA

3. A point source is placed at coordinates  $(0, 1)$  in  $xy$ -plane. A ray of light from the source is reflected on a plane mirror placed along the

$X$ -axis and perpendicular to the  $xy$ -plane. The reflected ray passes through the point  $(3, 3)$ . What is the path length of the ray from  $(0, 1)$  to  $(3, 3)$ ?

(a) 5 (b)  $\sqrt{13}$   
(c)  $2\sqrt{3}$  (d)  $1+2\sqrt{3}$

4. Two identical equiconvex lenses, each of focal length  $f$  are placed side by side in contact with each other with a layer of water in between



them as shown in the figure. If refractive index of the material of the lenses is greater than that of water, how the combined focal length  $F$  is related to  $f$ ?

(a)  $F > f$  (b)  $\frac{f}{2} < F < f$   
(c)  $F < \frac{f}{2}$  (d)  $F = f$

5. There is a small air bubble at the centre of a solid glass sphere of radius  $r$  and refractive index  $\mu$ . What will be the apparent distance of the bubble from the centre of the sphere, when viewed from outside?

(a)  $r$  (b)  $\frac{r}{\mu}$   
(c)  $r\left(1 - \frac{1}{\mu}\right)$  (d) Zero

6. If Young's double slit experiment is done with white light, which of the following statements will be true?

- (a) All the bright fringes will be coloured.  
 (b) All the bright fringes will be white.  
 (c) The central fringe will be white.  
 (d) No stable interference pattern will be visible.

7. How the linear velocity  $v$  of an electron in the Bohr orbit is related to its quantum number  $n$ ?

- (a)  $v \propto \frac{1}{n}$  (b)  $v \propto \frac{1}{n^2}$   
 (c)  $v \propto \frac{1}{\sqrt{n}}$  (d)  $v \propto n$

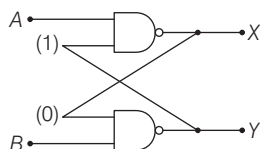
8. If the half-life of a radioactive nucleus is 3 days, nearly what fraction of the initial number of nuclei will decay on the third day? (Given,  $\sqrt[3]{0.25} \approx 0.63$ )

- (a) 0.63 (b) 0.5 (c) 0.37 (d) 0.13

9. An electron accelerated through a potential of 10000 V from rest has a de-Broglie wave length  $\lambda$ . What should be the accelerating potential, so that the wavelength is doubled?

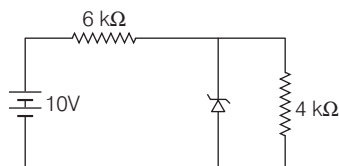
- (a) 20000 V (b) 40000 V (c) 5000 V (d) 2500 V

10. In the circuit shown, inputs  $A$  and  $B$  are in states 1 and 0 respectively. What is the only possible stable state of the outputs  $X$  and  $Y$ ?



- (a)  $X = 1, Y = 1$  (b)  $X = 1, Y = 0$   
 (c)  $X = 0, Y = 1$  (d)  $X = 0, Y = 0$

11. What will be the current flowing through the  $6 \text{ k}\Omega$  resistor in the circuit shown, where the breakdown voltage of the Zener is 6 V?



- (a)  $\frac{2}{3} \text{ mA}$  (b) 1 mA (c) 10 mA (d)  $\frac{3}{2} \text{ mA}$

12. In case of a simple harmonic motion, if the velocity is plotted along the  $X$ -axis and the displacement (from the equilibrium position) is plotted along the  $Y$ -axis, the resultant curve happens to be an ellipse with the ratio:  $\frac{\text{major axis (along } X)}{\text{minor axis (along } Y)} = 20\pi$

What is the frequency of the simple harmonic motion?

- (a) 100 Hz (b) 20 Hz (c) 10 Hz (d)  $\frac{1}{10} \text{ Hz}$

13. A block of mass  $m_2$  is placed on a horizontal table and another block of mass  $m_1$  is placed on top of it. An increasing horizontal force  $F = \alpha t$  is exerted on the upper block but the lower block never moves as a result. If the coefficient of friction between the blocks is  $\mu_1$  and that between the lower block and the table is  $\mu_2$ , then what is the maximum possible value of  $\mu_1 / \mu_2$ ?

- (a)  $\frac{m_2}{m_1}$  (b)  $1 + \frac{m_2}{m_1}$  (c)  $\frac{m_1}{m_2}$  (d)  $1 + \frac{m_1}{m_2}$

14. In a triangle  $ABC$ , the sides  $AB$  and  $AC$  are represented by the vectors  $3\hat{i} + \hat{j} + \hat{k}$  and  $\hat{i} + 2\hat{j} + \hat{k}$ , respectively. Calculate the angle  $\angle ABC$ .

- (a)  $\cos^{-1} \sqrt{\frac{5}{11}}$  (b)  $\cos^{-1} \sqrt{\frac{6}{11}}$   
 (c)  $\left( 90^\circ - \cos^{-1} \sqrt{\frac{5}{11}} \right)$  (d)  $\left( 180^\circ - \cos^{-1} \sqrt{\frac{5}{11}} \right)$

15. The velocity ( $v$ ) of a particle (under a force  $F$ ) depends on its distance ( $x$ ) from the origin (with  $x > 0$ )  $v \propto \frac{1}{\sqrt{x}}$ . Find how the magnitude

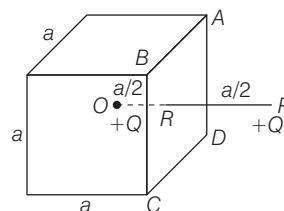
of the force ( $F$ ) on the particle depends on  $x$ ?

- (a)  $F \propto \frac{1}{x^{3/2}}$  (b)  $F \propto \frac{1}{x}$  (c)  $F \propto \frac{1}{x^2}$  (d)  $F \propto x$

16. The ratio of accelerations due to gravity  $g_1 : g_2$  on the surfaces of two planets is 5 : 2 and the ratio of their respective average densities  $\rho_1 : \rho_2$  is 2 : 1. What is the ratio of respective escape velocities  $v_1 : v_2$  from the surface of the planets?

- (a) 5 : 2 (b)  $\sqrt{5} : \sqrt{2}$  (c)  $5 : 2\sqrt{2}$  (d) 25 : 4

17. A spherical liquid drop is placed on a horizontal plane. A small disturbance causes the volume of the drop to oscillate. The time period of oscillation ( $T$ ) of the liquid drop depends on radius ( $r$ ) of the drop, density ( $\rho$ ) and surface tension ( $S$ ) of the liquid. Which among the following will be a possible expression for  $T$  (where,  $k$  is a dimensionless constant)?
- (a)  $k\sqrt{\frac{\rho r}{S}}$  (b)  $k\sqrt{\frac{\rho^2 r}{S}}$  (c)  $k\sqrt{\frac{\rho r^3}{S}}$  (d)  $k\sqrt{\frac{\rho r^3}{S^2}}$
18. The stress along the length of a rod (with rectangular cross-section) is 1% of the Young's modulus of its material. What is the approximate percentage of change of its volume? (Poisson's ratio of the material of the rod is 0.3.)
- (a) 3% (b) 1% (c) 0.7% (d) 0.4%
19. What will be the approximate terminal velocity of a rain drop of diameter  $1.8 \times 10^{-3}$  m, when density of rain water  $\approx 10^3 \text{ kg m}^{-3}$  and the coefficient of viscosity of air  $\approx 1.8 \times 10^{-5} \text{ N-sm}^{-2}$ ? (Neglect buoyancy of air)
- (a)  $49 \text{ ms}^{-1}$  (b)  $98 \text{ ms}^{-1}$  (c)  $392 \text{ ms}^{-1}$  (d)  $980 \text{ ms}^{-1}$
20. The water equivalent of a calorimeter is 10 g and it contains 50 g of water at  $15^\circ\text{C}$ . Some amount of ice, initially at  $-10^\circ\text{C}$  is dropped in it and half of the ice melts till equilibrium is reached. What was the initial amount of ice that was dropped (when specific heat of ice  $= 0.5 \text{ cal gm}^{-1}^\circ\text{C}^{-1}$ , specific heat of water  $= 1.0 \text{ cal gm}^{-1}^\circ\text{C}^{-1}$  and latent heat of melting of ice  $= 80 \text{ cal gm}^{-1}$ )?
- (a) 10 g (b) 18 g (c) 20 g (d) 30 g
21. One mole of a monoatomic ideal gas undergoes a quasistatic process, which is depicted by a straight line joining points  $(V_0, T_0)$  and  $(2V_0, 3T_0)$  in a  $V$ - $T$  diagram. What is the value of the heat capacity of the gas at the point  $(V_0, T_0)$ ?
- (a)  $R$  (b)  $\frac{3}{2}R$  (c)  $2R$  (d) 0
22. For an ideal gas with initial pressure and volume  $p_i$  and  $V_i$  respectively, a reversible isothermal expansion happens, when its volume becomes  $V_0$ . Then, it is compressed to its original volume  $V_i$  by a reversible adiabatic process. If the final pressure is  $p_f$ , then which of the following statement(s) is/are true?
- (a)  $p_f = p_i$  (b)  $p_f > p_i$  (c)  $p_f < p_i$  (d)  $\frac{p_f}{V_0} = \frac{p_i}{V_i}$
23. A point charge  $-q$  is carried from a point  $A$  to another point  $B$  on the axis of a charged ring of radius  $r$  carrying a charge  $+q$ . If the point  $A$  is at a distance  $\frac{4}{3}r$  from the centre of the ring and the point  $B$  is  $\frac{3}{4}r$  from the centre but on the opposite side, what is the net work that need to be done for this?
- (a)  $-\frac{7}{5} \cdot \frac{q^2}{4\pi\epsilon_0 r}$  (b)  $-\frac{1}{5} \cdot \frac{q^2}{4\pi\epsilon_0 r}$  (c)  $\frac{7}{5} \cdot \frac{q^2}{4\pi\epsilon_0 r}$  (d)  $\frac{1}{5} \cdot \frac{q^2}{4\pi\epsilon_0 r}$
24. Consider a region in free space bounded by the surfaces of an imaginary cube having sides of length  $a$  as shown in the figure. A charge  $+Q$  is placed at the centre  $O$  of the cube.  $P$  is such a point outside the cube that the line  $OP$  perpendicularly intersects the surface  $ABCD$  at  $R$  and also  $OR = RP = a/2$ . A charge  $+Q$  is placed at point  $P$  also. What is the total electric flux through the five faces of the cube other than  $ABCD$ ?



- (a)  $\frac{Q}{\epsilon_0}$  (b)  $\frac{5Q}{6\epsilon_0}$  (c)  $\frac{10Q}{6\epsilon_0}$  (d) zero

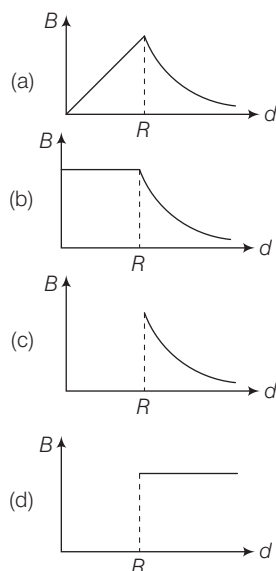
25. Four equal charges of value  $+Q$  are placed at any four vertices of a regular hexagon of side ' $a$ '. By suitably choosing the vertices, what can be the maximum possible magnitude of electric field at the centre of the hexagon?

(a)  $\frac{Q}{4\pi\epsilon_0 a^2}$  (b)  $\frac{\sqrt{2} Q}{4\pi\epsilon_0 a^2}$   
 (c)  $\frac{\sqrt{3} Q}{4\pi\epsilon_0 a^2}$  (d)  $\frac{2Q}{4\pi\epsilon_0 a^2}$

26. A proton of mass  $m$  moving with a speed  $v$  ( $\ll c$ , velocity of light in vacuum) completes a circular orbit in time  $T$  in a uniform magnetic field. If the speed of the proton is increased to  $\sqrt{2} v$ , what will be time needed to complete the circular orbit?

(a)  $\sqrt{2} T$  (b)  $\frac{T}{\sqrt{2}}$   
 (c)  $\frac{T}{\sqrt{2}}$  (d)  $\frac{T}{2}$

27. A uniform current is flowing along the length of an infinite, straight, thin, hollow cylinder of radius  $R$ . The magnetic field  $B$  produced at a perpendicular distance  $d$  from the axis of the cylinder is plotted in a graph. Which of the following figures looks like the plot?



28. A circular loop of radius  $r$  of conducting wire connected with a voltage source of zero internal resistance produces a magnetic field

$B$  at its centre. If instead, a circular loop of radius  $2r$ , made of same material, having the same cross-section is connected to the same voltage source, what will be the magnetic field at its centre?

(a)  $\frac{B}{2}$  (b)  $\frac{B}{4}$  (c)  $2B$  (d)  $B$

29. An alternating current is flowing through a series  $L$ - $C$ - $R$  circuit. It is found that the current reaches a value of 1 mA at both 200 Hz and 800 Hz frequency. What is the resonance frequency of the circuit?

(a) 600 Hz (b) 300 Hz  
 (c) 500 Hz (d) 400 Hz

30. An electric bulb, a capacitor, a battery and a switch are all in series in a circuit. How does the intensity of light vary when the switch is turn on?

- (a) Continues to increase gradually  
 (b) Gradually increases for sometime and then becomes steady  
 (c) Sharply rises initially and then gradually decreases  
 (d) Gradually increases for sometime and then gradually decreases

### Category-II (Q. Nos. 31 to 35)

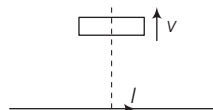
Only one answer is correct. Correct answer will fetch full marks 2. Incorrect answer or any combination of more than one answer will fetch  $-\frac{1}{2}$  marks. No answer will fetch 0 marks.

31. A light charged particle is revolving in a circle of radius  $r$  in electrostatic attraction of a static heavy particle with opposite charge. How does the magnetic field  $B$  at the centre of the circle due to the moving charge depend on  $r$ ?

(a)  $B \propto \frac{1}{r}$  (b)  $B \propto \frac{1}{r^2}$   
 (c)  $B \propto \frac{1}{r^{3/2}}$  (d)  $B \propto \frac{1}{r^{5/2}}$

32. As shown in the figure, a rectangular loop of a conducting wire is moving away with a constant velocity  $v$  in a perpendicular direction from a very long straight conductor carrying a steady current  $I$ . When the

breadth of the rectangular loop is very small compared to its distance from the straight conductor, how does the emf,  $E$  induced in the loop vary with time  $t$ ?

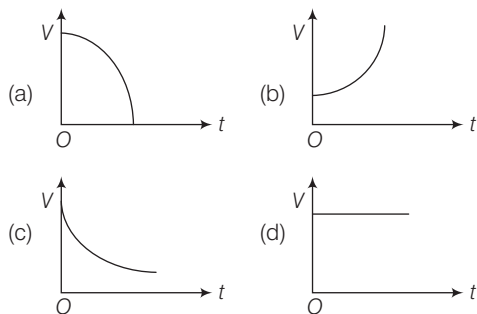


- (a)  $E \propto \frac{1}{t^2}$  (b)  $E \propto \frac{1}{t}$  (c)  $E \propto -\ln(t)$  (d)  $E \propto \frac{1}{t^3}$

33. A solid spherical ball and a hollow spherical ball of two different materials of densities  $\rho_1$  and  $\rho_2$  respectively have same outer radii and same mass. What will be the ratio, the moment of inertia (about an axis passing through the centre) of the hollow sphere to that of the solid sphere?

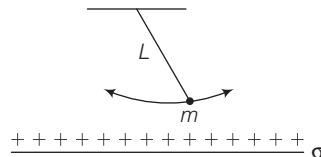
- (a)  $\frac{\rho_2}{\rho_1} \left( 1 - \frac{\rho_2}{\rho_1} \right)^{\frac{5}{3}}$  (b)  $\frac{\rho_2}{\rho_1} \left[ 1 - \left( 1 - \frac{\rho_2}{\rho_1} \right)^{\frac{5}{3}} \right]$   
 (c)  $\frac{\rho_2}{\rho_1} \left( 1 - \frac{\rho_1}{\rho_2} \right)^{\frac{5}{3}}$  (d)  $\frac{\rho_2}{\rho_1} \left[ 1 - \left( 1 - \frac{\rho_1}{\rho_2} \right)^{\frac{5}{3}} \right]$

34. The insulated plates of a charged parallel plate capacitor (with small separation between the plates) are approaching each other due to electrostatic attraction. Assuming no other force to be operative and no radiation taking place, which of the following graphs approximately shows the variation with time ( $t$ ) of the potential difference ( $V$ ) between the plates?



35. The bob of a pendulum of mass  $m$ , suspended by an inextensible string of length  $L$  as shown in the figure carries a small charge  $q$ . An infinite horizontal plane

conductor with uniform surface charge density  $\sigma$  is placed below it. What will be the time period of the pendulum for small amplitude oscillations?

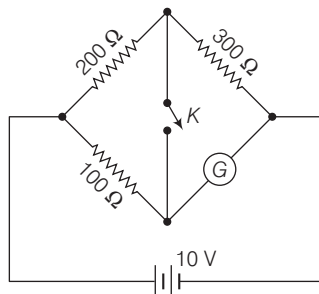


- (a)  $2\pi \sqrt{\frac{L}{g - \frac{mq}{\epsilon_0 \sigma}}}$  (b)  $\sqrt{\frac{L}{g - \frac{mq\sigma}{\epsilon_0}}}$   
 (c)  $\frac{1}{2\pi} \sqrt{\frac{L}{g - \frac{q\sigma}{\epsilon_0 m}}}$  (d)  $2\pi \sqrt{\frac{L}{g - \frac{q\sigma}{\epsilon_0 m}}}$

### Category-III (Q. Nos. 36 to 40)

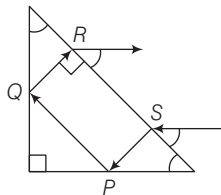
One or more answer(s) is (are) correct. Correct answer(s) will fetch full marks 2. Any combination containing one or more incorrect answer will fetch 0 marks. Also, no answer will fetch 0 marks. If all correct answers are not marked and also no incorrect answer is marked then score =  $2 \times \text{number of correct answers marked} \div \text{actual number of correct answers}$ .

36. A non-zero current passes through the galvanometer  $G$  shown in the circuit when the key  $K$  is closed and its value does not change when the key is opened. Then, which of the following statement(s) is/are true?



- (a) The galvanometer resistance is infinite.  
 (b) The current through the galvanometer is 40 mA.  
 (c) After the key is closed, the current through the 200  $\Omega$  resistor is same as the current through the 300  $\Omega$  resistor.  
 (d) The galvanometer resistance is 150  $\Omega$ .

37. A ray of light is incident on a right angled isosceles prism parallel to its base as shown in the figure. Refractive index of the material of the prism is  $\sqrt{2}$ . Then, which of the following statement(s) is/are true?



- (a) The reflection at  $P$  is total internal.  
 (b) The reflection at  $Q$  is total internal.  
 (c) The ray emerging at  $R$  is parallel to the ray incident at  $S$ .  
 (d) Total deviation of the ray is  $150^\circ$ .
38. The intensity of a sound appears to an observer to be periodic. Which of the following can be the cause of it?
- (a) The intensity of the source is periodic  
 (b) The source is moving towards the observer  
 (c) The observer is moving away from the source  
 (d) The source is producing a sound composed of two nearby frequencies

39. Which of the following statements(s) is/are true?

"Internal energy of an ideal gas ....."

- (a) decreases in an isothermal process.  
 (b) remains constant in an isothermal process.  
 (c) increases in an isobaric process.  
 (d) decreases in an isobaric expansion.
40. Two positive charges  $Q$  and  $4Q$  are placed at points  $A$  and  $B$  respectively, where  $B$  is at a distance  $d$  units to the right of  $A$ . The total electric potential due to these charges is minimum at  $P$  on the line through  $A$  and  $B$ . What is (are) the distance (s) of  $P$  from  $A$ ?

- (a)  $\frac{d}{3}$  units to the right of  $A$   
 (b)  $\frac{d}{3}$  units to the left of  $A$   
 (c)  $\frac{d}{5}$  units to the right of  $A$   
 (d)  $d$  units to the left of  $A$

## Chemistry

### Category-I (Q. Nos. 41 to 70)

Only one answer is correct. Correct will fetch full marks 1. Incorrect answer or any combination of more than one answer will fetch  $-\frac{1}{4}$  marks. No answer will fetch 0 marks.

41.  $\text{Cl}_2\text{O}_7$  is the anhydride of
- (a)  $\text{HOCl}$  (b)  $\text{HClO}_2$   
 (c)  $\text{HClO}_3$  (d)  $\text{HClO}_4$
42. The main reason that  $\text{SiCl}_4$  is easily hydrolysed as compared to  $\text{CCl}_4$  is that
- (a)  $\text{Si}-\text{Cl}$  bond is weaker than  $\text{C}-\text{Cl}$  bond  
 (b)  $\text{SiCl}_4$  can form hydrogen bonds  
 (c)  $\text{SiCl}_4$  is covalent  
 (d)  $\text{Si}$  can extend its coordination number beyond four
43. Silver chloride dissolves in excess of ammonium hydroxide solution. The cation present in the resulting solution is
- (a)  $[\text{Ag}(\text{NH}_3)_6]^+$  (b)  $[\text{Ag}(\text{NH}_3)_4]^+$   
 (c)  $\text{Ag}^+$  (d)  $[\text{Ag}(\text{NH}_3)_2]^+$

44. The ease of hydrolysis in the compounds  $\text{CH}_3\text{COCl}$  (I),  $\text{CH}_3-\text{CO}-\text{O}-\text{COCH}_3$  (II),  $\text{CH}_3\text{COOC}_2\text{H}_5$  (III) and  $\text{CH}_3\text{CONH}_2$  (IV) is of the order

- (a)  $\text{I} > \text{II} > \text{III} > \text{IV}$   
 (b)  $\text{IV} > \text{III} > \text{II} > \text{I}$   
 (c)  $\text{I} > \text{II} > \text{IV} > \text{III}$   
 (d)  $\text{II} > \text{I} > \text{IV} > \text{III}$

45.  $\text{CH}_3-\text{C}\equiv\text{C MgBr}$  can be prepared by the reaction of

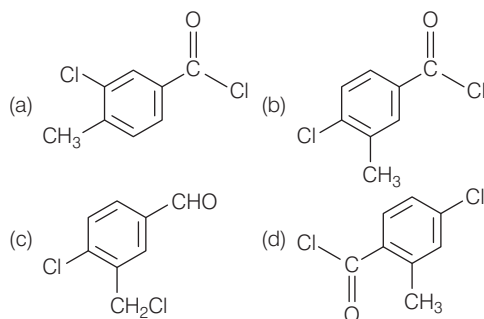
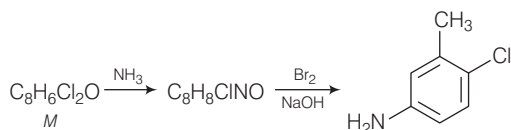
- (a)  $\text{CH}_3-\text{C}\equiv\text{C}-\text{Br}$  with  $\text{MgBr}_2$   
 (b)  $\text{CH}_3-\text{C}\equiv\text{CH}$  with  $\text{MgBr}_2$   
 (c)  $\text{CH}_3-\text{C}\equiv\text{CH}$  with  $\text{KBr}$  and  $\text{Mg}$  metal  
 (d)  $\text{CH}_3-\text{C}\equiv\text{CH}$  with  $\text{CH}_3\text{MgBr}$

46. The number of alkene (s) which can produce 2-butanol by the successive treatment of
- (i)  $\text{B}_2\text{H}_6$  in tetrahydrofuran solvent and  
 (ii) alkaline  $\text{H}_2\text{O}_2$  solution is

- (a) 1 (b) 2  
 (c) 3 (d) 4

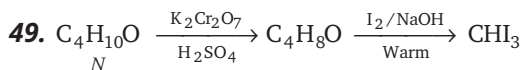


47. Identify 'M' in the following sequence of reactions

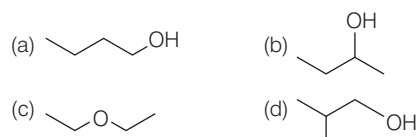


48. Methoxybenzene on treatment with HI produces

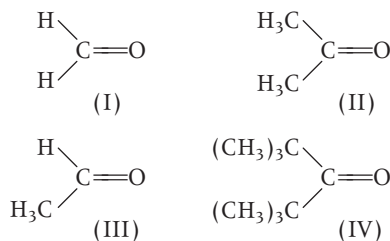
- (a) iodobenzene and methanol  
(b) phenol and methyl iodide  
(c) iodobenzene and methyl iodide  
(d) phenol and methanol



Here, N is



50. The correct order of reactivity for the addition reaction of the following carbonyl compounds with ethylmagnesium iodide is



- (a) I > III > II > IV  
(b) IV > III > II > I  
(c) I > II > IV > III  
(d) III > II > I > IV

51. If aniline is treated with conc.  $\text{H}_2\text{SO}_4$  and heated at  $200^\circ\text{C}$ , the product is

- (a) anilinium sulphate  
(b) benzenesulphonic acid  
(c) *m*-aminobenzenesulphonic acid  
(d) sulphanilic acid

52. Which of the following electronic configuration is not possible?

- (a)  $n = 3, l = 0, m = 0$   
(b)  $n = 3, l = 1, m = -1$   
(c)  $n = 2, l = 0, m = -1$   
(d)  $n = 2, l = 1, m = 0$

53. The number of unpaired electrons in Ni (atomic number = 28) are

- (a) 0 (b) 2 (c) 4 (d) 8

54. Which of the following has the strongest H-bond?

- (a)  $\text{O}-\text{H} \cdots \text{S}$  (b)  $\text{S}-\text{H} \cdots \text{O}$   
(c)  $\text{F}-\text{H} \cdots \text{F}$  (d)  $\text{F}-\text{H} \cdots \text{O}$

55. The half-life of  $\text{C}^{14}$  is 5760 years. For a 200 mg sample of  $\text{C}^{14}$ , the time taken to change to 25 mg is

- (a) 11520 years (b) 23040 years  
(c) 5760 years (d) 17280 years

56. Ferric ion forms a prussian blue precipitate due to the formation of

- (a)  $\text{K}_4[\text{Fe}(\text{CN})_6]$  (b)  $\text{K}_3[\text{Fe}(\text{CN})_6]$   
(c)  $\text{Fe}(\text{CNS})_3$  (d)  $\text{Fe}_4[\text{Fe}(\text{CN})_6]_3$

57. The nucleus  ${}^{64}_{29}\text{Cu}$  accepts an orbital electron to yield,

- (a)  ${}^{65}_{28}\text{Ni}$  (b)  ${}^{64}_{30}\text{Zn}$  (c)  ${}^{64}_{28}\text{Ni}$  (d)  ${}^{65}_{30}\text{Zn}$

58. How many moles of electrons will weigh one kilogram?

- (a)  $6.023 \times 10^{23}$  (b)  $\frac{1}{9.108} \times 10^{31}$   
(c)  $\frac{6.023}{9.108} \times 10^{54}$  (d)  $\frac{1}{9.108 \times 6.023} \times 10^8$

59. Equal weights of ethane and hydrogen are mixed in an empty container at  $25^\circ\text{C}$ . The fraction of total pressure exerted by hydrogen is

- (a) 1 : 2 (b) 1 : 1  
(c) 1 : 16 (d) 15 : 16



60. The heat of neutralisation of a strong base and a strong acid is 13.7 kcal. The heat released when 0.6 mole HCl solution is added to 0.25 mole of NaOH is

(a) 3.425 kcal (b) 8.22 kcal  
(c) 11.645 kcal (d) 13.7 kcal

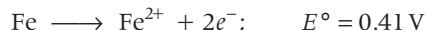
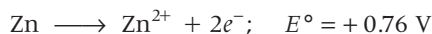
61. A compound formed by elements  $X$  and  $Y$  crystallises in the cubic structure, where  $X$  atoms are at the corners of a cube and  $Y$  atoms are at the centre of the body. The formula of the compounds is

(a)  $XY$  (b)  $XY_2$   
(c)  $X_2Y_3$  (d)  $XY_3$

62. What amount of electricity can deposit 1 mole of Al metal at cathode when passed through molten  $AlCl_3$ ?

(a) 0.3 F (b) 1 F  
(c) 3 F (d)  $1/3$  F

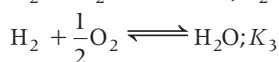
63. Given the standard half-cell potentials ( $E^\circ$ ) of the following as



Then the standard e.m.f. of the cell with the reaction  $Fe^{2+} + Zn \longrightarrow Zn^{2+} + Fe$  is

(a)  $-0.35 \text{ V}$  (b)  $+0.35 \text{ V}$   
(c)  $+1.17 \text{ V}$  (d)  $-1.17 \text{ V}$

64. The following equilibrium constants are given



The equilibrium constant for the oxidation of 2 mole of  $NH_3$  to give NO is

(a)  $K_1 \cdot \frac{K_2}{K_3}$  (b)  $K_2 \cdot \frac{K_3}{K_1}$   
(c)  $K_2 \cdot \frac{K_3^2}{K_1}$  (d)  $K_2^2 \cdot \frac{K_3}{K_1}$

65. Which one of the following is a condensation polymer?

(a) PVC (b) Teflon  
(c) Dacron (d) Polystyrene

66. Which of the following is present in maximum amount in 'acid rain'?

(a)  $HNO_3$  (b)  $H_2SO_4$   
(c) HCl (d)  $H_2CO_3$

67. Which of the set of oxides are arranged in the proper order of basic, amphoteric, acidic?

(a)  $SO_2, P_2O_5, CO$  (b)  $BaO, Al_2O_3, SO_2$   
(c)  $CaO, SiO_2, Al_2O_3$  (d)  $CO_2, Al_2O_3, CO$

68. Out of the following outer electronic configurations of atoms, the highest oxidation state is achieved by which one?

(a)  $(n-1)d^8ns^2$  (b)  $(n-1)d^5ns^2$   
(c)  $(n-1)d^3ns^2$  (d)  $(n-1)d^5ns^1$

69. At room temperature, the reaction between water and fluorine produces

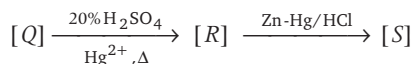
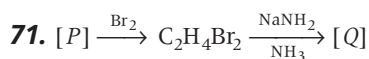
(a) HF and  $H_2O_2$  (b) HF,  $O_2$  and  $F_2O_2$   
(c)  $F^-$ ,  $O_2$  and  $H^+$  (d) HOF and HF

70. Which of the following is least thermally stable?

(a)  $MgCO_3$  (b)  $CaCO_3$   
(c)  $SrCO_3$  (d)  $BeCO_3$

### Category-II (Q. Nos. 71 to 75)

Only one answer is correct. Correct answer will fetch full marks 2. *Incorrect answer or any combination of more than one answer will fetch - 1/2 marks. No answer will fetch 0 marks.*



The species  $P$ ,  $Q$ ,  $R$  and  $S$  respectively are

(a) ethene, ethyne, ethanal, ethane  
(b) ethane, ethyne, ethanal, ethene  
(c) ethene, ethyne, ethanal, ethanol  
(d) ethyne, ethane, ethene, ethanal

72. The number of possible organobromine compounds which can be obtained in the allylic bromination of 1-butene with N-bromosuccinimide is

(a) 1 (b) 2  
(c) 3 (d) 4

**73.** A metal  $M$  (specific heat 0.16) forms a metal chloride with 65% chlorine present in it. The formula of the metal chloride will be

- (a)  $MCl$  (b)  $MCl_2$   
(c)  $MCl_3$  (d)  $MCl_4$

**74.** During a reversible adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The

ratio  $\frac{C_p}{C_v}$  for the gas is

- (a)  $\frac{3}{2}$  (b)  $\frac{7}{2}$   
(c)  $\frac{5}{3}$  (d)  $\frac{9}{7}$

**75.**  $[X] + \text{dil. } H_2SO_4 \longrightarrow [Y] :$

Colourless, suffocating gas

$[Y] + K_2Cr_2O_7 + H_2SO_4 \longrightarrow$

Green colouration of solution

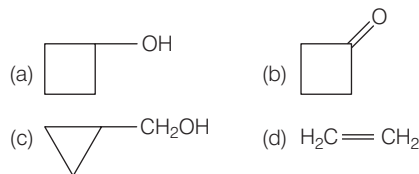
Then,  $[X]$  and  $[Y]$  are

- (a)  $SO_3^{2-}, SO_2$   
(b)  $Cl^-, HCl$   
(c)  $S^{2-}, H_2S$   
(d)  $CO_3^{2-}, CO_2$

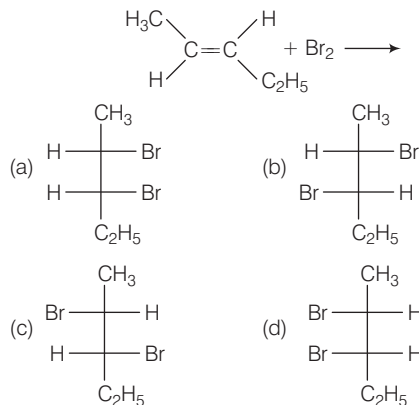
### Category-III (Q. Nos. 76 to 80)

One or more answer(s) is (are) correct. Correct answer(s) will fetch full marks 2. Any combination containing one or more incorrect answer will fetch 0 marks. Also no answer will fetch 0 marks. If all correct answers are not marked and also no incorrect answer is marked then score =  $2 \times \text{number of correct answers marked} \div \text{actual number of correct answers}$ .

**76.** The possible product(s) to be obtained from the reaction of cyclobutyl amine with  $HNO_2$  is/are



**77.** The major products obtained in the following reaction is/are



**78.** Which statements are correct for the peroxide ion?

- (a) It has five completely filled anti-bonding molecular orbitals  
(b) It is diamagnetic  
(c) It has bond order one  
(d) It is isoelectronic with neon

**79.** Among the following, the extensive variables are

- (a)  $H$  (Enthalpy) (b)  $p$  (Pressure)  
(c)  $E$  (Internal energy) (d)  $V$  (Volume)

**80.** White phosphorous  $P_4$  has the following characteristics

- (a)  $6P-P$  single bonds  
(b)  $4P-P$  single bonds  
(c) 4 lone pair of electrons  
(d)  $P-P-P$  angle of  $60^\circ$

# Mathematics

## Category-I (Q. Nos. 1 to 50)

Only one answer is correct. Correct answer will fetch full marks 1. Incorrect answer or any combination of more than one answer will fetch -1/4 marks. No answer will fetch 0 marks.

1. The approximate value of  $\sin 31^\circ$  is

- (a)  $> 0.5$  (b)  $> 0.6$   
(c)  $< 0.5$  (d)  $< 0.4$

2. Let  $f_1(x) = e^x$ ,  $f_2(x) = e^{f_1(x)}$ , ...,  $f_{n+1}(x) = e^{f_n(x)}$  for all  $n \geq 1$ . Then for any fixed

$n$ ,  $\frac{d}{dx} f_n(x)$  is

- (a)  $f_n(x)$   
(b)  $f_n(x)f_{n-1}(x)$   
(c)  $f_n(x)f_{n-1}(x) \dots f_1(x)$   
(d)  $f_n(x) \dots f_1(x)e^x$

3. The domain of definition of  $f(x) = \sqrt{\frac{1-|x|}{2-|x|}}$  is

- (a)  $(-\infty, -1) \cup (2, \infty)$   
(b)  $[-1, 1] \cup (2, \infty) \cup (-\infty, -2)$   
(c)  $(-\infty, 1) \cup (2, \infty)$   
(d)  $[-1, 1] \cup (2, \infty)$

Here  $(a, b) \equiv \{x : a < x < b\}$  and

$$[a, b] \equiv \{x : a \leq x \leq b\}$$

4. Let  $f : [a, b] \rightarrow R$  be differentiable on  $[a, b]$  and  $k \in R$ . Let  $f(a) = 0 = f(b)$ .

Also let  $J(x) = f'(x) + kf(x)$ . Then

- (a)  $J(x) > 0$  for all  $x \in [a, b]$   
(b)  $J(x) < 0$  for all  $x \in [a, b]$   
(c)  $J(x) = 0$  has at least one root in  $(a, b)$   
(d)  $J(x) = 0$  through  $(a, b)$

5. Let  $f(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 7$ .

Then  $\lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{h^3 + 3h}$

- (a) does not exist (b) is  $\frac{50}{3}$   
(c) is  $\frac{53}{3}$  (d) is  $\frac{22}{3}$

6. Let  $f : [a, b] \rightarrow R$  be such that  $f$  is differentiable in  $(a, b)$ ,  $f$  is continuous at  $x = a$  and  $x = b$  and moreover  $f(a) = 0 = f(b)$ . Then

- (a) there exists at least one point  $c$  in  $(a, b)$  such that  $f'(c) = f(c)$   
(b)  $f''(x) = f(x)$  does not hold at any point in  $(a, b)$   
(c) at every point of  $(a, b)$ ,  $f'(x) > f(x)$   
(d) at every point of  $(a, b)$ ,  $f'(x) < f(x)$

7. Let  $f : R \rightarrow R$  be a twice continuously differentiable function such that  $f(0) = f(1) = f'(0) = 0$ . Then

- (a)  $f''(0) = 0$   
(b)  $f''(c) = 0$  for some  $c \in R$   
(c) if  $c \neq 0$ , then  $f''(c) \neq 0$   
(d)  $f'(x) > 0$  for all  $x \neq 0$

8. If  $\int e^{\sin x} \cdot \left[ \frac{x \cos^3 x - \sin x}{\cos^2 x} \right] dx = e^{\sin x} f(x) + c$ ,

where  $c$  is constant of integration, then  $f(x)$  is equal to

- (a)  $\sec x - x$  (b)  $x - \sec x$   
(c)  $\tan x - x$  (d)  $x - \tan x$

9. If  $\int f(x) \sin x \cos x dx = \frac{1}{2(b^2 - a^2)} \log f(x) + c$ ,

where  $c$  is the constant of integration, then  $f(x)$  is equal to

- (a)  $\frac{2}{(b^2 - a^2) \sin 2x}$  (b)  $\frac{2}{ab \sin 2x}$   
(c)  $\frac{2}{(b^2 - a^2) \cos 2x}$  (d)  $\frac{2}{ab \cos 2x}$

10. If  $M = \int_0^{\pi/2} \frac{\cos x}{x+2} dx$ ,  $N = \int_0^{\pi/4} \frac{\sin x \cos x}{(x+1)^2} dx$ , then

the value of  $M - N$  is

- (a)  $\pi$  (b)  $\frac{\pi}{4}$  (c)  $\frac{2}{\pi - 4}$  (d)  $\frac{2}{\pi + 4}$

11. The value of the integral  $I = \int_{1/2014}^{2014} \frac{\tan^{-1} x}{x} dx$  is

- (a)  $\frac{\pi}{4} \log 2014$  (b)  $\frac{\pi}{2} \log 2014$   
(c)  $\pi \log 2014$  (d)  $\frac{1}{2} \log 2014$

12. Let  $I = \int_{\pi/4}^{\pi/3} \frac{\sin x}{x} dx$ . Then

- (a)  $\frac{1}{2} \leq I \leq 1$  (b)  $4 \leq I \leq 2\sqrt{30}$   
 (c)  $\frac{\sqrt{3}}{8} \leq I \leq \frac{\sqrt{2}}{6}$  (d)  $1 \leq I \leq \frac{2\sqrt{3}}{\sqrt{2}}$

13. The value of

$$I = \int_{\pi/2}^{5\pi/2} \frac{e^{\tan^{-1}(\sin x)}}{\pi/2 e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}} dx, \text{ is}$$

(a) 1 (b)  $\pi$  (c)  $e$  (d)  $\frac{\pi}{2}$

14. The value of

$$\lim_{n \rightarrow \infty} \frac{1}{n} \left\{ \sec^2 \frac{\pi}{4n} + \sec^2 \frac{2\pi}{4n} + \dots + \sec^2 \frac{n\pi}{4n} \right\} \text{ is}$$

(a)  $\log_e 2$  (b)  $\frac{\pi}{2}$  (c)  $\frac{4}{\pi}$  (d)  $e$

15. The differential equation representing the family of curves  $y^2 = 2d(x + \sqrt{d})$ , where  $d$  is a parameter, is of

- (a) order 2 (b) degree 2  
 (c) degree 3 (d) degree 4

16. Let  $y(x)$  be a solution of

$$(1 + x^2) \frac{dy}{dx} + 2xy - 4x^2 = 0 \text{ and } y(0) = -1. \text{ Then } y(1) \text{ is equal to}$$

(a)  $\frac{1}{2}$  (b)  $\frac{1}{3}$   
 (c)  $\frac{1}{6}$  (d)  $-1$

17. The law of motion of a body moving along a straight line is  $x = \frac{1}{2} vt$ ,  $x$  being its distance

from a fixed point on the line at time  $t$  and  $v$  is its velocity there. Then

- (a) acceleration  $f$  varies directly with  $x$   
 (b) acceleration  $f$  varies inversely with  $x$   
 (c) acceleration  $f$  is constant  
 (d) acceleration  $f$  varies directly with  $t$

18. Number of common tangents of  $y = x^2$  and  $y = -x^2 + 4x - 4$  is

- (a) 1 (b) 2  
 (c) 3 (d) 4

19. Given that  $n$  numbers of arithmetic means are inserted between two sets of numbers  $a, 2b$  and  $2a, b$  where  $a, b \in R$ . Suppose further that the  $m$ th means between these sets of numbers are same, then the ratio  $a : b$  equals

- (a)  $n - m + 1 : m$  (b)  $n - m + 1 : n$   
 (c)  $n : n - m + 1$  (d)  $m : n - m + 1$

20. If  $x + \log_{10}(1 + 2^x) = x \log_{10} 5 + \log_{10} 6$ , then the value of  $x$  is

- (a)  $\frac{1}{2}$  (b)  $\frac{1}{3}$   
 (c) 1 (d) 2

21. If  $Z_r = \sin \frac{2\pi r}{11} - i \cos \frac{2\pi r}{11}$ , then  $\sum_{r=0}^{10} Z_r$  is equal to

- (a)  $-1$  (b) 0 (c)  $i$  (d)  $-i$

22. If  $z_1$  and  $z_2$  be two non-zero complex

numbers such that  $\frac{z_1}{z_2} + \frac{z_2}{z_1} = 1$ , then the

origin and the points represented by  $z_1$  and  $z_2$

- (a) lie on a straight line  
 (b) form a right angled triangle  
 (c) form an equilateral triangle  
 (d) form an isosceles triangle

23. If  $b_1 b_2 = 2(c_1 + c_2)$  and  $b_1, b_2, c_1, c_2$  are all real numbers, then at least one of the equations  $x^2 + b_1 x + c_1 = 0$  and  $x^2 + b_2 x + c_2 = 0$  has

- (a) real roots  
 (b) purely imaginary roots  
 (c) roots of the form  $a + ib$  ( $a, b \in R, ab \neq 0$ )  
 (d) rational roots

24. The number of selection of  $n$  objects from  $2n$  objects of which  $n$  are identical and the rest are different, is

- (a)  $2^n$  (b)  $2^{n-1}$   
 (c)  $2^n - 1$  (d)  $2^{n-1} + 1$

25. If  $(2 \leq r \leq n)$ , then  ${}^nC_r + 2 \cdot {}^nC_{r+1} + {}^nC_{r+2}$  is equal to

- (a)  $2 \cdot {}^nC_{r+2}$  (b)  ${}^{n+1}C_{r+1}$   
 (c)  ${}^{n+2}C_{r+2}$  (d)  ${}^{n+1}C_r$

26. The number  $(101)^{100} - 1$  is divisible by

- (a)  $10^4$  (b)  $10^6$   
 (c)  $10^8$  (d)  $10^{12}$

27. If  $n$  is even positive integer, then the condition that the greatest term in the expansion of  $(1+x)^n$  may also have the greatest coefficient, is

- (a)  $\frac{n}{n+2} < x < \frac{n+2}{n}$  (b)  $\frac{n}{n+1} < x < \frac{n+1}{n}$   
 (c)  $\frac{n+1}{n+2} < x < \frac{n+2}{n+1}$  (d)  $\frac{n+2}{n+3} < x < \frac{n+3}{n+2}$

28. If  $\begin{vmatrix} -1 & 7 & 0 \\ 2 & 1 & -3 \\ 3 & 4 & 1 \end{vmatrix} = A$ , Then  $\begin{vmatrix} 13 & -11 & 5 \\ -7 & -1 & 25 \\ -21 & -3 & -15 \end{vmatrix}$  is

- (a)  $A^2$  (b)  $A^2 - A + I_3$   
 (c)  $A^2 - 3A + I_3$  (d)  $3A^2 + 5A - 4I_3$

( $I_3$  denotes the det of the identity matrix of order 3)

29. If  $a_r = (\cos 2r\pi + i \sin 2r\pi)^{1/9}$ , then the value of

$\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$  is equal to

- (a) 1 (b) -1  
 (c) 0 (d) 2

30. If  $S_r = \begin{vmatrix} 2r & x & n(n+1) \\ 6r^2 - 1 & y & n^2(2n+3) \\ 4r^3 - 2nr & z & n^3(n+1) \end{vmatrix}$ , then the

value of  $\sum_{r=1}^n S_r$  is independent of

- (a) only  $x$  (b) only  $y$   
 (c) only  $n$  (d)  $x, y, z$  and  $n$

31. If the following three linear equations have a non-trivial solution, then

$$\begin{aligned} x + 4ay + az &= 0 \\ x + 3by + bz &= 0 \\ x + 2cy + cz &= 0 \end{aligned}$$

- (a)  $a, b, c$  are in AP (b)  $a, b, c$  are in GP  
 (c)  $a, b, c$  are in HP (d)  $a + b + c = 0$

32. On  $R$ , a relation  $\rho$  is defined by  $x\rho y$  if and only if  $x - y$  is zero or irrational. Then,

- (a)  $\rho$  is equivalence relation  
 (b)  $\rho$  is reflexive but neither symmetric nor transitive  
 (c)  $\rho$  is reflexive and symmetric but not transitive  
 (d)  $\rho$  is symmetric and transitive but not reflexive

33. On the set  $R$  of real numbers, the relation  $\rho$  is defined by  $x\rho y, (x, y) \in R$ .

- (a) If  $|x - y| < 2$ , then  $\rho$  is reflexive but neither symmetric nor transitive.  
 (b) If  $x - y < 2$ , then  $\rho$  is reflexive and symmetric but not transitive.  
 (c) If  $|x| \geq y$ , then  $\rho$  is reflexive and transitive but not symmetric.  
 (d) If  $x > |y|$ , then  $\rho$  is transitive but neither reflexive nor symmetric.

34. If  $f: R \rightarrow R$  be defined by  $f(x) = e^x$  and

$g: R \rightarrow R$  be defined by  $g(x) = x^2$ . The mapping  $gof: R \rightarrow R$  be defined by  $(gof)(x) = g[f(x)] \forall x \in R$ . Then,

- (a)  $gof$  is bijective but  $f$  is not injective  
 (b)  $gof$  is injective and  $g$  is injective  
 (c)  $gof$  is injective but  $g$  is not bijective  
 (d)  $gof$  is surjective and  $g$  is surjective

35. In order to get a head at least once with probability  $\geq 0.9$ , the minimum number of times a unbiased coin needs to be tossed is

- (a) 3 (b) 4 (c) 5 (d) 6

36. A student appears for tests I, II and III. The student is successful if he passes in tests I, II or I, III. The probabilities of the student passing in tests I, II and III are respectively  $p, q$  and  $1/2$ . If the probability of the student to be successful is  $1/2$ . Then

- (a)  $p(1+q) = 1$  (b)  $q(1+p) = 1$   
 (c)  $pq = 1$  (d)  $\frac{1}{p} + \frac{1}{q} = 1$

37. If  $\sin 6\theta + \sin 4\theta + \sin 2\theta = 0$ , then general value of  $\theta$  is

- (a)  $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{3}$  (b)  $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{6}$   
 (c)  $\frac{n\pi}{4}, 2n\pi \pm \frac{\pi}{3}$  (d)  $\frac{n\pi}{4}, 2n\pi \pm \frac{\pi}{6}$

( $n$  is an integer)

38. If  $0 \leq A \leq \frac{\pi}{4}$ , then

$\tan^{-1}\left(\frac{1}{2}\tan 2A\right) + \tan^{-1}(\cot A) + \tan^{-1}(\cot^3 A)$

is equal to

- (a)  $\frac{\pi}{4}$  (b)  $\pi$  (c) 0 (d)  $\frac{\pi}{2}$

- 39.** Without changing the direction of the axes, the origin is transferred to the point  $(2, 3)$ . Then the equation  $x^2 + y^2 - 4x - 6y + 9 = 0$  changes to  
 (a)  $x^2 + y^2 + 4 = 0$   
 (b)  $x^2 + y^2 = 4$   
 (c)  $x^2 + y^2 - 8x - 12y + 48 = 0$   
 (d)  $x^2 + y^2 = 9$
- 40.** The angle between a pair of tangents drawn from a point  $P$  to the circle  $x^2 + y^2 + 4x - 6y + 9 \sin^2 \alpha + 13 \cos^2 \alpha = 0$  is  $2\alpha$ . The equation of the locus of the point  $P$  is  
 (a)  $x^2 + y^2 + 4x + 6y + 9 = 0$   
 (b)  $x^2 + y^2 - 4x + 6y + 9 = 0$   
 (c)  $x^2 + y^2 - 4x - 6y + 9 = 0$   
 (d)  $x^2 + y^2 + 4x - 6y + 9 = 0$
- 41.** The point  $Q$  is the image of the point  $P(1, 5)$  about the line  $y = x$  and  $R$  is the image of the point  $Q$  about the line  $y = -x$ . The circumcentre of the  $\Delta PQR$  is  
 (a)  $(5, 1)$  (b)  $(-5, 1)$   
 (c)  $(1, -5)$  (d)  $(0, 0)$
- 42.** The angular points of a triangle are  $A(-1, -7)$ ,  $B(5, 1)$  and  $C(1, 4)$ . The equation of the bisector of the angle  $\angle ABC$  is  
 (a)  $x = 7y + 2$  (b)  $7y = x + 2$   
 (c)  $y = 7x + 2$  (d)  $7x = y + 2$
- 43.** If one of the diameter of the circle, given by the equation  $x^2 + y^2 + 4x + 6y - 12 = 0$ , is a chord of a circle  $S$ , whose centre is  $(2, -3)$ , the radius of  $S$  is  
 (a)  $\sqrt{41}$  unit (b)  $3\sqrt{5}$  unit  
 (c)  $5\sqrt{2}$  unit (d)  $2\sqrt{5}$  unit
- 44.** A chord  $AB$  is drawn from the point  $A(0, 3)$  on the circle  $x^2 + 4x + (y - 3)^2 = 0$ , and is extended to  $M$  such that  $AM = 2AB$ . The locus of  $M$  is  
 (a)  $x^2 + y^2 - 8x - 6y + 9 = 0$   
 (b)  $x^2 + y^2 + 8x + 6y + 9 = 0$   
 (c)  $x^2 + y^2 + 8x - 6y + 9 = 0$   
 (d)  $x^2 + y^2 - 8x + 6y + 9 = 0$
- 45.** Let the eccentricity of the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  be reciprocal to that of the ellipse  $x^2 + 9y^2 = 9$ , then the ratio  $a^2 : b^2$  equals  
 (a)  $8 : 1$  (b)  $1 : 8$   
 (c)  $9 : 1$  (d)  $1 : 9$
- 46.** Let  $A, B$  be two distinct points on the parabola  $y^2 = 4x$ . If the axis of the parabola touches a circle of radius  $r$  having  $AB$  as diameter, the slope of the line  $AB$  is  
 (a)  $-\frac{1}{r}$  (b)  $\frac{1}{r}$  (c)  $\frac{2}{r}$  (d)  $-\frac{2}{r}$
- 47.** Let  $P(at^2, 2at)$ ,  $Q, R(ar^2, 2ar)$  be three points on a parabola  $y^2 = 4ax$ . If  $PQ$  is the focal chord and  $PK, QR$  are parallel where the co-ordinates of  $K$  is  $(2a, 0)$ , then the value of  $r$  is  
 (a)  $\frac{t}{1-t^2}$  (b)  $\frac{1-t^2}{t}$  (c)  $\frac{t^2+1}{t}$  (d)  $\frac{t^2-1}{t}$
- 48.** Let  $P$  be a point on the ellipse  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  and the line through  $P$  parallel to the  $Y$ -axis meets the circle  $x^2 + y^2 = 9$  at  $Q$ , where  $P, Q$  are on the same side of the  $X$ -axis. If  $R$  is a point on  $PQ$  such that  $\frac{PR}{RQ} = \frac{1}{2}$ , then the locus of  $R$  is  
 (a)  $\frac{x^2}{9} + \frac{9y^2}{49} = 1$  (b)  $\frac{x^2}{49} + \frac{y^2}{9} = 1$   
 (c)  $\frac{x^2}{9} + \frac{y^2}{49} = 1$  (d)  $\frac{9x^2}{49} + \frac{y^2}{9} = 1$
- 49.** A point  $P$  lies on a line through  $Q(1, -2, 3)$  and is parallel to the line  $\frac{x}{1} = \frac{y}{4} = \frac{z}{5}$ . If  $P$  lies on the plane  $2x + 3y - 4z + 22 = 0$ , then segment  $PQ$  equals  
 (a)  $\sqrt{42}$  units (b)  $\sqrt{32}$  units  
 (c)  $4$  units (d)  $5$  units
- 50.** The foot of the perpendicular drawn from the point  $(1, 8, 4)$  on the line joining the point  $(0, -11, 4)$  and  $(2, -3, 1)$  is  
 (a)  $(4, 5, 2)$  (b)  $(-4, 5, 2)$   
 (c)  $(4, -5, 2)$  (d)  $(4, 5, -2)$

### Category-II (Q. No. 51 to 65)

Carry 2 marks each if only one option is correct. In case of incorrect answer or any combination of more than one answer, 1/2 mark will be deducted.

- 51.** A ladder 20 ft long leans against a vertical wall. The top end slides downwards at the rate of 2 ft per second. The rate at which the lower end moves on a horizontal floor when it is 12 ft from the wall is

(a)  $-\frac{8}{3}$  (b)  $\frac{6}{5}$   
(c)  $\frac{3}{2}$  (d)  $\frac{17}{4}$

- 52.** For  $0 \leq p \leq 1$  and for any positive  $a, b$ ; let  $I(p) = (a + b)^p$ ,  $J(p) = a^p + b^p$ , then

(a)  $I(p) > J(p)$   
(b)  $I(p) \leq J(p)$   
(c)  $I(p) < J(p)$  in  $\left[0, \frac{p}{2}\right]$  and  $I(p) > J(p)$  in  $\left[\frac{p}{2}, \infty\right)$   
(d)  $I(p) < J(p)$  in  $\left[\frac{p}{2}, \infty\right)$  and  $J(p) < I(p)$  in  $\left[0, \frac{p}{2}\right]$

- 53.** Let  $\vec{\alpha} = \hat{i} + \hat{j} + \hat{k}$ ,  $\vec{\beta} = \hat{i} - \hat{j} - \hat{k}$  and

$\vec{\gamma} = -\hat{i} + \hat{j} - \hat{k}$  be three vectors. A vector  $\vec{\delta}$ , in the plane of  $\vec{\alpha}$  and  $\vec{\beta}$ , whose projection on  $\vec{\gamma}$  is  $\frac{1}{\sqrt{3}}$ , is given by

(a)  $-\hat{i} - 3\hat{j} - 3\hat{k}$  (b)  $\hat{i} - 3\hat{j} - 3\hat{k}$   
(c)  $-\hat{i} + 3\hat{j} + 3\hat{k}$  (d)  $\hat{i} + 3\hat{j} - 3\hat{k}$

- 54.** Let  $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$  be the three unit vectors such that  $\vec{\alpha} \cdot \vec{\beta} = \vec{\alpha} \cdot \vec{\gamma} = 0$  and the angle between  $\vec{\beta}$  and  $\vec{\gamma}$  is  $30^\circ$ . Then  $\vec{\alpha}$  is

(a)  $2(\vec{\beta} \times \vec{\gamma})$  (b)  $-2(\vec{\beta} \times \vec{\gamma})$   
(c)  $\pm 2(\vec{\beta} \times \vec{\gamma})$  (d)  $(\vec{\beta} \times \vec{\gamma})$

- 55.** Let  $z_1$  and  $z_2$  be complex numbers such that  $z_1 \neq z_2$  and  $|z_1| = |z_2|$ . If  $\text{Re}(z_1) > 0$  and  $\text{Im}(z_2) < 0$ , then  $\frac{z_1 + z_2}{z_1 - z_2}$  is

(a) one (b) real and positive  
(c) real and negative (d) purely imaginary

- 56.** From a collection of 20 consecutive natural numbers, four are selected such that they are not consecutive. The number of such selections is

(a)  $284 \times 17$  (b)  $285 \times 17$   
(c)  $284 \times 16$  (d)  $285 \times 16$

- 57.** The least positive integer  $n$  such that

$\begin{pmatrix} \cos \frac{\pi}{4} & \sin \frac{\pi}{4} \\ -\sin \frac{\pi}{4} & \cos \frac{\pi}{4} \end{pmatrix}^n$  is an identity matrix of

order 2 is

(a) 4 (b) 8  
(c) 12 (d) 16

- 58.** Let  $\rho$  be a relation defined on  $N$ , the set of natural numbers, as

$\rho = \{(x, y) \in N \times N : 2x + y = 41\}$ . Then

(a)  $\rho$  is an equivalence relation  
(b)  $\rho$  is only reflexive relation  
(c)  $\rho$  is only symmetric relation  
(d)  $\rho$  is not transitive

- 59.** If the polynomial

$f(x) = \begin{vmatrix} (1+x)^a & (2+x)^b & 1 \\ 1 & (1+x)^a & (2+x)^b \\ (2+x)^b & 1 & (1+x)^a \end{vmatrix}$ , then the

constant term of  $f(x)$  is

(a)  $2 - 3 \cdot 2^b + 2^{3b}$  (b)  $2 + 3 \cdot 2^b + 2^{3b}$   
(c)  $2 + 3 \cdot 2^b - 2^{3b}$  (d)  $2 - 3 \cdot 2^b - 2^{3b}$   
[ $a$  and  $b$  are positive integers]

- 60.** A line cuts the  $X$ -axis at  $A(5, 0)$  and the  $Y$ -axis at  $B(0, -3)$ . A variable line  $PQ$  is drawn perpendicular to  $AB$  cutting the  $X$ -axis at  $P$  and the  $Y$ -axis at  $Q$ . If  $AQ$  and  $BP$  meet at  $R$ , then the locus of  $R$  is

(a)  $x^2 + y^2 - 5x + 3y = 0$  (b)  $x^2 + y^2 + 5x + 3y = 0$   
(c)  $x^2 + y^2 + 5x - 3y = 0$  (d)  $x^2 + y^2 - 5x - 3y = 0$

- 61.** Let  $A$  be the centre of the circle  $x^2 + y^2 - 2x - 4y - 20 = 0$ . Let  $B(1, 7)$  and  $D(4, -2)$  be two points on the circle such that tangents at  $B$  and  $D$  meet at  $C$ . The area of the quadrilateral  $ABCD$  is

(a) 150 sq units (b) 50 sq units  
(c) 75 sq units (d) 70 sq units



62. Let  $f(x) = \begin{cases} -2\sin x, & \text{if } x \leq -\frac{\pi}{2} \\ A\sin x + B, & \text{if } -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \cos x, & \text{if } x \geq \frac{\pi}{2} \end{cases}$ . Then,

- (a)  $f$  is discontinuous for all  $A$  and  $B$   
 (b)  $f$  is continuous for all  $A = -1$  and  $B = 1$   
 (c)  $f$  is continuous for all  $A = 1$  and  $B = -1$   
 (d)  $f$  is continuous for all real values of  $A, B$

63. The normal to the curve  $y = x^2 - x + 1$ , drawn at the points with the abscissa  $x_1 = 0$ ,  $x_2 = -1$  and  $x_3 = 5/2$

- (a) are parallel to each other  
 (b) are pairwise perpendicular  
 (c) are concurrent  
 (d) are not concurrent

64. The equation  $x \log x = 3 - x$

- (a) has no root in  $(1, 3)$   
 (b) has exactly one root in  $(1, 3)$   
 (c)  $x \log x - (3 - x) > 0$  in  $[1, 3]$   
 (d)  $x \log x - (3 - x) < 0$  in  $[1, 3]$

65. Consider the parabola  $y^2 = 4x$ . Let  $P$  and  $Q$  be points on the parabola where  $P(4, -4)$  and  $Q(9, 6)$ . Let  $R$  be a point on the arc of the parabola between  $P$  and  $Q$ . Then, the area of  $\Delta PQR$  is largest when

- (a)  $\angle PQR = 90^\circ$  (b)  $R(4, 4)$   
 (c)  $R\left(\frac{1}{4}, 1\right)$  (d)  $R\left(1, \frac{1}{4}\right)$

### Category-III (Q. Nos. 66 to 75)

Carry 2 marks each and one or more option(s) is/are correct. If all correct answers are not marked and also no incorrect answer is marked then score =  $2 \times$  number of correct answers marked,  $\div$  actual number of correct answer. If any wrong option is marked or if, any combination including a wrong option is marked, the answer will be considered wrong, but there is no negative marking for the same and zero marks will be awarded.

66. Let  $I = \int_0^1 \frac{x^3 \cos 3x}{2 + x^2} dx$ . Then

- (a)  $-\frac{1}{2} < I < \frac{1}{2}$  (b)  $-\frac{1}{3} < I < \frac{1}{3}$   
 (c)  $-1 < I < 1$  (d)  $-\frac{3}{2} < I < \frac{3}{2}$

67. A particle is in motion along a curve  $12y = x^3$ . The rate of change of its ordinate exceeds that of abscissa in

- (a)  $-2 < x < 2$  (b)  $x = \pm 2$   
 (c)  $x < -2$  (d)  $x > 2$

68. The area of the region lying above  $X$ -axis, and included between the circle  $x^2 + y^2 = 2ax$  and the parabola  $y^2 = ax, a > 0$  is

- (a)  $8\pi a^2$  (b)  $a^2 \left( \frac{\pi}{4} - \frac{2}{3} \right)$   
 (c)  $\frac{16\pi a^2}{9}$  (d)  $\pi \left( \frac{27}{8} + 3a^2 \right)$

69. If the equation  $x^2 - cx + d = 0$  has roots equal to the fourth powers of the roots of  $x^2 + ax + b = 0$ , where  $a^2 > 4b$ , then the roots of  $x^2 - 4bx + 2b^2 - c = 0$  will be

- (a) both real  
 (b) both negative  
 (c) both positive  
 (d) one positive and one negative

70. On the occasion of Dipawali festival each student of a class sends greeting cards to others. If there are 20 students in the class, the number of cards sends by students is

- (a)  ${}^{20}C_2$  (b)  ${}^{20}P_2$  (c)  $2 \times {}^{20}C_2$  (d)  $2 \times {}^{20}P_2$

71. In a third order matrix  $A$ ,  $a_{ij}$  denotes the element in the  $i$ th row and  $j$ th column.

$$\begin{aligned} \text{If } a_{ij} &= 0 \text{ for } i = j \\ &= 1 \text{ for } i > j \\ &= -1 \text{ for } i < j \end{aligned}$$

Then the matrix is

- (a) skew symmetric (b) symmetric  
 (c) not invertible (d) non-singular

72. The area of the triangle formed by the intersection of a line parallel to  $X$ -axis and passing through  $P(h, k)$ , with the lines  $y = x$  and  $x + y = 2$  is  $h^2$ . The locus of the point  $P$  is

- (a)  $x = y - 1$  (b)  $x = -(y - 1)$   
 (c)  $x = 1 + y$  (d)  $x = -(1 + y)$

73. A hyperbola, having the transverse axis of length  $2\sin\theta$  is confocal with the ellipse  $3x^2 + 4y^2 = 12$ . Its equation is

- (a)  $x^2 \sin^2 \theta - y^2 \cos^2 \theta = 1$   
 (b)  $x^2 \operatorname{cosec}^2 \theta - y^2 \sec^2 \theta = 1$   
 (c)  $(x^2 + y^2) \sin^2 \theta = 1 + y^2$   
 (d)  $x^2 \operatorname{cosec}^2 \theta = x^2 + y^2 + \sin^2 \theta$

**74.** Let  $f(x) = \cos\left(\frac{\pi}{x}\right)$ ,  $x \neq 0$ , then assuming  $k$  as an integer,

- (a)  $f(x)$  increases in the interval  $\left(\frac{1}{2k+1}, \frac{1}{2k}\right)$   
 (b)  $f(x)$  decreases in the interval  $\left(\frac{1}{2k+1}, \frac{1}{2k}\right)$   
 (c)  $f(x)$  decreases in the interval  $\left(\frac{1}{2k+2}, \frac{1}{2k+1}\right)$   
 (d)  $f(x)$  increases in the interval  $\left(\frac{1}{2k+2}, \frac{1}{2k+1}\right)$

**75.** Consider the function  $y = \log_a(x + \sqrt{x^2 + 1})$ ,  $a > 0$ ,  $a \neq 1$ . The inverse of the function

- (a) does not exist  
 (b) is  $x = \log_{1/a}(y + \sqrt{y^2 + 1})$   
 (c) is  $x = \sinh(y \log a)$   
 (d) is  $x = \cosh\left(-y \log \frac{1}{a}\right)$

## Answers

### Physics

- |         |         |         |         |         |             |           |           |         |         |
|---------|---------|---------|---------|---------|-------------|-----------|-----------|---------|---------|
| 1. (d)  | 2. (c)  | 3. (a)  | 4. (b)  | 5. (d)  | 6. (c)      | 7. (a)    | 8. (d)    | 9. (d)  | 10. (c) |
| 11. (a) | 12. (c) | 13. (b) | 14. (a) | 15. (c) | 16. (c)     | 17. (c)   | 18. (d)   | 19. (b) | 20. (c) |
| 21. (c) | 22. (b) | 23. (b) | 24. (a) | 25. (c) | 26. (b)     | 27. (c)   | 28. (b)   | 29. (d) | 30. (c) |
| 31. (d) | 32. (a) | 33. (d) | 34. (a) | 35. (d) | 36. (b,c,d) | 37. (a,c) | 38. (a,d) | 39. (b) | 40. (a) |

### Chemistry

- |         |         |         |         |         |           |            |           |             |             |
|---------|---------|---------|---------|---------|-----------|------------|-----------|-------------|-------------|
| 41. (d) | 42. (d) | 43. (d) | 44. (a) | 45. (d) | 46. (b)   | 47. (b)    | 48. (b)   | 49. (b)     | 50. (a)     |
| 51. (d) | 52. (c) | 53. (b) | 54. (c) | 55. (d) | 56. (d)   | 57. (c)    | 58. (d)   | 59. (d)     | 60. (a)     |
| 61. (a) | 62. (c) | 63. (b) | 64. (b) | 65. (c) | 66. (b)   | 67. (b)    | 68. (b)   | 69. (c)     | 70. (d)     |
| 71. (a) | 72. (a) | 73. (b) | 74. (a) | 75. (a) | 76. (a,c) | 77. (a, d) | 78. (b,c) | 79. (a,c,d) | 80. (a,c,d) |

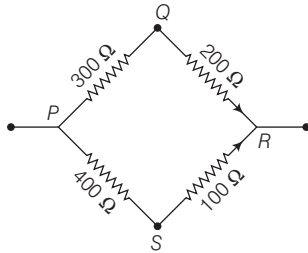
### Mathematics

- |           |           |         |           |         |           |           |         |           |           |
|-----------|-----------|---------|-----------|---------|-----------|-----------|---------|-----------|-----------|
| 1. (a)    | 2. (c)    | 3. (b)  | 4. (c)    | 5. (c)  | 6. (a)    | 7. (b)    | 8. (b)  | 9. (c)    | 10. (d)   |
| 11. (b)   | 12. (c)   | 13. (b) | 14. (c)   | 15. (c) | 16. (c)   | 17. (c)   | 18. (b) | 19. (d)   | 20. (c)   |
| 21. (b)   | 22. (c)   | 23. (a) | 24. (a)   | 25. (c) | 26. (a)   | 27. (a)   | 28. (a) | 29. (c)   | 30. (d)   |
| 31. (c)   | 32. (c)   | 33. (d) | 34. (c)   | 35. (b) | 36. (a)   | 37. (a)   | 38. (c) | 39. (b)   | 40. (d)   |
| 41. (d)   | 42. (b)   | 43. (a) | 44. (c)   | 45. (a) | 46. (c,d) | 47. (d)   | 48. (a) | 49. (a)   | 50. (d)   |
| 51. (a)   | 52. (b)   | 53. (c) | 54. (c)   | 55. (d) | 56. (a)   | 57. (b)   | 58. (d) | 59. (a)   | 60. (a)   |
| 61. (c)   | 62. (b)   | 63. (c) | 64. (b)   | 65. (c) | 66. (b)   | 67. (c,d) | 68. (b) | 69. (a,d) | 70. (b,c) |
| 71. (a,c) | 72. (a,b) | 73. (b) | 74. (a,c) | 75. (c) |           |           |         |           |           |

## Answer with Explanations

### Physics

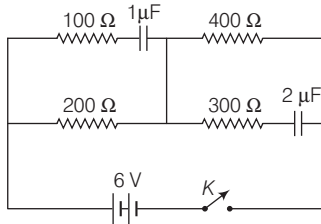
1. (d) For maximum equivalent resistance across the diagonal of the square, the given resistors connected as



Resistance of  $PQR$  arm,  $R_1 = 300 + 200 = 500 \Omega$   
Resistance of  $PSR$  arm,  $R_2 = 400 + 100 \Omega = 500 \Omega$   
The equivalent resistance between  $P$  and  $R$ .

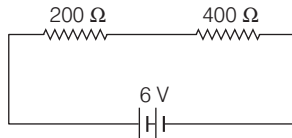
$$\begin{aligned} \frac{1}{R_{eq}} &= \frac{1}{R_1} + \frac{1}{R_2} \\ &= \frac{1}{500} + \frac{1}{500} = \frac{1+1}{500} \\ \therefore R_{eq} &= \frac{500}{2} = 250 \Omega \end{aligned}$$

2. (c)



In steady state, arm having capacitors does not flow current. So, we can neglect them.

$\therefore$  The given circuit reduces to

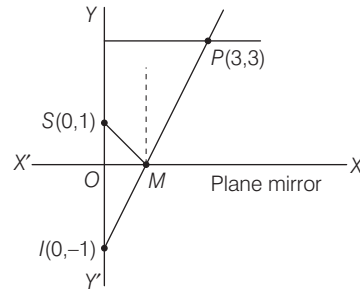


$$\therefore R_{net} = 200 + 400 = 600 \Omega$$

$\therefore$  Current in circuit,

$$\begin{aligned} I &= \frac{V}{R} = \frac{6}{600} = 0.01 \text{ A} \\ &= 10 \text{ mA} \end{aligned}$$

3. (a) Ray diagram for the question,



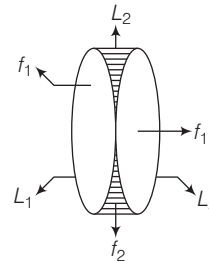
$I$  is image of source  $S$  by the plane mirror placed perpendicularly along  $X$ -axis.

$$\therefore SM = IM$$

$$\therefore PM + MS = PM + MI = PI$$

$$\begin{aligned} \therefore PI &= \sqrt{(3-0)^2 + (3+1)^2} \\ &= \sqrt{9+16} = \sqrt{25} = 5 \end{aligned}$$

4. (b) The given combination of lenses



Here,  $f_1$  = focal length of equiconvex lenses of glass.

$f_2$  = focal length of lens formed by water (concave).

The focal length of the combination.

$$\begin{aligned} \therefore \frac{1}{F} &= \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} = \frac{1}{f_1} - \frac{1}{f_2} + \frac{1}{f_1} = \frac{2}{f_1} - \frac{1}{f_2} \\ \frac{1}{F} &= \frac{2f_2 - f_1}{f_1 f_2} \Rightarrow F = \frac{f_1 f_2}{2f_2 - f_1} \quad [\because f_3 = f_1] \\ F &= \frac{f_1}{2 - \frac{f_1}{f_2}} \quad \dots(i) \end{aligned}$$

$$\text{Here, } \frac{1}{f_1} = (\mu_g - 1) \frac{2}{R}, \text{ for } L_1 \text{ and } L_3$$

$$\text{and } \frac{1}{f_2} = (\mu_w - 1) \left( -\frac{2}{R} \right), \text{ for } L_2$$

Given,  $\mu_w < \mu_g$

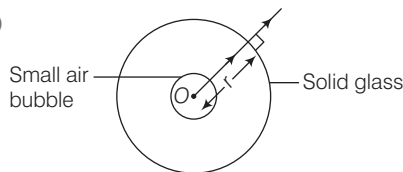
Thus,  $\frac{f_1}{f_2} < 1$

So,  $F > \frac{f_1}{2}$  ... (ii)

From Eqs. (i) and (ii), we get

$$\frac{f_1}{2} < F < f_1 \quad \text{or} \quad \frac{f}{2} < F < f \quad (\because f_1 = f)$$

5. (d)



As the object is at centre of the sphere.

$\therefore$  All rays will fall normally on surface, hence they do not deflect. Thus, a virtual image is formed at the centre O.

$\therefore$  Apparent depth = Real depth

6. (c) Young's double slit experiment with white light which consist wavelength from range 4000 Å to 7000 Å.

$\therefore$  At the centre of the screen, the path difference is zero for all wavelengths. The bright fringes of these wavelengths overlap at the centre. Thus, the white fringe at the centre is formed.

7. (a)  $\therefore$  Linear velocity of an electron in a orbit of H like atom,

$$v = 2.18 \times 10^6 \times \frac{Z}{n} \text{ m/s} = \frac{c_0}{137} \cdot \frac{Z}{n}$$

where, Z = number of protons in nucleus,

n = principal quantum number

and  $c_0$  = speed of light in free space,

Thus, we have

$$v \propto \frac{1}{n}$$

8. (d) Given,  $t_{1/2} = 3$  days

Number of active nuclei remaining after time t,

$$N = N_0 \left( \frac{1}{2} \right)^{\frac{t}{t_{1/2}}}$$

After  $t = 2$  days,

$$N_1 = N_0 \left( \frac{1}{2} \right)^{2/3} = \frac{N_0}{2^{2/3}} = \frac{N_0}{4^{1/3}} = 0.63 N_0$$

$\therefore N_1 = 0.63 N_0$

After  $t = 3$  days,

$$N_2 = \frac{N_0}{2} = 0.5 N_0$$

Number of nuclei that will decay on the 3rd day,

$$N_3 = N_2 - N_1 = 0.63 N_0 - 0.5 N_0 = 0.13 N_0$$

In the term, fraction is 0.13.

9. (d)  $\therefore$  Kinetic energy of a electron due to accelerated by a potential V,  $\text{KE} = eV$

$$\frac{1}{2} m_e v^2 = eV$$

$$\Rightarrow \frac{1 \times p^2}{2m_e} = eV \quad [\because p = mv]$$

$$\therefore p = \sqrt{2eVm_e}$$

$\therefore$  de-Broglie wavelength of a particle having momentum p,

$$\lambda = \frac{h}{p}$$

According to question,

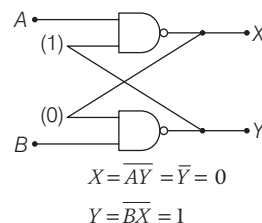
$$\frac{\lambda_1}{\lambda_2} = \frac{p_2}{p_1} = \frac{\sqrt{2eV_2m_e}}{\sqrt{2eV_1m_e}} = \sqrt{\frac{V_2}{V_1}}$$

$$\therefore \lambda_2 = 2\lambda_1 \quad (\text{given})$$

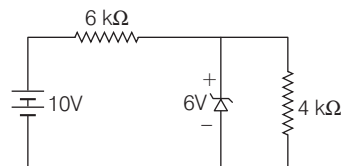
$$\Rightarrow \frac{\lambda_1}{2\lambda_1} = \sqrt{\frac{V_2}{10000}}$$

$$\therefore \text{Potential } V_2 = \frac{10^4}{4} = 2500 \text{ V}$$

10. (c)



11. (a)



In the given circuit, the zener diode is used as a voltage regulating device. The voltage across 6 kΩ resistance is  $(10 - 6) \text{ V} = 4 \text{ V}$

Current through 6 kΩ resistor,

$$I = \frac{4 \text{ V}}{6 \text{ k}\Omega} = \frac{4}{6 \times 10^3} = \frac{2}{3} \times 10^{-3} = \frac{2}{3} \text{ mA}$$

12. (c) In representation of simple harmonic motion in ellipse as velocity along X-axis and displacement along Y-axis.

$$\therefore \frac{\text{Major axis (along X-axis)}}{\text{Minor axis (along Y-axis)}} = 20\pi$$

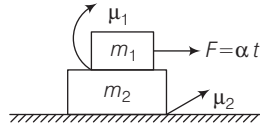
$$\Rightarrow \frac{\omega A}{A} = 20\pi \Rightarrow \omega = 20\pi$$

$$\Rightarrow 2\pi n = 20\pi$$

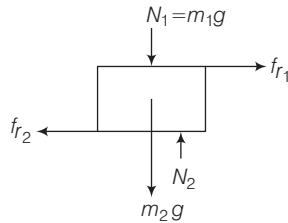
The frequency of the simple harmonic motion

$$\therefore n = 10\text{Hz}$$

13. (b) According to the question,



FBD of lower block of mass  $m_2$ ,



$$\therefore N_2 = m_2 g + N_1 = (m_1 + m_2) g$$

$\therefore$  Lower block never moves.

$$\therefore f_{r2} \geq f_{r1}$$

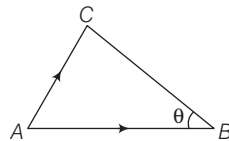
$$\mu_2 N_2 \geq \mu_1 N_1 \Rightarrow \mu_2 (m_1 + m_2) g \geq \mu_1 m_1 g$$

$$\Rightarrow \frac{m_1 + m_2}{m_1} \geq \frac{\mu_1}{\mu_2}$$

$$\therefore \frac{\mu_1}{\mu_2} \leq 1 + \frac{m_2}{m_1}$$

$$\therefore \left. \frac{\mu_1}{\mu_2} \right|_{\max} = 1 + \frac{m_2}{m_1}$$

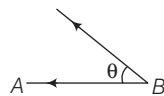
14. (a)



$$\mathbf{AB} = 3\hat{i} + \hat{j} + \hat{k} \text{ and } \mathbf{AC} = \hat{i} + 2\hat{j} + \hat{k}$$

$$\therefore \mathbf{BA} = -(3\hat{i} + \hat{j} + \hat{k})$$

$\angle ABC$  is angle between BA and BC



$$\begin{aligned} \mathbf{BC} &= \mathbf{AC} + \mathbf{BA} \\ &= \mathbf{AC} - \mathbf{AB} \quad (\because \mathbf{BA} = -\mathbf{AB}) \\ &= \hat{i} + 2\hat{j} + \hat{k} - (3\hat{i} + \hat{j} + \hat{k}) = -2\hat{i} + \hat{j} \end{aligned}$$

$$\therefore \mathbf{BA} \cdot \mathbf{BC} = |\mathbf{BA}| |\mathbf{BC}| \cos \theta$$

$$= (3\hat{i} + \hat{j} + \hat{k}) \cdot (-2\hat{i} + \hat{j})$$

$$= (\sqrt{3^2 + 1^2 + 1^2}) \cdot (\sqrt{2^2 + 1^2}) \cos \theta$$

$$\Rightarrow 6 - 1 = \sqrt{11} \times \sqrt{5} \cdot \cos \theta$$

$$\therefore \cos \theta = \frac{5}{\sqrt{55}} = \frac{1}{\sqrt{11}} = \theta = \cos^{-1} \left( \frac{1}{\sqrt{11}} \right)$$

15. (c) According to the question,

$$v \propto \frac{1}{\sqrt{x}}$$

or  $v = \frac{k}{\sqrt{x}} \quad \dots(i)$

$$\frac{dv}{dt} = \frac{d}{dt} \cdot \frac{k}{\sqrt{x}} = k \cdot \frac{-1}{2} \cdot x^{-\frac{1}{2}-1} \cdot \frac{dx}{dt}$$

$$a = -\frac{k}{2} \cdot x^{-\frac{3}{2}} \cdot \frac{k}{\sqrt{x}} \left[ \because \frac{dx}{dt} = v \text{ and } v = \frac{k}{\sqrt{x}} \text{ from Eq.(i)} \right]$$

$$= -\frac{k^2}{2} \cdot \frac{1}{x^2}$$

$$\therefore \text{Force, } F = \text{Mass} \times |\text{Acceleration}|$$

$$= m \frac{k^2}{2} \cdot \frac{1}{x^2} \quad (\text{magnitude})$$

$$F = \frac{k^2}{2} \cdot \frac{m}{x^2}$$

$$\therefore F \propto \frac{1}{x^2}$$

16. (c)  $\therefore$  Escape velocity from a planet,

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2GM}{R^2}} R$$

$$= \sqrt{2gR} \quad \dots(i)$$

According due to gravity

$$g = \frac{GM}{R^2} = \frac{G \frac{4}{3} \pi R^3 \cdot \rho}{R^2} = \frac{4}{3} G \pi R \rho$$

$$\therefore \text{Radius } R = \frac{3g}{4\pi G \rho} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$v_e = \sqrt{2g \cdot \frac{3g}{4\pi G \rho}} = \sqrt{\frac{3}{2} \frac{g^2}{\pi G \rho}}$$

Thus,

$$v_e \propto \frac{g}{\sqrt{\rho}}$$

$$\therefore \frac{v_{e1}}{v_{e2}} = \frac{g_1}{\sqrt{\rho_1}} \times \frac{\sqrt{\rho_2}}{g_2} = \frac{5}{2} \times \frac{1}{\sqrt{2}}$$

$$(\text{given, } \frac{g_1}{g_2} = \frac{5}{2} \text{ and } \frac{\rho_1}{\rho_2} = 2 : 1)$$

$$= \frac{5}{2\sqrt{2}}$$

17. (c) According to the question,  
time period,  $T \propto r^a \rho^b S^c$

$$T = k r^a \rho^b S^c \quad \dots(i)$$

Thus, putting dimension, we get

$$[T] = [L]^a [ML^{-3}]^b [MT^{-2}]^c$$

$$[T] = [M]^{b+c} \cdot [L]^{a-3b} \cdot [T]^{-2c}$$

Equating the dimensions of both sides, we get

$$b + c = 0, \quad a - 3b = 0$$

and  $-2c = 1$

$$\therefore c = -\frac{1}{2}$$

$$\therefore b = \frac{1}{2}$$

$$\text{and } a = 3b = \frac{3}{2}$$

Putting these value of  $a, b$  and  $c$  into Eq. (i), we get

$$T = k \cdot r^{\frac{3}{2}} \rho^{\frac{1}{2}} \cdot S^{-\frac{1}{2}}$$

$$= k \sqrt{\frac{\rho}{S} r^3}$$

18. (d)  $\therefore$  Stress,  $\frac{F}{\Delta A} = 1\% \text{ of } Y = \frac{Y}{100}$

$$\text{But Young's modulus, } Y = \frac{\text{stress}}{\text{strain}} = \frac{\frac{F}{\Delta A}}{\frac{\Delta l}{l}}$$

$$\therefore Y = \frac{\frac{100}{\Delta l}}{\frac{l}{l}} \quad \left( \text{putting } \frac{F}{\Delta A} = \frac{Y}{100} \right)$$

$$\therefore \frac{\Delta l}{l} = \frac{1}{100}$$

$$\text{Poisson's ratio, } \sigma = \frac{r}{\Delta l}$$

$$\therefore \frac{\Delta r}{r} = -\sigma \cdot \frac{\Delta l}{l} = \frac{-0.3}{100}$$

$$\frac{\Delta r}{r} = \frac{-0.3}{100}$$

$$\therefore \text{Change in volume, } \frac{\Delta V}{V} = \frac{2\Delta r}{r} + \frac{\Delta l}{l}$$

$$= \frac{2 \times (-0.3)}{100} + \frac{1}{100}$$

$$= \frac{1-0.6}{100} = \frac{0.4}{100}$$

$$\therefore \Delta V \% = 0.4\%$$

19. (b) Terminal velocity,  $v = \frac{2}{9} r^2 \frac{(\rho - \sigma)}{\eta} g$

Neglecting buoyancy effect of the fluid,

$$v = \frac{2}{9} \cdot \frac{\rho}{\eta} r^2 g$$

Putting the given values, we get

$$v = \frac{2}{9} \times \frac{10^3 \times (0.9 \times 10^{-3})^2}{1.8 \times 10^{-5}} \times 9.8 = 98 \text{ ms}^{-1}$$

20. (c) Let the mass of ice =  $m$

Applying calorimetry principle,

heat given = heat taken

$$(m_1 + m_2) s_1 (t_1 - t) = \frac{mL}{2} + ms_2(t - t_2)$$

Putting values, we get

$$(10 + 50) \times 1 \times (15 - 0) = \frac{m}{2} \times 80 + m \times 0.5 [0 - (-10)]$$

$$\Rightarrow 60 \times 15 = 40m + \frac{m}{2} \times 10 = 45m$$

$$\therefore m = \frac{60 \times 15}{45} = 20 \text{ g}$$

21. (c) Heat capacity of an ideal gas in a thermodynamic process,

$\therefore$  Ideal gas is monoatomic,

$$C_V = \frac{fR}{2} = \frac{3R}{2}$$

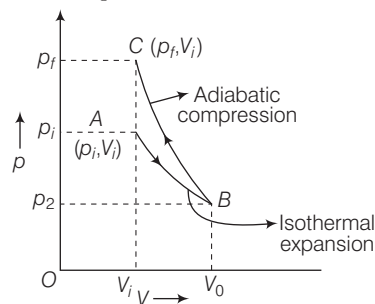
$$C_{\text{process}} = C_V + \frac{p}{n} \cdot \frac{dV}{dT}$$

$$= \frac{3}{2}R + \frac{p}{n} \cdot \frac{V_0}{2T}$$

$$\therefore C_{\text{process}} = \frac{3}{2}R + \frac{nRT}{n2T} \quad [\because pV = nRT]$$

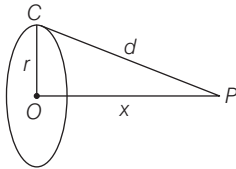
$$= \frac{3}{2}R + \frac{R}{2} = 2R$$

22. (b) In  $p$ - $V$  diagram, the slope of an adiabatic curve at any point is steeper than that of isothermal curve at that point.



Thus,  $p_f > p_i$

- 23. (b)** A charged circular ring of radius  $R$  is shown in figure.



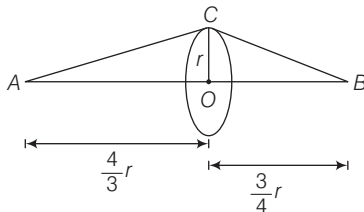
From the figure,  $d = \sqrt{x^2 + r^2}$  ... (i)

$\therefore$  Potential due to a uniform ring of positive charge  $q$  at point  $P$ ,

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{d}$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{\sqrt{x^2 + r^2}} \quad [\text{from Eq. (i)}]$$

Now,



$$\therefore CB = \sqrt{OC^2 + OB^2} = \sqrt{r^2 + \left(\frac{3}{4}\right)^2 r^2} = \frac{5}{4}r$$

$$\text{Similarly, } AC = \frac{5}{3}r$$

$\therefore$  Work done in bringing a point charge  $-q$  from  $A$  to  $B$ ,

$$W = -q(V_B - V_A)$$

$$= -q \left[ \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{CB} - \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{CA} \right]$$

$$= \frac{-1}{4\pi\epsilon_0} \cdot q^2 \left[ \frac{1}{\frac{5}{4}r} - \frac{1}{\frac{5}{3}r} \right]$$

$$= \frac{-1}{4\pi\epsilon_0} \cdot \frac{q^2}{5r} [4 - 3] = -\frac{1}{5} \cdot \frac{1}{4\pi\epsilon_0} \cdot \frac{q^2}{r}$$

- 24. (a)** For surface  $ABCD$  electric flux is zero. Because at surface  $ABCD$  net electric field is zero.

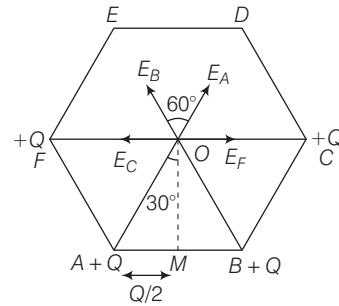
Using Gauss's law,

$$\oint \mathbf{E} \cdot d\mathbf{S} = \frac{Q_{\text{in}}}{\epsilon_0}$$

Electric flux through the five faces of the cube,

$$\Phi_E = \frac{Q}{\epsilon_0}$$

- 25. (c)**



(Regular hexagon)

$$\text{In } \triangle AOM, \sin 30^\circ = \frac{AM}{AO}$$

$$\therefore AO = \frac{\frac{a}{2}}{1/2} = a$$

For maximum electric field at centre  $O$  charges should be placed at  $F, A, B$  and  $C$ .

$\therefore$  Electric field due to charges at  $F$  and  $C$  is equal and opposite at  $O$ .

$\therefore$  Net electric field at centre  $O$  due to charges at  $A$  and  $B$ .

Angle between  $E_A$  and  $E_B$  is  $60^\circ$ .

$\therefore E_{\text{net}}$  at  $O$ ,

$$E_{\text{net}} = \sqrt{E_A^2 + E_B^2 + 2E_A E_B \cos 60^\circ} \quad (\because E_A = E_B = E)$$

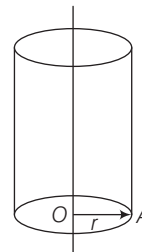
$$= \sqrt{E^2 + E^2 + 2E^2 \cdot \frac{1}{2}} \quad \left( \because \cos 60^\circ = \frac{1}{2} \right)$$

$$= E\sqrt{3} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{a^2} \sqrt{3}$$

- 26. (b)**  $\therefore$  When a charged particle moves at a circular path in uniform magnetic field, its time period is independent from its speed.

$\therefore$  Time period of proton will not change with change in its speed.

- 27. (c)**  $\therefore$  Magnetic field due to a hollow cylinder of radius  $r$ .

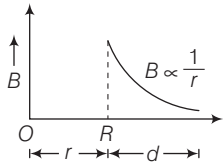


$$B_{\text{inside}} = 0 \quad (\text{for } d < r)$$

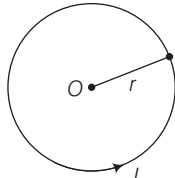
$$B_{\text{outside}} = \frac{\mu_0}{2\pi} \cdot \frac{i}{d} \quad (\text{for } d \geq r)$$



So, graph between  $B$  and  $d$ .



28. (b)



$$B = \frac{\mu_0}{2} \cdot \frac{I}{r}$$

$\therefore$

$$I_1 = \frac{V}{R_1}$$

Case (I)  $l_1 = 2\pi r$  (length of the loop),

$$R_1 = \rho \frac{l_1}{A} \text{ and } B_1 = \frac{\mu_0}{2} \cdot \frac{I_1}{r}$$

Case (II)  $l_2 = 2\pi \cdot 2r = 2l_1$

$$R_2 = \rho \cdot \frac{l_2}{A} = \rho \cdot \frac{2l_1}{A} = 2R_1$$

$$I_2 = \frac{V}{R_2} = \frac{V}{2R_1} = \frac{I_1}{2}$$

$$B_2 = \frac{\mu_0}{2} \cdot \frac{I_2}{2r} = \frac{\mu_0}{2} \cdot \frac{I_1}{2 \cdot 2r}$$

From Eqs. (i) and (ii),

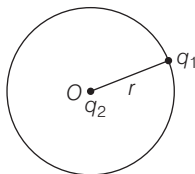
$$\frac{B_2}{B_1} = \frac{\frac{\mu_0}{2} \cdot \frac{I_1}{2 \cdot 2r}}{\frac{\mu_0}{2} \cdot \frac{I_1}{r}} = \frac{1}{4}$$

$$\text{Magnetic field } B_2 = \frac{B_1}{4}$$

29. (d)  $\therefore$  Resonance frequency,  $f_0 = \sqrt{f_1 f_2}$   
 $= \sqrt{200 \times 800} = 400 \text{ Hz}$

30. (c) Initially, there will be no voltage drop across capacitor, so intensity of bulb will rise sharply and gradually voltage drop across capacitor will increase as a result voltage drop across bulb decreases, so intensity of bulb will decrease.

31. (d)



$$F_{\text{centripetal}} = mr\omega^2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

$$\therefore \omega^2 = \frac{1}{4\pi\epsilon_0 m} \cdot \frac{q_1 q_2}{r^3} \Rightarrow \omega = \sqrt{\frac{q_1 q_2}{4\pi\epsilon_0 m r^3}}$$

$$\omega \propto \frac{1}{r^{3/2}} \quad \dots(i)$$

Current produced due to moving  $q_1$ ,

$$i = \frac{\omega}{2\pi} q_1 \quad \dots(ii)$$

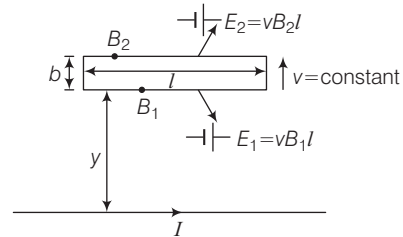
Thus, magnetic field due to motion of  $q_1$  in circular path at point O

$$B = \frac{\mu_0}{2} \cdot \frac{i}{r} = \frac{\mu_0}{2} \cdot \frac{\omega q_1}{2\pi r} \quad [\text{using Eq. (ii)}]$$

$$\text{Hence, } B \propto \frac{\omega}{r}$$

$$\text{Magnetic field } B \propto \frac{1}{r^{5/2}} \quad [\text{using Eq. (i)}]$$

32. (a)



$\therefore$  Motional emf in the loop,

$$\begin{aligned} E &= E_1 - E_2 = vl(B_1 - B_2) \\ &= vl \left[ \frac{\mu_0}{2\pi} \cdot \frac{I}{y} - \frac{\mu_0 I}{2\pi(y+b)} \right] \\ &= \frac{\mu_0}{2\pi} \cdot I \cdot vl \left[ \frac{y+b-y}{y(y+b)} \right] = \frac{\mu_0}{2\pi} \cdot \frac{vIl}{y(y+b)} b \end{aligned}$$

$$\therefore b \ll y$$

$$\therefore E = \frac{\mu_0}{2\pi} \cdot \frac{vIl}{y^2} b$$

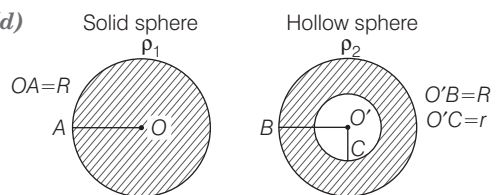
$$\therefore v = \text{constant}$$

$$(\therefore y = vt)$$

$$\text{Thus, } E = \frac{\mu_0}{2\pi} \cdot \frac{vIl}{v^2 t^2} b$$

$$\text{Hence, } E \propto \frac{1}{t^2}$$

33. (d)



$$\therefore M_1 = \rho_1 \frac{4}{3} \pi R^3 \text{ and } M_2 = \rho_2 \frac{4}{3} \pi (R^3 - r^3)$$

$$\therefore \rho_1 \frac{4}{3} \pi R^3 = \rho_2 \frac{4}{3} \pi (R^3 - r^3)$$

$$\Rightarrow \rho_1 R^3 = \rho_2 (R^3 - r^3)$$

$$\Rightarrow \frac{\rho_1}{\rho_2} = 1 - \frac{r^3}{R^3}$$

$$\therefore \frac{r}{R} = \left( 1 - \frac{\rho_1}{\rho_2} \right)^{\frac{1}{3}} \quad \dots(i)$$

Moment of inertia,

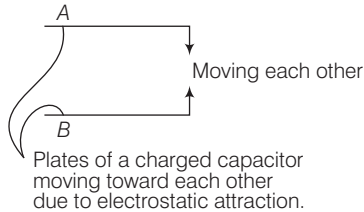
$$\frac{I_H}{I_S} = \frac{\frac{2}{5} M_2 \left[ \frac{R^5 - r^5}{R^3 - r^3} \right]}{\frac{2}{5} M_1 R^2}$$

$$= \frac{\rho_2 \frac{4}{3} \pi (R^3 - r^3) \left[ \frac{R^5 - r^5}{(R^3 - r^3)} \right]}{\rho_1 \frac{4}{3} \pi R^3 R^2}$$

$$= \frac{\rho_2}{\rho_1} \left[ \frac{R^5 - r^5}{R^5} \right] = \frac{\rho_2}{\rho_1} \left[ 1 - \left( \frac{r}{R} \right)^5 \right]$$

$$\therefore \frac{I_H}{I_S} = \frac{\rho_2}{\rho_1} \left[ 1 - \left( 1 - \frac{\rho_1}{\rho_2} \right)^{\frac{5}{3}} \right]$$

34. (a)



$$\text{Force} = \frac{1}{2} \cdot \frac{\sigma}{\epsilon_0} q$$

$\therefore$  Relative acceleration of plates,

$$a_{\text{rel}} = \frac{2F}{M}$$

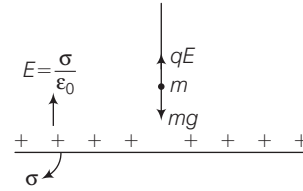
Potential difference,  $V = Ed$  ...(i)

where,  $d = d_0 - \frac{1}{2} a_{\text{rel}} t^2$  ( $\because$  plates are moving)

$$\therefore V = E \left[ d_0 - \frac{1}{2} a_{\text{rel}} t^2 \right]$$

This is an equation of a parabola with downward concavity.

35. (d)



$\therefore$  Apparent weight of the bob,  $w' = mg - qE$

$$mg' = mg - qE$$

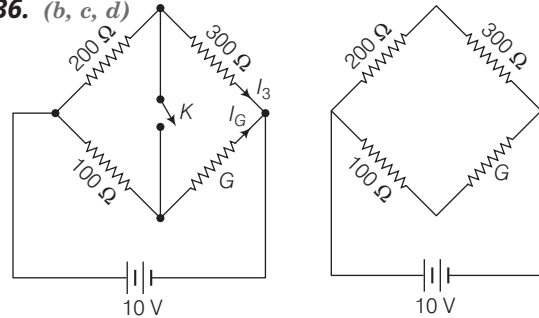
$$\therefore g' = g - \frac{qE}{m}$$

$\therefore$  Time period of a pendulum,

$$T = 2\pi \sqrt{\frac{L}{g_{\text{eff}}}}$$

$$= 2\pi \sqrt{\frac{L}{g - \frac{qE}{m}}} = 2\pi \sqrt{\frac{L}{\left( g - \frac{q\sigma}{\epsilon_0 m} \right)}}$$

36. (b, c, d)



$$\therefore I_G = \frac{10}{100 + G}$$

$$300 I_3 = \frac{10}{100 + G} G$$

$$I_3 = \frac{G}{30} \left[ \frac{1}{100 + G} \right]$$

$$\therefore I_3 + I_G = \frac{1}{(100 + G)} \left[ 10 + \frac{G}{30} \right] = \frac{(300 + G)}{(100 + G) \times 30}$$

According to the problem,

$$10 = \left( \frac{200}{3} + \frac{300G}{300 + G} \right) \left[ \frac{(300 + G)}{(100 + G) \times 30} \right] \quad [\because V = RI]$$

$$10 = \frac{60000 + 1100G}{3} \times \frac{1}{100 + G} \times \frac{1}{30}$$

$$\Rightarrow 900(100 + G) = 60000 + 1100G$$

$$\Rightarrow 30000 = 200G$$

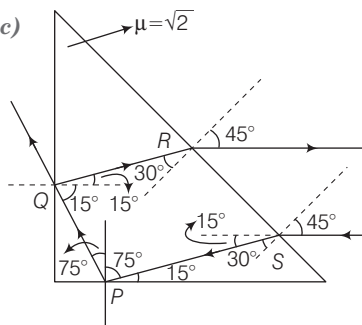
$$\therefore G = 150 \Omega$$

$$I_G = \frac{10}{100 + 150} = \frac{10}{250} = 40 \text{ mA}$$

$$\text{As } \frac{200}{100} = \frac{300}{150}$$

$$\text{So, } I_{200} = I_{300}$$

37. (a, c)



$$\text{Critical angle } \theta_c = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$$

Snell's law at points, ( $\therefore n_1 \sin \theta_1 = n_2 \sin \theta_2$ )

$$1 \sin 45^\circ = \sqrt{2} \sin \theta$$

$$\therefore \theta = 30^\circ$$

At P: incidence angle at P =  $75^\circ > \theta_c$ .

$\therefore$  TIR at point P.

At Q: incidence angle at Q =  $15^\circ < \theta_c$ .

$\therefore$  Partial reflection and refraction at Q.

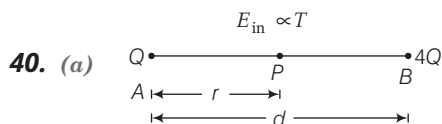
As  $SP \parallel QR$ , emergent ray at R is parallel to incident ray at S.

$\therefore$  Net deviation =  $180^\circ$ .

38. (a, d) The intensity of a sound source appears to be periodic due to

- (i) source intensity is periodic.
- (ii) source is producing a sound composed of two nearby frequencies.

39. (b)  $\therefore$  Internal energy of an ideal gas depends only on the temperature of gas.



$\therefore V_P$  is minimum.

$$\therefore E_P = 0 \Rightarrow E_A = E_B$$

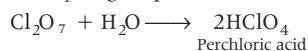
$$\Rightarrow \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{4Q}{(d-r)^2}$$

$$\Rightarrow \frac{1}{r} = \frac{2}{d-r} \Rightarrow d-r = 2r$$

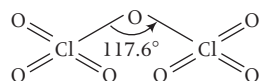
$\therefore$  Distance of P from the A  $r = \frac{d}{3}$  units to the right of A.

## Chemistry

41. (d)  $\text{Cl}_2\text{O}_7$  is the anhydride of  $\text{HClO}_4$ . It reacts with water slowly to give perchloric acid.



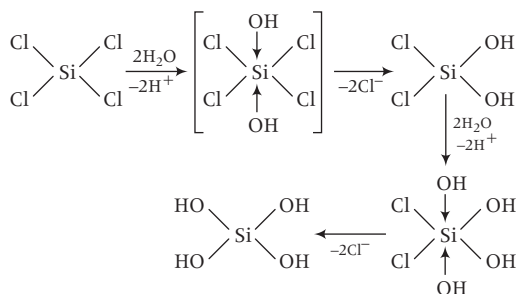
Its structure is as follows:



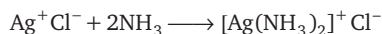
It is less reactive than other oxides of chlorine and does not react with P, S, coal or paper at room temperature.

42. (d) The main reason that  $\text{SiCl}_4$  is easily hydrolysed as compared to  $\text{CCl}_4$  is that Si can extend its coordination number beyond four because it has vacant  $d$ -orbital. On the other hand, carbon has no  $d$ -orbital thus, it cannot extend its coordination number beyond four, so its halides are not attacked (hydrolysed) by water. The vacant  $d$ -orbitals of Si can coordinate with water molecules and hence their halides are hydrolysed by water.

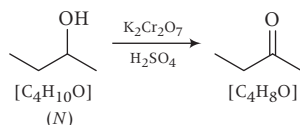
The hydrolysis of  $\text{SiCl}_4$  occurs due to coordination of  $\text{—OH}$  with empty  $3d$ -orbitals of Si-atom of  $\text{SiCl}_4$  molecule.



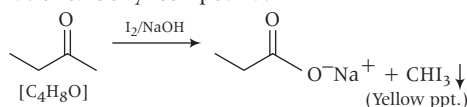
43. (d) Silver chloride dissolves in excess of ammonium hydroxide solution. The cation present in the resulting solution is  $[\text{Ag}(\text{NH}_3)_2]^+$ . In aqueous solution, silver chloride exist as  $\text{Ag}^+$  and  $\text{Cl}^-$ .  $\text{Ag}^+$  present in solution reacts with  $\text{NH}_3$  to form  $[\text{Ag}(\text{NH}_3)_2]^+$ .



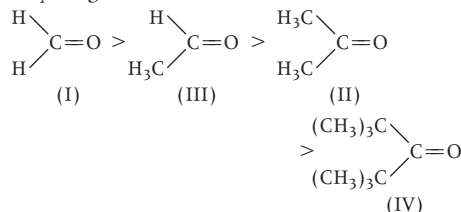




**Step II** It involves the iodoform reaction. Ketone formed in step I have one methyl group linked to the carbonyl carbon atom are oxidised by sodium hypohalite to sodium salts of corresponding carboxylic acids having one carbon atom less than that of carbonyl compound.



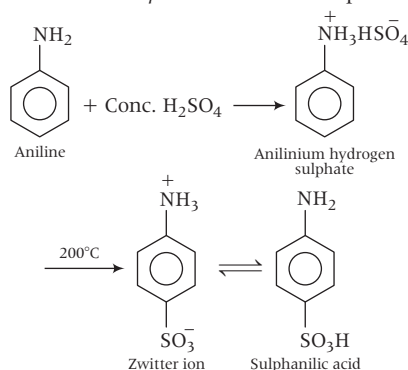
50. (a) The correct order of reactivity for the addition reaction of the given carbonyl compounds with ethylmagnesium iodide is I > III > II > IV.



This order can be explained on the basis of following two factors:

- Inductive effect** Greater the number of alkyl (electron releasing) groups attached to carbonyl group, greater will be the electron density on carbonyl carbon. Thus, it lowers the attack of nucleophile and hence, reactivity decreases.
- Steric effect** As the number of alkyl group attached to carbonyl carbon increases, the attack of nucleophile on carbonyl group becomes more and more difficult due to steric hinderance.

51. (d) When aniline is treated with conc.  $\text{H}_2\text{SO}_4$  followed by heating at  $200^\circ\text{C}$ , the product obtained is sulphanilic acid i.e. *p*-aminobenzene sulphonic acid



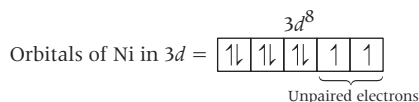
52. (c)  $n = 2, l = 0, m = -1$  is not a possible electronic configuration. With  $n = 2$  and  $l = 0$ , the orbital is  $2s$  having only one  $m_l$  value i.e. 0. As the value of  $m_l = -l$  to  $+l$

Other options are correct.

Orbital	
$n = 3, l = 0, m = 0$	$3s$
$n = 3, l = 1, m = -1$	$3p$
$n = 2, l = 1, m = 0$	$2p$

53. (b) The number of unpaired electrons in Ni (atomic number=28) are 2. It can be easily concluded from the electronic configuration of Ni.

Electronic configuration of Ni =  $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^8$



54. (c) The strongest H-bond is F—H-----F because in this case, a hydrogen is bonded to a most electronegative atom i.e. fluorine. In all other options, hydrogen is bonded to oxygen and sulphur that are less electronegative atom than fluorine.

The correct electronegativity order of elements are as follows:  $\text{F} > \text{O} > \text{S}$

55. (d) Given,

Half-life of  $\text{C}^{14}$ ,  $t_{1/2} = 5760$  years

Initial concentration of sample of  $\text{C}^{14}$ ,  $N_0 = 200$  mg

Final concentration of sample of  $\text{C}^{14}$ ,  $N_t = 25$  mg

The given decay is radioactive and all radioactive decay follows first order kinetics.

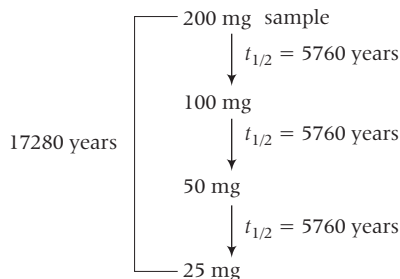
$$\begin{aligned}
 t_{1/2} &= \frac{0.693}{\lambda} \\
 \therefore \lambda &= \frac{0.693}{t_{1/2}} = \frac{0.693}{5760} \text{ yr}^{-1}
 \end{aligned}$$

We know that, for first order reaction

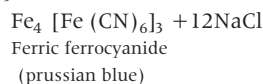
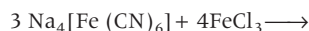
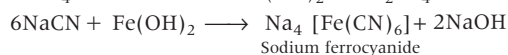
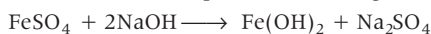
$$\begin{aligned}
 t &= \frac{2.303}{\lambda} \log \frac{N_0}{N_t} = \frac{2.303}{\frac{0.693}{5760}} \log \frac{[200]}{[25]} \\
 &= \frac{2.303 \times 5760}{0.693} \log 8 \\
 &= 17,286.78 \text{ yrs} \approx 17,280 \text{ yrs}
 \end{aligned}$$

Therefore, the time taken to change 200 mg to 25 mg is 17,286.78 yr, which is very close to option (d) i.e. 17280 yrs.

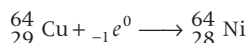
### Alternative method



56. (d) Ferric ion forms a prussian blue precipitate due to the formation of  $\text{Fe}_4[\text{Fe}(\text{CN})_6]_3$ . This complex is formed during the determine action of presence of nitrogen in the given sample. In this method, to portion of sodium fusion extract, freshly prepared ferrous sulphate,  $\text{FeSO}_4$  solution is added and warmed. Then about 2 to 3 drops of  $\text{FeCl}_3$  solution are added and acidified with conc. HCl. The appearance of a prussian blue colour indicate the presence of nitrogen.



57. (c) The nucleus  $^{64}_{29}\text{Cu}$  accepts an orbital electron to yield  $^{64}_{28}\text{Ni}$ . The atomic number of Cu is 29, which is equal to the number of electrons and also equal to the number of protons. When  $^{64}_{29}\text{Cu}$  accepts an orbital electron then electrons subtract from the atomic number, i.e.  $29-1=28$



58. (d) Mass of an electron =  $9.108 \times 10^{-31}$  kg

Mass of one mole of electron

$$= (9.108 \times 10^{-31} \times 6.023 \times 10^{23}) \text{ kg}$$

Then, number of mole of electron in 1 kg

$$= \frac{1}{9.108 \times 6.023 \times 10^{-8}}$$

$$= \frac{1}{9.108 \times 6.023} \times 10^8 \text{ mole of } e^-$$

59. (d) Given, equal weights of ethane and hydrogen are mixed in an empty container at  $25^\circ\text{C}$ .

$$\text{Initial gram weight} = \frac{\text{C}_2\text{H}_6}{wg} \quad \frac{\text{H}_2}{wg}$$

$$\text{Number of moles} = \frac{w}{30} \quad \frac{w}{2}$$

According to Henry's law,

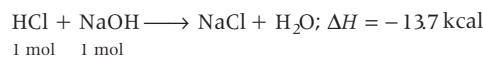
$$\frac{p_{\text{H}_2}}{p_{\text{total}}} = \chi_{\text{H}_2} = \frac{n_{\text{H}_2}}{n_{\text{H}_2} + n_{\text{C}_2\text{H}_6}} = \frac{\frac{w}{2}}{\frac{w}{2} + \frac{w}{30}}$$

$$\frac{p_{\text{H}_2}}{p_{\text{total}}} = \frac{\frac{w}{2}}{\frac{15w}{2} + w} = \frac{w}{2} \times \frac{30}{16w} = \frac{15}{16}$$

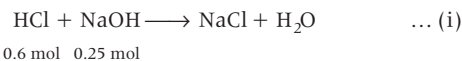
So, the fraction of total pressure exerted by hydrogen is 15 : 16.

60. (a) Given, the heat of neutralisation of a strong base and a strong acid is 13.7 kcal.

The reaction of neutralisation is as follows



According to question,



In equation (i), NaOH acts as a limiting reagent. For 1 mole of NaOH and 1 mole of HCl, heat of neutralisation = 13.7 kcal.

$\therefore$  For 0.25 mole of NaOH and 0.6 mole of HCl, heat of neutralisation =  $13.7 \times 0.25 \Rightarrow 3.425 \text{ kcal}$

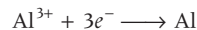
61. (a) Number of X atoms at the corners = 8

$$\text{Number of X atoms per unit cell} = 8 \times \frac{1}{8} = 1 \text{ atom}$$

Number of Y atoms at the centre of the body = 1 atom

Hence, the formula of the compound is XY.

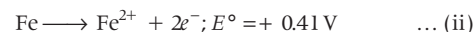
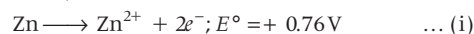
62. (c) The number of electrons involved in the reaction are three as shown below



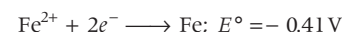
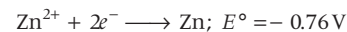
It means the conversion of every aluminium ion to aluminium atom requires three electrons.

Therefore, the amount of electricity required for one mole of  $\text{Al}^{3+}$  ions = 3F.

63. (b) Given,

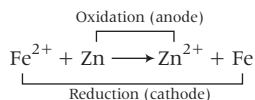


On reversing the above equation (i) and (ii), we get



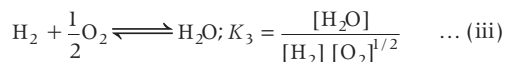
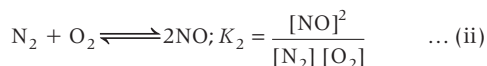
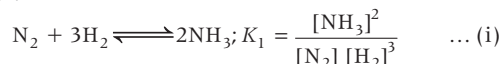
[where,  $E^\circ$  = standard reduction potential]

To find, the standard emf of the cell with the reaction.

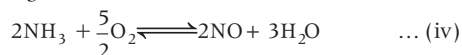


$$\begin{aligned} \text{So, } E_{\text{cell}}^{\circ} &= E_{\text{Fe}^{2+}/\text{Fe}}^{\circ} - E_{\text{Zn}^{2+}/\text{Zn}}^{\circ} \\ &= -0.41 \text{ V} + 0.76 \text{ V} = +0.35 \text{ V} \end{aligned}$$

64. (b) Given,

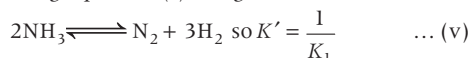


The chemical equation for the oxidation of 2 mol of  $\text{NH}_3$  to give NO is

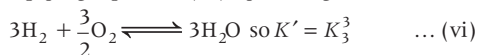


To get the equation (iv) from equation (i), (ii) and (iii) following steps are followed:

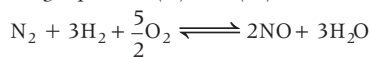
Reversing equation (i), we get



Multiplying equation (iii) by 3, we get



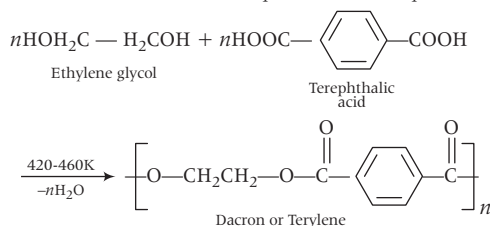
Adding equation (ii) and (vi)



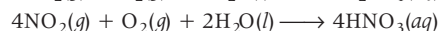
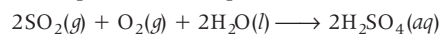
$$\text{so, } K' = K_2 \cdot K_3^3 \quad \dots (vii)$$

On combining (v) and (vii), we get the required equation having equilibrium constant  $K' = K_2 \cdot \frac{K_3^3}{K_1}$

65. (c) Dacron is a condensation polymer. It is also known as terylene. It is a polymer obtained by condensation reaction between ethylene glycol and terephthalic acid at 420-460 K in the presence of zinc acetate-antimony trioxide catalyst.



66. (b)  $\text{H}_2\text{SO}_4$  (sulphuric acid) is present in maximum amount in 'acid rain'. Oxides of nitrogen and sulphur, released into the atmosphere from thermal power plants, industries and automobiles are the main sources of acid rain. These oxides on oxidation followed by hydrolysis give sulphuric acid and nitric acid that alongwith HCl are responsible for the acidity of rain. The oxidation reaction is catalysed by particulate matter present in the polluted atmosphere.



67. (b) The correct set of oxides arranged in the proper order of basic, amphoteric, acidic are  $\text{BaO}$ ,  $\text{Al}_2\text{O}_3$ ,  $\text{SO}_2$ .

Oxides of non-metallic elements are **acidic** such as  $\text{CO}_2$ ,  $\text{NO}_2$ ,  $\text{SO}_2$  etc. Oxides of less electropositive elements (such as  $\text{BeO}$ ,  $\text{Al}_2\text{O}_3$ ,  $\text{Bi}_2\text{O}_3$ ,  $\text{ZnO}$  etc.) are **amphoteric** i.e. these behaves as acids toward strong bases and as bases towards strong acids. Oxides of electropositive elements ( $\text{Na}_2\text{O}$ ,  $\text{CaO}$ ,  $\text{Tl}_2\text{O}$ ,  $\text{BaO}$  etc.) are **basic** and contain discrete  $\text{O}^{2-}$  ions.

68. (b) Out of the given outer electronic configuration of atoms, the highest oxidation state is achieved by  $(n-1)d^5ns^2$  i.e. 7.

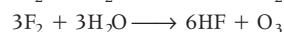
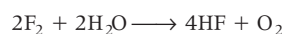
A large number of oxidation state is due to the fact the  $(n-1)d$ -electrons may get involved along with  $ns$  electrons in bonding as electrons in  $(n-1)d$ -orbitals are in an energy state comparable to  $ns$ -electrons.

Oxidation state of other options are as follows:

Electronic configuration	Oxidation state
$(n-1)d^8ns^2$	+ 2, + 3, + 4
$(n-1)d^3ns^2$	+ 2, + 3, + 4, + 5
$(n-1)d^5ns^1$	+ 2, + 3, + 4, + 5, + 6

69. (c) At room temperature, the reaction between water and fluorine produces  $\text{F}^-$ ,  $\text{O}_2$  and  $\text{H}^+$ .

Fluorine, being non-polar molecule, readily dissolves with water and forms mixture of oxygen and ozone as shown below



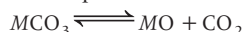
$\text{HF}$  exist as  $\text{H}^+$  and  $\text{F}^-$  in a solution.

70. (d)  $\text{BeCO}_3$  is least thermally stable. The thermal stability of carbonates increases down the group i.e from Be to Ba.



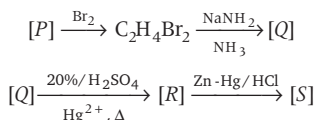


$\text{BeCO}_3$  is unstable to the extent that it is stable only in atmosphere of  $\text{CO}_2$ . These carbonates show reversible decomposition in closed container.

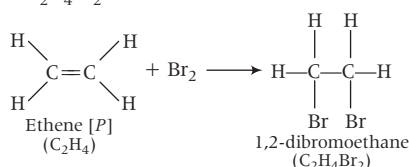


More stable the oxide is formed, lesser will be stability of carbonates.

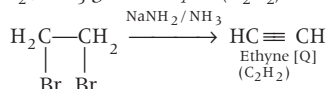
**71. (a)** Given reactions are



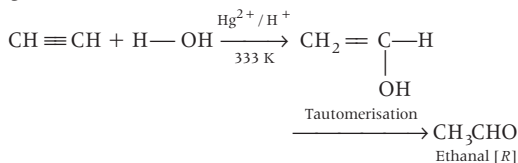
[P] is an alkene with two carbon atoms i.e. ethene. When it reacts with  $\text{Br}_2$ , addition reaction occurs and  $\text{C}_2\text{H}_4\text{Br}_2$  is formed.



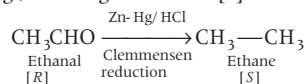
$\text{C}_2\text{H}_4\text{Br}_2$  (1, 2-dibromoethane) on reaction with  $\text{NaNH}_2/\text{NH}_3$  gives ethyne ( $\text{C}_2\text{H}_2$ ).



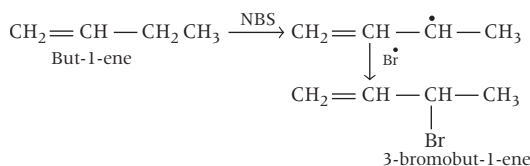
Ethyne [Q] in presence of 20%  $\text{H}_2\text{SO}_4$ ,  $\text{Hg}^{2+}$  at 333 K gives ethanal.



Ethanal [R] undergoes reduction in presence of  $\text{Zn-Hg/HCl}$  to give ethane [S]



**72. (a)** The number of possible organobromine compounds which can be obtained in the allylic bromination of but-1-ene with N-bromosuccinimide is 1.



**73. (b)** Given, specific heat = 0.16

Let metal chloride be  $\text{MCl}_x$  then,

$$\frac{6.4}{\text{specific heat}} = \text{Atomic weight of metal}$$

$$\frac{6.4}{0.16} = \text{Atomic weight of metal}$$

$$\text{Atomic weight} = 40$$

40 is the atomic weight of calcium. According to question, metal chloride ( $\text{MCl}_x$ ) have  $\approx 65\%$  chlorine present in it.

$$\frac{x \times \text{Atomic weight of chlorine}}{40 + x \times \text{Atomic weight of chlorine}} \times 100 = 65$$

$$\frac{x \times 35.5}{40 + x \times 35.5} \times 100 = 65$$

$$x = 209 \approx 2 \text{ (approx.)}$$

So, the formula of metal chloride will be  $\text{MCl}_2$ .

**74. (a)** For a reversible adiabatic process,

$$pT^{1-\gamma} = \text{constant} \quad \dots (i)$$

According to question, during a reversible adiabatic process the pressure of a gas is found to be proportional to the cube of its absolute temperature.

$$p \propto T^3$$

$$pT^{-3} = \text{constant} \quad \dots (ii)$$

Equating equation (i) and (ii), we get

$$\frac{\gamma}{1-\gamma} = -3$$

$$\gamma = -3(1-\gamma)$$

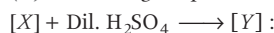
$$\gamma = -3 + 3\gamma$$

$$-2\gamma = -3$$

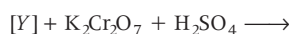
$$\gamma = \frac{3}{2}$$

As we know, the ratio of molar heat capacities at constant pressure and constant volume is represented by  $\gamma$ . So, the ratio of  $\frac{C_p}{C_v}$  for the gas is  $\frac{3}{2}$ .

**75. (a)** According to question,

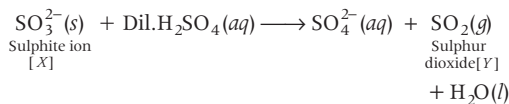


Colourless, suffocating gas

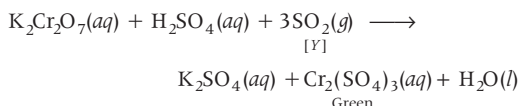


Green colouration of solution.

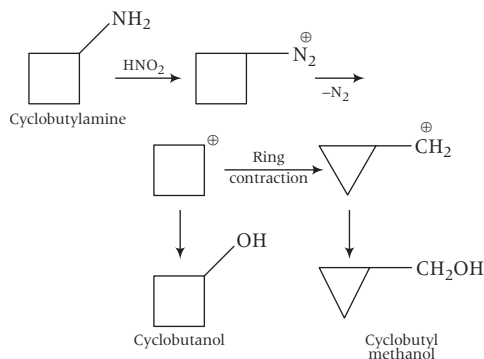
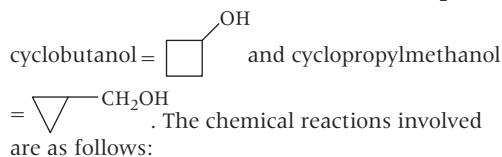
All sulphites when treated with dil.  $\text{H}_2\text{SO}_4$  gives colourless and suffocating sulphur dioxide gas.



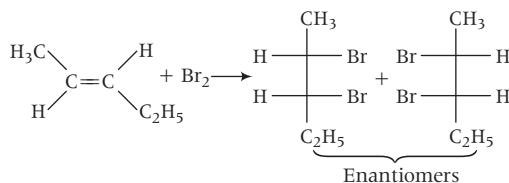
SO<sub>2</sub> gas [Y] turns acidified potassium dichromate paper green due to reduction of Cr (VI) to Cr (III).



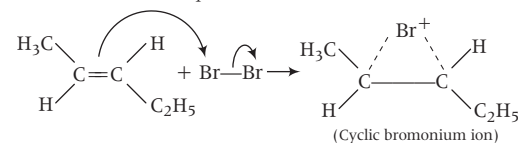
76. (a, c) The possible product(s) to be obtained from the reaction of cyclobutyl amine with HNO<sub>2</sub> is/are



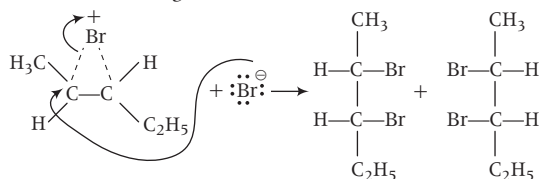
77. (a, d) The major product(s) obtained in the given reaction are



The mechanism for the above given reaction is as follows. When the  $\pi$ -electrons of the alkene approach a molecule of Br<sub>2</sub>, one of the bromine atom accepts them and releases the shared electrons to the other bromine atom. A cyclic bromonium ion is formed.

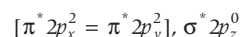
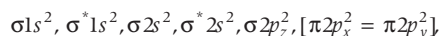


In next step, Br<sup>-</sup> attacks a carbon atom of the bromonium ion. This release the strain in the three membered ring and form a *vicinal* dibromide.



78. (b, c) The correct statements for peroxide ion (O<sub>2</sub><sup>2-</sup>) are that it is diamagnetic and it has bond order one.

Electronic configuration of O<sub>2</sub><sup>2-</sup> (peroxide ion) is



∴ Bond order =

No. of electrons present in bonding

$$- \frac{\text{No. of electrons present in non-bonding}}{2}$$

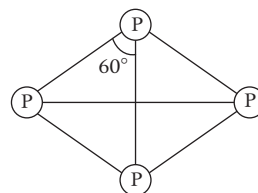
$$\therefore \text{BO} = \frac{10 - 8}{2} = \frac{2}{2} = 1$$

Number of unpaired electrons = 0

So, O<sub>2</sub><sup>2-</sup> is diamagnetic in nature.

79. (a, c, d) Among the given options, the extensive variables are *H* (enthalpy), *E* (internal energy), *V* (volume). These variables have values that depends upon the quantity or size of matter present in the system. Other examples are heat capacity, entropy, free energy, length and mass.

80. (a, c, d) White phosphorus (P<sub>4</sub>) has 6P—P single bonds, 4 lone pairs of electrons and P—P—P angle of 60°. It is a translucent white waxy solid. It has large atomic size and less electronegativity. So, it forms single bond instead of  $p\pi-p\pi$  multiple bond. It consists of discrete tetrahedral P<sub>4</sub> molecule as shown in the figure.



## Mathematics

1. (a) We know that,  $\sin 30^\circ = \frac{1}{2} = 0.5$

In 1<sup>st</sup> quadrant  $\sin x$  is increasing function.

$$\therefore \sin 31^\circ > \sin 30^\circ$$

$$\Rightarrow \sin 31^\circ > 0.5$$

2. (c) We have,  $f_1(x) = e^x$

$$f_2(x) = e^{f_1(x)}$$

$$\dots\dots\dots f_n(x) = e^{f_{n-1}(x)}$$

$$\dots\dots\dots f_{n+1}(x) = e^{f_n(x)}$$

$$\text{Now, } f_n(x) = e^{f_{n-1}(x)}$$

On taking log both sides, we get

$$\log \{f_n(x)\} = f_{n-1}(x) \log e$$

$$\Rightarrow \frac{d}{dx} \log (f_n(x)) = \frac{d}{dx} f_{n-1}(x) \quad (\because \log e = 1)$$

$$\Rightarrow \frac{1}{f_n(x)} \frac{d}{dx} f_n(x) = f'_{n-1}(x)$$

$$\Rightarrow \frac{d}{dx} f_n(x) = f_n(x) f'_{n-1}(x) \quad \dots (i)$$

$$\text{Now, } f'_1(x) = e^x$$

$$\text{and } f_2(x) = e^{f_1(x)}$$

$$\Rightarrow \log f_2(x) = f_1(x) \log e = f_1(x)$$

$$\Rightarrow \frac{1}{f_2(x)} \cdot f'_2(x) = f'_1(x)$$

$$\begin{aligned} \Rightarrow f'_2(x) &= f_2(x) \cdot f'_1(x) \\ &= f_2(x) \cdot e^x \quad (\because f'_1(x) = e^x) \\ &= f_2(x) \cdot f_1(x) \quad [\because e^x = f_1(x)] \dots (ii) \end{aligned}$$

From Eq. (i),

$$\frac{d}{dx} f_n(x) = f_n(x) \cdot f_{n-1}(x) \dots f_1(x) \text{ [using Eq. (ii)]}$$

3. (b) We have,  $f(x) = \sqrt{\frac{1-|x|}{2-|x|}}$

$f(x)$  is defined for all  $x$  satisfying

$$\frac{1-|x|}{2-|x|} \geq 0 \Rightarrow \frac{|x|-1}{|x|-2} \geq 0$$

$$\Rightarrow |x| \leq 1 \text{ or } |x| > 2$$

$$\Rightarrow x \in [-1, 1] \text{ or } x \in (-\infty, -2) \cup (2, \infty)$$

$$\Rightarrow x \in (-\infty, -2) \cup [-1, 1] \cup (2, \infty)$$

4. (c) We have,  $f: [a, b] \rightarrow \mathbb{R}$  be differentiable

on  $[a, b]$  and  $k \in \mathbb{R}$ , also  $f(a) = 0 = f(b)$

$$\text{and } J(x) = f'(x) + kf(x)$$

Let  $g(x) = kf(x)$  which is continuous in  $[a, b]$  and differentiable in  $(a, b)$  such that

$$g(a) = 0 = g(b)$$

Then, for every  $c \in (a, b)$ ,  $g'(c) = 0$

(by Rolle's theorem)

$$\text{Now, } g'(x) = kf'(x) + kxf''(x)$$

$$\Rightarrow g'(c) = kf'(c) + kcf''(c)$$

$$\Rightarrow kf'(c) + kcf''(c) = 0$$

$$\Rightarrow f(x) = 0, \text{ for every } x = c \in (a, b)$$

$\therefore J(x) = 0$  has atleast one root in  $(a, b)$ .

5. (c) We have,

$$f(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 7$$

$$\begin{aligned} \therefore f(1-h) &= 3(1-h)^{10} - 7(1-h)^8 \\ &\quad + 5(1-h)^6 - 21(1-h)^3 + 3(1-h)^2 - 7 \\ &= 3(1-10h+45h^2-120h^3+\dots+h^{10}) \\ &\quad - 7(1-8h+28h^2-56h^3+\dots+h^8) \\ &\quad + 5(1-6h+15h^2-20h^3+\dots+h^6) \\ &\quad - 21(1-3h+3h^2-h^3) - 7 \\ &\quad + 3(1-2h+h^2) - 7 \end{aligned}$$

$$\Rightarrow f(1-h) = -24 + 53h + h^2(-46) + h^3(-47) + \dots$$

$$\text{and } f(1) = -24$$

$$\begin{aligned} \therefore \lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{h^3 + 3h} &= \lim_{h \rightarrow 0} \frac{-24 + 53h + h^2(-46) + h^3(-47) + \dots - (-24)}{h(h^2 + 3)} \\ &= \lim_{h \rightarrow 0} \frac{53h + h^2(-46) + h^3(-47) + \dots}{h(h^2 + 3)} \\ &= \lim_{h \rightarrow 0} \frac{53 + h(-46) + h^2(-47) + \dots}{h^2 + 3} = \frac{53}{3} \end{aligned}$$

6. (a) Let  $g(x) = e^{-x} f(x)$

such that  $g(a) = 0, g(b) = 0$

and  $g(x)$  is continuous and differentiable.

Then, for atleast one value of  $c \in (a, b)$  such that  $g'(c) = 0$

$$\text{Now, } g'(x) = e^{-x} f'(x) + (-e^{-x}) f(x)$$

$$\Rightarrow g'(c) = e^{-c} f'(c) + (-e^{-c}) f(c) = 0$$

$$\Rightarrow e^{-c} f'(c) = e^{-c} f(c) \Rightarrow f'(c) = f(c)$$

7. (b) We have,

$f: R \rightarrow R$  be a twice continuously differentiable function such that  $f(0) = f(1) = f'(0) = 0$

Now, for atleast one value of  $c_1 \in (0, 1)$ ,

$$f'(c_1) = 0 \quad (\text{by Rolle's theorem})$$

Again,  $f'(0) = 0 = f'(c_1)$

$\Rightarrow f''(c) = 0$  for some  $c \in (0, c_1)$   
(by Rolle's theorem)

8. (b) We have,

$$\int e^{\sin x} \left( \frac{x \cos^3 x - \sin x}{\cos^2 x} \right) dx = e^{\sin x} f(x) + c$$

$$\Rightarrow \int e^{\sin x} (x \cos x - \sec x \tan x) dx = e^{\sin x} f(x) + c$$

$$\Rightarrow \int e^{\sin x} (x \cos x - 1 + 1 - \sec x \tan x) dx$$

$$= e^{\sin x} f(x) + c$$

$$\Rightarrow \int [e^{\sin x} \cos x (x - \sec x) + e^{\sin x} (1 - \sec x \tan x)] dx$$

$$= e^{\sin x} f(x) + c$$

$$\Rightarrow \int \frac{d}{dx} \{e^{\sin x} (x - \sec x)\} dx = e^{\sin x} f(x) + c$$

$$\Rightarrow e^{\sin x} (x - \sec x) = e^{\sin x} f(x) + c$$

$$\Rightarrow f(x) = x - \sec x$$

9. (c) We have,

$$\int f(x) \sin x \cos x dx = \frac{1}{2(b^2 - a^2)} \log(f(x)) + c$$

$$\Rightarrow f(x) \sin x \cos x = \frac{1}{2(b^2 - a^2)} \cdot \frac{1}{f(x)} \cdot f'(x)$$

$$\Rightarrow f(x) \sin 2x = \frac{1}{b^2 - a^2} \cdot \frac{f'(x)}{f(x)}$$

$$\Rightarrow \sin 2x = \frac{1}{b^2 - a^2} \cdot \frac{f'(x)}{(f(x))^2}$$

$$\Rightarrow \int \sin 2x dx = \frac{1}{b^2 - a^2} \int \frac{f'(x)}{(f(x))^2} dx$$

$$\Rightarrow \frac{-\cos 2x}{2} = \frac{1}{b^2 - a^2} \cdot \left( \frac{-1}{f(x)} \right)$$

$$\Rightarrow \frac{\cos 2x (b^2 - a^2)}{2} = \frac{1}{f(x)}$$

$$\Rightarrow f(x) = \frac{2}{(b^2 - a^2) \cos 2x}$$

10. (d) Given,  $M = \int_0^{\pi/2} \frac{\cos x}{(x+2)} dx$

and  $N = \int_0^{\pi/4} \frac{\sin x \cos x}{(x+1)^2} dx$

$$= \int_0^{\pi/4} \frac{1}{2} \cdot \frac{\sin 2x}{(x+1)^2} dx$$

Put  $2x = t \Rightarrow dx = \frac{dt}{2}$

$$\therefore N = \int_0^{\pi/2} \frac{\sin t}{4(t/2+1)^2} dt$$

$$= \int_0^{\pi/2} \frac{\sin t}{(t+2)^2} dt$$

$$\therefore M - N = \int_0^{\pi/2} (\cos x) \cdot \frac{1}{(x+2)} dx$$

$$- \int_0^{\pi/2} \frac{\sin x}{(x+2)^2} dx$$

$$= \left( \frac{\sin x}{x+2} \right)_0^{\pi/2} - \int_0^{\pi/2} - \frac{\sin x}{(x+2)^2} dx$$

$$- \int_0^{\pi/2} \frac{\sin x}{(x+2)^2} dx$$

$$= \frac{\sin \pi/2}{\pi/2+2} = \frac{1}{\pi/2+2} = \frac{2}{\pi+4}$$

11. (b) We have,

$$I = \int_{1/2014}^{2014} \frac{\tan^{-1} x}{x} dx \quad \dots (i)$$

Let  $x = \frac{1}{t}$

$$\Rightarrow dx = \frac{-1}{t^2} dt$$

Now,

$$I = \int_{2014}^{1/2014} \frac{\tan^{-1} (1/t)}{1/t} \left( \frac{-1}{t^2} dt \right)$$

$$= \int_{1/2014}^{2014} \frac{\cot^{-1} t}{t} dt$$

$$= \int_{1/2014}^{2014} \frac{\cot^{-1} x}{x} dx \quad \dots (ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_{1/2014}^{2014} \frac{\pi/2}{x} dx = \frac{\pi}{2} (\log x)_{1/2014}^{2014}$$

$$= \frac{\pi}{2} (\log 2014 - \log 1/2014)$$

$$\Rightarrow I = \frac{\pi}{4} \left( \log 2014 - \log \frac{1}{2014} \right)$$

$$= \frac{\pi}{4} (\log 2014 + \log 2014)$$

$$= \frac{\pi}{4} (2 \log 2014) = \frac{\pi}{2} \log 2014$$

12. (c) We have,

$$I = \int_{\pi/4}^{\pi/3} \frac{\sin x}{x} dx$$

Since,  $\frac{\sin x}{x}$  is a decreasing function.

$$\therefore \frac{\pi}{12} \times \frac{\sin \pi/3}{\pi/3} \leq I \leq \frac{\pi}{12} \times \frac{\sin \pi/4}{\pi/4}$$

$$\Rightarrow \frac{\sqrt{3}}{8} \leq I \leq \frac{\sqrt{2}}{6}$$

**13. (b)** Let

$$I = \int_{\pi/2}^{5\pi/2} \frac{e^{\tan^{-1}(\sin x)}}{e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}} dx$$

$$= \int_0^{5\pi/2} \frac{e^{\tan^{-1}(\sin x)}}{e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}} dx$$

$$- \int_0^{\pi/2} \frac{e^{\tan^{-1}(\sin x)}}{e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}} dx \dots (i)$$

$$I = \int_0^{5\pi/2} \frac{e^{\tan^{-1}(\cos x)}}{e^{\tan^{-1}(\cos x)} + e^{\tan^{-1}(\sin x)}} dx$$

$$- \int_0^{\pi/2} \frac{e^{\tan^{-1}(\cos x)}}{e^{\tan^{-1}(\cos x)} + e^{\tan^{-1}(\sin x)}} dx \dots (ii)$$

$$\left[ \text{using, } \int_a^b f(x) dx = \int_a^b f(a+b-x) dx \right]$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{5\pi/2} dx - \int_0^{\pi/2} dx$$

$$\Rightarrow 2I = (x)_0^{5\pi/2} - (x)_0^{\pi/2}$$

$$\Rightarrow 2I = \frac{5\pi}{2} - \frac{\pi}{2} = 2\pi \Rightarrow I = \pi$$

**14. (c)** We have,

$$\lim_{n \rightarrow \infty} \frac{1}{n} \left\{ \sec^2 \frac{\pi}{4n} + \sec^2 \frac{2\pi}{4n} + \dots + \sec^2 \frac{n\pi}{4n} \right\}$$

$$= \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n} \sec^2 \left( \frac{r\pi}{4n} \right) = \int_0^1 \sec^2 \left( \frac{\pi x}{4} \right)$$

$$= \frac{4}{\pi} \left[ \tan \left( \frac{\pi x}{4} \right) \right]_0^1$$

$$= \frac{4}{\pi} \times 1 = \frac{4}{\pi}$$

**15. (c)** Given,  $y^2 = 2d(x + \sqrt{d})$  ... (i)

$$\Rightarrow 2y y_1 = 2d \Rightarrow d = y y_1$$

From Eq. (i),

$$y^2 = 2y y_1 (x + \sqrt{y y_1})$$

$$\Rightarrow y^2 - 2y y_1 x = \sqrt{y y_1} \cdot 2y y_1$$

$$\Rightarrow (y^2 - 2y y_1 x)^2 = 4(y y_1)^3$$

So, degree of above equation is 3.

**16. (c)** We have,

$$(1+x^2) \frac{dy}{dx} + 2xy - 4x^2 = 0$$

$$\Rightarrow \frac{dy}{dx} + \left( \frac{2x}{1+x^2} \right) y = \frac{4x^2}{1+x^2}$$

$$\text{Here, IF} = e^{\int \frac{2x}{1+x^2}} = e^{\log(1+x^2)} = 1+x^2$$

$$\therefore y(1+x^2) = \int (1+x^2) \times \frac{4x^2}{(1+x^2)} dx + C$$

$$\Rightarrow y(1+x^2) = \int 4x^2 dx + C$$

$$\Rightarrow y(1+x^2) = \frac{4x^3}{3} + C$$

$$\Rightarrow y(1+x^2) = \frac{4x^3}{3} - 1 \quad [y(0) = -1]$$

$$\Rightarrow y = \frac{4x^3}{3(1+x^2)} - \frac{1}{1+x^2}$$

$$\therefore y(1) = \frac{4}{6} - \frac{1}{2} = \frac{1}{6}$$

**17. (c)** We have,

$$x = \frac{1}{2} vt$$

$$\Rightarrow x = \frac{1}{2} \frac{dx}{dt} t \quad \left[ \because v = \frac{dx}{dt} \right]$$

$$\Rightarrow \frac{2 dt}{t} = \frac{dx}{x}$$

$$\Rightarrow 2 \cdot \int \frac{dt}{t} = \int \frac{dx}{x}$$

$$\Rightarrow 2 \log |t| + \log |c| = \log |x|$$

$$\Rightarrow \log (t^2 \cdot c) = \log x$$

$$\Rightarrow x = t^2 c$$

$$\Rightarrow \frac{dx}{dt} = 2tc$$

$$\Rightarrow \frac{d^2 x}{dt^2} = 2c$$

$\Rightarrow$  acceleration  $f$  is constant.

**18. (b)** We have,

equation of parabola  $y = x^2$

Let  $P(\alpha, \alpha^2)$  is a point on the parabola,

$$\therefore y - \alpha^2 = 2\alpha (x - \alpha)$$

$$\therefore \left[ \because \frac{dy}{dx} = 2x \Rightarrow \frac{dy}{dx}_{(\alpha, \alpha^2)} = 2\alpha \right]$$

$$\Rightarrow y = 2\alpha x - \alpha^2$$

$$\text{Also, given } y = -x^2 + 4x - 4$$

$$\begin{aligned}\therefore -x^2 + 4x - 4 &= 2\alpha x - \alpha^2 \\ \Rightarrow x^2 + 2x(\alpha - 2) + (4 - \alpha^2) &= 0 \\ \text{Discriminant} &= 0 \\ 4(\alpha - 2)^2 - 4(4 - \alpha^2) &= 0 \\ \Rightarrow (\alpha - 2)^2 - (4 - \alpha^2) &= 0 \\ \Rightarrow \alpha^2 - 4\alpha + 4 - 4 + \alpha^2 &= 0 \\ \Rightarrow \alpha^2 - 2\alpha &= 0 \\ \Rightarrow \alpha &= 0, \alpha = 2\end{aligned}$$

$$\begin{aligned}19. (d) \quad 2b &= (n + 2)\text{th form} \\ &= a + (n + 2 - 1)d \\ \Rightarrow 2b &= a + (n + 1)d \\ \Rightarrow d &= \frac{2b - a}{n + 1} \\ \therefore m\text{th mean} &= a + m\left(\frac{2b - a}{n + 1}\right) \\ \text{and also, } b &= (n + 2)\text{th form} \\ &= 2a + (n + 2 - 1)d \\ &= 2a + (n + 1)d \\ \Rightarrow d &= \frac{b - 2a}{n + 1}\end{aligned}$$

$$\begin{aligned}\therefore m\text{th mean} &= 2a + m\left(\frac{b - 2a}{n + 1}\right) \\ \text{According to the question,} \\ a + m\left(\frac{2b - a}{n + 1}\right) &= 2a + m\left(\frac{b - 2a}{n + 1}\right) \\ \Rightarrow \frac{a}{b} &= \frac{m}{n + 1 - m}\end{aligned}$$

$$\begin{aligned}20. (c) \quad \text{We have,} \\ x + \log_{10}(1 + 2^x) &= x \log_{10} 5 + \log_{10} 6 \\ \Rightarrow \log_{10}(1 + 2^x) &= x \log_{10} 5 + \log_{10} 6 - x \\ &= \log_{10} 5^x + \log_{10} 6 - x \log_{10} 10 \\ &= \log_{10}(5^x \cdot 6) - \log_{10} 10^x \\ \Rightarrow \log_{10}(1 + 2^x) &= \log_{10}\left(\frac{5^x \cdot 6}{10^x}\right) \\ \Rightarrow 1 + 2^x &= \frac{5^x \cdot 6}{10^x} = \frac{6}{2^x} \\ \Rightarrow 2^x(1 + 2^x) &= 6 \\ \Rightarrow t(1 + t) &= 6 \quad (\text{let } 2^x = t) \\ \Rightarrow t^2 + t - 6 &= 0 \\ \Rightarrow (t + 3)(t - 2) &= 0 \\ \Rightarrow t + 3 &= 0\end{aligned}$$

$$\begin{aligned}\text{or } t - 2 &= 0 \\ \Rightarrow t &= 2 \quad [\because \text{neglect } t = -3] \\ \Rightarrow 2^x &= 2 \Rightarrow x = 1\end{aligned}$$

$$\begin{aligned}21. (b) \quad \text{We have, } Z_r &= \sin \frac{2\pi r}{11} - i \cos \frac{2\pi r}{11} \\ &= -i \left( \cos \frac{2\pi r}{11} + i \sin \frac{2\pi r}{11} \right) \\ &= -ie^{\frac{i2\pi r}{11}} \\ \therefore \sum_{r=0}^{10} Z_r &= -i \sum_{r=0}^{10} e^{\frac{i2\pi r}{11}} \\ &= -i \times 0 = 0\end{aligned}$$

$$\begin{aligned}22. (c) \quad \text{We know that, if } z_1, z_2 \text{ and } z_3 \text{ are the vertices} \\ \text{of an equilateral triangle. Then,} \\ z_1^2 + z_2^2 + z_3^2 - z_1 z_2 - z_2 z_3 - z_3 z_1 &= 0 \quad \dots(i) \\ \text{Now, but we have}\end{aligned}$$

$$\begin{aligned}\frac{z_1}{z_2} + \frac{z_2}{z_1} &= 1 \\ \Rightarrow z_1^2 + z_2^2 &= z_1 z_2 \\ \Rightarrow z_1^2 + z_2^2 - z_1 z_2 &= 0 \\ \text{Here, } z_3 &= 0 \\ \text{Hence, given points form an equilateral triangle.}\end{aligned}$$

$$\begin{aligned}23. (a) \quad \text{We have equations} \\ x^2 + b_1 x + c_1 &= 0 \\ D_1 &= b_1^2 - 4c_1 \\ \text{and } x^2 + b_2 x + c_2 &= 0 \\ D_2 &= b_2^2 - 4c_2 \\ \text{Now, } D_1 + D_2 &= b_1^2 + b_2^2 - 4(c_1 + c_2) \\ &= b_1^2 + b_2^2 - 2b_1 b_2 \quad [\because b_1 b_2 = 2(c_1 + c_2)] \\ &= (b_1 - b_2)^2 \geq 0\end{aligned}$$

$\Rightarrow$  At least one of  $D_1$  and  $D_2$  are non-negative real roots.

$$\begin{aligned}24. (a) \quad \text{Number of ways of selection of } n \text{ objects from} \\ 2n \text{ objects, where } n \text{ objects are identical in out} \\ \text{of } 2n \text{ objects.} \\ n \text{ identical and no different object} &= 1 \text{ ways} \\ &= {}^n C_0 \\ n - 1 \text{ identical and 1 different object} &= 1 \times {}^n C_1 \\ n - 2 \text{ identical and 2 different object} &= 1 \times {}^n C_2 \\ &\dots\dots\dots \\ 0 \text{ identical and } n \text{ different objects} &= 1 \times {}^n C_n \\ &= {}^n C_0 + {}^n C_1 + {}^n C_2 + \dots\dots\dots + {}^n C_n = 2^n\end{aligned}$$

$$\begin{aligned}
 25. (c) {}^nC_r + 2 \cdot {}^nC_{r+1} + {}^nC_{r+2} \\
 = {}^nC_r + {}^nC_{r+1} + {}^nC_{r+1} + {}^nC_{r+2} \\
 = {}^{n+1}C_{r+1} + {}^{n+1}C_{r+2} \\
 (\because {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}) \\
 = {}^{n+2}C_{r+2}
 \end{aligned}$$

$$\begin{aligned}
 26. (a) (101)^{100} - 1 &= (1 + 100)^{100} - 1 \\
 &= (1 + {}^{100}C_1 \cdot 100 + {}^{100}C_2 100^2 + \dots) - 1 \\
 &= {}^{100}C_1 100 + {}^{100}C_2 (100)^2 + \\
 &\quad {}^{100}C_3 (100)^3 + \dots + {}^{100}C_{100} (100)^{100} \\
 &= 10^4 (1 + {}^{100}C_2 + {}^{100}C_3 10^2 + \dots \\
 &\quad + {}^{100}C_{100} (100)^{98}) \\
 &= 10^4 (1 + \text{an integer multiple of } 10)
 \end{aligned}$$

27. (a) For greatest term of  $(1+x)^n$ , we have

$$\begin{aligned}
 \frac{n}{2} < \frac{n+1}{1+x} < \frac{n}{2} + 1 \\
 \Rightarrow \frac{n}{2} < \frac{n+1}{1+x} \text{ and } \frac{n+1}{1+x} < \frac{n}{2} + 1 \\
 \Rightarrow 1+x < \frac{n+1}{n/2} \text{ and } \frac{n+1}{n/2} < 1+x \\
 \Rightarrow x < \frac{n+1-n/2}{n/2} \\
 \text{and } \frac{n+1-(n/2-1)}{\frac{n+2}{2}} < x \\
 \Rightarrow x < \frac{n+2}{n} \text{ and } \frac{n}{n+2} < x \\
 \Rightarrow \frac{n}{n+2} < x < \frac{n+2}{n}
 \end{aligned}$$

28. (a) We have,

$$\begin{aligned}
 A &= \begin{vmatrix} -1 & 7 & 0 \\ 2 & 1 & -3 \\ 3 & 4 & 1 \end{vmatrix} \\
 &= -1(1+12) - 7(2+9) \\
 &= -13 - 77 = -90 \quad \dots (i)
 \end{aligned}$$

And let

$$\begin{aligned}
 B &= \begin{vmatrix} 13 & -11 & 5 \\ -7 & -1 & 25 \\ -21 & -3 & -15 \end{vmatrix} \\
 &= 3 \times 5 \begin{vmatrix} 13 & -11 & 1 \\ -7 & -1 & 5 \\ -7 & -1 & -1 \end{vmatrix}
 \end{aligned}$$

$$\begin{aligned}
 &= 15 \begin{vmatrix} 13 & -11 & 1 \\ -7 & -1 & 5 \\ 0 & 0 & -6 \end{vmatrix} (R_3 \rightarrow R_3 - R_2) \\
 &= 15 \{0 - 0 - 6(-13 - 77)\} \\
 &= 15 \{(-6)(-90)\} = 90 \times 90 \quad \dots (ii)
 \end{aligned}$$

From Eqs. (i) and (ii),

$$B = A^2$$

29. (c) We have,

$$a_r = (\cos 2r\pi + i \sin 2r\pi)^{1/9} = e^{\frac{2r\pi i}{9}}$$

$$\begin{aligned}
 \text{Now, } \begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} &= \begin{vmatrix} e^{\frac{2\pi i}{9}} & e^{\frac{4\pi i}{9}} & e^{\frac{6\pi i}{9}} \\ e^{\frac{8\pi i}{9}} & e^{\frac{10\pi i}{9}} & e^{\frac{12\pi i}{9}} \\ e^{\frac{14\pi i}{9}} & e^{\frac{16\pi i}{9}} & e^{\frac{18\pi i}{9}} \end{vmatrix} \\
 &= e^{\frac{2\pi i}{9}} \times e^{\frac{8\pi i}{9}} \times \begin{vmatrix} 1 & e^{\frac{2\pi i}{9}} & e^{\frac{4\pi i}{9}} \\ e^{\frac{14\pi i}{9}} & e^{\frac{16\pi i}{9}} & e^{\frac{18\pi i}{9}} \end{vmatrix}
 \end{aligned}$$

$$\begin{aligned}
 &= e^{\frac{2\pi i}{9}} \times e^{\frac{8\pi i}{9}} \times 0 \quad (\because R_1 \text{ and } R_2 \text{ are identical}) \\
 &= 0
 \end{aligned}$$

$$30. (d) \text{ We have, } S_r = \begin{vmatrix} 2r & x & n(n+1) \\ 6r^2 - 1 & y & n^2(2n+3) \\ 4r^3 - 2nr & z & n^3(n+1) \end{vmatrix}$$

$$\begin{aligned}
 \Rightarrow \sum_{r=1}^n S_r &= \begin{vmatrix} 2\sum_{r=1}^n r & x & n(n+1) \\ \sum_{r=1}^n (6r^2 - 1) & y & n^2(2n+3) \\ \sum_{r=1}^n (4r^3 - 2nr) & z & n^3(n+1) \end{vmatrix} \\
 &= \begin{vmatrix} n(n+1) & x & n(n+1) \\ n^2(2n+3) & y & n^2(2n+3) \\ n^3(n+1) & z & n^3(n+1) \end{vmatrix} \\
 &= 0 \quad (\because C_1 \text{ and } C_3 \text{ are identical})
 \end{aligned}$$

Hence,  $\sum_{r=1}^n S_r$  is independent of  $x, y, z$  and  $n$ .

31. (c) For non-trivial solution,

$$\text{We have, } \begin{vmatrix} 1 & 4a & a \\ 1 & 3b & b \\ 1 & 2c & c \end{vmatrix} = 0$$

$$\Rightarrow 1(3bc - 2bc) - (4ac - 2ac) + 1(4ab - 3ab) = 0$$



$$\begin{aligned}\Rightarrow & bc - 2ac + ab = 0 \\ \Rightarrow & bc + ab = 2ac \\ \Rightarrow & b(a + c) = 2ac \\ \Rightarrow & b = \frac{2ac}{a + c}\end{aligned}$$

$\Rightarrow a, b, c$  are in HP.

**32. (c)** On the set  $R$ ,

$xpy \Rightarrow x - y$  is zero or irrational number.

$$\begin{aligned}\text{Now,} & \quad xpx \\ \Rightarrow & \quad x - x = 0\end{aligned}$$

$\Rightarrow p$  is reflexive.

If  $xpy \Rightarrow x - y$  is zero or irrational.

$= -(y - x)$  is zero or irrational.

$\Rightarrow ypx$  is zero or irrational.

$\Rightarrow p$  is symmetric.

And if

$xpy \Rightarrow x - y$  is 0 or irrational.

$ypz \Rightarrow y - z$  is 0 or irrational.

Then,  $(x - y) + (y - z) = x - z$  may be 0 or rational.

$\Rightarrow p$  is not transitive.

**33. (d)** On the set  $R$  of real numbers

For reflexive,

$$\begin{aligned}xpx & \Rightarrow (x, x) \in R \\ \Rightarrow x & > |x| \text{ which is not true.}\end{aligned}$$

$\Rightarrow p$  is not reflexive.

For symmetric,

$$\begin{aligned}(x, y) \in R & \Rightarrow x > |y| \\ \text{and } (y, x) \in R & \Rightarrow y > |x|\end{aligned}$$

So,  $x > |y| \neq y > |x|$

$\Rightarrow p$  is not symmetric.

For transitive,

$$\begin{aligned}(x, y) \in R & \Rightarrow x > |y|, (y, z) \in R \Rightarrow y > |z| \\ \Rightarrow & \quad x > |z| \Rightarrow (x, z) \in R\end{aligned}$$

$\Rightarrow p$  is transitive.

**34. (c)** We have,  $f: R \rightarrow R$ , defined by  $f(x) = e^x$

and  $g: R \rightarrow R$  defined by  $g(x) = x^2$

Now, We have

$$\begin{aligned}(gof)(x) & = g(f(x)) \\ & = g(e^x) \\ & = (e^x)^2 \\ & = e^{2x}, \forall x \in R\end{aligned}$$

$\Rightarrow gof$  is injective and  $g$  is neither injective nor surjective.

$\Rightarrow gof$  is injective but  $g(x)$  is not bijective.

**35. (b)** We have,  $P(H) = \frac{1}{2}$ ,  $P(T) = \frac{1}{2}$

Now,  $P(X \geq 1) = 1 - P(X = 0)$

$$= 1 - {}^nC_0(p)^0(q)^n = 1 - \left(\frac{1}{2}\right)^n$$

$$\therefore 1 - \frac{1}{2^n} \geq 0.9$$

$$\Rightarrow 0.1 \geq \frac{1}{2^n}$$

$$\Rightarrow \frac{1}{10} \geq \frac{1}{2^n}$$

$$\Rightarrow 2^n \geq 10$$

$$\Rightarrow n \geq 4$$

$\therefore$  Minimum number of tossed = 4

**36. (a)** Let  $X$  be the event that student will be successful.

$X_1$  be the event that student will be pass in test-I.

$X_2$  be the event that student will be pass in test-II.

$X_3$  be the event that student will be pass in test-III.

$$\therefore P(X) = P(X_1 \cap X_2 \cap X'_3) + P(X_1 \cap X'_2 \cap X_3) + P(X_1 \cap X_2 \cap X_3)$$

$$\Rightarrow P(X) = P(X_1) \cdot P(X_2) \cdot P(X'_3) + P(X_1) \cdot P(X'_2) \cdot P(X_3) + P(X_1) \cdot P(X_2) \cdot P(X_3)$$

$$\Rightarrow \frac{1}{2} = p \cdot q \cdot \frac{1}{2} + p(1 - q) \frac{1}{2} + pq \frac{1}{2}$$

$$\Rightarrow \frac{1}{2} = p \cdot q \cdot \frac{1}{2} + p \cdot \frac{1}{2} - p \cdot q \cdot \frac{1}{2} + p \cdot q \cdot \frac{1}{2}$$

$$\Rightarrow \frac{1}{2} = p \cdot \frac{1}{2} + p \cdot q \cdot \frac{1}{2}$$

$$\Rightarrow (p + pq) = 1$$

$$\Rightarrow p(1 + q) = 1$$

**37. (a)** We have,  $\sin 6\theta + \sin 4\theta + \sin 2\theta = 0$

$$\Rightarrow \sin 6\theta + \sin 2\theta + \sin 4\theta = 0$$

$$\Rightarrow 2\sin 4\theta \cdot \cos 2\theta + \sin 4\theta = 0$$

$$\Rightarrow \sin 4\theta(2\cos 2\theta + 1) = 0$$

$$\Rightarrow \sin 4\theta = 0 \text{ or } 2\cos 2\theta + 1 = 0$$

$$\Rightarrow 4\theta = n\pi \text{ or } \cos 2\theta = \frac{-1}{2} = \cos \frac{2\pi}{3}$$

$$\Rightarrow \theta = \frac{n\pi}{4} \text{ or } 2\theta = 2n\pi \pm \frac{2\pi}{3}$$

$$\Rightarrow \theta = \frac{n\pi}{4} \text{ or } \theta = n\pi \pm \frac{\pi}{3}, \text{ where } n \in \mathbb{Z}$$

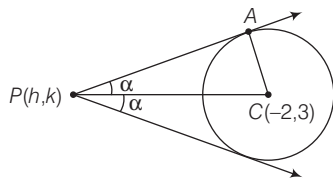
**38. (c)** We have,  $0 \leq A \leq \frac{\pi}{4}$

$$\tan^{-1}\left(\frac{1}{2} \tan 2A\right) + \tan^{-1}(\cot A) + \tan^{-1}(\cot^3 A)$$

$$\begin{aligned}
 &= \tan^{-1}\left(\frac{1}{2} \tan 2A\right) + \tan^{-1}\left(\frac{\cot A + \cot^3 A}{1 - \cot^4 A}\right) \\
 &= \tan^{-1}\left(\frac{1}{2} \cdot \frac{2 \tan A}{1 - \tan^2 A}\right) + \tan^{-1}\left(\frac{\tan A}{\tan^2 A - 1}\right) \\
 &= \tan^{-1}\left(\frac{\tan A}{1 - \tan^2 A}\right) - \tan^{-1}\left(\frac{\tan A}{1 - \tan^2 A}\right) \\
 &= 0
 \end{aligned}$$

39. (b) Put  $x = x' + 2$   
and  $y = y' + 3$   
 $\therefore x^2 + y^2 - 4x - 6y + 9 = 0$   
 $\Rightarrow (x' + 2)^2 + (y' + 3)^2 - 4(x' + 2) - 6(y' + 3) + 9 = 0$   
 $\Rightarrow x'^2 + 4 + 4x' + y'^2 + 9 + 6y' - 4x' - 8 - 6y' - 18 + 9 = 0$   
 $\Rightarrow x'^2 + y'^2 - 4 = 0$   
 $\Rightarrow x^2 + y^2 = 4$

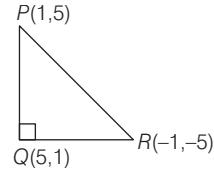
40. (d) We have equation of circle  
 $x^2 + y^2 + 4x - 6y + 9 \sin^2 \alpha + 13 \cos^2 \alpha = 0$   
Here,  $C \equiv (-2, 3)$   
Radius =  $\sqrt{(-2)^2 + (3)^2 - (9 \sin^2 \alpha + 13 \cos^2 \alpha)}$   
 $= \sqrt{4 + 9 - 9 \sin^2 \alpha - 13 \cos^2 \alpha}$   
 $= \sqrt{13 - 13(1 - \sin^2 \alpha) - 9 \sin^2 \alpha}$   
 $= \sqrt{13 \sin^2 \alpha - 9 \sin^2 \alpha}$   
 $= \sqrt{4 \sin^2 \alpha} = 2 \sin \alpha$



Here,  $\sin \alpha = \frac{AC}{PC}$   
 $\Rightarrow PC \sin \alpha = AC$   
 $\Rightarrow PC^2 \sin^2 \alpha = AC^2 = (2 \sin \alpha)^2$   
 $\Rightarrow [(h + 2)^2 + (k - 3)^2] \sin^2 \alpha = 4 \sin^2 \alpha$   
 $\Rightarrow (h + 2)^2 + (k - 3)^2 = 4$   
 $\Rightarrow h^2 + 4 + 4h + k^2 + 9 - 6k = 4$   
 $\Rightarrow h^2 + k^2 + 4h - 6k + 9 = 0$

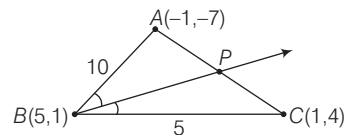
Hence, locus of a point is  
 $x^2 + y^2 + 4x - 6y + 9 = 0$

41. (d) Given, point  $P(1, 5)$  image of the point  $P(1, 5)$  about the line  $y = x$  is  $Q(5, 1)$  and image of the point  $Q$  on line  $y = -x$  is  $R(-1, -5)$



$\therefore$  Required circumcentre = Mid-point of  $P$  and  $R$   
 $= \left(\frac{1-1}{2}, \frac{5-5}{2}\right) = (0, 0)$

42. (b) Here,  $AB = \sqrt{(5+1)^2 + (1+7)^2} = \sqrt{36 + 64} = 10$



$BC = \sqrt{(1-5)^2 + (4-1)^2} = \sqrt{16 + 9} = 5$

By angle bisector theorem,

$AP : CP = 10 : 5 = 2 : 1$

$\therefore P\left(\frac{2 \times 1 + 1 \times (-1)}{2+1}, \frac{2 \times 4 + 1 \times (-7)}{2+1}\right) = P\left(\frac{1}{3}, \frac{1}{3}\right)$

Required equation of  $BP$  is

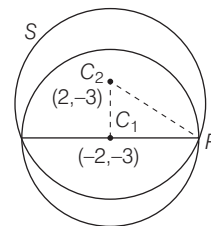
$y - 1 = \frac{\frac{1}{3} - 1}{\frac{1}{3} - 5}(x - 5)$

$\Rightarrow y - 1 = \frac{-2}{-14}(x - 5)$

$\Rightarrow 7y - 7 = x - 5$

$\Rightarrow 7y = x + 2$

43. (a) Given, equation of circle is  
 $x^2 + y^2 + 4x + 6y - 12 = 0$   
Centre  $C(-2, -3)$   
and radius =  $\sqrt{(-2)^2 + (-3)^2 + 12} = 5$

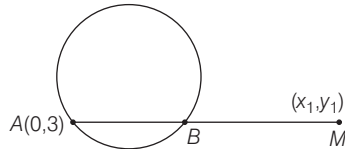


$x^2 + y^2 + 4x + 6y - 12 = 0$

$$\begin{aligned} \therefore C_1 C_2 &= \sqrt{(2+2)^2 + (-3+3)^2} \\ &= \sqrt{(4)^2 + (0)^2} = 4 \end{aligned}$$

$$\begin{aligned} \therefore \text{Radius of circle, } S &= \sqrt{(4)^2 + (3)^2} = \sqrt{16+25} = \sqrt{41} \text{ unit} \end{aligned}$$

**44. (c)** Given,  $AM = 2AB$



$\Rightarrow B$  is mid-point of  $AM$ .

$$\begin{aligned} \therefore \text{Coordinate of } B &\text{ is } \left( \frac{0+x_1}{2}, \frac{3+y_1}{2} \right) \\ &= \left( \frac{x_1}{2}, \frac{y_1+3}{2} \right) \end{aligned}$$

Since,  $B$  lies on the circle  $x^2 + 4x + (y-3)^2 = 0$

$$\therefore \left( \frac{x_1}{2} \right)^2 + 4 \left( \frac{x_1}{2} \right) + \left( \frac{y_1+3}{2} - 3 \right)^2 = 0$$

$$\Rightarrow \frac{x_1^2}{4} + 2x_1 + \left( \frac{y_1-3}{2} \right)^2 = 0$$

$$\Rightarrow \frac{x_1^2}{4} + 2x_1 + \frac{y_1^2 + 9 - 6y_1}{4} = 0$$

$$\Rightarrow x_1^2 + y_1^2 + 8x_1 - 6y_1 + 9 = 0$$

Hence, locus of a point is

$$x^2 + y^2 + 8x - 6y + 9 = 0$$

**45. (a)** Given equation of ellipses is  $x^2 + 9y^2 = 9$

$$\Rightarrow \frac{x^2}{9} + \frac{y^2}{1} = 1$$

Here,

$$a = 3, b = 1$$

$$c = \sqrt{(3)^2 - (1)^2} = \sqrt{8}$$

$\therefore$  Eccentricity of ellipse,  $e = \frac{c}{a}$

$$\Rightarrow e = \frac{\sqrt{8}}{3}$$

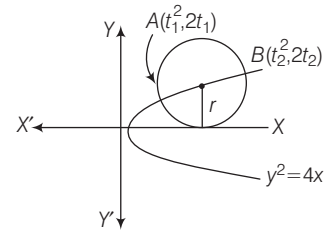
$\therefore$  Eccentricity of hyperbola =  $\frac{3}{\sqrt{8}}$

$$\Rightarrow 1 + \frac{b^2}{a^2} = \frac{9}{8}$$

$$\Rightarrow \frac{b^2}{a^2} = \frac{1}{8}$$

$$\Rightarrow a^2 : b^2 = 8 : 1$$

**46. (c, d)** Centre of circle =  $\left( \frac{t_1^2 + t_2^2}{2}, (t_1 + t_2) \right)$



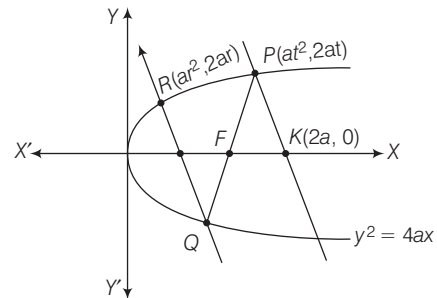
Since, circle touch the  $x$ -axis, so equation of tangent is  $y = 0$

$\therefore$  Radius = Perpendicular distance from centre to the tangent

$$\Rightarrow \text{Radius} = |t_1 + t_2| = r$$

$$\text{Slope of } AB = \frac{2}{t_1 + t_2} = \pm r$$

**47. (d)**



Here, coordinate of  $Q$  will be  $\left( \frac{a}{t^2}, \frac{-2a}{t} \right)$ .

$$\text{Slope of } QR = \frac{2}{r - \frac{1}{t}}$$

$$\text{Slope of } PK = \frac{2at}{at^2 - 2a} = \frac{2t}{t^2 - 2}$$

Since, Slope of  $QR$  = Slope of  $PK$

$$\therefore \frac{2}{r - \frac{1}{t}} = \frac{2t}{t^2 - 2}$$

$$\Rightarrow r = \frac{t^2 - 1}{t}$$

**48. (a)** Since, point  $P$  on the ellipse  $\frac{x^2}{9} + \frac{y^2}{4} = 1$

$$\therefore P(3\cos\theta, 2\sin\theta)$$

Now, equation of line parallel of  $Y$ -axis is

$$x = 3\cos\theta$$

and above line meets circle at  $Q$

$$\therefore Q(3\cos\theta, 3\sin\theta)$$

Given,  $\frac{PR}{RQ} = \frac{1}{2}$

$R(h, k)$   
 $P(3 \cos \theta, 2 \sin \theta) \quad Q(3 \cos \theta, 3 \sin \theta)$

$$\therefore h = \frac{3 \cos \theta + 6 \cos \theta}{3}, k = \frac{3 \sin \theta + 4 \sin \theta}{3}$$

$$\Rightarrow h = 3 \cos \theta, k = \frac{7}{3} \sin \theta$$

$$\Rightarrow \cos \theta = h/3, \sin \theta = \frac{3k}{7}$$

$$\text{Now, } \cos^2 \theta + \sin^2 \theta = h^2/9 + \frac{9k^2}{49} = 1$$

$$\text{Hence, locus of a point is } \frac{x^2}{9} + \frac{9y^2}{49} = 1$$

49. (a) Equation of line through  $Q(1, -2, 3)$  and

$$\text{parallel to the line } \frac{x}{1} = \frac{y}{4} = \frac{z}{5}$$

$$\text{is } \frac{x-1}{1} = \frac{y+2}{4} = \frac{z-3}{5} = \lambda \quad (\text{say})$$

Since, point  $P$  lies on above line.

$$\therefore P(\lambda + 1, 4\lambda - 2, 5\lambda + 3)$$

Since,  $P$  lies on the given plane.

$$\therefore 2(\lambda + 1) + 3(4\lambda - 2) - 4(5\lambda + 3) + 22 = 0$$

$$\Rightarrow 2\lambda + 2 + 12\lambda - 6 - 20\lambda - 12 + 22 = 0$$

$$\Rightarrow -6\lambda + 6 = 0$$

$$\Rightarrow \lambda = 1$$

$$\therefore P(2, 2, 8)$$

$$\therefore PQ = \sqrt{(2-1)^2 + (2+2)^2 + (3-8)^2}$$

$$\Rightarrow PQ = \sqrt{1+16+25} = \sqrt{42}$$

50. (d) Equation of line joining points  $(0, -11, 4)$  and  $(2, -3, 1)$  is

$$\frac{x-2}{2} = \frac{y+3}{8} = \frac{z-1}{-3} = \lambda \quad (\text{say})$$

$Q(1, 8, 4)$   
 $P(2\lambda + 2, 8\lambda - 3, -3\lambda + 1)$

Let  $P$  is any point of the above line then coordinate of  $P$  is  $(2\lambda + 2, 8\lambda - 3, -3\lambda + 1)$ .

$$\therefore \text{DR's of } PQ \text{ is } (2\lambda + 1, 8\lambda - 11, -3\lambda - 3)$$

$$\text{Now, } (2\lambda + 1)(2) + (8\lambda - 11)(8) + (-3\lambda - 3)(-3) = 0$$

$$\Rightarrow 4\lambda + 2 + 64\lambda - 88 + 9\lambda + 9 = 0$$

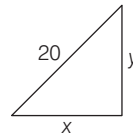
$$\Rightarrow 77\lambda - 77 = 0$$

$$\Rightarrow \lambda = 1$$

$\therefore$  Required foot of perpendicular,

$$P(4, 5, -2)$$

51. (a)



$$x^2 + y^2 = (20)^2 = 400$$

$$\text{We have, } \frac{dy}{dt} = 2 \text{ ft/sec}$$

$$\text{When } x = 12$$

$$\text{then } (12)^2 + y^2 = 400$$

$$\Rightarrow 144 + y^2 = 400$$

$$\Rightarrow y^2 = 400 - 144 = 256$$

$$\Rightarrow y = 16$$

$$\text{Now, } 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$\Rightarrow x \frac{dx}{dt} = -y \frac{dy}{dt}$$

$$\Rightarrow 12 \left( \frac{dx}{dt} \right) = -16(2)$$

$$\Rightarrow \frac{dx}{dt} = \frac{-8}{3}$$

52. (b) Here, let  $p = \frac{1}{m}$

$$\text{then } (a^p + b^p)^{1/p} = (a^{1/m} + b^{1/m})^m$$

$$= a + b + k, k \geq 0$$

$$\therefore a^p + b^p \geq (a + b)^p \geq (a + b)$$

$$\Rightarrow J(p) \geq I(p)$$

53. (c) We have,

$$\vec{\delta} = \vec{\alpha} + \lambda \vec{\beta}$$

$$= (\hat{i} + \hat{j} + \hat{k}) + \lambda (\hat{i} - \hat{j} - \hat{k})$$

$$\Rightarrow \vec{\delta} = (1 + \lambda)\hat{i} + (1 - \lambda)\hat{j} + (1 - \lambda)\hat{k}$$

$$\text{Given, } \frac{\vec{\delta} \cdot \vec{\gamma}}{|\vec{\gamma}|} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{(1 + \lambda)(-1) + (1 - \lambda)(1) + (1 - \lambda)(-1)}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \lambda = -2$$

$$\therefore \vec{\delta} = -\hat{i} + 3\hat{j} + 3\hat{k}$$

54. (c)  $\vec{\alpha} = \lambda(\vec{\beta} \times \vec{\gamma}) = \lambda(|\vec{\beta}| |\vec{\gamma}| \sin 30^\circ)$

$$\Rightarrow |\vec{\alpha}| = |\lambda| \left( |\vec{\beta}| |\vec{\gamma}| \cdot \frac{1}{2} \right)$$

$$\Rightarrow 1 = |\lambda| \cdot 1 \cdot 1 \cdot \frac{1}{2}$$

$$\Rightarrow |\lambda| = 2$$

$$\Rightarrow \lambda = \pm 2$$

$$\therefore \vec{\alpha} = \pm 2(\vec{\beta} \times \vec{\gamma})$$

55. (d) Let  $z_1 = x_1 + iy_1$  and  $z_2 = x_2 + iy_2$

$$\operatorname{Re}(z_1) > 0 \Rightarrow x_1 > 0$$

and  $\operatorname{Im}(z_2) < 0$

$$\Rightarrow y_2 < 0$$

Given,  $|z_1| = |z_2|$

$$\Rightarrow |z_1|^2 = |z_2|^2$$

$$\Rightarrow z_1 \bar{z}_1 = z_2 \bar{z}_2$$

Now,  $\left( \frac{z_1 + z_2}{z_1 - z_2} \right) + \left( \frac{\bar{z}_1 + \bar{z}_2}{\bar{z}_1 - \bar{z}_2} \right)$

$$= \left( \frac{z_1 + z_2}{z_1 - z_2} \right) + \left( \frac{\bar{z}_1 + \bar{z}_2}{\bar{z}_1 - \bar{z}_2} \right)$$

$$= \frac{z_1 \bar{z}_1 + z_2 \bar{z}_1 - z_1 \bar{z}_2 - z_2 \bar{z}_2 + z_1 \bar{z}_1 + z_1 \bar{z}_2 - z_2 \bar{z}_1 + z_2 \bar{z}_2}{(z_1 - z_2)(\bar{z}_1 - \bar{z}_2)}$$

$$= \frac{2(|z_1|^2 - |z_2|^2)}{(z_1 - z_2)(\bar{z}_1 - \bar{z}_2)} = 0 \quad (\because |z_1|^2 = |z_2|^2)$$

$$= \frac{z_1 + z_2}{z_1 - z_2} \text{ is purely imaginary.}$$

56. (a) Given, numbers are 1, 2, 3 ..... 20

Here, number of ways of selecting four consecutive numbers = 17

$\therefore$  Required number of selecting 4 non-consecutive numbers =  ${}^{20}C_4 - 17$

$$= \frac{20 \times 19 \times 18 \times 17}{4 \times 3 \times 2 \times 1} - 17$$

$$= 285 \times 17 - 17$$

$$= 284 \times 17$$

57. (b) We have,  $\begin{pmatrix} \cos \pi/4 & \sin \pi/4 \\ -\sin \pi/4 & \cos \pi/4 \end{pmatrix}^n$

$$\text{Let } A = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\Rightarrow A = \begin{pmatrix} k & k \\ -k & k \end{pmatrix} \quad \left( \text{where, } k = \frac{1}{\sqrt{2}} \right)$$

$$\Rightarrow A^2 = \begin{pmatrix} k & k \\ -k & k \end{pmatrix} \begin{pmatrix} k & k \\ -k & k \end{pmatrix} = \begin{pmatrix} 0 & 2k^2 \\ -2k^2 & 0 \end{pmatrix}$$

$$A^4 = \begin{pmatrix} 0 & 2k^2 \\ -2k^2 & 0 \end{pmatrix} \begin{pmatrix} 0 & 2k^2 \\ -2k^2 & 0 \end{pmatrix}$$

$$A^4 = \begin{pmatrix} -4k^4 & 0 \\ 0 & -4k^4 \end{pmatrix} = \begin{pmatrix} -4 \times \frac{1}{4} & 0 \\ 0 & -4 \times \frac{1}{4} \end{pmatrix}$$

$$\Rightarrow A^4 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\Rightarrow A^8 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

58. (d) We have,  $\rho = \{(x, y) \in N \times N : 2x + y = 41\}$

For reflexive,

$$xpx \Rightarrow 2x + x = 41$$

$$\Rightarrow 3x = 41$$

$$\Rightarrow x = \frac{41}{3} \notin N$$

So,  $\rho$  is not reflexive.

For symmetric,

$$xpy \Rightarrow 2x + y = 41$$

and  $ypx \Rightarrow 2y + x = 41$

$$\Rightarrow xpy \neq ypx$$

So,  $\rho$  is not symmetric.

For transitive,

$$xpy \Rightarrow 2x + y = 41$$

and  $ypz \Rightarrow 2y + z = 41$

$$\Rightarrow xpz$$

$\Rightarrow \rho$  is not transitive.

59. (a) Given,  $f(x) = \begin{vmatrix} (1+x)^a & (2+x)^b & 1 \\ 1 & (1+x)^a & (2+x)^b \\ (2+x)^b & 1 & (1+x)^a \end{vmatrix}$

For constant term put  $x = 0$

$$f(0) = \begin{vmatrix} 1 & 2^b & 1 \\ 1 & 1 & 2^b \\ 2^b & 1 & 1 \end{vmatrix}$$

$$= 1(1 - 2^b) - 2^b(1 - 2^{2b}) + 1(1 - 2^b)$$

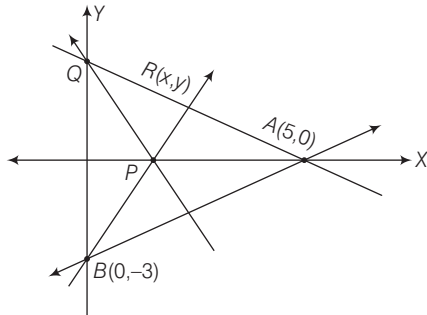
$$= 1 - 2^b - 2^b + 2^{3b} + 1 - 2^b$$

$$= 2 - 3 \cdot 2^b + 2^{3b}$$

- 60. (a)** Equation of line AB is

$$\frac{x}{5} + \frac{y}{-3} = 1$$

$$\Rightarrow 3x - 5y = 15$$



Perpendicular line to AB is

$$5x + 3y = \lambda$$

Coordinate of P is  $\left(\frac{\lambda}{5}, 0\right)$

and coordinate of Q is  $\left(0, \frac{\lambda}{3}\right)$

Now, equation of line AQ is

$$\frac{x}{5} + \frac{y}{\lambda/3} = 1$$

$$\Rightarrow \frac{x}{5} + \frac{3y}{\lambda} = 1$$

$$\Rightarrow \frac{3y}{\lambda} = 1 - \frac{x}{5}$$

$$\Rightarrow \frac{1}{\lambda} = \frac{1}{3y} \left(1 - \frac{x}{5}\right) \quad \dots (i)$$

and equation of line BP is

$$\frac{x}{\lambda/5} + \frac{y}{-3} = 1$$

$$\Rightarrow \frac{5x}{\lambda} - \frac{y}{3} = 1$$

$$\Rightarrow \frac{1}{\lambda} = \frac{1}{5x} \left(\frac{y}{3} + 1\right) \quad \dots (ii)$$

From Eqs. (i) and (ii),

$$\frac{1}{3y} \left(1 - \frac{x}{5}\right) = \frac{1}{5x} \left(\frac{y}{3} + 1\right)$$

$$\Rightarrow 5x \left(1 - \frac{x}{5}\right) = 3y \left(\frac{y}{3} + 1\right)$$

$$\Rightarrow 5x - x^2 = y^2 + 3y$$

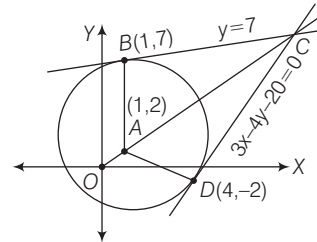
$$\Rightarrow x^2 + y^2 - 5x + 3y = 0$$

which is a circle.

- 61. (c)** Given, equation of circle is

$$x^2 + y^2 - 2x - 4y - 20 = 0$$

Center (1, 2) and radius =  $\sqrt{(1)^2 + (2)^2 + 20} = 5$



Coordinate of intersecting point of tangents at B and D is C(6, 7).

$\therefore$  Area of quadrilateral ABCD

$$= 2 \times \text{ar}(\triangle ABC)$$

$$= 2 \times \frac{1}{2} \times 15 \times 5 = 75 \text{ sq units}$$

- 62. (b)** At  $x = -\frac{\pi}{2}$

$$\text{LHL} = -2$$

$$\text{RHL} = -A + B$$

For continuity, LHL = RHL =  $f(-\pi/2)$

$$\Rightarrow -A + B = 2 \quad \dots (i)$$

At  $x = \pi/2$

$$\text{LHL} = A + B$$

$$\text{RHL} = 0$$

For continuity, LHL = RHL =  $f(\pi/2)$

$$\Rightarrow A + B = 0 \quad \dots (ii)$$

On solving Eqs. (i) and (ii), we get

$$A = -1 \text{ and } B = 1$$

- 63. (c)** Given equation of curve,

$$y = x^2 - x + 1$$

$$\Rightarrow \frac{dy}{dx} = 2x - 1$$

$$\text{Slope of normal} = \frac{1}{1 - 2x}$$

Now, at  $x_1 = 0, y_1 = 1$

$\therefore$  Slope of normal at (0, 1) = 1

$\therefore$  Equation of normal,  $y - 1 = 1(x - 0)$

$$\Rightarrow x - y + 1 = 0 \quad \dots (i)$$

At  $x_2 = -1, y_2 = 3$

$$\text{Slope of normal at } (-1, 3) = \frac{1}{3}$$

$$\text{Equation of normal, } y - 3 = \frac{1}{3}(x + 1)$$

$$\Rightarrow 3y - 9 = x + 1$$

$$\Rightarrow x - 3y + 10 = 0 \quad \dots (ii)$$

At  $x_3 = \frac{5}{2}, y_3 = \frac{19}{4}$

Slope of normal at  $\left(\frac{5}{2}, \frac{19}{4}\right) = -\frac{1}{4}$

Equation of normal,  $y - \frac{19}{4} = -\frac{1}{4}\left(x - \frac{5}{2}\right)$

$$\Rightarrow x + 4y = \frac{43}{2}$$

$$\Rightarrow 2x + 8y = 43 \quad \dots (iii)$$

Here, intersecting point of Eqs. (i) and (ii) is  $\left(\frac{7}{2}, \frac{9}{2}\right)$

and normal (iii) passes through it. Hence, normals are concurrent.

**64. (b)** Let  $f(x) = x \log x + x - 3$

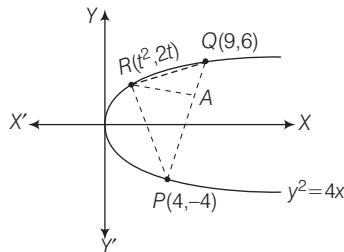
$$\Rightarrow f'(x) = x \cdot \frac{1}{x} + \log x + 1$$

$$\Rightarrow f'(x) = \log x + 2 > 0$$

$$\Rightarrow f(1) = -2 \text{ and } f(3) = 3 \log 3, f(1) \cdot f(3) < 0$$

Hence, exactly one root in  $x \in (1, 3)$  as  $f(x) > 0$

**65. (c)** Equation of PQ is  $2x - y = 12$



Perpendicular distance

$$AR = \left| \frac{2t^2 - 2t - 12}{\sqrt{5}} \right| = \frac{2}{\sqrt{5}} (t - 3)(t + 2)$$

AR is Maximum, at  $t = \frac{1}{2}$

$$\therefore R \text{ is } \left(\frac{1}{4}, 1\right)$$

**66. (b)**  $I = \int_0^1 \frac{x^3 \cos 3x}{2 + x^2} dx$

Here,  $-1 < \cos 3x < 1$

$$\Rightarrow -x^3 < x^3 \cos 3x < x^3$$

$$\Rightarrow \frac{-x^3}{x^2} < \frac{-x^3}{x} < \frac{-x^3}{2+x^2} < \frac{x^3 \cos 3x}{2+x^2} < \frac{x^3}{2+x^2} < \frac{x^3}{x} < \frac{x^3}{x^2}$$

$$\Rightarrow \int_0^1 -x^2 dx < I < \int_0^1 x^2 dx$$

$$\Rightarrow \left(\frac{-x^3}{3}\right)_0^1 < I < \left(\frac{x^3}{3}\right)_0^1$$

$$\Rightarrow \frac{-1}{3} < I < \frac{1}{3}$$

**67. (c, d)** Given, curve,  $12y = x^3$

and  $\frac{dy}{dt} > \frac{dx}{dt} \quad \dots (i)$

Now,  $12 \frac{dy}{dt} = 3x^2 \frac{dx}{dt} \quad \dots (ii)$

From Eqs. (i) and (ii), we get

$$3x^2 \frac{dx}{dt} > 12 \frac{dx}{dt}$$

$$\Rightarrow 3x^2 > 12$$

$$\Rightarrow x^2 - 4 > 0$$

$$\Rightarrow (x - 2)(x + 2) > 0$$

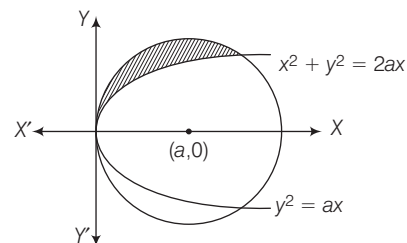
$$\Rightarrow x \in (-\infty, -2) \cup (2, \infty)$$

**68. (b)** Given, equation of circle

$$x^2 + y^2 = 2ax$$

$$\Rightarrow (x - a)^2 + y^2 = a^2$$

and equation of parabola is  $y^2 = ax, a > 0$



Intersection points of circle and parabola

$$\Rightarrow x^2 + ax = 2ax$$

$$\Rightarrow x^2 = ax$$

$$\Rightarrow x^2 - ax = 0$$

$$\Rightarrow x(x - a) = 0$$

$$\Rightarrow x = 0, a$$

Intersecting points are (0, 0) and (a, a).

$$\therefore \text{Required area} = \frac{\pi a^2}{4} - \int_0^a \sqrt{ax} dx$$

$$= \frac{\pi a^2}{4} - \sqrt{a} \left( \frac{x^{3/2}}{3/2} \right)_0^a$$

$$= \frac{\pi a^2}{4} - \frac{2a^2}{3} = a^2 \left( \frac{\pi}{4} - \frac{2}{3} \right)$$

- 69. (a, d)** Let  $\alpha$  and  $\beta$  are the roots of  $x^2 + ax + b = 0$   
and the roots of  $x^2 - cx + d = 0$  are  $\alpha^4$  and  $\beta^4$ .

Now,  $\alpha + \beta = -a, \alpha\beta = b$  ... (i)  
and  $\alpha^4 + \beta^4 = c, \alpha^4\beta^4 = d$  ... (ii)

From Eqs. (i) and (ii),

$$\begin{aligned} b^4 &= d \text{ and } \alpha^4 + \beta^4 = c \\ (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 &= c \\ \Rightarrow [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2 &= c \\ \Rightarrow (a^2 - 2b)^2 - 2b^2 &= c \\ \Rightarrow 2b^2 + c &= (a^2 - 2b)^2 \\ \Rightarrow 2b^2 + c &= a^4 + 4b^2 - 4a^2b \\ \Rightarrow 2b^2 - c &= 4a^2b - a^4 \\ \Rightarrow 2b^2 - c &= a^2(4b - a^2) \end{aligned}$$

and for equation  $x^2 - 4bx + 2b^2 - c = 0$

$$\begin{aligned} D &= (4b)^2 - 4(1)(2b^2 - c) \\ &= 16b^2 - 8b^2 + 4c = 8b^2 + 4c \\ &= 4(2b^2 + c) = 4(a^2 - 2b)^2 > 0 = \text{real root} \end{aligned}$$

Now,  $f(0) = 2b^2 - c = a^2(4b - a^2) < 0$  ( $\because a^2 > 4b$ )  
= roots are opposite in sign

- 70. (b, c)** Required number of ways =  ${}^{20}C_2 \times 2!$   
 $= {}^{20}P_2$

**71. (a, c)**

$$A = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

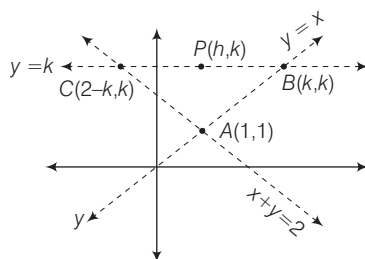
$$\Rightarrow A' = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix} = -A$$

$\Rightarrow A$  is a skew-symmetric matrix.

$$\begin{aligned} |A| &= 0 + 1(0 + 1) - 1(1 - 0) \\ &= 0 + 1 - 1 = 0 = \text{Singular} \end{aligned}$$

$\Rightarrow A$  is not invertible.

- 72. (a, b)** Given,  $\ar(\triangle ABC) = h^2$



$$\begin{aligned} \Rightarrow \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ k & k & 1 \\ 2-k & k & 1 \end{vmatrix} &= \pm h^2 \\ \Rightarrow 1(k-k) - 1(k-2+k) + 1(k^2-2k+k^2) &= \pm 2h^2 \\ \Rightarrow -(2k-2) + (2k^2-2k) &= \pm 2h^2 \\ \Rightarrow 2-2k+2k^2-2k &= \pm 2h^2 \\ \Rightarrow 2k^2-4k+2 &= \pm 2h^2 \\ \Rightarrow k^2-2k+1 &= \pm h^2 \end{aligned}$$

Hence, locus of a point is

$$\begin{aligned} \Rightarrow (k-1)^2 &= h^2 \\ y-1 &= \pm x \\ \Rightarrow x &= y-1 \text{ or } x = -(y-1) \end{aligned}$$

- 73. (b)** Given,  $2a_1 = 2 \sin \theta$

$$\begin{aligned} \Rightarrow a_1 &= \sin \theta \\ \text{and } 3x^2 + 4y^2 &= 12 \\ \Rightarrow \frac{x^2}{4} + \frac{y^2}{3} &= 1 \end{aligned}$$

Here,  $a^2 = 4$  and  $b^2 = 3$

$$\begin{aligned} \therefore b^2 &= a^2(1 - e^2) \\ \Rightarrow 3 &= 4(1 - e^2) \\ \Rightarrow e^2 &= 1 - \frac{3}{4} = \frac{1}{4} \\ \Rightarrow e &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{Focus, } F(ae, 0) &= F\left(2 \times \frac{1}{2}, 0\right) \\ &= F(1, 0) \end{aligned}$$

For hyperbola foci are same

$$\begin{aligned} a_1 e_1 &= ae = 1 \\ \therefore (\sin \theta) e_1 &= 1 \\ \Rightarrow e_1 &= \operatorname{cosec} \theta \\ \text{and } b_1^2 &= a_1^2(e_1^2 - 1) = a_1^2 e_1^2 - a_1^2 \\ \Rightarrow b_1^2 &= 1 - \sin^2 \theta = \cos^2 \theta \end{aligned}$$

$$\begin{aligned} \frac{x^2}{a_1^2} - \frac{y^2}{b_1^2} &= 1 \\ \Rightarrow \frac{x^2}{\sin^2 \theta} - \frac{y^2}{\cos^2 \theta} &= 1 \\ \Rightarrow x^2 \operatorname{cosec}^2 \theta - y^2 \sec^2 \theta &= 1 \end{aligned}$$



**74. (a, c)**  $f(x) = \cos\left(\frac{\pi}{x}\right)$

$$\Rightarrow f'(x) = -\sin\left(\frac{\pi}{x}\right)\left(-\frac{\pi}{x^2}\right) = \frac{\pi}{x^2} \sin \frac{\pi}{x}$$

For increasing function,  $f'(x) > 0$

$$\Rightarrow \sin\left(\frac{\pi}{x}\right) > 0 \Rightarrow 2k\pi < \frac{\pi}{x} < (2k+1)\pi$$

$$\Rightarrow \frac{1}{2k} > x > \frac{1}{2k+1}$$

For decreasing function,  $f'(x) < 0$

$$\Rightarrow \sin\left(\frac{\pi}{x}\right) < 0$$

$$\Rightarrow \frac{\pi}{x} \in [(2k+1)\pi, (2k+2)\pi] \Rightarrow x \in \left(\frac{1}{2k+2}, \frac{1}{2k+1}\right)$$

**75. (c)** Given,  $y = \log_a (x + \sqrt{x^2 + 1})$ ,  $a > 0$ ,  $a \neq 1$

$$\Rightarrow a^y = (x + \sqrt{x^2 + 1})$$

$$\Rightarrow a^{-y} = \frac{1}{x + \sqrt{x^2 + 1}} = \sqrt{x^2 + 1} - x$$

$$\Rightarrow a^y - a^{-y} = 2x \Rightarrow x = \frac{a^y - a^{-y}}{2}$$

$$\Rightarrow f^{-1}(y) = \frac{e^{y \log a} - e^{-y \log a}}{2}$$

$$\Rightarrow f^{-1}(y) = \sinh(y \log a) \quad \left( \because \frac{e^x - e^{-x}}{2} = \sinh(x) \right)$$