WBJEE 2022 Question Paper

West Bengal Joint Entrance Examinations

WBJEE 2022 Solved Paper

Mathematics

Question 1

The values of a, b, c for which the function

$$f(x)=\begin{cases}\frac{\sin(a+1)x+\sin x}{x}, x<0\\ c, x=0\\ \frac{(x+bx^2)^{\frac{1}{2}}-x^{\frac{1}{2}}}{bx^{\frac{1}{2}}}, x>0\end{cases}$$ **is continuous at x = 0, are**

Options:

A.

$a=\frac{3}{2}, b=-\frac{3}{2}, c=\frac{1}{2}$

B.

$a=-\frac{3}{2}, c=\frac{3}{2}, b$ is arbitrary non-zero real number.

C.

$a=-\frac{5}{2}, b=-\frac{3}{2}, c=\frac{3}{2}$

D.

$a=-2, b\in R-\{0\}, c=0$

Answer: D

Solution:

For continuous at x = 0,

$\lim_{x\to 0} f(x)=f(0)$

$RHL=\lim_{x\to 0^+}\frac{(x+bx^2)^{\frac{1}{2}}-x^{\frac{1}{2}}}{bx^{\frac{1}{2}}}$

$=\lim_{x\to 0^+}\frac{x^{\frac{1}{2}}\left\{(1+bx)^{\frac{1}{2}}-1\right\}}{bx^{\frac{1}{2}}}$

$= \lim_{x \to 0^+} \frac{(1+bx)^{\frac{1}{2}}-1}{b}$

= 0 (when b $\neq$ 0)

$LHL = \lim_{x \to 0^-} \frac{\sin(a+1)x+\sin x}{x} \left(\frac{0}{0}\right)$

$= \lim_{x \to 0^-} \frac{(\cos(a+1)x)(a+1)+\cos x}{1}$

$= a + 1 + 1$

$= a + 2$

And $f(0) = c$ (given)

We know, for continuous function

$LHL = RHL = f(0)$

$\therefore a + 2 = 0(b \neq 0) = c$

$\therefore$ c = 0, a = -2

and b = R $-$ {0}

Question 2

Domain of $y = \sqrt{\log_{10} \frac{3x-x^2}{2}}$ is

Options:

A.

x < 1

B.

2 < x

C.

1 $\leq$ x $\leq$ 2

D.

2 < x < 3

Answer: C

Solution:

$\log_{10}\left(\frac{3x-x^2}{2}\right) \geq 0$

$\Rightarrow \frac{3x-x^2}{2} \geq 10^0$

$\Rightarrow \frac{3x-x^2}{2} \geq 1$

$\Rightarrow 3x - x^2 \geq 2$

$\Rightarrow x^2 - 3x + 2 \leq 0$

$\Rightarrow (x-2)(x-1) \leq 0$

$\therefore x \in [1, 2]$ ...... (1)

Also, $\frac{3x-x^2}{2} > 0$

$\Rightarrow x^2 - 3x < 0$

$\Rightarrow x(x-3) < 0$

$\Rightarrow x \in (0, 3)$ ...... (2)

$\therefore$ Intersection of (1) and (2) is $x \in [1, 2]$

Question 3

Let $f(x) = a_0 + a_1|x| + a_2|x|^2 + a_3|x|^3$, where a_0, a_1, a_2, a_3 are real constants. Then f(x) is differentiable at x = 0

Options:

A.

whatever be a_0, a_1, a_2, a_3.

B.

for no values of a_0, a_1, a_2, a_3.

C.

only if $a_1 = 0$

D.

only if $a_1 = 0, a_3 = 0$

Answer: C

Solution:

Given,

$f(x) = a_0 + a_1|x| + a_2|x|^2 + a_3|x|^3$

$\therefore f(x) = \begin{cases} a_0 + a_1x + a_2x^2 + a_3x^3, & x \geq 0 \\ a_0 - a_1x + a_2x^2 - a_3x^3, & x < 0 \end{cases}$

$\Rightarrow f(x) = \begin{cases} a_1 + 2a_2x + 3a_3x^2, & x \geq 0 \\ -a_1 + 2a_2x - 3a_3x^2, & x < 0 \end{cases}$

$f(x)$ is differentiable at $x = 0$

$\therefore \mathrm{L.H.D} = \mathrm{R.H.D}$

$\Rightarrow -a_1 + 0 + 0 = a_1 + 0 + 0$

$\Rightarrow 2a_1 = 0$

$\Rightarrow a_1 = 0$

Question 4

If $y = e^{\tan^{-1}x}$, then

Options:

A.

$(1 + x^2)y_2 + (2x - 1)y_1 = 0$

B.

$(1 + x^2)y_2 + 2xy = 0$

C.

$(1 - x^2)y_2 - y_1 = 0$

D.

$(1 + x^2)y_2 + 3xy_1 + 4y = 0$

Answer: A

Solution:

$y = e^{\tan^{-1} x}$

Differentiating both sides with respect to x, we get

$\Rightarrow \frac{dy}{dx} = e^{\tan^{-1}(x)} \times \frac{1}{1+x^2}$

$\Rightarrow y_1(1+x^2) = e^{\tan^{-1} x}$

$\Rightarrow y_1(1+x^2) = y$

Differentiating both side with respect to x, we get

$\Rightarrow y_2(1+x^2) + y_1(2x) = y_1$

$\Rightarrow y_2(1+x^2) + y_1(2x-1) = 0$

Question 5

$\lim_{x \to 0} \left(\frac{1}{x} \ln \sqrt{\frac{1+x}{1-x}} \right)$ **is**

Options:

A.

$\frac{1}{2}$

B.

0

C.

1

D.

does not exist

Answer: C

Solution:

$$\lim_{x \to 0} \left(\frac{1}{x} \ln \sqrt{\frac{1+x}{1-x}} \right)$$

Put $x = \cos 2\theta$

$\Rightarrow 2\theta = \cos^{-1}(x)$

$\Rightarrow \theta = \frac{1}{2}\cos^{-1}(x)$

when $x \to 0$ then $\theta \to \frac{\pi}{4}$

$$\therefore \lim_{\theta \to \frac{\pi}{4}} \left(\frac{\ln \sqrt{\frac{1+\cos 2\theta}{1-\cos 2\theta}}}{\cos 2\theta} \right)$$

$$= \lim_{\theta \to \frac{\pi}{4}} \left(\frac{\ln \sqrt{\frac{2\cos^2\theta}{2\sin^2\theta}}}{\cos 2\theta} \right)$$

$$= \lim_{\theta \to \frac{\pi}{4}} \left(\frac{\ln (\cot^2\theta)^{\frac{1}{2}}}{\cos 2\theta} \right)$$

$$= \lim_{\theta \to \frac{\pi}{4}} \left(\frac{\ln |\cot \theta|}{\cos 2\theta} \right)$$

when $\theta \to \frac{\pi}{4}$ then $\cot \theta > 0$

$\therefore |\cot \theta| = \cot \theta$

$$= \lim_{\theta \to \frac{\pi}{4}} \frac{\ln(\cot \theta)}{\cos 2\theta} \ (\tfrac{0}{0} \text{ form})$$

Applying L' Hospital Rule,

$$= \lim_{\theta \to \frac{\pi}{4}} \frac{\frac{1}{\cot \theta} \times (-\cos ec^2\theta)}{-\sin 2\theta \times 2}$$

$$= \frac{\frac{1}{\cot \frac{\pi}{4}} \times -\cos ec^2 \frac{\pi}{4}}{-\sin \frac{\pi}{2} \times 2}$$

$$= \frac{\frac{1}{1} \times -(\sqrt{2})^2}{-1 \times 2} = 0$$

$$= \frac{2}{2} = 1$$

Question 6

Let f : [a, b] $\to$ R be continuous in [a, b], differentiable in (a, b) and f(a) = 0 = f(b). Then

Options:

A.

there exists at least one point $c \in (a, b)$ for which $f'(c) = f(c)$

B.

$f'(x) = f(x)$ does not hold at any point of (a, b)

C.

at every point of (a, b), $f'(x) > f(x)$

D.

at every point of (a, b), $f'(x) < f(x)$

Answer: A

Question 7

$I = \int \cos(\ln x) dx$. Then I =

Options:

A.

$\frac{x}{2}\{\cos(\ln x) + \sin(\ln x)\} + c$ (c denotes constant of integration)

B.

$x^2\{\cos(\ln x) - \sin(\ln x)\} + c$ (c denotes constant of integration)

C.

$x^2 \sin(\ln x) + c$ (c denotes constant of integration)

D.

$x \cos(\ln x) + c$ (c denotes constant of integration)

Answer: A

Solution:

$I = \int \cos(\ln x) dx$

Let, $\ln x = t$

$\Rightarrow x = e^t$

$\Rightarrow dx = e^t dt$

$\therefore I = \int \cos(t) \,.\, e^t dt$

Apply integration by parts theorem,

$= \cos t \,.\, \int e^t dt - \int \left(\frac{d}{dt}(\cos t) \,.\, \int e^t dt\right) dt$

$= \cos t \,.\, e^t + \int \sin t \,.\, e^t dt$

$I = \cos t \,.\, e^t + \sin t \,.\, e^t - \int (\cos t \,.\, e^t)\, dt$

$\Rightarrow I = \cos t \,.\, e^t + \sin t \,.\, e^t - I$

$\Rightarrow 2I = \cos t \,.\, e^t + \sin t \,.\, e^t + C$

$\Rightarrow 2I = e^t(\cos t + \sin t) + C$

$\Rightarrow 2I = x\,(\cos(\ln x) + \sin(\ln x)) + C$

$\Rightarrow I = \frac{x}{2}\,[\cos(\ln x) + \sin(\ln x)] + C$

Question 8

Let f be derivable in [0, 1], then

Options:

A.

there exists $c \in (0, 1)$ such that $\int_0^c f(x)dx = (1 - c)f(c)$

B.

there does not exist any point $d \in (0, 1)$ for which $\int_0^d f(x)dx = (1 - d)f(d)$

C.

$\int_0^c f(x)dx$ does not exist, for any $c \in (0, 1)$

D.

$\int_0^c f(x)dx$ is independent of c, $c \in (0, 1)$

Answer: A

Solution:

Let $f(x) = x$ which is derivable in [0, 1].

Option A :

$\int_0^c f(x)dx = (1-c)f(c)$

$\Rightarrow \int_0^c x\,dx = (1-c)\,.\,c$

$\Rightarrow \left[\frac{x^2}{2}\right]_0^c = (1-c)\,c$

$\Rightarrow \frac{c^2}{2} = (1-c)\,c$

$\therefore c = 0$

or $\frac{c}{2} = 1-c$

$\Rightarrow c = 2-2c$

$\Rightarrow 3c = 2$

$\Rightarrow c = \frac{2}{3}$

$\therefore$ c = 0 does not belongs to (0, 1) but $c = \frac{2}{3}$ belongs to (0, 1)

$\therefore$ Option A is correct.

Option B :

$\int_0^d f(x)dx = (1-d)f(d)$

$\Rightarrow \int_0^d x\,dx = (1-d)\,.\,d$

$\Rightarrow \frac{d^2}{2} = (1-d)d$

$\therefore d = 0$

or

$\frac{d}{2} = 1-d$

$\Rightarrow d = \frac{2}{3}$ (which belongs to in between (0, 1))

$\therefore$ Option B is incorrect.

Option C :

$\int_0^c f(x)dx$

$= \int_0^c x\,dx$

$= \left[\frac{x^2}{2}\right]_0^c$

$= \frac{c^2}{2}$

$\frac{c^2}{2}$ exist all values of c between 0 and 1.

$\therefore$ Option C is incorrect.

Option D :

$\int_0^c f(x) = dx$

$= \int_0^c x\, dx$

$= \left[\frac{x^2}{2}\right]_0^c$

$= \frac{c^2}{2}$

$\therefore \int_0^c f(x)\, dx$ is not independent of c.

$\therefore$ Option D is incorrect.

Question 9

Let $\int \frac{x^{\frac{1}{2}}}{\sqrt{1-x^3}} dx = \frac{2}{3} g(f(x)) + c$; then

(c denotes constant of integration)

Options:

A.

$f(x) = \sqrt{x}, g(x) = x^{\frac{3}{2}}$

B.

$f(x) = x^{\frac{3}{2}}, g(x) = \sin^{-1} x$

C.

$f(x) = \sqrt{x}, g(x) = \sin^{-1} x$

D.

$f(x) = \sin^{-1} x, g(x) = x^{\frac{3}{2}}$

Answer: B

Solution:

$I=\int \frac{x^{1/2}}{\sqrt{1-x^3}}dx$

Let $x^{3/2}=t$

$\Rightarrow \frac{3}{2}x^{1/2}dx=dt$

$\therefore I=\int \frac{2}{3}\cdot\frac{dt}{\sqrt{1-t^2}}$

$=\frac{2}{3}\sin^{-1}t+C$

$=\frac{2}{3}\sin^{-1}(x^{3/2})+C$

$=\frac{2}{3}g(f(x))+C$

$\therefore f(x)=x^{3/2}$ and $g(x)=\sin^{-1}(x)$

Question 10

The value of $\int\limits_0^{\frac{\pi}{2}} \frac{(\cos x)^{\sin x}}{(\cos x)^{\sin x}+(\sin x)^{\cos x}}dx$ is

Options:

A.

$\frac{\pi}{4}$

B.

0

C.

$\frac{\pi}{2}$

D.

$\frac{1}{2}$

Answer: A

Solution:

$I=\int\limits_0^{\frac{\pi}{2}} \frac{(\cos x)^{\sin x}}{(\cos x)^{\sin x}+(\sin x)^{\cos x}}dx$ ..... (1)

Replace x with $\left(\frac{\pi}{2}-x\right)$,

$$\therefore I=\int_0^{\frac{\pi}{2}} \frac{\left[\cos\left(\frac{\pi}{2}-x\right)\right]^{\sin\left(\frac{\pi}{2}-x\right)}}{\left[\cos\left(\frac{\pi}{2}-x\right)\right]^{\sin\left(\frac{\pi}{2}-x\right)}+\left[\sin\left(\frac{\pi}{2}-x\right)\right]^{\cos\left(\frac{\pi}{2}-x\right)}} dx$$

$$I=\int_0^{\frac{\pi}{2}} \frac{(\sin x)^{\cos x}}{(\sin x)^{\cos x}+(\cos x)^{\sin x}} dx \text{ ...... (2)}$$

Adding 1 and 2 we get,

$$2I=\int_0^{\frac{\pi}{2}} \frac{(\sin x)^{\cos x}+(\cos x)^{\sin x}}{(\sin x)^{\cos x}+(\cos x)^{\sin x}} dx$$

$$=\int_0^{\frac{\pi}{2}} 1\, dx$$

$$=\frac{\pi}{2}$$

$$\Rightarrow I=\frac{\pi}{4}$$

Question 11

Let $\lim\limits_{\in\to 0+} \int_{\in}^{x} \frac{bt\cos 4t-a\sin 4t}{t^2} dt=\frac{a\sin 4x}{x}-1,\left(0<x<\frac{\pi}{4}\right)$. Then a and b are given by

Options:

A.

$a=2, b=2$

B.

$a=\frac{1}{4}, b=1$

C.

$a=-1, b=4$

D.

$a=2, b=4$

Answer: B

Solution:

$\lim_{\in \to 0^+} \int_{\in}^{x} \frac{bt\cos 4t - a\sin 4t}{t^2} dt = \frac{a\sin 4x}{x} - 1$

$\Rightarrow \int_{0^+}^{x} \frac{bt\cos 4t - a\sin 4t}{t^2} dt = \frac{a\sin 4x}{x} - 1$

Differentiating both sides we get,

$\frac{bx\cos 4x - a\sin 4x}{x^2} = \frac{xa\cos 4x \times 4 - a\sin 4x}{x^2}$

$\Rightarrow bx\cos 4x - a\sin 4x = 4ax\cos 4x - a\sin 4x$

Comparing coefficient of cos 4x and sin 4x both side we get,

b = 4a an −a = −a

$\therefore \frac{b}{a} = 4$

By checking options, when $a = \frac{1}{4}$ and b = 1, then,

$\frac{b}{a} = 4$

Question 12

Let $f(x) = \int\limits_{\sin x}^{\cos x} e^{-t^2} dt$. Then $f'\left(\frac{\pi}{4}\right)$ equals

Options:

A.

$\sqrt{\frac{1}{e}}$

B.

$-\sqrt{\frac{2}{e}}$

C.

$\sqrt{\frac{2}{e}}$

D.

$-\sqrt{\frac{1}{e}}$

Answer: B

Solution:

$f(x) = \int_{\sin x}^{\cos x} e^{-t^2} dt$

Differentiating both sides using Newton Leibnitz formula,

$f'(x) = e^{-\cos^2 x} . (-\sin x) - e^{-\sin^2 x} . (\cos x)$

$\therefore f'\left(\frac{\pi}{4}\right) = e^{-\frac{1}{2}} . \left(-\frac{1}{\sqrt{2}}\right) - e^{-\frac{1}{2}} . \left(\frac{1}{\sqrt{2}}\right)$

$= -\frac{2}{\sqrt{2}} e^{-\frac{1}{2}}$

$= -\sqrt{2} e^{-\frac{1}{2}}$

$= -\sqrt{\frac{2}{e}}$

Question 13

If $x\frac{dy}{dx} + y = x\frac{f(xy)}{f'(xy)}$, then $|f(xy)|$ is equal to

Options:

A.

$Ce^{\frac{x^2}{2}}$ (where C is the constant of integration)

B.

Ce^{x^2} (where C is the constant of integration)

C.

Ce^{2x^2} (where C is the constant of integration)

D.

$Ce^{\frac{x^2}{3}}$ (where C is the constant of integration)

Answer: A

Solution:

Given, $x\frac{dy}{dx}+y=x\,.\,\frac{f(xy)}{f'(xy)}$

Let, $xy=t$

Differentiating both sides with respect to x,

$y+x\frac{dy}{dx}=\frac{dt}{dx}$

$\therefore\frac{dt}{dx}=x\,.\,\frac{f(t)}{f'(t)}$

$\Rightarrow\frac{f'(t)}{f(t)}dt=x\,dx$

Integrating both sides, we get

$\int\frac{f'(t)}{f(t)}dt=\int x\,dx$

$\Rightarrow\ln|f(t)|=\frac{x^2}{2}+C$

$\Rightarrow|f(t)|=e^{\frac{x^2}{2}+C}$

$\Rightarrow|f(xy)|=e^{\frac{x^2}{2}}.\,e^2$

$\Rightarrow|f(xy)|=e^{\frac{x^2}{2}}\,.\,C$ [e^C = constant = C]

Question 14

A curve passes through the point (3, 2) for which the segment of the tangent line contained between the co-ordinate axes is bisected at the point of contact. The equation of the curve is

Options:

A.

$y=x^2-7$

B.

$x=\frac{y^2}{2}+2$

C.

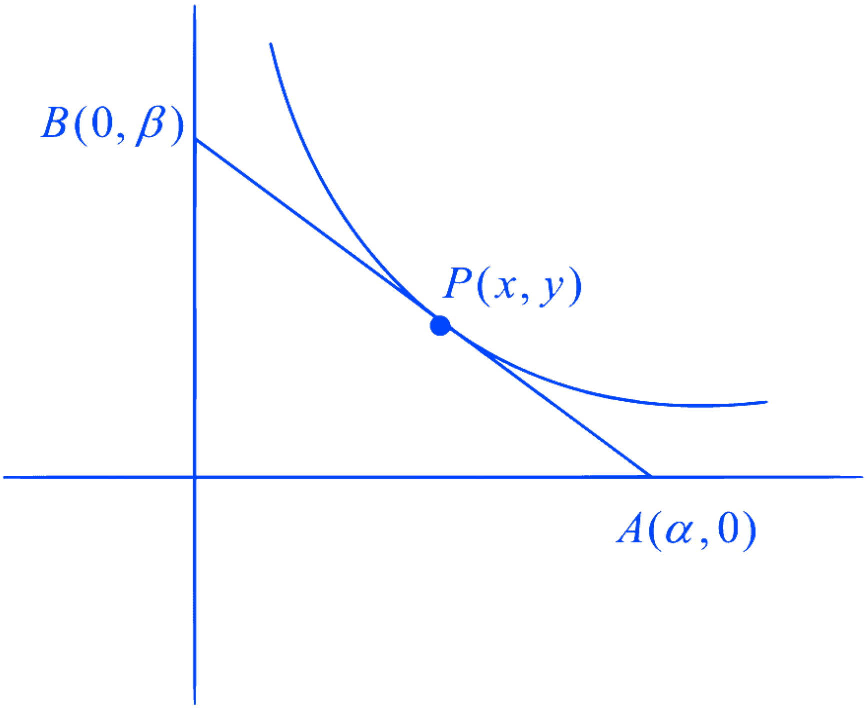
$xy=6$

D.

$x^2+y^2-5x+7y+11=0$

Answer: C

Solution:



According to the question, p(x, y) is the midpoint of line AB.

$\therefore \frac{\alpha+0}{2} = x \Rightarrow \alpha = 2x$

$\frac{\beta+0}{2} = y \Rightarrow \beta = 2y$

$\therefore$ Point A = (2x, 0)

and Point B = (0, 2y)

Slope of the tangent,

$\frac{dy}{dx} = \frac{2y-0}{0-2x}$

$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$

$\Rightarrow x\,dy + y\,dx = 0$

$\Rightarrow d(xy) = 0$

$\Rightarrow xy = c$

This curve goes through point (3, 2). So this point satisfy the equation.

∴ 3 . 2 = c

⇒ c = 6

∴ Equation of the curve,

xy = 6

Question 15

The solution of

$\cos y \frac{dy}{dx} = e^{x+\sin y} + x^2 e^{\sin y}$ is $f(x) + e^{-\sin y} = C$ (C is arbitrary real constant) where f(x) is equal to

Options:

A.

$e^x + \frac{1}{2}x^3$

B.

$e^{-x} + \frac{1}{3}x^3$

C.

$e^{-x} + \frac{1}{2}x^3$

D.

$e^x + \frac{1}{3}x^3$

Answer: D

Question 16

The point of contact of the tangent to the parabola $y^2 = 9x$ which passes through the point (4, 10) and makes an angle θ with the

positive side of the axis of the parabola where tanθ > 2, is

Options:

A.

$\left(\frac{4}{9}, 2\right)$

B.

(4, 6)

C.

(4, 5)

D.

$\left(\frac{1}{4}, \frac{1}{6}\right)$

Answer: A

Question 17

Let $f(x) = (x-2)^{17}(x+5)^{24}$. Then

Options:

A.

f does not have a critical point at x = 2

B.

f has a minimum at x = 2

C.

f has neither a maximum nor a minimum at x = 2

D.

f has a minimum at x = 2

Answer: C

Question 18

If $\overrightarrow{a} = \hat{i} + \hat{j} - \widehat{k}$, $\overrightarrow{b} = \hat{i} - \hat{j} + \widehat{k}$ and $\overrightarrow{c}$ is unit vector perpendicular to $\overrightarrow{a}$ and coplanar with $\overrightarrow{a}$ and $\overrightarrow{b}$, then unit vector $\overrightarrow{d}$ perpendicular to both $\overrightarrow{a}$ and $\overrightarrow{c}$ is

Options:

A.

$\pm\frac{1}{\sqrt{6}}\left(2\hat{i} - \hat{j} + \widehat{k}\right)$

B.

$\pm\frac{1}{\sqrt{2}}\left(\hat{j} + \widehat{k}\right)$

C.

$\pm\frac{1}{\sqrt{6}}\left(\hat{i} - 2\hat{j} + \widehat{k}\right)$

D.

$\pm\frac{1}{\sqrt{2}}\left(\hat{j} - \widehat{k}\right)$

Answer: B

Question 19

If the equation of one tangent to the circle with centre at (2, −1) from the origin is 3x + y = 0, then the equation of the other tangent through the origin is

Options:

A.

$3x - y = 0$

B.

$x + 3y = 0$

C.

$x - 3y = 0$

D.

$x + 2y = 0$

Answer: C

Question 20

Area of the figure bounded by the parabola $y^2 + 8x = 16$ and $y^2 - 24x = 48$ is

Options:

A.

$\frac{11}{9}$ sq. unit

B.

$\frac{32}{3}\sqrt{6}$ sq. unit

C.

$\frac{16}{3}$ sq. unit

D.

$\frac{24}{5}$ sq. unit

Answer: B

Question 21

A particle moving in a straight line starts from rest and the acceleration at any time t is $a - kt^2$ where a and k are positive constants. The maximum velocity attained by the particle is

Options:

A.

$\frac{2}{3}\sqrt{\frac{a^3}{k}}$

B.

$\frac{1}{3}\sqrt{\frac{a^3}{k}}$

C.

$\sqrt{\frac{a^3}{k}}$

D.

$2\sqrt{\frac{a^3}{k}}$

Answer: A

Question 22

If a, b, c are in G.P. and log a − log 2b, log 2b − log 3c, log 3c − log a are in A.P., then a, b, c are the lengths of the sides of a triangle which is

Options:

A.

acute angled

B.

obtuse angled

C.

right angled

D.

equilateral

Answer: B

Question 23

Let $a_n = (1^2 + 2^2 + \ldots\ldots n^2)^n$ and $b_n = n^n(n!)$. Then

Options:

A.

$a_n < b_n \forall n$

B.

$a_n > b_n \forall n$

C.

$a_n = b_n$ for infinitely many n

D.

$a_n < b_n$ if n be even and $a_n > b_n$ if n be odd

Answer: B

Question 24

The number of zeros at the end of $\underline{|100}$ is

Options:

A.

21

B.

22

C.

23

D.

24

Question 25

If $|z-25i| \leq 15$, then Maximum arg(z) $-$ Minimum arg(z) is equal to

(arg z is the principal value of argument of z)

Options:

A.

$2\cos^{-1}\left(\frac{3}{5}\right)$

B.

$2\cos^{-1}\left(\frac{4}{5}\right)$

C.

$\frac{\pi}{2}+\cos^{-1}\left(\frac{3}{5}\right)$

D.

$\sin^{-1}\left(\frac{3}{5}\right)-\cos^{-1}\left(\frac{3}{5}\right)$

Answer: B

Question 26

If z = x $-$ iy and $z^{\frac{1}{3}}=p+iq(x,y,p,q\in R)$, then $\frac{\left(\frac{x}{p}+\frac{y}{q}\right)}{(p^2+q^2)}$ is equal to

Options:

A.

2

B.

-1

C.

1

D.

-2

Answer: D

Question 27

If a, b are odd integers, then the roots of the equation $2ax^2 + (2a + b)x + b = 0, a \neq 0$ are

Options:

A.

rational

B.

irrational

C.

non-real

D.

equal

Answer: A

Question 28

There are n white and n black balls marked 1, 2, 3, ... n. The number of ways in which we can arrange these balls in a row so that neighbouring balls are of different colours is

Options:

A.

$(n!)^2$

B.

$(2n)!$

C.

$2(n!)^2$

D.

$\frac{(2n)!}{(n!)^2}$

Answer: C

Question 29

Let $f(n) = 2^{n+1}, g(n) = 1 + (n + 1)2^n$ for all $n \in N$. Then

Options:

A.

$f(n) > g(n)$

B.

$f(n) < g(n)$

C.

f(n) and g(n) are not comparable.

D.

$f(n) > g(n)$ if n be even and $f(n) < g(n)$ if n be odd.

Answer: B

Question 30

A is a set containing n elements. P and Q are two subsets of A. Then the number of ways of choosing P and Q so that P ∩ Q = φ is

Options:

A.

$2^{2n\_2n}C_n$

B.

2^n

C.

$3^n - 1$

D.

3^n

Answer: D

Question 31

Under which of the following condition(s) does(do) the system of equations $\begin{pmatrix} 1 & 2 & 4 \\ 2 & 1 & 2 \\ 1 & 2 & (a-4) \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \\ a \end{pmatrix}$ **possesses(possess) unique solution ?**

Options:

A.

∀ a ∈ R

B.

a = 8

C.

for all integral values of a

D.

a $\neq$ 8

Answer: D

Question 32

If $\Delta(x) = \begin{vmatrix} x-2 & (x-1)^2 & x^3 \\ x-1 & x^2 & (x+1)^3 \\ x & (x+1)^2 & (x+2)^3 \end{vmatrix}$**, then coefficient of x in Δx is**

Options:

A.

2

B.

-2

C.

3

D.

-4

Answer: B

Question 33

If $p = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ **is the adjoint of the** 3×3 **matrix A and det A = 4, then** α **is equal to**

Options:

A.

4

B.

11

C.

5

D.

0

Answer: B

Question 34

If $A = \begin{pmatrix} 1 & 1 \\ 0 & i \end{pmatrix}$ and $A^{2018} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $(a + d)$ equals

Options:

A.

1 + i

B.

0

C.

2

D.

2018

Answer: B

Question 35

Let S, T, U be three non-void sets and f : S → T, g : T → U and composed mapping g . f : S → U be defined. Let g . f be injective mapping. Then

Options:

A.

f, g both are injective.

B.

neither f nor g is injective.

C.

f is obviously injective.

D.

g is obviously injective.

Answer: C

Question 36

For the mapping $f : R - \{1\} \rightarrow R - \{2\}$, given by $f(x) = \frac{2x}{x-1}$, which of the following is correct?

Options:

A.

f is one-one but not onto

B.

f is onto but not one-one

C.

f is neither one-one nor onto

D.

f is both one-one and onto

Question 37

A, B, C are mutually exclusive events such that $P(A) = \frac{3x+1}{3}$, $P(B) = \frac{1-x}{4}$ and $P(C) = \frac{1-2x}{2}$. Then the set of possible values of x are in

Options:

A.

[0, 1]

B.

$\left[\frac{1}{3}, \frac{1}{2}\right]$

C.

$\left[\frac{1}{3}, \frac{2}{3}\right]$

D.

$\left[\frac{1}{3}, \frac{13}{3}\right]$

Answer: B

Question 38

A determinant is chosen at random from the set of all determinants of order 2 with elements 0 or 1 only. The probability that the determinant chosen is non-zero is

Options:

A.

$\frac{3}{16}$

B.

$\frac{3}{8}$

C.

$\frac{1}{4}$

D.

$\frac{5}{8}$

Answer: C

Question 39

If $(\cot\alpha_1)(\cot\alpha_2)\ldots\ldots(\cot\alpha_n)=1, 0<\alpha_1,\alpha_2,\ldots.\alpha_n<\pi/2$, then the maximum value of $(\cos\alpha_1)(\cos\alpha_2)\ldots..(\cos\alpha_n)$ is given by

Options:

A.

$\frac{1}{2^{n/2}}$

B.

$\frac{1}{2^n}$

C.

$\frac{1}{2n}$

D.

1

Answer: A

Question 40

If the algebraic sum of the distances from the points (2, 0), (0, 2) and (1, 1) to a variable straight line be zero, then the line passes through the fixed point

Options:

A.

$(-1, 1)$

B.

$(1, -1)$

C.

$(-1, -1)$

D.

$(1, 1)$

Answer: D

Question 41

The side AB of ΔABC is fixed and is of length 2a unit. The vertex moves in the plane such that the vertical angle is always constant and is α. Let x-axis be along AB and the origin be at A. Then the locus of the vertex is

Options:

A.

$x^2 + y^2 + 2ax \sin \alpha + a^2 \cos \alpha = 0$

B.

$x^2 + y^2 - 2ax - 2ay \cot \alpha = 0$

C.

$x^2 + y^2 - 2ax \cos \alpha - a^2 = 0$

D.

$x^2 + y^2 - ax \sin \alpha - ay \cos \alpha = 0$

Answer: B

Question 42

If the sum of the distances of a point from two perpendicular lines in a plane is 1 unit, then its locus is

Options:

A.

a square

B.

a circle

C.

a straight line

D.

two intersecting lines

Answer: A

Question 43

A line passes through the point $(-1, 1)$ and makes an angle $\sin^{-1}\left(\frac{3}{5}\right)$ in the positive direction of x-axis. If this line meets the curve $x^2 = 4y - 9$ at A and B, then |AB| is equal to

Options:

A.

$\frac{4}{5}$ unit

B.

$\frac{5}{4}$ unit

C.

$\frac{3}{5}$ unit

D.

$\frac{5}{3}$ unit

Answer: B

Question 44

Two circles $S_1 = px^2 + py^2 + 2g'x + 2f'y + d = 0$ and $S_2 = x^2 + y^2 + 2gx + 2fy + d' = 0$ have a common chord PQ. The equation of PQ is

Options:

A.

$S_1 - S_2 = 0$

B.

$S_1 + S_2 = 0$

C.

$S_1 - pS_2 = 0$

D.

$S_1 + pS_2 = 0$

Answer: C

Question 45

Let $P(3\sec\theta, 2\tan\theta)$ and $Q(3\sec\phi, 2\tan\phi)$ be two points on $\frac{x^2}{9} - \frac{y^2}{4} = 1$ such that $\theta + \phi = \frac{\pi}{2}, 0 < \theta, \phi < \frac{\pi}{2}$. Then the ordinate of the point of intersection of the normals at P and Q is

Options:

A.

$\frac{13}{2}$

B.

$-\frac{13}{2}$

C.

$\frac{5}{2}$

D.

$-\frac{5}{2}$

Answer: A

Question 46

Let P be a point on (2, 0) and Q be a variable point on $(y - 6)^2 = 2(x - 4)$. Then the locus of mid-point of PQ is

Options:

A.

$y^2 + x + 6y + 12 = 0$

B.

$y^2 - x + 6y + 12 = 0$

C.

$y^2 + x - 6y + 12 = 0$

D.

$y^2 - x - 6y + 12 = 0$

Answer: D

Question 47

AB is a chord of a parabola $y^2 = 4ax$, $(a > 0)$ with vertex A. BC is drawn perpendicular to AB meeting the axis at C. The projection of BC on the axis of the parabola is

Options:

A.

a unit

B.

2a unit

C.

8a unit

D.

4a unit

Answer: D

Question 48

AB is a variable chord of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. If AB subtends a right angle at the origin O, then $\frac{1}{OA^2} + \frac{1}{OB^2}$ equals to

Options:

A.

$\frac{1}{a^2} + \frac{1}{b^2}$

B.

$\frac{1}{a^2} - \frac{1}{b^2}$

C.

$a^2 + b^2$

D.

$a^2 - b^2$

Answer: A

Question 49

The equation of the plane through the intersection of the planes x + y + z = 1 and 2x + 3y − z + 4 = 0 and parallel to the x-axis is

Options:

A.

y + 3z + 6 = 0

B.

y + 3z − 6 = 0

C.

y − 3z + 6 = 0

D.

y − 3z − 6 = 0

Answer: C

Question 50

The line $x - 2y + 4z + 4 = 0, x + y + z - 8 = 0$ intersect the plane $x - y + 2z + 1 = 0$ at the point

Options:

A.

$(-2, 5, 1)$

B.

$(2, -5, 1)$

C.

$(2, 5, -1)$

D.

$(2, 5, 1)$

Answer: D

Question 51

If I is the greatest of $I_1 = \int_0^1 e^{-x}\cos^2 x\, dx$, $I_2 = \int_0^1 e^{-x^2}\cos^2 x\, dx$, $I_3 = \int_0^1 e^{-x^2} dx$, $I_4 = \int_0^1 e^{-x^2/2} dx$, then

Options:

A.

I = I_1

B.

I = I_2

C.

I = I_3

D.

I = I_4

Answer: D

Question 52

$\lim_{x \to \infty} \left(\frac{x^2+1}{x+1} - ax - b\right), (a, b \in R)$ **= 0. Then**

Options:

A.

a = 0, b = 1

B.

a = 1, b = −1

C.

a = −1, b = 1

D.

a = 0, b = 0

Answer: B

Question 53

If the transformation $z = \log\tan\frac{x}{2}$ reduces the differential equation $\frac{d^2y}{dx^2} + \cot x\frac{dy}{dx} + 4y\cos ec^2x = 0$ into the form $\frac{d^2y}{dz^2} + ky = 0$ then k is equal to

Options:

A.

-4

B.

4

C.

2

D.

-2

Answer: B

Question 54

From the point $(-1, -6)$, two tangents are drawn to $y^2 = 4x$. Then the angle between the two tangents is

Options:

A.

$\pi/3$

B.

$\pi/4$

C.

$\pi/6$

D.

$\pi/2$

Answer: D

Question 55

If $\overrightarrow{\alpha}$ is a unit vector, $\overrightarrow{\beta} = \hat{i} + \hat{j} - \widehat{k}$, $\overrightarrow{\gamma} = \hat{i} + \widehat{k}$ then the maximum value of $\left[\overrightarrow{\alpha}\overrightarrow{\beta}\overrightarrow{\gamma}\right]$ is

Options:

A.

3

B.

$\sqrt{3}$

C.

2

D.

$\sqrt{6}$

Answer: D

Question 56

The maximum value of $f(x) = e^{\sin x} + e^{\cos x}; x \in R$ is

Options:

A.

2e

B.

$2\sqrt{e}$

C.

$2e^{\frac{1}{\sqrt{2}}}$

D.

$2e^{-\frac{1}{\sqrt{2}}}$

Answer: C

Question 57

A straight line meets the co-ordinate axes at A and B. A circle is circumscribed about the triangle OAB, O being the origin. If m and n are the distances of the tangent to the circle at the origin from the points A and B respectively, the diameter of the circle is

Options:

A.

m(m + n)

B.

m + n

C.

n(m + n)

D.

$\frac{1}{2}(m + n)$

Answer: B

Question 58

Let the tangent and normal at any point $P(at^2, 2at)$, $(a > 0)$, on the parabola $y^2 = 4ax$ meet the axis of the parabola at T and G respectively. Then the radius of the circle through P, T and G is

Options:

A.

$a(1 + t^2)$

B.

$(1 + t)^2$

C.

$a(1 - t^2)$

D.

$(1 - t^2)$

Answer: A

Question 59

The value of a for which the sum of the squares of the roots of the equation $x^2 - (a-2)x - a - 1 = 0$ assumes the least value is

Options:

A.

0

B.

1

C.

2

D.

3

Answer: B

Question 60

If x satisfies the inequality $\log_{25} x^2 + (\log_5 x)^2 < 2$, then x belongs to

Options:

A.

$\left(\frac{1}{5}, 5\right)$

B.

$\left(\frac{1}{25}, 5\right)$

C.

$\left(\frac{1}{5}, 25\right)$

D.

$\left(\frac{1}{25}, 25\right)$

Answer: B

Question 61

The solution of $\det(A - \lambda I_2) = 0$ be 4 and 8 and $A = \begin{pmatrix} 2 & 2 \\ x & y \end{pmatrix}$. Then

(I_2 is identity matrix of order 2)

Options:

A.

$x = 4, y = 10$

B.

$x = 5, y = 8$

C.

$x = 3, y = 9$

D.

$x = -4, y = 10$

Answer: D

Question 62

If P_1P_2 and P_3P_4 are two focal chords of the parabola $y^2 = 4ax$ then the chords P_1P_3 and P_2P_4 intersect on the

Options:

A.

directrix of the parabola

B.

axis of the parabola

C.

latus-rectum of the parabola

D.

y-axis

Answer: A

Question 63

$f : X \to R, X = \{x|0 < x < 1\}$ is defined as $f(x) = \frac{2x-1}{1-|2x-1|}$. Then

Options:

A.

f is only injective

B.

f is only surjective

C.

f is bijective

D.

f is neither injective nor surjective

Answer: A

Question 64

Let f be a non-negative function defined in $[0, \pi/2]$, f' exists and be continuous for all x and $\int_0^x \sqrt{1-(f'(t))^2}dt = \int_0^x f(t)dt$ and f (0) = 0. Then

Options:

A.

$f\left(\frac{1}{2}\right)<\frac{1}{2}$ and $f\left(\frac{1}{3}\right)<\frac{1}{3}$

B.

$f\left(\frac{1}{2}\right)>\frac{1}{2}$ and $f\left(\frac{1}{3}\right)<\frac{1}{3}$

C.

$f\left(\frac{4}{3}\right)<\frac{4}{3}$ and $f\left(\frac{2}{3}\right)<\frac{2}{3}$

D.

$f\left(\frac{4}{3}\right)>\frac{4}{3}$ and $f\left(\frac{2}{3}\right)>\frac{2}{3}$

Answer: C

Question 65

PQ is a double ordinate of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ such that ΔOPQ is an equilateral triangle, O being the centre of the hyperbola. Then the eccentricity e of the hyperbola satisfies

Options:

A.

$1<e<\frac{2}{\sqrt{3}}$

B.

$e=\frac{2}{\sqrt{3}}$

C.

$e=2\sqrt{3}$

D.

$e>\frac{2}{\sqrt{3}}$

Answer: D

Question 66

From a balloon rising vertically with uniform velocity v ft/sec a piece of stone is let go. The height of the balloon above the ground when the stone reaches the ground after 4 sec is [g = 32 ft/sec^2]

Options:

A.

220 ft

B.

240 ft

C.

256 ft

D.

260 ft

Answer: C

Question 67

Let $f(x) = x^2 + x \sin x - \cos x$. Then

Options:

A.

f(x) = 0 has at least one real root

B.

f(x) = 0 has no real root

C.

f(x) = 0 has at least one positive root

D.

f(x) = 0 has at least one negative root

Answer: D

Question 68

Let z_1 and z_2 be two non-zero complex numbers. Then

Options:

A.

Principal value of $\arg(z_1z_2)$ may not be equal to Principal value of $\arg z_1$ + Principal value of $\arg z_2$

B.

Principal value of $\arg(z_1z_2)$ = Principal value of $\arg z_1$ + Principal value of $\arg z_2$

C.

Principal value of $\arg(z_1/z_2)$ = Principal value of $\arg z_1$ − Principal value of $\arg z_2$

D.

Principal value of $\arg(z_1/z_2)$ may not be $\arg z_1 - \arg z_2$

Answer: D

Question 69

Let $\Delta = \begin{vmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\theta\sin\phi & \sin\theta\cos\phi & 0 \end{vmatrix}$. Then

Options:

A.

Δ is independent of θ

B.

Δ is independent of φ

C.

Δ is a constant

D.

$\left(\frac{d\Delta}{d\theta}\right)_{\theta=\frac{\pi}{2}} = 0$

Answer: D

Question 70

Let R and S be two equivalence relations on a non-void set A. Then

Options:

A.

R ∪ S is equivalence relation

B.

R ∩ S is equivalence relation

C.

R ∩ S is not equivalence relation

D.

R ∪ S is not equivalence relation

Answer: B

Question 71

Chords of an ellipse are drawn through the positive end of the minor axis. Their midpoint lies on

Options:

A.

a circle

B.

a parabola

C.

an ellipse

D.

a hyperbola

Answer: C

Question 72

Consider the equation $y - y_1 = m(x - x_1)$. If m and x_1 are fixed and different lines are drawn for different values of y_1, then

Options:

A.

the lines will pass through a fixed point

B.

there will be a set of parallel lines

C.

all lines intersect the line $x = x_1$

D.

all lines will be parallel to the line $y = x_1$

Answer: C

Question 73

Let p(x) be a polynomial with real co-efficient, p(0) = 1 and p'(x) > 0 for all x ∈ R. Then

Options:

A.

p(x) has at least two real roots

B.

p(x) has only one positive real root

C.

p(x) may have negative real root

D.

p(x) has infinitely many real roots

Answer: C

Question 74

Twenty metres of wire is available to fence off a flower bed in the form of a circular sector. What must the radius of the circle be, if the area of the flower bed be greatest?

Options:

A.

10 m

B.

4 m

C.

5 m

D.

6 m

Answer: C

Question 75

The line y = x + 5 touches

Options:

A.

the parabola $y^2 = 20$ x

B.

the ellipse $9x^2 + 16y^2 = 144$

C.

the hyperbola $\frac{x^2}{29} - \frac{y^2}{4} = 1$

D.

the circle $x^2 + y^2 = 25$

Answer: C

Chemistry

Question 76

A sample of $MgCO_3$ is dissolved in dil. HCl and the solution is neutralized with ammonia and buffered with NH_4Cl / NH_4OH. Disodium hydrogen phosphate reagent is added to the resulting solution. A white precipitate is formed. What is the formula of the precipitate?

Options:

A.

$Mg_3(PO_4)_2$

B.

$Mg(NH_4)PO_4$

C.

$MgHPO_4$

D.

$Mg_2P_2O_7$

Answer: B

Question 77

XeF_2, NO_2, HCN, ClO_2, CO_2.

Identify the non-linear molecule-pair from the above mentioned molecules.

Options:

A.

XeF_2, ClO_2

B.

CO_2, NO_2

C.

HCN, NO_2

D.

ClO_2, NO_2

Answer: D

Question 78

The number of atoms in body centred and face centred cubic unit cell respectively are

Options:

A.

2 and 4

B.

4 and 3

C.

1 and 2

D.

4 and 6

Answer: A

Question 79

The number of unpaired electron in Mn^{2+} ion is

Options:

A.

2

B.

3

C.

5

D.

6

Answer: C

Question 80

The average speed of H_2 at T_1K is equal to that of O_2 at T_2K. The ratio $T_1 : T_2$ is

Options:

A.

1 : 16

B.

16 : 1

C.

1 : 4

D.

1 : 1

Answer: A

Question 81

Sodium nitroprusside is :

Options:

A.

$Na_4[Fe(CN)_5NO_2]$

B.

$Na_2[Fe(CN)_5NO]$

C.

$Na_3[Fe(CN)_5NO]$

D.

$Na_4[Fe(CN)_5NO_3]$

Answer: B

Question 82

Choose the correct statement for the $[Ni(CN)_4]^{2-}$ complex ion (Atomic no. of Ni = 28)

Options:

A.

The complex is square planar and paramagnetic.

B.

The complex is tetrahedral and diamagnetic.

C.

The complex is square planar and diamagnetic.

D.

The complex is tetrahedral and paramagnetic.

Answer: C

Question 83

The boiling point of the water is higher than liquid HF. The reason is that

Options:

A.

Hydrogen bonds are stronger in water.

B.

Hydrogen bonds are stronger in HF.

C.

Hydrogen bonds are larger in number in HF.

D.

Hydrogen bonds are larger in number in water.

Answer: D

Question 84

The metal-pair that can produce nascent hydrogen in alkaline medium is :

Options:

A.

Zn, Al

B.

Fe, Ni

C.

Al, Mg

D.

Mg, Zn

Answer: A

Question 85

The correct bond order of B-F bond in BF_3 molecule is :

Options:

A.

1

B.

$1\frac{1}{2}$

C.

2

D.

$1\frac{1}{3}$

Answer: D

Question 86

Which of the following is radioactive?

Options:

A.

Hydrogen

B.

Deuterium

C.

Tritium

D.

none

Answer: C

Question 87

The correct order of acidity of the following hydra acids is

Options:

A.

HF > HCl > HBr > HI

B.

HF < HCl < HBr < HI

C.

HF < HCl > HBr > HI

D.

HF > HCl < HBr > HI

Answer: B

Question 88

To a solution of colourless sodium salt, a solution of lead nitrate was added to have a white precipitate which dissolves in warm water and reprecipitates on cooling. Which of the following acid radical is present in the salt?

Options:

A.

Cl^-

B.

SO_4^{2-}

C.

S^{2-}

D.

NO_3^-

Answer: A

Question 89

Oxidation states of Cr in $K_2Cr_2O_7$ and CrO_5 are respectively

Options:

A.

+6, +5

B.

+6, +10

C.

+6, +6

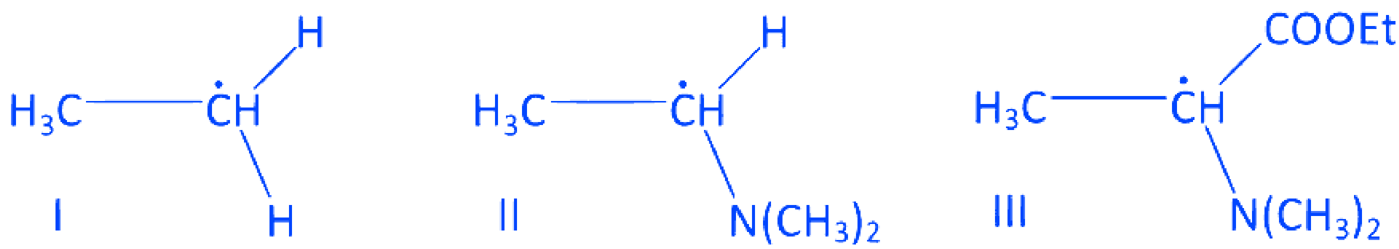
D.

None of these

Answer: C

Question 90

The correct order of relative stability for the given free radicals is :



Options:

A.

II > I > III

B.

II > III > I

C.

III > I > II

D.

III > II > I

Answer: D

Question 91

Image

Hybridisation of the negative carbons in (1) and (2) are

Options:

A.

sp^2 and sp^3

B.

sp^3 and sp^2

C.

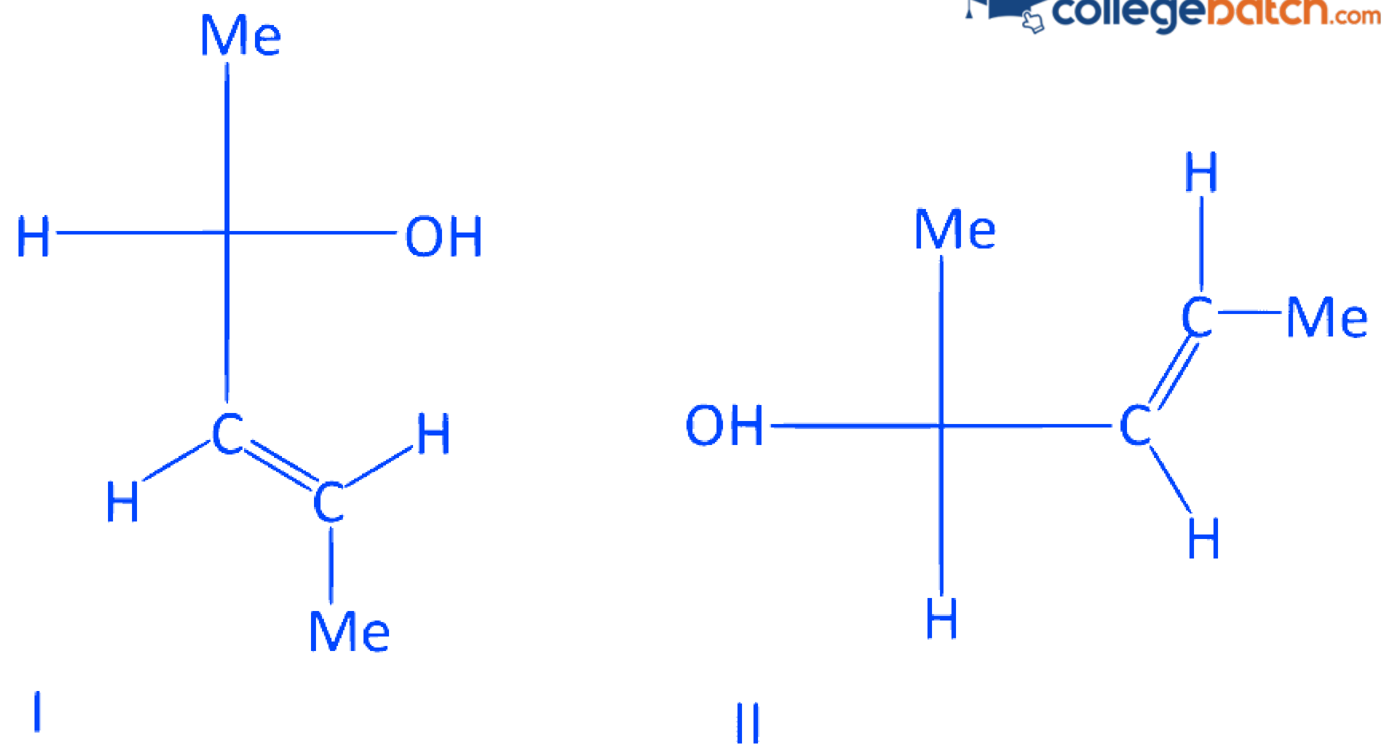
both sp^2

D.

both sp^3

Answer: B

Question 92



The correct relationship between molecules I and II is

Options:

A.

Enantiomer

B.

Homomer

C.

Diastereomer

D.

Constitutional isomer

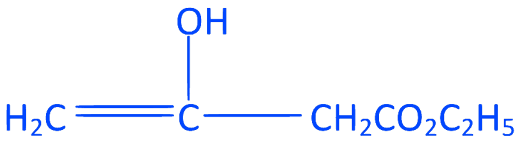
Answer: B

Question 93

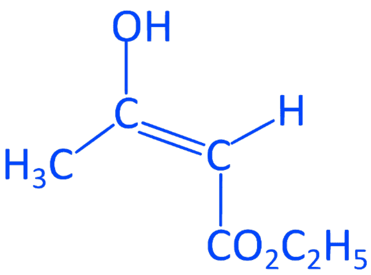
The enol form in which ethyle-3-oxobutanoate exists is

Options:

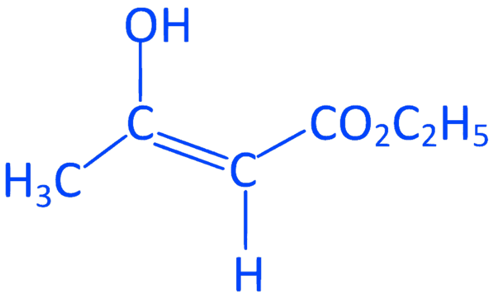
A.



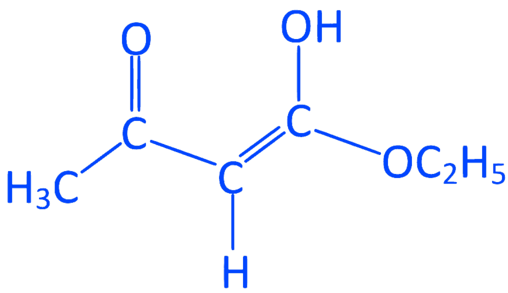
B.



C.



D.



Answer: C

Question 94

How many monobriminated product(s) (including stereoisomers) would form in the free radical bromination of n-butane?

Options:

A.

2

B.

1

C.

3

D.

4

Answer: C

Question 95

What is the correct order of acidity of salicylic acid, 4-hydroxybenzoic acid, and 2, 6-dihydroxybenzoic acid ?

Options:

A.

2, 6-dihydroxybenzoic acid > salicylic acid > 4-hydroxybenzoic acid

B.

2, 6-dihydroxybenzoic acid > 4-hydroxybenzoic acid > salicylic acid

C.

salicylic acid > 2, 6-dihydroxybenzoic acid > 4-hydroxybenzoic acid

D.

salicylic acid > 4-hydroxybenzoic acid > 2, 6-dihydroxybenzoic acid

Answer: A

Question 96

How much solid oxalic acid (Molecular weight 126) has to be weighed to prepare 100 ml. exactly 0.1 (N) oxalic acid solution in water?

Options:

A.

1.26 g

B.

0.126 g

C.

0.63 g

D.

0.063 g

Answer: C

Question 97

The major product of the following reaction is

$$F_3C - CH = CH_2 + HBr \rightarrow$$

Options:

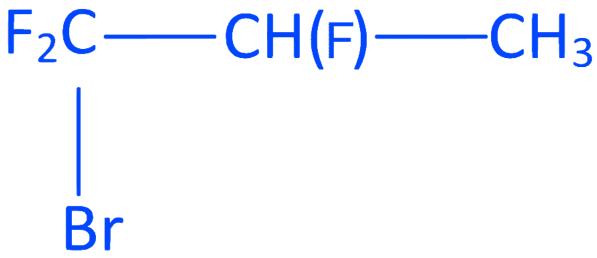
A.

$F_3C - CH_2 - CH_2Br$

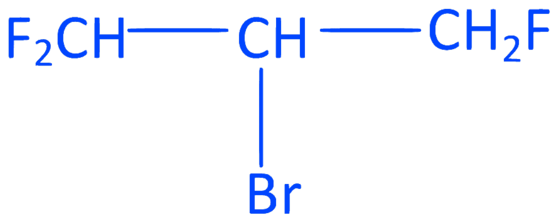
B.

$F_3C - CH(Br) - CH_3$

C.



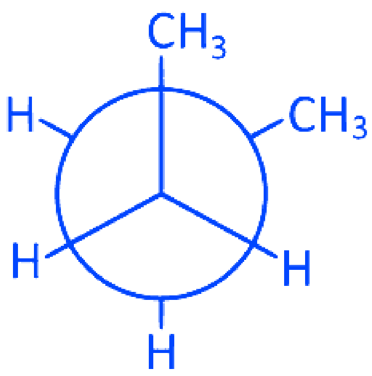
D.



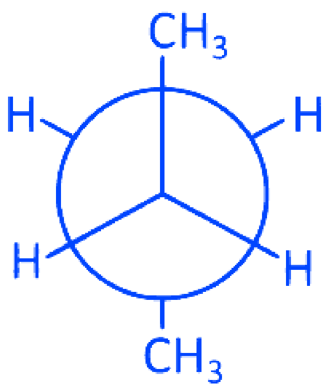
Answer: A

Question 98

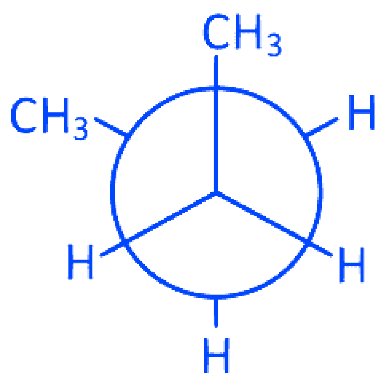
The correct order of relative stability of the given conformers of n-butane is



I



II



III

Options:

A.

II > I = III

B.

II > III > I

C.

II > I > III

D.

I = III > II

Answer: A

Question 99

$$C_6H_6(liq) + \frac{15}{2}O_2(g) \rightarrow 6CO_2(g) + 3H_2O(liq)$$

Benzene burns in oxygen according to the above equation. What is the volume of oxygen (at STP) needed for complete combustion of 39 gram of liquid benzene?

Options:

A.

11.2 litre

B.

22.4 litre

C.

84 litre

D.

168 litre

Answer: C

Question 100

Avogadro's law is valid for

Options:

A.

all gases

B.

ideal gas

C.

Van der Waals gas

D.

real gas

Answer: B

Question 101

A metal (M) forms two oxides. The ratio M:O (by weight) in the two oxides are 25:4 and 25:6. The minimum value of atomic mass of M is

Options:

A.

50

B.

100

C.

150

D.

200

Answer: B

Question 102

The de-Broglie wavelength (λ) for electron (e), proton (p) and He^{2+} ion (α) are in the following order. Speed of e, p and α are the same

Options:

A.

$\alpha > p > e$

B.

$e > p > \alpha$

C.

$e > \alpha > p$

D.

$\alpha < p > e$

Answer: B

Question 103

1 mL of water has 25 drops. Let N_0 be the Avogadro number. What is the number of molecules present in 1 drop of water ? (Density of water = 1 g/mL)

Options:

A.

$\frac{0.02}{9} N_0$

B.

$\frac{18}{25} N_0$

C.

$\frac{25}{18} N_0$

D.

$\frac{0.04}{25} N_0$

Answer: A

Question 104

In Bohr model of atom, radius of hydrogen atom in ground state is r_1 and radius of He^+ ion in ground state is r_2. Which of the following is correct?

Options:

A.

$\frac{r_1}{r_2} = 4$

B.

$\frac{r_1}{r_2} = \frac{1}{2}$

C.

$\frac{r_1}{r_2} = \frac{1}{4}$

D.

$\frac{r_2}{r_1} = \frac{1}{2}$

Answer: D

Question 105

Which one of the following is the correct set of four quantum numbers (n, 1, m, s) ?

Options:

A.

$\left(3, 0, -1, +\frac{1}{2}\right)$

B.

$\left(4, 3, -2, -\frac{1}{2}\right)$

C.

$\left(3, 1, -2, -\frac{1}{2}\right)$

D.

$\left(4, 2, -3, +\frac{1}{2}\right)$

Answer: B

Question 106

Let $(C_{rms})_{H_2}$ is the r.m.s. speed of H_2 at 150 K. At what temperature, the most probable speed of helium $[C_{mp})_{He}]$ will be half of $(C_{rms})_{H_2}$?

Options:

A.

75 K

B.

112.5 K

C.

225 K

D.

900 K

Answer: B

Question 107

The correct pair of electron affinity order is

Options:

A.

O > S, F > Cl

B.

O < S, Cl > F

C.

S > O, F > Cl

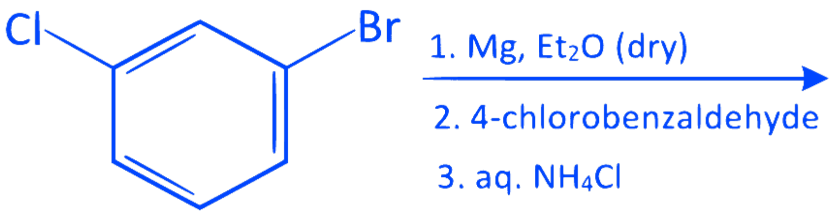
D.

S < O, Cl > F

Answer: B

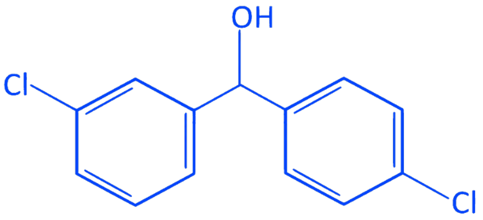
Question 108

The product of the following reaction is :

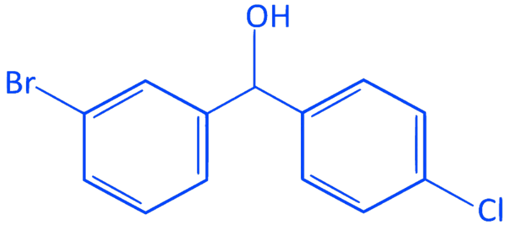


Options:

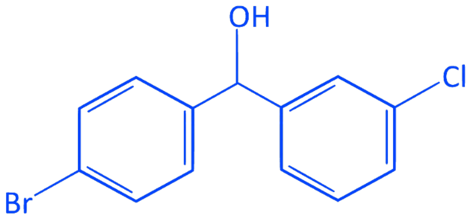
A.



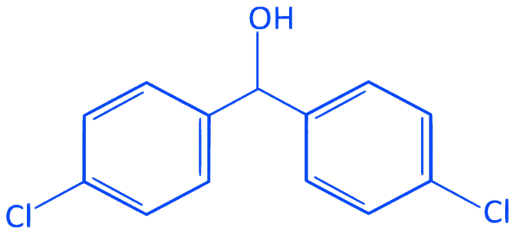
B.



C.



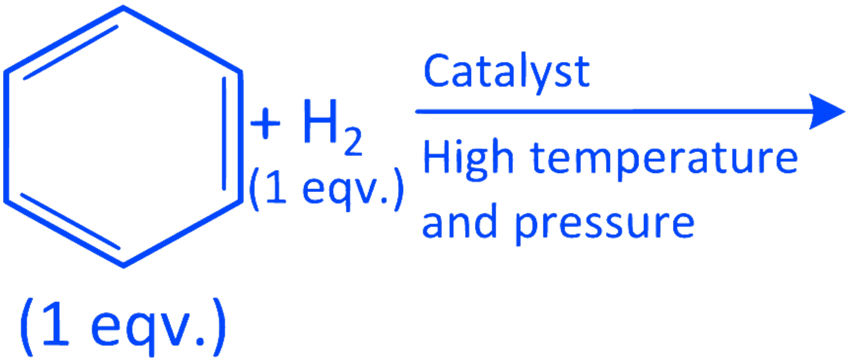
D.



Answer: A

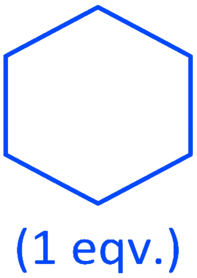
Question 109

The product of the following hydrogenation reaction is:

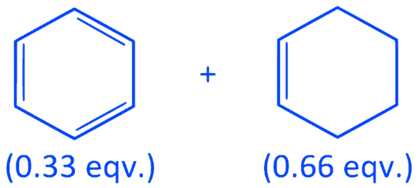


Options:

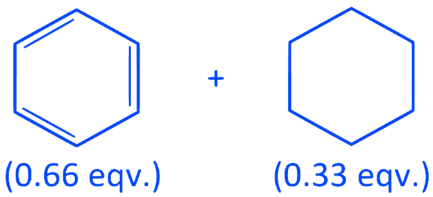
A.



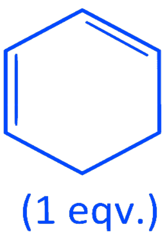
B.



C.



D.



Answer: A

Question 110

Pick the correct statement.

Options:

A.

Relative lowering of vapour pressure is independent of T.

B.

Osmotic pressure always depends on the nature of solute.

C.

Elevation of boiling point is independent of nature of the solvent.

D.

Lowering of freezing point is proportional to the molar concentration of solute.

Answer: A

Question 111

During the preparation of NH_3 in Haber's process, the promoter(s) used is/are -

Options:

A.

PtO_2

B.

Mo

C.

Mix of Al_2O_3 and K_2O

D.

Fe and Mn

Answer: C

Question 112

The correct statement(s) about B_2H_6 is /are :

Options:

A.

All B atoms are sp^3 hybridised.

B.

It is paramagnetic.

C.

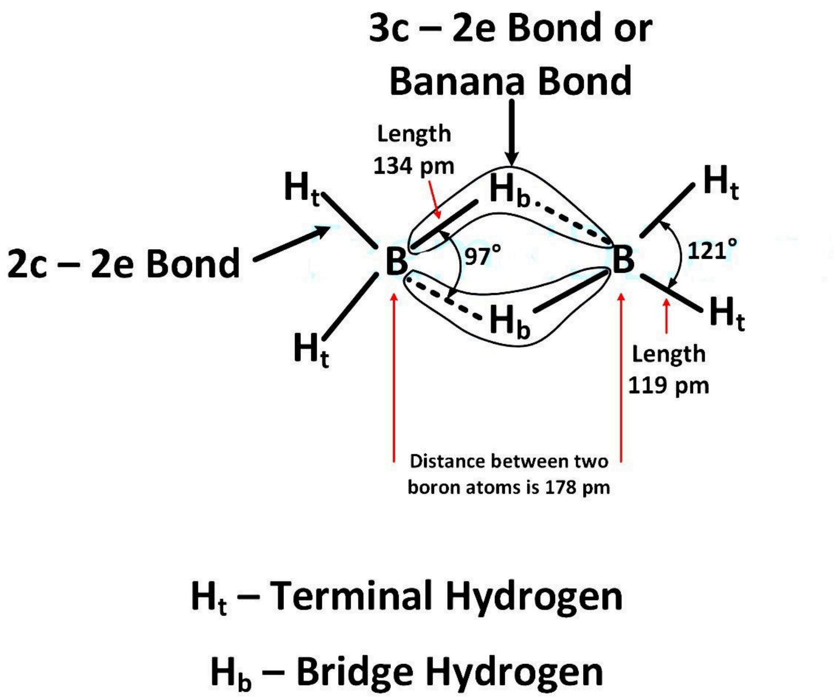
It contains 3C - 4C bonding.

D.

There are two types of H present.

Answer: D

Solution:



It has two 3-centre-2-electron bonds and four 2-centre-2-electron bonds.

Question 113

Which of the following would produce enantiomeric products when reacted with methyl magnesium iodide?

Options:

A.

Benzaldehyde

B.

Propiophenone

C.

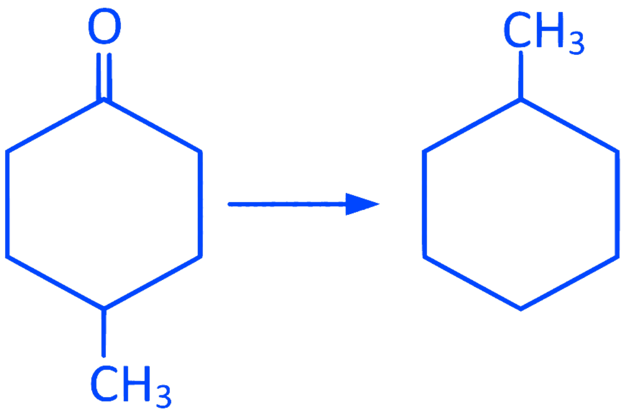
Acetone

D.

Acetaldehyde

Answer: B

Question 114



The above conversion can be carried out by,

Options:

A.

Zn - Hg/Conc. HCl

B.

i. H_2NNH_2 ii. NaOH in ethylene glycol, Δ

C.

i. $HSCH_2CH_2SH/H^{\oplus}$ ii. H_2/Ni

D.

Bromine water

Answer: C

Question 115

Which of the statements are incorrect?

Options:

A.

pH of a solution of salt of strong acid and weak base is less than 7.

B.

pH of a solution of a weak acid and weak base is basic if $K_b < K_a$.

C.

pH of an aqueous solution of 10^{-8} (M) HCl is 8.

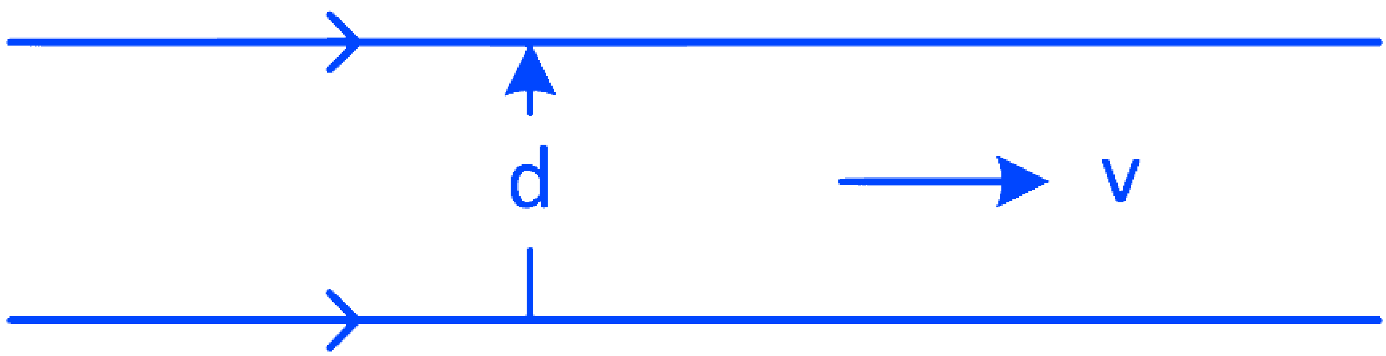
D.

Conjugate acid of NH_2^- is NH_3.

Answer: C

Physics

Question 116



Two infinite line-charges parallel to each other are moving with a constant velocity v in the same direction as shown in the figure. The separation between two line-charges is d. The magnetic attraction

balances the electric repulsion when, [c = speed of light in free space]

Options:

A.

$v = \sqrt{2}c$

B.

$v = \frac{c}{\sqrt{2}}$

C.

$v = c$

D.

$v = \frac{c}{2}$

Answer: C

Solution:

Electric field due to line charge is given by

$E = \frac{\lambda}{2\pi\varepsilon_0 d}$

where, λ = linear charge density.

$\lambda = \frac{q}{l} \Rightarrow q = \lambda l$

Electric force due to one wire on another wire,

$F_E = q\,.\,E$

$= \lambda l\,.\,\frac{\lambda}{2\pi\varepsilon_0 d}$

$\Rightarrow F_E = \frac{\lambda^2 l}{2\pi\varepsilon_0 d}$

Now, current due to moving charged wire is given as

$I = \frac{dq}{dt} = \frac{d}{dt}(\lambda dl)\ [\because v = \frac{dl}{dt}]$

$= \lambda v$

Magnetic force on one wire due to another wire,

$F_B = \frac{\mu_0 I_1 I_2 l}{2\pi d}$

Since, both forces balance each other, thus

$F_E = F_B$

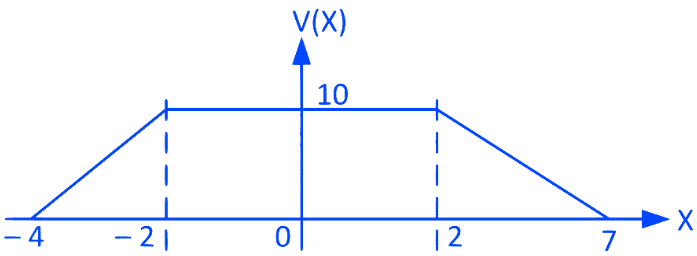
$\Rightarrow \frac{\lambda^2 l}{2\pi\varepsilon_0 d} = \frac{\mu_0(\lambda v)(\lambda v)l}{2\pi d}$ $[\because I_1 = I_2 = I]$

$\Rightarrow \frac{1}{\varepsilon_0\mu_0} = v^2$

or, $v = \frac{1}{\sqrt{\varepsilon_0\mu_0}} = c$

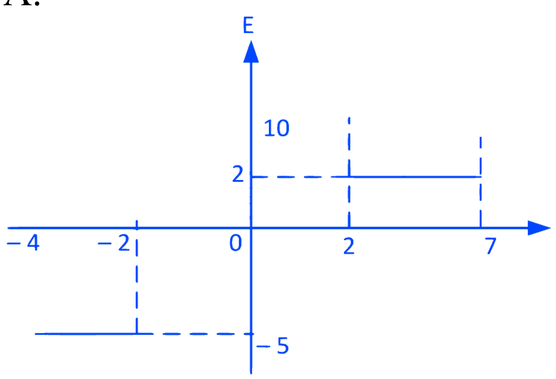
Question 117

The electric potential for an electric field directed parallel to X-axis is shown in the figure. Choose the correct plot of electric field strength.

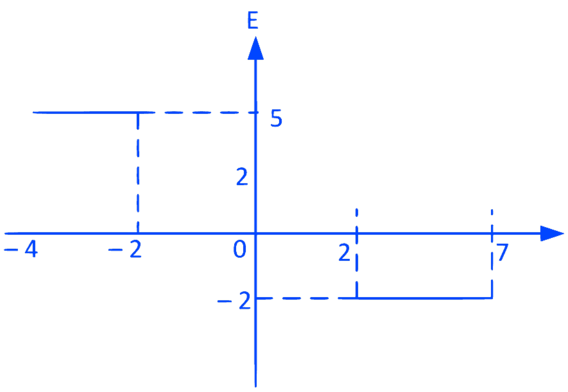


Options:

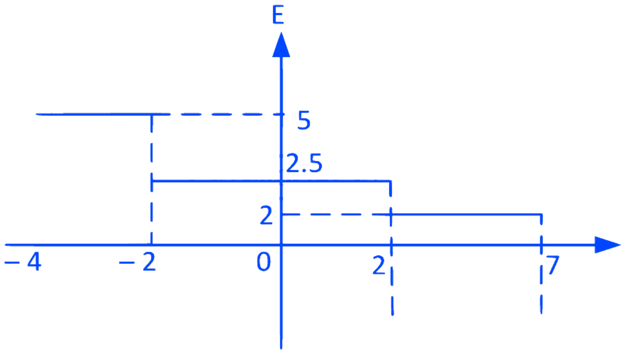
A.



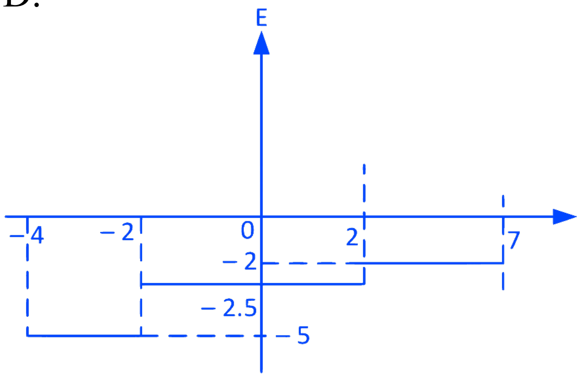
B.



C.



D.



Answer: A

Solution:

We know that, $E=-\frac{dV}{dr}$

Thus, electric field from $(-4 \text{ to } -2)$,

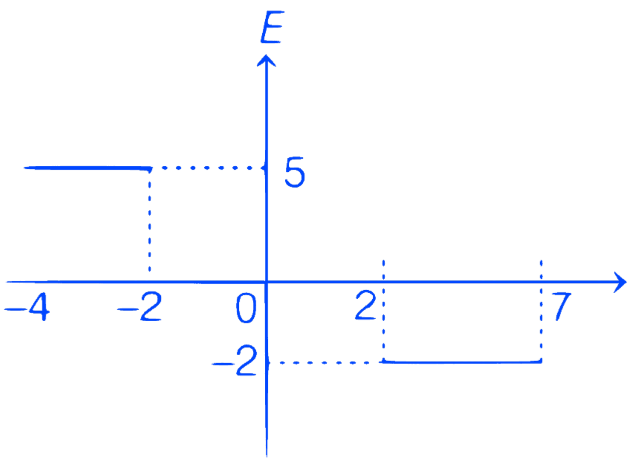
$E=-\left(\frac{10-0}{4-(-2)}\right)=\frac{10}{2}=5$ N/C

Electric field from $(-2 \text{ to } 2)$,

$E=-\left(\frac{10-10}{2-(-2)}\right)=0$

Electric field from $(2 \text{ to } 7)$,

$E=-\left(\frac{10-0}{2-7}\right)=-2$ N/C



Question 118

An electron revolves around the nucleus in a circular path with angular momentum $\overrightarrow{L}$. A uniform magnetic field $\overrightarrow{B}$ is applied perpendicular to the plane of its orbit. If the electron experiences a torque $\overrightarrow{T}$, then

Options:

A.

$\overrightarrow{T} || \overrightarrow{L}$

B.

$\overrightarrow{T}$ is anti-parallel to $\overrightarrow{L}$

C.

$\overrightarrow{T} . \overrightarrow{L} = 0$

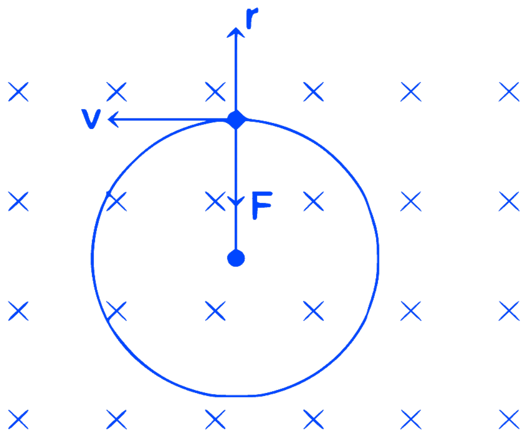
D.

Angle between $\overrightarrow{T}$ and $\overrightarrow{L}$ is $45°$

Answer: C

Solution:

According to the question, given situation can be represented by the figure below.



From the Lorentz force, we have

$F = q(v \times B)$

Hence, force is acting directly towards the centre.

Thus, both angular momentum ($\mathbf{L}$) and torque ($\mathbf{T}$) will have the same direction. i.e. $\mathbf{T} \parallel \mathbf{L}$.

Question 119

A straight wire is placed in a magnetic field that varies with distance x from origin as $\overrightarrow{B} = B_0\left(2 - \frac{x}{a}\right)\widehat{k}$. Ends of wire are at (a, 0) and (2a, 0) and it carries a current I. If force on wire is $\overrightarrow{F} = IB_0\left(\frac{ka}{2}\right)\hat{j}$, then value of k is

Options:

A.

1

B.

5

C.

-1

D.

$\frac{1}{2}$

Answer: A

Solution:

Given, $B = B_0\left(2 - \frac{x}{a}\right)\widehat{k}$

Force on a current carrying wire is given as

$dF = IB \,.\, dl$

$\Rightarrow \int dF = \int IB_0\left(2 - \frac{x}{a}\right)dx$

$\Rightarrow F = IB_0 \int_a^{2a} \left(2 - \frac{x}{a}\right)dx$

$= IB_0 \left|2x - \frac{x^2}{2a}\right|_a^{2a} = IB_0\left(\frac{a}{2}\right)$

Therefore, the value of k is 1.

Question 120

In a closed circuit there is only a coil of inductance L and resistance 100 Ω. The coil is situated in a uniform magnetic field. All on a sudden, the magnetic flux linked with the circuit changes by 5 Weber. What amount of charge will flow in the circuit as a result?

Options:

A.

500 C

B.

0.05 C

C.

20 C

D.

Value of L is to be known to find the charge flown

Answer: B

Solution:

Given, change in flux, $d\phi = 5$ Wb

Emf due to change in flux is given as

$E = \frac{d\phi}{dt} \Rightarrow E \,.\, dt = 5$

Now, it is given that, $R = 100\,\Omega$

Thus, current, $I = \frac{V}{R} = \frac{E}{R}$ ..... (i)

Also, $I = \frac{dq}{dt}$ .... (ii)

where, dq is the charge flowing per unit time.

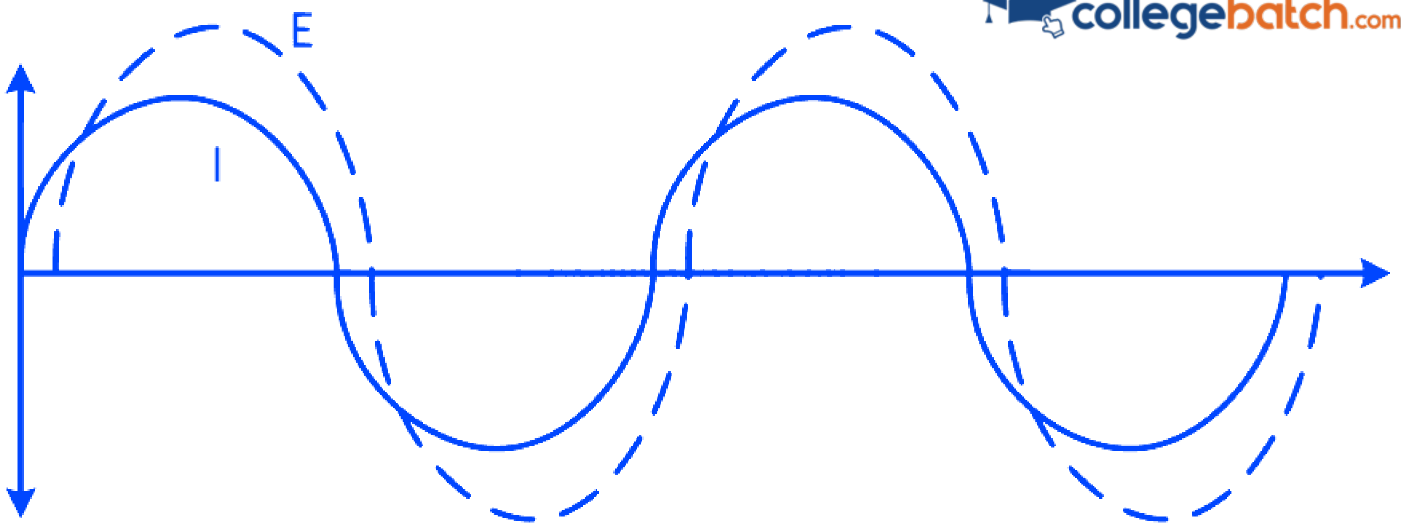
Thus, from Eqs. (i) and (ii), we have

$\frac{E}{R} = \frac{dq}{dt} \Rightarrow \frac{E\,.\,dt}{R} = dq$

$\Rightarrow \frac{5}{100} = dq \Rightarrow dq = 0.05\,C$

Question 121

When an AC source of emf E with frequency ω = 100 Hz is connected across a circuit, the phase difference between E and current I in the circuit is observed to be $\frac{\pi}{4}$ as shown in the figure. If the circuit consist of only RC or RL in series, then



Options:

A.

R = 1 kΩ, C = 5 μF

B.

R = 1 kΩ, L = 10 H

C.

R = 1 kΩ, L = 1 H

D.

R = 1 kΩ, C = 10 μF

Answer: D

Solution:

At any moment, the phase difference between current and voltage is given by

$\tan \phi = \frac{X_L}{R}$

Since, voltage leads current by $\frac{\pi}{4}$, thus given circuit will be RL series circuit

$\therefore \tan \frac{\pi}{4} = \frac{X_L}{R} \Rightarrow R = X_L = \omega L$

From option (b), $R = 1\, k\Omega = 1000\, \Omega$

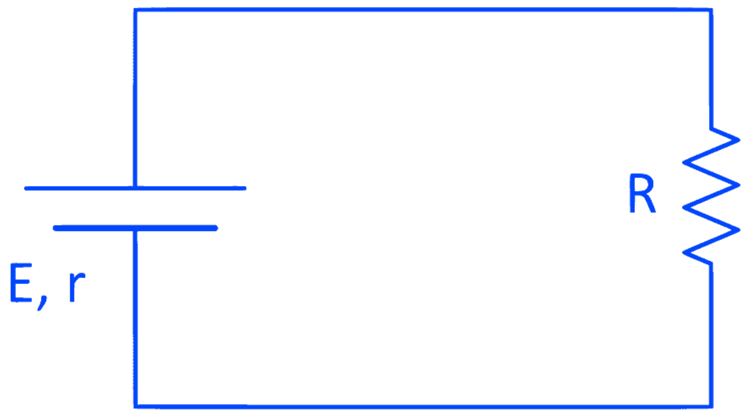
$\therefore X_L = \omega L$

$= 100 \times 10$ $[\because \omega = 100$ rad/s$]$

$= 1000\, \Omega$

Thus, the condition is satisfied. Hence, option (b) is correct.

Question 122



A battery of emf E and internal resistance r is connected with an external resistance R as shown in the figure. The battery will act as a constant voltage source if

Options:

A.

r << R

B.

r >> R

C.

r = R

D.

It will never act as a constant voltage source.

Answer: A

Solution:

The constant voltage source provides a constant voltage to the load resistance regardless of variations or changes, in the load resistance. For this to happen, the source must have an internal resistance which is very low as compared to the resistance of the load.

i.e., $r << R$

Question 123

If the kinetic energies of an electron, an alpha particle and a proton having same de-Broglie wavelength are $\varepsilon_1, \varepsilon_2$ and ε_3 respectively, then

Options:

A.

$\varepsilon_1 > \varepsilon_3 > \varepsilon_2$

B.

$\varepsilon_1 = \varepsilon_3 = \varepsilon_2$

C.

$\varepsilon_1 < \varepsilon_3 < \varepsilon_2$

D.

$\varepsilon_1 > \varepsilon_2 > \varepsilon_3$

Answer: A

Solution:

de-Broglie wavelength of a charged particle having energy ε is given by

$\lambda = \frac{h}{\sqrt{2m\varepsilon}}$

Thus, $\lambda_e = \frac{h}{\sqrt{2m_e\varepsilon_1}}$

$\lambda_\alpha = \frac{h}{\sqrt{2m_\alpha\varepsilon_2}}$

$\lambda_p = \frac{h}{\sqrt{2m_p\varepsilon_3}}$

Since, $\lambda_e = \lambda_\alpha = \lambda_p$ and $m_\alpha = m_p = m_e$

Thus, we can say

$\frac{1}{\sqrt{m_\alpha\varepsilon_2}} = \frac{1}{\sqrt{m_p\varepsilon_3}} = \frac{1}{\sqrt{m_e\varepsilon_1}}$

$\therefore \varepsilon_1 > \varepsilon_3 > \varepsilon_2$

Question 124

In a Young's double slit experiment, the intensity of light at a point on the screen where the path difference between the interfering waves is λ, (λ being the wavelength of light used) is I. The intensity at a point where the path difference is $\frac{\lambda}{4}$ will be (assume two waves have same amplitude)

Options:

A.

zero

B.

I

C.

$\frac{I}{2}$

D.

$\frac{I}{4}$

Answer: C

Solution:

Intensity at any point on the screen is given by

$I = 4I_0\cos^2\frac{\phi}{2}$

where, I_0 is the intensity of either wave and ϕ is the phase difference between two waves.

Phase difference $(\phi) = \frac{2\pi}{\lambda} \times$ path difference

$= \frac{2\pi}{\lambda} \times \lambda = 2\pi$

$\therefore I = 4I_0\cos^2\left(\frac{2\pi}{2}\right)$

$= 4I_0\cos^2(\pi) = 4I_0 = I$

$\Rightarrow I_0 = \frac{I}{4}$ ...... (i)

When path difference is $\frac{\lambda}{4}$, then

$\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$

$\therefore I = 4I_0\cos^2\left(\frac{\pi}{4}\right) = 2I_0 = 2 \times \frac{I}{4} = \frac{I}{2}$

Question 125

In Young's double slit experiment with a monochromatic light, maximum intensity is 4 times the minimum intensity in the interference pattern. What is the ratio of the intensities of the two interfering waves?

Options:

A.

1/9

B.

1/3

C.

1/16

D.

1/2

Answer: A

Solution:

Given, $\frac{I_{max}}{I_{min}} = \frac{4}{1}$

Let I_1 and I_2 be the intensities of interfering waves,

then $\frac{(\sqrt{I_2}+\sqrt{I_1})^2}{(\sqrt{I_2}-\sqrt{I_1})^2} = \frac{I_{\max}}{I_{\min}}$

$\Rightarrow \left(\frac{\sqrt{\frac{I_2}{I_1}}+1}{\sqrt{\frac{I_2}{I_1}}-1}\right)^2 = \frac{4}{1}$

$\Rightarrow \frac{\sqrt{\frac{I_2}{I_1}}+1}{\sqrt{\frac{I_2}{I_1}}-1} = \frac{2}{1} \Rightarrow \sqrt{\frac{I_2}{I_1}} = \frac{3}{1}$

or, $\frac{I_1}{I_2} = \frac{1}{9}$

Question 126

The human eye has an approximate angular resolution of $\theta = 5.8 \times 10^{-4}$ rad and typical photo printer prints a minimum of 300 dpi (dots per inch, 1 inch = 2.54 cm). At what minimal distance d should a printed page be held so that one does not see the individual dots?

Options:

A.

20.32 cm

B.

29.50 cm

C.

14.59 cm

D.

6.85 cm

Answer: C

Solution:

Given, angular resolution of human eye,

$\theta = 5.8 \times 10^{-4}$ rad

The linear distance between two successive dots in a printer,

$l = \frac{2.54}{300} = 0.846 \times 10^{-2}$ cm

At a distance d cm, the gap l will subtend an angle which is given by

$d = \frac{l}{\theta} = \frac{0.846 \times 10^{-2}}{5.8 \times 10^{-4}} = 14.59$ cm

----

Question 127

Suppose in a hypothetical world the angular momentum is quantized to be even integral multiples of $\frac{h}{2\pi}$. The largest possible wavelength emitted by hydrogen atoms in visible range in a world according to Bohr's model will be,

(Consider hc = 1242 Mev-fm)

Options:

A.

153 nm

B.

409 nm

C.

121 nm

D.

487 nm

Answer: D

Solution:

The angular momentum is quantised to even multiple of $\frac{h}{2\pi}$. Hence, the quantum numbers that are allowed are n = 2 and n = 4 .... So,

$E = 13.6\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$= 13.6\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$

$= 13.6\left(\frac{1}{4} - \frac{1}{16}\right)$

$= 2.55$ eV

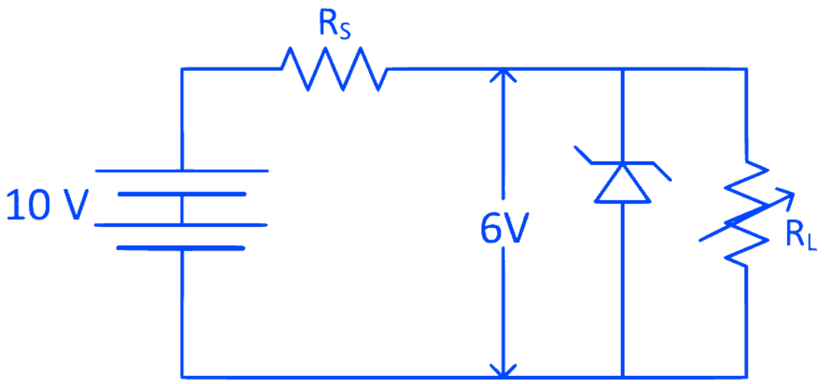
Now, given that, $hc = 1242$ MeV-nm

Hence, $\lambda = \frac{hc}{2.55\ eV} = \frac{1242\ eV - nm}{2.55\ eV}$

= 487 nm

Question 128

A Zener diode having break down voltage Vz = 6V is used in a voltage regulator circuit as shown in the figure. The minimum current required to pass through the Zener to act as a voltage regulator is 10 mA and maximum allowed current through Zener is 40 mA. The maximum value of R_s for Zener to act as a voltage regulator is



Options:

A.

100 Ω

B.

400 Ω

C.

0.4 Ω

D.

950 Ω

Answer: B

Solution:

Given, $V_s = 10\,V$

$V_z = 6\,V$

Maximum current $(I_{z\ \max}) = 40\,mA = 40 \times 10^{-3}\,A$

Minimum current $(I_{z\ \min}) = 10\,mA = 10 \times 10^{-3}\,A$

Maximum value of series resistance R_S in the voltage regulator circuit is given as

$R_{\max} = \frac{V_s - V_z}{I_{z\ \min}}$

$= \frac{10-6}{10 \times 10^{-3}}$

$= 400\,\Omega$

Question 129

The expression $\overline{A}(A+B) + (B+AA)(A+\overline{B})$ simplifies to

Options:

A.

A + B

B.

AB

C.

$\overline{A+B}$

D.

$\overline{A} + \overline{B}$

Answer: A

Solution:

Given expression, $\overline{A}(A+B)+(B+AA)(A+\overline{B})$

$= \overline{A}(A+B)+(B+A)(A+\overline{B})$

$= A\overline{A}+\overline{A}B+AB+B\overline{B}+AA+A\overline{B}$

$= A+A\overline{B}+B(\overline{A}+\overline{A})$ $[\because A\overline{A}=B\overline{B}=0$ and $A.A=A]$

$= A+A\overline{B}+B(\overline{A}+A)$

$= A(1+\overline{B})+B(1)$ $[\because = A+\overline{A}=1]$

$= A+B$ $[\because 1+\overline{B}=1]$

Question 130

Given : The percentage error in the measurements of A, B, C and D are respectively, 4%, 2%, 3% and 1%. The relative error in $Z=\frac{A^4B^{\frac{1}{3}}}{CD^{\frac{3}{2}}}$ is

Options:

A.

$\frac{127}{2}\%$

B.

$\frac{127}{5}\%$

C.

$\frac{127}{6}\%$

D.

$\frac{127}{7}\%$

Answer: C

Solution:

The percentage error in Z is given as

$\frac{\Delta Z}{Z}\% = 4\frac{\Delta A}{A} + \frac{1}{3}\frac{\Delta B}{B} + \frac{\Delta C}{C} + \frac{3}{2}\frac{\Delta D}{D}$

$= 4 \times 4 + \frac{1}{3} \times 2 + 3 + \frac{3}{2} \times 1$

$= 16 + \frac{2}{3} + 3 + \frac{3}{2}$

$= \frac{127}{6}\%$

Question 131

The Entropy (S) of a black hole can be written as $S = \beta k_B A$, where k_B is the Boltzmann constant and A is the area of the black hole. The β has dimension of

Options:

A.

L^2

B.

ML^2L^{-1}

C.

L^{-2}

D.

dimensionless

Answer: C

Solution:

Given, $S = \beta k_B A$

where, dimensional formula of $S = [M^1 L^2 T^{-2} K^{-1}]$

Dimensional formula of $k_B = [M^1L^2T^{-2}K^{-1}]$

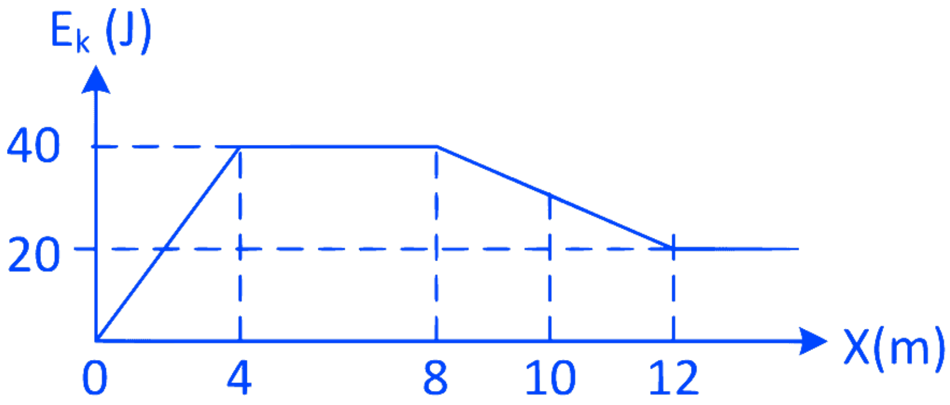
Dimensional formula of $A = [L^2]$

Thus, $[M^1L^2T^{-2}K^{-1}] = \beta[M^1L^2T^{-2}K^{-1}][L^2]$

$\Rightarrow \beta = [L^{-2}]$

Question 132

The kinetic energy (E_k) of a particle moving along X-axis varies with its position (X) as shown in the figure. The force acting on the particle at X = 10 m is



Options:

A.

$5\hat{i}$ N

B.

0 N

C.

$97.5\hat{i}$ N

D.

$-5\hat{i}$ N

Answer: D

Solution:

According to work-energy theorem,

change in kinetic energy = work done

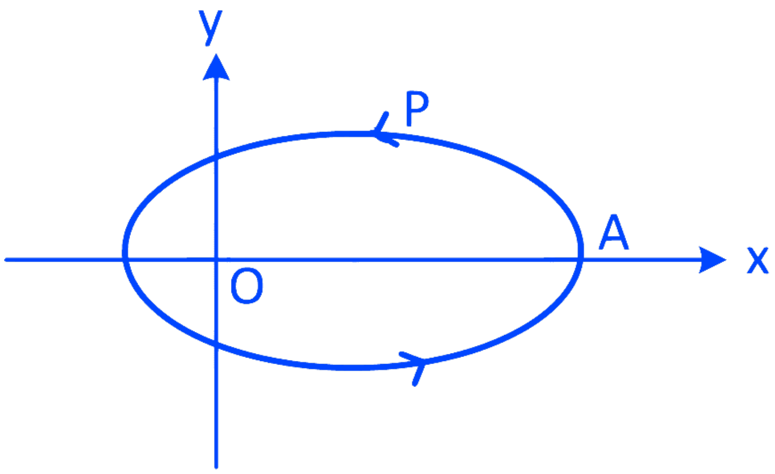
= force × distance

Thus, force $= \frac{\text{change in energy}}{\text{change in distance}}$

At x = 10 m, we have

$F = \frac{20-40}{12-8}\hat{i} = -5\hat{i}\,N$

Question 133



A particle is moving in an elliptical orbit as shown in figure. If $\vec{p}$, $\vec{L}$ and $\vec{r}$ denote the linear momentum, angular momentum and position vector of the particle (from focus O) respectively at a point A, then the direction of $\vec{\alpha} = \vec{p} \times \vec{L}$ is along.

Options:

A.

+ve x axis

B.

−ve x axis

C.

+ve y axis

D.

−ve y axis

Answer: A

Solution:

At position A, direction of $\vec{p}$ is along + ve Y-axis.

At position A, direction of $\vec{L}$ is along + ve Z-axis because $\vec{L} = \vec{r} \times \vec{p}$ and $\vec{r}$ is along + ve X-axsis.

Thus, $\vec{\alpha} = \vec{p} \times \vec{L}$

$= \hat{i} \times \hat{k}$

$= -\hat{j}$ or −ve Y-axis.

Question 134

A particle is subjected to two simple harmonic motions in the same direction having equal amplitudes and equal frequency. If the resultant amplitude is equal to the amplitude of the individual motion, the phase difference (δ) between the two motion is

Options:

A.

$\delta = \frac{\pi}{3}$

B.

$\delta=\frac{2\pi}{3}$

C.

$\delta=\pi$

D.

$\delta=\frac{\pi}{2}$

Answer: B

Solution:

According to the question,

$A=\sqrt{A^2+A^2+2AA\cos\delta}$

$\Rightarrow\cos\delta=-\frac{1}{2}$

or, $\delta=120^\circ$

or, $=\frac{2\pi}{3}$

Question 135

A body of mass m is thrown with velocity u from the origin of a co-ordinate axes at an angle θ with the horizon. The magnitude of the angular momentum of the particle about the origin at time t when it is at the maximum height of the trajectory is proportional to

Options:

A.

u

B.

u^2

C.

u^3

D.

independent of u

Answer: C

Solution:

Maximum height of projectile,

$h = \frac{u^2 \sin^2\theta}{2g}$ ...... (i)

At maximum height velocity of projectile,

$v = u\cos\theta$

$\therefore$ Momentum at highest point $= mu\cos\theta$

$\therefore$ Angular momentum about origin,

$p = mu\cos\theta \times h$

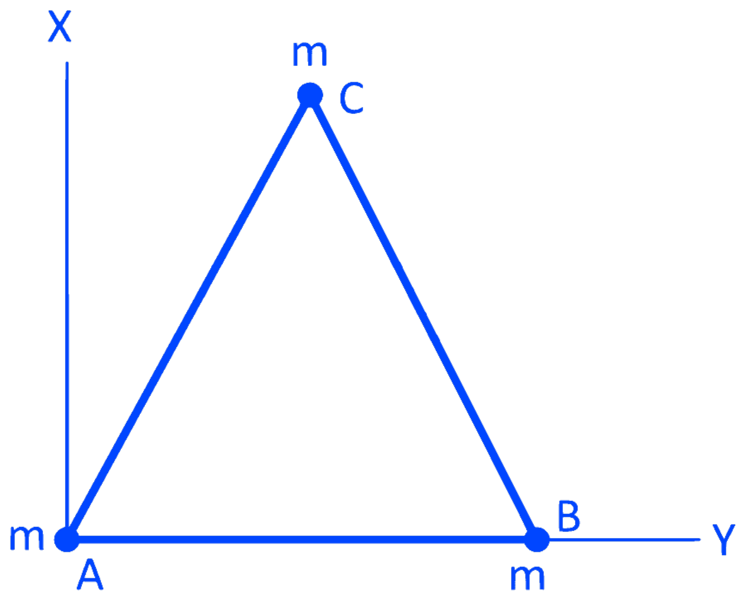
$= mu\cos\theta \times \frac{u^2 \sin^2\theta}{2g}$

$= \frac{mu^3 \cos\theta \sin^2\theta}{2g}$

$\Rightarrow p \propto u^3$

Question 136

Three particles, each of mass 'm' grams situated at the vertices of an equilateral ΔABC of side 'a' cm (as shown in the figure). The moment of inertia of the system about a line AX perpendicular to AB and in the plane of ABC in g-cm^2 units will be



Options:

A.

$2\ ma^2$

B.

$\frac{3}{2}\ ma^2$

C.

$\frac{3}{4}\ ma^2$

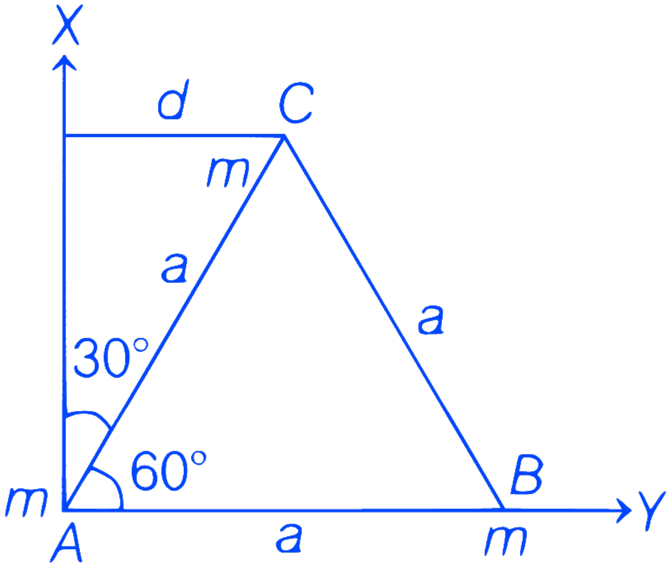
D.

$\frac{5}{4}\ ma^2$

Answer: D

Solution:

Moment of inertia about line $AX = ma^2 + md^2$



We know that,

$\sin\theta = \frac{d}{a}$

$\Rightarrow \frac{1}{2} = \frac{d}{a}\ (\because \theta = 30°)$

$\Rightarrow d = \frac{a}{2}$

$\therefore I_{Ax} = ma^2 + m\left(\frac{a}{2}\right)^2$

$= ma^2 + \frac{ma^2}{4} = \frac{5}{4}ma^2$

Question 137

A body of mass m is thrown vertically upward with speed $\sqrt{3}$ v_e, where v_e is the escape velocity of a body from earth surface. The final velocity of the body is

Options:

A.

0

B.

2 v_e

C.

$\sqrt{3}$ v_e

D.

$\sqrt{2}$ v_e

Answer: D

Solution:

Given, initial speed, $v = \sqrt{3}\, v_e$

We know that, $v_e = \sqrt{\frac{2GM}{R}}$

At the surface of earth,

Total energy $= PE + KE$

$= -\frac{GMm}{R} + \frac{1}{2}mv^2$

$= -\frac{GMm}{R} + \frac{1}{2}m\left(\sqrt{3}\sqrt{\frac{2GM}{R}}\right)^2$

$= -\frac{GMm}{R} + \frac{1}{2}3m \times \frac{2GM}{R}$

$= -\frac{GMm}{R} + \frac{3GMm}{R} = \frac{2GMm}{R}$

At infinity, total energy $= PE + KE$

$= 0 + \frac{1}{2}\, mv^2$

From the principle of conservation of energy,

$\frac{2GMm}{R} = \frac{1}{2}mv^2$

$\Rightarrow \frac{4GM}{R} = v^2$

$\Rightarrow v = \sqrt{\frac{4GM}{R}}$

$= \sqrt{2 \times \frac{2GM}{R}} = \sqrt{2}\, v_e$

Question 138

If a string, suspended from the ceiling is given a downward force F_1, its length becomes L_1. Its length is L_2, if the downward force is F_2. What is its actual length?

Options:

A.

$\frac{L_1+L_2}{2}$

B.

$\sqrt{L_1 L_2}$

C.

$\frac{F_2 L_1+F_1 L_2}{F_2+F_1}$

D.

$\frac{F_2 L_1-F_1 L_2}{F_2-F_1}$

Answer: D

Solution:

According to the Hooke's law, for deformation of string of length l,

$\frac{F_1}{A} \propto \frac{\Delta l_1}{l}$ .... (i)

and in second case,

$\frac{F_2}{A} \propto \frac{\Delta l_2}{l}$ ...... (ii)

From Eqs. (i) and (ii), we have

$\frac{F_1}{F_2} = \frac{\Delta l_1}{\Delta l_2} = \frac{l_1 - l}{l_2 - l}$

$\Rightarrow (F_2 - F_1)l = F_2 l_1 - F_1 l_2$

$\Rightarrow l = \frac{F_2 l_1 - F_1 l_2}{F_2 - F_1}$

Question 139

27 drops of mercury coalesce to form a bigger drop. What is the relative increase in surface energy?

Options:

A.

$\frac{3}{2}$

B.

$\frac{2}{3}$

C.

$-\frac{2}{3}$

D.

8

Answer: C

Solution:

Let r be the radius of small mercury drop.

Thus, total volume of 27 drops $= 27 \times \frac{4}{3} \pi r^3$

Total volume of bigger drop $= \frac{4}{3} \pi R^3$

$\therefore$ According to given situation,

$\frac{4}{3} \pi R^3 = 27 \times \frac{4}{3} \pi r^3$

$\Rightarrow R = 3r$

Now, surface energy = surface tension $\times$ area

$= T \times A$

Surface energy of 27 drops $= T \times 27 \times (4\pi r^2)$

$= 27 \times 4\pi r^2 T$

Surface energy of bigger drop $= T \times 4\pi R^2$

$= T \times 4\pi(3r)^2$

$= 9 \times 4\pi r^2 T$

Increase in surface energy $= 9 \times 4\pi r^2 T - 27 \times 4\pi r^2 T$

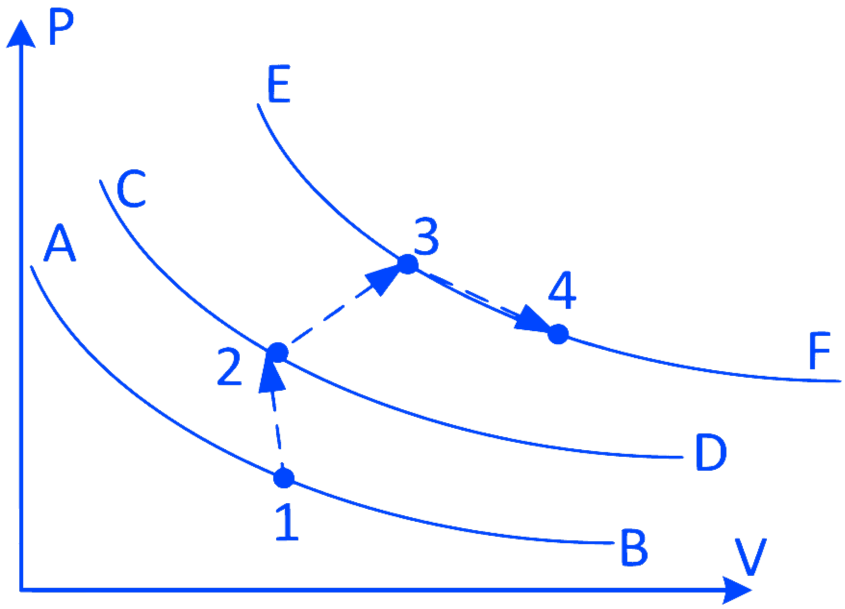
$= -18 \times 4\pi r^2 T$

$\therefore$ Relative increase in surface energy

$= \frac{-18 \times 4\pi r^2 T}{27 \times 4\pi r^2 T} = -\frac{2}{3}$

Question 140

Certain amount of an ideal gas is taken from its initial state 1 to final state 4 through the paths $1 \rightarrow 2 \rightarrow 3 \rightarrow 4$ as shown in figure. AB, CD, EF are all isotherms. If v_p is the most probable speed of the molecules, then



Options:

A.

v_p at 3 = v_p at 4 > v_p at 2 > v_p at 1

B.

v_p at 3 > v_p at 1 > v_p at 2 > v_p at 4

C.

v_p at 3 > v_p at 2 > v_p at 4 > v_p at 1

D.

v_p at 2 = v_p at 3 > v_p at 1 > v_p at 4

Answer: A

Solution:

For the given isotherm, the temperature of the curve EF is greater than the curve CD and AB.

As we know that, most probable speed of the molecule is given as

$v_p = \sqrt{\frac{2RT}{M}}$

Thus, the v_p at 3 and 4 will be same.

Hence, v_p at 3 = v_p at 4 > v_p at 2 > v_p at 1.

Question 141

Consider a thermodynamic process where integral energy $U = AP^2V$ (A = constant). If the process is performed adiabatically, then

Options:

A.

$AP^2(V + 1)$ = constant

B.

$(AP + 1)^2V$ = constant

C.

$(AP + 1)V^2$ = constant

D.

$\frac{V}{(AP+1)^2}$ = constant

Answer: B

Solution:

$U = Ap^2V$, $A =$ constant

For adiabatic process, $dQ = 0$

$\therefore$ From first law of thermodynamics,

$dQ = dU + dW$

$\therefore dU + dW = 0 \, (\because dQ = 0)$

$\Rightarrow d(Ap^2V) + pdV = 0$

$\Rightarrow A(2p \, . \, Vdp + p^2dV) + pdV = 0$

$\Rightarrow (Ap^2 + p)dV + 2ApVdp = 0$

$\Rightarrow (Ap^2 + p)dV = -2ApVdp$

$\Rightarrow \frac{dV}{-2AV} = \frac{pdp}{p(Ap+1)}$

$\Rightarrow \frac{dp}{(Ap+1)} + \frac{1}{2A} \cdot \frac{dv}{v} = 0$

Integrating on both sides,

$\Rightarrow \int \frac{dp}{(Ap+1)} + \frac{1}{2A} \int \frac{dV}{V} = \int 0$

$\Rightarrow \frac{\ln(Ap+1)}{A} + \frac{1}{2A} \ln V = \ln C$

$\Rightarrow \ln(Ap + 1) + \ln V^{1/2} = \ln C$

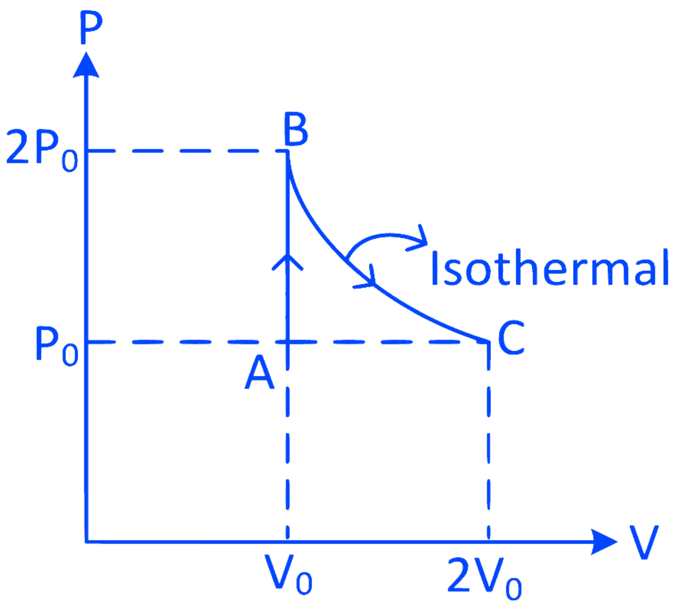
$\Rightarrow \ln[V^{1/2}(Ap + 1)] = \ln C$

$\Rightarrow V^{1/2}(Ap + 1) = C$

$\Rightarrow V(Ap + 1)^2 = C$ (constant)

Question 142

One mole of a diatomic ideal gas undergoes a process shown in P-V diagram. The total heat given to the gas (ln 2 = 0.7) is



Options:

A.

2.5 P_0V_0

B.

3.9 P_0V_0

C.

1.1 P_0V_0

D.

1.4 P_0V_0

Answer: B

Solution:

The given process BC is isothermal and AB is isochoric.

From A to B, we have

$p_0 \rightarrow 2p_0$

as $p \propto T$

Hence, $T_B = 2T_A = 2T_0$

$Q_{AB} = \Delta U_{AB} = W_{AB} = C_V \Delta T + 0$

$\Rightarrow Q_{AB} = \frac{5}{2} R(2T_0 - T_0) = \frac{5}{2} p_0 V_0$

$(\because C_V = \frac{5}{2} R$ for diatomic gas and $pV = RT)$

From B to C, we have,

$Q_{BC} = \Delta U_{BC} + W_{BC} = 0 + R(2T_0) \ln \frac{2V_0}{V_0}$

$= 2RT_0 \ln 2$

$= 2 \ln 2 p_0 V_0$

Hence, $Q_{total} = Q_{AB} + Q_{BC}$

$= (2.5 + 2 \ln 2) p_0 V_0$

$= 3.9\, p_0 V_0$

Question 143

Two charges, each equal to −q are kept at (−a, 0) and (a, 0). A charge q is placed at the origin. If q is given a small displacement along y direction, the force acting on q is proportional to

Options:

A.

y

B.

−y

C.

$\frac{1}{y}$

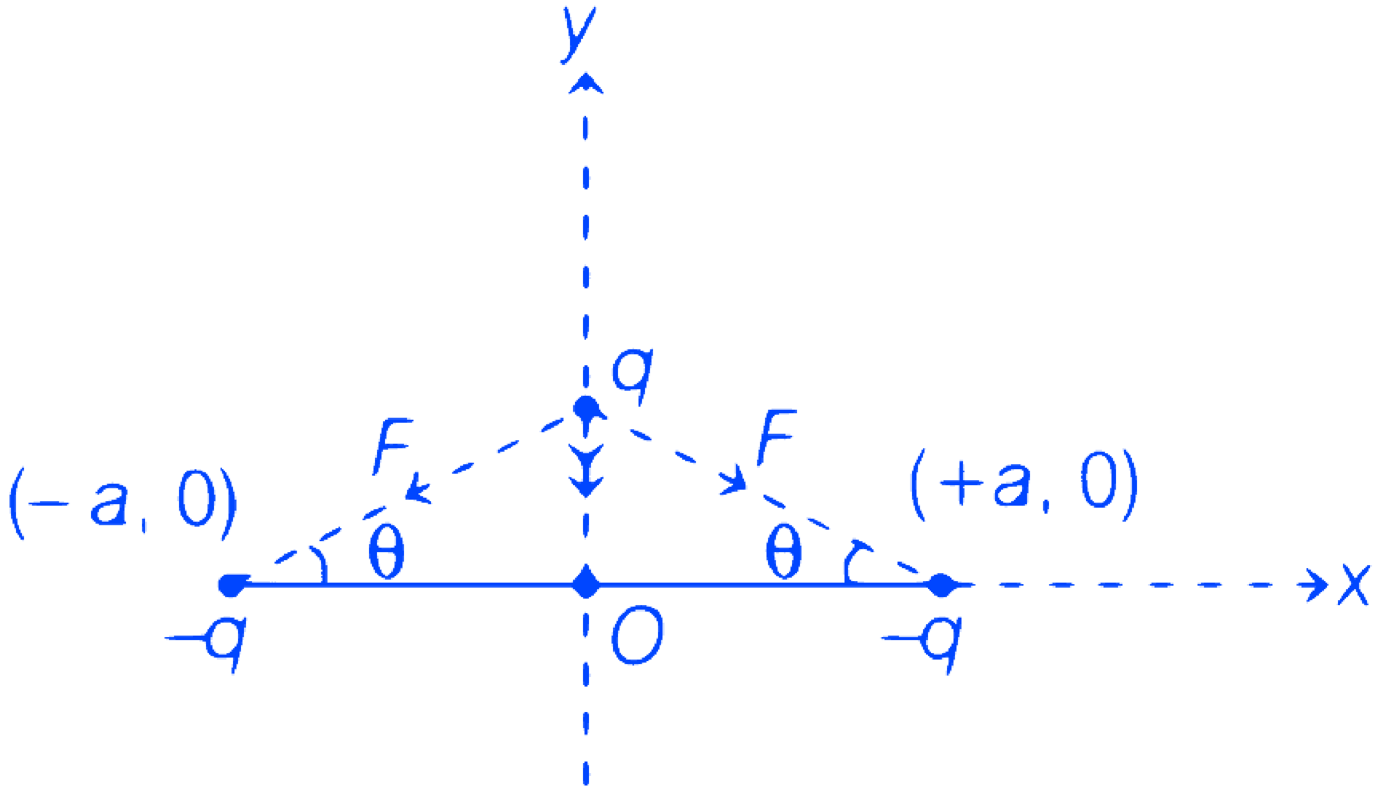
D.

$-\frac{1}{y}$

Answer: B

Solution:

Consider the diagram given below.



When the charge q is displaced momentarily, it starts oscillating. The net force acting on q is given by $F_{net} = 2F \sin\theta$

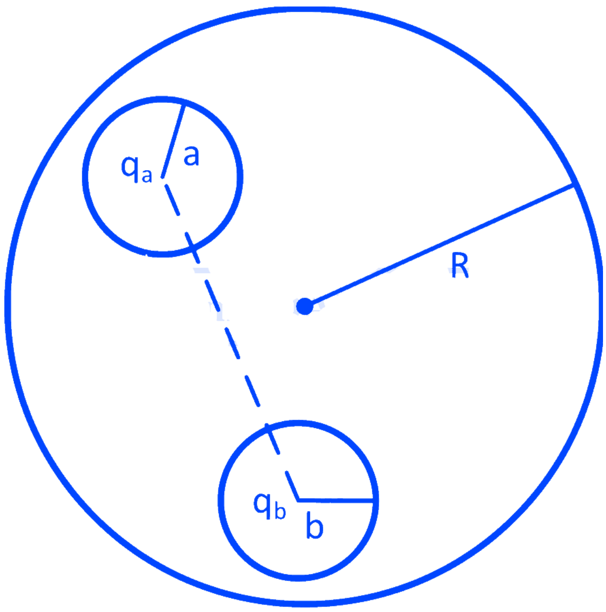
$$= -\frac{2\,kq \times q}{\left(\sqrt{y^2+a^2}\right)^2} \cdot \frac{y}{\sqrt{y^2+a^2}} = -\frac{2kq^2 y}{\left(\sqrt{y^2+a^2}\right)^{\frac{3}{2}}}$$

For $a >> y$, we have

$$F_{net} = -\frac{2kq^2 y}{a^3}$$

or, $F_{net} \propto -y$

Question 144



A neutral conducting solid sphere of radius R has two spherical cavities of radius a and b as shown in the figure. Centre to centre distance between two cavities is c. q_a and q_b charges are placed at the centres of cavities respectively. The force between q_a and q_b is

Options:

A.

$\frac{1}{4\pi\varepsilon_0}\frac{q_a q_b}{c^2}$

B.

$\frac{1}{4\pi\varepsilon_0} q_a q_b \left(\frac{1}{a^2}+\frac{1}{b^2}\right)$

C.

zero

D.

insufficient data

Answer: A

Solution:

We can find the force between q_a and q_b by calculating electric field first.

From Gauss's law,

$\oint E \,.\, ds = \frac{Q_{enclosed}}{\varepsilon_0} \Rightarrow E \,.\, 4\pi c^2 = \frac{q_a}{\varepsilon_0}$

(∵ For cavity of electric field is to be calculated upto q_b)

$\Rightarrow E_a = \frac{q_a}{4\pi\varepsilon_0 c^2}$

Now, force on q_b due to q_a is given by

$F = E_a \cdot q_b = \frac{q_a q_b}{4\pi\varepsilon_0 c^2}$

Question 145

Consider two concentric conducting sphere of radii R and 2R respectively. The inner sphere is given a charge +Q. The other sphere is grounded. The potential at $r = \frac{3R}{2}$ is

Options:

A.

$\frac{1}{4\pi\varepsilon_0} \frac{Q}{6R}$

B.

0

C.

$\frac{1}{4\pi\varepsilon_0} \frac{2Q}{3R}$

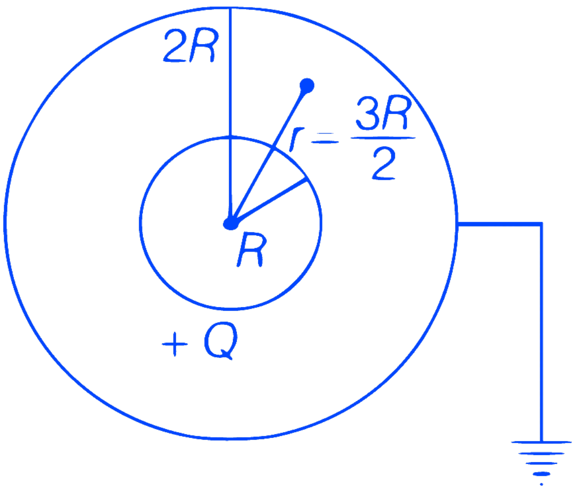
D.

$\frac{1}{4\pi\varepsilon_0} \frac{Q}{R}$

Answer: A

Solution:

Consider the figure given below.



Due to grounding, charge on outer sphere = 0

Now, potential due to inner sphere,

$V = \frac{kQ}{r} = \frac{kQ}{\frac{3R}{2}}$

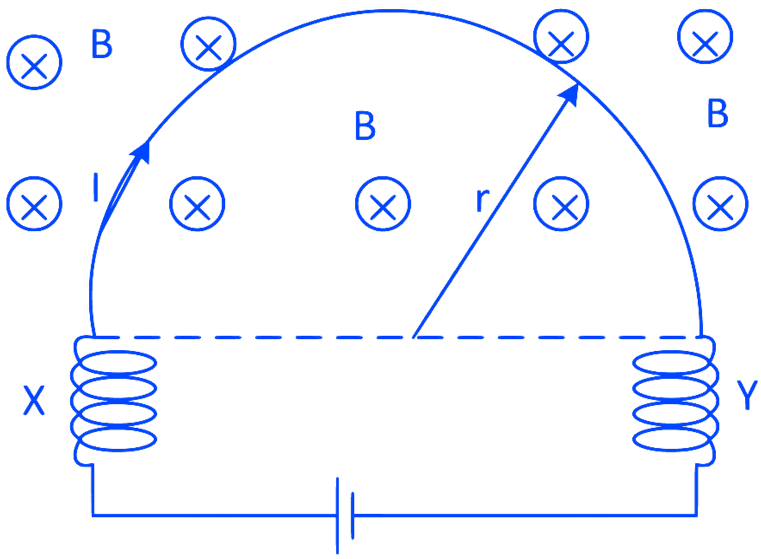
$= \frac{2kQ}{3R}$

or, $\frac{1}{4\pi\varepsilon_0}\frac{2Q}{3R}$

Question 146

A horizontal semi-circular wire of radius r is connected to a battery through two similar springs X and Y to an electric cell, which sends current I through it. A vertically downward uniform magnetic field

B is applied on the wire, as shown in the figure. What is the force acting on each spring?



Options:

A.

$2\pi rBI$

B.

$\frac{1}{2}\pi rBI$

C.

BIr

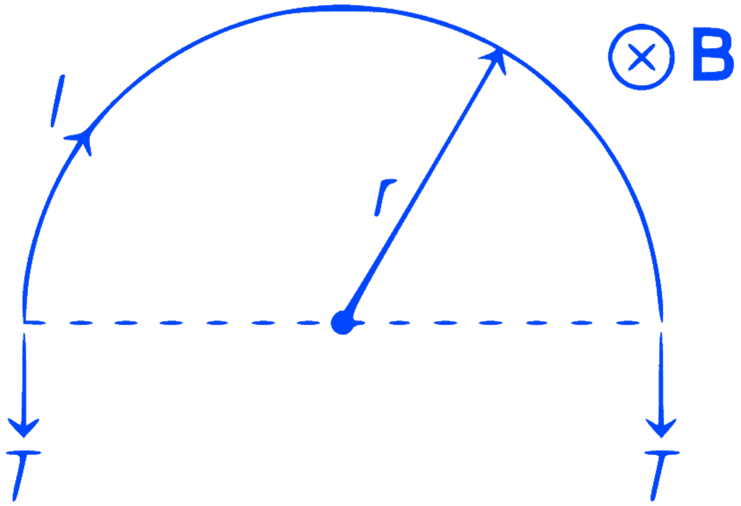
D.

$2BIr$

Answer: C

Solution:

Considering the semi-circular wire of radius r. Force acting on it is shown below



Let T be the force exerted by each spring.

$\therefore T + T = IBl$

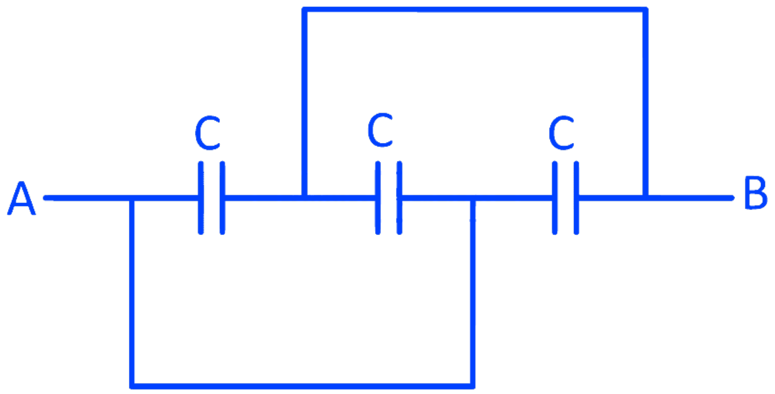
$2T = IBl$

$2T = IB \, . \, \pi r \; (\because l = \pi r)$

$\therefore T = \frac{IB\pi r}{2}$

Question 147

Find the equivalent capacitance between A and B of the following arrangement :



Options:

A.

C

B.

3C

C.

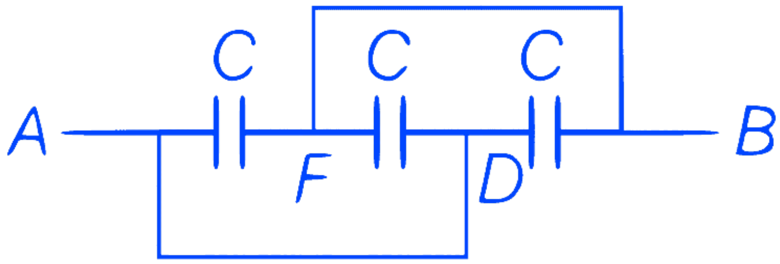
$\frac{2C}{3}$

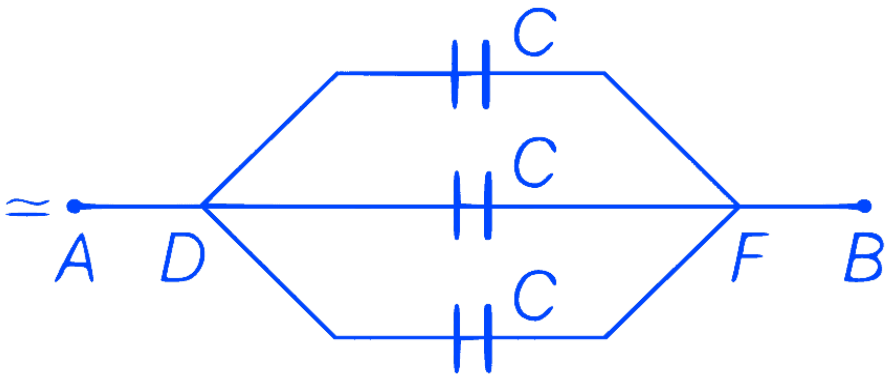
D.

$\frac{3C}{2}$

Answer: B

Solution:





Here, all the three capacitors are connected in parallel. So, equivalent capacitance will be

$C_{eq} = C + C + C = 3C$

Question 148

A golf ball of mass 50 gm placed on a tee, is struck by a golf-club. The speed of the golf ball as it leaves the tee is 100 m/s, the time of contact on the ball is 0.02 s. If the force decreases to zero linearly with time, then the force at the beginning of the contact is

Options:

A.

100 N

B.

200 N

C.

250 N

D.

500 N

Answer: D

Solution:

Given, initial velocity, $u=0$ m/s

Final velocity, $v=100$ m/s

Mass of ball, $m=50\times10^{-3}$ kg

Time of contact $=0.02\ s$

Impulse, $F=\frac{m(v-u)}{t}$

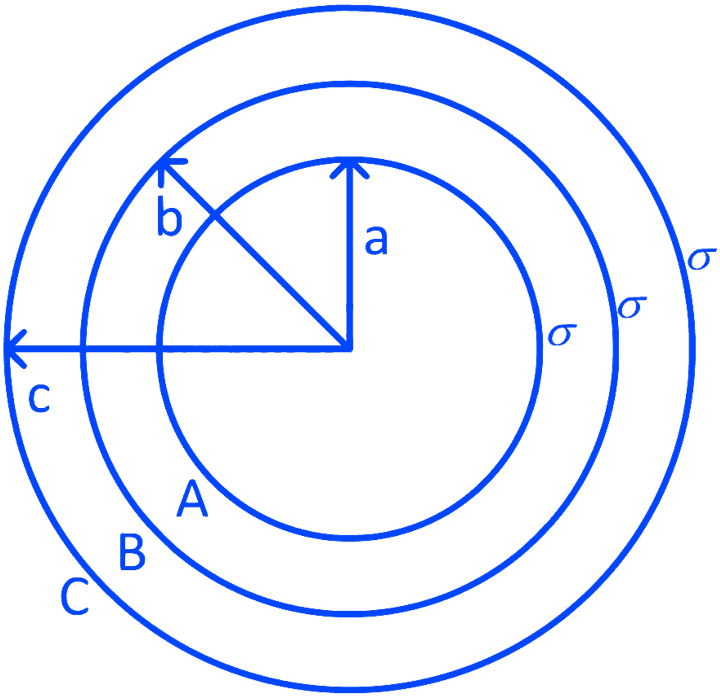
$=\frac{50\times10^{-3}(100-0)}{0.02}$

$=\frac{5\times100}{2}$

$=250\ N$

Question 149

Three concentric metallic shells A, B and C of radii a, b and c (a < b < c) have surface charge densities $+\sigma$, $-\sigma$ and $+\sigma$ respectively. The potential of shell B is



Options:

A.

$(a+b+c)\frac{\sigma}{\varepsilon_0}$

B.

$\frac{\sigma c}{\varepsilon_0}$

C.

$\left(\frac{a^2}{c}-\frac{b^2}{c}+c\right)\frac{\sigma}{\varepsilon_0}$

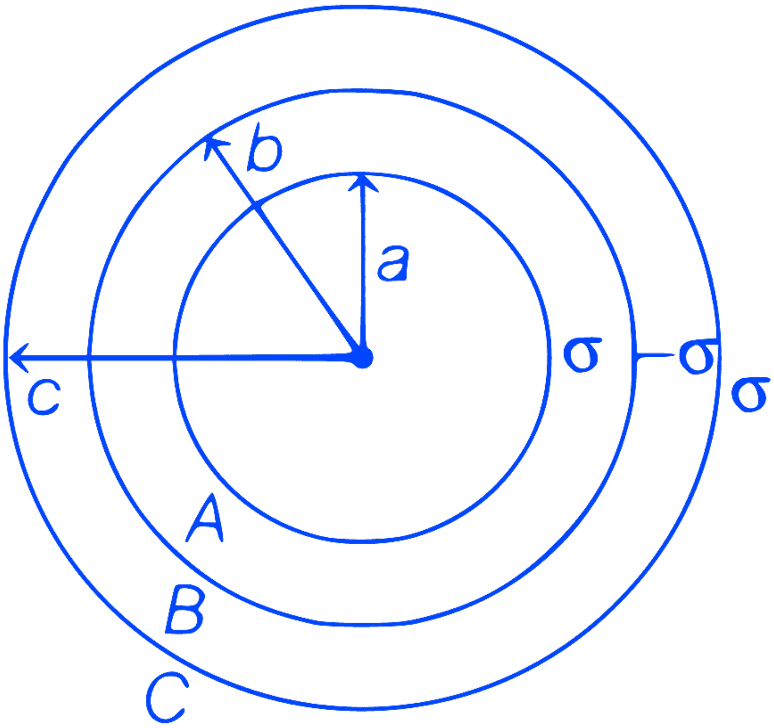
D.

$\left(\frac{a^2}{c} - b + c\right)\frac{\sigma}{\varepsilon_0}$

Answer: D

Solution:

Net potential at any point $(r = b)$ of shell B



= potential due to shell A + potential due to shell B + potential due to shell C.

$V = V_{A,\,out} + V_{B,\,surface} + V_{C,\,in}$

$= V_{A,\,out} + V_{B,\,surface} + V_{C,\,surface}$ (for shell, $V_{in} = V_{surface}$)

$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_a}{b} + \frac{q_b}{b} + \frac{q_c}{c}\right]$

$= \frac{1}{4\pi\varepsilon_0}\left[\frac{\sigma 4\pi a^2}{b} + \frac{(-\sigma)4\pi b^2}{b} + \frac{(\sigma)4\pi c^2}{c}\right]$

$= \frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{b} - b + c\right]$

Question 150

One mole of an ideal monoatomic gas expands along the polytrope PV^3 = constant from V_1 to V_2 at a constant pressure P_1. The temperature during the process is such that molar specific heat $C_V = \frac{3R}{2}$. The total heat absorbed during the process can be expressed as

Options:

A.

$P_1V_1\left(\frac{V_1^2}{V_2^2} + 1\right)$

B.

$P_1V_1\left(\frac{V_1^2}{V_2^2} - 1\right)$

C.

$P_1V_1\left(\frac{V_1^3}{V_2^2} - 1\right)$

D.

$P_1V_1\left(\frac{V_1}{V_2^2} - 1\right)$

Answer: B

Solution:

Given $pV^3 = C$, $C_V = \frac{3R}{2}$, $n = 1$ mol

For polytropic process,

$pV^x = C$

$\therefore x = 3$

$C = C_V + \frac{R}{1-x} = \frac{3R}{2} + \frac{R}{1-3} = R$

Heat supplied $(Q) = nC\Delta T$

$= 1 \times R \times (T_2 - T_1) = R(T_2 - T_1)$

Now, As $pV = nRT$ ...... (i)

$\therefore pV = RT\ (\because n = 1)$

$\therefore R = \frac{pV}{T} = \frac{p_1 T_1}{T_1}$

Also, $pV^3 = C$

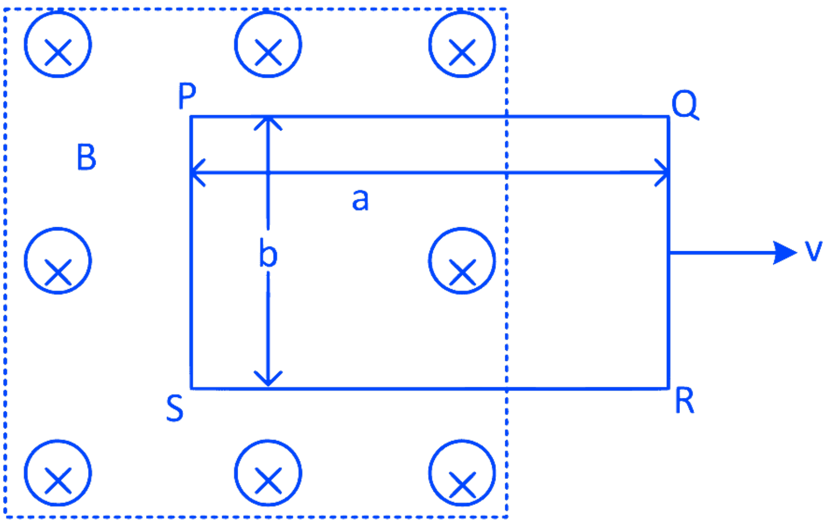
$\Rightarrow \left(\frac{RT}{V}\right)V^3 = C$ [from Eq. (i)]

$\Rightarrow TV^2 = C \Rightarrow T_1V_1^2 = T_2V_2^2$

$\therefore T_2 = \frac{T_1V_1^2}{V_2^2}$

Heat supplied $(Q) = \frac{p_1V_1}{T_1}\left[\frac{T_1V_1^2}{V_2^2} - T_1\right] = p_1V_1\left[\frac{V_1^2}{V_2^2} - 1\right]$

Question 151



As shown in figure, a rectangular loop of length 'a' and width 'b' and made of a conducting material of uniform cross-section is kept

in a horizontal plane where a uniform magnetic field of intensity B is acting vertically downward. Resistance per unit length of the loop is λ Ω/m. If the loop is pulled with uniform velocity 'v' in horizontal direction, which of the following statement is/are true?

Options:

A.

Current in the loop I = $\frac{Bbv}{\lambda(2b+2a)}$

B.

Current will be in clockwise direction, looking from the top.

C.

$V_P - V_S = V_Q - V_R$, where V is the potential.

D.

There cannot be any induction in part SR.

Answer: D

Solution:

Given, resistance per unit length of loop

$= \lambda \Omega/m$

(a) Current in the loop, $I = \frac{E}{R}$

$I = \frac{Blv}{R}$ ($\because$ emf $E = Blv$) .... (i)

We know that,

$\frac{R}{2(a+b)} = \lambda \Rightarrow R = \lambda(2a + 2b)$ .... (ii)

Using Eqs. (i) and (ii), we get

$I = \frac{Bbv}{\lambda(2a+2b)}$

So, option (a) is correct.

(b) Using right hand thumb rule, direction of current in the loop will be clockwise, looking from the top.

But as the flux is changing, so according to Lenz's law, direction of current will be anti-clockwise.

So, option (b) is incorrect.

(c) As the arm PS and QR has same potential, then $V_P - V_S = V_Q - V_R$ is also same.

So, option (c) is correct.

(d) As the arm SR is parallel to the velocity. So, there will not be any current or induction.

So, option (d) is correct.

Question 152

A sample of hydrogen atom in its ground state is radiated with photons of 10.2 eV energies. The radiation from the sample is absorbed by excited ionized He^+. Thenwhich of the following statement/s is/are true?

Options:

A.

He^+ electron moves from n = 2 to n = 4.

B.

In the He^+ emission spectra, there will be 6 lines.

C.

Smallest wavelength of He^+ spectrum is obtained when transition taken place from n = 4 to n = 3.

D.

He^+ electron moves from n = 2 to n = 3.

Answer: B

Solution:

Given, energy of photons $= \mathbf{10.2}$ eV

Energy for nth state,

$E_n = -\mathbf{13.6}\, Z^2 \times \left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$

$= -\mathbf{13.6} \times (2)^2 \left(\frac{1}{2^2} - \frac{1}{4^2}\right) (\because Z = 2)$

$= -13.6 \times 4 \times \left(\frac{4-1}{16}\right)$

$= -3.4 \times \frac{3}{4}$

$= -10.2$ eV

or, $E = 10.2$ eV

So, option (a) is correct.

(b) Number of lines in emission spectra

$= \frac{n(n-1)}{2} = \frac{4\times3}{2} = 6\ [\because n = 4]$

So, option (b) is correct.

(c) For smallest wavelength of He^+ spectrum the final state should be $n_f = \infty$

So, option (c) is incorrect.

(d) He^+ electron cannot jump from $n = 2$ to $n = 3$ as the energy of photon is less than required.

So, option (d) is incorrect.

Question 153

A particle is moving in x-y plane according to $\vec{r} = b\cos\omega t\hat{i} + b\sin\omega t\hat{j}$, where ω is constant. Which of the following statement(s) is/are true?

Options:

A.

$\frac{E}{\omega}$ is a constant where E is the total energy of the particle.

B.

The trajectory of the particle in x-y plane is a circle.

C.

In a_x - a_y plane, trajectory of the particle is an ellipse (a_x, a_y denotes the components of acceleration)

D.

$\vec{a} = \omega^2\vec{v}$

Answer: B

Solution:

Given, trajectory of the particle,

$\vec{r} = b\cos\omega t\hat{i} + b\sin\omega t\hat{j}$

It's component can be written as

$x = b\cos\omega t$ ..... (i)

$y = b\sin\omega t$ ...... (ii)

Both Eqs. (i) and (ii) together represent the circle in x-y plane. Thus, the trajectory of the particle in x-y plane is circle.

Now,

$\frac{d\vec{r}}{dt} = \omega b(-\sin\omega t\hat{i} + \cos\omega t\hat{j})$

$\frac{d^2\vec{r}}{dt^2} = -b\omega^2(\cos\omega t\hat{i} + \sin\omega t\hat{j})$

$= -\omega^2(\vec{r})$

Therefore, $\vec{a} = -\omega^2\vec{r}$

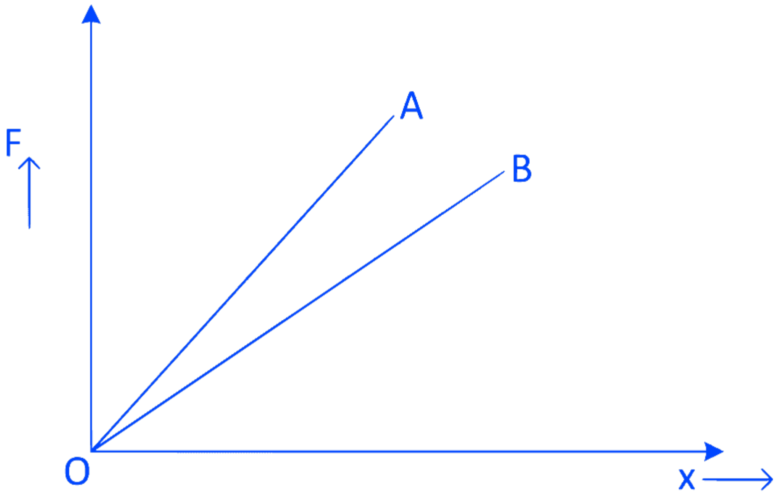
Hence, option (c) and (d) are incorrect.

Energy of the particle in circular motion is given as $\frac{1}{2}m\omega^2A^2$ (where, A is the maximum displacement from the centre), which is constant.

Hence, $\frac{E}{\omega}$ is also constant.

So, option (a) and (b) are correct.

Question 154



Two wires A and B of same length are made of same material. Load (F) vs. elongation (x) graph for these two wires is shown in the figure. Then which of the following statement(s) is/are true?

Options:

A.

The cross-section area of A is greater than that of B.

B.

Young's modulus of A is greater than Young's modulus of B.

C.

The cross-sectional area of B is greater than that of A.

D.

Young's modulus of both A and B are same.

Answer: D

Solution:

Load F can be given as

$F = \left(\frac{AY}{L}\right)\Delta x$

where, A = cross-sectional area

Y = Young's modulus

L = length

Δx = elongation in length.

i.e., F versus Δx graph is a straight line of slope $\frac{YA}{L}$.

$(slope)_A > (slope)_B$

$\left(\frac{YA}{L}\right)_A > \left(\frac{YA}{L}\right)_B$

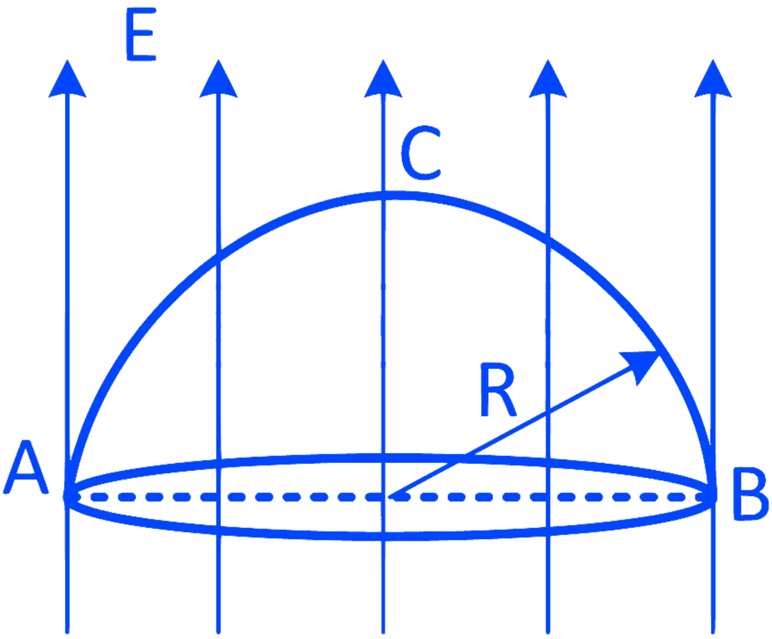
or, $(A)_A > (A)_B$

As we know that, they are of same material.

Hence, $Y_B = Y_A$

So, option (a) and (d) are correct.

Question 155



A hemisphere of radius R is placed in a uniform electric field E so that its axis is parallel to the field. Which of the following statement(s) is/are true?

Options:

A.

Flux through the curved surface of hemisphere is $\pi R^2 E$.

B.

Flux through the circular surface of hemisphere is $\pi R^2 E$.

C.

Total flux enclosed is zero.

D.

Work done in moving a point charge q from A to B via the path ACB depends upon R.

Answer: C

Solution:

According to Gauss's theorem

Net flux through the surface $= \frac{q_{\text{enclosed}}}{\varepsilon_0}$

Here, $q_{\text{enclosed}} = 0$

So, $\phi_{\text{curved surface}} + \phi_{\text{circular surface}} = 0$

$\phi_{\text{curved surface}} = -\phi_{\text{circular surface}}$

$= -E\,.\,S = -ES\cos 180°$

$= ES\ (\because \cos 180° = -1)$

$= E\,.\,\pi R^2 = \pi R^2 E$

As, electrostatic field is conservative in nature.

So, work done is path independent.

So, option (a) and (c) are correct.
